

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.2-d-x-
 $^m-a+b-x^n+c-x^{-2-n-p}$

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [549]. This is test number [32].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. https://github.com/stblake/algebraic_integration. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (549)	0.00 (0)
Mathematica	100.00 (549)	0.00 (0)
Fricas	97.45 (535)	2.55 (14)
Maple	90.35 (496)	9.65 (53)
Giac	75.05 (412)	24.95 (137)
Mupad	65.57 (360)	34.43 (189)
Maxima	55.19 (303)	44.81 (246)
Sympy	51.37 (282)	% 48.63 (267)
IntegrateAlgebraic	39.89 (219)	60.11 (330)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

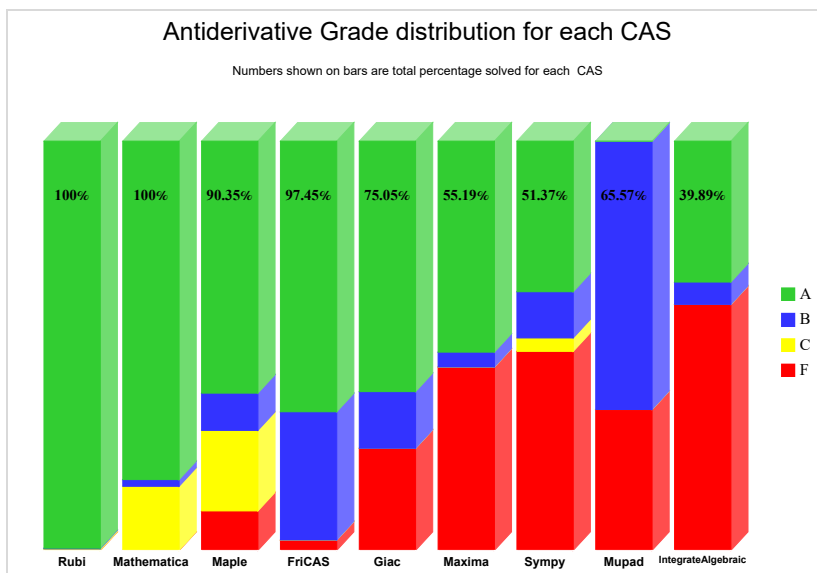
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

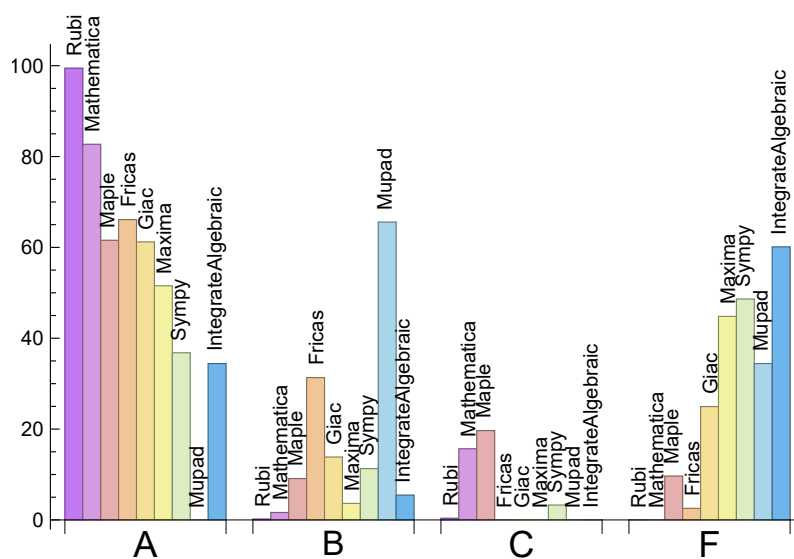
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.45	0.18	0.36	0.00
Mathematica	82.70	1.64	15.66	0.00
Fricas	66.12	31.33	0.00	2.55
Maple	61.57	9.11	19.67	9.65
Giac	61.20	13.84	0.00	24.95
Maxima	51.55	3.64	0.00	44.81
Sympy	36.79	11.29	3.28	48.63
IntegrateAlgebraic	34.43	5.46	0.00	60.11
Mupad	N/A	65.57	0.00	34.43

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	53	100.00 %	0.00 %	0.00 %
Fricas	14	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	330	100.00 %	0.00 %	0.00 %
Giac	137	88.32 %	2.92 %	8.76 %
Maxima	246	52.44 %	14.63 %	32.93 %
Sympy	267	74.91 %	23.97 %	1.12 %
Mupad	189	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

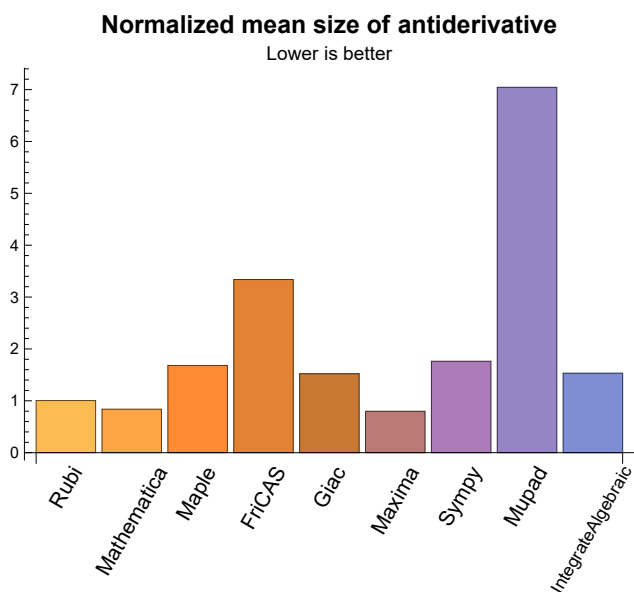
1.3 Performance

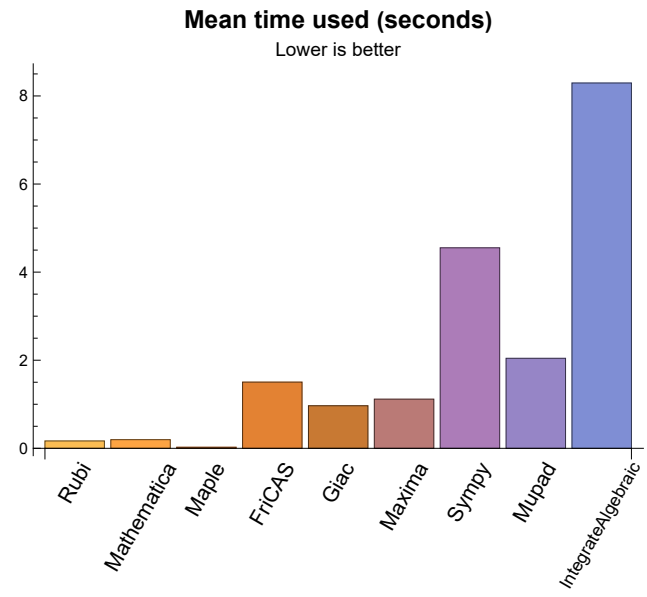
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	162.59	1.00	126.00	1.00
Mathematica	0.20	104.81	0.84	81.00	0.89
Maple	0.02	283.22	1.68	78.00	0.79
Maxima	1.12	80.96	0.80	55.00	0.76
Fricas	1.50	774.34	3.34	143.00	1.50
Sympy	4.55	183.60	1.76	59.00	0.94
Giac	0.97	256.17	1.52	91.00	0.85
Mupad	2.04	1652.88	7.04	119.00	0.99
IntegrateAlgebraic	8.29	229.20	1.53	101.00	0.88

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {271, 273, 275, 280, 282, 283, 285, 286, 288, 292, 294}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each `integrate` call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the `integrate` command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by

failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

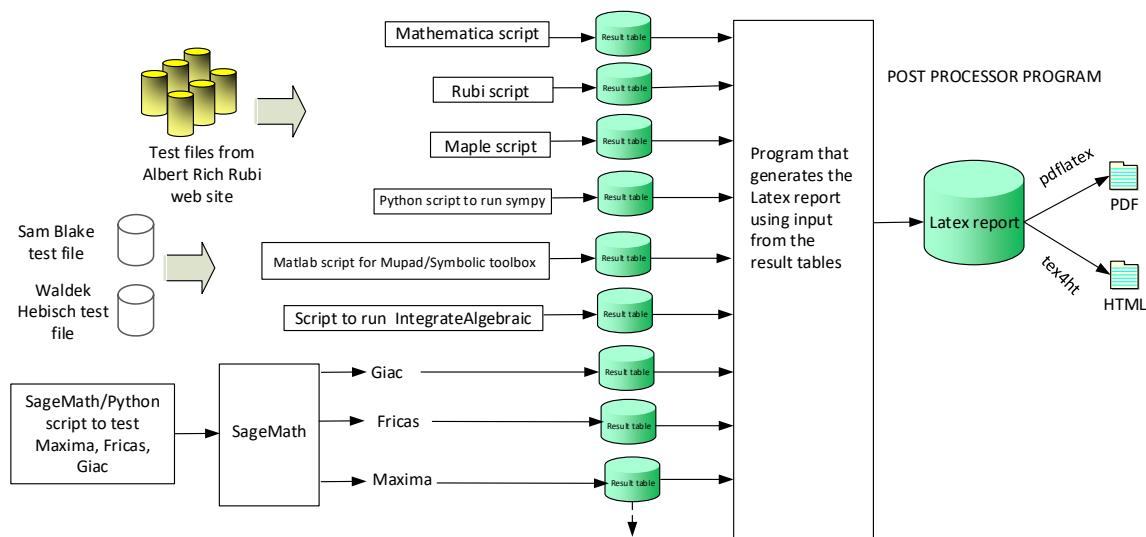
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. integer. 1 if result was verified or 0 if not verified.
The following field present only in Rubi and Mathematica Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549 }

B grade: { 141 }

C grade: { 163, 414 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 270, 272, 274, 281, 284, 287, 289, 290, 291, 293, 296, 308, 309, 310, 311, 312, 313, 314, 316, 325, 326,

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B grade: { 61, 141, 494, 495, 496, 497, 498, 499, 549 }

C grade: { 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 157, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 171, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 273, 275, 276, 277, 278, 279, 280, 282, 283, 285, 286, 288, 292, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 315, 317, 318, 319, 320, 321, 322, 323, 324, 348, 349, 350, 396, 397, 414, 436, 437, 438, 439, 440, 441, 470, 471, 483, 484, 485 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 111, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 164, 167, 170, 209, 210, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 270, 271, 272, 273, 274, 275, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 308, 310, 312, 313, 314, 316, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 375, 376, 379, 380, 381, 382, 383, 384, 385, 386, 387, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 438, 439, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 468, 487, 488 }

B grade: { 61, 82, 107, 109, 110, 112, 115, 116, 141, 215, 254, 276, 278, 309, 311, 315, 317, 348, 350, 361, 362, 367, 368, 369, 370, 371, 372, 373, 374, 377, 378, 388, 389, 395, 473, 474, 475, 476, 477, 478, 479, 486, 494, 495, 496, 497, 498, 499, 548, 549 }

C grade: { 130, 131, 132, 133, 134, 135, 136, 137, 157, 159, 160, 162, 163, 165, 166, 168, 169, 171, 262, 263, 264, 265, 266, 267, 268, 269, 299, 300, 301, 302, 303, 304, 305, 306, 307, 318, 319, 320, 321, 322, 323, 324, 396, 397, 436, 437, 440, 441, 451, 458, 472, 480, 481, 482, 483, 484, 485, 491, 492, 493, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520,

521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547 }

F grade: { 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 211, 212, 213, 214, 411, 412, 413, 414, 442, 443, 467, 469, 470, 471, 489, 490 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 83, 84, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 164, 167, 170, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 289, 291, 293, 295, 297, 308, 310, 312, 314, 316, 325, 326, 327, 329, 331, 332, 333, 334, 344, 345, 346, 347, 348, 349, 379, 380, 381, 382, 383, 384, 385, 386, 387, 394, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 442, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 472, 491, 492, 493 }

B grade: { 9, 21, 42, 45, 48, 79, 82, 85, 88, 141, 328, 330, 494, 495, 496, 497, 498, 499, 548, 549 }

C grade: { }

F grade: { 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 157, 159, 160, 162, 163, 165, 166, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 292, 294, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 311, 313, 315, 317, 318, 319, 320, 321, 322, 323, 324, 335, 336, 337, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 414, 417, 436, 437, 438, 439, 440, 441, 443, 467, 468, 469, 470, 471, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 164, 167, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 240, 242, 253, 255, 257, 259, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 289, 290, 291, 293, 295, 296, 297, 300, 303, 306, 308, 310, 314, 316, 325, 327, 331, 333, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 399, 401, 402, 403, 404, 405, 406, 407, 408, 411, 412, 413, 414, 415, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 486, 487, 488, 489, 503, 504, 526, 528, 529 }

B grade: { 82, 130, 131, 132, 133, 134, 135, 136, 137, 141, 157, 159, 160, 162, 163, 165, 166, 168, 169, 171, 211, 215, 237, 239, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 256, 258, 260, 262, 263, 264, 265, 266, 267, 268, 269, 286, 288, 292, 294, 298, 299, 301, 302, 304, 305, 307, 309, 311, 312, 313, 315, 317, 318, 319, 320, 321, 322, 323, 324, 326, 328, 329, 330, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 393, 395, 396, 397, 409, 410, 476, 480, 481, 482, 483, 484, 485, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 527, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549 }

C grade: { }

F grade: { 398, 400, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428 }

2.1.6 SymPy

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 89, 90, 91, 92, 93, 94, 95, 96, 97, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 256, 258, 260, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 284, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 327, 329, 331, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 379, 380, 381, 382, 383, 384, 385, 386, 387, 395, 396, 399, 429, 430, 431, 432, 434, 435, 438, 439, 493, 500, 502, 505, 507, 525, 527, 530, 532 }

B grade: { 125, 126, 127, 128, 129, 253, 255, 257, 259, 326, 328, 330, 332, 334, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 365, 366, 370, 371, 372, 373, 374, 375, 376, 494, 495, 496, 497, 498, 499, 501, 503, 504, 506, 508, 509, 510, 511, 518, 520, 526, 528, 529, 531, 533, 534, 535, 536, 542, 544, 548, 549 }

C grade: { 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 280, 282, 283, 285, 286, 288 }

F grade: { 1, 2, 3, 4, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 261, 262, 263, 268, 269, 358, 359, 360, 367, 368, 369, 377, 378, 388, 389, 390, 391, 392, 393, 394, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 433, 436, 437, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 512, 513, 514, 515, 516, 517, 519, 521, 522, 523, 524, 537, 538, 539, 540, 541, 543, 545, 546, 547 }

2.1.7 Giac

A grade: { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 120, 122, 123, 124, 125, 126, 127, 128, 129, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 161, 164, 167, 170, 175, 176, 186, 187, 199, 200, 209, 210, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 257, 259, 261, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 394, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 413, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 432, 436, 437, 438, 451, 452, 453, 454, 459, 460, 461, 476, 501, 503, 506, 509, 511, 515, 524, 528, 548, 549 }

B grade: { 45, 61, 82, 118, 119, 121, 141, 157, 159, 160, 162, 163, 165, 166, 168, 215, 216, 254, 256, 258, 260, 290, 296, 328, 330, 395, 411, 412, 458, 482, 492, 493, 494, 495, 496, 497, 498, 499, 500, 502, 504, 507, 508, 510, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 529, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547 }

C grade: { }

F grade: { 1, 2, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115,

116, 117, 130, 131, 132, 133, 134, 135, 136, 137, 169, 171, 172, 173, 174, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 201, 202, 203, 204, 205, 206, 207, 208, 211, 212, 213, 214, 262, 263, 264, 265, 266, 267, 268, 269, 388, 389, 390, 391, 392, 393, 396, 397, 398, 414, 415, 429, 430, 431, 433, 434, 435, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 483, 484, 485, 486, 487, 488, 489, 490, 491, 505, 530 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 27, 30, 43, 44, 45, 46, 47, 48, 49, 61, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 94, 97, 100, 108, 111, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 199, 200, 201, 202, 208, 209, 210, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 390, 391, 394, 395, 396, 397, 399, 404, 407, 408, 409, 410, 411, 412, 413, 414, 421, 432, 476, 486, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549 }

C grade: { }

F grade: { 7, 8, 10, 11, 13, 14, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 89, 90, 92, 93, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 203, 204, 205, 206, 207, 211, 212, 213, 214, 388, 389, 392, 393, 398, 400, 401, 402, 403, 405, 406, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 487, 488, 489, 490 }

2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 80, 81, 83, 84, 86, 87, 89, 90, 92, 93, 94, 95, 96, 98, 99, 101, 102, 104, 105, 107, 109, 110, 112, 113, 115, 116, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 388, 389, 390, 391, 392, 393, 394, 398, 399, 405, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 444, 445, 446, 447, 448, 449, 450, 455, 462, 465, 466, 473, 474, 475, 476, 477, 478, 479, 486, 487, 488, 489, 490 }

B grade: { 12, 15, 21, 39, 45, 48, 61, 76, 79, 82, 85, 88, 91, 97, 100, 103, 106, 108, 111, 114, 117, 400, 406, 407, 408, 409, 410, 415, 416, 427 }

C grade: { }

F grade: { 5, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 395, 396, 397, 401, 402, 403, 404, 411, 412, 413, 414, 436, 437, 438, 439, 440, 441, 442, 443, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 463, 464, 467, 468, 469, 470, 471, 472, 480, 481, 482, 483, 484, 485, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	42	39	46	53	0	0	40	57
N.S.	1	1.00	0.81	0.75	0.88	1.02	0.00	0.00	0.77	1.10
time (sec)	N/A	0.048	0.024	0.006	0.443	1.973	0.000	0.000	1.226	0.279
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	29	14	28	0	0	29	25
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.00	1.16	1.00
time (sec)	N/A	0.005	0.008	0.004	0.458	1.908	0.000	0.000	1.152	0.203
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	27	17	21	0	14	21	23
N.S.	1	1.00	1.00	1.17	0.74	0.91	0.00	0.61	0.91	1.00
time (sec)	N/A	0.005	0.008	0.004	0.449	1.167	0.000	24.996	1.151	0.407

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	46	46	38	54	0	52	51	46
N.S.	1	1.00	0.60	0.60	0.49	0.70	0.00	0.68	0.66	0.60
time (sec)	N/A	0.057	0.010	0.004	0.465	1.064	0.000	23.789	1.282	0.950

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	38	37	46	48	38	51	0
N.S.	1	1.00	1.00	0.79	0.77	0.96	1.00	0.79	1.06	0.00
time (sec)	N/A	0.023	0.017	0.007	0.946	1.546	0.147	0.316	0.095	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	83	13	12	23	59	39
N.S.	1	1.00	0.49	0.46	1.05	0.16	0.15	0.29	0.75	0.49
time (sec)	N/A	0.023	0.012	0.003	0.490	1.342	0.107	0.395	1.259	6.282

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	13	13	12	29	-1	39
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01	0.49
time (sec)	N/A	0.023	0.008	0.001	0.466	1.328	0.109	0.371	0.000	6.451

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	13	13	12	29	-1	39
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01	0.49
time (sec)	N/A	0.024	0.007	0.003	0.463	1.671	0.106	0.388	0.000	6.463

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	38	35	52	13	12	22	33	38
N.S.	1	1.00	1.06	0.97	1.44	0.36	0.33	0.61	0.92	1.06
time (sec)	N/A	0.028	0.009	0.003	0.462	1.667	0.105	0.362	1.229	6.097
Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	13	13	12	29	-1	39
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01	0.49
time (sec)	N/A	0.018	0.009	0.007	0.451	1.010	0.105	0.326	0.000	8.160
Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	36	33	10	10	8	20	-1	36
N.S.	1	1.00	0.49	0.45	0.14	0.14	0.11	0.27	-0.01	0.49
time (sec)	N/A	0.012	0.007	0.001	0.440	1.062	0.103	0.379	0.000	11.163
Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	37	34	96	11	10	28	109	197
N.S.	1	1.00	0.49	0.45	1.28	0.15	0.13	0.37	1.45	2.63
time (sec)	N/A	0.020	0.009	0.010	0.462	1.179	0.129	0.289	1.377	0.243

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	38	36	14	14	8	29	-1	38
N.S.	1	1.00	0.49	0.47	0.18	0.18	0.10	0.38	-0.01	0.49
time (sec)	N/A	0.021	0.009	0.001	0.463	1.072	0.125	0.323	0.000	16.226

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	37	34	15	15	8	26	-1	39
N.S.	1	1.00	0.50	0.46	0.20	0.20	0.11	0.35	-0.01	0.53
time (sec)	N/A	0.020	0.007	0.003	0.456	1.532	0.134	0.403	0.000	19.038

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	39	38	99	17	10	43	112	749
N.S.	1	1.00	0.52	0.51	1.32	0.23	0.13	0.57	1.49	9.99
time (sec)	N/A	0.021	0.009	0.009	0.465	0.885	0.162	0.346	1.384	0.615

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	37	34	13	13	14	30	33	39
N.S.	1	1.00	0.48	0.44	0.17	0.17	0.18	0.39	0.43	0.51
time (sec)	N/A	0.021	0.008	0.003	0.454	1.145	0.168	0.354	1.208	19.206

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.022	0.008	0.003	0.456	0.953	0.179	0.348	1.176	18.832

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	37	34	86	13	14	30	33	118
N.S.	1	1.00	0.47	0.43	1.09	0.16	0.18	0.38	0.42	1.49
time (sec)	N/A	0.022	0.008	0.003	0.488	1.338	0.185	0.336	1.184	0.429

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.022	0.008	0.003	0.457	1.909	0.193	0.339	1.267	19.284

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.021	0.008	0.003	0.453	1.066	0.197	0.379	1.164	19.244

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	117	15	15	31	35	753
N.S.	1	1.00	0.49	0.46	1.48	0.19	0.19	0.39	0.44	9.53
time (sec)	N/A	0.022	0.008	0.004	0.473	1.370	0.209	0.357	1.151	3.559

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	39	36	15	15	15	31	35	39
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44	0.49
time (sec)	N/A	0.022	0.009	0.004	0.452	1.404	0.217	0.307	1.162	20.698

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.043	0.020	0.007	0.472	1.220	0.000	0.385	0.000	11.614

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	167	61	58	114	35	0	67	-1	61
N.S.	1	1.40	0.51	0.49	0.96	0.29	0.00	0.56	-0.01	0.51
time (sec)	N/A	0.053	0.015	0.008	0.463	0.759	0.000	0.303	0.000	10.095

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.043	0.016	0.007	0.448	1.320	0.000	0.295	0.000	9.620

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.041	0.015	0.008	0.452	1.542	0.000	0.359	0.000	9.203

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	61	58	83	35	0	45	46	61
N.S.	1	1.00	0.78	0.74	1.06	0.45	0.00	0.58	0.59	0.78
time (sec)	N/A	0.050	0.015	0.007	0.487	1.482	0.000	0.437	1.249	8.621

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.041	0.016	0.007	0.469	1.125	0.000	0.368	0.000	8.311

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.041	0.015	0.007	0.480	1.338	0.000	0.447	0.000	8.242
Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	60	57	52	35	0	44	36	60
N.S.	1	1.00	1.67	1.58	1.44	0.97	0.00	1.22	1.00	1.67
time (sec)	N/A	0.030	0.016	0.007	0.453	1.473	0.000	0.413	1.220	8.020
Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	35	35	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.038	0.015	0.005	0.443	1.515	0.000	0.374	0.000	10.189
Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	59	56	32	32	0	64	-1	59
N.S.	1	1.00	0.36	0.35	0.20	0.20	0.00	0.40	-0.01	0.36
time (sec)	N/A	0.033	0.014	0.003	0.715	3.026	0.000	0.360	0.000	12.398

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	60	57	152	32	0	65	-1	256
N.S.	1	1.00	0.38	0.36	0.95	0.20	0.00	0.41	-0.01	1.60
time (sec)	N/A	0.048	0.020	0.010	0.517	1.095	0.000	0.426	0.000	0.442
Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	61	58	37	37	0	67	-1	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	-0.01	0.37
time (sec)	N/A	0.042	0.016	0.007	0.496	1.382	0.000	0.337	0.000	17.203
Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	61	58	37	37	0	65	-1	61
N.S.	1	1.00	0.37	0.36	0.23	0.23	0.00	0.40	-0.01	0.37
time (sec)	N/A	0.042	0.018	0.006	0.498	1.469	0.000	0.384	0.000	19.881
Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	62	59	156	38	0	85	-1	317
N.S.	1	1.00	0.39	0.37	0.97	0.24	0.00	0.53	-0.01	1.97
time (sec)	N/A	0.048	0.020	0.010	0.528	1.174	0.000	0.365	0.000	0.744

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	61	58	37	37	0	69	-1	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	-0.01	0.37
time (sec)	N/A	0.042	0.015	0.006	0.525	1.152	0.000	0.362	0.000	20.334

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	61	58	37	37	0	68	-1	61
N.S.	1	1.00	0.37	0.36	0.23	0.23	0.00	0.42	-0.01	0.37
time (sec)	N/A	0.041	0.017	0.006	0.497	1.420	0.000	0.293	0.000	20.008

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	61	60	220	39	0	86	-1	1164
N.S.	1	1.00	0.38	0.37	1.36	0.24	0.00	0.53	-0.01	7.19
time (sec)	N/A	0.047	0.015	0.012	0.700	1.299	0.000	0.340	0.000	1.679

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	61	58	37	37	0	70	-1	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	-0.01	0.37
time (sec)	N/A	0.041	0.014	0.006	0.738	1.917	0.000	0.470	0.000	20.101

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	61	58	37	37	0	67	-1	61
N.S.	1	1.00	0.38	0.36	0.23	0.23	0.00	0.41	-0.01	0.38
time (sec)	N/A	0.041	0.014	0.007	0.677	1.256	0.000	0.391	0.000	20.489

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	63	60	253	39	0	85	-1	293
N.S.	1	1.00	0.39	0.37	1.57	0.24	0.00	0.53	-0.01	1.82
time (sec)	N/A	0.046	0.021	0.013	0.724	1.311	0.000	0.377	0.000	1.166

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	0.92	0.37
time (sec)	N/A	0.040	0.013	0.007	0.665	1.525	0.000	0.361	1.214	20.137

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.041	0.016	0.007	0.718	1.191	0.000	0.419	1.224	20.142

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	59	56	148	35	0	68	151	310
N.S.	1	1.00	1.44	1.37	3.61	0.85	0.00	1.66	3.68	7.56
time (sec)	N/A	0.018	0.013	0.007	0.546	1.419	0.000	0.357	1.208	1.349
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.041	0.013	0.010	0.689	1.365	0.000	0.358	1.191	18.763
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.041	0.016	0.007	0.686	2.232	0.000	0.360	1.207	18.640
Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	61	58	179	37	0	69	151	356
N.S.	1	1.00	0.73	0.69	2.13	0.44	0.00	0.82	1.80	4.24
time (sec)	N/A	0.040	0.014	0.012	0.522	0.850	0.000	0.414	1.209	1.016

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	61	58	37	37	0	69	151	61
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90	0.37
time (sec)	N/A	0.040	0.014	0.007	0.766	1.648	0.000	0.304	1.215	20.486
Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.064	0.025	0.013	0.830	0.976	0.000	0.326	0.000	27.589
Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.058	0.020	0.010	0.603	1.987	0.000	0.322	0.000	24.777
Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	83	80	145	57	0	105	-1	83
N.S.	1	1.00	0.52	0.50	0.91	0.36	0.00	0.66	-0.01	0.52
time (sec)	N/A	0.120	0.021	0.012	0.725	1.450	0.000	0.351	0.000	21.673

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.061	0.024	0.010	0.805	1.557	0.000	0.443	0.000	19.229
Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.057	0.020	0.008	0.674	1.140	0.000	0.401	0.000	18.134
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	83	80	114	57	0	105	-1	83
N.S.	1	1.00	0.70	0.67	0.96	0.48	0.00	0.88	-0.01	0.70
time (sec)	N/A	0.090	0.021	0.009	0.666	1.150	0.000	0.377	0.000	14.505
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.060	0.021	0.008	0.774	1.141	0.000	0.339	0.000	13.722

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.059	0.020	0.009	0.714	1.054	0.000	0.345	0.000	12.977

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	83	80	83	57	0	67	-1	83
N.S.	1	1.00	1.06	1.03	1.06	0.73	0.00	0.86	-0.01	1.06
time (sec)	N/A	0.056	0.021	0.009	0.625	1.126	0.000	0.291	0.000	11.576

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	57	57	0	105	-1	83
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.059	0.021	0.009	0.691	1.328	0.000	0.416	0.000	10.545

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	83	80	56	56	0	104	-1	83
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.057	0.020	0.008	0.511	1.158	0.000	0.317	0.000	10.380

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	82	79	52	57	0	66	36	82
N.S.	1	1.00	2.28	2.19	1.44	1.58	0.00	1.83	1.00	2.28
time (sec)	N/A	0.029	0.021	0.009	0.660	0.939	0.000	0.373	1.244	9.630
Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	83	80	56	56	0	104	-1	83
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.054	0.020	0.005	1.101	0.814	0.000	0.368	0.000	11.176
Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	81	78	53	53	0	101	-1	81
N.S.	1	1.00	0.33	0.32	0.21	0.21	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.051	0.019	0.004	1.016	1.287	0.000	0.376	0.000	12.919
Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	82	79	206	55	0	104	-1	314
N.S.	1	1.00	0.33	0.31	0.82	0.22	0.00	0.41	-0.00	1.25
time (sec)	N/A	0.069	0.025	0.008	1.156	1.170	0.000	0.352	0.000	0.580

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	83	80	59	59	0	105	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00	0.33
time (sec)	N/A	0.061	0.021	0.008	0.820	1.219	0.000	0.334	0.000	16.653

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	83	80	59	59	0	103	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.41	-0.00	0.33
time (sec)	N/A	0.059	0.021	0.008	1.231	1.217	0.000	0.362	0.000	18.808

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	85	82	214	61	0	124	-1	364
N.S.	1	1.00	0.34	0.33	0.85	0.24	0.00	0.49	-0.00	1.44
time (sec)	N/A	0.074	0.026	0.011	1.196	0.985	0.000	0.308	0.000	0.980

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	249	83	80	59	59	0	107	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	-0.00	0.33
time (sec)	N/A	0.059	0.023	0.007	0.972	1.343	0.000	0.351	0.000	18.555

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	83	80	59	59	0	106	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00	0.33
time (sec)	N/A	0.058	0.020	0.007	0.672	0.715	0.000	0.341	0.000	18.311

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	85	82	282	61	0	126	-1	368
N.S.	1	1.00	0.34	0.33	1.12	0.24	0.00	0.50	-0.00	1.46
time (sec)	N/A	0.073	0.024	0.013	1.245	1.309	0.000	0.367	0.000	1.164

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	83	80	59	59	0	107	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	-0.00	0.33
time (sec)	N/A	0.058	0.021	0.008	1.099	1.142	0.000	0.326	0.000	18.571

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	83	80	59	59	0	105	-1	83
N.S.	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	-0.00	0.34
time (sec)	N/A	0.059	0.021	0.008	1.326	1.285	0.000	0.336	0.000	18.466

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	85	82	313	61	0	127	-1	366
N.S.	1	1.00	0.34	0.33	1.24	0.24	0.00	0.50	-0.00	1.45
time (sec)	N/A	0.071	0.028	0.012	0.797	1.716	0.000	0.397	0.000	1.821

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	83	80	59	59	0	108	-1	83
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	-0.00	0.33
time (sec)	N/A	0.062	0.017	0.006	1.013	1.197	0.000	0.397	0.000	18.356

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	247	83	80	59	59	0	106	-1	83
N.S.	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	-0.00	0.34
time (sec)	N/A	0.060	0.016	0.006	0.974	1.079	0.000	0.350	0.000	18.198

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	85	82	342	61	0	125	-1	2031
N.S.	1	1.00	0.34	0.33	1.36	0.24	0.00	0.50	-0.00	8.06
time (sec)	N/A	0.071	0.019	0.013	1.097	1.172	0.000	0.378	0.000	3.023

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	83	80	59	59	0	108	-1	83
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	-0.00	0.33
time (sec)	N/A	0.059	0.017	0.006	0.986	1.179	0.000	0.335	0.000	19.396

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	83	80	59	59	0	105	-1	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00	0.33
time (sec)	N/A	0.060	0.018	0.007	0.990	0.678	0.000	0.346	0.000	19.620

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	85	82	374	61	0	123	-1	2386
N.S.	1	1.00	0.34	0.33	1.49	0.24	0.00	0.49	-0.00	9.51
time (sec)	N/A	0.069	0.028	0.014	1.097	1.234	0.000	0.370	0.000	3.577

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	251	251	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	0.92	0.33
time (sec)	N/A	0.058	0.017	0.009	0.985	1.156	0.000	0.340	1.260	20.035

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.058	0.018	0.008	1.163	1.205	0.000	0.448	1.323	20.552

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	81	78	210	57	0	106	231	442
N.S.	1	1.00	1.98	1.90	5.12	1.39	0.00	2.59	5.63	10.78
time (sec)	N/A	0.018	0.017	0.007	1.097	1.141	0.000	0.291	1.222	2.887

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.061	0.020	0.008	1.045	1.100	0.000	0.361	1.305	21.316

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.056	0.017	0.009	0.983	1.111	0.000	0.318	1.241	21.387

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	83	80	241	59	0	107	231	488
N.S.	1	1.00	0.99	0.95	2.87	0.70	0.00	1.27	2.75	5.81
time (sec)	N/A	0.040	0.018	0.007	1.159	1.019	0.000	0.322	1.217	1.671
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.058	0.018	0.010	0.984	1.107	0.000	0.330	1.220	23.704
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	83	80	59	59	0	107	231	83
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91	0.33
time (sec)	N/A	0.056	0.018	0.008	0.832	1.094	0.000	0.340	1.230	23.518
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	83	80	272	59	0	107	231	532
N.S.	1	1.00	0.65	0.62	2.12	0.46	0.00	0.84	1.80	4.16
time (sec)	N/A	0.057	0.020	0.010	1.021	1.275	0.000	0.395	1.222	1.537

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	131	113	109	123	32	146	-1	144
N.S.	1	1.00	0.55	0.47	0.45	0.51	0.13	0.61	-0.00	0.60
time (sec)	N/A	0.124	0.056	0.010	2.178	0.854	0.212	0.371	0.000	7.362
Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	235	128	110	106	106	22	143	-1	139
N.S.	1	1.00	0.54	0.47	0.45	0.45	0.09	0.61	-0.00	0.59
time (sec)	N/A	0.114	0.032	0.007	1.365	1.174	0.203	0.409	0.000	6.524
Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	35	32	15	13	10	22	33	149
N.S.	1	1.00	0.80	0.73	0.34	0.30	0.23	0.50	0.75	3.39
time (sec)	N/A	0.035	0.008	0.007	0.932	1.073	0.173	0.340	1.392	0.272
Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	109	97	98	304	24	124	-1	135
N.S.	1	1.00	0.54	0.48	0.49	1.50	0.12	0.61	-0.00	0.67
time (sec)	N/A	0.085	0.025	0.004	1.277	1.755	0.179	0.417	0.000	8.355

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	109	97	98	299	20	122	-1	135
N.S.	1	1.00	0.54	0.48	0.49	1.48	0.10	0.60	-0.00	0.67
time (sec)	N/A	0.117	0.023	0.004	2.733	1.312	0.190	0.365	0.000	10.094
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	42	39	43	18	15	32	48	94
N.S.	1	1.00	0.52	0.49	0.54	0.22	0.19	0.40	0.60	1.18
time (sec)	N/A	0.034	0.012	0.008	1.146	1.118	0.276	0.343	1.387	0.201
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	133	111	106	103	29	131	-1	142
N.S.	1	1.00	0.56	0.47	0.45	0.43	0.12	0.55	-0.00	0.60
time (sec)	N/A	0.108	0.031	0.007	1.742	1.075	0.234	0.371	0.000	15.054
Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	240	140	117	106	143	32	125	-1	144
N.S.	1	0.99	0.58	0.48	0.44	0.59	0.13	0.51	-0.00	0.59
time (sec)	N/A	0.107	0.034	0.013	2.211	1.298	0.261	0.334	0.000	17.846

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	122	54	51	73	33	31	50	75	383
N.S.	1	0.98	0.43	0.41	0.58	0.26	0.25	0.40	0.60	3.06
time (sec)	N/A	0.051	0.016	0.014	1.195	1.116	0.362	0.367	1.399	0.679

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	280	235	299	149	512	0	0	-1	169
N.S.	1	1.00	0.84	1.07	0.53	1.83	0.00	0.00	-0.00	0.60
time (sec)	N/A	0.135	0.075	0.025	1.810	1.379	0.000	0.000	0.000	11.029

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	235	299	146	503	0	0	-1	166
N.S.	1	1.00	0.85	1.08	0.53	1.82	0.00	0.00	-0.00	0.60
time (sec)	N/A	0.133	0.066	0.014	1.700	1.306	0.000	0.000	0.000	11.086

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	24	16	26	0	0	34	137
N.S.	1	1.00	0.71	0.63	0.42	0.68	0.00	0.00	0.89	3.61
time (sec)	N/A	0.029	0.011	0.006	0.888	1.410	0.000	0.000	1.193	0.565

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	277	277	237	301	147	514	0	0	-1	166
N.S.	1	1.00	0.86	1.09	0.53	1.86	0.00	0.00	-0.00	0.60
time (sec)	N/A	0.140	0.069	0.010	1.361	0.736	0.000	0.000	0.000	13.198
Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	286	286	235	299	145	499	0	0	-1	164
N.S.	1	1.00	0.82	1.05	0.51	1.74	0.00	0.00	-0.00	0.57
time (sec)	N/A	0.147	0.068	0.010	1.916	1.343	0.000	0.000	0.000	15.693
Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	74	107	88	90	0	0	-1	758
N.S.	1	1.00	0.50	0.73	0.60	0.61	0.00	0.00	-0.01	5.16
time (sec)	N/A	0.083	0.030	0.017	1.153	1.240	0.000	0.000	0.000	2.571
Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	260	316	148	201	0	0	-1	177
N.S.	1	1.00	0.82	1.00	0.47	0.64	0.00	0.00	-0.00	0.56
time (sec)	N/A	0.160	0.084	0.020	1.949	1.397	0.000	0.000	0.000	21.141

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	266	322	150	242	0	0	-1	177
N.S.	1	1.00	0.84	1.02	0.47	0.77	0.00	0.00	-0.00	0.56
time (sec)	N/A	0.159	0.088	0.018	1.801	1.398	0.000	0.000	0.000	24.318

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	97	133	117	119	0	0	-1	796
N.S.	1	1.00	0.52	0.71	0.62	0.63	0.00	0.00	-0.01	4.23
time (sec)	N/A	0.097	0.033	0.024	0.994	1.267	0.000	0.000	0.000	3.838

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	359	359	218	519	195	723	0	0	-1	189
N.S.	1	1.00	0.61	1.45	0.54	2.01	0.00	0.00	-0.00	0.53
time (sec)	N/A	0.185	0.128	0.023	1.348	0.779	0.000	0.000	0.000	24.917

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	39	32	43	58	0	0	42	229
N.S.	1	1.00	0.50	0.41	0.55	0.74	0.00	0.00	0.54	2.94
time (sec)	N/A	0.053	0.016	0.009	0.857	1.181	0.000	0.000	1.284	0.765

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	368	368	229	521	195	734	0	0	-1	191
N.S.	1	1.00	0.62	1.42	0.53	1.99	0.00	0.00	-0.00	0.52
time (sec)	N/A	0.191	0.135	0.022	1.766	1.302	0.000	0.000	0.000	26.107
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	221	519	193	723	0	0	-1	189
N.S.	1	1.00	0.61	1.44	0.54	2.01	0.00	0.00	-0.00	0.52
time (sec)	N/A	0.180	0.132	0.019	2.284	1.230	0.000	0.000	0.000	26.811
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	24	16	48	0	0	34	209
N.S.	1	1.00	0.71	0.63	0.42	1.26	0.00	0.00	0.89	5.50
time (sec)	N/A	0.030	0.010	0.009	0.512	1.147	0.000	0.000	1.259	0.828
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	359	359	219	521	191	734	0	0	-1	188
N.S.	1	1.00	0.61	1.45	0.53	2.04	0.00	0.00	-0.00	0.52
time (sec)	N/A	0.193	0.120	0.011	2.184	1.270	0.000	0.000	0.000	28.089

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	364	364	211	519	189	719	0	0	-1	186
N.S.	1	1.00	0.58	1.43	0.52	1.98	0.00	0.00	-0.00	0.51
time (sec)	N/A	0.199	0.111	0.008	2.152	1.243	0.000	0.000	0.000	34.135

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	96	193	132	178	0	0	-1	3896
N.S.	1	1.00	0.43	0.87	0.59	0.80	0.00	0.00	-0.00	17.47
time (sec)	N/A	0.124	0.044	0.024	0.875	1.022	0.000	0.000	0.000	127.754

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	398	398	242	536	192	311	0	0	-1	199
N.S.	1	1.00	0.61	1.35	0.48	0.78	0.00	0.00	-0.00	0.50
time (sec)	N/A	0.216	0.129	0.025	2.333	0.966	0.000	0.000	0.000	35.250

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	398	398	234	542	194	352	0	0	-1	199
N.S.	1	1.00	0.59	1.36	0.49	0.88	0.00	0.00	-0.00	0.50
time (sec)	N/A	0.211	0.135	0.025	2.059	1.304	0.000	0.000	0.000	42.116

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	119	219	163	207	0	0	-1	2850
N.S.	1	1.00	0.44	0.81	0.61	0.77	0.00	0.00	-0.00	10.59
time (sec)	N/A	0.143	0.051	0.021	0.851	1.267	0.000	0.000	0.000	81.815

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	111	453	243	369	0	900	-1	0
N.S.	1	1.00	0.35	1.45	0.78	1.18	0.00	2.88	-0.00	0.00
time (sec)	N/A	0.138	0.091	0.008	0.952	1.906	0.000	0.673	0.000	3.285

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	131	199	119	159	0	384	-1	0
N.S.	1	1.00	0.64	0.97	0.58	0.78	0.00	1.87	-0.00	0.00
time (sec)	N/A	0.085	0.065	0.015	1.037	1.197	0.000	0.524	0.000	1.804

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	53	56	35	35	0	83	-1	0
N.S.	1	1.00	0.55	0.58	0.36	0.36	0.00	0.86	-0.01	0.00
time (sec)	N/A	0.038	0.023	0.003	1.034	1.277	0.000	0.452	0.000	1.211

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	110	150	115	163	0	375	207	0
N.S.	1	1.00	0.64	0.87	0.67	0.95	0.00	2.18	1.20	0.00
time (sec)	N/A	0.113	0.056	0.014	0.808	1.168	0.000	0.530	1.310	0.527

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	77	96	79	108	0	235	137	0
N.S.	1	1.00	0.59	0.74	0.61	0.83	0.00	1.81	1.05	0.00
time (sec)	N/A	0.079	0.033	0.007	1.222	1.129	0.000	0.439	1.219	0.492

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	51	60	54	70	0	132	85	0
N.S.	1	1.00	0.61	0.71	0.64	0.83	0.00	1.57	1.01	0.00
time (sec)	N/A	0.055	0.020	0.007	0.628	1.116	0.000	0.479	1.193	0.288

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	32	40	30	37	0	58	46	0
N.S.	1	1.00	0.78	0.98	0.73	0.90	0.00	1.41	1.12	0.00
time (sec)	N/A	0.028	0.006	0.007	0.947	1.389	0.000	0.482	1.155	0.171

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	78	111	0	254	316	75	1758	0
N.S.	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	21.70	0.00
time (sec)	N/A	0.083	0.052	0.006	0.000	1.290	2.695	1.138	1.981	0.001
Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	62	60	0	197	223	59	1199	0
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	19.03	0.00
time (sec)	N/A	0.058	0.023	0.003	0.000	0.833	1.369	1.186	1.800	0.001
Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	42	37	0	129	131	36	174	0
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	4.58	0.00
time (sec)	N/A	0.036	0.010	0.002	0.000	1.309	0.638	0.983	1.232	0.001
Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	66	66	0	223	253	66	1362	0
N.S.	1	1.00	0.96	0.96	0.00	3.23	3.67	0.96	19.74	0.00
time (sec)	N/A	0.070	0.023	0.007	0.000	1.407	6.731	1.003	1.922	0.001

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	92	119	0	293	345	93	4281	0
N.S.	1	1.00	1.03	1.34	0.00	3.29	3.88	1.04	48.10	0.00
time (sec)	N/A	0.125	0.029	0.009	0.000	1.382	113.787	1.137	2.026	0.001

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	636	636	70	61	0	5601	279	0	4069	0
N.S.	1	1.00	0.11	0.10	0.00	8.81	0.44	0.00	6.40	0.00
time (sec)	N/A	1.249	0.030	0.141	0.000	4.092	14.581	0.000	12.146	0.001

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	631	631	70	59	0	5260	196	0	2280	0
N.S.	1	1.00	0.11	0.09	0.00	8.34	0.31	0.00	3.61	0.00
time (sec)	N/A	1.024	0.030	0.005	0.000	3.185	6.903	0.000	3.397	0.001

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	558	558	44	43	0	3799	175	0	2695	0
N.S.	1	1.00	0.08	0.08	0.00	6.81	0.31	0.00	4.83	0.00
time (sec)	N/A	0.520	0.017	0.004	0.000	1.778	2.182	0.000	8.111	0.001

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	558	558	42	43	0	2551	122	0	2129	0
N.S.	1	1.00	0.08	0.08	0.00	4.57	0.22	0.00	3.82	0.00
time (sec)	N/A	0.574	0.016	0.003	0.000	1.485	1.800	0.000	7.712	0.001

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	558	558	43	41	0	2875	158	0	1543	0
N.S.	1	1.00	0.08	0.07	0.00	5.15	0.28	0.00	2.77	0.00
time (sec)	N/A	0.472	0.017	0.004	0.000	1.701	1.528	0.000	5.388	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	558	558	45	40	0	3978	155	0	2597	0
N.S.	1	1.00	0.08	0.07	0.00	7.13	0.28	0.00	4.65	0.00
time (sec)	N/A	0.595	0.017	0.002	0.000	2.024	4.353	0.000	8.495	0.001

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	610	610	71	61	0	5266	252	0	2978	0
N.S.	1	1.00	0.12	0.10	0.00	8.63	0.41	0.00	4.88	0.00
time (sec)	N/A	0.818	0.031	0.006	0.000	3.992	3.188	0.000	6.889	0.001

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	612	612	75	62	0	5771	241	0	4063	0
N.S.	1	1.00	0.12	0.10	0.00	9.43	0.39	0.00	6.64	0.00
time (sec)	N/A	0.815	0.034	0.005	0.000	3.764	52.501	0.000	10.654	0.001

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	28	27	27	29	29	27	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.83	0.77	0.00
time (sec)	N/A	0.024	0.006	0.005	0.489	1.288	0.126	0.394	1.253	0.001

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	23	22	22	22	24	22	0
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.86	0.79	0.00
time (sec)	N/A	0.019	0.005	0.006	0.487	1.126	0.124	0.343	0.046	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	19	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.81	0.00
time (sec)	N/A	0.014	0.004	0.004	0.519	1.205	0.116	0.364	0.055	0.001

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	21	21	18	17	17	15	19	16	0
N.S.	1	2.10	2.10	1.80	1.70	1.70	1.50	1.90	1.60	0.00
time (sec)	N/A	0.014	0.004	0.003	0.465	1.211	0.108	0.328	0.383	0.000
Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	31	23	21	20	24	21	0
N.S.	1	1.00	1.00	1.15	0.85	0.78	0.74	0.89	0.78	0.00
time (sec)	N/A	0.019	0.006	0.010	0.451	1.253	0.141	0.353	1.261	0.000
Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	36	28	35	29	36	26	0
N.S.	1	1.00	1.00	1.06	0.82	1.03	0.85	1.06	0.76	0.00
time (sec)	N/A	0.032	0.006	0.010	0.682	1.271	0.170	0.301	1.233	0.001
Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	41	35	40	34	41	32	0
N.S.	1	1.00	1.00	1.00	0.85	0.98	0.83	1.00	0.78	0.00
time (sec)	N/A	0.036	0.005	0.008	0.588	1.209	0.188	0.365	0.043	0.001

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	118	94	94	102	144	96	124	0
N.S.	1	1.00	0.95	0.76	0.76	0.82	1.16	0.77	1.00	0.00
time (sec)	N/A	0.114	0.054	0.010	1.230	1.299	0.619	0.365	0.244	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	114	92	92	90	129	94	119	0
N.S.	1	1.00	0.93	0.75	0.75	0.74	1.06	0.77	0.98	0.00
time (sec)	N/A	0.094	0.028	0.007	1.331	1.117	0.611	0.432	1.420	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	111	89	89	99	134	91	118	0
N.S.	1	1.00	0.93	0.75	0.75	0.83	1.13	0.76	0.99	0.00
time (sec)	N/A	0.082	0.027	0.007	1.305	1.180	0.607	0.336	0.186	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	111	85	85	88	126	87	104	0
N.S.	1	1.00	0.98	0.75	0.75	0.78	1.12	0.77	0.92	0.00
time (sec)	N/A	0.074	0.026	0.007	1.227	1.506	0.615	0.454	0.160	0.001

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	107	84	84	106	134	86	114	0
N.S.	1	1.00	0.96	0.75	0.75	0.95	1.20	0.77	1.02	0.00
time (sec)	N/A	0.068	0.024	0.006	1.607	1.468	0.600	0.303	1.371	0.001

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	106	84	84	102	110	86	113	0
N.S.	1	1.00	0.95	0.75	0.75	0.91	0.98	0.77	1.01	0.00
time (sec)	N/A	0.069	0.023	0.007	1.706	1.306	0.589	0.335	1.363	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	108	84	84	84	119	86	113	0
N.S.	1	1.00	0.96	0.75	0.75	0.75	1.06	0.77	1.01	0.00
time (sec)	N/A	0.069	0.025	0.005	1.597	1.018	1.857	0.363	1.364	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	107	84	84	124	124	86	110	0
N.S.	1	1.00	0.96	0.75	0.75	1.11	1.11	0.77	0.98	0.00
time (sec)	N/A	0.065	0.023	0.007	1.283	1.419	1.819	0.339	0.225	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	118	89	89	117	139	91	119	0
N.S.	1	1.00	0.99	0.75	0.75	0.98	1.17	0.76	1.00	0.00
time (sec)	N/A	0.082	0.041	0.007	1.085	1.182	1.830	0.414	1.380	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	113	89	89	126	128	91	118	0
N.S.	1	1.00	0.95	0.75	0.75	1.06	1.08	0.76	0.99	0.00
time (sec)	N/A	0.078	0.049	0.008	1.275	1.161	1.742	0.411	1.362	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	118	94	96	112	141	98	124	0
N.S.	1	1.00	0.94	0.75	0.76	0.89	1.12	0.78	0.98	0.00
time (sec)	N/A	0.103	0.048	0.011	1.198	1.160	1.836	0.363	0.189	0.001

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	118	94	96	153	136	98	121	0
N.S.	1	1.00	0.94	0.75	0.76	1.21	1.08	0.78	0.96	0.00
time (sec)	N/A	0.099	0.059	0.014	1.658	1.155	1.776	0.358	1.399	0.001

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	412	412	59	44	0	1028	26	638	320	0
N.S.	1	1.00	0.14	0.11	0.00	2.50	0.06	1.55	0.78	0.00
time (sec)	N/A	0.428	0.013	0.013	0.000	1.382	0.180	0.525	1.821	0.001

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	33	32	32	37	32	34	0
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87	0.00
time (sec)	N/A	0.035	0.011	0.005	1.041	1.180	0.128	0.433	1.214	0.002

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	411	41	40	0	1583	26	824	304	0
N.S.	1	1.00	0.10	0.10	0.00	3.85	0.06	2.00	0.74	0.00
time (sec)	N/A	0.281	0.009	0.007	0.000	1.730	0.181	0.688	1.717	0.001

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	411	39	40	0	1031	24	637	327	0
N.S.	1	1.00	0.09	0.10	0.00	2.51	0.06	1.55	0.80	0.00
time (sec)	N/A	0.278	0.009	0.007	0.000	1.425	0.179	0.574	1.840	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	27	18	20	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87	0.00
time (sec)	N/A	0.023	0.006	0.001	1.135	1.128	0.116	0.495	1.217	0.001

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	40	38	0	1583	26	812	304	0
N.S.	1	1.00	0.11	0.10	0.00	4.22	0.07	2.17	0.81	0.00
time (sec)	N/A	0.250	0.010	0.007	0.000	1.498	0.180	0.570	0.451	0.001

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	375	42	37	0	1027	20	629	327	0
N.S.	1	2.02	0.23	0.20	0.00	5.52	0.11	3.38	1.76	0.00
time (sec)	N/A	0.242	0.008	0.007	0.000	1.292	0.184	0.499	1.788	0.001

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	55	35	38	34	41	35	36	0
N.S.	1	1.00	1.34	0.85	0.93	0.83	1.00	0.85	0.88	0.00
time (sec)	N/A	0.039	0.012	0.006	1.447	1.230	0.148	0.352	1.231	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	61	50	0	1598	24	826	286	0
N.S.	1	1.00	0.15	0.12	0.00	3.84	0.06	1.99	0.69	0.00
time (sec)	N/A	0.300	0.013	0.009	0.000	1.650	0.198	0.550	1.656	0.001
Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	418	418	65	50	0	1066	31	642	324	0
N.S.	1	1.00	0.16	0.12	0.00	2.55	0.07	1.54	0.78	0.00
time (sec)	N/A	0.342	0.013	0.009	0.000	1.446	0.206	0.551	1.717	0.000
Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	51	40	43	51	48	45	41	0
N.S.	1	1.00	1.06	0.83	0.90	1.06	1.00	0.94	0.85	0.00
time (sec)	N/A	0.052	0.013	0.007	1.130	1.173	0.172	0.417	0.064	0.000
Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	423	423	54	51	0	1623	39	836	318	0
N.S.	1	1.00	0.13	0.12	0.00	3.84	0.09	1.98	0.75	0.00
time (sec)	N/A	0.369	0.014	0.010	0.000	1.620	0.218	0.549	1.592	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	381	381	38	33	0	1996	24	0	513	0
N.S.	1	1.00	0.10	0.09	0.00	5.24	0.06	0.00	1.35	0.00
time (sec)	N/A	0.403	0.010	0.007	0.000	3.213	0.154	0.000	2.615	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	27	18	20	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87	0.00
time (sec)	N/A	0.024	0.008	0.003	1.360	1.188	0.113	0.504	0.046	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	399	399	37	36	0	1435	24	0	351	0
N.S.	1	1.00	0.09	0.09	0.00	3.60	0.06	0.00	0.88	0.00
time (sec)	N/A	0.305	0.009	0.006	0.000	1.506	0.148	0.000	2.614	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	208	0	0	451	0	0	543	209
N.S.	1	1.00	0.90	0.00	0.00	1.95	0.00	0.00	2.35	0.90
time (sec)	N/A	0.300	0.160	0.053	0.000	1.353	0.000	0.000	2.939	0.800

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	164	0	0	367	0	0	315	170
N.S.	1	1.00	0.96	0.00	0.00	2.15	0.00	0.00	1.84	0.99
time (sec)	N/A	0.152	0.134	0.046	0.000	1.750	0.000	0.000	1.866	0.582
Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	136	0	0	303	0	0	193	132
N.S.	1	1.00	0.89	0.00	0.00	1.98	0.00	0.00	1.26	0.86
time (sec)	N/A	0.136	0.069	0.032	0.000	1.220	0.000	0.000	1.591	0.424
Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	99	0	0	237	0	98	87	107
N.S.	1	1.00	0.92	0.00	0.00	2.19	0.00	0.91	0.81	0.99
time (sec)	N/A	0.085	0.074	0.028	0.000	1.219	0.000	0.491	1.390	0.323
Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	87	0	0	197	0	76	72	85
N.S.	1	1.00	1.05	0.00	0.00	2.37	0.00	0.92	0.87	1.02
time (sec)	N/A	0.060	0.010	0.026	0.000	1.340	0.000	0.556	1.390	0.245

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	106	0	0	566	0	0	88	111
N.S.	1	1.00	0.97	0.00	0.00	5.19	0.00	0.00	0.81	1.02
time (sec)	N/A	0.111	0.042	0.036	0.000	1.367	0.000	0.000	1.362	0.290

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	112	0	0	601	0	0	91	114
N.S.	1	1.00	1.00	0.00	0.00	5.37	0.00	0.00	0.81	1.02
time (sec)	N/A	0.114	0.048	0.043	0.000	1.568	0.000	0.000	1.553	0.300

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	89	0	0	215	0	0	-1	91
N.S.	1	1.00	1.01	0.00	0.00	2.44	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.073	0.043	0.039	0.000	1.366	0.000	0.000	0.000	0.384

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	108	0	0	259	0	0	-1	108
N.S.	1	1.00	0.93	0.00	0.00	2.23	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.095	0.074	0.050	0.000	1.363	0.000	0.000	0.000	0.654

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	139	0	0	325	0	0	-1	141
N.S.	1	1.00	0.86	0.00	0.00	2.02	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.147	0.088	0.060	0.000	1.400	0.000	0.000	0.000	0.837
Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	173	0	0	389	0	0	-1	176
N.S.	1	1.00	0.87	0.00	0.00	1.95	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.227	0.116	0.061	0.000	1.522	0.000	0.000	0.000	1.197
Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	241	0	0	641	0	0	-1	314
N.S.	1	1.00	0.82	0.00	0.00	2.19	0.00	0.00	-0.00	1.07
time (sec)	N/A	0.402	0.349	0.049	0.000	1.555	0.000	0.000	0.000	1.398
Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	192	0	0	535	0	0	-1	255
N.S.	1	1.00	0.86	0.00	0.00	2.40	0.00	0.00	-0.00	1.14
time (sec)	N/A	0.206	0.192	0.032	0.000	1.245	0.000	0.000	0.000	1.134

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	175	0	0	451	0	0	-1	209
N.S.	1	1.00	0.86	0.00	0.00	2.21	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.188	0.159	0.031	0.000	1.296	0.000	0.000	0.000	0.883

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	149	0	0	361	0	172	223	162
N.S.	1	1.00	0.99	0.00	0.00	2.41	0.00	1.15	1.49	1.08
time (sec)	N/A	0.116	0.146	0.036	0.000	1.212	0.000	0.561	1.575	0.661

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	126	0	0	297	0	135	115	132
N.S.	1	1.00	1.02	0.00	0.00	2.40	0.00	1.09	0.93	1.06
time (sec)	N/A	0.086	0.085	0.033	0.000	1.383	0.000	0.611	1.444	0.495

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	143	0	0	727	0	0	-1	151
N.S.	1	1.00	0.92	0.00	0.00	4.69	0.00	0.00	-0.01	0.97
time (sec)	N/A	0.179	0.143	0.042	0.000	1.821	0.000	0.000	0.000	0.703

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	134	0	0	713	0	0	-1	137
N.S.	1	1.00	0.89	0.00	0.00	4.75	0.00	0.00	-0.01	0.91
time (sec)	N/A	0.170	0.111	0.065	0.000	1.554	0.000	0.000	0.000	0.727

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	134	0	0	713	0	0	-1	134
N.S.	1	1.00	0.89	0.00	0.00	4.72	0.00	0.00	-0.01	0.89
time (sec)	N/A	0.162	0.172	0.056	0.000	1.457	0.000	0.000	0.000	0.796

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	149	0	0	771	0	0	-1	148
N.S.	1	1.00	0.91	0.00	0.00	4.73	0.00	0.00	-0.01	0.91
time (sec)	N/A	0.176	0.206	0.056	0.000	1.596	0.000	0.000	0.000	0.991

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	138	0	0	319	0	0	-1	139
N.S.	1	1.00	1.04	0.00	0.00	2.40	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.108	0.164	0.051	0.000	1.404	0.000	0.000	0.000	1.109

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	167	0	0	383	0	0	-1	176
N.S.	1	1.00	1.03	0.00	0.00	2.36	0.00	0.00	-0.01	1.09
time (sec)	N/A	0.146	0.140	0.065	0.000	1.673	0.000	0.000	0.000	1.551

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	216	206	0	0	473	0	0	-1	221
N.S.	1	1.00	0.95	0.00	0.00	2.19	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.207	0.210	0.078	0.000	2.090	0.000	0.000	0.000	2.272

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	243	0	0	557	0	0	-1	270
N.S.	1	1.00	0.95	0.00	0.00	2.18	0.00	0.00	-0.00	1.06
time (sec)	N/A	0.312	0.486	0.093	0.000	2.626	0.000	0.000	0.000	2.484

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	137	0	0	303	0	0	-1	138
N.S.	1	1.00	0.80	0.00	0.00	1.77	0.00	0.00	-0.01	0.81
time (sec)	N/A	0.216	0.111	0.036	0.000	0.641	0.000	0.000	0.000	0.415

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	104	0	0	241	0	0	-1	101
N.S.	1	1.00	0.86	0.00	0.00	1.99	0.00	0.00	-0.01	0.83
time (sec)	N/A	0.105	0.050	0.040	0.000	1.357	0.000	0.000	0.000	0.315
Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	88	0	0	203	0	0	-1	91
N.S.	1	1.00	0.85	0.00	0.00	1.95	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.089	0.033	0.027	0.000	1.456	0.000	0.000	0.000	0.288
Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	0	0	161	0	61	55	70
N.S.	1	1.00	1.00	0.00	0.00	2.37	0.00	0.90	0.81	1.03
time (sec)	N/A	0.057	0.016	0.024	0.000	1.347	0.000	0.674	1.492	0.188
Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	0	0	118	0	40	34	41
N.S.	1	1.00	1.00	0.00	0.00	2.74	0.00	0.93	0.79	0.95
time (sec)	N/A	0.034	0.006	0.020	0.000	1.214	0.000	0.676	1.575	0.140

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	0	0	124	0	0	36	48
N.S.	1	1.00	1.00	0.00	0.00	2.82	0.00	0.00	0.82	1.09
time (sec)	N/A	0.040	0.006	0.020	0.000	1.101	0.000	0.000	1.571	0.126

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	72	0	0	179	0	0	56	76
N.S.	1	1.00	1.00	0.00	0.00	2.49	0.00	0.00	0.78	1.06
time (sec)	N/A	0.060	0.024	0.033	0.000	1.360	0.000	0.000	1.560	0.215

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	92	0	0	221	0	0	-1	91
N.S.	1	1.00	0.85	0.00	0.00	2.05	0.00	0.00	-0.01	0.84
time (sec)	N/A	0.101	0.059	0.037	0.000	1.414	0.000	0.000	0.000	0.343

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	112	0	0	263	0	0	-1	110
N.S.	1	1.00	0.77	0.00	0.00	1.81	0.00	0.00	-0.01	0.76
time (sec)	N/A	0.158	0.078	0.042	0.000	1.275	0.000	0.000	0.000	0.531

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	141	0	0	327	0	0	-1	141
N.S.	1	1.00	0.73	0.00	0.00	1.70	0.00	0.00	-0.01	0.73
time (sec)	N/A	0.233	0.098	0.046	0.000	1.776	0.000	0.000	0.000	0.785
Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	181	0	0	591	0	0	-1	175
N.S.	1	1.00	0.93	0.00	0.00	3.03	0.00	0.00	-0.01	0.90
time (sec)	N/A	0.229	0.181	0.065	0.000	1.906	0.000	0.000	0.000	0.958
Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	137	0	0	459	0	0	-1	131
N.S.	1	1.00	1.00	0.00	0.00	3.35	0.00	0.00	-0.01	0.96
time (sec)	N/A	0.111	0.119	0.055	0.000	1.371	0.000	0.000	0.000	0.667
Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	107	0	0	387	0	0	84	99
N.S.	1	1.00	0.89	0.00	0.00	3.22	0.00	0.00	0.70	0.82
time (sec)	N/A	0.090	0.098	0.028	0.000	1.338	0.000	0.000	1.658	0.520

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	41	38	0	68	0	45	38	39
N.S.	1	1.00	1.05	0.97	0.00	1.74	0.00	1.15	0.97	1.00
time (sec)	N/A	0.029	0.099	0.006	0.000	0.945	0.000	1.321	1.429	0.394

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	37	0	67	0	45	37	38
N.S.	1	1.00	1.00	0.97	0.00	1.76	0.00	1.18	0.97	1.00
time (sec)	N/A	0.025	0.024	0.006	0.000	1.198	0.000	1.407	1.370	0.325

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	92	0	0	389	0	0	-1	101
N.S.	1	1.00	1.00	0.00	0.00	4.23	0.00	0.00	-0.01	1.10
time (sec)	N/A	0.078	0.123	0.062	0.000	1.666	0.000	0.000	0.000	0.540

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	137	0	0	485	0	0	-1	132
N.S.	1	1.00	0.96	0.00	0.00	3.42	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.124	0.083	0.087	0.000	1.527	0.000	0.000	0.000	0.682

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	179	0	0	615	0	0	-1	175
N.S.	1	1.00	0.90	0.00	0.00	3.11	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.204	0.126	0.098	0.000	1.846	0.000	0.000	0.000	1.015
Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	256	256	223	0	0	705	0	0	-1	224
N.S.	1	1.00	0.87	0.00	0.00	2.75	0.00	0.00	-0.00	0.88
time (sec)	N/A	0.286	0.177	0.133	0.000	2.103	0.000	0.000	0.000	1.354
Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	70	301	110	241	1510	449	260	0
N.S.	1	1.00	0.69	2.98	1.09	2.39	14.95	4.45	2.57	0.00
time (sec)	N/A	0.061	0.071	0.008	1.102	1.078	5.824	0.465	1.524	1.170
Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	35	78	50	71	314	119	89	0
N.S.	1	1.00	0.67	1.50	0.96	1.37	6.04	2.29	1.71	0.00
time (sec)	N/A	0.021	0.029	0.003	1.163	1.308	1.468	0.346	1.358	0.214

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	24	25	24	31	22	24	25	0
N.S.	1	1.00	0.80	0.83	0.80	1.03	0.73	0.80	0.83	0.00
time (sec)	N/A	0.012	0.014	0.009	2.156	0.931	0.126	0.340	0.047	0.000
Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	18	19	18	23	15	18	18	0
N.S.	1	1.00	0.82	0.86	0.82	1.05	0.68	0.82	0.82	0.00
time (sec)	N/A	0.010	0.006	0.007	0.984	0.938	0.106	0.279	1.315	0.000
Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	24	15	19	21	0
N.S.	1	1.00	1.00	0.87	0.83	1.04	0.65	0.83	0.91	0.00
time (sec)	N/A	0.009	0.009	0.006	2.015	0.942	0.118	0.372	1.365	0.000
Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	8	9	11	0
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.00	0.00
time (sec)	N/A	0.003	0.002	0.003	0.887	0.858	0.094	0.355	0.018	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	20	20	19	23	15	19	20	0
N.S.	1	1.00	0.87	0.87	0.83	1.00	0.65	0.83	0.87	0.00
time (sec)	N/A	0.007	0.005	0.004	2.173	1.119	0.116	0.392	0.026	0.000
Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	21	24	32	19	29	20	0
N.S.	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83	0.00
time (sec)	N/A	0.012	0.010	0.013	0.974	0.782	0.125	0.361	0.044	0.000
Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	25	31	26	25	25	0
N.S.	1	1.00	1.00	0.83	0.83	1.03	0.87	0.83	0.83	0.00
time (sec)	N/A	0.012	0.011	0.010	1.976	1.021	0.147	0.335	0.040	0.001
Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	33	44	31	33	31	0
N.S.	1	1.00	1.00	0.85	1.00	1.33	0.94	1.00	0.94	0.00
time (sec)	N/A	0.016	0.012	0.015	0.877	1.129	0.153	0.355	0.054	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	33	28	30	36	29	31	30	0
N.S.	1	1.00	0.89	0.76	0.81	0.97	0.78	0.84	0.81	0.00
time (sec)	N/A	0.016	0.011	0.012	2.487	1.097	0.170	0.296	0.046	0.001

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	94	69	83	132	90	83	45	0
N.S.	1	1.00	0.90	0.66	0.80	1.27	0.87	0.80	0.43	0.00
time (sec)	N/A	0.055	0.068	0.010	2.022	1.018	0.176	0.339	1.369	0.001

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	93	70	84	129	90	84	47	0
N.S.	1	1.00	0.94	0.71	0.85	1.30	0.91	0.85	0.47	0.00
time (sec)	N/A	0.051	0.061	0.008	2.075	1.366	0.179	0.293	1.326	0.001

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	90	68	82	126	82	82	45	0
N.S.	1	1.00	0.93	0.70	0.85	1.30	0.85	0.85	0.46	0.00
time (sec)	N/A	0.051	0.065	0.006	1.986	1.298	0.170	0.357	0.082	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	92	70	84	128	83	84	46	0
N.S.	1	1.00	0.93	0.71	0.85	1.29	0.84	0.85	0.46	0.00
time (sec)	N/A	0.050	0.049	0.007	2.065	1.108	0.174	0.345	0.045	0.001

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	91	68	82	127	88	82	44	0
N.S.	1	1.00	0.94	0.70	0.85	1.31	0.91	0.85	0.45	0.00
time (sec)	N/A	0.047	0.046	0.004	2.002	1.132	0.200	0.336	1.305	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	98	75	88	130	97	88	49	0
N.S.	1	1.00	0.92	0.71	0.83	1.23	0.92	0.83	0.46	0.00
time (sec)	N/A	0.052	0.069	0.009	2.025	1.275	0.207	0.358	1.314	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	96	73	90	140	99	87	51	0
N.S.	1	1.00	0.91	0.69	0.85	1.32	0.93	0.82	0.48	0.00
time (sec)	N/A	0.052	0.071	0.010	2.198	1.305	0.224	0.443	1.365	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	103	80	95	145	102	96	55	0
N.S.	1	1.00	0.91	0.71	0.84	1.28	0.90	0.85	0.49	0.00
time (sec)	N/A	0.054	0.076	0.012	2.012	1.279	0.232	0.297	0.093	0.001

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	101	78	95	145	102	94	55	0
N.S.	1	1.00	0.89	0.69	0.84	1.28	0.90	0.83	0.49	0.00
time (sec)	N/A	0.055	0.076	0.010	1.932	1.235	0.234	0.364	0.104	0.001

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	39	41	34	46	34	35	26	0
N.S.	1	1.00	1.22	1.28	1.06	1.44	1.06	1.09	0.81	0.00
time (sec)	N/A	0.013	0.022	0.009	1.032	1.141	0.122	0.326	0.047	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	22	19	18	23	15	19	20	0
N.S.	1	1.00	0.85	0.73	0.69	0.88	0.58	0.73	0.77	0.00
time (sec)	N/A	0.013	0.007	0.006	1.128	1.268	0.104	0.410	0.048	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	33	36	29	40	26	30	21	0
N.S.	1	1.00	1.32	1.44	1.16	1.60	1.04	1.20	0.84	0.00
time (sec)	N/A	0.010	0.011	0.009	0.914	1.168	0.118	0.496	1.269	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	11	10	9	9	8	9	11	0
N.S.	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	0.85	0.00
time (sec)	N/A	0.003	0.002	0.003	0.884	1.146	0.095	0.499	0.022	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	33	36	29	40	26	30	21	0
N.S.	1	1.00	1.32	1.44	1.16	1.60	1.04	1.20	0.84	0.00
time (sec)	N/A	0.008	0.007	0.010	0.918	1.260	0.119	0.471	0.035	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	47	24	32	19	30	22	0
N.S.	1	1.00	0.93	1.68	0.86	1.14	0.68	1.07	0.79	0.00
time (sec)	N/A	0.015	0.010	0.015	0.943	0.981	0.126	0.381	0.058	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	41	50	37	54	36	38	26	0
N.S.	1	1.00	1.28	1.56	1.16	1.69	1.12	1.19	0.81	0.00
time (sec)	N/A	0.013	0.017	0.017	0.953	1.162	0.150	0.414	0.044	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	35	54	35	50	29	36	32	0
N.S.	1	1.00	0.95	1.46	0.95	1.35	0.78	0.97	0.86	0.00
time (sec)	N/A	0.019	0.012	0.018	0.828	0.863	0.152	0.453	0.052	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	49	55	42	59	41	42	32	0
N.S.	1	1.00	1.26	1.41	1.08	1.51	1.05	1.08	0.82	0.00
time (sec)	N/A	0.017	0.014	0.020	0.937	1.227	0.176	0.328	0.051	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	38	43	28	49	32	30	26	0
N.S.	1	1.00	1.12	1.26	0.82	1.44	0.94	0.88	0.76	0.00
time (sec)	N/A	0.009	0.016	0.013	2.127	1.190	0.151	0.280	1.285	0.001

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	35	42	29	46	32	31	23	0
N.S.	1	1.00	1.21	1.45	1.00	1.59	1.10	1.07	0.79	0.00
time (sec)	N/A	0.008	0.014	0.015	2.027	1.232	0.153	0.400	0.034	0.001

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	31	42	27	43	26	29	21	0
N.S.	1	1.00	1.15	1.56	1.00	1.59	0.96	1.07	0.78	0.00
time (sec)	N/A	0.007	0.013	0.013	1.918	1.268	0.147	0.512	0.034	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	33	42	29	45	27	31	23	0
N.S.	1	1.00	1.14	1.45	1.00	1.55	0.93	1.07	0.79	0.00
time (sec)	N/A	0.008	0.012	0.010	1.711	1.217	0.149	0.335	0.032	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	33	42	27	44	31	29	21	0
N.S.	1	1.00	1.22	1.56	1.00	1.63	1.15	1.07	0.78	0.00
time (sec)	N/A	0.005	0.009	0.013	2.132	1.299	0.156	0.366	0.029	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	40	47	35	55	37	37	26	0
N.S.	1	1.00	1.11	1.31	0.97	1.53	1.03	1.03	0.72	0.00
time (sec)	N/A	0.010	0.016	0.014	2.086	1.255	0.179	0.375	0.042	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	38	47	37	63	39	34	28	0
N.S.	1	1.00	1.06	1.31	1.03	1.75	1.08	0.94	0.78	0.00
time (sec)	N/A	0.009	0.018	0.013	1.983	1.221	0.184	0.457	1.292	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	51	52	42	68	44	43	34	0
N.S.	1	1.00	1.19	1.21	0.98	1.58	1.02	1.00	0.79	0.00
time (sec)	N/A	0.012	0.020	0.020	2.434	1.358	0.201	0.451	0.044	0.001

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	52	42	68	44	41	34	0
N.S.	1	1.00	1.00	1.21	0.98	1.58	1.02	0.95	0.79	0.00
time (sec)	N/A	0.012	0.019	0.016	1.940	1.178	0.211	0.452	0.047	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	78	111	0	254	316	75	3916	0
N.S.	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	48.35	0.00
time (sec)	N/A	0.084	0.051	0.006	0.000	1.365	4.113	17.069	2.687	0.001
Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	210	360	0	1071	134	2043	5659	0
N.S.	1	1.00	1.09	1.88	0.00	5.58	0.70	10.64	29.47	0.00
time (sec)	N/A	0.339	0.123	0.030	0.000	1.439	4.419	18.166	3.071	0.001
Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	62	60	0	197	223	59	2654	0
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	42.13	0.00
time (sec)	N/A	0.058	0.024	0.004	0.000	1.422	2.180	17.121	2.608	0.001
Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	171	216	0	567	76	1036	1220	0
N.S.	1	1.00	1.08	1.36	0.00	3.57	0.48	6.52	7.67	0.00
time (sec)	N/A	0.126	0.086	0.018	0.000	1.249	2.509	18.741	2.810	0.001

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	42	37	0	129	131	36	260	0
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	6.84	0.00
time (sec)	N/A	0.034	0.009	0.003	0.000	1.295	0.770	17.345	1.370	0.001

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	133	120	0	619	88	1030	1105	0
N.S.	1	1.00	0.86	0.78	0.00	4.02	0.57	6.69	7.18	0.00
time (sec)	N/A	0.094	0.081	0.013	0.000	1.254	3.359	19.496	2.271	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	66	66	0	223	253	68	1690	0
N.S.	1	1.00	0.96	0.96	0.00	3.23	3.67	0.99	24.49	0.00
time (sec)	N/A	0.068	0.023	0.008	0.000	1.240	14.469	16.281	2.185	0.001

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	75	240	0	1134	153	2055	5451	0
N.S.	1	1.00	0.41	1.30	0.00	6.16	0.83	11.17	29.62	0.00
time (sec)	N/A	0.224	0.030	0.025	0.000	1.404	15.344	15.891	2.416	0.001

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	92	119	0	293	0	94	8817	0
N.S.	1	1.00	1.03	1.34	0.00	3.29	0.00	1.06	99.07	0.00
time (sec)	N/A	0.124	0.032	0.010	0.000	1.879	0.000	14.501	2.788	0.001

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	381	381	70	63	0	6296	0	0	12709	0
N.S.	1	1.00	0.18	0.17	0.00	16.52	0.00	0.00	33.36	0.00
time (sec)	N/A	0.647	0.040	0.038	0.000	5.663	0.000	0.000	3.491	0.001

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	376	376	70	59	0	5082	0	0	10382	0
N.S.	1	1.00	0.19	0.16	0.00	13.52	0.00	0.00	27.61	0.00
time (sec)	N/A	0.574	0.037	0.007	0.000	2.612	0.000	0.000	3.969	0.001

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	325	44	43	0	3912	230	0	8033	0
N.S.	1	1.00	0.14	0.13	0.00	12.04	0.71	0.00	24.72	0.00
time (sec)	N/A	0.307	0.024	0.003	0.000	1.935	61.577	0.000	3.511	0.001

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	325	42	43	0	2479	126	0	8169	0
N.S.	1	1.00	0.13	0.13	0.00	7.63	0.39	0.00	25.14	0.00
time (sec)	N/A	0.296	0.021	0.003	0.000	1.531	2.917	0.000	3.633	0.001

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	315	315	43	43	0	2746	172	0	6067	0
N.S.	1	1.00	0.14	0.14	0.00	8.72	0.55	0.00	19.26	0.00
time (sec)	N/A	0.290	0.023	0.003	0.000	1.660	4.099	0.000	2.336	0.001

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	315	315	45	40	0	3929	177	0	10337	0
N.S.	1	1.00	0.14	0.13	0.00	12.47	0.56	0.00	32.82	0.00
time (sec)	N/A	0.304	0.025	0.003	0.000	1.868	19.792	0.000	3.416	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	363	71	63	0	5125	0	0	10509	0
N.S.	1	1.00	0.20	0.17	0.00	14.12	0.00	0.00	28.95	0.00
time (sec)	N/A	0.412	0.037	0.009	0.000	3.474	0.000	0.000	2.809	0.001

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	365	365	75	62	0	6324	0	0	16497	0
N.S.	1	1.00	0.21	0.17	0.00	17.33	0.00	0.00	45.20	0.00
time (sec)	N/A	0.399	0.042	0.012	0.000	4.075	0.000	0.000	5.568	0.001
Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	36	35	35	42	35	37	0
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.95	0.80	0.84	0.00
time (sec)	N/A	0.037	0.011	0.004	2.423	1.417	0.139	0.342	0.046	0.001
Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	54	54	98	43	42	40	51	42	43	0
N.S.	1	1.00	1.81	0.80	0.78	0.74	0.94	0.78	0.80	0.00
time (sec)	N/A	0.058	0.173	0.006	2.394	1.226	0.140	0.315	0.042	0.000
Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	37	30	32	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86	0.00
time (sec)	N/A	0.031	0.008	0.003	2.435	1.212	0.133	0.390	0.040	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	75	75	94	62	61	61	76	61	51	0
N.S.	1	1.00	1.25	0.83	0.81	0.81	1.01	0.81	0.68	0.00
time (sec)	N/A	0.077	0.119	0.004	2.490	0.973	0.206	0.389	0.093	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	26	18	17	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74	0.00
time (sec)	N/A	0.021	0.006	0.001	2.470	1.140	0.118	0.361	1.302	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	75	75	79	62	61	61	76	61	51	0
N.S.	1	1.00	1.05	0.83	0.81	0.81	1.01	0.81	0.68	0.00
time (sec)	N/A	0.064	0.048	0.004	2.589	1.501	0.202	0.309	1.275	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	138	87	36	32	41	36	34	0
N.S.	1	1.00	3.54	2.23	0.92	0.82	1.05	0.92	0.87	0.00
time (sec)	N/A	0.035	0.084	0.008	3.047	1.261	0.155	0.383	1.295	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	100	57	42	45	53	42	43	0
N.S.	1	1.00	1.85	1.06	0.78	0.83	0.98	0.78	0.80	0.00
time (sec)	N/A	0.052	0.049	0.006	2.392	1.084	0.164	0.378	0.036	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	141	94	41	49	48	46	41	0
N.S.	1	1.00	2.94	1.96	0.85	1.02	1.00	0.96	0.85	0.00
time (sec)	N/A	0.051	0.102	0.010	2.791	1.085	0.184	0.306	0.063	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	142	95	73	84	88	73	62	0
N.S.	1	1.00	1.60	1.07	0.82	0.94	0.99	0.82	0.70	0.00
time (sec)	N/A	0.096	0.110	0.009	2.372	1.458	0.249	0.266	0.039	0.001

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	141	141	139	110	0	212	192	109	100	0
N.S.	1	1.00	0.99	0.78	0.00	1.50	1.36	0.77	0.71	0.00
time (sec)	N/A	0.097	0.275	0.045	0.000	0.947	0.709	0.395	0.103	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	68	67	0	70	82	66	38	0
N.S.	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.43	0.00
time (sec)	N/A	0.058	0.020	0.013	0.000	1.211	0.179	0.326	1.310	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	140	135	109	0	211	197	108	99	0
N.S.	1	1.00	0.96	0.78	0.00	1.51	1.41	0.77	0.71	0.00
time (sec)	N/A	0.107	0.165	0.015	0.000	1.525	0.725	0.404	0.067	0.001

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	140	135	109	0	211	214	108	97	0
N.S.	1	1.00	0.96	0.78	0.00	1.51	1.53	0.77	0.69	0.00
time (sec)	N/A	0.083	0.151	0.010	0.000	1.447	0.713	0.335	1.311	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	68	67	0	70	82	66	40	0
N.S.	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.45	0.00
time (sec)	N/A	0.052	0.017	0.010	0.000	1.124	0.182	0.387	0.037	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	145	145	140	114	0	224	218	113	102	0
N.S.	1	1.00	0.97	0.79	0.00	1.54	1.50	0.78	0.70	0.00
time (sec)	N/A	0.112	0.208	0.013	0.000	1.318	0.738	0.330	0.048	0.001

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	147	147	148	114	0	240	197	113	104	0
N.S.	1	1.00	1.01	0.78	0.00	1.63	1.34	0.77	0.71	0.00
time (sec)	N/A	0.099	0.294	0.013	0.000	1.231	0.748	0.377	0.030	0.001

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	95	75	0	90	94	100	52	0
N.S.	1	1.00	0.97	0.77	0.00	0.92	0.96	1.02	0.53	0.00
time (sec)	N/A	0.084	0.034	0.013	0.000	1.196	0.217	0.404	0.038	0.001

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	C	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	154	154	171	119	0	246	209	120	110	0
N.S.	1	1.00	1.11	0.77	0.00	1.60	1.36	0.78	0.71	0.00
time (sec)	N/A	0.142	0.344	0.013	0.000	1.201	0.764	0.426	0.032	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	38	37	37	42	37	39	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85	0.00
time (sec)	N/A	0.040	0.012	0.006	2.112	1.135	0.140	0.342	0.047	0.000
Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	55	44	0	47	48	99	29	0
N.S.	1	1.00	0.96	0.77	0.00	0.82	0.84	1.74	0.51	0.00
time (sec)	N/A	0.043	0.015	0.010	0.000	1.173	0.127	0.321	1.305	0.001
Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	33	32	32	37	32	34	0
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87	0.00
time (sec)	N/A	0.033	0.008	0.004	1.968	1.163	0.137	0.423	1.281	0.000
Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	82	82	98	65	0	171	70	76	53	0
N.S.	1	1.00	1.20	0.79	0.00	2.09	0.85	0.93	0.65	0.00
time (sec)	N/A	0.071	0.132	0.007	0.000	1.165	0.209	0.348	0.050	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	26	18	17	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74	0.00
time (sec)	N/A	0.022	0.007	0.001	1.937	1.702	0.122	0.410	1.285	0.000
Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	82	82	83	65	0	171	70	64	53	0
N.S.	1	1.00	1.01	0.79	0.00	2.09	0.85	0.78	0.65	0.00
time (sec)	N/A	0.057	0.052	0.008	0.000	1.169	0.208	0.407	0.045	0.000
Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	55	35	38	34	41	38	36	0
N.S.	1	1.00	1.34	0.85	0.93	0.83	1.00	0.93	0.88	0.00
time (sec)	N/A	0.038	0.013	0.006	1.947	1.115	0.157	0.346	1.292	0.000
Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	55	44	0	50	49	99	29	0
N.S.	1	1.00	0.96	0.77	0.00	0.88	0.86	1.74	0.51	0.00
time (sec)	N/A	0.042	0.015	0.010	0.000	1.242	0.153	0.314	1.273	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	51	40	43	51	48	48	41	0
N.S.	1	1.00	1.06	0.83	0.90	1.06	1.00	1.00	0.85	0.00
time (sec)	N/A	0.054	0.014	0.007	1.968	1.224	0.190	0.404	0.066	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	56	75	0	193	83	56	63	0
N.S.	1	1.00	0.58	0.78	0.00	2.01	0.86	0.58	0.66	0.00
time (sec)	N/A	0.094	0.018	0.011	0.000	1.326	0.261	0.415	0.056	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	59	44	0	716	26	254	209	0
N.S.	1	1.00	0.17	0.12	0.00	2.01	0.07	0.71	0.59	0.00
time (sec)	N/A	0.333	0.015	0.016	0.000	1.430	3.202	0.347	0.157	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	275	275	41	32	0	215	165	205	53	0
N.S.	1	1.00	0.15	0.12	0.00	0.78	0.60	0.75	0.19	0.00
time (sec)	N/A	0.243	0.011	0.013	0.000	1.267	0.219	0.398	0.096	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	347	347	39	40	0	567	24	253	474	0
N.S.	1	1.00	0.11	0.12	0.00	1.63	0.07	0.73	1.37	0.00
time (sec)	N/A	0.209	0.011	0.012	0.000	1.324	3.187	0.480	1.328	0.000
Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	355	355	40	40	0	567	26	253	286	0
N.S.	1	1.00	0.11	0.11	0.00	1.60	0.07	0.71	0.81	0.00
time (sec)	N/A	0.200	0.011	0.013	0.000	1.302	3.281	0.457	0.079	0.000
Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	275	275	42	30	0	215	165	205	53	0
N.S.	1	1.00	0.15	0.11	0.00	0.78	0.60	0.75	0.19	0.00
time (sec)	N/A	0.211	0.011	0.006	0.000	1.461	0.220	0.400	0.037	0.000
Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	61	52	0	732	29	258	253	0
N.S.	1	1.00	0.17	0.14	0.00	2.03	0.08	0.72	0.70	0.00
time (sec)	N/A	0.239	0.016	0.016	0.000	1.557	3.293	0.462	1.292	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	370	370	65	50	0	756	31	258	213	0
N.S.	1	1.00	0.18	0.14	0.00	2.04	0.08	0.70	0.58	0.00
time (sec)	N/A	0.236	0.014	0.014	0.000	0.964	3.252	0.353	1.288	0.000
Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	287	287	54	43	0	238	182	217	63	0
N.S.	1	1.00	0.19	0.15	0.00	0.83	0.63	0.76	0.22	0.00
time (sec)	N/A	0.241	0.017	0.011	0.000	2.126	0.273	0.383	1.296	0.001
Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	377	377	54	51	0	614	37	265	486	0
N.S.	1	1.00	0.14	0.14	0.00	1.63	0.10	0.70	1.29	0.00
time (sec)	N/A	0.286	0.015	0.019	0.000	1.731	3.366	0.381	0.064	0.001
Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	57	38	50	62	60	50	64	0
N.S.	1	1.00	0.92	0.61	0.81	1.00	0.97	0.81	1.03	0.00
time (sec)	N/A	0.056	0.033	0.004	1.281	1.476	0.145	0.497	0.134	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	97	117	0	154	54	66	130	0
N.S.	1	1.00	1.08	1.30	0.00	1.71	0.60	0.73	1.44	0.00
time (sec)	N/A	0.142	0.152	0.050	0.000	1.290	0.207	0.561	1.343	0.000
Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	53	33	45	56	53	45	59	0
N.S.	1	1.00	0.96	0.60	0.82	1.02	0.96	0.82	1.07	0.00
time (sec)	N/A	0.033	0.023	0.003	1.169	1.398	0.135	0.497	1.358	0.000
Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	75	110	0	165	49	47	117	0
N.S.	1	1.00	0.93	1.36	0.00	2.04	0.60	0.58	1.44	0.00
time (sec)	N/A	0.084	0.048	0.024	0.000	1.447	0.201	0.551	0.119	0.000
Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	38	19	31	43	42	31	30	0
N.S.	1	1.00	1.65	0.83	1.35	1.87	1.83	1.35	1.30	0.00
time (sec)	N/A	0.026	0.010	0.001	1.482	1.590	0.118	0.601	1.333	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	74	60	0	128	49	41	125	0
N.S.	1	1.00	0.99	0.80	0.00	1.71	0.65	0.55	1.67	0.00
time (sec)	N/A	0.058	0.037	0.016	0.000	1.364	0.204	0.419	0.054	0.000
Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	55	35	51	58	58	51	42	0
N.S.	1	1.00	0.96	0.61	0.89	1.02	1.02	0.89	0.74	0.00
time (sec)	N/A	0.034	0.030	0.009	1.505	1.450	0.158	0.484	1.412	0.000
Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	65	117	0	158	56	68	130	0
N.S.	1	1.00	0.73	1.31	0.00	1.78	0.63	0.76	1.46	0.00
time (sec)	N/A	0.072	0.017	0.028	0.000	0.955	0.238	0.456	1.304	0.000
Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	60	42	56	76	65	63	49	0
N.S.	1	1.00	0.91	0.64	0.85	1.15	0.98	0.95	0.74	0.00
time (sec)	N/A	0.067	0.035	0.010	1.292	1.184	0.188	0.551	1.358	0.001

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	73	122	0	180	65	77	136	0
N.S.	1	1.00	0.75	1.26	0.00	1.86	0.67	0.79	1.40	0.00
time (sec)	N/A	0.134	0.018	0.027	0.000	0.786	0.271	0.509	0.124	0.001
Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	460	440	58	46	0	1012	29	240	216	0
N.S.	1	0.96	0.13	0.10	0.00	2.20	0.06	0.52	0.47	0.00
time (sec)	N/A	0.416	0.015	0.013	0.000	1.678	1.591	0.725	1.438	0.001
Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	431	431	41	40	0	725	26	239	149	0
N.S.	1	1.00	0.10	0.09	0.00	1.68	0.06	0.55	0.35	0.00
time (sec)	N/A	0.295	0.012	0.013	0.000	1.577	1.530	0.663	1.465	0.000
Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	451	451	39	40	0	843	24	239	454	0
N.S.	1	1.00	0.09	0.09	0.00	1.87	0.05	0.53	1.01	0.00
time (sec)	N/A	0.280	0.011	0.016	0.000	1.104	1.482	0.765	0.196	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	427	431	40	40	0	955	26	239	275	0
N.S.	1	1.01	0.09	0.09	0.00	2.24	0.06	0.56	0.64	0.00
time (sec)	N/A	0.259	0.010	0.010	0.000	1.862	1.510	0.593	0.086	0.001
Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	414	414	42	37	0	733	26	239	403	0
N.S.	1	1.00	0.10	0.09	0.00	1.77	0.06	0.58	0.97	0.00
time (sec)	N/A	0.257	0.010	0.010	0.000	1.763	1.519	0.528	0.083	0.001
Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	61	52	0	1017	32	244	292	0
N.S.	1	1.00	0.15	0.12	0.00	2.44	0.08	0.59	0.70	0.00
time (sec)	N/A	0.285	0.016	0.013	0.000	1.500	1.609	0.603	1.291	0.000
Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	466	466	65	50	0	1057	34	244	492	0
N.S.	1	1.00	0.14	0.11	0.00	2.27	0.07	0.52	1.06	0.00
time (sec)	N/A	0.367	0.016	0.012	0.000	1.795	1.624	0.702	0.184	0.001

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	56	38	50	62	58	53	64	0
N.S.	1	1.00	0.90	0.61	0.81	1.00	0.94	0.85	1.03	0.00
time (sec)	N/A	0.046	0.032	0.004	1.393	0.681	0.143	0.416	0.120	0.000
Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	103	67	92	114	170	97	90	0
N.S.	1	1.00	1.14	0.74	1.02	1.27	1.89	1.08	1.00	0.00
time (sec)	N/A	0.072	0.054	0.006	1.359	1.162	0.380	0.463	1.328	0.000
Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	53	33	45	57	53	48	59	0
N.S.	1	1.00	0.96	0.60	0.82	1.04	0.96	0.87	1.07	0.00
time (sec)	N/A	0.032	0.022	0.003	1.479	1.040	0.135	0.425	0.102	0.000
Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	91	62	87	109	165	92	77	0
N.S.	1	1.00	1.12	0.77	1.07	1.35	2.04	1.14	0.95	0.00
time (sec)	N/A	0.053	0.034	0.003	1.917	1.168	0.371	0.445	1.381	0.001

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	38	19	31	43	42	33	30	0
N.S.	1	1.00	1.65	0.83	1.35	1.87	1.83	1.43	1.30	0.00
time (sec)	N/A	0.027	0.010	0.003	1.413	1.277	0.118	0.537	1.570	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	91	62	87	107	165	92	83	0
N.S.	1	1.00	1.21	0.83	1.16	1.43	2.20	1.23	1.11	0.00
time (sec)	N/A	0.038	0.029	0.004	1.450	1.228	0.366	0.433	1.302	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	55	64	51	59	58	54	42	0
N.S.	1	1.00	0.96	1.12	0.89	1.04	1.02	0.95	0.74	0.00
time (sec)	N/A	0.030	0.031	0.010	1.445	1.278	0.158	0.477	0.429	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	103	67	92	125	172	97	88	0
N.S.	1	1.00	1.16	0.75	1.03	1.40	1.93	1.09	0.99	0.00
time (sec)	N/A	0.058	0.057	0.007	1.462	1.109	0.406	0.384	0.064	0.001

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	61	71	56	76	66	66	49	0
N.S.	1	1.00	0.92	1.08	0.85	1.15	1.00	1.00	0.74	0.00
time (sec)	N/A	0.063	0.036	0.012	1.395	1.279	0.190	0.505	1.348	0.000
Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	111	72	99	130	199	104	95	0
N.S.	1	1.00	1.14	0.74	1.02	1.34	2.05	1.07	0.98	0.00
time (sec)	N/A	0.090	0.071	0.009	1.325	1.200	0.439	0.409	1.385	0.000
Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	160	205	0	304	58	148	246	0
N.S.	1	1.00	0.94	1.21	0.00	1.79	0.34	0.87	1.45	0.00
time (sec)	N/A	0.114	0.266	0.061	0.000	1.312	1.245	0.676	1.445	0.001
Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	160	206	0	255	53	147	147	0
N.S.	1	1.00	0.96	1.23	0.00	1.53	0.32	0.88	0.88	0.00
time (sec)	N/A	0.078	0.150	0.030	0.000	1.268	1.222	0.748	0.194	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	132	206	0	271	49	147	269	0
N.S.	1	1.00	0.76	1.19	0.00	1.57	0.28	0.85	1.55	0.00
time (sec)	N/A	0.075	0.191	0.040	0.000	1.424	1.203	0.615	1.472	0.000
Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	166	131	110	0	255	53	147	269	0
N.S.	1	1.14	0.90	0.76	0.00	1.76	0.37	1.01	1.86	0.00
time (sec)	N/A	0.060	0.044	0.029	0.000	1.080	1.185	0.624	0.081	0.000
Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	160	206	0	251	53	147	245	0
N.S.	1	1.00	0.95	1.22	0.00	1.49	0.31	0.87	1.45	0.00
time (sec)	N/A	0.060	0.156	0.029	0.000	1.490	1.212	0.482	0.079	0.000
Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	174	211	0	313	63	152	250	0
N.S.	1	1.00	1.01	1.23	0.00	1.82	0.37	0.88	1.45	0.00
time (sec)	N/A	0.089	0.267	0.029	0.000	1.312	1.250	0.542	1.339	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	166	209	0	327	63	152	268	0
N.S.	1	1.00	0.91	1.15	0.00	1.80	0.35	0.84	1.47	0.00
time (sec)	N/A	0.118	0.262	0.030	0.000	1.401	1.281	0.669	0.204	0.000
Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	189	216	0	300	73	159	257	0
N.S.	1	1.00	1.09	1.25	0.00	1.73	0.42	0.92	1.49	0.00
time (sec)	N/A	0.140	0.274	0.039	0.000	1.193	1.290	0.539	1.487	0.001
Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	189	216	0	332	70	159	291	0
N.S.	1	1.00	1.00	1.14	0.00	1.76	0.37	0.84	1.54	0.00
time (sec)	N/A	0.163	0.268	0.043	0.000	1.205	1.298	0.556	0.210	0.000
Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	17	16	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.76	0.00
time (sec)	N/A	0.013	0.004	0.005	0.587	1.276	0.116	0.282	0.063	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	23	22	22	19	22	22	0
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.73	0.85	0.85	0.00
time (sec)	N/A	0.019	0.005	0.006	0.861	1.204	0.130	0.364	1.315	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	37	30	32	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86	0.00
time (sec)	N/A	0.034	0.013	0.003	2.046	1.199	0.143	2.721	1.346	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	18	18	27	18	20	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87	0.00
time (sec)	N/A	0.022	0.006	0.001	2.092	1.217	0.131	2.962	1.333	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	197	66	36	32	41	33	34	0
N.S.	1	1.00	5.05	1.69	0.92	0.82	1.05	0.85	0.87	0.00
time (sec)	N/A	0.035	0.037	0.039	2.033	1.164	0.165	0.408	0.060	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	208	73	41	49	48	45	41	0
N.S.	1	1.00	4.33	1.52	0.85	1.02	1.00	0.94	0.85	0.00
time (sec)	N/A	0.051	0.042	0.024	2.070	1.273	0.202	0.251	1.369	0.001
Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	197	66	0	32	41	33	34	0
N.S.	1	1.00	5.05	1.69	0.00	0.82	1.05	0.85	0.87	0.00
time (sec)	N/A	0.036	0.020	0.022	0.000	1.061	0.166	0.350	0.033	0.000
Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	140	236	0	466	605	145	183	0
N.S.	1	1.00	0.95	1.61	0.00	3.17	4.12	0.99	1.24	0.00
time (sec)	N/A	0.137	0.122	0.006	0.000	1.303	1.249	0.291	0.159	0.001
Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	112	190	0	383	498	113	151	0
N.S.	1	1.00	0.95	1.61	0.00	3.25	4.22	0.96	1.28	0.00
time (sec)	N/A	0.105	0.085	0.003	0.000	1.337	1.012	0.392	1.396	0.001

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	84	132	0	297	381	86	112	0
N.S.	1	1.00	0.94	1.48	0.00	3.34	4.28	0.97	1.26	0.00
time (sec)	N/A	0.086	0.104	0.005	0.000	1.413	0.835	0.257	0.131	0.001

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	73	101	0	235	306	67	172	0
N.S.	1	1.00	1.04	1.44	0.00	3.36	4.37	0.96	2.46	0.00
time (sec)	N/A	0.049	0.064	0.003	0.000	1.434	0.597	0.274	1.418	0.001

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	57	56	0	185	216	55	112	0
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00	0.00
time (sec)	N/A	0.036	0.031	0.003	0.000	1.349	0.318	0.291	0.168	0.001

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	38	35	0	120	124	34	46	0
N.S.	1	1.00	1.06	0.97	0.00	3.33	3.44	0.94	1.28	0.00
time (sec)	N/A	0.032	0.007	0.001	0.000	1.243	0.218	0.345	0.046	0.001

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	61	62	0	211	564	62	213	0
N.S.	1	1.00	0.98	1.00	0.00	3.40	9.10	1.00	3.44	0.00
time (sec)	N/A	0.048	0.067	0.006	0.000	1.406	4.080	0.297	1.715	0.001

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	77	112	0	269	0	79	339	0
N.S.	1	1.00	0.95	1.38	0.00	3.32	0.00	0.98	4.19	0.00
time (sec)	N/A	0.102	0.079	0.006	0.000	0.889	0.000	0.386	1.814	0.001

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	102	150	0	358	0	105	447	0
N.S.	1	1.00	0.98	1.44	0.00	3.44	0.00	1.01	4.30	0.00
time (sec)	N/A	0.153	0.136	0.007	0.000	1.361	0.000	0.314	1.866	0.001

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	131	214	0	445	0	136	524	0
N.S.	1	1.00	0.96	1.56	0.00	3.25	0.00	0.99	3.82	0.00
time (sec)	N/A	0.193	0.099	0.010	0.000	1.471	0.000	0.382	1.923	0.001

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	163	434	0	1029	1012	188	382	0
N.S.	1	1.00	0.83	2.21	0.00	5.25	5.16	0.96	1.95	0.00
time (sec)	N/A	0.205	0.222	0.013	0.000	1.332	2.369	0.330	1.822	0.001

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	132	352	0	837	842	161	261	0
N.S.	1	1.00	0.88	2.35	0.00	5.58	5.61	1.07	1.74	0.00
time (sec)	N/A	0.148	0.177	0.012	0.000	1.620	1.727	0.369	1.798	0.001

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	109	209	0	635	729	125	279	0
N.S.	1	1.00	0.96	1.83	0.00	5.57	6.39	1.10	2.45	0.00
time (sec)	N/A	0.100	0.140	0.010	0.000	1.284	1.323	0.362	1.859	0.001

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	81	97	0	387	280	88	135	0
N.S.	1	1.00	1.14	1.37	0.00	5.45	3.94	1.24	1.90	0.00
time (sec)	N/A	0.042	0.092	0.007	0.000	1.200	0.597	0.379	1.370	0.001

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	69	70	0	338	253	76	110	0
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67	0.00
time (sec)	N/A	0.034	0.064	0.003	0.000	1.171	0.562	0.371	1.369	0.001

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	70	68	0	341	265	76	119	0
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80	0.00
time (sec)	N/A	0.033	0.077	0.003	0.000	1.264	0.579	0.395	0.083	0.001

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	107	237	0	781	0	126	620	0
N.S.	1	1.00	0.99	2.19	0.00	7.23	0.00	1.17	5.74	0.00
time (sec)	N/A	0.145	0.185	0.013	0.000	1.488	0.000	0.359	2.096	0.001

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	131	328	0	975	0	171	775	0
N.S.	1	1.00	0.89	2.22	0.00	6.59	0.00	1.16	5.24	0.00
time (sec)	N/A	0.182	0.266	0.014	0.000	1.787	0.000	0.423	2.134	0.001

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	175	418	0	1226	0	229	914	0
N.S.	1	1.00	0.87	2.07	0.00	6.07	0.00	1.13	4.52	0.00
time (sec)	N/A	0.239	0.343	0.017	0.000	2.004	0.000	0.330	2.298	0.001
Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	260	1040	0	1926	1714	282	705	0
N.S.	1	1.00	1.09	4.37	0.00	8.09	7.20	1.18	2.96	0.00
time (sec)	N/A	0.291	0.370	0.017	0.000	1.006	4.196	0.379	2.000	0.001
Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	221	530	0	1603	1510	245	620	0
N.S.	1	1.00	1.16	2.79	0.00	8.44	7.95	1.29	3.26	0.00
time (sec)	N/A	0.279	0.308	0.016	0.000	1.679	3.208	0.398	2.203	0.001
Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	174	260	0	953	547	202	343	0
N.S.	1	1.00	1.57	2.34	0.00	8.59	4.93	1.82	3.09	0.00
time (sec)	N/A	0.067	0.174	0.010	0.000	0.755	1.411	0.416	0.195	0.001

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	126	223	0	872	513	163	271	0
N.S.	1	1.00	1.18	2.08	0.00	8.15	4.79	1.52	2.53	0.00
time (sec)	N/A	0.050	0.194	0.012	0.000	1.110	1.209	0.308	1.429	0.002

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	131	262	0	887	570	154	313	0
N.S.	1	1.00	1.14	2.28	0.00	7.71	4.96	1.34	2.72	0.00
time (sec)	N/A	0.068	0.140	0.010	0.000	1.103	1.282	0.436	1.500	0.001

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	102	130	0	788	481	135	253	0
N.S.	1	1.00	0.99	1.26	0.00	7.65	4.67	1.31	2.46	0.00
time (sec)	N/A	0.041	0.096	0.005	0.000	1.186	1.092	0.409	1.430	0.001

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	101	97	129	0	785	474	136	285	0
N.S.	1	0.98	0.94	1.25	0.00	7.62	4.60	1.32	2.77	0.00
time (sec)	N/A	0.041	0.096	0.004	0.000	1.696	1.111	0.337	1.424	0.001

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	178	781	0	1985	0	239	1089	0
N.S.	1	1.00	0.96	4.22	0.00	10.73	0.00	1.29	5.89	0.00
time (sec)	N/A	0.220	0.353	0.017	0.000	3.243	0.000	0.315	2.456	0.001

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	221	954	0	2280	0	309	1255	0
N.S.	1	1.00	0.92	3.99	0.00	9.54	0.00	1.29	5.25	0.00
time (sec)	N/A	0.277	0.437	0.023	0.000	3.188	0.000	0.468	2.554	0.001

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	31	30	30	34	32	26	0
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.85	0.80	0.65	0.00
time (sec)	N/A	0.022	0.006	0.006	0.423	1.723	0.121	0.255	0.046	0.001

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	26	25	25	27	27	21	0
N.S.	1	1.00	1.00	0.79	0.76	0.76	0.82	0.82	0.64	0.00
time (sec)	N/A	0.020	0.004	0.004	0.418	0.759	0.119	0.280	1.310	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	21	20	20	20	22	16	0
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.77	0.85	0.62	0.00
time (sec)	N/A	0.015	0.004	0.006	0.418	1.221	0.116	0.277	0.077	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	17	19	13	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62	0.00
time (sec)	N/A	0.011	0.003	0.006	0.420	1.403	0.111	0.275	0.066	0.001

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	18	17	17	15	19	8	0
N.S.	1	1.00	0.91	0.78	0.74	0.74	0.65	0.83	0.35	0.00
time (sec)	N/A	0.014	0.003	0.006	0.418	1.044	0.111	0.224	1.373	0.001

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	22	21	21	24	24	17	0
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.89	0.89	0.63	0.00
time (sec)	N/A	0.017	0.004	0.007	0.421	1.302	0.149	0.250	1.385	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	27	26	30	31	29	22	0
N.S.	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65	0.00
time (sec)	N/A	0.031	0.004	0.009	0.427	1.296	0.163	0.308	0.044	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	32	31	39	36	34	26	0
N.S.	1	1.00	1.00	0.78	0.76	0.95	0.88	0.83	0.63	0.00
time (sec)	N/A	0.035	0.005	0.007	0.421	1.301	0.173	0.298	1.312	0.001

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	37	36	44	41	39	32	0
N.S.	1	1.00	1.00	0.77	0.75	0.92	0.85	0.81	0.67	0.00
time (sec)	N/A	0.042	0.005	0.007	0.645	1.143	0.185	0.294	0.047	0.001

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	213	701	0	959	0	0	-1	209
N.S.	1	1.00	1.04	3.44	0.00	4.70	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.231	0.522	0.019	0.000	1.592	0.000	0.000	0.000	8.555

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	163	334	0	709	0	0	-1	154
N.S.	1	1.00	1.12	2.30	0.00	4.89	0.00	0.00	-0.01	1.06
time (sec)	N/A	0.134	0.250	0.007	0.000	1.362	0.000	0.000	0.000	7.173
Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	128	121	0	590	0	0	100	125
N.S.	1	1.00	1.22	1.15	0.00	5.62	0.00	0.00	0.95	1.19
time (sec)	N/A	0.083	0.081	0.007	0.000	1.253	0.000	0.000	0.126	5.604
Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	89	88	0	171	0	0	53	93
N.S.	1	1.00	1.33	1.31	0.00	2.55	0.00	0.00	0.79	1.39
time (sec)	N/A	0.041	0.058	0.005	0.000	1.295	0.000	0.000	1.452	4.695
Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	138	197	0	465	0	0	-1	150
N.S.	1	1.00	1.04	1.48	0.00	3.50	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.099	0.171	0.008	0.000	1.347	0.000	0.000	0.000	5.928

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	256	376	0	1081	0	0	-1	273
N.S.	1	1.00	1.16	1.71	0.00	4.91	0.00	0.00	-0.00	1.24
time (sec)	N/A	0.194	0.378	0.012	0.000	1.863	0.000	0.000	0.000	7.914

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	32	40	8	8	0	29	134	32
N.S.	1	1.00	0.44	0.55	0.11	0.11	0.00	0.40	1.84	0.44
time (sec)	N/A	0.036	0.019	0.013	0.785	1.599	0.000	0.341	0.112	4.983

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	202	343	0	1059	129	2109	3026	0
N.S.	1	1.00	1.13	1.92	0.00	5.92	0.72	11.78	16.91	0.00
time (sec)	N/A	0.291	0.117	0.011	0.000	0.956	2.119	1.663	2.079	0.001

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	631	631	70	59	0	5260	196	0	2280	0
N.S.	1	1.00	0.11	0.09	0.00	8.34	0.31	0.00	3.61	0.00
time (sec)	N/A	1.170	0.036	0.006	0.000	3.179	6.965	0.000	4.540	0.001

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	376	376	70	59	0	5082	0	0	10382	0
N.S.	1	1.00	0.19	0.16	0.00	13.52	0.00	0.00	27.61	0.00
time (sec)	N/A	0.666	0.043	0.004	0.000	2.634	0.000	0.000	3.776	0.001
Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	106	84	0	0	0	0	-1	109
N.S.	1	1.00	1.00	0.79	0.00	0.00	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.092	0.056	0.007	0.000	0.000	0.000	0.000	0.000	0.235
Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	29	52	54	53	51	49	44	55
N.S.	1	1.00	0.72	1.30	1.35	1.32	1.28	1.22	1.10	1.38
time (sec)	N/A	0.019	0.025	0.001	0.869	1.068	0.330	0.332	0.038	0.027
Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	50	50	23	0	0	45	-1	151
N.S.	1	1.00	0.67	0.67	0.31	0.00	0.00	0.60	-0.01	2.01
time (sec)	N/A	0.040	0.030	0.011	0.956	0.000	0.000	0.397	0.000	0.242

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	56	109	114	84	0	140	-1	0
N.S.	1	1.00	0.41	0.80	0.83	0.61	0.00	1.02	-0.01	0.00
time (sec)	N/A	0.081	0.049	0.016	0.906	1.467	0.000	0.511	0.000	2.556

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	56	87	114	61	0	102	-1	0
N.S.	1	1.00	0.41	0.64	0.83	0.45	0.00	0.74	-0.01	0.00
time (sec)	N/A	0.071	0.035	0.002	0.890	1.117	0.000	0.534	0.000	2.402

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	65	65	114	32	0	64	-1	0
N.S.	1	1.00	0.47	0.47	0.83	0.23	0.00	0.47	-0.01	0.00
time (sec)	N/A	0.056	0.032	0.003	0.914	1.395	0.000	0.368	0.000	1.683

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	43	43	114	10	0	26	71	0
N.S.	1	1.00	0.49	0.49	1.30	0.11	0.00	0.30	0.81	0.00
time (sec)	N/A	0.040	0.009	0.003	0.884	0.950	0.000	0.389	1.561	0.817

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	65	103	36	33	0	61	-1	204
N.S.	1	1.00	0.44	0.70	0.24	0.22	0.00	0.41	-0.01	1.39
time (sec)	N/A	0.070	0.036	0.018	0.876	1.371	0.000	0.489	0.000	0.359
Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	72	92	55	113	0	64	-1	1439
N.S.	1	1.00	0.55	0.71	0.42	0.87	0.00	0.49	-0.01	11.07
time (sec)	N/A	0.072	0.045	0.008	0.617	1.231	0.000	0.532	0.000	1.315
Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	58	54	53	136	0	43	53	284
N.S.	1	1.00	0.43	0.40	0.39	1.01	0.00	0.32	0.39	2.10
time (sec)	N/A	0.075	0.036	0.009	0.716	1.137	0.000	0.545	2.804	0.773
Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	58	54	53	209	0	43	53	356
N.S.	1	1.00	0.42	0.39	0.39	1.53	0.00	0.31	0.39	2.60
time (sec)	N/A	0.078	0.035	0.007	0.538	1.309	0.000	0.589	3.228	0.894

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	58	54	53	275	0	43	53	434
N.S.	1	1.00	0.42	0.39	0.39	2.01	0.00	0.31	0.39	3.17
time (sec)	N/A	0.079	0.035	0.007	0.721	1.574	0.000	0.649	3.654	1.004

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	58	54	53	343	0	43	53	507
N.S.	1	1.00	0.42	0.39	0.39	2.50	0.00	0.31	0.39	3.70
time (sec)	N/A	0.078	0.037	0.007	0.543	1.412	0.000	0.743	4.344	1.220

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-2)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	468	468	207	0	362	579	0	1564	777	0
N.S.	1	1.00	0.44	0.00	0.77	1.24	0.00	3.34	1.66	0.00
time (sec)	N/A	0.224	0.216	0.009	0.704	1.657	0.000	0.627	3.515	0.544

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-2)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	315	315	143	0	198	297	0	745	390	0
N.S.	1	1.00	0.45	0.00	0.63	0.94	0.00	2.37	1.24	0.00
time (sec)	N/A	0.140	0.174	0.007	0.640	1.315	0.000	0.481	2.166	0.189

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	83	0	77	110	0	229	138	0
N.S.	1	1.00	0.58	0.00	0.54	0.77	0.00	1.61	0.97	0.00
time (sec)	N/A	0.068	0.049	0.006	0.798	1.498	0.000	0.483	1.542	0.157
Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	162	101	0	0	82	0	0	69	0
N.S.	1	1.11	0.69	0.00	0.00	0.56	0.00	0.00	0.47	0.00
time (sec)	N/A	0.099	0.082	0.011	0.000	1.667	0.000	0.000	1.647	5.793
Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	93	114	114	147	0	0	-1	1582
N.S.	1	1.00	0.53	0.65	0.65	0.84	0.00	0.00	-0.01	8.99
time (sec)	N/A	0.104	0.069	0.016	0.576	11.031	0.000	0.000	0.000	1.263
Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	121	174	119	0	0	237	-1	2887
N.S.	1	1.00	0.45	0.65	0.44	0.00	0.00	0.88	-0.00	10.77
time (sec)	N/A	0.151	0.096	0.016	0.878	0.000	0.000	0.866	0.000	2.595

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	66	68	0	0	0	80	-1	77
N.S.	1	1.00	0.37	0.38	0.00	0.00	0.00	0.45	-0.01	0.43
time (sec)	N/A	0.092	0.034	0.030	0.000	0.000	0.000	0.390	0.000	5.928

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	391	391	125	115	79	0	0	173	-1	133
N.S.	1	1.00	0.32	0.29	0.20	0.00	0.00	0.44	-0.00	0.34
time (sec)	N/A	0.187	0.068	0.028	0.598	0.000	0.000	0.541	0.000	6.543

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	99	91	57	0	0	128	-1	109
N.S.	1	1.00	0.34	0.31	0.20	0.00	0.00	0.44	-0.00	0.37
time (sec)	N/A	0.137	0.057	0.010	0.635	0.000	0.000	0.505	0.000	6.281

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	77	69	30	0	0	79	-1	85
N.S.	1	1.00	0.41	0.37	0.16	0.00	0.00	0.42	-0.01	0.45
time (sec)	N/A	0.091	0.029	0.005	0.663	0.000	0.000	0.412	0.000	5.979

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	49	50	10	0	0	34	39	54
N.S.	1	1.00	0.56	0.57	0.11	0.00	0.00	0.39	0.44	0.61
time (sec)	N/A	0.053	0.014	0.005	0.733	0.000	0.000	0.397	1.426	5.303
Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	86	78	44	0	0	77	-1	94
N.S.	1	1.00	0.45	0.41	0.23	0.00	0.00	0.41	-0.01	0.49
time (sec)	N/A	0.118	0.040	0.006	0.743	0.000	0.000	0.592	0.000	4.797
Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	300	300	126	141	97	0	0	121	-1	135
N.S.	1	1.00	0.42	0.47	0.32	0.00	0.00	0.40	-0.00	0.45
time (sec)	N/A	0.187	0.086	0.011	1.068	0.000	0.000	0.660	0.000	6.436
Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	410	410	152	199	139	0	0	141	-1	159
N.S.	1	1.00	0.37	0.49	0.34	0.00	0.00	0.34	-0.00	0.39
time (sec)	N/A	0.268	0.130	0.013	0.854	0.000	0.000	0.717	0.000	9.554

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	98	94	57	0	0	126	-1	109
N.S.	1	1.00	0.34	0.33	0.20	0.00	0.00	0.44	-0.00	0.38
time (sec)	N/A	0.138	0.052	0.036	0.995	0.000	0.000	0.486	0.000	6.280

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	103	91	52	0	0	125	-1	109
N.S.	1	1.00	0.35	0.31	0.18	0.00	0.00	0.43	-0.00	0.37
time (sec)	N/A	0.137	0.048	0.026	0.940	0.000	0.000	0.481	0.000	6.305

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	98	152	99	302	0	84	-1	2765
N.S.	1	1.00	0.44	0.68	0.45	1.36	0.00	0.38	-0.00	12.45
time (sec)	N/A	0.126	0.087	0.012	1.022	0.906	0.000	0.590	0.000	2.806

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	391	391	124	116	79	0	0	172	-1	135
N.S.	1	1.00	0.32	0.30	0.20	0.00	0.00	0.44	-0.00	0.35
time (sec)	N/A	0.180	0.066	0.032	0.947	0.000	0.000	0.461	0.000	6.779

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	38	62	45	38	42	0	-1	41
N.S.	1	1.00	0.83	1.35	0.98	0.83	0.91	0.00	-0.02	0.89
time (sec)	N/A	0.036	0.028	0.030	0.934	1.645	19.826	0.000	0.000	0.063
Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	33	32	24	26	0	-1	34
N.S.	1	1.00	0.93	1.18	1.14	0.86	0.93	0.00	-0.04	1.21
time (sec)	N/A	0.026	0.014	0.024	0.872	2.059	13.890	0.000	0.000	0.070
Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	18	19	15	37	0	-1	18
N.S.	1	1.00	1.00	1.20	1.27	1.00	2.47	0.00	-0.07	1.20
time (sec)	N/A	0.012	0.003	0.022	0.885	1.126	11.280	0.000	0.000	0.051
Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	22	26	27	22	66	25	20	34
N.S.	1	1.00	0.96	1.13	1.17	0.96	2.87	1.09	0.87	1.48
time (sec)	N/A	0.019	0.007	0.020	0.896	1.463	16.485	0.251	1.369	0.054

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-2)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	49	69	58	59	0	0	-1	59
N.S.	1	1.00	0.86	1.21	1.02	1.04	0.00	0.00	-0.02	1.04
time (sec)	N/A	0.042	0.066	0.024	0.911	1.386	0.000	0.000	0.000	0.074
Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	62	88	71	72	73	0	-1	72
N.S.	1	1.00	0.82	1.16	0.93	0.95	0.96	0.00	-0.01	0.95
time (sec)	N/A	0.047	0.078	0.031	0.910	1.473	52.911	0.000	0.000	0.101
Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	75	105	84	85	88	0	-1	85
N.S.	1	1.00	0.81	1.13	0.90	0.91	0.95	0.00	-0.01	0.91
time (sec)	N/A	0.054	0.095	0.032	0.921	1.224	105.734	0.000	0.000	0.117
Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	34	54	0	272	0	203	-1	0
N.S.	1	1.00	0.14	0.23	0.00	1.15	0.00	0.86	-0.00	0.00
time (sec)	N/A	0.206	0.009	0.121	0.000	0.920	0.000	0.348	0.000	0.110

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	34	54	0	212	0	136	-1	0
N.S.	1	1.00	0.21	0.34	0.00	1.32	0.00	0.85	-0.01	0.00
time (sec)	N/A	0.132	0.009	0.069	0.000	1.331	0.000	0.419	0.000	0.097
Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	32	79	0	151	36	38	-1	0
N.S.	1	1.00	0.64	1.58	0.00	3.02	0.72	0.76	-0.02	0.00
time (sec)	N/A	0.035	0.007	0.073	0.000	1.283	12.999	0.375	0.000	0.086
Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	34	97	0	161	58	0	-1	0
N.S.	1	1.00	0.50	1.43	0.00	2.37	0.85	0.00	-0.01	0.00
time (sec)	N/A	0.041	0.008	0.095	0.000	1.394	24.919	0.000	0.000	0.118
Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	34	73	0	171	0	0	-1	0
N.S.	1	1.00	0.19	0.41	0.00	0.97	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.142	0.008	0.079	0.000	1.425	0.000	0.000	0.000	0.103

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	34	73	0	259	0	0	-1	0
N.S.	1	1.00	0.13	0.29	0.00	1.03	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.225	0.008	0.092	0.000	1.385	0.000	0.000	0.000	0.122

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	38	0	43	59	0	0	-1	0
N.S.	1	1.00	1.03	0.00	1.16	1.59	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.040	0.017	0.098	1.106	1.340	0.000	0.000	0.000	0.161

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	43	0	0	59	0	0	-1	0
N.S.	1	1.00	1.13	0.00	0.00	1.55	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.039	0.019	0.070	0.000	1.150	0.000	0.000	0.000	0.152

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	40	208	74	74	0	0	-1	96
N.S.	1	1.00	0.36	1.86	0.66	0.66	0.00	0.00	-0.01	0.86
time (sec)	N/A	0.042	0.056	0.047	0.922	1.303	0.000	0.000	0.000	0.066

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	40	135	48	48	0	0	-1	70
N.S.	1	1.00	0.36	1.21	0.43	0.43	0.00	0.00	-0.01	0.62
time (sec)	N/A	0.040	0.037	0.026	0.922	1.329	0.000	0.000	0.000	0.057
Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	44	64	22	22	0	0	-1	44
N.S.	1	1.00	0.44	0.65	0.22	0.22	0.00	0.00	-0.01	0.44
time (sec)	N/A	0.031	0.023	0.023	0.890	1.250	0.000	0.000	0.000	0.051
Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	46	71	32	24	0	0	-1	55
N.S.	1	1.00	0.51	0.79	0.36	0.27	0.00	0.00	-0.01	0.61
time (sec)	N/A	0.044	0.032	0.030	0.894	1.313	0.000	0.000	0.000	0.075
Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	35	37	41	41	0	0	-1	42
N.S.	1	1.00	0.73	0.77	0.85	0.85	0.00	0.00	-0.02	0.88
time (sec)	N/A	0.027	0.016	0.026	0.915	1.304	0.000	0.000	0.000	0.057

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	40	37	69	69	0	0	-1	42
N.S.	1	1.00	0.45	0.42	0.78	0.78	0.00	0.00	-0.01	0.48
time (sec)	N/A	0.052	0.032	0.030	0.947	1.183	0.000	0.000	0.000	0.141

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	40	37	97	97	0	0	-1	42
N.S.	1	1.00	0.45	0.42	1.10	1.10	0.00	0.00	-0.01	0.48
time (sec)	N/A	0.051	0.034	0.032	0.975	1.210	0.000	0.000	0.000	0.148

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	55	132	47	57	0	173	-1	0
N.S.	1	1.00	0.51	1.22	0.44	0.53	0.00	1.60	-0.01	0.00
time (sec)	N/A	0.042	0.033	0.043	0.940	1.448	0.000	0.350	0.000	0.193

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	46	61	25	28	0	53	-1	0
N.S.	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	-0.01	0.00
time (sec)	N/A	0.028	0.022	0.016	0.949	1.105	0.000	0.258	0.000	0.177

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	46	61	25	28	0	53	-1	0
N.S.	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	-0.01	0.00
time (sec)	N/A	0.025	0.021	0.015	0.883	1.154	0.000	0.295	0.000	0.106
Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	39	56	19	20	0	25	-1	0
N.S.	1	1.00	0.44	0.64	0.22	0.23	0.00	0.28	-0.01	0.00
time (sec)	N/A	0.019	0.014	0.016	1.043	1.323	0.000	0.219	0.000	0.087
Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	37	54	13	15	0	0	-1	41
N.S.	1	1.00	0.44	0.64	0.15	0.18	0.00	0.00	-0.01	0.48
time (sec)	N/A	0.026	0.015	0.017	0.797	1.360	0.000	0.000	0.000	0.041
Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	42	61	22	23	0	0	-1	0
N.S.	1	1.00	0.45	0.65	0.23	0.24	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.033	0.025	0.017	0.665	1.285	0.000	0.000	0.000	0.207

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	47	61	22	23	0	0	-1	0
N.S.	1	1.00	0.49	0.64	0.23	0.24	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.029	0.026	0.020	0.911	1.333	0.000	0.000	0.000	0.243

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	238	90	532	276	390	0	2719	-1	0
N.S.	1	1.00	0.38	2.24	1.16	1.64	0.00	11.42	-0.00	0.00
time (sec)	N/A	0.098	0.107	0.059	0.993	1.013	0.000	0.955	0.000	0.219

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	123	146	108	144	0	292	-1	0
N.S.	1	1.00	0.58	0.69	0.51	0.68	0.00	1.38	-0.00	0.00
time (sec)	N/A	0.062	0.078	0.017	0.928	1.101	0.000	0.434	0.000	0.148

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	124	145	109	145	0	292	-1	0
N.S.	1	1.00	0.59	0.69	0.52	0.69	0.00	1.38	-0.00	0.00
time (sec)	N/A	0.058	0.070	0.017	0.937	1.323	0.000	0.373	0.000	0.124

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	122	138	101	130	0	263	-1	0
N.S.	1	1.00	0.59	0.67	0.49	0.63	0.00	1.28	-0.00	0.00
time (sec)	N/A	0.052	0.075	0.016	0.945	1.361	0.000	0.489	0.000	0.085
Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	68	127	43	44	0	0	-1	69
N.S.	1	1.00	0.35	0.65	0.22	0.22	0.00	0.00	-0.01	0.35
time (sec)	N/A	0.052	0.037	0.019	0.902	0.803	0.000	0.000	0.000	0.044
Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	124	147	101	131	0	0	-1	0
N.S.	1	1.00	0.58	0.69	0.48	0.62	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.070	0.091	0.025	0.945	1.343	0.000	0.000	0.000	0.192
Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	124	145	101	134	0	0	-1	0
N.S.	1	1.00	0.57	0.67	0.46	0.61	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.070	0.087	0.026	1.172	1.330	0.000	0.000	0.000	0.159

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	42	66	27	22	0	0	-1	55
N.S.	1	1.00	0.49	0.78	0.32	0.26	0.00	0.00	-0.01	0.65
time (sec)	N/A	0.036	0.015	0.020	1.066	1.217	0.000	0.000	0.000	0.043

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	76	104	70	106	0	0	-1	78
N.S.	1	1.00	0.48	0.65	0.44	0.67	0.00	0.00	-0.01	0.49
time (sec)	N/A	0.083	0.063	0.021	1.162	1.486	0.000	0.000	0.000	0.047

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	58	0	0	79	0	0	-1	0
N.S.	1	1.00	1.12	0.00	0.00	1.52	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.020	0.023	0.279	0.000	1.090	0.000	0.000	0.000	0.142

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	32	51	0	45	0	0	-1	0
N.S.	1	1.00	0.74	1.19	0.00	1.05	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.013	0.062	0.043	0.000	1.279	0.000	0.000	0.000	0.090

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	80	0	0	103	0	0	-1	0
N.S.	1	1.00	0.62	0.00	0.00	0.79	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.055	0.062	0.282	0.000	0.928	0.000	0.000	0.000	0.144
Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	59	0	0	82	0	0	-1	0
N.S.	1	1.00	0.58	0.00	0.00	0.80	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.044	0.045	0.111	0.000	1.131	0.000	0.000	0.000	0.074
Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	124	75	0	0	165	0	0	-1	0
N.S.	1	1.06	0.64	0.00	0.00	1.41	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.064	0.035	0.052	0.000	1.076	0.000	0.000	0.000	0.222
Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	54	148	59	78	0	0	-1	0
N.S.	1	1.00	0.52	1.44	0.57	0.76	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.065	0.029	0.073	1.233	1.078	0.000	0.000	0.000	0.232

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	93	973	0	353	0	0	-1	126
N.S.	1	1.00	0.84	8.77	0.00	3.18	0.00	0.00	-0.01	1.14
time (sec)	N/A	0.122	0.194	0.164	0.000	1.183	0.000	0.000	0.000	0.248

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	80	664	0	285	0	0	-1	104
N.S.	1	1.00	0.92	7.63	0.00	3.28	0.00	0.00	-0.01	1.20
time (sec)	N/A	0.074	0.127	0.127	0.000	1.077	0.000	0.000	0.000	0.162

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	62	402	0	231	0	0	-1	87
N.S.	1	1.00	0.91	5.91	0.00	3.40	0.00	0.00	-0.01	1.28
time (sec)	N/A	0.051	0.066	0.101	0.000	1.579	0.000	0.000	0.000	0.123

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	113	0	159	0	39	39	57
N.S.	1	1.00	1.00	2.90	0.00	4.08	0.00	1.00	1.00	1.46
time (sec)	N/A	0.033	0.059	0.063	0.000	1.223	0.000	0.414	1.467	0.080

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	135	658	0	333	0	0	-1	120
N.S.	1	1.00	1.38	6.71	0.00	3.40	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.126	0.665	0.148	0.000	1.408	0.000	0.000	0.000	0.191

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	112	958	0	429	0	0	-1	149
N.S.	1	1.00	0.89	7.60	0.00	3.40	0.00	0.00	-0.01	1.18
time (sec)	N/A	0.171	0.333	0.174	0.000	1.409	0.000	0.000	0.000	0.332

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	143	1300	0	522	0	0	-1	182
N.S.	1	1.00	0.87	7.93	0.00	3.18	0.00	0.00	-0.01	1.11
time (sec)	N/A	0.231	0.443	0.201	0.000	1.359	0.000	0.000	0.000	0.319

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	353	353	340	280	0	4426	0	0	-1	0
N.S.	1	1.00	0.96	0.79	0.00	12.54	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.627	0.898	0.747	0.000	2.560	0.000	0.000	0.000	0.056

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	610	610	526	260	0	4699	0	0	-1	0
N.S.	1	1.00	0.86	0.43	0.00	7.70	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.152	0.763	0.461	0.000	1.984	0.000	0.000	0.000	0.056

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	145	114	0	801	0	1035	-1	0
N.S.	1	1.00	0.86	0.67	0.00	4.74	0.00	6.12	-0.01	0.00
time (sec)	N/A	0.192	0.258	0.194	0.000	1.363	0.000	1.846	0.000	0.058

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	127	268	0	1229	0	0	-1	0
N.S.	1	1.00	0.62	1.31	0.00	6.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.395	0.179	0.348	0.000	1.249	0.000	0.000	0.000	0.059

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	699	699	127	534	0	6279	0	0	-1	0
N.S.	1	1.00	0.18	0.76	0.00	8.98	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.493	0.128	0.750	0.000	7.485	0.000	0.000	0.000	0.098

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	414	414	127	630	0	5712	0	0	-1	0
N.S.	1	1.00	0.31	1.52	0.00	13.80	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.787	0.125	1.215	0.000	4.182	0.000	0.000	0.000	0.071
Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	397	0	259	0	0	224	98
N.S.	1	1.00	1.00	5.36	0.00	3.50	0.00	0.00	3.03	1.32
time (sec)	N/A	0.066	0.139	0.096	0.000	1.005	0.000	0.000	1.608	0.148
Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	110	125	0	658	0	0	-1	125
N.S.	1	1.00	0.92	1.05	0.00	5.53	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.095	0.148	0.115	0.000	1.606	0.000	0.000	0.000	0.349
Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	158	209	0	827	0	0	-1	169
N.S.	1	1.00	0.91	1.21	0.00	4.78	0.00	0.00	-0.01	0.98
time (sec)	N/A	0.159	0.282	0.047	0.000	1.803	0.000	0.000	0.000	0.892

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	0	0	148	0	0	-1	51
N.S.	1	1.00	1.00	0.00	0.00	3.15	0.00	0.00	-0.02	1.09
time (sec)	N/A	0.033	0.063	0.036	0.000	1.336	0.000	0.000	0.000	0.171

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	94	0	0	449	0	0	-1	107
N.S.	1	1.00	0.96	0.00	0.00	4.58	0.00	0.00	-0.01	1.09
time (sec)	N/A	0.073	0.310	0.013	0.000	1.703	0.000	0.000	0.000	0.592

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	137	3798	273	2303	0	0	1734	0
N.S.	1	1.00	0.75	20.87	1.50	12.65	0.00	0.00	9.53	0.00
time (sec)	N/A	0.156	0.355	0.113	1.299	1.391	0.000	0.000	2.156	0.177

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	86	1065	152	706	0	5454	543	0
N.S.	1	1.00	0.74	9.10	1.30	6.03	0.00	46.62	4.64	0.00
time (sec)	N/A	0.069	0.151	0.071	1.265	1.255	0.000	0.803	1.617	0.139

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	41	205	65	142	1239	557	83	0
N.S.	1	1.00	0.71	3.53	1.12	2.45	21.36	9.60	1.43	0.00
time (sec)	N/A	0.024	0.065	0.046	1.124	1.278	42.999	0.403	1.406	0.077
Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	150	298	142	175	178	169	141	0
N.S.	1	1.00	3.26	6.48	3.09	3.80	3.87	3.67	3.07	0.00
time (sec)	N/A	0.054	0.039	0.002	1.018	1.156	0.106	0.408	0.076	0.000
Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	401	1314	403	571	559	493	383	0
N.S.	1	1.00	4.51	14.76	4.53	6.42	6.28	5.54	4.30	0.00
time (sec)	N/A	0.187	0.114	0.003	1.061	1.145	0.194	0.410	1.479	0.000
Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	797	7550	872	1335	1314	1109	777	0
N.S.	1	1.00	5.78	54.71	6.32	9.67	9.52	8.04	5.63	0.00
time (sec)	N/A	0.373	0.275	0.003	1.129	1.244	0.356	0.617	1.657	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	154	349	166	229	240	213	164	0
N.S.	1	1.00	2.80	6.35	3.02	4.16	4.36	3.87	2.98	0.00
time (sec)	N/A	0.053	0.010	0.001	1.077	1.089	0.112	0.300	0.077	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	405	1413	439	715	722	615	419	0
N.S.	1	1.00	3.89	13.59	4.22	6.88	6.94	5.91	4.03	0.00
time (sec)	N/A	0.164	0.074	0.000	1.034	1.016	0.212	0.433	1.472	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	801	7697	920	1635	1654	1360	825	0
N.S.	1	1.00	5.04	48.41	5.79	10.28	10.40	8.55	5.19	0.00
time (sec)	N/A	0.315	0.042	0.003	1.164	0.727	0.392	0.514	1.650	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	219	158	0	1231	178	1194	3988	0
N.S.	1	1.00	1.13	0.82	0.00	6.38	0.92	6.19	20.66	0.00
time (sec)	N/A	0.438	0.138	0.069	0.000	2.179	3.065	0.431	2.324	0.001

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	77	151	0	434	280	130	278	0
N.S.	1	1.00	0.95	1.86	0.00	5.36	3.46	1.60	3.43	0.00
time (sec)	N/A	0.129	0.044	0.003	0.000	1.008	1.638	0.405	1.764	0.001
Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	175	140	0	703	104	1285	590	0
N.S.	1	1.00	1.07	0.85	0.00	4.29	0.63	7.84	3.60	0.00
time (sec)	N/A	0.155	0.091	0.006	0.000	1.247	1.350	0.455	1.737	0.001
Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	46	129	0	272	168	53	61	0
N.S.	1	1.00	1.07	3.00	0.00	6.33	3.91	1.23	1.42	0.00
time (sec)	N/A	0.061	0.016	0.007	0.000	1.302	1.036	0.452	0.094	0.001
Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	128	184	0	468	320	274	2173	0
N.S.	1	1.00	1.36	1.96	0.00	4.98	3.40	2.91	23.12	0.00
time (sec)	N/A	0.132	0.078	0.011	0.000	1.299	6.044	1.147	2.504	0.001

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	206	168	0	1339	211	0	3844	0
N.S.	1	1.00	1.06	0.86	0.00	6.87	1.08	0.00	19.71	0.00
time (sec)	N/A	0.286	0.358	0.010	0.000	1.361	4.269	0.000	2.389	0.001
Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	154	213	0	810	464	102	4950	0
N.S.	1	1.00	1.27	1.76	0.00	6.69	3.83	0.84	40.91	0.00
time (sec)	N/A	0.198	0.134	0.013	0.000	1.096	146.464	0.402	5.855	0.001
Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	235	188	0	2044	347	1243	5214	0
N.S.	1	1.00	1.05	0.84	0.00	9.12	1.55	5.55	23.28	0.00
time (sec)	N/A	0.497	0.209	0.010	0.000	1.224	12.774	0.495	2.833	0.001
Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	263	323	0	2454	573	1304	7327	0
N.S.	1	1.00	0.97	1.20	0.00	9.09	2.12	4.83	27.14	0.00
time (sec)	N/A	0.579	0.468	0.020	0.000	1.110	8.966	0.524	4.755	0.002

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	100	276	0	1021	495	171	427	0
N.S.	1	1.00	1.03	2.85	0.00	10.53	5.10	1.76	4.40	0.00
time (sec)	N/A	0.135	0.131	0.021	0.000	1.026	4.875	0.565	1.770	0.001
Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	247	319	0	2474	578	1312	7200	0
N.S.	1	1.00	0.97	1.26	0.00	9.74	2.28	5.17	28.35	0.00
time (sec)	N/A	0.388	0.975	0.022	0.000	1.324	18.532	0.529	3.953	0.001
Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	96	98	270	0	1042	495	172	417	0
N.S.	1	0.98	1.00	2.76	0.00	10.63	5.05	1.76	4.26	0.00
time (sec)	N/A	0.122	0.122	0.023	0.000	0.872	4.675	0.462	1.720	0.001
Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	299	299	271	364	0	3228	0	1357	9056	0
N.S.	1	1.00	0.91	1.22	0.00	10.80	0.00	4.54	30.29	0.00
time (sec)	N/A	0.702	0.889	0.018	0.000	1.726	0.000	0.463	4.846	0.001

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	235	693	0	2476	0	454	11072	0
N.S.	1	1.00	1.45	4.28	0.00	15.28	0.00	2.80	68.35	0.00
time (sec)	N/A	0.294	0.473	0.033	0.000	1.789	0.000	1.253	11.354	0.001

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	348	348	339	1304	0	4330	0	847	10556	0
N.S.	1	1.00	0.97	3.75	0.00	12.44	0.00	2.43	30.33	0.00
time (sec)	N/A	1.653	1.632	0.032	0.000	1.861	0.000	0.787	6.384	0.001

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	284	1014	0	4562	0	224	12436	0
N.S.	1	1.00	1.33	4.76	0.00	21.42	0.00	1.05	58.38	0.00
time (sec)	N/A	0.390	0.520	0.040	0.000	2.509	0.000	0.430	12.317	0.002

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	408	408	384	1518	0	5734	0	1987	12239	0
N.S.	1	1.00	0.94	3.72	0.00	14.05	0.00	4.87	30.00	0.00
time (sec)	N/A	3.676	2.983	0.036	0.000	2.383	0.000	0.548	8.725	0.002

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	341	341	328	704	0	6633	0	1688	12677	0
N.S.	1	1.00	0.96	2.06	0.00	19.45	0.00	4.95	37.18	0.00
time (sec)	N/A	0.951	4.434	0.050	0.000	2.043	0.000	0.746	7.019	0.002
Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	146	544	0	3739	1671	365	1182	0
N.S.	1	1.00	0.97	3.63	0.00	24.93	11.14	2.43	7.88	0.00
time (sec)	N/A	0.196	0.206	0.046	0.000	1.531	14.453	0.693	3.855	0.002
Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	363	382	885	0	7701	0	2295	14584	0
N.S.	1	1.00	1.05	2.44	0.00	21.21	0.00	6.32	40.18	0.00
time (sec)	N/A	1.039	4.861	0.051	0.000	2.258	0.000	0.866	7.429	0.002
Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	150	147	541	0	3708	1646	365	1157	0
N.S.	1	0.99	0.97	3.56	0.00	24.39	10.83	2.40	7.61	0.00
time (sec)	N/A	0.185	0.184	0.050	0.000	1.470	13.928	0.633	3.802	0.002

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	437	437	463	1010	0	8554	0	2487	16086	0
N.S.	1	1.00	1.06	2.31	0.00	19.57	0.00	5.69	36.81	0.00
time (sec)	N/A	5.361	6.169	0.052	0.000	2.546	0.000	0.563	7.800	0.001

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	391	4477	0	9908	0	1012	19440	0
N.S.	1	1.00	1.53	17.56	0.00	38.85	0.00	3.97	76.24	0.00
time (sec)	N/A	0.486	3.949	0.080	0.000	4.114	0.000	1.716	17.982	0.001

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	484	484	560	6821	0	10260	0	1412	18112	0
N.S.	1	1.00	1.16	14.09	0.00	21.20	0.00	2.92	37.42	0.00
time (sec)	N/A	1.227	6.237	0.071	0.000	3.917	0.000	1.220	14.376	0.001

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	325	491	5575	0	15165	0	377	21465	0
N.S.	1	1.00	1.51	17.15	0.00	46.66	0.00	1.16	66.05	0.00
time (sec)	N/A	0.585	6.179	0.086	0.000	7.534	0.000	0.639	22.450	0.001

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	222	164	0	1346	219	1245	4605	0
N.S.	1	1.00	1.10	0.81	0.00	6.66	1.08	6.16	22.80	0.00
time (sec)	N/A	0.362	0.049	0.005	0.000	1.322	3.623	0.536	1.341	0.001
Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	80	154	0	446	332	162	287	0
N.S.	1	1.00	0.92	1.77	0.00	5.13	3.82	1.86	3.30	0.00
time (sec)	N/A	0.127	0.044	0.005	0.000	1.187	1.950	0.429	0.443	0.001
Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	178	143	0	799	124	1325	683	0
N.S.	1	1.00	1.05	0.84	0.00	4.70	0.73	7.79	4.02	0.00
time (sec)	N/A	0.153	0.101	0.002	0.000	1.103	1.595	0.466	1.792	0.001
Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	47	130	0	274	189	62	477	0
N.S.	1	1.00	1.07	2.95	0.00	6.23	4.30	1.41	10.84	0.00
time (sec)	N/A	0.063	0.016	0.005	0.000	1.220	1.187	0.397	1.622	0.001

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	131	190	0	474	348	285	2520	0
N.S.	1	1.00	1.27	1.84	0.00	4.60	3.38	2.77	24.47	0.00
time (sec)	N/A	0.138	0.069	0.007	0.000	1.184	7.157	1.193	3.462	0.002

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	209	174	0	1477	258	0	4339	0
N.S.	1	1.00	1.02	0.85	0.00	7.24	1.26	0.00	21.27	0.00
time (sec)	N/A	0.273	0.342	0.007	0.000	1.448	4.899	0.000	3.872	0.002

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	157	222	0	828	532	348	5947	0
N.S.	1	1.00	1.18	1.67	0.00	6.23	4.00	2.62	44.71	0.00
time (sec)	N/A	0.195	0.136	0.008	0.000	1.935	156.628	1.106	6.981	0.002

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	238	197	0	2212	411	1249	5771	0
N.S.	1	1.00	1.01	0.83	0.00	9.37	1.74	5.29	24.45	0.00
time (sec)	N/A	0.488	0.103	0.010	0.000	1.316	14.012	0.570	3.173	0.002

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	266	695	0	2578	641	1370	8025	0
N.S.	1	1.00	0.95	2.49	0.00	9.24	2.30	4.91	28.76	0.00
time (sec)	N/A	0.539	0.461	0.018	0.000	1.256	9.821	0.603	5.092	0.002
Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	103	500	0	1077	556	211	460	0
N.S.	1	1.00	1.00	4.85	0.00	10.46	5.40	2.05	4.47	0.00
time (sec)	N/A	0.138	0.126	0.020	0.000	1.209	5.286	0.515	1.903	0.002
Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	263	263	250	693	0	2600	646	1378	7835	0
N.S.	1	1.00	0.95	2.63	0.00	9.89	2.46	5.24	29.79	0.00
time (sec)	N/A	0.368	0.939	0.017	0.000	1.316	19.757	0.675	4.398	0.002
Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	99	484	0	1066	525	211	442	0
N.S.	1	1.00	1.01	4.94	0.00	10.88	5.36	2.15	4.51	0.00
time (sec)	N/A	0.127	0.123	0.024	0.000	1.115	5.068	0.445	1.905	0.002

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	238	714	0	2486	0	476	13434	0
N.S.	1	1.00	1.37	4.10	0.00	14.29	0.00	2.74	77.21	0.00
time (sec)	N/A	0.295	0.436	0.030	0.000	2.338	0.000	1.432	11.689	0.002

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	342	1346	0	4520	0	999	12008	0
N.S.	1	1.00	0.95	3.74	0.00	12.56	0.00	2.78	33.36	0.00
time (sec)	N/A	1.599	1.471	0.028	0.000	1.763	0.000	1.021	7.289	0.001

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	228	228	287	1047	0	4604	0	687	14830	0
N.S.	1	1.00	1.26	4.59	0.00	20.19	0.00	3.01	65.04	0.00
time (sec)	N/A	0.369	0.515	0.034	0.000	5.904	0.000	1.296	13.520	0.002

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	423	423	387	1569	0	5954	0	2002	13781	0
N.S.	1	1.00	0.91	3.71	0.00	14.08	0.00	4.73	32.58	0.00
time (sec)	N/A	3.550	3.025	0.030	0.000	2.131	0.000	0.714	10.447	0.002

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	353	353	331	3432	0	6770	0	1844	13840	0
N.S.	1	1.00	0.94	9.72	0.00	19.18	0.00	5.22	39.21	0.00
time (sec)	N/A	0.870	4.445	0.049	0.000	1.412	0.000	0.799	7.520	0.002
Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	149	2181	0	3843	1794	447	1267	0
N.S.	1	1.00	0.94	13.72	0.00	24.17	11.28	2.81	7.97	0.00
time (sec)	N/A	0.200	0.209	0.046	0.000	1.443	14.583	0.781	4.019	0.002
Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	385	4751	0	7838	0	2527	16025	0
N.S.	1	1.00	1.03	12.67	0.00	20.90	0.00	6.74	42.73	0.00
time (sec)	N/A	0.972	4.483	0.049	0.000	2.199	0.000	0.794	7.944	0.002
Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	148	2132	0	3748	1707	445	1199	0
N.S.	1	1.00	0.97	13.93	0.00	24.50	11.16	2.91	7.84	0.00
time (sec)	N/A	0.192	0.179	0.053	0.000	1.591	13.965	0.690	3.994	0.002

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	394	4606	0	9926	0	1044	22621	0
N.S.	1	1.00	1.46	17.06	0.00	36.76	0.00	3.87	83.78	0.00
time (sec)	N/A	0.496	3.925	0.075	0.000	7.394	0.000	1.700	18.492	0.001
Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	499	499	575	7019	0	10518	0	1658	20580	0
N.S.	1	1.00	1.15	14.07	0.00	21.08	0.00	3.32	41.24	0.00
time (sec)	N/A	1.094	6.210	0.070	0.000	3.605	0.000	1.419	15.398	0.002
Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-1)	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	509	5737	0	15231	0	1735	25334	0
N.S.	1	1.00	1.48	16.73	0.00	44.41	0.00	5.06	73.86	0.00
time (sec)	N/A	0.589	6.157	0.082	0.000	22.607	0.000	1.628	24.912	0.002
Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	105	104	104	107	28	29	0
N.S.	1	1.00	1.00	3.09	3.06	3.06	3.15	0.82	0.85	0.00
time (sec)	N/A	0.036	0.011	0.003	0.543	1.171	0.101	0.349	1.584	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	188	175	174	174	187	46	46	0
N.S.	1	1.00	3.36	3.12	3.11	3.11	3.34	0.82	0.82	0.00
time (sec)	N/A	0.096	0.010	0.003	0.774	0.996	0.150	0.415	1.602	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [350] had the largest ratio of [.8000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	1	1	1.00	15	0.067
3	A	1	1	1.00	15	0.067
4	A	3	3	1.00	15	0.200
5	A	8	8	1.00	11	0.727
6	A	3	2	1.00	26	0.077
7	A	3	2	1.00	26	0.077
8	A	3	2	1.00	26	0.077
9	A	2	2	1.00	26	0.077
10	A	3	2	1.00	24	0.083
11	A	2	1	1.00	22	0.045
12	A	3	2	1.00	26	0.077
13	A	3	2	1.00	26	0.077
14	A	3	2	1.00	26	0.077
15	A	3	2	1.00	26	0.077
16	A	3	2	1.00	26	0.077
17	A	3	2	1.00	26	0.077
18	A	3	2	1.00	26	0.077
19	A	3	2	1.00	26	0.077
20	A	3	2	1.00	26	0.077
21	A	3	2	1.00	26	0.077
22	A	3	2	1.00	26	0.077
23	A	3	2	1.00	26	0.077
24	A	4	3	1.40	26	0.115
25	A	3	2	1.00	26	0.077
26	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	4	3	1.00	26	0.115
28	A	3	2	1.00	26	0.077
29	A	3	2	1.00	26	0.077
30	A	2	2	1.00	26	0.077
31	A	3	2	1.00	24	0.083
32	A	3	2	1.00	22	0.091
33	A	4	3	1.00	26	0.115
34	A	3	2	1.00	26	0.077
35	A	3	2	1.00	26	0.077
36	A	4	3	1.00	26	0.115
37	A	3	2	1.00	26	0.077
38	A	3	2	1.00	26	0.077
39	A	4	3	1.00	26	0.115
40	A	3	2	1.00	26	0.077
41	A	3	2	1.00	26	0.077
42	A	4	3	1.00	26	0.115
43	A	3	2	1.00	26	0.077
44	A	3	2	1.00	26	0.077
45	A	2	2	1.00	26	0.077
46	A	3	2	1.00	26	0.077
47	A	3	2	1.00	26	0.077
48	A	4	4	1.00	26	0.154
49	A	3	2	1.00	26	0.077
50	A	3	2	1.00	26	0.077
51	A	3	2	1.00	26	0.077
52	A	4	3	1.00	26	0.115
53	A	3	2	1.00	26	0.077
54	A	3	2	1.00	26	0.077
55	A	4	3	1.00	26	0.115
56	A	3	2	1.00	26	0.077
57	A	3	2	1.00	26	0.077
58	A	4	3	1.00	26	0.115
59	A	3	2	1.00	26	0.077
60	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	26	0.077
62	A	3	2	1.00	24	0.083
63	A	3	2	1.00	22	0.091
64	A	4	3	1.00	26	0.115
65	A	3	2	1.00	26	0.077
66	A	3	2	1.00	26	0.077
67	A	4	3	1.00	26	0.115
68	A	3	2	1.00	26	0.077
69	A	3	2	1.00	26	0.077
70	A	4	3	1.00	26	0.115
71	A	3	2	1.00	26	0.077
72	A	3	2	1.00	26	0.077
73	A	4	3	1.00	26	0.115
74	A	3	2	1.00	26	0.077
75	A	3	2	1.00	26	0.077
76	A	4	3	1.00	26	0.115
77	A	3	2	1.00	26	0.077
78	A	3	2	1.00	26	0.077
79	A	4	3	1.00	26	0.115
80	A	3	2	1.00	26	0.077
81	A	3	2	1.00	26	0.077
82	A	2	2	1.00	26	0.077
83	A	3	2	1.00	26	0.077
84	A	3	2	1.00	26	0.077
85	A	4	4	1.00	26	0.154
86	A	3	2	1.00	26	0.077
87	A	3	2	1.00	26	0.077
88	A	5	4	1.00	26	0.154
89	A	8	8	1.00	26	0.308
90	A	8	8	1.00	26	0.308
91	A	3	3	1.00	26	0.115
92	A	7	7	1.00	24	0.292
93	A	7	7	1.00	22	0.318
94	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	8	8	1.00	26	0.308
96	A	8	8	0.99	26	0.308
97	A	4	3	0.98	26	0.115
98	A	9	9	1.00	26	0.346
99	A	9	9	1.00	26	0.346
100	A	2	2	1.00	26	0.077
101	A	9	8	1.00	24	0.333
102	A	9	8	1.00	22	0.364
103	A	4	3	1.00	26	0.115
104	A	10	9	1.00	26	0.346
105	A	10	9	1.00	26	0.346
106	A	4	3	1.00	26	0.115
107	A	11	9	1.00	26	0.346
108	A	4	3	1.00	26	0.115
109	A	11	9	1.00	26	0.346
110	A	11	9	1.00	26	0.346
111	A	2	2	1.00	26	0.077
112	A	11	8	1.00	24	0.333
113	A	11	8	1.00	22	0.364
114	A	4	3	1.00	26	0.115
115	A	12	9	1.00	26	0.346
116	A	12	9	1.00	26	0.346
117	A	4	3	1.00	26	0.115
118	A	3	2	1.00	28	0.071
119	A	3	2	1.00	28	0.071
120	A	3	2	1.00	28	0.071
121	A	4	3	1.00	24	0.125
122	A	4	3	1.00	24	0.125
123	A	4	3	1.00	24	0.125
124	A	2	2	1.00	24	0.083
125	A	6	6	1.00	18	0.333
126	A	5	5	1.00	18	0.278
127	A	3	3	1.00	18	0.167
128	A	7	7	1.00	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	8	7	1.00	18	0.389
130	A	14	8	1.00	18	0.444
131	A	14	8	1.00	18	0.444
132	A	13	7	1.00	18	0.389
133	A	13	7	1.00	18	0.389
134	A	13	7	1.00	16	0.438
135	A	13	7	1.00	14	0.500
136	A	14	8	1.00	18	0.444
137	A	14	8	1.00	18	0.444
138	A	6	4	1.00	16	0.250
139	A	5	4	1.00	16	0.250
140	A	4	3	1.00	16	0.188
141	B	4	3	2.10	16	0.188
142	A	6	5	1.00	16	0.312
143	A	4	3	1.00	16	0.188
144	A	4	3	1.00	16	0.188
145	A	15	10	1.00	16	0.625
146	A	15	10	1.00	16	0.625
147	A	14	9	1.00	16	0.562
148	A	14	9	1.00	16	0.562
149	A	13	8	1.00	16	0.500
150	A	13	8	1.00	16	0.500
151	A	13	8	1.00	14	0.571
152	A	13	8	1.00	12	0.667
153	A	14	9	1.00	16	0.562
154	A	14	9	1.00	16	0.562
155	A	15	10	1.00	16	0.625
156	A	15	10	1.00	16	0.625
157	A	14	8	1.00	16	0.500
158	A	5	5	1.00	16	0.312
159	A	13	7	1.00	16	0.438
160	A	13	7	1.00	16	0.438
161	A	3	3	1.00	16	0.188
162	A	13	7	1.00	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	C	13	7	2.02	12	0.583
164	A	7	7	1.00	16	0.438
165	A	14	8	1.00	16	0.500
166	A	14	8	1.00	16	0.500
167	A	8	7	1.00	16	0.438
168	A	16	10	1.00	16	0.625
169	A	13	7	1.00	10	0.700
170	A	3	3	1.00	14	0.214
171	A	13	7	1.00	14	0.500
172	A	7	7	1.00	20	0.350
173	A	6	6	1.00	20	0.300
174	A	6	6	1.00	20	0.300
175	A	5	5	1.00	20	0.250
176	A	4	4	1.00	20	0.200
177	A	7	6	1.00	20	0.300
178	A	7	6	1.00	20	0.300
179	A	4	4	1.00	20	0.200
180	A	5	5	1.00	20	0.250
181	A	6	6	1.00	20	0.300
182	A	7	7	1.00	20	0.350
183	A	8	7	1.00	20	0.350
184	A	7	6	1.00	20	0.300
185	A	7	6	1.00	20	0.300
186	A	6	5	1.00	20	0.250
187	A	5	4	1.00	20	0.200
188	A	8	7	1.00	20	0.350
189	A	8	7	1.00	20	0.350
190	A	8	7	1.00	20	0.350
191	A	8	7	1.00	20	0.350
192	A	5	4	1.00	20	0.200
193	A	6	5	1.00	20	0.250
194	A	7	6	1.00	20	0.300
195	A	8	7	1.00	20	0.350
196	A	6	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	5	5	1.00	20	0.250
198	A	5	5	1.00	20	0.250
199	A	4	4	1.00	20	0.200
200	A	3	3	1.00	20	0.150
201	A	3	3	1.00	20	0.150
202	A	4	4	1.00	20	0.200
203	A	5	5	1.00	20	0.250
204	A	6	6	1.00	20	0.300
205	A	7	6	1.00	20	0.300
206	A	6	6	1.00	20	0.300
207	A	5	5	1.00	20	0.250
208	A	5	5	1.00	20	0.250
209	A	2	2	1.00	20	0.100
210	A	2	2	1.00	20	0.100
211	A	5	5	1.00	20	0.250
212	A	5	5	1.00	20	0.250
213	A	6	6	1.00	20	0.300
214	A	7	6	1.00	20	0.300
215	A	2	1	1.00	20	0.050
216	A	2	1	1.00	18	0.056
217	A	5	5	1.00	16	0.312
218	A	4	3	1.00	16	0.188
219	A	4	4	1.00	16	0.250
220	A	2	2	1.00	16	0.125
221	A	4	4	1.00	14	0.286
222	A	4	3	1.00	16	0.188
223	A	5	5	1.00	16	0.312
224	A	4	3	1.00	16	0.188
225	A	6	5	1.00	16	0.312
226	A	12	9	1.00	16	0.562
227	A	11	8	1.00	16	0.500
228	A	11	8	1.00	16	0.500
229	A	11	8	1.00	16	0.500
230	A	11	8	1.00	12	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	12	9	1.00	16	0.562
232	A	12	9	1.00	16	0.562
233	A	13	9	1.00	16	0.562
234	A	13	9	1.00	16	0.562
235	A	5	5	1.00	16	0.312
236	A	4	3	1.00	16	0.188
237	A	4	4	1.00	16	0.250
238	A	2	2	1.00	16	0.125
239	A	4	4	1.00	14	0.286
240	A	4	3	1.00	16	0.188
241	A	5	5	1.00	16	0.312
242	A	4	3	1.00	16	0.188
243	A	6	5	1.00	16	0.312
244	A	6	6	1.00	16	0.375
245	A	5	5	1.00	16	0.312
246	A	5	5	1.00	16	0.312
247	A	5	5	1.00	16	0.312
248	A	5	5	1.00	12	0.417
249	A	6	6	1.00	16	0.375
250	A	6	6	1.00	16	0.375
251	A	7	6	1.00	16	0.375
252	A	7	6	1.00	16	0.375
253	A	6	6	1.00	18	0.333
254	A	5	4	1.00	18	0.222
255	A	5	5	1.00	18	0.278
256	A	4	3	1.00	18	0.167
257	A	3	3	1.00	18	0.167
258	A	4	3	1.00	16	0.188
259	A	7	7	1.00	18	0.389
260	A	5	4	1.00	18	0.222
261	A	8	7	1.00	18	0.389
262	A	8	5	1.00	18	0.278
263	A	8	5	1.00	18	0.278
264	A	7	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	7	4	1.00	18	0.222
266	A	7	4	1.00	18	0.222
267	A	7	4	1.00	14	0.286
268	A	8	5	1.00	18	0.278
269	A	8	5	1.00	18	0.278
270	A	6	6	1.00	14	0.429
271	A	7	5	1.00	14	0.357
272	A	5	5	1.00	14	0.357
273	A	10	7	1.00	14	0.500
274	A	3	3	1.00	14	0.214
275	A	10	6	1.00	12	0.500
276	A	7	7	1.00	14	0.500
277	A	7	5	1.00	14	0.357
278	A	8	7	1.00	14	0.500
279	A	13	10	1.00	14	0.714
280	A	20	7	1.00	14	0.500
281	A	9	6	1.00	14	0.429
282	A	19	7	1.00	14	0.500
283	A	19	6	1.00	14	0.429
284	A	9	6	1.00	10	0.600
285	A	20	8	1.00	14	0.571
286	A	20	7	1.00	14	0.500
287	A	12	9	1.00	14	0.643
288	A	22	10	1.00	14	0.714
289	A	6	6	1.00	16	0.375
290	A	5	4	1.00	16	0.250
291	A	5	5	1.00	16	0.312
292	A	10	7	1.00	16	0.438
293	A	3	3	1.00	16	0.188
294	A	10	6	1.00	14	0.429
295	A	7	7	1.00	16	0.438
296	A	5	4	1.00	16	0.250
297	A	8	7	1.00	16	0.438
298	A	13	10	1.00	16	0.625

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
299	A	20	7	1.00	16	0.438
300	A	19	6	1.00	16	0.375
301	A	19	7	1.00	16	0.438
302	A	19	6	1.00	16	0.375
303	A	19	6	1.00	12	0.500
304	A	22	8	1.00	16	0.500
305	A	20	7	1.00	16	0.438
306	A	22	9	1.00	16	0.562
307	A	22	10	1.00	16	0.625
308	A	5	4	1.00	16	0.250
309	A	5	4	1.00	16	0.250
310	A	4	3	1.00	16	0.188
311	A	4	3	1.00	16	0.188
312	A	3	3	1.00	16	0.188
313	A	4	3	1.00	14	0.214
314	A	6	5	1.00	16	0.312
315	A	5	4	1.00	16	0.250
316	A	7	5	1.00	16	0.312
317	A	6	5	1.00	16	0.312
318	A	20	8	0.96	16	0.500
319	A	19	7	1.00	16	0.438
320	A	19	7	1.00	16	0.438
321	A	19	7	1.01	16	0.438
322	A	19	7	1.00	12	0.583
323	A	20	8	1.00	16	0.500
324	A	20	8	1.00	16	0.500
325	A	5	4	1.00	16	0.250
326	A	5	4	1.00	16	0.250
327	A	4	3	1.00	16	0.188
328	A	4	3	1.00	16	0.188
329	A	3	3	1.00	16	0.188
330	A	4	3	1.00	14	0.214
331	A	6	5	1.00	16	0.312
332	A	5	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
333	A	7	5	1.00	16	0.312
334	A	6	5	1.00	16	0.312
335	A	8	5	1.00	16	0.312
336	A	7	4	1.00	16	0.250
337	A	7	4	1.00	16	0.250
338	A	7	4	1.14	16	0.250
339	A	7	4	1.00	12	0.333
340	A	8	5	1.00	16	0.312
341	A	8	5	1.00	16	0.312
342	A	9	6	1.00	16	0.375
343	A	9	6	1.00	16	0.375
344	A	4	3	1.00	16	0.188
345	A	5	4	1.00	16	0.250
346	A	5	5	1.00	14	0.357
347	A	3	3	1.00	14	0.214
348	A	7	7	1.00	14	0.500
349	A	8	7	1.00	14	0.500
350	A	8	8	1.00	10	0.800
351	A	7	6	1.00	18	0.333
352	A	7	6	1.00	18	0.333
353	A	7	6	1.00	16	0.375
354	A	6	6	1.00	14	0.429
355	A	5	5	1.00	18	0.278
356	A	3	3	1.00	18	0.167
357	A	7	7	1.00	18	0.389
358	A	8	7	1.00	18	0.389
359	A	8	7	1.00	18	0.389
360	A	8	7	1.00	18	0.389
361	A	8	7	1.00	16	0.438
362	A	8	7	1.00	14	0.500
363	A	7	7	1.00	18	0.389
364	A	4	4	1.00	18	0.222
365	A	4	4	1.00	18	0.222
366	A	4	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	A	8	7	1.00	18	0.389
368	A	8	7	1.00	18	0.389
369	A	8	7	1.00	18	0.389
370	A	9	8	1.00	14	0.571
371	A	8	8	1.00	18	0.444
372	A	5	4	1.00	18	0.222
373	A	5	5	1.00	18	0.278
374	A	5	5	1.00	18	0.278
375	A	5	5	1.00	18	0.278
376	A	5	4	0.98	18	0.222
377	A	9	8	1.00	18	0.444
378	A	9	8	1.00	18	0.444
379	A	6	4	1.00	18	0.222
380	A	6	4	1.00	16	0.250
381	A	5	4	1.00	14	0.286
382	A	4	3	1.00	18	0.167
383	A	4	3	1.00	18	0.167
384	A	6	5	1.00	18	0.278
385	A	4	3	1.00	18	0.167
386	A	4	3	1.00	18	0.167
387	A	4	3	1.00	18	0.167
388	A	9	7	1.00	16	0.438
389	A	8	7	1.00	16	0.438
390	A	7	6	1.00	16	0.375
391	A	4	4	1.00	16	0.250
392	A	5	5	1.00	16	0.312
393	A	6	6	1.00	16	0.375
394	A	4	3	1.00	22	0.136
395	A	5	4	1.00	14	0.286
396	A	15	9	1.00	14	0.643
397	A	9	6	1.00	14	0.429
398	A	7	6	1.00	20	0.300
399	A	4	3	1.00	23	0.130
400	A	4	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	4	3	1.00	26	0.115
402	A	4	3	1.00	26	0.115
403	A	3	2	1.00	26	0.077
404	A	4	3	1.00	26	0.115
405	A	4	3	1.00	26	0.115
406	A	4	3	1.00	26	0.115
407	A	4	3	1.00	26	0.115
408	A	4	3	1.00	26	0.115
409	A	4	3	1.00	26	0.115
410	A	4	3	1.00	26	0.115
411	A	4	3	1.00	28	0.107
412	A	4	3	1.00	26	0.115
413	A	4	3	1.00	24	0.125
414	C	7	3	1.11	77	0.039
415	A	4	3	1.00	26	0.115
416	A	4	3	1.00	26	0.115
417	A	5	4	1.00	24	0.167
418	A	5	4	1.00	26	0.154
419	A	5	4	1.00	26	0.154
420	A	5	4	1.00	26	0.154
421	A	4	3	1.00	26	0.115
422	A	5	4	1.00	26	0.154
423	A	5	4	1.00	26	0.154
424	A	5	4	1.00	26	0.154
425	A	5	4	1.00	26	0.154
426	A	5	4	1.00	26	0.154
427	A	4	3	1.00	26	0.115
428	A	5	4	1.00	26	0.154
429	A	4	3	1.00	23	0.130
430	A	4	3	1.00	23	0.130
431	A	2	2	1.00	23	0.087
432	A	5	5	1.00	21	0.238
433	A	4	3	1.00	23	0.130
434	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
435	A	4	3	1.00	23	0.130
436	A	12	9	1.00	25	0.360
437	A	9	9	1.00	25	0.360
438	A	5	5	1.00	25	0.200
439	A	6	6	1.00	25	0.240
440	A	11	11	1.00	25	0.440
441	A	14	11	1.00	25	0.440
442	A	1	1	1.00	26	0.038
443	A	1	1	1.00	28	0.036
444	A	4	3	1.00	32	0.094
445	A	4	3	1.00	32	0.094
446	A	3	2	1.00	32	0.062
447	A	4	3	1.00	32	0.094
448	A	2	2	1.00	32	0.062
449	A	4	3	1.00	32	0.094
450	A	4	3	1.00	32	0.094
451	A	5	4	1.00	30	0.133
452	A	3	2	1.00	28	0.071
453	A	3	2	1.00	26	0.077
454	A	2	1	1.00	24	0.042
455	A	3	2	1.00	28	0.071
456	A	3	2	1.00	28	0.071
457	A	3	2	1.00	28	0.071
458	A	9	4	1.00	30	0.133
459	A	3	2	1.00	28	0.071
460	A	3	2	1.00	26	0.077
461	A	3	2	1.00	24	0.083
462	A	4	3	1.00	28	0.107
463	A	3	2	1.00	28	0.071
464	A	3	2	1.00	28	0.071
465	A	5	5	1.00	28	0.179
466	A	4	3	1.00	28	0.107
467	A	2	2	1.00	36	0.056
468	A	2	2	1.00	31	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	3	3	1.00	34	0.088
470	A	3	3	1.00	33	0.091
471	A	3	3	1.06	35	0.086
472	A	4	3	1.00	30	0.100
473	A	7	6	1.00	24	0.250
474	A	6	6	1.00	24	0.250
475	A	5	5	1.00	24	0.208
476	A	3	3	1.00	22	0.136
477	A	8	7	1.00	24	0.292
478	A	8	7	1.00	24	0.292
479	A	8	7	1.00	24	0.292
480	A	8	5	1.00	26	0.192
481	A	14	8	1.00	26	0.308
482	A	4	3	1.00	26	0.115
483	A	6	5	1.00	26	0.192
484	A	16	10	1.00	26	0.385
485	A	10	7	1.00	26	0.269
486	A	7	7	1.00	20	0.350
487	A	7	6	1.00	22	0.273
488	A	8	7	1.00	22	0.318
489	A	3	3	1.00	22	0.136
490	A	5	5	1.00	22	0.227
491	A	14	3	1.00	22	0.136
492	A	10	3	1.00	22	0.136
493	A	6	3	1.00	20	0.150
494	A	3	2	1.00	28	0.071
495	A	4	3	1.00	30	0.100
496	A	4	3	1.00	30	0.100
497	A	3	2	1.00	31	0.065
498	A	4	3	1.00	33	0.091
499	A	4	3	1.00	33	0.091
500	A	5	4	1.00	30	0.133
501	A	6	6	1.00	30	0.200
502	A	4	3	1.00	30	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
503	A	4	4	1.00	28	0.143
504	A	8	8	1.00	30	0.267
505	A	5	4	1.00	30	0.133
506	A	9	8	1.00	30	0.267
507	A	6	5	1.00	30	0.167
508	A	5	4	1.00	30	0.133
509	A	5	5	1.00	30	0.167
510	A	5	4	1.00	30	0.133
511	A	5	5	0.98	28	0.179
512	A	5	4	1.00	22	0.182
513	A	9	8	1.00	30	0.267
514	A	6	5	1.00	30	0.167
515	A	9	8	1.00	30	0.267
516	A	7	5	1.00	30	0.167
517	A	6	5	1.00	30	0.167
518	A	6	6	1.00	30	0.200
519	A	6	5	1.00	30	0.167
520	A	6	5	0.99	28	0.179
521	A	6	5	1.00	22	0.227
522	A	10	9	1.00	30	0.300
523	A	7	6	1.00	30	0.200
524	A	10	9	1.00	30	0.300
525	A	5	4	1.00	33	0.121
526	A	6	6	1.00	33	0.182
527	A	4	3	1.00	33	0.091
528	A	4	4	1.00	31	0.129
529	A	8	8	1.00	33	0.242
530	A	5	4	1.00	33	0.121
531	A	9	8	1.00	33	0.242
532	A	6	5	1.00	33	0.152
533	A	5	4	1.00	33	0.121
534	A	5	5	1.00	33	0.152
535	A	5	4	1.00	33	0.121
536	A	5	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
537	A	9	8	1.00	33	0.242
538	A	6	5	1.00	33	0.152
539	A	9	8	1.00	33	0.242
540	A	7	5	1.00	33	0.152
541	A	6	5	1.00	33	0.152
542	A	6	6	1.00	33	0.182
543	A	6	5	1.00	33	0.152
544	A	6	5	1.00	31	0.161
545	A	10	9	1.00	33	0.273
546	A	7	6	1.00	33	0.182
547	A	10	9	1.00	33	0.273
548	A	3	2	1.00	24	0.083
549	A	4	3	1.00	26	0.115

Chapter 3

Listing of integrals

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3.7	$\int x^4 \sqrt{a^2 + 2abx^3 + b^2x^6} dx$	228
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3.11	$\int \sqrt{a^2 + 2abx^3 + b^2x^6} dx$	243
3.12	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x} dx$	246
3.13	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2} dx$	250
3.14	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3} dx$	254
3.15	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4} dx$	258
3.16	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5} dx$	262
3.17	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6} dx$	266
3.18	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7} dx$	270

3.19	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8} dx$	274
3.20	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9} dx$	278
3.21	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}} dx$	282
3.22	$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}} dx$	286
3.23	$\int x^9 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	290
3.24	$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	294
3.25	$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	298
3.26	$\int x^6 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	302
3.27	$\int x^5 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	306
3.28	$\int x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	310
3.29	$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	314
3.30	$\int x^2 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	318
3.31	$\int x (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	321
3.32	$\int (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	325
3.33	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x} dx$	329
3.34	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2} dx$	333
3.35	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3} dx$	337
3.36	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4} dx$	341
3.37	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5} dx$	345
3.38	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6} dx$	349
3.39	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7} dx$	353
3.40	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^8} dx$	357
3.41	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^9} dx$	361
3.42	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}} dx$	365
3.43	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}} dx$	369
3.44	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}} dx$	373

3.45	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}} dx$	377
3.46	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}} dx$	381
3.47	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{15}} dx$	385
3.48	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}} dx$	389
3.49	$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}} dx$	393
3.50	$\int x^{13} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	397
3.51	$\int x^{12} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	401
3.52	$\int x^{11} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	405
3.53	$\int x^{10} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	409
3.54	$\int x^9 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	413
3.55	$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	417
3.56	$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	421
3.57	$\int x^6 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	425
3.58	$\int x^5 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	429
3.59	$\int x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	433
3.60	$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	437
3.61	$\int x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	441
3.62	$\int x (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	444
3.63	$\int (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	448
3.64	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x} dx$	452
3.65	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2} dx$	456
3.66	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3} dx$	460
3.67	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4} dx$	464
3.68	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5} dx$	468
3.69	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6} dx$	472
3.70	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7} dx$	476

3.71	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8} dx$	480
3.72	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9} dx$	484
3.73	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}} dx$	488
3.74	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}} dx$	492
3.75	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}} dx$	496
3.76	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}} dx$	500
3.77	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}} dx$	505
3.78	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}} dx$	509
3.79	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}} dx$	513
3.80	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}} dx$	518
3.81	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}} dx$	522
3.82	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}} dx$	526
3.83	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}} dx$	530
3.84	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}} dx$	534
3.85	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}} dx$	538
3.86	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}} dx$	542
3.87	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}} dx$	546
3.88	$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}} dx$	550
3.89	$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}} dx$	555
3.90	$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}} dx$	560
3.91	$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}} dx$	565
3.92	$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}} dx$	569
3.93	$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}} dx$	575
3.94	$\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}} dx$	581

3.95	$\int \frac{1}{x^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} dx$	585
3.96	$\int \frac{1}{x^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} dx$	591
3.97	$\int \frac{1}{x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}} dx$	597
3.98	$\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$	601
3.99	$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$	607
3.100	$\int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$	613
3.101	$\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$	616
3.102	$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$	622
3.103	$\int \frac{1}{x(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$	628
3.104	$\int \frac{1}{x^2(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$	632
3.105	$\int \frac{1}{x^3(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$	638
3.106	$\int \frac{1}{x^4(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$	644
3.107	$\int \frac{x^6}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	648
3.108	$\int \frac{x^5}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	654
3.109	$\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	658
3.110	$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	664
3.111	$\int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	670
3.112	$\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	673
3.113	$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	679
3.114	$\int \frac{1}{x(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	685
3.115	$\int \frac{1}{x^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	694
3.116	$\int \frac{1}{x^3(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$	700

3.117	$\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{5/2}} dx$	706
3.118	$\int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$	713
3.119	$\int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$	717
3.120	$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6} dx$	721
3.121	$\int x^{11} (a^2 + 2abx^3 + b^2x^6)^p dx$	725
3.122	$\int x^8 (a^2 + 2abx^3 + b^2x^6)^p dx$	730
3.123	$\int x^5 (a^2 + 2abx^3 + b^2x^6)^p dx$	734
3.124	$\int x^2 (a^2 + 2abx^3 + b^2x^6)^p dx$	738
3.125	$\int \frac{x^8}{a+bx^3+cx^6} dx$	741
3.126	$\int \frac{x^5}{a+bx^3+cx^6} dx$	746
3.127	$\int \frac{x^2}{a+bx^3+cx^6} dx$	751
3.128	$\int \frac{1}{x(a+bx^3+cx^6)} dx$	755
3.129	$\int \frac{1}{x^4(a+bx^3+cx^6)} dx$	760
3.130	$\int \frac{x^7}{a+bx^3+cx^6} dx$	767
3.131	$\int \frac{x^6}{a+bx^3+cx^6} dx$	777
3.132	$\int \frac{x^4}{a+bx^3+cx^6} dx$	786
3.133	$\int \frac{x^3}{a+bx^3+cx^6} dx$	794
3.134	$\int \frac{x}{a+bx^3+cx^6} dx$	801
3.135	$\int \frac{1}{a+bx^3+cx^6} dx$	808
3.136	$\int \frac{1}{x^2(a+bx^3+cx^6)} dx$	816
3.137	$\int \frac{1}{x^3(a+bx^3+cx^6)} dx$	825
3.138	$\int \frac{x^{11}}{3+4x^3+x^6} dx$	835
3.139	$\int \frac{x^8}{3+4x^3+x^6} dx$	839
3.140	$\int \frac{x^5}{3+4x^3+x^6} dx$	843
3.141	$\int \frac{x^2}{3+4x^3+x^6} dx$	846
3.142	$\int \frac{1}{x(3+4x^3+x^6)} dx$	849
3.143	$\int \frac{1}{x^4(3+4x^3+x^6)} dx$	853
3.144	$\int \frac{1}{x^7(3+4x^3+x^6)} dx$	857

3.145	$\int \frac{x^{10}}{3+4x^3+x^6} dx$	861
3.146	$\int \frac{x^9}{3+4x^3+x^6} dx$	866
3.147	$\int \frac{x^7}{3+4x^3+x^6} dx$	871
3.148	$\int \frac{x^6}{3+4x^3+x^6} dx$	876
3.149	$\int \frac{x^4}{3+4x^3+x^6} dx$	881
3.150	$\int \frac{x^3}{3+4x^3+x^6} dx$	886
3.151	$\int \frac{x}{3+4x^3+x^6} dx$	891
3.152	$\int \frac{1}{3+4x^3+x^6} dx$	896
3.153	$\int \frac{1}{x^2(3+4x^3+x^6)} dx$	901
3.154	$\int \frac{1}{x^3(3+4x^3+x^6)} dx$	906
3.155	$\int \frac{1}{x^5(3+4x^3+x^6)} dx$	911
3.156	$\int \frac{1}{x^6(3+4x^3+x^6)} dx$	916
3.157	$\int \frac{x^6}{1-x^3+x^6} dx$	921
3.158	$\int \frac{x^5}{1-x^3+x^6} dx$	927
3.159	$\int \frac{x^4}{1-x^3+x^6} dx$	931
3.160	$\int \frac{x^3}{1-x^3+x^6} dx$	938
3.161	$\int \frac{x^2}{1-x^3+x^6} dx$	944
3.162	$\int \frac{x}{1-x^3+x^6} dx$	948
3.163	$\int \frac{1}{1-x^3+x^6} dx$	955
3.164	$\int \frac{1}{x(1-x^3+x^6)} dx$	961
3.165	$\int \frac{1}{x^2(1-x^3+x^6)} dx$	965
3.166	$\int \frac{1}{x^3(1-x^3+x^6)} dx$	972
3.167	$\int \frac{1}{x^4(1-x^3+x^6)} dx$	978
3.168	$\int \frac{1}{x^5(1-x^3+x^6)} dx$	983
3.169	$\int \frac{1}{2+x^3+x^6} dx$	991
3.170	$\int \frac{x^2}{2+x^3+x^6} dx$	998
3.171	$\int \frac{x^3}{2+x^3+x^6} dx$	1002
3.172	$\int x^{14} \sqrt{a+bx^3+cx^6} dx$	1008

3.173	$\int x^{11} \sqrt{a + bx^3 + cx^6} dx$.1013
3.174	$\int x^8 \sqrt{a + bx^3 + cx^6} dx$.1018
3.175	$\int x^5 \sqrt{a + bx^3 + cx^6} dx$.1023
3.176	$\int x^2 \sqrt{a + bx^3 + cx^6} dx$.1027
3.177	$\int \frac{\sqrt{a+bx^3+cx^6}}{x} dx$.1031
3.178	$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx$.1036
3.179	$\int \frac{\sqrt{a+bx^3+cx^6}}{x^7} dx$.1041
3.180	$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}} dx$.1045
3.181	$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}} dx$.1049
3.182	$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}} dx$.1054
3.183	$\int x^{14} (a + bx^3 + cx^6)^{3/2} dx$.1059
3.184	$\int x^{11} (a + bx^3 + cx^6)^{3/2} dx$.1064
3.185	$\int x^8 (a + bx^3 + cx^6)^{3/2} dx$.1069
3.186	$\int x^5 (a + bx^3 + cx^6)^{3/2} dx$.1074
3.187	$\int x^2 (a + bx^3 + cx^6)^{3/2} dx$.1078
3.188	$\int \frac{(a+bx^3+cx^6)^{3/2}}{x} dx$.1082
3.189	$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4} dx$.1087
3.190	$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7} dx$.1092
3.191	$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}} dx$.1097
3.192	$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}} dx$.1102
3.193	$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}} dx$.1106
3.194	$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}} dx$.1111
3.195	$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}} dx$.1116
3.196	$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{14}} dx$.1121
3.197	$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{11}} dx$.1126
3.198	$\int \frac{\sqrt{a+bx^3+cx^6}}{x^8} dx$.1130

3.199	$\int \frac{x^5}{\sqrt{a+bx^3+cx^6}} dx$1134
3.200	$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}} dx$1138
3.201	$\int \frac{1}{x\sqrt{a+bx^3+cx^6}} dx$1142
3.202	$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}} dx$1146
3.203	$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}} dx$1150
3.204	$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}} dx$1154
3.205	$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}} dx$1159
3.206	$\int \frac{1}{x^{14}(a+bx^3+cx^6)^{3/2}} dx$1164
3.207	$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}} dx$1169
3.208	$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}} dx$1174
3.209	$\int \frac{x^5}{(a+bx^3+cx^6)^{3/2}} dx$1179
3.210	$\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}} dx$1182
3.211	$\int \frac{1}{x(a+bx^3+cx^6)^{3/2}} dx$1185
3.212	$\int \frac{1}{x^4(a+bx^3+cx^6)^{3/2}} dx$1190
3.213	$\int \frac{1}{x^7(a+bx^3+cx^6)^{3/2}} dx$1195
3.214	$\int \frac{1}{x^{10}(a+bx^3+cx^6)^{3/2}} dx$1200
3.215	$\int (dx)^m (a + bx^3 + cx^6)^2 dx$1205
3.216	$\int (dx)^m (a + bx^3 + cx^6) dx$1210
3.217	$\int \frac{x^9}{1+2x^4+x^8} dx$1213
3.218	$\int \frac{x^7}{1+2x^4+x^8} dx$1217
3.219	$\int \frac{x^5}{1+2x^4+x^8} dx$1221
3.220	$\int \frac{x^3}{1+2x^4+x^8} dx$1225
3.221	$\int \frac{x}{1+2x^4+x^8} dx$1228
3.222	$\int \frac{1}{x(1+2x^4+x^8)} dx$1232
3.223	$\int \frac{1}{x^3(1+2x^4+x^8)} dx$1236

3.224	$\int \frac{1}{x^5(1+2x^4+x^8)} dx$.1240
3.225	$\int \frac{1}{x^7(1+2x^4+x^8)} dx$.1244
3.226	$\int \frac{x^8}{1+2x^4+x^8} dx$.1248
3.227	$\int \frac{x^6}{1+2x^4+x^8} dx$.1253
3.228	$\int \frac{x^4}{1+2x^4+x^8} dx$.1258
3.229	$\int \frac{x^2}{1+2x^4+x^8} dx$.1263
3.230	$\int \frac{1}{1+2x^4+x^8} dx$.1268
3.231	$\int \frac{1}{x^2(1+2x^4+x^8)} dx$.1273
3.232	$\int \frac{1}{x^4(1+2x^4+x^8)} dx$.1278
3.233	$\int \frac{1}{x^6(1+2x^4+x^8)} dx$.1283
3.234	$\int \frac{1}{x^8(1+2x^4+x^8)} dx$.1288
3.235	$\int \frac{x^9}{1-2x^4+x^8} dx$.1293
3.236	$\int \frac{x^7}{1-2x^4+x^8} dx$.1297
3.237	$\int \frac{x^5}{1-2x^4+x^8} dx$.1301
3.238	$\int \frac{x^3}{1-2x^4+x^8} dx$.1305
3.239	$\int \frac{x}{1-2x^4+x^8} dx$.1308
3.240	$\int \frac{1}{x(1-2x^4+x^8)} dx$.1312
3.241	$\int \frac{1}{x^3(1-2x^4+x^8)} dx$.1316
3.242	$\int \frac{1}{x^5(1-2x^4+x^8)} dx$.1320
3.243	$\int \frac{1}{x^7(1-2x^4+x^8)} dx$.1324
3.244	$\int \frac{x^8}{1-2x^4+x^8} dx$.1328
3.245	$\int \frac{x^6}{1-2x^4+x^8} dx$.1332
3.246	$\int \frac{x^4}{1-2x^4+x^8} dx$.1336
3.247	$\int \frac{x^2}{1-2x^4+x^8} dx$.1340
3.248	$\int \frac{1}{1-2x^4+x^8} dx$.1344
3.249	$\int \frac{1}{x^2(1-2x^4+x^8)} dx$.1348
3.250	$\int \frac{1}{x^4(1-2x^4+x^8)} dx$.1352

3.251	$\int \frac{1}{x^6(1-2x^4+x^8)} dx$.1356
3.252	$\int \frac{1}{x^8(1-2x^4+x^8)} dx$.1360
3.253	$\int \frac{x^{11}}{a+bx^4+cx^8} dx$.1364
3.254	$\int \frac{x^9}{a+bx^4+cx^8} dx$.1370
3.255	$\int \frac{x^7}{a+bx^4+cx^8} dx$.1378
3.256	$\int \frac{x^5}{a+bx^4+cx^8} dx$.1383
3.257	$\int \frac{x^3}{a+bx^4+cx^8} dx$.1388
3.258	$\int \frac{x}{a+bx^4+cx^8} dx$.1392
3.259	$\int \frac{1}{x(a+bx^4+cx^8)} dx$.1397
3.260	$\int \frac{1}{x^3(a+bx^4+cx^8)} dx$.1402
3.261	$\int \frac{1}{x^5(a+bx^4+cx^8)} dx$.1410
3.262	$\int \frac{x^{10}}{a+bx^4+cx^8} dx$.1419
3.263	$\int \frac{x^8}{a+bx^4+cx^8} dx$.1432
3.264	$\int \frac{x^6}{a+bx^4+cx^8} dx$.1444
3.265	$\int \frac{x^4}{a+bx^4+cx^8} dx$.1454
3.266	$\int \frac{x^2}{a+bx^4+cx^8} dx$.1463
3.267	$\int \frac{1}{a+bx^4+cx^8} dx$.1471
3.268	$\int \frac{1}{x^2(a+bx^4+cx^8)} dx$.1482
3.269	$\int \frac{1}{x^4(a+bx^4+cx^8)} dx$.1494
3.270	$\int \frac{x^{11}}{1+x^4+x^8} dx$.1509
3.271	$\int \frac{x^9}{1+x^4+x^8} dx$.1513
3.272	$\int \frac{x^7}{1+x^4+x^8} dx$.1517
3.273	$\int \frac{x^5}{1+x^4+x^8} dx$.1521
3.274	$\int \frac{x^3}{1+x^4+x^8} dx$.1525
3.275	$\int \frac{x}{1+x^4+x^8} dx$.1529
3.276	$\int \frac{1}{x(1+x^4+x^8)} dx$.1533
3.277	$\int \frac{1}{x^3(1+x^4+x^8)} dx$.1537
3.278	$\int \frac{1}{x^5(1+x^4+x^8)} dx$.1541

3.279	$\int \frac{1}{x^7(1+x^4+x^8)} dx$.1546
3.280	$\int \frac{x^8}{1+x^4+x^8} dx$.1551
3.281	$\int \frac{x^6}{1+x^4+x^8} dx$.1556
3.282	$\int \frac{x^4}{1+x^4+x^8} dx$.1560
3.283	$\int \frac{x^2}{1+x^4+x^8} dx$.1565
3.284	$\int \frac{1}{1+x^4+x^8} dx$.1570
3.285	$\int \frac{1}{x^2(1+x^4+x^8)} dx$.1574
3.286	$\int \frac{1}{x^4(1+x^4+x^8)} dx$.1579
3.287	$\int \frac{1}{x^6(1+x^4+x^8)} dx$.1584
3.288	$\int \frac{1}{x^8(1+x^4+x^8)} dx$.1589
3.289	$\int \frac{x^{11}}{1-x^4+x^8} dx$.1595
3.290	$\int \frac{x^9}{1-x^4+x^8} dx$.1599
3.291	$\int \frac{x^7}{1-x^4+x^8} dx$.1603
3.292	$\int \frac{x^5}{1-x^4+x^8} dx$.1607
3.293	$\int \frac{x^3}{1-x^4+x^8} dx$.1611
3.294	$\int \frac{x}{1-x^4+x^8} dx$.1615
3.295	$\int \frac{1}{x(1-x^4+x^8)} dx$.1619
3.296	$\int \frac{1}{x^3(1-x^4+x^8)} dx$.1623
3.297	$\int \frac{1}{x^5(1-x^4+x^8)} dx$.1627
3.298	$\int \frac{1}{x^7(1-x^4+x^8)} dx$.1632
3.299	$\int \frac{x^8}{1-x^4+x^8} dx$.1637
3.300	$\int \frac{x^6}{1-x^4+x^8} dx$.1643
3.301	$\int \frac{x^4}{1-x^4+x^8} dx$.1648
3.302	$\int \frac{x^2}{1-x^4+x^8} dx$.1654
3.303	$\int \frac{1}{1-x^4+x^8} dx$.1660
3.304	$\int \frac{1}{x^2(1-x^4+x^8)} dx$.1665
3.305	$\int \frac{1}{x^4(1-x^4+x^8)} dx$.1671

3.306	$\int \frac{1}{x^6(1-x^4+x^8)} dx$.1677
3.307	$\int \frac{1}{x^8(1-x^4+x^8)} dx$.1682
3.308	$\int \frac{x^{11}}{1+3x^4+x^8} dx$.1688
3.309	$\int \frac{x^9}{1+3x^4+x^8} dx$.1692
3.310	$\int \frac{x^7}{1+3x^4+x^8} dx$.1696
3.311	$\int \frac{x^5}{1+3x^4+x^8} dx$.1700
3.312	$\int \frac{x^3}{1+3x^4+x^8} dx$.1704
3.313	$\int \frac{x}{1+3x^4+x^8} dx$.1708
3.314	$\int \frac{1}{x(1+3x^4+x^8)} dx$.1712
3.315	$\int \frac{1}{x^3(1+3x^4+x^8)} dx$.1716
3.316	$\int \frac{1}{x^5(1+3x^4+x^8)} dx$.1720
3.317	$\int \frac{1}{x^7(1+3x^4+x^8)} dx$.1724
3.318	$\int \frac{x^8}{1+3x^4+x^8} dx$.1728
3.319	$\int \frac{x^6}{1+3x^4+x^8} dx$.1734
3.320	$\int \frac{x^4}{1+3x^4+x^8} dx$.1739
3.321	$\int \frac{x^2}{1+3x^4+x^8} dx$.1745
3.322	$\int \frac{1}{1+3x^4+x^8} dx$.1751
3.323	$\int \frac{1}{x^2(1+3x^4+x^8)} dx$.1757
3.324	$\int \frac{1}{x^4(1+3x^4+x^8)} dx$.1763
3.325	$\int \frac{x^{11}}{1-3x^4+x^8} dx$.1769
3.326	$\int \frac{x^9}{1-3x^4+x^8} dx$.1773
3.327	$\int \frac{x^7}{1-3x^4+x^8} dx$.1777
3.328	$\int \frac{x^5}{1-3x^4+x^8} dx$.1781
3.329	$\int \frac{x^3}{1-3x^4+x^8} dx$.1785
3.330	$\int \frac{x}{1-3x^4+x^8} dx$.1789
3.331	$\int \frac{1}{x(1-3x^4+x^8)} dx$.1793
3.332	$\int \frac{1}{x^3(1-3x^4+x^8)} dx$.1797
3.333	$\int \frac{1}{x^5(1-3x^4+x^8)} dx$.1801

3.334	$\int \frac{1}{x^7(1-3x^4+x^8)} dx$	1805
3.335	$\int \frac{x^8}{1-3x^4+x^8} dx$	1809
3.336	$\int \frac{x^6}{1-3x^4+x^8} dx$	1814
3.337	$\int \frac{x^4}{1-3x^4+x^8} dx$	1818
3.338	$\int \frac{x^2}{1-3x^4+x^8} dx$	1822
3.339	$\int \frac{1}{1-3x^4+x^8} dx$	1826
3.340	$\int \frac{1}{x^2(1-3x^4+x^8)} dx$	1830
3.341	$\int \frac{1}{x^4(1-3x^4+x^8)} dx$	1835
3.342	$\int \frac{1}{x^6(1-3x^4+x^8)} dx$	1840
3.343	$\int \frac{1}{x^8(1-3x^4+x^8)} dx$	1845
3.344	$\int \frac{x^3}{2+3x^4+x^8} dx$	1851
3.345	$\int \frac{x^{11}}{2+3x^4+x^8} dx$	1854
3.346	$\int \frac{x^9}{2+x^5+x^{10}} dx$	1858
3.347	$\int \frac{x^4}{2+x^5+x^{10}} dx$	1862
3.348	$\int \frac{1}{x(1+x^5+x^{10})} dx$	1866
3.349	$\int \frac{1}{x^6(1+x^5+x^{10})} dx$	1870
3.350	$\int \frac{1}{x+x^6+x^{11}} dx$	1875
3.351	$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}} dx$	1880
3.352	$\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}} dx$	1885
3.353	$\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}} dx$	1890
3.354	$\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}} dx$	1895
3.355	$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x} dx$	1900
3.356	$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2} dx$	1904
3.357	$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3} dx$	1908
3.358	$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4} dx$	1913

3.359	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^5} dx$1918
3.360	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^6} dx$1923
3.361	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x} dx$1928
3.362	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} dx$1934
3.363	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x} dx$1940
3.364	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^2} dx$1946
3.365	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^3} dx$1951
3.366	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^4} dx$1955
3.367	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^5} dx$1959
3.368	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^6} dx$1965
3.369	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^7} dx$1971
3.370	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3} dx$1977
3.371	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x} dx$1985
3.372	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^2} dx$1992
3.373	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^3} dx$1997
3.374	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^4} dx$2002
3.375	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^5} dx$2007
3.376	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^6} dx$2012
3.377	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^7} dx$2017

3.378	$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^8} dx$.2024
3.379	$\int \frac{x^2}{15 + \frac{2}{x^2} + \frac{13}{x}} dx$.2031
3.380	$\int \frac{x}{15 + \frac{2}{x^2} + \frac{13}{x}} dx$.2035
3.381	$\int \frac{1}{15 + \frac{2}{x^2} + \frac{13}{x}} dx$.2039
3.382	$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x} dx$.2043
3.383	$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^2} dx$.2046
3.384	$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^3} dx$.2049
3.385	$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^4} dx$.2053
3.386	$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^5} dx$.2057
3.387	$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^6} dx$.2061
3.388	$\int \left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{5/2} dx$.2065
3.389	$\int \left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2} dx$.2071
3.390	$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} dx$.2076
3.391	$\int \frac{1}{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} dx$.2081
3.392	$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2}} dx$.2085
3.393	$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{5/2}} dx$.2091
3.394	$\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}} dx$.2097
3.395	$\int \frac{1}{c + \frac{a}{x^4} + \frac{b}{x^2}} dx$.2101
3.396	$\int \frac{1}{c + \frac{a}{x^6} + \frac{b}{x^3}} dx$.2108
3.397	$\int \frac{1}{c + \frac{a}{x^8} + \frac{b}{x^4}} dx$.2118
3.398	$\int \frac{\sqrt{a + b\sqrt{x} + cx}}{x} dx$.2130

3.399	$\int \left(\frac{b^2}{4c} + b\sqrt{x} + cx \right)^2 dx \dots\dots\dots$.2135
3.400	$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}} dx \dots\dots\dots$.2139
3.401	$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^{7/2} dx \dots\dots\dots$.2143
3.402	$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^{5/2} dx \dots\dots\dots$.2147
3.403	$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^{3/2} dx \dots\dots\dots$.2151
3.404	$\int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} dx \dots\dots\dots$.2155
3.405	$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} dx \dots\dots\dots$.2159
3.406	$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3} \right)^{3/2}} dx \dots\dots\dots$.2163
3.407	$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3} \right)^{5/2}} dx \dots\dots\dots$.2168
3.408	$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3} \right)^{7/2}} dx \dots\dots\dots$.2172
3.409	$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3} \right)^{9/2}} dx \dots\dots\dots$.2176
3.410	$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3} \right)^{11/2}} dx \dots\dots\dots$.2180
3.411	$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p x^2 dx \dots\dots\dots$.2184
3.412	$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p x dx \dots\dots\dots$.2190
3.413	$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p dx \dots\dots\dots$.2194
3.414	$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3} \right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3} \right)^p}{3a^3x} \right) dx \dots\dots\dots$.2198
3.415	$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x} \right)^{3/2}} dx \dots\dots\dots$.2202
3.416	$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x} \right)^{5/2}} dx \dots\dots\dots$.2207
3.417	$\int \left(a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}} \right)^{3/2} dx \dots\dots\dots$.2212
3.418	$\int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}} \right)^{7/2} dx \dots\dots\dots$.2216
3.419	$\int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}} \right)^{5/2} dx \dots\dots\dots$.2220
3.420	$\int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}} \right)^{3/2} dx \dots\dots\dots$.2225
3.421	$\int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} dx \dots\dots\dots$.2229

3.422	$\int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} dx$	2233
3.423	$\int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}\right)^{3/2}} dx$	2237
3.424	$\int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}\right)^{5/2}} dx$	2242
3.425	$\int \left(a^2 + \frac{b^2}{\sqrt{x}} + \frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx$	2247
3.426	$\int \left(a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx$	2251
3.427	$\int \frac{1}{\left(a^2 + 2ab\sqrt[5]{x} + b^2x^{2/5}\right)^{5/2}} dx$	2255
3.428	$\int \left(a^2 + \frac{b^2}{\sqrt[3]{x}} + \frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx$	2261
3.429	$\int \frac{x^{-1+4n}}{bx^n + cx^{2n}} dx$	2265
3.430	$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}} dx$	2269
3.431	$\int \frac{x^{-1+2n}}{bx^n + cx^{2n}} dx$	2273
3.432	$\int \frac{x^{-1+n}}{bx^n + cx^{2n}} dx$	2276
3.433	$\int \frac{x^{-1-n}}{bx^n + cx^{2n}} dx$	2280
3.434	$\int \frac{x^{-1-2n}}{bx^n + cx^{2n}} dx$	2284
3.435	$\int \frac{x^{-1-3n}}{bx^n + cx^{2n}} dx$	2288
3.436	$\int \frac{x^{-1+\frac{n}{4}}}{bx^n + cx^{2n}} dx$	2292
3.437	$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}} dx$	2297
3.438	$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}} dx$	2302
3.439	$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}} dx$	2306
3.440	$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}} dx$	2310
3.441	$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}} dx$	2315
3.442	$\int x^{-1-n(-1+p)} (bx^n + cx^{2n})^p dx$	2320
3.443	$\int x^{-1-n(1+2p)} (bx^n + cx^{2n})^p dx$	2323
3.444	$\int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^{5/2} dx$	2326
3.445	$\int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$	2330

3.446	$\int x^{-1+2n} \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$.2334
3.447	$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}} dx$.2338
3.448	$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}} dx$.2342
3.449	$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}} dx$.2345
3.450	$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}} dx$.2349
3.451	$\int (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$.2353
3.452	$\int x^2 \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$.2357
3.453	$\int x \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$.2360
3.454	$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$.2363
3.455	$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x} dx$.2366
3.456	$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2} dx$.2370
3.457	$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3} dx$.2374
3.458	$\int (dx)^m (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$.2378
3.459	$\int x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$.2384
3.460	$\int x (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$.2388
3.461	$\int (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$.2392
3.462	$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x} dx$.2396
3.463	$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2} dx$.2400
3.464	$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3} dx$.2404
3.465	$\int \frac{1}{x \sqrt{a^2+2abx^n+b^2x^{2n}}} dx$.2408
3.466	$\int \frac{1}{x(a^2+2abx^n+b^2x^{2n})^{3/2}} dx$.2412
3.467	$\int \left(a^2 + b^2x^{-\frac{2}{1+2p}} + 2abx^{-\frac{1}{1+2p}} \right)^p dx$.2416
3.468	$\int (a^2 + 2abx^n + b^2x^{2n})^{\frac{-1-n}{2n}} dx$.2419
3.469	$\int \left(a^2 + b^2x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}} \right)^p dx$.2422
3.470	$\int (a^2 + 2abx^n + b^2x^{2n})^{\frac{-1-2n}{2n}} dx$.2426
3.471	$\int (dx)^{-1-2n(1+p)} (a^2 + 2abx^n + b^2x^{2n})^p dx$.2430

3.472	$\int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^p dx$.2434
3.473	$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}} dx$.2438
3.474	$\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}} dx$.2443
3.475	$\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}} dx$.2448
3.476	$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}} dx$.2452
3.477	$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}} dx$.2456
3.478	$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}} dx$.2461
3.479	$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}} dx$.2467
3.480	$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}} dx$.2473
3.481	$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}} dx$.2479
3.482	$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}} dx$.2486
3.483	$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}} dx$.2491
3.484	$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}} dx$.2496
3.485	$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}} dx$.2505
3.486	$\int \frac{1}{x(a+bx^n+cx^{2n})} dx$.2513
3.487	$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x} dx$.2518
3.488	$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x} dx$.2523
3.489	$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}} dx$.2528
3.490	$\int \frac{1}{x(a+bx^n+cx^{2n})^{3/2}} dx$.2532
3.491	$\int (dx)^m (a + bx^n + cx^{2n})^3 dx$.2537
3.492	$\int (dx)^m (a + bx^n + cx^{2n})^2 dx$.2544
3.493	$\int (dx)^m (a + bx^n + cx^{2n}) dx$.2552
3.494	$\int (d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4) dx$.2557
3.495	$\int (d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)^2 dx$.2561
3.496	$\int (d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)^3 dx$.2566
3.497	$\int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4) dx$.2572
3.498	$\int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^2 dx$.2576

3.499	$\int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^3 dx$.2581
3.500	$\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4} dx$.2588
3.501	$\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4} dx$.2595
3.502	$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} dx$.2600
3.503	$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4} dx$.2605
3.504	$\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} dx$.2609
3.505	$\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} dx$.2615
3.506	$\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)} dx$.2622
3.507	$\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)} dx$.2630
3.508	$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2638
3.509	$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2648
3.510	$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2653
3.511	$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2663
3.512	$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2668
3.513	$\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2679
3.514	$\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2691
3.515	$\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2704
3.516	$\int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2717
3.517	$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.2733
3.518	$\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.2749
3.519	$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.2757
3.520	$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.2775
3.521	$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.2782

3.522	$\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.2802
3.523	$\int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.2825
3.524	$\int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.2847
3.525	$\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4} dx$.2871
3.526	$\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4} dx$.2878
3.527	$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4} dx$.2883
3.528	$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4} dx$.2888
3.529	$\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)} dx$.2892
3.530	$\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)} dx$.2898
3.531	$\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)} dx$.2905
3.532	$\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)} dx$.2914
3.533	$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2923
3.534	$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2934
3.535	$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2940
3.536	$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2950
3.537	$\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2955
3.538	$\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2968
3.539	$\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2982
3.540	$\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)^2} dx$.2997
3.541	$\int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.3014
3.542	$\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.3032
3.543	$\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.3041
3.544	$\int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$.3061

- 3.545 $\int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^3} dx \dots\dots\dots .3069$
- 3.546 $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^3} dx \dots\dots\dots .3093$
- 3.547 $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^3} dx \dots\dots\dots .3116$
- 3.548 $\int (2+3x)^6 (1+(2+3x)^7+(2+3x)^{14}) dx \dots\dots\dots .3143$
- 3.549 $\int (2+3x)^6 (1+(2+3x)^7+(2+3x)^{14})^2 dx \dots\dots\dots .3147$

3.1 $\int (ax^3 + bx^6)^{5/3} dx$

Optimal. Leaf size=52

$$\frac{(ax^3 + bx^6)^{8/3}}{11bx^5} - \frac{3a(ax^3 + bx^6)^{8/3}}{88b^2x^8}$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2002, 2014}

$$\frac{(ax^3 + bx^6)^{8/3}}{11bx^5} - \frac{3a(ax^3 + bx^6)^{8/3}}{88b^2x^8}$$

Antiderivative was successfully verified.

[In] Int[(a*x^3 + b*x^6)^(5/3), x]

[Out] (-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)/(11*b*x^5)

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]

Rule 2014

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int (ax^3 + bx^6)^{5/3} dx &= \frac{(ax^3 + bx^6)^{8/3}}{11bx^5} - \frac{(3a) \int \frac{(ax^3 + bx^6)^{5/3}}{x^3} dx}{11b} \\ &= -\frac{3a(ax^3 + bx^6)^{8/3}}{88b^2x^8} + \frac{(ax^3 + bx^6)^{8/3}}{11bx^5} \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.81

$$\frac{x(a+bx^3)^3(8bx^3-3a)}{88b^2\sqrt[3]{x^3(a+bx^3)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^3 + b*x^6)^(5/3),x]

[Out] (x*(a + b*x^3)^3*(-3*a + 8*b*x^3))/(88*b^2*(x^3*(a + b*x^3))^(1/3))

IntegrateAlgebraic [A] time = 0.28, size = 57, normalized size = 1.10

$$\frac{(ax^3 + bx^6)^{2/3}(-3a^3 + 2a^2bx^3 + 13ab^2x^6 + 8b^3x^9)}{88b^2x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^3 + b*x^6)^(5/3),x]

[Out] ((a*x^3 + b*x^6)^(2/3)*(-3*a^3 + 2*a^2*b*x^3 + 13*a*b^2*x^6 + 8*b^3*x^9))/(88*b^2*x^2)

fricas [A] time = 1.97, size = 53, normalized size = 1.02

$$\frac{(8b^3x^9 + 13ab^2x^6 + 2a^2bx^3 - 3a^3)(bx^6 + ax^3)^{2/3}}{88b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="fricas")

[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^6 + a*x^3)^(2/3)/(b^2*x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^6 + ax^3)^{5/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="giac")

[Out] integrate((b*x^6 + a*x^3)^(5/3), x)

maple [A] time = 0.01, size = 39, normalized size = 0.75

$$\frac{(bx^3 + a)(-8bx^3 + 3a)(bx^6 + ax^3)^{\frac{5}{3}}}{88b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^6+a*x^3)^(5/3),x)`

[Out] `-1/88*(b*x^3+a)*(-8*b*x^3+3*a)*(b*x^6+a*x^3)^(5/3)/b^2/x^5`

maxima [A] time = 0.44, size = 46, normalized size = 0.88

$$\frac{(8b^3x^9 + 13ab^2x^6 + 2a^2bx^3 - 3a^3)(bx^3 + a)^{\frac{2}{3}}}{88b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^6+a*x^3)^(5/3),x, algorithm="maxima")`

[Out] `1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^3 + a)^(2/3)/b^2`

mupad [B] time = 1.23, size = 40, normalized size = 0.77

$$\frac{(bx^3 + a)^2 (bx^6 + ax^3)^{\frac{2}{3}} (3a - 8bx^3)}{88b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^3 + b*x^6)^(5/3),x)`

[Out] `-((a + b*x^3)^2*(a*x^3 + b*x^6)^(2/3)*(3*a - 8*b*x^3))/(88*b^2*x^2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^3 + bx^6)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**6+a*x**3)**(5/3),x)`

[Out] `Integral((a*x**3 + b*x**6)**(5/3), x)`

$$3.2 \quad \int (ax^3 + bx^6)^{2/3} dx$$

Optimal. Leaf size=25

$$\frac{(ax^3 + bx^6)^{5/3}}{5bx^5}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2000}

$$\frac{(ax^3 + bx^6)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

[In] Int[(a*x^3 + b*x^6)^(2/3), x]

[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\int (ax^3 + bx^6)^{2/3} dx = \frac{(ax^3 + bx^6)^{5/3}}{5bx^5}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{(x^3(a + bx^3))^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^3 + b*x^6)^(2/3), x]

[Out] (x^3*(a + b*x^3))^(5/3)/(5*b*x^5)

IntegrateAlgebraic [A] time = 0.20, size = 25, normalized size = 1.00

$$\frac{(ax^3 + bx^6)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^3 + b*x^6)^(2/3),x]

[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)

fricas [A] time = 1.91, size = 28, normalized size = 1.12

$$\frac{(bx^6 + ax^3)^{\frac{2}{3}}(bx^3 + a)}{5bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="fricas")

[Out] 1/5*(b*x^6 + a*x^3)^(2/3)*(b*x^3 + a)/(b*x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^6 + ax^3)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="giac")

[Out] integrate((b*x^6 + a*x^3)^(2/3), x)

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$\frac{(bx^3 + a)(bx^6 + ax^3)^{\frac{2}{3}}}{5bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^6+a*x^3)^(2/3),x)

[Out] 1/5*(b*x^3+a)/b/x^2*(b*x^6+a*x^3)^(2/3)

maxima [A] time = 0.46, size = 14, normalized size = 0.56

$$\frac{(bx^3 + a)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^6+a*x^3)^(2/3),x, algorithm="maxima")`

[Out] $1/5*(b*x^3 + a)^{5/3}/b$

mupad [B] time = 1.15, size = 29, normalized size = 1.16

$$\frac{\left(\frac{a}{5b} + \frac{x^3}{5}\right) (bx^6 + ax^3)^{2/3}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^3 + b*x^6)^(2/3),x)`

[Out] $((a/(5*b) + x^3/5)*(a*x^3 + b*x^6)^{2/3})/x^2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^3 + bx^6)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**6+a*x**3)**(2/3),x)`

[Out] `Integral((a*x**3 + b*x**6)**(2/3), x)`

$$3.3 \quad \int \frac{1}{(ax^3+bx^6)^{2/3}} dx$$

Optimal. Leaf size=23

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2000}

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Antiderivative was successfully verified.

[In] Int[(a*x^3 + b*x^6)^(-2/3), x]

[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rubi steps

$$\int \frac{1}{(ax^3+bx^6)^{2/3}} dx = -\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{\sqrt[3]{x^3(a+bx^3)}}{ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^3 + b*x^6)^(-2/3), x]

[Out] -((x^3*(a + b*x^3))^(1/3)/(a*x^2))

IntegrateAlgebraic [A] time = 0.41, size = 23, normalized size = 1.00

$$-\frac{\sqrt[3]{ax^3 + bx^6}}{ax^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^3 + b*x^6)^(-2/3), x]

[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))

fricas [A] time = 1.17, size = 21, normalized size = 0.91

$$-\frac{(bx^6 + ax^3)^{\frac{1}{3}}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^6+a*x^3)^(2/3), x, algorithm="fricas")

[Out] -(b*x^6 + a*x^3)^(1/3)/(a*x^2)

giac [A] time = 25.00, size = 14, normalized size = 0.61

$$-\frac{\left(b + \frac{a}{x^3}\right)^{\frac{1}{3}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^6+a*x^3)^(2/3), x, algorithm="giac")

[Out] -(b + a/x^3)^(1/3)/a

maple [A] time = 0.00, size = 27, normalized size = 1.17

$$-\frac{(bx^3 + a)x}{(bx^6 + ax^3)^{\frac{2}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^6+a*x^3)^(2/3), x)

[Out] -x*(b*x^3+a)/a/(b*x^6+a*x^3)^(2/3)

maxima [A] time = 0.45, size = 17, normalized size = 0.74

$$\frac{(bx^3 + a)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="maxima")

[Out] -(b*x^3 + a)^(1/3)/(a*x)

mupad [B] time = 1.15, size = 21, normalized size = 0.91

$$\frac{(bx^6 + ax^3)^{1/3}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^3 + b*x^6)^(2/3),x)

[Out] -(a*x^3 + b*x^6)^(1/3)/(a*x^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^3 + bx^6)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**6+a*x**3)**(2/3),x)

[Out] Integral((a*x**3 + b*x**6)**(-2/3), x)

$$3.4 \quad \int \frac{1}{(ax^3+bx^6)^{5/3}} dx$$

Optimal. Leaf size=77

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2} - \frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5} + \frac{1}{2ax^2(ax^3+bx^6)^{2/3}}$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2001, 2016, 2000}

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2} - \frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5} + \frac{1}{2ax^2(ax^3+bx^6)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^3 + b*x^6)^(-5/3), x]

[Out] 1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) - (3*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^5) + (9*b*(a*x^3 + b*x^6)^(1/3))/(4*a^3*x^2)

Rule 2000

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]

Rule 2001

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j + b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b, j, n}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && LtQ[p, -1]

Rule 2016

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/

(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(ax^3 + bx^6)^{5/3}} dx &= \frac{1}{2ax^2 (ax^3 + bx^6)^{2/3}} + \frac{3 \int \frac{1}{x^3 (ax^3 + bx^6)^{2/3}} dx}{a} \\ &= \frac{1}{2ax^2 (ax^3 + bx^6)^{2/3}} - \frac{3\sqrt[3]{ax^3 + bx^6}}{4a^2x^5} - \frac{(9b) \int \frac{1}{(ax^3 + bx^6)^{2/3}} dx}{4a^2} \\ &= \frac{1}{2ax^2 (ax^3 + bx^6)^{2/3}} - \frac{3\sqrt[3]{ax^3 + bx^6}}{4a^2x^5} + \frac{9b\sqrt[3]{ax^3 + bx^6}}{4a^3x^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.60

$$\frac{-a^2 + 6abx^3 + 9b^2x^6}{4a^3x^2 (x^3 (a + bx^3))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^3 + b*x^6)^(-5/3), x]

[Out] (-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(x^3*(a + b*x^3))^(2/3))

IntegrateAlgebraic [A] time = 0.95, size = 46, normalized size = 0.60

$$\frac{-a^2 + 6abx^3 + 9b^2x^6}{4a^3x^2 (ax^3 + bx^6)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^3 + b*x^6)^(-5/3), x]

[Out] (-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(a*x^3 + b*x^6)^(2/3))

fricas [A] time = 1.06, size = 54, normalized size = 0.70

$$\frac{(9b^2x^6 + 6abx^3 - a^2)(bx^6 + ax^3)^{\frac{1}{3}}}{4(a^3bx^8 + a^4x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="fricas")

[Out] $1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)*(b*x^6 + a*x^3)^{(1/3)}/(a^3*b*x^8 + a^4*x^5)$

giac [A] time = 23.79, size = 52, normalized size = 0.68

$$\frac{b^2}{2a^3\left(b + \frac{a}{x^3}\right)^{\frac{2}{3}}} - \frac{a^9\left(b + \frac{a}{x^3}\right)^{\frac{4}{3}} - 8a^9\left(b + \frac{a}{x^3}\right)^{\frac{1}{3}}b}{4a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="giac")

[Out] $1/2*b^2/(a^3*(b + a/x^3)^{(2/3)}) - 1/4*(a^9*(b + a/x^3)^{(4/3)} - 8*a^9*(b + a/x^3)^{(1/3)*b})/a^{12}$

maple [A] time = 0.00, size = 46, normalized size = 0.60

$$\frac{(bx^3 + a)(-9b^2x^6 - 6bx^3a + a^2)x}{4(bx^6 + ax^3)^{\frac{5}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^6+a*x^3)^(5/3),x)

[Out] $-1/4*x*(b*x^3+a)*(-9*b^2*x^6-6*a*b*x^3+a^2)/a^3/(b*x^6+a*x^3)^{(5/3)}$

maxima [A] time = 0.47, size = 38, normalized size = 0.49

$$\frac{9b^2x^6 + 6abx^3 - a^2}{4(bx^3 + a)^{\frac{2}{3}}a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="maxima")

[Out] $1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)/((b*x^3 + a)^{(2/3)}*a^3*x^4)$

mupad [B] time = 1.28, size = 51, normalized size = 0.66

$$\frac{(bx^6 + ax^3)^{1/3}(-a^2 + 6abx^3 + 9b^2x^6)}{4a^3x^5(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*x^3 + b*x^6)^(5/3), x)`

[Out] $((a*x^3 + b*x^6)^{(1/3)}*(9*b^2*x^6 - a^2 + 6*a*b*x^3))/(4*a^3*x^5*(a + b*x^3))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^3 + bx^6)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**6+a*x**3)**(5/3), x)`

[Out] `Integral((a*x**3 + b*x**6)**(-5/3), x)`

$$3.5 \quad \int \frac{1}{-x^3+x^6} dx$$

Optimal. Leaf size=48

$$\frac{1}{2x^2} - \frac{1}{6} \log(x^2 + x + 1) + \frac{1}{3} \log(1 - x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {1593, 325, 200, 31, 634, 618, 204, 628}

$$\frac{1}{2x^2} - \frac{1}{6} \log(x^2 + x + 1) + \frac{1}{3} \log(1 - x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-x^3 + x^6)^(-1), x]

[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,

b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x^3 + x^6} dx &= \int \frac{1}{x^3(-1 + x^3)} dx \\
&= \frac{1}{2x^2} + \int \frac{1}{-1 + x^3} dx \\
&= \frac{1}{2x^2} + \frac{1}{3} \int \frac{1}{-1 + x} dx + \frac{1}{3} \int \frac{-2 - x}{1 + x + x^2} dx \\
&= \frac{1}{2x^2} + \frac{1}{3} \log(1 - x) - \frac{1}{6} \int \frac{1 + 2x}{1 + x + x^2} dx - \frac{1}{2} \int \frac{1}{1 + x + x^2} dx \\
&= \frac{1}{2x^2} + \frac{1}{3} \log(1 - x) - \frac{1}{6} \log(1 + x + x^2) + \text{Subst}\left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2x\right) \\
&= \frac{1}{2x^2} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3} \log(1 - x) - \frac{1}{6} \log(1 + x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$\frac{1}{2x^2} - \frac{1}{6} \log(x^2 + x + 1) + \frac{1}{3} \log(1 - x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-x^3 + x^6)^(-1), x]

[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-x^3 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-x^3 + x^6)^(-1), x]

[Out] IntegrateAlgebraic[(-x^3 + x^6)^(-1), x]

fricas [A] time = 1.55, size = 46, normalized size = 0.96

$$\frac{2\sqrt{3}x^2 \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x^2 \log(x^2 + x + 1) - 2x^2 \log(x-1) - 3}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-x^3),x, algorithm="fricas")

[Out] $-1/6*(2*\sqrt{3}*x^2*\arctan(1/3*\sqrt{3}*(2*x + 1)) + x^2*\log(x^2 + x + 1) - 2*x^2*\log(x - 1) - 3)/x^2$

giac [A] time = 0.32, size = 38, normalized size = 0.79

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{2x^2} - \frac{1}{6}\log(x^2+x+1) + \frac{1}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-x^3),x, algorithm="giac")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) + 1/2/x^2 - 1/6*\log(x^2 + x + 1) + 1/3*\log(\text{abs}(x - 1))$

maple [A] time = 0.01, size = 38, normalized size = 0.79

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x-1)}{3} - \frac{\ln(x^2+x+1)}{6} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6-x^3),x)

[Out] $-1/6*\ln(x^2+x+1)-1/3*\arctan(1/3*(2*x+1)*3^{(1/2)})*3^{(1/2)}+1/3*\ln(x-1)+1/2/x^2$

maxima [A] time = 0.95, size = 37, normalized size = 0.77

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{2x^2} - \frac{1}{6}\log(x^2+x+1) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-x^3),x, algorithm="maxima")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) + 1/2/x^2 - 1/6*\log(x^2 + x + 1) + 1/3*\log(x - 1)$

mupad [B] time = 0.09, size = 51, normalized size = 1.06

$$\frac{\ln(x-1)}{3} + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} \text{li}}{2}\right) \left(-\frac{1}{6} + \frac{\sqrt{3} \text{li}}{6}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} \text{li}}{2}\right) \left(\frac{1}{6} + \frac{\sqrt{3} \text{li}}{6}\right) + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(x^3 - x^6),x)`

[Out] `log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + 1/(2*x^2)`

sympy [A] time = 0.15, size = 48, normalized size = 1.00

$$\frac{\log(x-1)}{3} - \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**6-x**3),x)`

[Out] `log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + 1/(2*x**2)`

3.6 $\int x^5 \sqrt{a^2 + 2abx^3 + b^2x^6} dx$

Optimal. Leaf size=79

$$\frac{ax^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{6(a + bx^3)} + \frac{bx^9 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{bx^9 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{ax^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{6(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt{a^2 + 2abx^3 + b^2x^6} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^5 (ab + b^2x^3) dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (abx^5 + b^2x^8) dx}{ab + b^2x^3} \\
&= \frac{ax^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{6(a + bx^3)} + \frac{bx^9 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (3ax^6 + 2bx^9)}{18(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))

IntegrateAlgebraic [A] time = 6.28, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (3ax^6 + 2bx^9)}{18(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))

fricas [A] time = 1.34, size = 13, normalized size = 0.16

$$\frac{1}{9}bx^9 + \frac{1}{6}ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/9*b*x^9 + 1/6*a*x^6

giac [A] time = 0.40, size = 23, normalized size = 0.29

$$\frac{1}{18} (2bx^9 + 3ax^6) \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/18*(2*b*x^9 + 3*a*x^6)*sgn(b*x^3 + a)

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{(2bx^3 + 3a) \sqrt{(bx^3 + a)^2} x^6}{18bx^3 + 18a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*((b*x^3+a)^2)^(1/2),x)

[Out] 1/18*x^6*(2*b*x^3+3*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)

maxima [A] time = 0.49, size = 83, normalized size = 1.05

$$-\frac{\sqrt{b^2x^6 + 2abx^3 + a^2} ax^3}{6b} - \frac{\sqrt{b^2x^6 + 2abx^3 + a^2} a^2}{6b^2} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")

[Out] -1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*x^3/b - 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2/b^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/b^2

mupad [B] time = 1.26, size = 59, normalized size = 0.75

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (8b^2 (a^2 + b^2x^6) - 12a^2b^2 + 4ab^3x^3)}{72b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*((a + b*x^3)^2)^(1/2),x)

[Out] ((a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)*(8*b^2*(a^2 + b^2*x^6) - 12*a^2*b^2 + 4*a*b^3*x^3))/(72*b^4)

sympy [A] time = 0.11, size = 12, normalized size = 0.15

$$\frac{ax^6}{6} + \frac{bx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*((b*x**3+a)**2)**(1/2),x)

[Out] a*x**6/6 + b*x**9/9

$$3.7 \quad \int x^4 \sqrt{a^2 + 2abx^3 + b^2x^6} dx$$

Optimal. Leaf size=79

$$\frac{bx^8 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{ax^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{bx^8 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{ax^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^4 \sqrt{a^2 + 2abx^3 + b^2x^6} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^4 (ab + b^2x^3) dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (abx^4 + b^2x^7) dx}{ab + b^2x^3} \\
&= \frac{ax^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{bx^8 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (8ax^5 + 5bx^8)}{40(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))

IntegrateAlgebraic [A] time = 6.45, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (8ax^5 + 5bx^8)}{40(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))

fricas [A] time = 1.33, size = 13, normalized size = 0.16

$$\frac{1}{8}bx^8 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/8*b*x^8 + 1/5*a*x^5

giac [A] time = 0.37, size = 29, normalized size = 0.37

$$\frac{1}{8}bx^8\operatorname{sgn}(bx^3+a) + \frac{1}{5}ax^5\operatorname{sgn}(bx^3+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/8*b*x^8*sgn(b*x^3 + a) + 1/5*a*x^5*sgn(b*x^3 + a)

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{(5bx^3 + 8a)\sqrt{(bx^3 + a)^2}x^5}{40bx^3 + 40a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*((b*x^3+a)^2)^(1/2),x)

[Out] 1/40*x^5*(5*b*x^3+8*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)

maxima [A] time = 0.47, size = 13, normalized size = 0.16

$$\frac{1}{8}bx^8 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/8*b*x^8 + 1/5*a*x^5

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \sqrt{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*((a + b*x^3)^2)^(1/2),x)

[Out] int(x^4*((a + b*x^3)^2)^(1/2), x)

sympy [A] time = 0.11, size = 12, normalized size = 0.15

$$\frac{ax^5}{5} + \frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**8/8
```

3.8 $\int x^3 \sqrt{a^2 + 2abx^3 + b^2x^6} dx$

Optimal. Leaf size=79

$$\frac{bx^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{ax^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{bx^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{ax^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{a^2 + 2abx^3 + b^2x^6} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^3 (ab + b^2x^3) dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (abx^3 + b^2x^6) dx}{ab + b^2x^3} \\
&= \frac{ax^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{bx^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (7ax^4 + 4bx^7)}{28(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))

IntegrateAlgebraic [A] time = 6.46, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (7ax^4 + 4bx^7)}{28(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))

fricas [A] time = 1.67, size = 13, normalized size = 0.16

$$\frac{1}{7}bx^7 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/7*b*x^7 + 1/4*a*x^4

giac [A] time = 0.39, size = 29, normalized size = 0.37

$$\frac{1}{7}bx^7\operatorname{sgn}(bx^3+a) + \frac{1}{4}ax^4\operatorname{sgn}(bx^3+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/7*b*x^7*sgn(b*x^3 + a) + 1/4*a*x^4*sgn(b*x^3 + a)

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{(4bx^3 + 7a)\sqrt{(bx^3 + a)^2}x^4}{28bx^3 + 28a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((b*x^3+a)^2)^(1/2),x)

[Out] 1/28*x^4*(4*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)

maxima [A] time = 0.46, size = 13, normalized size = 0.16

$$\frac{1}{7}bx^7 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/7*b*x^7 + 1/4*a*x^4

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((a + b*x^3)^2)^(1/2),x)

[Out] int(x^3*((a + b*x^3)^2)^(1/2), x)

sympy [A] time = 0.11, size = 12, normalized size = 0.15

$$\frac{ax^4}{4} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**7/7
```

$$3.9 \quad \int x^2 \sqrt{a^2 + 2abx^3 + b^2x^6} dx$$

Optimal. Leaf size=36

$$\frac{(a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}}{6b}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1352, 609}

$$\frac{(a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}}{6b}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] ((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)

Rule 609

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x) * (a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{a^2 + 2abx^3 + b^2x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \sqrt{a^2 + 2abx + b^2x^2} dx, x, x^3 \right) \\ &= \frac{(a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}}{6b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a + bx^3)^2 (2ax^3 + bx^6)}}{6(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(2*a*x^3 + b*x^6))/(6*(a + b*x^3))

IntegrateAlgebraic [A] time = 6.10, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a + bx^3)^2} (2ax^3 + bx^6)}{6(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(2*a*x^3 + b*x^6))/(6*(a + b*x^3))

fricas [A] time = 1.67, size = 13, normalized size = 0.36

$$\frac{1}{6}bx^6 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/6*b*x^6 + 1/3*a*x^3

giac [A] time = 0.36, size = 22, normalized size = 0.61

$$\frac{1}{6}(bx^6 + 2ax^3)\operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/6*(b*x^6 + 2*a*x^3)*sgn(b*x^3 + a)

maple [A] time = 0.00, size = 35, normalized size = 0.97

$$\frac{(bx^3 + 2a)\sqrt{(bx^3 + a)^2}x^3}{6bx^3 + 6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*((b*x^3+a)^2)^(1/2),x)

[Out] $1/6*x^3*(b*x^3+2*a)*((b*x^3+a)^2)^{(1/2)}/(b*x^3+a)$

maxima [B] time = 0.46, size = 52, normalized size = 1.44

$$\frac{1}{6} \sqrt{b^2x^6 + 2abx^3 + a^2} x^3 + \frac{\sqrt{b^2x^6 + 2abx^3 + a^2} a}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $1/6*\text{sqrt}(b^2*x^6 + 2*a*b*x^3 + a^2)*x^3 + 1/6*\text{sqrt}(b^2*x^6 + 2*a*b*x^3 + a^2)*a/b$

mupad [B] time = 1.23, size = 33, normalized size = 0.92

$$\left(\frac{a}{6b} + \frac{x^3}{6}\right) \sqrt{a^2 + 2abx^3 + b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*((a + b*x^3)^2)^(1/2),x)`

[Out] $(a/(6*b) + x^3/6)*(a^2 + b^2*x^6 + 2*a*b*x^3)^{(1/2)}$

sympy [A] time = 0.11, size = 12, normalized size = 0.33

$$\frac{ax^3}{3} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*((b*x**3+a)**2)**(1/2),x)`

[Out] $a*x**3/3 + b*x**6/6$

3.10 $\int x\sqrt{a^2 + 2abx^3 + b^2x^6} dx$

Optimal. Leaf size=79

$$\frac{bx^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{ax^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1355, 14}

$$\frac{bx^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{ax^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_ + (b_)*(x_)^(n_)) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x\sqrt{a^2 + 2abx^3 + b^2x^6} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x(ab + b^2x^3) dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (abx + b^2x^4) dx}{ab + b^2x^3} \\
&= \frac{ax^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{bx^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (5ax^2 + 2bx^5)}{10(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))

IntegrateAlgebraic [A] time = 8.16, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (5ax^2 + 2bx^5)}{10(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))

fricas [A] time = 1.01, size = 13, normalized size = 0.16

$$\frac{1}{5}bx^5 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/5*b*x^5 + 1/2*a*x^2

giac [A] time = 0.33, size = 29, normalized size = 0.37

$$\frac{1}{5}bx^5\operatorname{sgn}(bx^3+a) + \frac{1}{2}ax^2\operatorname{sgn}(bx^3+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/5*b*x^5*sgn(b*x^3 + a) + 1/2*a*x^2*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 36, normalized size = 0.46

$$\frac{(2bx^3 + 5a)\sqrt{(bx^3 + a)^2}x^2}{10bx^3 + 10a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x^3+a)^2)^(1/2),x)

[Out] 1/10*x^2*(2*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)

maxima [A] time = 0.45, size = 13, normalized size = 0.16

$$\frac{1}{5}bx^5 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/5*b*x^5 + 1/2*a*x^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{(bx^3+a)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((a + b*x^3)^2)^(1/2),x)

[Out] int(x*((a + b*x^3)^2)^(1/2), x)

sympy [A] time = 0.10, size = 12, normalized size = 0.15

$$\frac{ax^2}{2} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**5/5
```

3.11 $\int \sqrt{a^2 + 2abx^3 + b^2x^6} dx$

Optimal. Leaf size=74

$$\frac{ax\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{bx^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)}$$

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1343}

$$\frac{bx^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{ax\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] (a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))

Rule 1343

Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^p], x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{a^2 + 2abx^3 + b^2x^6} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (2ab + 2b^2x^3) dx}{2ab + 2b^2x^3} \\ &= \frac{ax\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{bx^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (4ax + bx^4)}{4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))

IntegrateAlgebraic [A] time = 11.16, size = 36, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (4ax + bx^4)}{4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] (Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))

fricas [A] time = 1.06, size = 10, normalized size = 0.14

$$\frac{1}{4}bx^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/4*b*x^4 + a*x

giac [A] time = 0.38, size = 20, normalized size = 0.27

$$\frac{1}{4}(bx^4 + 4ax)\operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/4*(b*x^4 + 4*a*x)*sgn(b*x^3 + a)

maple [A] time = 0.00, size = 33, normalized size = 0.45

$$\frac{(bx^3 + 4a)\sqrt{(bx^3 + a)^2}x}{4bx^3 + 4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2),x)

[Out] $\frac{1}{4}x(bx^3+4a)((bx^3+a)^2)^{1/2}/(bx^3+a)$

maxima [A] time = 0.44, size = 10, normalized size = 0.14

$$\frac{1}{4}bx^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x^3+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{4}bx^4 + ax$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x^3)^2)^(1/2),x)`

[Out] `int(((a + b*x^3)^2)^(1/2), x)`

sympy [A] time = 0.10, size = 8, normalized size = 0.11

$$ax + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x**3+a)**2)**(1/2),x)`

[Out] `a*x + b*x**4/4`

$$3.12 \quad \int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x} dx$$

Optimal. Leaf size=75

$$\frac{bx^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{a \log(x)\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{bx^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{a \log(x)\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]

[Out] (b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab+b^2x^3}{x} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{ab}{x} + b^2x^2\right) dx}{ab + b^2x^3} \\
&= \frac{bx^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6} \log(x)}{a + bx^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.49

$$\frac{\sqrt{(a + bx^3)^2} (3a \log(x) + bx^3)}{3(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x, x]

[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3 + 3*a*Log[x]))/(3*(a + b*x^3))

IntegrateAlgebraic [B] time = 0.24, size = 197, normalized size = 2.63

$$\frac{1}{6}\sqrt{a^2 + 2abx^3 + b^2x^6} + \frac{1}{6}a \log(\sqrt{a^2 + 2abx^3 + b^2x^6} - a - \sqrt{b^2x^3}) - \frac{a(\sqrt{b^2} + b) \log(\sqrt{a^2 + 2abx^3 + b^2x^6} + a - \sqrt{b^2x^3})}{6b} - \frac{a\sqrt{b^2} \log(b\sqrt{a^2 + 2abx^3 + b^2x^6} - ab - b\sqrt{b^2x^3})}{6b} - \frac{1}{6}\sqrt{b^2x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x, x]

[Out] -1/6*(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/6 + (a*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (a*(b + Sqrt[b^2])*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b) - (a*Sqrt[b^2]*Log[-(a*b) - b*Sqrt[b^2]*x^3 + b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b)

fricas [A] time = 1.18, size = 11, normalized size = 0.15

$$\frac{1}{3}bx^3 + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="fricas")

[Out] 1/3*b*x^3 + a*log(x)

giac [A] time = 0.29, size = 28, normalized size = 0.37

$$\frac{1}{3} b x^3 \operatorname{sgn}(b x^3 + a) + a \log(|x|) \operatorname{sgn}(b x^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="giac")

[Out] 1/3*b*x^3*sgn(b*x^3 + a) + a*log(abs(x))*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 34, normalized size = 0.45

$$\frac{\sqrt{(b x^3 + a)^2} (b x^3 + 3 a \ln(x))}{3 b x^3 + 3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x,x)

[Out] 1/3*((b*x^3+a)^2)^(1/2)*(b*x^3+3*a*ln(x))/(b*x^3+a)

maxima [A] time = 0.46, size = 96, normalized size = 1.28

$$\frac{1}{3} (-1)^{2 b^2 x^3 + 2 a b} a \log(2 b^2 x^3 + 2 a b) - \frac{1}{3} (-1)^{2 a b x^3 + 2 a^2} a \log\left(\frac{2 a b x}{|x|} + \frac{2 a^2}{x^2 |x|}\right) + \frac{1}{3} \sqrt{b^2 x^6 + 2 a b x^3 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="maxima")

[Out] 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)

mupad [B] time = 1.38, size = 109, normalized size = 1.45

$$\frac{\sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{3} - \frac{\ln\left(a b + \frac{a^2}{x^3} + \frac{\sqrt{a^2} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{x^3}\right) \sqrt{a^2}}{3} + \frac{a b \ln\left(a b + \sqrt{(b x^3 + a)^2} \sqrt{b^2 + b^2 x^3}\right)}{3 \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^2)^(1/2)/x,x)

[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/3 - (log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)*(a^2)^(1/2))/3 + (a*b*log(a*b + ((a + b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3))/(3*(b^2)^(1/2))

sympy [A] time = 0.13, size = 10, normalized size = 0.13

$$a \log(x) + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x**3/3
```

$$3.13 \quad \int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2} dx$$

Optimal. Leaf size=77

$$\frac{bx^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} - \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{bx^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} - \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]

[Out] -((a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (b*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab + b^2x^3}{x^2} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{ab}{x^2} + b^2x\right) dx}{ab + b^2x^3} \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} + \frac{bx^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.49

$$\frac{(bx^3 - 2a)\sqrt{(a + bx^3)^2}}{2x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]

[Out] ((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))

IntegrateAlgebraic [A] time = 16.23, size = 38, normalized size = 0.49

$$\frac{(bx^3 - 2a)\sqrt{(a + bx^3)^2}}{2x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]

[Out] ((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))

fricas [A] time = 1.07, size = 14, normalized size = 0.18

$$\frac{bx^3 - 2a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="fricas")

[Out] 1/2*(b*x^3 - 2*a)/x

giac [A] time = 0.32, size = 29, normalized size = 0.38

$$\frac{1}{2}bx^2\operatorname{sgn}(bx^3+a) - \frac{a\operatorname{sgn}(bx^3+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="giac")

[Out] 1/2*b*x^2*sgn(b*x^3 + a) - a*sgn(b*x^3 + a)/x

maple [A] time = 0.00, size = 36, normalized size = 0.47

$$\frac{(-bx^3+2a)\sqrt{(bx^3+a)^2}}{2(bx^3+a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x^2,x)

[Out] -1/2*(-b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)

maxima [A] time = 0.46, size = 14, normalized size = 0.18

$$\frac{bx^3-2a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="maxima")

[Out] 1/2*(b*x^3 - 2*a)/x

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(bx^3+a)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^3)^2)^(1/2)/x^2,x)

[Out] int(((a + b*x^3)^2)^(1/2)/x^2, x)

sympy [A] time = 0.13, size = 8, normalized size = 0.10

$$-\frac{a}{x} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a/x + b*x**2/2
```

$$3.14 \quad \int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3} dx$$

Optimal. Leaf size=74

$$\frac{bx\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{bx\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab+b^2x^3}{x^3} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(b^2 + \frac{ab}{x^3}\right) dx}{ab + b^2x^3} \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{bx\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.50

$$-\frac{(a - 2bx^3) \sqrt{(a + bx^3)^2}}{2x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]

[Out] -1/2*((a - 2*b*x^3)*Sqrt[(a + b*x^3)^2])/(x^2*(a + b*x^3))

IntegrateAlgebraic [A] time = 19.04, size = 39, normalized size = 0.53

$$\frac{\sqrt{(a + bx^3)^2} (2bx^3 - a)}{2x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-a + 2*b*x^3))/(2*x^2*(a + b*x^3))

fricas [A] time = 1.53, size = 15, normalized size = 0.20

$$\frac{2bx^3 - a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="fricas")

[Out] 1/2*(2*b*x^3 - a)/x^2

giac [A] time = 0.40, size = 26, normalized size = 0.35

$$bx\operatorname{sgn}(bx^3 + a) - \frac{a\operatorname{sgn}(bx^3 + a)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="giac")

[Out] b*x*sgn(b*x^3 + a) - 1/2*a*sgn(b*x^3 + a)/x^2

maple [A] time = 0.00, size = 34, normalized size = 0.46

$$\frac{(-2bx^3 + a)\sqrt{(bx^3 + a)^2}}{2(bx^3 + a)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x^3,x)

[Out] -1/2*(-2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)

maxima [A] time = 0.46, size = 15, normalized size = 0.20

$$\frac{2bx^3 - a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="maxima")

[Out] 1/2*(2*b*x^3 - a)/x^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(bx^3 + a)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^3)^2)^(1/2)/x^3,x)

[Out] int(((a + b*x^3)^2)^(1/2)/x^3, x)

sympy [A] time = 0.13, size = 8, normalized size = 0.11

$$-\frac{a}{2x^2} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*x
```

$$3.15 \quad \int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4} dx$$

Optimal. Leaf size=75

$$\frac{b \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3 (a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{b \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^4} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{ab + b^2x^3} \int \frac{ab + b^2x^3}{x^4} dx \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{ab + b^2x^3} \int \left(\frac{ab}{x^4} + \frac{b^2}{x} \right) dx \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)} + \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6} \log(x)}{a + bx^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.52

$$-\frac{\sqrt{(a + bx^3)^2} (a - 3bx^3 \log(x))}{3x^3 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4, x]

[Out] -1/3*(Sqrt[(a + b*x^3)^2]*(a - 3*b*x^3*Log[x]))/(x^3*(a + b*x^3))

IntegrateAlgebraic [B] time = 0.61, size = 749, normalized size = 9.99

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4, x]

[Out] ((2*a*b*Sqrt[b^2]*x^3)/3 - (2*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/3 - (2*a*b^2*x^3*ArcTanh[(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/a])/3 - (2*b^3*x^6*ArcTanh[(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/a])/3 + (2*b*Sqrt[b^2]*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*ArcTanh[(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/a])/3)/((-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((2*a^2*Sqrt[b^2])/3 - (a*b*Sqrt[b^2]*x^3*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3 - ((b^2)^(3/2)*x^6*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3 + (b^2*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3 - (a*b*Sqrt[b^2]*x^3*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3 - ((b^2)^(3/2)*x^6*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3 + (b^2*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/3)/((-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]))

$2*a*b*x^3 + b^2*x^6])*(a - \text{Sqrt}[b^2]*x^3 + \text{Sqrt}[a^2 + 2*a*b*x^3 + b^2*x^6])$
 $)$

fricas [A] time = 0.89, size = 17, normalized size = 0.23

$$\frac{3bx^3 \log(x) - a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="fricas")

[Out] 1/3*(3*b*x^3*log(x) - a)/x^3

giac [A] time = 0.35, size = 43, normalized size = 0.57

$$b \log(|x|) \operatorname{sgn}(bx^3 + a) - \frac{bx^3 \operatorname{sgn}(bx^3 + a) + a \operatorname{sgn}(bx^3 + a)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="giac")

[Out] b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^3

maple [A] time = 0.01, size = 38, normalized size = 0.51

$$\frac{\sqrt{(bx^3 + a)^2} (3bx^3 \ln(x) - a)}{3(bx^3 + a)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x^4,x)

[Out] 1/3*((b*x^3+a)^2)^(1/2)*(3*b*ln(x)*x^3-a)/(b*x^3+a)/x^3

maxima [A] time = 0.46, size = 99, normalized size = 1.32

$$\frac{1}{3} (-1)^{2b^2x^3+2ab} b \log(2b^2x^3 + 2ab) - \frac{1}{3} (-1)^{2abx^3+2a^2} b \log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right) - \frac{\sqrt{b^2x^6 + 2abx^3 + a^2}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="maxima")

[Out] $\frac{1}{3}(-1)^{(2b^2x^3 + 2ab)} b \log(2b^2x^3 + 2ab) - \frac{1}{3}(-1)^{(2abx^3 + 2a^2)} b \log(2abx/\text{abs}(x) + 2a^2/(x^2\text{abs}(x))) - \frac{1}{3}\sqrt{b^2x^6 + 2abx^3 + a^2}/x^3$

mupad [B] time = 1.38, size = 112, normalized size = 1.49

$$\frac{\ln\left(ab + \sqrt{(bx^3 + a)^2} \sqrt{b^2 + b^2x^3}\right) \sqrt{b^2}}{3} - \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3} - \frac{ab \ln\left(ab + \frac{a^2}{x^3} + \frac{\sqrt{a^2} \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3}\right)}{3\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x^3)^2)^(1/2)/x^4, x)`

[Out] $(\log(ab + ((a + b^2x^3)^2)^{1/2} * (b^2)^{1/2} + b^2x^3 * (b^2)^{1/2}))/3 - (a^2 + b^2x^6 + 2abx^3)^{1/2}/(3x^3) - (ab \log(ab + a^2/x^3 + ((a^2)^{1/2} * (a^2 + b^2x^6 + 2abx^3)^{1/2})/x^3))/(3 * (a^2)^{1/2})$

sympy [A] time = 0.16, size = 10, normalized size = 0.13

$$-\frac{a}{3x^3} + b \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x**3+a)**2)**(1/2)/x**4, x)`

[Out] `-a/(3*x**3) + b*log(x)`

$$3.16 \quad \int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5} dx$$

Optimal. Leaf size=77

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} - \frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab + b^2x^3}{x^5} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{ab}{x^5} + \frac{b^2}{x^2}\right) dx}{ab + b^2x^3} \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.48

$$-\frac{\sqrt{(a + bx^3)^2} (a + 4bx^3)}{4x^4 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]

[Out] -1/4*(Sqrt[(a + b*x^3)^2]*(a + 4*b*x^3))/(x^4*(a + b*x^3))

IntegrateAlgebraic [A] time = 19.21, size = 39, normalized size = 0.51

$$\frac{(-a - 4bx^3) \sqrt{(a + bx^3)^2}}{4x^4 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]

[Out] ((-a - 4*b*x^3)*Sqrt[(a + b*x^3)^2])/(4*x^4*(a + b*x^3))

fricas [A] time = 1.15, size = 13, normalized size = 0.17

$$-\frac{4bx^3 + a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="fricas")

[Out] -1/4*(4*b*x^3 + a)/x^4

giac [A] time = 0.35, size = 30, normalized size = 0.39

$$-\frac{4bx^3\operatorname{sgn}(bx^3+a)+a\operatorname{sgn}(bx^3+a)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="giac")

[Out] -1/4*(4*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^4

maple [A] time = 0.00, size = 34, normalized size = 0.44

$$-\frac{(4bx^3+a)\sqrt{(bx^3+a)^2}}{4(bx^3+a)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x^5,x)

[Out] -1/4*(4*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)

maxima [A] time = 0.45, size = 13, normalized size = 0.17

$$-\frac{4bx^3+a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="maxima")

[Out] -1/4*(4*b*x^3 + a)/x^4

mupad [B] time = 1.21, size = 33, normalized size = 0.43

$$-\frac{(4bx^3+a)\sqrt{(bx^3+a)^2}}{4x^4(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^3)^2)^(1/2)/x^5,x)

[Out] -((a + 4*b*x^3)*((a + b*x^3)^2)^(1/2))/(4*x^4*(a + b*x^3))

sympy [A] time = 0.17, size = 14, normalized size = 0.18

$$\frac{-a - 4bx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
```

```
[Out] (-a - 4*b*x**3)/(4*x**4)
```

$$3.17 \quad \int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6} dx$$

Optimal. Leaf size=79

$$\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^6} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab+b^2x^3}{x^6} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{ab}{x^6} + \frac{b^2}{x^3}\right) dx}{ab + b^2x^3} \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a + bx^3)^2} (2a + 5bx^3)}{10x^5 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]

[Out] -1/10*(Sqrt[(a + b*x^3)^2]*(2*a + 5*b*x^3))/(x^5*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.83, size = 39, normalized size = 0.49

$$\frac{(-2a - 5bx^3) \sqrt{(a + bx^3)^2}}{10x^5 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]

[Out] ((-2*a - 5*b*x^3)*Sqrt[(a + b*x^3)^2])/(10*x^5*(a + b*x^3))

fricas [A] time = 0.95, size = 15, normalized size = 0.19

$$-\frac{5bx^3 + 2a}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="fricas")

[Out] -1/10*(5*b*x^3 + 2*a)/x^5

giac [A] time = 0.35, size = 31, normalized size = 0.39

$$\frac{5bx^3\operatorname{sgn}(bx^3+a)+2a\operatorname{sgn}(bx^3+a)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="giac")

[Out] -1/10*(5*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^5

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{(5bx^3+2a)\sqrt{(bx^3+a)^2}}{10(bx^3+a)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x^6,x)

[Out] -1/10*(5*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)

maxima [A] time = 0.46, size = 15, normalized size = 0.19

$$\frac{5bx^3+2a}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="maxima")

[Out] -1/10*(5*b*x^3 + 2*a)/x^5

mupad [B] time = 1.18, size = 35, normalized size = 0.44

$$\frac{(5bx^3+2a)\sqrt{(bx^3+a)^2}}{10x^5(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^3)^2)^(1/2)/x^6,x)

[Out] -((2*a + 5*b*x^3)*((a + b*x^3)^2)^(1/2))/(10*x^5*(a + b*x^3))

sympy [A] time = 0.18, size = 15, normalized size = 0.19

$$\frac{-2a - 5bx^3}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**6,x)
```

```
[Out] (-2*a - 5*b*x**3)/(10*x**5)
```

$$3.18 \quad \int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7} dx$$

Optimal. Leaf size=79

$$\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^7} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab + b^2x^3}{x^7} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{ab}{x^7} + \frac{b^2}{x^4}\right) dx}{ab + b^2x^3} \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.47

$$-\frac{\sqrt{(a + bx^3)^2} (a + 2bx^3)}{6x^6 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7, x]

[Out] -1/6*(Sqrt[(a + b*x^3)^2]*(a + 2*b*x^3))/(x^6*(a + b*x^3))

IntegrateAlgebraic [A] time = 0.43, size = 118, normalized size = 1.49

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (-ab - 2b^2x^3) + \sqrt{b^2} (a^2 + 3abx^3 + 2b^2x^6)}{6x^6 (ab + b^2x^3) - 6\sqrt{b^2} x^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7, x]

[Out] ((-(a*b) - 2*b^2*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6] + Sqrt[b^2]*(a^2 + 3*a*b*x^3 + 2*b^2*x^6))/(6*x^6*(a*b + b^2*x^3) - 6*Sqrt[b^2]*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

fricas [A] time = 1.34, size = 13, normalized size = 0.16

$$\frac{2bx^3 + a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="fricas")

[Out] $-1/6*(2*b*x^3 + a)/x^6$

giac [A] time = 0.34, size = 30, normalized size = 0.38

$$\frac{2bx^3 \operatorname{sgn}(bx^3 + a) + a \operatorname{sgn}(bx^3 + a)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="giac")`

[Out] $-1/6*(2*b*x^3*\operatorname{sgn}(b*x^3 + a) + a*\operatorname{sgn}(b*x^3 + a))/x^6$

maple [A] time = 0.00, size = 34, normalized size = 0.43

$$-\frac{(2bx^3 + a)\sqrt{(bx^3 + a)^2}}{6(bx^3 + a)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((b*x^3+a)^2)^(1/2)/x^7,x)`

[Out] $-1/6*(2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)$

maxima [A] time = 0.49, size = 86, normalized size = 1.09

$$\frac{\sqrt{b^2x^6 + 2abx^3 + a^2}b^2}{6a^2} + \frac{\sqrt{b^2x^6 + 2abx^3 + a^2}b}{6ax^3} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="maxima")`

[Out] $1/6*\sqrt{b^2*x^6 + 2*a*b*x^3 + a^2}*b^2/a^2 + 1/6*\sqrt{b^2*x^6 + 2*a*b*x^3 + a^2}*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^6)$

mupad [B] time = 1.18, size = 33, normalized size = 0.42

$$-\frac{(2bx^3 + a)\sqrt{(bx^3 + a)^2}}{6x^6(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x^3)^2)^(1/2)/x^7,x)`

[Out] $-\frac{(a + 2bx^3)(a + bx^3)^2}{6x^6(a + bx^3)}$

sympy [A] time = 0.19, size = 14, normalized size = 0.18

$$\frac{-a - 2bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x**3+a)**2)**(1/2)/x**7,x)

[Out] $(-a - 2bx^3)/(6x^6)$

$$3.19 \quad \int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8} dx$$

Optimal. Leaf size=79

$$\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^8} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab + b^2x^3}{x^8} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{ab}{x^8} + \frac{b^2}{x^5}\right) dx}{ab + b^2x^3} \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a + bx^3)^2} (4a + 7bx^3)}{28x^7(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]

[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a + 7*b*x^3))/(x^7*(a + b*x^3))

IntegrateAlgebraic [A] time = 19.28, size = 39, normalized size = 0.49

$$\frac{(-4a - 7bx^3)\sqrt{(a + bx^3)^2}}{28x^7(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]

[Out] ((-4*a - 7*b*x^3)*Sqrt[(a + b*x^3)^2])/(28*x^7*(a + b*x^3))

fricas [A] time = 1.91, size = 15, normalized size = 0.19

$$-\frac{7bx^3 + 4a}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="fricas")

[Out] -1/28*(7*b*x^3 + 4*a)/x^7

giac [A] time = 0.34, size = 31, normalized size = 0.39

$$-\frac{7bx^3\operatorname{sgn}(bx^3+a)+4a\operatorname{sgn}(bx^3+a)}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="giac")

[Out] -1/28*(7*b*x^3*sgn(b*x^3 + a) + 4*a*sgn(b*x^3 + a))/x^7

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{(7bx^3+4a)\sqrt{(bx^3+a)^2}}{28(bx^3+a)x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x^8,x)

[Out] -1/28*(7*b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)

maxima [A] time = 0.46, size = 15, normalized size = 0.19

$$-\frac{7bx^3+4a}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="maxima")

[Out] -1/28*(7*b*x^3 + 4*a)/x^7

mupad [B] time = 1.27, size = 35, normalized size = 0.44

$$-\frac{(7bx^3+4a)\sqrt{(bx^3+a)^2}}{28x^7(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^3)^2)^(1/2)/x^8,x)

[Out] -((4*a + 7*b*x^3)*((a + b*x^3)^2)^(1/2))/(28*x^7*(a + b*x^3))

sympy [A] time = 0.19, size = 15, normalized size = 0.19

$$\frac{-4a - 7bx^3}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x**3+a)**2)**(1/2)/x**8,x)

[Out] (-4*a - 7*b*x**3)/(28*x**7)

$$3.20 \quad \int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9} dx$$

Optimal. Leaf size=79

$$\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^9} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab + b^2x^3}{x^9} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{ab}{x^9} + \frac{b^2}{x^6}\right) dx}{ab + b^2x^3} \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a + bx^3)^2} (5a + 8bx^3)}{40x^8(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]

[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a + 8*b*x^3))/(x^8*(a + b*x^3))

IntegrateAlgebraic [A] time = 19.24, size = 39, normalized size = 0.49

$$\frac{(-5a - 8bx^3)\sqrt{(a + bx^3)^2}}{40x^8(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]

[Out] ((-5*a - 8*b*x^3)*Sqrt[(a + b*x^3)^2])/(40*x^8*(a + b*x^3))

fricas [A] time = 1.07, size = 15, normalized size = 0.19

$$-\frac{8bx^3 + 5a}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="fricas")

[Out] -1/40*(8*b*x^3 + 5*a)/x^8

giac [A] time = 0.38, size = 31, normalized size = 0.39

$$\frac{8bx^3\operatorname{sgn}(bx^3+a)+5a\operatorname{sgn}(bx^3+a)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="giac")

[Out] -1/40*(8*b*x^3*sgn(b*x^3 + a) + 5*a*sgn(b*x^3 + a))/x^8

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{(8bx^3+5a)\sqrt{(bx^3+a)^2}}{40(bx^3+a)x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x^9,x)

[Out] -1/40*(8*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)

maxima [A] time = 0.45, size = 15, normalized size = 0.19

$$\frac{8bx^3+5a}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="maxima")

[Out] -1/40*(8*b*x^3 + 5*a)/x^8

mupad [B] time = 1.16, size = 35, normalized size = 0.44

$$\frac{(8bx^3+5a)\sqrt{(bx^3+a)^2}}{40x^8(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^3)^2)^(1/2)/x^9,x)

[Out] -((5*a + 8*b*x^3)*((a + b*x^3)^2)^(1/2))/(40*x^8*(a + b*x^3))

sympy [A] time = 0.20, size = 15, normalized size = 0.19

$$\frac{-5a - 8bx^3}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x**3+a)**2)**(1/2)/x**9,x)

[Out] (-5*a - 8*b*x**3)/(40*x**8)

$$3.21 \quad \int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}} dx$$

Optimal. Leaf size=79

$$\frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6(a + bx^3)} - \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3))

Rule 14

```
Int[(u_)*((c_)*(x_)^(m_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab + b^2x^3}{x^{10}} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{ab}{x^{10}} + \frac{b^2}{x^7} \right) dx}{ab + b^2x^3} \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a + bx^3)^2} (2a + 3bx^3)}{18x^9 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]

[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(2*a + 3*b*x^3))/(x^9*(a + b*x^3))

IntegrateAlgebraic [B] time = 3.56, size = 753, normalized size = 9.53

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]

[Out] (2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-2*a^15*b^3 - 55*a^14*b^4*x^3 - 704*a^13*b^5*x^6 - 5563*a^12*b^6*x^9 - 30344*a^11*b^7*x^12 - 121000*a^10*b^8*x^15 - 364320*a^9*b^9*x^18 - 843216*a^8*b^10*x^21 - 1512192*a^7*b^11*x^24 - 2100736*a^6*b^12*x^27 - 2241536*a^5*b^13*x^30 - 1803520*a^4*b^14*x^33 - 1058816*a^3*b^15*x^36 - 428032*a^2*b^16*x^39 - 106496*a*b^17*x^42 - 12288*b^18*x^45) + 2*Sqrt[b^2]*(2*a^16*b^2 + 57*a^15*b^3*x^3 + 759*a^14*b^4*x^6 + 6267*a^13*b^5*x^9 + 35907*a^12*b^6*x^12 + 151344*a^11*b^7*x^15 + 485320*a^10*b^8*x^18 + 1207536*a^9*b^9*x^21 + 2355408*a^8*b^10*x^24 + 3612928*a^7*b^11*x^27 + 4342272*a^6*b^12*x^30 + 4045056*a^5*b^13*x^33 + 2862336*a^4*b^14*x^36 + 1486848*a^3*b^15*x^39 + 534528*a^2*b^16*x^42 + 118784*a*b^17*x^45 + 12288*b^18*x^48))/(9*Sqrt[b^2]*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-4*a^14*b^2 - 104*a^13*b^3*x^3 - 1252*a^12*b^4*x^6 - 9248*a^11*b^5*x^9 - 46816*a^10*b^6*x^12 - 171776*a^9*b^7*x^15 - 470976*a^8*b^8*x^18 - 979968*a^7*b^9*x^21 - 1554432*a^6*b^10*x^24 - 1869824*a^5*b^11*x^27 - 1678336*a^4*b^12*x^30 - 1089536

$*a^3*b^{13}*x^{33} - 483328*a^2*b^{14}*x^{36} - 131072*a*b^{15}*x^{39} - 16384*b^{16}*x^{42} + 9*x^9*(4*a^{15}*b^3 + 108*a^{14}*b^4*x^3 + 1356*a^{13}*b^5*x^6 + 10500*a^{12}*b^6*x^9 + 56064*a^{11}*b^7*x^{12} + 218592*a^{10}*b^8*x^{15} + 642752*a^9*b^9*x^{18} + 1450944*a^8*b^{10}*x^{21} + 2534400*a^7*b^{11}*x^{24} + 3424256*a^6*b^{12}*x^{27} + 3548160*a^5*b^{13}*x^{30} + 2767872*a^4*b^{14}*x^{33} + 1572864*a^3*b^{15}*x^{36} + 614400*a^2*b^{16}*x^{39} + 147456*a*b^{17}*x^{42} + 16384*b^{18}*x^{45}))$

fricas [A] time = 1.37, size = 15, normalized size = 0.19

$$\frac{3bx^3 + 2a}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="fricas")

[Out] -1/18*(3*b*x^3 + 2*a)/x^9

giac [A] time = 0.36, size = 31, normalized size = 0.39

$$\frac{3bx^3\operatorname{sgn}(bx^3 + a) + 2a\operatorname{sgn}(bx^3 + a)}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="giac")

[Out] -1/18*(3*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^9

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$\frac{(3bx^3 + 2a)\sqrt{(bx^3 + a)^2}}{18(bx^3 + a)x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x^10,x)

[Out] -1/18*(3*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)

maxima [B] time = 0.47, size = 117, normalized size = 1.48

$$-\frac{\sqrt{b^2x^6 + 2abx^3 + a^2}b^3}{6a^3} - \frac{\sqrt{b^2x^6 + 2abx^3 + a^2}b^2}{6a^2x^3} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}b}{6a^3x^6} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}}{9a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="maxima")

[Out] $-\frac{1}{6}\sqrt{b^2x^6 + 2abx^3 + a^2} \frac{b^3}{a^3} - \frac{1}{6}\sqrt{b^2x^6 + 2abx^3 + a^2} \frac{b^2}{a^2x^3} + \frac{1}{6}(b^2x^6 + 2abx^3 + a^2)^{3/2} \frac{b}{a^3x^6} - \frac{1}{9}(b^2x^6 + 2abx^3 + a^2)^{3/2} \frac{1}{a^2x^9}$

mupad [B] time = 1.15, size = 35, normalized size = 0.44

$$-\frac{(3bx^3 + 2a)\sqrt{(bx^3 + a)^2}}{18x^9(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^3)^2)^(1/2)/x^10,x)

[Out] $-\frac{(2a + 3bx^3)((a + bx^3)^2)^{1/2}}{18x^9(a + bx^3)}$

sympy [A] time = 0.21, size = 15, normalized size = 0.19

$$\frac{-2a - 3bx^3}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x**3+a)**2)**(1/2)/x**10,x)

[Out] $(-2*a - 3*b*x**3)/(18*x**9)$

$$3.22 \quad \int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{11}} dx$$

Optimal. Leaf size=79

$$\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{11}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{ab + b^2x^3}{x^{11}} dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{ab}{x^{11}} + \frac{b^2}{x^8} \right) dx}{ab + b^2x^3} \\
&= -\frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a + bx^3)^2} (7a + 10bx^3)}{70x^{10}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]

[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a + 10*b*x^3))/(x^10*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.70, size = 39, normalized size = 0.49

$$\frac{(-7a - 10bx^3) \sqrt{(a + bx^3)^2}}{70x^{10}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]

[Out] ((-7*a - 10*b*x^3)*Sqrt[(a + b*x^3)^2])/(70*x^10*(a + b*x^3))

fricas [A] time = 1.40, size = 15, normalized size = 0.19

$$-\frac{10bx^3 + 7a}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="fricas")

[Out] -1/70*(10*b*x^3 + 7*a)/x^10

giac [A] time = 0.31, size = 31, normalized size = 0.39

$$-\frac{10bx^3\operatorname{sgn}(bx^3+a)+7a\operatorname{sgn}(bx^3+a)}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="giac")

[Out] -1/70*(10*b*x^3*sgn(b*x^3 + a) + 7*a*sgn(b*x^3 + a))/x^10

maple [A] time = 0.00, size = 36, normalized size = 0.46

$$-\frac{(10bx^3+7a)\sqrt{(bx^3+a)^2}}{70(bx^3+a)x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^3+a)^2)^(1/2)/x^11,x)

[Out] -1/70*(10*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)

maxima [A] time = 0.45, size = 15, normalized size = 0.19

$$-\frac{10bx^3+7a}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="maxima")

[Out] -1/70*(10*b*x^3 + 7*a)/x^10

mupad [B] time = 1.16, size = 35, normalized size = 0.44

$$-\frac{(10bx^3+7a)\sqrt{(bx^3+a)^2}}{70x^{10}(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x^3)^2)^(1/2)/x^11,x)

[Out] -((7*a + 10*b*x^3)*((a + b*x^3)^2)^(1/2))/(70*x^10*(a + b*x^3))

sympy [A] time = 0.22, size = 15, normalized size = 0.19

$$\frac{-7a - 10bx^3}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x**3+a)**2)**(1/2)/x**11,x)

[Out] (-7*a - 10*b*x**3)/(70*x**10)

$$3.23 \quad \int x^9 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=167

$$\frac{3ab^2x^{16}\sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)} + \frac{3a^2bx^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{b^3x^{19}\sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{a^3x^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^{19}\sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{3ab^2x^{16}\sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)} + \frac{3a^2bx^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{a^3x^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a^2*b*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (3*a*b^2*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^3*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^9 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^9 (ab + b^2x^3)^3 dx}{b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^3b^3x^9 + 3a^2b^4x^{12} + 3ab^5x^{15} + b^6x^{18}) dx}{b^2 (ab + b^2x^3)} \\
&= \frac{a^3x^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)} + \frac{3a^2bx^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{3ab^2x^{16}\sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)} + \frac{b^3x^{19}\sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^{10} \sqrt{(a + bx^3)^2} (1976a^3 + 4560a^2bx^3 + 3705ab^2x^6 + 1040b^3x^9)}{19760(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (x^10*sqrt[(a + b*x^3)^2]*(1976*a^3 + 4560*a^2*b*x^3 + 3705*a*b^2*x^6 + 1040*b^3*x^9))/(19760*(a + b*x^3))

IntegrateAlgebraic [A] time = 11.61, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (1976a^3x^{10} + 4560a^2bx^{13} + 3705ab^2x^{16} + 1040b^3x^{19})}{19760(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (sqrt[(a + b*x^3)^2]*(1976*a^3*x^10 + 4560*a^2*b*x^13 + 3705*a*b^2*x^16 + 1040*b^3*x^19))/(19760*(a + b*x^3))

fricas [A] time = 1.22, size = 35, normalized size = 0.21

$$\frac{1}{19} b^3 x^{19} + \frac{3}{16} ab^2 x^{16} + \frac{3}{13} a^2 b x^{13} + \frac{1}{10} a^3 x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] $1/19*b^3*x^{19} + 3/16*a*b^2*x^{16} + 3/13*a^2*b*x^{13} + 1/10*a^3*x^{10}$

giac [A] time = 0.39, size = 67, normalized size = 0.40

$$\frac{1}{19} b^3 x^{19} \operatorname{sgn}(bx^3 + a) + \frac{3}{16} ab^2 x^{16} \operatorname{sgn}(bx^3 + a) + \frac{3}{13} a^2 b x^{13} \operatorname{sgn}(bx^3 + a) + \frac{1}{10} a^3 x^{10} \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

[Out] $1/19*b^3*x^{19}*sgn(b*x^3 + a) + 3/16*a*b^2*x^{16}*sgn(b*x^3 + a) + 3/13*a^2*b*x^{13}*sgn(b*x^3 + a) + 1/10*a^3*x^{10}*sgn(b*x^3 + a)$

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(1040b^3x^9 + 3705ab^2x^6 + 4560a^2bx^3 + 1976a^3) \left((bx^3 + a)^2 \right)^{\frac{3}{2}} x^{10}}{19760 (bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)`

[Out] $1/19760*x^{10}*(1040*b^3*x^9+3705*a*b^2*x^6+4560*a^2*b*x^3+1976*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3$

maxima [A] time = 0.47, size = 35, normalized size = 0.21

$$\frac{1}{19} b^3 x^{19} + \frac{3}{16} ab^2 x^{16} + \frac{3}{13} a^2 b x^{13} + \frac{1}{10} a^3 x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")`

[Out] $1/19*b^3*x^{19} + 3/16*a*b^2*x^{16} + 3/13*a^2*b*x^{13} + 1/10*a^3*x^{10}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)`

[Out] `int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(x**9*((a + b*x**3)**2)**(3/2), x)

$$3.24 \quad \int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{18b^3} - \frac{2a\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^4}{15b^3} + \frac{a^2\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^3}{12b^3}$$

Rubi [A] time = 0.05, antiderivative size = 167, normalized size of antiderivative = 1.40, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^3x^{18}\sqrt{a^2 + 2abx^3 + b^2x^6}}{18(a + bx^3)} + \frac{ab^2x^{15}\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{a^2bx^{12}\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{a^3x^9\sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^2*b*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (a*b^2*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^18*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*(a + b*x^3))

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^8 (ab + b^2x^3)^3 dx}{b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int x^2 (ab + b^2x)^3 dx, x, x^3\right)}{3b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int (a^3b^3x^2 + 3a^2b^4x^3 + 3ab^5x^4 + b^6x^5) dx, x, x^3\right)}{3b^2 (ab + b^2x^3)} \\
&= \frac{a^3x^9\sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{a^2bx^{12}\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{ab^2x^{15}\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.51

$$\frac{x^9 \sqrt{(a + bx^3)^2} (20a^3 + 45a^2bx^3 + 36ab^2x^6 + 10b^3x^9)}{180(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (x^9*Sqrt[(a + b*x^3)^2]*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9))/(180*(a + b*x^3))

IntegrateAlgebraic [A] time = 10.09, size = 61, normalized size = 0.51

$$\frac{x^9 \sqrt{(a + bx^3)^2} (20a^3 + 45a^2bx^3 + 36ab^2x^6 + 10b^3x^9)}{180(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (x^9*Sqrt[(a + b*x^3)^2]*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9))/(180*(a + b*x^3))

fricas [A] time = 0.76, size = 35, normalized size = 0.29

$$\frac{1}{18} b^3 x^{18} + \frac{1}{5} ab^2 x^{15} + \frac{1}{4} a^2 b x^{12} + \frac{1}{9} a^3 x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] 1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9

giac [A] time = 0.30, size = 67, normalized size = 0.56

$$\frac{1}{18} b^3 x^{18} \operatorname{sgn}(bx^3 + a) + \frac{1}{5} ab^2 x^{15} \operatorname{sgn}(bx^3 + a) + \frac{1}{4} a^2 b x^{12} \operatorname{sgn}(bx^3 + a) + \frac{1}{9} a^3 x^9 \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] 1/18*b^3*x^18*sgn(b*x^3 + a) + 1/5*a*b^2*x^15*sgn(b*x^3 + a) + 1/4*a^2*b*x^12*sgn(b*x^3 + a) + 1/9*a^3*x^9*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 58, normalized size = 0.49

$$\frac{(10b^3x^9 + 36ab^2x^6 + 45a^2bx^3 + 20a^3) \left((bx^3 + a)^2 \right)^{\frac{3}{2}} x^9}{180 (bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)

[Out] 1/180*x^9*(10*b^3*x^9+36*a*b^2*x^6+45*a^2*b*x^3+20*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3

maxima [A] time = 0.46, size = 114, normalized size = 0.96

$$\frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}} a^2 x^3}{12b^2} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}} x^3}{18b^2} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}} a^3}{12b^3} - \frac{7(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}} a}{90b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3/b^2 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3/b^3 - 7/90*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a/b^3

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

[Out] `int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2), x)`

[Out] `Integral(x**8*((a + b*x**3)**2)**(3/2), x)`

$$3.25 \quad \int x^7 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=167

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)} + \frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)} + \frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)} + \frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)} + \frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)} + \frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)} + \frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a^2*b*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (3*a*b^2*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^7 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^7 (ab + b^2x^3)^3 dx}{b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^3b^3x^7 + 3a^2b^4x^{10} + 3ab^5x^{13} + b^6x^{16}) dx}{b^2 (ab + b^2x^3)} \\
&= \frac{a^3x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{3a^2bx^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{3ab^2x^{14}\sqrt{a^2 + 2abx^3 + b^2x^6}}{14(a + bx^3)} + \frac{b^3x^{17}}{17(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^8 \sqrt{(a + bx^3)^2} (1309a^3 + 2856a^2bx^3 + 2244ab^2x^6 + 616b^3x^9)}{10472(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (x^8*Sqrt[(a + b*x^3)^2]*(1309*a^3 + 2856*a^2*b*x^3 + 2244*a*b^2*x^6 + 616*b^3*x^9))/(10472*(a + b*x^3))

IntegrateAlgebraic [A] time = 9.62, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (1309a^3x^8 + 2856a^2bx^{11} + 2244ab^2x^{14} + 616b^3x^{17})}{10472(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (Sqrt[(a + b*x^3)^2]*(1309*a^3*x^8 + 2856*a^2*b*x^11 + 2244*a*b^2*x^14 + 616*b^3*x^17))/(10472*(a + b*x^3))

fricas [A] time = 1.32, size = 35, normalized size = 0.21

$$\frac{1}{17} b^3 x^{17} + \frac{3}{14} ab^2 x^{14} + \frac{3}{11} a^2 b x^{11} + \frac{1}{8} a^3 x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] $1/17*b^3*x^{17} + 3/14*a*b^2*x^{14} + 3/11*a^2*b*x^{11} + 1/8*a^3*x^8$

giac [A] time = 0.29, size = 67, normalized size = 0.40

$$\frac{1}{17} b^3 x^{17} \operatorname{sgn}(bx^3 + a) + \frac{3}{14} ab^2 x^{14} \operatorname{sgn}(bx^3 + a) + \frac{3}{11} a^2 b x^{11} \operatorname{sgn}(bx^3 + a) + \frac{1}{8} a^3 x^8 \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

[Out] $1/17*b^3*x^{17}*\operatorname{sgn}(b*x^3 + a) + 3/14*a*b^2*x^{14}*\operatorname{sgn}(b*x^3 + a) + 3/11*a^2*b*x^{11}*\operatorname{sgn}(b*x^3 + a) + 1/8*a^3*x^8*\operatorname{sgn}(b*x^3 + a)$

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(616b^3x^9 + 2244ab^2x^6 + 2856a^2bx^3 + 1309a^3)\left((bx^3 + a)^2\right)^{\frac{3}{2}}x^8}{10472(bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)`

[Out] $1/10472*x^8*(616*b^3*x^9+2244*a*b^2*x^6+2856*a^2*b*x^3+1309*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3$

maxima [A] time = 0.45, size = 35, normalized size = 0.21

$$\frac{1}{17} b^3 x^{17} + \frac{3}{14} ab^2 x^{14} + \frac{3}{11} a^2 b x^{11} + \frac{1}{8} a^3 x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")`

[Out] $1/17*b^3*x^{17} + 3/14*a*b^2*x^{14} + 3/11*a^2*b*x^{11} + 1/8*a^3*x^8$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)`

[Out] `int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(x**7*((a + b*x**3)**2)**(3/2), x)

$$3.26 \quad \int x^6 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=167

$$\frac{3ab^2x^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{3a^2bx^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)} + \frac{b^3x^{16}\sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)} + \frac{a^3x^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^{16}\sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)} + \frac{3ab^2x^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{3a^2bx^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)} + \frac{a^3x^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a^2*b*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a*b^2*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^6 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^6 (ab + b^2x^3)^3 dx}{b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^3b^3x^6 + 3a^2b^4x^9 + 3ab^5x^{12} + b^6x^{15}) dx}{b^2 (ab + b^2x^3)} \\
&= \frac{a^3x^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{3a^2bx^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)} + \frac{3ab^2x^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{b^3x^{16}\sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^7 \sqrt{(a + bx^3)^2} (1040a^3 + 2184a^2bx^3 + 1680ab^2x^6 + 455b^3x^9)}{7280(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (x^7*Sqrt[(a + b*x^3)^2]*(1040*a^3 + 2184*a^2*b*x^3 + 1680*a*b^2*x^6 + 455*b^3*x^9))/(7280*(a + b*x^3))

IntegrateAlgebraic [A] time = 9.20, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (1040a^3x^7 + 2184a^2bx^{10} + 1680ab^2x^{13} + 455b^3x^{16})}{7280(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (Sqrt[(a + b*x^3)^2]*(1040*a^3*x^7 + 2184*a^2*b*x^10 + 1680*a*b^2*x^13 + 455*b^3*x^16))/(7280*(a + b*x^3))

fricas [A] time = 1.54, size = 35, normalized size = 0.21

$$\frac{1}{16} b^3 x^{16} + \frac{3}{13} ab^2 x^{13} + \frac{3}{10} a^2 b x^{10} + \frac{1}{7} a^3 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] $1/16*b^3*x^{16} + 3/13*a*b^2*x^{13} + 3/10*a^2*b*x^{10} + 1/7*a^3*x^7$

giac [A] time = 0.36, size = 67, normalized size = 0.40

$$\frac{1}{16} b^3 x^{16} \operatorname{sgn}(bx^3 + a) + \frac{3}{13} ab^2 x^{13} \operatorname{sgn}(bx^3 + a) + \frac{3}{10} a^2 b x^{10} \operatorname{sgn}(bx^3 + a) + \frac{1}{7} a^3 x^7 \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

[Out] $1/16*b^3*x^{16}*sgn(b*x^3 + a) + 3/13*a*b^2*x^{13}*sgn(b*x^3 + a) + 3/10*a^2*b*x^{10}*sgn(b*x^3 + a) + 1/7*a^3*x^7*sgn(b*x^3 + a)$

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(455b^3x^9 + 1680ab^2x^6 + 2184a^2bx^3 + 1040a^3) \left((bx^3 + a)^2 \right)^{\frac{3}{2}} x^7}{7280 (bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)`

[Out] $1/7280*x^7*(455*b^3*x^9+1680*a*b^2*x^6+2184*a^2*b*x^3+1040*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3$

maxima [A] time = 0.45, size = 35, normalized size = 0.21

$$\frac{1}{16} b^3 x^{16} + \frac{3}{13} ab^2 x^{13} + \frac{3}{10} a^2 b x^{10} + \frac{1}{7} a^3 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")`

[Out] $1/16*b^3*x^{16} + 3/13*a*b^2*x^{13} + 3/10*a^2*b*x^{10} + 1/7*a^3*x^7$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)`

[Out] `int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(x**6*((a + b*x**3)**2)**(3/2), x)

$$3.27 \quad \int x^5 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=78

$$\frac{(a + bx^3)^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15b^2} - \frac{a(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12b^2}$$

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{(a + bx^3)^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15b^2} - \frac{a(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12b^2}$$

Antiderivative was successfully verified.

[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] -(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*b^2) + ((a + b*x^3)^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^5 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^5 (ab + b^2x^3)^3 dx}{b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst} \left(\int x (ab + b^2x)^3 dx, x, x^3 \right)}{3b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst} \left(\int \left(-\frac{a(ab+b^2x)^3}{b} + \frac{(ab+b^2x)^4}{b^2} \right) dx, x, x^3 \right)}{3b^2 (ab + b^2x^3)} \\
&= -\frac{a (a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12b^2} + \frac{(a + bx^3)^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.78

$$\frac{x^6 \sqrt{(a + bx^3)^2} (10a^3 + 20a^2bx^3 + 15ab^2x^6 + 4b^3x^9)}{60(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (x^6*Sqrt[(a + b*x^3)^2]*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9))/(60*(a + b*x^3))

IntegrateAlgebraic [A] time = 8.62, size = 61, normalized size = 0.78

$$\frac{x^6 \sqrt{(a + bx^3)^2} (10a^3 + 20a^2bx^3 + 15ab^2x^6 + 4b^3x^9)}{60(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (x^6*Sqrt[(a + b*x^3)^2]*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9))/(60*(a + b*x^3))

fricas [A] time = 1.48, size = 35, normalized size = 0.45

$$\frac{1}{15} b^3 x^{15} + \frac{1}{4} ab^2 x^{12} + \frac{1}{3} a^2 b x^9 + \frac{1}{6} a^3 x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] 1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6

giac [A] time = 0.44, size = 45, normalized size = 0.58

$$\frac{1}{60} (4b^3x^{15} + 15ab^2x^{12} + 20a^2bx^9 + 10a^3x^6) \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] 1/60*(4*b^3*x^15 + 15*a*b^2*x^12 + 20*a^2*b*x^9 + 10*a^3*x^6)*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 58, normalized size = 0.74

$$\frac{(4b^3x^9 + 15ab^2x^6 + 20a^2bx^3 + 10a^3) \left((bx^3 + a)^2 \right)^{\frac{3}{2}} x^6}{60 (bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)

[Out] 1/60*x^6*(4*b^3*x^9+15*a*b^2*x^6+20*a^2*b*x^3+10*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3

maxima [A] time = 0.49, size = 83, normalized size = 1.06

$$-\frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}} ax^3}{12b} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}} a^2}{12b^2} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] -1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*x^3/b - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2/b^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/b^2

mupad [B] time = 1.25, size = 46, normalized size = 0.59

$$\frac{(a^2 + 2abx^3 + b^2x^6)^{3/2} (-a^2 + 3abx^3 + 4b^2x^6)}{60b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

[Out] $((a^2 + b^2*x^6 + 2*a*b*x^3)^{(3/2)}*(4*b^2*x^6 - a^2 + 3*a*b*x^3))/(60*b^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2), x)`

[Out] `Integral(x**5*((a + b*x**3)**2)**(3/2), x)`

$$3.28 \quad \int x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=167

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)} + \frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)} + \frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)} + \frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)} + \frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a^2*b*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a*b^2*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^4 (ab + b^2x^3)^3 dx}{b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^3b^3x^4 + 3a^2b^4x^7 + 3ab^5x^{10} + b^6x^{13}) dx}{b^2 (ab + b^2x^3)} \\
&= \frac{a^3x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{3a^2bx^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{3ab^2x^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{b^6x^{14}\sqrt{a^2 + 2abx^3 + b^2x^6}}{14(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^5 \sqrt{(a + bx^3)^2} (616a^3 + 1155a^2bx^3 + 840ab^2x^6 + 220b^3x^9)}{3080(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (x^5*Sqrt[(a + b*x^3)^2]*(616*a^3 + 1155*a^2*b*x^3 + 840*a*b^2*x^6 + 220*b^3*x^9))/(3080*(a + b*x^3))

IntegrateAlgebraic [A] time = 8.31, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (616a^3x^5 + 1155a^2bx^8 + 840ab^2x^{11} + 220b^3x^{14})}{3080(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (Sqrt[(a + b*x^3)^2]*(616*a^3*x^5 + 1155*a^2*b*x^8 + 840*a*b^2*x^11 + 220*b^3*x^14))/(3080*(a + b*x^3))

fricas [A] time = 1.13, size = 35, normalized size = 0.21

$$\frac{1}{14} b^3 x^{14} + \frac{3}{11} ab^2 x^{11} + \frac{3}{8} a^2 b x^8 + \frac{1}{5} a^3 x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] $1/14*b^3*x^{14} + 3/11*a*b^2*x^{11} + 3/8*a^2*b*x^8 + 1/5*a^3*x^5$

giac [A] time = 0.37, size = 67, normalized size = 0.40

$$\frac{1}{14} b^3 x^{14} \operatorname{sgn}(b x^3 + a) + \frac{3}{11} a b^2 x^{11} \operatorname{sgn}(b x^3 + a) + \frac{3}{8} a^2 b x^8 \operatorname{sgn}(b x^3 + a) + \frac{1}{5} a^3 x^5 \operatorname{sgn}(b x^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

[Out] $1/14*b^3*x^{14}*sgn(b*x^3 + a) + 3/11*a*b^2*x^{11}*sgn(b*x^3 + a) + 3/8*a^2*b*x^8*sgn(b*x^3 + a) + 1/5*a^3*x^5*sgn(b*x^3 + a)$

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(220b^3x^9 + 840ab^2x^6 + 1155a^2bx^3 + 616a^3)\left((bx^3 + a)^2\right)^{\frac{3}{2}}x^5}{3080(bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)`

[Out] $1/3080*x^5*(220*b^3*x^9+840*a*b^2*x^6+1155*a^2*b*x^3+616*a^3)*((b*x^3+a)^2)^{3/2}/(b*x^3+a)^3$

maxima [A] time = 0.47, size = 35, normalized size = 0.21

$$\frac{1}{14} b^3 x^{14} + \frac{3}{11} a b^2 x^{11} + \frac{3}{8} a^2 b x^8 + \frac{1}{5} a^3 x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")`

[Out] $1/14*b^3*x^{14} + 3/11*a*b^2*x^{11} + 3/8*a^2*b*x^8 + 1/5*a^3*x^5$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 (a^2 + 2 a b x^3 + b^2 x^6)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)`

[Out] `int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(x**4*((a + b*x**3)**2)**(3/2), x)

$$3.29 \quad \int x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=167

$$\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)} + \frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} + \frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)} + \frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)} + \frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)} + \frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} + \frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (3*a^2*b*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a*b^2*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^3 (ab + b^2x^3)^3 dx}{b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^3b^3x^3 + 3a^2b^4x^6 + 3ab^5x^9 + b^6x^{12}) dx}{b^2 (ab + b^2x^3)} \\
&= \frac{a^3x^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{3a^2bx^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{3ab^2x^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)} + \frac{b^3x^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^4 \sqrt{(a + bx^3)^2} (455a^3 + 780a^2bx^3 + 546ab^2x^6 + 140b^3x^9)}{1820(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (x^4*Sqrt[(a + b*x^3)^2]*(455*a^3 + 780*a^2*b*x^3 + 546*a*b^2*x^6 + 140*b^3*x^9))/(1820*(a + b*x^3))

IntegrateAlgebraic [A] time = 8.24, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (455a^3x^4 + 780a^2bx^7 + 546ab^2x^{10} + 140b^3x^{13})}{1820(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (Sqrt[(a + b*x^3)^2]*(455*a^3*x^4 + 780*a^2*b*x^7 + 546*a*b^2*x^10 + 140*b^3*x^13))/(1820*(a + b*x^3))

fricas [A] time = 1.34, size = 35, normalized size = 0.21

$$\frac{1}{13} b^3 x^{13} + \frac{3}{10} ab^2 x^{10} + \frac{3}{7} a^2 b x^7 + \frac{1}{4} a^3 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] $1/13*b^3*x^{13} + 3/10*a*b^2*x^{10} + 3/7*a^2*b*x^7 + 1/4*a^3*x^4$

giac [A] time = 0.45, size = 67, normalized size = 0.40

$$\frac{1}{13} b^3 x^{13} \operatorname{sgn}(bx^3 + a) + \frac{3}{10} ab^2 x^{10} \operatorname{sgn}(bx^3 + a) + \frac{3}{7} a^2 b x^7 \operatorname{sgn}(bx^3 + a) + \frac{1}{4} a^3 x^4 \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

[Out] $1/13*b^3*x^{13}*\operatorname{sgn}(b*x^3 + a) + 3/10*a*b^2*x^{10}*\operatorname{sgn}(b*x^3 + a) + 3/7*a^2*b*x^7*\operatorname{sgn}(b*x^3 + a) + 1/4*a^3*x^4*\operatorname{sgn}(b*x^3 + a)$

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(140b^3x^9 + 546ab^2x^6 + 780a^2bx^3 + 455a^3)\left((bx^3 + a)^2\right)^{\frac{3}{2}}x^4}{1820(bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)`

[Out] $1/1820*x^4*(140*b^3*x^9+546*a*b^2*x^6+780*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3$

maxima [A] time = 0.48, size = 35, normalized size = 0.21

$$\frac{1}{13} b^3 x^{13} + \frac{3}{10} ab^2 x^{10} + \frac{3}{7} a^2 b x^7 + \frac{1}{4} a^3 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")`

[Out] $1/13*b^3*x^{13} + 3/10*a*b^2*x^{10} + 3/7*a^2*b*x^7 + 1/4*a^3*x^4$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)`

[Out] `int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(x**3*((a + b*x**3)**2)**(3/2), x)

$$3.30 \quad \int x^2 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=36

$$\frac{(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^{3/2}}{12b}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1352, 609}

$$\frac{(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^{3/2}}{12b}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(12*b)

Rule 609

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x) * (a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^2 (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{1}{3} \text{Subst} \left(\int (a^2 + 2abx + b^2x^2)^{3/2} dx, x, x^3 \right) \\ &= \frac{(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^{3/2}}{12b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.67

$$\frac{x^3 \sqrt{(a + bx^3)^2} (4a^3 + 6a^2bx^3 + 4ab^2x^6 + b^3x^9)}{12(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (x^3*sqrt[(a + b*x^3)^2]*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9))/(12*(a + b*x^3))

IntegrateAlgebraic [A] time = 8.02, size = 60, normalized size = 1.67

$$\frac{x^3 \sqrt{(a + bx^3)^2} (4a^3 + 6a^2bx^3 + 4ab^2x^6 + b^3x^9)}{12(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (x^3*sqrt[(a + b*x^3)^2]*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9))/(12*(a + b*x^3))

fricas [A] time = 1.47, size = 35, normalized size = 0.97

$$\frac{1}{12} b^3 x^{12} + \frac{1}{3} ab^2 x^9 + \frac{1}{2} a^2 b x^6 + \frac{1}{3} a^3 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="fricas")

[Out] 1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3

giac [A] time = 0.41, size = 44, normalized size = 1.22

$$\frac{1}{12} \left(2 (bx^6 + 2ax^3)a^2 + (bx^6 + 2ax^3)^2 b \right) \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="giac")

[Out] 1/12*(2*(b*x^6 + 2*a*x^3)*a^2 + (b*x^6 + 2*a*x^3)^2*b)*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 57, normalized size = 1.58

$$\frac{(b^3x^9 + 4ab^2x^6 + 6a^2bx^3 + 4a^3) \left((bx^3 + a)^2 \right)^{\frac{3}{2}} x^3}{12(bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)`

[Out] $1/12*x^3*(b^3*x^9+4*a*b^2*x^6+6*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3$

maxima [A] time = 0.45, size = 52, normalized size = 1.44

$$\frac{1}{12} (b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}x^3 + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}a}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")`

[Out] $1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a/b$

mupad [B] time = 1.22, size = 36, normalized size = 1.00

$$\frac{(b^2x^3 + ab)(a^2 + 2abx^3 + b^2x^6)^{3/2}}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)`

[Out] $((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2))/(12*b^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)`

[Out] `Integral(x**2*((a + b*x**3)**2)**(3/2), x)`

$$3.31 \quad \int x \left(a^2 + 2abx^3 + b^2x^6 \right)^{3/2} dx$$

Optimal. Leaf size=167

$$\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)} + \frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)} + \frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1355, 270}

$$\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)} + \frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)} + \frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a^2*b*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a*b^2*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x(a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x(ab + b^2x^3)^3 dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^3b^3x + 3a^2b^4x^4 + 3ab^5x^7 + b^6x^{10}) dx}{b^2(ab + b^2x^3)} \\
&= \frac{a^3x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{3a^2bx^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{3ab^2x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{x^2\sqrt{(a + bx^3)^2} (220a^3 + 264a^2bx^3 + 165ab^2x^6 + 40b^3x^9)}{440(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (x^2*sqrt[(a + b*x^3)^2]*(220*a^3 + 264*a^2*b*x^3 + 165*a*b^2*x^6 + 40*b^3*x^9))/(440*(a + b*x^3))

IntegrateAlgebraic [A] time = 10.19, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (220a^3x^2 + 264a^2bx^5 + 165ab^2x^8 + 40b^3x^{11})}{440(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]

[Out] (sqrt[(a + b*x^3)^2]*(220*a^3*x^2 + 264*a^2*b*x^5 + 165*a*b^2*x^8 + 40*b^3*x^11))/(440*(a + b*x^3))

fricas [A] time = 1.51, size = 35, normalized size = 0.21

$$\frac{1}{11}b^3x^{11} + \frac{3}{8}ab^2x^8 + \frac{3}{5}a^2bx^5 + \frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] $1/11*b^3*x^{11} + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2$

giac [A] time = 0.37, size = 67, normalized size = 0.40

$$\frac{1}{11} b^3 x^{11} \operatorname{sgn}(bx^3 + a) + \frac{3}{8} ab^2 x^8 \operatorname{sgn}(bx^3 + a) + \frac{3}{5} a^2 b x^5 \operatorname{sgn}(bx^3 + a) + \frac{1}{2} a^3 x^2 \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

[Out] $1/11*b^3*x^{11}*sgn(b*x^3 + a) + 3/8*a*b^2*x^8*sgn(b*x^3 + a) + 3/5*a^2*b*x^5*sgn(b*x^3 + a) + 1/2*a^3*x^2*sgn(b*x^3 + a)$

maple [A] time = 0.00, size = 58, normalized size = 0.35

$$\frac{(40b^3x^9 + 165ab^2x^6 + 264a^2bx^3 + 220a^3) \left((bx^3 + a)^2 \right)^{\frac{3}{2}} x^2}{440 (bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)`

[Out] $1/440*x^2*(40*b^3*x^9+165*a*b^2*x^6+264*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3$

maxima [A] time = 0.44, size = 35, normalized size = 0.21

$$\frac{1}{11} b^3 x^{11} + \frac{3}{8} ab^2 x^8 + \frac{3}{5} a^2 b x^5 + \frac{1}{2} a^3 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")`

[Out] $1/11*b^3*x^{11} + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)`

[Out] `int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(x*((a + b*x**3)**2)**(3/2), x)

$$3.32 \quad \int (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=162

$$\frac{3ab^2x^7(a^2 + 2abx^3 + b^2x^6)^{3/2}}{7(a + bx^3)^3} + \frac{3a^2bx^4(a^2 + 2abx^3 + b^2x^6)^{3/2}}{4(a + bx^3)^3} + \frac{b^3x^{10}(a^2 + 2abx^3 + b^2x^6)^{3/2}}{10(a + bx^3)^3} + \frac{a^3x(a^2 + 2abx^3 + b^2x^6)^{3/2}}{(a + bx^3)^3}$$

Rubi [A] time = 0.03, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1343, 194}

$$\frac{b^3x^{10}(a^2 + 2abx^3 + b^2x^6)^{3/2}}{10(a + bx^3)^3} + \frac{3ab^2x^7(a^2 + 2abx^3 + b^2x^6)^{3/2}}{7(a + bx^3)^3} + \frac{3a^2bx^4(a^2 + 2abx^3 + b^2x^6)^{3/2}}{4(a + bx^3)^3} + \frac{a^3x(a^2 + 2abx^3 + b^2x^6)^{3/2}}{(a + bx^3)^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (a^3*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(a + b*x^3)^3 + (3*a^2*b*x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(4*(a + b*x^3)^3) + (3*a*b^2*x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(7*(a + b*x^3)^3) + (b^3*x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(10*(a + b*x^3)^3)

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 1343

Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2} \int (2ab + 2b^2x^3)^3 dx}{(2ab + 2b^2x^3)^3} \\
&= \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2} \int (8a^3b^3 + 24a^2b^4x^3 + 24ab^5x^6 + 8b^6x^9) dx}{(2ab + 2b^2x^3)^3} \\
&= \frac{a^3x(a^2 + 2abx^3 + b^2x^6)^{3/2}}{(a + bx^3)^3} + \frac{3a^2bx^4(a^2 + 2abx^3 + b^2x^6)^{3/2}}{4(a + bx^3)^3} + \frac{3ab^2x^7(a^2 + 2abx^3 + b^2x^6)^{3/2}}{7(a + bx^3)^3} + \frac{b^3x^{10}(a^2 + 2abx^3 + b^2x^6)^{3/2}}{10(a + bx^3)^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.36

$$\frac{x\sqrt{(a + bx^3)^2} (140a^3 + 105a^2bx^3 + 60ab^2x^6 + 14b^3x^9)}{140(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (x*Sqrt[(a + b*x^3)^2]*(140*a^3 + 105*a^2*b*x^3 + 60*a*b^2*x^6 + 14*b^3*x^9))/(140*(a + b*x^3))

IntegrateAlgebraic [A] time = 12.40, size = 59, normalized size = 0.36

$$\frac{\sqrt{(a + bx^3)^2} (140a^3x + 105a^2bx^4 + 60ab^2x^7 + 14b^3x^{10})}{140(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (Sqrt[(a + b*x^3)^2]*(140*a^3*x + 105*a^2*b*x^4 + 60*a*b^2*x^7 + 14*b^3*x^10))/(140*(a + b*x^3))

fricas [A] time = 3.03, size = 32, normalized size = 0.20

$$\frac{1}{10} b^3 x^{10} + \frac{3}{7} ab^2 x^7 + \frac{3}{4} a^2 b x^4 + a^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x

giac [A] time = 0.36, size = 64, normalized size = 0.40

$$\frac{1}{10} b^3 x^{10} \operatorname{sgn}(b x^3 + a) + \frac{3}{7} a b^2 x^7 \operatorname{sgn}(b x^3 + a) + \frac{3}{4} a^2 b x^4 \operatorname{sgn}(b x^3 + a) + a^3 x \operatorname{sgn}(b x^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] 1/10*b^3*x^10*sgn(b*x^3 + a) + 3/7*a*b^2*x^7*sgn(b*x^3 + a) + 3/4*a^2*b*x^4*sgn(b*x^3 + a) + a^3*x*sgn(b*x^3 + a)

maple [A] time = 0.00, size = 56, normalized size = 0.35

$$\frac{(14b^3x^9 + 60ab^2x^6 + 105a^2bx^3 + 140a^3) \left((bx^3 + a)^2 \right)^{\frac{3}{2}} x}{140(bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)

[Out] 1/140*x*(14*b^3*x^9+60*a*b^2*x^6+105*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/2)/(b*x^3+a)^3

maxima [A] time = 0.71, size = 32, normalized size = 0.20

$$\frac{1}{10} b^3 x^{10} + \frac{3}{7} a b^2 x^7 + \frac{3}{4} a^2 b x^4 + a^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^2 + 2abx^3 + b^2x^6)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2), x)`

[Out] `Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2), x)`

$$3.33 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x} dx$$

Optimal. Leaf size=160

$$\frac{ab^2x^6\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{a^2bx^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{b^3x^9\sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{a^3\log(x)\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Rubi [A] time = 0.05, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^3x^9\sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{ab^2x^6\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{a^2bx^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{a^3\log(x)\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]

[Out] (a^2*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^2*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^3}{x} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^3}{x} dx, x, x^3\right)}{3b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \left(3a^2b^4 + \frac{a^3b^3}{x} + 3ab^5x + b^6x^2\right) dx, x, x^3\right)}{3b^2(ab + b^2x^3)} \\
&= \frac{a^2bx^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{ab^2x^6\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{b^3x^9\sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.38

$$\frac{\sqrt{(a + bx^3)^2} (18a^3 \log(x) + bx^3 (18a^2 + 9abx^3 + 2b^2x^6))}{18(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]

[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(18*a^2 + 9*a*b*x^3 + 2*b^2*x^6) + 18*a^3*Log[x]))/(18*(a + b*x^3))

IntegrateAlgebraic [A] time = 0.44, size = 256, normalized size = 1.60

$$\frac{1}{36}\sqrt{a^2 + 2abx^3 + b^2x^6} (11a^2 + 7abx^3 + 2b^2x^6) + \frac{1}{36}(-18a^2\sqrt{b^2x^3} - 9ab\sqrt{b^2x^6} - 2(b^2)^{3/2}x^9) + \frac{1}{6}a^3 \log(\sqrt{a^2 + 2abx^3 + b^2x^6} - a - \sqrt{b^2x^3}) - \frac{a^3(\sqrt{b^2} + b) \log(\sqrt{a^2 + 2abx^3 + b^2x^6} + a - \sqrt{b^2x^3})}{6b} - \frac{a^3\sqrt{b^2} \log(b\sqrt{a^2 + 2abx^3 + b^2x^6} - ab - b\sqrt{b^2x^3})}{6b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]

[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(11*a^2 + 7*a*b*x^3 + 2*b^2*x^6))/36 + (-18*a^2*Sqrt[b^2]*x^3 - 9*a*b*Sqrt[b^2]*x^6 - 2*(b^2)^(3/2)*x^9)/36 + (a^3*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (a^3*(b + Sqrt[b^2])*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b) - (a^3*Sqrt[b^2]*Log[-(a*b) - b*Sqrt[b^2]*x^3 + b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b)

fricas [A] time = 1.09, size = 32, normalized size = 0.20

$$\frac{1}{9} b^3 x^9 + \frac{1}{2} a b^2 x^6 + a^2 b x^3 + a^3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="fricas")

[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)

giac [A] time = 0.43, size = 65, normalized size = 0.41

$$\frac{1}{9} b^3 x^9 \operatorname{sgn}(b x^3 + a) + \frac{1}{2} a b^2 x^6 \operatorname{sgn}(b x^3 + a) + a^2 b x^3 \operatorname{sgn}(b x^3 + a) + a^3 \log(|x|) \operatorname{sgn}(b x^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="giac")

[Out] 1/9*b^3*x^9*sgn(b*x^3 + a) + 1/2*a*b^2*x^6*sgn(b*x^3 + a) + a^2*b*x^3*sgn(b*x^3 + a) + a^3*log(abs(x))*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 57, normalized size = 0.36

$$\frac{\left((b x^3 + a)^2 \right)^{\frac{3}{2}} \left(2 b^3 x^9 + 9 a b^2 x^6 + 18 a^2 b x^3 + 18 a^3 \ln(x) \right)}{18 (b x^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x)

[Out] 1/18*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+9*a*b^2*x^6+18*a^2*b*x^3+18*a^3*ln(x))/(b*x^3+a)^3

maxima [A] time = 0.52, size = 152, normalized size = 0.95

$$\frac{1}{6} \sqrt{b^2 x^6 + 2 a b x^3 + a^2} a b x^3 + \frac{1}{3} (-1)^{2 b^2 x^3 + 2 a b} a^3 \log(2 b^2 x^3 + 2 a b) - \frac{1}{3} (-1)^{2 a b x^3 + 2 a^2} a^3 \log\left(\frac{2 a b x}{|x|} + \frac{2 a^2}{x^2 |x|}\right) + \frac{1}{2} \sqrt{b^2 x^6 + 2 a b x^3 + a^2} a^2 + \frac{1}{9} (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="maxima")

[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*log(2*a*b*x/a

$\text{bs}(x) + 2*a^2/(x^2*\text{abs}(x))) + 1/2*\text{sqrt}(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2 + 1/9$
 $*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)`

[Out] `Integral(((a + b*x**3)**2)**(3/2)/x, x)`

$$3.34 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2} dx$$

Optimal. Leaf size=165

$$\frac{3ab^2x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{3a^2bx^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{b^3x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{3ab^2x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{3a^2bx^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]

[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (3*a^2*b*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a*b^2*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p]))], Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^3}{x^2} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^3b^3}{x^2} + 3a^2b^4x + 3ab^5x^4 + b^6x^7 \right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} + \frac{3a^2bx^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{3ab^2x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-40a^3 + 60a^2bx^3 + 24ab^2x^6 + 5b^3x^9)}{40x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/(40*x*(a + b*x^3))

IntegrateAlgebraic [A] time = 17.20, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-40a^3 + 60a^2bx^3 + 24ab^2x^6 + 5b^3x^9)}{40x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/(40*x*(a + b*x^3))

fricas [A] time = 1.38, size = 37, normalized size = 0.22

$$\frac{5b^3x^9 + 24ab^2x^6 + 60a^2bx^3 - 40a^3}{40x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x

giac [A] time = 0.34, size = 67, normalized size = 0.41

$$\frac{1}{8} b^3 x^8 \operatorname{sgn}(b x^3 + a) + \frac{3}{5} a b^2 x^5 \operatorname{sgn}(b x^3 + a) + \frac{3}{2} a^2 b x^2 \operatorname{sgn}(b x^3 + a) - \frac{a^3 \operatorname{sgn}(b x^3 + a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="giac")

[Out] 1/8*b^3*x^8*sgn(b*x^3 + a) + 3/5*a*b^2*x^5*sgn(b*x^3 + a) + 3/2*a^2*b*x^2*sgn(b*x^3 + a) - a^3*sgn(b*x^3 + a)/x

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(-5b^3x^9 - 24ab^2x^6 - 60a^2bx^3 + 40a^3) \left((bx^3 + a)^2 \right)^{\frac{3}{2}}}{40(bx^3 + a)^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x)

[Out] -1/40*(-5*b^3*x^9-24*a*b^2*x^6-60*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)/x/(b*x^3+a)^3

maxima [A] time = 0.50, size = 37, normalized size = 0.22

$$\frac{5b^3x^9 + 24ab^2x^6 + 60a^2bx^3 - 40a^3}{40x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="maxima")

[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)`

[Out] `Integral(((a + b*x**3)**2)**(3/2)/x**2, x)`

$$3.35 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} dx$$

Optimal. Leaf size=163

$$\frac{3a^2bx\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{3ab^2x^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{b^3x^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{3ab^2x^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{3a^2bx\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]

[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a^2*b*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (3*a*b^2*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^3}{x^3} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(3a^2b^4 + \frac{a^3b^3}{x^3} + 3ab^5x^3 + b^6x^6\right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{3a^2bx\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{3ab^2x^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-14a^3 + 84a^2bx^3 + 21ab^2x^6 + 4b^3x^9)}{28x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/(28*x^2*(a + b*x^3))

IntegrateAlgebraic [A] time = 19.88, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-14a^3 + 84a^2bx^3 + 21ab^2x^6 + 4b^3x^9)}{28x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/(28*x^2*(a + b*x^3))

fricas [A] time = 1.47, size = 37, normalized size = 0.23

$$\frac{4b^3x^9 + 21ab^2x^6 + 84a^2bx^3 - 14a^3}{28x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="fricas")

[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2

giac [A] time = 0.38, size = 65, normalized size = 0.40

$$\frac{1}{7} b^3 x^7 \operatorname{sgn}(b x^3 + a) + \frac{3}{4} a b^2 x^4 \operatorname{sgn}(b x^3 + a) + 3 a^2 b x \operatorname{sgn}(b x^3 + a) - \frac{a^3 \operatorname{sgn}(b x^3 + a)}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="giac")

[Out] 1/7*b^3*x^7*sgn(b*x^3 + a) + 3/4*a*b^2*x^4*sgn(b*x^3 + a) + 3*a^2*b*x*sgn(b*x^3 + a) - 1/2*a^3*sgn(b*x^3 + a)/x^2

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$\frac{(-4b^3x^9 - 21ab^2x^6 - 84a^2bx^3 + 14a^3) \left((bx^3 + a)^2 \right)^{\frac{3}{2}}}{28(bx^3 + a)^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x)

[Out] -1/28*(-4*b^3*x^9-21*a*b^2*x^6-84*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/x^2/(b*x^3+a)^3

maxima [A] time = 0.50, size = 37, normalized size = 0.23

$$\frac{4b^3x^9 + 21ab^2x^6 + 84a^2bx^3 - 14a^3}{28x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="maxima")

[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)`

[Out] `Integral(((a + b*x**3)**2)**(3/2)/x**3, x)`

$$3.36 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^4} dx$$

Optimal. Leaf size=161

$$\frac{ab^2x^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{3a^2b \log(x)\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{b^3x^6\sqrt{a^2 + 2abx^3 + b^2x^6}}{6(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)}$$

Rubi [A] time = 0.05, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^3x^6\sqrt{a^2 + 2abx^3 + b^2x^6}}{6(a + bx^3)} + \frac{ab^2x^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)} + \frac{3a^2b \log(x)\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4, x]

[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (a*b^2*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^4} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^3}{x^4} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^3}{x^2} dx, x, x^3\right)}{3b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \left(3ab^5 + \frac{a^3b^3}{x^2} + \frac{3a^2b^4}{x} + b^6x\right) dx, x, x^3\right)}{3b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)} + \frac{ab^2x^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{b^3x^6\sqrt{a^2 + 2abx^3 + b^2x^6}}{6(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.39

$$\frac{\sqrt{(a + bx^3)^2} (-2a^3 + 18a^2bx^3 \log(x) + 6ab^2x^6 + b^3x^9)}{6x^3(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4, x]

[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^3 + 6*a*b^2*x^6 + b^3*x^9 + 18*a^2*b*x^3*Log[x]))/(6*x^3*(a + b*x^3))

IntegrateAlgebraic [A] time = 0.74, size = 317, normalized size = 1.97

$$\frac{1}{2}a^2\sqrt{b^2}\log(\sqrt{a^2 + 2abx^3 + b^2x^6} - a - \sqrt{b^2}x^3) - \frac{1}{2}a^2\sqrt{b^2}\log(\sqrt{a^2 + 2abx^3 + b^2x^6} + a - \sqrt{b^2}x^3) + a^2b \operatorname{tanh}^{-1}\left(\frac{\sqrt{b^2}x^3}{a} - \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{a}\right) + \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}(-8a^3b - 21a^2b^2x^3 + 24ab^3x^6 + 4b^4x^9) + \sqrt{b^2}(8a^4 + 29a^3bx^3 - 3a^2b^2x^6 - 28ab^3x^9 - 4b^4x^{12})}{24x^3(ab + b^2x^3) - 24\sqrt{b^2}x^3\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4, x]

[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^3*b - 21*a^2*b^2*x^3 + 24*a*b^3*x^6 + 4*b^4*x^9) + Sqrt[b^2]*(8*a^4 + 29*a^3*b*x^3 - 3*a^2*b^2*x^6 - 28*a*b^3*x^9 - 4*b^4*x^12))/(24*x^3*(a*b + b^2*x^3) - 24*Sqrt[b^2]*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + a^2*b*ArcTanh[(Sqrt[b^2]*x^3)/a - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/a] - (a^2*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])

$\sqrt[3]{b^2 x^6} \sqrt{a^2 + 2 a b x^3 + b^2 x^6} \log(a - \sqrt{b^2} x^3 + \sqrt{a^2 + 2 a b x^3 + b^2 x^6}) / 2 - (a^2 \sqrt{b^2} \log[a - \sqrt{b^2} x^3 + \sqrt{a^2 + 2 a b x^3 + b^2 x^6}]) / 2$

fricas [A] time = 1.17, size = 38, normalized size = 0.24

$$\frac{b^3 x^9 + 6 a b^2 x^6 + 18 a^2 b x^3 \log(x) - 2 a^3}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="fricas")

[Out] 1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3

giac [A] time = 0.37, size = 85, normalized size = 0.53

$$\frac{1}{6} b^3 x^6 \operatorname{sgn}(b x^3 + a) + a b^2 x^3 \operatorname{sgn}(b x^3 + a) + 3 a^2 b \log(|x|) \operatorname{sgn}(b x^3 + a) - \frac{3 a^2 b x^3 \operatorname{sgn}(b x^3 + a) + a^3 \operatorname{sgn}(b x^3 + a)}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="giac")

[Out] 1/6*b^3*x^6*sgn(b*x^3 + a) + a*b^2*x^3*sgn(b*x^3 + a) + 3*a^2*b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(3*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^3

maple [A] time = 0.01, size = 59, normalized size = 0.37

$$\frac{\left((b x^3 + a)^2 \right)^{\frac{3}{2}} (b^3 x^9 + 6 a b^2 x^6 + 18 a^2 b x^3 \ln(x) - 2 a^3)}{6 (b x^3 + a)^3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x)

[Out] 1/6*((b*x^3+a)^2)^(3/2)*(b^3*x^9+6*a*b^2*x^6+18*a^2*b*ln(x)*x^3-2*a^3)/(b*x^3+a)^3/x^3

maxima [A] time = 0.53, size = 156, normalized size = 0.97

$$\frac{1}{2} \sqrt{b^2 x^6 + 2 a b x^3 + a^2} b^2 x^3 + (-1)^{2 b^2 x^3 + 2 a b} a^2 b \log(2 b^2 x^3 + 2 a b) - (-1)^{2 a b x^3 + 2 a^2} a^2 b \log\left(\frac{2 a b x}{|x|} + \frac{2 a^2}{x^2 |x|}\right) + \frac{3}{2} \sqrt{b^2 x^6 + 2 a b x^3 + a^2} a b - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{3}{2}}}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="maxima")

[Out] $\frac{1}{2}\sqrt{b^2x^6 + 2abx^3 + a^2}b^2x^3 + (-1)^{(2b^2x^3 + 2ab)}a^2b \log(2b^2x^3 + 2ab) - (-1)^{(2abx^3 + 2a^2)}a^2b \log(2abx/abs(x) + 2a^2/(x^2abs(x))) + \frac{3}{2}\sqrt{b^2x^6 + 2abx^3 + a^2}ab - \frac{1}{3}(b^2x^6 + 2abx^3 + a^2)^{(3/2)}/x^3$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4, x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4, x)`

[Out] `Integral(((a + b*x**3)**2)**(3/2)/x**4, x)`

$$3.37 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5} dx$$

Optimal. Leaf size=165

$$\frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} + \frac{3ab^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{b^3x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{3ab^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (3*a*b^2*x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^5} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^3}{x^5} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^3b^3}{x^5} + \frac{3a^2b^4}{x^2} + 3ab^5x + b^6x^4 \right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} + \frac{3ab^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-5a^3 - 60a^2bx^3 + 30ab^2x^6 + 4b^3x^9)}{20x^4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(20*x^4*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.33, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-5a^3 - 60a^2bx^3 + 30ab^2x^6 + 4b^3x^9)}{20x^4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(20*x^4*(a + b*x^3))

fricas [A] time = 1.15, size = 37, normalized size = 0.22

$$\frac{4b^3x^9 + 30ab^2x^6 - 60a^2bx^3 - 5a^3}{20x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="fricas")

[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4

giac [A] time = 0.36, size = 69, normalized size = 0.42

$$\frac{1}{5} b^3 x^5 \operatorname{sgn}(b x^3 + a) + \frac{3}{2} a b^2 x^2 \operatorname{sgn}(b x^3 + a) - \frac{12 a^2 b x^3 \operatorname{sgn}(b x^3 + a) + a^3 \operatorname{sgn}(b x^3 + a)}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="giac")

[Out] 1/5*b^3*x^5*sgn(b*x^3 + a) + 3/2*a*b^2*x^2*sgn(b*x^3 + a) - 1/4*(12*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^4

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$-\frac{(-4b^3x^9 - 30ab^2x^6 + 60a^2bx^3 + 5a^3)\left((bx^3 + a)^2\right)^{\frac{3}{2}}}{20(bx^3 + a)^3 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x)

[Out] -1/20*(-4*b^3*x^9-30*a*b^2*x^6+60*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^4/(b*x^3+a)^3

maxima [A] time = 0.53, size = 37, normalized size = 0.22

$$\frac{4 b^3 x^9 + 30 a b^2 x^6 - 60 a^2 b x^3 - 5 a^3}{20 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="maxima")

[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2 a b x^3 + b^2 x^6)^{3/2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)`

[Out] `Integral(((a + b*x**3)**2)**(3/2)/x**5, x)`

$$3.38 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^6} dx$$

Optimal. Leaf size=163

$$\frac{3ab^2x\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{b^3x^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^3x^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{3ab^2x\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6, x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a*b^2*x*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^6} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^3}{x^6} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(3ab^5 + \frac{a^3b^3}{x^6} + \frac{3a^2b^4}{x^3} + b^6x^3\right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{3ab^2x\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-4a^3 - 30a^2bx^3 + 60ab^2x^6 + 5b^3x^9)}{20x^5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(20*x^5*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-4a^3 - 30a^2bx^3 + 60ab^2x^6 + 5b^3x^9)}{20x^5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(20*x^5*(a + b*x^3))

fricas [A] time = 1.42, size = 37, normalized size = 0.23

$$\frac{5b^3x^9 + 60ab^2x^6 - 30a^2bx^3 - 4a^3}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="fricas")

[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5

giac [A] time = 0.29, size = 68, normalized size = 0.42

$$\frac{1}{4} b^3 x^4 \operatorname{sgn}(b x^3 + a) + 3 a b^2 x \operatorname{sgn}(b x^3 + a) - \frac{15 a^2 b x^3 \operatorname{sgn}(b x^3 + a) + 2 a^3 \operatorname{sgn}(b x^3 + a)}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="giac")

[Out] 1/4*b^3*x^4*sgn(b*x^3 + a) + 3*a*b^2*x*sgn(b*x^3 + a) - 1/10*(15*a^2*b*x^3*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^5

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$-\frac{\left(-5b^3x^9 - 60ab^2x^6 + 30a^2bx^3 + 4a^3\right)\left((bx^3 + a)^2\right)^{\frac{3}{2}}}{20(bx^3 + a)^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x)

[Out] -1/20*(-5*b^3*x^9-60*a*b^2*x^6+30*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^5/(b*x^3+a)^3

maxima [A] time = 0.50, size = 37, normalized size = 0.23

$$\frac{5b^3x^9 + 60ab^2x^6 - 30a^2bx^3 - 4a^3}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="maxima")

[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)`

[Out] `Integral(((a + b*x**3)**2)**(3/2)/x**6, x)`

$$3.39 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^7} dx$$

Optimal. Leaf size=162

$$\frac{a^2 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3 (a + bx^3)} + \frac{3ab^2 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{b^3 x^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} - \frac{a^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6 (a + bx^3)}$$

Rubi [A] time = 0.05, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$-\frac{a^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6 (a + bx^3)} - \frac{a^2 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3 (a + bx^3)} + \frac{b^3 x^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{3ab^2 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7, x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*x^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (3*a*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^7} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^3}{x^7} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^3}{x^3} dx, x, x^3\right)}{3b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \left(b^6 + \frac{a^3b^3}{x^3} + \frac{3a^2b^4}{x^2} + \frac{3ab^5}{x}\right) dx, x, x^3\right)}{3b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6(a + bx^3)} - \frac{a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3(a + bx^3)} + \frac{b^3x^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \dots
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.38

$$-\frac{\sqrt{(a + bx^3)^2} (a^3 + 6a^2bx^3 - 18ab^2x^6 \log(x) - 2b^3x^9)}{6x^6(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7, x]

[Out] -1/6*(Sqrt[(a + b*x^3)^2]*(a^3 + 6*a^2*b*x^3 - 2*b^3*x^9 - 18*a*b^2*x^6*Log[x]))/(x^6*(a + b*x^3))

IntegrateAlgebraic [B] time = 1.68, size = 1164, normalized size = 7.19

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7, x]

[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^3*b) - 6*a^2*b^2*x^3 + a*b^3*x^6 + 2*b^4*x^9) + Sqrt[b^2]*(a^4 + 7*a^3*b*x^3 + 5*a^2*b^2*x^6 - 3*a*b^3*x^9 - 2*b^4*x^12))/(6*x^6*(a*b + b^2*x^3) - 6*Sqrt[b^2]*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a*b^2*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/2 - (a*b*Sqrt[b^2]*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/2 - (a^5*b^2*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])

$$\begin{aligned} & / (2 * (-a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6])^2 * (a - \text{Sqrt}[b^2] \\ & * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6])^2) - (a^5 * b * \text{Sqrt}[b^2] * \text{Log}[a - \text{Sqrt}[\\ & b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6]]) / (2 * (-a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[\\ & a^2 + 2 * a * b * x^3 + b^2 * x^6])^2 * (a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b \\ & ^2 * x^6])^2) + (a^3 * b^2 * (-\text{Sqrt}[b^2] * x^3) + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6]) \\ & ^2 * \text{Log}[a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6]]) / ((-a - \text{Sqrt}[b^ \\ & 2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6])^2 * (a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + \\ & 2 * a * b * x^3 + b^2 * x^6])^2) + (a^3 * b * \text{Sqrt}[b^2] * (-\text{Sqrt}[b^2] * x^3) + \text{Sqrt}[a^2 + \\ & 2 * a * b * x^3 + b^2 * x^6])^2 * \text{Log}[a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 \\ & * x^6]]) / ((-a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6])^2 * (a - \text{Sqrt} \\ & [b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6])^2) - (a * b^2 * (-\text{Sqrt}[b^2] * x^3) \\ & + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6])^4 * \text{Log}[a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a \\ & * b * x^3 + b^2 * x^6]]) / (2 * (-a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6 \\ &]) ^2 * (a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6])^2) - (a * b * \text{Sqrt}[b \\ & ^2] * (-\text{Sqrt}[b^2] * x^3) + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6])^4 * \text{Log}[a - \text{Sqrt}[b^2 \\ &] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6]]) / (2 * (-a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 \\ & + 2 * a * b * x^3 + b^2 * x^6])^2 * (a - \text{Sqrt}[b^2] * x^3 + \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * \\ & x^6])^2) \end{aligned}$$

fricas [A] time = 1.30, size = 39, normalized size = 0.24

$$\frac{2b^3x^9 + 18ab^2x^6 \log(x) - 6a^2bx^3 - a^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="fricas")

[Out] 1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6

giac [A] time = 0.34, size = 86, normalized size = 0.53

$$\frac{1}{3} b^3 x^3 \text{sgn}(bx^3 + a) + 3ab^2 \log(|x|) \text{sgn}(bx^3 + a) - \frac{9ab^2x^6 \text{sgn}(bx^3 + a) + 6a^2bx^3 \text{sgn}(bx^3 + a) + a^3 \text{sgn}(bx^3 + a)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="giac")

[Out] 1/3*b^3*x^3*sgn(b*x^3 + a) + 3*a*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(9*a*b^2*x^6*sgn(b*x^3 + a) + 6*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^6

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left((bx^3 + a)^2\right)^{\frac{3}{2}} \left(2b^3x^9 + 18ab^2x^6 \ln(x) - 6a^2bx^3 - a^3\right)}{6(bx^3 + a)^3 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x)`

[Out] $1/6*((b*x^3+a)^2)^{(3/2)}*(2*b^3*x^9+18*a*b^2*\ln(x)*x^6-6*a^2*b*x^3-a^3)/(b*x^3+a)^3/x^6$

maxima [A] time = 0.70, size = 220, normalized size = 1.36

$$\frac{\sqrt{b^2x^6+2abx^3+a^2}b^3x^3}{2a} + (-1)^{2i^2+3+2ab} ab^2 \log(2b^2x^3+2ab) - (-1)^{2abx^3+2a^2} ab^2 \log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right) + \frac{3}{2} \sqrt{b^2x^6+2abx^3+a^2} b^2 + \frac{(b^2x^6+2abx^3+a^2)^{\frac{3}{2}} b^2}{6a^2} - \frac{(b^2x^6+2abx^3+a^2)^{\frac{3}{2}} b}{6ax^3} - \frac{(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="maxima")`

[Out] $1/2*\sqrt{b^2*x^6 + 2*a*b*x^3 + a^2}*b^3*x^3/a + (-1)^{(2*b^2*x^3 + 2*a*b)}*a*b^2*\log(2*b^2*x^3 + 2*a*b) - (-1)^{(2*a*b*x^3 + 2*a^2)}*a*b^2*\log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 3/2*\sqrt{b^2*x^6 + 2*a*b*x^3 + a^2}*b^2 + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^{(3/2)}*b^2/a^2 - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^{(3/2)}*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^{(5/2)}/(a^2*x^6)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)`

[Out] `Integral(((a + b*x**3)**2)**(3/2)/x**7, x)`

$$3.40 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8} dx$$

Optimal. Leaf size=165

$$\frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} + \frac{b^3x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} + \frac{b^3x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8, x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^3*x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^8} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^3}{x^8} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^3b^3}{x^8} + \frac{3a^2b^4}{x^5} + \frac{3ab^5}{x^2} + b^6x \right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a + bx^3)^2} (4a^3 + 21a^2bx^3 + 84ab^2x^6 - 14b^3x^9)}{28x^7(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]

[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 21*a^2*b*x^3 + 84*a*b^2*x^6 - 14*b^3*x^9))/(x^7*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.10, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-4a^3 - 21a^2bx^3 - 84ab^2x^6 + 14b^3x^9)}{28x^7(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 21*a^2*b*x^3 - 84*a*b^2*x^6 + 14*b^3*x^9))/(28*x^7*(a + b*x^3))

fricas [A] time = 1.92, size = 37, normalized size = 0.22

$$\frac{14b^3x^9 - 84ab^2x^6 - 21a^2bx^3 - 4a^3}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="fricas")

[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7

giac [A] time = 0.47, size = 70, normalized size = 0.42

$$\frac{1}{2} b^3 x^2 \operatorname{sgn}(b x^3 + a) - \frac{84 a b^2 x^6 \operatorname{sgn}(b x^3 + a) + 21 a^2 b x^3 \operatorname{sgn}(b x^3 + a) + 4 a^3 \operatorname{sgn}(b x^3 + a)}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="giac")

[Out] 1/2*b^3*x^2*sgn(b*x^3 + a) - 1/28*(84*a*b^2*x^6*sgn(b*x^3 + a) + 21*a^2*b*x^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^7

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(-14 b^3 x^9 + 84 a b^2 x^6 + 21 a^2 b x^3 + 4 a^3) \left((b x^3 + a)^2 \right)^{\frac{3}{2}}}{28 (b x^3 + a)^3 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x)

[Out] -1/28*(-14*b^3*x^9+84*a*b^2*x^6+21*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^7/(b*x^3+a)^3

maxima [A] time = 0.74, size = 37, normalized size = 0.22

$$\frac{14 b^3 x^9 - 84 a b^2 x^6 - 21 a^2 b x^3 - 4 a^3}{28 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="maxima")

[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2 a b x^3 + b^2 x^6)^{3/2}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)`

[Out] `Integral(((a + b*x**3)**2)**(3/2)/x**8, x)`

$$3.41 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^9} dx$$

Optimal. Leaf size=162

$$\frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{b^3x\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{b^3x\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9, x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^3*x*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p]))], Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^9} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^3}{x^9} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(b^6 + \frac{a^3b^3}{x^9} + \frac{3a^2b^4}{x^6} + \frac{3ab^5}{x^3}\right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.38

$$-\frac{\sqrt{(a + bx^3)^2} (5a^3 + 24a^2bx^3 + 60ab^2x^6 - 40b^3x^9)}{40x^8(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9, x]

[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a^3 + 24*a^2*b*x^3 + 60*a*b^2*x^6 - 40*b^3*x^9))/(x^8*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.49, size = 61, normalized size = 0.38

$$\frac{\sqrt{(a + bx^3)^2} (-5a^3 - 24a^2bx^3 - 60ab^2x^6 + 40b^3x^9)}{40x^8(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9, x]

[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 24*a^2*b*x^3 - 60*a*b^2*x^6 + 40*b^3*x^9))/(40*x^8*(a + b*x^3))

fricas [A] time = 1.26, size = 37, normalized size = 0.23

$$\frac{40b^3x^9 - 60ab^2x^6 - 24a^2bx^3 - 5a^3}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="fricas")

[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8

giac [A] time = 0.39, size = 67, normalized size = 0.41

$$b^3 x \operatorname{sgn}(bx^3 + a) - \frac{60 ab^2 x^6 \operatorname{sgn}(bx^3 + a) + 24 a^2 b x^3 \operatorname{sgn}(bx^3 + a) + 5 a^3 \operatorname{sgn}(bx^3 + a)}{40 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="giac")

[Out] b^3*x*sgn(b*x^3 + a) - 1/40*(60*a*b^2*x^6*sgn(b*x^3 + a) + 24*a^2*b*x^3*sgn(b*x^3 + a) + 5*a^3*sgn(b*x^3 + a))/x^8

maple [A] time = 0.01, size = 58, normalized size = 0.36

$$\frac{(-40b^3x^9 + 60ab^2x^6 + 24a^2bx^3 + 5a^3)\left((bx^3 + a)^2\right)^{\frac{3}{2}}}{40(bx^3 + a)^3 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x)

[Out] -1/40*(-40*b^3*x^9+60*a*b^2*x^6+24*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^8/(b*x^3+a)^3

maxima [A] time = 0.68, size = 37, normalized size = 0.23

$$\frac{40 b^3 x^9 - 60 a b^2 x^6 - 24 a^2 b x^3 - 5 a^3}{40 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="maxima")

[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2 a b x^3 + b^2 x^6)^{3/2}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**9,x)`

[Out] `Integral(((a + b*x**3)**2)**(3/2)/x**9, x)`

$$3.42 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{10}} dx$$

Optimal. Leaf size=161

$$\frac{a^2 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^6 (a + bx^3)} - \frac{ab^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3 (a + bx^3)} + \frac{b^3 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9 (a + bx^3)}$$

Rubi [A] time = 0.05, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{a^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9 (a + bx^3)} - \frac{a^2 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^6 (a + bx^3)} - \frac{ab^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3 (a + bx^3)} + \frac{b^3 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^6*(a + b*x^3)) - (a*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{10}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^3}{x^{10}} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^4} dx, x, x^3\right)}{3b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int \left(\frac{a^3b^3}{x^4} + \frac{3a^2b^4}{x^3} + \frac{3ab^5}{x^2} + \frac{b^6}{x}\right) dx, x, x^3\right)}{3b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9(a + bx^3)} - \frac{a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^6(a + bx^3)} - \frac{ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3(a + bx^3)} +
\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.39

$$-\frac{\sqrt{(a + bx^3)^2} (a(2a^2 + 9abx^3 + 18b^2x^6) - 18b^3x^9 \log(x))}{18x^9(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]

[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(a*(2*a^2 + 9*a*b*x^3 + 18*b^2*x^6) - 18*b^3*x^9*Log[x]))/(x^9*(a + b*x^3))

IntegrateAlgebraic [A] time = 1.17, size = 293, normalized size = 1.82

$$-\frac{1}{6}(b^2)^{3/2} \log(\sqrt{a^2 + 2abx^3 + b^2x^6} - a - \sqrt{b^2x^3}) - \frac{1}{6}(b^2)^{3/2} \log(\sqrt{a^2 + 2abx^3 + b^2x^6} + a - \sqrt{b^2x^3}) + \frac{1}{3}b^3 \tanh^{-1}\left(\frac{\sqrt{b^2x^3}}{a} - \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{a}\right) + \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6}(-2a^2b - 9ab^2x^3 - 18b^3x^6) + a\sqrt{b^2}(2a^3 + 11a^2bx^3 + 27ab^2x^6 + 18b^3x^9)}{18x^9(ab + b^2x^3) - 18\sqrt{b^2x^3}\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]

[Out] (a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-2*a^2*b - 9*a*b^2*x^3 - 18*b^3*x^6) + a*Sqrt[b^2]*(2*a^3 + 11*a^2*b*x^3 + 27*a*b^2*x^6 + 18*b^3*x^9))/(18*x^9*(a + b + b^2*x^3) - 18*Sqrt[b^2]*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^3*ArcTanh[(Sqrt[b^2]*x^3)/a - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/a])/3 - ((b^2)^(3/2)*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - ((b^2)^(3/2)*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6

fricas [A] time = 1.31, size = 39, normalized size = 0.24

$$\frac{18 b^3 x^9 \log(x) - 18 a b^2 x^6 - 9 a^2 b x^3 - 2 a^3}{18 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="fricas")

[Out] 1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9

giac [A] time = 0.38, size = 85, normalized size = 0.53

$$b^3 \log(|x|) \operatorname{sgn}(bx^3 + a) - \frac{11 b^3 x^9 \operatorname{sgn}(bx^3 + a) + 18 a b^2 x^6 \operatorname{sgn}(bx^3 + a) + 9 a^2 b x^3 \operatorname{sgn}(bx^3 + a) + 2 a^3 \operatorname{sgn}(bx^3 + a)}{18 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="giac")

[Out] b^3*log(abs(x))*sgn(b*x^3 + a) - 1/18*(11*b^3*x^9*sgn(b*x^3 + a) + 18*a*b^2*x^6*sgn(b*x^3 + a) + 9*a^2*b*x^3*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^9

maple [A] time = 0.01, size = 60, normalized size = 0.37

$$\frac{\left((b x^3 + a)^2 \right)^{\frac{3}{2}} \left(18 b^3 x^9 \ln(x) - 18 a b^2 x^6 - 9 a^2 b x^3 - 2 a^3 \right)}{18 (b x^3 + a)^3 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x)

[Out] 1/18*((b*x^3+a)^2)^(3/2)*(18*b^3*ln(x)*x^9-18*a*b^2*x^6-9*a^2*b*x^3-2*a^3)/(b*x^3+a)^3/x^9

maxima [B] time = 0.72, size = 253, normalized size = 1.57

$$\frac{\sqrt{b^2 x^6 + 2 a b x^3 + a^2} b^4 x^3}{6 a^2} + \frac{1}{3} (-1)^{2 \sqrt{b^2 x^3 + 2 a b}} b^3 \log(2 b^2 x^3 + 2 a b) - \frac{1}{3} (-1)^{2 a b x^3 + 2 a^2} b^2 \log\left(\frac{2 a b x}{|x|} + \frac{2 a^2}{x^2 |x|}\right) + \frac{\sqrt{b^2 x^6 + 2 a b x^3 + a^2} b^3}{2 a} - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{3}{2}} b^3}{18 a^3} - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{3}{2}} b^2}{6 a^2 x^3} + \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} b}{18 a^3 x^6} - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}}}{9 a^2 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="maxima")

[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*b^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^3*log(2*a*b

$\frac{x}{\text{abs}(x) + 2a^2/(x^2\text{abs}(x))} + \frac{1}{2}\sqrt{b^2x^6 + 2abx^3 + a^2} \frac{b^3/a}{x^10} - \frac{1}{18}(b^2x^6 + 2abx^3 + a^2)^{3/2} \frac{b^3/a^3}{x^10} - \frac{1}{6}(b^2x^6 + 2abx^3 + a^2)^{3/2} \frac{b^2/a^2x^3}{x^10} + \frac{1}{18}(b^2x^6 + 2abx^3 + a^2)^{5/2} \frac{b/a^3x^6}{x^10} - \frac{1}{9}(b^2x^6 + 2abx^3 + a^2)^{5/2} \frac{1/a^2x^9}{x^10}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10, x)

[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10, x)

[Out] Integral(((a + b*x**3)**2)**(3/2)/x**10, x)

$$3.43 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{11}} dx$$

Optimal. Leaf size=165

$$\frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11, x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p]))], Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{11}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^3}{x^{11}} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^3b^3}{x^{11}} + \frac{3a^2b^4}{x^8} + \frac{3ab^5}{x^5} + \frac{b^6}{x^2} \right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (14a^3 + 60a^2bx^3 + 105ab^2x^6 + 140b^3x^9)}{140x^{10}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]

[Out] -1/140*(Sqrt[(a + b*x^3)^2]*(14*a^3 + 60*a^2*b*x^3 + 105*a*b^2*x^6 + 140*b^3*x^9))/(x^10*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.14, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-14a^3 - 60a^2bx^3 - 105ab^2x^6 - 140b^3x^9)}{140x^{10}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 - 60*a^2*b*x^3 - 105*a*b^2*x^6 - 140*b^3*x^9))/(140*x^10*(a + b*x^3))

fricas [A] time = 1.52, size = 37, normalized size = 0.22

$$-\frac{140b^3x^9 + 105ab^2x^6 + 60a^2bx^3 + 14a^3}{140x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="fricas")

[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10

giac [A] time = 0.36, size = 69, normalized size = 0.42

$$\frac{140 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 105 a b^2 x^6 \operatorname{sgn}(b x^3 + a) + 60 a^2 b x^3 \operatorname{sgn}(b x^3 + a) + 14 a^3 \operatorname{sgn}(b x^3 + a)}{140 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="giac")

[Out] -1/140*(140*b^3*x^9*sgn(b*x^3 + a) + 105*a*b^2*x^6*sgn(b*x^3 + a) + 60*a^2*b*x^3*sgn(b*x^3 + a) + 14*a^3*sgn(b*x^3 + a))/x^10

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(140 b^3 x^9 + 105 a b^2 x^6 + 60 a^2 b x^3 + 14 a^3) \left((b x^3 + a)^2 \right)^{\frac{3}{2}}}{140 (b x^3 + a)^3 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x)

[Out] -1/140*(140*b^3*x^9+105*a*b^2*x^6+60*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/x^10/(b*x^3+a)^3

maxima [A] time = 0.66, size = 37, normalized size = 0.22

$$\frac{140 b^3 x^9 + 105 a b^2 x^6 + 60 a^2 b x^3 + 14 a^3}{140 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="maxima")

[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10

mupad [B] time = 1.21, size = 151, normalized size = 0.92

$$\frac{a^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{10 x^{10} (b x^3 + a)} - \frac{b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{x (b x^3 + a)} - \frac{3 a b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{4 x^4 (b x^3 + a)} - \frac{3 a^2 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{7 x^7 (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^11,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**11, x)
```

$$3.44 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{12}} dx$$

Optimal. Leaf size=167

$$\frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p]))], Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{12}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^3}{x^{12}} dx}{b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^3b^3}{x^{12}} + \frac{3a^2b^4}{x^9} + \frac{3ab^5}{x^6} + \frac{b^6}{x^3} \right) dx}{b^2 (ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11} (a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8 (a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5 (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a + bx^3)^2} (40a^3 + 165a^2bx^3 + 264ab^2x^6 + 220b^3x^9)}{440x^{11} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]

[Out] -1/440*(Sqrt[(a + b*x^3)^2]*(40*a^3 + 165*a^2*b*x^3 + 264*a*b^2*x^6 + 220*b^3*x^9))/(x^11*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.14, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-40a^3 - 165a^2bx^3 - 264ab^2x^6 - 220b^3x^9)}{440x^{11} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 - 165*a^2*b*x^3 - 264*a*b^2*x^6 - 220*b^3*x^9))/(440*x^11*(a + b*x^3))

fricas [A] time = 1.19, size = 37, normalized size = 0.22

$$-\frac{220b^3x^9 + 264ab^2x^6 + 165a^2bx^3 + 40a^3}{440x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="fricas")

[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11

giac [A] time = 0.42, size = 69, normalized size = 0.41

$$\frac{220 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 264 a b^2 x^6 \operatorname{sgn}(b x^3 + a) + 165 a^2 b x^3 \operatorname{sgn}(b x^3 + a) + 40 a^3 \operatorname{sgn}(b x^3 + a)}{440 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="giac")

[Out] -1/440*(220*b^3*x^9*sgn(b*x^3 + a) + 264*a*b^2*x^6*sgn(b*x^3 + a) + 165*a^2*b*x^3*sgn(b*x^3 + a) + 40*a^3*sgn(b*x^3 + a))/x^11

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{\left(220 b^3 x^9 + 264 a b^2 x^6 + 165 a^2 b x^3 + 40 a^3\right) \left(\left(b x^3 + a\right)^2\right)^{\frac{3}{2}}}{440 \left(b x^3 + a\right)^3 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x)

[Out] -1/440*(220*b^3*x^9+264*a*b^2*x^6+165*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)/x^11/(b*x^3+a)^3

maxima [A] time = 0.72, size = 37, normalized size = 0.22

$$\frac{220 b^3 x^9 + 264 a b^2 x^6 + 165 a^2 b x^3 + 40 a^3}{440 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="maxima")

[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11

mupad [B] time = 1.22, size = 151, normalized size = 0.90

$$\frac{a^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{11 x^{11} (b x^3 + a)} - \frac{b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{2 x^2 (b x^3 + a)} - \frac{3 a b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{5 x^5 (b x^3 + a)} - \frac{3 a^2 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{8 x^8 (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^12,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**12, x)
```


$$3.45 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{13}} dx$$

Optimal. Leaf size=41

$$\frac{(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12ax^{12}}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 264}

$$\frac{(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]

[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*a*x^12)

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{13}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^3}{x^{13}} dx}{b^2 (ab + b^2x^3)} \\ &= -\frac{(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12ax^{12}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.44

$$\frac{\sqrt{(a + bx^3)^2} (a^3 + 4a^2bx^3 + 6ab^2x^6 + 4b^3x^9)}{12x^{12}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]

[Out] -1/12*(Sqrt[(a + b*x^3)^2]*(a^3 + 4*a^2*b*x^3 + 6*a*b^2*x^6 + 4*b^3*x^9))/(x^12*(a + b*x^3))

IntegrateAlgebraic [B] time = 1.35, size = 310, normalized size = 7.56

$$\frac{2b^3\sqrt{a^2+2abx^3+b^2x^6}(-a^6b-7a^5b^2x^3-21a^4b^3x^6-35a^3b^4x^9-34a^2b^5x^{12}-18ab^6x^{15}-4b^7x^{18})+2\sqrt{b^2}b^3(a^7+8a^6bx^3+28a^5b^2x^6+56a^4b^3x^9+69a^3b^4x^{12}+52a^2b^5x^{15}+22ab^6x^{18}+4b^7x^{21})}{3\sqrt{b^2}x^{12}\sqrt{a^2+2abx^3+b^2x^6}(-8a^3b^3-24a^2b^4x^3-24ab^5x^6-8b^6x^9)+3x^{12}(8a^4b^4+32a^3b^5x^3+48a^2b^6x^6+32ab^7x^9+8b^8x^{12})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]

[Out] (2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^6*b) - 7*a^5*b^2*x^3 - 21*a^4*b^3*x^6 - 35*a^3*b^4*x^9 - 34*a^2*b^5*x^12 - 18*a*b^6*x^15 - 4*b^7*x^18) + 2*b^3*Sqrt[b^2]*(a^7 + 8*a^6*b*x^3 + 28*a^5*b^2*x^6 + 56*a^4*b^3*x^9 + 69*a^3*b^4*x^12 + 52*a^2*b^5*x^15 + 22*a*b^6*x^18 + 4*b^7*x^21))/(3*Sqrt[b^2]*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^3*b^3 - 24*a^2*b^4*x^3 - 24*a*b^5*x^6 - 8*b^6*x^9) + 3*x^12*(8*a^4*b^4 + 32*a^3*b^5*x^3 + 48*a^2*b^6*x^6 + 32*a*b^7*x^9 + 8*b^8*x^12))

fricas [A] time = 1.42, size = 35, normalized size = 0.85

$$\frac{4b^3x^9 + 6ab^2x^6 + 4a^2bx^3 + a^3}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="fricas")

[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12

giac [B] time = 0.36, size = 68, normalized size = 1.66

$$\frac{4b^3x^9\operatorname{sgn}(bx^3+a)+6ab^2x^6\operatorname{sgn}(bx^3+a)+4a^2bx^3\operatorname{sgn}(bx^3+a)+a^3\operatorname{sgn}(bx^3+a)}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="giac")

[Out] $-1/12*(4*b^3*x^9*\text{sgn}(b*x^3 + a) + 6*a*b^2*x^6*\text{sgn}(b*x^3 + a) + 4*a^2*b*x^3*\text{sgn}(b*x^3 + a) + a^3*\text{sgn}(b*x^3 + a))/x^{12}$

maple [A] time = 0.01, size = 56, normalized size = 1.37

$$-\frac{(4b^3x^9 + 6ab^2x^6 + 4a^2bx^3 + a^3)\left((bx^3 + a)^2\right)^{\frac{3}{2}}}{12(bx^3 + a)^3 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x)

[Out] $-1/12*(4*b^3*x^9+6*a*b^2*x^6+4*a^2*b*x^3+a^3)*((b*x^3+a)^2)^(3/2)/x^{12}/(b*x^3+a)^3$

maxima [B] time = 0.55, size = 148, normalized size = 3.61

$$\frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}b^4}{12a^4} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}b^3}{12a^3x^3} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^2}{12a^4x^6} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b}{12a^3x^9} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}}{12a^2x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="maxima")

[Out] $1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4/a^4 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/(a^3*x^3) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^4*x^6) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^9) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^{12})$

mupad [B] time = 1.21, size = 151, normalized size = 3.68

$$\frac{a^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12x^{12}(bx^3 + a)} - \frac{b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(bx^3 + a)} - \frac{ab^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^6(bx^3 + a)} - \frac{a^2 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^9(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^13,x)

[Out] $-(a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(12*x^{12}*(a + b*x^3)) - (b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^6*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^9*(a + b*x^3))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)

[Out] Integral(((a + b*x**3)**2)**(3/2)/x**13, x)

$$3.46 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{14}} dx$$

Optimal. Leaf size=167

$$\frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{14}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^3}{x^{14}} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^3b^3}{x^{14}} + \frac{3a^2b^4}{x^{11}} + \frac{3ab^5}{x^8} + \frac{b^6}{x^5} \right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (140a^3 + 546a^2bx^3 + 780ab^2x^6 + 455b^3x^9)}{1820x^{13}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]

[Out] -1/1820*(Sqrt[(a + b*x^3)^2]*(140*a^3 + 546*a^2*b*x^3 + 780*a*b^2*x^6 + 455*b^3*x^9))/(x^13*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.76, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-140a^3 - 546a^2bx^3 - 780ab^2x^6 - 455b^3x^9)}{1820x^{13}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-140*a^3 - 546*a^2*b*x^3 - 780*a*b^2*x^6 - 455*b^3*x^9))/(1820*x^13*(a + b*x^3))

fricas [A] time = 1.36, size = 37, normalized size = 0.22

$$\frac{455b^3x^9 + 780ab^2x^6 + 546a^2bx^3 + 140a^3}{1820x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="fricas")

[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13

giac [A] time = 0.36, size = 69, normalized size = 0.41

$$\frac{455 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 780 a b^2 x^6 \operatorname{sgn}(b x^3 + a) + 546 a^2 b x^3 \operatorname{sgn}(b x^3 + a) + 140 a^3 \operatorname{sgn}(b x^3 + a)}{1820 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="giac")

[Out] -1/1820*(455*b^3*x^9*sgn(b*x^3 + a) + 780*a*b^2*x^6*sgn(b*x^3 + a) + 546*a^2*b*x^3*sgn(b*x^3 + a) + 140*a^3*sgn(b*x^3 + a))/x^13

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(455 b^3 x^9 + 780 a b^2 x^6 + 546 a^2 b x^3 + 140 a^3) \left((b x^3 + a)^2 \right)^{\frac{3}{2}}}{1820 (b x^3 + a)^3 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x)

[Out] -1/1820*(455*b^3*x^9+780*a*b^2*x^6+546*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/2)/x^13/(b*x^3+a)^3

maxima [A] time = 0.69, size = 37, normalized size = 0.22

$$\frac{455 b^3 x^9 + 780 a b^2 x^6 + 546 a^2 b x^3 + 140 a^3}{1820 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="maxima")

[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13

mupad [B] time = 1.19, size = 151, normalized size = 0.90

$$\frac{a^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{13 x^{13} (b x^3 + a)} - \frac{b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{4 x^4 (b x^3 + a)} - \frac{3 a b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{7 x^7 (b x^3 + a)} - \frac{3 a^2 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{10 x^{10} (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^14,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(13*x^13*(a + b*x^3)) - (b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**14, x)
```


$$3.47 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{15}} dx$$

Optimal. Leaf size=167

$$\frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{14x^{14}(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{14x^{14}(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (3*a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p]))], Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{15}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^3}{x^{15}} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^3b^3}{x^{15}} + \frac{3a^2b^4}{x^{12}} + \frac{3ab^5}{x^9} + \frac{b^6}{x^6} \right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{14x^{14}(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a + bx^3)^2} (220a^3 + 840a^2bx^3 + 1155ab^2x^6 + 616b^3x^9)}{3080x^{14}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]

[Out] -1/3080*(Sqrt[(a + b*x^3)^2]*(220*a^3 + 840*a^2*b*x^3 + 1155*a*b^2*x^6 + 616*b^3*x^9))/(x^14*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.64, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-220a^3 - 840a^2bx^3 - 1155ab^2x^6 - 616b^3x^9)}{3080x^{14}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-220*a^3 - 840*a^2*b*x^3 - 1155*a*b^2*x^6 - 616*b^3*x^9))/(3080*x^14*(a + b*x^3))

fricas [A] time = 2.23, size = 37, normalized size = 0.22

$$\frac{616b^3x^9 + 1155ab^2x^6 + 840a^2bx^3 + 220a^3}{3080x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="fricas")

[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14

giac [A] time = 0.36, size = 69, normalized size = 0.41

$$\frac{616 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 1155 a b^2 x^6 \operatorname{sgn}(b x^3 + a) + 840 a^2 b x^3 \operatorname{sgn}(b x^3 + a) + 220 a^3 \operatorname{sgn}(b x^3 + a)}{3080 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="giac")

[Out] -1/3080*(616*b^3*x^9*sgn(b*x^3 + a) + 1155*a*b^2*x^6*sgn(b*x^3 + a) + 840*a^2*b*x^3*sgn(b*x^3 + a) + 220*a^3*sgn(b*x^3 + a))/x^14

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(616 b^3 x^9 + 1155 a b^2 x^6 + 840 a^2 b x^3 + 220 a^3) \left((b x^3 + a)^2 \right)^{\frac{3}{2}}}{3080 (b x^3 + a)^3 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x)

[Out] -1/3080*(616*b^3*x^9+1155*a*b^2*x^6+840*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3/2)/x^14/(b*x^3+a)^3

maxima [A] time = 0.69, size = 37, normalized size = 0.22

$$\frac{616 b^3 x^9 + 1155 a b^2 x^6 + 840 a^2 b x^3 + 220 a^3}{3080 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="maxima")

[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14

mupad [B] time = 1.21, size = 151, normalized size = 0.90

$$\frac{a^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{14 x^{14} (b x^3 + a)} - \frac{b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{5 x^5 (b x^3 + a)} - \frac{3 a b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{8 x^8 (b x^3 + a)} - \frac{3 a^2 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{11 x^{11} (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^15,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(14*x^14*(a + b*x^3)) - (b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{15}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**15, x)
```

$$3.48 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{16}} dx$$

Optimal. Leaf size=84

$$\frac{b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{60a^2x^{12}} - \frac{(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15ax^{15}}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 266, 45, 37}

$$\frac{b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{60a^2x^{12}} - \frac{(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15ax^{15}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]

[Out] -((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*a*x^15) + (b*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(60*a^2*x^12)

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 45

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*Simplify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.) + (c_.)*(x_.)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{16}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^3}{x^{16}} dx}{b^2(ab + b^2x^3)} \\ &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^3}{x^6} dx, x, x^3\right)}{3b^2(ab + b^2x^3)} \\ &= -\frac{(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15ax^{15}} - \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^3}{x^5} dx, x, x\right)}{15ab(ab + b^2x^3)} \\ &= -\frac{(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15ax^{15}} + \frac{b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{60a^2x^{12}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.73

$$-\frac{\sqrt{(a + bx^3)^2} (4a^3 + 15a^2bx^3 + 20ab^2x^6 + 10b^3x^9)}{60x^{15}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16, x]

[Out] -1/60*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 15*a^2*b*x^3 + 20*a*b^2*x^6 + 10*b^3*x^9))/(x^15*(a + b*x^3))

IntegrateAlgebraic [B] time = 1.02, size = 356, normalized size = 4.24

$$\frac{4b^4\sqrt{a^2 + 2abx^3 + b^2x^6} (-4a^7b - 31a^6b^2x^3 - 104a^5b^3x^6 - 196a^4b^4x^9 - 224a^3b^5x^{12} - 155a^2b^6x^{15} - 60ab^7x^{18} - 10b^8x^{21}) + 4\sqrt{b^2} b^4 (4a^8 + 35a^7bx^3 + 135a^6b^2x^6 + 300a^5b^3x^9 + 420a^4b^4x^{12} + 379a^3b^5x^{15} + 215a^2b^6x^{18} + 70ab^7x^{21} + 10b^8x^{24})}{15\sqrt{b^2} x^{15} \sqrt{a^2 + 2abx^3 + b^2x^6} (-16a^4b^4 - 64a^3b^5x^3 - 96a^2b^6x^6 - 64ab^7x^9 - 16b^8x^{12}) + 15x^{15} (16a^2b^5 + 80a^4b^6x^3 + 160a^3b^7x^6 + 160a^2b^8x^9 + 80ab^9x^{12} + 16b^{10}x^{15})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]

[Out] (4*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-4*a^7*b - 31*a^6*b^2*x^3 - 104*a^5*b^3*x^6 - 196*a^4*b^4*x^9 - 224*a^3*b^5*x^12 - 155*a^2*b^6*x^15 - 60*a*b^7*x^18 - 10*b^8*x^21) + 4*b^4*Sqrt[b^2]*(4*a^8 + 35*a^7*b*x^3 + 135*a^6*b^2*x^6 + 300*a^5*b^3*x^9 + 420*a^4*b^4*x^12 + 379*a^3*b^5*x^15 + 215*a^2*b^6*x^18 + 70*a*b^7*x^21 + 10*b^8*x^24))/(15*Sqrt[b^2]*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-16*a^4*b^4 - 64*a^3*b^5*x^3 - 96*a^2*b^6*x^6 - 64*a*b^7*x^9 - 16*b^8*x^12) + 15*x^15*(16*a^5*b^5 + 80*a^4*b^6*x^3 + 160*a^3*b^7*x^6 + 160*a^2*b^8*x^9 + 80*a*b^9*x^12 + 16*b^10*x^15))

fricas [A] time = 0.85, size = 37, normalized size = 0.44

$$\frac{10b^3x^9 + 20ab^2x^6 + 15a^2bx^3 + 4a^3}{60x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="fricas")

[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15

giac [A] time = 0.41, size = 69, normalized size = 0.82

$$\frac{10b^3x^9\operatorname{sgn}(bx^3 + a) + 20ab^2x^6\operatorname{sgn}(bx^3 + a) + 15a^2bx^3\operatorname{sgn}(bx^3 + a) + 4a^3\operatorname{sgn}(bx^3 + a)}{60x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="giac")

[Out] -1/60*(10*b^3*x^9*sgn(b*x^3 + a) + 20*a*b^2*x^6*sgn(b*x^3 + a) + 15*a^2*b*x^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^15

maple [A] time = 0.01, size = 58, normalized size = 0.69

$$\frac{(10b^3x^9 + 20ab^2x^6 + 15a^2bx^3 + 4a^3)\left((bx^3 + a)^2\right)^{\frac{3}{2}}}{60(bx^3 + a)^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x)

[Out] -1/60*(10*b^3*x^9+20*a*b^2*x^6+15*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^15/(b*x^3+a)^3

maxima [B] time = 0.52, size = 179, normalized size = 2.13

$$-\frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}b^5}{12a^5} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}b^4}{12a^4x^3} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^3}{12a^5x^6} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^2}{12a^4x^9} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b}{12a^3x^{12}} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}}{15a^2x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="maxima")

[Out] $-\frac{1}{12} \frac{(b^2x^6 + 2abx^3 + a^2)^{3/2} b^5}{a^5} - \frac{1}{12} \frac{(b^2x^6 + 2abx^3 + a^2)^{3/2} b^4}{a^4 x^3} + \frac{1}{12} \frac{(b^2x^6 + 2abx^3 + a^2)^{5/2} b^3}{a^5 x^6} - \frac{1}{12} \frac{(b^2x^6 + 2abx^3 + a^2)^{5/2} b^2}{a^4 x^9} + \frac{1}{12} \frac{(b^2x^6 + 2abx^3 + a^2)^{5/2} b}{a^3 x^{12}} - \frac{1}{15} \frac{(b^2x^6 + 2abx^3 + a^2)^{5/2}}{a^2 x^{15}}$

mupad [B] time = 1.21, size = 151, normalized size = 1.80

$$-\frac{a^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15x^{15}(bx^3 + a)} - \frac{b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6(bx^3 + a)} - \frac{ab^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^9(bx^3 + a)} - \frac{a^2 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^{12}(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^16,x)

[Out] $-\frac{(a^3(a^2 + b^2x^6 + 2abx^3)^{1/2})}{(15x^{15}(a + bx^3))} - \frac{(b^3(a^2 + b^2x^6 + 2abx^3)^{1/2})}{(6x^6(a + bx^3))} - \frac{(ab^2(a^2 + b^2x^6 + 2abx^3)^{1/2})}{(3x^9(a + bx^3))} - \frac{(a^2b(a^2 + b^2x^6 + 2abx^3)^{1/2})}{(4x^{12}(a + bx^3))}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)

[Out] Integral(((a + b*x**3)**2)**(3/2)/x**16, x)

$$3.49 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{17}} dx$$

Optimal. Leaf size=167

$$\frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{16x^{16}(a + bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{16x^{16}(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17, x]

[Out] -(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (3*a^2*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{3/2}}{x^{17}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^3}{x^{17}} dx}{b^2(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^3b^3}{x^{17}} + \frac{3a^2b^4}{x^{14}} + \frac{3ab^5}{x^{11}} + \frac{b^6}{x^8} \right) dx}{b^2(ab + b^2x^3)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{16x^{16}(a + bx^3)} - \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)} - \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a + bx^3)^2} (455a^3 + 1680a^2bx^3 + 2184ab^2x^6 + 1040b^3x^9)}{7280x^{16}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17, x]

[Out] -1/7280*(Sqrt[(a + b*x^3)^2]*(455*a^3 + 1680*a^2*b*x^3 + 2184*a*b^2*x^6 + 1040*b^3*x^9))/(x^16*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.49, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a + bx^3)^2} (-455a^3 - 1680a^2bx^3 - 2184ab^2x^6 - 1040b^3x^9)}{7280x^{16}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17, x]

[Out] (Sqrt[(a + b*x^3)^2]*(-455*a^3 - 1680*a^2*b*x^3 - 2184*a*b^2*x^6 - 1040*b^3*x^9))/(7280*x^16*(a + b*x^3))

fricas [A] time = 1.65, size = 37, normalized size = 0.22

$$\frac{1040 b^3 x^9 + 2184 a b^2 x^6 + 1680 a^2 b x^3 + 455 a^3}{7280 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="fricas")

[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16

giac [A] time = 0.30, size = 69, normalized size = 0.41

$$\frac{1040 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 2184 a b^2 x^6 \operatorname{sgn}(b x^3 + a) + 1680 a^2 b x^3 \operatorname{sgn}(b x^3 + a) + 455 a^3 \operatorname{sgn}(b x^3 + a)}{7280 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="giac")

[Out] -1/7280*(1040*b^3*x^9*sgn(b*x^3 + a) + 2184*a*b^2*x^6*sgn(b*x^3 + a) + 1680*a^2*b*x^3*sgn(b*x^3 + a) + 455*a^3*sgn(b*x^3 + a))/x^16

maple [A] time = 0.01, size = 58, normalized size = 0.35

$$\frac{(1040 b^3 x^9 + 2184 a b^2 x^6 + 1680 a^2 b x^3 + 455 a^3) \left((b x^3 + a)^2 \right)^{\frac{3}{2}}}{7280 (b x^3 + a)^3 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x)

[Out] -1/7280*(1040*b^3*x^9+2184*a*b^2*x^6+1680*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^(3/2)/x^16/(b*x^3+a)^3

maxima [A] time = 0.77, size = 37, normalized size = 0.22

$$\frac{1040 b^3 x^9 + 2184 a b^2 x^6 + 1680 a^2 b x^3 + 455 a^3}{7280 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="maxima")

[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16

mupad [B] time = 1.22, size = 151, normalized size = 0.90

$$\frac{a^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{16 x^{16} (b x^3 + a)} - \frac{b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{7 x^7 (b x^3 + a)} - \frac{3 a b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{10 x^{10} (b x^3 + a)} - \frac{3 a^2 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{13 x^{13} (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^17,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(13*x^13*(a + b*x^3))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^{17}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**17, x)
```

$$3.50 \quad \int x^{13} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=255

$$\frac{b^5 x^{29} \sqrt{a^2 + 2abx^3 + b^2x^6}}{29(a + bx^3)} + \frac{5ab^4 x^{26} \sqrt{a^2 + 2abx^3 + b^2x^6}}{26(a + bx^3)} + \frac{10a^2 b^3 x^{23} \sqrt{a^2 + 2abx^3 + b^2x^6}}{23(a + bx^3)} + \frac{a^5 x^{14} \sqrt{a^2 + 2abx^3 + b^2x^6}}{14(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5 x^{29} \sqrt{a^2 + 2abx^3 + b^2x^6}}{29(a + bx^3)} + \frac{5ab^4 x^{26} \sqrt{a^2 + 2abx^3 + b^2x^6}}{26(a + bx^3)} + \frac{10a^2 b^3 x^{23} \sqrt{a^2 + 2abx^3 + b^2x^6}}{23(a + bx^3)} + \frac{a^3 b^2 x^{20} \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{5a^4 b x^{17} \sqrt{a^2 + 2abx^3 + b^2x^6}}{17(a + bx^3)} + \frac{a^5 x^{14} \sqrt{a^2 + 2abx^3 + b^2x^6}}{14(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] (a^5*x^14*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (5*a^4*b*x^17*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^3*b^2*x^20*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^23*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a + b*x^3)) + (5*a*b^4*x^26*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*(a + b*x^3)) + (b^5*x^29*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(29*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^{13} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^{13} (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^5b^5x^{13} + 5a^4b^6x^{16} + 10a^3b^7x^{19} + 10a^2b^8x^{22} + 5ab^9x^{25} + b^{10}x^{28}) dx}{b^4 (ab + b^2x^3)} \\
&= \frac{a^5x^{14}\sqrt{a^2 + 2abx^3 + b^2x^6}}{14 (a + bx^3)} + \frac{5a^4bx^{17}\sqrt{a^2 + 2abx^3 + b^2x^6}}{17 (a + bx^3)} + \frac{a^3b^2x^{20}\sqrt{a^2 + 2abx^3 + b^2x^6}}{2 (a + bx^3)} + \dots
\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.33

$$\frac{x^{14} \sqrt{(a + bx^3)^2} (147407a^5 + 606970a^4bx^3 + 1031849a^3b^2x^6 + 897260a^2b^3x^9 + 396865ab^4x^{12} + 71162b^5x^{15})}{2063698 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^14*Sqrt[(a + b*x^3)^2]*(147407*a^5 + 606970*a^4*b*x^3 + 1031849*a^3*b^2*x^6 + 897260*a^2*b^3*x^9 + 396865*a*b^4*x^12 + 71162*b^5*x^15))/(2063698*(a + b*x^3))

IntegrateAlgebraic [A] time = 27.59, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (147407a^5x^{14} + 606970a^4bx^{17} + 1031849a^3b^2x^{20} + 897260a^2b^3x^{23} + 396865ab^4x^{26} + 71162b^5x^{29})}{2063698 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (Sqrt[(a + b*x^3)^2]*(147407*a^5*x^14 + 606970*a^4*b*x^17 + 1031849*a^3*b^2*x^20 + 897260*a^2*b^3*x^23 + 396865*a*b^4*x^26 + 71162*b^5*x^29))/(2063698*(a + b*x^3))

fricas [A] time = 0.98, size = 57, normalized size = 0.22

$$\frac{1}{29} b^5 x^{29} + \frac{5}{26} ab^4 x^{26} + \frac{10}{23} a^2 b^3 x^{23} + \frac{1}{2} a^3 b^2 x^{20} + \frac{5}{17} a^4 b x^{17} + \frac{1}{14} a^5 x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹³*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="fricas")

[Out] 1/29*b⁵*x²⁹ + 5/26*a*b⁴*x²⁶ + 10/23*a²*b³*x²³ + 1/2*a³*b²*x²⁰ + 5/17*a⁴*b*x¹⁷ + 1/14*a⁵*x¹⁴

giac [A] time = 0.33, size = 105, normalized size = 0.41

$$\frac{1}{29} b^5 x^{29} \operatorname{sgn}(b x^3 + a) + \frac{5}{26} a b^4 x^{26} \operatorname{sgn}(b x^3 + a) + \frac{10}{23} a^2 b^3 x^{23} \operatorname{sgn}(b x^3 + a) + \frac{1}{2} a^3 b^2 x^{20} \operatorname{sgn}(b x^3 + a) + \frac{5}{17} a^4 b x^{17} \operatorname{sgn}(b x^3 + a) + \frac{1}{14} a^5 x^{14} \operatorname{sgn}(b x^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹³*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="giac")

[Out] 1/29*b⁵*x²⁹*sgn(b*x³ + a) + 5/26*a*b⁴*x²⁶*sgn(b*x³ + a) + 10/23*a²*b³*x²³*sgn(b*x³ + a) + 1/2*a³*b²*x²⁰*sgn(b*x³ + a) + 5/17*a⁴*b*x¹⁷*sgn(b*x³ + a) + 1/14*a⁵*x¹⁴*sgn(b*x³ + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(71162b^5x^{15} + 396865ab^4x^{12} + 897260a^2b^3x^9 + 1031849a^3b^2x^6 + 606970a^4bx^3 + 147407a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^{14}}{2063698 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹³*(b²*x⁶+2*a*b*x³+a²)^(5/2),x)

[Out] 1/2063698*x¹⁴*(71162*b⁵*x¹⁵+396865*a*b⁴*x¹²+897260*a²*b³*x⁹+1031849*a³*b²*x⁶+606970*a⁴*b*x³+147407*a⁵)*((b*x³+a)²)^(5/2)/(b*x³+a)⁵

maxima [A] time = 0.83, size = 57, normalized size = 0.22

$$\frac{1}{29} b^5 x^{29} + \frac{5}{26} a b^4 x^{26} + \frac{10}{23} a^2 b^3 x^{23} + \frac{1}{2} a^3 b^2 x^{20} + \frac{5}{17} a^4 b x^{17} + \frac{1}{14} a^5 x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹³*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="maxima")

[Out] 1/29*b⁵*x²⁹ + 5/26*a*b⁴*x²⁶ + 10/23*a²*b³*x²³ + 1/2*a³*b²*x²⁰ + 5/17*a⁴*b*x¹⁷ + 1/14*a⁵*x¹⁴

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{13} (a^2 + 2 a b x^3 + b^2 x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13} \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**13*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**13*((a + b*x**3)**2)**(5/2), x)`

$$3.51 \quad \int x^{12} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=255

$$\frac{b^5 x^{28} \sqrt{a^2 + 2abx^3 + b^2x^6}}{28(a + bx^3)} + \frac{ab^4 x^{25} \sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{5a^2 b^3 x^{22} \sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{a^5 x^{13} \sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5 x^{28} \sqrt{a^2 + 2abx^3 + b^2x^6}}{28(a + bx^3)} + \frac{ab^4 x^{25} \sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{5a^2 b^3 x^{22} \sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{10a^3 b^2 x^{19} \sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{5a^4 b x^{16} \sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)} + \frac{a^5 x^{13} \sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^4*b*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (10*a^3*b^2*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a^2*b^3*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (a*b^4*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^5*x^28*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(28*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^{12} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^{12} (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^5b^5x^{12} + 5a^4b^6x^{15} + 10a^3b^7x^{18} + 10a^2b^8x^{21} + 5ab^9x^{24} + b^{10}x^{27}) dx}{b^4 (ab + b^2x^3)} \\
&= \frac{a^5x^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{5a^4bx^{16}\sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)} + \frac{10a^3b^2x^{19}\sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{10a^2b^3x^{22}\sqrt{a^2 + 2abx^3 + b^2x^6}}{22(a + bx^3)} + \frac{5ab^4x^{25}\sqrt{a^2 + 2abx^3 + b^2x^6}}{25(a + bx^3)} + \frac{b^5x^{28}\sqrt{a^2 + 2abx^3 + b^2x^6}}{28(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{13} \sqrt{(a + bx^3)^2} (117040a^5 + 475475a^4bx^3 + 800800a^3b^2x^6 + 691600a^2b^3x^9 + 304304ab^4x^{12} + 54340b^5x^{15})}{1521520(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^13*Sqrt[(a + b*x^3)^2]*(117040*a^5 + 475475*a^4*b*x^3 + 800800*a^3*b^2*x^6 + 691600*a^2*b^3*x^9 + 304304*a*b^4*x^12 + 54340*b^5*x^15))/(1521520*(a + b*x^3))

IntegrateAlgebraic [A] time = 24.78, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (117040a^5x^{13} + 475475a^4bx^{16} + 800800a^3b^2x^{19} + 691600a^2b^3x^{22} + 304304ab^4x^{25} + 54340b^5x^{28})}{1521520(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (Sqrt[(a + b*x^3)^2]*(117040*a^5*x^13 + 475475*a^4*b*x^16 + 800800*a^3*b^2*x^19 + 691600*a^2*b^3*x^22 + 304304*a*b^4*x^25 + 54340*b^5*x^28))/(1521520*(a + b*x^3))

fricas [A] time = 1.99, size = 57, normalized size = 0.22

$$\frac{1}{28} b^5 x^{28} + \frac{1}{5} ab^4 x^{25} + \frac{5}{11} a^2 b^3 x^{22} + \frac{10}{19} a^3 b^2 x^{19} + \frac{5}{16} a^4 b x^{16} + \frac{1}{13} a^5 x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹²*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="fricas")

[Out] 1/28*b⁵*x²⁸ + 1/5*a*b⁴*x²⁵ + 5/11*a²*b³*x²² + 10/19*a³*b²*x¹⁹ + 5/16*a⁴*b*x¹⁶ + 1/13*a⁵*x¹³

giac [A] time = 0.32, size = 105, normalized size = 0.41

$$\frac{1}{28} b^5 x^{28} \operatorname{sgn}(bx^3 + a) + \frac{1}{5} ab^4 x^{25} \operatorname{sgn}(bx^3 + a) + \frac{5}{11} a^2 b^3 x^{22} \operatorname{sgn}(bx^3 + a) + \frac{10}{19} a^3 b^2 x^{19} \operatorname{sgn}(bx^3 + a) + \frac{5}{16} a^4 b x^{16} \operatorname{sgn}(bx^3 + a) + \frac{1}{13} a^5 x^{13} \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹²*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="giac")

[Out] 1/28*b⁵*x²⁸*sgn(b*x³ + a) + 1/5*a*b⁴*x²⁵*sgn(b*x³ + a) + 5/11*a²*b³*x²²*sgn(b*x³ + a) + 10/19*a³*b²*x¹⁹*sgn(b*x³ + a) + 5/16*a⁴*b*x¹⁶*sgn(b*x³ + a) + 1/13*a⁵*x¹³*sgn(b*x³ + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(54340b^5x^{15} + 304304ab^4x^{12} + 691600a^2b^3x^9 + 800800a^3b^2x^6 + 475475a^4bx^3 + 117040a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^{13}}{1521520 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹²*(b²*x⁶+2*a*b*x³+a²)^(5/2),x)

[Out] 1/1521520*x¹³*(54340*b⁵*x¹⁵+304304*a*b⁴*x¹²+691600*a²*b³*x⁹+800800*a³*b²*x⁶+475475*a⁴*b*x³+117040*a⁵)*((b*x³+a)²)^(5/2)/(b*x³+a)⁵

maxima [A] time = 0.60, size = 57, normalized size = 0.22

$$\frac{1}{28} b^5 x^{28} + \frac{1}{5} ab^4 x^{25} + \frac{5}{11} a^2 b^3 x^{22} + \frac{10}{19} a^3 b^2 x^{19} + \frac{5}{16} a^4 b x^{16} + \frac{1}{13} a^5 x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹²*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="maxima")

[Out] 1/28*b⁵*x²⁸ + 1/5*a*b⁴*x²⁵ + 5/11*a²*b³*x²² + 10/19*a³*b²*x¹⁹ + 5/16*a⁴*b*x¹⁶ + 1/13*a⁵*x¹³

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{12} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12} \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**12*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**12*((a + b*x**3)**2)**(5/2), x)`

$$3.52 \quad \int x^{11} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^8}{27b^4} - \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^7}{8b^4} + \frac{a^2\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^6}{7b^4} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{18b^4}$$

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^8}{27b^4} - \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^7}{8b^4} + \frac{a^2\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^6}{7b^4} - \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{18b^4}$$

Antiderivative was successfully verified.

[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] -(a^3*(a + b*x^3)^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^4) + (a^2*(a + b*x^3)^6*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*b^4) - (a*(a + b*x^3)^7*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*b^4) + ((a + b*x^3)^8*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(27*b^4)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^{11} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^{11} (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int x^3 (ab + b^2x)^5 dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int \left(-\frac{a^3(ab+b^2x)^5}{b^3} + \frac{3a^2(ab+b^2x)^6}{b^4} - \frac{3a(ab+b^2x)^7}{b^5} + \frac{(ab+b^2x)^8}{b^6}\right) dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= -\frac{a^3 (a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18b^4} + \frac{a^2 (a + bx^3)^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7b^4} - \frac{a (a + bx^3)^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18b^4} + \frac{(ab + b^2x^3)^8}{18b^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.52

$$\frac{x^{12} \sqrt{(a + bx^3)^2} (126a^5 + 504a^4bx^3 + 840a^3b^2x^6 + 720a^2b^3x^9 + 315ab^4x^{12} + 56b^5x^{15})}{1512(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^12*Sqrt[(a + b*x^3)^2]*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720*a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15))/(1512*(a + b*x^3))

IntegrateAlgebraic [A] time = 21.67, size = 83, normalized size = 0.52

$$\frac{x^{12} \sqrt{(a + bx^3)^2} (126a^5 + 504a^4bx^3 + 840a^3b^2x^6 + 720a^2b^3x^9 + 315ab^4x^{12} + 56b^5x^{15})}{1512(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^12*Sqrt[(a + b*x^3)^2]*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720*a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15))/(1512*(a + b*x^3))

fricas [A] time = 1.45, size = 57, normalized size = 0.36

$$\frac{1}{27} b^5 x^{27} + \frac{5}{24} ab^4 x^{24} + \frac{10}{21} a^2 b^3 x^{21} + \frac{5}{9} a^3 b^2 x^{18} + \frac{1}{3} a^4 b x^{15} + \frac{1}{12} a^5 x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="fricas")

[Out] 1/27*b⁵*x²⁷ + 5/24*a*b⁴*x²⁴ + 10/21*a²*b³*x²¹ + 5/9*a³*b²*x¹⁸ + 1/3*a⁴*b*x¹⁵ + 1/12*a⁵*x¹²

giac [A] time = 0.35, size = 105, normalized size = 0.66

$$\frac{1}{27} b^5 x^{27} \operatorname{sgn}(bx^3 + a) + \frac{5}{24} ab^4 x^{24} \operatorname{sgn}(bx^3 + a) + \frac{10}{21} a^2 b^3 x^{21} \operatorname{sgn}(bx^3 + a) + \frac{5}{9} a^3 b^2 x^{18} \operatorname{sgn}(bx^3 + a) + \frac{1}{3} a^4 b x^{15} \operatorname{sgn}(bx^3 + a) + \frac{1}{12} a^5 x^{12} \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="giac")

[Out] 1/27*b⁵*x²⁷*sgn(b*x³ + a) + 5/24*a*b⁴*x²⁴*sgn(b*x³ + a) + 10/21*a²*b³*x²¹*sgn(b*x³ + a) + 5/9*a³*b²*x¹⁸*sgn(b*x³ + a) + 1/3*a⁴*b*x¹⁵*sgn(b*x³ + a) + 1/12*a⁵*x¹²*sgn(b*x³ + a)

maple [A] time = 0.01, size = 80, normalized size = 0.50

$$\frac{(56b^5x^{15} + 315ab^4x^{12} + 720a^2b^3x^9 + 840a^3b^2x^6 + 504a^4bx^3 + 126a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^{12}}{1512 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹*(b²*x⁶+2*a*b*x³+a²)^(5/2),x)

[Out] 1/1512*x¹²*(56*b⁵*x¹⁵+315*a*b⁴*x¹²+720*a²*b³*x⁹+840*a³*b²*x⁶+504*a⁴*b*x³+126*a⁵)*((b*x³+a)²)^(5/2)/(b*x³+a)⁵

maxima [A] time = 0.72, size = 145, normalized size = 0.91

$$\frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}x^6}{27b^2} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}a^3x^3}{18b^3} - \frac{11(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}ax^3}{216b^3} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}a^4}{18b^4} + \frac{83(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}a^2}{1512b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="maxima")

[Out] 1/27*(b²*x⁶ + 2*a*b*x³ + a²)^(7/2)*x⁶/b² - 1/18*(b²*x⁶ + 2*a*b*x³ + a²)^(5/2)*a³*x³/b³ - 11/216*(b²*x⁶ + 2*a*b*x³ + a²)^(7/2)*a*x³/b³ - 1/18*(b²*x⁶ + 2*a*b*x³ + a²)^(5/2)*a⁴/b⁴ + 83/1512*(b²*x⁶ + 2*a*b*x³ + a²)^(7/2)*a²/b⁴

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{11} (a^2 + 2 a b x^3 + b^2 x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11} \left((a + b x^3)^2 \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**11*((a + b*x**3)**2)**(5/2), x)`

$$3.53 \quad \int x^{10} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=255

$$\frac{b^5 x^{26} \sqrt{a^2 + 2abx^3 + b^2x^6}}{26(a + bx^3)} + \frac{5ab^4 x^{23} \sqrt{a^2 + 2abx^3 + b^2x^6}}{23(a + bx^3)} + \frac{a^2 b^3 x^{20} \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{a^5 x^{11} \sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5 x^{26} \sqrt{a^2 + 2abx^3 + b^2x^6}}{26(a + bx^3)} + \frac{5ab^4 x^{23} \sqrt{a^2 + 2abx^3 + b^2x^6}}{23(a + bx^3)} + \frac{a^2 b^3 x^{20} \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{10a^3 b^2 x^{17} \sqrt{a^2 + 2abx^3 + b^2x^6}}{17(a + bx^3)} + \frac{5a^4 b x^{14} \sqrt{a^2 + 2abx^3 + b^2x^6}}{14(a + bx^3)} + \frac{a^5 x^{11} \sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^4*b*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (10*a^3*b^2*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a + b*x^3)) + (b^5*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^{10} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^{10} (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^5b^5x^{10} + 5a^4b^6x^{13} + 10a^3b^7x^{16} + 10a^2b^8x^{19} + 5ab^9x^{22} + b^{10}x^{25}) dx}{b^4 (ab + b^2x^3)} \\
&= \frac{a^5x^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11 (a + bx^3)} + \frac{5a^4bx^{14}\sqrt{a^2 + 2abx^3 + b^2x^6}}{14 (a + bx^3)} + \frac{10a^3b^2x^{17}\sqrt{a^2 + 2abx^3 + b^2x^6}}{17 (a + bx^3)} + \frac{10a^2b^3x^{20}\sqrt{a^2 + 2abx^3 + b^2x^6}}{20 (a + bx^3)} + \frac{5ab^4x^{23}\sqrt{a^2 + 2abx^3 + b^2x^6}}{23 (a + bx^3)} + \frac{b^5x^{26}\sqrt{a^2 + 2abx^3 + b^2x^6}}{26 (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{11} \sqrt{(a + bx^3)^2} (71162a^5 + 279565a^4bx^3 + 460460a^3b^2x^6 + 391391a^2b^3x^9 + 170170ab^4x^{12} + 30107b^5x^{15})}{782782 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^11*sqrt[(a + b*x^3)^2]*(71162*a^5 + 279565*a^4*b*x^3 + 460460*a^3*b^2*x^6 + 391391*a^2*b^3*x^9 + 170170*a*b^4*x^12 + 30107*b^5*x^15))/(782782*(a + b*x^3))

IntegrateAlgebraic [A] time = 19.23, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (71162a^5x^{11} + 279565a^4bx^{14} + 460460a^3b^2x^{17} + 391391a^2b^3x^{20} + 170170ab^4x^{23} + 30107b^5x^{26})}{782782 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (sqrt[(a + b*x^3)^2]*(71162*a^5*x^11 + 279565*a^4*b*x^14 + 460460*a^3*b^2*x^17 + 391391*a^2*b^3*x^20 + 170170*a*b^4*x^23 + 30107*b^5*x^26))/(782782*(a + b*x^3))

fricas [A] time = 1.56, size = 57, normalized size = 0.22

$$\frac{1}{26} b^5 x^{26} + \frac{5}{23} ab^4 x^{23} + \frac{1}{2} a^2 b^3 x^{20} + \frac{10}{17} a^3 b^2 x^{17} + \frac{5}{14} a^4 b x^{14} + \frac{1}{11} a^5 x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁰*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="fricas")

[Out] 1/26*b⁵*x²⁶ + 5/23*a*b⁴*x²³ + 1/2*a²*b³*x²⁰ + 10/17*a³*b²*x¹⁷ + 5/14*a⁴*b*x¹⁴ + 1/11*a⁵*x¹¹

giac [A] time = 0.44, size = 105, normalized size = 0.41

$$\frac{1}{26} b^5 x^{26} \operatorname{sgn}(bx^3 + a) + \frac{5}{23} ab^4 x^{23} \operatorname{sgn}(bx^3 + a) + \frac{1}{2} a^2 b^3 x^{20} \operatorname{sgn}(bx^3 + a) + \frac{10}{17} a^3 b^2 x^{17} \operatorname{sgn}(bx^3 + a) + \frac{5}{14} a^4 b x^{14} \operatorname{sgn}(bx^3 + a) + \frac{1}{11} a^5 x^{11} \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁰*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="giac")

[Out] 1/26*b⁵*x²⁶*sgn(b*x³ + a) + 5/23*a*b⁴*x²³*sgn(b*x³ + a) + 1/2*a²*b³*x²⁰*sgn(b*x³ + a) + 10/17*a³*b²*x¹⁷*sgn(b*x³ + a) + 5/14*a⁴*b*x¹⁴*sgn(b*x³ + a) + 1/11*a⁵*x¹¹*sgn(b*x³ + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(30107b^5x^{15} + 170170ab^4x^{12} + 391391a^2b^3x^9 + 460460a^3b^2x^6 + 279565a^4bx^3 + 71162a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^{11}}{782782 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁰*(b²*x⁶+2*a*b*x³+a²)^(5/2),x)

[Out] 1/782782*x¹¹*(30107*b⁵*x¹⁵+170170*a*b⁴*x¹²+391391*a²*b³*x⁹+460460*a³*b²*x⁶+279565*a⁴*b*x³+71162*a⁵)*((b*x³+a)²)^(5/2)/(b*x³+a)⁵

maxima [A] time = 0.81, size = 57, normalized size = 0.22

$$\frac{1}{26} b^5 x^{26} + \frac{5}{23} ab^4 x^{23} + \frac{1}{2} a^2 b^3 x^{20} + \frac{10}{17} a^3 b^2 x^{17} + \frac{5}{14} a^4 b x^{14} + \frac{1}{11} a^5 x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁰*(b²*x⁶+2*a*b*x³+a²)^(5/2),x, algorithm="maxima")

[Out] 1/26*b⁵*x²⁶ + 5/23*a*b⁴*x²³ + 1/2*a²*b³*x²⁰ + 10/17*a³*b²*x¹⁷ + 5/14*a⁴*b*x¹⁴ + 1/11*a⁵*x¹¹

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{10} (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{10} \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**10*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**10*((a + b*x**3)**2)**(5/2), x)`

$$3.54 \quad \int x^9 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=255

$$\frac{b^5 x^{25} \sqrt{a^2 + 2abx^3 + b^2x^6}}{25(a + bx^3)} + \frac{5ab^4 x^{22} \sqrt{a^2 + 2abx^3 + b^2x^6}}{22(a + bx^3)} + \frac{10a^2 b^3 x^{19} \sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{a^5 x^{10} \sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5 x^{25} \sqrt{a^2 + 2abx^3 + b^2x^6}}{25(a + bx^3)} + \frac{5ab^4 x^{22} \sqrt{a^2 + 2abx^3 + b^2x^6}}{22(a + bx^3)} + \frac{10a^2 b^3 x^{19} \sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{5a^3 b^2 x^{16} \sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{5a^4 b x^{13} \sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{a^5 x^{10} \sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (a^5*x^10*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (5*a^4*b*x^13*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^3*b^2*x^16*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^2*b^3*x^19*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a*b^4*x^22*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a + b*x^3)) + (b^5*x^25*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(25*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^9 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^9 (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^5b^5x^9 + 5a^4b^6x^{12} + 10a^3b^7x^{15} + 10a^2b^8x^{18} + 5ab^9x^{21} + b^{10}x^{24}) dx}{b^4 (ab + b^2x^3)} \\
&= \frac{a^5x^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)} + \frac{5a^4bx^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{5a^3b^2x^{16}\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{5a^2b^3x^{19}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{5ab^4x^{22}\sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)} + \frac{b^5x^{25}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{10} \sqrt{(a + bx^3)^2} (54340a^5 + 209000a^4bx^3 + 339625a^3b^2x^6 + 286000a^2b^3x^9 + 123500ab^4x^{12} + 21736b^5x^{15})}{543400(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^10*sqrt[(a + b*x^3)^2]*(54340*a^5 + 209000*a^4*b*x^3 + 339625*a^3*b^2*x^6 + 286000*a^2*b^3*x^9 + 123500*a*b^4*x^12 + 21736*b^5*x^15))/(543400*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.13, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (54340a^5x^{10} + 209000a^4bx^{13} + 339625a^3b^2x^{16} + 286000a^2b^3x^{19} + 123500ab^4x^{22} + 21736b^5x^{25})}{543400(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (sqrt[(a + b*x^3)^2]*(54340*a^5*x^10 + 209000*a^4*b*x^13 + 339625*a^3*b^2*x^16 + 286000*a^2*b^3*x^19 + 123500*a*b^4*x^22 + 21736*b^5*x^25))/(543400*(a + b*x^3))

fricas [A] time = 1.14, size = 57, normalized size = 0.22

$$\frac{1}{25} b^5 x^{25} + \frac{5}{22} ab^4 x^{22} + \frac{10}{19} a^2 b^3 x^{19} + \frac{5}{8} a^3 b^2 x^{16} + \frac{5}{13} a^4 b x^{13} + \frac{1}{10} a^5 x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5/13*a^4*b*x^13 + 1/10*a^5*x^10

giac [A] time = 0.40, size = 105, normalized size = 0.41

$$\frac{1}{25} b^5 x^{25} \operatorname{sgn}(bx^3 + a) + \frac{5}{22} ab^4 x^{22} \operatorname{sgn}(bx^3 + a) + \frac{10}{19} a^2 b^3 x^{19} \operatorname{sgn}(bx^3 + a) + \frac{5}{8} a^3 b^2 x^{16} \operatorname{sgn}(bx^3 + a) + \frac{5}{13} a^4 b x^{13} \operatorname{sgn}(bx^3 + a) + \frac{1}{10} a^5 x^{10} \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] 1/25*b^5*x^25*sgn(b*x^3 + a) + 5/22*a*b^4*x^22*sgn(b*x^3 + a) + 10/19*a^2*b^3*x^19*sgn(b*x^3 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^3 + a) + 5/13*a^4*b*x^13*sgn(b*x^3 + a) + 1/10*a^5*x^10*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(21736b^5x^{15} + 123500ab^4x^{12} + 286000a^2b^3x^9 + 339625a^3b^2x^6 + 209000a^4bx^3 + 54340a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^{10}}{543400 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] 1/543400*x^10*(21736*b^5*x^15+123500*a*b^4*x^12+286000*a^2*b^3*x^9+339625*a^3*b^2*x^6+209000*a^4*b*x^3+54340*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

maxima [A] time = 0.67, size = 57, normalized size = 0.22

$$\frac{1}{25} b^5 x^{25} + \frac{5}{22} ab^4 x^{22} + \frac{10}{19} a^2 b^3 x^{19} + \frac{5}{8} a^3 b^2 x^{16} + \frac{5}{13} a^4 b x^{13} + \frac{1}{10} a^5 x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5/13*a^4*b*x^13 + 1/10*a^5*x^10

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^9 (a^2 + 2 a b x^3 + b^2 x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**9*((a + b*x**3)**2)**(5/2), x)`

$$3.55 \quad \int x^8 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^7}{24b^3} - \frac{2a\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^6}{21b^3} + \frac{a^2\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{18b^3}$$

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^7}{24b^3} - \frac{2a\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^6}{21b^3} + \frac{a^2\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{18b^3}$$

Antiderivative was successfully verified.

[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] (a^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3) - (2*a*(a + b*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^3) + ((a + b*x^3)^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*b^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^8 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^8 (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int x^2 (ab + b^2x)^5 dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int \left(\frac{a^2(ab+b^2x)^5}{b^2} - \frac{2a(ab+b^2x)^6}{b^3} + \frac{(ab+b^2x)^7}{b^4}\right) dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= \frac{a^2 (a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18b^3} - \frac{2a (a + bx^3)^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{21b^3} + \frac{(a + bx^3)^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{21b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.70

$$\frac{x^9 \sqrt{(a + bx^3)^2} (56a^5 + 210a^4bx^3 + 336a^3b^2x^6 + 280a^2b^3x^9 + 120ab^4x^{12} + 21b^5x^{15})}{504(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^9*Sqrt[(a + b*x^3)^2]*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a^2*b^3*x^9 + 120*a*b^4*x^12 + 21*b^5*x^15))/(504*(a + b*x^3))

IntegrateAlgebraic [A] time = 14.51, size = 83, normalized size = 0.70

$$\frac{x^9 \sqrt{(a + bx^3)^2} (56a^5 + 210a^4bx^3 + 336a^3b^2x^6 + 280a^2b^3x^9 + 120ab^4x^{12} + 21b^5x^{15})}{504(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^9*Sqrt[(a + b*x^3)^2]*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a^2*b^3*x^9 + 120*a*b^4*x^12 + 21*b^5*x^15))/(504*(a + b*x^3))

fricas [A] time = 1.15, size = 57, normalized size = 0.48

$$\frac{1}{24} b^5 x^{24} + \frac{5}{21} a b^4 x^{21} + \frac{5}{9} a^2 b^3 x^{18} + \frac{2}{3} a^3 b^2 x^{15} + \frac{5}{12} a^4 b x^{12} + \frac{1}{9} a^5 x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/12*a^4*b*x^12 + 1/9*a^5*x^9

giac [A] time = 0.38, size = 105, normalized size = 0.88

$$\frac{1}{24} b^5 x^{24} \operatorname{sgn}(b x^3 + a) + \frac{5}{21} a b^4 x^{21} \operatorname{sgn}(b x^3 + a) + \frac{5}{9} a^2 b^3 x^{18} \operatorname{sgn}(b x^3 + a) + \frac{2}{3} a^3 b^2 x^{15} \operatorname{sgn}(b x^3 + a) + \frac{5}{12} a^4 b x^{12} \operatorname{sgn}(b x^3 + a) + \frac{1}{9} a^5 x^9 \operatorname{sgn}(b x^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] 1/24*b^5*x^24*sgn(b*x^3 + a) + 5/21*a*b^4*x^21*sgn(b*x^3 + a) + 5/9*a^2*b^3*x^18*sgn(b*x^3 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^3 + a) + 5/12*a^4*b*x^12*sgn(b*x^3 + a) + 1/9*a^5*x^9*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 80, normalized size = 0.67

$$\frac{(21b^5x^{15} + 120ab^4x^{12} + 280a^2b^3x^9 + 336a^3b^2x^6 + 210a^4bx^3 + 56a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^9}{504 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] 1/504*x^9*(21*b^5*x^15+120*a*b^4*x^12+280*a^2*b^3*x^9+336*a^3*b^2*x^6+210*a^4*b*x^3+56*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

maxima [A] time = 0.67, size = 114, normalized size = 0.96

$$\frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}} a^2 x^3}{18b^2} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}} x^3}{24b^2} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}} a^3}{18b^3} - \frac{3(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}} a}{56b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^2*x^3/b^2 + 1/24*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^3/b^3 - 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a/b^3

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**8*((a + b*x**3)**2)**(5/2), x)`

$$3.56 \quad \int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=255

$$\frac{b^5 x^{23} \sqrt{a^2 + 2abx^3 + b^2x^6}}{23(a + bx^3)} + \frac{ab^4 x^{20} \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{10a^2 b^3 x^{17} \sqrt{a^2 + 2abx^3 + b^2x^6}}{17(a + bx^3)} + \frac{a^5 x^8 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5 x^{23} \sqrt{a^2 + 2abx^3 + b^2x^6}}{23(a + bx^3)} + \frac{ab^4 x^{20} \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{10a^2 b^3 x^{17} \sqrt{a^2 + 2abx^3 + b^2x^6}}{17(a + bx^3)} + \frac{5a^3 b^2 x^{14} \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{5a^4 b x^{11} \sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{a^5 x^8 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (a^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a^4*b*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^3*b^2*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^7 (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^5b^5x^7 + 5a^4b^6x^{10} + 10a^3b^7x^{13} + 10a^2b^8x^{16} + 5ab^9x^{19} + b^{10}x^{22}) dx}{b^4 (ab + b^2x^3)} \\
&= \frac{a^5x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{5a^4bx^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{5a^3b^2x^{14}\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{5a^2b^3x^{17}\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{5ab^4x^{20}\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{b^5x^{23}\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^8 \sqrt{(a + bx^3)^2} (30107a^5 + 109480a^4bx^3 + 172040a^3b^2x^6 + 141680a^2b^3x^9 + 60214ab^4x^{12} + 10472b^5x^{15})}{240856(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^8*sqrt[(a + b*x^3)^2]*(30107*a^5 + 109480*a^4*b*x^3 + 172040*a^3*b^2*x^6 + 141680*a^2*b^3*x^9 + 60214*a*b^4*x^12 + 10472*b^5*x^15))/(240856*(a + b*x^3))

IntegrateAlgebraic [A] time = 13.72, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (30107a^5x^8 + 109480a^4bx^{11} + 172040a^3b^2x^{14} + 141680a^2b^3x^{17} + 60214ab^4x^{20} + 10472b^5x^{23})}{240856(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (sqrt[(a + b*x^3)^2]*(30107*a^5*x^8 + 109480*a^4*b*x^11 + 172040*a^3*b^2*x^14 + 141680*a^2*b^3*x^17 + 60214*a*b^4*x^20 + 10472*b^5*x^23))/(240856*(a + b*x^3))

fricas [A] time = 1.14, size = 57, normalized size = 0.22

$$\frac{1}{23} b^5 x^{23} + \frac{1}{4} ab^4 x^{20} + \frac{10}{17} a^2 b^3 x^{17} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{11} a^4 b x^{11} + \frac{1}{8} a^5 x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/11*a^4*b*x^11 + 1/8*a^5*x^8

giac [A] time = 0.34, size = 105, normalized size = 0.41

$$\frac{1}{23} b^5 x^{23} \operatorname{sgn}(bx^3 + a) + \frac{1}{4} ab^4 x^{20} \operatorname{sgn}(bx^3 + a) + \frac{10}{17} a^2 b^3 x^{17} \operatorname{sgn}(bx^3 + a) + \frac{5}{7} a^3 b^2 x^{14} \operatorname{sgn}(bx^3 + a) + \frac{5}{11} a^4 b x^{11} \operatorname{sgn}(bx^3 + a) + \frac{1}{8} a^5 x^8 \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] 1/23*b^5*x^23*sgn(b*x^3 + a) + 1/4*a*b^4*x^20*sgn(b*x^3 + a) + 10/17*a^2*b^3*x^17*sgn(b*x^3 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^3 + a) + 5/11*a^4*b*x^11*sgn(b*x^3 + a) + 1/8*a^5*x^8*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(10472b^5x^{15} + 60214ab^4x^{12} + 141680a^2b^3x^9 + 172040a^3b^2x^6 + 109480a^4bx^3 + 30107a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^8}{240856 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] 1/240856*x^8*(10472*b^5*x^15+60214*a*b^4*x^12+141680*a^2*b^3*x^9+172040*a^3*b^2*x^6+109480*a^4*b*x^3+30107*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

maxima [A] time = 0.77, size = 57, normalized size = 0.22

$$\frac{1}{23} b^5 x^{23} + \frac{1}{4} ab^4 x^{20} + \frac{10}{17} a^2 b^3 x^{17} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{11} a^4 b x^{11} + \frac{1}{8} a^5 x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/11*a^4*b*x^11 + 1/8*a^5*x^8

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**7*((a + b*x**3)**2)**(5/2), x)`

$$3.57 \quad \int x^6 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=255

$$\frac{b^5 x^{22} \sqrt{a^2 + 2abx^3 + b^2x^6}}{22(a + bx^3)} + \frac{5ab^4 x^{19} \sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{5a^2 b^3 x^{16} \sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{a^5 x^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5 x^{22} \sqrt{a^2 + 2abx^3 + b^2x^6}}{22(a + bx^3)} + \frac{5ab^4 x^{19} \sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{5a^2 b^3 x^{16} \sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{10a^3 b^2 x^{13} \sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{a^4 b x^{10} \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{a^5 x^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^4*b*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^3*b^2*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^2*b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int x^6 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^6 (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\ &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^5b^5x^6 + 5a^4b^6x^9 + 10a^3b^7x^{12} + 10a^2b^8x^{15} + 5ab^9x^{18} + b^{10}x^{21}) dx}{b^4 (ab + b^2x^3)} \\ &= \frac{a^5x^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{a^4bx^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{10a^3b^2x^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{5a^2b^3x^{16}\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{5ab^4x^{19}\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{b^5x^{22}\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^7 \sqrt{(a + bx^3)^2} (21736a^5 + 76076a^4bx^3 + 117040a^3b^2x^6 + 95095a^2b^3x^9 + 40040ab^4x^{12} + 6916b^5x^{15})}{152152(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^7*sqrt[(a + b*x^3)^2]*(21736*a^5 + 76076*a^4*b*x^3 + 117040*a^3*b^2*x^6 + 95095*a^2*b^3*x^9 + 40040*a*b^4*x^12 + 6916*b^5*x^15))/(152152*(a + b*x^3))

IntegrateAlgebraic [A] time = 12.98, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (21736a^5x^7 + 76076a^4bx^{10} + 117040a^3b^2x^{13} + 95095a^2b^3x^{16} + 40040ab^4x^{19} + 6916b^5x^{22})}{152152(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (sqrt[(a + b*x^3)^2]*(21736*a^5*x^7 + 76076*a^4*b*x^10 + 117040*a^3*b^2*x^13 + 95095*a^2*b^3*x^16 + 40040*a*b^4*x^19 + 6916*b^5*x^22))/(152152*(a + b*x^3))

fricas [A] time = 1.05, size = 57, normalized size = 0.22

$$\frac{1}{22} b^5 x^{22} + \frac{5}{19} ab^4 x^{19} + \frac{5}{8} a^2 b^3 x^{16} + \frac{10}{13} a^3 b^2 x^{13} + \frac{1}{2} a^4 b x^{10} + \frac{1}{7} a^5 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1/2*a^4*b*x^10 + 1/7*a^5*x^7

giac [A] time = 0.34, size = 105, normalized size = 0.41

$$\frac{1}{22} b^5 x^{22} \operatorname{sgn}(bx^3 + a) + \frac{5}{19} ab^4 x^{19} \operatorname{sgn}(bx^3 + a) + \frac{5}{8} a^2 b^3 x^{16} \operatorname{sgn}(bx^3 + a) + \frac{10}{13} a^3 b^2 x^{13} \operatorname{sgn}(bx^3 + a) + \frac{1}{2} a^4 b x^{10} \operatorname{sgn}(bx^3 + a) + \frac{1}{7} a^5 x^7 \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] 1/22*b^5*x^22*sgn(b*x^3 + a) + 5/19*a*b^4*x^19*sgn(b*x^3 + a) + 5/8*a^2*b^3*x^16*sgn(b*x^3 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^3 + a) + 1/2*a^4*b*x^10*sgn(b*x^3 + a) + 1/7*a^5*x^7*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(6916b^5x^{15} + 40040ab^4x^{12} + 95095a^2b^3x^9 + 117040a^3b^2x^6 + 76076a^4bx^3 + 21736a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^7}{152152 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] 1/152152*x^7*(6916*b^5*x^15+40040*a*b^4*x^12+95095*a^2*b^3*x^9+117040*a^3*b^2*x^6+76076*a^4*b*x^3+21736*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

maxima [A] time = 0.71, size = 57, normalized size = 0.22

$$\frac{1}{22} b^5 x^{22} + \frac{5}{19} ab^4 x^{19} + \frac{5}{8} a^2 b^3 x^{16} + \frac{10}{13} a^3 b^2 x^{13} + \frac{1}{2} a^4 b x^{10} + \frac{1}{7} a^5 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1/2*a^4*b*x^10 + 1/7*a^5*x^7

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**6*((a + b*x**3)**2)**(5/2), x)`

$$3.58 \quad \int x^5 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=78

$$\frac{(a + bx^3)^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{21b^2} - \frac{a(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18b^2}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{(a + bx^3)^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{21b^2} - \frac{a(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18b^2}$$

Antiderivative was successfully verified.

[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] -(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/((18*b^2) + ((a + b*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]))/(21*b^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^5 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^5 (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst} \left(\int x (ab + b^2x)^5 dx, x, x^3 \right)}{3b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst} \left(\int \left(-\frac{a(ab+b^2x)^5}{b} + \frac{(ab+b^2x)^6}{b^2} \right) dx, x, x^3 \right)}{3b^4 (ab + b^2x^3)} \\
&= -\frac{a (a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18b^2} + \frac{(a + bx^3)^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{21b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.06

$$\frac{x^6 \sqrt{(a + bx^3)^2} (21a^5 + 70a^4bx^3 + 105a^3b^2x^6 + 84a^2b^3x^9 + 35ab^4x^{12} + 6b^5x^{15})}{126(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^6*Sqrt[(a + b*x^3)^2]*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2*b^3*x^9 + 35*a*b^4*x^12 + 6*b^5*x^15))/(126*(a + b*x^3))

IntegrateAlgebraic [A] time = 11.58, size = 83, normalized size = 1.06

$$\frac{x^6 \sqrt{(a + bx^3)^2} (21a^5 + 70a^4bx^3 + 105a^3b^2x^6 + 84a^2b^3x^9 + 35ab^4x^{12} + 6b^5x^{15})}{126(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^6*Sqrt[(a + b*x^3)^2]*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2*b^3*x^9 + 35*a*b^4*x^12 + 6*b^5*x^15))/(126*(a + b*x^3))

fricas [A] time = 1.13, size = 57, normalized size = 0.73

$$\frac{1}{21} b^5 x^{21} + \frac{5}{18} ab^4 x^{18} + \frac{2}{3} a^2 b^3 x^{15} + \frac{5}{6} a^3 b^2 x^{12} + \frac{5}{9} a^4 b x^9 + \frac{1}{6} a^5 x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")`

[Out] $1/21*b^5*x^{21} + 5/18*a*b^4*x^{18} + 2/3*a^2*b^3*x^{15} + 5/6*a^3*b^2*x^{12} + 5/9*a^4*b*x^9 + 1/6*a^5*x^6$

giac [A] time = 0.29, size = 67, normalized size = 0.86

$$\frac{1}{126} \left(6b^5x^{21} + 35ab^4x^{18} + 84a^2b^3x^{15} + 105a^3b^2x^{12} + 70a^4bx^9 + 21a^5x^6 \right) \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

[Out] $1/126*(6*b^5*x^{21} + 35*a*b^4*x^{18} + 84*a^2*b^3*x^{15} + 105*a^3*b^2*x^{12} + 70*a^4*b*x^9 + 21*a^5*x^6)*\operatorname{sgn}(b*x^3 + a)$

maple [A] time = 0.01, size = 80, normalized size = 1.03

$$\frac{\left(6b^5x^{15} + 35a^4b^4x^{12} + 84a^2b^3x^9 + 105a^3b^2x^6 + 70a^4bx^3 + 21a^5 \right) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^6}{126 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)`

[Out] $1/126*x^6*(6*b^5*x^{15}+35*a*b^4*x^{12}+84*a^2*b^3*x^9+105*a^3*b^2*x^6+70*a^4*b*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5$

maxima [A] time = 0.63, size = 83, normalized size = 1.06

$$-\frac{\left(b^2x^6 + 2abx^3 + a^2 \right)^{\frac{5}{2}} ax^3}{18b} - \frac{\left(b^2x^6 + 2abx^3 + a^2 \right)^{\frac{5}{2}} a^2}{18b^2} + \frac{\left(b^2x^6 + 2abx^3 + a^2 \right)^{\frac{7}{2}}}{21b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")`

[Out] $-1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a*x^3/b - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^2/b^2 + 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/b^2$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**5*((a + b*x**3)**2)**(5/2), x)`

$$3.59 \quad \int x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=255

$$\frac{b^5 x^{20} \sqrt{a^2 + 2abx^3 + b^2x^6}}{20(a + bx^3)} + \frac{5ab^4 x^{17} \sqrt{a^2 + 2abx^3 + b^2x^6}}{17(a + bx^3)} + \frac{5a^2 b^3 x^{14} \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{a^5 x^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5 x^{20} \sqrt{a^2 + 2abx^3 + b^2x^6}}{20(a + bx^3)} + \frac{5ab^4 x^{17} \sqrt{a^2 + 2abx^3 + b^2x^6}}{17(a + bx^3)} + \frac{5a^2 b^3 x^{14} \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{10a^3 b^2 x^{11} \sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{5a^4 b x^8 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{a^5 x^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (5*a^4*b*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^3*b^2*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^2*b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (b^5*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^4 (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^5b^5x^4 + 5a^4b^6x^7 + 10a^3b^7x^{10} + 10a^2b^8x^{13} + 5ab^9x^{16} + b^{10}x^{19}) dx}{b^4 (ab + b^2x^3)} \\
&= \frac{a^5x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{5a^4bx^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{10a^3b^2x^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \dots
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^5 \sqrt{(a + bx^3)^2} (10472a^5 + 32725a^4bx^3 + 47600a^3b^2x^6 + 37400a^2b^3x^9 + 15400ab^4x^{12} + 2618b^5x^{15})}{52360(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^5*Sqrt[(a + b*x^3)^2]*(10472*a^5 + 32725*a^4*b*x^3 + 47600*a^3*b^2*x^6 + 37400*a^2*b^3*x^9 + 15400*a*b^4*x^12 + 2618*b^5*x^15))/(52360*(a + b*x^3))

IntegrateAlgebraic [A] time = 10.54, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (10472a^5x^5 + 32725a^4bx^8 + 47600a^3b^2x^{11} + 37400a^2b^3x^{14} + 15400ab^4x^{17} + 2618b^5x^{20})}{52360(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (Sqrt[(a + b*x^3)^2]*(10472*a^5*x^5 + 32725*a^4*b*x^8 + 47600*a^3*b^2*x^11 + 37400*a^2*b^3*x^14 + 15400*a*b^4*x^17 + 2618*b^5*x^20))/(52360*(a + b*x^3))

fricas [A] time = 1.33, size = 57, normalized size = 0.22

$$\frac{1}{20} b^5 x^{20} + \frac{5}{17} ab^4 x^{17} + \frac{5}{7} a^2 b^3 x^{14} + \frac{10}{11} a^3 b^2 x^{11} + \frac{5}{8} a^4 b x^8 + \frac{1}{5} a^5 x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5/8*a^4*b*x^8 + 1/5*a^5*x^5

giac [A] time = 0.42, size = 105, normalized size = 0.41

$$\frac{1}{20} b^5 x^{20} \operatorname{sgn}(b x^3 + a) + \frac{5}{17} a b^4 x^{17} \operatorname{sgn}(b x^3 + a) + \frac{5}{7} a^2 b^3 x^{14} \operatorname{sgn}(b x^3 + a) + \frac{10}{11} a^3 b^2 x^{11} \operatorname{sgn}(b x^3 + a) + \frac{5}{8} a^4 b x^8 \operatorname{sgn}(b x^3 + a) + \frac{1}{5} a^5 x^5 \operatorname{sgn}(b x^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] 1/20*b^5*x^20*sgn(b*x^3 + a) + 5/17*a*b^4*x^17*sgn(b*x^3 + a) + 5/7*a^2*b^3*x^14*sgn(b*x^3 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^3 + a) + 5/8*a^4*b*x^8*sgn(b*x^3 + a) + 1/5*a^5*x^5*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(2618b^5x^{15} + 15400ab^4x^{12} + 37400a^2b^3x^9 + 47600a^3b^2x^6 + 32725a^4bx^3 + 10472a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^5}{52360 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] 1/52360*x^5*(2618*b^5*x^15+15400*a*b^4*x^12+37400*a^2*b^3*x^9+47600*a^3*b^2*x^6+32725*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

maxima [A] time = 0.69, size = 57, normalized size = 0.22

$$\frac{1}{20} b^5 x^{20} + \frac{5}{17} a b^4 x^{17} + \frac{5}{7} a^2 b^3 x^{14} + \frac{10}{11} a^3 b^2 x^{11} + \frac{5}{8} a^4 b x^8 + \frac{1}{5} a^5 x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5/8*a^4*b*x^8 + 1/5*a^5*x^5

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 (a^2 + 2 a b x^3 + b^2 x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**4*((a + b*x**3)**2)**(5/2), x)`

$$3.60 \quad \int x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=252

$$\frac{b^5 x^{19} \sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{5ab^4 x^{16} \sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)} + \frac{10a^2 b^3 x^{13} \sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{a^5 x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5 x^{19} \sqrt{a^2 + 2abx^3 + b^2x^6}}{19(a + bx^3)} + \frac{5ab^4 x^{16} \sqrt{a^2 + 2abx^3 + b^2x^6}}{16(a + bx^3)} + \frac{10a^2 b^3 x^{13} \sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{a^5 x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{5a^4 b x^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{a^5 x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (a^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a^4*b*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^3*b^2*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a*b^4*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x^3 (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^5b^5x^3 + 5a^4b^6x^6 + 10a^3b^7x^9 + 10a^2b^8x^{12} + 5ab^9x^{15} + b^{10}x^{18}) dx}{b^4 (ab + b^2x^3)} \\
&= \frac{a^5x^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{5a^4bx^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{a^3b^2x^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^4 \sqrt{(a + bx^3)^2} (6916a^5 + 19760a^4bx^3 + 27664a^3b^2x^6 + 21280a^2b^3x^9 + 8645ab^4x^{12} + 1456b^5x^{15})}{27664(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^4*sqrt[(a + b*x^3)^2]*(6916*a^5 + 19760*a^4*b*x^3 + 27664*a^3*b^2*x^6 + 21280*a^2*b^3*x^9 + 8645*a*b^4*x^12 + 1456*b^5*x^15))/(27664*(a + b*x^3))

IntegrateAlgebraic [A] time = 10.38, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (6916a^5x^4 + 19760a^4bx^7 + 27664a^3b^2x^{10} + 21280a^2b^3x^{13} + 8645ab^4x^{16} + 1456b^5x^{19})}{27664(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (sqrt[(a + b*x^3)^2]*(6916*a^5*x^4 + 19760*a^4*b*x^7 + 27664*a^3*b^2*x^10 + 21280*a^2*b^3*x^13 + 8645*a*b^4*x^16 + 1456*b^5*x^19))/(27664*(a + b*x^3))

fricas [A] time = 1.16, size = 56, normalized size = 0.22

$$\frac{1}{19} b^5 x^{19} + \frac{5}{16} ab^4 x^{16} + \frac{10}{13} a^2 b^3 x^{13} + a^3 b^2 x^{10} + \frac{5}{7} a^4 b x^7 + \frac{1}{4} a^5 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] $1/19*b^5*x^{19} + 5/16*a*b^4*x^{16} + 10/13*a^2*b^3*x^{13} + a^3*b^2*x^{10} + 5/7*a^4*b*x^7 + 1/4*a^5*x^4$

giac [A] time = 0.32, size = 104, normalized size = 0.41

$$\frac{1}{19}b^5x^{19}\operatorname{sgn}(bx^3+a) + \frac{5}{16}ab^4x^{16}\operatorname{sgn}(bx^3+a) + \frac{10}{13}a^2b^3x^{13}\operatorname{sgn}(bx^3+a) + a^3b^2x^{10}\operatorname{sgn}(bx^3+a) + \frac{5}{7}a^4bx^7\operatorname{sgn}(bx^3+a) + \frac{1}{4}a^5x^4\operatorname{sgn}(bx^3+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

[Out] $1/19*b^5*x^{19}*\operatorname{sgn}(b*x^3 + a) + 5/16*a*b^4*x^{16}*\operatorname{sgn}(b*x^3 + a) + 10/13*a^2*b^3*x^{13}*\operatorname{sgn}(b*x^3 + a) + a^3*b^2*x^{10}*\operatorname{sgn}(b*x^3 + a) + 5/7*a^4*b*x^7*\operatorname{sgn}(b*x^3 + a) + 1/4*a^5*x^4*\operatorname{sgn}(b*x^3 + a)$

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(1456b^5x^{15} + 8645ab^4x^{12} + 21280a^2b^3x^9 + 27664a^3b^2x^6 + 19760a^4bx^3 + 6916a^5)\left((bx^3 + a)^2\right)^{\frac{5}{2}}x^4}{27664(bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)`

[Out] $1/27664*x^4*(1456*b^5*x^{15}+8645*a*b^4*x^{12}+21280*a^2*b^3*x^9+27664*a^3*b^2*x^6+19760*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5$

maxima [A] time = 0.51, size = 56, normalized size = 0.22

$$\frac{1}{19}b^5x^{19} + \frac{5}{16}ab^4x^{16} + \frac{10}{13}a^2b^3x^{13} + a^3b^2x^{10} + \frac{5}{7}a^4bx^7 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")`

[Out] $1/19*b^5*x^{19} + 5/16*a*b^4*x^{16} + 10/13*a^2*b^3*x^{13} + a^3*b^2*x^{10} + 5/7*a^4*b*x^7 + 1/4*a^5*x^4$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)`

```
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x^3 \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**(5/2), x)
```


$$3.61 \quad \int x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=36

$$\frac{(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^{5/2}}{18b}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1352, 609}

$$\frac{(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^{5/2}}{18b}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(18*b)

Rule 609

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x) * (a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{1}{3} \text{Subst} \left(\int (a^2 + 2abx + b^2x^2)^{5/2} dx, x, x^3 \right) \\ &= \frac{(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^{5/2}}{18b} \end{aligned}$$

Mathematica [B] time = 0.02, size = 82, normalized size = 2.28

$$\frac{x^3 \sqrt{(a + bx^3)^2 (6a^5 + 15a^4bx^3 + 20a^3b^2x^6 + 15a^2b^3x^9 + 6ab^4x^{12} + b^5x^{15})}}{18(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b^3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))

IntegrateAlgebraic [B] time = 9.63, size = 82, normalized size = 2.28

$$\frac{x^3 \sqrt{(a + bx^3)^2} (6a^5 + 15a^4bx^3 + 20a^3b^2x^6 + 15a^2b^3x^9 + 6ab^4x^{12} + b^5x^{15})}{18(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b^3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))

fricas [A] time = 0.94, size = 57, normalized size = 1.58

$$\frac{1}{18} b^5 x^{18} + \frac{1}{3} a b^4 x^{15} + \frac{5}{6} a^2 b^3 x^{12} + \frac{10}{9} a^3 b^2 x^9 + \frac{5}{6} a^4 b x^6 + \frac{1}{3} a^5 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] 1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*x^9 + 5/6*a^4*b*x^6 + 1/3*a^5*x^3

giac [B] time = 0.37, size = 66, normalized size = 1.83

$$\frac{1}{18} \left(3(bx^6 + 2ax^3)a^4 + 3(bx^6 + 2ax^3)^2 a^2 b + (bx^6 + 2ax^3)^3 b^2 \right) \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="giac")

[Out] 1/18*(3*(b*x^6 + 2*a*x^3)*a^4 + 3*(b*x^6 + 2*a*x^3)^2*a^2*b + (b*x^6 + 2*a*x^3)^3*b^2)*sgn(b*x^3 + a)

maple [B] time = 0.01, size = 79, normalized size = 2.19

$$\frac{(b^5 x^{15} + 6a b^4 x^{12} + 15a^2 b^3 x^9 + 20a^3 b^2 x^6 + 15a^4 b x^3 + 6a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x^3}{18(bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)`

[Out] $\frac{1}{18}x^3(b^5x^{15}+6a^2b^4x^{12}+15a^2b^3x^9+20a^3b^2x^6+15a^4b^2x^3+6a^5)((b^2x^3+a)^2)^{(5/2)}/(b^2x^3+a)^5$

maxima [A] time = 0.66, size = 52, normalized size = 1.44

$$\frac{1}{18} (b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}} x^3 + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}} a}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")`

[Out] $\frac{1}{18}(b^2x^6 + 2a^2bx^3 + a^2)^{(5/2)}x^3 + \frac{1}{18}(b^2x^6 + 2a^2bx^3 + a^2)^{(5/2)}a/b$

mupad [B] time = 1.24, size = 36, normalized size = 1.00

$$\frac{(b^2x^3 + ab)(a^2 + 2abx^3 + b^2x^6)^{5/2}}{18b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)`

[Out] $((a^2b + b^2x^3)(a^2 + b^2x^6 + 2abx^3)^{(5/2)})/(18b^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)`

[Out] `Integral(x**2*((a + b*x**3)**2)**(5/2), x)`

$$3.62 \quad \int x \left(a^2 + 2abx^3 + b^2x^6 \right)^{5/2} dx$$

Optimal. Leaf size=252

$$\frac{b^5 x^{17} \sqrt{a^2 + 2abx^3 + b^2x^6}}{17(a + bx^3)} + \frac{5ab^4 x^{14} \sqrt{a^2 + 2abx^3 + b^2x^6}}{14(a + bx^3)} + \frac{10a^2 b^3 x^{11} \sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{a^5 x^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}$$

Rubi [A] time = 0.05, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1355, 270}

$$\frac{b^5 x^{17} \sqrt{a^2 + 2abx^3 + b^2x^6}}{17(a + bx^3)} + \frac{5ab^4 x^{14} \sqrt{a^2 + 2abx^3 + b^2x^6}}{14(a + bx^3)} + \frac{10a^2 b^3 x^{11} \sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{5a^3 b^2 x^8 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{a^4 b x^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{a^5 x^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]

[Out] (a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (a^4*b*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a*b^4*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x(a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int x(ab + b^2x^3)^5 dx}{b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (a^5b^5x + 5a^4b^6x^4 + 10a^3b^7x^7 + 10a^2b^8x^{10} + 5ab^9x^{13} + \dots)}{b^4(ab + b^2x^3)} \\
&= \frac{a^5x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{a^4bx^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{5a^3b^2x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \dots
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^2 \sqrt{(a + bx^3)^2} (2618a^5 + 5236a^4bx^3 + 6545a^3b^2x^6 + 4760a^2b^3x^9 + 1870ab^4x^{12} + 308b^5x^{15})}{5236(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x^2*sqrt[(a + b*x^3)^2]*(2618*a^5 + 5236*a^4*b*x^3 + 6545*a^3*b^2*x^6 + 4760*a^2*b^3*x^9 + 1870*a*b^4*x^12 + 308*b^5*x^15))/(5236*(a + b*x^3))

IntegrateAlgebraic [A] time = 11.18, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (2618a^5x^2 + 5236a^4bx^5 + 6545a^3b^2x^8 + 4760a^2b^3x^{11} + 1870ab^4x^{14} + 308b^5x^{17})}{5236(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (sqrt[(a + b*x^3)^2]*(2618*a^5*x^2 + 5236*a^4*b*x^5 + 6545*a^3*b^2*x^8 + 4760*a^2*b^3*x^11 + 1870*a*b^4*x^14 + 308*b^5*x^17))/(5236*(a + b*x^3))

fricas [A] time = 0.81, size = 56, normalized size = 0.22

$$\frac{1}{17} b^5 x^{17} + \frac{5}{14} ab^4 x^{14} + \frac{10}{11} a^2 b^3 x^{11} + \frac{5}{4} a^3 b^2 x^8 + a^4 b x^5 + \frac{1}{2} a^5 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] $\frac{1}{17}b^5x^{17} + \frac{5}{14}ab^4x^{14} + \frac{10}{11}a^2b^3x^{11} + \frac{5}{4}a^3b^2x^8 + a^4bx^5 + \frac{1}{2}a^5x^2$

giac [A] time = 0.37, size = 104, normalized size = 0.41

$$\frac{1}{17}b^5x^{17}\operatorname{sgn}(bx^3+a) + \frac{5}{14}ab^4x^{14}\operatorname{sgn}(bx^3+a) + \frac{10}{11}a^2b^3x^{11}\operatorname{sgn}(bx^3+a) + \frac{5}{4}a^3b^2x^8\operatorname{sgn}(bx^3+a) + a^4bx^5\operatorname{sgn}(bx^3+a) + \frac{1}{2}a^5x^2\operatorname{sgn}(bx^3+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

[Out] $\frac{1}{17}b^5x^{17}\operatorname{sgn}(bx^3+a) + \frac{5}{14}ab^4x^{14}\operatorname{sgn}(bx^3+a) + \frac{10}{11}a^2b^3x^{11}\operatorname{sgn}(bx^3+a) + \frac{5}{4}a^3b^2x^8\operatorname{sgn}(bx^3+a) + a^4bx^5\operatorname{sgn}(bx^3+a) + \frac{1}{2}a^5x^2\operatorname{sgn}(bx^3+a)$

maple [A] time = 0.00, size = 80, normalized size = 0.32

$$\frac{(308b^5x^{15} + 1870ab^4x^{12} + 4760a^2b^3x^9 + 6545a^3b^2x^6 + 5236a^4bx^3 + 2618a^5)\left((bx^3+a)^2\right)^{\frac{5}{2}}x^2}{5236(bx^3+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)`

[Out] $\frac{1}{5236}x^2(308b^5x^{15}+1870ab^4x^{12}+4760a^2b^3x^9+6545a^3b^2x^6+5236a^4bx^3+2618a^5)\left((bx^3+a)^2\right)^{\frac{5}{2}}/(bx^3+a)^5$

maxima [A] time = 1.10, size = 56, normalized size = 0.22

$$\frac{1}{17}b^5x^{17} + \frac{5}{14}ab^4x^{14} + \frac{10}{11}a^2b^3x^{11} + \frac{5}{4}a^3b^2x^8 + a^4bx^5 + \frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")`

[Out] $\frac{1}{17}b^5x^{17} + \frac{5}{14}ab^4x^{14} + \frac{10}{11}a^2b^3x^{11} + \frac{5}{4}a^3b^2x^8 + a^4bx^5 + \frac{1}{2}a^5x^2$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x(a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left((a + bx^3)^2 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x*((a + b*x**3)**2)**(5/2), x)`

$$3.63 \quad \int (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=247

$$\frac{b^5x^{16}(a^2 + 2abx^3 + b^2x^6)^{5/2}}{16(a + bx^3)^5} + \frac{5ab^4x^{13}(a^2 + 2abx^3 + b^2x^6)^{5/2}}{13(a + bx^3)^5} + \frac{a^2b^3x^{10}(a^2 + 2abx^3 + b^2x^6)^{5/2}}{(a + bx^3)^5} + \frac{a^5x(a^2 + 2abx^3 + b^2x^6)^{5/2}}{(a + bx^3)^5}$$

Rubi [A] time = 0.05, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1343, 194}

$$\frac{b^5x^{16}(a^2 + 2abx^3 + b^2x^6)^{5/2}}{16(a + bx^3)^5} + \frac{5ab^4x^{13}(a^2 + 2abx^3 + b^2x^6)^{5/2}}{13(a + bx^3)^5} + \frac{a^2b^3x^{10}(a^2 + 2abx^3 + b^2x^6)^{5/2}}{(a + bx^3)^5} + \frac{10a^3b^2x^7(a^2 + 2abx^3 + b^2x^6)^{5/2}}{7(a + bx^3)^5} + \frac{5a^4bx^4(a^2 + 2abx^3 + b^2x^6)^{5/2}}{4(a + bx^3)^5} + \frac{a^5x(a^2 + 2abx^3 + b^2x^6)^{5/2}}{(a + bx^3)^5}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (a^5*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a^4*b*x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(4*(a + b*x^3)^5) + (10*a^3*b^2*x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(7*(a + b*x^3)^5) + (a^2*b^3*x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a*b^4*x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(13*(a + b*x^3)^5) + (b^5*x^16*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(16*(a + b*x^3)^5)

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 1343

Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2} \int (2ab + 2b^2x^3)^5 dx}{(2ab + 2b^2x^3)^5} \\
&= \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2} \int (32a^5b^5 + 160a^4b^6x^3 + 320a^3b^7x^6 + 320a^2b^8x^9 + 160a^1b^9x^{12}) dx}{(2ab + 2b^2x^3)^5} \\
&= \frac{a^5x(a^2 + 2abx^3 + b^2x^6)^{5/2}}{(a + bx^3)^5} + \frac{5a^4bx^4(a^2 + 2abx^3 + b^2x^6)^{5/2}}{4(a + bx^3)^5} + \frac{10a^3b^2x^7(a^2 + 2abx^3 + b^2x^6)^{5/2}}{7(a + bx^3)^5} + \frac{5a^2b^3x^{10}(a^2 + 2abx^3 + b^2x^6)^{5/2}}{4(a + bx^3)^5} + \frac{10ab^4x^{13}(a^2 + 2abx^3 + b^2x^6)^{5/2}}{7(a + bx^3)^5} + \frac{b^5x^{16}(a^2 + 2abx^3 + b^2x^6)^{5/2}}{4(a + bx^3)^5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 0.33

$$\frac{x\sqrt{(a + bx^3)^2} (1456a^5 + 1820a^4bx^3 + 2080a^3b^2x^6 + 1456a^2b^3x^9 + 560ab^4x^{12} + 91b^5x^{15})}{1456(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x*Sqrt[(a + b*x^3)^2]*(1456*a^5 + 1820*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1456*a^2*b^3*x^9 + 560*a*b^4*x^12 + 91*b^5*x^15))/(1456*(a + b*x^3))

IntegrateAlgebraic [A] time = 12.92, size = 81, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (1456a^5x + 1820a^4bx^4 + 2080a^3b^2x^7 + 1456a^2b^3x^{10} + 560ab^4x^{13} + 91b^5x^{16})}{1456(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (Sqrt[(a + b*x^3)^2]*(1456*a^5*x + 1820*a^4*b*x^4 + 2080*a^3*b^2*x^7 + 1456*a^2*b^3*x^10 + 560*a*b^4*x^13 + 91*b^5*x^16))/(1456*(a + b*x^3))

fricas [A] time = 1.29, size = 53, normalized size = 0.21

$$\frac{1}{16} b^5 x^{16} + \frac{5}{13} ab^4 x^{13} + a^2 b^3 x^{10} + \frac{10}{7} a^3 b^2 x^7 + \frac{5}{4} a^4 b x^4 + a^5 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4*b*x^4 + a^5*x

giac [A] time = 0.38, size = 101, normalized size = 0.41

$$\frac{1}{16} b^5 x^{16} \operatorname{sgn}(bx^3 + a) + \frac{5}{13} ab^4 x^{13} \operatorname{sgn}(bx^3 + a) + a^2 b^3 x^{10} \operatorname{sgn}(bx^3 + a) + \frac{10}{7} a^3 b^2 x^7 \operatorname{sgn}(bx^3 + a) + \frac{5}{4} a^4 b x^4 \operatorname{sgn}(bx^3 + a) + a^5 x \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] 1/16*b^5*x^16*sgn(b*x^3 + a) + 5/13*a*b^4*x^13*sgn(b*x^3 + a) + a^2*b^3*x^10*sgn(b*x^3 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^3 + a) + 5/4*a^4*b*x^4*sgn(b*x^3 + a) + a^5*x*sgn(b*x^3 + a)

maple [A] time = 0.00, size = 78, normalized size = 0.32

$$\frac{(91b^5x^{15} + 560ab^4x^{12} + 1456a^2b^3x^9 + 2080a^3b^2x^6 + 1820a^4bx^3 + 1456a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}} x}{1456 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] 1/1456*x*(91*b^5*x^15+560*a*b^4*x^12+1456*a^2*b^3*x^9+2080*a^3*b^2*x^6+1820*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5

maxima [A] time = 1.02, size = 53, normalized size = 0.21

$$\frac{1}{16} b^5 x^{16} + \frac{5}{13} ab^4 x^{13} + a^2 b^3 x^{10} + \frac{10}{7} a^3 b^2 x^7 + \frac{5}{4} a^4 b x^4 + a^5 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4*b*x^4 + a^5*x

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^2 + 2abx^3 + b^2x^6)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2), x)`

$$3.64 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x} dx$$

Optimal. Leaf size=251

$$\frac{b^5 x^{15} \sqrt{a^2 + 2abx^3 + b^2x^6}}{15(a + bx^3)} + \frac{5ab^4 x^{12} \sqrt{a^2 + 2abx^3 + b^2x^6}}{12(a + bx^3)} + \frac{10a^2 b^3 x^9 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{a^5 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^5 x^{15} \sqrt{a^2 + 2abx^3 + b^2x^6}}{15(a + bx^3)} + \frac{5ab^4 x^{12} \sqrt{a^2 + 2abx^3 + b^2x^6}}{12(a + bx^3)} + \frac{10a^2 b^3 x^9 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{5a^3 b^2 x^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{5a^4 b x^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{a^5 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]

[Out] (5*a^4*b*x^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^3*b^2*x^6*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (10*a^2*b^3*x^9*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (5*a*b^4*x^12*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (b^5*x^15*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*(a + b*x^3)) + (a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^5}{x} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^5}{x} dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \left(5a^4b^6 + \frac{a^5b^5}{x} + 10a^3b^7x + 10a^2b^8x^2 + 5ab^9x^3 + b^{10}x^4\right) dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= \frac{5a^4bx^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{5a^3b^2x^6\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{10a^2b^3x^9\sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{5ab^4x^{12}\sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{b^5x^{15}\sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (180a^5 \log(x) + bx^3 (300a^4 + 300a^3bx^3 + 200a^2b^2x^6 + 75ab^3x^9 + 12b^4x^{12}))}{180(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x, x]

[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(300*a^4 + 300*a^3*b*x^3 + 200*a^2*b^2*x^6 + 75*a*b^3*x^9 + 12*b^4*x^12) + 180*a^5*Log[x]))/(180*(a + b*x^3))

IntegrateAlgebraic [A] time = 0.58, size = 314, normalized size = 1.25

$$\frac{1}{66} a^5 \log(\sqrt{a^2 + 2abx^3 + b^2x^6} - a - \sqrt{a^2 + 2abx^3 + b^2x^6}) - \frac{a^5 (\sqrt{a^2 + b}) \log(\sqrt{a^2 + 2abx^3 + b^2x^6} + a - \sqrt{a^2 + 2abx^3 + b^2x^6})}{66} - \frac{a^5 \sqrt{a^2} \log(b \sqrt{a^2 + 2abx^3 + b^2x^6} - ab - b \sqrt{a^2 + 2abx^3 + b^2x^6})}{66} + \frac{1}{360} \sqrt{a^2 + 2abx^3 + b^2x^6} (137a^4 + 163a^3bx^3 + 137a^2b^2x^6 + 63ab^3x^9 + 12b^4x^{12}) + \frac{1}{360} (-300a^4\sqrt{b^2}x^3 - 300a^3b\sqrt{b^2}x^6 - 200a^2(b^2)^{3/2}x^9 - 75ab^3\sqrt{b^2}x^{12} - 12b^4\sqrt{b^2}x^{15})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x, x]

[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(137*a^4 + 163*a^3*b*x^3 + 137*a^2*b^2*x^6 + 63*a*b^3*x^9 + 12*b^4*x^12))/360 + (-300*a^4*Sqrt[b^2]*x^3 - 300*a^3*b*Sqrt[b^2]*x^6 - 200*a^2*(b^2)^(3/2)*x^9 - 75*a*b^3*Sqrt[b^2]*x^12 - 12*b^4*Sqrt[b^2]*x^15)

$\text{qrt}[b^2*x^{15}]/360 + (a^5*\text{Log}[-a - \text{Sqrt}[b^2]*x^3 + \text{Sqrt}[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (a^5*(b + \text{Sqrt}[b^2])*\text{Log}[a - \text{Sqrt}[b^2]*x^3 + \text{Sqrt}[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b) - (a^5*\text{Sqrt}[b^2]*\text{Log}[-(a*b) - b*\text{Sqrt}[b^2]*x^3 + b*\text{qrt}[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*b)$

fricas [A] time = 1.17, size = 55, normalized size = 0.22

$$\frac{1}{15} b^5 x^{15} + \frac{5}{12} a b^4 x^{12} + \frac{10}{9} a^2 b^3 x^9 + \frac{5}{3} a^3 b^2 x^6 + \frac{5}{3} a^4 b x^3 + a^5 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="fricas")

[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3*a^4*b*x^3 + a^5*log(x)

giac [A] time = 0.35, size = 104, normalized size = 0.41

$$\frac{1}{15} b^5 x^{15} \text{sgn}(bx^3 + a) + \frac{5}{12} a b^4 x^{12} \text{sgn}(bx^3 + a) + \frac{10}{9} a^2 b^3 x^9 \text{sgn}(bx^3 + a) + \frac{5}{3} a^3 b^2 x^6 \text{sgn}(bx^3 + a) + \frac{5}{3} a^4 b x^3 \text{sgn}(bx^3 + a) + a^5 \log(|x|) \text{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="giac")

[Out] 1/15*b^5*x^15*sgn(b*x^3 + a) + 5/12*a*b^4*x^12*sgn(b*x^3 + a) + 10/9*a^2*b^3*x^9*sgn(b*x^3 + a) + 5/3*a^3*b^2*x^6*sgn(b*x^3 + a) + 5/3*a^4*b*x^3*sgn(b*x^3 + a) + a^5*log(abs(x))*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 79, normalized size = 0.31

$$\frac{\left((bx^3 + a)^2 \right)^{\frac{5}{2}} \left(12b^5x^{15} + 75a b^4x^{12} + 200a^2b^3x^9 + 300a^3b^2x^6 + 300a^4bx^3 + 180a^5 \ln(x) \right)}{180 (bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x)

[Out] 1/180*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+75*a*b^4*x^12+200*a^2*b^3*x^9+300*a^3*b^2*x^6+300*a^4*b*x^3+180*a^5*ln(x))/(b*x^3+a)^5

maxima [A] time = 1.16, size = 206, normalized size = 0.82

$$\frac{1}{6} \sqrt{b^2x^6 + 2abx^3 + a^2} a^3 bx^3 + \frac{1}{3} (-1)^{2i\pi x^3 + 2ab} a^5 \log(2b^2x^3 + 2ab) - \frac{1}{3} (-1)^{2abx^3 + 2a^2} a^5 \log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right) + \frac{1}{12} (b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}} abx^3 + \frac{1}{2} \sqrt{b^2x^6 + 2abx^3 + a^2} a^4 + \frac{7}{36} (b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}} a^2 + \frac{1}{15} (b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="maxima")

[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^5*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*b*x^3 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^4 + 7/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x,x)

[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x, x)

$$3.65 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^2} dx$$

Optimal. Leaf size=251

$$\frac{b^5x^{14}\sqrt{a^2 + 2abx^3 + b^2x^6}}{14(a + bx^3)} + \frac{5ab^4x^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{5a^2b^3x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} +$$

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^{14}\sqrt{a^2 + 2abx^3 + b^2x^6}}{14(a + bx^3)} + \frac{5ab^4x^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{5a^2b^3x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{2a^3b^2x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{5a^4bx^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]

[Out] -((a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (5*a^4*b*x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (2*a^3*b^2*x^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^8*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a*b^4*x^11*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^5*x^14*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^2} dx}{b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5b^5}{x^2} + 5a^4b^6x + 10a^3b^7x^4 + 10a^2b^8x^7 + 5ab^9x^{10} + b^{10}x^{13} \right) dx}{b^4(ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} + \frac{5a^4bx^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{2a^3b^2x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-308a^5 + 770a^4bx^3 + 616a^3b^2x^6 + 385a^2b^3x^9 + 140ab^4x^{12} + 22b^5x^{15})}{308x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2*b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))

IntegrateAlgebraic [A] time = 16.65, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-308a^5 + 770a^4bx^3 + 616a^3b^2x^6 + 385a^2b^3x^9 + 140ab^4x^{12} + 22b^5x^{15})}{308x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2*b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))

fricas [A] time = 1.22, size = 59, normalized size = 0.24

$$\frac{22b^5x^{15} + 140ab^4x^{12} + 385a^2b^3x^9 + 616a^3b^2x^6 + 770a^4bx^3 - 308a^5}{308x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="fricas")

[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 770*a^4*b*x^3 - 308*a^5)/x

giac [A] time = 0.33, size = 105, normalized size = 0.42

$$\frac{1}{14} b^5 x^{14} \operatorname{sgn}(b x^3 + a) + \frac{5}{11} a b^4 x^{11} \operatorname{sgn}(b x^3 + a) + \frac{5}{4} a^2 b^3 x^8 \operatorname{sgn}(b x^3 + a) + 2 a^3 b^2 x^5 \operatorname{sgn}(b x^3 + a) + \frac{5}{2} a^4 b x^2 \operatorname{sgn}(b x^3 + a) - \frac{a^5 \operatorname{sgn}(b x^3 + a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="giac")

[Out] 1/14*b^5*x^14*sgn(b*x^3 + a) + 5/11*a*b^4*x^11*sgn(b*x^3 + a) + 5/4*a^2*b^3*x^8*sgn(b*x^3 + a) + 2*a^3*b^2*x^5*sgn(b*x^3 + a) + 5/2*a^4*b*x^2*sgn(b*x^3 + a) - a^5*sgn(b*x^3 + a)/x

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(-22b^5x^{15} - 140ab^4x^{12} - 385a^2b^3x^9 - 616a^3b^2x^6 - 770a^4bx^3 + 308a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}}}{308 (bx^3 + a)^5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x)

[Out] -1/308*(-22*b^5*x^15-140*a*b^4*x^12-385*a^2*b^3*x^9-616*a^3*b^2*x^6-770*a^4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x/(b*x^3+a)^5

maxima [A] time = 0.82, size = 59, normalized size = 0.24

$$\frac{22 b^5 x^{15} + 140 a b^4 x^{12} + 385 a^2 b^3 x^9 + 616 a^3 b^2 x^6 + 770 a^4 b x^3 - 308 a^5}{308 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="maxima")

[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 770*a^4*b*x^3 - 308*a^5)/x

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2 a b x^3 + b^2 x^6)^{5/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2, x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2, x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**2, x)`

$$3.66 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3} dx$$

Optimal. Leaf size=251

$$\frac{b^5x^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{ab^4x^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{10a^2b^3x^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} +$$

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^{13}\sqrt{a^2 + 2abx^3 + b^2x^6}}{13(a + bx^3)} + \frac{ab^4x^{10}\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{10a^2b^3x^7\sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{5a^3b^2x^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{5a^4bx\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]

[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (5*a^4*b*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a*b^4*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^3} dx}{b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(5a^4b^6 + \frac{a^5b^5}{x^3} + 10a^3b^7x^3 + 10a^2b^8x^6 + 5ab^9x^9 + b^{10}x^{12}\right)}{b^4(ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{5a^4bx\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{5a^3b^2x^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-91a^5 + 910a^4bx^3 + 455a^3b^2x^6 + 260a^2b^3x^9 + 91ab^4x^{12} + 14b^5x^{15})}{182x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*b^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.81, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-91a^5 + 910a^4bx^3 + 455a^3b^2x^6 + 260a^2b^3x^9 + 91ab^4x^{12} + 14b^5x^{15})}{182x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*b^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))

fricas [A] time = 1.22, size = 59, normalized size = 0.24

$$\frac{14b^5x^{15} + 91ab^4x^{12} + 260a^2b^3x^9 + 455a^3b^2x^6 + 910a^4bx^3 - 91a^5}{182x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="fricas")

[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 910*a^4*b*x^3 - 91*a^5)/x^2

giac [A] time = 0.36, size = 103, normalized size = 0.41

$$\frac{1}{13}b^5x^{13}\operatorname{sgn}(bx^3+a) + \frac{1}{2}ab^4x^{10}\operatorname{sgn}(bx^3+a) + \frac{10}{7}a^2b^3x^7\operatorname{sgn}(bx^3+a) + \frac{5}{2}a^3b^2x^4\operatorname{sgn}(bx^3+a) + 5a^4bx\operatorname{sgn}(bx^3+a) - \frac{a^5\operatorname{sgn}(bx^3+a)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="giac")

[Out] 1/13*b^5*x^13*sgn(b*x^3 + a) + 1/2*a*b^4*x^10*sgn(b*x^3 + a) + 10/7*a^2*b^3*x^7*sgn(b*x^3 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^3 + a) + 5*a^4*b*x*sgn(b*x^3 + a) - 1/2*a^5*sgn(b*x^3 + a)/x^2

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(-14b^5x^{15} - 91ab^4x^{12} - 260a^2b^3x^9 - 455a^3b^2x^6 - 910a^4bx^3 + 91a^5)\left((bx^3 + a)^2\right)^{\frac{5}{2}}}{182(bx^3 + a)^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x)

[Out] -1/182*(-14*b^5*x^15-91*a*b^4*x^12-260*a^2*b^3*x^9-455*a^3*b^2*x^6-910*a^4*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^2/(b*x^3+a)^5

maxima [A] time = 1.23, size = 59, normalized size = 0.24

$$\frac{14b^5x^{15} + 91ab^4x^{12} + 260a^2b^3x^9 + 455a^3b^2x^6 + 910a^4bx^3 - 91a^5}{182x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="maxima")

[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 910*a^4*b*x^3 - 91*a^5)/x^2

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**3, x)`

$$3.67 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^4} dx$$

Optimal. Leaf size=252

$$\frac{b^5 x^{12} \sqrt{a^2 + 2abx^3 + b^2x^6}}{12(a + bx^3)} + \frac{5ab^4 x^9 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{5a^2 b^3 x^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)} + \dots$$

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^5 x^{12} \sqrt{a^2 + 2abx^3 + b^2x^6}}{12(a + bx^3)} + \frac{5ab^4 x^9 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9(a + bx^3)} + \frac{5a^2 b^3 x^6 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{10a^3 b^2 x^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)} + \frac{5a^4 b \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^3*b^2*x^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^2*b^3*x^6*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^9*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (b^5*x^12*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(2*n_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^4} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^5}{x^4} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^5}{x^2} dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \left(10a^3b^7 + \frac{a^5b^5}{x^2} + \frac{5a^4b^6}{x} + 10a^2b^8x + 5ab^9x^2 + b^{10}\right) dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3 (a + bx^3)} + \frac{10a^3b^2x^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{5a^2b^3x^6\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a + bx^3)^2} (-12a^5 + 180a^4bx^3 \log(x) + 120a^3b^2x^6 + 60a^2b^3x^9 + 20ab^4x^{12} + 3b^5x^{15})}{36x^3 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4, x]

[Out] (Sqrt[(a + b*x^3)^2]*(-12*a^5 + 120*a^3*b^2*x^6 + 60*a^2*b^3*x^9 + 20*a*b^4*x^12 + 3*b^5*x^15 + 180*a^4*b*x^3*Log[x]))/(36*x^3*(a + b*x^3))

IntegrateAlgebraic [A] time = 0.98, size = 364, normalized size = 1.44

$$\frac{\frac{5}{6}a^4\sqrt{b}\log(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b}x^3)-\frac{5}{6}a^4\sqrt{b}\log(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b}x^3)+\frac{5}{6}a^4b\operatorname{tanh}^{-1}\left(\frac{\sqrt{b}x^3}{a-\sqrt{a^2+2abx^3+b^2x^6}}\right)-\frac{\sqrt{a^2+2abx^3+b^2x^6}(-192a^5b-395a^4b^2x^3+1920a^3b^3x^6+960a^2b^4x^9+320ab^5x^{12}+48b^6x^{15})}{576x^3(ab+b^2x^3)}+\sqrt{b}(192a^6+587a^5bx^3-1525a^4b^2x^6-2880a^3b^3x^9-1280a^2b^4x^{12}-368a^2b^5x^{15}-48b^6x^{18})}{576x^3(ab+b^2x^3)-576\sqrt{b}x^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4, x]

[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-192*a^5*b - 395*a^4*b^2*x^3 + 1920*a^3*b^3*x^6 + 960*a^2*b^4*x^9 + 320*a*b^5*x^12 + 48*b^6*x^15) + Sqrt[b^2]*(192*a^6 + 587*a^5*b*x^3 - 1525*a^4*b^2*x^6 - 2880*a^3*b^3*x^9 - 1280*a^2*b^4*x^12 - 368*a^2*b^5*x^15 - 48*b^6*x^18)) / (576*x^3*(ab + b^2*x^3) - 576*sqrt(b)*x^3*sqrt(a^2 + 2*ab*x^3 + b^2*x^6))

$$2 - 368*a*b^5*x^15 - 48*b^6*x^18)/(576*x^3*(a*b + b^2*x^3) - 576*\text{Sqrt}[b^2]*x^3*\text{Sqrt}[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*a^4*b*\text{ArcTanh}[(\text{Sqrt}[b^2]*x^3)/a - \text{Sqrt}[a^2 + 2*a*b*x^3 + b^2*x^6]/a])/3 - (5*a^4*\text{Sqrt}[b^2]*\text{Log}[-a - \text{Sqrt}[b^2]*x^3 + \text{Sqrt}[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (5*a^4*\text{Sqrt}[b^2]*\text{Log}[a - \text{Sqrt}[b^2]*x^3 + \text{Sqrt}[a^2 + 2*a*b*x^3 + b^2*x^6]])/6$$

fricas [A] time = 0.98, size = 61, normalized size = 0.24

$$\frac{3b^5x^{15} + 20ab^4x^{12} + 60a^2b^3x^9 + 120a^3b^2x^6 + 180a^4bx^3 \log(x) - 12a^5}{36x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="fricas")

[Out] 1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*a^4*b*x^3*log(x) - 12*a^5)/x^3

giac [A] time = 0.31, size = 124, normalized size = 0.49

$$\frac{1}{12}b^5x^{12}\text{sgn}(bx^3+a) + \frac{5}{9}ab^4x^9\text{sgn}(bx^3+a) + \frac{5}{3}a^2b^3x^6\text{sgn}(bx^3+a) + \frac{10}{3}a^3b^2x^3\text{sgn}(bx^3+a) + 5a^4b \log(|x|)\text{sgn}(bx^3+a) - \frac{5a^4bx^3\text{sgn}(bx^3+a) + a^5\text{sgn}(bx^3+a)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="giac")

[Out] 1/12*b^5*x^12*sgn(b*x^3 + a) + 5/9*a*b^4*x^9*sgn(b*x^3 + a) + 5/3*a^2*b^3*x^6*sgn(b*x^3 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^3 + a) + 5*a^4*b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(5*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^3

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left((bx^3 + a)^2\right)^{\frac{5}{2}} \left(3b^5x^{15} + 20ab^4x^{12} + 60a^2b^3x^9 + 120a^3b^2x^6 + 180a^4bx^3 \ln(x) - 12a^5\right)}{36(bx^3 + a)^5 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x)

[Out] 1/36*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+20*a*b^4*x^12+60*a^2*b^3*x^9+120*a^3*b^2*x^6+180*a^4*b*ln(x)*x^3-12*a^5)/(b*x^3+a)^5/x^3

maxima [A] time = 1.20, size = 214, normalized size = 0.85

$$\frac{5}{6}\sqrt{b^2x^6+2abx^3+a^2}a^2b^2x^3 + \frac{5}{3}(-1)^{2b^2x^3+2ab}a^4b \log(2b^2x^3+2ab) - \frac{5}{3}(-1)^{2abx^3+2a^2}a^4b \log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right) + \frac{5}{12}(b^2x^6+2abx^3+a^2)^{\frac{3}{2}}b^2x^3 + \frac{5}{2}\sqrt{b^2x^6+2abx^3+a^2}a^2a^3b + \frac{35}{36}(b^2x^6+2abx^3+a^2)^{\frac{3}{2}}ab - \frac{(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="maxima")

[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2*x^3 + 5/3*(-1)^(2*b^2*x^3 + 2*a*b)*a^4*b*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^4*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2*x^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b + 35/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*b - 1/3*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2 a b x^3 + b^2 x^6)^{5/2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4,x)

[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + b x^3)^2\right)^{\frac{5}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**4, x)

$$3.68 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} dx$$

Optimal. Leaf size=249

$$\frac{b^5x^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{5ab^4x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{2a^2b^3x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^{11}\sqrt{a^2 + 2abx^3 + b^2x^6}}{11(a + bx^3)} + \frac{5ab^4x^8\sqrt{a^2 + 2abx^3 + b^2x^6}}{8(a + bx^3)} + \frac{2a^2b^3x^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{5a^3b^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^3*b^2*x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (2*a^2*b^3*x^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a*b^4*x^8*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^5*x^11*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[Exp andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^5} dx}{b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5b^5}{x^5} + \frac{5a^4b^6}{x^2} + 10a^3b^7x + 10a^2b^8x^4 + 5ab^9x^7 + b^{10}x^{10} \right) dx}{b^4(ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} + \frac{5a^3b^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-22a^5 - 440a^4bx^3 + 440a^3b^2x^6 + 176a^2b^3x^9 + 55ab^4x^{12} + 8b^5x^{15})}{88x^4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*b^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.55, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-22a^5 - 440a^4bx^3 + 440a^3b^2x^6 + 176a^2b^3x^9 + 55ab^4x^{12} + 8b^5x^{15})}{88x^4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*b^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))

fricas [A] time = 1.34, size = 59, normalized size = 0.24

$$\frac{8b^5x^{15} + 55ab^4x^{12} + 176a^2b^3x^9 + 440a^3b^2x^6 - 440a^4bx^3 - 22a^5}{88x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="fricas")

[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*a^4*b*x^3 - 22*a^5)/x^4

giac [A] time = 0.35, size = 107, normalized size = 0.43

$$\frac{1}{11}b^5x^{11}\operatorname{sgn}(bx^3+a) + \frac{5}{8}ab^4x^8\operatorname{sgn}(bx^3+a) + 2a^2b^3x^5\operatorname{sgn}(bx^3+a) + 5a^3b^2x^2\operatorname{sgn}(bx^3+a) - \frac{20a^4bx^3\operatorname{sgn}(bx^3+a) + a^5\operatorname{sgn}(bx^3+a)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="giac")

[Out] 1/11*b^5*x^11*sgn(b*x^3 + a) + 5/8*a*b^4*x^8*sgn(b*x^3 + a) + 2*a^2*b^3*x^5*sgn(b*x^3 + a) + 5*a^3*b^2*x^2*sgn(b*x^3 + a) - 1/4*(20*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^4

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(-8b^5x^{15} - 55ab^4x^{12} - 176a^2b^3x^9 - 440a^3b^2x^6 + 440a^4bx^3 + 22a^5)\left((bx^3 + a)^2\right)^{\frac{5}{2}}}{88(bx^3 + a)^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x)

[Out] -1/88*(-8*b^5*x^15-55*a*b^4*x^12-176*a^2*b^3*x^9-440*a^3*b^2*x^6+440*a^4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^4/(b*x^3+a)^5

maxima [A] time = 0.97, size = 59, normalized size = 0.24

$$\frac{8b^5x^{15} + 55ab^4x^{12} + 176a^2b^3x^9 + 440a^3b^2x^6 - 440a^4bx^3 - 22a^5}{88x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="maxima")

[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*a^4*b*x^3 - 22*a^5)/x^4

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5, x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5, x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**5, x)`

$$3.69 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^6} dx$$

Optimal. Leaf size=251

$$\frac{b^5 x^{10} \sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)} + \frac{5ab^4 x^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{5a^2 b^3 x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5 x^{10} \sqrt{a^2 + 2abx^3 + b^2x^6}}{10(a + bx^3)} + \frac{5ab^4 x^7 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7(a + bx^3)} + \frac{5a^2 b^3 x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{10a^3 b^2 x \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (10*a^3*b^2*x*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^7*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (b^5*x^10*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[Exp andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^6} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^6} dx}{b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(10a^3b^7 + \frac{a^5b^5}{x^6} + \frac{5a^4b^6}{x^3} + 10a^2b^8x^3 + 5ab^9x^6 + b^{10}x^9\right) dx}{b^4(ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{10a^3b^2x\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-14a^5 - 175a^4bx^3 + 700a^3b^2x^6 + 175a^2b^3x^9 + 50ab^4x^{12} + 7b^5x^{15})}{70x^5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*b^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.31, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-14a^5 - 175a^4bx^3 + 700a^3b^2x^6 + 175a^2b^3x^9 + 50ab^4x^{12} + 7b^5x^{15})}{70x^5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*b^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))

fricas [A] time = 0.71, size = 59, normalized size = 0.24

$$\frac{7b^5x^{15} + 50ab^4x^{12} + 175a^2b^3x^9 + 700a^3b^2x^6 - 175a^4bx^3 - 14a^5}{70x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="fricas")

[Out] $1/70*(7*b^5*x^{15} + 50*a*b^4*x^{12} + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*a^4*b*x^3 - 14*a^5)/x^5$

giac [A] time = 0.34, size = 106, normalized size = 0.42

$$\frac{1}{10} b^5 x^{10} \operatorname{sgn}(b x^3 + a) + \frac{5}{7} a b^4 x^7 \operatorname{sgn}(b x^3 + a) + \frac{5}{2} a^2 b^3 x^4 \operatorname{sgn}(b x^3 + a) + 10 a^3 b^2 x \operatorname{sgn}(b x^3 + a) - \frac{25 a^4 b x^3 \operatorname{sgn}(b x^3 + a) + 2 a^5 \operatorname{sgn}(b x^3 + a)}{10 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="giac")

[Out] $1/10*b^5*x^{10}*sgn(b*x^3 + a) + 5/7*a*b^4*x^7*sgn(b*x^3 + a) + 5/2*a^2*b^3*x^4*sgn(b*x^3 + a) + 10*a^3*b^2*x*sgn(b*x^3 + a) - 1/10*(25*a^4*b*x^3*sgn(b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^5$

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(-7b^5x^{15} - 50ab^4x^{12} - 175a^2b^3x^9 - 700a^3b^2x^6 + 175a^4bx^3 + 14a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}}}{70(bx^3 + a)^5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x)

[Out] $-1/70*(-7*b^5*x^{15}-50*a*b^4*x^{12}-175*a^2*b^3*x^9-700*a^3*b^2*x^6+175*a^4*b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^5/(b*x^3+a)^5$

maxima [A] time = 0.67, size = 59, normalized size = 0.24

$$\frac{7b^5x^{15} + 50ab^4x^{12} + 175a^2b^3x^9 + 700a^3b^2x^6 - 175a^4bx^3 - 14a^5}{70x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="maxima")

[Out] $1/70*(7*b^5*x^{15} + 50*a*b^4*x^{12} + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*a^4*b*x^3 - 14*a^5)/x^5$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6, x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6, x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**6, x)`

$$3.70 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^7} dx$$

Optimal. Leaf size=252

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)}$$

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)} + \frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} + \frac{10a^3b^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^2*b^3*x^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^6*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b^5*x^9*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (10*a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^7} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^5}{x^7} dx}{b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^5}{x^3} dx, x, x^3\right)}{3b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \left(10a^2b^8 + \frac{a^5b^5}{x^3} + \frac{5a^4b^6}{x^2} + \frac{10a^3b^7}{x} + 5ab^9x + b^{10}x^2\right) dx, x, x^3\right)}{3b^4(ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)} + \frac{10a^2b^3x^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a + bx^3)^2} (-3a^5 - 30a^4bx^3 + 180a^3b^2x^6 \log(x) + 60a^2b^3x^9 + 15ab^4x^{12} + 2b^5x^{15})}{18x^6(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7, x]

[Out] (Sqrt[(a + b*x^3)^2]*(-3*a^5 - 30*a^4*b*x^3 + 60*a^2*b^3*x^9 + 15*a*b^4*x^12 + 2*b^5*x^15 + 180*a^3*b^2*x^6*Log[x]))/(18*x^6*(a + b*x^3))

IntegrateAlgebraic [A] time = 1.16, size = 368, normalized size = 1.46

$$-\frac{5}{3}a^3b\sqrt{b}\log(\sqrt{a^2+2abx^3+b^2x^6}-a-\sqrt{b^2x^3})-\frac{5}{3}a^3b\sqrt{b}\log(\sqrt{a^2+2abx^3+b^2x^6}+a-\sqrt{b^2x^3})+\frac{10}{3}a^3b^2\operatorname{tanh}^{-1}\left(\frac{\sqrt{b^2x^3}}{a}-\frac{\sqrt{a^2+2abx^3+b^2x^6}}{a}\right)+\frac{\sqrt{a^2+2abx^3+b^2x^6}(-6a^5b-60a^4b^2x^3+53a^3b^3x^6+120a^2b^4x^9+30ab^5x^{12}+4b^6x^{15})+\sqrt{b^2}(6a^6+66a^5b^2x^3+7a^4b^3x^6-173a^3b^4x^9-150a^2b^5x^{12}-34ab^6x^{15}-4b^7x^{18})}{36a^4(ab+b^2x^3)-36\sqrt{b^2}x^6\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7, x]

[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-6*a^5*b - 60*a^4*b^2*x^3 + 53*a^3*b^3*x^6 + 120*a^2*b^4*x^9 + 30*a*b^5*x^12 + 4*b^6*x^15) + Sqrt[b^2]*(6*a^6 + 66*a^5*b*x^3 + 7*a^4*b^2*x^6 - 173*a^3*b^3*x^9 - 150*a^2*b^4*x^12 - 34*a*b^5*x^15 - 4*b^6*x^18))

$$\frac{15 - 4b^6x^{18}}{(36x^6(ab + b^2x^3) - 36\sqrt{b^2}x^6\sqrt{a^2 + 2abx^3 + b^2x^6}) + (10a^3b^2\text{ArcTanh}(\frac{\sqrt{b^2}x^3}{a - \sqrt{a^2 + 2abx^3 + b^2x^6}})/a - \sqrt{a^2 + 2abx^3 + b^2x^6})/3 - (5a^3b\sqrt{b^2}\text{Log}[-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}])/3 - (5a^3b\sqrt{b^2}\text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}])/3}$$

fricas [A] time = 1.31, size = 61, normalized size = 0.24

$$\frac{2b^5x^{15} + 15ab^4x^{12} + 60a^2b^3x^9 + 180a^3b^2x^6 \log(x) - 30a^4bx^3 - 3a^5}{18x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="fricas")

[Out] 1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x) - 30*a^4*b*x^3 - 3*a^5)/x^6

giac [A] time = 0.37, size = 126, normalized size = 0.50

$$\frac{1}{9}b^5x^9\text{sgn}(bx^3+a) + \frac{5}{6}ab^4x^6\text{sgn}(bx^3+a) + \frac{10}{3}a^2b^3x^3\text{sgn}(bx^3+a) + 10a^3b^2\log(|x|)\text{sgn}(bx^3+a) - \frac{30a^3b^2x^6\text{sgn}(bx^3+a) + 10a^4bx^3\text{sgn}(bx^3+a) + a^5\text{sgn}(bx^3+a)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="giac")

[Out] 1/9*b^5*x^9*sgn(b*x^3 + a) + 5/6*a*b^4*x^6*sgn(b*x^3 + a) + 10/3*a^2*b^3*x^3*sgn(b*x^3 + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(30*a^3*b^2*x^6*sgn(b*x^3 + a) + 10*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^6

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left((bx^3 + a)^2\right)^{\frac{5}{2}} \left(2b^5x^{15} + 15ab^4x^{12} + 60a^2b^3x^9 + 180a^3b^2x^6 \ln(x) - 30a^4bx^3 - 3a^5\right)}{18(bx^3 + a)^5 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x)

[Out] 1/18*((b*x^3+a)^2)^(5/2)*(2*b^5*x^15+15*a*b^4*x^12+60*a^2*b^3*x^9+180*a^3*b^2*ln(x)*x^6-30*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^6

maxima [A] time = 1.24, size = 282, normalized size = 1.12

$$\frac{5\sqrt{b^2x^6 + 2abx^3 + a^2}ab^3x^3 + \frac{10}{3}(-1)^{2abx^3+2ab}a^3b^2\log(2b^2x^3 + 2ab) - \frac{10}{3}(-1)^{2abx^3+2ab}a^3b^2\log\left(\frac{2abx^3 + 2a^2}{|x| + \sqrt{b^2x^6 + 2abx^3 + a^2}}\right) + \frac{5(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^3x^3}{6a} + 5\sqrt{b^2x^6 + 2abx^3 + a^2}a^2b^2 + \frac{35}{18}(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^2 + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^2}{6a^2} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b}{2ax^3} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}}{6a^2x^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="maxima")

[Out] 5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*b)*a^3*b^2*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*b^2*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3*x^3/a + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2 + 35/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2 + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/a^2 - 1/2*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^6)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7,x)

[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{5/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**7, x)

$$3.71 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^8} dx$$

Optimal. Leaf size=248

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8(a+bx^3)} + \frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]

[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^2*b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[Exp andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^8} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^8} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5 b^5}{x^8} + \frac{5a^4 b^6}{x^5} + \frac{10a^3 b^7}{x^2} + 10a^2 b^8 x + 5ab^9 x^4 + b^{10} x^7 \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7 (a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4 (a + bx^3)} - \frac{10a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-8a^5 - 70a^4bx^3 - 560a^3b^2x^6 + 280a^2b^3x^9 + 56ab^4x^{12} + 7b^5x^{15})}{56x^7 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^3*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.57, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-8a^5 - 70a^4bx^3 - 560a^3b^2x^6 + 280a^2b^3x^9 + 56ab^4x^{12} + 7b^5x^{15})}{56x^7 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^3*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))

fricas [A] time = 1.14, size = 59, normalized size = 0.24

$$\frac{7b^5x^{15} + 56ab^4x^{12} + 280a^2b^3x^9 - 560a^3b^2x^6 - 70a^4bx^3 - 8a^5}{56x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="fricas")

[Out] $\frac{1}{56}*(7*b^5*x^{15} + 56*a*b^4*x^{12} + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a^4*b*x^3 - 8*a^5)/x^7$

giac [A] time = 0.33, size = 107, normalized size = 0.43

$$\frac{1}{8}b^5x^8\operatorname{sgn}(bx^3+a) + ab^4x^5\operatorname{sgn}(bx^3+a) + 5a^2b^3x^2\operatorname{sgn}(bx^3+a) - \frac{280a^3b^2x^6\operatorname{sgn}(bx^3+a) + 35a^4bx^3\operatorname{sgn}(bx^3+a) + 4a^5\operatorname{sgn}(bx^3+a)}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="giac")

[Out] $\frac{1}{8}b^5x^8\operatorname{sgn}(bx^3+a) + a*b^4x^5\operatorname{sgn}(bx^3+a) + 5*a^2*b^3*x^2*\operatorname{sgn}(bx^3+a) - \frac{1}{28}*(280*a^3*b^2*x^6*\operatorname{sgn}(bx^3+a) + 35*a^4*b*x^3*\operatorname{sgn}(bx^3+a) + 4*a^5*\operatorname{sgn}(bx^3+a))/x^7$

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(-7b^5x^{15} - 56ab^4x^{12} - 280a^2b^3x^9 + 560a^3b^2x^6 + 70a^4bx^3 + 8a^5)\left((bx^3+a)^2\right)^{\frac{5}{2}}}{56(bx^3+a)^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x)

[Out] $-1/56*(-7*b^5*x^{15}-56*a*b^4*x^{12}-280*a^2*b^3*x^9+560*a^3*b^2*x^6+70*a^4*b*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^7/(b*x^3+a)^5$

maxima [A] time = 1.10, size = 59, normalized size = 0.24

$$\frac{7b^5x^{15} + 56ab^4x^{12} + 280a^2b^3x^9 - 560a^3b^2x^6 - 70a^4bx^3 - 8a^5}{56x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="maxima")

[Out] $\frac{1}{56}*(7*b^5*x^{15} + 56*a*b^4*x^{12} + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a^4*b*x^3 - 8*a^5)/x^7$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8, x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8, x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**8, x)`

$$3.72 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^9} dx$$

Optimal. Leaf size=247

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} + \frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} + \frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)} - \frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} + \frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)} + \frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]

[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[Exp andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^9} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^9} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(10a^2b^8 + \frac{a^5b^5}{x^9} + \frac{5a^4b^6}{x^6} + \frac{10a^3b^7}{x^3} + 5ab^9x^3 + b^{10}x^6\right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8 (a + bx^3)} - \frac{a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5 (a + bx^3)} - \frac{5a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2 (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a + bx^3)^2} (-7a^5 - 56a^4bx^3 - 280a^3b^2x^6 + 560a^2b^3x^9 + 70ab^4x^{12} + 8b^5x^{15})}{56x^8 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 560*a^2*b^3*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.47, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a + bx^3)^2} (-7a^5 - 56a^4bx^3 - 280a^3b^2x^6 + 560a^2b^3x^9 + 70ab^4x^{12} + 8b^5x^{15})}{56x^8 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 560*a^2*b^3*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3))

fricas [A] time = 1.29, size = 59, normalized size = 0.24

$$\frac{8b^5x^{15} + 70ab^4x^{12} + 560a^2b^3x^9 - 280a^3b^2x^6 - 56a^4bx^3 - 7a^5}{56x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="fricas")

[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a^4*b*x^3 - 7*a^5)/x^8

giac [A] time = 0.34, size = 105, normalized size = 0.43

$$\frac{1}{7}b^5x^7\operatorname{sgn}(bx^3+a) + \frac{5}{4}ab^4x^4\operatorname{sgn}(bx^3+a) + 10a^2b^3x\operatorname{sgn}(bx^3+a) - \frac{40a^3b^2x^6\operatorname{sgn}(bx^3+a) + 8a^4bx^3\operatorname{sgn}(bx^3+a) + a^5\operatorname{sgn}(bx^3+a)}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="giac")

[Out] 1/7*b^5*x^7*sgn(b*x^3 + a) + 5/4*a*b^4*x^4*sgn(b*x^3 + a) + 10*a^2*b^3*x*sgn(b*x^3 + a) - 1/8*(40*a^3*b^2*x^6*sgn(b*x^3 + a) + 8*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^8

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(-8b^5x^{15} - 70ab^4x^{12} - 560a^2b^3x^9 + 280a^3b^2x^6 + 56a^4bx^3 + 7a^5)\left((bx^3 + a)^2\right)^{\frac{5}{2}}}{56(bx^3 + a)^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x)

[Out] -1/56*(-8*b^5*x^15-70*a*b^4*x^12-560*a^2*b^3*x^9+280*a^3*b^2*x^6+56*a^4*b*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^8/(b*x^3+a)^5

maxima [A] time = 1.33, size = 59, normalized size = 0.24

$$\frac{8b^5x^{15} + 70ab^4x^{12} + 560a^2b^3x^9 - 280a^3b^2x^6 - 56a^4bx^3 - 7a^5}{56x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="maxima")

[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a^4*b*x^3 - 7*a^5)/x^8

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9, x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**9, x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**9, x)`

$$3.73 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{10}} dx$$

Optimal. Leaf size=252

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} + \frac{10a^2b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}$$

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)} + \frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} + \frac{10a^2b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10, x]

[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{10}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^5}{x^{10}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^5}{x^4} dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \left(5ab^9 + \frac{a^5b^5}{x^4} + \frac{5a^4b^6}{x^3} + \frac{10a^3b^7}{x^2} + \frac{10a^2b^8}{x} + b^{10}x\right) dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9 (a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{6x^6 (a + bx^3)} - \frac{10a^3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3 (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a + bx^3)^2} (-2a^5 - 15a^4bx^3 - 60a^3b^2x^6 + 180a^2b^3x^9 \log(x) + 30ab^4x^{12} + 3b^5x^{15})}{18x^9 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10, x]

[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^5 - 15*a^4*b*x^3 - 60*a^3*b^2*x^6 + 30*a*b^4*x^12 + 3*b^5*x^15 + 180*a^2*b^3*x^9*Log[x]))/(18*x^9*(a + b*x^3))

IntegrateAlgebraic [A] time = 1.82, size = 366, normalized size = 1.45

$$\frac{\frac{5}{3}a^2(b^3)^{3/2} \log(\sqrt{a^2 + 2abx^3 + b^2x^6} - a - \sqrt{b^2x^3}) - \frac{5}{3}a^2(b^3)^{3/2} \log(\sqrt{a^2 + 2abx^3 + b^2x^6} + a - \sqrt{b^2x^3}) + \frac{10}{3}a^2b^3 \operatorname{tanh}^{-1}\left(\frac{\sqrt{b^2x^3}}{a} - \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{a}\right) + \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (-8a^5b - 60a^4b^2x^3 - 240a^3b^3x^6 - 391a^2b^4x^9 + 120ab^5x^{12} + 12b^6x^{15}) + \sqrt{b^2} (8a^6 + 68a^5b^3 + 300a^4b^2x^6 + 631a^3b^3x^9 + 271a^2b^4x^{12} - 132ab^5x^{15} - 12b^6x^{18})}{72a^2 (ab + b^2x^3) - 72\sqrt{b^2} a \sqrt{a^2 + 2abx^3 + b^2x^6}}}{1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10, x]

[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^5*b - 60*a^4*b^2*x^3 - 240*a^3*b^3*x^6 - 391*a^2*b^4*x^9 + 120*a*b^5*x^12 + 12*b^6*x^15) + Sqrt[b^2]*(8*a^6 + 68*a^5*b*x^3 + 300*a^4*b^2*x^6 + 631*a^3*b^3*x^9 + 271*a^2*b^4*x^12 - 132*a*b^5*x^15 - 12*b^6*x^18))/72/a^2/(ab + b^2*x^3) - 72*sqrt(b^2)*a*sqrt(a^2 + 2*a*b*x^3 + b^2*x^6)

$$\frac{b^5 x^{15} - 12 b^6 x^{18}}{(72 x^9 (a b + b^2 x^3) - 72 \sqrt{b^2} x^9 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}) + (10 a^2 b^3 \operatorname{ArcTanh}(\frac{\sqrt{b^2} x^3}{a} - \sqrt{a^2 + 2 a b x^3 + b^2 x^6}) / a) / 3 - (5 a^2 (b^2)^{(3/2)} \operatorname{Log}[-a - \sqrt{b^2} x^3 + \sqrt{a^2 + 2 a b x^3 + b^2 x^6}]) / 3 - (5 a^2 (b^2)^{(3/2)} \operatorname{Log}[a - \sqrt{b^2} x^3 + \sqrt{a^2 + 2 a b x^3 + b^2 x^6}]) / 3}$$

fricas [A] time = 1.72, size = 61, normalized size = 0.24

$$\frac{3 b^5 x^{15} + 30 a b^4 x^{12} + 180 a^2 b^3 x^9 \log(x) - 60 a^3 b^2 x^6 - 15 a^4 b x^3 - 2 a^5}{18 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="fricas")

[Out] 1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6 - 15*a^4*b*x^3 - 2*a^5)/x^9

giac [A] time = 0.40, size = 127, normalized size = 0.50

$$\frac{1}{6} b^5 x^6 \operatorname{sgn}(b x^3 + a) + \frac{5}{3} a b^4 x^3 \operatorname{sgn}(b x^3 + a) + 10 a^2 b^3 \log(|x|) \operatorname{sgn}(b x^3 + a) - \frac{110 a^2 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 60 a^3 b^2 x^6 \operatorname{sgn}(b x^3 + a) + 15 a^4 b x^3 \operatorname{sgn}(b x^3 + a) + 2 a^5 \operatorname{sgn}(b x^3 + a)}{18 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="giac")

[Out] 1/6*b^5*x^6*sgn(b*x^3 + a) + 5/3*a*b^4*x^3*sgn(b*x^3 + a) + 10*a^2*b^3*log(abs(x))*sgn(b*x^3 + a) - 1/18*(110*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*x^6*sgn(b*x^3 + a) + 15*a^4*b*x^3*sgn(b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^9

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left((b x^3 + a)^2 \right)^{\frac{5}{2}} \left(3 b^5 x^{15} + 30 a b^4 x^{12} + 180 a^2 b^3 x^9 \ln(x) - 60 a^3 b^2 x^6 - 15 a^4 b x^3 - 2 a^5 \right)}{18 (b x^3 + a)^5 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x)

[Out] 1/18*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+30*a*b^4*x^12+180*a^2*b^3*ln(x))*x^9-60*a^3*b^2*x^6-15*a^4*b*x^3-2*a^5)/(b*x^3+a)^5/x^9

maxima [A] time = 0.80, size = 313, normalized size = 1.24

$$\frac{5 \sqrt{6 x^6 + 2 a b x^3 + a^2} b^4 x^3 + \frac{10}{3} (-1)^{2 b^2 + 2 a b} a^2 b^3 \log(2 b^2 x^3 + 2 a b) - \frac{10}{3} (-1)^{2 a b^2 + 2 a^2} a^2 b^3 \log\left(\frac{2 a b x}{|x|} + \frac{2 a^2}{x^2 |x|}\right) + \frac{5 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} b^4 x^3}{6 a^2} + 5 \sqrt{6 x^6 + 2 a b x^3 + a^2} a b^3 + \frac{35 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{3}{2}} b^3}{18 a} + \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} b^3}{18 a^3} - \frac{11 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} b^2}{18 a^2 x^3} - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} b}{18 a^2 x^6} - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}}}{9 a^2 x^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="maxima")

[Out] $\frac{5}{3}\sqrt{b^2x^6 + 2abx^3 + a^2}b^4x^3 + \frac{10}{3}(-1)^{(2b^2x^3 + 2ab)}a^2b^3\log(2b^2x^3 + 2ab) - \frac{10}{3}(-1)^{(2abx^3 + 2a^2)}a^2b^3\log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right) + \frac{5}{6}(b^2x^6 + 2abx^3 + a^2)^{(3/2)}b^4x^3/a^2 + 5\sqrt{b^2x^6 + 2abx^3 + a^2}ab^3 + \frac{35}{18}(b^2x^6 + 2abx^3 + a^2)^{(3/2)}b^3/a + \frac{1}{18}(b^2x^6 + 2abx^3 + a^2)^{(5/2)}b^3/a^3 - \frac{11}{18}(b^2x^6 + 2abx^3 + a^2)^{(5/2)}b^2/(a^2x^3) - \frac{1}{18}(b^2x^6 + 2abx^3 + a^2)^{(7/2)}b/(a^3x^6) - \frac{1}{9}(b^2x^6 + 2abx^3 + a^2)^{(7/2)}/(a^2x^9)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10,x)

[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{5/2}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**10, x)

$$3.74 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{11}} dx$$

Optimal. Leaf size=253

$$\frac{b^5 x^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)} + \frac{5ab^4 x^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} - \frac{10a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{5a^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10}(a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{5a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^4(a + bx^3)} - \frac{10a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} + \frac{5ab^4 x^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} + \frac{b^5 x^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11, x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (10*a^2*b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a*b^4*x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^5*x^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{11}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{11}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5 b^5}{x^{11}} + \frac{5a^4 b^6}{x^8} + \frac{10a^3 b^7}{x^5} + \frac{10a^2 b^8}{x^2} + 5ab^9 x + b^{10} x^4 \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{10x^{10} (a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7 (a + bx^3)} - \frac{5a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^4 (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (7a^5 + 50a^4bx^3 + 175a^3b^2x^6 + 700a^2b^3x^9 - 175ab^4x^{12} - 14b^5x^{15})}{70x^{10} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]

[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a^5 + 50*a^4*b*x^3 + 175*a^3*b^2*x^6 + 700*a^2*b^3*x^9 - 175*a*b^4*x^12 - 14*b^5*x^15))/(x^10*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.36, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-7a^5 - 50a^4bx^3 - 175a^3b^2x^6 - 700a^2b^3x^9 + 175ab^4x^{12} + 14b^5x^{15})}{70x^{10} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 50*a^4*b*x^3 - 175*a^3*b^2*x^6 - 700*a^2*b^3*x^9 + 175*a*b^4*x^12 + 14*b^5*x^15))/(70*x^10*(a + b*x^3))

fricas [A] time = 1.20, size = 59, normalized size = 0.23

$$\frac{14 b^5 x^{15} + 175 a b^4 x^{12} - 700 a^2 b^3 x^9 - 175 a^3 b^2 x^6 - 50 a^4 b x^3 - 7 a^5}{70 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="fricas")

[Out] $1/70*(14*b^5*x^{15} + 175*a*b^4*x^{12} - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50*a^4*b*x^3 - 7*a^5)/x^{10}$

giac [A] time = 0.40, size = 108, normalized size = 0.43

$$\frac{1}{5}b^5x^5\operatorname{sgn}(bx^3+a) + \frac{5}{2}ab^4x^2\operatorname{sgn}(bx^3+a) - \frac{700a^2b^3x^9\operatorname{sgn}(bx^3+a) + 175a^3b^2x^6\operatorname{sgn}(bx^3+a) + 50a^4bx^3\operatorname{sgn}(bx^3+a) + 7a^5\operatorname{sgn}(bx^3+a)}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="giac")

[Out] $1/5*b^5*x^5*\operatorname{sgn}(b*x^3 + a) + 5/2*a*b^4*x^2*\operatorname{sgn}(b*x^3 + a) - 1/70*(700*a^2*b^3*x^9*\operatorname{sgn}(b*x^3 + a) + 175*a^3*b^2*x^6*\operatorname{sgn}(b*x^3 + a) + 50*a^4*b*x^3*\operatorname{sgn}(b*x^3 + a) + 7*a^5*\operatorname{sgn}(b*x^3 + a))/x^{10}$

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(-14b^5x^{15} - 175ab^4x^{12} + 700a^2b^3x^9 + 175a^3b^2x^6 + 50a^4bx^3 + 7a^5)\left((bx^3 + a)^2\right)^{\frac{5}{2}}}{70(bx^3 + a)^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x)

[Out] $-1/70*(-14*b^5*x^{15}-175*a*b^4*x^{12}+700*a^2*b^3*x^9+175*a^3*b^2*x^6+50*a^4*b*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/x^{10}/(b*x^3+a)^5$

maxima [A] time = 1.01, size = 59, normalized size = 0.23

$$\frac{14b^5x^{15} + 175ab^4x^{12} - 700a^2b^3x^9 - 175a^3b^2x^6 - 50a^4bx^3 - 7a^5}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="maxima")

[Out] $1/70*(14*b^5*x^{15} + 175*a*b^4*x^{12} - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50*a^4*b*x^3 - 7*a^5)/x^{10}$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11, x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{11}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**11, x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**11, x)`

$$3.75 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{12}} dx$$

Optimal. Leaf size=247

$$\frac{b^5 x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)} + \frac{5ab^4 x \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{5a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2(a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{5a^4 b}{11x^{11}(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)} - \frac{2a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5(a + bx^3)} - \frac{5a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^2(a + bx^3)} + \frac{5ab^4 x \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} + \frac{b^5 x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]

[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (2*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{12}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{12}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(5ab^9 + \frac{a^5b^5}{x^{12}} + \frac{5a^4b^6}{x^9} + \frac{10a^3b^7}{x^6} + \frac{10a^2b^8}{x^3} + b^{10}x^3 \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11} (a + bx^3)} - \frac{5a^4b \sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8 (a + bx^3)} - \frac{2a^3b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5 (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a + bx^3)^2} (8a^5 + 55a^4bx^3 + 176a^3b^2x^6 + 440a^2b^3x^9 - 440ab^4x^{12} - 22b^5x^{15})}{88x^{11} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]

[Out] -1/88*(Sqrt[(a + b*x^3)^2]*(8*a^5 + 55*a^4*b*x^3 + 176*a^3*b^2*x^6 + 440*a^2*b^3*x^9 - 440*a*b^4*x^12 - 22*b^5*x^15))/(x^11*(a + b*x^3))

IntegrateAlgebraic [A] time = 18.20, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a + bx^3)^2} (-8a^5 - 55a^4bx^3 - 176a^3b^2x^6 - 440a^2b^3x^9 + 440ab^4x^{12} + 22b^5x^{15})}{88x^{11} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 55*a^4*b*x^3 - 176*a^3*b^2*x^6 - 440*a^2*b^3*x^9 + 440*a*b^4*x^12 + 22*b^5*x^15))/(88*x^11*(a + b*x^3))

fricas [A] time = 1.08, size = 59, normalized size = 0.24

$$\frac{22b^5x^{15} + 440ab^4x^{12} - 440a^2b^3x^9 - 176a^3b^2x^6 - 55a^4bx^3 - 8a^5}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="fricas")

[Out] $\frac{1}{88}*(22*b^5*x^{15} + 440*a*b^4*x^{12} - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55*a^4*b*x^3 - 8*a^5)/x^{11}$

giac [A] time = 0.35, size = 106, normalized size = 0.43

$$\frac{1}{4}b^5x^4\operatorname{sgn}(bx^3+a) + 5ab^4x\operatorname{sgn}(bx^3+a) - \frac{440a^2b^3x^9\operatorname{sgn}(bx^3+a) + 176a^3b^2x^6\operatorname{sgn}(bx^3+a) + 55a^4bx^3\operatorname{sgn}(bx^3+a) + 8a^5\operatorname{sgn}(bx^3+a)}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="giac")

[Out] $\frac{1}{4}*b^5*x^4*\operatorname{sgn}(b*x^3 + a) + 5*a*b^4*x*\operatorname{sgn}(b*x^3 + a) - 1/88*(440*a^2*b^3*x^9*\operatorname{sgn}(b*x^3 + a) + 176*a^3*b^2*x^6*\operatorname{sgn}(b*x^3 + a) + 55*a^4*b*x^3*\operatorname{sgn}(b*x^3 + a) + 8*a^5*\operatorname{sgn}(b*x^3 + a))/x^{11}$

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(-22b^5x^{15} - 440ab^4x^{12} + 440a^2b^3x^9 + 176a^3b^2x^6 + 55a^4bx^3 + 8a^5)\left((bx^3 + a)^2\right)^{\frac{5}{2}}}{88(bx^3 + a)^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x)

[Out] $-1/88*(-22*b^5*x^{15}-440*a*b^4*x^{12}+440*a^2*b^3*x^9+176*a^3*b^2*x^6+55*a^4*b*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^{11}/(b*x^3+a)^5$

maxima [A] time = 0.97, size = 59, normalized size = 0.24

$$\frac{22b^5x^{15} + 440ab^4x^{12} - 440a^2b^3x^9 - 176a^3b^2x^6 - 55a^4bx^3 - 8a^5}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="maxima")

[Out] $\frac{1}{88}*(22*b^5*x^{15} + 440*a*b^4*x^{12} - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55*a^4*b*x^3 - 8*a^5)/x^{11}$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12,x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**12,x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**12, x)`

$$3.76 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{13}} dx$$

Optimal. Leaf size=252

$$\frac{b^5 x^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{5ab^4 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{10a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12x^{12}(a + bx^3)}$$

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12x^{12}(a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9(a + bx^3)} - \frac{5a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^6(a + bx^3)} - \frac{10a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3(a + bx^3)} + \frac{b^5 x^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3(a + bx^3)} + \frac{5ab^4 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (10*a^2*b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b^5*x^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{13}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^5}{x^{13}} dx}{b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^5}{x^5} dx, x, x^3\right)}{3b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \left(b^{10} + \frac{a^5b^5}{x^5} + \frac{5a^4b^6}{x^4} + \frac{10a^3b^7}{x^3} + \frac{10a^2b^8}{x^2} + \frac{5ab^9}{x}\right) dx, x, x^3\right)}{3b^4(ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{12x^{12}(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9(a + bx^3)} - \frac{5a^3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^6(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a + bx^3)^2} (3a^5 + 20a^4bx^3 + 60a^3b^2x^6 + 120a^2b^3x^9 - 180ab^4x^{12} \log(x) - 12b^5x^{15})}{36x^{12}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]

[Out] -1/36*(Sqrt[(a + b*x^3)^2]*(3*a^5 + 20*a^4*b*x^3 + 60*a^3*b^2*x^6 + 120*a^2*b^3*x^9 - 12*b^5*x^15 - 180*a*b^4*x^12*Log[x]))/(x^12*(a + b*x^3))

IntegrateAlgebraic [B] time = 3.02, size = 2031, normalized size = 8.06

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]

[Out] (-2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^8*b + 29*a^7*b^2*x^3 + 129*a^6*b^3*x^6 + 363*a^5*b^4*x^9 + 554*a^4*b^5*x^12 + 390*a^3*b^6*x^15 + 66*a^2*b^7*x^18 - 42*a*b^8*x^21 - 12*b^9*x^24) - 2*b^3*Sqrt[b^2]*(-3*a^9 - 32*a^8*b

$$\begin{aligned}
& *x^3 - 158*a^7*b^2*x^6 - 492*a^6*b^3*x^9 - 917*a^5*b^4*x^{12} - 944*a^4*b^5*x^{15} - 456*a^3*b^6*x^{18} - 24*a^2*b^7*x^{21} + 54*a*b^8*x^{24} + 12*b^9*x^{27})/(9 \\
& *Sqrt[b^2]*x^{12}*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^3*b^3 - 24*a^2*b^4*x^3 - 24*a*b^5*x^6 - 8*b^6*x^9) + 9*x^{12}*(8*a^4*b^4 + 32*a^3*b^5*x^3 + 48*a^2 \\
& *b^6*x^6 + 32*a*b^7*x^9 + 8*b^8*x^{12})) + (5*a*b^4*Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (5*a*b^3*Sqrt[b^2]*Log[-a - Sqrt[b^2] \\
& *x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/6 - (5*a^9*b^4*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + \\
& 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) - (5*a^9*b^3*Sqrt[b^2]*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + \\
& b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) + (10*a^7*b^4*(-(Sqrt \\
& [b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2*Log[a - Sqrt[b^2]*x^3 + Sqr \\
& t[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^ \\
& 3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) + \\
& (10*a^7*b^3*Sqrt[b^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^ \\
& 2*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*(-a - Sqrt[b \\
& ^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 \\
& + 2*a*b*x^3 + b^2*x^6])^4) - (5*a^5*b^4*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a* \\
& b*x^3 + b^2*x^6])^4*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6] \\
&])/((-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2] \\
& *x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) - (5*a^5*b^3*Sqrt[b^2]*(-(Sqrt[b \\
& ^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*Log[a - Sqrt[b^2]*x^3 + Sqrt[\\
& a^2 + 2*a*b*x^3 + b^2*x^6]])/((-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + \\
& b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) + (10* \\
& a^3*b^4*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^6*Log[a - Sqrt \\
& [b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*(-a - Sqrt[b^2]*x^3 + Sqrt \\
& [a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + \\
& b^2*x^6])^4) + (10*a^3*b^3*Sqrt[b^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x \\
& ^3 + b^2*x^6])^6*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/ \\
& (3*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]* \\
& x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) - (5*a*b^4*(-(Sqrt[b^2]*x^3) + Sq \\
& rt[a^2 + 2*a*b*x^3 + b^2*x^6])^8*Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x \\
& ^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4 \\
& *(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^4) - (5*a*b^3*Sqrt[b \\
& ^2]*(-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^8*Log[a - Sqrt[b^2 \\
&]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(6*(-a - Sqrt[b^2]*x^3 + Sqrt[a^2 \\
& + 2*a*b*x^3 + b^2*x^6])^4*(a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2* \\
& x^6])^4)
\end{aligned}$$

fricas [A] time = 1.17, size = 61, normalized size = 0.24

$$\frac{12b^5x^{15} + 180ab^4x^{12}\log(x) - 120a^2b^3x^9 - 60a^3b^2x^6 - 20a^4bx^3 - 3a^5}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="fricas")

[Out] 1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x^6 - 20*a^4*b*x^3 - 3*a^5)/x^12

giac [A] time = 0.38, size = 125, normalized size = 0.50

$$\frac{1}{3} b^5 x^3 \operatorname{sgn}(bx^3 + a) + 5 ab^4 \log(|x|) \operatorname{sgn}(bx^3 + a) - \frac{125 ab^4 x^{12} \operatorname{sgn}(bx^3 + a) + 120 a^2 b^3 x^9 \operatorname{sgn}(bx^3 + a) + 60 a^3 b^2 x^6 \operatorname{sgn}(bx^3 + a) + 20 a^4 b x^3 \operatorname{sgn}(bx^3 + a) + 3 a^5 \operatorname{sgn}(bx^3 + a)}{36 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="giac")

[Out] 1/3*b^5*x^3*sgn(b*x^3 + a) + 5*a*b^4*log(abs(x))*sgn(b*x^3 + a) - 1/36*(125*a*b^4*x^12*sgn(b*x^3 + a) + 120*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*x^6*sgn(b*x^3 + a) + 20*a^4*b*x^3*sgn(b*x^3 + a) + 3*a^5*sgn(b*x^3 + a))/x^12

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left((bx^3 + a)^2 \right)^{\frac{5}{2}} \left(12b^5x^{15} + 180ab^4x^{12} \ln(x) - 120a^2b^3x^9 - 60a^3b^2x^6 - 20a^4bx^3 - 3a^5 \right)}{36 (bx^3 + a)^5 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x)

[Out] 1/36*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+180*a*b^4*ln(x)*x^12-120*a^2*b^3*x^9-60*a^3*b^2*x^6-20*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^12

maxima [A] time = 1.10, size = 342, normalized size = 1.36

$$\frac{5\sqrt{bx^6+2abx^3+a^2}b^5x^3}{6a} + \frac{5}{3}(-1)^{2b^2x^3+2ab}ab^4\log(2b^2x^3+2ab) - \frac{5}{3}(-1)^{2b^2x^3+2ab}ab^4\log\left(\frac{2abx+2a^2}{|x|-|a|}\right) + \frac{5(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}b^5x^3}{12a^2} + \frac{5}{2}\sqrt{bx^6+2abx^3+a^2}b^4 + \frac{35(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}b^4}{36a^2} + \frac{(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}b^4}{9a^4} - \frac{2(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}b^4}{9a^2x^3} - \frac{(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}b^4}{9a^4x^6} + \frac{(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}b^4}{36a^2x^9} - \frac{(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}}{12a^2x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="maxima")

[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5*x^3/a + 5/3*(-1)^(2*b^2*x^3 + 2*a*b)*a*b^4*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a*b^4*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^5*x^3/a^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4 + 35/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4/a^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/a^4 - 2/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a^3*x^3) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^6) + 1/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^9) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^12)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13, x)

[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{5/2}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13, x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**13, x)

$$3.77 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{14}} dx$$

Optimal. Leaf size=253

$$\frac{b^5 x^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)} - \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} - \frac{5a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^4(a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)} - \frac{a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)} - \frac{a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^{10}(a + bx^3)} - \frac{10a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{5a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^4(a + bx^3)} - \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} + \frac{b^5 x^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (10*a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^2*b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (5*a*b^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^5*x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{14}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{14}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5 b^5}{x^{14}} + \frac{5a^4 b^6}{x^{11}} + \frac{10a^3 b^7}{x^8} + \frac{10a^2 b^8}{x^5} + \frac{5ab^9}{x^2} + b^{10} x \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13} (a + bx^3)} - \frac{a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^{10} (a + bx^3)} - \frac{10a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7 (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a + bx^3)^2} (14a^5 + 91a^4bx^3 + 260a^3b^2x^6 + 455a^2b^3x^9 + 910ab^4x^{12} - 91b^5x^{15})}{182x^{13} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]

[Out] -1/182*(Sqrt[(a + b*x^3)^2]*(14*a^5 + 91*a^4*b*x^3 + 260*a^3*b^2*x^6 + 455*a^2*b^3*x^9 + 910*a*b^4*x^12 - 91*b^5*x^15))/(x^13*(a + b*x^3))

IntegrateAlgebraic [A] time = 19.40, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-14a^5 - 91a^4bx^3 - 260a^3b^2x^6 - 455a^2b^3x^9 - 910ab^4x^{12} + 91b^5x^{15})}{182x^{13} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 91*a^4*b*x^3 - 260*a^3*b^2*x^6 - 455*a^2*b^3*x^9 - 910*a*b^4*x^12 + 91*b^5*x^15))/(182*x^13*(a + b*x^3))

fricas [A] time = 1.18, size = 59, normalized size = 0.23

$$\frac{91 b^5 x^{15} - 910 a b^4 x^{12} - 455 a^2 b^3 x^9 - 260 a^3 b^2 x^6 - 91 a^4 b x^3 - 14 a^5}{182 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="fricas")

[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 91*a^4*b*x^3 - 14*a^5)/x^13

giac [A] time = 0.33, size = 108, normalized size = 0.43

$$\frac{1}{2} b^5 x^2 \operatorname{sgn}(b x^3 + a) - \frac{910 a b^4 x^{12} \operatorname{sgn}(b x^3 + a) + 455 a^2 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 260 a^3 b^2 x^6 \operatorname{sgn}(b x^3 + a) + 91 a^4 b x^3 \operatorname{sgn}(b x^3 + a) + 14 a^5 \operatorname{sgn}(b x^3 + a)}{182 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="giac")

[Out] 1/2*b^5*x^2*sgn(b*x^3 + a) - 1/182*(910*a*b^4*x^12*sgn(b*x^3 + a) + 455*a^2*b^3*x^9*sgn(b*x^3 + a) + 260*a^3*b^2*x^6*sgn(b*x^3 + a) + 91*a^4*b*x^3*sgn(b*x^3 + a) + 14*a^5*sgn(b*x^3 + a))/x^13

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(-91b^5x^{15} + 910ab^4x^{12} + 455a^2b^3x^9 + 260a^3b^2x^6 + 91a^4bx^3 + 14a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}}}{182 (bx^3 + a)^5 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x)

[Out] -1/182*(-91*b^5*x^15+910*a*b^4*x^12+455*a^2*b^3*x^9+260*a^3*b^2*x^6+91*a^4*b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/x^13/(b*x^3+a)^5

maxima [A] time = 0.99, size = 59, normalized size = 0.23

$$\frac{91 b^5 x^{15} - 910 a b^4 x^{12} - 455 a^2 b^3 x^9 - 260 a^3 b^2 x^6 - 91 a^4 b x^3 - 14 a^5}{182 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="maxima")

[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 91*a^4*b*x^3 - 14*a^5)/x^13

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2 a b x^3 + b^2 x^6)^{5/2}}{x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14, x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{14}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**14, x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**14, x)
```

$$3.78 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{15}} dx$$

Optimal. Leaf size=248

$$\frac{b^5x\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{5ab^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} - \frac{2a^2b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5(a + bx^3)} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{14x^{14}(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{5a^3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^8(a + bx^3)} - \frac{2a^2b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5(a + bx^3)} - \frac{5ab^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{b^5x\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Rubi [A] time = 0.06, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{14x^{14}(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{5a^3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^8(a + bx^3)} - \frac{2a^2b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5(a + bx^3)} - \frac{5ab^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} + \frac{b^5x\sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15, x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (2*a^2*b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a*b^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^5*x*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{15}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^5}{x^{15}} dx}{b^4(ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(b^{10} + \frac{a^5b^5}{x^{15}} + \frac{5a^4b^6}{x^{12}} + \frac{10a^3b^7}{x^9} + \frac{10a^2b^8}{x^6} + \frac{5ab^9}{x^3} \right) dx}{b^4(ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{14x^{14}(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{5a^3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^8(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (22a^5 + 140a^4bx^3 + 385a^3b^2x^6 + 616a^2b^3x^9 + 770ab^4x^{12} - 308b^5x^{15})}{308x^{14}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]

[Out] -1/308*(Sqrt[(a + b*x^3)^2]*(22*a^5 + 140*a^4*b*x^3 + 385*a^3*b^2*x^6 + 616*a^2*b^3*x^9 + 770*a*b^4*x^12 - 308*b^5*x^15))/(x^14*(a + b*x^3))

IntegrateAlgebraic [A] time = 19.62, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-22a^5 - 140a^4bx^3 - 385a^3b^2x^6 - 616a^2b^3x^9 - 770ab^4x^{12} + 308b^5x^{15})}{308x^{14}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 140*a^4*b*x^3 - 385*a^3*b^2*x^6 - 616*a^2*b^3*x^9 - 770*a*b^4*x^12 + 308*b^5*x^15))/(308*x^14*(a + b*x^3))

fricas [A] time = 0.68, size = 59, normalized size = 0.24

$$\frac{308b^5x^{15} - 770ab^4x^{12} - 616a^2b^3x^9 - 385a^3b^2x^6 - 140a^4bx^3 - 22a^5}{308x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="fricas")

[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 - 140*a^4*b*x^3 - 22*a^5)/x^14

giac [A] time = 0.35, size = 105, normalized size = 0.42

$$b^5 \operatorname{sgn}(bx^3 + a) - \frac{770 ab^4 x^{12} \operatorname{sgn}(bx^3 + a) + 616 a^2 b^3 x^9 \operatorname{sgn}(bx^3 + a) + 385 a^3 b^2 x^6 \operatorname{sgn}(bx^3 + a) + 140 a^4 b x^3 \operatorname{sgn}(bx^3 + a) + 22 a^5 \operatorname{sgn}(bx^3 + a)}{308 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="giac")

[Out] b^5*x*sgn(b*x^3 + a) - 1/308*(770*a*b^4*x^12*sgn(b*x^3 + a) + 616*a^2*b^3*x^9*sgn(b*x^3 + a) + 385*a^3*b^2*x^6*sgn(b*x^3 + a) + 140*a^4*b*x^3*sgn(b*x^3 + a) + 22*a^5*sgn(b*x^3 + a))/x^14

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{\left(-308b^5x^{15} + 770ab^4x^{12} + 616a^2b^3x^9 + 385a^3b^2x^6 + 140a^4bx^3 + 22a^5\right)\left(bx^3 + a\right)^{\frac{5}{2}}}{308\left(bx^3 + a\right)^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x)

[Out] -1/308*(-308*b^5*x^15+770*a*b^4*x^12+616*a^2*b^3*x^9+385*a^3*b^2*x^6+140*a^4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^14/(b*x^3+a)^5

maxima [A] time = 0.99, size = 59, normalized size = 0.24

$$\frac{308 b^5 x^{15} - 770 a b^4 x^{12} - 616 a^2 b^3 x^9 - 385 a^3 b^2 x^6 - 140 a^4 b x^3 - 22 a^5}{308 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="maxima")

[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 - 140*a^4*b*x^3 - 22*a^5)/x^14

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2 + 2 a b x^3 + b^2 x^6\right)^{5/2}}{x^{15}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15, x)`

[Out] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{15}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**15, x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**15, x)`

$$3.79 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{16}} dx$$

Optimal. Leaf size=251

$$\frac{b^5 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3} - \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3 (a + bx^3)} - \frac{5a^2b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^6 (a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15x^{15} (a + bx^3)}$$

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$-\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{15x^{15} (a + bx^3)} - \frac{5a^4b \sqrt{a^2 + 2abx^3 + b^2x^6}}{12x^{12} (a + bx^3)} - \frac{10a^3b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9 (a + bx^3)} - \frac{5a^2b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^6 (a + bx^3)} - \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^3 (a + bx^3)} + \frac{b^5 \log(x) \sqrt{a^2 + 2abx^3 + b^2x^6}}{a + bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*x^15*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (10*a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^2*b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (5*a*b^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{16}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^5}{x^{16}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^6} dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int \left(\frac{a^5b^5}{x^6} + \frac{5a^4b^6}{x^5} + \frac{10a^3b^7}{x^4} + \frac{10a^2b^8}{x^3} + \frac{5ab^9}{x^2} + \frac{b^{10}}{x}\right) dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
&= -\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{15x^{15} (a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{12x^{12} (a + bx^3)} - \frac{10a^3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9 (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a + bx^3)^2} (a(12a^4 + 75a^3bx^3 + 200a^2b^2x^6 + 300ab^3x^9 + 300b^4x^{12}) - 180b^5x^{15} \log(x))}{180x^{15} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16, x]

[Out] -1/180*(Sqrt[(a + b*x^3)^2]*(a*(12*a^4 + 75*a^3*b*x^3 + 200*a^2*b^2*x^6 + 300*a*b^3*x^9 + 300*b^4*x^12) - 180*b^5*x^15*Log[x]))/(x^15*(a + b*x^3))

IntegrateAlgebraic [B] time = 3.58, size = 2386, normalized size = 9.51

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16, x]

[Out] (4*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-12*a^8*b - 123*a^7*b^2*x^3 - 572*a^6*b^3*x^6 - 1598*a^5*b^4*x^9 - 3012*a^4*b^5*x^12 - 3875*a^3*b^6*x^15 - 3200*a^2*b^7*x^18 - 1500*a*b^8*x^21 - 300*b^9*x^24) + 4*a*b^4*Sqrt[b^2]*(12*

$$\begin{aligned}
& a^9 + 135a^8b^2x^3 + 695a^7b^2x^6 + 2170a^6b^3x^9 + 4610a^5b^4x^{12} \\
& + 6887a^4b^5x^{15} + 7075a^3b^6x^{18} + 4700a^2b^7x^{21} + 1800ab^8x^{24} + 300b^9x^{27}) / (45\sqrt{b^2}x^{15}\sqrt{a^2 + 2abx^3 + b^2x^6} * (- \\
& 16a^4b^4 - 64a^3b^5x^3 - 96a^2b^6x^6 - 64ab^7x^9 - 16b^8x^{12}) \\
& + 45x^{15}(16a^5b^5 + 80a^4b^6x^3 + 160a^3b^7x^6 + 160a^2b^8x^9 \\
& + 80ab^9x^{12} + 16b^{10}x^{15})) + (b^5\text{Log}[-a - \sqrt{b^2}x^3 + \sqrt{a^2 + \\
& 2abx^3 + b^2x^6}]) / 6 - (b^4\sqrt{b^2}\text{Log}[-a - \sqrt{b^2}x^3 + \sqrt{a^2 + \\
& 2abx^3 + b^2x^6}]) / 6 + (a^{10}b^5\text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + \\
& 2abx^3 + b^2x^6}]) / (6(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) + (a^{10}b^4\sqrt{b^2}\text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (6(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) - (5a^8b^5 * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^2 \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (6(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) - (5a^8b^4\sqrt{b^2} * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^2 \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (6(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) + (5a^6b^5 * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^4 \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (3(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) + (5a^6b^4\sqrt{b^2} * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^4 \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (3(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) + (5a^4b^5 * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^6 \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (3(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) - (5a^4b^4\sqrt{b^2} * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^6 \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (3(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) + (5a^2b^5 * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^8 \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (6(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) + (5a^2b^4\sqrt{b^2} * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^8 \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (6(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) - (b^5 * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^{10} \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (6(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5) - (b^4\sqrt{b^2} * (-\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^{10} \text{Log}[a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6}]) / (6(-a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5 * (a - \sqrt{b^2}x^3 + \sqrt{a^2 + 2abx^3 + b^2x^6})^5)
\end{aligned}$$

t[a^2 + 2*a*b*x^3 + b^2*x^6])^5)

fricas [A] time = 1.23, size = 61, normalized size = 0.24

$$\frac{180 b^5 x^{15} \log(x) - 300 a b^4 x^{12} - 300 a^2 b^3 x^9 - 200 a^3 b^2 x^6 - 75 a^4 b x^3 - 12 a^5}{180 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="fricas")

[Out] 1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^2*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15

giac [A] time = 0.37, size = 123, normalized size = 0.49

$$b^5 \log(|x|) \operatorname{sgn}(bx^3 + a) - \frac{137 b^5 x^{15} \operatorname{sgn}(bx^3 + a) + 300 a b^4 x^{12} \operatorname{sgn}(bx^3 + a) + 300 a^2 b^3 x^9 \operatorname{sgn}(bx^3 + a) + 200 a^3 b^2 x^6 \operatorname{sgn}(bx^3 + a) + 75 a^4 b x^3 \operatorname{sgn}(bx^3 + a) + 12 a^5 \operatorname{sgn}(bx^3 + a)}{180 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="giac")

[Out] b^5*log(abs(x))*sgn(b*x^3 + a) - 1/180*(137*b^5*x^15*sgn(b*x^3 + a) + 300*a*b^4*x^12*sgn(b*x^3 + a) + 300*a^2*b^3*x^9*sgn(b*x^3 + a) + 200*a^3*b^2*x^6*sgn(b*x^3 + a) + 75*a^4*b*x^3*sgn(b*x^3 + a) + 12*a^5*sgn(b*x^3 + a))/x^15

maple [A] time = 0.01, size = 82, normalized size = 0.33

$$\frac{\left((b x^3 + a)^2\right)^{\frac{5}{2}} \left(180 b^5 x^{15} \ln(x) - 300 a b^4 x^{12} - 300 a^2 b^3 x^9 - 200 a^3 b^2 x^6 - 75 a^4 b x^3 - 12 a^5\right)}{180 (b x^3 + a)^5 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x)

[Out] 1/180*((b*x^3+a)^2)^(5/2)*(180*b^5*ln(x)*x^15-300*a*b^4*x^12-300*a^2*b^3*x^9-200*a^3*b^2*x^6-75*a^4*b*x^3-12*a^5)/(b*x^3+a)^5/x^15

maxima [B] time = 1.10, size = 374, normalized size = 1.49

$$\frac{\sqrt{b^2 x^6 + 2 a b x^3 + a^2} x^{15} \log\left(\frac{2 a b x^3 + 2 a^2}{|x| + \frac{2 a^2}{|x|}}\right) + \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{12 a^4} + \frac{\sqrt{b^2 x^6 + 2 a b x^3 + a^2} x^{15}}{2 a} + \frac{7 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{36 a^3} - \frac{2 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{45 a^2} - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{9 a^2 x^3} - \frac{2 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{45 a^2 x^6} - \frac{11 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{180 a^2 x^9} + \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{20 a^2 x^{12}} - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{15 a^2 x^{15}}}{6 a^2} + \frac{1}{3} (-1)^{b^2 x^6 + 2 a b x^3 + a^2} b^5 \log(2 b^2 x^3 + 2 a b) - \frac{1}{3} (-1)^{b^2 x^6 + 2 a b x^3 + a^2} b^5 \log\left(\frac{2 a b x^3 + 2 a^2}{|x| + \frac{2 a^2}{|x|}}\right) + \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{12 a^4} + \frac{\sqrt{b^2 x^6 + 2 a b x^3 + a^2} x^{15}}{2 a} + \frac{7 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{36 a^3} - \frac{2 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{45 a^2} - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{9 a^2 x^3} - \frac{2 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{45 a^2 x^6} - \frac{11 (b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{180 a^2 x^9} + \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{20 a^2 x^{12}} - \frac{(b^2 x^6 + 2 a b x^3 + a^2)^{\frac{5}{2}} x^{15}}{15 a^2 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="maxima")

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^6*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a
*b)*b^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^5*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^
6*x^3/a^4 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5/a + 7/36*(b^2*x^6 + 2*a
*b*x^3 + a^2)^(3/2)*b^5/a^3 - 2/45*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^5/a^
5 - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/(a^4*x^3) + 2/45*(b^2*x^6 + 2
*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^6) - 11/180*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7
/2)*b^2/(a^4*x^9) + 1/20*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^12) - 1
/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^15)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{5/2}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**16, x)
```

$$3.80 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{17}} dx$$

Optimal. Leaf size=251

$$\frac{b^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)} - \frac{5ab^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{10a^2b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{16x^{16}(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{16x^{16}(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13}(a + bx^3)} - \frac{a^3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10}(a + bx^3)} - \frac{10a^2b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7(a + bx^3)} - \frac{5ab^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4(a + bx^3)} - \frac{b^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{x(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (10*a^2*b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a*b^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{17}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{17}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5 b^5}{x^{17}} + \frac{5a^4 b^6}{x^{14}} + \frac{10a^3 b^7}{x^{11}} + \frac{10a^2 b^8}{x^8} + \frac{5ab^9}{x^5} + \frac{b^{10}}{x^2} \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{16x^{16} (a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13} (a + bx^3)} - \frac{a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10} (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a + bx^3)^2} (91a^5 + 560a^4bx^3 + 1456a^3b^2x^6 + 2080a^2b^3x^9 + 1820ab^4x^{12} + 1456b^5x^{15})}{1456x^{16} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]

[Out] -1/1456*(Sqrt[(a + b*x^3)^2]*(91*a^5 + 560*a^4*b*x^3 + 1456*a^3*b^2*x^6 + 2080*a^2*b^3*x^9 + 1820*a*b^4*x^12 + 1456*b^5*x^15))/(x^16*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.03, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-91a^5 - 560a^4bx^3 - 1456a^3b^2x^6 - 2080a^2b^3x^9 - 1820ab^4x^{12} - 1456b^5x^{15})}{1456x^{16} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 - 560*a^4*b*x^3 - 1456*a^3*b^2*x^6 - 2080*a^2*b^3*x^9 - 1820*a*b^4*x^12 - 1456*b^5*x^15))/(1456*x^16*(a + b*x^3))

fricas [A] time = 1.16, size = 59, normalized size = 0.24

$$-\frac{1456 b^5 x^{15} + 1820 a b^4 x^{12} + 2080 a^2 b^3 x^9 + 1456 a^3 b^2 x^6 + 560 a^4 b x^3 + 91 a^5}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="fricas")

[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16

giac [A] time = 0.34, size = 107, normalized size = 0.43

$$\frac{1456 b^5 x^{15} \operatorname{sgn}(b x^3 + a) + 1820 a b^4 x^{12} \operatorname{sgn}(b x^3 + a) + 2080 a^2 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 1456 a^3 b^2 x^6 \operatorname{sgn}(b x^3 + a) + 560 a^4 b x^3 \operatorname{sgn}(b x^3 + a) + 91 a^5 \operatorname{sgn}(b x^3 + a)}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="giac")

[Out] -1/1456*(1456*b^5*x^15*sgn(b*x^3 + a) + 1820*a*b^4*x^12*sgn(b*x^3 + a) + 2080*a^2*b^3*x^9*sgn(b*x^3 + a) + 1456*a^3*b^2*x^6*sgn(b*x^3 + a) + 560*a^4*b*x^3*sgn(b*x^3 + a) + 91*a^5*sgn(b*x^3 + a))/x^16

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(1456 b^5 x^{15} + 1820 a b^4 x^{12} + 2080 a^2 b^3 x^9 + 1456 a^3 b^2 x^6 + 560 a^4 b x^3 + 91 a^5) \left((b x^3 + a)^2 \right)^{\frac{5}{2}}}{1456 (b x^3 + a)^5 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x)

[Out] -1/1456*(1456*b^5*x^15+1820*a*b^4*x^12+2080*a^2*b^3*x^9+1456*a^3*b^2*x^6+560*a^4*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^16/(b*x^3+a)^5

maxima [A] time = 0.99, size = 59, normalized size = 0.24

$$\frac{1456 b^5 x^{15} + 1820 a b^4 x^{12} + 2080 a^2 b^3 x^9 + 1456 a^3 b^2 x^6 + 560 a^4 b x^3 + 91 a^5}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="maxima")

[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16

mupad [B] time = 1.26, size = 231, normalized size = 0.92

$$\frac{a^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{16 x^{16} (b x^3 + a)} - \frac{b^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{x (b x^3 + a)} - \frac{5 a b^4 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{4 x^4 (b x^3 + a)} - \frac{5 a^4 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{13 x^{13} (b x^3 + a)} - \frac{10 a^2 b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{7 x^7 (b x^3 + a)} - \frac{a^3 b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{x^{10} (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^17,x)`

[Out] $-\frac{a^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{16x^{16}(a + bx^3)} - \frac{b^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{x(a + bx^3)} - \frac{5a^4b^4(a^2 + b^2x^6 + 2abx^3)^{1/2}}{4x^4(a + bx^3)} - \frac{5a^4b(a^2 + b^2x^6 + 2abx^3)^{1/2}}{13x^{13}(a + bx^3)} - \frac{10a^2b^3(a^2 + b^2x^6 + 2abx^3)^{1/2}}{7x^7(a + bx^3)} - \frac{a^3b^2(a^2 + b^2x^6 + 2abx^3)^{1/2}}{x^{10}(a + bx^3)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{17}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**17,x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**17, x)`

$$3.81 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{18}} dx$$

Optimal. Leaf size=253

$$\frac{b^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)} - \frac{ab^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5(a + bx^3)} - \frac{5a^2b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^8(a + bx^3)} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{17x^{17}(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{14x^{14}(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{17x^{17}(a + bx^3)} - \frac{5a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{14x^{14}(a + bx^3)} - \frac{10a^3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{5a^2b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^8(a + bx^3)} - \frac{ab^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{x^5(a + bx^3)} - \frac{b^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^2(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]

[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{18}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{18}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5 b^5}{x^{18}} + \frac{5a^4 b^6}{x^{15}} + \frac{10a^3 b^7}{x^{12}} + \frac{10a^2 b^8}{x^9} + \frac{5ab^9}{x^6} + \frac{b^{10}}{x^3} \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{17x^{17} (a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{14x^{14} (a + bx^3)} - \frac{10a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11} (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a + bx^3)^2} (308a^5 + 1870a^4bx^3 + 4760a^3b^2x^6 + 6545a^2b^3x^9 + 5236ab^4x^{12} + 2618b^5x^{15})}{5236x^{17} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]

[Out] -1/5236*(Sqrt[(a + b*x^3)^2]*(308*a^5 + 1870*a^4*b*x^3 + 4760*a^3*b^2*x^6 + 6545*a^2*b^3*x^9 + 5236*a*b^4*x^12 + 2618*b^5*x^15))/(x^17*(a + b*x^3))

IntegrateAlgebraic [A] time = 20.55, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-308a^5 - 1870a^4bx^3 - 4760a^3b^2x^6 - 6545a^2b^3x^9 - 5236ab^4x^{12} - 2618b^5x^{15})}{5236x^{17} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 - 1870*a^4*b*x^3 - 4760*a^3*b^2*x^6 - 6545*a^2*b^3*x^9 - 5236*a*b^4*x^12 - 2618*b^5*x^15))/(5236*x^17*(a + b*x^3))

fricas [A] time = 1.21, size = 59, normalized size = 0.23

$$-\frac{2618 b^5 x^{15} + 5236 a b^4 x^{12} + 6545 a^2 b^3 x^9 + 4760 a^3 b^2 x^6 + 1870 a^4 b x^3 + 308 a^5}{5236 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="fricas")

[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17

giac [A] time = 0.45, size = 107, normalized size = 0.42

$$\frac{2618 b^5 x^{15} \operatorname{sgn}(b x^3 + a) + 5236 a b^4 x^{12} \operatorname{sgn}(b x^3 + a) + 6545 a^2 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 4760 a^3 b^2 x^6 \operatorname{sgn}(b x^3 + a) + 1870 a^4 b x^3 \operatorname{sgn}(b x^3 + a) + 308 a^5 \operatorname{sgn}(b x^3 + a)}{5236 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="giac")

[Out] -1/5236*(2618*b^5*x^15*sgn(b*x^3 + a) + 5236*a*b^4*x^12*sgn(b*x^3 + a) + 6545*a^2*b^3*x^9*sgn(b*x^3 + a) + 4760*a^3*b^2*x^6*sgn(b*x^3 + a) + 1870*a^4*b*x^3*sgn(b*x^3 + a) + 308*a^5*sgn(b*x^3 + a))/x^17

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(2618 b^5 x^{15} + 5236 a b^4 x^{12} + 6545 a^2 b^3 x^9 + 4760 a^3 b^2 x^6 + 1870 a^4 b x^3 + 308 a^5) \left((b x^3 + a)^2 \right)^{\frac{5}{2}}}{5236 (b x^3 + a)^5 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x)

[Out] -1/5236*(2618*b^5*x^15+5236*a*b^4*x^12+6545*a^2*b^3*x^9+4760*a^3*b^2*x^6+1870*a^4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x^17/(b*x^3+a)^5

maxima [A] time = 1.16, size = 59, normalized size = 0.23

$$\frac{2618 b^5 x^{15} + 5236 a b^4 x^{12} + 6545 a^2 b^3 x^9 + 4760 a^3 b^2 x^6 + 1870 a^4 b x^3 + 308 a^5}{5236 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="maxima")

[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17

mupad [B] time = 1.32, size = 231, normalized size = 0.91

$$\frac{a^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{17 x^{17} (b x^3 + a)} - \frac{b^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{2 x^2 (b x^3 + a)} - \frac{a b^4 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{x^5 (b x^3 + a)} - \frac{5 a^4 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{14 x^{14} (b x^3 + a)} - \frac{5 a^2 b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{4 x^8 (b x^3 + a)} - \frac{10 a^3 b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{11 x^{11} (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^18,x)`

[Out] $-\frac{a^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{17x^{17}(a + bx^3)} - \frac{b^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{2x^2(a + bx^3)} - \frac{ab^4(a^2 + b^2x^6 + 2abx^3)^{1/2}}{x^5(a + bx^3)} - \frac{5a^4b(a^2 + b^2x^6 + 2abx^3)^{1/2}}{14x^{14}(a + bx^3)} - \frac{5a^2b^3(a^2 + b^2x^6 + 2abx^3)^{1/2}}{4x^8(a + bx^3)} - \frac{10a^3b^2(a^2 + b^2x^6 + 2abx^3)^{1/2}}{11x^{11}(a + bx^3)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{18}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**18,x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**18, x)`

$$3.82 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{19}} dx$$

Optimal. Leaf size=41

$$-\frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18ax^{18}}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 264}

$$-\frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18ax^{18}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19, x]

[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*a*x^18)

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{19}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{19}} dx}{b^4 (ab + b^2x^3)} \\ &= -\frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{18ax^{18}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.98

$$\frac{\sqrt{(a + bx^3)^2} (a^5 + 6a^4bx^3 + 15a^3b^2x^6 + 20a^2b^3x^9 + 15ab^4x^{12} + 6b^5x^{15})}{18x^{18}(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]

[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(a^5 + 6*a^4*b*x^3 + 15*a^3*b^2*x^6 + 20*a^2*b^3*x^9 + 15*a*b^4*x^12 + 6*b^5*x^15))/(x^18*(a + b*x^3))

IntegrateAlgebraic [B] time = 2.89, size = 442, normalized size = 10.78

$$\frac{16b^5\sqrt{a^2+2abx^3+b^2x^6}(-a^{10}b-11a^9b^2x^3-55a^8b^3x^6-165a^7b^4x^9-330a^6b^5x^{12}-462a^5b^6x^{15}-461a^4b^7x^{18}-325a^3b^8x^{21}-155a^2b^9x^{24}-45a^1b^{10}x^{27}-6b^{11}x^{30})+16\sqrt{b^2}(a^{11}+12a^{10}b^1x^3+66a^9b^2x^6+220a^8b^3x^9+495a^7b^4x^{12}+792a^6b^5x^{15}+923a^5b^6x^{18}+786a^4b^7x^{21}+480a^3b^8x^{24}+200a^2b^9x^{27}+51ab^{10}x^{30}+6b^{11}x^{33})}{9\sqrt{b^2}x^{18}\sqrt{a^2+2abx^3+b^2x^6}(-32a^5b^5-160a^4b^6x^3-320a^3b^7x^6-320a^2b^8x^9-160a^1b^9x^{12}-32b^{10}x^{15})+9x^{18}(32a^6b^6+192a^5b^7x^3+480a^4b^8x^6+640a^3b^9x^9+480a^2b^{10}x^{12}+192ab^{11}x^{15}+32b^{12}x^{18})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]

[Out] (16*b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^10*b) - 11*a^9*b^2*x^3 - 55*a^8*b^3*x^6 - 165*a^7*b^4*x^9 - 330*a^6*b^5*x^12 - 462*a^5*b^6*x^15 - 461*a^4*b^7*x^18 - 325*a^3*b^8*x^21 - 155*a^2*b^9*x^24 - 45*a^1*b^10*x^27 - 6*b^11*x^30) + 16*b^5*Sqrt[b^2]*(a^11 + 12*a^10*b*x^3 + 66*a^9*b^2*x^6 + 220*a^8*b^3*x^9 + 495*a^7*b^4*x^12 + 792*a^6*b^5*x^15 + 923*a^5*b^6*x^18 + 786*a^4*b^7*x^21 + 480*a^3*b^8*x^24 + 200*a^2*b^9*x^27 + 51*a*b^10*x^30 + 6*b^11*x^33))/(9*Sqrt[b^2]*x^18*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-32*a^5*b^5 - 160*a^4*b^6*x^3 - 320*a^3*b^7*x^6 - 320*a^2*b^8*x^9 - 160*a^1*b^9*x^12 - 32*b^10*x^15) + 9*x^18*(32*a^6*b^6 + 192*a^5*b^7*x^3 + 480*a^4*b^8*x^6 + 640*a^3*b^9*x^9 + 480*a^2*b^10*x^12 + 192*a*b^11*x^15 + 32*b^12*x^18))

fricas [B] time = 1.14, size = 57, normalized size = 1.39

$$\frac{6b^5x^{15} + 15ab^4x^{12} + 20a^2b^3x^9 + 15a^3b^2x^6 + 6a^4bx^3 + a^5}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="fricas")

[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^4*b*x^3 + a^5)/x^18

giac [B] time = 0.29, size = 106, normalized size = 2.59

$$\frac{6b^5x^{15}\operatorname{sgn}(bx^3+a) + 15ab^4x^{12}\operatorname{sgn}(bx^3+a) + 20a^2b^3x^9\operatorname{sgn}(bx^3+a) + 15a^3b^2x^6\operatorname{sgn}(bx^3+a) + 6a^4bx^3\operatorname{sgn}(bx^3+a) + a^5\operatorname{sgn}(bx^3+a)}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="giac")

[Out] -1/18*(6*b^5*x^15*sgn(b*x^3 + a) + 15*a*b^4*x^12*sgn(b*x^3 + a) + 20*a^2*b^3*x^9*sgn(b*x^3 + a) + 15*a^3*b^2*x^6*sgn(b*x^3 + a) + 6*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^18

maple [B] time = 0.01, size = 78, normalized size = 1.90

$$\frac{(6b^5x^{15} + 15ab^4x^{12} + 20a^2b^3x^9 + 15a^3b^2x^6 + 6a^4bx^3 + a^5)\left((bx^3 + a)^2\right)^{\frac{5}{2}}}{18(bx^3 + a)^5x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x)

[Out] -1/18*(6*b^5*x^15+15*a*b^4*x^12+20*a^2*b^3*x^9+15*a^3*b^2*x^6+6*a^4*b*x^3+a^5)*((b*x^3+a)^2)^(5/2)/x^18/(b*x^3+a)^5

maxima [B] time = 1.10, size = 210, normalized size = 5.12

$$\frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^6}{18a^6} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^5}{18a^5x^3} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b^4}{18a^6x^6} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b^3}{18a^5x^9} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b^2}{18a^4x^{12}} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b}{18a^3x^{15}} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}}{18a^2x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="maxima")

[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^6/a^6 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^5/(a^5*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^9) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^18)

mupad [B] time = 1.22, size = 231, normalized size = 5.63

$$\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{18x^{18}(bx^3+a)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(bx^3+a)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(bx^3+a)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^{15}(bx^3+a)} - \frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(bx^3+a)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{6x^{12}(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^19,x)

[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (b^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^15*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^12*(a + b*x^3))

$(3x^{15}(a + bx^3))^{-1/2} - (10a^2b^3(a^2 + b^2x^6 + 2abx^3))^{-1/2} / (9x^9(a + bx^3)) - (5a^3b^2(a^2 + b^2x^6 + 2abx^3))^{-1/2} / (6x^{12}(a + bx^3))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{19}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**19, x)

$$3.83 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}} dx$$

Optimal. Leaf size=253

$$\frac{b^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4 (a + bx^3)} - \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7 (a + bx^3)} - \frac{a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10} (a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{19x^{19} (a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{16x^{16} (a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{19x^{19} (a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{16x^{16} (a + bx^3)} - \frac{10a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13} (a + bx^3)} - \frac{a^2 b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^{10} (a + bx^3)} - \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7 (a + bx^3)} - \frac{b^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^4 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]

[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{20}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{20}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5 b^5}{x^{20}} + \frac{5a^4 b^6}{x^{17}} + \frac{10a^3 b^7}{x^{14}} + \frac{10a^2 b^8}{x^{11}} + \frac{5ab^9}{x^8} + \frac{b^{10}}{x^5} \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{19x^{19} (a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{16x^{16} (a + bx^3)} - \frac{10a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13} (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (1456a^5 + 8645a^4bx^3 + 21280a^3b^2x^6 + 27664a^2b^3x^9 + 19760ab^4x^{12} + 6916b^5x^{15})}{27664x^{19} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]

[Out] -1/27664*(Sqrt[(a + b*x^3)^2]*(1456*a^5 + 8645*a^4*b*x^3 + 21280*a^3*b^2*x^6 + 27664*a^2*b^3*x^9 + 19760*a*b^4*x^12 + 6916*b^5*x^15))/(x^19*(a + b*x^3))

IntegrateAlgebraic [A] time = 21.32, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-1456a^5 - 8645a^4bx^3 - 21280a^3b^2x^6 - 27664a^2b^3x^9 - 19760ab^4x^{12} - 6916b^5x^{15})}{27664x^{19} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-1456*a^5 - 8645*a^4*b*x^3 - 21280*a^3*b^2*x^6 - 27664*a^2*b^3*x^9 - 19760*a*b^4*x^12 - 6916*b^5*x^15))/(27664*x^19*(a + b*x^3))

fricas [A] time = 1.10, size = 59, normalized size = 0.23

$$\frac{6916b^5x^{15} + 19760ab^4x^{12} + 27664a^2b^3x^9 + 21280a^3b^2x^6 + 8645a^4bx^3 + 1456a^5}{27664x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="fricas")

[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19

giac [A] time = 0.36, size = 107, normalized size = 0.42

$$\frac{6916 b^5 x^{15} \operatorname{sgn}(b x^3 + a) + 19760 a b^4 x^{12} \operatorname{sgn}(b x^3 + a) + 27664 a^2 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 21280 a^3 b^2 x^6 \operatorname{sgn}(b x^3 + a) + 8645 a^4 b x^3 \operatorname{sgn}(b x^3 + a) + 1456 a^5 \operatorname{sgn}(b x^3 + a)}{27664 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="giac")

[Out] -1/27664*(6916*b^5*x^15*sgn(b*x^3 + a) + 19760*a*b^4*x^12*sgn(b*x^3 + a) + 27664*a^2*b^3*x^9*sgn(b*x^3 + a) + 21280*a^3*b^2*x^6*sgn(b*x^3 + a) + 8645*a^4*b*x^3*sgn(b*x^3 + a) + 1456*a^5*sgn(b*x^3 + a))/x^19

maple [A] time = 0.01, size = 80, normalized size = 0.32

$$\frac{(6916 b^5 x^{15} + 19760 a b^4 x^{12} + 27664 a^2 b^3 x^9 + 21280 a^3 b^2 x^6 + 8645 a^4 b x^3 + 1456 a^5) \left((b x^3 + a)^2 \right)^{\frac{5}{2}}}{27664 (b x^3 + a)^5 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x)

[Out] -1/27664*(6916*b^5*x^15+19760*a*b^4*x^12+27664*a^2*b^3*x^9+21280*a^3*b^2*x^6+8645*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/x^19/(b*x^3+a)^5

maxima [A] time = 1.04, size = 59, normalized size = 0.23

$$\frac{6916 b^5 x^{15} + 19760 a b^4 x^{12} + 27664 a^2 b^3 x^9 + 21280 a^3 b^2 x^6 + 8645 a^4 b x^3 + 1456 a^5}{27664 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="maxima")

[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19

mupad [B] time = 1.31, size = 231, normalized size = 0.91

$$\frac{a^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{19 x^{19} (b x^3 + a)} - \frac{b^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{4 x^4 (b x^3 + a)} - \frac{5 a b^4 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{7 x^7 (b x^3 + a)} - \frac{5 a^4 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{16 x^{16} (b x^3 + a)} - \frac{a^2 b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{x^{10} (b x^3 + a)} - \frac{10 a^3 b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{13 x^{13} (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^20,x)`

[Out] $-\frac{a^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{19x^{19}(a + bx^3)} - \frac{b^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{4x^4(a + bx^3)} - \frac{5a^4b(a^2 + b^2x^6 + 2abx^3)^{1/2}}{7x^7(a + bx^3)} - \frac{5a^4b(a^2 + b^2x^6 + 2abx^3)^{1/2}}{16x^{16}(a + bx^3)} - \frac{a^2b^3(a^2 + b^2x^6 + 2abx^3)^{1/2}}{x^{10}(a + bx^3)} - \frac{10a^3b^2(a^2 + b^2x^6 + 2abx^3)^{1/2}}{13x^{13}(a + bx^3)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{20}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**20,x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**20, x)`

$$3.84 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{21}} dx$$

Optimal. Leaf size=255

$$\frac{b^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5 (a + bx^3)} - \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8 (a + bx^3)} - \frac{10a^2b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11} (a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{20x^{20} (a + bx^3)} - \frac{5a^4b \sqrt{a^2 + 2abx^3 + b^2x^6}}{17x^{17} (a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{20x^{20} (a + bx^3)} - \frac{5a^4b \sqrt{a^2 + 2abx^3 + b^2x^6}}{17x^{17} (a + bx^3)} - \frac{5a^3b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^{14} (a + bx^3)} - \frac{10a^2b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11} (a + bx^3)} - \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8 (a + bx^3)} - \frac{b^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{5x^5 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*x^20*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (10*a^2*b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a*b^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{21}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{21}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5 b^5}{x^{21}} + \frac{5a^4 b^6}{x^{18}} + \frac{10a^3 b^7}{x^{15}} + \frac{10a^2 b^8}{x^{12}} + \frac{5ab^9}{x^9} + \frac{b^{10}}{x^6} \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{20x^{20} (a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{17x^{17} (a + bx^3)} - \frac{5a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^{14} (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (2618a^5 + 15400a^4bx^3 + 37400a^3b^2x^6 + 47600a^2b^3x^9 + 32725ab^4x^{12} + 10472b^5x^{15})}{52360x^{20} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21, x]

[Out] -1/52360*(Sqrt[(a + b*x^3)^2]*(2618*a^5 + 15400*a^4*b*x^3 + 37400*a^3*b^2*x^6 + 47600*a^2*b^3*x^9 + 32725*a*b^4*x^12 + 10472*b^5*x^15))/(x^20*(a + b*x^3))

IntegrateAlgebraic [A] time = 21.39, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-2618a^5 - 15400a^4bx^3 - 37400a^3b^2x^6 - 47600a^2b^3x^9 - 32725ab^4x^{12} - 10472b^5x^{15})}{52360x^{20} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21, x]

[Out] (Sqrt[(a + b*x^3)^2]*(-2618*a^5 - 15400*a^4*b*x^3 - 37400*a^3*b^2*x^6 - 47600*a^2*b^3*x^9 - 32725*a*b^4*x^12 - 10472*b^5*x^15))/(52360*x^20*(a + b*x^3))

fricas [A] time = 1.11, size = 59, normalized size = 0.23

$$-\frac{10472 b^5 x^{15} + 32725 a b^4 x^{12} + 47600 a^2 b^3 x^9 + 37400 a^3 b^2 x^6 + 15400 a^4 b x^3 + 2618 a^5}{52360 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="fricas")

[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20

giac [A] time = 0.32, size = 107, normalized size = 0.42

$$\frac{10472 b^5 x^{15} \operatorname{sgn}(b x^3 + a) + 32725 a b^4 x^{12} \operatorname{sgn}(b x^3 + a) + 47600 a^2 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 37400 a^3 b^2 x^6 \operatorname{sgn}(b x^3 + a) + 15400 a^4 b x^3 \operatorname{sgn}(b x^3 + a) + 2618 a^5 \operatorname{sgn}(b x^3 + a)}{52360 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="giac")

[Out] -1/52360*(10472*b^5*x^15*sgn(b*x^3 + a) + 32725*a*b^4*x^12*sgn(b*x^3 + a) + 47600*a^2*b^3*x^9*sgn(b*x^3 + a) + 37400*a^3*b^2*x^6*sgn(b*x^3 + a) + 15400*a^4*b*x^3*sgn(b*x^3 + a) + 2618*a^5*sgn(b*x^3 + a))/x^20

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(10472 b^5 x^{15} + 32725 a b^4 x^{12} + 47600 a^2 b^3 x^9 + 37400 a^3 b^2 x^6 + 15400 a^4 b x^3 + 2618 a^5) \left((b x^3 + a)^2 \right)^{\frac{5}{2}}}{52360 (b x^3 + a)^5 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x)

[Out] -1/52360*(10472*b^5*x^15+32725*a*b^4*x^12+47600*a^2*b^3*x^9+37400*a^3*b^2*x^6+15400*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/x^20/(b*x^3+a)^5

maxima [A] time = 0.98, size = 59, normalized size = 0.23

$$\frac{10472 b^5 x^{15} + 32725 a b^4 x^{12} + 47600 a^2 b^3 x^9 + 37400 a^3 b^2 x^6 + 15400 a^4 b x^3 + 2618 a^5}{52360 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="maxima")

[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20

mupad [B] time = 1.24, size = 231, normalized size = 0.91

$$\frac{a^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{20 x^{20} (b x^3 + a)} - \frac{b^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{5 x^5 (b x^3 + a)} - \frac{5 a b^4 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{8 x^8 (b x^3 + a)} - \frac{5 a^4 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{17 x^{17} (b x^3 + a)} - \frac{10 a^2 b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{11 x^{11} (b x^3 + a)} - \frac{5 a^3 b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{7 x^{14} (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^21,x)`

[Out]
$$-\frac{a^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{20x^{20}(a + bx^3)} - \frac{b^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{5x^5(a + bx^3)} - \frac{5a^4b(a^2 + b^2x^6 + 2abx^3)^{1/2}}{8x^8(a + bx^3)} - \frac{5a^4b(a^2 + b^2x^6 + 2abx^3)^{1/2}}{17x^{17}(a + bx^3)} - \frac{10a^2b^3(a^2 + b^2x^6 + 2abx^3)^{1/2}}{11x^{11}(a + bx^3)} - \frac{5a^3b^2(a^2 + b^2x^6 + 2abx^3)^{1/2}}{7x^{14}(a + bx^3)}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{21}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**21,x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**21, x)`

$$3.85 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}} dx$$

Optimal. Leaf size=84

$$\frac{b(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{126a^2x^{18}} - \frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{21ax^{21}}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 266, 45, 37}

$$\frac{b(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{126a^2x^{18}} - \frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{21ax^{21}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]

[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*a*x^21) + (b*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(126*a^2*x^18)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
  ((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
  a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
  1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
  ((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
  implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
  + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
  LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
  (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
  Q[m, 1] || !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
  Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
  , m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{22}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^5}{x^{22}} dx}{b^4 (ab + b^2x^3)} \\ &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^8} dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\ &= -\frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{21ax^{21}} - \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^7} dx, x, x^3\right)}{21ab^3 (ab + b^2x^3)} \\ &= -\frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{21ax^{21}} + \frac{b (a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{126a^2x^{18}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.99

$$\frac{\sqrt{(a + bx^3)^2} (6a^5 + 35a^4bx^3 + 84a^3b^2x^6 + 105a^2b^3x^9 + 70ab^4x^{12} + 21b^5x^{15})}{126x^{21} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]

[Out] -1/126*(Sqrt[(a + b*x^3)^2]*(6*a^5 + 35*a^4*b*x^3 + 84*a^3*b^2*x^6 + 105*a^2*b^3*x^9 + 70*a*b^4*x^12 + 21*b^5*x^15))/(x^21*(a + b*x^3))

IntegrateAlgebraic [B] time = 1.67, size = 488, normalized size = 5.81

324*sqrt(2)*sqrt(5)*sqrt(13)*sqrt(17)*sqrt(19)*sqrt(23)*sqrt(29)*sqrt(31)*sqrt(37)*sqrt(41)*sqrt(43)*sqrt(47)*sqrt(53)*sqrt(59)*sqrt(61)*sqrt(67)*sqrt(71)*sqrt(73)*sqrt(79)*sqrt(83)*sqrt(89)*sqrt(97)*sqrt(101)*sqrt(103)*sqrt(107)*sqrt(113)*sqrt(127)*sqrt(131)*sqrt(137)*sqrt(143)*sqrt(149)*sqrt(157)*sqrt(163)*sqrt(167)*sqrt(173)*sqrt(179)*sqrt(181)*sqrt(187)*sqrt(193)*sqrt(197)*sqrt(211)*sqrt(223)*sqrt(227)*sqrt(229)*sqrt(233)*sqrt(239)*sqrt(241)*sqrt(247)*sqrt(251)*sqrt(257)*sqrt(263)*sqrt(269)*sqrt(271)*sqrt(277)*sqrt(281)*sqrt(283)*sqrt(287)*sqrt(293)*sqrt(299)*sqrt(307)*sqrt(311)*sqrt(313)*sqrt(317)*sqrt(323)*sqrt(329)*sqrt(331)*sqrt(337)*sqrt(341)*sqrt(343)*sqrt(347)*sqrt(353)*sqrt(359)*sqrt(361)*sqrt(367)*sqrt(371)*sqrt(373)*sqrt(379)*sqrt(383)*sqrt(389)*sqrt(397)*sqrt(401)*sqrt(403)*sqrt(407)*sqrt(413)*sqrt(427)*sqrt(431)*sqrt(433)*sqrt(437)*sqrt(443)*sqrt(449)*sqrt(457)*sqrt(461)*sqrt(463)*sqrt(467)*sqrt(473)*sqrt(479)*sqrt(481)*sqrt(487)*sqrt(491)*sqrt(493)*sqrt(497)

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]

[Out] (32*b^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-6*a^11*b - 71*a^10*b^2*x^3 - 384*a^9*b^3*x^6 - 1254*a^8*b^4*x^9 - 2750*a^7*b^5*x^12 - 4257*a^6*b^6*x^15 - 4752*a^5*b^7*x^18 - 3829*a^4*b^8*x^21 - 2184*a^3*b^9*x^24 - 840*a^2*b^10*x^27 - 196*a*b^11*x^30 - 21*b^12*x^33) + 32*b^6*Sqrt[b^2]*(6*a^12 + 77*a^11*b*x^3 + 455*a^10*b^2*x^6 + 1638*a^9*b^3*x^9 + 4004*a^8*b^4*x^12 + 7007*a^7*b^5*x^15 + 9009*a^6*b^6*x^18 + 8581*a^5*b^7*x^21 + 6013*a^4*b^8*x^24 + 3024*a^3*b^9*x^27 + 1036*a^2*b^10*x^30 + 217*a*b^11*x^33 + 21*b^12*x^36))/(63*Sqrt[b^2]*x^21*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-64*a^6*b^6 - 384*a^5*b^7*x^3 - 960*a^4*b^8*x^6 - 1280*a^3*b^9*x^9 - 960*a^2*b^10*x^12 - 384*a*b^11*x^15 - 64*b^12*x^18) + 63*x^21*(64*a^7*b^7 + 448*a^6*b^8*x^3 + 1344*a^5*b^9*x^6 + 2240*a^4*b^10*x^9 + 2240*a^3*b^11*x^12 + 1344*a^2*b^12*x^15 + 448*a*b^13*x^18 + 64*b^14*x^21))

fricas [A] time = 1.02, size = 59, normalized size = 0.70

$$\frac{21 b^5 x^{15} + 70 a b^4 x^{12} + 105 a^2 b^3 x^9 + 84 a^3 b^2 x^6 + 35 a^4 b x^3 + 6 a^5}{126 x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="fricas")

[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 35*a^4*b*x^3 + 6*a^5)/x^21

giac [A] time = 0.32, size = 107, normalized size = 1.27

$$\frac{21 b^5 x^{15} \operatorname{sgn}(b x^3 + a) + 70 a b^4 x^{12} \operatorname{sgn}(b x^3 + a) + 105 a^2 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 84 a^3 b^2 x^6 \operatorname{sgn}(b x^3 + a) + 35 a^4 b x^3 \operatorname{sgn}(b x^3 + a) + 6 a^5 \operatorname{sgn}(b x^3 + a)}{126 x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="giac")

[Out] -1/126*(21*b^5*x^15*sgn(b*x^3 + a) + 70*a*b^4*x^12*sgn(b*x^3 + a) + 105*a^2*b^3*x^9*sgn(b*x^3 + a) + 84*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(b*x^3 + a) + 6*a^5*sgn(b*x^3 + a))/x^21

maple [A] time = 0.01, size = 80, normalized size = 0.95

$$\frac{(21 b^5 x^{15} + 70 a b^4 x^{12} + 105 a^2 b^3 x^9 + 84 a^3 b^2 x^6 + 35 a^4 b x^3 + 6 a^5) \left((b x^3 + a)^2 \right)^{\frac{5}{2}}}{126 (b x^3 + a)^5 x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x)

[Out] $-1/126*(21*b^5*x^15+70*a*b^4*x^12+105*a^2*b^3*x^9+84*a^3*b^2*x^6+35*a^4*b*x^3+6*a^5)*((b*x^3+a)^2)^(5/2)/x^21/(b*x^3+a)^5$

maxima [B] time = 1.16, size = 241, normalized size = 2.87

$$-\frac{(b^2x^6+2abx^3+a^2)^{5/2}b^7}{18a^7} - \frac{(b^2x^6+2abx^3+a^2)^{5/2}b^6}{18a^6x^3} + \frac{(b^2x^6+2abx^3+a^2)^{7/2}b^5}{18a^7x^6} - \frac{(b^2x^6+2abx^3+a^2)^{7/2}b^4}{18a^6x^9} + \frac{(b^2x^6+2abx^3+a^2)^{7/2}b^3}{18a^5x^{12}} - \frac{(b^2x^6+2abx^3+a^2)^{7/2}b^2}{18a^4x^{15}} + \frac{(b^2x^6+2abx^3+a^2)^{7/2}b}{18a^3x^{18}} - \frac{(b^2x^6+2abx^3+a^2)^{7/2}}{21a^2x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="maxima")

[Out] $-1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^7/a^7 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^6/(a^6*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a^7*x^6) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^9) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^{12}) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^{15}) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^{18}) - 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^{21})$

mupad [B] time = 1.22, size = 231, normalized size = 2.75

$$\frac{a^5 \sqrt{a^2+2abx^3+b^2x^6}}{21x^{21}(bx^3+a)} - \frac{b^5 \sqrt{a^2+2abx^3+b^2x^6}}{6x^6(bx^3+a)} - \frac{5ab^4 \sqrt{a^2+2abx^3+b^2x^6}}{9x^9(bx^3+a)} - \frac{5a^4b \sqrt{a^2+2abx^3+b^2x^6}}{18x^{18}(bx^3+a)} - \frac{5a^2b^3 \sqrt{a^2+2abx^3+b^2x^6}}{6x^{12}(bx^3+a)} - \frac{2a^3b^2 \sqrt{a^2+2abx^3+b^2x^6}}{3x^{15}(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^22,x)

[Out] $-(a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(21*x^{21}*(a + b*x^3)) - (b^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(18*x^{18}*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^{12}*(a + b*x^3)) - (2*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^{15}*(a + b*x^3))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{22}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**22, x)

$$3.86 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{23}} dx$$

Optimal. Leaf size=255

$$\frac{b^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7 (a + bx^3)} - \frac{ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^{10} (a + bx^3)} - \frac{10a^2b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13} (a + bx^3)} - \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{22x^{22} (a + bx^3)} - \frac{5a^4b \sqrt{a^2 + 2abx^3 + b^2x^6}}{19x^{19} (a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{22x^{22} (a + bx^3)} - \frac{5a^4b \sqrt{a^2 + 2abx^3 + b^2x^6}}{19x^{19} (a + bx^3)} - \frac{5a^3b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^{16} (a + bx^3)} - \frac{10a^2b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{13x^{13} (a + bx^3)} - \frac{ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{2x^{10} (a + bx^3)} - \frac{b^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^7 (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*x^22*(a + b*x^3)) - (5*a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^16*(a + b*x^3)) - (10*a^2*b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a*b^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (b^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{23}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{23}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5 b^5}{x^{23}} + \frac{5a^4 b^6}{x^{20}} + \frac{10a^3 b^7}{x^{17}} + \frac{10a^2 b^8}{x^{14}} + \frac{5ab^9}{x^{11}} + \frac{b^{10}}{x^8} \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{22x^{22} (a + bx^3)} - \frac{5a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{19x^{19} (a + bx^3)} - \frac{5a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^{16} (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (6916a^5 + 40040a^4bx^3 + 95095a^3b^2x^6 + 117040a^2b^3x^9 + 76076ab^4x^{12} + 21736b^5x^{15})}{152152x^{22} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]

[Out] -1/152152*(Sqrt[(a + b*x^3)^2]*(6916*a^5 + 40040*a^4*b*x^3 + 95095*a^3*b^2*x^6 + 117040*a^2*b^3*x^9 + 76076*a*b^4*x^12 + 21736*b^5*x^15))/(x^22*(a + b*x^3))

IntegrateAlgebraic [A] time = 23.70, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-6916a^5 - 40040a^4bx^3 - 95095a^3b^2x^6 - 117040a^2b^3x^9 - 76076ab^4x^{12} - 21736b^5x^{15})}{152152x^{22} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-6916*a^5 - 40040*a^4*b*x^3 - 95095*a^3*b^2*x^6 - 117040*a^2*b^3*x^9 - 76076*a*b^4*x^12 - 21736*b^5*x^15))/(152152*x^22*(a + b*x^3))

fricas [A] time = 1.11, size = 59, normalized size = 0.23

$$-\frac{21736 b^5 x^{15} + 76076 a b^4 x^{12} + 117040 a^2 b^3 x^9 + 95095 a^3 b^2 x^6 + 40040 a^4 b x^3 + 6916 a^5}{152152 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="fricas")

[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22

giac [A] time = 0.33, size = 107, normalized size = 0.42

$$\frac{21736 b^5 x^{15} \operatorname{sgn}(b x^3 + a) + 76076 a b^4 x^{12} \operatorname{sgn}(b x^3 + a) + 117040 a^2 b^3 x^9 \operatorname{sgn}(b x^3 + a) + 95095 a^3 b^2 x^6 \operatorname{sgn}(b x^3 + a) + 40040 a^4 b x^3 \operatorname{sgn}(b x^3 + a) + 6916 a^5 \operatorname{sgn}(b x^3 + a)}{152152 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="giac")

[Out] -1/152152*(21736*b^5*x^15*sgn(b*x^3 + a) + 76076*a*b^4*x^12*sgn(b*x^3 + a) + 117040*a^2*b^3*x^9*sgn(b*x^3 + a) + 95095*a^3*b^2*x^6*sgn(b*x^3 + a) + 40040*a^4*b*x^3*sgn(b*x^3 + a) + 6916*a^5*sgn(b*x^3 + a))/x^22

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(21736 b^5 x^{15} + 76076 a b^4 x^{12} + 117040 a^2 b^3 x^9 + 95095 a^3 b^2 x^6 + 40040 a^4 b x^3 + 6916 a^5) \left((b x^3 + a)^2 \right)^{\frac{5}{2}}}{152152 (b x^3 + a)^5 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x)

[Out] -1/152152*(21736*b^5*x^15+76076*a*b^4*x^12+117040*a^2*b^3*x^9+95095*a^3*b^2*x^6+40040*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/x^22/(b*x^3+a)^5

maxima [A] time = 0.98, size = 59, normalized size = 0.23

$$\frac{21736 b^5 x^{15} + 76076 a b^4 x^{12} + 117040 a^2 b^3 x^9 + 95095 a^3 b^2 x^6 + 40040 a^4 b x^3 + 6916 a^5}{152152 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="maxima")

[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22

mapad [B] time = 1.22, size = 231, normalized size = 0.91

$$\frac{a^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{22 x^{22} (b x^3 + a)} - \frac{b^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{7 x^7 (b x^3 + a)} - \frac{a b^4 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{2 x^{10} (b x^3 + a)} - \frac{5 a^4 b \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{19 x^{19} (b x^3 + a)} - \frac{10 a^2 b^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{13 x^{13} (b x^3 + a)} - \frac{5 a^3 b^2 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{8 x^{16} (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^23,x)`

[Out] $-\frac{a^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{22x^{22}(a + bx^3)} - \frac{b^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{7x^7(a + bx^3)} - \frac{ab^4(a^2 + b^2x^6 + 2abx^3)^{1/2}}{2x^{10}(a + bx^3)} - \frac{5a^4b(a^2 + b^2x^6 + 2abx^3)^{1/2}}{19x^{19}(a + bx^3)} - \frac{10a^2b^3(a^2 + b^2x^6 + 2abx^3)^{1/2}}{13x^{13}(a + bx^3)} - \frac{5a^3b^2(a^2 + b^2x^6 + 2abx^3)^{1/2}}{8x^{16}(a + bx^3)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{23}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**23,x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**23, x)`

$$3.87 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{24}} dx$$

Optimal. Leaf size=255

$$\frac{b^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)} - \frac{5ab^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{5a^2b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^{14}(a + bx^3)} - \frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{23x^{23}(a + bx^3)} - \frac{a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^{26}(a + bx^3)}$$

Rubi [A] time = 0.06, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{a^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{23x^{23}(a + bx^3)} - \frac{a^4b\sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^{20}(a + bx^3)} - \frac{10a^3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}}{17x^{17}(a + bx^3)} - \frac{5a^2b^3\sqrt{a^2 + 2abx^3 + b^2x^6}}{7x^{14}(a + bx^3)} - \frac{5ab^4\sqrt{a^2 + 2abx^3 + b^2x^6}}{11x^{11}(a + bx^3)} - \frac{b^5\sqrt{a^2 + 2abx^3 + b^2x^6}}{8x^8(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]

[Out] -(a^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*x^23*(a + b*x^3)) - (a^4*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^20*(a + b*x^3)) - (10*a^3*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^2*b^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (5*a*b^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (b^5*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{24}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab + b^2x^3)^5}{x^{24}} dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(\frac{a^5 b^5}{x^{24}} + \frac{5a^4 b^6}{x^{21}} + \frac{10a^3 b^7}{x^{18}} + \frac{10a^2 b^8}{x^{15}} + \frac{5ab^9}{x^{12}} + \frac{b^{10}}{x^9} \right) dx}{b^4 (ab + b^2x^3)} \\
&= -\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{23x^{23} (a + bx^3)} - \frac{a^4 b \sqrt{a^2 + 2abx^3 + b^2x^6}}{4x^{20} (a + bx^3)} - \frac{10a^3 b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{17x^{17} (a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (10472a^5 + 60214a^4bx^3 + 141680a^3b^2x^6 + 172040a^2b^3x^9 + 109480ab^4x^{12} + 30107b^5x^{15})}{240856x^{23} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]

[Out] -1/240856*(Sqrt[(a + b*x^3)^2]*(10472*a^5 + 60214*a^4*b*x^3 + 141680*a^3*b^2*x^6 + 172040*a^2*b^3*x^9 + 109480*a*b^4*x^12 + 30107*b^5*x^15))/(x^23*(a + b*x^3))

IntegrateAlgebraic [A] time = 23.52, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a + bx^3)^2} (-10472a^5 - 60214a^4bx^3 - 141680a^3b^2x^6 - 172040a^2b^3x^9 - 109480ab^4x^{12} - 30107b^5x^{15})}{240856x^{23} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]

[Out] (Sqrt[(a + b*x^3)^2]*(-10472*a^5 - 60214*a^4*b*x^3 - 141680*a^3*b^2*x^6 - 172040*a^2*b^3*x^9 - 109480*a*b^4*x^12 - 30107*b^5*x^15))/(240856*x^23*(a + b*x^3))

fricas [A] time = 1.09, size = 59, normalized size = 0.23

$$\frac{30107 b^5 x^{15} + 109480 a b^4 x^{12} + 172040 a^2 b^3 x^9 + 141680 a^3 b^2 x^6 + 60214 a^4 b x^3 + 10472 a^5}{240856 x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="fricas")

[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23

giac [A] time = 0.34, size = 107, normalized size = 0.42

$$\frac{30107b^5x^{15}\operatorname{sgn}(bx^3+a) + 109480ab^4x^{12}\operatorname{sgn}(bx^3+a) + 172040a^2b^3x^9\operatorname{sgn}(bx^3+a) + 141680a^3b^2x^6\operatorname{sgn}(bx^3+a) + 60214a^4bx^3\operatorname{sgn}(bx^3+a) + 10472a^5\operatorname{sgn}(bx^3+a)}{240856x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="giac")

[Out] -1/240856*(30107*b^5*x^15*sgn(b*x^3 + a) + 109480*a*b^4*x^12*sgn(b*x^3 + a) + 172040*a^2*b^3*x^9*sgn(b*x^3 + a) + 141680*a^3*b^2*x^6*sgn(b*x^3 + a) + 60214*a^4*b*x^3*sgn(b*x^3 + a) + 10472*a^5*sgn(b*x^3 + a))/x^23

maple [A] time = 0.01, size = 80, normalized size = 0.31

$$\frac{(30107b^5x^{15} + 109480ab^4x^{12} + 172040a^2b^3x^9 + 141680a^3b^2x^6 + 60214a^4bx^3 + 10472a^5)\left((bx^3 + a)^2\right)^{\frac{5}{2}}}{240856(bx^3 + a)^5x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x)

[Out] -1/240856*(30107*b^5*x^15+109480*a*b^4*x^12+172040*a^2*b^3*x^9+141680*a^3*b^2*x^6+60214*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/x^23/(b*x^3+a)^5

maxima [A] time = 0.83, size = 59, normalized size = 0.23

$$\frac{30107b^5x^{15} + 109480ab^4x^{12} + 172040a^2b^3x^9 + 141680a^3b^2x^6 + 60214a^4bx^3 + 10472a^5}{240856x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="maxima")

[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23

mapad [B] time = 1.23, size = 231, normalized size = 0.91

$$\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}(bx^3+a)} - \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(bx^3+a)} - \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(bx^3+a)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}(bx^3+a)} - \frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(bx^3+a)} - \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(bx^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^24,x)`

[Out] $-\frac{a^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{23x^{23}(a + bx^3)} - \frac{b^5(a^2 + b^2x^6 + 2abx^3)^{1/2}}{8x^8(a + bx^3)} - \frac{5a^4b^4(a^2 + b^2x^6 + 2abx^3)^{1/2}}{11x^{11}(a + bx^3)} - \frac{a^4b^4(a^2 + b^2x^6 + 2abx^3)^{1/2}}{4x^{20}(a + bx^3)} - \frac{5a^2b^3(a^2 + b^2x^6 + 2abx^3)^{1/2}}{7x^{14}(a + bx^3)} - \frac{10a^3b^2(a^2 + b^2x^6 + 2abx^3)^{1/2}}{17x^{17}(a + bx^3)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{24}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**24,x)`

[Out] `Integral(((a + b*x**3)**2)**(5/2)/x**24, x)`

$$3.88 \quad \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{25}} dx$$

Optimal. Leaf size=128

$$-\frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{24ax^{24}} + \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{84a^2x^{21}} - \frac{b^2\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{504a^3x^{18}}$$

Rubi [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 266, 45, 37}

$$-\frac{b^2\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{504a^3x^{18}} + \frac{b\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{84a^2x^{21}} - \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} (a + bx^3)^5}{24ax^{24}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]

[Out] -((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(24*a*x^24) + (b*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(84*a^2*x^21) - (b^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(504*a^3*x^18)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
+ d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
(EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] || !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p]))], Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
 \int \frac{(a^2 + 2abx^3 + b^2x^6)^{5/2}}{x^{25}} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \frac{(ab+b^2x^3)^5}{x^{25}} dx}{b^4 (ab + b^2x^3)} \\
 &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^5}{x^9} dx, x, x^3\right)}{3b^4 (ab + b^2x^3)} \\
 &= -\frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{24ax^{24}} - \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \operatorname{Subst}\left(\int \frac{(ab+b^2x)^5}{x^8} dx, x, x^3\right)}{12ab^3 (ab + b^2x^3)} \\
 &= -\frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{24ax^{24}} + \frac{b(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{84a^2x^{21}} + \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{84a^2x^{21}} \\
 &= -\frac{(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{24ax^{24}} + \frac{b(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{84a^2x^{21}} - \frac{b^2(a + bx^3)^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{84a^2x^{21}}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 0.65

$$\frac{\sqrt{(a + bx^3)^2} (21a^5 + 120a^4bx^3 + 280a^3b^2x^6 + 336a^2b^3x^9 + 210ab^4x^{12} + 56b^5x^{15})}{504x^{24} (a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25, x]

[Out] -1/504*(Sqrt[(a + b*x^3)^2]*(21*a^5 + 120*a^4*b*x^3 + 280*a^3*b^2*x^6 + 336*a^2*b^3*x^9 + 210*a*b^4*x^12 + 56*b^5*x^15))/(x^24*(a + b*x^3))

IntegrateAlgebraic [B] time = 1.54, size = 532, normalized size = 4.16

$16\sqrt{2}\sqrt{23a^2+7b^2}(-21a^9-267a^8b-196a^7b^2-551a^6b^3-1337a^5b^4-2302a^4b^5-2902a^3b^6-2702a^2b^7-1844a^2b^8-900a^2b^9-602ab^{10}-56b^{11})+16\sqrt{2}(21a^{13}+288a^{12}b+1828a^{11}b^2+7112a^{10}b^3+18928a^9b^4+36400a^8b^5+52052a^7b^6+56056a^6b^7+45473a^5b^8+27448a^4b^9+11984a^3b^{10}+3584a^2b^{11}+658ab^{12}+56b^{13})\sqrt{b^2}+63\sqrt{2}\sqrt{23a^2+7b^2}(21a^{13}+288a^{12}b+1828a^{11}b^2+7112a^{10}b^3+18928a^9b^4+36400a^8b^5+52052a^7b^6+56056a^6b^7+45473a^5b^8+27448a^4b^9+11984a^3b^{10}+3584a^2b^{11}+658ab^{12}+56b^{13})\sqrt{a^2+2abx^3+b^2x^6}(-128a^7b^7-896a^6b^8x^3-2688a^5b^9x^6-4480a^4b^{10}x^9-4480a^3b^{11}x^{12}-2688a^2b^{12}x^{15}-896ab^{13}x^{18}-128b^{14}x^{21})+63x^{24}(128a^8b^8+1024a^7b^9x^3+3584a^6b^{10}x^6+7168a^5b^{11}x^9+8960a^4b^{12}x^{12}+7168a^3b^{13}x^{15}+3584a^2b^{14}x^{18}+1024ab^{15}x^{21}+128b^{16}x^{24})$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]

[Out] (16*b^7*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-21*a^12*b - 267*a^11*b^2*x^3 - 1561*a^10*b^3*x^6 - 5551*a^9*b^4*x^9 - 13377*a^8*b^5*x^12 - 23023*a^7*b^6*x^15 - 29029*a^6*b^7*x^18 - 27027*a^5*b^8*x^21 - 18446*a^4*b^9*x^24 - 9002*a^3*b^10*x^27 - 2982*a^2*b^11*x^30 - 602*a*b^12*x^33 - 56*b^13*x^36) + 16*b^7*sqrt[b^2]*(21*a^13 + 288*a^12*b*x^3 + 1828*a^11*b^2*x^6 + 7112*a^10*b^3*x^9 + 18928*a^9*b^4*x^12 + 36400*a^8*b^5*x^15 + 52052*a^7*b^6*x^18 + 56056*a^6*b^7*x^21 + 45473*a^5*b^8*x^24 + 27448*a^4*b^9*x^27 + 11984*a^3*b^10*x^30 + 3584*a^2*b^11*x^33 + 658*a*b^12*x^36 + 56*b^13*x^39))/(63*sqrt[b^2]*x^24*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-128*a^7*b^7 - 896*a^6*b^8*x^3 - 2688*a^5*b^9*x^6 - 4480*a^4*b^10*x^9 - 4480*a^3*b^11*x^12 - 2688*a^2*b^12*x^15 - 896*a*b^13*x^18 - 128*b^14*x^21) + 63*x^24*(128*a^8*b^8 + 1024*a^7*b^9*x^3 + 3584*a^6*b^10*x^6 + 7168*a^5*b^11*x^9 + 8960*a^4*b^12*x^12 + 7168*a^3*b^13*x^15 + 3584*a^2*b^14*x^18 + 1024*a*b^15*x^21 + 128*b^16*x^24))

fricas [A] time = 1.27, size = 59, normalized size = 0.46

$$\frac{56b^5x^{15} + 210ab^4x^{12} + 336a^2b^3x^9 + 280a^3b^2x^6 + 120a^4bx^3 + 21a^5}{504x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="fricas")

[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 + 120*a^4*b*x^3 + 21*a^5)/x^24

giac [A] time = 0.39, size = 107, normalized size = 0.84

$$\frac{56b^5x^{15}\operatorname{sgn}(bx^3+a) + 210ab^4x^{12}\operatorname{sgn}(bx^3+a) + 336a^2b^3x^9\operatorname{sgn}(bx^3+a) + 280a^3b^2x^6\operatorname{sgn}(bx^3+a) + 120a^4bx^3\operatorname{sgn}(bx^3+a) + 21a^5\operatorname{sgn}(bx^3+a)}{504x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="giac")

[Out] -1/504*(56*b^5*x^15*sgn(b*x^3 + a) + 210*a*b^4*x^12*sgn(b*x^3 + a) + 336*a^2*b^3*x^9*sgn(b*x^3 + a) + 280*a^3*b^2*x^6*sgn(b*x^3 + a) + 120*a^4*b*x^3*sgn(b*x^3 + a) + 21*a^5*sgn(b*x^3 + a))/x^24

maple [A] time = 0.01, size = 80, normalized size = 0.62

$$\frac{(56b^5x^{15} + 210ab^4x^{12} + 336a^2b^3x^9 + 280a^3b^2x^6 + 120a^4bx^3 + 21a^5) \left((bx^3 + a)^2 \right)^{\frac{5}{2}}}{504 (bx^3 + a)^5 x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x)

[Out] -1/504*(56*b^5*x^15+210*a*b^4*x^12+336*a^2*b^3*x^9+280*a^3*b^2*x^6+120*a^4*b*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/x^24/(b*x^3+a)^5

maxima [B] time = 1.02, size = 272, normalized size = 2.12

$$\frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^8}{18a^8} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{5}{2}}b^7}{18a^7x^3} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b^6}{18a^6x^6} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b^5}{18a^5x^9} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b^4}{18a^4x^{12}} + \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b^3}{18a^3x^{15}} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b^2}{18a^2x^{18}} + \frac{3(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}b}{56a^3x^{21}} - \frac{(b^2x^6 + 2abx^3 + a^2)^{\frac{7}{2}}}{24a^2x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="maxima")

[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^8/a^8 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^7/(a^7*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^6/(a^8*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a^7*x^9) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^18) + 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^21) - 1/24*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^24)

mupad [B] time = 1.22, size = 231, normalized size = 1.80

$$\frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{24x^{24}(bx^3 + a)} - \frac{b^5 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^9(bx^3 + a)} - \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6}}{12x^{12}(bx^3 + a)} - \frac{5a^4b \sqrt{a^2 + 2abx^3 + b^2x^6}}{21x^{21}(bx^3 + a)} - \frac{2a^2b^3 \sqrt{a^2 + 2abx^3 + b^2x^6}}{3x^{15}(bx^3 + a)} - \frac{5a^3b^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}{9x^{18}(bx^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^25,x)

[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(24*x^24*(a + b*x^3)) - (b^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (2*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^15*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^18*(a + b*x^3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{25}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**25, x)

$$3.89 \quad \int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}} dx$$

Optimal. Leaf size=240

$$\frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}(a+bx^3)\log(a)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.12, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1355, 321, 292, 31, 634, 617, 204, 628}

$$\frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}(a+bx^3)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] (x^2*(a + b*x^3))/(2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a^2 + 2abx^3 + b^2x^6}} dx &= \frac{(ab + b^2x^3) \int \frac{x^4}{ab+b^2x^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2(a + bx^3)}{2b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a(ab + b^2x^3)) \int \frac{x}{ab+b^2x^3} dx}{b\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2(a + bx^3)}{2b\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a^{2/3}(ab + b^2x^3)) \int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x} dx}{3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a^{2/3}(ab + b^2x^3)) \int \frac{1}{a}}{3b^2\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2(a + bx^3)}{2b\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{a^{2/3}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a^{2/3}(ab + b^2x^3)) \int \frac{1}{a}}{6b^{8/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2(a + bx^3)}{2b\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{a^{2/3}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{a^{2/3}(a + bx^3) \log(a^{2/3})}{6b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2(a + bx^3)}{2b\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{a^{2/3}(a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{a^{2/3}(a + bx^3) \log(\sqrt[3]{a})}{3b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 0.55

$$\frac{(a + bx^3) \left(-a^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2) + 2a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x) + 2\sqrt{3} a^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 3b^{2/3}x^2 \right)}{6b^{5/3} \sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] ((a + b*x^3)*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [A] time = 7.36, size = 144, normalized size = 0.60

$$\frac{(a + bx^3) \left(-\frac{a^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{6b^{5/3}} + \frac{a^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}} + \frac{a^{2/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} + \frac{x^2}{2b} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] ((a + b*x^3)*(x^2/(2*b) + (a^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(5/3)) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(5/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 0.85, size = 123, normalized size = 0.51

$$\frac{3x^2 - 2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}a}{3a}\right) - \left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \log\left(ax^2 - bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}} + a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right) + 2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} \log\left(ax + b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((b*x^3+a)^2)^(1/2), x, algorithm="fricas")

[Out] 1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3))/b

giac [A] time = 0.37, size = 146, normalized size = 0.61

$$\frac{x^2 \operatorname{sgn}(bx^3 + a)}{2b} + \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) \operatorname{sgn}(bx^3 + a)}{3b} + \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right) \operatorname{sgn}(bx^3 + a)}{3b^3} - \frac{(-ab^2)^{\frac{2}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) \operatorname{sgn}(bx^3 + a)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((b*x^3+a)^2)^(1/2), x, algorithm="giac")

[Out] 1/2*x^2*sgn(b*x^3 + a)/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^3 - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^3

maple [A] time = 0.01, size = 113, normalized size = 0.47

$$\frac{(bx^3 + a) \left(3\left(\frac{a}{b}\right)^{\frac{1}{3}} bx^2 + 2\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(-2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 2a \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - a \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \right)}{6\sqrt{(bx^3 + a)^2} \left(\frac{a}{b}\right)^{\frac{1}{3}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/((b*x^3+a)^2)^(1/2),x)`

[Out] $\frac{1}{6}*(b*x^3+a)*(3*x^2*b*(a/b)^(1/3)+2*\arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3)))/(a/b)^(1/3))*3^(1/2)*a+2*\ln(x+(a/b)^(1/3))*a-\ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a)/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(1/3)$

maxima [A] time = 2.18, size = 109, normalized size = 0.45

$$\frac{x^2}{2b} - \frac{\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{a \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{2}*x^2/b - \frac{1}{3}*\sqrt{3}*a*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^(1/3)))/(a/b)^(1/3))/((b^2*(a/b)^(1/3)) - \frac{1}{6}*a*\log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(1/3)) + \frac{1}{3}*a*\log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{(bx^3 + a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/((a + b*x^3)^2)^(1/2),x)`

[Out] `int(x^4/((a + b*x^3)^2)^(1/2), x)`

sympy [A] time = 0.21, size = 32, normalized size = 0.13

$$\text{RootSum}\left(27t^3b^5 - a^2, \left(t \mapsto t \log\left(\frac{9t^2b^3}{a} + x\right)\right)\right) + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/((b*x**3+a)**2)**(1/2),x)`

[Out] `RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) + x**2/(2*b)`

$$3.90 \quad \int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}} dx$$

Optimal. Leaf size=235

$$\frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\log(a^{2/3})}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.11, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1355, 321, 200, 31, 634, 617, 204, 628}

$$\frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] (x*(a + b*x^3))/(b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a^2 + 2abx^3 + b^2x^6}} dx &= \frac{(ab + b^2x^3) \int \frac{x^3}{ab + b^2x^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x(a + bx^3)}{b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a(ab + b^2x^3)) \int \frac{1}{ab + b^2x^3} dx}{b\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x(a + bx^3)}{b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(\sqrt[3]{a}(ab + b^2x^3)) \int \frac{1}{\sqrt[3]{a}\sqrt[3]{b} + b^{2/3}x} dx}{3b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(\sqrt[3]{a}(ab + b^2x^3)) \int \frac{1}{a^{2/3}b^{1/3}} dx}{3b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x(a + bx^3)}{b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{\sqrt[3]{a}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(\sqrt[3]{a}(ab + b^2x^3)) \int \frac{1}{a^{2/3}b^{1/3}} dx}{6b^{7/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x(a + bx^3)}{b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{\sqrt[3]{a}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{a}(a + bx^3) \log(a^{2/3} - \sqrt[3]{b}x)}{6b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x(a + bx^3)}{b\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{a}(a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{\sqrt[3]{a}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 128, normalized size = 0.54

$$\frac{(a + bx^3) \left(\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - 2\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{b}x) + 2\sqrt{3} \sqrt[3]{a} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 6\sqrt[3]{b}x \right)}{6b^{4/3}\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] ((a + b*x^3)*(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(4/3)*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [A] time = 6.52, size = 139, normalized size = 0.59

$$\frac{(a + bx^3) \left(\frac{\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6b^{4/3}} - \frac{\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{4/3}} + \frac{\sqrt[3]{a} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}} + \frac{x}{b} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] ((a + b*x^3)*(x/b + (a^(1/3))*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(4/3)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(4/3)) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 1.17, size = 106, normalized size = 0.45

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6x}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((b*x^3+a)^2)^(1/2), x, algorithm="fricas")

[Out] 1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b

giac [A] time = 0.41, size = 143, normalized size = 0.61

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}(bx^3+a)}{3b} + \frac{x\operatorname{sgn}(bx^3+a)}{b} - \frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}(bx^3+a)}{3b^2} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}(bx^3+a)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((b*x^3+a)^2)^(1/2), x, algorithm="giac")

[Out] 1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b + x*sgn(b*x^3 + a)/b - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)))/(-a/b)^(1/3)*sgn(b*x^3 + a)/b^2 - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^2

maple [A] time = 0.01, size = 110, normalized size = 0.47

$$\frac{(bx^3 + a)\left(2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 2a\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + a\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 6\left(\frac{a}{b}\right)^{\frac{2}{3}}bx\right)}{6\sqrt{(bx^3 + a)^2}\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((b*x^3+a)^2)^(1/2),x)`

[Out] $\frac{1}{6}*(b*x^3+a)*(6*x*b*(a/b)^(2/3)+2*3^(1/2)*a*\arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))-2*a*\ln(x+(a/b)^(1/3))+a*\ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(2/3)$

maxima [A] time = 1.36, size = 106, normalized size = 0.45

$$\frac{x}{b} - \frac{\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{a \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $\frac{x}{b} - \frac{1}{3}\sqrt{3}*a*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) + \frac{1}{6}*a*\log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - \frac{1}{3}*a*\log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{(bx^3 + a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((a + b*x^3)^2)^(1/2),x)`

[Out] `int(x^3/((a + b*x^3)^2)^(1/2), x)`

sympy [A] time = 0.20, size = 22, normalized size = 0.09

$$\text{RootSum}\left(27t^3b^4 + a, \left(t \mapsto t \log(-3tb + x)\right)\right) + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/((b*x**3+a)**2)**(1/2),x)`

[Out] `RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(-3*_t*b + x))) + x/b`

$$3.91 \quad \int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}} dx$$

Optimal. Leaf size=44

$$\frac{(a + bx^3) \log(a + bx^3)}{3b\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1352, 608, 31}

$$\frac{(a + bx^3) \log(a + bx^3)}{3b\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 608

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[(b/2 + c*x)/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a^2 + 2abx^3 + b^2x^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}} dx, x, x^3 \right) \\ &= \frac{(ab + b^2x^3) \text{Subst} \left(\int \frac{1}{ab + b^2x} dx, x, x^3 \right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\ &= \frac{(a + bx^3) \log(a + bx^3)}{3b\sqrt{a^2 + 2abx^3 + b^2x^6}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.80

$$\frac{(a + bx^3) \log(a + bx^3)}{3b\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [B] time = 0.27, size = 149, normalized size = 3.39

$$-\frac{\log\left(\sqrt{a^2 + 2abx^3 + b^2x^6} - a - \sqrt{b^2}x^3\right)}{6\sqrt{b^2}} - \frac{\log\left(\sqrt{a^2 + 2abx^3 + b^2x^6} + a - \sqrt{b^2}x^3\right)}{6\sqrt{b^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b^2}x^3}{a} - \frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{a}\right)}{3b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] -1/3*ArcTanh[(Sqrt[b^2]*x^3)/a - Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/a]/b - Log[-a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]]/(6*Sqrt[b^2]) - Log[a - Sqrt[b^2]*x^3 + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]]/(6*Sqrt[b^2])

fricas [A] time = 1.07, size = 13, normalized size = 0.30

$$\frac{\log(bx^3 + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x^3+a)^2)^(1/2), x, algorithm="fricas")

[Out] $1/3 \cdot \log(b \cdot x^3 + a) / b$

giac [A] time = 0.34, size = 22, normalized size = 0.50

$$\frac{\log(|bx^3 + a|) \operatorname{sgn}(bx^3 + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")`

[Out] $1/3 \cdot \log(\operatorname{abs}(b \cdot x^3 + a)) \cdot \operatorname{sgn}(b \cdot x^3 + a) / b$

maple [A] time = 0.01, size = 32, normalized size = 0.73

$$\frac{(bx^3 + a) \ln(bx^3 + a)}{3 \sqrt{(bx^3 + a)^2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((b*x^3+a)^2)^(1/2),x)`

[Out] $1/3 \cdot (b \cdot x^3 + a) \cdot \ln(b \cdot x^3 + a) / b / ((b \cdot x^3 + a)^2)^{(1/2)}$

maxima [A] time = 0.93, size = 15, normalized size = 0.34

$$\frac{\log\left(x^3 + \frac{a}{b}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $1/3 \cdot \log(x^3 + a/b) / b$

mupad [B] time = 1.39, size = 33, normalized size = 0.75

$$\frac{\ln(b^2 x^3 + ab) \operatorname{sign}(2b^2 x^3 + 2ab)}{3 \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^3)^2)^(1/2),x)`

[Out] $(\log(ab + b^2 \cdot x^3) \cdot \operatorname{sign}(2 \cdot a \cdot b + 2 \cdot b^2 \cdot x^3)) / (3 \cdot (b^2)^{(1/2)})$

sympy [A] time = 0.17, size = 10, normalized size = 0.23

$$\frac{\log(a + bx^3)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/((b*x**3+a)**2)**(1/2),x)

[Out] log(a + b*x**3)/(3*b)

$$3.92 \quad \int \frac{x}{\sqrt{a^2 + 2abx^3 + b^2x^6}} dx$$

Optimal. Leaf size=202

$$\frac{(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1355, 292, 31, 634, 617, 204, 628}

$$\frac{(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) - ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a^2 + 2abx^3 + b^2x^6}} dx &= \frac{(ab + b^2x^3) \int \frac{x}{ab+b^2x^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{(ab + b^2x^3) \int \frac{1}{\sqrt[3]{a} \sqrt[3]{b} + b^{2/3}x} dx}{3\sqrt[3]{a} b \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{\sqrt[3]{a} \sqrt[3]{b} + b^{2/3}x}{a^{2/3}b^{2/3} - \sqrt[3]{a} bx + b^{4/3}x^2} dx}{3\sqrt[3]{a} b \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{-\sqrt[3]{a} b + 2b^{4/3}x}{a^{2/3}b^{2/3} - \sqrt[3]{a} bx + b^{4/3}x^2} dx}{6\sqrt[3]{a} b^{5/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{1}{a^{2/3}b^{2/3} - \sqrt[3]{a} bx + b^{4/3}x^2} dx}{2b^{2/3}} \\
&= -\frac{(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{1}{a^{2/3}b^{2/3} - \sqrt[3]{a} bx + b^{4/3}x^2} dx}{2b^{2/3}} \\
&= -\frac{(a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} \sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a} b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 109, normalized size = 0.54

$$\frac{(a + bx^3) \left(\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2) - 2 \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 2\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right) \right)}{6\sqrt[3]{a} b^{2/3} \sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] ((a + b*x^3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(1/3)*b^(2/3)*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [A] time = 8.36, size = 135, normalized size = 0.67

$$\frac{(a + bx^3) \left(\frac{\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{6\sqrt[3]{a} b^{2/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3\sqrt[3]{a} b^{2/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} \sqrt[3]{a} b^{2/3}} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] ((a + b*x^3)*(-ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2/3))) - Log[a^(1/3) + b^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(1/3)*b^(2/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 1.76, size = 304, normalized size = 1.50

$$\frac{3\sqrt{3}ab\sqrt{\frac{(-ab)^{\frac{1}{3}}}{a}} \log\left(\frac{2b^2x^3 - ab + 3\sqrt{\frac{(-ab)^{\frac{1}{3}}}{a}} \sqrt{\frac{(-ab)^{\frac{1}{3}}}{a}} \sqrt{\frac{(-ab)^{\frac{1}{3}}}{a}}}{b^3x^3 + a}\right) + (-ab^2)^{\frac{1}{3}} \log(b^2x^2 + (-ab^2)^{\frac{1}{3}}bx + (-ab^2)^{\frac{1}{3}}) - 2(-ab^2)^{\frac{1}{3}} \log(bx - (-ab^2)^{\frac{1}{3}})}{6ab^2} - \frac{6\sqrt{3}ab\sqrt{\frac{(-ab)^{\frac{1}{3}}}{a}} \arctan\left(\frac{\sqrt{\frac{(-ab)^{\frac{1}{3}}}{a}} \sqrt{\frac{(-ab)^{\frac{1}{3}}}{a}} \sqrt{\frac{(-ab)^{\frac{1}{3}}}{a}}}{b}\right) + (-ab^2)^{\frac{1}{3}} \log(b^2x^2 + (-ab^2)^{\frac{1}{3}}bx + (-ab^2)^{\frac{1}{3}}) - 2(-ab^2)^{\frac{1}{3}} \log(bx - (-ab^2)^{\frac{1}{3}})}{6ab^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x^3+a)^2)^(1/2), x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^2)]

giac [A] time = 0.42, size = 124, normalized size = 0.61

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right) \operatorname{sgn}(bx^3 + a)}{3(-ab^2)^{\frac{1}{3}}} - \frac{\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) \operatorname{sgn}(bx^3 + a)}{6(-ab^2)^{\frac{1}{3}}} - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right) \operatorname{sgn}(bx^3 + a)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x^3+a)^2)^(1/2), x, algorithm="giac")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sgn(b*x^3 + a)/(-a*b^2)^(1/3) - 1/6*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/(-a*b^2)^(1/3) - 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/a

maple [A] time = 0.00, size = 97, normalized size = 0.48

$$\frac{(bx^3 + a) \left(2\sqrt{3} \arctan \left(\frac{\sqrt{3} \left(-2x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right) + 2 \ln \left(x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right) - \ln \left(x^2 - \left(\frac{a}{b} \right)^{\frac{1}{3}} x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) \right)}{6 \sqrt{(bx^3 + a)^2} \left(\frac{a}{b} \right)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((b*x^3+a)^2)^(1/2),x)

[Out] $-1/6*(b*x^3+a)*(2*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})+2*\ln(x+(a/b)^{(1/3)})-\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}))/((b*x^3+a)^2)^{(1/2)}/b/(a/b)^{(1/3)}$

maxima [A] time = 1.28, size = 98, normalized size = 0.49

$$\frac{\sqrt{3} \arctan \left(\frac{\sqrt{3} \left(2x - \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3b \left(\frac{a}{b} \right)^{\frac{1}{3}}} + \frac{\log \left(x^2 - x \left(\frac{a}{b} \right)^{\frac{1}{3}} + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right)}{6b \left(\frac{a}{b} \right)^{\frac{1}{3}}} - \frac{\log \left(x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3b \left(\frac{a}{b} \right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")

[Out] $1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b*(a/b)^{(1/3)}) + 1/6*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b*(a/b)^{(1/3)}) - 1/3*\log(x + (a/b)^{(1/3)})/(b*(a/b)^{(1/3)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{(bx^3 + a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x^3)^2)^(1/2),x)

[Out] int(x/((a + b*x^3)^2)^(1/2), x)

sympy [A] time = 0.18, size = 24, normalized size = 0.12

$$\text{RootSum} \left(27t^3ab^2 + 1, \left(t \mapsto t \log(9t^2ab + x) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

$$3.93 \quad \int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}} dx$$

Optimal. Leaf size=202

$$\frac{(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1343, 200, 31, 634, 617, 204, 628}

$$\frac{(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{6a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^p, x_Symbol] := Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a^2 + 2abx^3 + b^2x^6}} dx &= \frac{(2ab + 2b^2x^3) \int \frac{1}{2ab+2b^2x^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(2ab + 2b^2x^3) \int \frac{1}{\sqrt[3]{2} \sqrt[3]{a} \sqrt[3]{b} + \sqrt[3]{2} b^{2/3} x} dx}{3 \cdot 2^{2/3} a^{2/3} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(2ab + 2b^2x^3) \int \frac{2 \sqrt[3]{2} \sqrt[3]{a} \sqrt[3]{b} - \sqrt[3]{2} b^{2/3} x}{2^{2/3} a^{2/3} b^{2/3} - 2^{2/3} \sqrt[3]{a} b x + 2^{2/3} b^{4/3} x^2} dx}{3 \cdot 2^{2/3} a^{2/3} b^{2/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} \sqrt[3]{b} \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(2ab + 2b^2x^3) \int \frac{-2^{2/3} \sqrt[3]{a} b + 2 \cdot 2^{2/3} b^{4/3} x}{2^{2/3} a^{2/3} b^{2/3} - 2^{2/3} \sqrt[3]{a} b x + 2^{2/3} b^{4/3} x^2} dx}{12a^{2/3} b^{4/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= \frac{(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} \sqrt[3]{b} \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} \sqrt[3]{b} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(2ab + 2b^2x^3) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} \sqrt[3]{b} \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{(a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} \sqrt[3]{b} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} \sqrt[3]{b} \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} \sqrt[3]{b} \sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 109, normalized size = 0.54

$$\frac{(a + bx^3) \left(\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2) - 2 \log(\sqrt[3]{a} + \sqrt[3]{b} x) + 2\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right) \right)}{6a^{2/3} \sqrt[3]{b} \sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] -1/6*((a + b*x^3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/ (a^(2/3)*b^(1/3)*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [A] time = 10.09, size = 135, normalized size = 0.67

$$\frac{(a + bx^3) \left(-\frac{\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} \sqrt[3]{b}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} \sqrt[3]{b}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} \sqrt[3]{b}} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]

[Out] ((a + b*x^3)*(-ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(1/3))) + Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(2/3)*b^(1/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 1.31, size = 299, normalized size = 1.48

$$\left[\frac{3 \sqrt{\frac{3}{2}} ab \sqrt{\frac{(a^2)^{\frac{1}{3}}}{b}} \log\left(\frac{2 abx^2 - 3(a^2)^{\frac{1}{3}} ax - a^2 + 3 \sqrt{\frac{3}{2}} \frac{2 abx^2 + (a^2)^{\frac{1}{3}} x - (a^2)^{\frac{1}{3}} a}{bx^3 + a}} \sqrt{\frac{(a^2)^{\frac{1}{3}}}{b}}\right)}{6 a^2 b} - \frac{(a^2)^{\frac{1}{3}} \log\left(abx^2 - (a^2)^{\frac{1}{3}} x + (a^2)^{\frac{1}{3}} a\right) + 2 (a^2)^{\frac{1}{3}} \log\left(abx + (a^2)^{\frac{1}{3}}\right)}{6 a^2 b}, \frac{6 \sqrt{\frac{3}{2}} ab \sqrt{\frac{(a^2)^{\frac{1}{3}}}{b}} \arctan\left(\frac{\sqrt{\frac{3}{2}} \left(2 (a^2)^{\frac{1}{3}} x - (a^2)^{\frac{1}{3}} a\right) \sqrt{\frac{(a^2)^{\frac{1}{3}}}{b}}}{a^2}\right)}{6 a^2 b} - \frac{(a^2)^{\frac{1}{3}} \log\left(abx^2 - (a^2)^{\frac{1}{3}} x + (a^2)^{\frac{1}{3}} a\right) + 2 (a^2)^{\frac{1}{3}} \log\left(abx + (a^2)^{\frac{1}{3}}\right)}{6 a^2 b} \right] \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2 - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]

giac [A] time = 0.36, size = 122, normalized size = 0.60

$$-\frac{1}{6} \left(\frac{2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a} - \frac{2 \sqrt{3} (-ab^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{(-ab^2)^{\frac{1}{3}} \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab} \right) \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b))*sgn(b*x^3 + a)

maple [A] time = 0.00, size = 97, normalized size = 0.48

$$\frac{(bx^3 + a) \left(-2\sqrt{3} \arctan \left(\frac{\sqrt{3} \left(-2x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right) + 2 \ln \left(x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right) - \ln \left(x^2 - \left(\frac{a}{b} \right)^{\frac{1}{3}} x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) \right)}{6 \sqrt{(bx^3 + a)^2} \left(\frac{a}{b} \right)^{\frac{2}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x^3+a)^2)^(1/2),x)

[Out] 1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))+2*ln(x+(a/b)^(1/3))-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2)/b/(a/b)^(2/3)

maxima [A] time = 2.73, size = 98, normalized size = 0.49

$$\frac{\sqrt{3} \arctan \left(\frac{\sqrt{3} \left(2x - \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3b \left(\frac{a}{b} \right)^{\frac{2}{3}}} - \frac{\log \left(x^2 - x \left(\frac{a}{b} \right)^{\frac{1}{3}} + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right)}{6b \left(\frac{a}{b} \right)^{\frac{2}{3}}} + \frac{\log \left(x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3b \left(\frac{a}{b} \right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{(bx^3 + a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x^3)^2)^(1/2),x)

[Out] int(1/((a + b*x^3)^2)^(1/2), x)

sympy [A] time = 0.19, size = 20, normalized size = 0.10

$$\text{RootSum} \left(27t^3 a^2 b - 1, \left(t \mapsto t \log(3ta + x) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

$$3.94 \quad \int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}} dx$$

Optimal. Leaf size=80

$$\frac{\log(x)(a+bx^3)}{a\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1355, 266, 36, 29, 31}

$$\frac{\log(x)(a+bx^3)}{a\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]), x]

[Out] ((a + b*x^3)*Log[x])/(a*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a^2 + 2abx^3 + b^2x^6}} dx &= \frac{(ab + b^2x^3) \int \frac{1}{x(ab+b^2x^3)} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\ &= \frac{(ab + b^2x^3) \operatorname{Subst}\left(\int \frac{1}{x(ab+b^2x)} dx, x, x^3\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\ &= \frac{(ab + b^2x^3) \operatorname{Subst}\left(\int \frac{1}{x} dx, x, x^3\right)}{3ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(b(ab + b^2x^3)) \operatorname{Subst}\left(\int \frac{1}{ab+b^2x} dx, x, x^3\right)}{3a\sqrt{a^2 + 2abx^3 + b^2x^6}} \\ &= \frac{(a + bx^3) \log(x)}{a\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \log(a + bx^3)}{3a\sqrt{a^2 + 2abx^3 + b^2x^6}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.52

$$\frac{(a + bx^3)(3 \log(x) - \log(a + bx^3))}{3a\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]), x]
```

```
[Out] ((a + b*x^3)*(3*Log[x] - Log[a + b*x^3]))/(3*a*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [A] time = 0.20, size = 94, normalized size = 1.18

$$\frac{\log\left(-a\sqrt{a^2 + 2abx^3 + b^2x^6} + a^2 + a\sqrt{b^2x^3}\right)}{3a} - \frac{\log\left(\sqrt{a^2 + 2abx^3 + b^2x^6} + a - \sqrt{b^2x^3}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]), x]
```

[Out] $-1/3 \cdot \text{Log}[a - \text{Sqrt}[b^2] \cdot x^3 + \text{Sqrt}[a^2 + 2 \cdot a \cdot b \cdot x^3 + b^2 \cdot x^6]]/a + \text{Log}[a^2 + a \cdot \text{Sqrt}[b^2] \cdot x^3 - a \cdot \text{Sqrt}[a^2 + 2 \cdot a \cdot b \cdot x^3 + b^2 \cdot x^6]]/(3 \cdot a)$

fricas [A] time = 1.12, size = 18, normalized size = 0.22

$$-\frac{\log(bx^3 + a) - 3 \log(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")`

[Out] $-1/3 \cdot (\log(bx^3 + a) - 3 \cdot \log(x))/a$

giac [A] time = 0.34, size = 32, normalized size = 0.40

$$-\frac{1}{3} \left(\frac{\log(|bx^3 + a|)}{a} - \frac{3 \log(|x|)}{a} \right) \text{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")`

[Out] $-1/3 \cdot (\log(\text{abs}(bx^3 + a))/a - 3 \cdot \log(\text{abs}(x))/a) \cdot \text{sgn}(bx^3 + a)$

maple [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{(bx^3 + a)(3 \ln(x) - \ln(bx^3 + a))}{3 \sqrt{(bx^3 + a)^2} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/((b*x^3+a)^2)^(1/2),x)`

[Out] $1/3 \cdot (bx^3 + a) \cdot (3 \cdot \ln(x) - \ln(bx^3 + a)) / ((bx^3 + a)^2)^{(1/2)} / a$

maxima [A] time = 1.15, size = 43, normalized size = 0.54

$$-\frac{(-1)^{2abx^3+2a^2} \log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $-1/3 \cdot (-1)^{(2 \cdot a \cdot b \cdot x^3 + 2 \cdot a^2)} \cdot \log(2 \cdot a \cdot b \cdot x / \text{abs}(x) + 2 \cdot a^2 / (x^2 \cdot \text{abs}(x))) / a$

mupad [B] time = 1.39, size = 48, normalized size = 0.60

$$\frac{\ln\left(ab + \frac{a^2}{x^3} + \frac{\sqrt{a^2} \sqrt{a^2 + 2abx^3 + b^2x^6}}{x^3}\right)}{3\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*((a + b*x^3)^2)^(1/2)),x)

[Out] -log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)/(3*(a^2)^(1/2))

sympy [A] time = 0.28, size = 15, normalized size = 0.19

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^3\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x**3+a)**2)**(1/2),x)

[Out] log(x)/a - log(a/b + x**3)/(3*a)

$$3.95 \quad \int \frac{1}{x^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} dx$$

Optimal. Leaf size=238

$$-\frac{a + bx^3}{ax\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{b} (a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{b} (a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{\sqrt[3]{b} (a + bx^3) \log(\sqrt[3]{a} - \sqrt[3]{b}x)}{6a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Rubi [A] time = 0.11, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1355, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{a + bx^3}{ax\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{b} (a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{\sqrt[3]{b} (a + bx^3) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{b} (a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]

[Out] -((a + b*x^3)/(a*x*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + (b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))]/(sqrt[3]*a^(4/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 325

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} dx &= \frac{(ab + b^2x^3) \int \frac{1}{x^2(ab+b^2x^3)} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{ax\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(b(ab + b^2x^3)) \int \frac{x}{ab+b^2x^3} dx}{a\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{ax\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{1}{\sqrt[3]{a} \sqrt[3]{b} + b^{2/3}x} dx}{3a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(ab + b^2x^3) \int \frac{\sqrt[3]{a}}{a^{2/3}b^{2/3}}}{3a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{ax\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{b} (a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(ab + b^2x^3) \int \frac{1}{a^{2/3}b^{2/3}}}{6a^{4/3}b^{2/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{ax\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{b} (a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{\sqrt[3]{b} (a + bx^3) \log}{6a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{ax\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{b} (a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{4/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{\sqrt[3]{b} (a + bx^3) \log}{3a^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 133, normalized size = 0.56

$$\frac{(a + bx^3) \left(\sqrt[3]{b} x \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2) - 2\sqrt[3]{b} x \log(\sqrt[3]{a} + \sqrt[3]{b} x) - 2\sqrt{3} \sqrt[3]{b} x \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right) + 6\sqrt[3]{a} \right)}{6a^{4/3}x\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]

[Out] -1/6*((a + b*x^3)*(6*a^(1/3) - 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*x*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(4/3)*x*sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [A] time = 15.05, size = 142, normalized size = 0.60

$$\frac{(a + bx^3) \left(-\frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{4/3}} + \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{4/3}} + \frac{\sqrt[3]{b} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b} x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} - \frac{1}{ax} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]

[Out] ((a + b*x^3)*(-1/(a*x)) + (b^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 1.08, size = 103, normalized size = 0.43

$$\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + x\left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(bx^2 - ax\left(\frac{b}{a}\right)^{\frac{2}{3}} + a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 2x\left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(bx + a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right) + 6}{6ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)

giac [A] time = 0.37, size = 131, normalized size = 0.55

$$\frac{1}{6} \left(\frac{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a^2} + \frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b} - \frac{(-ab^2)^{\frac{2}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b} - \frac{6}{ax} \operatorname{sgn}(bx^3 + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/6*(2*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 6/ax*sgn(b*x^3 + a))

$2)^{(2/3)} * \log(x^2 + x * (-a/b)^{(1/3)} + (-a/b)^{(2/3)}) / (a^2 * b) - 6 / (a * x) * \operatorname{sgn}(b * x^3 + a)$

maple [A] time = 0.01, size = 111, normalized size = 0.47

$$\frac{(bx^3 + a) \left(-2\sqrt{3} x \arctan \left(\frac{\sqrt{3} \left(-2x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right) - 2x \ln \left(x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right) + x \ln \left(x^2 - \left(\frac{a}{b} \right)^{\frac{1}{3}} x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) + 6 \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{6 \sqrt{(bx^3 + a)^2} \left(\frac{a}{b} \right)^{\frac{1}{3}} ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/((b*x^3+a)^2)^(1/2),x)`

[Out] $-1/6 * (b * x^3 + a) * (-2 * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (-2 * x + (a/b)^{(1/3)}) / (a/b)^{(1/3)}) * x + \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * x - 2 * \ln(x + (a/b)^{(1/3)}) * x + 6 * (a/b)^{(1/3)}) / ((b * x^3 + a)^2)^{(1/2)} / a / x / (a/b)^{(1/3)}$

maxima [A] time = 1.74, size = 106, normalized size = 0.45

$$\frac{\sqrt{3} \arctan \left(\frac{\sqrt{3} \left(2x - \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3 a \left(\frac{a}{b} \right)^{\frac{1}{3}}} - \frac{\log \left(x^2 - x \left(\frac{a}{b} \right)^{\frac{1}{3}} + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right)}{6 a \left(\frac{a}{b} \right)^{\frac{1}{3}}} + \frac{\log \left(x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 a \left(\frac{a}{b} \right)^{\frac{1}{3}}} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $-1/3 * \sqrt{3} * \arctan(1/3 * \sqrt{3} * (2 * x - (a/b)^{(1/3)}) / (a/b)^{(1/3)}) / (a * (a/b)^{(1/3)}) - 1/6 * \log(x^2 - x * (a/b)^{(1/3)} + (a/b)^{(2/3)}) / (a * (a/b)^{(1/3)}) + 1/3 * \log(x + (a/b)^{(1/3)}) / (a * (a/b)^{(1/3)}) - 1 / (a * x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \sqrt{(bx^3 + a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*((a + b*x^3)^2)^(1/2)),x)`

[Out] `int(1/(x^2*((a + b*x^3)^2)^(1/2)), x)`

sympy [A] time = 0.23, size = 29, normalized size = 0.12

$$\text{RootSum}\left(27t^3a^4 - b, \left(t \mapsto t \log\left(\frac{9t^2a^3}{b} + x\right)\right)\right) - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)

[Out] RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) - 1/(a*x
)

$$3.96 \quad \int \frac{1}{x^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} dx$$

Optimal. Leaf size=243

$$\frac{-a - bx^3}{2ax^2\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b^{2/3}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{b^{2/3}(a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{b^{2/3}(a + bx^3) \log}{6a^{5/3}\sqrt{a^2}}$$

Rubi [A] time = 0.11, antiderivative size = 240, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1355, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{a + bx^3}{2ax^2\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b^{2/3}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{b^{2/3}(a + bx^3) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{b^{2/3}(a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]), x]

[Out] -(a + b*x^3)/(2*a*x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))]/(sqrt[3]*a^(5/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(5/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*a^(5/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 325

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} dx &= \frac{(ab + b^2x^3) \int \frac{1}{x^3(ab+b^2x^3)} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{2ax^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(b(ab + b^2x^3)) \int \frac{1}{ab+b^2x^3} dx}{a \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{2ax^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(\sqrt[3]{b}(ab + b^2x^3)) \int \frac{1}{\sqrt[3]{a} \sqrt[3]{b+b^2/3x}} dx}{3a^{5/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(\sqrt[3]{b}(ab + b^2x^3)) \int \frac{1}{\sqrt[3]{a} \sqrt[3]{b+b^2/3x}} dx}{3a^{5/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{2ax^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b^{2/3}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{1}{ab+b^2x^3} dx}{6a^{5/3} \sqrt[3]{b} \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{2ax^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b^{2/3}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{b^{2/3}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{6a^{5/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{2ax^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{b^{2/3}(a + bx^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{5/3} \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b^{2/3}(a + bx^3) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{5/3} \sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 140, normalized size = 0.58

$$\frac{(a + bx^3) \left(-b^{2/3} x^2 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2) + 3a^{2/3} + 2b^{2/3} x^2 \log(\sqrt[3]{a} + \sqrt[3]{b} x) - 2\sqrt{3} b^{2/3} x^2 \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right) \right)}{6a^{5/3} x^2 \sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]), x]

[Out] -1/6*((a + b*x^3)*(3*a^(2/3) - 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*x^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(5/3)*x^2*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [A] time = 17.85, size = 144, normalized size = 0.59

$$\frac{(a + bx^3) \left(\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{5/3}} - \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{5/3}} + \frac{b^{2/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{5/3}} - \frac{1}{2ax^2} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]

[Out] ((a + b*x^3)*(-1/2*1/(a*x^2) + (b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(5/3)) - (b^(2/3)*Log[a^(1/3) + b^(1/3)*x]/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3))))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 1.30, size = 143, normalized size = 0.59

$$\frac{2\sqrt{3}x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3}{6ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 3)/(a*x^2)

giac [A] time = 0.33, size = 125, normalized size = 0.51

$$\frac{1}{6}\left(\frac{2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a^2}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2}-\frac{3}{ax^2}\operatorname{sgn}\left(bx^3+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/6*(2*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/(a*x^2))*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 117, normalized size = 0.48

$$\frac{(bx^3 + a) \left(2\sqrt{3} x^2 \arctan \left(\frac{\sqrt{3} \left(-2x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right) - 2x^2 \ln \left(x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right) + x^2 \ln \left(x^2 - \left(\frac{a}{b} \right)^{\frac{1}{3}} x + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right) - 3 \left(\frac{a}{b} \right)^{\frac{2}{3}} \right)}{6 \sqrt{(bx^3 + a)^2} \left(\frac{a}{b} \right)^{\frac{2}{3}} a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/((b*x^3+a)^2)^(1/2), x)

[Out] 1/6*(b*x^3+a)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*x^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^2-2*ln(x+(a/b)^(1/3))*x^2-3*(a/b)^(2/3)/((b*x^3+a)^2)^(1/2)/x^2/a/(a/b)^(2/3)

maxima [A] time = 2.21, size = 106, normalized size = 0.44

$$-\frac{\sqrt{3} \arctan \left(\frac{\sqrt{3} \left(2x - \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3 a \left(\frac{a}{b} \right)^{\frac{2}{3}}} + \frac{\log \left(x^2 - x \left(\frac{a}{b} \right)^{\frac{1}{3}} + \left(\frac{a}{b} \right)^{\frac{2}{3}} \right)}{6 a \left(\frac{a}{b} \right)^{\frac{2}{3}}} - \frac{\log \left(x + \left(\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 a \left(\frac{a}{b} \right)^{\frac{2}{3}}} - \frac{1}{2 a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2), x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \sqrt{(bx^3 + a)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*((a + b*x^3)^2)^(1/2)), x)

[Out] int(1/(x^3*((a + b*x^3)^2)^(1/2)), x)

sympy [A] time = 0.26, size = 32, normalized size = 0.13

$$\text{RootSum} \left(27t^3 a^5 + b^2, \left(t \mapsto t \log \left(-\frac{3ta^2}{b} + x \right) \right) \right) - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**5 + b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(2*a*x**2)
```

$$3.97 \quad \int \frac{1}{x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}} dx$$

Optimal. Leaf size=125

$$\frac{-a - bx^3}{3ax^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b \log(x)(a + bx^3)}{a^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{b(a + bx^3) \log(a + bx^3)}{3a^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 44}

$$-\frac{a + bx^3}{3ax^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b \log(x)(a + bx^3)}{a^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{b(a + bx^3) \log(a + bx^3)}{3a^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]

[Out] -(a + b*x^3)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b*(a + b*x^3)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3])/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt{a^2 + 2abx^3 + b^2x^6}} dx &= \frac{(ab + b^2x^3) \int \frac{1}{x^4(ab+b^2x^3)} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(ab + b^2x^3) \text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)} dx, x, x^3\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(ab + b^2x^3) \text{Subst}\left(\int \left(\frac{1}{abx^2} - \frac{1}{a^2x} + \frac{b}{a^2(a+bx)}\right) dx, x, x^3\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{a + bx^3}{3ax^3\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b(a + bx^3)\log(x)}{a^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{b(a + bx^3)\log(a + bx^3)}{3a^2\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.43

$$\frac{(a + bx^3) \left(-bx^3 \log(a + bx^3) + a + 3bx^3 \log(x) \right)}{3a^2x^3\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]), x]

[Out] -1/3*((a + b*x^3)*(a + 3*b*x^3*Log[x] - b*x^3*Log[a + b*x^3]))/(a^2*x^3*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [B] time = 0.68, size = 383, normalized size = 3.06

$$\frac{\left(\sqrt{a^2 + 2abx^3 + b^2x^6} - \sqrt{b^2x^3}\right)^2 \left(\frac{b \log(\sqrt{a^2 + 2abx^3 + b^2x^6} + a - \sqrt{b^2x^3})}{3a^2} - \frac{b \log(a^3 + a^2\sqrt{b^2x^3} - a^2\sqrt{a^2 + 2abx^3 + b^2x^6})}{3a^2}\right)}{-2\sqrt{b^2x^3}\sqrt{a^2 + 2abx^3 + b^2x^6} + a^2 + 2abx^3 + 2b^2x^6} + \frac{-2\sqrt{a^2 + 2abx^3 + b^2x^6}(-a^2b - 4ab^2x^3 - 4b^3x^6) - 2\sqrt{b^2} (a^3 + 5a^2bx^3 + 8ab^2x^6 + 4b^3x^9)}{3a\sqrt{b^2}\sqrt{a^2 + 2abx^3 + b^2x^6} (2a^2x^3 + 8abx^6 + 8b^2x^9) + 3a(-2a^3bx^3 - 10a^2b^2x^6 - 16ab^3x^9 - 8b^4x^{12})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]), x]

[Out] (-2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-(a^2*b) - 4*a*b^2*x^3 - 4*b^3*x^6) - 2*Sqrt[b^2]*(a^3 + 5*a^2*b*x^3 + 8*a*b^2*x^6 + 4*b^3*x^9))/(3*a*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(2*a^2*x^3 + 8*a*b*x^6 + 8*b^2*x^9) + 3*a*(-2*a^3*b*x^3 - 10*a^2*b^2*x^6 - 16*a*b^3*x^9 - 8*b^4*x^12)) + ((-(Sqrt[b^2]*x^3) + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])^2*((b*Log[a - Sqrt[b^2]*x^3] + Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]))/(3*a^2) - (b*Log[a^3 + a^2*Sqrt[b^2]*x^3 - a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]])/(3*a^2)))/(a^2 + 2*a*b*x^3 + 2*b^2*x^6 - 2*Sqrt[b^2]*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

fricas [A] time = 1.12, size = 33, normalized size = 0.26

$$\frac{bx^3 \log(bx^3 + a) - 3bx^3 \log(x) - a}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")

[Out] 1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)

giac [A] time = 0.37, size = 50, normalized size = 0.40

$$\frac{1}{3} \left(\frac{b \log(|bx^3 + a|)}{a^2} - \frac{3b \log(|x|)}{a^2} + \frac{bx^3 - a}{a^2x^3} \right) \operatorname{sgn}(bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/3*(b*log(abs(b*x^3 + a))/a^2 - 3*b*log(abs(x))/a^2 + (b*x^3 - a)/(a^2*x^3))*sgn(b*x^3 + a)

maple [A] time = 0.01, size = 51, normalized size = 0.41

$$\frac{(bx^3 + a)(3bx^3 \ln(x) - bx^3 \ln(bx^3 + a) + a)}{3\sqrt{(bx^3 + a)^2} a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/((b*x^3+a)^2)^(1/2),x)

[Out] -1/3*(b*x^3+a)*(3*b*x^3*ln(x)-b*ln(b*x^3+a)*x^3+a)/((b*x^3+a)^2)^(1/2)/a^2/x^3

maxima [A] time = 1.20, size = 73, normalized size = 0.58

$$\frac{(-1)^{2abx^3+2a^2} b \log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right)}{3a^2} - \frac{\sqrt{b^2x^6 + 2abx^3 + a^2}}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")

[Out] 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^2 - 1/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)/(a^2*x^3)

mupad [B] time = 1.40, size = 75, normalized size = 0.60

$$\frac{a b \operatorname{atanh}\left(\frac{a^2 + b a x^3}{\sqrt{a^2} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}\right)}{3 (a^2)^{3/2}} - \frac{\sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{3 a^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*((a + b*x^3)^2)^(1/2)),x)`

[Out] `(a*b*atanh((a^2 + a*b*x^3)/((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))))/(3*(a^2)^(3/2)) - (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*a^2*x^3)`

sympy [A] time = 0.36, size = 31, normalized size = 0.25

$$-\frac{1}{3ax^3} - \frac{b \log(x)}{a^2} + \frac{b \log\left(\frac{a}{b} + x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)`

[Out] `-1/(3*a*x**3) - b*log(x)/a**2 + b*log(a/b + x**3)/(3*a**2)`

$$3.98 \quad \int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal. Leaf size=280

$$\frac{x^2}{9ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{4/3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3)\tan^{-1}}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Rubi [A] time = 0.14, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1355, 288, 290, 292, 31, 634, 617, 204, 628}

$$\frac{x^2}{9ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{4/3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{54a^{4/3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] x^2/(9*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(6*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I

LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx &= \frac{(b^2(ab + b^2x^3)) \int \frac{x^4}{(ab+b^2x^3)^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{x^2}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{x}{(ab+b^2x^3)^2} dx}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{1}{ab}}{9ab\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(ab + b^2x^3) \int \frac{1}{\sqrt{3}}}{27a^{4/3}b^2\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \log(a + bx^3)}{27a^{4/3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \log(a + bx^3)}{27a^{4/3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{a}}{\sqrt[3]{a}}}{\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 235, normalized size = 0.84

$$\frac{-3a^{4/3}b^{2/3}x^2 + 2abx^3 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}) + b^2x^6 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}) + a^2 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}) + 6\sqrt[3]{a}b^{5/3}x^5 - 2(a + bx^3)^2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 2\sqrt{3}(a + bx^3)^2 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{a}}{\sqrt[3]{a}}}{\sqrt[3]{a}}\right)}{54a^{4/3}b^{5/3}(a + bx^3)\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (-3*a^(4/3)*b^(2/3)*x^2 + 6*a^(1/3)*b^(5/3)*x^5 - 2*Sqrt[3]*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + b^2*x^6*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]

$(1/3)*b^{(1/3)}*x + b^{(2/3)}*x^2]/(54*a^{(4/3)}*b^{(5/3)}*(a + b*x^3)*\text{Sqrt}[(a + b*x^3)^2])$

IntegrateAlgebraic [A] time = 11.03, size = 169, normalized size = 0.60

$$\frac{(a + bx^3) \left(\frac{\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54a^{4/3}b^{5/3}} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{4/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b} x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}} + \frac{2bx^5 - ax^2}{18ab(a+bx^3)^2} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] $((a + b*x^3)*((-a*x^2) + 2*b*x^5)/(18*a*b*(a + b*x^3)^2) - \text{ArcTan}[1/\text{Sqrt}[3] - (2*b^{(1/3)}*x)/(\text{Sqrt}[3]*a^{(1/3)})]/(9*\text{Sqrt}[3]*a^{(4/3)}*b^{(5/3)}) - \text{Log}[a^{(1/3)} + b^{(1/3)}*x]/(27*a^{(4/3)}*b^{(5/3)}) + \text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(54*a^{(4/3)}*b^{(5/3)})]/\text{Sqrt}[(a + b*x^3)^2]$

fricas [A] time = 1.38, size = 512, normalized size = 1.83

$$\frac{\frac{a^{4/3} b^{5/3} \sqrt{3} \log\left(\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2}{54 a^{4/3} b^{5/3}}\right) - \frac{a^{4/3} b^{5/3} \log\left(\frac{\sqrt[3]{a} + \sqrt[3]{b} x}{27 a^{4/3} b^{5/3}}\right)}{27 a^{4/3} b^{5/3}} - \frac{a^{4/3} b^{5/3} \arctan\left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9 \sqrt{3} a^{4/3} b^{5/3}} + \frac{2 b x^5 - a x^2}{18 a b (a + b x^3)^2}}{\sqrt{(a + b x^3)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="fricas")

[Out] $[1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*\text{sqrt}(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*\text{sqrt}((-a*b^2)^{(1/3)}/a)*\log((2*b^2*x^3 - a*b + 3*\text{sqrt}(1/3)*(a*b*x + 2*(-a*b^2)^{(2/3)}*x^2 + (-a*b^2)^{(1/3)}*a)*\text{sqrt}((-a*b^2)^{(1/3)}/a) - 3*(-a*b^2)^{(2/3)}*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)}*b*x + (-a*b^2)^{(2/3)}) - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})]/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*\text{sqrt}(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*\text{sqrt}(-(-a*b^2)^{(1/3)}/a)*\arctan(\text{sqrt}(1/3)*(2*b*x + (-a*b^2)^{(1/3)})*\text{sqrt}(-(-a*b^2)^{(1/3)}/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^{(2/3)}*\log(b^2*x^2 + (-a*b^2)^{(1/3)}*b*x + (-a*b^2)^{(2/3)}) - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^{(2/3)}*\log(b*x - (-a*b^2)^{(1/3)})]/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.02, size = 299, normalized size = 1.07

$$\frac{-2\sqrt{3}b^2x^6 \arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 2b^2x^6 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + b^2x^6 \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5 - 4\sqrt{3}abx^3 \arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 4abx^3 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 2abx^3 \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 3\left(\frac{a}{b}\right)^{\frac{1}{3}}abx^2 - 2\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 2a^2 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + a^2 \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{3}{2}}ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)

[Out] $\frac{1}{54}(-2*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^6*b^2 - 2*\ln(x+(a/b)^{(1/3)})*x^6*b^2 + \ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^6*b^2 + 6*(a/b)^{(1/3)}*x^5*b^2 - 4*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^3*a*b - 4*\ln(x+(a/b)^{(1/3)})*x^3*a*b + 2*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^3*a*b - 3*(a/b)^{(1/3)}*x^2*a*b - 2*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*a^2 - 2*\ln(x+(a/b)^{(1/3)})*a^2 + \ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*a^2)*(b*x^3+a)/(a/b)^{(1/3)}/b^2/a/((b*x^3+a)^2)^{(3/2)}$

maxima [A] time = 1.81, size = 149, normalized size = 0.53

$$\frac{2bx^5 - ax^2}{18(ab^3x^6 + 2a^2b^2x^3 + a^3b)} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{18}(2*b*x^5 - a*x^2)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + \frac{1}{27}*\sqrt{3}*a*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^2*(a/b)^{(1/3)}) + \frac{1}{54}*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^2*(a/b)^{(1/3)}) - \frac{1}{27}*\log(x + (a/b)^{(1/3)})/(a*b^2*(a/b)^{(1/3)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)

[Out] `int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left((a + bx^3)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2), x)`

[Out] `Integral(x**4/((a + b*x**3)**2)**(3/2), x)`

$$3.99 \quad \int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal. Leaf size=276

$$\frac{x}{18ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{5/3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Rubi [A] time = 0.13, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{x}{18ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{5/3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{54a^{5/3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] x/(18*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(6*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F

reeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1355

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx &= \frac{(b^2(ab + b^2x^3)) \int \frac{x^3}{(ab+b^2x^3)^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{x}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{1}{(ab+b^2x^3)^2} dx}{6\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x}{18ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{1}{(ab+b^2x^3)^2} dx}{9ab\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x}{18ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(ab + b^2x^3) \int \frac{1}{(ab+b^2x^3)^2} dx}{27a^{5/3}b^{5/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x}{18ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3) \log(a + bx^3)}{27a^{5/3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x}{18ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3) \log(a + bx^3)}{27a^{5/3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x}{18ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3) \log(a + bx^3)}{27a^{5/3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x}{18ab\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(a + bx^3) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx^3}}{\sqrt{3}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 235, normalized size = 0.85

$$\frac{3a^{2/3}b^{4/3}x^4 - 2abx^3 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - b^2x^6 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - 6a^{5/3}\sqrt[3]{b}x - a^2 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + 2(a + bx^3)^2 \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 2\sqrt{3}(a + bx^3)^2 \tan^{-1}\left(\frac{1 - 2\sqrt[3]{bx^3}}{\sqrt{3}}\right)}{54a^{5/3}b^{4/3}(a + bx^3)\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (-6*a^(5/3)*b^(1/3)*x + 3*a^(2/3)*b^(4/3)*x^4 - 2*sqrt(3)*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt(3)] + 2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - b^2*x^6*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]

3)*b^(1/3)*x + b^(2/3)*x^2]]/(54*a^(5/3)*b^(4/3)*(a + b*x^3)*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [A] time = 11.09, size = 166, normalized size = 0.60

$$\frac{(a + bx^3) \left(-\frac{\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2})}{54a^{5/3}b^{4/3}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{5/3}b^{4/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}} + \frac{bx^4 - 2ax}{18ab(a + bx^3)^2} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] ((a + b*x^3)*((-2*a*x + b*x^4)/(18*a*b*(a + b*x^3)^2) - ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x]/(27*a^(5/3)*b^(4/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(54*a^(5/3)*b^(4/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 1.31, size = 503, normalized size = 1.82

$$\frac{\sqrt[3]{3ab^2 - 6a^2b} + 3\sqrt[3]{(ab^2 + 2a^2b^2 + a^3)\sqrt[3]{\frac{a^2}{b^3}} \log\left(\frac{2a^2 - (a^2)b^2 + a^2\sqrt[3]{(a^2b^2 + a^3)\sqrt[3]{\frac{a^2}{b^3}}}}{2a^2 + 2ab^2 + a^3}\right) \log\left(\frac{ab^2 - (a^2)^2 + (a^2)^2}{2a^2 + 2ab^2 + a^3}\right) \log\left(\frac{ab^2 + (a^2)^2}{2a^2 + 2ab^2 + a^3}\right) \log\left(\frac{ab^2 - (a^2)^2 + (a^2)^2}{2a^2 + 2ab^2 + a^3}\right) \log\left(\frac{ab^2 + (a^2)^2}{2a^2 + 2ab^2 + a^3}\right)}{54(a^{5/3}b^{4/3} + 2a^{5/3}b^{4/3})} + \frac{\sqrt[3]{(a^2b^2 - a^2b^2) + (a^2b^2 - a^2b^2)\sqrt[3]{\frac{a^2}{b^3}}}}{54(a^{5/3}b^{4/3} + 2a^{5/3}b^{4/3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="fricas")

[Out] [1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.01, size = 299, normalized size = 1.08

$$\frac{-2\sqrt{3}b^2x^4 \arctan\left(\frac{\sqrt{3}\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 2b^2x^6 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - b^2x^6 \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 4\sqrt{3}abx^3 \arctan\left(\frac{\sqrt{3}\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 4abx^3 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 2abx^3 \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^4 - 2\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 2a^2 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - a^2 \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 6\left(\frac{a}{b}\right)^{\frac{2}{3}}abx}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{3}{2}}ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)

[Out] $\frac{1}{54}(-2*3^{(1/2)}*b^2*x^6*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)}) + 2*b^2*x^6*\ln(x+(a/b)^{(1/3)}) - b^2*x^6*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) + 3*(a/b)^{(2/3)}*x^4*b^2 - 4*3^{(1/2)}*a*b*x^3*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)}) + 4*a*b*x^3*\ln(x+(a/b)^{(1/3)}) - 2*a*b*x^3*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) - 6*(a/b)^{(2/3)}*x*a*b - 2*3^{(1/2)}*a^2*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)}) + 2*a^2*\ln(x+(a/b)^{(1/3)}) - a^2*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})) * (b*x^3+a)/(a/b)^{(2/3)}/b^2/a/((b*x^3+a)^2)^{(3/2)}$

maxima [A] time = 1.70, size = 146, normalized size = 0.53

$$\frac{bx^4 - 2ax}{18(ab^3x^6 + 2a^2b^2x^3 + a^3b)} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{18}(bx^4 - 2ax)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + \frac{1}{27}*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^2*(a/b)^{(2/3)}) - \frac{1}{54}*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^2*(a/b)^{(2/3)}) + \frac{1}{27}*\log(x + (a/b)^{(1/3)})/(a*b^2*(a/b)^{(2/3)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)

[Out] `int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left((a + bx^3)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2), x)`

[Out] `Integral(x**3/((a + b*x**3)**2)**(3/2), x)`

$$3.100 \quad \int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal. Leaf size=38

$$-\frac{1}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1352, 607}

$$-\frac{1}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] -1/(6*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 607

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x + c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx, x, x^3 \right) \\ &= -\frac{1}{6b(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$\frac{a + bx^3}{6b \left((a + bx^3)^2 \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] -1/6*(a + b*x^3)/(b*((a + b*x^3)^2)^(3/2))

IntegrateAlgebraic [B] time = 0.57, size = 137, normalized size = 3.61

$$\frac{\sqrt{b^2} (a - bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6} + a^2b + b^3x^6}{3b\sqrt{b^2} x^6 (2a^2b^2 + 4ab^3x^3 + 2b^4x^6) + 3bx^6 (-2ab^3 - 2b^4x^3) \sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (a^2*b + b^3*x^6 + Sqrt[b^2]*(a - b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/ (3*b*x^6*(-2*a*b^3 - 2*b^4*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6] + 3*b*Sqrt[b^2]*x^6*(2*a^2*b^2 + 4*a*b^3*x^3 + 2*b^4*x^6))

fricas [A] time = 1.41, size = 26, normalized size = 0.68

$$\frac{1}{6(b^3x^6 + 2ab^2x^3 + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="fricas")

[Out] -1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.01, size = 24, normalized size = 0.63

$$-\frac{bx^3 + a}{6\left((bx^3 + a)^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)`

[Out] `-1/6*(b*x^3+a)/b/((b*x^3+a)^2)^(3/2)`

maxima [A] time = 0.89, size = 16, normalized size = 0.42

$$-\frac{1}{6\left(x^3 + \frac{a}{b}\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")`

[Out] `-1/6/((x^3 + a/b)^2*b^3)`

mupad [B] time = 1.19, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{6b(bx^3 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)`

[Out] `-(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(6*b*(a + b*x^3)^3)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((a + bx^3)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)`

[Out] `Integral(x**2/((a + b*x**3)**2)**(3/2), x)`

$$3.101 \quad \int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal. Leaf size=277

$$\frac{x^2}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.14, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1355, 290, 292, 31, 634, 617, 204, 628}

$$\frac{x^2}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (2*x^2)/(9*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(6*a*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x]/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m + n*(p+1) + 1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx &= \frac{(b^2(ab + b^2x^3)) \int \frac{x}{(ab+b^2x^3)^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2}{6a(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(2b(ab + b^2x^3)) \int \frac{x}{(ab+b^2x^3)^2} dx}{3a\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{2x^2}{9a^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{6a(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(2(ab + b^2x^3)) \int}{9a^2\sqrt{a^2 + 2abx^3}} \\
&= \frac{2x^2}{9a^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{6a(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{(2(ab + b^2x^3)) \int}{27a^{7/3}b\sqrt{a^2 + 2}} \\
&= \frac{2x^2}{9a^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{6a(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{2(a + bx^3) \log}{27a^{7/3}b^{2/3}\sqrt{a^2 + 2}} \\
&= \frac{2x^2}{9a^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{6a(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{2(a + bx^3) \log}{27a^{7/3}b^{2/3}\sqrt{a^2 + 2}} \\
&= \frac{2x^2}{9a^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{6a(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{2(a + bx^3) \tan^{-1}}{9\sqrt{3} a^{7/3} b^{2/3} \sqrt{a^2 + 2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 237, normalized size = 0.86

$$\frac{21a^{4/3}b^{2/3}x^2 + 4abx^3 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}) + 2b^2x^6 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}) + 2a^2 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx + b^{2/3}x^2}) + 12\sqrt[3]{a}b^{5/3}x^5 - 4(a + bx^3)^2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 4\sqrt{3}(a + bx^3)^2 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt{3}}}{\sqrt{3}}\right)}{54a^{7/3}b^{2/3}(a + bx^3)\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (21*a^(4/3)*b^(2/3)*x^2 + 12*a^(1/3)*b^(5/3)*x^5 - 4*Sqrt[3]*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 2*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^2*x^6*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(2/3)*(a + b*x^3)*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [A] time = 13.20, size = 166, normalized size = 0.60

$$\frac{(a + bx^3) \left(\frac{\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{27a^{7/3}b^{2/3}} - \frac{2 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{7/3}b^{2/3}} - \frac{2 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b} x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}} + \frac{x^2(7a+4bx^3)}{18a^2(a+bx^3)^2} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] ((a + b*x^3)*((x^2*(7*a + 4*b*x^3))/(18*a^2*(a + b*x^3)^2) - (2*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)) - (2*Log[a^(1/3) + b^(1/3)*x]/(27*a^(7/3)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(27*a^(7/3)*b^(2/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 0.74, size = 514, normalized size = 1.86

$$\frac{12\sqrt{3}a^2 + 21a^{5/3}b + 6\sqrt{3}a^{2/3}b^2 + 2a^{1/3}b^3}{34(b^2x^6 + 2abx^3 + a^2)^{3/2}} \log\left(\frac{2b^2x^3 - ab + 3\sqrt{3}a^{1/3}(abx + 2(-ab^2)^{2/3}x^2 + (-ab^2)^{1/3}a)}{(-ab^2)^{1/3}/a}\right) - 3(-ab^2)^{2/3}x/(b^2x^6 + 2abx^3 + a^2) + 2(b^2x^6 + 2abx^3 + a^2)(-ab^2)^{2/3} \log(b^2x^2 + (-ab^2)^{1/3}bx + (-ab^2)^{2/3}) - 4(b^2x^6 + 2abx^3 + a^2)(-ab^2)^{2/3} \log(bx - (-ab^2)^{1/3})}{(a^3b^4x^6 + 2a^4b^3x^3 + a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="fricas")

[Out] [1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.01, size = 301, normalized size = 1.09

$$\frac{\left(-4\sqrt{3}b^2x^6\arctan\left(\frac{\sqrt{3}\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\frac{3x^2}{b}}\right)-4b^2x^6\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+2b^2x^6\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+12\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5-8\sqrt{3}abx^3\arctan\left(\frac{\sqrt{3}\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\frac{3x^2}{b}}\right)-8abx^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+4abx^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+21\left(\frac{a}{b}\right)^{\frac{1}{3}}abx^2-4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\frac{3x^2}{b}}\right)-4a^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+2a^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)\left(bx^3+a\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)

[Out] $\frac{1}{54}*(-4*3^{(1/2)}*b^2*x^6*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})-4*b^2*x^6*\ln(x+(a/b)^{(1/3)})+2*b^2*x^6*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+12*(a/b)^{(1/3)}*b^2*x^5-8*3^{(1/2)}*a*b*x^3*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})-8*a*b*x^3*\ln(x+(a/b)^{(1/3)})+4*a*b*x^3*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+21*(a/b)^{(1/3)}*a*b*x^2-4*3^{(1/2)}*a^2*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})-4*a^2*\ln(x+(a/b)^{(1/3)})+2*a^2*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}))*(b*x^3+a)/(a/b)^{(1/3)}/b/a^2/((b*x^3+a)^2)^{(3/2)}$

maxima [A] time = 1.36, size = 147, normalized size = 0.53

$$\frac{4bx^5 + 7ax^2}{18(a^2b^2x^6 + 2a^3bx^3 + a^4)} + \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{18}*(4*b*x^5 + 7*a*x^2)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + \frac{2}{27}*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a^2*b*(a/b)^{(1/3)}) + \frac{1}{27}*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a^2*b*(a/b)^{(1/3)}) - \frac{2}{27}*\log(x + (a/b)^{(1/3)})/(a^2*b*(a/b)^{(1/3)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)

[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left((a + bx^3)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(x/((a + b*x**3)**2)**(3/2), x)

$$3.102 \quad \int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal. Leaf size=286

$$\frac{5x(a+bx^3)^2}{18a^2(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{x(a+bx^3)}{6a(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5(a+bx^3)^3 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{8/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{3/2}} - \frac{5(a+bx^3)^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{3/2}}$$

Rubi [A] time = 0.15, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1343, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5x(a+bx^3)^2}{18a^2(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{x(a+bx^3)}{6a(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5(a+bx^3)^3 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{27a^{8/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{3/2}} - \frac{5(a+bx^3)^3 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{54a^{8/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{3/2}} - \frac{5(a+bx^3)^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2), x]

[Out] (x*(a + b*x^3))/(6*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*x*(a + b*x^3)^2)/(18*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*(a + b*x^3)^3*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R

$\int \frac{t[b, 3]*x}{(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x}, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTan}[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 617

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] := \text{With}\{q = 1 - 4*Simplify[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1343

$\text{Int}[(a_ + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(2n_)})^{(p_)}, x_Symbol] := \text{Dist}[(a + b*x^n + c*x^{(2n)})^p/(b + 2*c*x^n)^{(2*p)}, \text{Int}[(b + 2*c*x^n)^{(2*p)}, x], x] /; \text{FreeQ}\{a, b, c, n, p\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx &= \frac{(2ab + 2b^2x^3)^3 \int \frac{1}{(2ab+2b^2x^3)^3} dx}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} \\
&= \frac{x(a + bx^3)}{6a(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{(5(2ab + 2b^2x^3)^3) \int \frac{1}{(2ab+2b^2x^3)^2} dx}{12ab(a^2 + 2abx^3 + b^2x^6)^{3/2}} \\
&= \frac{x(a + bx^3)}{6a(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{5x(a + bx^3)^2}{18a^2(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{(5(2ab + 2b^2x^3)^3) \int}{36a^2b^2(a^2 + 2abx^3)} \\
&= \frac{x(a + bx^3)}{6a(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{5x(a + bx^3)^2}{18a^2(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{(5(2ab + 2b^2x^3)^3) \int}{108 \cdot 2^{2/3} a^{8/3} b^{8/3} (a^2 -)} \\
&= \frac{x(a + bx^3)}{6a(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{5x(a + bx^3)^2}{18a^2(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{5(a + bx^3)^3 \log(\sqrt[3]{a})}{27a^{8/3} \sqrt[3]{b} (a^2 + 2abx^3)} \\
&= \frac{x(a + bx^3)}{6a(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{5x(a + bx^3)^2}{18a^2(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{5(a + bx^3)^3 \log(\sqrt[3]{a})}{27a^{8/3} \sqrt[3]{b} (a^2 + 2abx^3)} \\
&= \frac{x(a + bx^3)}{6a(a^2 + 2abx^3 + b^2x^6)^{3/2}} + \frac{5x(a + bx^3)^2}{18a^2(a^2 + 2abx^3 + b^2x^6)^{3/2}} - \frac{5(a + bx^3)^3 \tan^{-1}}{9\sqrt{3} a^{8/3} \sqrt[3]{b} (a^2 + 2abx^3)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 235, normalized size = 0.82

$$\frac{15a^{2/3}b^{4/3}x^4 - 10abx^3 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3}x^2) - 5b^2x^6 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3}x^2) + 24a^{5/3} \sqrt[3]{b} x - 5a^2 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3}x^2) + 10(a + bx^3)^2 \log(\sqrt[3]{a} + \sqrt[3]{b} x) - 10\sqrt{3}(a + bx^3)^2 \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{8/3} \sqrt[3]{b} (a + bx^3) \sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2), x]

[Out] (24*a^(5/3)*b^(1/3)*x + 15*a^(2/3)*b^(4/3)*x^4 - 10*Sqrt[3]*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 5*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 10*a*b*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 5*b^2*x^6*Log[a^(2/3)

$- a^{1/3} b^{1/3} x + b^{2/3} x^2) / (54 a^{8/3} b^{1/3} (a + b x^3) \sqrt{(a + b x^3)^2})$

IntegrateAlgebraic [A] time = 15.69, size = 164, normalized size = 0.57

$$\frac{(a + b x^3) \left(-\frac{5 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54 a^{8/3} \sqrt[3]{b}} + \frac{5 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27 a^{8/3} \sqrt[3]{b}} - \frac{5 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9 \sqrt{3} a^{8/3} \sqrt[3]{b}} + \frac{x(8a + 5bx^3)}{18a^2(a + bx^3)^2} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2), x]

[Out] ((a + b*x^3)*((x*(8*a + 5*b*x^3))/(18*a^2*(a + b*x^3)^2) - (5*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)) + (5*Log[a^(1/3) + b^(1/3)*x])/(27*a^(8/3)*b^(1/3)) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 1.34, size = 499, normalized size = 1.74

$$\frac{15 \sqrt{3} a^{5/3} b^{1/3} x^4 + 24 a^{2/3} b^{2/3} x^3 + 15 \sqrt{3} a^{5/3} b^{1/3} x^2 + 24 a^{2/3} b^{2/3} x + 15 \sqrt{3} a^{5/3} b^{1/3}}{54 (a^{8/3} b^{1/3} + 2 a^{5/3} b^{2/3} x + a^2 b^{1/3} x^2 + a^{5/3} b^{2/3} x^3 + a^2 b^{1/3} x^4 + a^{5/3} b^{2/3} x^5 + a^2 b^{1/3} x^6)} \left(\frac{5 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{54 a^{8/3} \sqrt[3]{b}} + \frac{5 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27 a^{8/3} \sqrt[3]{b}} - \frac{5 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{9 \sqrt{3} a^{8/3} \sqrt[3]{b}} + \frac{x(8a + 5bx^3)}{18a^2(a + bx^3)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="fricas")

[Out] [1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3 + a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.01, size = 299, normalized size = 1.05

$$\frac{\left(-10\sqrt{5}b^2x^6\arctan\left(\frac{\sqrt{5}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+10b^2x^6\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-5b^2x^6\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-20\sqrt{5}abx^3\arctan\left(\frac{\sqrt{5}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+20abx^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-10abx^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+15\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2x^4-10\sqrt{5}a^2\arctan\left(\frac{\sqrt{5}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+10a^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-5a^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+24\left(\frac{a}{b}\right)^{\frac{2}{3}}\operatorname{arct}\left(\frac{b^2x^4}{(bx^3+a)^{\frac{2}{3}}}\right)\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{2}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)

[Out] $\frac{1}{54}\left(-10\sqrt{3}^{\frac{1}{2}}b^2x^6\arctan\left(\frac{1}{3}\sqrt{3}^{\frac{1}{2}}(-2x+(a/b)^{\frac{1}{3}})\right)/(a/b)^{\frac{1}{3}}\right)+10b^2x^6\ln\left(x+(a/b)^{\frac{1}{3}}\right)-5b^2x^6\ln\left(x^2-(a/b)^{\frac{1}{3}}x+(a/b)^{\frac{2}{3}}\right)+15(a/b)^{\frac{2}{3}}b^2x^4-20\sqrt{3}^{\frac{1}{2}}a^2\arctan\left(\frac{1}{3}\sqrt{3}^{\frac{1}{2}}(-2x+(a/b)^{\frac{1}{3}})\right)/(a/b)^{\frac{1}{3}}+20a^2\ln\left(x+(a/b)^{\frac{1}{3}}\right)-10a^2\ln\left(x^2-(a/b)^{\frac{1}{3}}x+(a/b)^{\frac{2}{3}}\right)+24(a/b)^{\frac{2}{3}}a^2\arctan\left(\frac{1}{3}\sqrt{3}^{\frac{1}{2}}(-2x+(a/b)^{\frac{1}{3}})\right)/(a/b)^{\frac{1}{3}}+10a^2\ln\left(x+(a/b)^{\frac{1}{3}}\right)-5a^2\ln\left(x^2-(a/b)^{\frac{1}{3}}x+(a/b)^{\frac{2}{3}}\right)\right)\cdot(bx^3+a)/(a/b)^{\frac{2}{3}}/b/a^2/((bx^3+a)^2)^{\frac{3}{2}}$

maxima [A] time = 1.92, size = 145, normalized size = 0.51

$$\frac{5bx^4+8ax}{18(a^2b^2x^6+2a^3bx^3+a^4)} + \frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{18}\left(\frac{5b^2x^4+8a^2x}{a^2b^2x^6+2a^3bx^3+a^4}\right) + \frac{5}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\right)/(a/b)^{\frac{2}{3}} - \frac{5}{54}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)/(a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}) + \frac{5}{27}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)/(a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}})$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)

[Out] `int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2), x)`

[Out] `Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-3/2), x)`

$$3.103 \quad \int \frac{1}{x(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal. Leaf size=147

$$\frac{1}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\log(x)(a+bx^3)}{a^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 44}

$$\frac{1}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\log(x)(a+bx^3)}{a^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)), x]

[Out] 1/(3*a^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*(a + b*x^3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx &= \frac{(b^2(ab + b^2x^3)) \int \frac{1}{x(ab+b^2x^3)^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^2(ab + b^2x^3)) \text{Subst}\left(\int \frac{1}{x(ab+b^2x^3)^3} dx, x, x^3\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^2(ab + b^2x^3)) \text{Subst}\left(\int \left(\frac{1}{a^3b^3x} - \frac{1}{ab^2(a+bx)^3} - \frac{1}{a^2b^2(a+bx)^2} - \frac{1}{a^3b^2(a+bx)}\right) dx, x, x^3\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{1}{3a^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6a(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(a + bx^3)\log(a + bx^3)}{a^3\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.50

$$\frac{a(3a + 2bx^3) + 6\log(x)(a + bx^3)^2 - 2(a + bx^3)^2\log(a + bx^3)}{6a^3(a + bx^3)\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)), x]

[Out] (a*(3*a + 2*b*x^3) + 6*(a + b*x^3)^2*Log[x] - 2*(a + b*x^3)^2*Log[a + b*x^3])/(6*a^3*(a + b*x^3)*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [B] time = 2.57, size = 758, normalized size = 5.16

$$\frac{2(\sqrt{a^2 + 2abx^3 + b^2x^6})^2 \operatorname{arctan}\left(\frac{bx^3}{\sqrt{a^2 + 2abx^3 + b^2x^6}}\right) + \sqrt{a^2 + 2abx^3 + b^2x^6} \left[(a^9 + 9a^8bx^3 + 48a^7b^2x^6 + 2304a^6b^3x^9 + 1248a^5b^4x^{12} + 3008a^4b^5x^{15} + 4608a^3b^6x^{18} + 4352a^2b^7x^{21} + 2304ab^8x^{24} + 512b^9x^{27}) \sqrt{a^2 + 2abx^3 + b^2x^6} - (a^{11} + 3a^9b^2x^6 + 50a^8b^3x^9 + 368a^7b^4x^{12} + 1568a^6b^5x^{15} + 4256a^5b^6x^{18} + 7616a^4b^7x^{21} + 8960a^3b^8x^{24}) \right]}{6a^3(a + bx^3)\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)), x]

[Out] (Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-a^10*b) + a^9*b^2*x^3 + 2*a^8*b^3*x^6 + 48*a^7*b^4*x^9 + 320*a^6*b^5*x^12 + 1248*a^5*b^6*x^15 + 3008*a^4*b^7*x^18 + 4608*a^3*b^8*x^21 + 4352*a^2*b^9*x^24 + 2304*a*b^10*x^27 + 512*b^11*x^30) + Sqrt[b^2]*(-a^11 - 3*a^9*b^2*x^6 - 50*a^8*b^3*x^9 - 368*a^7*b^4*x^12 - 1568*a^6*b^5*x^15 - 4256*a^5*b^6*x^18 - 7616*a^4*b^7*x^21 - 8960*a^3*b^8*x^24)

$$4 - 6656a^2b^9x^{27} - 2816ab^{10}x^{30} - 512b^{11}x^{33}) / (3a^2b\sqrt{b^2}x^6\sqrt{a^2 + 2abx^3 + b^2x^6} * (-2a^9b - 34a^8b^2x^3 - 256a^7b^3x^6 - 1120a^6b^4x^9 - 3136a^5b^5x^{12} - 5824a^4b^6x^{15} - 7168a^3b^7x^{18} - 5632a^2b^8x^{21} - 2560ab^9x^{24} - 512b^{10}x^{27}) + 3a^2b^2x^6(2a^{10}b^2 + 36a^9b^3x^3 + 290a^8b^4x^6 + 1376a^7b^5x^9 + 4256a^6b^6x^{12} + 8960a^5b^7x^{15} + 12992a^4b^8x^{18} + 12800a^3b^9x^{21} + 8192a^2b^{10}x^{24} + 3072ab^{11}x^{27} + 512b^{12}x^{30})) + (2 * (- (\sqrt{b^2}x^3) + \sqrt{a^2 + 2abx^3 + b^2x^6})^4 * \text{ArcTanh}[(\sqrt{b^2}x^3)/a - \sqrt{a^2 + 2abx^3 + b^2x^6}/a]) / (3a^3(a^4 + 4a^3bx^3 + 12a^2b^2x^6 + 16ab^3x^9 + 8b^4x^{12} - 4a^2\sqrt{b^2}x^3\sqrt{a^2 + 2abx^3 + b^2x^6} - 8ab\sqrt{b^2}x^6\sqrt{a^2 + 2abx^3 + b^2x^6} - 8(b^2)^{(3/2)}x^9\sqrt{a^2 + 2abx^3 + b^2x^6})))$$

fricas [A] time = 1.24, size = 90, normalized size = 0.61

$$\frac{2abx^3 + 3a^2 - 2(b^2x^6 + 2abx^3 + a^2) \log(bx^3 + a) + 6(b^2x^6 + 2abx^3 + a^2) \log(x)}{6(a^3b^2x^6 + 2a^4bx^3 + a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] 1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.02, size = 107, normalized size = 0.73

$$\frac{(6b^2x^6 \ln(x) - 2b^2x^6 \ln(bx^3 + a) + 12abx^3 \ln(x) - 4abx^3 \ln(bx^3 + a) + 2abx^3 + 6a^2 \ln(x) - 2a^2 \ln(bx^3 + a) + 3a^2)(bx^3 + a)}{6\left((bx^3 + a)^2\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)

[Out] 1/6*(6*ln(x)*x^6*b^2-2*ln(b*x^3+a)*x^6*b^2+12*ln(x)*x^3*a*b-4*ln(b*x^3+a)*x^3*a*b+2*a*b*x^3+6*ln(x)*a^2-2*ln(b*x^3+a)*a^2+3*a^2)*(b*x^3+a)/a^3/((b*x^3+a)^2)^(3/2)

maxima [A] time = 1.15, size = 88, normalized size = 0.60

$$-\frac{(-1)^{2abx^3+2a^2} \log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right)}{3a^3} + \frac{1}{3\sqrt{b^2x^6 + 2abx^3 + a^2} a^2} + \frac{1}{6\left(x^3 + \frac{a}{b}\right)^2 ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^3 + 1/3/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2) + 1/6/((x^3 + a/b)^2*a*b^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)

[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left((a + bx^3)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)

[Out] Integral(1/(x*((a + b*x**3)**2)**(3/2)), x)

$$3.104 \quad \int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}} dx$$

Optimal. Leaf size=316

$$\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)} + \frac{14\sqrt[3]{b}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{14\sqrt[3]{b}(a+bx^3)}{9\sqrt{3}a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.16, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1355, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{14(a+bx^3)}{9a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)} + \frac{14\sqrt[3]{b}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7\sqrt[3]{b}(a+bx^3)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{14\sqrt[3]{b}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)), x]

[Out] 7/(18*a^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (14*(a + b*x^3))/(9*a^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m + n*(p+1) + 1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

]

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx &= \frac{(b^2 (ab + b^2x^3)) \int \frac{1}{x^2(ab+b^2x^3)^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{1}{6ax (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(7b (ab + b^2x^3)) \int \frac{1}{x^2(ab+b^2x^3)^2} dx}{6a\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7}{18a^2x\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(14 (ab + b^2x^3)) \int \frac{1}{x^2(ab+b^2x^3)} dx}{9a^2\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7}{18a^2x\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{14 (a + bx^3)}{9a^3x\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7}{18a^2x\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{14 (a + bx^3)}{9a^3x\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7}{18a^2x\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{14 (a + bx^3)}{9a^3x\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7}{18a^2x\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{14 (a + bx^3)}{9a^3x\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7}{18a^2x\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{14 (a + bx^3)}{9a^3x\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 260, normalized size = 0.82

$$\frac{-14b^{7/3}x^7 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}) - 28ab^{4/3}x^4 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}) - 147a^{4/3}bx^3 - 54a^{7/3} - 14a^2 \sqrt[3]{b} x \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}) - 84 \sqrt[3]{a} b^2 x^6 + 28 \sqrt[3]{b} x (a + bx^3)^2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) + 28 \sqrt{3} \sqrt[3]{b} x (a + bx^3)^2 \tan^{-1}\left(\frac{1 - \sqrt[3]{bx}}{\sqrt[3]{a}}\right)}{54a^{10/3}x(a + bx^3)\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)), x]

[Out] (-54*a^(7/3) - 147*a^(4/3)*b*x^3 - 84*a^(1/3)*b^2*x^6 + 28*sqrt[3]*b^(1/3)*x*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 28*b^(1/3)*x*

$(a + b*x^3)^2 * \text{Log}[a^{(1/3)} + b^{(1/3)}*x] - 14*a^2*b^{(1/3)}*x * \text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2] - 28*a*b^{(4/3)}*x^4 * \text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2] - 14*b^{(7/3)}*x^7 * \text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2] / (54*a^{(10/3)}*x*(a + b*x^3)*\text{Sqrt}[(a + b*x^3)^2])$

IntegrateAlgebraic [A] time = 21.14, size = 177, normalized size = 0.56

$$\frac{(a + bx^3) \left(-\frac{7\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{27a^{10/3}} + \frac{14\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{27a^{10/3}} + \frac{14\sqrt[3]{b} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b} x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3} a^{10/3}} + \frac{-18a^2 - 49abx^3 - 28b^2x^6}{18a^3x(a+bx^3)^2} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]

[Out] $((a + b*x^3)*((-18*a^2 - 49*a*b*x^3 - 28*b^2*x^6)/(18*a^3*x*(a + b*x^3)^2) + (14*b^{(1/3)}*ArcTan[1/Sqrt[3] - (2*b^{(1/3)}*x)/(Sqrt[3]*a^{(1/3)})])/(9*Sqrt[3]*a^{(10/3)}) + (14*b^{(1/3)}*Log[a^{(1/3)} + b^{(1/3)}*x]/(27*a^{(10/3)}) - (7*b^{(1/3)}*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]/(27*a^{(10/3)})))/Sqrt[(a + b*x^3)^2]$

fricas [A] time = 1.40, size = 201, normalized size = 0.64

$$\frac{84b^2x^6 + 147abx^3 + 28\sqrt{3}(b^2x^7 + 2abx^4 + a^2x)\left(\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + 14(b^2x^7 + 2abx^4 + a^2x)\left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(bx^2 - ax\left(\frac{b}{a}\right)^{\frac{2}{3}} + a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right) - 28(b^2x^7 + 2abx^4 + a^2x)\left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(bx + a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right) + 54a^2}{54(a^3b^2x^7 + 2a^4bx^4 + a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] $-1/54*(84*b^2*x^6 + 147*a*b*x^3 + 28*\text{sqrt}(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^{(1/3)}*\text{arctan}(2/3*\text{sqrt}(3)*x*(b/a)^{(1/3)} - 1/3*\text{sqrt}(3)) + 14*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^{(1/3)}*\log(b*x^2 - a*x*(b/a)^{(2/3)} + a*(b/a)^{(1/3)}) - 28*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^{(1/3)}*\log(b*x + a*(b/a)^{(2/3)}) + 54*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.02, size = 316, normalized size = 1.00

$$\frac{-28\sqrt{3}b^2x^7 \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 28b^2x^7 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 14b^2x^7 \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 84\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^6 - 56\sqrt{3}abx^4 \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 56abx^4 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 28abx^4 \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 147\left(\frac{a}{b}\right)^{\frac{1}{3}}abx^3 - 28\sqrt{3}a^2x \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - 28a^2x \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 14a^2x \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 54\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}(b^2x^3+a)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x)

[Out] $-1/54*(-28*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^7*b^2-28*\ln(x+(a/b)^{(1/3)})*x^7*b^2+14*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^7*b^2+84*(a/b)^{(1/3)}*x^6*b^2-56*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^4*a*b-56*\ln(x+(a/b)^{(1/3)})*x^4*a*b+28*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^4*a*b+147*(a/b)^{(1/3)}*x^3*a*b-28*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x*a^2-28*\ln(x+(a/b)^{(1/3)})*x*a^2+14*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x*a^2+54*(a/b)^{(1/3)}*a^2)*(b*x^3+a)/(a/b)^{(1/3)}/x/a^3/((b*x^3+a)^2)^{(3/2)}$

maxima [A] time = 1.95, size = 148, normalized size = 0.47

$$\frac{28b^2x^6 + 49abx^3 + 18a^2}{18(a^3b^2x^7 + 2a^4bx^4 + a^5x)} - \frac{14\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{7 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{14 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="maxima")

[Out] $-1/18*(28*b^2*x^6 + 49*a*b*x^3 + 18*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x) - 14/27*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a^3*(a/b)^{(1/3)}) - 7/27*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a^3*(a/b)^{(1/3)}) + 14/27*\log(x + (a/b)^{(1/3)})/(a^3*(a/b)^{(1/3)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)

[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left((a + bx^3)^2 \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2), x)

[Out] Integral(1/(x**2*((a + b*x**3)**2)**(3/2)), x)

$$3.105 \quad \int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{3/2}} dx$$

Optimal. Leaf size=316

$$\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)} - \frac{20b^{2/3}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{20b^{2/3}(a+bx^3)\log(\sqrt[3]{a}-\sqrt[3]{b}x)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{20b^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.16, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1355, 290, 325, 200, 31, 634, 617, 204, 628}

$$\frac{10(a+bx^3)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)} - \frac{20b^{2/3}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{10b^{2/3}(a+bx^3)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{20b^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)), x]

[Out] 4/(9*a^2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3))/(9*a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (20*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (20*b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx &= \frac{(b^2 (ab + b^2x^3)) \int \frac{1}{x^3(ab+b^2x^3)^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{1}{6ax^2 (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(4b (ab + b^2x^3)) \int \frac{1}{x^3(ab+b^2x^3)^2} dx}{3a\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{4}{9a^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax^2 (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(20(ab + b^2x^3)) \int \frac{1}{x^3(ab+b^2x^3)} dx}{9a^2\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{4}{9a^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax^2 (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{10(ab + b^2x^3)}{9a^3x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{4}{9a^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax^2 (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{10(ab + b^2x^3)}{9a^3x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{4}{9a^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax^2 (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{10(ab + b^2x^3)}{9a^3x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{4}{9a^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax^2 (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{10(ab + b^2x^3)}{9a^3x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{4}{9a^2x^2\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{6ax^2 (a + bx^3) \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{10(ab + b^2x^3)}{9a^3x^2\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 266, normalized size = 0.84

$$\frac{20b^{8/3}x^8 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}) + 40ab^{5/3}x^5 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}) - 60a^{2/3}b^2x^6 - 27a^{8/3} + 20a^2b^{2/3}x^2 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}) - 40b^{2/3}x^2 (a + bx^3)^2 \log(\sqrt[3]{a} + \sqrt[3]{bx}) + 40\sqrt{3}b^{2/3}x^2 (a + bx^3)^2 \tan^{-1}\left(\frac{1 - \frac{3\sqrt{3}}{5}}{\sqrt{5}}\right)}{54a^{11/3}x^2 (a + bx^3) \sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)), x]

[Out] (-27*a^(8/3) - 96*a^(5/3)*b*x^3 - 60*a^(2/3)*b^2*x^6 + 40*sqrt[3]*b^(2/3)*x^2*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] - 40*b^(2/3)*x

$\int 2(a + bx^3)^2 \log[a^{1/3} + b^{1/3}x] + 20a^2b^{2/3}x^2 \log[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] + 40ab^{5/3}x^5 \log[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] + 20b^{8/3}x^8 \log[a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2] / (54a^{11/3}x^2(a + bx^3)\sqrt{(a + bx^3)^2})$

IntegrateAlgebraic [A] time = 24.32, size = 177, normalized size = 0.56

$$\frac{(a + bx^3) \left(\frac{10b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2})}{27a^{11/3}} - \frac{20b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{bx})}{27a^{11/3}} + \frac{20b^{2/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}} + \frac{-9a^2 - 32abx^3 - 20b^2x^6}{18a^3x^2(a + bx^3)^2} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]

[Out] $((a + bx^3) * ((-9a^2 - 32a*b*x^3 - 20b^2*x^6) / (18a^3*x^2*(a + bx^3)^2) + (20b^{2/3} * \text{ArcTan}[1/\text{Sqrt}[3] - (2b^{1/3} * x) / (\text{Sqrt}[3] * a^{1/3})]) / (9 * \text{Sqrt}[3] * a^{11/3}) - (20b^{2/3} * \text{Log}[a^{1/3} + b^{1/3} * x]) / (27 * a^{11/3}) + (10 * b^{2/3} * \text{Log}[a^{2/3} - a^{1/3} * b^{1/3} * x + b^{2/3} * x^2]) / (27 * a^{11/3}))) / \text{Sqrt}[(a + bx^3)^2]$

fricas [A] time = 1.40, size = 242, normalized size = 0.77

$$\frac{60b^2x^6 + 96abx^3 - 40\sqrt{3}(b^2x^8 + 2abx^5 + a^2x^2) \left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}a\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}} - \sqrt{3}b}{3b}\right) + 20(b^2x^8 + 2abx^5 + a^2x^2) \left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}} \log\left(b^2x^2 + abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} + a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right) - 40(b^2x^8 + 2abx^5 + a^2x^2) \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \log\left(bx - a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right) + 27a^2}{54(a^3b^2x^8 + 2a^4bx^5 + a^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] $-1/54 * (60 * b^2 * x^6 + 96 * a * b * x^3 - 40 * \text{sqrt}(3) * (b^2 * x^8 + 2 * a * b * x^5 + a^2 * x^2) * (-b^2/a^2)^{1/3} * \arctan(1/3 * (2 * \text{sqrt}(3) * a * x * (-b^2/a^2)^{2/3} - \text{sqrt}(3) * b) / b) + 20 * (b^2 * x^8 + 2 * a * b * x^5 + a^2 * x^2) * (-b^2/a^2)^{1/3} * \log(b^2 * x^2 + a * b * x * (-b^2/a^2)^{1/3} + a^2 * (-b^2/a^2)^{2/3}) - 40 * (b^2 * x^8 + 2 * a * b * x^5 + a^2 * x^2) * (-b^2/a^2)^{1/3} * \log(b * x - a * (-b^2/a^2)^{1/3}) + 27 * a^2) / (a^3 * b^2 * x^8 + 2 * a^4 * b * x^5 + a^5 * x^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.02, size = 322, normalized size = 1.02

$$\frac{-40\sqrt{5}b^2a^6 \arctan\left(\frac{\sqrt{5}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\sqrt{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 40b^2a^6 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 20b^2a^6 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 80\sqrt{5}abx^5 \arctan\left(\frac{\sqrt{5}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\sqrt{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 80abx^5 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 40abx^5 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 60\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2a^6 - 40\sqrt{5}a^2x^2 \arctan\left(\frac{\sqrt{5}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\sqrt{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + 40a^2x^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) - 20a^2x^2 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + 96\left(\frac{a}{b}\right)^{\frac{2}{3}}abx^3 + 27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x)

[Out]
$$-1/54*(-40*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*(a/b)^{(1/3)}*x^8*b^2+40*\ln(x+(a/b)^{(1/3)})*x^8*b^2-20*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^8*b^2+60*(a/b)^{(2/3)}*x^6*b^2-80*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^5*a*b+80*\ln(x+(a/b)^{(1/3)})*x^5*a*b-40*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^5*a*b+96*(a/b)^{(2/3)}*x^3*a*b-40*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^2*a^2+40*\ln(x+(a/b)^{(1/3)})*x^2*a^2-20*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^2*a^2+27*(a/b)^{(2/3)}*a^2)*(b*x^3+a)/(a/b)^{(2/3)}/x^2/a^3/((b*x^3+a)^2)^{(3/2)}$$

maxima [A] time = 1.80, size = 150, normalized size = 0.47

$$-\frac{20b^2x^6 + 32abx^3 + 9a^2}{18(a^3b^2x^8 + 2a^4bx^5 + a^5x^2)} - \frac{20\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{10\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{20\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="maxima")

[Out]
$$-1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*a^2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^2) - 20/27*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a^3*(a/b)^{(2/3)}) + 10/27*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a^3*(a/b)^{(2/3)}) - 20/27*\log(x + (a/b)^{(1/3)})/(a^3*(a/b)^{(2/3)})$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)

[Out] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left((a + bx^3)^2 \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2), x)

[Out] Integral(1/(x**3*((a + b*x**3)**2)**(3/2)), x)

$$3.106 \quad \int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{3/2}} dx$$

Optimal. Leaf size=188

$$-\frac{b}{6a^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{3b\log(x)(a+bx^3)}{a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b(a+bx^3)\log(a+bx^3)}{a^4\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.10, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 44}

$$-\frac{b}{6a^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{3b\log(x)(a+bx^3)}{a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b(a+bx^3)\log(a+bx^3)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]

[Out] (-2*b)/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(6*a^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^3*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (3*b*(a + b*x^3)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx &= \frac{(b^2 (ab + b^2x^3)) \int \frac{1}{x^4(ab+b^2x^3)^3} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^2 (ab + b^2x^3)) \text{Subst} \left(\int \frac{1}{x^2(ab+b^2x)^3} dx, x, x^3 \right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^2 (ab + b^2x^3)) \text{Subst} \left(\int \left(\frac{1}{a^3b^3x^2} - \frac{3}{a^4b^2x} + \frac{1}{a^2b(a+bx)^3} + \frac{2}{a^3b(a+bx)^2} + \frac{3}{a^4b(a+bx)} \right) dx, x, x^3 \right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{2b}{3a^3\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b}{6a^2(a + bx^3)\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{a}{3a^3x^3\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 0.52

$$\frac{-a(2a^2 + 9abx^3 + 6b^2x^6) - 18bx^3 \log(x)(a + bx^3)^2 + 6bx^3(a + bx^3)^2 \log(a + bx^3)}{6a^4x^3(a + bx^3)\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]

[Out] $(-(a*(2*a^2 + 9*a*b*x^3 + 6*b^2*x^6)) - 18*b*x^3*(a + b*x^3)^2*\text{Log}[x] + 6*b*x^3*(a + b*x^3)^2*\text{Log}[a + b*x^3])/(6*a^4*x^3*(a + b*x^3)*\text{Sqrt}[(a + b*x^3)^2])$

IntegrateAlgebraic [B] time = 3.84, size = 796, normalized size = 4.23

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]

[Out] $(-a^{16}b) + 2a^{15}b^2x^3 + 61a^{14}b^3x^6 + 864a^{13}b^4x^9 + 7540a^{12}b^5x^{12} + 45344a^{11}b^6x^{15} + 199056a^{10}b^7x^{18} + 658944a^9b^8x^{21} + 1674816a^8b^9x^{24} + 3294720a^7b^{10}x^{27} + 5015296a^6b^{11}x^{30} + 5857280a^5b^{12}x^{33} + 5151744a^4b^{13}x^{36} + 3301376a^3b^{14}x^{39} + 14$

$$54080a^2b^{15}x^{42} + 393216a^3b^{16}x^{45} + 49152b^{17}x^{48} + \sqrt{b^2} \sqrt{a^2 + 2abx^3 + b^2x^6} (-a^{15} - a^{14}bx^3 - 60a^{13}b^2x^6 - 804a^{12}b^3x^9 - 6736a^{11}b^4x^{12} - 38608a^{10}b^5x^{15} - 160448a^9b^6x^{18} - 498496a^8b^7x^{21} - 1176320a^7b^8x^{24} - 2118400a^6b^9x^{27} - 2896896a^5b^{10}x^{30} - 2960384a^4b^{11}x^{33} - 2191360a^3b^{12}x^{36} - 1110016a^2b^{13}x^{39} - 344064ab^{14}x^{42} - 49152b^{15}x^{45}) / (3a^3x^6 \sqrt{a^2 + 2abx^3 + b^2x^6} (-2a^{14}b^2 - 54a^{13}b^3x^3 - 676a^{12}b^4x^6 - 5200a^{11}b^5x^9 - 27456a^{10}b^6x^{12} - 105248a^9b^7x^{15} - 302016a^8b^8x^{18} - 658944a^7b^9x^{21} - 1098240a^6b^{10}x^{24} - 1391104a^5b^{11}x^{27} - 1317888a^4b^{12}x^{30} - 905216a^3b^{13}x^{33} - 425984a^2b^{14}x^{36} - 122880ab^{15}x^{39} - 16384b^{16}x^{42}) + 3a^3 \sqrt{b^2} x^6 (2a^{15}b + 56a^{14}b^2x^3 + 730a^{13}b^3x^6 + 5876a^{12}b^4x^9 + 32656a^{11}b^5x^{12} + 132704a^{10}b^6x^{15} + 407264a^9b^7x^{18} + 960960a^8b^8x^{21} + 1757184a^7b^9x^{24} + 2489344a^6b^{10}x^{27} + 2708992a^5b^{11}x^{30} + 2223104a^4b^{12}x^{33} + 1331200a^3b^{13}x^{36} + 548864a^2b^{14}x^{39} + 139264ab^{15}x^{42} + 16384b^{16}x^{45}) - (2b \operatorname{ArcTanh}(\sqrt{b^2}x^3/a - \sqrt{a^2 + 2abx^3 + b^2x^6}/a)) / a^4$$

fricas [A] time = 1.27, size = 119, normalized size = 0.63

$$\frac{6ab^2x^6 + 9a^2bx^3 + 2a^3 - 6(b^3x^9 + 2ab^2x^6 + a^2bx^3) \log(bx^3 + a) + 18(b^3x^9 + 2ab^2x^6 + a^2bx^3) \log(x)}{6(a^4b^2x^9 + 2a^5bx^6 + a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")

[Out] $-1/6*(6a^3b^2x^6 + 9a^2b^3x^3 + 2a^3 - 6(b^3x^9 + 2a^2b^2x^6 + a^2b^2x^3) \log(bx^3 + a) + 18(b^3x^9 + 2a^2b^2x^6 + a^2b^2x^3) \log(x)) / (a^4b^2x^9 + 2a^5b^2x^6 + a^6x^3)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.02, size = 133, normalized size = 0.71

$$\frac{(18b^3x^9 \ln(x) - 6b^3x^9 \ln(bx^3 + a) + 36a^2b^2x^6 \ln(x) - 12a^2b^2x^6 \ln(bx^3 + a) + 6a^2b^2x^6 + 18a^2bx^3 \ln(x) - 6a^2bx^3 \ln(bx^3 + a) + 9a^2bx^3 + 2a^3)(bx^3 + a)}{6((bx^3 + a)^2)^{\frac{3}{2}} a^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)`

[Out] $-1/6*(18*b^3*x^9*\ln(x)-6*\ln(b*x^3+a)*x^9*b^3+36*a*b^2*x^6*\ln(x)-12*\ln(b*x^3+a)*x^6*a*b^2+6*a*b^2*x^6+18*a^2*b*x^3*\ln(x)-6*\ln(b*x^3+a)*x^3*a^2*b+9*a^2*b*x^3+2*a^3)*(b*x^3+a)/x^3/a^4/((b*x^3+a)^2)^(3/2)$

maxima [A] time = 0.99, size = 117, normalized size = 0.62

$$\frac{(-1)^{2abx^3+2a^2} b \log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right)}{a^4} - \frac{b}{\sqrt{b^2x^6 + 2abx^3 + a^2} a^3} - \frac{1}{6\left(x^3 + \frac{a}{b}\right)^2 a^2 b} - \frac{1}{3\sqrt{b^2x^6 + 2abx^3 + a^2} a^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")`

[Out] $(-1)^{(2*a*b*x^3 + 2*a^2)*b*\log(2*a*b*x/\text{abs}(x) + 2*a^2/(x^2*\text{abs}(x)))/a^4 - b}/(\text{sqrt}(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3) - 1/6/((x^3 + a/b)^2*a^2*b) - 1/3/(\text{sqrt}(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*x^3)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)`

[Out] `int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left((a + bx^3)^2 \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)`

[Out] `Integral(1/(x**4*((a + b*x**3)**2)**(3/2)), x)`

$$3.107 \quad \int \frac{x^6}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=359

$$\frac{x}{162ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.19, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, number of rules / integrand size = 0.346, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{x^4}{12b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{162ab^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5(a+bx^3)\log(\sqrt{a}+\sqrt{bx})}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log(a^{2/3}-\sqrt{a}\sqrt{bx}+b^{2/3}x^2)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\tan^{-1}\left(\frac{\sqrt{a}-2\sqrt{bx}}{\sqrt{3}\sqrt{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (5*x)/(486*a^2*b^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^4/(12*b*(a + b*x^3)^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(27*b^2*(a + b*x^3)^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(162*a*b^2*(a + b*x^3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))])/(2*43*sqrt[3]*a^(8/3)*b^(7/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(8/3)*b^(7/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1458*a^(8/3)*b^(7/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R

$\text{t}[b, 3]*x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3]*\text{Rt}[b, 3]*x + \text{Rt}[b, 3]^2*x^2), x], x] /; \text{FreeQ}\{a, b\}, x]$

Rule 204

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 288

$\text{Int}[(c \cdot x)^m \cdot (a + (b \cdot x)^n)^p, x_Symbol] := \text{Simp}[(c^{n-1} \cdot (c \cdot x)^{m-n+1} \cdot (a + b \cdot x^n)^{p+1})/(b \cdot n \cdot (p+1)), x] - \text{Dist}[(c^{n \cdot (m-n+1)})/(b \cdot n \cdot (p+1)), \text{Int}[(c \cdot x)^{m-n} \cdot (a + b \cdot x^n)^{p+1}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m+1, n] \ \&\& \ !\text{LtQ}[(m+n \cdot (p+1)+1)/n, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 617

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_Symbol] := \text{With}\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c]) /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x)^2), x_Symbol] := \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 634

$\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x)^2), x_Symbol] := \text{Dist}[(2 \cdot c \cdot d - b \cdot e)/(2 \cdot c), \text{Int}[1/(a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[(b + 2 \cdot c \cdot x)/(a + b \cdot x + c \cdot x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4 \cdot a \cdot c]$

Rule 1355

$\text{Int}[(d \cdot x)^m \cdot (a + (b \cdot x)^n + (c \cdot x)^{2n})^p, x_Symbol] := \text{Dist}[(a + b \cdot x^n + c \cdot x^{2n})^{\text{FracPart}[p]}/(c^{\text{IntPart}[p]} \cdot (b/2 + c \cdot x^n)^{2 \cdot \text{FracPart}[p]}), \text{Int}[(d \cdot x)^m \cdot (b/2 + c \cdot x^n)^{2 \cdot p}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{EqQ}[n, 2 \cdot n] \ \&\& \ \text{EqQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ \text{IntegerQ}[p - 1/2]$

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx &= \frac{(b^4(ab + b^2x^3)) \int \frac{x^6}{(ab+b^2x^3)^5} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{x^4}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(b^2(ab + b^2x^3)) \int \frac{x^3}{(ab+b^2x^3)^4} dx}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{x^4}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{27b^2(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= -\frac{x^4}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{27b^2(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= \frac{5x}{486a^2b^2\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^4}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{27b^2(a + b)} \\
&= \frac{5x}{486a^2b^2\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^4}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{27b^2(a + b)} \\
&= \frac{5x}{486a^2b^2\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^4}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{27b^2(a + b)} \\
&= \frac{5x}{486a^2b^2\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^4}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{27b^2(a + b)} \\
&= \frac{5x}{486a^2b^2\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^4}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{27b^2(a + b)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 218, normalized size = 0.61

$$\frac{(a + bx^3) \left(-\frac{10(a+bx^3)^4 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx+b^2x^3})}{a^{8/3}} + \frac{20(a+bx^3)^4 \log(\sqrt[3]{a} + \sqrt[3]{bx})}{a^{8/3}} + \frac{20\sqrt{3}(a+bx^3)^4 \tan^{-1}\left(\frac{2\sqrt[3]{bx}-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{8/3}} + \frac{30\sqrt[3]{bx}(a+bx^3)^3}{a^2} + \frac{18\sqrt[3]{bx}(a+bx^3)^2}{a} - 351\sqrt[3]{bx}(a+bx^3) + 243a\sqrt[3]{bx} \right)}{2916b^{7/3}((a+bx^3)^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] ((a + b*x^3)*(243*a*b^(1/3)*x - 351*b^(1/3)*x*(a + b*x^3) + (18*b^(1/3)*x*(a + b*x^3)^2)/a + (30*b^(1/3)*x*(a + b*x^3)^3)/a^2 + (20*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(8/3) + (20*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - (10*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3))/(2916*b^(7/3)*((a + b*x^3)^2)^(5/2))

IntegrateAlgebraic [A] time = 24.92, size = 189, normalized size = 0.53

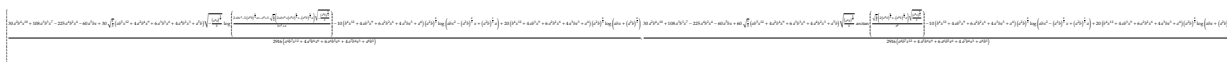
$$\frac{(a + bx^3) \left(-\frac{5 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{1458 a^{8/3} b^{7/3}} + \frac{5 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{729 a^{8/3} b^{7/3}} - \frac{5 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{243 \sqrt{3} a^{8/3} b^{7/3}} + \frac{-20 a^3 x - 75 a^2 b x^4 + 36 a b^2 x^7 + 10 b^3 x^{10}}{972 a^2 b^2 (a + b x^3)^4} \right)}{\sqrt{(a + b x^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] ((a + b*x^3)*((-20*a^3*x - 75*a^2*b*x^4 + 36*a*b^2*x^7 + 10*b^3*x^10)/(972*a^2*b^2*(a + b*x^3)^4) - (5*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/ (243*Sqrt[3]*a^(8/3)*b^(7/3)) + (5*Log[a^(1/3) + b^(1/3)*x])/ (729*a^(8/3)*b^(7/3)) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/ (1458*a^(8/3)*b^(7/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 0.78, size = 723, normalized size = 2.01



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] [1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 30*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9

$$+ 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*\text{sqrt}((a^2*b)^{(1/3)}/b)*\text{arctan}(\text{sqrt}(1/3)*(2*(a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\text{sqrt}((a^2*b)^{(1/3)}/b)/a^2) - 10*(b^4*x^{12} + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 20*(b^4*x^{12} + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}))/((a^4*b^7*x^{12} + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3)]$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

maple [B] time = 0.02, size = 519, normalized size = 1.45

$$\frac{10b^3x^{10} + 36ab^2x^7 - 75a^2bx^4 - 20a^3x}{972(a^2b^6x^{12} + 4a^3b^5x^9 + 6a^4b^4x^6 + 4a^5b^3x^3 + a^6b^2)} + \frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] $\frac{1}{2916}(-20*3^{(1/2)}*\text{arctan}(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^{12} + b^4 + 20*\ln(x+(a/b)^{(1/3)})*x^{12}*b^4 - 10*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^{12} + 2*b^4 + 30*(a/b)^{(2/3)}*x^{10}*b^4 - 80*3^{(1/2)}*\text{arctan}(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^9*a*b^3 + 80*\ln(x+(a/b)^{(1/3)})*x^9*a*b^3 - 40*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^9*a*b^3 + 108*(a/b)^{(2/3)}*x^7*a*b^3 - 120*3^{(1/2)}*\text{arctan}(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^6*a^2*b^2 + 120*\ln(x+(a/b)^{(1/3)})*x^6*a^2*b^2 - 60*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^6*a^2*b^2 - 225*(a/b)^{(2/3)}*x^4*a^2*b^2 - 80*3^{(1/2)}*\text{arctan}(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*x^3*a^3*b + 80*\ln(x+(a/b)^{(1/3)})*x^3*a^3*b - 40*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*x^3*a^3*b - 60*(a/b)^{(2/3)}*x*a^3*b - 20*3^{(1/2)}*\text{arctan}(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)})/(a/b)^{(1/3)})*a^4 + 20*\ln(x+(a/b)^{(1/3)})*a^4 - 10*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*a^4)*(b*x^3+a)/(a/b)^{(2/3)}/b^3/a^2/((b*x^3+a)^2)^{(5/2)}$

maxima [A] time = 1.35, size = 195, normalized size = 0.54

$$\frac{10b^3x^{10} + 36ab^2x^7 - 75a^2bx^4 - 20a^3x}{972(a^2b^6x^{12} + 4a^3b^5x^9 + 6a^4b^4x^6 + 4a^5b^3x^3 + a^6b^2)} + \frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729a^2b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/972*(10*b^3*x^10 + 36*a*b^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/(a^2*b^6*x^12 + 4*a^3*b^5*x^9 + 6*a^4*b^4*x^6 + 4*a^5*b^3*x^3 + a^6*b^2) + 5/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)) - 5/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) + 5/729*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)

[Out] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left((a + bx^3)^2\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] Integral(x**6/((a + b*x**3)**2)**(5/2), x)

$$3.108 \quad \int \frac{x^5}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=78

$$\frac{a}{12b^2 (a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{1}{9b^2 (a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 43}

$$\frac{a}{12b^2 (a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{1}{9b^2 (a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] a/(12*b^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - 1/(9*b^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx &= \frac{(b^4(ab + b^2x^3)) \int \frac{x^5}{(ab+b^2x^3)^5} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^4(ab + b^2x^3)) \text{Subst}\left(\int \frac{x}{(ab+b^2x)^5} dx, x, x^3\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^4(ab + b^2x^3)) \text{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5} + \frac{1}{b^6(a+bx)^4}\right) dx, x, x^3\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{a}{12b^2(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{1}{9b^2(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.50

$$\frac{-a - 4bx^3}{36b^2(a + bx^3)^3 \sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (-a - 4*b*x^3)/(36*b^2*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [B] time = 0.77, size = 229, normalized size = 2.94

$$\frac{-2(3a^5b - ab^5x^{12} - 4b^6x^{15}) - 2\sqrt{b^2} \sqrt{a^2 + 2abx^3 + b^2x^6} (3a^4 - 3a^3bx^3 + 3a^2b^2x^6 - 3ab^3x^9 + 4b^4x^{12})}{9x^{12} \sqrt{a^2 + 2abx^3 + b^2x^6} (-8a^3b^7 - 24a^2b^8x^3 - 24ab^9x^6 - 8b^{10}x^9) + 9\sqrt{b^2} x^{12} (8a^4b^6 + 32a^3b^7x^3 + 48a^2b^8x^6 + 32ab^9x^9 + 8b^{10}x^{12})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (-2*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^4 - 3*a^3*b*x^3 + 3*a^2*b^2*x^6 - 3*a*b^3*x^9 + 4*b^4*x^12) - 2*(3*a^5*b - a*b^5*x^12 - 4*b^6*x^15))/(9*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^3*b^7 - 24*a^2*b^8*x^3 - 24*a*b^9*x^6 - 8*b^10*x^9) + 9*Sqrt[b^2]*x^12*(8*a^4*b^6 + 32*a^3*b^7*x^3 + 48*a^2*b^8*x^6 + 32*a*b^9*x^9 + 8*b^10*x^12))

fricas [A] time = 1.18, size = 58, normalized size = 0.74

$$\frac{4bx^3 + a}{36(b^6x^{12} + 4ab^5x^9 + 6a^2b^4x^6 + 4a^3b^3x^3 + a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] -1/36*(4*b*x^3 + a)/(b^6*x^12 + 4*a*b^5*x^9 + 6*a^2*b^4*x^6 + 4*a^3*b^3*x^3 + a^4*b^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.01, size = 32, normalized size = 0.41

$$-\frac{(bx^3 + a)(4bx^3 + a)}{36\left((bx^3 + a)^2\right)^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] -1/36*(b*x^3+a)*(4*b*x^3+a)/b^2/((b*x^3+a)^2)^(5/2)

maxima [A] time = 0.86, size = 43, normalized size = 0.55

$$-\frac{1}{9\left(b^2x^6 + 2abx^3 + a^2\right)^{\frac{3}{2}}b^2} + \frac{a}{12\left(x^3 + \frac{a}{b}\right)^4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] -1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2) + 1/12*a/((x^3 + a/b)^4*b^6)

mupad [B] time = 1.28, size = 42, normalized size = 0.54

$$-\frac{(4bx^3 + a)\sqrt{a^2 + 2abx^3 + b^2x^6}}{36b^2(bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

[Out] $-\frac{(a + 4bx^3)(a^2 + b^2x^6 + 2abx^3)^{1/2}}{36b^2(a + bx^3)^5}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left((a + bx^3)^2\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(x**5/((a + b*x**3)**2)**(5/2), x)`

$$3.109 \quad \int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=368

$$\frac{7x^2}{324a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.19, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1355, 288, 290, 292, 31, 634, 617, 204, 628}

$$\frac{7x^2}{243a^2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7x^2}{324a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\log(\sqrt[3]{a} + \sqrt[3]{bx})}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7(a+bx^3)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{bx} + b^{2/3}x^2)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{7(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (7*x^2)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(12*b*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(54*a*b*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (7*x^2)/(324*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (7*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1458*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x]

/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx &= \frac{(b^4(ab + b^2x^3)) \int \frac{x^4}{(ab+b^2x^3)^5} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{x^2}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(b^2(ab + b^2x^3)) \int \frac{x}{(ab+b^2x^3)^4} dx}{6\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{x^2}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= -\frac{x^2}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= \frac{7x^2}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7x^2}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7x^2}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7x^2}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{7x^2}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x^2}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 229, normalized size = 0.62

$$\frac{(a + bx^3) \left(-243a^{10/3}b^{2/3}x^2 + 63a^{4/3}b^{2/3}x^2(a + bx^3)^2 + 54a^{7/3}b^{2/3}x^2(a + bx^3) + 14(a + bx^3)^4 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2) + 84\sqrt[3]{a} b^{2/3}x^2(a + bx^3)^3 - 28(a + bx^3)^4 \log(\sqrt[3]{a} + \sqrt[3]{bx}) + 28\sqrt[3]{3} (a + bx^3)^4 \tan^{-1}\left(\frac{2\sqrt[3]{bx} - \sqrt[3]{a}}{\sqrt[3]{3} \sqrt[3]{a}}\right) \right)}{2916a^{10/3}b^{5/3}(a + bx^3)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] ((a + b*x^3)*(-243*a^(10/3)*b^(2/3)*x^2 + 54*a^(7/3)*b^(2/3)*x^2*(a + b*x^3) + 63*a^(4/3)*b^(2/3)*x^2*(a + b*x^3)^2 + 84*a^(1/3)*b^(2/3)*x^2*(a + b*x^3)^3 + 28*sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))] - 28*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 14*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2916*a^(10/3)*b^(5/3)*((a + b*x^3)^2)^(5/2))

IntegrateAlgebraic [A] time = 26.11, size = 191, normalized size = 0.52

$$\frac{(a + bx^3) \left(\frac{7 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2)}{1458a^{10/3}b^{5/3}} - \frac{7 \log(\sqrt[3]{a} + \sqrt[3]{bx})}{729a^{10/3}b^{5/3}} - \frac{7 \tan^{-1}\left(\frac{1}{\sqrt[3]{3}} - \frac{2\sqrt[3]{bx}}{\sqrt[3]{3} \sqrt[3]{a}}\right)}{243\sqrt[3]{3} a^{10/3}b^{5/3}} + \frac{-14a^3x^2 + 144a^2bx^5 + 105ab^2x^8 + 28b^3x^{11}}{972a^3b(a + bx^3)^4} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] ((a + b*x^3)*((-14*a^3*x^2 + 144*a^2*b*x^5 + 105*a*b^2*x^8 + 28*b^3*x^11)/(972*a^3*b*(a + b*x^3)^4) - (7*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(sqrt[3]*a^(1/3))])/(243*sqrt[3]*a^(10/3)*b^(5/3)) - (7*Log[a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)) + (7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1458*a^(10/3)*b^(5/3)))/sqrt[(a + b*x^3)^2]

fricas [A] time = 1.30, size = 734, normalized size = 1.99

$$\frac{(a + bx^3) \left(\frac{7 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2)}{1458a^{10/3}b^{5/3}} - \frac{7 \log(\sqrt[3]{a} + \sqrt[3]{bx})}{729a^{10/3}b^{5/3}} - \frac{7 \tan^{-1}\left(\frac{1}{\sqrt[3]{3}} - \frac{2\sqrt[3]{bx}}{\sqrt[3]{3} \sqrt[3]{a}}\right)}{243\sqrt[3]{3} a^{10/3}b^{5/3}} + \frac{-14a^3x^2 + 144a^2bx^5 + 105ab^2x^8 + 28b^3x^{11}}{972a^3b(a + bx^3)^4} \right)}{\sqrt{(a + bx^3)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] [1/2916*(84*a*b^5*x^11 + 315*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2 + 42*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*

$$a^3 b x^3 + a^4) (-a b^2)^{2/3} \log(b^2 x^2 + (-a b^2)^{1/3} b x + (-a b^2)^{2/3}) - 28(b^4 x^{12} + 4 a b^3 x^9 + 6 a^2 b^2 x^6 + 4 a^3 b x^3 + a^4) (-a b^2)^{2/3} \log(b x - (-a b^2)^{1/3}) / (a^4 b^7 x^{12} + 4 a^5 b^6 x^9 + 6 a^6 b^5 x^6 + 4 a^7 b^4 x^3 + a^8 b^3), 1/2916 (84 a b^5 x^{11} + 315 a^2 b^4 x^8 + 432 a^3 b^3 x^5 - 42 a^4 b^2 x^2 + 84 \sqrt{1/3} (a b^5 x^{12} + 4 a^2 b^4 x^9 + 6 a^3 b^3 x^6 + 4 a^4 b^2 x^3 + a^5 b) \sqrt{-(-a b^2)^{1/3} / a} \arctan(\sqrt{1/3} (2 b x + (-a b^2)^{1/3}) \sqrt{-(-a b^2)^{1/3} / a}) / b) + 14 (b^4 x^{12} + 4 a b^3 x^9 + 6 a^2 b^2 x^6 + 4 a^3 b x^3 + a^4) (-a b^2)^{2/3} \log(b^2 x^2 + (-a b^2)^{1/3} b x + (-a b^2)^{2/3}) - 28 (b^4 x^{12} + 4 a b^3 x^9 + 6 a^2 b^2 x^6 + 4 a^3 b x^3 + a^4) (-a b^2)^{2/3} \log(b x - (-a b^2)^{1/3}) / (a^4 b^7 x^{12} + 4 a^5 b^6 x^9 + 6 a^6 b^5 x^6 + 4 a^7 b^4 x^3 + a^8 b^3)]$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

maple [B] time = 0.02, size = 521, normalized size = 1.42

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] $1/2916 * (-28 * 3^{1/2} * b^4 * x^{12} * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3})) / (a/b)^{1/3}) - 28 * b^4 * x^{12} * \ln(x + (a/b)^{1/3}) + 14 * b^4 * x^{12} * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) + 84 * (a/b)^{1/3} * x^{11} * b^4 - 112 * 3^{1/2} * a * b^3 * x^9 * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3})) / (a/b)^{1/3} - 112 * a * b^3 * x^9 * \ln(x + (a/b)^{1/3}) + 56 * a * b^3 * x^9 * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) + 315 * (a/b)^{1/3} * x^8 * a * b^3 - 168 * 3^{1/2} * a^2 * b^2 * x^6 * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3})) / (a/b)^{1/3} - 168 * a^2 * b^2 * x^6 * \ln(x + (a/b)^{1/3}) + 84 * a^2 * b^2 * x^6 * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) + 432 * (a/b)^{1/3} * x^5 * a^2 * b^2 - 112 * 3^{1/2} * a^3 * b * x^3 * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3})) / (a/b)^{1/3} - 112 * a^3 * b * x^3 * \ln(x + (a/b)^{1/3}) + 56 * a^3 * b * x^3 * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) - 42 * (a/b)^{1/3} * x^2 * a^3 * b - 28 * 3^{1/2} * a^4 * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3})) / (a/b)^{1/3} - 28 * a^4 * \ln(x + (a/b)^{1/3}) + 14 * a^4 * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3})) * (b * x^3 + a) / (a/b)^{1/3} / b^2 / a^3 / ((b * x^3 + a)^2)^{5/2}$

maxima [A] time = 1.77, size = 195, normalized size = 0.53

$$\frac{28b^3x^{11} + 105ab^2x^8 + 144a^2bx^5 - 14a^3x^2}{972(a^3b^5x^{12} + 4a^4b^4x^9 + 6a^5b^3x^6 + 4a^6b^2x^3 + a^7b)} + \frac{7\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{7 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{7 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/972*(28*b^3*x^11 + 105*a*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/(a^3*b^5*x^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 7/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3)) + 7/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3)) - 7/729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)

[Out] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left((a + bx^3)^2\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] Integral(x**4/((a + b*x**3)**2)**(5/2), x)

$$3.110 \quad \int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=360

$$\frac{x}{81a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.18, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1355, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5x}{243a^2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{81a^2b(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{10(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{bx})}{729a^{1/3}b^4\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5(a+bx^3)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2)}{729a^{1/3}b^4\sqrt{a^2+2abx^3+b^2x^6}} - \frac{10(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx}}{\sqrt[3]{3}\sqrt[3]{a}}\right)}{243\sqrt[3]{3}a^{1/3}b^4\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (5*x)/(243*a^3*b*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(12*b*(a + b*x^3)^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(108*a*b*(a + b*x^3)^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(81*a^2*b*(a + b*x^3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))]/(243*sqrt[3]*a^(11/3)*b^(4/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x]/(729*a^(11/3)*b^(4/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(729*a^(11/3)*b^(4/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R

$\text{t}[b, 3]*x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3]*\text{Rt}[b, 3]*x + \text{Rt}[b, 3]^2*x^2), x], x] /; \text{FreeQ}\{a, b\}, x]$

Rule 204

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 288

$\text{Int}[(c \cdot x)^m \cdot (a + (b \cdot x)^n)^p, x_Symbol] \rightarrow \text{Simp}[(c^{n-1} \cdot (c \cdot x)^{m-n+1} \cdot (a + b \cdot x^n)^{p+1})/(b \cdot n \cdot (p+1)), x] - \text{Dist}[(c^{n \cdot (m-n+1)})/(b \cdot n \cdot (p+1)), \text{Int}[(c \cdot x)^{m-n} \cdot (a + b \cdot x^n)^{p+1}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m+1, n] \ \&\& \ !\text{LtQ}[(m+n \cdot (p+1)+1)/n, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 617

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c]) /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 634

$\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Dist}[(2 \cdot c \cdot d - b \cdot e)/(2 \cdot c), \text{Int}[1/(a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[(b + 2 \cdot c \cdot x)/(a + b \cdot x + c \cdot x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4 \cdot a \cdot c]$

Rule 1355

$\text{Int}[(d \cdot x)^m \cdot (a + (b \cdot x)^n + (c \cdot x)^{2n})^p, x_Symbol] \rightarrow \text{Dist}[(a + b \cdot x^n + c \cdot x^{2n})^{\text{FracPart}[p]}/(c^{\text{IntPart}[p]} \cdot (b/2 + c \cdot x^n)^{2 \cdot \text{FracPart}[p]}), \text{Int}[(d \cdot x)^m \cdot (b/2 + c \cdot x^n)^{2 \cdot p}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{EqQ}[n2, 2 \cdot n] \ \&\& \ \text{EqQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ \text{IntegerQ}[p - 1/2]$

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx &= \frac{(b^4(ab + b^2x^3)) \int \frac{x^3}{(ab+b^2x^3)^5} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{x}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(b^2(ab + b^2x^3)) \int \frac{1}{(ab+b^2x^3)^4} dx}{12\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{x}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x}{108ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \\
&= -\frac{x}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x}{108ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \\
&= \frac{5x}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x}{108ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{5x}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x}{108ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{5x}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x}{108ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{5x}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x}{108ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{5x}{243a^3b\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{x}{12b(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x}{108ab(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 221, normalized size = 0.61

$$\frac{(a + bx^3) \left(-20(a + bx^3)^4 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}) + 60a^{2/3} \sqrt[3]{bx} (a + bx^3)^3 + 36a^{5/3} \sqrt[3]{bx} (a + bx^3)^2 + 27a^{8/3} \sqrt[3]{bx} (a + bx^3) - 243a^{11/3} \sqrt[3]{bx} + 40(a + bx^3)^4 \log(\sqrt[3]{a} + \sqrt[3]{bx}) + 40\sqrt{3} (a + bx^3)^4 \tan^{-1}\left(\frac{2\sqrt[3]{bx} - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right) \right)}{2916a^{11/3}b^{4/3} (a + bx^3)^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] ((a + b*x^3)*(-243*a^(11/3)*b^(1/3)*x + 27*a^(8/3)*b^(1/3)*x*(a + b*x^3) + 36*a^(5/3)*b^(1/3)*x*(a + b*x^3)^2 + 60*a^(2/3)*b^(1/3)*x*(a + b*x^3)^3 + 40*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 40*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 20*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(11/3)*b^(4/3)*((a + b*x^3)^2)^(5/2))

IntegrateAlgebraic [A] time = 26.81, size = 189, normalized size = 0.52

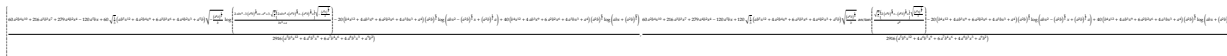
$$\frac{(a + bx^3) \left(-\frac{5 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2})}{729a^{11/3}b^{4/3}} + \frac{10 \log(\sqrt[3]{a} + \sqrt[3]{bx})}{729a^{11/3}b^{4/3}} - \frac{10 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}} + \frac{-40a^3x + 93a^2bx^4 + 72ab^2x^7 + 20b^3x^{10}}{972a^3b(a+bx^3)^4} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] ((a + b*x^3)*((-40*a^3*x + 93*a^2*b*x^4 + 72*a*b^2*x^7 + 20*b^3*x^10)/(972*a^3*b*(a + b*x^3)^4) - (10*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]))/(243*Sqrt[3]*a^(11/3)*b^(4/3)) + (10*Log[a^(1/3) + b^(1/3)*x])/(729*a^(11/3)*b^(4/3)) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(11/3)*b^(4/3)))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 1.23, size = 723, normalized size = 2.01



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] [1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b)))/(b*x^3 + a) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 120*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) -

$$20*(b^4*x^{12} + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 40*(b^4*x^{12} + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)})))/(a^5*b^6*x^{12} + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2)]$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

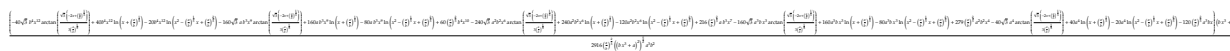
sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

maple [B] time = 0.02, size = 519, normalized size = 1.44



Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] $\frac{1}{2916}*(-40*3^{(1/2)}*b^4*x^{12}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+40*b^4*x^{12}*\ln(x+(a/b)^{(1/3)})-20*b^4*x^{12}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+60*(a/b)^{(2/3)}*b^4*x^{10}-160*3^{(1/2)}*a*b^3*x^9*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+160*a*b^3*x^9*\ln(x+(a/b)^{(1/3)})-80*a*b^3*x^9*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+216*(a/b)^{(2/3)}*a*b^3*x^7-240*3^{(1/2)}*a^2*b^2*x^6*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+240*a^2*b^2*x^6*\ln(x+(a/b)^{(1/3)})-120*a^2*b^2*x^6*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+279*(a/b)^{(2/3)}*a^2*b^2*x^4-160*3^{(1/2)}*a^3*b*x^3*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+160*a^3*b*x^3*\ln(x+(a/b)^{(1/3)})-80*a^3*b*x^3*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})-120*(a/b)^{(2/3)}*a^3*b*x-40*3^{(1/2)}*a^4*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+40*a^4*\ln(x+(a/b)^{(1/3)})-20*a^4*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}))*(b*x^3+a)/(a/b)^{(2/3)}/b^2/a^3/((b*x^3+a)^2)^{(5/2)}$

maxima [A] time = 2.28, size = 193, normalized size = 0.54

$$\frac{20 b^3 x^{10} + 72 a b^2 x^7 + 93 a^2 b x^4 - 40 a^3 x}{972 (a^3 b^5 x^{12} + 4 a^4 b^4 x^9 + 6 a^5 b^3 x^6 + 4 a^6 b^2 x^3 + a^7 b)} + \frac{10 \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^3 b^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729 a^3 b^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{10 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729 a^3 b^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/972*(20*b^3*x^10 + 72*a*b^2*x^7 + 93*a^2*b*x^4 - 40*a^3*x)/(a^3*b^5*x^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 10/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) - 5/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 10/729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)

[Out] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{((a + bx^3)^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)

[Out] Integral(x**3/((a + b*x**3)**2)**(5/2), x)

$$3.111 \quad \int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=38

$$-\frac{1}{12b(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1352, 607}

$$-\frac{1}{12b(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] -1/(12*b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))

Rule 607

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(2*(a + b*x + c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx, x, x^3 \right) \\ &= -\frac{1}{12b(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$\frac{a + bx^3}{12b \left((a + bx^3)^2 \right)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] -1/12*(a + b*x^3)/(b*((a + b*x^3)^2)^(5/2))

IntegrateAlgebraic [B] time = 0.83, size = 209, normalized size = 5.50

$$\frac{-2(-a^4b - b^5x^{12}) - 2\sqrt{b^2}\sqrt{a^2 + 2abx^3 + b^2x^6}(-a^3 + a^2bx^3 - ab^2x^6 + b^3x^9)}{3bx^{12}\sqrt{a^2 + 2abx^3 + b^2x^6}(-8a^3b^5 - 24a^2b^6x^3 - 24ab^7x^6 - 8b^8x^9) + 3b\sqrt{b^2}x^{12}(8a^4b^4 + 32a^3b^5x^3 + 48a^2b^6x^6 + 32ab^7x^9 + 8b^8x^{12})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (-2*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-a^3 + a^2*b*x^3 - a*b^2*x^6 + b^3*x^9) - 2*(-(a^4*b) - b^5*x^12))/(3*b*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^3*b^5 - 24*a^2*b^6*x^3 - 24*a*b^7*x^6 - 8*b^8*x^9) + 3*b*Sqrt[b^2]*x^12*(8*a^4*b^4 + 32*a^3*b^5*x^3 + 48*a^2*b^6*x^6 + 32*a*b^7*x^9 + 8*b^8*x^12))

fricas [A] time = 1.15, size = 48, normalized size = 1.26

$$\frac{1}{12(b^5x^{12} + 4ab^4x^9 + 6a^2b^3x^6 + 4a^3b^2x^3 + a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] -1/12/(b^5*x^12 + 4*a*b^4*x^9 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^3 + a^4*b)

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0*x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.01, size = 24, normalized size = 0.63

$$-\frac{bx^3 + a}{12 \left((bx^3 + a)^2 \right)^{\frac{5}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)`

[Out] `-1/12*(b*x^3+a)/b/((b*x^3+a)^2)^(5/2)`

maxima [A] time = 0.51, size = 16, normalized size = 0.42

$$-\frac{1}{12 \left(x^3 + \frac{a}{b} \right)^4 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")`

[Out] `-1/12/((x^3 + a/b)^4*b^5)`

mupad [B] time = 1.26, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^2 + 2abx^3 + b^2x^6}}{12b(bx^3 + a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)`

[Out] `-(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(12*b*(a + b*x^3)^5)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((a + bx^3)^2 \right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)`

[Out] `Integral(x**2/((a + b*x**3)**2)**(5/2), x)`

$$3.112 \quad \int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=359

$$\frac{5x^2}{54a^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{35(a+bx^3)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{35(a+bx^3)\log(\sqrt[3]{a} - \sqrt[3]{b}x)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{35(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt[3]{b}}\right)}{243\sqrt[3]{a^{13}b^{2/3}}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.19, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1355, 290, 292, 31, 634, 617, 204, 628}

$$\frac{35x^2}{324a^3(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} - \frac{35(a+bx^3)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{35(a+bx^3)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{35(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt[3]{b}}\right)}{243\sqrt[3]{a^{13}b^{2/3}}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (35*x^2)/(243*a^4*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(12*a*(a + b*x^3)^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*x^2)/(54*a^2*(a + b*x^3)^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (35*x^2)/(324*a^3*(a + b*x^3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (35*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))]/(243*sqrt[3]*a^(13/3)*b^(2/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (35*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x]/(729*a^(13/3)*b^(2/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (35*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(1458*a^(13/3)*b^(2/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m + n*(p+1) + 1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

]

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx &= \frac{(b^4(ab + b^2x^3)) \int \frac{x}{(ab+b^2x^3)^5} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2}{12a(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{(5b^3(ab + b^2x^3)) \int \frac{x}{(ab+b^2x^3)^4} dx}{6a\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{x^2}{12a(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{5x^2}{54a^2(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= \frac{x^2}{12a(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{5x^2}{54a^2(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{35x^2}{243a^4 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= \frac{35x^2}{243a^4 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{12a(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54a^2(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= \frac{35x^2}{243a^4 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{12a(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54a^2(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= \frac{35x^2}{243a^4 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{12a(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54a^2(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots \\
&= \frac{35x^2}{243a^4 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{12a(a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{x^2}{54a^2(a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}} + \dots
\end{aligned}$$

Mathematica [A] time = 0.12, size = 219, normalized size = 0.61

$$\frac{(a + bx^3) \left(\frac{70(a+bx^3)^4 \log\left(\frac{a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx+b^{2/3}x^2}}{b^{2/3}}\right)}{b^{2/3}} + 315a^{4/3}x^2(a + bx^3)^2 + 270a^{7/3}x^2(a + bx^3) + 243a^{10/3}x^2 - \frac{140(a+bx^3)^4 \log\left(\frac{\sqrt[3]{a} + \sqrt[3]{bx}}{b^{2/3}}\right)}{b^{2/3}} + \frac{140\sqrt{5}(a+bx^3)^4 \tan^{-1}\left(\frac{2\sqrt[3]{bx} - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + 420\sqrt[3]{a}x^2(a + bx^3)^3 \right)}{2916a^{13/3} \left((a + bx^3)^2 \right)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] ((a + b*x^3)*(243*a^(10/3)*x^2 + 270*a^(7/3)*x^2*(a + b*x^3) + 315*a^(4/3)*x^2*(a + b*x^3)^2 + 420*a^(1/3)*x^2*(a + b*x^3)^3 + (140*sqrt(3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(sqrt(3)*a^(1/3))])/b^(2/3) - (140*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (70*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(2916*a^(13/3)*((a + b*x^3)^2)^(5/2))

IntegrateAlgebraic [A] time = 28.09, size = 188, normalized size = 0.52

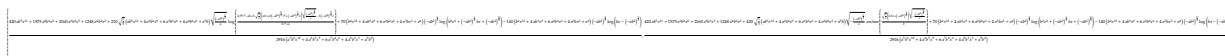
$$\frac{(a + bx^3) \left(\frac{35 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx} + b^{2/3}x^2)}{1458a^{13/3}b^{2/3}} - \frac{35 \log(\sqrt[3]{a} + \sqrt[3]{bx})}{729a^{13/3}b^{2/3}} - \frac{35 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{13/3}b^{2/3}} + \frac{x^2(416a^3 + 720a^2bx^3 + 525ab^2x^6 + 140b^3x^9)}{972a^4(a+bx^3)^4} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] ((a + b*x^3)*((x^2*(416*a^3 + 720*a^2*b*x^3 + 525*a*b^2*x^6 + 140*b^3*x^9))/(972*a^4*(a + b*x^3)^4) - (35*ArcTan[1/sqrt(3) - (2*b^(1/3)*x)/(sqrt(3)*a^(1/3))])/((243*sqrt(3)*a^(13/3)*b^(2/3)) - (35*Log[a^(1/3) + b^(1/3)*x])/(729*a^(13/3)*b^(2/3)) + (35*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1458*a^(13/3)*b^(2/3))))/sqrt[(a + b*x^3)^2]

fricas [A] time = 1.27, size = 734, normalized size = 2.04



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] [1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 210*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 420*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a

) / b) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

maple [B] time = 0.01, size = 521, normalized size = 1.45



Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] $\frac{1}{2916}(-140 \cdot 3^{1/2} \cdot b^4 \cdot x^{12} \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (-2 \cdot x + (a/b)^{1/3})) / (a/b)^{(1/3)} - 140 \cdot b^4 \cdot x^{12} \cdot \ln(x + (a/b)^{1/3}) + 70 \cdot b^4 \cdot x^{12} \cdot \ln(x^2 - (a/b)^{1/3} \cdot x + (a/b)^{(2/3)}) + 420 \cdot (a/b)^{1/3} \cdot b^4 \cdot x^{11} - 560 \cdot 3^{1/2} \cdot a \cdot b^3 \cdot x^9 \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (-2 \cdot x + (a/b)^{1/3})) / (a/b)^{(1/3)} - 560 \cdot a \cdot b^3 \cdot x^9 \cdot \ln(x + (a/b)^{1/3}) + 280 \cdot a \cdot b^3 \cdot x^9 \cdot \ln(x^2 - (a/b)^{1/3} \cdot x + (a/b)^{(2/3)}) + 1575 \cdot (a/b)^{1/3} \cdot a \cdot b^3 \cdot x^8 - 840 \cdot 3^{1/2} \cdot a^2 \cdot b^2 \cdot x^6 \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (-2 \cdot x + (a/b)^{1/3})) / (a/b)^{(1/3)} - 840 \cdot a^2 \cdot b^2 \cdot x^6 \cdot \ln(x + (a/b)^{1/3}) + 420 \cdot a^2 \cdot b^2 \cdot x^6 \cdot \ln(x^2 - (a/b)^{1/3} \cdot x + (a/b)^{(2/3)}) + 2160 \cdot (a/b)^{1/3} \cdot a^2 \cdot b^2 \cdot x^5 - 560 \cdot 3^{1/2} \cdot a^3 \cdot b \cdot x^3 \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (-2 \cdot x + (a/b)^{1/3})) / (a/b)^{(1/3)} - 560 \cdot a^3 \cdot b \cdot x^3 \cdot \ln(x + (a/b)^{1/3}) + 280 \cdot a^3 \cdot b \cdot x^3 \cdot \ln(x^2 - (a/b)^{1/3} \cdot x + (a/b)^{(2/3)}) + 1248 \cdot (a/b)^{1/3} \cdot a^3 \cdot b \cdot x^2 - 140 \cdot 3^{1/2} \cdot a^4 \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (-2 \cdot x + (a/b)^{1/3})) / (a/b)^{(1/3)} - 140 \cdot a^4 \cdot \ln(x + (a/b)^{1/3}) + 70 \cdot a^4 \cdot \ln(x^2 - (a/b)^{1/3} \cdot x + (a/b)^{(2/3})) \cdot (b \cdot x^3 + a) / (a/b)^{(1/3)} / b / a^4 / ((b \cdot x^3 + a)^2)^{(5/2)}$

maxima [A] time = 2.18, size = 191, normalized size = 0.53

$$\frac{140 b^3 x^{11} + 525 a b^2 x^8 + 720 a^2 b x^5 + 416 a^3 x^2}{972 (a^4 b^4 x^{12} + 4 a^5 b^3 x^9 + 6 a^6 b^2 x^6 + 4 a^7 b x^3 + a^8)} + \frac{35 \sqrt{3} \arctan\left(\frac{\sqrt{3} \left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^4 b \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{35 \log\left(x^2 - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458 a^4 b \left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{35 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729 a^4 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/972*(140*b^3*x^11 + 525*a*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/(a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 35/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3)) + 35/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(1/3)) - 35/729*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)

[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{((a + bx^3)^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] Integral(x/((a + b*x**3)**2)**(5/2), x)

$$3.113 \quad \int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=364

$$\frac{11x(a+bx^3)^2}{108a^2(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{x(a+bx^3)}{12a(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{110(a+bx^3)^5 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{729a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}} - \frac{110(a+bx^3)^5}{243\sqrt{3}a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}}$$

Rubi [A] time = 0.20, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1343, 199, 200, 31, 634, 617, 204, 628}

$$\frac{55x(a+bx^3)^4}{243a^4(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{11x(a+bx^3)^3}{81a^3(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{x(a+bx^3)}{12a(a^2+2abx^3+b^2x^6)^{5/2}} + \frac{110(a+bx^3)^5 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{729a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}} - \frac{55(a+bx^3)^5 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{729a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}} - \frac{110(a+bx^3)^5 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{14/3}\sqrt[3]{b}(a^2+2abx^3+b^2x^6)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2), x]

[Out] (x*(a + b*x^3))/(12*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3)^2)/(108*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3)^3)/(81*a^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (55*x*(a + b*x^3)^4)/(243*a^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (110*(a + b*x^3)^5*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (110*(a + b*x^3)^5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (55*(a + b*x^3)^5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^p, x_Symbol] := Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx &= \frac{(2ab + 2b^2x^3)^5 \int \frac{1}{(2ab+2b^2x^3)^5} dx}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{(11(2ab + 2b^2x^3)^5) \int \frac{1}{(2ab+2b^2x^3)^4} dx}{24ab(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{(11(2ab + 2b^2x^3))}{54a^2b^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)}{81a^3(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)}{81a^3(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)}{81a^3(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)}{81a^3(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)}{81a^3(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)}{81a^3(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)}{81a^3(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)}{81a^3(a^2 + 2abx^3 + b^2x^6)^{5/2}} \\
&= \frac{x(a + bx^3)}{12a(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)^2}{108a^2(a^2 + 2abx^3 + b^2x^6)^{5/2}} + \frac{11x(a + bx^3)}{81a^3(a^2 + 2abx^3 + b^2x^6)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 211, normalized size = 0.58

$$\frac{(a + bx^3) \left(-\frac{220(a+bx^3)^4 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx+bx^2})}{\sqrt[3]{b}} + 660a^{2/3}x(a + bx^3)^3 + 396a^{5/3}x(a + bx^3)^2 + 297a^{8/3}x(a + bx^3) + 243a^{11/3}x + \frac{440(a+bx^3)^4 \log(\sqrt[3]{a} + \sqrt[3]{bx})}{\sqrt[3]{b}} + \frac{440\sqrt{5}(a+bx^3)^4 \tan^{-1}\left(\frac{2\sqrt[3]{bx}-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} \right)}{2916a^{14/3}((a + bx^3)^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2), x]

[Out] ((a + b*x^3)*(243*a^(11/3)*x + 297*a^(8/3)*x*(a + b*x^3) + 396*a^(5/3)*x*(a + b*x^3)^2 + 660*a^(2/3)*x*(a + b*x^3)^3 + (440*sqrt(3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(sqrt(3)*a^(1/3))])/b^(1/3) + (440*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (220*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(2916*a^(14/3)*((a + b*x^3)^2)^(5/2))

IntegrateAlgebraic [A] time = 34.13, size = 186, normalized size = 0.51

$$\frac{(a + bx^3) \left(-\frac{55 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{729 a^{14/3} \sqrt[3]{b}} + \frac{110 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{729 a^{14/3} \sqrt[3]{b}} - \frac{110 \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{243 \sqrt{3} a^{14/3} \sqrt[3]{b}} + \frac{x(532 a^3 + 1023 a^2 b x^3 + 792 a b^2 x^6 + 220 b^3 x^9)}{972 a^4 (a + b x^3)^4} \right)}{\sqrt{(a + b x^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2), x]

[Out] ((a + b*x^3)*((x*(532*a^3 + 1023*a^2*b*x^3 + 792*a*b^2*x^6 + 220*b^3*x^9))/(972*a^4*(a + b*x^3)^4) - (110*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(sqrt(3)*a^(1/3))])/((243*sqrt(3)*a^(14/3)*b^(1/3)) + (110*Log[a^(1/3) + b^(1/3)*x])/(729*a^(14/3)*b^(1/3)) - (55*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/3))))/sqrt[(a + b*x^3)^2]

fricas [A] time = 1.24, size = 719, normalized size = 1.98

$$\frac{(a + b x^3) \left(\frac{x(532 a^3 + 1023 a^2 b x^3 + 792 a b^2 x^6 + 220 b^3 x^9)}{972 a^4 (a + b x^3)^4} - \frac{110 \operatorname{ArcTan}\left(\frac{1}{\sqrt{3}} - \frac{2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{243 \sqrt{3} a^{14/3} \sqrt[3]{b}} + \frac{110 \log(a^{1/3} + b^{1/3} x)}{729 a^{14/3} \sqrt[3]{b}} - \frac{55 \log(a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2)}{729 a^{14/3} \sqrt[3]{b}} \right)}{\sqrt{(a + b x^3)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] [1/2916*(660*a^2*b^4*x^10 + 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*b*x + 660*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b)))/(b*x^3 + a) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 + 6*a^8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b), 1/2916*(660*a^2*b^4*x^10 + 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*b*x + 1320*sqrt(1/3)*(a*b^5*x^12 +

$$4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*\sqrt{(a^2*b)^{(1/3)}/b)}*\arctan(\sqrt{1/3}*(2*(a^2*b)^{(2/3)}*x - (a^2*b)^{(1/3)}*a)*\sqrt{(a^2*b)^{(1/3)}/b)}/a^2) - 220*(b^4*x^{12} + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^{(2/3)}*\log(a*b*x^2 - (a^2*b)^{(2/3)}*x + (a^2*b)^{(1/3)}*a) + 440*(b^4*x^{12} + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^{(2/3)}*\log(a*b*x + (a^2*b)^{(2/3)}))/(a^6*b^5*x^{12} + 4*a^7*b^4*x^9 + 6*a^8*b^3*x^6 + 4*a^9*b^2*x^3 + a^{10}*b)]$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

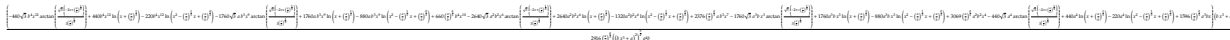
sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.01, size = 519, normalized size = 1.43



Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] $\frac{1}{2916}*(-440*3^{(1/2)}*b^4*x^{12}*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+440*b^4*x^{12}*\ln(x+(a/b)^{(1/3)})-220*b^4*x^{12}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+660*(a/b)^{(2/3)}*b^4*x^{10}-1760*3^{(1/2)}*a*b^3*x^9*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+1760*a*b^3*x^9*\ln(x+(a/b)^{(1/3)})-880*a*b^3*x^9*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+2376*(a/b)^{(2/3)}*a*b^3*x^7-2640*3^{(1/2)}*a^2*b^2*x^6*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+2640*a^2*b^2*x^6*\ln(x+(a/b)^{(1/3)})-1320*a^2*b^2*x^6*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+3069*(a/b)^{(2/3)}*a^2*b^2*x^4-1760*3^{(1/2)}*a^3*b*x^3*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+1760*a^3*b*x^3*\ln(x+(a/b)^{(1/3)})-880*a^3*b*x^3*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+1596*(a/b)^{(2/3)}*a^3*b*x-440*3^{(1/2)}*a^4*\arctan(1/3*3^{(1/2)}*(-2*x+(a/b)^{(1/3)}))/(a/b)^{(1/3)}+440*a^4*\ln(x+(a/b)^{(1/3)})-220*a^4*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*(b*x^3+a)/(a/b)^{(2/3)}/b/a^4/((b*x^3+a)^2)^{(5/2)}$

maxima [A] time = 2.15, size = 189, normalized size = 0.52

$$\frac{220 b^3 x^{10} + 792 a b^2 x^7 + 1023 a^2 b x^4 + 532 a^3 x}{972 (a^4 b^4 x^{12} + 4 a^5 b^3 x^9 + 6 a^6 b^2 x^6 + 4 a^7 b x^3 + a^8)} + \frac{110 \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^4 b \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{55 \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729 a^4 b \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{110 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729 a^4 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/972*(220*b^3*x^10 + 792*a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/(a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 110/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3)) - 55/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) + 110/729*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)

[Out] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-5/2), x)

$$3.114 \quad \int \frac{1}{x(a^2+2abx^3+b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=223

$$\frac{1}{9a^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\log(x)(a+bx^3)}{a^5\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.12, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 44}

$$\frac{1}{6a^3(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{9a^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\log(x)(a+bx^3)}{a^5\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log(a+bx^3)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]

[Out] 1/(3*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(9*a^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx &= \frac{(b^4(ab + b^2x^3)) \int \frac{1}{x(ab+b^2x^3)^5} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^4(ab + b^2x^3)) \text{Subst}\left(\int \frac{1}{x(ab+b^2x)^5} dx, x, x^3\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^4(ab + b^2x^3)) \text{Subst}\left(\int \left(\frac{1}{a^5b^5x} - \frac{1}{ab^4(a+bx)^5} - \frac{1}{a^2b^4(a+bx)^4} - \frac{1}{a^3b^4(a+bx)^3} - \frac{1}{a^4b^4(a+bx)^2}\right) dx, x, x^3\right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{1}{3a^4\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{12a(a + bx^3)^3\sqrt{a^2 + 2abx^3 + b^2x^6}} + \frac{1}{9a^2(a + bx^3)^2\sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 0.43

$$\frac{a(25a^3 + 52a^2bx^3 + 42ab^2x^6 + 12b^3x^9) + 36 \log(x)(a + bx^3)^4 - 12(a + bx^3)^4 \log(a + bx^3)}{36a^5(a + bx^3)^3 \sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

[Out] (a*(25*a^3 + 52*a^2*b*x^3 + 42*a*b^2*x^6 + 12*b^3*x^9) + 36*(a + b*x^3)^4*Log[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])

IntegrateAlgebraic [B] time = 127.75, size = 3896, normalized size = 17.47

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

[Out] (-32*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^87 - 3*a^86*b*x^3 + 3*a^85*b^2*x^6 - 3*a^84*b^3*x^9 + 28*a^83*b^4*x^12 + 4074*a^82*b^5*x^15 + 328392*a^81*b^6*x^18 + 17416344*a^80*b^7*x^21 + 684119040*a^79*b^8*x^24 + 21226212480*a^78*b^9*x^27 + 541798692864*a^77*b^10*x^30 + 11700119129088*a^76*b^11*x^33 + 21226212480*a^75*b^12*x^36 + 21226212480*a^74*b^13*x^39 + 17416344*a^73*b^14*x^42 + 4074*a^72*b^15*x^45 + 28*a^71*b^16*x^48 + 3*a^70*b^17*x^51 + 3*a^69*b^18*x^54 + 3*a^68*b^19*x^57 + 3*a^67*b^20*x^60 + 3*a^66*b^21*x^63 + 3*a^65*b^22*x^66 + 3*a^64*b^23*x^69 + 3*a^63*b^24*x^72 + 3*a^62*b^25*x^75 + 3*a^61*b^26*x^78 + 3*a^60*b^27*x^81 + 3*a^59*b^28*x^84 + 3*a^58*b^29*x^87 + 3*a^57*b^30*x^90 + 3*a^56*b^31*x^93 + 3*a^55*b^32*x^96 + 3*a^54*b^33*x^99 + 3*a^53*b^34*x^102 + 3*a^52*b^35*x^105 + 3*a^51*b^36*x^108 + 3*a^50*b^37*x^111 + 3*a^49*b^38*x^114 + 3*a^48*b^39*x^117 + 3*a^47*b^40*x^120 + 3*a^46*b^41*x^123 + 3*a^45*b^42*x^126 + 3*a^44*b^43*x^129 + 3*a^43*b^44*x^132 + 3*a^42*b^45*x^135 + 3*a^41*b^46*x^138 + 3*a^40*b^47*x^141 + 3*a^39*b^48*x^144 + 3*a^38*b^49*x^147 + 3*a^37*b^50*x^150 + 3*a^36*b^51*x^153 + 3*a^35*b^52*x^156 + 3*a^34*b^53*x^159 + 3*a^33*b^54*x^162 + 3*a^32*b^55*x^165 + 3*a^31*b^56*x^168 + 3*a^30*b^57*x^171 + 3*a^29*b^58*x^174 + 3*a^28*b^59*x^177 + 3*a^27*b^60*x^180 + 3*a^26*b^61*x^183 + 3*a^25*b^62*x^186 + 3*a^24*b^63*x^189 + 3*a^23*b^64*x^192 + 3*a^22*b^65*x^195 + 3*a^21*b^66*x^198 + 3*a^20*b^67*x^201 + 3*a^19*b^68*x^204 + 3*a^18*b^69*x^207 + 3*a^17*b^70*x^210 + 3*a^16*b^71*x^213 + 3*a^15*b^72*x^216 + 3*a^14*b^73*x^219 + 3*a^13*b^74*x^222 + 3*a^12*b^75*x^225 + 3*a^11*b^76*x^228 + 3*a^10*b^77*x^231 + 3*a^9*b^78*x^234 + 3*a^8*b^79*x^237 + 3*a^7*b^80*x^240 + 3*a^6*b^81*x^243 + 3*a^5*b^82*x^246 + 3*a^4*b^83*x^249 + 3*a^3*b^84*x^252 + 3*a^2*b^85*x^255 + 3*a*b^86*x^258 + b^87*x^261) + 36*(a + b*x^3)^4*Log[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])

$$\begin{aligned}
& 11x^{33} + 218177660073984a^{75}b^{12}x^{36} + 3568299926191104a^{74}b^{13}x^{39} \\
& + 51815594461802496a^{73}b^{14}x^{42} + 674671446755364864a^{72}b^{15}x^{45} + 79 \\
& 41063256075862016a^{71}b^{16}x^{48} + 85067029477553700864a^{70}b^{17}x^{51} + 83 \\
& 4123879538833555456a^{69}b^{18}x^{54} + 7523474448566234578944a^{68}b^{19}x^{57} \\
& + 62685534036478928879616a^{67}b^{20}x^{60} + 484263949539613361700864a^{66}b^{21} \\
& x^{63} + 3479930411745645615906816a^{65}b^{22}x^{66} + 2332783513405509482997 \\
& 3504a^{64}b^{23}x^{69} + 146249615315137820985655296a^{63}b^{24}x^{72} + 85943205 \\
& 9122570824614150144a^{62}b^{25}x^{75} + 4743516432354559922458853376a^{61}b^{26} \\
& x^{78} + 24634603647181064055495327744a^{60}b^{27}x^{81} + 12057328734669237530 \\
& 6185998336a^{59}b^{28}x^{84} + 556993306425737520204547620864a^{58}b^{29}x^{87} + \\
& 2431710423525963544007685439488a^{57}b^{30}x^{90} + 1004499081167460608101908 \\
& 7945728a^{56}b^{31}x^{93} + 39302890124942958048947321438208a^{55}b^{32}x^{96} + \\
& 145797964967106670900820916043776a^{54}b^{33}x^{99} + 513218808138628154416518 \\
& 496518144a^{53}b^{34}x^{102} + 1715578603609902451791538956533760a^{52}b^{35}x^{105} \\
& + 5449694039783497785024773904924672a^{51}b^{36}x^{108} + 1646075331496819 \\
& 2305329000761262080a^{50}b^{37}x^{111} + 47301435355834570726904893777379328a^{49} \\
& b^{38}x^{114} + 129374002716573907248459868192899072a^{48}b^{39}x^{117} + 336 \\
& 930636646549696381527413823111168a^{47}b^{40}x^{120} + 83579628335498463107133 \\
& 5990534602752a^{46}b^{41}x^{123} + 1975361005020021175749282788611719168a^{45} \\
& b^{42}x^{126} + 4449110127870895658771635322027507712a^{44}b^{43}x^{129} + 955103 \\
& 7404260216847866175737338789888a^{43}b^{44}x^{132} + 1954450841640742220634082 \\
& 9051147517952a^{42}b^{45}x^{135} + 38125736076483998462868401157222432768a^{41} \\
& b^{46}x^{138} + 70897347674846732883050381793665482752a^{40}b^{47}x^{141} + 1256 \\
& 71127881529846697796348885994569728a^{39}b^{48}x^{144} + 212317935948353209887 \\
& 693339475650281472a^{38}b^{49}x^{147} + 34183097624385207952098724987670416588 \\
& 8a^{37}b^{50}x^{150} + 524340224188047425154680645061540052992a^{36}b^{51}x^{153} \\
& + 766073281583947528164240486507938316288a^{35}b^{52}x^{156} + 10657006133710 \\
& 13117842709614376603615232a^{34}b^{53}x^{159} + 141102016868974492528729515994 \\
& 3475232768a^{33}b^{54}x^{162} + 1777302527925165461341561537146824687616a^{32} \\
& b^{55}x^{165} + 2128564412652177688937195991485965664256a^{31}b^{56}x^{168} + 242 \\
& 2395406390342874120214563859260768256a^{30}b^{57}x^{171} + 2617807829735370087 \\
& 962525106611234537472a^{29}b^{58}x^{174} + 26842957773725159953799290973956268 \\
& 35968a^{28}b^{59}x^{177} + 2609450207845959606905876323113652715520a^{27}b^{60} \\
& x^{180} + 2402576508782798093149416471150968438784a^{26}b^{61}x^{183} + 20929165 \\
& 07617038227414227202520498831360a^{25}b^{62}x^{186} + 172289432255020078577246 \\
& 2871077208457216a^{24}b^{63}x^{189} + 1338524199162955156171842117301576925184 \\
& a^{23}b^{64}x^{192} + 979986658223088202764220685370425081856a^{22}b^{65}x^{195} \\
& + 675048463780981537495401786462871486464a^{21}b^{66}x^{198} + 436701745764302 \\
& 470805819212429950713856a^{20}b^{67}x^{201} + 26478490145474068609917612066427 \\
& 2666624a^{19}b^{68}x^{204} + 150133777416926053796721610962266750976a^{18}b^{69} \\
& x^{207} + 79403252661685074394167343394234302464a^{17}b^{70}x^{210} + 390597799 \\
& 66378067682904413566775853056a^{16}b^{71}x^{213} + 178132859737480923949878737 \\
& 83737483264a^{15}b^{72}x^{216} + 7503690680834688849004036334253244416a^{14}b^{73} \\
& x^{219} + 2907226777830056762419313398234742784a^{13}b^{74}x^{222} + 10309151 \\
& 12359687157926188398124466176a^{12}b^{75}x^{225} + 332671302995605405712650389
\end{aligned}$$

$373845504a^{11}b^{76}x^{228} + 97031595508821255286739673941016576a^{10}b^{77}x^{231} + 25374257060868870577491562299654144a^9b^{78}x^{234} + 5890738842117298555432264066400256a^8b^{79}x^{237} + 1199289224076687944555612256337920a^7b^{80}x^{240} + 210809734722622322011205101682688a^6b^{81}x^{243} + 31346364513998779407019781652480a^5b^{82}x^{246} + 3833971628290179974483895386112a^4b^{83}x^{249} + 370352006987302418412887605248a^3b^{84}x^{252} + 26492400411034983734511140864a^2b^{85}x^{255} + 1247611445842297308296773632a^1b^{86}x^{258} + 29014219670751100192948224b^{87}x^{261} - 32(3a^{88}b - 25a^{84}b^5x^{12} - 4102a^{83}b^6x^{15} - 332466a^{82}b^7x^{18} - 17744736a^{81}b^8x^{21} - 701535384a^{80}b^9x^{24} - 21910331520a^{79}b^{10}x^{27} - 563024905344a^{78}b^{11}x^{30} - 12241917821952a^{77}b^{12}x^{33} - 229877779203072a^{76}b^{13}x^{36} - 3786477586265088a^{75}b^{14}x^{39} - 55383894387993600a^{74}b^{15}x^{42} - 726487041217167360a^{73}b^{16}x^{45} - 8615734702831226880a^{72}b^{17}x^{48} - 93008092733629562880a^{71}b^{18}x^{51} - 919190909016387256320a^{70}b^{19}x^{54} - 8357598328105068134400a^{69}b^{20}x^{57} - 70209008485045163458560a^{68}b^{21}x^{60} - 546949483576092290580480a^{67}b^{22}x^{63} - 3964194361285258977607680a^{66}b^{23}x^{66} - 26807765545800740445880320a^{65}b^{24}x^{69} - 169577450449192915815628800a^{64}b^{25}x^{72} - 1005681674437708645599805440a^{63}b^{26}x^{75} - 5602948491477130747073003520a^{62}b^{27}x^{78} - 29378120079535623977954181120a^{61}b^{28}x^{81} - 145207890993873439361681326080a^{60}b^{29}x^{84} - 677566593772429895510733619200a^{59}b^{30}x^{87} - 2988703729951701064212233060352a^{58}b^{31}x^{90} - 12476701235200569625026773385216a^{57}b^{32}x^{93} - 49347880936617564129966409383936a^{56}b^{33}x^{96} - 185100855092049628949768237481984a^{55}b^{34}x^{99} - 659016773105734825317339412561920a^{54}b^{35}x^{102} - 2228797411748530606208057453051904a^{53}b^{36}x^{105} - 7165272643393400236816312861458432a^{52}b^{37}x^{108} - 21910447354751690090353774666186752a^{51}b^{38}x^{111} - 63762188670802763032233894538641408a^{50}b^{39}x^{114} - 176675438072408477975364761970278400a^{49}b^{40}x^{117} - 466304639363123603629987282016010240a^{48}b^{41}x^{120} - 1172726920001534327452863404357713920a^{47}b^{42}x^{123} - 2811157288375005806820618779146321920a^{46}b^{43}x^{126} - 6424471132890916834520918110639226880a^{45}b^{44}x^{129} - 14000147532131112506637811059366297600a^{44}b^{45}x^{132} - 29095545820667639054207004788486307840a^{43}b^{46}x^{135} - 57670244492891420669209230208369950720a^{42}b^{47}x^{138} - 109023083751330731345918782950887915520a^{41}b^{48}x^{141} - 196568475556376579580846730679660052480a^{40}b^{49}x^{144} - 337989063829883056585489688361644851200a^{39}b^{50}x^{147} - 554148912192205289408680589352354447360a^{38}b^{51}x^{150} - 866171200431899504675667894938244218880a^{37}b^{52}x^{153} - 1290413505771994953318921131569478369280a^{36}b^{53}x^{156} - 1831773894954960646006950100884541931520a^{35}b^{54}x^{159} - 2476720782060758043130004774320078848000a^{34}b^{55}x^{162} - 3188322696614910386628856697090299920384a^{33}b^{56}x^{165} - 3905866940577343150278757528632790351872a^{32}b^{57}x^{168} - 4550959819042520563057410555345226432512a^{31}b^{58}x^{171} - 5040203236125712962082739670470495305728a^{30}b^{59}x^{174} - 5302103607107886083342454204006861373440a^{29}b^{60}x^{177} - 5293745985218475602285805420509279551488a^{28}b^{61}x^{180} - 5012026716628757700055292794264621154304a^{27}b^{62}x^{183} - 4495493016399836320563643673671467270144a^{26}b^{62}$

$3x^{186} - 3815810830167239013186690073597707288576a^{25}b^{64}x^{189} - 306141$
 $8521713155941944304988378785382400a^{24}b^{65}x^{192} - 2318510857386043358936$
 $062802672002007040a^{23}b^{66}x^{195} - 16550351220040697402596224718332965683$
 $20a^{22}b^{67}x^{198} - 1111750209545284008301220998892822200320a^{21}b^{68}x^{201}$
 $01 - 701486647219043156904995333094223380480a^{20}b^{69}x^{204} - 414918678871$
 $666739895897731626539417600a^{19}b^{70}x^{207} - 22953703007861112819088895435$
 $6501053440a^{18}b^{71}x^{210} - 118463032628063142077071756961010155520a^{17}b^{72}$
 $x^{213} - 56873065940126160077892287350513336320a^{16}b^{73}x^{216} - 253169$
 $76654582781243991910117990727680a^{15}b^{74}x^{219} - 104109174586647456114233$
 $49732487987200a^{14}b^{75}x^{222} - 3938141890189743920345501796359208960a^{13}$
 $b^{76}x^{225} - 1363586415355292563638838787498311680a^{12}b^{77}x^{228} - 42970$
 $2898504426660999390063314862080a^{11}b^{78}x^{231} - 1224058525696901258642312$
 $36240670720a^{10}b^{79}x^{234} - 31264995902986169132923826366054400a^9b^{80}$
 $x^{237} - 7090028066193986499987876322738176a^8b^{81}x^{240} - 141009895879931$
 $0266566817358020608a^7b^{82}x^{243} - 242156099236621101418224883335168a^6b^{83}$
 $x^{246} - 35180336142288959381503677038592a^5b^{84}x^{249} - 420432363527$
 $7482392896782991360a^4b^{85}x^{252} - 396844407398337402147398746112a^3b^{86}$
 $x^{255} - 27740011856877281042807914496a^2b^{87}x^{258} - 127662566551304840$
 $8489721856a^1b^{88}x^{261} - 29014219670751100192948224b^{89}x^{264}) / (9a^4b^8$
 $\text{Sqrt}[a^2 + 2abx^3 + b^2x^6] * (128a^{84}b^4x^{12} + 21120a^{83}b^5x^{15} +$
 $1721472a^{82}b^6x^{18} + 92406656a^{81}b^7x^{21} + 3674439936a^{80}b^8x^{24} +$
 $115431837696a^{79}b^9x^{27} + 2983781007360a^{78}b^{10}x^{30} + 65264862633984$
 $a^{77}b^{11}x^{33} + 1232951257055232a^{76}b^{12}x^{36} + 20432990676713472a^{75}$
 $b^{13}x^{39} + 300716130445885440a^{74}b^{14}x^{42} + 3969238746518323200a^{73}b^{15}$
 $x^{45} + 47370458487200808960a^{72}b^{16}x^{48} + 514638307833890734080a^{71}$
 $b^{17}x^{51} + 5118983772087653498880a^{70}b^{18}x^{54} + 46847576245279590973440$
 $a^{69}b^{19}x^{57} + 396147929845363900416000a^{68}b^{20}x^{60} + 310672195963271$
 $5288412160a^{67}b^{21}x^{63} + 22669023436505501851975680a^{66}b^{22}x^{66} + 154$
 $345386528573397590343680a^{65}b^{23}x^{69} + 983080641935232659210895360a^{64}$
 $b^{24}x^{72} + 5870874650091141471613747200a^{63}b^{25}x^{75} + 32939251658045736$
 $840163491840a^{62}b^{26}x^{78} + 173944179132957033591283384320a^{61}b^{27}x^{81}$
 $+ 865962850098397774705978245120a^{60}b^{28}x^{84} + 407022939653487004583170$
 $1987328a^{59}b^{29}x^{87} + 18086007219228693734214252625920a^{58}b^{30}x^{90} +$
 $76065544328296704454991881961472a^{57}b^{31}x^{93} + 3031243771788375588352918$
 $35334656a^{56}b^{32}x^{96} + 1145674020859022985294059076059136a^{55}b^{33}x^{99}$
 $+ 4110416795701119981416309783003136a^{54}b^{34}x^{102} + 1400985167651859764$
 $4792572002959360a^{53}b^{35}x^{105} + 45394822957658124638063332524294144a^{52}$
 $b^{36}x^{108} + 139917789751193392435587350802726912a^{51}b^{37}x^{111} + 410459$
 $873858094378326686815750193152a^{50}b^{38}x^{114} + 11465881887909049103717479$
 $76404008960a^{49}b^{39}x^{117} + 3051146630923992568193710004423884800a^{48}b^{40}$
 $x^{120} + 7737324336869628583877642484858224640a^{47}b^{41}x^{123} + 18703305$
 $019991137633578654068606238720a^{46}b^{42}x^{126} + 43107103684238284774282301$
 $309630545920a^{45}b^{43}x^{129} + 94746125967065558286369665929351004160a^{44}$
 $b^{44}x^{132} + 198615000002424286648098587431403520000a^{43}b^{45}x^{135} + 3971$
 $30692504847361152873125569091338240a^{42}b^{46}x^{138} + 757418302509245017132$

$$\begin{aligned}
& 523963169657323520a^{41}b^{47}x^{141} + 13778644786531817944002773914070758195 \\
& 20a^{40}b^{48}x^{144} + 2390618413362513124890580419918786723840a^{39}b^{49}x^{147} + 3955385015068042115189991389350644940800a^{38}b^{50}x^{150} + 62396517372 \\
& 15173635140320005662399528960a^{37}b^{51}x^{153} + 938251880913198683328764636 \\
& 9303054254080a^{36}b^{52}x^{156} + 13444227011136623928559678656620034785280a^{35}b^{53}x^{159} + 18350748809159245654215695088179610648576a^{34}b^{54}x^{162} \\
& + 23850139665038281564949273182080325386240a^{33}b^{55}x^{165} + 2950101144237 \\
& 0924678193433541945719783424a^{32}b^{56}x^{168} + 3470977963912952651679986663 \\
& 9524772708352a^{31}b^{57}x^{171} + 38820803891602154416762573083734716710912a^{30}b^{58}x^{174} + 41244886350924311041670535470940728328192a^{29}b^{59}x^{177} \\
& + 41593718455288022589388471161144339333120a^{28}b^{60}x^{180} + 3977933777171 \\
& 4841625416239365458407456768a^{27}b^{61}x^{183} + 3604434839423862440091822325 \\
& 5712630308864a^{26}b^{62}x^{186} + 30909917019058539460410388414124412370944a^{25}b^{63}x^{189} + 25056451871195099922047049138792730460160a^{24}b^{64}x^{192} \\
& + 19174574965327876626062519403677535436800a^{23}b^{65}x^{195} + 1383173129744 \\
& 2951886921500151546055229440a^{22}b^{66}x^{198} + 9389906208869529129738482371 \\
& 217451909120a^{21}b^{67}x^{201} + 5988100438665955998606600464981368504320a^{20}b^{68}x^{204} + 3579959214285221715179046877486708162560a^{19}b^{69}x^{207} + 2 \\
& 001897592698205886195342610222022656000a^{18}b^{70}x^{210} + 10444153669223527 \\
& 00482947391151763619840a^{17}b^{71}x^{213} + 506902354822510510083032763037195 \\
& 960320a^{16}b^{72}x^{216} + 228129429525827274249013529575497400320a^{15}b^{73}x^{219} + 94849133060508440059978288214562570240a^{14}b^{74}x^{222} + 3627706898 \\
& 6647020067202576304360652800a^{13}b^{75}x^{225} + 1270106131021082442296761676 \\
& 9422786560a^{12}b^{76}x^{228} + 4047255710028362687341349707846778880a^{11}b^{77}x^{231} + 1165860312209840554524188084357038080a^{10}b^{78}x^{234} + 301141170 \\
& 995507566848412970190372864a^9b^{79}x^{237} + 690623155850836457839339931015 \\
& 57760a^8b^{80}x^{240} + 13891106442322941803683011662708736a^7b^{81}x^{243} + \\
& 2412615694461848378203398972899328a^6b^{82}x^{246} + 3544937629702993310826 \\
& 28280352768a^5b^{83}x^{249} + 42847967496007858756144393617408a^4b^{84}x^{252} + 4090618117313628445869793607680a^3b^{85}x^{255} + 2892137416780469667233 \\
& 07896832a^2b^{86}x^{258} + 13462597927228510489527975936a^1b^{87}x^{261} + 3094 \\
& 85009821345068724781056b^{88}x^{264} + 9a^4b^8\sqrt{b^2}(-128a^{85}b^3x^{12} \\
& - 21248a^{84}b^4x^{15} - 1742592a^{83}b^5x^{18} - 94128128a^{82}b^6x^{21} - 3 \\
& 766846592a^{81}b^7x^{24} - 119106277632a^{80}b^8x^{27} - 3099212845056a^{79}b^9x^{30} - 68248643641344a^{78}b^{10}x^{33} - 1298216119689216a^{77}b^{11}x^{36} - \\
& 21665941933768704a^{76}b^{12}x^{39} - 321149121122598912a^{75}b^{13}x^{42} - 426 \\
& 9954876964208640a^{74}b^{14}x^{45} - 51339697233719132160a^{73}b^{15}x^{48} - 562 \\
& 008766321091543040a^{72}b^{16}x^{51} - 5633622079921544232960a^{71}b^{17}x^{54} - \\
& 51966560017367244472320a^{70}b^{18}x^{57} - 442995506090643491389440a^{69}b^{19}x^{60} - 3502869889478079188828160a^{68}b^{20}x^{63} - 25775745396138217140387 \\
& 840a^{67}b^{21}x^{66} - 177014409965078899442319360a^{66}b^{22}x^{69} - 113742602 \\
& 8463806056801239040a^{65}b^{23}x^{72} - 6853955292026374130824642560a^{64}b^{24}x^{75} - 38810126308136878311777239040a^{63}b^{25}x^{78} - 20688343079100277043 \\
& 1446876160a^{62}b^{26}x^{81} - 1039907029231354808297261629440a^{61}b^{27}x^{84} \\
& - 4936192246633267820537680232448a^{60}b^{28}x^{87} - 221562366157635637800459
\end{aligned}$$

$54613248a^{59}b^{29}x^{90} - 94151551547525398189206134587392a^{58}b^{30}x^{93} -$
 $379189921507134263290283717296128a^{57}b^{31}x^{96} - 14487983980378605441293$
 $50911393792a^{56}b^{32}x^{99} - 5256090816560142966710368859062272a^{55}b^{33}x$
 $^{102} - 18120268472219717626208881785962496a^{54}b^{34}x^{105} - 59404674634176$
 $722282855904527253504a^{53}b^{35}x^{108} - 18531261270885151707365068332702105$
 $6a^{52}b^{36}x^{111} - 550377663609287770762274166552920064a^{51}b^{37}x^{114} -$
 $1557048062648999288698434792154202112a^{50}b^{38}x^{117} - 4197734819714897478$
 $565457980827893760a^{49}b^{39}x^{120} - 10788470967793621152071352489282109440$
 $a^{48}b^{40}x^{123} - 26440629356860766217456296553464463360a^{47}b^{41}x^{126} -$
 $61810408704229422407860955378236784640a^{46}b^{42}x^{129} - 13785322965130384$
 $3060651967238981550080a^{45}b^{43}x^{132} - 2933611259694898449344682533607545$
 $24160a^{44}b^{44}x^{135} - 595745692507271647800971713000494858240a^{43}b^{45}x$
 $^{138} - 1154548995014092378285397088738748661760a^{42}b^{46}x^{141} - 213528278$
 $1162426811532801354576733143040a^{41}b^{47}x^{144} - 3768482892015694919290857$
 $811325862543360a^{40}b^{48}x^{147} - 6346003428430555240080571809269431664640a$
 $^{39}b^{49}x^{150} - 10195036752283215750330311395013044469760a^{38}b^{50}x^{153}$
 $- 15622170546347160468427966374965453783040a^{37}b^{51}x^{156} - 228267458202$
 $68610761847325025923089039360a^{36}b^{52}x^{159} - 317949758202958695827753737$
 $44799645433856a^{35}b^{53}x^{162} - 42200888474197527219164968270259936034816a$
 $^{34}b^{54}x^{165} - 53351151107409206243142706724026045169664a^{33}b^{55}x^{168}$
 $- 64210791081500451194993300181470492491776a^{32}b^{56}x^{171} - 735305835307$
 $31680933562439723259489419264a^{31}b^{57}x^{174} - 800656902425264654584331085$
 $54675445039104a^{30}b^{58}x^{177} - 82838604806212333631059006632085067661312a$
 $^{29}b^{59}x^{180} - 81373056227002864214804710526602746789888a^{28}b^{60}x^{183}$
 $- 75823686165953466026334462621171037765632a^{27}b^{61}x^{186} - 669542654132$
 $97163861328611669837042679808a^{26}b^{62}x^{189} - 559663688902536393824574375$
 $52917142831104a^{25}b^{63}x^{192} - 44231026836522976548109568542470265896960a$
 $^{24}b^{64}x^{195} - 33006306262770828512984019555223590666240a^{23}b^{65}x^{198}$
 $- 23221637506312481016659982522763507138560a^{22}b^{66}x^{201} - 153780066475$
 $35485128345082836198820413440a^{21}b^{67}x^{204} - 956805965295117771378564734$
 $2468076666880a^{20}b^{68}x^{207} - 5581856806983427601374389487708730818560a^{19}$
 $b^{69}x^{210} - 3046312959620558586678290001373786275840a^{18}b^{70}x^{213} -$
 $1551317721744863210565980154188959580160a^{17}b^{71}x^{216} - 7350317843483377$
 $84332046292612693360640a^{16}b^{72}x^{219} - 322978562586335714308991817790059$
 $970560a^{15}b^{73}x^{222} - 131126202047155460127180864518923223040a^{14}b^{74}x$
 $^{225} - 48978130296857844490170193073783439360a^{13}b^{75}x^{228} - 1674831702$
 $0239187110308966477269565440a^{12}b^{76}x^{231} - 5213116022238203241865537792$
 $203816960a^{11}b^{77}x^{234} - 1467001483205348121372601054547410944a^{10}b^{78}$
 $x^{237} - 370203486580591212632346963291930624a^9b^{79}x^{240} - 829534220274$
 $06587587617004764266496a^8b^{80}x^{243} - 1630372213678479018188641063560806$
 $4a^7b^{81}x^{246} - 2767109457432147709286027253252096a^6b^{82}x^{249} - 3973$
 $41730466307189838772673970176a^5b^{83}x^{252} - 4693858561332148720201418722$
 $5088a^4b^{84}x^{255} - 4379831858991675412593101504512a^3b^{85}x^{258} - 3026$
 $76339605275477212835872768a^2b^{86}x^{261} - 13772082937049855558252756992a$
 $b^{87}x^{264} - 309485009821345068724781056b^{88}x^{267})) + (2*\text{ArcTanh}[(\text{Sqrt}[b$

$\wedge 2] * x^3) / a - \text{Sqrt}[a^2 + 2 * a * b * x^3 + b^2 * x^6] / a]) / (3 * a^5)$

fricas [A] time = 1.02, size = 178, normalized size = 0.80

$$\frac{12ab^3x^9 + 42a^2b^2x^6 + 52a^3bx^3 + 25a^4 - 12(b^4x^{12} + 4ab^3x^9 + 6a^2b^2x^6 + 4a^3bx^3 + a^4)\log(bx^3 + a) + 36(b^4x^{12} + 4ab^3x^9 + 6a^2b^2x^6 + 4a^3bx^3 + a^4)\log(x)}{36(a^5b^4x^{12} + 4a^6b^3x^9 + 6a^7b^2x^6 + 4a^8bx^3 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/36*(12*a*b^3*x^9 + 42*a^2*b^2*x^6 + 52*a^3*b*x^3 + 25*a^4 - 12*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(b*x^3 + a) + 36*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(x))/(a^5*b^4*x^12 + 4*a^6*b^3*x^9 + 6*a^7*b^2*x^6 + 4*a^8*b*x^3 + a^9)

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.02, size = 193, normalized size = 0.87

$$\frac{(36b^4x^{12}\ln(x) - 12b^4x^{12}\ln(bx^3 + a) + 144ab^3x^9\ln(x) - 48ab^3x^9\ln(bx^3 + a) + 12a^2b^2x^6\ln(x) - 72a^2b^2x^6\ln(bx^3 + a) + 42a^3bx^3\ln(x) - 48a^3bx^3\ln(bx^3 + a) + 52a^4\ln(x) - 12a^4\ln(bx^3 + a) + 25a^4)(bx^3 + a)}{36((bx^3 + a)^2)^{\frac{5}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] 1/36*(36*ln(x)*x^12*b^4-12*ln(b*x^3+a)*x^12*b^4+144*ln(x)*x^9*a*b^3-48*ln(b*x^3+a)*x^9*a*b^3+12*x^9*a*b^3+216*ln(x)*x^6*a^2*b^2-72*ln(b*x^3+a)*x^6*a^2*b^2+42*x^6*a^2*b^2+144*ln(x)*x^3*a^3*b-48*ln(b*x^3+a)*x^3*a^3*b+52*x^3*a^3*b+36*ln(x)*a^4-12*ln(b*x^3+a)*a^4+25*a^4)*(b*x^3+a)/a^5/((b*x^3+a)^2)^(5/2)

maxima [A] time = 0.88, size = 132, normalized size = 0.59

$$-\frac{(-1)^{2abx^3+2a^2}\log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right)}{3a^5} + \frac{1}{9(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}a^2} + \frac{1}{3\sqrt{b^2x^6 + 2abx^3 + a^2}a^4} + \frac{1}{6\left(x^3 + \frac{a}{b}\right)^2a^3b^2} + \frac{1}{12\left(x^3 + \frac{a}{b}\right)^4ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] $-\frac{1}{3}(-1)^{(2abx^3 + 2a^2)} \log\left(\frac{2abx}{\text{abs}(x)} + \frac{2a^2}{x^2 \text{abs}(x)}\right) / a^5$
 $+ \frac{1}{9} / ((b^2x^6 + 2abx^3 + a^2)^{(3/2)} a^2) + \frac{1}{3} / (\text{sqrt}(b^2x^6 + 2abx^3 + a^2) a^4) + \frac{1}{6} / ((x^3 + a/b)^{2a^3b^2}) + \frac{1}{12} / ((x^3 + a/b)^4 a^4 b^4)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)

[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left((a + bx^3)^2 \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] Integral(1/(x*((a + b*x**3)**2)**(5/2)), x)

$$3.115 \quad \int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=398

$$\frac{13}{108a^2x\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^2} + \frac{1}{12ax\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^3} + \frac{455\sqrt[3]{b}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.22, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1355, 290, 325, 292, 31, 634, 617, 204, 628}

$$\frac{455(a+bx^3)}{243a^5\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{65}{324a^3x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455\sqrt[3]{b}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{455\sqrt[3]{b}(a+bx^3)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+bx^2)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{455\sqrt[3]{b}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt[3]{b}}\right)}{243\sqrt[3]{a}\sqrt[3]{b}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

[Out] 455/(972*a^4*x*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x*(a + b*x^3)^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 13/(108*a^2*x*(a + b*x^3)^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 65/(324*a^3*x*(a + b*x^3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (455*(a + b*x^3))/(243*a^5*x*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))])/(243*sqrt[3]*a^(16/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (455*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1458*a^(16/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^n)^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b}

, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 292

Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 325

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

Mathematica [A] time = 0.13, size = 242, normalized size = 0.61

$$\frac{(a+bx^3)\left(-910\sqrt[3]{b}(a+bx^3)^4\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)-243a^{10/3}bx^2-1179a^{4/3}bx^2(a+bx^3)^2-594a^{7/3}bx^2(a+bx^3)-\frac{2916\sqrt[3]{a}(a+bx^3)^4}{x}+1820\sqrt[3]{b}(a+bx^3)^4\log(\sqrt[3]{a}+\sqrt[3]{b}x)-1820\sqrt[3]{b}(a+bx^3)^4\tan^{-1}\left(\frac{2\sqrt[3]{bx}-\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{b}}\right)-2544\sqrt[3]{a}bx^2(a+bx^3)^3\right)}{2916a^{16/3}(a+bx^3)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

[Out] ((a + b*x^3)*(-243*a^(10/3)*b*x^2 - 594*a^(7/3)*b*x^2*(a + b*x^3) - 1179*a^(4/3)*b*x^2*(a + b*x^3)^2 - 2544*a^(1/3)*b*x^2*(a + b*x^3)^3 - (2916*a^(1/3)*(a + b*x^3)^4)/x - 1820*sqrt[3]*b^(1/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))] + 1820*b^(1/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 910*b^(1/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2916*a^(16/3)*(a + b*x^3)^2)^(5/2))

IntegrateAlgebraic [A] time = 35.25, size = 199, normalized size = 0.50

$$\frac{(a+bx^3)\left(-\frac{455\sqrt[3]{b}\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{1458a^{16/3}}+\frac{455\sqrt[3]{b}\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{729a^{16/3}}+\frac{455\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt[3]{a}}-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{243\sqrt[3]{a}^{16/3}}+\frac{-972a^4-5408a^3bx^3-9360a^2b^2x^6-6825ab^3x^9-1820b^4x^{12}}{972a^5x(a+bx^3)^4}\right)}{\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

[Out] ((a + b*x^3)*((-972*a^4 - 5408*a^3*b*x^3 - 9360*a^2*b^2*x^6 - 6825*a*b^3*x^9 - 1820*b^4*x^12)/(972*a^5*x*(a + b*x^3)^4) + (455*b^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(sqrt[3]*a^(1/3))])/(243*sqrt[3]*a^(16/3)) + (455*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)) - (455*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(1458*a^(16/3)))/sqrt[(a + b*x^3)^2]

fricas [A] time = 0.97, size = 311, normalized size = 0.78

$$\frac{5460b^4x^{12} + 20475ab^3x^9 + 28080a^2b^2x^6 + 16224a^3bx^3 + 2916a^4 + 1820\sqrt[3]{b}(b^4x^{13} + 4ab^3x^{10} + 6a^2b^2x^7 + 4a^3bx^4 + a^4x)\left(\frac{1}{3}\right)\arctan\left(\frac{2\sqrt[3]{bx}-\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{b}}\right) + 910(b^4x^{13} + 4ab^3x^{10} + 6a^2b^2x^7 + 4a^3bx^4 + a^4x)\left(\frac{1}{3}\right)\log\left(bx^2 - ax\left(\frac{b}{a}\right)^{2/3} + a\left(\frac{b}{a}\right)^{1/3}\right) - 1820(b^4x^{13} + 4ab^3x^{10} + 6a^2b^2x^7 + 4a^3bx^4 + a^4x)\left(\frac{1}{3}\right)\log\left(bx + a\left(\frac{b}{a}\right)^{1/3}\right)}{2916(b^4x^{13} + 4ab^3x^{10} + 6a^2b^2x^7 + 4a^3bx^4 + a^4x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] -1/2916*(5460*b^4*x^12 + 20475*a*b^3*x^9 + 28080*a^2*b^2*x^6 + 16224*a^3*b*x^3 + 2916*a^4 + 1820*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 910*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 1820*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b*x + a

$$\frac{(b/a)^{2/3}}{(a^5 b^4 x^{13} + 4 a^6 b^3 x^{10} + 6 a^7 b^2 x^7 + 4 a^8 b x^4 + a^9 x)}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

maple [B] time = 0.02, size = 536, normalized size = 1.35

([...])

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out]
$$-1/2916 * (-1820 * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3}) / (a/b)^{1/3}) * x^{13} * b^4 - 1820 * \ln(x + (a/b)^{1/3}) * x^{13} * b^4 + 910 * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) * x^{13} * b^4 + 5460 * (a/b)^{1/3} * x^{12} * b^4 - 7280 * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3}) / (a/b)^{1/3}) * x^{10} * a * b^3 - 7280 * \ln(x + (a/b)^{1/3}) * x^{10} * a * b^3 + 3640 * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) * x^{10} * a * b^3 + 20475 * (a/b)^{1/3} * x^9 * a * b^3 - 10920 * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3}) / (a/b)^{1/3}) * x^7 * a^2 * b^2 - 10920 * \ln(x + (a/b)^{1/3}) * x^7 * a^2 * b^2 + 5460 * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) * x^7 * a^2 * b^2 + 28080 * (a/b)^{1/3} * x^6 * a^2 * b^2 - 7280 * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3}) / (a/b)^{1/3}) * x^4 * a^3 * b - 7280 * \ln(x + (a/b)^{1/3}) * x^4 * a^3 * b + 3640 * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) * x^4 * a^3 * b + 16224 * (a/b)^{1/3} * x^3 * a^3 * b - 1820 * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (-2 * x + (a/b)^{1/3}) / (a/b)^{1/3}) * x * a^4 - 1820 * \ln(x + (a/b)^{1/3}) * x * a^4 + 910 * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) * x * a^4 + 2916 * (a/b)^{1/3} * a^4 * (b * x^3 + a) / (a/b)^{1/3} / x / a^5 / ((b * x^3 + a)^2)^{5/2}$$

maxima [A] time = 2.33, size = 192, normalized size = 0.48

$$\frac{-1820 b^4 x^{12} + 6825 a b^3 x^9 + 9360 a^2 b^2 x^6 + 5408 a^3 b x^3 + 972 a^4}{972 (a^5 b^4 x^{13} + 4 a^6 b^3 x^{10} + 6 a^7 b^2 x^7 + 4 a^8 b x^4 + a^9 x)} - \frac{455 \sqrt{3} \arctan\left(\frac{\sqrt{3} \left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{455 \log\left(x^2 - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458 a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{455 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out]
$$-1/972 * (1820 * b^4 * x^{12} + 6825 * a * b^3 * x^9 + 9360 * a^2 * b^2 * x^6 + 5408 * a^3 * b * x^3 + 972 * a^4) / (a^5 * b^4 * x^{13} + 4 * a^6 * b^3 * x^{10} + 6 * a^7 * b^2 * x^7 + 4 * a^8 * b * x^4 + a^9 x)$$

$\sqrt[9]{x} - \frac{455}{729}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{1/3}\right)/\left(\frac{a}{b}\right)^{1/3}\right) / \left(a^5\left(\frac{a}{b}\right)^{1/3}\right) - \frac{455}{1458}\log\left(x^2 - x\left(\frac{a}{b}\right)^{1/3} + \left(\frac{a}{b}\right)^{2/3}\right) / \left(a^5\left(\frac{a}{b}\right)^{1/3}\right) + \frac{455}{729}\log\left(x + \left(\frac{a}{b}\right)^{1/3}\right) / \left(a^5\left(\frac{a}{b}\right)^{1/3}\right)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \left(a^2 + 2abx^3 + b^2x^6\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)

[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left((a + bx^3)^2\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)

[Out] Integral(1/(x**2*((a + b*x**3)**2)**(5/2)), x)

$$3.116 \quad \int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=398

$$\frac{7}{54a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{770b^{2/3}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{bx^3})}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.21, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1355, 290, 325, 200, 31, 634, 617, 204, 628}

$$\frac{385(a+bx^3)}{243a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{154}{243a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{77}{324a^2\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)} + \frac{7}{54a^2\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^2} + \frac{1}{12ax^2\sqrt{a^2+2abx^3+b^2x^6}(a+bx^3)^3} + \frac{770b^{2/3}(a+bx^3)\log(\sqrt[3]{a}+\sqrt[3]{bx^3})}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{385b^{2/3}(a+bx^3)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx^3}+b^{2/3}x^2)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{770b^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}+\sqrt[3]{bx^3}}{\sqrt[3]{3}\sqrt[3]{a^2+2abx^3+b^2x^6}}\right)}{243\sqrt[3]{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

[Out] 154/(243*a^4*x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x^2*(a + b*x^3)^3*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 7/(54*a^2*x^2*(a + b*x^3)^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 77/(324*a^3*x^2*(a + b*x^3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (385*(a + b*x^3))/(243*a^5*x^2*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (770*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))])/(243*sqrt[3]*a^(17/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (770*b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(17/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (385*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(17/3)*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 31

Int[((a_) + (b_.)*(x_))^(n), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```


Mathematica [A] time = 0.14, size = 234, normalized size = 0.59

$$\frac{(a + bx^3) \left(1540b^{2/3} (a + bx^3)^4 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}) - 3162a^{2/3}bx (a + bx^3)^3 - 1314a^{5/3}bx (a + bx^3)^2 - 621a^{8/3}bx (a + bx^3) - \frac{1458a^{2/3}(a+bx^3)^4}{x^2} - 243a^{11/3}bx - 3080b^{2/3} (a + bx^3)^4 \log(\sqrt[3]{a} + \sqrt[3]{bx}) - 3080\sqrt{3}b^{2/3} (a + bx^3)^4 \tan^{-1}\left(\frac{2\sqrt[3]{bx} - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right) \right)}{2916a^{17/3} (a + bx^3)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

[Out] ((a + b*x^3)*(-243*a^(11/3)*b*x - 621*a^(8/3)*b*x*(a + b*x^3) - 1314*a^(5/3)*b*x*(a + b*x^3)^2 - 3162*a^(2/3)*b*x*(a + b*x^3)^3 - (1458*a^(2/3)*(a + b*x^3)^4)/x^2 - 3080*Sqrt[3]*b^(2/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] - 3080*b^(2/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 1540*b^(2/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(17/3)*((a + b*x^3)^2)^(5/2))

IntegrateAlgebraic [A] time = 42.12, size = 199, normalized size = 0.50

$$\frac{(a + bx^3) \left(\frac{385b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2})}{729a^{17/3}} - \frac{770b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{bx})}{729a^{17/3}} + \frac{770b^{2/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{17/3}} + \frac{-486a^4 - 3724a^3bx^3 - 7161a^2b^2x^6 - 5544ab^3x^9 - 1540b^4x^{12}}{972a^5x^2(a+bx^3)^4} \right)}{\sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

[Out] ((a + b*x^3)*((-486*a^4 - 3724*a^3*b*x^3 - 7161*a^2*b^2*x^6 - 5544*a*b^3*x^9 - 1540*b^4*x^12)/(972*a^5*x^2*(a + b*x^3)^4) + (770*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(17/3)) - (770*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(17/3)) + (385*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(17/3))))/Sqrt[(a + b*x^3)^2]

fricas [A] time = 1.30, size = 352, normalized size = 0.88

$$\frac{4620b^{4/3} + 16632ab^3x^9 + 21483a^2b^2x^6 + 11172a^3bx^3 + 1458a^4 - 3080\sqrt{3}(b^{4/3}x^{14} + 4ab^{3/3}x^{11} + 6a^2b^{2/3}x^8 + 4a^3bx^5 + a^4x^2) \left(-\frac{2}{\sqrt{3}} \arctan\left(\frac{2\sqrt{3} - \frac{2}{\sqrt{3}} - \sqrt{3}}{x}\right) + 1540(b^{4/3}x^{14} + 4ab^{3/3}x^{11} + 6a^2b^{2/3}x^8 + 4a^3bx^5 + a^4x^2) \left(-\frac{2}{\sqrt{3}} \right) \log\left(b^2x^2 + abx\left(-\frac{2}{\sqrt{3}}\right) + a^2\left(-\frac{2}{\sqrt{3}}\right)^2\right) - 3080(b^{4/3}x^{14} + 4ab^{3/3}x^{11} + 6a^2b^{2/3}x^8 + 4a^3bx^5 + a^4x^2) \left(-\frac{2}{\sqrt{3}} \right) \log\left(bx - a\left(-\frac{2}{\sqrt{3}}\right)\right) \right)}{2916(a^{17/3}x^{14} + 4a^{16/3}x^{11} + 6a^{15/3}x^8 + 4a^{14/3}x^5 + a^{13/3}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

[Out] -1/2916*(4620*b^4*x^12 + 16632*a*b^3*x^9 + 21483*a^2*b^2*x^6 + 11172*a^3*b*x^3 + 1458*a^4 - 3080*sqrt(3)*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 1540*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 3080*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*

$$a^3 b x^5 + a^4 x^2) * (-b^2/a^2)^{(1/3)} * \log(bx - a * (-b^2/a^2)^{(1/3)}) / (a^5 b^4 x^{14} + 4 a^6 b^3 x^{11} + 6 a^7 b^2 x^8 + 4 a^8 b x^5 + a^9 x^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x

maple [B] time = 0.02, size = 542, normalized size = 1.36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out]
$$-1/2916 * (-3080 * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (-2 * x + (a/b)^{(1/3)}) / (a/b)^{(1/3)}) * x^{14} * b^4 + 3080 * \ln(x + (a/b)^{(1/3)}) * x^{14} * b^4 - 1540 * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * x^{14} * b^4 + 4620 * (a/b)^{(2/3)} * x^{12} * b^4 - 12320 * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (-2 * x + (a/b)^{(1/3)}) / (a/b)^{(1/3)}) * x^{11} * a * b^3 + 12320 * \ln(x + (a/b)^{(1/3)}) * x^{11} * a * b^3 - 6160 * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * x^{11} * a * b^3 + 16632 * (a/b)^{(2/3)} * x^9 * a * b^3 - 18480 * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (-2 * x + (a/b)^{(1/3)}) / (a/b)^{(1/3)}) * x^8 * a^2 * b^2 + 18480 * \ln(x + (a/b)^{(1/3)}) * x^8 * a^2 * b^2 - 9240 * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * x^8 * a^2 * b^2 + 21483 * (a/b)^{(2/3)} * x^6 * a^2 * b^2 - 12320 * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (-2 * x + (a/b)^{(1/3)}) / (a/b)^{(1/3)}) * x^5 * a^3 * b + 12320 * \ln(x + (a/b)^{(1/3)}) * x^5 * a^3 * b - 6160 * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * x^5 * a^3 * b + 11172 * (a/b)^{(2/3)} * x^3 * a^3 * b - 3080 * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (-2 * x + (a/b)^{(1/3)}) / (a/b)^{(1/3)}) * x^2 * a^4 + 3080 * \ln(x + (a/b)^{(1/3)}) * x^2 * a^4 - 1540 * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * x^2 * a^4 + 1458 * (a/b)^{(2/3)} * a^4 * (b * x^3 + a) / (a/b)^{(2/3)} / x^2 / a^5 / ((b * x^3 + a)^2)^{(5/2)}$$

maxima [A] time = 2.06, size = 194, normalized size = 0.49

$$\frac{1540 b^4 x^{12} + 5544 a b^3 x^9 + 7161 a^2 b^2 x^6 + 3724 a^3 b x^3 + 486 a^4}{972 (a^5 b^4 x^{14} + 4 a^6 b^3 x^{11} + 6 a^7 b^2 x^8 + 4 a^8 b x^5 + a^9 x^2)} - \frac{770 \sqrt{3} \arctan\left(\frac{\sqrt{3} \left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{385 \log\left(x^2 - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{770 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729 a^5 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out]
$$-1/972*(1540*b^4*x^{12} + 5544*a*b^3*x^9 + 7161*a^2*b^2*x^6 + 3724*a^3*b*x^3 + 486*a^4)/(a^5*b^4*x^{14} + 4*a^6*b^3*x^{11} + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a^9*x^2) - 770/729*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{1/3}))/((a/b)^{1/3})/(a^5*(a/b)^{2/3}) + 385/729*\log(x^2 - x*(a/b)^{1/3} + (a/b)^{2/3})/(a^5*(a/b)^{2/3}) - 770/729*\log(x + (a/b)^{1/3})/(a^5*(a/b)^{2/3})$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)`

[Out] `int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left((a + bx^3)^2 \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(1/(x**3*((a + b*x**3)**2)**(5/2)), x)`

$$3.117 \quad \int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{5/2}} dx$$

Optimal. Leaf size=269

$$\frac{b}{12a^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5b\log(x)(a+bx^3)}{a^6\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5b(a+bx^3)\log(a+bx^3)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}} - \frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}$$

Rubi [A] time = 0.14, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 266, 44}

$$\frac{b}{2a^4(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2b}{9a^3(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b}{12a^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{5b\log(x)(a+bx^3)}{a^6\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5b(a+bx^3)\log(a+bx^3)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]

[Out] (-4*b)/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(12*a^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*b)/(9*a^3*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(2*a^4*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*b*(a + b*x^3)*Log[x])/(a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*b*(a + b*x^3)*Log[a + b*x^3])/(3*a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(2*n_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx &= \frac{(b^4 (ab + b^2x^3)) \int \frac{1}{x^4(ab+b^2x^3)^5} dx}{\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^4 (ab + b^2x^3)) \text{Subst} \left(\int \frac{1}{x^2(ab+b^2x)^5} dx, x, x^3 \right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= \frac{(b^4 (ab + b^2x^3)) \text{Subst} \left(\int \left(\frac{1}{a^5b^5x^2} - \frac{5}{a^6b^4x} + \frac{1}{a^2b^3(a+bx)^5} + \frac{2}{a^3b^3(a+bx)^4} + \frac{3}{a^4b^3(a+bx)^3} \right) dx, x, x^3 \right)}{3\sqrt{a^2 + 2abx^3 + b^2x^6}} \\
&= -\frac{4b}{3a^5\sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{b}{12a^2 (a + bx^3)^3 \sqrt{a^2 + 2abx^3 + b^2x^6}} - \frac{1}{9a^3 (a + bx^3)^2 \sqrt{a^2 + 2abx^3 + b^2x^6}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 119, normalized size = 0.44

$$\frac{-a(12a^4 + 125a^3bx^3 + 260a^2b^2x^6 + 210ab^3x^9 + 60b^4x^{12}) - 180bx^3 \log(x)(a + bx^3)^4 + 60bx^3 (a + bx^3)^4 \log(a + bx^3)}{36a^6x^3 (a + bx^3)^3 \sqrt{(a + bx^3)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

```
[Out] (-a*(12*a^4 + 125*a^3*b*x^3 + 260*a^2*b^2*x^6 + 210*a*b^3*x^9 + 60*b^4*x^12) - 180*b*x^3*(a + b*x^3)^4*Log[x] + 60*b*x^3*(a + b*x^3)^4*Log[a + b*x^3])/
(36*a^6*x^3*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

IntegrateAlgebraic [B] time = 81.82, size = 2850, normalized size = 10.59

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)), x]

```
[Out] (-2*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(3*a^63 - 3*a^62*b*x^3 + 3*a^61*b^2*x^6 + 9*a^60*b^3*x^9 + 1484*a^59*b^4*x^12 + 89634*a^58*b^5*x^15 + 35
```

$47176a^{57}b^6x^{18} + 103238584a^{56}b^7x^{21} + 2356838400a^{55}b^8x^{24} +$
 $43953744576a^{54}b^9x^{27} + 688626915840a^{53}b^{10}x^{30} + 9250107586560a^{52}b^{11}x^{33} + 108195839032320a^{51}b^{12}x^{36} + 1115441253588480a^{50}b^{13}x^{39} +$
 $10234958239733760a^{49}b^{14}x^{42} + 84253639955097600a^{48}b^{15}x^{45} + 626363783779123200a^{47}b^{16}x^{48} + 4228757765620531200a^{46}b^{17}x^{51} + 26049151725611581440a^{45}b^{18}x^{54} + 147000114743606968320a^{44}b^{19}x^{57} +$
 $762584480826139607040a^{43}b^{20}x^{60} + 3647550970994505154560a^{42}b^{21}x^{63} + 16127976156648442429440a^{41}b^{22}x^{66} + 66068580639182534737920a^{40}b^{23}x^{69} + 251239072713580814008320a^{39}b^{24}x^{72} + 888344859983663773777920a^{38}b^{25}x^{75} + 2924829168272464863559680a^{37}b^{26}x^{78} + 8977874965576903525662720a^{36}b^{27}x^{81} + 25718516470160904041791488a^{35}b^{28}x^{84} + 68815279531254829887258624a^{34}b^{29}x^{87} + 172101662127535626374873088a^{33}b^{30}x^{90} + 402506093314484489003466752a^{32}b^{31}x^{93} + 880658309537180769413234688a^{31}b^{32}x^{96} + 1802963185421958307337207808a^{30}b^{33}x^{99} + 3454182779430019135581454336a^{29}b^{34}x^{102} + 6192350885862042683119239168a^{28}b^{35}x^{105} + 10385517381568557089430700032a^{27}b^{36}x^{108} + 16289592602873600309238693888a^{26}b^{37}x^{111} + 23882708470211058375623442432a^{25}b^{38}x^{114} + 32708573827319329343938756608a^{24}b^{39}x^{117} + 41810503900180714625288896512a^{23}b^{40}x^{120} + 49833178729962384051916505088a^{22}b^{41}x^{123} + 55314738880465273152142835712a^{21}b^{42}x^{126} + 57100495733850880840216608768a^{20}b^{43}x^{129} + 54726941348404022924701335552a^{19}b^{44}x^{132} + 48606655891944070942417747968a^{18}b^{45}x^{135} + 39917608008706788034432991232a^{17}b^{46}x^{138} + 30234237658617949398937632768a^{16}b^{47}x^{141} + 21058000972731372092000305152a^{15}b^{48}x^{144} + 13441018440774824592175792128a^{14}b^{49}x^{147} + 7830928399966132744489009152a^{13}b^{50}x^{150} + 4145087005408504523991810048a^{12}b^{51}x^{153} + 1982450466391073623482826752a^{11}b^{52}x^{156} + 851082047429972494978646016a^{10}b^{53}x^{159} + 325392797217001837833486336a^9b^{54}x^{162} + 109726425313175461183881216a^8b^{55}x^{165} + 32243624862748916500660224a^7b^{56}x^{168} + 8130508717424292308975616a^6b^{57}x^{171} + 1723944790864168421949440a^5b^{58}x^{174} + 298927614217018297810944a^4b^{59}x^{177} + 40705334872025491046400a^3b^{60}x^{180} + 4082062702248617574400a^2b^{61}x^{183} + 268054249821091921920a^1b^{62}x^{186} + 8646911284551352320b^{63}x^{189}) - 2*(3a^64b - 12a^61b^4x^9 - 1493a^60b^5x^{12} - 91118a^59b^6x^{15} - 3636810a^58b^7x^{18} - 106785760a^57b^8x^{21} - 2460076984a^56b^9x^{24} - 46310582976a^55b^{10}x^{27} - 732580660416a^54b^{11}x^{30} - 9938734502400a^53b^{12}x^{33} - 117445946618880a^52b^{13}x^{36} - 1223637092620800a^51b^{14}x^{39} - 11350399493322240a^50b^{15}x^{42} - 94488598194831360a^49b^{16}x^{45} - 710617423734220800a^48b^{17}x^{48} - 4855121549399654400a^47b^{18}x^{51} - 30277909491232112640a^46b^{19}x^{54} - 173049266469218549760a^45b^{20}x^{57} - 909584595569746575360a^44b^{21}x^{60} - 4410135451820644761600a^43b^{22}x^{63} - 19775527127642947584000a^42b^{23}x^{66} - 82196556795830977167360a^41b^{24}x^{69} - 317307653352763348746240a^40b^{25}x^{72} - 1139583932697244587786240a^39b^{26}x^{75} - 3813174028256128637337600a^38b^{27}x^{78} - 11902704133849368389222400a^37b^{28}x^{81} - 34696391435737807567454208a^36b^{29}x^{84} - 94533796001415733929050112a^35b^{30}x^{87} - 2409169$

$41658790456262131712*a^{34}*b^{31}*x^{90} - 574607755442020115378339840*a^{33}*b^{32}$
 $*x^{93} - 1283164402851665258416701440*a^{32}*b^{33}*x^{96} - 268362149495913907675$
 $0442496*a^{31}*b^{34}*x^{99} - 5257145964851977442918662144*a^{30}*b^{35}*x^{102} - 964$
 $6533665292061818700693504*a^{29}*b^{36}*x^{105} - 16577868267430599772549939200*a$
 $^{28}*b^{37}*x^{108} - 26675109984442157398669393920*a^{27}*b^{38}*x^{111} - 4017230107$
 $3084658684862136320*a^{26}*b^{39}*x^{114} - 56591282297530387719562199040*a^{25}*b^{40}$
 $*x^{117} - 74519077727500043969227653120*a^{24}*b^{41}*x^{120} - 9164368263014309$
 $8677205401600*a^{23}*b^{42}*x^{123} - 105147917610427657204059340800*a^{22}*b^{43}*x^{126}$
 $- 112415234614316153992359444480*a^{21}*b^{44}*x^{129} - 11182743708225490376$
 $4917944320*a^{20}*b^{45}*x^{132} - 103333597240348093867119083520*a^{19}*b^{46}*x^{135}$
 $- 88524263900650858976850739200*a^{18}*b^{47}*x^{138} - 701518456673247374333706$
 $24000*a^{17}*b^{48}*x^{141} - 51292238631349321490937937920*a^{16}*b^{49}*x^{144} - 344$
 $99019413506196684176097280*a^{15}*b^{50}*x^{147} - 21271946840740957336664801280*$
 $a^{14}*b^{51}*x^{150} - 11976015405374637268480819200*a^{13}*b^{52}*x^{153} - 612753747$
 $1799578147474636800*a^{12}*b^{53}*x^{156} - 2833532513821046118461472768*a^{11}*b^{54}$
 $*x^{159} - 1176474844646974332812132352*a^{10}*b^{55}*x^{162} - 435119222530177299$
 $017367552*a^9*b^{56}*x^{165} - 141970050175924377684541440*a^8*b^{57}*x^{168} - 403$
 $74133580173208809635840*a^7*b^{58}*x^{171} - 9854453508288460730925056*a^6*b^{59}$
 $*x^{174} - 2022872405081186719760384*a^5*b^{60}*x^{177} - 33963294908904378885734$
 $4*a^4*b^{61}*x^{180} - 44787397574274108620800*a^3*b^{62}*x^{183} - 435011695206970$
 $9496320*a^2*b^{63}*x^{186} - 276701161105643274240*a*b^{64}*x^{189} - 8646911284551$
 $352320*b^{65}*x^{192}))/ (9*a^5*x^{12}*sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*(-8*a^60*b^4$
 $- 936*a^59*b^5*x^3 - 53832*a^58*b^6*x^6 - 2028600*a^57*b^7*x^9 - 56333328$
 $*a^56*b^8*x^{12} - 1229251968*a^55*b^9*x^{15} - 21948838912*a^54*b^{10}*x^{18} - 32$
 $9736109824*a^53*b^{11}*x^{21} - 4253142643200*a^52*b^{12}*x^{24} - 47832699273216*a$
 $^{51}*b^{13}*x^{27} - 474726976948224*a^50*b^{14}*x^{30} - 4198198392760320*a^49*b^{15}$
 $*x^{33} - 33343446890557440*a^48*b^{16}*x^{36} - 239403280048128000*a^47*b^{17}*x^{39}$
 $- 1562459053231964160*a^46*b^{18}*x^{42} - 9312555821128089600*a^45*b^{19}*x^{45}$
 $- 50890640751160197120*a^44*b^{20}*x^{48} - 255857896219504803840*a^43*b^{21}*x^{51}$
 $- 1186944307588300800000*a^42*b^{22}*x^{54} - 5093751076470545448960*a^41*b^{23}$
 $*x^{57} - 20266419321282499706880*a^40*b^{24}*x^{60} - 74898737599223200481280*$
 $a^{39}*b^{25}*x^{63} - 257538426184575127388160*a^{38}*b^{26}*x^{66} - 8250678812240322$
 $23232000*a^{37}*b^{27}*x^{69} - 2465674112032552349859840*a^{36}*b^{28}*x^{72} - 688041$
 $4330081729610514432*a^{35}*b^{29}*x^{75} - 17942536074222169724289024*a^{34}*b^{30}*x$
 $^{78} - 4375510777792062666047488*a^{33}*b^{31}*x^{81} - 9983129760884363561533440$
 $*a^{32}*b^{32}*x^{84} - 213181721876762624526385152*a^{31}*b^{33}*x^{87} - 42615515677$
 $5629047172431872*a^{30}*b^{34}*x^{90} - 797530838364556999815856128*a^{29}*b^{35}*x^{93}$
 $- 1397189047727722213310201856*a^{28}*b^{36}*x^{96} - 2290841170528074923297996$
 $800*a^{27}*b^{37}*x^{99} - 3514050815948758604845154304*a^{26}*b^{38}*x^{102} - 5040462$
 $441364212140879118336*a^{25}*b^{39}*x^{105} - 6756005325714055365069373440*a^{24}*b$
 $^{40}*x^{108} - 8454788378368570414312980480*a^{23}*b^{41}*x^{111} - 9868868050118054$
 $737084416000*a^{22}*b^{42}*x^{114} - 10731458193725856494093598720*a^{21}*b^{43}*x^{117}$
 $- 10855754855129860210792857600*a^{20}*b^{44}*x^{120} - 10198877550514546149257$
 $379840*a^{19}*b^{45}*x^{123} - 8881831640526360274670714880*a^{18}*b^{46}*x^{126} - 715$
 $3973538484081655808000000*a^{17}*b^{47}*x^{129} - 5315869468116817954964766720*a^{16}$

$$\begin{aligned}
& 16*b^{48}*x^{132} - 3633267961926811266066677760*a^{15}*b^{49}*x^{135} - 227628260141 \\
& 4251560210268160*a^{14}*b^{50}*x^{138} - 1302044770810745855693291520*a^{13}*b^{51}*x \\
& ^{141} - 676809132086273815609344000*a^{12}*b^{52}*x^{144} - 3179452545867346780933 \\
& 32480*a^{11}*b^{53}*x^{147} - 134101230649370624636485632*a^{10}*b^{54}*x^{150} - 50381 \\
& 470381939849118613504*a^9*b^{55}*x^{153} - 16697916908414960085762048*a^8*b^{56}* \\
& x^{156} - 4823541649938374354534400*a^7*b^{57}*x^{159} - 119588743031903034225459 \\
& 2*a^6*b^{58}*x^{162} - 249357249723288646582272*a^5*b^{59}*x^{165} - 42526806734116 \\
& 233412608*a^4*b^{60}*x^{168} - 5696585154262430908416*a^3*b^{61}*x^{171} - 56204923 \\
& 3495837900800*a^2*b^{62}*x^{174} - 36317027395115679744*a*b^{63}*x^{177} - 11529215 \\
& 04606846976*b^{64}*x^{180}) + 9*a^5*\text{Sqrt}[b^2]*x^{12}*(8*a^61*b^3 + 944*a^60*b^4*x \\
& ^3 + 54768*a^59*b^5*x^6 + 2082432*a^58*b^6*x^9 + 58361928*a^57*b^7*x^{12} + 1 \\
& 285585296*a^56*b^8*x^{15} + 23178090880*a^55*b^9*x^{18} + 351684948736*a^54*b^1 \\
& 0*x^{21} + 4582878753024*a^53*b^11*x^{24} + 52085841916416*a^52*b^12*x^{27} + 522 \\
& 559676221440*a^51*b^13*x^{30} + 4672925369708544*a^50*b^14*x^{33} + 37541645283 \\
& 317760*a^49*b^15*x^{36} + 272746726938685440*a^48*b^16*x^{39} + 180186233328009 \\
& 2160*a^47*b^17*x^{42} + 10875014874360053760*a^46*b^18*x^{45} + 602031965722882 \\
& 86720*a^45*b^19*x^{48} + 306748536970665000960*a^44*b^20*x^{51} + 1442802203807 \\
& 805603840*a^43*b^21*x^{54} + 6280695384058846248960*a^42*b^22*x^{57} + 25360170 \\
& 397753045155840*a^41*b^23*x^{60} + 95165156920505700188160*a^40*b^24*x^{63} + 3 \\
& 32437163783798327869440*a^39*b^25*x^{66} + 1082606307408607350620160*a^38*b^2 \\
& 6*x^{69} + 3290741993256584573091840*a^37*b^27*x^{72} + 93460884421142819603742 \\
& 72*a^36*b^28*x^{75} + 24822950404303899334803456*a^35*b^29*x^{78} + 61697643852 \\
& 014232390336512*a^34*b^30*x^{81} + 143586405386635698281381888*a^33*b^31*x^{84} \\
& + 313013019485606260141719552*a^32*b^32*x^{87} + 639336878652391671698817024 \\
& *a^31*b^33*x^{90} + 1223685995140186046988288000*a^30*b^34*x^{93} + 21947198860 \\
& 92279213126057984*a^29*b^35*x^{96} + 3688030218255797136608198656*a^28*b^36*x \\
& ^99 + 5804891986476833528143151104*a^27*b^37*x^{102} + 8554513257312970745724 \\
& 272640*a^26*b^38*x^{105} + 11796467767078267505948491776*a^25*b^39*x^{108} + 15 \\
& 210793704082625779382353920*a^24*b^40*x^{111} + 18323656428486625151397396480 \\
& *a^23*b^41*x^{114} + 20600326243843911231178014720*a^22*b^42*x^{117} + 21587213 \\
& 048855716704886456320*a^21*b^43*x^{120} + 21054632405644406360050237440*a^20* \\
& b^44*x^{123} + 19080709191040906423928094720*a^19*b^45*x^{126} + 16035805179010 \\
& 441930478714880*a^18*b^46*x^{129} + 12469843006600899610772766720*a^17*b^47*x \\
& ^132 + 8949137430043629221031444480*a^16*b^48*x^{135} + 590955056334106282627 \\
& 6945920*a^15*b^49*x^{138} + 3578327372224997415903559680*a^14*b^50*x^{141} + 19 \\
& 78853902897019671302635520*a^13*b^51*x^{144} + 994754386673008493702676480*a^ \\
& 12*b^52*x^{147} + 452046485236105302729818112*a^11*b^53*x^{150} + 1844827010313 \\
& 10473755099136*a^10*b^54*x^{153} + 67079387290354809204375552*a^9*b^55*x^{156} \\
& + 21521458558353334440296448*a^8*b^56*x^{159} + 6019429080257404696788992*a^7 \\
& *b^57*x^{162} + 1445244680042318988836864*a^6*b^58*x^{165} + 291884056457404879 \\
& 994880*a^5*b^59*x^{168} + 48223391888378664321024*a^4*b^60*x^{171} + 6258634387 \\
& 758268809216*a^3*b^61*x^{174} + 598366260890953580544*a^2*b^62*x^{177} + 374699 \\
& 48899722526720*a*b^63*x^{180} + 1152921504606846976*b^64*x^{183})) - (10*b*\text{ArcT} \\
& \text{anh}[(\text{Sqrt}[b^2]*x^3)/a - \text{Sqrt}[a^2 + 2*a*b*x^3 + b^2*x^6]/a)]/(3*a^6)
\end{aligned}$$

fricas [A] time = 1.27, size = 207, normalized size = 0.77

$$\frac{60ab^4x^{12} + 210a^2b^3x^9 + 260a^3b^2x^6 + 125a^4bx^3 + 12a^5 - 60(b^5x^{15} + 4ab^4x^{12} + 6a^2b^3x^9 + 4a^3b^2x^6 + a^4bx^3)\log(bx^3 + a) + 180(b^5x^{15} + 4ab^4x^{12} + 6a^2b^3x^9 + 4a^3b^2x^6 + a^4bx^3)\log(x)}{36(a^6b^4x^{15} + 4a^7b^3x^{12} + 6a^8b^2x^9 + 4a^9bx^6 + a^{10}x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] $-1/36*(60*a*b^4*x^{12} + 210*a^2*b^3*x^9 + 260*a^3*b^2*x^6 + 125*a^4*b*x^3 + 12*a^5 - 60*(b^5*x^{15} + 4*a*b^4*x^{12} + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^4*b*x^3)*\log(b*x^3 + a) + 180*(b^5*x^{15} + 4*a*b^4*x^{12} + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^4*b*x^3)*\log(x))/(a^6*b^4*x^{15} + 4*a^7*b^3*x^{12} + 6*a^8*b^2*x^9 + 4*a^9*b*x^6 + a^{10}*x^3)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")

[Out] *sage0*x*

maple [A] time = 0.02, size = 219, normalized size = 0.81

$$\frac{(180b^5x^{15}\ln(x) - 60b^5x^{15}\ln(bx^3 + a) + 720ab^4x^{12}\ln(x) - 240a^2b^3x^9\ln(bx^3 + a) + 60a^3b^2x^6\ln(bx^3 + a) + 1080a^4bx^3\ln(x) - 360a^5\ln(bx^3 + a) + 210a^2b^3x^9 + 720a^3b^2x^6 - 240a^4bx^3 - 240a^5\ln(bx^3 + a) + 260a^3b^2x^6 + 180a^4bx^3 - 60a^5\ln(bx^3 + a) + 125a^4bx^3 + 12a^5)(bx^3 + a)}{36((bx^3 + a)^2)^{\frac{5}{2}}a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)

[Out] $-1/36*(180*b^5*x^{15}*\ln(x) - 60*\ln(b*x^3+a)*x^{15}*b^5 + 720*a*b^4*x^{12}*\ln(x) - 240*\ln(b*x^3+a)*x^{12}*a*b^4 + 60*a*b^4*x^{12} + 1080*a^2*b^3*x^9*\ln(x) - 360*\ln(b*x^3+a)*x^9*a^2*b^3 + 210*a^2*b^3*x^9 + 720*a^3*b^2*x^6*\ln(x) - 240*\ln(b*x^3+a)*x^6*a^3*b^2 + 260*a^3*b^2*x^6 + 180*a^4*b*x^3*\ln(x) - 60*\ln(b*x^3+a)*x^3*a^4*b + 125*a^4*b*x^3 + 12*a^5)*(b*x^3+a)/x^3/a^6/((b*x^3+a)^2)^(5/2)$

maxima [A] time = 0.85, size = 163, normalized size = 0.61

$$\frac{5(-1)^{2abx^3+2a^2}b\log\left(\frac{2abx}{|x|} + \frac{2a^2}{x^2|x|}\right)}{3a^6} - \frac{5b}{9(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}a^3} - \frac{5b}{3\sqrt{b^2x^6 + 2abx^3 + a^2}a^5} - \frac{1}{3(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}a^2x^3} - \frac{5}{6\left(x^3 + \frac{a}{b}\right)^2a^4b} - \frac{1}{12\left(x^3 + \frac{a}{b}\right)^4a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] $\frac{5}{3}(-1)^{(2abx^3 + 2a^2)b \log(2abx/\text{abs}(x) + 2a^2/(x^2\text{abs}(x)))}/a^6$
 $- \frac{5}{9}b/((b^2x^6 + 2abx^3 + a^2)^{(3/2)}a^3) - \frac{5}{3}b/(\text{sqrt}(b^2x^6 + 2abx^3 + a^2)a^5) - \frac{1}{3}/((b^2x^6 + 2abx^3 + a^2)^{(3/2)}a^2x^3) - \frac{5}{6}/((x^3 + a/b)^2a^4b) - \frac{1}{12}/((x^3 + a/b)^4a^2b^3)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)`

[Out] `int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left((a + bx^3)^2 \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2), x)`

[Out] `Integral(1/(x**4*((a + b*x**3)**2)**(5/2)), x)`

$$3.118 \quad \int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Optimal. Leaf size=313

$$\frac{b^5 \sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+16}}{d^{16}(m+16)(a+bx^3)} + \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+13}}{d^{13}(m+13)(a+bx^3)} + \frac{10a^2b^3 \sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+10}}{d^{10}(m+10)(a+bx^3)} + \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+7}}{d^7(m+7)(a+bx^3)} + \frac{10a^3b^2 \sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+4}}{d^4(m+4)(a+bx^3)} + \frac{10a^2b^2 \sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+10}}{d^{10}(m+10)(a+bx^3)} + \frac{5ab^4 \sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+13}}{d^{13}(m+13)(a+bx^3)} + \frac{b^5 \sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+16}}{d^{16}(m+16)(a+bx^3)} + \frac{a^5 \sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+1}}{d(m+1)(a+bx^3)}$$

Rubi [A] time = 0.14, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+4}}{d^4(m+4)(a+bx^3)} + \frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+7}}{d^7(m+7)(a+bx^3)} + \frac{10a^2b^2\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+10}}{d^{10}(m+10)(a+bx^3)} + \frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+13}}{d^{13}(m+13)(a+bx^3)} + \frac{b^5\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+16}}{d^{16}(m+16)(a+bx^3)} + \frac{a^5\sqrt{a^2+2abx^3+b^2x^6}(dx)^{m+1}}{d(m+1)(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (a^5*(d*x)^(1+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1+m)*(a + b*x^3)) + (5*a^4*b*(d*x)^(4+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4+m)*(a + b*x^3)) + (10*a^3*b^2*(d*x)^(7+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7+m)*(a + b*x^3)) + (10*a^2*b^2*(d*x)^(10+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^10*(10+m)*(a + b*x^3)) + (5*a*b^4*(d*x)^(13+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^13*(13+m)*(a + b*x^3)) + (b^5*(d*x)^(16+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^16*(16+m)*(a + b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (dx)^m (ab + b^2x^3)^5 dx}{b^4 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(a^5 b^5 (dx)^m + \frac{5a^4 b^6 (dx)^{3+m}}{d^3} + \frac{10a^3 b^7 (dx)^{6+m}}{d^6} + \frac{10a^2 b^8 (dx)^{9+m}}{d^9} \right)}{b^4 (ab + b^2x^3)} \\
&= \frac{a^5 (dx)^{1+m} \sqrt{a^2 + 2abx^3 + b^2x^6}}{d(1+m)(a + bx^3)} + \frac{5a^4 b (dx)^{4+m} \sqrt{a^2 + 2abx^3 + b^2x^6}}{d^4(4+m)(a + bx^3)} + \frac{10a^3 b^2 (dx)^{7+m} \sqrt{a^2 + 2abx^3 + b^2x^6}}{d^7(7+m)(a + bx^3)} + \frac{10a^2 b^3 (dx)^{10+m} \sqrt{a^2 + 2abx^3 + b^2x^6}}{d^{10}(10+m)(a + bx^3)} + \frac{5a b^4 (dx)^{13+m} \sqrt{a^2 + 2abx^3 + b^2x^6}}{d^{13}(13+m)(a + bx^3)} + \frac{b^5 (dx)^{16+m} \sqrt{a^2 + 2abx^3 + b^2x^6}}{d^{16}(16+m)(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 111, normalized size = 0.35

$$\frac{x \left((a + bx^3)^2 \right)^{5/2} (dx)^m \left(\frac{a^5}{m+1} + \frac{5a^4 bx^3}{m+4} + \frac{10a^3 b^2 x^6}{m+7} + \frac{10a^2 b^3 x^9}{m+10} + \frac{5ab^4 x^{12}}{m+13} + \frac{b^5 x^{15}}{m+16} \right)}{(a + bx^3)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] (x*(d*x)^m*((a + b*x^3)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (10*a^3*b^2*x^6)/(7 + m) + (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 + m) + (b^5*x^15)/(16 + m)))/(a + b*x^3)^5

IntegrateAlgebraic [F] time = 3.28, size = 0, normalized size = 0.00

$$\int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2), x]

fricas [A] time = 1.91, size = 369, normalized size = 1.18

[[0]a^10 - 33]a^9 + 445]a^8 + 2485]a^7 + 5754]a^6 + 3640]a^5 + 5]a^4]a^3 + 38]a^2]a + 51]a]a^2 + 2962]a]a^2 + 4988]a]a + 448]a^2]a^3 + 5]a^2]a^4 + 41]a^2]a^5 + 95]a^2]a^6 + 365]a^2]a^7 + 892]a^2]a^8 + 1624]a^2]a^9 + 5]a^2]a^10 + 44]a^2]a^11 + 697]a^2]a^12 + 4726]a^2]a^13 + 22392]a^2]a^14 + 83252]a^2]a^15 + 5]a^2]a^16 + 47]a^2]a^17 + 837]a^2]a^18 + 6337]a^2]a^19 + 20326]a^2]a^20 + 148560]a^2]a^21 + 5]a^2]a^22 + 95]a^2]a^23 + 165]a^2]a^24 + 24824]a^2]a^25]]]a^26]]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2), x, algorithm="fricas")

```
[Out] ((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^5)
)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 696
8*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^2*b
^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a^3*b
^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a^3*b^
2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3 + 633
7*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m^4 + 95
5*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*(d*x)^m/(m^6 + 51*m^
5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

giac [B] time = 0.67, size = 900, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^16*sgn(b*x^3 + a) + 35*(d*x)^m*b^5*m^4*x^16*sgn(b*x^3 +
a) + 445*(d*x)^m*b^5*m^3*x^16*sgn(b*x^3 + a) + 5*(d*x)^m*a*b^4*m^5*x^13*sgn
(b*x^3 + a) + 2485*(d*x)^m*b^5*m^2*x^16*sgn(b*x^3 + a) + 190*(d*x)^m*a*b^4*
m^4*x^13*sgn(b*x^3 + a) + 5714*(d*x)^m*b^5*m*x^16*sgn(b*x^3 + a) + 2555*(d*
x)^m*a*b^4*m^3*x^13*sgn(b*x^3 + a) + 3640*(d*x)^m*b^5*x^16*sgn(b*x^3 + a) +
10*(d*x)^m*a^2*b^3*m^5*x^10*sgn(b*x^3 + a) + 14810*(d*x)^m*a*b^4*m^2*x^13*
sgn(b*x^3 + a) + 410*(d*x)^m*a^2*b^3*m^4*x^10*sgn(b*x^3 + a) + 34840*(d*x)^
m*a*b^4*m*x^13*sgn(b*x^3 + a) + 5950*(d*x)^m*a^2*b^3*m^3*x^10*sgn(b*x^3 + a
) + 22400*(d*x)^m*a*b^4*x^13*sgn(b*x^3 + a) + 10*(d*x)^m*a^3*b^2*m^5*x^7*sg
n(b*x^3 + a) + 36550*(d*x)^m*a^2*b^3*m^2*x^10*sgn(b*x^3 + a) + 440*(d*x)^m*
a^3*b^2*m^4*x^7*sgn(b*x^3 + a) + 89240*(d*x)^m*a^2*b^3*m*x^10*sgn(b*x^3 + a
) + 6970*(d*x)^m*a^3*b^2*m^3*x^7*sgn(b*x^3 + a) + 58240*(d*x)^m*a^2*b^3*x^1
0*sgn(b*x^3 + a) + 5*(d*x)^m*a^4*b*m^5*x^4*sgn(b*x^3 + a) + 47260*(d*x)^m*a
^3*b^2*m^2*x^7*sgn(b*x^3 + a) + 235*(d*x)^m*a^4*b*m^4*x^4*sgn(b*x^3 + a) +
123920*(d*x)^m*a^3*b^2*m*x^7*sgn(b*x^3 + a) + 4085*(d*x)^m*a^4*b*m^3*x^4*sg
n(b*x^3 + a) + 83200*(d*x)^m*a^3*b^2*x^7*sgn(b*x^3 + a) + (d*x)^m*a^5*m^5*x
*sgn(b*x^3 + a) + 31685*(d*x)^m*a^4*b*m^2*x^4*sgn(b*x^3 + a) + 50*(d*x)^m*a
^5*m^4*x*sgn(b*x^3 + a) + 100630*(d*x)^m*a^4*b*m*x^4*sgn(b*x^3 + a) + 955*(
d*x)^m*a^5*m^3*x*sgn(b*x^3 + a) + 72800*(d*x)^m*a^4*b*x^4*sgn(b*x^3 + a) +
8650*(d*x)^m*a^5*m^2*x*sgn(b*x^3 + a) + 36824*(d*x)^m*a^5*m*x*sgn(b*x^3 + a
) + 58240*(d*x)^m*a^5*x*sgn(b*x^3 + a))/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3
+ 45474*m^2 + 95064*m + 58240)
```

maple [A] time = 0.01, size = 453, normalized size = 1.45

(b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^5)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 6968*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^2*b^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a^3*b^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a^3*b^2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3 + 6337*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m^4 + 955*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x*(d*x)^m/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)`

[Out] $x*(b^5*m^5*x^{15}+35*b^5*m^4*x^{15}+445*b^5*m^3*x^{15}+5*a*b^4*m^5*x^{12}+2485*b^5*m^2*x^{15}+190*a*b^4*m^4*x^{12}+5714*b^5*m*x^{15}+2555*a*b^4*m^3*x^{12}+3640*b^5*x^{15}+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^{12}+410*a^2*b^3*m^4*x^9+34840*a*b^4*m*x^{12}+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^{12}+10*a^3*b^2*m^5*x^6+36550*a^2*b^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^6+58240*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^3+123920*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*a^4*b*m^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+8650*a^5*m^2+36824*a^5*m+58240*a^5)*(d*x)^m*((b*x^3+a)^2)^(5/2)/(m+1)/(m+4)/(m+7)/(m+10)/(m+13)/(16+m)/(b*x^3+a)^5$

maxima [A] time = 0.95, size = 243, normalized size = 0.78

((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^16 + 5*(m^5 + 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^m*x^13 + 10*(m^5 + 41*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a^2*b^3*d^m*x^10 + 10*(m^5 + 44*m^4 + 697*m^3 + 4726*m^2 + 12392*m + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 + 47*m^4 + 817*m^3 + 6337*m^2 + 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m^4 + 955*m^3 + 8650*m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")`

[Out] $((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^{16} + 5*(m^5 + 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^m*x^{13} + 10*(m^5 + 41*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a^2*b^3*d^m*x^{10} + 10*(m^5 + 44*m^4 + 697*m^3 + 4726*m^2 + 12392*m + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 + 47*m^4 + 817*m^3 + 6337*m^2 + 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m^4 + 955*m^3 + 8650*m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)`

[Out] `int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left((a + bx^3)^2 \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)`

[Out] `Integral((d*x)**m*((a + b*x**3)**2)**(5/2), x)`

$$3.119 \quad \int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Optimal. Leaf size=205

$$\frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+7}}{d^7(m+7)(a+bx^3)} + \frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+4}}{d^4(m+4)(a+bx^3)} + \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+10}}{d^{10}(m+10)(a+bx^3)} + \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+13}}{d^{13}(m+13)(a+bx^3)}$$

Rubi [A] time = 0.09, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$\frac{3a^2b\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+4}}{d^4(m+4)(a+bx^3)} + \frac{3ab^2\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+7}}{d^7(m+7)(a+bx^3)} + \frac{b^3\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+10}}{d^{10}(m+10)(a+bx^3)} + \frac{a^3\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+13}}{d^{13}(m+13)(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (a^3*(d*x)^(1+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1+m)*(a+b*x^3)) + (3*a^2*b*(d*x)^(4+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4+m)*(a+b*x^3)) + (3*a*b^2*(d*x)^(7+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7+m)*(a+b*x^3)) + (b^3*(d*x)^(10+m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^10*(10+m)*(a+b*x^3))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a+b*x^n+c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2+c*x^n)^(2*FracPart[p]))], Int[(d*x)^m*(b/2+c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2-4*a*c, 0] && IntegerQ[p-1/2]

Rubi steps

$$\begin{aligned}
\int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (dx)^m (ab + b^2x^3)^3 dx}{b^2 (ab + b^2x^3)} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(a^3b^3(dx)^m + \frac{3a^2b^4(dx)^{3+m}}{d^3} + \frac{3ab^5(dx)^{6+m}}{d^6} + \frac{b^6(dx)^{9+m}}{d^9} \right) dx}{b^2 (ab + b^2x^3)} \\
&= \frac{a^3(dx)^{1+m}\sqrt{a^2 + 2abx^3 + b^2x^6}}{d(1+m)(a + bx^3)} + \frac{3a^2b(dx)^{4+m}\sqrt{a^2 + 2abx^3 + b^2x^6}}{d^4(4+m)(a + bx^3)} + \frac{3ab^2(dx)^{7+m}\sqrt{a^2 + 2abx^3 + b^2x^6}}{d^7(7+m)(a + bx^3)} + \frac{b^3(dx)^{10+m}\sqrt{a^2 + 2abx^3 + b^2x^6}}{d^{10}(10+m)(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 0.64

$$\frac{x\sqrt{(a+bx^3)^2}(dx)^m(a^3(m^3+21m^2+138m+280)+3a^2b(m^3+18m^2+87m+70)x^3+3ab^2(m^3+15m^2+54m+40)x^6+b^3(m^3+12m^2+39m+28)x^9)}{(m+1)(m+4)(m+7)(m+10)(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a^3*(280 + 138*m + 21*m^2 + m^3) + 3*a^2*b*(70 + 87*m + 18*m^2 + m^3)*x^3 + 3*a*b^2*(40 + 54*m + 15*m^2 + m^3)*x^6 + b^3*(28 + 39*m + 12*m^2 + m^3)*x^9))/((1 + m)*(4 + m)*(7 + m)*(10 + m)*(a + b*x^3))

IntegrateAlgebraic [F] time = 1.80, size = 0, normalized size = 0.00

$$\int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2), x]

fricas [A] time = 1.20, size = 159, normalized size = 0.78

$$\frac{((b^3m^3 + 12b^3m^2 + 39b^3m + 28b^3)x^{10} + 3(ab^2m^3 + 15ab^2m^2 + 54ab^2m + 40ab^2)x^7 + 3(a^2bm^3 + 18a^2bm^2 + 87a^2bm + 70a^2b)x^4 + (a^3m^3 + 21a^3m^2 + 138a^3m + 280a^3)x)(dx)^m}{m^4 + 22m^3 + 159m^2 + 418m + 280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2), x, algorithm="fricas")

[Out] ((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2*m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b*m + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)(dx)^m

+ 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*(d*x)^m/
(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)

giac [B] time = 0.52, size = 384, normalized size = 1.87

(60^7*P^6*Q^6*sgn(b*x^3+a)^12+12*60^7*P^6*Q^6*sgn(b*x^3+a)+38*60^7*P^6*Q^6*sgn(b*x^3+a)+51*60^7*P^6*Q^6*sgn(b*x^3+a)-28*60^7*P^6*Q^6*sgn(b*x^3+a)+45*60^7*P^6*Q^6*sgn(b*x^3+a)+162*60^7*P^6*Q^6*sgn(b*x^3+a)+132*60^7*P^6*Q^6*sgn(b*x^3+a)+122*60^7*P^6*Q^6*sgn(b*x^3+a)+54*60^7*P^6*Q^6*sgn(b*x^3+a)+281*60^7*P^6*Q^6*sgn(b*x^3+a)+60^7*P^6*Q^6*sgn(b*x^3+a)-221*60^7*P^6*Q^6*sgn(b*x^3+a)+21*60^7*P^6*Q^6*sgn(b*x^3+a)+138*60^7*P^6*Q^6*sgn(b*x^3+a)+280*60^7*P^6*Q^6*sgn(b*x^3+a))/m^4+22*m^3+159*m^2+418*m+280

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")

[Out] ((d*x)^m*b^3*m^3*x^10*sgn(b*x^3 + a) + 12*(d*x)^m*b^3*m^2*x^10*sgn(b*x^3 + a) + 39*(d*x)^m*b^3*m*x^10*sgn(b*x^3 + a) + 3*(d*x)^m*a*b^2*m^3*x^7*sgn(b*x^3 + a) + 28*(d*x)^m*b^3*x^10*sgn(b*x^3 + a) + 45*(d*x)^m*a*b^2*m^2*x^7*sgn(b*x^3 + a) + 162*(d*x)^m*a*b^2*m*x^7*sgn(b*x^3 + a) + 3*(d*x)^m*a^2*b*m^3*x^4*sgn(b*x^3 + a) + 120*(d*x)^m*a*b^2*x^7*sgn(b*x^3 + a) + 54*(d*x)^m*a^2*b*m^2*x^4*sgn(b*x^3 + a) + 261*(d*x)^m*a^2*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m*a^3*m^3*x*sgn(b*x^3 + a) + 210*(d*x)^m*a^2*b*x^4*sgn(b*x^3 + a) + 21*(d*x)^m*a^3*m^2*x*sgn(b*x^3 + a) + 138*(d*x)^m*a^3*m*x*sgn(b*x^3 + a) + 280*(d*x)^m*a^3*x*sgn(b*x^3 + a))/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)

maple [A] time = 0.02, size = 199, normalized size = 0.97

(b^2*m^3*x^9+12*b^2*m^2*x^9+39*b^2*m*x^9+3*a*b^2*m^3*x^6+28*b^2*x^9+45*a*b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^3+261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*((b*x^3+a)^2)^(3/2)*x(dx)^m/(m+10)(m+7)(m+4)(m+1)(b*x^3+a)^3

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)

[Out] x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9+45*a*b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^3+261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*(d*x)^m*((b*x^3+a)^2)^(3/2)/(m+10)/(m+7)/(m+4)/(m+1)/(b*x^3+a)^3

maxima [A] time = 1.04, size = 119, normalized size = 0.58

((m^3+12*m^2+39*m+28)*b^3*d^m*x^10+3*(m^3+15*m^2+54*m+40)*a*b^2*d^m*x^7+3*(m^3+18*m^2+87*m+70)*a^2*b*d^m*x^4+(m^3+21*m^2+138*m+280)*a^3*d^m*x)/(m^4+22*m^3+159*m^2+418*m+280)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")

[Out] ((m^3 + 12*m^2 + 39*m + 28)*b^3*d^m*x^10 + 3*(m^3 + 15*m^2 + 54*m + 40)*a*b^2*d^m*x^7 + 3*(m^3 + 18*m^2 + 87*m + 70)*a^2*b*d^m*x^4 + (m^3 + 21*m^2 + 138*m + 280)*a^3*d^m*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (dx)^m (a^2 + 2abx^3 + b^2x^6)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)

[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left((a + bx^3)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2), x)

[Out] Integral((d*x)**m*((a + b*x**3)**2)**(3/2), x)

$$3.120 \quad \int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6} dx$$

Optimal. Leaf size=97

$$\frac{b\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+4}}{d^4(m+4)(a+bx^3)} + \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+1}}{d(m+1)(a+bx^3)}$$

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 14}

$$\frac{b\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+4}}{d^4(m+4)(a+bx^3)} + \frac{a\sqrt{a^2 + 2abx^3 + b^2x^6} (dx)^{m+1}}{d(m+1)(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3)) + (b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a + b*x^3))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_.) + (c_)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6} dx &= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (dx)^m (ab + b^2x^3) dx}{ab + b^2x^3} \\
&= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(ab(dx)^m + \frac{b^2(dx)^{3+m}}{d^3} \right) dx}{ab + b^2x^3} \\
&= \frac{a(dx)^{1+m} \sqrt{a^2 + 2abx^3 + b^2x^6}}{d(1+m)(a + bx^3)} + \frac{b(dx)^{4+m} \sqrt{a^2 + 2abx^3 + b^2x^6}}{d^4(4+m)(a + bx^3)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.55

$$\frac{x \sqrt{(a + bx^3)^2} (dx)^m (a(m + 4) + b(m + 1)x^3)}{(m + 1)(m + 4)(a + bx^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a*(4 + m) + b*(1 + m)*x^3))/((1 + m)*(4 + m)*(a + b*x^3))

IntegrateAlgebraic [F] time = 1.21, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6], x]

fricas [A] time = 1.28, size = 35, normalized size = 0.36

$$\frac{((bm + b)x^4 + (am + 4a)x)(dx)^m}{m^2 + 5m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2), x, algorithm="fricas")

[Out] ((b*m + b)*x^4 + (a*m + 4*a)*x)*(d*x)^m/(m^2 + 5*m + 4)

giac [A] time = 0.45, size = 83, normalized size = 0.86

$$\frac{(dx)^m b m x^4 \operatorname{sgn}(b x^3 + a) + (dx)^m b x^4 \operatorname{sgn}(b x^3 + a) + (dx)^m a m x \operatorname{sgn}(b x^3 + a) + 4 (dx)^m a x \operatorname{sgn}(b x^3 + a)}{m^2 + 5 m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")

[Out] ((d*x)^m*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m*b*x^4*sgn(b*x^3 + a) + (d*x)^m*a*m*x*sgn(b*x^3 + a) + 4*(d*x)^m*a*x*sgn(b*x^3 + a))/(m^2 + 5*m + 4)

maple [A] time = 0.00, size = 56, normalized size = 0.58

$$\frac{(b m x^3 + b x^3 + a m + 4 a) \sqrt{(b x^3 + a)^2} x (d x)^m}{(m + 4) (m + 1) (b x^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)

[Out] x*(b*m*x^3+b*x^3+a*m+4*a)*(d*x)^m*((b*x^3+a)^2)^(1/2)/(m+4)/(m+1)/(b*x^3+a)

maxima [A] time = 1.03, size = 35, normalized size = 0.36

$$\frac{(b d^m (m + 1) x^4 + a d^m (m + 4) x) x^m}{m^2 + 5 m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x, algorithm="maxima")

[Out] (b*d^m*(m + 1)*x^4 + a*d^m*(m + 4)*x)*x^m/(m^2 + 5*m + 4)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d x)^m \sqrt{a^2 + 2 a b x^3 + b^2 x^6} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)

[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d x)^m \sqrt{(a + b x^3)^2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**3)**2), x)
```

$$3.121 \quad \int x^{11} (a^2 + 2abx^3 + b^2x^6)^p dx$$

Optimal. Leaf size=172

$$\frac{(a + bx^3)^4 (a^2 + 2abx^3 + b^2x^6)^p}{6b^4(p + 2)} - \frac{a(a + bx^3)^3 (a^2 + 2abx^3 + b^2x^6)^p}{b^4(2p + 3)} + \frac{a^2(a + bx^3)^2 (a^2 + 2abx^3 + b^2x^6)^p}{2b^4(p + 1)} - \frac{a^3(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^p}{3b^4(2p + 1)}$$

Rubi [A] time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1356, 266, 43}

$$\frac{(a + bx^3)^4 (a^2 + 2abx^3 + b^2x^6)^p}{6b^4(p + 2)} - \frac{a(a + bx^3)^3 (a^2 + 2abx^3 + b^2x^6)^p}{b^4(2p + 3)} + \frac{a^2(a + bx^3)^2 (a^2 + 2abx^3 + b^2x^6)^p}{2b^4(p + 1)} - \frac{a^3(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^p}{3b^4(2p + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] -(a^3*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^4*(1 + 2*p)) + (a^2*(a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(2*b^4*(1 + p)) - (a*(a + b*x^3)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(3 + 2*p)) + ((a + b*x^3)^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^4*(2 + p))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1356

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
\int x^{11} (a^2 + 2abx^3 + b^2x^6)^p dx &= \left(\left(1 + \frac{bx^3}{a} \right)^{-2p} (a^2 + 2abx^3 + b^2x^6)^p \right) \int x^{11} \left(1 + \frac{bx^3}{a} \right)^{2p} dx \\
&= \frac{1}{3} \left(\left(1 + \frac{bx^3}{a} \right)^{-2p} (a^2 + 2abx^3 + b^2x^6)^p \right) \text{Subst} \left(\int x^3 \left(1 + \frac{bx}{a} \right)^{2p} dx, x, x^3 \right) \\
&= \frac{1}{3} \left(\left(1 + \frac{bx^3}{a} \right)^{-2p} (a^2 + 2abx^3 + b^2x^6)^p \right) \text{Subst} \left(\int \left(-\frac{a^3 \left(1 + \frac{bx}{a} \right)^{2p}}{b^3} + \frac{3a^3 \left(1 + \frac{bx}{a} \right)^{2p}}{b^3} \right) dx, x, x^3 \right) \\
&= -\frac{a^3 (a + bx^3) (a^2 + 2abx^3 + b^2x^6)^p}{3b^4(1 + 2p)} + \frac{a^2 (a + bx^3)^2 (a^2 + 2abx^3 + b^2x^6)^p}{2b^4(1 + p)} - \frac{a^3 (a + bx^3)^3 (a^2 + 2abx^3 + b^2x^6)^p}{b^4(1 + 2p)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 110, normalized size = 0.64

$$\frac{(a + bx^3) \left((a + bx^3)^2 \right)^p \left(-3a^3 + 3a^2b(2p + 1)x^3 - 3ab^2(2p^2 + 3p + 1)x^6 + b^3(4p^3 + 12p^2 + 11p + 3)x^9 \right)}{6b^4(p + 1)(p + 2)(2p + 1)(2p + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[x¹¹*(a² + 2*a*b*x³ + b²*x⁶)^p,x]

[Out] ((a + b*x³)*(a + b*x³)²)^p*(-3*a³ + 3*a²*b*(1 + 2*p)*x³ - 3*a*b²*(1 + 3*p + 2*p²)*x⁶ + b³*(3 + 11*p + 12*p² + 4*p³)*x⁹)/(6*b⁴*(1 + p)*(2 + p)*(1 + 2*p)*(3 + 2*p))

IntegrateAlgebraic [F] time = 0.53, size = 0, normalized size = 0.00

$$\int x^{11} (a^2 + 2abx^3 + b^2x^6)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x¹¹*(a² + 2*a*b*x³ + b²*x⁶)^p,x]

[Out] Defer[IntegrateAlgebraic][x¹¹*(a² + 2*a*b*x³ + b²*x⁶)^p, x]

fricas [A] time = 1.17, size = 163, normalized size = 0.95

$$\frac{((4b^4p^3 + 12b^4p^2 + 11b^4p + 3b^4)x^{12} + 2(2ab^3p^3 + 3ab^3p^2 + ab^3p)x^9 + 6a^3bpx^3 - 3(2a^2b^2p^2 + a^2b^2p)x^6 - 3a^4)(b^2x^6 + 2abx^3 + a^2)^p}{6(4b^4p^4 + 20b^4p^3 + 35b^4p^2 + 25b^4p + 6b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(b²*x⁶+2*a*b*x³+a²)^p,x, algorithm="fricas")

[Out] 1/6*((4*b⁴*p³ + 12*b⁴*p² + 11*b⁴*p + 3*b⁴)*x¹² + 2*(2*a*b³*p³ + 3*a*b³*p² + a*b³*p)*x⁹ + 6*a³*b*p*x³ - 3*(2*a²*b²*p² + a²*b²*p)*x⁶ - 3*a⁴)*(b²*x⁶ + 2*a*b*x³ + a²)^p/(4*b⁴*p⁴ + 20*b⁴*p³ + 35*b⁴*p² + 25*b⁴*p + 6*b⁴)

giac [B] time = 0.53, size = 375, normalized size = 2.18

$$\frac{4(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} + 12(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} + 11(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} + 4(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} + 3(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} + 6(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} + 2(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} + 2(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} - 6(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} - 3(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} + 6(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1} - 3(b^{2p} + 2ab^{2p} + a^2b^{2p})^{p+1}}{6(4p^4 + 20p^3 + 35p^2 + 25p + 6)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(b²*x⁶+2*a*b*x³+a²)^p,x, algorithm="giac")

[Out] 1/6*(4*(b²*x⁶ + 2*a*b*x³ + a²)^p*b⁴*p³*x¹² + 12*(b²*x⁶ + 2*a*b*x³ + a²)^p*b⁴*p²*x¹² + 11*(b²*x⁶ + 2*a*b*x³ + a²)^p*b⁴*p*x¹² + 4*(b²*x⁶ + 2*a*b*x³ + a²)^p*a*b³*p³*x⁹ + 3*(b²*x⁶ + 2*a*b*x³ + a²)^p*b⁴*x¹² + 6*(b²*x⁶ + 2*a*b*x³ + a²)^p*a*b³*p²*x⁹ + 2*(b²*x⁶ + 2*a*b*x³ + a²)^p*a*b³*p*x⁹ - 6*(b²*x⁶ + 2*a*b*x³ + a²)^p*a²*b²*p²*x⁶ - 3*(b²*x⁶ + 2*a*b*x³ + a²)^p*a²*b²*p*x⁶ + 6*(b²*x⁶ + 2*a*b*x³ + a²)^p*a³*b*p*x³ - 3*(b²*x⁶ + 2*a*b*x³ + a²)^p*a⁴)/(4*b⁴*p⁴ + 20*b⁴*p³ + 35*b⁴*p² + 25*b⁴*p + 6*b⁴)

maple [A] time = 0.01, size = 150, normalized size = 0.87

$$\frac{(-4b^3p^3x^9 - 12b^3p^2x^9 - 11b^3p^2x^9 - 3b^3x^9 + 6a^2b^2p^2x^6 + 9a^2b^2p^2x^6 + 3a^2b^2x^6 - 6a^2bp^2x^3 - 3a^2bx^3 + 3a^3)(bx^3 + a)(b^2x^6 + 2abx^3 + a^2)^p}{6(4p^4 + 20p^3 + 35p^2 + 25p + 6)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹*(b²*x⁶+2*a*b*x³+a²)^p,x)

[Out] -1/6*(b²*x⁶+2*a*b*x³+a²)^p*(-4*b³*p³*x⁹-12*b³*p²*x⁹-11*b³*p*x⁹-3*b³*x⁹+6*a*b²*p²*x⁶+9*a*b²*p*x⁶+3*a*b²*x⁶-6*a²*b*p*x³-3*a²*b*x³+3*a³)*(b*x³+a)/b⁴/(4*p⁴+20*p³+35*p²+25*p+6)

maxima [A] time = 0.81, size = 115, normalized size = 0.67

$$\frac{((4p^3 + 12p^2 + 11p + 3)b^4x^{12} + 2(2p^3 + 3p^2 + p)ab^3x^9 - 3(2p^2 + p)a^2b^2x^6 + 6a^3bpx^3 - 3a^4)(bx^3 + a)^{2p}}{6(4p^4 + 20p^3 + 35p^2 + 25p + 6)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(b²*x⁶+2*a*b*x³+a²)^p,x, algorithm="maxima")

[Out] $\frac{1}{6} \cdot ((4p^3 + 12p^2 + 11p + 3) \cdot b^4 x^{12} + 2 \cdot (2p^3 + 3p^2 + p) \cdot a \cdot b^3 x^9 - 3 \cdot (2p^2 + p) \cdot a^2 \cdot b^2 x^6 + 6 \cdot a^3 \cdot b \cdot p x^3 - 3 \cdot a^4) \cdot (b x^3 + a)^{(2p)} / ((4p^4 + 20p^3 + 35p^2 + 25p + 6) \cdot b^4)$

mupad [B] time = 1.31, size = 207, normalized size = 1.20

$$(a^2 + 2abx^3 + b^2x^6)^p \left(\frac{x^{12}(4p^3 + 12p^2 + 11p + 3)}{6(4p^4 + 20p^3 + 35p^2 + 25p + 6)} - \frac{a^4}{2b^4(4p^4 + 20p^3 + 35p^2 + 25p + 6)} + \frac{a^3 p x^3}{b^3(4p^4 + 20p^3 + 35p^2 + 25p + 6)} + \frac{a p x^9}{3b(4p^4 + 20p^3 + 35p^2 + 25p + 6)} - \frac{a^2 p x^6 (2p + 1)}{2b^2(4p^4 + 20p^3 + 35p^2 + 25p + 6)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{11} \cdot (a^2 + b^2 x^6 + 2abx^3)^p, x)$

[Out] $(a^2 + b^2 x^6 + 2abx^3)^p \cdot ((x^{12} \cdot (11p + 12p^2 + 4p^3 + 3)) / (6 \cdot (25p + 35p^2 + 20p^3 + 4p^4 + 6)) - a^4 / (2 \cdot b^4 \cdot (25p + 35p^2 + 20p^3 + 4p^4 + 6)) + (a^3 p x^3) / (b^3 \cdot (25p + 35p^2 + 20p^3 + 4p^4 + 6)) + (a p x^9 \cdot (3p + 2p^2 + 1)) / (3 \cdot b \cdot (25p + 35p^2 + 20p^3 + 4p^4 + 6)) - (a^2 p x^6 \cdot (2p + 1)) / (2 \cdot b^2 \cdot (25p + 35p^2 + 20p^3 + 4p^4 + 6)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{11} \cdot (b^2 x^6 + 2abx^3 + a^2)^p, x)$

[Out] $\text{Piecewise}((x^{12} \cdot (a^2)^p / 12, \text{Eq}(b, 0)), (6 \cdot a^{3p} \cdot \log(-(-1)^{(1/3)} \cdot a^{(1/3)} \cdot (1/b)^{(1/3)} + x) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 6 \cdot a^{3p} \cdot \log(4 \cdot (-1)^{(2/3)} \cdot a^{(2/3)} \cdot (1/b)^{(2/3)} + 4 \cdot (-1)^{(1/3)} \cdot a^{(1/3)} \cdot x \cdot (1/b)^{(1/3)} + 4 \cdot x^2) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) - 12 \cdot a^{3p} \cdot \log(2) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 11 \cdot a^{3p} / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 18 \cdot a^{2p} \cdot b \cdot x^3 \cdot \log(-(-1)^{(1/3)} \cdot a^{(1/3)} \cdot (1/b)^{(1/3)} + x) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 18 \cdot a^{2p} \cdot b \cdot x^3 \cdot \log(4 \cdot (-1)^{(2/3)} \cdot a^{(2/3)} \cdot (1/b)^{(2/3)} + 4 \cdot (-1)^{(1/3)} \cdot a^{(1/3)} \cdot x \cdot (1/b)^{(1/3)} + 4 \cdot x^2) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) - 36 \cdot a^{2p} \cdot b \cdot x^3 \cdot \log(2) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 27 \cdot a^{2p} \cdot b \cdot x^3 / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 18 \cdot a \cdot b^{2p} \cdot x^6 \cdot \log(-(-1)^{(1/3)} \cdot a^{(1/3)} \cdot (1/b)^{(1/3)} + x) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 18 \cdot a \cdot b^{2p} \cdot x^6 \cdot \log(4 \cdot (-1)^{(2/3)} \cdot a^{(2/3)} \cdot (1/b)^{(2/3)} + 4 \cdot (-1)^{(1/3)} \cdot a^{(1/3)} \cdot x \cdot (1/b)^{(1/3)} + 4 \cdot x^2) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) - 36 \cdot a \cdot b^{2p} \cdot x^6 \cdot \log(2) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 18 \cdot a \cdot b^{2p} \cdot x^6 / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 6 \cdot b^{3p} \cdot x^9 \cdot \log(-(-1)^{(1/3)} \cdot a^{(1/3)} \cdot (1/b)^{(1/3)} + x) / (18 \cdot a^{3p} \cdot b^{4p} + 54 \cdot a^{2p} \cdot b^{5p} \cdot x^3 + 54 \cdot a \cdot b^{6p} \cdot x^6 + 18 \cdot b^{7p} \cdot x^9) + 6 \cdot b^{3p}$


```

*x**9*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1
/b)**(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 1
8*b**7*x**9) - 12*b**3*x**9*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a
*b**6*x**6 + 18*b**7*x**9), Eq(p, -2)), (Integral(x**11/((a + b*x**3)**2)**
(3/2), x), Eq(p, -3/2)), (6*a**3*log((-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x
)/(6*a*b**4 + 6*b**5*x**3) + 6*a**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3)
+ 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(6*a*b**4 + 6*b**5*x**3)
- 12*a**3*log(2)/(6*a*b**4 + 6*b**5*x**3) + 6*a**3/(6*a*b**4 + 6*b**5*x**3
) + 6*a**2*b*x**3*log((-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(6*a*b**4 + 6
*b**5*x**3) + 6*a**2*b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1
)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(6*a*b**4 + 6*b**5*x**3) - 12*a
*2*b*x**3*log(2)/(6*a*b**4 + 6*b**5*x**3) - 3*a*b**2*x**6/(6*a*b**4 + 6*b**
5*x**3) + b**3*x**9/(6*a*b**4 + 6*b**5*x**3), Eq(p, -1)), (Integral(x**11/s
qrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-3*a**4*(a**2 + 2*a*b*x**3 + b**2*
x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b
**4) + 6*a**3*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 12
0*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) - 6*a**2*b**2*p**2*x**6
*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**
4*p**2 + 150*b**4*p + 36*b**4) - 3*a**2*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b
**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36
*b**4) + 4*a*b**3*p**3*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**
4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 6*a*b**3*p**2*x
**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*
b**4*p**2 + 150*b**4*p + 36*b**4) + 2*a*b**3*p*x**9*(a**2 + 2*a*b*x**3 + b
**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36
*b**4) + 4*b**4*p**3*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4
+ 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 12*b**4*p**2*x**
12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b
**4*p**2 + 150*b**4*p + 36*b**4) + 11*b**4*p*x**12*(a**2 + 2*a*b*x**3 + b**
2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*
b**4) + 3*b**4*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120
*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4), True))

```

$$3.122 \quad \int x^8 (a^2 + 2abx^3 + b^2x^6)^p dx$$

Optimal. Leaf size=130

$$\frac{(a + bx^3)^3 (a^2 + 2abx^3 + b^2x^6)^p}{3b^3(2p + 3)} - \frac{a(a + bx^3)^2 (a^2 + 2abx^3 + b^2x^6)^p}{3b^3(p + 1)} + \frac{a^2(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^p}{3b^3(2p + 1)}$$

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1356, 266, 43}

$$\frac{(a + bx^3)^3 (a^2 + 2abx^3 + b^2x^6)^p}{3b^3(2p + 3)} - \frac{a(a + bx^3)^2 (a^2 + 2abx^3 + b^2x^6)^p}{3b^3(p + 1)} + \frac{a^2(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^p}{3b^3(2p + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] (a^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + 2*p)) - (a*(a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + p)) + ((a + b*x^3)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(3 + 2*p))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1356

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(2*n_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
\int x^8 (a^2 + 2abx^3 + b^2x^6)^p dx &= \left(\left(1 + \frac{bx^3}{a} \right)^{-2p} (a^2 + 2abx^3 + b^2x^6)^p \right) \int x^8 \left(1 + \frac{bx^3}{a} \right)^{2p} dx \\
&= \frac{1}{3} \left(\left(1 + \frac{bx^3}{a} \right)^{-2p} (a^2 + 2abx^3 + b^2x^6)^p \right) \text{Subst} \left(\int x^2 \left(1 + \frac{bx}{a} \right)^{2p} dx, x, x^3 \right) \\
&= \frac{1}{3} \left(\left(1 + \frac{bx^3}{a} \right)^{-2p} (a^2 + 2abx^3 + b^2x^6)^p \right) \text{Subst} \left(\int \left(\frac{a^2 \left(1 + \frac{bx}{a} \right)^{2p}}{b^2} - \frac{2a^2 \left(1 + \frac{bx}{a} \right)^{2p}}{b^2} \right) dx, x, x^3 \right) \\
&= \frac{a^2 (a + bx^3) (a^2 + 2abx^3 + b^2x^6)^p}{3b^3(1 + 2p)} - \frac{a (a + bx^3)^2 (a^2 + 2abx^3 + b^2x^6)^p}{3b^3(1 + p)} + \frac{(a + bx^3)^3 (a^2 + 2abx^3 + b^2x^6)^p}{3b^3(1 + p)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.59

$$\frac{(a + bx^3) \left((a + bx^3)^2 \right)^p (a^2 - ab(2p + 1)x^3 + b^2(2p^2 + 3p + 1)x^6)}{3b^3(p + 1)(2p + 1)(2p + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(a^2 - a*b*(1 + 2*p)*x^3 + b^2*(1 + 3*p + 2*p^2)*x^6))/(3*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))

IntegrateAlgebraic [F] time = 0.49, size = 0, normalized size = 0.00

$$\int x^8 (a^2 + 2abx^3 + b^2x^6)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] Defer[IntegrateAlgebraic][x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p, x]

fricas [A] time = 1.13, size = 108, normalized size = 0.83

$$\frac{\left((2b^3p^2 + 3b^3p + b^3)x^9 - 2a^2bpx^3 + (2ab^2p^2 + ab^2p)x^6 + a^3 \right) (b^2x^6 + 2abx^3 + a^2)^p}{3(4b^3p^3 + 12b^3p^2 + 11b^3p + 3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")

[Out] $\frac{1}{3} * ((2*b^3*p^2 + 3*b^3*p + b^3)*x^9 - 2*a^2*b*p*x^3 + (2*a*b^2*p^2 + a*b^2*p)*x^6 + a^3)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p / (4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)$

giac [A] time = 0.44, size = 235, normalized size = 1.81

$$\frac{2(b^2x^6 + 2abx^3 + a^2)^p b^3 p^2 x^9 + 3(b^2x^6 + 2abx^3 + a^2)^p b^3 p x^9 + (b^2x^6 + 2abx^3 + a^2)^p b^3 x^9 + 2(b^2x^6 + 2abx^3 + a^2)^p ab^2 p^2 x^6 + (b^2x^6 + 2abx^3 + a^2)^p ab^2 p x^6 - 2(b^2x^6 + 2abx^3 + a^2)^p a^2 b p x^3 + (b^2x^6 + 2abx^3 + a^2)^p a^3}{3(4b^3p^3 + 12b^3p^2 + 11b^3p + 3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")

[Out] $\frac{1}{3} * (2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p * b^3 * p^2 * x^9 + 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p * b^3 * p * x^9 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p * b^3 * x^9 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p * a * b^2 * p^2 * x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p * a * b^2 * p * x^6 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p * a^2 * b * p * x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p * a^3) / (4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)$

maple [A] time = 0.01, size = 96, normalized size = 0.74

$$\frac{(bx^3 + a)(2b^2p^2x^6 + 3b^2px^6 + b^2x^6 - 2abpx^3 - abx^3 + a^2)(b^2x^6 + 2abx^3 + a^2)^p}{3(4p^3 + 12p^2 + 11p + 3)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x)

[Out] $\frac{1}{3} * (b*x^3+a) * (2*b^2*p^2*x^6+3*b^2*p*x^6+b^2*x^6-2*a*b*p*x^3-a*b*x^3+a^2) * (b^2*x^6+2*a*b*x^3+a^2)^p / b^3 / (4*p^3+12*p^2+11*p+3)$

maxima [A] time = 1.22, size = 79, normalized size = 0.61

$$\frac{((2p^2 + 3p + 1)b^3x^9 + (2p^2 + p)ab^2x^6 - 2a^2bpx^3 + a^3)(bx^3 + a)^{2p}}{3(4p^3 + 12p^2 + 11p + 3)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")

[Out] $\frac{1}{3} * ((2*p^2 + 3*p + 1)*b^3*x^9 + (2*p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + a^3)*(b*x^3 + a)^{(2*p)} / ((4*p^3 + 12*p^2 + 11*p + 3)*b^3)$

mupad [B] time = 1.22, size = 137, normalized size = 1.05

$$(a^2 + 2abx^3 + b^2x^6)^p \left(\frac{x^9 \left(\frac{2p^2}{3} + p + \frac{1}{3} \right)}{4p^3 + 12p^2 + 11p + 3} + \frac{a^3}{3b^3(4p^3 + 12p^2 + 11p + 3)} - \frac{2a^2px^3}{3b^2(4p^3 + 12p^2 + 11p + 3)} + \frac{apx^6(2p+1)}{3b(4p^3 + 12p^2 + 11p + 3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8(a^2 + b^2x^6 + 2abx^3)^p, x)$

[Out] $(a^2 + b^2x^6 + 2abx^3)^p \left(\frac{x^9(p + (2p^2)/3 + 1/3)}{(11p + 12p^2 + 4p^3 + 3) + a^3/(3b^3(11p + 12p^2 + 4p^3 + 3))} - \frac{(2a^2x^6 + 2abx^3)/(3b^3(11p + 12p^2 + 4p^3 + 3))}{(11p + 12p^2 + 4p^3 + 3)} + \frac{(a^2x^6 + 2abx^3)/(3b^3(11p + 12p^2 + 4p^3 + 3))}{(11p + 12p^2 + 4p^3 + 3)} \right)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\left\{ \begin{array}{ll} \frac{x^9(p^2)^p}{9} & \text{for } b = 0 \\ \int \frac{x^8}{(a+bx^3)^2} dx & \text{for } p = -\frac{3}{2} \\ \frac{2a^2 \log\left(-\sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + x\right) - 2a^2 \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1} \sqrt[3]{a} x \sqrt[3]{\frac{1}{b}} + 4x^2\right)}{3ab^3 + 3b^4x^3} - \frac{2a^2}{3ab^3 + 3b^4x^3} + \frac{4a^2 \log(2)}{3ab^3 + 3b^4x^3} - \frac{2abx^3 \log\left(-\sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + x\right) - 2abx^3 \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1} \sqrt[3]{a} x \sqrt[3]{\frac{1}{b}} + 4x^2\right)}{3ab^3 + 3b^4x^3} + \frac{4abx^3 \log(2)}{3ab^3 + 3b^4x^3} + \frac{b^2x^6}{3ab^3 + 3b^4x^3} & \text{for } p = -1 \\ \int \frac{x^8}{\sqrt{(a+bx^3)^2}} dx & \text{for } p = -\frac{1}{2} \\ \frac{a^2(a^2 + 2abx^3 + b^2x^6)^p}{12b^3p^3 + 36b^3p^2 + 33b^3p + 9b^3} - \frac{2a^2bx^3(a^2 + 2abx^3 + b^2x^6)^p}{12b^3p^3 + 36b^3p^2 + 33b^3p + 9b^3} + \frac{2ab^2p^2x^6(a^2 + 2abx^3 + b^2x^6)^p}{12b^3p^3 + 36b^3p^2 + 33b^3p + 9b^3} + \frac{ab^2px^6(a^2 + 2abx^3 + b^2x^6)^p}{12b^3p^3 + 36b^3p^2 + 33b^3p + 9b^3} + \frac{2b^3p^2x^9(a^2 + 2abx^3 + b^2x^6)^p}{12b^3p^3 + 36b^3p^2 + 33b^3p + 9b^3} + \frac{3b^3px^9(a^2 + 2abx^3 + b^2x^6)^p}{12b^3p^3 + 36b^3p^2 + 33b^3p + 9b^3} + \frac{b^3x^9(a^2 + 2abx^3 + b^2x^6)^p}{12b^3p^3 + 36b^3p^2 + 33b^3p + 9b^3} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{**8}*(b^{**2}*x^{**6}+2*a*b*x^{**3}+a^{**2})^{**p}, x)$

[Out] $\text{Piecewise}\left(\left(x^{**9}*(a^{**2})^{**p}/9, \text{Eq}(b, 0)\right), \left(\text{Integral}(x^{**8}/((a + b*x^{**3})^{**2})^{**p}, x), \text{Eq}(p, -3/2)\right), \left(-2*a^{**2}*\log(-(-1)^{**}(1/3)*a^{**}(1/3)*(1/b)^{**}(1/3) + x)/(3*a*b^{**3} + 3*b^{**4}*x^{**3}) - 2*a^{**2}*\log(4*(-1)^{**}(2/3)*a^{**}(2/3)*(1/b)^{**}(2/3) + 4*(-1)^{**}(1/3)*a^{**}(1/3)*x*(1/b)^{**}(1/3) + 4*x^{**2})/(3*a*b^{**3} + 3*b^{**4}*x^{**3}) - 2*a^{**2}/(3*a*b^{**3} + 3*b^{**4}*x^{**3}) + 4*a^{**2}*\log(2)/(3*a*b^{**3} + 3*b^{**4}*x^{**3}) - 2*a*b*x^{**3}*\log(-(-1)^{**}(1/3)*a^{**}(1/3)*(1/b)^{**}(1/3) + x)/(3*a*b^{**3} + 3*b^{**4}*x^{**3}) - 2*a*b*x^{**3}*\log(4*(-1)^{**}(2/3)*a^{**}(2/3)*(1/b)^{**}(2/3) + 4*(-1)^{**}(1/3)*a^{**}(1/3)*x*(1/b)^{**}(1/3) + 4*x^{**2})/(3*a*b^{**3} + 3*b^{**4}*x^{**3}) + 4*a*b*x^{**3}*\log(2)/(3*a*b^{**3} + 3*b^{**4}*x^{**3}) + b^{**2}*x^{**6}/(3*a*b^{**3} + 3*b^{**4}*x^{**3}), \text{Eq}(p, -1)\right), \left(\text{Integral}(x^{**8}/\text{sqrt}((a + b*x^{**3})^{**2}), x), \text{Eq}(p, -1/2)\right), \left(a^{**3}*(a^{**2} + 2*a*b*x^{**3} + b^{**2}*x^{**6})^{**p}/(12*b^{**3}*p^{**3} + 36*b^{**3}*p^{**2} + 33*b^{**3}*p + 9*b^{**3}) - 2*a^{**2}*b*p*x^{**3}*(a^{**2} + 2*a*b*x^{**3} + b^{**2}*x^{**6})^{**p}/(12*b^{**3}*p^{**3} + 36*b^{**3}*p^{**2} + 33*b^{**3}*p + 9*b^{**3}) + 2*a*b^{**2}*p^{**2}*x^{**6}*(a^{**2} + 2*a*b*x^{**3} + b^{**2}*x^{**6})^{**p}/(12*b^{**3}*p^{**3} + 36*b^{**3}*p^{**2} + 33*b^{**3}*p + 9*b^{**3}) + a*b^{**2}*p*x^{**6}*(a^{**2} + 2*a*b*x^{**3} + b^{**2}*x^{**6})^{**p}/(12*b^{**3}*p^{**3} + 36*b^{**3}*p^{**2} + 33*b^{**3}*p + 9*b^{**3}) + 2*b^{**3}*p^{**2}*x^{**9}*(a^{**2} + 2*a*b*x^{**3} + b^{**2}*x^{**6})^{**p}/(12*b^{**3}*p^{**3} + 36*b^{**3}*p^{**2} + 33*b^{**3}*p + 9*b^{**3}) + 3*b^{**3}*p*x^{**9}*(a^{**2} + 2*a*b*x^{**3} + b^{**2}*x^{**6})^{**p}/(12*b^{**3}*p^{**3} + 36*b^{**3}*p^{**2} + 33*b^{**3}*p + 9*b^{**3}) + b^{**3}*x^{**9}*(a^{**2} + 2*a*b*x^{**3} + b^{**2}*x^{**6})^{**p}/(12*b^{**3}*p^{**3} + 36*b^{**3}*p^{**2} + 33*b^{**3}*p + 9*b^{**3}), \text{True})\right)$

3.123 $\int x^5 (a^2 + 2abx^3 + b^2x^6)^p dx$

Optimal. Leaf size=84

$$\frac{(a + bx^3)^2 (a^2 + 2abx^3 + b^2x^6)^p}{6b^2(p + 1)} - \frac{a(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^p}{3b^2(2p + 1)}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1356, 266, 43}

$$\frac{(a + bx^3)^2 (a^2 + 2abx^3 + b^2x^6)^p}{6b^2(p + 1)} - \frac{a(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^p}{3b^2(2p + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] -(a*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^2*(1 + 2*p)) + ((a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^2*(1 + p))

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int x^5 (a^2 + 2abx^3 + b^2x^6)^p dx &= \left(\left(1 + \frac{bx^3}{a}\right)^{-2p} (a^2 + 2abx^3 + b^2x^6)^p \right) \int x^5 \left(1 + \frac{bx^3}{a}\right)^{2p} dx \\
&= \frac{1}{3} \left(\left(1 + \frac{bx^3}{a}\right)^{-2p} (a^2 + 2abx^3 + b^2x^6)^p \right) \text{Subst} \left(\int x \left(1 + \frac{bx}{a}\right)^{2p} dx, x, x^3 \right) \\
&= \frac{1}{3} \left(\left(1 + \frac{bx^3}{a}\right)^{-2p} (a^2 + 2abx^3 + b^2x^6)^p \right) \text{Subst} \left(\int \left(-\frac{a \left(1 + \frac{bx}{a}\right)^{2p}}{b} + \frac{a \left(1 + \frac{bx}{a}\right)^{2p+1}}{b} \right) dx, x, x^3 \right) \\
&= -\frac{a(a+bx^3)(a^2+2abx^3+b^2x^6)^p}{3b^2(1+2p)} + \frac{(a+bx^3)^2(a^2+2abx^3+b^2x^6)^p}{6b^2(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.61

$$\frac{(a+bx^3)\left((a+bx^3)^2\right)^p(b(2p+1)x^3-a)}{6b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-a + b*(1 + 2*p)*x^3))/(6*b^2*(1 + p)*(1 + 2*p))

IntegrateAlgebraic [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x^5 (a^2 + 2abx^3 + b^2x^6)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] Defer[IntegrateAlgebraic][x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p, x]

fricas [A] time = 1.12, size = 70, normalized size = 0.83

$$\frac{\left((2b^2p+b^2)x^6+2abpx^3-a^2\right)\left(b^2x^6+2abx^3+a^2\right)^p}{6\left(2b^2p^2+3b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")

[Out] $\frac{1}{6} * ((2 * b^{2 * p} + b^2) * x^6 + 2 * a * b * p * x^3 - a^2) * (b^2 * x^6 + 2 * a * b * x^3 + a^2)^p / (2 * b^{2 * p} + 3 * b^2 * p + b^2)$

giac [A] time = 0.48, size = 132, normalized size = 1.57

$$\frac{2(b^2x^6 + 2abx^3 + a^2)^p b^2 p x^6 + (b^2x^6 + 2abx^3 + a^2)^p b^2 x^6 + 2(b^2x^6 + 2abx^3 + a^2)^p ab p x^3 - (b^2x^6 + 2abx^3 + a^2)^p a^2}{6(2b^2p^2 + 3b^2p + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")

[Out] $\frac{1}{6} * (2 * (b^2 * x^6 + 2 * a * b * x^3 + a^2)^p * b^{2 * p} * x^6 + (b^2 * x^6 + 2 * a * b * x^3 + a^2)^p * b^{2 * p} * x^6 + 2 * (b^2 * x^6 + 2 * a * b * x^3 + a^2)^p * a * b * p * x^3 - (b^2 * x^6 + 2 * a * b * x^3 + a^2)^p * a^2) / (2 * b^{2 * p} + 3 * b^2 * p + b^2)$

maple [A] time = 0.01, size = 60, normalized size = 0.71

$$\frac{(-2x^3pb - bx^3 + a)(bx^3 + a)(b^2x^6 + 2abx^3 + a^2)^p}{6(2p^2 + 3p + 1)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x)

[Out] $-1/6 * (b^2 * x^6 + 2 * a * b * x^3 + a^2)^p * (-2 * b * p * x^3 - b * x^3 + a) * (b * x^3 + a) / b^2 / (2 * p^2 + 3 * p + 1)$

maxima [A] time = 0.63, size = 54, normalized size = 0.64

$$\frac{(b^2(2p + 1)x^6 + 2abpx^3 - a^2)(bx^3 + a)^{2p}}{6(2p^2 + 3p + 1)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")

[Out] $\frac{1}{6} * (b^2 * (2 * p + 1) * x^6 + 2 * a * b * p * x^3 - a^2) * (b * x^3 + a)^{(2 * p)} / ((2 * p^2 + 3 * p + 1) * b^2)$

mupad [B] time = 1.19, size = 85, normalized size = 1.01

$$(a^2 + 2abx^3 + b^2x^6)^p \left(\frac{x^6(2p + 1)}{6(2p^2 + 3p + 1)} - \frac{a^2}{6b^2(2p^2 + 3p + 1)} + \frac{apx^3}{3b(2p^2 + 3p + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)`

[Out] $(a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^6*(2*p + 1))/(6*(3*p + 2*p^2 + 1)) - a^2/(6*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^3)/(3*b*(3*p + 2*p^2 + 1)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\left\{ \begin{array}{ll} \frac{x^6(a^2)^p}{6} & \text{for } b = 0 \\ \frac{a \log\left(-\sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + x\right)}{3ab^2+3b^3x^3} + \frac{a \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1} \sqrt[3]{a} x \sqrt[3]{\frac{1}{b}} + 4x^2\right)}{3ab^2+3b^3x^3} - \frac{2a \log(2)}{3ab^2+3b^3x^3} + \frac{a}{3ab^2+3b^3x^3} + \frac{bx^3 \log\left(-\sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + x\right)}{3ab^2+3b^3x^3} + \frac{bx^3 \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1} \sqrt[3]{a} x \sqrt[3]{\frac{1}{b}} + 4x^2\right)}{3ab^2+3b^3x^3} - \frac{2bx^3 \log(2)}{3ab^2+3b^3x^3} & \text{for } p = -1 \\ \int \frac{x^5}{\sqrt{(a+bx^3)^2}} dx & \text{for } p = -\frac{1}{2} \\ -\frac{a^2(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} + \frac{2abpx^3(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} + \frac{2b^2px^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} + \frac{b^2x^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)`

[Out] `Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (a*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(3*a*b**2 + 3*b**3*x**3) + a*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**2 + 3*b**3*x**3) - 2*a*log(2)/(3*a*b**2 + 3*b**3*x**3) + a/(3*a*b**2 + 3*b**3*x**3) + b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(3*a*b**2 + 3*b**3*x**3) + b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**2 + 3*b**3*x**3) - 2*b*x**3*log(2)/(3*a*b**2 + 3*b**3*x**3), Eq(p, -1)), (Integral(x**5/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-a**2*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*a*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + b**2*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2), True))`

$$3.124 \quad \int x^2 (a^2 + 2abx^3 + b^2x^6)^p dx$$

Optimal. Leaf size=41

$$\frac{(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^p}{3b(2p + 1)}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1352, 609}

$$\frac{(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^p}{3b(2p + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b*(1 + 2*p))

Rule 609

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^p], x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p], x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^2 (a^2 + 2abx^3 + b^2x^6)^p dx &= \frac{1}{3} \text{Subst} \left(\int (a^2 + 2abx + b^2x^2)^p dx, x, x^3 \right) \\ &= \frac{(a + bx^3)(a^2 + 2abx^3 + b^2x^6)^p}{3b(1 + 2p)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{(a + bx^3) \left((a + bx^3)^2 \right)^p}{3b(2p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] ((a + b*x^3)*((a + b*x^3)^2)^p)/(3*b*(1 + 2*p))

IntegrateAlgebraic [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x^2 (a^2 + 2abx^3 + b^2x^6)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]

[Out] Defer[IntegrateAlgebraic][x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p, x]

fricas [A] time = 1.39, size = 37, normalized size = 0.90

$$\frac{(bx^3 + a)(b^2x^6 + 2abx^3 + a^2)^p}{3(2bp + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")

[Out] 1/3*(b*x^3 + a)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(2*b*p + b)

giac [A] time = 0.48, size = 58, normalized size = 1.41

$$\frac{(b^2x^6 + 2abx^3 + a^2)^p bx^3 + (b^2x^6 + 2abx^3 + a^2)^p a}{3(2bp + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")

[Out] 1/3*((b^2*x^6 + 2*a*b*x^3 + a^2)^p*b*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a)/(2*b*p + b)

maple [A] time = 0.01, size = 40, normalized size = 0.98

$$\frac{(bx^3 + a)(b^2x^6 + 2abx^3 + a^2)^p}{3(2p + 1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x)`

[Out] `1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)`

maxima [A] time = 0.95, size = 30, normalized size = 0.73

$$\frac{(bx^3 + a)(bx^3 + a)^{2p}}{3b(2p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")`

[Out] `1/3*(b*x^3 + a)*(b*x^3 + a)^(2*p)/(b*(2*p + 1))`

mupad [B] time = 1.16, size = 46, normalized size = 1.12

$$\left(\frac{x^3}{3(2p+1)} + \frac{a}{3b(2p+1)} \right) (a^2 + 2abx^3 + b^2x^6)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)`

[Out] `(x^3/(3*(2*p + 1)) + a/(3*b*(2*p + 1)))*(a^2 + b^2*x^6 + 2*a*b*x^3)^p`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^3}{3\sqrt{a^2}} & \text{for } b = 0 \wedge p = -\frac{1}{2} \\ \frac{x^3(a^2)^p}{3} & \text{for } b = 0 \\ \int \frac{x^2}{\sqrt{(a+bx^3)^2}} dx & \text{for } p = -\frac{1}{2} \\ \frac{a(a^2+2abx^3+b^2x^6)^p}{6bp+3b} + \frac{bx^3(a^2+2abx^3+b^2x^6)^p}{6bp+3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)`

[Out] `Piecewise((x**3/(3*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**3*(a**2)**p/3, Eq(b, 0)), (Integral(x**2/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (a*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b*p + 3*b) + b*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b*p + 3*b), True))`

$$3.125 \quad \int \frac{x^8}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=81

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx^3 + cx^6)}{6c^2} + \frac{x^3}{3c}$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx^3 + cx^6)}{6c^2} + \frac{x^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a + b*x^3 + c*x^6), x]

[Out] x^3/(3*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^2 *Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(6*c^2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^8}{a + bx^3 + cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{a + bx + cx^2} dx, x, x^3 \right) \\ &= \frac{x^3}{3c} + \frac{\text{Subst} \left(\int \frac{-a-bx}{a+bx+cx^2} dx, x, x^3 \right)}{3c} \\ &= \frac{x^3}{3c} - \frac{b \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^3 \right)}{6c^2} + \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^3 \right)}{6c^2} \\ &= \frac{x^3}{3c} - \frac{b \log(a + bx^3 + cx^6)}{6c^2} - \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^3 \right)}{3c^2} \\ &= \frac{x^3}{3c} - \frac{(b^2 - 2ac) \tanh^{-1} \left(\frac{b+2cx^3}{\sqrt{b^2-4ac}} \right)}{3c^2 \sqrt{b^2 - 4ac}} - \frac{b \log(a + bx^3 + cx^6)}{6c^2} \end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.96

$$\frac{2(b^2-2ac) \tan^{-1} \left(\frac{b+2cx^3}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}} - \frac{b \log(a + bx^3 + cx^6) + 2cx^3}{6c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a + b*x^3 + c*x^6),x]

[Out] (2*c*x^3 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + b*x^3 + c*x^6])/(6*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(a + b*x^3 + c*x^6),x]

[Out] IntegrateAlgebraic[x^8/(a + b*x^3 + c*x^6), x]

fricas [A] time = 1.29, size = 254, normalized size = 3.14

$$\frac{2(b^2c - 4ac^2)x^3 - (b^2 - 2ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^6 + 2bcx^3 + b^2 - 2ac + (2cx^3 + b)\sqrt{b^2 - 4ac}}{cx^6 + bx^3 + a}\right) - (b^3 - 4abc) \log(cx^6 + bx^3 + a)}{6(b^2c^2 - 4ac^3)} - \frac{2(b^2c - 4ac^2)x^3 - 2(b^2 - 2ac)\sqrt{-b^2 + 4ac} \arctan\left(\frac{(2cx^3 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) - (b^3 - 4abc) \log(cx^6 + bx^3 + a)}{6(b^2c^2 - 4ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(2*(b^2*c - 4*a*c^2)*x^3 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^3 - 4*a*b*c)*log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1/6*(2*(b^2*c - 4*a*c^2)*x^3 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]

giac [A] time = 1.14, size = 75, normalized size = 0.93

$$\frac{x^3}{3c} - \frac{b \log(cx^6 + bx^3 + a)}{6c^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^3 + b}{\sqrt{-b^2 + 4ac}}\right)}{3\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] 1/3*x^3/c - 1/6*b*log(c*x^6 + b*x^3 + a)/c^2 + 1/3*(b^2 - 2*a*c)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)

maple [A] time = 0.01, size = 111, normalized size = 1.37

$$\frac{x^3}{3c} - \frac{2a \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}c} + \frac{b^2 \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}c^2} - \frac{b \ln(cx^6 + bx^3 + a)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*x^3/c-1/6*b*ln(c*x^6+b*x^3+a)/c^2-2/3/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a+1/3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.98, size = 1758, normalized size = 21.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6),x)
```

```
[Out] x^3/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a*c^3 - 9*b^2*c^2)) + (atan((4*c^3*x^3*(4*a*c - b^2)^(3/2)*((b*((b^5 + a^2*b*c^2 - 2*a*b^3*c)/c^3 + ((3*b^3 - 12*a*b*c)*((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((2*a*c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2))*((36*a*c^3 - 9*b^2*c^2))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (3*b^2*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(4*c*(4*a*c - b^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))))/(6*c^2*(4*a*c - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))) - (b^2*(2*a*c - b^2)^3)/(4*c^3*(4*a*c - b^2)^(3/2)) + (((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)))/(4*a^2*c*(4*a*c - b^2)^(1/2)))/(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)
```


$$\begin{aligned}
& - (c^2(2ac - b^2)(4ac - b^2)((3b^3 - 12abc) * (((36a^2c^5 - 7 \\
& 2ab^2c^4)/c^3 - (54abc^3(3b^3 - 12abc))/(36ac^3 - 9b^2c^2)) * \\
& (2ac - b^2))/(6c^2(4ac - b^2)^{(1/2)}) - (9abc(3b^3 - 12abc) * (2 \\
& ac - b^2))/((4ac - b^2)^{(1/2)} * (36ac^3 - 9b^2c^2)))/((2 * (36ac^3 - \\
& 9b^2c^2)) - (((15ab^3c^2 - 12a^2b^2c^3)/c^3 - ((3b^3 - 12abc) * ((3 \\
& 6a^2c^5 - 72ab^2c^4)/c^3 - (54abc^3(3b^3 - 12abc))/(36ac^3 - \\
& 9b^2c^2)))/(2 * (36ac^3 - 9b^2c^2))) * (2ac - b^2))/(6c^2(4ac - b^ \\
& 2)^{(1/2)}) + (ab * (2ac - b^2)^3)/(2c^3(4ac - b^2)^{(3/2)})))/(a^2(b^6 - \\
& 8a^3c^3 + 12a^2b^2c^2 - 6ab^4c)) + (b * c^2(4ac - b^2)^{(3/2)} * ((a \\
& b^4 - a^2b^2c)/c^3 + ((3b^3 - 12abc) * ((15ab^3c^2 - 12a^2b^2c^3)/c \\
& ^3 - ((3b^3 - 12abc) * ((36a^2c^5 - 72ab^2c^4)/c^3 - (54abc^3(3 \\
& b^3 - 12abc))/(36ac^3 - 9b^2c^2)))/(2 * (36ac^3 - 9b^2c^2)))))/(2 * (\\
& 36ac^3 - 9b^2c^2)) + (((((36a^2c^5 - 72ab^2c^4)/c^3 - (54abc^3 * \\
& (3b^3 - 12abc))/(36ac^3 - 9b^2c^2)) * (2ac - b^2))/(6c^2(4ac - \\
& b^2)^{(1/2)}) - (9abc(3b^3 - 12abc) * (2ac - b^2))/((4ac - b^2)^{(1/ \\
& 2)} * (36ac^3 - 9b^2c^2))) * (2ac - b^2))/(6c^2(4ac - b^2)^{(1/2)}) - (3 \\
& ab(3b^3 - 12abc) * (2ac - b^2)^2)/(2c * (4ac - b^2) * (36ac^3 - 9b \\
& ^2c^2)))/((a^2(b^6 - 8a^3c^3 + 12a^2b^2c^2 - 6ab^4c))) * (2ac - b \\
& ^2))/(3c^2(4ac - b^2)^{(1/2)})
\end{aligned}$$

sympy [B] time = 2.70, size = 316, normalized size = 3.90

$$\left(\frac{-b}{6c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6c^2(4ac - b^2)} \right) \log \left(x^3 + \frac{-ab - 12ac^2 \left(\frac{-b}{6c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6c^2(4ac - b^2)} \right) + 3b^2c \left(\frac{-b}{6c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6c^2(4ac - b^2)} \right)}{2ac - b^2} \right) + \left(\frac{-b}{6c^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6c^2(4ac - b^2)} \right) \log \left(x^3 + \frac{-ab - 12ac^2 \left(\frac{-b}{6c^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6c^2(4ac - b^2)} \right) + 3b^2c \left(\frac{-b}{6c^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6c^2(4ac - b^2)} \right)}{2ac - b^2} \right) + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(c*x**6+b*x**3+a), x)

[Out] $(-b/(6c**2) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(6c**2*(4*a*c - b**2)))* \log(x**3 + (-a*b - 12*a*c**2*(-b/(6c**2) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(6c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6c**2) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(6c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(6c**2) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(6c**2*(4*a*c - b**2)))* \log(x**3 + (-a*b - 12*a*c**2*(-b/(6c**2) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(6c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6c**2) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(6c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**3/(3*c)$

$$3.126 \quad \int \frac{x^5}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log(a+bx^3+cx^6)}{6c}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log(a+bx^3+cx^6)}{6c}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b*x^3 + c*x^6), x]

[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x^3 + c*x^6]/(6*c)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1357

$\text{Int}[(x_)^{(m_.)*((a_) + (c_.)*(x_)^{(n2_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^5}{a + bx^3 + cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{a + bx + cx^2} dx, x, x^3 \right) \\ &= \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^3 \right)}{6c} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^3 \right)}{6c} \\ &= \frac{\log(a + bx^3 + cx^6)}{6c} + \frac{b \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^3 \right)}{3c} \\ &= \frac{b \tanh^{-1} \left(\frac{b+2cx^3}{\sqrt{b^2 - 4ac}} \right)}{3c\sqrt{b^2 - 4ac}} + \frac{\log(a + bx^3 + cx^6)}{6c} \end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$\frac{\log(a + bx^3 + cx^6) - \frac{2b \tan^{-1} \left(\frac{b+2cx^3}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}}}{6c}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b*x^3 + c*x^6), x]

[Out] ((-2*b*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a + b*x^3 + c*x^6])/(6*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(a + b*x^3 + c*x^6),x]

[Out] IntegrateAlgebraic[x^5/(a + b*x^3 + c*x^6), x]

fricas [A] time = 0.83, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^6 + 2bcx^3 + b^2 - 2ac + (2cx^3 + b)\sqrt{b^2 - 4ac}}{cx^6 + bx^3 + a}\right) + (b^2 - 4ac) \log(cx^6 + bx^3 + a)}{6(b^2c - 4ac^2)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(-\frac{(2cx^3 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) + (b^2 - 4ac) \log(cx^6 + bx^3 + a)}{6(b^2c - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + (b^2 - 4*a*c)*log(c*x^6 + b*x^3 + a))/(b^2*c - 4*a*c^2), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^6 + b*x^3 + a))/(b^2*c - 4*a*c^2)]

giac [A] time = 1.19, size = 59, normalized size = 0.94

$$-\frac{b \arctan\left(\frac{2cx^3+b}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}c} + \frac{\log(cx^6+bx^3+a)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/6*log(c*x^6 + b*x^3 + a)/c

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b \arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}c} + \frac{\ln(cx^6+bx^3+a)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^6+b*x^3+a),x)

[Out] 1/6*ln(c*x^6+b*x^3+a)/c-1/3*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.80, size = 1199, normalized size = 19.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6),x)
```

```
[Out] (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) + (b*atan
n((4*x^3*((b*(b^2 - ((12*b^2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2
)))/(36*a*c^2 - 9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)))*(12*a*
c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(45*b^2*c^2 - (27*b^2*c^3*(12
*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^
2*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))))/(6*c*(4*
a*c - b^2)^(1/2)) + (3*b^4*c*(12*a*c - 3*b^2))/(4*(36*a*c^2 - 9*b^2*c)*(4*a
*c - b^2)))/(4*a^2*c) + ((2*a*c - b^2)*(b^5/(4*(4*a*c - b^2)^(3/2)) + ((12
*a*c - 3*b^2)*((b*(45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9
*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^2*(12*a*c - 3*b^2))/(2*(36*a
*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))))/(2*(36*a*c^2 - 9*b^2*c)) - (b*(12*b^
2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(1
2*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)))))/(6*c*(4*a*c - b^2)^(1/2)))/(4*a
^2*c*(4*a*c - b^2)^(1/2))*(4*a*c - b^2)^(3/2))/b^3 + ((4*a*c - b^2)^(3/2)*
(a*b + (((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c)*(12*a*c - 3*b^2))/(
2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2
))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c -
3*b^2))/((36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b^3*c*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)))
)/(a^2*b^2*c) + ((2*a*c - b^2)*(4*a*c - b^2)*((a*b^4)/(2*(4*a*c - b^2)^(3/2
)) + (((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
)/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c - 3*b^2))/((36*a*c^2 - 9
*b^2*c)*(4*a*c - b^2)^(1/2)))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) +
(b*((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(12*
a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c))/(6*c*(4*a*c - b^2)^(1/2
)))))/(a^2*b^3*c)))/(3*c*(4*a*c - b^2)^(1/2))
```

sympy [B] time = 1.37, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)} + \frac{1}{6c}\right) \log\left(x^3 + \frac{-12ac\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)} + \frac{1}{6c}\right) + 2a + 3b^2\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)} + \frac{1}{6c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)} + \frac{1}{6c}\right) \log\left(x^3 + \frac{-12ac\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)} + \frac{1}{6c}\right) + 2a + 3b^2\left(\frac{b\sqrt{-4ac+b^2}}{6c(4ac-b^2)} + \frac{1}{6c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**6+b*x**3+a), x)

[Out] (-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b) + (b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c*(b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b)

$$3.127 \quad \int \frac{x^2}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=38

$$\frac{2 \tanh^{-1} \left(\frac{b+2cx^3}{\sqrt{b^2-4ac}} \right)}{3\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1352, 618, 206}

$$\frac{2 \tanh^{-1} \left(\frac{b+2cx^3}{\sqrt{b^2-4ac}} \right)}{3\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*x^3 + c*x^6),x]

[Out] (-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{a + bx^3 + cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^3 \right) \\ &= - \left(\frac{2}{3} \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^3 \right) \right) \\ &= - \frac{2 \tanh^{-1} \left(\frac{b+2cx^3}{\sqrt{b^2-4ac}} \right)}{3\sqrt{b^2-4ac}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.11

$$\frac{2 \tan^{-1} \left(\frac{b+2cx^3}{\sqrt{4ac-b^2}} \right)}{3\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*x^3 + c*x^6), x]

[Out] (2*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/(3*Sqrt[-b^2 + 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a + b*x^3 + c*x^6), x]

[Out] IntegrateAlgebraic[x^2/(a + b*x^3 + c*x^6), x]

fricas [A] time = 1.31, size = 129, normalized size = 3.39

$$\left[\frac{\log \left(\frac{2c^2x^6 + 2bcx^3 + b^2 - 2ac - (2cx^3 + b)\sqrt{b^2 - 4ac}}{cx^6 + bx^3 + a} \right)}{3\sqrt{b^2 - 4ac}}, - \frac{2\sqrt{-b^2 + 4ac} \arctan \left(-\frac{(2cx^3 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac} \right)}{3(b^2 - 4ac)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] $\frac{1}{3} \log\left(\frac{(2cx^2 + b^2 - 2ac - (2cx^3 + b)\sqrt{b^2 - 4ac})}{(cx^6 + bx^3 + a)}\right) - \frac{2}{3} \sqrt{-b^2 + 4ac} \arctan\left(\frac{-(2cx^3 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right)$

giac [A] time = 0.98, size = 36, normalized size = 0.95

$$\frac{2 \arctan\left(\frac{2cx^3 + b}{\sqrt{-b^2 + 4ac}}\right)}{3\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="giac")`

[Out] $\frac{2}{3} \arctan\left(\frac{2cx^3 + b}{\sqrt{-b^2 + 4ac}}\right) / \sqrt{-b^2 + 4ac}$

maple [A] time = 0.00, size = 37, normalized size = 0.97

$$\frac{2 \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(c*x^6+b*x^3+a),x)`

[Out] $\frac{2}{3} (4ac - b^2)^{-1/2} \arctan\left(\frac{2cx^3 + b}{(4ac - b^2)^{1/2}}\right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.23, size = 174, normalized size = 4.58

$$\frac{2 \operatorname{atan}\left(\frac{\frac{x^3(4ac-b^2)^4}{2} + ab(4ac-b^2)^3 + ab^3(4ac-b^2)^2 + b^2x^3(4ac-b^2)^3 + \frac{b^4x^3(4ac-b^2)^2}{2}}{b^2(32a^3c^2\sqrt{4ac-b^2} - 4a^2b^2c\sqrt{4ac-b^2}) - 64a^4c^3\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*x^3 + c*x^6),x)`

[Out] $-(2*\operatorname{atan}\left(\frac{(x^3*(4*a*c - b^2)^4)/2 + a*b*(4*a*c - b^2)^3 + a*b^3*(4*a*c - b^2)^2 + b^2*x^3*(4*a*c - b^2)^3 + (b^4*x^3*(4*a*c - b^2)^2)/2}{b^2*(32*a^3*c^2*(4*a*c - b^2)^{1/2} - 4*a^2*b^2*c*(4*a*c - b^2)^{1/2})} - 64*a^4*c^3*(4*a*c - b^2)^{1/2}\right))/(3*(4*a*c - b^2)^{1/2})$

sympy [B] time = 0.64, size = 131, normalized size = 3.45

$$\frac{\sqrt{-\frac{1}{4ac-b^2}} \log\left(x^3 + \frac{-4ac\sqrt{-\frac{1}{4ac-b^2}} + b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c}\right)}{3} + \frac{\sqrt{-\frac{1}{4ac-b^2}} \log\left(x^3 + \frac{4ac\sqrt{-\frac{1}{4ac-b^2}} - b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**6+b*x**3+a),x)`

[Out] $-\operatorname{sqrt}(-1/(4*a*c - b**2))*\log(x**3 + (-4*a*c*\operatorname{sqrt}(-1/(4*a*c - b**2)) + b**2*\operatorname{sqrt}(-1/(4*a*c - b**2)) + b)/(2*c))/3 + \operatorname{sqrt}(-1/(4*a*c - b**2))*\log(x**3 + (4*a*c*\operatorname{sqrt}(-1/(4*a*c - b**2)) - b**2*\operatorname{sqrt}(-1/(4*a*c - b**2)) + b)/(2*c))/3$

$$3.128 \quad \int \frac{1}{x(a+bx^3+cx^6)} dx$$

Optimal. Leaf size=69

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}} - \frac{\log(a+bx^3+cx^6)}{6a} + \frac{\log(x)}{a}$$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}} - \frac{\log(a+bx^3+cx^6)}{6a} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^3 + c*x^6)),x]

[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]]/(3*a*Sqrt[b^2 - 4*a*c])) + Log[x]/a - Log[a + b*x^3 + c*x^6]/(6*a)

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a + bx^3 + cx^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(a + bx + cx^2)} dx, x, x^3 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^3 \right)}{3a} + \frac{\text{Subst} \left(\int \frac{-b-cx}{a+bx+cx^2} dx, x, x^3 \right)}{3a} \\
&= \frac{\log(x)}{a} - \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^3 \right)}{6a} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^3 \right)}{6a} \\
&= \frac{\log(x)}{a} - \frac{\log(a + bx^3 + cx^6)}{6a} + \frac{b \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^3 \right)}{3a} \\
&= \frac{b \tanh^{-1} \left(\frac{b+2cx^3}{\sqrt{b^2-4ac}} \right)}{3a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a + bx^3 + cx^6)}{6a}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a} - \frac{\text{RootSum}\left[\#1^6 c + \#1^3 b + a \&, \frac{\#1^3 c \log(x-\#1) + b \log(x-\#1)}{2\#1^3 c + b} \&\right]}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3 + c*x^6)),x]

[Out] Log[x]/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]*#1^3)/(b + 2*c*#1^3) &]/(3*a)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^3 + cx^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a + b*x^3 + c*x^6)),x]

[Out] IntegrateAlgebraic[1/(x*(a + b*x^3 + c*x^6)), x]

fricas [A] time = 1.41, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^6 + 2bcx^3 + b^2 - 2ac + (2cx^3 + b)\sqrt{b^2 - 4ac}}{cx^6 + bx^3 + a}\right) - (b^2 - 4ac) \log(cx^6 + bx^3 + a) + 6(b^2 - 4ac) \log(x)}{6(ab^2 - 4a^2c)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(\frac{-2cx^3 + b}{b^2 - 4ac}\right) - (b^2 - 4ac) \log(cx^6 + bx^3 + a) + 6(b^2 - 4ac) \log(x)}{6(ab^2 - 4a^2c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^2 - 4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]

giac [A] time = 1.00, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2cx^3 + b}{\sqrt{-b^2 + 4ac}}\right)}{3\sqrt{-b^2 + 4ac}a} - \frac{\log(cx^6 + bx^3 + a)}{6a} + \frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] $-1/3*b*\arctan((2*c*x^3 + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c}*a) - 1/6*\log(c*x^6 + b*x^3 + a)/a + \log(\text{abs}(x))/a$

maple [A] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}a} + \frac{\ln(x)}{a} - \frac{\ln(cx^6+bx^3+a)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(c*x^6+b*x^3+a),x)`

[Out] $\ln(x)/a - 1/6*\ln(c*x^6+b*x^3+a)/a - 1/3/a*b/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.92, size = 1362, normalized size = 19.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*x^3 + c*x^6)),x)`

[Out] $\log(x)/a + (\log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)) - (b*\text{atan}((3*(4*a*c - b^2)^2*(4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))))/(216*a^3*(4*a*c - b^2)^{(3/2)} + (9*b^4*c^3*(12*a*c - 3*b^2)^3)/(16*(9*a*b^2 - 36*a^2*c)^3*(4*a*c - b^2)^{(1/2)} - (3*b^6*c^3*(12*a*c - 3*b^2))/(16*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^{(3/2)} - (b*(12*a*c - 3*b^2)^2*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))))/(8*a*(9*a*b^2 - 36*a^2*c)^2*(4*a*c - b^2)^{(1/2)})))/(b^3*c^6*(49*a*c - 12*b^2) - (3*(4*a*c - b^2)^{(3/2)}*(4*b^5 + 29*a^2*b*c^2 - 23*a*b^3*c)*(((12*a*c - 3*b^2)^3*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))))/(8*(9*a*b^2 - 36*a^2*c)^3) - (b^7*c^3)/(48*a^3*(4*a*c - b^2)^2) - (b^2*(12*a*c - 3$

$$\begin{aligned}
& b^2) * (27 * b^3 * c^3 - (27 * a * b^3 * c^3 * (12 * a * c - 3 * b^2)) / (2 * (9 * a * b^2 - 36 * a^2 * c))) \\
&) / (24 * a^2 * (9 * a * b^2 - 36 * a^2 * c) * (4 * a * c - b^2)) + (9 * b^5 * c^3 * (12 * a * c - 3 * b^2) \\
&)^2 / (16 * a * (9 * a * b^2 - 36 * a^2 * c)^2 * (4 * a * c - b^2))) / (b^3 * c^6 * (49 * a * c - 12 * b^2) \\
&) + (48 * a^4 * x^3 * (((4 * b^4 + 7 * a^2 * c^2 - 15 * a * b^2 * c) * ((b^3 * (63 * b^2 * c^4 - ((\\
& 108 * b^4 * c^3 - 378 * a * b^2 * c^4) * (12 * a * c - 3 * b^2)) / (2 * (9 * a * b^2 - 36 * a^2 * c)))) / (\\
& 216 * a^3 * (4 * a * c - b^2)^{(3/2)}) + (b * (108 * b^4 * c^3 - 378 * a * b^2 * c^4) * (12 * a * c - 3 \\
& * b^2)^3) / (48 * a * (9 * a * b^2 - 36 * a^2 * c)^3 * (4 * a * c - b^2)^{(1/2)}) - (b^3 * (108 * b^4 * \\
& c^3 - 378 * a * b^2 * c^4) * (12 * a * c - 3 * b^2)) / (144 * a^3 * (9 * a * b^2 - 36 * a^2 * c) * (4 * a * c \\
& - b^2)^{(3/2)}) - (b * (63 * b^2 * c^4 - ((108 * b^4 * c^3 - 378 * a * b^2 * c^4) * (12 * a * c - \\
& 3 * b^2)) / (2 * (9 * a * b^2 - 36 * a^2 * c))) * (12 * a * c - 3 * b^2)^2) / (8 * a * (9 * a * b^2 - 36 * a^2 * c)^2 * (4 * a * c - b^2)^{(1/2)})) / (16 * a^4 * c^3 * (49 * a * c - 12 * b^2)) - ((4 * b^5 + 29 \\
& * a^2 * b * c^2 - 23 * a * b^3 * c) * (((63 * b^2 * c^4 - ((108 * b^4 * c^3 - 378 * a * b^2 * c^4) * (12 \\
& * a * c - 3 * b^2)) / (2 * (9 * a * b^2 - 36 * a^2 * c)))) * (12 * a * c - 3 * b^2)^3) / (8 * (9 * a * b^2 - \\
& 36 * a^2 * c)^3) - (b^4 * (108 * b^4 * c^3 - 378 * a * b^2 * c^4)) / (1296 * a^4 * (4 * a * c - b^2)^2) + (b^2 * (108 * b^4 * c^3 - 378 * a * b^2 * c^4) * (12 * a * c - 3 * b^2)^2) / (48 * a^2 * (9 * a * b^2 - 36 * a^2 * c)^2 * (4 * a * c - b^2)) - (b^2 * (63 * b^2 * c^4 - ((108 * b^4 * c^3 - 378 * a * b^2 * c^4) * (12 * a * c - 3 * b^2)) / (2 * (9 * a * b^2 - 36 * a^2 * c))) * (12 * a * c - 3 * b^2)) / (24 * a^2 * (9 * a * b^2 - 36 * a^2 * c) * (4 * a * c - b^2))) / (16 * a^4 * c^3 * (4 * a * c - b^2)^{(1/2)} * (49 * a * c - 12 * b^2)) * (4 * a * c - b^2)^2) / (b^3 * c^3))) / (3 * a * (4 * a * c - b^2)^{(1/2)})
\end{aligned}$$

sympy [B] time = 6.73, size = 253, normalized size = 3.67

$$\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)} - \frac{1}{6a} \right) \log \left(x^3 + \frac{-12a^2c \left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)} - \frac{1}{6a} \right) + 3ab^2 \left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)} - \frac{1}{6a} \right) - 2ac + b^2}{bc} \right) + \left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)} - \frac{1}{6a} \right) \log \left(x^3 + \frac{-12a^2c \left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)} - \frac{1}{6a} \right) + 3ab^2 \left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)} - \frac{1}{6a} \right) - 2ac + b^2}{bc} \right) + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**6+b*x**3+a),x)

[Out] $(-b\sqrt{-4ac+b^2})/(6a(4ac-b^2)) - 1/(6a)) * \log(x^3 + (-12a^2 * c * (-b\sqrt{-4ac+b^2})/(6a(4ac-b^2)) - 1/(6a)) + 3a * b^2 * (-b\sqrt{-4ac+b^2})/(6a(4ac-b^2)) - 1/(6a)) - 2ac + b^2) / (b * c)) + (b\sqrt{-4ac+b^2})/(6a(4ac-b^2)) - 1/(6a)) * \log(x^3 + (-12a^2 * c * (b\sqrt{-4ac+b^2})/(6a(4ac-b^2)) - 1/(6a)) + 3a * b^2 * (b\sqrt{-4ac+b^2})/(6a(4ac-b^2)) - 1/(6a)) - 2ac + b^2) / (b * c)) + \log(x)/a$

$$3.129 \quad \int \frac{1}{x^4(a+bx^3+cx^6)} dx$$

Optimal. Leaf size=89

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx^3 + cx^6)}{6a^2} - \frac{b \log(x)}{a^2} - \frac{1}{3ax^3}$$

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx^3 + cx^6)}{6a^2} - \frac{b \log(x)}{a^2} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3 + c*x^6)),x]

[Out] -1/(3*a*x^3) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^3 + c*x^6])/(6*a^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)(a + bx + cx^2)}{(a + bx + cx^2)^2}, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2cd - b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 709

$\text{Int}[\frac{(d + ex)^m}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Simp}[e(d + ex)^{m+1}/((m+1)(cd^2 - bde + ae^2)), x] + \text{Dist}[1/(cd^2 - bde + ae^2), \text{Int}[\frac{(d + ex)^{m+1} \text{Simp}[cd - b^2 - 4ac, x]}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{NeQ}[2cd - b^2 - 4ac, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 800

$\text{Int}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 1357

$\text{Int}[x^m (a + cx^{n2} + bx^n)^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)(a + bx + cx^2)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, m, n, p, x\} \ \&\& \ \text{EqQ}[n2, 2n] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(a+bx^3+cx^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(a+bx+cx^2)} dx, x, x^3 \right) \\
&= -\frac{1}{3ax^3} + \frac{\text{Subst} \left(\int \frac{-b-cx}{x(a+bx+cx^2)} dx, x, x^3 \right)}{3a} \\
&= -\frac{1}{3ax^3} + \frac{\text{Subst} \left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)} \right) dx, x, x^3 \right)}{3a} \\
&= -\frac{1}{3ax^3} - \frac{b \log(x)}{a^2} + \frac{\text{Subst} \left(\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx, x, x^3 \right)}{3a^2} \\
&= -\frac{1}{3ax^3} - \frac{b \log(x)}{a^2} + \frac{b \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^3 \right)}{6a^2} + \frac{(b^2-2ac) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^3 \right)}{6a^2} \\
&= -\frac{1}{3ax^3} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx^3+cx^6)}{6a^2} - \frac{(b^2-2ac) \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2cx \right)}{3a^2} \\
&= -\frac{1}{3ax^3} - \frac{(b^2-2ac) \tanh^{-1} \left(\frac{b+2cx^3}{\sqrt{b^2-4ac}} \right)}{3a^2 \sqrt{b^2-4ac}} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx^3+cx^6)}{6a^2}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 92, normalized size = 1.03

$$\frac{\text{RootSum} \left[\#1^6 c + \#1^3 b + a \&, \frac{\#1^3 bc \log(x-\#1) - ac \log(x-\#1) + b^2 \log(x-\#1)}{2\#1^3 c + b} \& \right]}{3a^2} - \frac{b \log(x)}{a^2} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)),x]

[Out] -1/3*1/(a*x^3) - (b*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 & , (b^2*Log[x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^3)/(b + 2*c*#1^3) &]/(3*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx^3+cx^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3 + c*x^6)),x]

[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^3 + c*x^6)), x]

fricas [A] time = 1.38, size = 293, normalized size = 3.29

$$\frac{\left((b^2 - 2ac)\sqrt{b^2 - 4ac}x^3 \log\left(\frac{2cx^6 + 2bx^3 + b^2 - 2ac + (2cx^3 + b)\sqrt{b^2 - 4ac}}{cx^6 + bx^3 + a}\right) - (b^3 - 4abc)x^3 \log(cx^6 + bx^3 + a) + 6(b^3 - 4abc)x^3 \log(x) + 2ab^2 - 8a^2c \right)}{6(a^2b^2 - 4a^2c)x^3} - \frac{2(b^2 - 2ac)\sqrt{-b^2 + 4ac}x^3 \arctan\left(\frac{(2cx^3 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) - (b^3 - 4abc)x^3 \log(cx^6 + bx^3 + a) + 6(b^3 - 4abc)x^3 \log(x) + 2ab^2 - 8a^2c}{6(a^2b^2 - 4a^2c)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] [-1/6*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^3 - 4*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3), -1/6*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3)]

giac [A] time = 1.14, size = 93, normalized size = 1.04

$$\frac{b \log(cx^6 + bx^3 + a)}{6a^2} - \frac{b \log(|x|)}{a^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^3 + b}{\sqrt{-b^2 + 4ac}}\right)}{3\sqrt{-b^2 + 4ac}a^2} + \frac{bx^3 - a}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] 1/6*b*log(c*x^6 + b*x^3 + a)/a^2 - b*log(abs(x))/a^2 + 1/3*(b^2 - 2*a*c)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*x^3 - a)/(a^2*x^3)

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{2c \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}a} + \frac{b^2 \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}a^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln(cx^6 + bx^3 + a)}{6a^2} - \frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(c*x^6+b*x^3+a),x)

[Out] -1/3/a/x^3 - b*ln(x)/a^2 + 1/6*b*ln(c*x^6 + b*x^3 + a)/a^2 - 2/3/a/(4*a*c - b^2)^(1/2)*arctan((2*c*x^3 + b)/(4*a*c - b^2)^(1/2))*c + 1/3/a^2/(4*a*c - b^2)^(1/2)*arctan((2*c*x^3 + b)/(4*a*c - b^2)^(1/2))*b^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.03, size = 4281, normalized size = 48.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^3 + c*x^6)),x)

[Out] (atan((48*a^8*x^3*((4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*((3*b^3 - 12*a*b*c)*((3*b^3 - 12*a*b*c)*((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))/(2*(36*a^3*c - 9*a^2*b^2)) + (((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2)/(6*a^2*(4*a*c - b^2)^(1/2)))/(2*(36*a^3*c - 9*a^2*b^2)) - (((((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2)/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2)/(6*a^2*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))))/(2*(36*a^3*c - 9*a^2*b^2)))/(2*(36*a^3*c - 9*a^2*b^2)) + (12*b*c^6)/a^3))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)^3*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(432*a^10*(4*a*c - b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2)))/(16*a^4*c^3*(a^2*c^2 - 12*b^4 + 48*a*b^2*c)) + ((4*b^6 - 2*a^3*c^3 + 33*a^2*b^2*c^2 - 24*a*b^4*c)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))

$$\begin{aligned}
& \left. \right) \Big/ (6a^2(4ac - b^2)^{1/2}) - ((3b^3 - 12abc)(2ac - b^2)(108a^4b^4c^3 - 378a^5b^2c^4)) \Big/ (12a^6(4ac - b^2)^{1/2}(36a^3c - 9a^2b^2)) \\
& \cdot (2ac - b^2) \Big/ (6a^2(4ac - b^2)^{1/2}) - ((3b^3 - 12abc)(2ac - b^2)^2(108a^4b^4c^3 - 378a^5b^2c^4)) \Big/ (72a^8(4ac - b^2)(36a^3c - 9a^2b^2)) \\
& \Big/ (2(36a^3c - 9a^2b^2)) - c^7/a^4 - ((3b^3 - 12abc) \cdot ((3b^3 - 12abc) \cdot ((42a^3c^6 + 33a^2b^2c^5)/a^4 + ((3b^3 - 12abc) \cdot ((252a^4b^4c^5 - 18a^3b^3c^4)/a^4 - ((3b^3 - 12abc) \cdot (108a^4b^4c^3 - 378a^5b^2c^4)) \Big/ (2a^4(36a^3c - 9a^2b^2)))) \Big/ (2(36a^3c - 9a^2b^2))) \Big/ (2(36a^3c - 9a^2b^2)) + (12b^6c^6)/a^3) \Big/ (2(36a^3c - 9a^2b^2)) + ((2ac - b^2) \cdot ((3b^3 - 12abc) \cdot ((2ac - b^2) \cdot ((252a^4b^4c^5 - 18a^3b^3c^4)/a^4 - ((3b^3 - 12abc) \cdot (108a^4b^4c^3 - 378a^5b^2c^4)) \Big/ (2a^4(36a^3c - 9a^2b^2)))) \Big/ (6a^2(4ac - b^2)^{1/2}) - ((3b^3 - 12abc)(2ac - b^2)(108a^4b^4c^3 - 378a^5b^2c^4)) \Big/ (12a^6(4ac - b^2)^{1/2}(36a^3c - 9a^2b^2))) \Big/ (2(36a^3c - 9a^2b^2)) + (((42a^3c^6 + 33a^2b^2c^5)/a^4 + ((3b^3 - 12abc) \cdot ((252a^4b^4c^5 - 18a^3b^3c^4)/a^4 - ((3b^3 - 12abc) \cdot (108a^4b^4c^3 - 378a^5b^2c^4)) \Big/ (2a^4(36a^3c - 9a^2b^2)))) \Big/ (2(36a^3c - 9a^2b^2))) \cdot (2ac - b^2) \Big/ (6a^2(4ac - b^2)^{1/2})) \Big/ (6a^2(4ac - b^2)^{1/2}) + ((2ac - b^2)^4(108a^4b^4c^3 - 378a^5b^2c^4)) \Big/ (1296a^{12}(4ac - b^2)^2) \Big/ (16a^4c^3(4ac - b^2)^{1/2}(a^2c^2 - 12b^4 + 48ab^2c)) \cdot (4ac - b^2)^2 \Big/ (8a^3c^6 - b^6c^3 + 6ab^4c^4 - 12a^2b^2c^5) + (3a^4(4ac - b^2)^2(4b^5 + 9a^2b^3c^2 - 16ab^3c) \cdot ((3b^3 - 12abc) \cdot ((3b^3 - 12abc) \cdot ((27a^3b^4c^3 - 27a^4b^2c^4)/a^4 - (27ab^3c^3(3b^3 - 12abc)) \Big/ (2(36a^3c - 9a^2b^2))) \cdot (2ac - b^2)) \Big/ (6a^2(4ac - b^2)^{1/2}) - (9b^3c^3(3b^3 - 12abc)(2ac - b^2)) \Big/ (4a(4ac - b^2)^{1/2}(36a^3c - 9a^2b^2))) \Big/ (2(36a^3c - 9a^2b^2)) - ((2ac - b^2) \cdot ((9a^3b^4c^5 - 27a^2b^3c^4)/a^4 - ((3b^3 - 12abc) \cdot ((27a^3b^4c^3 - 27a^4b^2c^4)/a^4 - (27ab^3c^3(3b^3 - 12abc)) \Big/ (2(36a^3c - 9a^2b^2)))) \Big/ (2(36a^3c - 9a^2b^2))) \Big/ (6a^2(4ac - b^2)^{1/2})) \Big/ (2(36a^3c - 9a^2b^2)) - (((a^2c^6 - 9ab^2c^5)/a^4 + ((3b^3 - 12abc) \cdot ((9a^3b^4c^5 - 27a^2b^3c^4)/a^4 - ((3b^3 - 12abc) \cdot ((27a^3b^4c^3 - 27a^4b^2c^4)/a^4 - (27ab^3c^3(3b^3 - 12abc)) \Big/ (2(36a^3c - 9a^2b^2)))) \Big/ (2(36a^3c - 9a^2b^2))) \Big/ (2(36a^3c - 9a^2b^2))) \cdot (2ac - b^2) \Big/ (6a^2(4ac - b^2)^{1/2}) - (((((((27a^3b^4c^3 - 27a^4b^2c^4)/a^4 - (27ab^3c^3(3b^3 - 12abc)) \Big/ (2(36a^3c - 9a^2b^2))) \cdot (2ac - b^2)) \Big/ (6a^2(4ac - b^2)^{1/2}) - (9b^3c^3(3b^3 - 12abc)(2ac - b^2)) \Big/ (4a(4ac - b^2)^{1/2}(36a^3c - 9a^2b^2))) \cdot (2ac - b^2) \Big/ (6a^2(4ac - b^2)^{1/2}) - (3b^3c^3(3b^3 - 12abc)(2ac - b^2)^2) \Big/ (8a^3(4ac - b^2)(36a^3c - 9a^2b^2))) \cdot (2ac - b^2) \Big/ (6a^2(4ac - b^2)^{1/2}) + (b^3c^3(3b^3 - 12abc)(2ac - b^2)^3) \Big/ (16a^5(4ac - b^2)^{3/2}(36a^3c - 9a^2b^2))) \Big/ (c^3(a^2c^2 - 12b^4 + 48ab^2c) \cdot (8a^3c^6 - b^6c^3 + 6ab^4c^4 - 12a^2b^2c^5)) + (3a^4(4ac - b^2)^{3/2}(4b^6 - 2a^3c^3 + 33a^2b^2c^2 - 24ab^4c) \cdot ((3b^3 - 12abc) \cdot (((((((27a^3b^4c^3 - 27a^4b^2c^4)/a^4 - (27ab^3c^3(3b^3 - 12abc)) \Big/ (2(36a^3c - 9a^2b^2))) \cdot (2ac - b^2)) \Big/ (6
\end{aligned}$$

$$\begin{aligned} & a^2(4ac - b^2)^{1/2} - (9b^3c^3(3b^3 - 12abc)(2ac - b^2))/(4 \\ & a(4ac - b^2)^{1/2}(36a^3c - 9a^2b^2))(2ac - b^2)/(6a^2(4ac \\ & c - b^2)^{1/2}) - (3b^3c^3(3b^3 - 12abc)(2ac - b^2)^2)/(8a^3(4 \\ & ac - b^2)(36a^3c - 9a^2b^2)))/(2(36a^3c - 9a^2b^2)) - (b^6)/a \\ & ^4 + ((3b^3 - 12abc)((a^2c^6 - 9ab^2c^5)/a^4 + ((3b^3 - 12abc) \\ & *((9a^3b^5c - 27a^2b^3c^4)/a^4 - ((3b^3 - 12abc)((27a^3b^4c^3 \\ & - 27a^4b^2c^4)/a^4 - (27ab^3c^3(3b^3 - 12abc))/(2(36a^3c - 9 \\ & a^2b^2)))))/(2(36a^3c - 9a^2b^2)))/(2(36a^3c - 9a^2b^2)))/(2(\\ & 36a^3c - 9a^2b^2)) + ((2ac - b^2)((3b^3 - 12abc)((27a^3b^4 \\ & c^3 - 27a^4b^2c^4)/a^4 - (27ab^3c^3(3b^3 - 12abc))/(2(36a^3c \\ & - 9a^2b^2)))(2ac - b^2))/(6a^2(4ac - b^2)^{1/2}) - (9b^3c^3(3 \\ & b^3 - 12abc)(2ac - b^2))/(4a(4ac - b^2)^{1/2}(36a^3c - 9a^2b \\ & ^2)))/(2(36a^3c - 9a^2b^2)) - ((2ac - b^2)((9a^3b^5c - 27a^2b \\ & ^3c^4)/a^4 - ((3b^3 - 12abc)((27a^3b^4c^3 - 27a^4b^2c^4)/a^4 - \\ & (27ab^3c^3(3b^3 - 12abc))/(2(36a^3c - 9a^2b^2)))))/(2(36a^3c \\ & - 9a^2b^2)))/(6a^2(4ac - b^2)^{1/2}))/((6a^2(4ac - b^2)^{1/2}) \\ & + (b^3c^3(2ac - b^2)^4)/(48a^7(4ac - b^2)^2))/(c^3(a^2c^2 - 12b \\ & ^4 + 48ab^2c)(8a^3c^6 - b^6c^3 + 6ab^4c^4 - 12a^2b^2c^5))(2 \\ & ac - b^2)/(3a^2(4ac - b^2)^{1/2}) - (b \log(x))/a^2 - (\log(a + bx^3 + \\ & cx^6)(3b^3 - 12abc))/(2(36a^3c - 9a^2b^2)) - 1/(3ax^3) \end{aligned}$$

sympy [B] time = 113.79, size = 345, normalized size = 3.88

$$\left(\frac{b}{6a^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6a^2(4ac - b^2)} \right) \log \left(x^3 + \frac{-12a^3c \left(\frac{b}{6a^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6a^2(4ac - b^2)} \right) + 3a^2b^2 \left(\frac{b}{6a^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6a^2(4ac - b^2)} \right) + 3abc - b^3}{2ac^2 - b^2c} \right) + \left(\frac{b}{6a^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6a^2(4ac - b^2)} \right) \log \left(x^3 + \frac{-12a^3c \left(\frac{b}{6a^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6a^2(4ac - b^2)} \right) + 3a^2b^2 \left(\frac{b}{6a^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{6a^2(4ac - b^2)} \right) + 3abc - b^3}{2ac^2 - b^2c} \right) - \frac{1}{3ax^3} - \frac{b \log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(c*x**6+b*x**3+a),x)

[Out] (b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2)))*1
 og(x**3 + (-12*a**3*c*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a
 2*(4*a*c - b2))) + 3*a**2*b**2*(b/(6*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) +
 (b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a**2*(4*a*c - b**2)))*1
 og(x**3 + (-12*a**3*c*(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*a
 2*(4*a*c - b2))) + 3*a**2*b**2*(b/(6*a**2) + sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(6*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) -
 1/(3*a*x**3) - b*log(x)/a**2

$$3.130 \quad \int \frac{x^7}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=636

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2-4ac}}}$$

Rubi [A] time = 1.25, antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, number of rules / integrand size = 0.444, Rules used = {1367, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2-4ac}}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2-4ac}}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2-4ac}}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(\sqrt{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2-4ac}}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b - \sqrt{b^2-4ac}}}\right)}{2^{2/3} \sqrt[3]{c} \sqrt[3]{b - \sqrt{b^2-4ac}}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{1 - \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b - \sqrt{b^2-4ac}}}\right)}{2^{2/3} \sqrt[3]{c} \sqrt[3]{b - \sqrt{b^2-4ac}}} + \frac{x^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b*x^3 + c*x^6), x]

[Out] $x^2/(2*c) + ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTan}[(1 - (2*2^{(1/3)} * c^{(1/3)} * x) / (b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) / \text{Sqrt}[3]]) / (2^{(2/3)} * \text{Sqrt}[3] * c^{(5/3)} * (b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) + ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTan}[(1 - (2*2^{(1/3)} * c^{(1/3)} * x) / (b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) / \text{Sqrt}[3]]) / (2^{(2/3)} * \text{Sqrt}[3] * c^{(5/3)} * (b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) + ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c]) * \text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)} * c^{(1/3)} * x]) / (3 * 2^{(2/3)} * c^{(5/3)} * (b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) + ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c]) * \text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)} * c^{(1/3)} * x]) / (3 * 2^{(2/3)} * c^{(5/3)} * (b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) - ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c]) * \text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} - 2^{(1/3)} * c^{(1/3)} * (b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} * x + 2^{(2/3)} * c^{(2/3)} * x^2]) / (6 * 2^{(2/3)} * c^{(5/3)} * (b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) - ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c]) * \text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} - 2^{(1/3)} * c^{(1/3)} * (b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} * x + 2^{(2/3)} * c^{(2/3)} * x^2]) / (6 * 2^{(2/3)} * c^{(5/3)} * (b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})$

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(n-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1367

Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^(p), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]

Rule 1510

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_)))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b

, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^7}{a + bx^3 + cx^6} dx &= \frac{x^2}{2c} - \frac{\int \frac{x(2a+2bx^3)}{a+bx^3+cx^6} dx}{2c} \\
 &= \frac{x^2}{2c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{x}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{x}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2c} \\
 &= \frac{x^2}{2c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} c^{4/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x}{\frac{(b-\sqrt{b^2-4ac})^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c} \sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + c^{2/3}} dx}{3 \cdot 2^{2/3} c^{4/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} \\
 &= \frac{x^2}{2c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b + \sqrt{b^2 - 4ac}}} \\
 &= \frac{x^2}{2c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b + \sqrt{b^2 - 4ac}}} \\
 &= \frac{x^2}{2c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3} c^{5/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3} c^{5/3} \sqrt[3]{b + \sqrt{b^2 - 4ac}}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.11

$$\frac{3x^2 - 2\text{RootSum}\left[\#1^6 c + \#1^3 b + a\&, \frac{\#1^3 b \log(x-\#1) + a \log(x-\#1)}{2\#1^4 c + \#1 b}\&\right]}{6c}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b*x^3 + c*x^6),x]

[Out] (3*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) &])/(6*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(a + b*x^3 + c*x^6),x]

[Out] IntegrateAlgebraic[x^7/(a + b*x^3 + c*x^6), x]

fricas [B] time = 4.09, size = 5601, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/6*(4*\sqrt{3}*(1/2)^{(1/3)}*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(1/3)}*\arctan(-1/3*((1/2)^{(5/6)}*(\sqrt{3}*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*b^2*c^7 - 32*a^3*c^8)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})) - \sqrt{3}*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(1/3)}*\sqrt{((2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 + (1/2)^{(2/3)}*((b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})) - (b^{10} - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(2/3)} - (1/2)^{(1/3)}*(a^2*b^7 - 9*a^3*b^5*c + 25*a^4*b^3*c^2 - 20*a^5*b*c^3 - (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(1/3)})/(a^3*b^5 - 5*a^4*b^3 \end{aligned}$$

$$\begin{aligned}
& *c + 5*a^5*b*c^2)) - (1/2)^{(1/3)}*(\text{sqrt}(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*b^2*c^7 - 32*a^3*c^8)*x*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))) - \text{sqrt}(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3)*x*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))))/(b^2*c^5 - 4*a*c^6))^{(1/3)} + \text{sqrt}(3)*(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)/(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2) - 4*\text{sqrt}(3)*(1/2)^{(1/3)}*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))))/(b^2*c^5 - 4*a*c^6))^{(1/3)}*\arctan(-1/3*((1/2)^{(5/6)}*(\text{sqrt}(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*b^2*c^7 - 32*a^3*c^8)*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))) + \text{sqrt}(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))))/(b^2*c^5 - 4*a*c^6))^{(1/3)}*\text{sqrt}((2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 - (1/2)^{(2/3)}*((b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))) + (b^{10} - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))))/(b^2*c^5 - 4*a*c^6))^{(2/3)} - (1/2)^{(1/3)}*(a^2*b^7 - 9*a^3*b^5*c + 25*a^4*b^3*c^2 - 20*a^5*b*c^3 + (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))))/(b^2*c^5 - 4*a*c^6))^{(1/3)}(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2) - (1/2)^{(1/3)}*(\text{sqrt}(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*b^2*c^7 - 32*a^3*c^8)*x*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))) + \text{sqrt}(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3)*x*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))))/(b^2*c^5 - 4*a*c^6))^{(1/3)} - \text{sqrt}(3)*(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)/(a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2) + (1/2)^{(1/3)}*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))))/(b^2*c^5 - 4*a*c^6))^{(1/3)}*\log(2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 + (1/2)^{(2/3)}*((b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*\text{sqrt}((b
\end{aligned}$$

$$\begin{aligned}
& ^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c \\
& ^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})) - (b^{10} - 12*a*b^8*c \\
& + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x*((b^4 - 3*a*b^2*c + \\
& a^2*c^2 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50 \\
& *a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} \\
& - 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(2/3)} - (1/2)^{(1/3)}*(a^2*b^7 - 9*a^3* \\
& b^5*c + 25*a^4*b^3*c^2 - 20*a^5*b*c^3 - (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a \\
& ^4*b*c^7)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^ \\
& 4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*((b \\
& ^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35 \\
& *a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + \\
& 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(1/3)} + (1/2)^{(1/3)} \\
& *c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8* \\
& c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4* \\
& c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(1/3)}*\log(2*(a \\
& ^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x^2 - (1/2)^{(2/3)}*((b^8*c^5 - 13*a*b^6* \\
& c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*x*\sqrt{(b^{10} - 10*a*b^ \\
& 8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^ \\
& 4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})) + (b^{10} - 12*a*b^8*c + 52*a^2*b^6* \\
& c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^ \\
& 2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 \\
& + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13} \\
&)))/(b^2*c^5 - 4*a*c^6))^{(2/3)} - (1/2)^{(1/3)}*(a^2*b^7 - 9*a^3*b^5*c + 25*a^ \\
& 4*b^3*c^2 - 20*a^5*b*c^3 + (a^2*b^5*c^5 - 8*a^3*b^3*c^6 + 16*a^4*b*c^7)*\sqrt{ \\
& t((b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b \\
& ^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*((b^4 - 3*a*b^2* \\
& c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 \\
& - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^ \\
& ^{12} - 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(1/3)} - 2*(1/2)^{(1/3)}*c*((b^4 - \\
& 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2* \\
& b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a \\
& ^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(1/3)}*\log((1/2)^{(2/3)}*(b^ \\
& 10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4 - (b^8*c \\
& ^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - 112*a^3*b^2*c^8 + 64*a^4*c^9)*\sqrt{(b^ \\
& 10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^ \\
& 10 - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*((b^4 - 3*a*b^2*c + a \\
& ^2*c^2 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50* \\
& a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - \\
& 64*a^3*c^{13})))/(b^2*c^5 - 4*a*c^6))^{(2/3)} + 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a \\
& ^5*b*c^2)*x - 2*(1/2)^{(1/3)}*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a \\
& *c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^ \\
& 2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^2*c^ \\
& 5 - 4*a*c^6))^{(1/3)}*\log((1/2)^{(2/3)}*(b^{10} - 12*a*b^8*c + 52*a^2*b^6*c^2 - 9 \\
& 5*a^3*b^4*c^3 + 60*a^4*b^2*c^4 + (b^8*c^5 - 13*a*b^6*c^6 + 60*a^2*b^4*c^7 - \\
& 112*a^3*b^2*c^8 + 64*a^4*c^9)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 5
\end{aligned}$$

$$\frac{0*a^3*b^4*c^3 + 25*a^4*b^2*c^4}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))} * ((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*\sqrt{(b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))}) / (b^2*c^5 - 4*a*c^6))^{2/3} + 2*(a^3*b^5 - 5*a^4*b^3*c + 5*a^5*b*c^2)*x - 3*x^2)/c$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate(x^7/(c*x^6 + b*x^3 + a), x)

maple [C] time = 0.14, size = 61, normalized size = 0.10

$$\frac{x^2}{2c} - \frac{\left(\text{RootOf}(c_Z^6 + b_Z^3 + a)^4 b + \text{RootOf}(c_Z^6 + b_Z^3 + a) a\right) \ln\left(-\text{RootOf}(c_Z^6 + b_Z^3 + a) + x\right)}{3c \left(2 \text{RootOf}(c_Z^6 + b_Z^3 + a)^5 c + \text{RootOf}(c_Z^6 + b_Z^3 + a)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(c*x^6+b*x^3+a),x)

[Out] 1/2*x^2/c - 1/3/c*sum((_R^4*b+_R*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{2c} - \frac{\int \frac{bx^4+ax}{cx^6+bx^3+a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] 1/2*x^2/c - integrate((b*x^4 + a*x)/(c*x^6 + b*x^3 + a), x)/c

mupad [B] time = 12.15, size = 4069, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(a + b*x^3 + c*x^6),x)

[Out] $\log\left(\left(2^{\frac{1}{3}}\right)\left(\left(2^{\frac{2}{3}}\right)\left(27a^2c^2x(b^4 + 8a^2c^2 - 6ab^2c) + (27\cdot 2^{\frac{1}{3}})ab^3c^3(4ac - b^2)^2(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{2}{3}}\right)/2\left(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{1}{3}}\right)/6 - (9ab(b^6 - 12a^3c^3 + 19a^2b^2c^2 - 8ab^4c))/c^2\left(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{2}{3}}\right)/18 + (a^4xx(ac - b^2))/c^2\left(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(54(64a^3c^8 - b^6c^5 + 12ab^4c^6 - 48a^2b^2c^7)\right)^{\frac{1}{3}} + \log\left(\left(2^{\frac{1}{3}}\right)\left(\left(2^{\frac{2}{3}}\right)\left(27a^2c^2x(b^4 + 8a^2c^2 - 6ab^2c) + (27\cdot 2^{\frac{1}{3}})ab^3c^3(4ac - b^2)^2(-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} + 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{2}{3}}\right)/2\left(-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} + 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{1}{3}}\right)/6 - (9ab(b^6 - 12a^3c^3 + 19a^2b^2c^2 - 8ab^4c))/c^2\left(-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} + 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{2}{3}}\right)/18 + (a^4xx(ac - b^2))/c^2\left(-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} + 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(54(64a^3c^8 - b^6c^5 + 12ab^4c^6 - 48a^2b^2c^7)\right)^{\frac{1}{3}} + x^2/(2c) - \log\left(\left(a^4xx(ac - b^2)\right)/c^2 - \left(2^{\frac{1}{3}}\right)\left(3^{\frac{1}{2}}\right)\left(1i - 1\right)\left(\left(2^{\frac{2}{3}}\right)\left(3^{\frac{1}{2}}\right)\left(1i + 1\right)\left(27a^2c^2x(b^4 + 8a^2c^2 - 6ab^2c) + (27\cdot 2^{\frac{1}{3}})ab^3c^3(3^{\frac{1}{2}}\right)\left(1i - 1\right)\left(4ac - b^2\right)^2(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{2}{3}}\right)/4\left(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{1}{3}}\right)/12 + (9ab(b^6 - 12a^3c^3 + 19a^2b^2c^2 - 8ab^4c))/c^2\left(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{2}{3}}\right)/36\left(\left(3^{\frac{1}{2}}\right)\left(1i\right)\right)/2 + 1/2\left(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(54(64a^3c^8 - b^6c^5 + 12ab^4c^6 - 48a^2b^2c^7)\right)^{\frac{1}{3}} + \log\left(\left(a^4xx(ac - b^2)\right)/c^2 - \left(2^{\frac{1}{3}}\right)\left(3^{\frac{1}{2}}\right)\left(1i + 1\right)\left(\left(2^{\frac{2}{3}}\right)\left(3^{\frac{1}{2}}\right)\left(1i - 1\right)\left(27a^2c^2x(b^4 + 8a^2c^2 - 6ab^2c) + (27\cdot 2^{\frac{1}{3}})ab^3c^3(3^{\frac{1}{2}}\right)\left(1i - 1\right)\left(4ac - b^2\right)^2(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{2}{3}}\right)/4\left(-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{\frac{1}{2}} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{\frac{1}{2}} - 5ab^3c^3(-4ac - b^2)^3)^{\frac{1}{2}}\right)/\left(c^5(4ac - b^2)^3\right)^{\frac{1}{3}}\right)$

$$\begin{aligned}
& c^2 - 6ab^2c) - (27 \cdot 2^{1/3} a^2 b^2 c^3 (3^{1/2} i + 1) (4ac - b^2)^2 (-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2}) / (c^5(4ac - b^2)^3)^{2/3} / 4) * (-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2}) / (c^5(4ac - b^2)^3)^{1/3} / 12 - (9ab(b^6 - 12a^3c^3 + 19a^2b^2c^2 - 8ab^4c) / c^2) * (-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2}) / (c^5(4ac - b^2)^3)^{2/3} / 36) * ((3^{1/2} i) / 2 - 1/2) * (-b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2}) / (54(64a^3c^8 - b^6c^5 + 12ab^4c^6 - 48a^2b^2c^7)))^{1/3} - \log((a^4 x (ac - b^2)) / c^2 - (2^{1/3} (3^{1/2} i - 1) ((2^{2/3} (3^{1/2} i + 1) (27a^2 c x (b^4 + 8a^2c^2 - 6ab^2c) + (27 \cdot 2^{1/3} a^2 b^2 c^3 (3^{1/2} i - 1) (4ac - b^2)^2 (-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (c^5(4ac - b^2)^3)^{2/3} / 4) * (-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (c^5(4ac - b^2)^3)^{1/3} / 12 + (9ab(b^6 - 12a^3c^3 + 19a^2b^2c^2 - 8ab^4c) / c^2) * (-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (c^5(4ac - b^2)^3)^{2/3} / 36) * ((3^{1/2} i) / 2 + 1/2) * (-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (54(64a^3c^8 - b^6c^5 + 12ab^4c^6 - 48a^2b^2c^7)))^{1/3} + \log((a^4 x (ac - b^2)) / c^2 - (2^{1/3} (3^{1/2} i + 1) ((2^{2/3} (3^{1/2} i - 1) (27a^2 c x (b^4 + 8a^2c^2 - 6ab^2c) - (27 \cdot 2^{1/3} a^2 b^2 c^3 (3^{1/2} i + 1) (4ac - b^2)^2 (-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (c^5(4ac - b^2)^3)^{2/3} / 4) * (-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (c^5(4ac - b^2)^3)^{1/3} / 12 - (9ab(b^6 - 12a^3c^3 + 19a^2b^2c^2 - 8ab^4c) / c^2) * (-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (c^5(4ac - b^2)^3)^{2/3} / 36) * ((3^{1/2} i) / 2 - 1/2) * (-b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (54(64a^3c^8 - b^6c^5 + 12ab^4c^6 - 48a^2b^2c^7)))^{1/3}
\end{aligned}$$

sympy [A] time = 14.58, size = 279, normalized size = 0.44

$\text{RootSum}\left(t^6(46656a^3c^8 - 34992a^2b^2c^7 + 8748ab^4c^6 - 729b^6c^5) + t^3(432a^4c^4 - 1512a^3b^2c^3 + 1107a^2b^4c^2 - 297ab^6c + 27b^8) + a^5, \left(t + \log\left(x + \frac{-15552t^5a^4c^9 + 27216t^5a^3b^2c^8 - 14580t^5a^2b^4c^7 + 3159t^5ab^6c^6 - 243t^5b^8c^5 + 594t^5a^4b^2c^4 - 864t^5a^3b^4c^3 + 468t^5a^2b^6c^2 - 108t^5ab^8c + 9t^5b^{10}}{5a^5b^2c^2 - 5a^4b^3c + a^3b^5}\right)\right)\right) + \frac{x^2}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(c*x**6+b*x**3+a),x)

[Out] $\text{RootSum}(_t**6*(46656*a**3*c**8 - 34992*a**2*b**2*c**7 + 8748*a*b**4*c**6 - 729*b**6*c**5) + _t**3*(432*a**4*c**4 - 1512*a**3*b**2*c**3 + 1107*a**2*b**4*c**2 - 297*a*b**6*c + 27*b**8) + a**5, \text{Lambda}(_t, _t*\log(x + (-15552*_t**5*a**4*c**9 + 27216*_t**5*a**3*b**2*c**8 - 14580*_t**5*a**2*b**4*c**7 + 3159*_t**5*a*b**6*c**6 - 243*_t**5*b**8*c**5 - 72*_t**2*a**5*c**5 + 594*_t**2*a**4*b**2*c**4 - 864*_t**2*a**3*b**4*c**3 + 468*_t**2*a**2*b**6*c**2 - 108*_t**2*a*b**8*c + 9*_t**2*b**10)/(5*a**5*b*c**2 - 5*a**4*b**3*c + a**3*b**5))) + x**2/(2*c)$

$$3.131 \quad \int \frac{x^6}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=631

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x\right)}{6\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \dots$$

Rubi [A] time = 1.02, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, number of rules / integrand size = 0.444, Rules used = {1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \dots$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b*x^3 + c*x^6), x]

[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R

$t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[\{a, b\}, x]$

Rule 204

$Int[((a_) + (b_)*(x_)^2)^{-1}, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[\{a, b\}, x] \&\& PosQ[a/b] \&\& (LtQ[a, 0] || LtQ[b, 0])$

Rule 617

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] := With[\{q = 1 - 4*Simplify[(a*c)/b^2]\}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] \&\& (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[\{a, b, c\}, x] \&\& NeQ[b^2 - 4*a*c, 0]$

Rule 628

$Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& EqQ[2*c*d - b*e, 0]$

Rule 634

$Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& NeQ[2*c*d - b*e, 0] \&\& NeQ[b^2 - 4*a*c, 0] \&\& !NiceSqrtQ[b^2 - 4*a*c]$

Rule 1367

$Int[((d_)*(x_)^m)*((a_) + (c_)*(x_)^{n2_}) + (b_)*(x_)^{n_})^p, x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[\{a, b, c, d, p\}, x] \&\& EqQ[n2, 2*n] \&\& NeQ[b^2 - 4*a*c, 0] \&\& IGtQ[n, 0] \&\& GtQ[m, 2*n - 1] \&\& NeQ[m + 2*n*p + 1, 0] \&\& IntegerQ[p]$

Rule 1422

$Int[((d_) + (e_)*(x_)^n)/((a_) + (b_)*(x_)^n + (c_)*(x_)^{n2_}), x_Symbol] := With[\{q = Rt[b^2 - 4*a*c, 2]\}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[\{a, b, c, d, e, n\}, x] \&\& EqQ[n2, 2*n]$

&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^6}{a + bx^3 + cx^6} dx &= \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6} dx}{c} \\
 &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2c} \\
 &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\sqrt[3]{2}c\left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}} - \sqrt[3]{c}x}{\frac{(b-\sqrt{b^2-4ac})^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}} dx}{3\sqrt[3]{2}c\left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
 &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}} - \sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}} \\
 &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}} - \sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}} \\
 &= \frac{x}{c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{c}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1^3b \log(x-\#1) + a \log(x-\#1)}{2\#1^5c + \#1^2b}\&\right]}{3c}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b*x^3 + c*x^6),x]

[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) &]/(3*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(a + b*x^3 + c*x^6),x]

[Out] IntegrateAlgebraic[x^6/(a + b*x^3 + c*x^6), x]

fricas [B] time = 3.18, size = 5260, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8))*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/6)*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3)*sqrt((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*

$$\begin{aligned} & \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})} - (ab^6 - 8a^2b^4c + 18a^3b^2c^2 - 8a^4c^3)x * (-b^3 - 2ab^2c + (b^2c^4 - 4a^3c^5) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5)^{1/3} \\ & \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5)^{1/3} \\ & \sqrt{(a^2b^4 - 4a^3b^2c + 2a^4c^2))} + 2\sqrt{3} * (a^3b^4 - 4a^4b^2c + 2a^5c^2) / (a^3b^4 - 4a^4b^2c + 2a^5c^2) - 4\sqrt{3} * (1/2)^{1/3} * c * (-b^3 - 2ab^2c - (b^2c^4 - 4a^3c^5) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5)^{1/3} * \arctan(-1/6 * (2 * (1/2)^{2/3}) * (\sqrt{3} * (b^8c^4 - 13ab^6c^5 + 60a^2b^4c^6 - 112a^3b^2c^7 + 64a^4c^8) * x * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) + \sqrt{3} * (b^9 - 11ab^7c + 42a^2b^5c^2 - 62a^3b^3c^3 + 24a^4b^2c^4) * x) * (-b^3 - 2ab^2c - (b^2c^4 - 4a^3c^5) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5)^{2/3} - (1/2)^{1/6} * (\sqrt{3} * (b^8c^4 - 13ab^6c^5 + 60a^2b^4c^6 - 112a^3b^2c^7 + 64a^4c^8) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) + \sqrt{3} * (b^9 - 11ab^7c + 42a^2b^5c^2 - 62a^3b^3c^3 + 24a^4b^2c^4) * (-b^3 - 2ab^2c - (b^2c^4 - 4a^3c^5) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5))^{2/3} * \sqrt{(2 * (a^2b^4 - 4a^3b^2c + 2a^4c^2) * x^2 + (1/2)^{2/3} * (b^8 - 10ab^6c + 34a^2b^4c^2 - 44a^3b^2c^3 + 16a^4c^4 + (b^7c^4 - 12ab^5c^5 + 48a^2b^3c^6 - 64a^3b^2c^7) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})})) * (-b^3 - 2ab^2c - (b^2c^4 - 4a^3c^5) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5))^{2/3} - (1/2)^{1/3} * ((ab^5c^4 - 8a^2b^3c^5 + 16a^3b^2c^6) * x * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) + (ab^6 - 8a^2b^4c + 18a^3b^2c^2 - 8a^4c^3) * x) * (-b^3 - 2ab^2c - (b^2c^4 - 4a^3c^5) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5))^{1/3} / (a^2b^4 - 4a^3b^2c + 2a^4c^2) - 2\sqrt{3} * (a^3b^4 - 4a^4b^2c + 2a^5c^2) / (a^3b^4 - 4a^4b^2c + 2a^5c^2) - (1/2)^{1/3} * c * (-b^3 - 2ab^2c + (b^2c^4 - 4a^3c^5) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5)^{1/3} \\ & \sqrt{(a^2b^4 - 4a^3b^2c + 2a^4c^2) * x^2 + (1/2)^{2/3} * (b^8 - 10ab^6c + 34a^2b^4c^2 - 44a^3b^2c^3 + 16a^4c^4 - (b^7c^4 - 12ab^5c^5 + 48a^2b^3c^6 - 64a^3b^2c^7) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})})) * (-b^3 - 2ab^2c + (b^2c^4 - 4a^3c^5) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5))^{1/3} * \log(2 * (a^2b^4 - 4a^3b^2c + 2a^4c^2) * x^2 + (1/2)^{2/3} * (b^8 - 10ab^6c + 34a^2b^4c^2 - 44a^3b^2c^3 + 16a^4c^4 - (b^7c^4 - 12ab^5c^5 + 48a^2b^3c^6 - 64a^3b^2c^7) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})})) * (-b^3 - 2ab^2c + (b^2c^4 - 4a^3c^5) * \sqrt{(b^8 - 8ab^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/(b^6c^8 - 12ab^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})}) / (b^2c^4 - 4a^3c^5))^{1/3} \end{aligned}$$

$$\begin{aligned}
& *b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11))/ (b^2*c^4 - 4*a*c^5))^{(2/3)} + (1/2)^{(1/3)}*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11))} - (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/ (b^2*c^4 - 4*a*c^5))^{(1/3)}) - (1/2)^{(1/3)}*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/ (b^2*c^4 - 4*a*c^5))^{(1/3)}*\log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^{(2/3)}*(b^8 - 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/ (b^2*c^4 - 4*a*c^5))^{(2/3)} - (1/2)^{(1/3)}*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11))} + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/ (b^2*c^4 - 4*a*c^5))^{(1/3)}) + 2*(1/2)^{(1/3)}*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/ (b^2*c^4 - 4*a*c^5))^{(1/3)}*\log(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^{(1/3)}*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/ (b^2*c^4 - 4*a*c^5))^{(1/3)}) + 2*(1/2)^{(1/3)}*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/ (b^2*c^4 - 4*a*c^5))^{(1/3)}*\log(2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x + (1/2)^{(1/3)}*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/ (b^2*c^4 - 4*a*c^5))^{(1/3)}) + 6*x)/c
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate(x^6/(c*x^6 + b*x^3 + a), x)

maple [C] time = 0.00, size = 59, normalized size = 0.09

$$\frac{x}{c} + \frac{\left(-\text{RootOf}(c_Z^6 + b_Z^3 + a)^3 b - a\right) \ln\left(-\text{RootOf}(c_Z^6 + b_Z^3 + a) + x\right)}{3c \left(2 \text{RootOf}(c_Z^6 + b_Z^3 + a)^5 c + \text{RootOf}(c_Z^6 + b_Z^3 + a)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(c*x^6+b*x^3+a),x)

[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c} - \frac{\int \frac{bx^3+a}{cx^6+bx^3+a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c

mupad [B] time = 3.40, size = 2280, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(a + b*x^3 + c*x^6),x)

[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(4*a*c

$$\begin{aligned}
& - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} + 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (54(64 \\
& a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} + x/c + \log((3a^2x(b^4 + 2a^2c^2 - 4ab^2c)) / c + (3^{2/3})a((b^7 + b^4(-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (c^4(4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b(-4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c) / (4c(4ac - b^2)) * ((b^7 + b^4(-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (54(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} + \log((3a^2x(b^4 + 2a^2c^2 - 4ab^2c)) / c + (3^{2/3})a(3^{(1/2)}i - 1)((b^7 + b^4(-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (c^4(4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b(-4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c) / (8c(4ac - b^2)) * ((3^{(1/2)}i) / 2 - 1/2) * ((b^7 + b^4(-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (54(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} - \log((3a^2x(b^4 + 2a^2c^2 - 4ab^2c)) / c - (3^{2/3})a(3^{(1/2)}i + 1)((b^7 + b^4(-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (c^4(4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b(-4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c) / (8c(4ac - b^2)) * ((3^{(1/2)}i) / 2 + 1/2) * ((b^7 + b^4(-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (54(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} + \log((3a^2x(b^4 + 2a^2c^2 - 4ab^2c)) / c - (3^{2/3})a(3^{(1/2)}i - 1)((b^4(-4ac - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} + 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (c^4(4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b(-4ac - b^2)^3)^{(1/2)} + b^4 + 16a^2c^2 - 8ab^2c) / (8c(4ac - b^2)) * ((3^{(1/2)}i) / 2 - 1/2) * ((b^4(-4ac - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} + 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (54(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} - \log((3a^2x(b^4 + 2a^2c^2 - 4ab^2c)) / c + (3^{2/3})a(3^{(1/2)}i + 1)((b^4(-4ac - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} + 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (c^4(4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b(-4ac - b^2)^3)^{(1/2)} + b^4 + 16a^2c^2 - 8ab^2c) / (8c(4ac - b^2)) * ((3^{(1/2)}i) / 2 + 1/2) * ((b^4(-4ac - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2(-4ac - b^2)^3)^{(1/2)} + 10ab^5c - 4ab^2c(-4ac - b^2)^3)^{(1/2)} / (54(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)}
\end{aligned}$$

sympy [A] time = 6.90, size = 196, normalized size = 0.31

$$\text{RootSum}\left(i^6(46656a^3c^7 - 34992a^2b^2c^6 + 8748ab^4c^5 - 729b^6c^4) + i^3(864a^3bc^3 - 864a^2b^3c^2 + 270ab^5c - 27b^7) + a^4, \left(t \mapsto t \log\left(x + \frac{1296t^4a^2bc^6 - 648t^4ab^3c^5 + 81t^4b^5c^4 - 121a^3c^3 + 39t^2b^2c^2 - 211ab^4c + 31b^6}{2a^3c^2 - 4a^2b^2c + ab^4}\right)\right)\right) + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(c*x**6+b*x**3+a), x)

[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 - 729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t**4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) + x/c

$$3.132 \quad \int \frac{x^4}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=558

$$\frac{\left(b - \sqrt{b^2 - 4ac}\right)^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \left(\sqrt{b^2 - 4ac} + b\right)^{2/3} \log\left(\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b + \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} + \frac{\left(b - \sqrt{b^2 - 4ac}\right)^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \left(\sqrt{b^2 - 4ac} + b\right)^{2/3} \log\left(\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b + \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}}$$

Rubi [A] time = 0.52, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1374, 292, 31, 634, 617, 204, 628}

$$\frac{(b - \sqrt{b^2 - 4ac})^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} + \frac{(b + \sqrt{b^2 - 4ac})^{2/3} \log\left(\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b + \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} + \frac{(b - \sqrt{b^2 - 4ac})^{2/3} \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} + \frac{(b + \sqrt{b^2 - 4ac})^{2/3} \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} + \frac{(b - \sqrt{b^2 - 4ac})^{2/3} \tan^{-1}\left(\frac{1 - \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}\right)}{2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} + \frac{(b + \sqrt{b^2 - 4ac})^{2/3} \tan^{-1}\left(\frac{1 - \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}\right)}{2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b*x^3 + c*x^6), x]

[Out] ((b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_)^(m_))/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{a + bx^3 + cx^6} dx &= -\left(\frac{1}{2}\left(-1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{x}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx\right) + \frac{1}{2}\left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{x}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx \\
&= \frac{(b - \sqrt{b^2 - 4ac})^{2/3} \int \frac{1}{\frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} \sqrt[3]{c} \sqrt{b^2 - 4ac}} - \frac{(b + \sqrt{b^2 - 4ac})^{2/3} \int \frac{1}{\frac{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} \sqrt[3]{c} \sqrt{b^2 - 4ac}} \\
&= \frac{(b - \sqrt{b^2 - 4ac})^{2/3} \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} - \frac{(b + \sqrt{b^2 - 4ac})^{2/3} \log\left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} \\
&= \frac{(b - \sqrt{b^2 - 4ac})^{2/3} \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} - \frac{(b + \sqrt{b^2 - 4ac})^{2/3} \log\left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt{b^2 - 4ac}} \\
&= \frac{(b - \sqrt{b^2 - 4ac})^{2/3} \tan^{-1}\left(\frac{1 - \frac{2 \sqrt[3]{2} \sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3} c^{2/3} \sqrt{b^2 - 4ac}} - \frac{(b + \sqrt{b^2 - 4ac})^{2/3} \tan^{-1}\left(\frac{1 - \frac{2 \sqrt[3]{2} \sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3} c^{2/3} \sqrt{b^2 - 4ac}} + \dots
\end{aligned}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.08

$$\frac{1}{3} \text{RootSum}\left[\#1^6 c + \#1^3 b + a \&, \frac{\#1^2 \log(x - \#1)}{2\#1^3 c + b} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b*x^3 + c*x^6),x]

[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1^2)/(b + 2*c*#1^3) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(a + b*x^3 + c*x^6),x]

[Out] IntegrateAlgebraic[x^4/(a + b*x^3 + c*x^6), x]

fricas [B] time = 1.78, size = 3799, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out]
$$\frac{2}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(-\left(\frac{b^2c^2 - 4ac^3}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} + b\right)^{\frac{1}{3}}\arctan\left(-\frac{1}{3}\left(\frac{1}{2}\right)^{\frac{5}{6}}\left(\sqrt{3}\left(\frac{b^5c^2 - 8ab^3c^3 + 16a^2b^2c^4}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} - \sqrt{3}\left(\frac{b^4 - 6ab^2c + 8a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{2(ab^2 - 2a^2c)x^2 + (1/2)^{\frac{2}{3}}(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}}\right) - (b^5 - 6ab^3c + 8a^2b^2c^2)x\right)\left(-\left(\frac{b^2c^2 - 4ac^3}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} + b\right)^{\frac{2}{3}} - 2\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\frac{ab^4c^2 - 8a^2b^2c^3 + 16a^3c^4}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}}\left(-\left(\frac{b^2c^2 - 4ac^3}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} + b\right)^{\frac{1}{3}}\right)/\left(\frac{ab^2 - 2a^2c}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)^{\frac{1}{3}} - \left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\sqrt{3}\left(\frac{b^5c^2 - 8ab^3c^3 + 16a^2b^2c^4}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} - \sqrt{3}\left(\frac{b^4 - 6ab^2c + 8a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)x\right)\left(-\left(\frac{b^2c^2 - 4ac^3}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} + b\right)^{\frac{1}{3}} - \sqrt{3}\left(\frac{ab^2 - 2a^2c}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)^{\frac{1}{3}} - \frac{2}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\left(\frac{b^2c^2 - 4ac^3}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} - b\right)^{\frac{1}{3}}\arctan\left(-\frac{1}{3}\left(\frac{1}{2}\right)^{\frac{5}{6}}\left(\sqrt{3}\left(\frac{b^5c^2 - 8ab^3c^3 + 16a^2b^2c^4}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} + \sqrt{3}\left(\frac{b^4 - 6ab^2c + 8a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{2(ab^2 - 2a^2c)x^2 - (1/2)^{\frac{2}{3}}(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}}\right) + (b^5 - 6ab^3c + 8a^2b^2c^2)x\right)\left(\left(\frac{b^2c^2 - 4ac^3}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} - b\right)^{\frac{2}{3}} + 2\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\frac{ab^4c^2 - 8a^2b^2c^3 + 16a^3c^4}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}}\left(\left(\frac{b^2c^2 - 4ac^3}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}\right)\sqrt{\frac{b^4 - 4ab^2c + 4a^2c^2}{b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7}} - b\right)^{\frac{1}{3}}\right)$$

$$\begin{aligned}
& c^2 - 4ac^3)^{1/3}) / (ab^2 - 2a^2c) * ((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) - b) / (b^2c^2 - 4ac^3)^{1/3} - (1/2)^{1/3} * (\sqrt{3} * (b^5c^2 - 8ab^3c^3 + 16a^2b^2c^4) * x * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) + \sqrt{3} * (b^4 - 6ab^2c + 8a^2c^2) * x) * ((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) - b) / (b^2c^2 - 4ac^3)^{1/3} + \sqrt{3} * (ab^2 - 2a^2c) / (ab^2 - 2a^2c) - 1/6 * (1/2)^{1/3} * (-((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) + b) / (b^2c^2 - 4ac^3))^{1/3} * \log(-2 * (ab^2 - 2a^2c) * x^2 - (1/2)^{2/3} * ((b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5) * x * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) - (b^5 - 6ab^3c + 8a^2b^2c^2) * x) * (-((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) + b) / (b^2c^2 - 4ac^3))^{2/3} + 2 * (1/2)^{1/3} * (ab^4c^2 - 8a^2b^2c^3 + 16a^3c^4) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) * (-((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) + b) / (b^2c^2 - 4ac^3))^{1/3} - 1/6 * (1/2)^{1/3} * (((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) - b) / (b^2c^2 - 4ac^3))^{1/3} * \log(-2 * (ab^2 - 2a^2c) * x^2 + (1/2)^{2/3} * ((b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5) * x * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) + (b^5 - 6ab^3c + 8a^2b^2c^2) * x) * ((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) - b) / (b^2c^2 - 4ac^3))^{2/3} - 2 * (1/2)^{1/3} * (ab^4c^2 - 8a^2b^2c^3 + 16a^3c^4) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) * (((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) - b) / (b^2c^2 - 4ac^3))^{1/3} + 1/3 * (1/2)^{1/3} * (-((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) + b) / (b^2c^2 - 4ac^3))^{1/3} * \log(-(1/2)^{2/3} * (b^5 - 6ab^3c + 8a^2b^2c^2 - (b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7))) * (-((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) + b) / (b^2c^2 - 4ac^3))^{2/3} - 2 * (ab^2 - 2a^2c) * x) + 1/3 * (1/2)^{1/3} * (((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) - b) / (b^2c^2 - 4ac^3))^{1/3} * \log(-(1/2)^{2/3} * (b^5 - 6ab^3c + 8a^2b^2c^2 + (b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7))) * (((b^2c^2 - 4ac^3) * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / (b^6c^4 - 12ab^4c^5 + 48a^2b^2c^6 - 64a^3c^7)) - b) / (b^2c^2 - 4ac^3))^{2/3} - 2 * (ab^2 - 2a^2c) * x)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)

maple [C] time = 0.00, size = 43, normalized size = 0.08

$$\frac{\text{RootOf}(c_Z^6 + b_Z^3 + a)^4 \ln(-\text{RootOf}(c_Z^6 + b_Z^3 + a) + x)}{6 \text{RootOf}(c_Z^6 + b_Z^3 + a)^5 c + 3 \text{RootOf}(c_Z^6 + b_Z^3 + a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^6+b*x^3+a),x)

[Out] 1/3*sum(_R^4/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)

mupad [B] time = 8.11, size = 2695, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a + b*x^3 + c*x^6),x)

[Out] log((2^(1/3)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2)))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2)))/(c^2*(4*a*c - b^2)^3))/2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2)))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a^2*b^2*c^2 + 9*a*b^4*c)/18 +

$$\begin{aligned}
& a^2 b^3 c^3 x \left((b^5 + b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c \right. \\
& \left. - 2ac(-4ac - b^2)^3)^{1/2} \right) / (54(64a^3 c^5 - b^6 c^2 + 12a^2 b^4 c^3 - 48a^2 b^2 c^4))^{1/3} + \log\left((2^{1/3} (b^5 - b^2(-4ac - b^2)^3)^{1/2} \right. \\
& \left. + 16a^2 b^3 c^2 - 8a^2 b^3 c + 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{2/3} \left(36a^3 c^3 - (2^{2/3} (54a^2 c^3 x (4ac - b^2) - (27 \right. \\
& \left. 2^{1/3}) a^2 b^3 c^3 (4ac - b^2)^2 (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c + 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{2/3} \\
& \left. \right) / 2 \left((b^5 - b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c + 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{1/3} \Big/ 6 - 45a^2 b^2 c^2 + 9a^2 b^4 c \\
& \Big/ 18 + a^2 b^3 c^3 x \left((b^5 - b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c + 2ac(-4ac - b^2)^3)^{1/2} \right) / (54(64a^3 c^5 - b^6 c^2 + 12a^2 b^4 c^3 - 48a^2 b^2 c^4))^{1/3} \\
& - \log\left((2^{1/3} (3^{1/2} i - 1) (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c - 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{2/3} \left(36a^3 c^3 - 45a^2 b^2 c^2 + 9a^2 b^4 c \right. \\
& \left. + (2^{2/3} (3^{1/2} i + 1) (54a^2 c^3 x (4ac - b^2) - (27 2^{1/3}) a^2 b^3 c^3 (3^{1/2} i - 1) (4ac - b^2)^2 (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c - 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{2/3} \Big/ 4 \\
& \left((b^5 + b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c - 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{1/3} \Big/ 12 \Big/ 36 + a^2 b^3 c^3 x \left((3^{1/2} i) / 2 + 1/2 \right) \left((b^5 + b^2(-4ac - b^2)^3)^{1/2} \right. \\
& \left. + 16a^2 b^3 c^2 - 8a^2 b^3 c - 2ac(-4ac - b^2)^3)^{1/2} \right) / (54(64a^3 c^5 - b^6 c^2 + 12a^2 b^4 c^3 - 48a^2 b^2 c^4))^{1/3} + \log\left((2^{1/3} (3^{1/2} i + 1) (b^5 + b^2(-4ac - b^2)^3)^{1/2} \right. \\
& \left. + 16a^2 b^3 c^2 - 8a^2 b^3 c - 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{2/3} \left(36a^3 c^3 - 45a^2 b^2 c^2 + 9a^2 b^4 c - (2^{2/3} (3^{1/2} i - 1) (54a^2 c^3 x (4ac - b^2) + (27 2^{1/3}) a^2 b^3 c^3 (3^{1/2} i + 1) \right. \\
& \left. (4ac - b^2)^2 (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c - 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{2/3} \Big/ 4 \left((b^5 + b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c - 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{1/3} \Big/ 12 \Big/ 36 - a^2 b^3 c^3 x \\
& \left((3^{1/2} i) / 2 - 1/2 \right) \left((b^5 + b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c - 2ac(-4ac - b^2)^3)^{1/2} \right) / (54(64a^3 c^5 - b^6 c^2 + 12a^2 b^4 c^3 - 48a^2 b^2 c^4))^{1/3} - \log\left((2^{1/3} (3^{1/2} i - 1) (b^5 - b^2(-4ac - b^2)^3)^{1/2} \right. \\
& \left. + 16a^2 b^3 c^2 - 8a^2 b^3 c + 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{2/3} \left(36a^3 c^3 - 45a^2 b^2 c^2 + 9a^2 b^4 c + (2^{2/3} (3^{1/2} i + 1) (54a^2 c^3 x (4ac - b^2) - (27 2^{1/3}) a^2 b^3 c^3 (3^{1/2} i - 1) (4ac - b^2)^2 (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c + 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{2/3} \Big/ 4 \left((b^5 - b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c + 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{1/3} \Big/ 12 \Big/ 36 + a^2 b^3 c^3 x \left((3^{1/2} i) / 2 + 1/2 \right) \left((b^5 - b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c + 2ac(-4ac - b^2)^3)^{1/2} \right) / (54(64a^3 c^5 - b^6 c^2 + 12a^2 b^4 c^3 - 48a^2 b^2 c^4))^{1/3} + \log\left((2^{1/3} (3^{1/2} i + 1) (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 16a^2 b^3 c^2 - 8a^2 b^3 c + 2ac(-4ac - b^2)^3)^{1/2} \right) / (c^2(4ac - b^2)^3)^{2/3}
\end{aligned}$$

$$\begin{aligned} &)*(36*a^3*c^3 - 45*a^2*b^2*c^2 + 9*a*b^4*c - (2^{(2/3)}*(3^{(1/2)}*1i - 1)*(54* \\ & a^2*c^3*x*(4*a*c - b^2) + (27*2^{(1/3)}*a*b*c^3*(3^{(1/2)}*1i + 1)*(4*a*c - b^2 \\ &)^2*((b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c \\ & *(-(4*a*c - b^2)^3)^{(1/2)))/(c^2*(4*a*c - b^2)^3)^{(2/3))/4)*((b^5 - b^2*(-(\\ & 4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3) \\ & ^{(1/2)))/(c^2*(4*a*c - b^2)^3)^{(1/3))/12))/36 - a^2*b*c*x)*((3^{(1/2)}*1i)/2 \\ & - 1/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c + 2* \\ & a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48 \\ & *a^2*b^2*c^4))^{(1/3)} \end{aligned}$$

sympy [A] time = 2.18, size = 175, normalized size = 0.31

$$\text{RootSum}\left(t^6(46656a^3c^5 - 34992a^2b^2c^4 + 8748ab^4c^3 - 729b^6c^2) + t^3(-432a^2bc^2 + 216ab^3c - 27b^5) + a^2, \left(t \mapsto t \log\left(x + \frac{15552t^5a^3c^5 - 11664t^5a^2b^2c^4 + 2916t^5ab^4c^3 - 243t^5b^6c^2 - 108t^2a^2bc^2 + 63t^2ab^3c - 9t^2b^5}{2a^2c - ab^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**6+b*x**3+a), x)

[Out] RootSum(_t**6*(46656*a**3*c**5 - 34992*a**2*b**2*c**4 + 8748*a*b**4*c**3 - 729*b**6*c**2) + _t**3*(-432*a**2*b*c**2 + 216*a*b**3*c - 27*b**5) + a**2, Lambda(_t, _t*log(x + (15552*_t**5*a**3*c**5 - 11664*_t**5*a**2*b**2*c**4 + 2916*_t**5*a*b**4*c**3 - 243*_t**5*b**6*c**2 - 108*_t**2*a**2*b*c**2 + 63*_t**2*a*b**3*c - 9*_t**2*b**5)/(2*a**2*c - a*b**2))))

$$3.133 \quad \int \frac{x^3}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \sqrt[3]{\sqrt{b^2 - 4ac} + b} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{\sqrt{b^2 - 4ac} + b} + \left(\sqrt{b^2 - 4ac} + b\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}}$$

Rubi [A] time = 0.57, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt{b - \sqrt{b^2 - 4ac}} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}} - \frac{\sqrt{\sqrt{b^2 - 4ac} + b} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{\sqrt{b^2 - 4ac} + b} + \left(\sqrt{b^2 - 4ac} + b\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}} - \frac{\sqrt{b - \sqrt{b^2 - 4ac}} \log\left(\frac{\sqrt{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}}\right)}{3 \sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}} + \frac{\sqrt{\sqrt{b^2 - 4ac} + b} \log\left(\frac{\sqrt{\sqrt{b^2 - 4ac} + b} + \sqrt[3]{2} \sqrt[3]{c} x}{3 \sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}}\right)}{3 \sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}} + \frac{\sqrt{b - \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{1 - \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}}\right)}{\sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}} - \frac{\sqrt{\sqrt{b^2 - 4ac} + b} \tan^{-1}\left(\frac{1 - \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}}\right)}{\sqrt[3]{2} \sqrt[3]{c} \sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*x^3 + c*x^6), x]

[Out] ((b - Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F

reeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1374

Int[((d_)*(x_)^m)/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a + bx^3 + cx^6} dx &= -\left(\frac{1}{2}\left(-1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx\right) + \frac{1}{2}\left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx \\
&= -\frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}} \int \frac{1}{\frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\sqrt[3]{2}\sqrt{b^2 - 4ac}} - \frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}} \int \frac{2^{2/3}\sqrt[3]{b - \sqrt{b^2 - 4ac}} - \sqrt[3]{c}x}{\frac{(b - \sqrt{b^2 - 4ac})^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b - \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} + c} dx}{3\sqrt[3]{2}\sqrt{b^2 - 4ac}} \\
&= -\frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}} \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2 - 4ac}} + \frac{\sqrt[3]{b + \sqrt{b^2 - 4ac}} \log\left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2 - 4ac}} \\
&= -\frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}} \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2 - 4ac}} + \frac{\sqrt[3]{b + \sqrt{b^2 - 4ac}} \log\left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2 - 4ac}} \\
&= \frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2 - 4ac}} - \frac{\sqrt[3]{b + \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2 - 4ac}} - \frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2 - 4ac}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1 \log(x - \#1)}{2\#1^3c + b}\& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*x^3 + c*x^6),x]

[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1)/(b + 2*c*#1^3) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(a + b*x^3 + c*x^6),x]

[Out] IntegrateAlgebraic[x^3/(a + b*x^3 + c*x^6), x]

fricas [B] time = 1.49, size = 2551, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out]
$$-2/3\sqrt{3}\cdot(1/2)^{1/3}\cdot\left(\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} + 1\right)/(b^2c - 4a^2c^2)^{1/3}\cdot\arctan\left(-1/3\cdot\left((1/2)^{2/3}\cdot\left(\sqrt{3}\cdot(b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4)\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} - \sqrt{3}\right)\cdot(b^4 - 4ab^2c)\cdot\left(\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} + 1\right)/(b^2c - 4a^2c^2)^{2/3}\sqrt{-\left((1/2)^{1/3}\cdot(b^4c - 8ab^2c^2 + 16a^2c^3)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)}\right)\cdot\left(\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} + 1\right)/(b^2c - 4a^2c^2)^{1/3} - b^2x - (1/2)^{2/3}\cdot(b^3 - 4ab^2c)\cdot\left(\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} + 1\right)/(b^2c - 4a^2c^2)^{2/3}\right)/b - \sqrt{3}\cdot ab - (1/2)^{2/3}\cdot\left(\sqrt{3}\cdot(b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)}\right)\cdot x - \sqrt{3}\cdot(b^4 - 4ab^2c)\cdot x\cdot\left(\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} + 1\right)/(b^2c - 4a^2c^2)^{2/3}\right)/(ab) + 2/3\sqrt{3}\cdot(1/2)^{1/3}\cdot\left(-\left(\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} - 1\right)/(b^2c - 4a^2c^2)\right)^{1/3}\cdot\arctan\left(-1/3\cdot\left((1/2)^{2/3}\cdot\left(\sqrt{3}\cdot(b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} + \sqrt{3}\cdot(b^4 - 4ab^2c)\right)\cdot\left(-\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} - 1\right)/(b^2c - 4a^2c^2)\right)^{2/3}\sqrt{\left(\left(1/2\right)^{1/3}\cdot(b^4c - 8ab^2c^2 + 16a^2c^3)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)}\right)\cdot\left(-\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} - 1\right)/(b^2c - 4a^2c^2)\right)^{1/3} + b^2x^2 + (1/2)^{2/3}\cdot(b^3 - 4ab^2c)\cdot\left(-\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} - 1\right)/(b^2c - 4a^2c^2)^{2/3}\right)/b + \sqrt{3}\cdot ab - (1/2)^{2/3}\cdot\left(\sqrt{3}\cdot(b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)}\right)\cdot x + \sqrt{3}\cdot(b^4 - 4ab^2c)\cdot x\cdot\left(-\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} - 1\right)/(b^2c - 4a^2c^2)^{2/3}\right)/(ab) - 1/6\cdot(1/2)^{1/3}\cdot\left(\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} + 1\right)/(b^2c - 4a^2c^2)^{1/3}\cdot\log\left(-\left(1/2\right)^{1/3}\cdot(b^4c - 8ab^2c^2 + 16a^2c^3)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)}\right)\cdot\left(\left(b^2c - 4a^2c^2\right)\sqrt{b^2/(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)} + 1\right)$$

$$\begin{aligned} &)/(b^2*c - 4*a*c^2)^{(1/3)} + b*x^2 + (1/2)^{(2/3)}*(b^3 - 4*a*b*c)*(((b^2*c - 4*a*c^2)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)} \\ & + 1)/(b^2*c - 4*a*c^2)^{(2/3)}) - 1/6*(1/2)^{(1/3)}*(-((b^2*c - 4*a*c^2)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)} \\ & - 1)/(b^2*c - 4*a*c^2))^{(1/3)}*\log((1/2)^{(1/3)}*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)})*x*(-((b^2*c - 4*a*c^2)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)} \\ & - 1)/(b^2*c - 4*a*c^2))^{(1/3)} + b*x^2 + (1/2)^{(2/3)}*(b^3 - 4*a*b*c)*(-((b^2*c - 4*a*c^2)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)} \\ & - 1)/(b^2*c - 4*a*c^2))^{(2/3)}) + 1/3*(1/2)^{(1/3)}*(((b^2*c - 4*a*c^2)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)} + 1)/(b^2*c - 4*a*c^2))^{(1/3)}*\log((1/2)^{(1/3)}*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)})*(((b^2*c - 4*a*c^2)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)} + 1)/(b^2*c - 4*a*c^2))^{(1/3)} + b*x) + 1/3*(1/2)^{(1/3)}*(-((b^2*c - 4*a*c^2)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)} - 1)/(b^2*c - 4*a*c^2))^{(1/3)}*\log(-((1/2)^{(1/3)}*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)})*(-((b^2*c - 4*a*c^2)*\sqrt{b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)} - 1)/(b^2*c - 4*a*c^2))^{(1/3)} + b*x) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)

maple [C] time = 0.00, size = 43, normalized size = 0.08

$$\frac{\text{RootOf}(c_Z^6 + b_Z^3 + a)^3 \ln(-\text{RootOf}(c_Z^6 + b_Z^3 + a) + x)}{6 \text{RootOf}(c_Z^6 + b_Z^3 + a)^5 c + 3 \text{RootOf}(c_Z^6 + b_Z^3 + a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^6+b*x^3+a),x)

[Out] 1/3*sum(_R^3/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)

mupad [B] time = 7.71, size = 2129, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b*x^3 + c*x^6),x)

[Out]
$$\log\left(\frac{(2^{2/3}*(-(b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c))/(c(4ac - b^2)^3))^{1/3}*(9ab^3c^2 - 36a^2b^3c^3 + (9*2^{1/3})ac^3*(4ac - b^2)^2*(x - (2^{2/3})b*(-(b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c))/(c(4ac - b^2)^3))^{1/3})/2}{(b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c)/(c(4ac - b^2)^3)}\right)^{1/2} + 3ac^2x(2ac - b^2)*((b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c)/(54(b^6c - 64a^3c^4 - 12ab^4c^2 + 48a^2b^2c^3)))^{1/3} + \log\left(\frac{(2^{2/3}*(9ab^3c^2 - 36a^2b^3c^3 + (9*2^{1/3})ac^3*(x - (2^{2/3})b*((b*(-(4ac - b^2)^3)^{1/2} - b^4 - 16a^2c^2 + 8ab^2c))/(c(4ac - b^2)^3))^{1/3})/2}{(4ac - b^2)^2*((b*(-(4ac - b^2)^3)^{1/2} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3))^{2/3}}\right)^{1/2} * ((b*(-(4ac - b^2)^3)^{1/2} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3))^{1/3} / 6 + 3ac^2x(2ac - b^2)*((b*(-(4ac - b^2)^3)^{1/2} - b^4 - 16a^2c^2 + 8ab^2c)/(54(b^6c - 64a^3c^4 - 12ab^4c^2 + 48a^2b^2c^3)))^{1/3} + \log\left(\frac{(2^{2/3}*(3^{1/2}*1i - 1)*(36a^2b^3c^3 - 9ab^3c^2 + (2^{1/3})*(3^{1/2})*1i + 1)*(81ac^3x(4ac - b^2)^2 - (81*2^{2/3})ab^3c^3*(3^{1/2}*1i - 1)*(4ac - b^2)^2*(-(b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c))/(c(4ac - b^2)^3))^{1/3}}{4} * ((b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c)/(c(4ac - b^2)^3))^{2/3} / 36 * ((b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c)/(c(4ac - b^2)^3))^{1/3} / 12 - 3ac^2x(2ac - b^2)*((3^{1/2}*1i)/2 - 1/2)*((b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c)/(54(b^6c - 64a^3c^4 - 12ab^4c^2 + 48a^2b^2c^3)))^{1/3} - \log\left(\frac{(2^{2/3}*(3^{1/2}*1i + 1)*(9ab^3c^2 - 36a^2b^3c^3 + (2^{1/3})*(3^{1/2}*1i - 1)*(81ac^3x(4ac - b^2)^2 + (81*2^{2/3})ab^3c^3*(3^{1/2}*1i + 1)*(4ac - b^2)^2*(-(b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c))/(c(4ac - b^2)^3))^{1/3}}{4} * ((b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c)/(c(4ac - b^2)^3))^{2/3} / 36 * ((b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c)/(c(4ac - b^2)^3))^{1/3} / 12 - 3ac^2x(2ac - b^2)*((3^{1/2}*1i)/2 + 1/2)*((b*(-(4ac - b^2)^3)^{1/2} + b^4 + 16a^2c^2 - 8ab^2c)/(54(b^6c - 64a^3c^4 - 12ab^4c^2 + 48a^2b^2c^3)))^{1/3} + \log\left(\frac{(2^{2/3}*(3^{1/2}*1i - 1)*(36a^2b^3c^3 - 9ab^3c^2 + (2^{1/3})*(81ac^3x(4ac - b^2)^2 - (81*2^{2/3})ab^3c^3*(3^{1/2}*1i - 1)*(4ac - b^2)^2*((b*(-(4ac - b^2)^3)^{1/2} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3))^{1/3})/2}{(81*2^{2/3})ab^3c^3*(3^{1/2}*1i - 1)*(4ac - b^2)^2*((b*(-(4ac - b^2)^3)^{1/2} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3))^{1/3}}\right)^{1/2}$$

$$\begin{aligned} &)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3)^{(1/3)}/4*(3^{(1/2)}*i + 1)*((b*(-(4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3)^{(2/3)}/36)*((b*(-(4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3)^{(1/3)}/12 - 3ac^2*x*(2ac - b^2))* \\ &((3^{(1/2)}*i)/2 - 1/2)*(-b*(-(4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c)/(54*(b^6c - 64a^3c^4 - 12ab^4c^2 + 48a^2b^2c^3))^{(1/3)} \\ &- \log((2^{(2/3)}*(3^{(1/2)}*i + 1)*(9ab^3c^2 - 36a^2b^2c^3 + (2^{(1/3)}*(81a^3c^3*x*(4ac - b^2)^2 + (81*2^{(2/3)}*ab^2c^3*(3^{(1/2)}*i + 1)*(4ac - b^2)^2*((b*(-(4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3))^{(1/3)}/4)*(3^{(1/2)}*i - 1)*((b*(-(4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3)^{(2/3)}/36)*((b*(-(4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c)/(c(4ac - b^2)^3)^{(1/3)}/12 - 3ac^2*x*(2ac - b^2))*((3^{(1/2)}*i)/2 + 1/2)*(-b*(-(4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c)/(54*(b^6c - 64a^3c^4 - 12ab^4c^2 + 48a^2b^2c^3))^{(1/3)} \end{aligned}$$

sympy [A] time = 1.80, size = 122, normalized size = 0.22

$$\text{RootSum}\left(t^6(46656a^3c^4 - 34992a^2b^2c^3 + 8748ab^4c^2 - 729b^6c) + t^3(432a^2c^2 - 216ab^2c + 27b^4) + a, \left(t \mapsto t \log\left(x + \frac{2592t^4a^2c^3 - 1296t^4ab^2c^2 + 162t^4b^4c + 12tac - 3tb^2}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**6+b*x**3+a),x)

[Out] RootSum(_t**6*(46656*a**3*c**4 - 34992*a**2*b**2*c**3 + 8748*a*b**4*c**2 - 729*b**6*c) + _t**3*(432*a**2*c**2 - 216*a*b**2*c + 27*b**4) + a, Lambda(_t, _t*log(x + (2592*_t**4*a**2*c**3 - 1296*_t**4*a*b**2*c**2 + 162*_t**4*b**4*c + 12*_t*a*c - 3*_t*b**2)/b)))

$$3.134 \quad \int \frac{x}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{c} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + (b - \sqrt{b^2 - 4ac})^{2/3} + 2^{2/3} c^{2/3} x^2\right) \sqrt[3]{c} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{\sqrt{b^2 - 4ac} + b} + \sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{\sqrt{b^2 - 4ac} + b}\right)}{3 \cdot 2^{2/3} \sqrt{b^2 - 4ac} \sqrt[3]{b - \sqrt{b^2 - 4ac}} - 3 \cdot 2^{2/3} \sqrt{b^2 - 4ac} \sqrt[3]{\sqrt{b^2 - 4ac} + b}}$$

Rubi [A] time = 0.47, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, number of rules / integrand size = 0.438, Rules used = {1375, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{c} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + (b - \sqrt{b^2 - 4ac})^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{3 \cdot 2^{2/3} \sqrt{b^2 - 4ac} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt[3]{c} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{\sqrt{b^2 - 4ac} + b} + (\sqrt{b^2 - 4ac} + b)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{3 \cdot 2^{2/3} \sqrt{b^2 - 4ac} \sqrt[3]{\sqrt{b^2 - 4ac} + b}} - \frac{\sqrt[3]{2} \sqrt[3]{c} \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \sqrt{b^2 - 4ac} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt[3]{2} \sqrt[3]{c} \log\left(\sqrt[3]{\sqrt{b^2 - 4ac} + b} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \sqrt{b^2 - 4ac} \sqrt[3]{\sqrt{b^2 - 4ac} + b}} - \frac{\sqrt[3]{2} \sqrt[3]{c} \tan^{-1}\left(\frac{1 - \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt[3]{2} \sqrt[3]{c} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt[3]{2} \sqrt[3]{c} \tan^{-1}\left(\frac{1 - \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt[3]{2} \sqrt[3]{c} \sqrt[3]{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^3 + c*x^6), x]

[Out] -((2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3)]/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3)]/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - (2^(1/3)*c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]/(3*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/((3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/((3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_)*(x_)^(m_))/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a + bx^3 + cx^6} dx &= \frac{c \int \frac{x}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{\sqrt{b^2-4ac}} - \frac{c \int \frac{x}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{\sqrt{b^2-4ac}} \\
&= \frac{(\sqrt[3]{2} c^{2/3}) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c} x} dx}{3\sqrt{b^2-4ac} \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{(\sqrt[3]{2} c^{2/3}) \int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c} x}{\frac{(b-\sqrt{b^2-4ac})^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c} \sqrt[3]{b-\sqrt{b^2-4ac}} x}{\sqrt[3]{2}} + c^{2/3} x^2} dx}{3\sqrt{b^2-4ac} \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{(\sqrt[3]{2} c^{2/3}) \int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c} x} dx}{3\sqrt{b^2-4ac} \sqrt[3]{b+\sqrt{b^2-4ac}}} \\
&= -\frac{\sqrt[3]{2} \sqrt[3]{c} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3\sqrt{b^2-4ac} \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2} \sqrt[3]{c} \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3\sqrt{b^2-4ac} \sqrt[3]{b+\sqrt{b^2-4ac}}} \\
&= -\frac{\sqrt[3]{2} \sqrt[3]{c} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3\sqrt{b^2-4ac} \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2} \sqrt[3]{c} \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3\sqrt{b^2-4ac} \sqrt[3]{b+\sqrt{b^2-4ac}}} \\
&= -\frac{\sqrt[3]{2} \sqrt[3]{c} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3} \sqrt{b^2-4ac} \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2} \sqrt[3]{c} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3} \sqrt{b^2-4ac} \sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\sqrt[3]{2} \sqrt[3]{c} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.08

$$\frac{1}{3} \text{RootSum}\left[\#1^6 c + \#1^3 b + a \&, \frac{\log(x - \#1)}{2\#1^4 c + \#1 b} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^3 + c*x^6), x]

[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1 + 2*c*#1^4) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a + b*x^3 + c*x^6),x]

[Out] IntegrateAlgebraic[x/(a + b*x^3 + c*x^6), x]

fricas [B] time = 1.70, size = 2875, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -2/3\sqrt{3}\cdot(1/2)^{1/3}\cdot(-((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} + 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{1/3}\cdot\arctan(-1/3\cdot(2\cdot\sqrt{3}\cdot(1/2)^{1/3}\cdot(a\cdot b^4 - 8\cdot a^2\cdot b^2\cdot c + 16\cdot a^3\cdot c^2)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)}))\cdot x\cdot(-((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} + 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{1/3} - 2\cdot\sqrt{3}\cdot(1/2)^{5/6}\cdot(a\cdot b^4 - 8\cdot a^2\cdot b^2\cdot c + 16\cdot a^3\cdot c^2)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)}\cdot(-((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} + 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{1/3}\cdot\sqrt{((2\cdot b\cdot c\cdot x^2 + (1/2)^{2/3}\cdot((a\cdot b^6 - 12\cdot a^2\cdot b^4\cdot c + 48\cdot a^3\cdot b^2\cdot c^2 - 64\cdot a^4\cdot c^3)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)}))\cdot x - (b^4 - 4\cdot a\cdot b^2\cdot c)\cdot x)\cdot(-((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} + 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{2/3} + (1/2)^{1/3}\cdot(b^3 - 4\cdot a\cdot b\cdot c - (a\cdot b^5 - 8\cdot a^2\cdot b^3\cdot c + 16\cdot a^3\cdot b\cdot c^2)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)}))\cdot(-((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} + 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{1/3}}/(b\cdot c)) + \sqrt{3}\cdot b)/b + 2/3\cdot\sqrt{3}\cdot(1/2)^{1/3}\cdot(((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} - 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{1/3}\cdot\arctan(-1/3\cdot(2\cdot\sqrt{3}\cdot(1/2)^{1/3}\cdot(a\cdot b^4 - 8\cdot a^2\cdot b^2\cdot c + 16\cdot a^3\cdot c^2)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)}))\cdot x\cdot(((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} - 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{1/3} - 2\cdot\sqrt{3}\cdot(1/2)^{5/6}\cdot(a\cdot b^4 - 8\cdot a^2\cdot b^2\cdot c + 16\cdot a^3\cdot c^2)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)}\cdot(((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} - 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{1/3}\cdot\sqrt{((2\cdot b\cdot c\cdot x^2 - (1/2)^{2/3}\cdot((a\cdot b^6 - 12\cdot a^2\cdot b^4\cdot c + 48\cdot a^3\cdot b^2\cdot c^2 - 64\cdot a^4\cdot c^3)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)}))\cdot x + (b^4 - 4\cdot a\cdot b^2\cdot c)\cdot x)\cdot(((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} - 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{2/3} + (1/2)^{1/3}\cdot(b^3 - 4\cdot a\cdot b\cdot c + (a\cdot b^5 - 8\cdot a^2\cdot b^3\cdot c + 16\cdot a^3\cdot b\cdot c^2)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)}))\cdot(((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} - 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{1/3}}/(b\cdot c)) - \sqrt{3}\cdot b)/b - 1/6\cdot(1/2)^{1/3}\cdot(-((a\cdot b^2 - 4\cdot a^2\cdot c)\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)} + 1)/(a\cdot b^2 - 4\cdot a^2\cdot c))^{1/3}\cdot\sqrt{b^2/(a^2\cdot b^6 - 12\cdot a^3\cdot b^4\cdot c + 48\cdot a^4\cdot b^2\cdot c^2 - 64\cdot a^5\cdot c^3)}} \end{aligned}$$

$$2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^{(1/3)}*\log(2*b*c*x^2 + (1/2)^{(2/3)}*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)}*x - (b^4 - 4*a*b^2*c)*x)*(-((a*b^2 - 4*a^2*c)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} + 1)/(a*b^2 - 4*a^2*c))^{(2/3)} + (1/2)^{(1/3)}*(b^3 - 4*a*b*c - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)})))*(-((a*b^2 - 4*a^2*c)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} + 1)/(a*b^2 - 4*a^2*c))^{(1/3)} - 1/6*(1/2)^{(1/3)}*(((a*b^2 - 4*a^2*c)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} - 1)/(a*b^2 - 4*a^2*c))^{(1/3)}*\log(2*b*c*x^2 - (1/2)^{(2/3)}*((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)}*x + (b^4 - 4*a*b^2*c)*x)*(((a*b^2 - 4*a^2*c)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} - 1)/(a*b^2 - 4*a^2*c))^{(2/3)} + (1/2)^{(1/3)}*(b^3 - 4*a*b*c + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)})))*(((a*b^2 - 4*a^2*c)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} - 1)/(a*b^2 - 4*a^2*c))^{(1/3)} + 1/3*(1/2)^{(1/3)}*(-((a*b^2 - 4*a^2*c)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} + 1)/(a*b^2 - 4*a^2*c))^{(1/3)}*\log(2*b*c*x + (1/2)^{(2/3)}*(b^4 - 4*a*b^2*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)})))*(-((a*b^2 - 4*a^2*c)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} + 1)/(a*b^2 - 4*a^2*c))^{(2/3)} + 1/3*(1/2)^{(1/3)}*(((a*b^2 - 4*a^2*c)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} - 1)/(a*b^2 - 4*a^2*c))^{(1/3)}*\log(2*b*c*x + (1/2)^{(2/3)}*(b^4 - 4*a*b^2*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)})))*(((a*b^2 - 4*a^2*c)*\sqrt{b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} - 1)/(a*b^2 - 4*a^2*c))^{(2/3)}))$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate(x/(c*x^6 + b*x^3 + a), x)

maple [C] time = 0.00, size = 41, normalized size = 0.07

$$\frac{\text{RootOf}(c_Z^6 + b_Z^3 + a) \ln(-\text{RootOf}(c_Z^6 + b_Z^3 + a) + x)}{6 \text{RootOf}(c_Z^6 + b_Z^3 + a)^5 c + 3 \text{RootOf}(c_Z^6 + b_Z^3 + a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^6+b*x^3+a),x)

[Out] 1/3*sum(_R/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate(x/(c*x^6 + b*x^3 + a), x)

mupad [B] time = 5.39, size = 1543, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*x^3 + c*x^6),x)

[Out] $\log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^{(1/3)}*a*b*c^3*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(a*(4*a*c - b^2)^3))^{(2/3)})/2*(b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/((54*a*(4*a*c - b^2)^3))*(-(b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^{(1/3)} + \log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^{(1/3)}*a*b*c^3*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^{(1/2)} - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(a*(4*a*c - b^2)^3))^{(2/3)})/2*(b*(-(4*a*c - b^2)^3)^{(1/2)} - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/((54*a*(4*a*c - b^2)^3))*((b*(-(4*a*c - b^2)^3)^{(1/2)} - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^{(1/3)} - \log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^{(1/3)}*a*b*c^3*(3^{(1/2)}*1i - 1)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(a*(4*a*c - b^2)^3))^{(2/3)})/4*(b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/((54*a*(4*a*c - b^2)^3))*((3^{(1/2)}*1i)/2 + 1/2)*(-(b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^{(1/3)} + \log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^{(1/3)}*a*b*c^3*(3^{(1/2)}*1i + 1)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(a*(4*a*c - b^2)^3))^{(2/3)})/4*(b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/((54*a*(4*a*c - b^2)^3))*((3^{(1/2)}*1i)/2 - 1/2)*(-(b*(-(4*a*c - b^2)^3)^{(1/2)} + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^{(1/3)} - \log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^{(1/3)}*a*b*c^3*(3^{(1/2)}*1i - 1)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^{(1/2)} - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(a*(4*a*c - b^2)^3))^{(2/3)})/4*(b*(-(4*a*c - b^2)^3)^{(1/2)} - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/((54*a*(4*a*c - b^2)^3))^{(2/3)})/4*(b*(-(4*a*c - b^2)^3)^{(1/2)} - b^4 - 16*a^2*c^2 + 8*a*b^2*c))^{(1/3)}$

$$\frac{4ac - b^2)^{3/2} - b^4 - 16a^2c^2 + 8ab^2c}{(54a(4ac - b^2)^3)^{1/2} \left(\frac{3^{1/2}i + 1}{2} + \frac{1}{2} \right) \left((b(-4ac - b^2)^{3/2} - b^4 - 16a^2c^2 + 8ab^2c) / (54(a^6b - 64a^4c^3 - 12a^2b^4c + 48a^3b^2c^2)) \right)^{1/3} + \log(c^4x + ((27c^3x(b^4 + 8a^2c^2 - 6ab^2c) - (27^{1/3})ab^3c^3(3^{1/2}i + 1)(4ac - b^2)^2(-b(-4ac - b^2)^{3/2} - b^4 - 16a^2c^2 + 8ab^2c)) / (a(4ac - b^2)^3))^{1/4} \left((b(-4ac - b^2)^{3/2} - b^4 - 16a^2c^2 + 8ab^2c) / (54a(4ac - b^2)^3) \right)^{1/2} - 1/2) \left((b(-4ac - b^2)^{3/2} - b^4 - 16a^2c^2 + 8ab^2c) / (54(a^6b - 64a^4c^3 - 12a^2b^4c + 48a^3b^2c^2)) \right)^{1/3}}$$

sympy [A] time = 1.53, size = 158, normalized size = 0.28

$$\text{RootSum}\left(t^6(46656a^4c^3 - 34992a^3b^2c^2 + 8748a^2b^4c - 729ab^6) + t^3(-432a^2c^2 + 216ab^2c - 27b^4) + c, \left(t \mapsto t \log\left(x + \frac{-15552t^5a^4c^3 + 11664t^5a^3b^2c^2 - 2916t^5a^2b^4c + 243t^5ab^6 + 72t^2a^2c^2 - 54t^2ab^2c + 9t^2b^4}{bc}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**6+b*x**3+a), x)

[Out] RootSum(_t**6*(46656*a**4*c**3 - 34992*a**3*b**2*c**2 + 8748*a**2*b**4*c - 729*a*b**6) + _t**3*(-432*a**2*c**2 + 216*a*b**2*c - 27*b**4) + c, Lambda(_t, _t*log(x + (-15552*_t**5*a**4*c**3 + 11664*_t**5*a**3*b**2*c**2 - 2916*_t**5*a**2*b**4*c + 243*_t**5*a*b**6 + 72*_t**2*a**2*c**2 - 54*_t**2*a*b**2*c + 9*_t**2*b**4)/(b*c))))

$$3.135 \quad \int \frac{1}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=558

$$\frac{c^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + (b - \sqrt{b^2 - 4ac})^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{3\sqrt[3]{2} \sqrt{b^2 - 4ac} (b - \sqrt{b^2 - 4ac})^{2/3}} + \frac{c^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{\sqrt{b^2 - 4ac} + b} + (b + \sqrt{b^2 - 4ac})^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{3\sqrt[3]{2} \sqrt{b^2 - 4ac} (\sqrt{b^2 - 4ac} + b)^{2/3}}$$

Rubi [A] time = 0.60, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$\frac{c^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + (b - \sqrt{b^2 - 4ac})^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{3\sqrt[3]{2} \sqrt{b^2 - 4ac} (b - \sqrt{b^2 - 4ac})^{2/3}} + \frac{c^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{\sqrt{b^2 - 4ac} + b} + (b + \sqrt{b^2 - 4ac})^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{3\sqrt[3]{2} \sqrt{b^2 - 4ac} (\sqrt{b^2 - 4ac} + b)^{2/3}} + \frac{2^{2/3} c^{2/3} \log\left(\frac{\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b^2 - 4ac}}\right)}{3\sqrt[3]{b^2 - 4ac} (b - \sqrt{b^2 - 4ac})^{2/3}} + \frac{2^{2/3} c^{2/3} \log\left(\frac{\sqrt[3]{\sqrt{b^2 - 4ac} + b} + \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b^2 - 4ac}}\right)}{3\sqrt[3]{b^2 - 4ac} (\sqrt{b^2 - 4ac} + b)^{2/3}} + \frac{2^{2/3} c^{2/3} \tan^{-1}\left(\frac{1 - \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b^2 - 4ac}}\right)}{\sqrt[3]{b^2 - 4ac} (b - \sqrt{b^2 - 4ac})^{2/3}} + \frac{2^{2/3} c^{2/3} \tan^{-1}\left(\frac{1 + \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b^2 - 4ac}}\right)}{\sqrt[3]{b^2 - 4ac} (\sqrt{b^2 - 4ac} + b)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(-1), x]

[Out] -((2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3)]/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3)]/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R

$\int \frac{t[b, 3]*x}{(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[\{a, b\}, x]$

Rule 204

$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^2}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}]^{-1}, x_Symbol] := -\text{Simp}[\text{ArcTan}[\frac{Rt[-b, 2]*x}{Rt[-a, 2]}] / (Rt[-a, 2]*Rt[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 617

$\text{Int}[\frac{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}]^{-1}, x_Symbol] := \text{With}[\{q = 1 - 4*Simplify[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] := \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] := \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1347

$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^{(n_.)} + (c_.)*(x_.)^{(n2_.)}}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}]^{-1}, x_Symbol] := \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[c/q, \text{Int}[1/(b/2 - q/2 + c*x^n), x], x] - \text{Dist}[c/q, \text{Int}[1/(b/2 + q/2 + c*x^n), x], x]] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{a + bx^3 + cx^6} dx &= \frac{c \int \frac{1}{\frac{b-1}{2} \sqrt{b^2-4ac} + cx^3} dx}{\sqrt{b^2-4ac}} - \frac{c \int \frac{1}{\frac{b+1}{2} \sqrt{b^2-4ac} + cx^3} dx}{\sqrt{b^2-4ac}} \\
&= \frac{(2^{2/3}c) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\sqrt{b^2-4ac} (b - \sqrt{b^2-4ac})^{2/3}} + \frac{(2^{2/3}c) \int \frac{2^{2/3} \sqrt[3]{b-\sqrt{b^2-4ac}} - \sqrt[3]{c}x}{(b-\sqrt{b^2-4ac})^{2/3} - \frac{\sqrt[3]{c}}{\sqrt[3]{2}} \sqrt[3]{b-\sqrt{b^2-4ac}}x + c^{2/3}x^2} dx}{3\sqrt{b^2-4ac} (b - \sqrt{b^2-4ac})^{2/3}} - \frac{(2^{2/3}c) \int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\sqrt{b^2-4ac} (b + \sqrt{b^2-4ac})^{2/3}} \\
&= \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac} (b - \sqrt{b^2-4ac})^{2/3}} - \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac} (b + \sqrt{b^2-4ac})^{2/3}} \\
&= \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac} (b - \sqrt{b^2-4ac})^{2/3}} - \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac} (b + \sqrt{b^2-4ac})^{2/3}} \\
&= \frac{2^{2/3}c^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac} (b - \sqrt{b^2-4ac})^{2/3}} + \frac{2^{2/3}c^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac} (b + \sqrt{b^2-4ac})^{2/3}} + \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac} (b - \sqrt{b^2-4ac})^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.08

$$\frac{1}{3} \text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\log(x - \#1)}{2\#1^5c + \#1^2b} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)^(-1), x]

[Out] RootSum[a + b*#1^3 + c*#1^6 &, Log[x - #1]/(b*#1^2 + 2*c*#1^5) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + bx^3 + cx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(-1), x]

[Out] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(-1), x]

fricas [B] time = 2.02, size = 3978, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out]
$$\frac{2}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\frac{(a^2b^2 - 4a^3c)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} + b\right)\left(\frac{(a^2b^2 - 4a^3c)}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)}\right)^{\frac{1}{3}}\arctan\left(\frac{-1/6(2(1/2)^{\frac{2}{3}}(\sqrt{3}(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3))x\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} - \sqrt{3}(b^5 - 6ab^3c + 8a^2b^2c^2)x)}{(a^2b^2 - 4a^3c)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}\right) - \left(\frac{1}{2}\right)^{\frac{1}{6}}\left(\sqrt{3}(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} - \sqrt{3}(b^5 - 6ab^3c + 8a^2b^2c^2)\sqrt{(2(b^2c^2 - 2ac^3)x^2 + (1/2)^{\frac{2}{3}}(b^6 - 8ab^4c + 20a^2b^2c^2 - 16a^3c^3 - (a^2b^7 - 12a^3b^5c + 48a^4b^3c^2 - 64a^5b^2c^3)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3))}\right)\left(\frac{(a^2b^2 - 4a^3c)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} + b\right)\left(\frac{(a^2b^2 - 4a^3c)}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)}\right)^{\frac{2}{3}} - \left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\frac{(a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3)x\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} - (b^4c - 6ab^2c^2 + 8a^2c^3)x\right)\left(\frac{(a^2b^2 - 4a^3c)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} + b\right)\left(\frac{(a^2b^2 - 4a^3c)}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)}\right)^{\frac{1}{3}}\left(\frac{(b^2c^2 - 2ac^3)}{(b^2c^2 - 2ac^3)}\right)\left(\frac{(a^2b^2 - 4a^3c)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} + b\right)\left(\frac{(a^2b^2 - 4a^3c)}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)}\right)^{\frac{2}{3}} + 2\sqrt{3}\frac{(b^2c - 2ac^2)}{(b^2c - 2ac^2)} - \frac{2}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(-\left(\frac{(a^2b^2 - 4a^3c)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} - b\right)\left(\frac{(a^2b^2 - 4a^3c)}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)}\right)^{\frac{1}{3}}\arctan\left(\frac{-1/6(2(1/2)^{\frac{2}{3}}(\sqrt{3}(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3))x\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} + \sqrt{3}(b^5 - 6ab^3c + 8a^2b^2c^2)x)}{(a^2b^2 - 4a^3c)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}\right) - \left(\frac{(a^2b^2 - 4a^3c)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} - b\right)\left(\frac{(a^2b^2 - 4a^3c)}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)}\right)^{\frac{2}{3}} - \left(\frac{1}{2}\right)^{\frac{1}{6}}\left(\sqrt{3}(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3)\sqrt{(b^4 - 4ab^2c + 4a^2c^2)}}{(a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3)} + \sqrt{3}(b^5 - 6ab^3c + 8a^2b^2c^2)\sqrt{(2(b^2c^2 - 2ac^3))}\right)$$

$$\begin{aligned}
& *a*c^3)*x^2 + (1/2)^{(2/3)}*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 + \\
& (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))* \\
& -((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^{(2/3)} + (1/ \\
& 2)^{(1/3)}*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + (b \\
& ^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*x)*(-((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - \\
& b)/(a^2*b^2 - 4*a^3*c))^{(1/3)})/(b^2*c^2 - 2*a*c^3))*(-((a^2*b^2 - 4*a^3*c) \\
& *\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^{(2/3)} - 2*\sqrt{3}*(b^2*c - 2*a*c \\
& ^2))/(b^2*c - 2*a*c^2)) - 1/6*(1/2)^{(1/3)}*((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) \\
& + b)/(a^2*b^2 - 4*a^3*c))^{(1/3)}*\log(-2*(b^2*c^2 - 2*a*c^3)*x^2 - (1/2) \\
& ^{(2/3)}*(b^6 - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 - (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a \\
& ^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c) \\
& *\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^{(2/3)} + (1/2)^{(1/3)}*((a^2*b^5*c \\
& - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - (b^4*c - 6*a*b^2*c^2 + \\
& 8*a^2*c^3)*x)*(((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c) \\
& ^{(1/3)}) - 1/6*(1/2)^{(1/3)}*(-((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a \\
& ^2*b^2 - 4*a^3*c))^{(1/3)}*\log(-2*(b^2*c^2 - 2*a*c^3)*x^2 - (1/2)^{(2/3)}*(b^6 \\
& - 8*a*b^4*c + 20*a^2*b^2*c^2 - 16*a^3*c^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12* \\
& a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 \\
& - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^{(2/3)} - (1/2)^{(1/3)}*((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3) \\
& *x*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3) \\
& *x)*(-((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12 \\
& *a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^{(1/3)} \\
& + 1/3*(1/2)^{(1/3)}*((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4* \\
& a^3*c))^{(1/3)}*\log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^{(1/3)}*(b^4 - 6*a*b^2*c + 8 \\
& *a^2*c^2 - (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*\sqrt{(b^4 - 4*a*b^2*c + 4 \\
& *a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 \\
& ^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - 12*a^5*b^4*c + \\
& 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^{(1/3)}) + 1/3*(1/2) \\
& ^{(1/3)}*(-((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)}/(a^4*b^6 - \\
& 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^{(1/3)}
\end{aligned}$$

) $\log(-2*(b^2*c - 2*a*c^2)*x + (1/2)^{(1/3)}*(b^4 - 6*a*b^2*c + 8*a^2*c^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)})))*(-((a^2*b^2 - 4*a^3*c)*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)})) - b)/(a^2*b^2 - 4*a^3*c))^{(1/3)}$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate(1/(c*x^6 + b*x^3 + a), x)

maple [C] time = 0.00, size = 40, normalized size = 0.07

$$\frac{\ln\left(-\text{RootOf}\left(c_Z^6 + b_Z^3 + a\right) + x\right)}{6\text{RootOf}\left(c_Z^6 + b_Z^3 + a\right)^5 c + 3\text{RootOf}\left(c_Z^6 + b_Z^3 + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^6+b*x^3+a),x)

[Out] 1/3*sum(1/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate(1/(c*x^6 + b*x^3 + a), x)

mupad [B] time = 8.49, size = 2597, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*x^3 + c*x^6),x)

[Out] $\log(6*c^5*x + (2^{(2/3)}*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(a^2*(4*a*c - b^2)^3))^{(1/3)}$

$$\begin{aligned} & (1/2))/((54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^{(1/3)} - \\ & \log(6*c^5*x - (2^{(2/3)}*(3^{(1/2)}*1i + 1)*(-b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(a^2*(4*a*c - b^2)^3))^{(1/3)}*(36*a*c^5 - 9*b^2*c^4 + (2^{(1/3)}*(3^{(1/2)}*1i - 1)*(81*b*c^3*x*(4*a*c - b^2)^2 - (81*2^{(2/3)}*a*b*c^3*(3^{(1/2)}*1i + 1)*(4*a*c - b^2)^2*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(a^2*(4*a*c - b^2)^3))^{(1/3)))/4)*(-b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(a^2*(4*a*c - b^2)^3))^{(2/3))/36))/12)*((3^{(1/2)}*1i)/2 + 1/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^{(1/3)} \end{aligned}$$

sympy [A] time = 4.35, size = 155, normalized size = 0.28

$$\text{RootSum}\left(t^6(46656a^5c^3 - 34992a^4b^2c^2 + 8748a^3b^4c - 729a^2b^6) + t^3(432a^2bc^2 - 216ab^3c + 27b^5) + c^2, \left(t \mapsto t \log\left(x + \frac{-1296t^4a^4bc^2 + 648t^4a^3b^3c - 81t^4a^2b^5 + 12ta^2c^2 - 15tab^2c + 3tb^4}{2ac^2 - b^2c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**6+b*x**3+a),x)

[Out] RootSum(_t**6*(46656*a**5*c**3 - 34992*a**4*b**2*c**2 + 8748*a**3*b**4*c - 729*a**2*b**6) + _t**3*(432*a**2*b*c**2 - 216*a*b**3*c + 27*b**5) + c**2, Lambda(_t, _t*log(x + (-1296*_t**4*a**4*b*c**2 + 648*_t**4*a**3*b**3*c - 81*_t**4*a**2*b**5 + 12*_t*a**2*c**2 - 15*_t*a*b**2*c + 3*_t*b**4)/(2*a*c**2 - b**2*c))))

$$3.136 \quad \int \frac{1}{x^2(a+bx^3+cx^6)} dx$$

Optimal. Leaf size=610

$$\frac{\sqrt[3]{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right) \sqrt[3]{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b + \sqrt{b^2-4ac}} + \left(b + \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right) \sqrt[3]{c} \left(1 + \frac{b}{\sqrt{b^2-4ac}} \right) \log \left(\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right) \sqrt[3]{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \log \left(\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b + \sqrt{b^2-4ac}} + \left(b + \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{6 \cdot 2^{2/3} a \sqrt[3]{b - \sqrt{b^2-4ac}}}$$

Rubi [A] time = 0.82, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1368, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{6 \cdot 2^{2/3} a \sqrt[3]{b - \sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b + \sqrt{b^2-4ac}} + \left(b + \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{6 \cdot 2^{2/3} a \sqrt[3]{b + \sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \log \left(\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{3 \cdot 2^{2/3} a \sqrt[3]{b - \sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \log \left(\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b + \sqrt{b^2-4ac}} + \left(b + \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{3 \cdot 2^{2/3} a \sqrt[3]{b + \sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \operatorname{arctan} \left(\frac{1 - \sqrt[3]{\frac{b^2-4ac}{c}}}{\sqrt[3]{\frac{b^2-4ac}{c}}} \right)}{2^{2/3} \sqrt[3]{c} a \sqrt[3]{b - \sqrt{b^2-4ac}}} - \frac{\sqrt[3]{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \operatorname{arctan} \left(\frac{1 - \sqrt[3]{\frac{b^2-4ac}{c}}}{\sqrt[3]{\frac{b^2-4ac}{c}}} \right)}{2^{2/3} \sqrt[3]{c} a \sqrt[3]{b + \sqrt{b^2-4ac}}} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^3 + c*x^6)),x]

[Out] $-(1/(a*x)) + (c^{(1/3)}*(1 + b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(1 - (2*2^{(1/3)})*c^{(1/3)}*x)/(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/\text{Sqrt}[3]])/(2^{(2/3)}*\text{Sqrt}[3]*a*(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) + (c^{(1/3)}*(1 - b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(1 - (2*2^{(1/3)})*c^{(1/3)}*x)/(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/\text{Sqrt}[3]])/(2^{(2/3)}*\text{Sqrt}[3]*a*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) + (c^{(1/3)}*(1 + b/\text{Sqrt}[b^2 - 4*a*c])*\text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)}*c^{(1/3)}*x])/(3*2^{(2/3)}*a*(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) + (c^{(1/3)}*(1 - b/\text{Sqrt}[b^2 - 4*a*c])*\text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)}*c^{(1/3)}*x])/(3*2^{(2/3)}*a*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) - (c^{(1/3)}*(1 + b/\text{Sqrt}[b^2 - 4*a*c])*\text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} - 2^{(1/3)}*c^{(1/3)}*(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}*x + 2^{(2/3)}*c^{(2/3)}*x^2])/(6*2^{(2/3)}*a*(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) - (c^{(1/3)}*(1 - b/\text{Sqrt}[b^2 - 4*a*c])*\text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} - 2^{(1/3)}*c^{(1/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}*x + 2^{(2/3)}*c^{(2/3)}*x^2])/(6*2^{(2/3)}*a*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(−1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1368

Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*x^n + c*x^(2*n))^(p+1))/(a*d*(m+1)), x] - Dist[1/(a*d^n*(m+1)), Int[(d*x)^(m+n)*(b*(m+n*(p+1)+1) + c*(m+2*n*(p+1)+1)*x^n*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1510

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_)))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b

, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2(a+bx^3+cx^6)} dx &= -\frac{1}{ax} + \frac{\int \frac{x(-b-cx^3)}{a+bx^3+cx^6} dx}{a} \\
 &= -\frac{1}{ax} - \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{x}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2a} - \frac{\left(c\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{x}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2a} \\
 &= -\frac{1}{ax} + \frac{\left(c^{2/3}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} a \sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(c^{2/3}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} - \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} a \sqrt[3]{b-\sqrt{b^2-4ac}}} \\
 &= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} a \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} - \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} a \sqrt[3]{b-\sqrt{b^2-4ac}}} \\
 &= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} a \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} - \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} a \sqrt[3]{b-\sqrt{b^2-4ac}}} \\
 &= -\frac{1}{ax} + \frac{\sqrt[3]{c}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.12

$$\frac{\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1^3c \log(x-\#1) + b \log(x-\#1)}{2\#1^4c + \#1b}\& \right]}{3a} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)),x]

[Out] $-(1/(a*x)) - \text{RootSum}[a + b*\#1^3 + c*\#1^6 \& , (b*\text{Log}[x - \#1] + c*\text{Log}[x - \#1] * \#1^3)/(b*\#1 + 2*c*\#1^4) \&]/(3*a)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a + bx^3 + cx^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(a + b*x^3 + c*x^6)),x]

[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^3 + c*x^6)), x]

fricas [B] time = 3.99, size = 5266, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{6} * (4 * \sqrt{3}) * (1/2)^{(1/3)} * a * x * ((b^3 - 2 * a * b * c + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))}) / (a^4 * b^2 - 4 * a^5 * c))^{(1/3)} * \text{rctan}(1/3 * ((1/2)^{(5/6)} * (\sqrt{3}) * (a^4 * b^5 - 8 * a^5 * b^3 * c + 16 * a^6 * b * c^2) * \sqrt{((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))} - \sqrt{3}) * (b^6 - 8 * a * b^4 * c + 18 * a^2 * b^2 * c^2 - 8 * a^3 * c^3)) * ((b^3 - 2 * a * b * c + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))}) / (a^4 * b^2 - 4 * a^5 * c))^{(1/3)} * \sqrt{((2 * (b^4 * c^3 - 4 * a * b^2 * c^4 + 2 * a^2 * c^5) * x^2 + (1/2)^{(2/3)} * ((a^4 * b^8 - 13 * a^5 * b^6 * c + 60 * a^6 * b^4 * c^2 - 112 * a^7 * b^2 * c^3 + 64 * a^8 * c^4) * x * \sqrt{((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))} - (b^9 - 11 * a * b^7 * c + 42 * a^2 * b^5 * c^2 - 62 * a^3 * b^3 * c^3 + 24 * a^4 * b * c^4) * x) * ((b^3 - 2 * a * b * c + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))}) / (a^4 * b^2 - 4 * a^5 * c))^{(2/3)} - (1/2)^{(1/3)} * (b^7 * c - 8 * a * b^5 * c^2 + 18 * a^2 * b^3 * c^3 - 8 * a^3 * b * c^4 - (a^4 * b^6 * c - 10 * a^5 * b^4 * c^2 + 32 * a^6 * b^2 * c^3 - 32 * a^7 * c^4) * \sqrt{((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))}) * ((b^3 - 2 * a * b * c + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))}) / (a^4 * b^2 - 4 * a^5 * c))^{(1/3)}) / (b^4 * c^3 - 4 * a * b^2 * c^4 + 2 * a^2 * c^5) - (1/2)^{(1/3)} * (\sqrt{3}) * (a^4 * b^5 - 8 * a^5 * b^3 * c + 16 * a^6 * b * c^2) * x * \sqrt{((b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))}$

$$\begin{aligned}
& \sqrt[3]{b^2c^3 + 4a^4c^4} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3) - \sqrt{3} * (b^6 - 8a^2b^4c + 18a^3b^2c^2 - 8a^3c^3) * x * ((b^3 - 2a^2b^2c + (a^4b^2 - 4a^5c) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) / (a^4b^2 - 4a^5c)^{1/3} + \sqrt{3} * (b^4c - 4a^2b^2c^2 + 2a^2c^3) / (b^4c - 4a^2b^2c^2 + 2a^2c^3) - 4 * \sqrt{3} * (1/2)^{1/3} * a * x * ((b^3 - 2a^2b^2c - (a^4b^2 - 4a^5c) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) / (a^4b^2 - 4a^5c)^{1/3} * \arctan(1/3 * ((1/2)^{5/6} * (\sqrt{3} * (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) + \sqrt{3} * (b^6 - 8a^2b^4c + 18a^3b^2c^2 - 8a^3c^3) * ((b^3 - 2a^2b^2c - (a^4b^2 - 4a^5c) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) / (a^4b^2 - 4a^5c)^{1/3} * \sqrt{(2 * (b^4c^3 - 4a^2b^2c^4 + 2a^2c^5) * x^2 - (1/2)^{2/3} * ((a^4b^8 - 13a^5b^6c + 60a^6b^4c^2 - 112a^7b^2c^3 + 64a^8c^4) * x * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) + (b^9 - 11a^2b^7c + 42a^2b^5c^2 - 62a^3b^3c^3 + 24a^4b^2c^4) * x) * ((b^3 - 2a^2b^2c - (a^4b^2 - 4a^5c) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) / (a^4b^2 - 4a^5c)^{2/3} - (1/2)^{1/3} * (b^7c - 8a^2b^5c^2 + 18a^2b^3c^3 - 8a^3b^2c^4 + (a^4b^6c - 10a^5b^4c^2 + 32a^6b^2c^3 - 32a^7c^4) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) * ((b^3 - 2a^2b^2c - (a^4b^2 - 4a^5c) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) / (a^4b^2 - 4a^5c)^{1/3} / (b^4c^3 - 4a^2b^2c^4 + 2a^2c^5) - (1/2)^{1/3} * (\sqrt{3} * (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2) * x * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) + \sqrt{3} * (b^6 - 8a^2b^4c + 18a^3b^2c^2 - 8a^3c^3) * x * ((b^3 - 2a^2b^2c - (a^4b^2 - 4a^5c) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) / (a^4b^2 - 4a^5c)^{1/3} - \sqrt{3} * (b^4c - 4a^2b^2c^2 + 2a^2c^3) / (b^4c - 4a^2b^2c^2 + 2a^2c^3) - (1/2)^{1/3} * a * x * ((b^3 - 2a^2b^2c + (a^4b^2 - 4a^5c) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) / (a^4b^2 - 4a^5c)^{1/3} * \log(2 * (b^4c^3 - 4a^2b^2c^4 + 2a^2c^5) * x^2 + (1/2)^{2/3} * ((a^4b^8 - 13a^5b^6c + 60a^6b^4c^2 - 112a^7b^2c^3 + 64a^8c^4) * x * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) - (b^9 - 11a^2b^7c + 42a^2b^5c^2 - 62a^3b^3c^3 + 24a^4b^2c^4) * x) * ((b^3 - 2a^2b^2c + (a^4b^2 - 4a^5c) * \sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3))) / (a^4b^2 - 4a^5c)^{2/3} - (1/2)^{1/3} *
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{3} \right) * (b^7 * c - 8 * a * b^5 * c^2 + 18 * a^2 * b^3 * c^3 - 8 * a^3 * b * c^4 - (a^4 * b^6 * c - 10 * a^5 * b^4 * c^2 + 32 * a^6 * b^2 * c^3 - 32 * a^7 * c^4) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3)) * ((b^3 - 2 * a * b * c + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))) / (a^4 * b^2 - 4 * a^5 * c)^{(1/3)} - \left(\frac{1}{2} \right)^{(1/3)} * a * x * ((b^3 - 2 * a * b * c - (a^4 * b^2 - 4 * a^5 * c) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))) / (a^4 * b^2 - 4 * a^5 * c)^{(1/3)} * \log(2 * (b^4 * c^3 - 4 * a * b^2 * c^4 + 2 * a^2 * c^5) * x^2 - \left(\frac{1}{2} \right)^{(2/3)} * ((a^4 * b^8 - 13 * a^5 * b^6 * c + 60 * a^6 * b^4 * c^2 - 112 * a^7 * b^2 * c^3 + 64 * a^8 * c^4) * x * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3)) + (b^9 - 11 * a * b^7 * c + 42 * a^2 * b^5 * c^2 - 62 * a^3 * b^3 * c^3 + 24 * a^4 * b * c^4) * x) * ((b^3 - 2 * a * b * c - (a^4 * b^2 - 4 * a^5 * c) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))) / (a^4 * b^2 - 4 * a^5 * c)^{(2/3)} - \left(\frac{1}{2} \right)^{(1/3)} * (b^7 * c - 8 * a * b^5 * c^2 + 18 * a^2 * b^3 * c^3 - 8 * a^3 * b * c^4 + (a^4 * b^6 * c - 10 * a^5 * b^4 * c^2 + 32 * a^6 * b^2 * c^3 - 32 * a^7 * c^4) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3)) * ((b^3 - 2 * a * b * c - (a^4 * b^2 - 4 * a^5 * c) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))) / (a^4 * b^2 - 4 * a^5 * c)^{(1/3)} + 2 * \left(\frac{1}{2} \right)^{(1/3)} * a * x * ((b^3 - 2 * a * b * c + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))) / (a^4 * b^2 - 4 * a^5 * c)^{(1/3)} * \log\left(\left(\frac{1}{2}\right)^{(2/3)} * (b^9 - 11 * a * b^7 * c + 42 * a^2 * b^5 * c^2 - 62 * a^3 * b^3 * c^3 + 24 * a^4 * b * c^4 - (a^4 * b^8 - 13 * a^5 * b^6 * c + 60 * a^6 * b^4 * c^2 - 112 * a^7 * b^2 * c^3 + 64 * a^8 * c^4) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3)) * ((b^3 - 2 * a * b * c + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))) / (a^4 * b^2 - 4 * a^5 * c)^{(2/3)} + 2 * (b^4 * c^3 - 4 * a * b^2 * c^4 + 2 * a^2 * c^5) * x) + 2 * \left(\frac{1}{2} \right)^{(1/3)} * a * x * ((b^3 - 2 * a * b * c - (a^4 * b^2 - 4 * a^5 * c) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))) / (a^4 * b^2 - 4 * a^5 * c)^{(1/3)} * \log\left(\left(\frac{1}{2}\right)^{(2/3)} * (b^9 - 11 * a * b^7 * c + 42 * a^2 * b^5 * c^2 - 62 * a^3 * b^3 * c^3 + 24 * a^4 * b * c^4 + (a^4 * b^8 - 13 * a^5 * b^6 * c + 60 * a^6 * b^4 * c^2 - 112 * a^7 * b^2 * c^3 + 64 * a^8 * c^4) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3)) * ((b^3 - 2 * a * b * c - (a^4 * b^2 - 4 * a^5 * c) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3))) / (a^4 * b^2 - 4 * a^5 * c)^{(2/3)} + 2 * (b^4 * c^3 - 4 * a * b^2 * c^4 + 2 * a^2 * c^5) * x) - 6) / (a * x)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^2), x)

maple [C] time = 0.01, size = 61, normalized size = 0.10

$$\frac{\left(\text{RootOf}(c_Z^6 + b_Z^3 + a)^4 c + \text{RootOf}(c_Z^6 + b_Z^3 + a) b\right) \ln(-\text{RootOf}(c_Z^6 + b_Z^3 + a) + x)}{3a \left(2 \text{RootOf}(c_Z^6 + b_Z^3 + a)^5 c + \text{RootOf}(c_Z^6 + b_Z^3 + a)^2 b\right)} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^6+b*x^3+a),x)

[Out] -1/3/a*sum((R^4*c+R*b)/(2*R^5*c+R^2*b)*ln(-R+x),R=RootOf(Z^6*c+Z^3*b+a))-1/a/x

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{cx^4+bx}{cx^6+bx^3+a} dx}{a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] -integrate((c*x^4 + b*x)/(c*x^6 + b*x^3 + a), x)/a - 1/(a*x)

mupad [B] time = 6.89, size = 2978, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x^3 + c*x^6)),x)

[Out] log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3))*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2))^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3))^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3))^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3))^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7 + b^4*(-(4*a*c -

$$\begin{aligned}
& b^2)^3)^{(1/2)} - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(a^4*(4*a*c - b^2)^3))^{(1/3)}/6)*((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^{(1/3)} + \log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^{(2/3)}*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^{(1/3)})*a^{10}*b*c^3*(4*a*c - b^2)^2*((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(a^4*(4*a*c - b^2)^3))^{(2/3)}/2)*((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(a^4*(4*a*c - b^2)^3))^{(1/3)}/6)*(-(b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^{(1/3)} - 1/(a*x) + \log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^{(2/3)}*(3^{(1/2)}*1i - 1)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) - (27*2^{(1/3)})*a^{10}*b*c^3*(3^{(1/2)}*1i + 1)*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(a^4*(4*a*c - b^2)^3))^{(2/3)}/4)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(a^4*(4*a*c - b^2)^3))^{(1/3)}/12)*((3^{(1/2)}*1i)/2 - 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^{(1/3)} - \log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 + (2^{(2/3)}*(3^{(1/2)}*1i + 1)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^{(1/3)})*a^{10}*b*c^3*(3^{(1/2)}*1i - 1)*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(a^4*(4*a*c - b^2)^3))^{(2/3)}/4)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(a^4*(4*a*c - b^2)^3))^{(1/3)}/12)*((3^{(1/2)}*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^{(1/3)} - \log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 + (2^{(2/3)}*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^{(1/3)})*a^{10}*b*c^3*(3^{(1/2)}*1i - 1)*(4*a*c - b^2)^2*((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(a^4*(4*a*c - b^2)^3))^{(2/3)}/4)*(3^{(1/2)}*1i + 1)*((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(a^4*(4*a*c - b^2)^3))^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(a^4*(4
\end{aligned}$$

$$\begin{aligned} & *a*c - b^2)^3)^{(1/3))/12)*((3^{(1/2)}*1i)/2 + 1/2)*(-(b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^{(1/3)} + \log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^{(2/3)}*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) - (27*2^{(1/3)}*a^{10}*b*c^3*(3^{(1/2)}*1i + 1)*(4*a*c - b^2)^2*((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)))/(a^4*(4*a*c - b^2)^3))^{(2/3)))/4)*(3^{(1/2)}*1i - 1)*((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)))/(a^4*(4*a*c - b^2)^3))^{(1/3))/12)*((3^{(1/2)}*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^{(1/3)} \end{aligned}$$

sympy [A] time = 3.19, size = 252, normalized size = 0.41

$$\text{RootSum}\left(t^6(46656t^3 - 34992t^2 + 8748t^5b^4c - 729t^6b^6) + t^3(-864a^3b^3c^3 + 864a^2b^3c^2 - 270ab^5c + 27b^7) + c^4, \left(t \mapsto t \log\left(x + \frac{-15552t^5a^8c^4 + 27216t^5a^7b^2c^3 - 14580t^5a^6b^4c^2 + 3159t^5a^5b^6c - 243t^5a^4b^8 + 252t^5a^4bc^4 - 567t^5a^3b^3c^3 + 378t^5a^2b^5c^2 - 99t^5ab^7c + 9t^5b^9}{2t^2c^5 - 4ab^2c^4 + b^4c^3}\right)\right) - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**6+b*x**3+a),x)

[Out] RootSum(_t**6*(46656*a**7*c**3 - 34992*a**6*b**2*c**2 + 8748*a**5*b**4*c - 729*a**4*b**6) + _t**3*(-864*a**3*b**c**3 + 864*a**2*b**3*c**2 - 270*a*b**5*c + 27*b**7) + c**4, Lambda(_t, _t*log(x + (-15552*_t**5*a**8*c**4 + 27216*_t**5*a**7*b**2*c**3 - 14580*_t**5*a**6*b**4*c**2 + 3159*_t**5*a**5*b**6*c - 243*_t**5*a**4*b**8 + 252*_t**2*a**4*b**c**4 - 567*_t**2*a**3*b**3*c**3 + 378*_t**2*a**2*b**5*c**2 - 99*_t**2*a*b**7*c + 9*_t**2*b**9)/(2*a**2*c**5 - 4*a*b**2*c**4 + b**4*c**3)))) - 1/(a*x)

$$3.137 \quad \int \frac{1}{x^3(a+bx^3+cx^6)} dx$$

Optimal. Leaf size=612

$$\frac{c^{2/3} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right) c^{2/3} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \log \left(- \right)}{6\sqrt[3]{2} a \left(b - \sqrt{b^2-4ac} \right)^{2/3}} + \frac{c^{2/3} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right) c^{2/3} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \log \left(- \right)}{6\sqrt[3]{2} a \left(b - \sqrt{b^2-4ac} \right)^{2/3}}$$

Rubi [A] time = 0.82, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1368, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{c^{2/3} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{6\sqrt[3]{2} a \left(b - \sqrt{b^2-4ac} \right)^{2/3}} + \frac{c^{2/3} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{6\sqrt[3]{2} a \left(b - \sqrt{b^2-4ac} \right)^{2/3}} + \frac{c^{2/3} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \log \left(\sqrt{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x \right)}{3\sqrt[3]{2} a \left(b - \sqrt{b^2-4ac} \right)^{2/3}} + \frac{c^{2/3} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \log \left(\sqrt{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x \right)}{3\sqrt[3]{2} a \left(b - \sqrt{b^2-4ac} \right)^{2/3}} + \frac{c^{2/3} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{\sqrt[3]{2} \sqrt[3]{c} x}{\sqrt{b - \sqrt{b^2-4ac}}} \right)}{\sqrt[3]{2} \sqrt[3]{a} \left(b - \sqrt{b^2-4ac} \right)^{2/3}} + \frac{c^{2/3} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt[3]{2} \sqrt[3]{c} x}{\sqrt{b - \sqrt{b^2-4ac}}} \right)}{\sqrt[3]{2} \sqrt[3]{a} \left(b - \sqrt{b^2-4ac} \right)^{2/3}} - \frac{1}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*x^3 + c*x^6)),x]

[Out] $-1/(2*a*x^2) + (c^{(2/3)}*(1 + b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(1 - (2*2^{(1/3)})*c^{(1/3)}*x)/(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/\text{Sqrt}[3]])/(2^{(1/3)}*\text{Sqrt}[3]*a*(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)}) + (c^{(2/3)}*(1 - b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(1 - (2*2^{(1/3)})*c^{(1/3)}*x)/(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/\text{Sqrt}[3]])/(2^{(1/3)}*\text{Sqrt}[3]*a*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)}) - (c^{(2/3)}*(1 + b/\text{Sqrt}[b^2 - 4*a*c])* \text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)}*c^{(1/3)}*x])/(3*2^{(1/3)}*a*(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)}) - (c^{(2/3)}*(1 - b/\text{Sqrt}[b^2 - 4*a*c])* \text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)}*c^{(1/3)}*x])/(3*2^{(1/3)}*a*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)}) + (c^{(2/3)}*(1 + b/\text{Sqrt}[b^2 - 4*a*c])* \text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} - 2^{(1/3)}*c^{(1/3)}*(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}*x + 2^{(2/3)}*c^{(2/3)}*x^2])/(6*2^{(1/3)}*a*(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)}) + (c^{(2/3)}*(1 - b/\text{Sqrt}[b^2 - 4*a*c])* \text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} - 2^{(1/3)}*c^{(1/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}*x + 2^{(2/3)}*c^{(2/3)}*x^2])/(6*2^{(1/3)}*a*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)})$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)⁽⁻¹⁾, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R

$t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[\{a, b\}, x]$

Rule 204

$Int[((a_) + (b_)*(x_)^2)^{-1}, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[\{a, b\}, x] \&\& PosQ[a/b] \&\& (LtQ[a, 0] || LtQ[b, 0])$

Rule 617

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] := With[\{q = 1 - 4*Simplify[(a*c)/b^2]\}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] \&\& (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[\{a, b, c\}, x] \&\& NeQ[b^2 - 4*a*c, 0]$

Rule 628

$Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& EqQ[2*c*d - b*e, 0]$

Rule 634

$Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& NeQ[2*c*d - b*e, 0] \&\& NeQ[b^2 - 4*a*c, 0] \&\& !NiceSqrtQ[b^2 - 4*a*c]$

Rule 1368

$Int[((d_)*(x_)^m)*((a_) + (c_)*(x_)^{n2_}) + (b_)*(x_)^{n_})^{p_}, x_Symbol] := Simp[((d*x)^{m+1}*(a + b*x^n + c*x^{2*n})^{p+1})/(a*d*(m+1)), x] - Dist[1/(a*d^n*(m+1)), Int[(d*x)^{m+n}*(b*(m+n*(p+1)+1) + c*(m+2*n*(p+1)+1)*x^n*(a + b*x^n + c*x^{2*n})^p, x], x] /; FreeQ[\{a, b, c, d, p\}, x] \&\& EqQ[n2, 2*n] \&\& NeQ[b^2 - 4*a*c, 0] \&\& IGtQ[n, 0] \&\& LtQ[m, -1] \&\& IntegerQ[p]$

Rule 1422

$Int[((d_) + (e_)*(x_)^{n_})/((a_) + (b_)*(x_)^{n_}) + (c_)*(x_)^{n2_}), x_Symbol] := With[\{q = Rt[b^2 - 4*a*c, 2]\}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[\{a, b, c, d, e, n\}, x] \&\& EqQ[n2, 2*n] \&\& NeQ[b^2 - 4*a*c, 0] \&\& NeQ[c*d^2 - b*d*e + a*e^2, 0] \&\& (PosQ[b^2 - 4*a$

*c] || !IGtQ[n/2, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 (a + bx^3 + cx^6)} dx &= -\frac{1}{2ax^2} + \frac{\int \frac{-2b-2cx^3}{a+bx^3+cx^6} dx}{2a} \\
 &= -\frac{1}{2ax^2} - \frac{\left(c \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2a} - \frac{\left(c \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2a} \\
 &= -\frac{1}{2ax^2} - \frac{\left(c \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{3\sqrt{b-\sqrt{b^2-4ac}}}{\sqrt{2}} + \sqrt[3]{c}x} dx}{3\sqrt[3]{2}a \left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(c \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\left(b - \sqrt{b^2-4ac}\right)^{2/3}} dx}{3\sqrt[3]{2}a \left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
 &= -\frac{1}{2ax^2} - \frac{c^{2/3} \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3\sqrt[3]{2}a \left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3\sqrt[3]{2}a \left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
 &= -\frac{1}{2ax^2} - \frac{c^{2/3} \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3\sqrt[3]{2}a \left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3\sqrt[3]{2}a \left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
 &= -\frac{1}{2ax^2} + \frac{c^{2/3} \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a \left(b + \sqrt{b^2-4ac}\right)^{2/3}}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.12

$$\frac{\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1^3c \log(x-\#1) + b \log(x-\#1)}{2\#1^5c + \#1^2b}\&\right]}{3a} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)),x]

[Out] $-\frac{1}{2} \frac{1}{a x^2} - \frac{\text{RootSum}[a + b \#1^3 + c \#1^6 \& , (b \text{Log}[x - \#1] + c \text{Log}[x - \#1]) \#1^3] / (b \#1^2 + 2 c \#1^5) \&]}{3 a}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b x^3 + c x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(a + b*x^3 + c*x^6)),x]

[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^3 + c*x^6)), x]

fricas [B] time = 3.76, size = 5771, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] $\frac{1}{6} (4 \sqrt{3}) (1/2)^{1/3} a x^2 (-b^4 - 3 a b^2 c + a^2 c^2 + (a^5 b^2 - 4 a^6 c) \sqrt{(b^{10} - 10 a b^8 c + 35 a^2 b^6 c^2 - 50 a^3 b^4 c^3 + 25 a^4 b^2 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3)}) / (a^5 b^2 - 4 a^6 c)^{1/3} \arctan(-1/6 (2 (1/2)^{2/3} (\sqrt{3} (a^5 b^8 - 13 a^6 b^6 c + 60 a^7 b^4 c^2 - 112 a^8 b^2 c^3 + 64 a^9 c^4) x \sqrt{(b^{10} - 10 a b^8 c + 35 a^2 b^6 c^2 - 50 a^3 b^4 c^3 + 25 a^4 b^2 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3)}) - \sqrt{3} (b^{10} - 12 a b^8 c + 52 a^2 b^6 c^2 - 95 a^3 b^4 c^3 + 60 a^4 b^2 c^4) x) (-b^4 - 3 a b^2 c + a^2 c^2 + (a^5 b^2 - 4 a^6 c) \sqrt{(b^{10} - 10 a b^8 c + 35 a^2 b^6 c^2 - 50 a^3 b^4 c^3 + 25 a^4 b^2 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3)}) / (a^5 b^2 - 4 a^6 c)^{2/3} - (1/2)^{1/6} (\sqrt{3} (a^5 b^8 - 13 a^6 b^6 c + 60 a^7 b^4 c^2 - 112 a^8 b^2 c^3 + 64 a^9 c^4) \sqrt{(b^{10} - 10 a b^8 c + 35 a^2 b^6 c^2 - 50 a^3 b^4 c^3 + 25 a^4 b^2 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3)}) - \sqrt{3} (b^{10} - 12 a b^8 c + 52 a^2 b^6 c^2 - 95 a^3 b^4 c^3 + 60 a^4 b^2 c^4) x) (-b^4 - 3 a b^2 c + a^2 c^2 + (a^5 b^2 - 4 a^6 c) \sqrt{(b^{10} - 10 a b^8 c + 35 a^2 b^6 c^2 - 50 a^3 b^4 c^3 + 25 a^4 b^2 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3)}) / (a^5 b^2 - 4 a^6 c)^{2/3} \sqrt{(2 (b^5 c^4 - 5 a b^3 c^5 + 5 a^2 b c^6) x^2 + (1/2)^{2/3} (b^{11} - 13 a b^9 c + 63 a^2 b^7 c^2 - 138 a^3 b^5 c^3 + 130 a^4 b^3 c^4 - 40 a^5 b c^5 - (a^5 b^9 - 14 a^6 b^7 c + 72 a^7 b^5 c^2 - 160 a^8 b^3 c^3 + 128 a^9 b c^4) \sqrt{(b^{10} - 10 a b^8 c + 35 a^2 b^6 c^2 - 50 a^3 b^4 c^3 + 25 a^4 b^2 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3)}) (-b^4 - 3 a b^2 c + a^2 c^2 + (a^5 b^2 - 4 a^6 c) \sqrt{(b^{10} - 10 a b^8 c + 35 a^2 b^6 c^2 - 50 a^3 b^4 c^3 +$

$$\begin{aligned}
& 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)) \\
&)/(a^5b^2 - 4a^6c)^{(2/3)} + (1/2)^{(1/3)}*((a^5b^6c^2 - 10a^6b^4c^3 + \\
& 32a^7b^2c^4 - 32a^8c^5)*x*\sqrt{(b^{10} - 10a*b^8c + 35a^2b^6c^2 - \\
& 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - \\
& 64a^{13}c^3)) - (b^8c^2 - 9a*b^6c^3 + 25a^2b^4c^4 - 20a^3b^2c^5)*x) \\
& *(-b^4 - 3a*b^2c + a^2c^2 + (a^5b^2 - 4a^6c)*\sqrt{(b^{10} - 10a*b^8c + \\
& 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + \\
& 48a^{12}b^2c^2 - 64a^{13}c^3)))/(a^5b^2 - 4a^6c)^{(1/3)))/(b^5c^4 - \\
& 5a*b^3c^5 + 5a^2b*c^6)) + 2*\sqrt{3}*(b^5c^3 - 5a*b^3c^4 + 5a^2b*c^5))/ \\
& (b^5c^3 - 5a*b^3c^4 + 5a^2b*c^5)) - 4*\sqrt{3}*(1/2)^{(1/3)}* \\
& a*x^2*(-b^4 - 3a*b^2c + a^2c^2 - (a^5b^2 - 4a^6c)*\sqrt{(b^{10} - 10a*b^8c + \\
& 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + \\
& 48a^{12}b^2c^2 - 64a^{13}c^3)))/(a^5b^2 - 4a^6c)^{(1/3)}*\arctan \\
& (-1/6*(2*(1/2)^{(2/3)}*(\sqrt{3}*(a^5b^8 - 13a^6b^6c + 60a^7b^4c^2 - \\
& 112a^8b^2c^3 + 64a^9c^4))*\sqrt{(b^{10} - 10a*b^8c + 35a^2b^6c^2 - \\
& 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - \\
& 64a^{13}c^3)) + \sqrt{3}*(b^{10} - 12a*b^8c + 52a^2b^6c^2 - 95a^3b^4c^3 + \\
& 60a^4b^2c^4)*x)*(-b^4 - 3a*b^2c + a^2c^2 - (a^5b^2 - 4a^6c)*\sqrt{(b^{10} - \\
& 10a*b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + \\
& 48a^{12}b^2c^2 - 64a^{13}c^3)))/(a^5b^2 - 4a^6c)^{(2/3)} - (1/2)^{(1/6)}*(\sqrt{3}*(a^5b^8 - \\
& 13a^6b^6c + 60a^7b^4c^2 - 112a^8b^2c^3 + 64a^9c^4))*\sqrt{(b^{10} - 10a*b^8c + \\
& 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - \\
& 64a^{13}c^3)) + \sqrt{3}*(b^{10} - 12a*b^8c + 52a^2b^6c^2 - 95a^3b^4c^3 + \\
& 60a^4b^2c^4))*(-b^4 - 3a*b^2c + a^2c^2 - (a^5b^2 - 4a^6c)*\sqrt{(b^{10} - \\
& 10a*b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + \\
& 48a^{12}b^2c^2 - 64a^{13}c^3)))/(a^5b^2 - 4a^6c)^{(2/3)}*\sqrt{(2*(b^5c^4 - 5a*b^3c^5 + \\
& 5a^2b*c^6))*x^2 + (1/2)^{(2/3)}*(b^{11} - 13a*b^9c + 63a^2b^7c^2 - 138a^3b^5c^3 + \\
& 130a^4b^3c^4 - 40a^5b*c^5 + (a^5b^9 - 14a^6b^7c + 72a^7b^5c^2 - 160a^8b^3c^3 + \\
& 128a^9b*c^4))*\sqrt{(b^{10} - 10a*b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/ \\
& (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)))*(-b^4 - 3a*b^2c + a^2c^2 - \\
& (a^5b^2 - 4a^6c)*\sqrt{(b^{10} - 10a*b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/ \\
& (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)))/(a^5b^2 - 4a^6c)^{(2/3)} \\
&) - (1/2)^{(1/3)}*((a^5b^6c^2 - 10a^6b^4c^3 + 32a^7b^2c^4 - 32a^8c^5)*x*\sqrt{(b^{10} - \\
& 10a*b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + \\
& 48a^{12}b^2c^2 - 64a^{13}c^3)) + (b^8c^2 - 9a*b^6c^3 + 25a^2b^4c^4 - 20a^3b^2c^5)*x) \\
& *(-b^4 - 3a*b^2c + a^2c^2 - (a^5b^2 - 4a^6c)*\sqrt{(b^{10} - 10a*b^8c + 35a^2b^6c^2 - \\
& 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)))/ \\
& (a^5b^2 - 4a^6c)^{(1/3)))/(b^5c^4 - 5a*b^3c^5 + 5a^2b*c^6)) - 2*\sqrt{3}*(b^5c^3 - \\
& 5a*b^3c^4 + 5a^2b*c^5))/ (b^5c^3 - 5a*b^3c^4 + 5a^2b*c^5)) - (1/2)^{(1/3)}* \\
& a*x^2*(-b^4 - 3a*b^2c + a^2c^2 + (a^5b^2 - 4a^6c)*\sqrt{(b^{10} - 10a*b^8c + 35a^2b^6c^2 - \\
& 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))
\end{aligned}$$

$$\begin{aligned}
&^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)))/(a^5b^2 - 4a^6c))^{1/3} \log(2*(b^5c^4 - 5ab^3c^5 + 5a^2b^2c^6) * x^2 + (1/2)^{(2/3)}*(b^{11} - 13a^2b^9c + 63a^4b^7c^2 - 138a^3b^5c^3 + 130a^4b^3c^4 - 40a^5b^2c^5 - (a^5b^9 - 14a^6b^7c + 72a^7b^5c^2 - 160a^8b^3c^3 + 128a^9b^2c^4) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) * (-b^4 - 3ab^2c + a^2c^2 + (a^5b^2 - 4a^6c) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{2/3} + (1/2)^{(1/3)} * ((a^5b^6c^2 - 10a^6b^4c^3 + 32a^7b^2c^4 - 32a^8c^5) * x * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)}) - (b^8c^2 - 9a^2b^6c^3 + 25a^2b^4c^4 - 20a^3b^2c^5) * x) * (-b^4 - 3ab^2c + a^2c^2 + (a^5b^2 - 4a^6c) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{1/3} - (1/2)^{(1/3)} * a * x^2 * (-b^4 - 3ab^2c + a^2c^2 - (a^5b^2 - 4a^6c) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{1/3} * \log(2*(b^5c^4 - 5ab^3c^5 + 5a^2b^2c^6) * x^2 + (1/2)^{(2/3)}*(b^{11} - 13a^2b^9c + 63a^4b^7c^2 - 138a^3b^5c^3 + 130a^4b^3c^4 - 40a^5b^2c^5 + (a^5b^9 - 14a^6b^7c + 72a^7b^5c^2 - 160a^8b^3c^3 + 128a^9b^2c^4) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) * (-b^4 - 3ab^2c + a^2c^2 - (a^5b^2 - 4a^6c) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{2/3} - (1/2)^{(1/3)} * ((a^5b^6c^2 - 10a^6b^4c^3 + 32a^7b^2c^4 - 32a^8c^5) * x * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)}) + (b^8c^2 - 9a^2b^6c^3 + 25a^2b^4c^4 - 20a^3b^2c^5) * x) * (-b^4 - 3ab^2c + a^2c^2 - (a^5b^2 - 4a^6c) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{1/3} + 2*(1/2)^{(1/3)} * a * x^2 * (-b^4 - 3ab^2c + a^2c^2 + (a^5b^2 - 4a^6c) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{1/3} * \log(2*(b^5c^2 - 5ab^3c^3 + 5a^2b^2c^4) * x + (1/2)^{(1/3)}*(b^8 - 9a^2b^6c + 25a^2b^4c^2 - 20a^3b^2c^3 - (a^5b^6 - 10a^6b^4c + 32a^7b^2c^2 - 32a^8c^3) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) * (-b^4 - 3ab^2c + a^2c^2 + (a^5b^2 - 4a^6c) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{1/3} + 2*(1/2)^{(1/3)} * a * x^2 * (-b^4 - 3ab^2c + a^2c^2 - (a^5b^2 - 4a^6c) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{1/3} * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{1/3} + 2*(1/2)^{(1/3)} * a * x^2 * (-b^4 - 3ab^2c + a^2c^2 - (a^5b^2 - 4a^6c) * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{1/3} * \sqrt{(b^{10} - 10a^2b^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})) / (a^5b^2 - 4a^6c))^{1/3}
\end{aligned}$$

$$\frac{3 + 25a^4b^2c^4}{(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)} \left/ \frac{(a^5b^2 - 4a^6c)^{1/3} \log(2(b^5c^2 - 5ab^3c^3 + 5a^2b^2c^4)x + (1/2)^{1/3}(b^8 - 9ab^6c + 25a^2b^4c^2 - 20a^3b^2c^3 + (a^5b^6 - 10a^6b^4c + 32a^7b^2c^2 - 32a^8c^3) \sqrt{(b^{10} - 10ab^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)}) \sqrt{(b^4 - 3ab^2c + a^2c^2 - (a^5b^2 - 4a^6c) \sqrt{(b^{10} - 10ab^8c + 35a^2b^6c^2 - 50a^3b^4c^3 + 25a^4b^2c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)})} \right) - 3 \right/ (ax^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^3), x)

maple [C] time = 0.00, size = 62, normalized size = 0.10

$$\frac{\left(-\text{RootOf}(c_Z^6 + b_Z^3 + a)^3 c - b\right) \ln\left(-\text{RootOf}(c_Z^6 + b_Z^3 + a) + x\right)}{3a \left(2 \text{RootOf}(c_Z^6 + b_Z^3 + a)^5 c + \text{RootOf}(c_Z^6 + b_Z^3 + a)^2 b\right)} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(c*x^6+b*x^3+a),x)

[Out] 1/3/a*sum((-_R^3*c-b)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))-1/2/a/x^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx^3+b}{cx^6+bx^3+a} dx}{a} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] -integrate((c*x^3 + b)/(c*x^6 + b*x^3 + a), x)/a - 1/2/(a*x^2)

mupad [B] time = 10.65, size = 4063, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^3*(a + b*x^3 + c*x^6)), x)$

[Out] $\log\left(\left(2^{2/3}\right)\left(b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3\right)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3\right)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2} / (a^5(4ac - b^2)^3)^{1/3} * (72a^8b^6c^6 + (2^{1/3})(81a^8c^3x(ac - b^2)(4ac - b^2)^2 + (81*2^{2/3})a^{10}b^2c^3(4ac - b^2)^2((b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2}) / (a^5(4ac - b^2)^3)^{1/3}) / 2 * ((b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2} / (a^5(4ac - b^2)^3)^{1/3}) / 18 + 9a^6b^5c^4 - 54a^7b^3c^5) / 6 - 3a^6c^6x(2ac - b^2)) * (- (b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2} / (54(a^5b^6 - 64a^8c^3 - 12a^6b^4c + 48a^7b^2c^2))^{1/3} + \log\left(\left(2^{2/3}\right)\left(b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3\right)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3\right)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2} / (a^5(4ac - b^2)^3)^{1/3} * (72a^8b^6c^6 + (2^{1/3})(81a^8c^3x(ac - b^2)(4ac - b^2)^2 + (81*2^{2/3})a^{10}b^2c^3(4ac - b^2)^2((b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (a^5(4ac - b^2)^3)^{1/3}) / 2 * ((b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2} / (a^5(4ac - b^2)^3)^{1/3}) / 18 + 9a^6b^5c^4 - 54a^7b^3c^5) / 6 - 3a^6c^6x(2ac - b^2)) * (- (b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2} / (54(a^5b^6 - 64a^8c^3 - 12a^6b^4c + 48a^7b^2c^2))^{1/3} - 1/(2ax^2) + \log\left(\left(2^{2/3}\right)\left(3^{1/2}i - 1\right)\left(b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3\right)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3\right)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2} / (a^5(4ac - b^2)^3)^{1/3} * (72a^8b^6c^6 + 9a^6b^5c^4 - 54a^7b^3c^5 - (2^{1/3})(3^{1/2}i + 1)(81a^8c^3x(ac - b^2)(4ac - b^2)^2 + (81*2^{2/3})a^{10}b^2c^3(3^{1/2}i - 1)(4ac - b^2)^2((b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2}) / (a^5(4ac - b^2)^3)^{1/3}) / 4 * ((b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2} / (a^5(4ac - b^2)^3)^{1/3}) / 36) / 12 - 3a^6c^6x(2ac - b^2)) * ((3^{1/2}i) / 2 - 1/2) * (- (b^8 + 16$

$$\begin{aligned}
& a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - \\
& 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2} \\
& / (54(a^5b^6 - 64a^8c^3 - 12a^6b^4c + 48a^7b^2c^2))^{1/3} - \log((2^{2/3}(3^{1/2}i + 1)((b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2}) / (a^5(4ac - b^2)^3))^{1/3} \\
& * (72a^8b^6c^6 + 9a^6b^5c^4 - 54a^7b^3c^5 + (2^{1/3}(3^{1/2}i - 1)(81a^8c^3x(ac - b^2)(4ac - b^2)^2 - (81*2^{2/3})a^{10}b^2c^3(3^{1/2}i + 1)(4ac - b^2)^2((b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2}) / (a^5(4ac - b^2)^3))^{1/3} \\
& / 4 * ((b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2} / (a^5(4ac - b^2)^3))^{2/3} \\
& / 36) / 12 + 3a^6c^6x(2ac - b^2)) * ((3^{1/2}i) / 2 + 1/2) * (- (b^8 + 16a^4c^4 + b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c + 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2} / (54(a^5b^6 - 64a^8c^3 - 12a^6b^4c + 48a^7b^2c^2))^{1/3} \\
& + \log((2^{2/3}(3^{1/2}i - 1)((b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (a^5(4ac - b^2)^3))^{1/3} \\
& * (72a^8b^6c^6 + 9a^6b^5c^4 - 54a^7b^3c^5 - (2^{1/3}(3^{1/2}i + 1)(81a^8c^3x(ac - b^2)(4ac - b^2)^2 + (81*2^{2/3})a^{10}b^2c^3(3^{1/2}i - 1)(4ac - b^2)^2((b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (a^5(4ac - b^2)^3))^{1/3} \\
& / 4 * ((b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2} / (a^5(4ac - b^2)^3))^{2/3} \\
& / 36) / 12 - 3a^6c^6x(2ac - b^2)) * ((3^{1/2}i) / 2 - 1/2) * (- (b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2} / (54(a^5b^6 - 64a^8c^3 - 12a^6b^4c + 48a^7b^2c^2))^{1/3} \\
& - \log((2^{2/3}(3^{1/2}i + 1)((b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2}) / (a^5(4ac - b^2)^3))^{1/3} \\
& * (72a^8b^6c^6 + 9a^6b^5c^4 - 54a^7b^3c^5 + (2^{1/3}(3^{1/2}i - 1)(81a^8c^3x(ac - b^2)(4ac - b^2)^2 - (81*2^{2/3})a^{10}b^2c^3(3^{1/2}i + 1)(4ac - b^2)^2((b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} - 5ab^3c(-4ac - b^2)^3)^{1/2}) / (a^5(4ac - b^2)^3))^{1/3} \\
& / 4 * ((b^8 + 16a^4c^4 - b^5(-4ac - b^2)^3)^{1/2} + 41a^2b^4c^2 - 56a^3b^2c^3 - 11ab^6c - 5a^2b^2c^2(-4ac - b^2)^3)^{1/2} + 5ab^3c(-4ac - b^2)^3)^{1/2} / (a^5(4ac - b^2)^3))^{2/3} \\
& / 36) / 12 + 3a^6c^6x(2ac - b^2)) * ((3^{1/2}i) / 2 + 1/2) * (- (b^8 + 16a^4c^4
\end{aligned}$$

$$\begin{aligned} &^4 - b^5 * (- (4 * a * c - b^2)^3)^{(1/2)} + 41 * a^2 * b^4 * c^2 - 56 * a^3 * b^2 * c^3 - 11 * a * \\ &b^6 * c - 5 * a^2 * b * c^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 5 * a * b^3 * c * (- (4 * a * c - b^2)^3) \\ &^{(1/2)} / (54 * (a^5 * b^6 - 64 * a^8 * c^3 - 12 * a^6 * b^4 * c + 48 * a^7 * b^2 * c^2))^{(1/3)} \end{aligned}$$

sympy [A] time = 52.50, size = 241, normalized size = 0.39

$$\text{RootSum}\left(t^6(46656a^8c^3 - 34992a^7b^2c^2 + 8748a^6b^4c - 729a^5b^6) + t^3(-432a^4c^4 + 1512a^3b^2c^3 - 1107a^2b^4c^2 + 297ab^6c - 27b^8) + c^5, \left(t \mapsto t \log\left(x + \frac{-2592t^4a^8c^3 + 2592t^4a^7b^2c^2 - 810t^4a^6b^4c + 81t^4a^5b^6 + 121a^4c^4 - 75ta^3b^2c^3 + 78ta^2b^4c^2 - 27tab^6c + 3tb^8}{5a^2b^4c^4 - 5ab^3c^3 + b^5c^2}\right)\right)\right) - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(c*x**6+b*x**3+a),x)

[Out] RootSum(_t**6*(46656*a**8*c**3 - 34992*a**7*b**2*c**2 + 8748*a**6*b**4*c - 729*a**5*b**6) + _t**3*(-432*a**4*c**4 + 1512*a**3*b**2*c**3 - 1107*a**2*b**4*c**2 + 297*a*b**6*c - 27*b**8) + c**5, Lambda(_t, _t*log(x + (-2592*_t**4*a**8*c**3 + 2592*_t**4*a**7*b**2*c**2 - 810*_t**4*a**6*b**4*c + 81*_t**4*a**5*b**6 + 12*_t*a**4*c**4 - 75*_t*a**3*b**2*c**3 + 78*_t*a**2*b**4*c**2 - 27*_t*a*b**6*c + 3*_t*b**8)/(5*a**2*b*c**4 - 5*a*b**3*c**3 + b**5*c**2)))) - 1/(2*a*x**2)

$$3.138 \quad \int \frac{x^{11}}{3+4x^3+x^6} dx$$

Optimal. Leaf size=35

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{1}{6} \log(x^3 + 1) + \frac{9}{2} \log(x^3 + 3)$$

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 701, 632, 31}

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{1}{6} \log(x^3 + 1) + \frac{9}{2} \log(x^3 + 3)$$

Antiderivative was successfully verified.

[In] Int[x^11/(3 + 4*x^3 + x^6),x]

[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 701

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -

4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{3 + 4x^3 + x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{3 + 4x + x^2} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-4 + x + \frac{12 + 13x}{3 + 4x + x^2} \right) dx, x, x^3 \right) \\
 &= -\frac{4x^3}{3} + \frac{x^6}{6} + \frac{1}{3} \text{Subst} \left(\int \frac{12 + 13x}{3 + 4x + x^2} dx, x, x^3 \right) \\
 &= -\frac{4x^3}{3} + \frac{x^6}{6} - \frac{1}{6} \text{Subst} \left(\int \frac{1}{1 + x} dx, x, x^3 \right) + \frac{9}{2} \text{Subst} \left(\int \frac{1}{3 + x} dx, x, x^3 \right) \\
 &= -\frac{4x^3}{3} + \frac{x^6}{6} - \frac{1}{6} \log(1 + x^3) + \frac{9}{2} \log(3 + x^3)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{1}{6} \log(x^3 + 1) + \frac{9}{2} \log(x^3 + 3)$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(3 + 4*x^3 + x^6), x]

[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{3 + 4x^3 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^11/(3 + 4*x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^11/(3 + 4*x^3 + x^6), x]

fricas [A] time = 1.29, size = 27, normalized size = 0.77

$$\frac{1}{6} x^6 - \frac{4}{3} x^3 + \frac{9}{2} \log(x^3 + 3) - \frac{1}{6} \log(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁶+4*x³+3),x, algorithm="fricas")

[Out] 1/6*x⁶ - 4/3*x³ + 9/2*log(x³ + 3) - 1/6*log(x³ + 1)

giac [A] time = 0.39, size = 29, normalized size = 0.83

$$\frac{1}{6}x^6 - \frac{4}{3}x^3 + \frac{9}{2}\log(|x^3 + 3|) - \frac{1}{6}\log(|x^3 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁶+4*x³+3),x, algorithm="giac")

[Out] 1/6*x⁶ - 4/3*x³ + 9/2*log(abs(x³ + 3)) - 1/6*log(abs(x³ + 1))

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{\ln(x^3 + 1)}{6} + \frac{9\ln(x^3 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(x⁶+4*x³+3),x)

[Out] -4/3*x³+1/6*x⁶-1/6*ln(x³+1)+9/2*ln(x³+3)

maxima [A] time = 0.49, size = 27, normalized size = 0.77

$$\frac{1}{6}x^6 - \frac{4}{3}x^3 + \frac{9}{2}\log(x^3 + 3) - \frac{1}{6}\log(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁶+4*x³+3),x, algorithm="maxima")

[Out] 1/6*x⁶ - 4/3*x³ + 9/2*log(x³ + 3) - 1/6*log(x³ + 1)

mupad [B] time = 1.25, size = 27, normalized size = 0.77

$$\frac{9\ln(x^3 + 3)}{2} - \frac{\ln(x^3 + 1)}{6} - \frac{4x^3}{3} + \frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(4*x³ + x⁶ + 3),x)

[Out] (9*log(x³ + 3))/2 - log(x³ + 1)/6 - (4*x³)/3 + x⁶/6

sympy [A] time = 0.13, size = 29, normalized size = 0.83

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{\log(x^3 + 1)}{6} + \frac{9\log(x^3 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(x**6+4*x**3+3),x)

[Out] x**6/6 - 4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2

$$3.139 \quad \int \frac{x^8}{3+4x^3+x^6} dx$$

Optimal. Leaf size=28

$$\frac{x^3}{3} + \frac{1}{6} \log(x^3 + 1) - \frac{3}{2} \log(x^3 + 3)$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 703, 632, 31}

$$\frac{x^3}{3} + \frac{1}{6} \log(x^3 + 1) - \frac{3}{2} \log(x^3 + 3)$$

Antiderivative was successfully verified.

[In] Int[x^8/(3 + 4*x^3 + x^6),x]

[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{3 + 4x^3 + x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{3 + 4x + x^2} dx, x, x^3 \right) \\
 &= \frac{x^3}{3} + \frac{1}{3} \text{Subst} \left(\int \frac{-3 - 4x}{3 + 4x + x^2} dx, x, x^3 \right) \\
 &= \frac{x^3}{3} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1 + x} dx, x, x^3 \right) - \frac{3}{2} \text{Subst} \left(\int \frac{1}{3 + x} dx, x, x^3 \right) \\
 &= \frac{x^3}{3} + \frac{1}{6} \log(1 + x^3) - \frac{3}{2} \log(3 + x^3)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{x^3}{3} + \frac{1}{6} \log(x^3 + 1) - \frac{3}{2} \log(x^3 + 3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(3 + 4*x^3 + x^6), x]
```

```
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{3 + 4x^3 + x^6} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(3 + 4*x^3 + x^6), x]
```

```
[Out] IntegrateAlgebraic[x^8/(3 + 4*x^3 + x^6), x]
```

fricas [A] time = 1.13, size = 22, normalized size = 0.79

$$\frac{1}{3} x^3 - \frac{3}{2} \log(x^3 + 3) + \frac{1}{6} \log(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)

giac [A] time = 0.34, size = 24, normalized size = 0.86

$$\frac{1}{3}x^3 - \frac{3}{2}\log(|x^3 + 3|) + \frac{1}{6}\log(|x^3 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="giac")

[Out] 1/3*x^3 - 3/2*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))

maple [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{x^3}{3} + \frac{\ln(x^3 + 1)}{6} - \frac{3\ln(x^3 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^6+4*x^3+3),x)

[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)

maxima [A] time = 0.49, size = 22, normalized size = 0.79

$$\frac{1}{3}x^3 - \frac{3}{2}\log(x^3 + 3) + \frac{1}{6}\log(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)

mupad [B] time = 0.05, size = 22, normalized size = 0.79

$$\frac{\ln(x^3 + 1)}{6} - \frac{3\ln(x^3 + 3)}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(4*x^3 + x^6 + 3),x)

[Out] log(x^3 + 1)/6 - (3*log(x^3 + 3))/2 + x^3/3

sympy [A] time = 0.12, size = 22, normalized size = 0.79

$$\frac{x^3}{3} + \frac{\log(x^3 + 1)}{6} - \frac{3 \log(x^3 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(x**6+4*x**3+3),x)

[Out] x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2

$$3.140 \quad \int \frac{x^5}{3+4x^3+x^6} dx$$

Optimal. Leaf size=21

$$\frac{1}{2} \log(x^3 + 3) - \frac{1}{6} \log(x^3 + 1)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1357, 632, 31}

$$\frac{1}{2} \log(x^3 + 3) - \frac{1}{6} \log(x^3 + 1)$$

Antiderivative was successfully verified.

[In] Int[x^5/(3 + 4*x^3 + x^6), x]

[Out] -Log[1 + x^3]/6 + Log[3 + x^3]/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{3+4x^3+x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{3+4x+x^2} dx, x, x^3 \right) \\
&= -\left(\frac{1}{6} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^3 \right) \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{3+x} dx, x, x^3 \right) \\
&= -\frac{1}{6} \log(1+x^3) + \frac{1}{2} \log(3+x^3)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{2} \log(x^3 + 3) - \frac{1}{6} \log(x^3 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(3 + 4*x^3 + x^6), x]

[Out] -1/6*Log[1 + x^3] + Log[3 + x^3]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{3+4x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(3 + 4*x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^5/(3 + 4*x^3 + x^6), x]

fricas [A] time = 1.21, size = 17, normalized size = 0.81

$$\frac{1}{2} \log(x^3 + 3) - \frac{1}{6} \log(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+4*x^3+3), x, algorithm="fricas")

[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)

giac [A] time = 0.36, size = 19, normalized size = 0.90

$$\frac{1}{2} \log(|x^3 + 3|) - \frac{1}{6} \log(|x^3 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁵/(x⁶+4*x³+3),x, algorithm="giac")

[Out] 1/2*log(abs(x³ + 3)) - 1/6*log(abs(x³ + 1))

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\ln(x^3 + 1)}{6} + \frac{\ln(x^3 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x⁵/(x⁶+4*x³+3),x)

[Out] -1/6*ln(x³+1)+1/2*ln(x³+3)

maxima [A] time = 0.52, size = 17, normalized size = 0.81

$$\frac{1}{2} \log(x^3 + 3) - \frac{1}{6} \log(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁵/(x⁶+4*x³+3),x, algorithm="maxima")

[Out] 1/2*log(x³ + 3) - 1/6*log(x³ + 1)

mupad [B] time = 0.05, size = 17, normalized size = 0.81

$$\frac{\ln(x^3 + 3)}{2} - \frac{\ln(x^3 + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x⁵/(4*x³ + x⁶ + 3),x)

[Out] log(x³ + 3)/2 - log(x³ + 1)/6

sympy [A] time = 0.12, size = 15, normalized size = 0.71

$$-\frac{\log(x^3 + 1)}{6} + \frac{\log(x^3 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(x**6+4*x**3+3),x)

[Out] -log(x**3 + 1)/6 + log(x**3 + 3)/2

$$3.141 \quad \int \frac{x^2}{3+4x^3+x^6} dx$$

Optimal. Leaf size=10

$$-\frac{1}{3} \tanh^{-1}(x^3 + 2)$$

Rubi [B] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 2.10, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1352, 616, 31}

$$\frac{1}{6} \log(x^3 + 1) - \frac{1}{6} \log(x^3 + 3)$$

Antiderivative was successfully verified.

[In] Int[x^2/(3 + 4*x^3 + x^6), x]

[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{3+4x^3+x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{3+4x+x^2} dx, x, x^3 \right) \\
&= \frac{1}{6} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^3 \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{3+x} dx, x, x^3 \right) \\
&= \frac{1}{6} \log(1+x^3) - \frac{1}{6} \log(3+x^3)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 21, normalized size = 2.10

$$\frac{1}{6} \log(x^3 + 1) - \frac{1}{6} \log(x^3 + 3)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(3 + 4*x^3 + x^6), x]

[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{3+4x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(3 + 4*x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^2/(3 + 4*x^3 + x^6), x]

fricas [B] time = 1.21, size = 17, normalized size = 1.70

$$-\frac{1}{6} \log(x^3 + 3) + \frac{1}{6} \log(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+4*x^3+3), x, algorithm="fricas")

[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)

giac [B] time = 0.33, size = 19, normalized size = 1.90

$$-\frac{1}{6} \log(|x^3 + 3|) + \frac{1}{6} \log(|x^3 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="giac")

[Out] -1/6*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))

maple [B] time = 0.00, size = 18, normalized size = 1.80

$$\frac{\ln(x^3 + 1)}{6} - \frac{\ln(x^3 + 3)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^6+4*x^3+3),x)

[Out] 1/6*ln(x^3+1)-1/6*ln(x^3+3)

maxima [B] time = 0.47, size = 17, normalized size = 1.70

$$-\frac{1}{6} \log(x^3 + 3) + \frac{1}{6} \log(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)

mupad [B] time = 0.38, size = 16, normalized size = 1.60

$$\frac{\operatorname{atanh}\left(\frac{9}{2(8x^3+6)} + \frac{5}{4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(4*x^3 + x^6 + 3),x)

[Out] atanh(9/(2*(8*x^3 + 6)) + 5/4)/3

sympy [A] time = 0.11, size = 15, normalized size = 1.50

$$\frac{\log(x^3 + 1)}{6} - \frac{\log(x^3 + 3)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**6+4*x**3+3),x)

[Out] log(x**3 + 1)/6 - log(x**3 + 3)/6

$$3.142 \quad \int \frac{1}{x(3+4x^3+x^6)} dx$$

Optimal. Leaf size=27

$$-\frac{1}{6} \log(x^3 + 1) + \frac{1}{18} \log(x^3 + 3) + \frac{\log(x)}{3}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{6} \log(x^3 + 1) + \frac{1}{18} \log(x^3 + 3) + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(3 + 4*x^3 + x^6)),x]

[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(n-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(3+4x^3+x^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(3+4x+x^2)} dx, x, x^3 \right) \\ &= \frac{1}{9} \text{Subst} \left(\int \frac{1}{x} dx, x, x^3 \right) + \frac{1}{9} \text{Subst} \left(\int \frac{-4-x}{3+4x+x^2} dx, x, x^3 \right) \\ &= \frac{\log(x)}{3} + \frac{1}{18} \text{Subst} \left(\int \frac{1}{3+x} dx, x, x^3 \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^3 \right) \\ &= \frac{\log(x)}{3} - \frac{1}{6} \log(1+x^3) + \frac{1}{18} \log(3+x^3) \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{1}{6} \log(x^3 + 1) + \frac{1}{18} \log(x^3 + 3) + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + 4*x^3 + x^6)),x]
```

```
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(3+4x^3+x^6)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(3 + 4*x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(3 + 4*x^3 + x^6)), x]
```

fricas [A] time = 1.25, size = 21, normalized size = 0.78

$$\frac{1}{18} \log(x^3 + 3) - \frac{1}{6} \log(x^3 + 1) + \frac{1}{3} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/3*log(x)

giac [A] time = 0.35, size = 24, normalized size = 0.89

$$\frac{1}{18} \log(|x^3 + 3|) - \frac{1}{6} \log(|x^3 + 1|) + \frac{1}{3} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="giac")

[Out] 1/18*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1)) + 1/3*log(abs(x))

maple [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{\ln(x)}{3} - \frac{\ln(x+1)}{6} + \frac{\ln(x^3+3)}{18} - \frac{\ln(x^2-x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^6+4*x^3+3),x)

[Out] 1/3*ln(x)-1/6*ln(x+1)+1/18*ln(x^3+3)-1/6*ln(x^2-x+1)

maxima [A] time = 0.45, size = 23, normalized size = 0.85

$$\frac{1}{18} \log(x^3 + 3) - \frac{1}{6} \log(x^3 + 1) + \frac{1}{9} \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/9*log(x^3)

mupad [B] time = 1.26, size = 21, normalized size = 0.78

$$\frac{\ln(x^3 + 3)}{18} - \frac{\ln(x^3 + 1)}{6} + \frac{\ln(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(4*x^3 + x^6 + 3)),x)

[Out] log(x^3 + 3)/18 - log(x^3 + 1)/6 + log(x)/3

sympy [A] time = 0.14, size = 20, normalized size = 0.74

$$\frac{\log(x)}{3} - \frac{\log(x^3 + 1)}{6} + \frac{\log(x^3 + 3)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x**6+4*x**3+3),x)

[Out] log(x)/3 - log(x**3 + 1)/6 + log(x**3 + 3)/18

$$3.143 \quad \int \frac{1}{x^4(3+4x^3+x^6)} dx$$

Optimal. Leaf size=34

$$-\frac{1}{9x^3} + \frac{1}{6} \log(x^3 + 1) - \frac{1}{54} \log(x^3 + 3) - \frac{4 \log(x)}{9}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1357, 709, 800}

$$-\frac{1}{9x^3} + \frac{1}{6} \log(x^3 + 1) - \frac{1}{54} \log(x^3 + 3) - \frac{4 \log(x)}{9}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(3 + 4*x^3 + x^6)),x]

[Out] -1/(9*x^3) - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

Int((((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(3+4x^3+x^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(3+4x+x^2)} dx, x, x^3 \right) \\
&= -\frac{1}{9x^3} + \frac{1}{9} \text{Subst} \left(\int \frac{-4-x}{x(3+4x+x^2)} dx, x, x^3 \right) \\
&= -\frac{1}{9x^3} + \frac{1}{9} \text{Subst} \left(\int \left(-\frac{4}{3x} + \frac{3}{2(1+x)} - \frac{1}{6(3+x)} \right) dx, x, x^3 \right) \\
&= -\frac{1}{9x^3} - \frac{4 \log(x)}{9} + \frac{1}{6} \log(1+x^3) - \frac{1}{54} \log(3+x^3)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{1}{9x^3} + \frac{1}{6} \log(x^3 + 1) - \frac{1}{54} \log(x^3 + 3) - \frac{4 \log(x)}{9}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(3 + 4*x^3 + x^6)),x]

[Out] -1/9*1/x^3 - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(3+4x^3+x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(3 + 4*x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x^4*(3 + 4*x^3 + x^6)), x]

fricas [A] time = 1.27, size = 35, normalized size = 1.03

$$-\frac{x^3 \log(x^3 + 3) - 9x^3 \log(x^3 + 1) + 24x^3 \log(x) + 6}{54x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] -1/54*(x^3*log(x^3 + 3) - 9*x^3*log(x^3 + 1) + 24*x^3*log(x) + 6)/x^3

giac [A] time = 0.30, size = 36, normalized size = 1.06

$$\frac{4x^3 - 3}{27x^3} - \frac{1}{54} \log(|x^3 + 3|) + \frac{1}{6} \log(|x^3 + 1|) - \frac{4}{9} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="giac")

[Out] 1/27*(4*x^3 - 3)/x^3 - 1/54*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1)) - 4/9*log(abs(x))

maple [A] time = 0.01, size = 36, normalized size = 1.06

$$-\frac{4 \ln(x)}{9} + \frac{\ln(x+1)}{6} - \frac{\ln(x^3+3)}{54} + \frac{\ln(x^2-x+1)}{6} - \frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(x^6+4*x^3+3),x)

[Out] -1/9/x^3-4/9*ln(x)+1/6*ln(x+1)-1/54*ln(x^3+3)+1/6*ln(x^2-x+1)

maxima [A] time = 0.68, size = 28, normalized size = 0.82

$$-\frac{1}{9x^3} - \frac{1}{54} \log(x^3 + 3) + \frac{1}{6} \log(x^3 + 1) - \frac{4}{27} \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] -1/9/x^3 - 1/54*log(x^3 + 3) + 1/6*log(x^3 + 1) - 4/27*log(x^3)

mupad [B] time = 1.23, size = 26, normalized size = 0.76

$$\frac{\ln(x^3 + 1)}{6} - \frac{\ln(x^3 + 3)}{54} - \frac{4 \ln(x)}{9} - \frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(4*x^3 + x^6 + 3)),x)

[Out] log(x^3 + 1)/6 - log(x^3 + 3)/54 - (4*log(x))/9 - 1/(9*x^3)

sympy [A] time = 0.17, size = 29, normalized size = 0.85

$$-\frac{4 \log(x)}{9} + \frac{\log(x^3 + 1)}{6} - \frac{\log(x^3 + 3)}{54} - \frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6+4*x**3+3),x)
```

```
[Out] -4*log(x)/9 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```


$$3.144 \quad \int \frac{1}{x^7(3+4x^3+x^6)} dx$$

Optimal. Leaf size=41

$$-\frac{1}{18x^6} + \frac{4}{27x^3} - \frac{1}{6} \log(x^3 + 1) + \frac{1}{162} \log(x^3 + 3) + \frac{13 \log(x)}{27}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1357, 709, 800}

$$\frac{4}{27x^3} - \frac{1}{18x^6} - \frac{1}{6} \log(x^3 + 1) + \frac{1}{162} \log(x^3 + 3) + \frac{13 \log(x)}{27}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(3 + 4*x^3 + x^6)),x]

[Out] -1/(18*x^6) + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/162

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7(3+4x^3+x^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^3(3+4x+x^2)} dx, x, x^3 \right) \\
&= -\frac{1}{18x^6} + \frac{1}{9} \text{Subst} \left(\int \frac{-4-x}{x^2(3+4x+x^2)} dx, x, x^3 \right) \\
&= -\frac{1}{18x^6} + \frac{1}{9} \text{Subst} \left(\int \left(-\frac{4}{3x^2} + \frac{13}{9x} - \frac{3}{2(1+x)} + \frac{1}{18(3+x)} \right) dx, x, x^3 \right) \\
&= -\frac{1}{18x^6} + \frac{4}{27x^3} + \frac{13 \log(x)}{27} - \frac{1}{6} \log(1+x^3) + \frac{1}{162} \log(3+x^3)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{1}{18x^6} + \frac{4}{27x^3} - \frac{1}{6} \log(x^3 + 1) + \frac{1}{162} \log(x^3 + 3) + \frac{13 \log(x)}{27}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(3 + 4*x^3 + x^6)),x]

[Out] -1/18*1/x^6 + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/162

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7(3+4x^3+x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^7*(3 + 4*x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x^7*(3 + 4*x^3 + x^6)), x]

fricas [A] time = 1.21, size = 40, normalized size = 0.98

$$\frac{x^6 \log(x^3 + 3) - 27x^6 \log(x^3 + 1) + 78x^6 \log(x) + 24x^3 - 9}{162x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $1/162*(x^6*\log(x^3 + 3) - 27*x^6*\log(x^3 + 1) + 78*x^6*\log(x) + 24*x^3 - 9)/x^6$

giac [A] time = 0.37, size = 41, normalized size = 1.00

$$-\frac{13x^6 - 8x^3 + 3}{54x^6} + \frac{1}{162} \log(|x^3 + 3|) - \frac{1}{6} \log(|x^3 + 1|) + \frac{13}{27} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="giac")`

[Out] $-1/54*(13*x^6 - 8*x^3 + 3)/x^6 + 1/162*\log(\text{abs}(x^3 + 3)) - 1/6*\log(\text{abs}(x^3 + 1)) + 13/27*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{13 \ln(x)}{27} - \frac{\ln(x+1)}{6} + \frac{\ln(x^3+3)}{162} - \frac{\ln(x^2-x+1)}{6} + \frac{4}{27x^3} - \frac{1}{18x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(x^6+4*x^3+3),x)`

[Out] $-1/18/x^6+4/27/x^3+13/27*\ln(x)-1/6*\ln(x+1)+1/162*\ln(x^3+3)-1/6*\ln(x^2-x+1)$

maxima [A] time = 0.59, size = 35, normalized size = 0.85

$$\frac{8x^3 - 3}{54x^6} + \frac{1}{162} \log(x^3 + 3) - \frac{1}{6} \log(x^3 + 1) + \frac{13}{81} \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="maxima")`

[Out] $1/54*(8*x^3 - 3)/x^6 + 1/162*\log(x^3 + 3) - 1/6*\log(x^3 + 1) + 13/81*\log(x^3)$

mupad [B] time = 0.04, size = 32, normalized size = 0.78

$$\frac{\ln(x^3 + 3)}{162} - \frac{\ln(x^3 + 1)}{6} + \frac{13 \ln(x)}{27} + \frac{4x^3 - 1}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(4*x^3 + x^6 + 3)),x)`

[Out] $\log(x^3 + 3)/162 - \log(x^3 + 1)/6 + (13*\log(x))/27 + ((4*x^3)/27 - 1/18)/x^6$

sympy [A] time = 0.19, size = 34, normalized size = 0.83

$$\frac{13 \log(x)}{27} - \frac{\log(x^3 + 1)}{6} + \frac{\log(x^3 + 3)}{162} + \frac{8x^3 - 3}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(x**6+4*x**3+3),x)`

[Out] $13*\log(x)/27 - \log(x^3 + 1)/6 + \log(x^3 + 3)/162 + (8*x^3 - 3)/(54*x^6)$

$$3.145 \quad \int \frac{x^{10}}{3+4x^3+x^6} dx$$

Optimal. Leaf size=124

$$\frac{x^5}{5} - 2x^2 - \frac{1}{12} \log(x^2 - x + 1) + \frac{3}{4} 3^{2/3} \log(x^2 - \sqrt[3]{3}x + 3^{2/3}) + \frac{1}{6} \log(x+1) - \frac{3}{2} 3^{2/3} \log(x + \sqrt[3]{3}) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{9}{2} \sqrt[6]{3}$$

Rubi [A] time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1367, 1502, 1510, 292, 31, 634, 618, 204, 628, 617}

$$\frac{x^5}{5} - 2x^2 - \frac{1}{12} \log(x^2 - x + 1) + \frac{3}{4} 3^{2/3} \log(x^2 - \sqrt[3]{3}x + 3^{2/3}) + \frac{1}{6} \log(x+1) - \frac{3}{2} 3^{2/3} \log(x + \sqrt[3]{3}) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{9}{2} \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^10/(3 + 4*x^3 + x^6), x]

[Out] -2*x^2 + x^5/5 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (9*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3*3^(2/3)*Log[3^(1/3) + x])/2 - Log[1 - x + x^2]/12 + (3*3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1367

Int[((d_.)*(x_)^(m_.))*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]

Rule 1502

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_.))*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^p, x_Symbol] := Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 -
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{3+4x^3+x^6} dx &= \frac{x^5}{5} - \frac{1}{5} \int \frac{x^4(15+20x^3)}{3+4x^3+x^6} dx \\
&= -2x^2 + \frac{x^5}{5} + \frac{1}{10} \int \frac{x(120+130x^3)}{3+4x^3+x^6} dx \\
&= -2x^2 + \frac{x^5}{5} - \frac{1}{2} \int \frac{x}{1+x^3} dx + \frac{27}{2} \int \frac{x}{3+x^3} dx \\
&= -2x^2 + \frac{x^5}{5} + \frac{1}{6} \int \frac{1}{1+x} dx - \frac{1}{6} \int \frac{1+x}{1-x+x^2} dx - \frac{1}{2} (3^{3^{2/3}}) \int \frac{1}{\sqrt[3]{3}+x} dx + \frac{1}{2} (3^{3^{2/3}}) \int \frac{1}{\sqrt[3]{3}-x} dx \\
&= -2x^2 + \frac{x^5}{5} + \frac{1}{6} \log(1+x) - \frac{3}{2} 3^{2/3} \log(\sqrt[3]{3}+x) - \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{4} \int \frac{1}{1-x+x^2} dx \\
&= -2x^2 + \frac{x^5}{5} + \frac{1}{6} \log(1+x) - \frac{3}{2} 3^{2/3} \log(\sqrt[3]{3}+x) - \frac{1}{12} \log(1-x+x^2) + \frac{3}{4} 3^{2/3} \log(3^{2/3}-x) \\
&= -2x^2 + \frac{x^5}{5} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{9}{2} \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + \frac{1}{6} \log(1+x) - \frac{3}{2} 3^{2/3} \log(\sqrt[3]{3}+x)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.95

$$\frac{1}{60} \left(12x^5 - 120x^2 - 5 \log(x^2 - x + 1) + 45 \cdot 3^{2/3} \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) + 10 \log(x + 1) - 90 \cdot 3^{2/3} \log(3^{2/3}x + 3) - 270 \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right) - 10\sqrt{3} \tan^{-1}\left(\frac{2x - 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^10/(3 + 4*x^3 + x^6), x]

[Out] (-120*x^2 + 12*x^5 - 270*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Log[1 + x] - 90*3^(2/3)*Log[3 + 3^(2/3)*x] - 5*Log[1 - x + x^2] + 45*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/60

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{3+4x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^10/(3 + 4*x^3 + x^6),x]

[Out] IntegrateAlgebraic[x^10/(3 + 4*x^3 + x^6), x]

fricas [A] time = 1.30, size = 102, normalized size = 0.82

$$\frac{1}{5}x^5 - 2x^2 + \frac{3}{2}\sqrt{3}(-9)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}(2(-9)^{\frac{1}{3}}x+3)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4}(-9)^{\frac{1}{3}}\log(3x^2 - (-9)^{\frac{2}{3}}x - 3(-9)^{\frac{1}{3}}) + \frac{3}{2}(-9)^{\frac{1}{3}}\log(3x + (-9)^{\frac{2}{3}}) - \frac{1}{12}\log(x^2 - x + 1) + \frac{1}{6}\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $1/5*x^5 - 2*x^2 + 3/2*\sqrt{3}*(-9)^{(1/3)}*\arctan(1/9*\sqrt{3}*(2*(-9)^{(1/3)}*x + 3)) - 1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 3/4*(-9)^{(1/3)}*\log(3*x^2 - (-9)^{(2/3)}*x - 3*(-9)^{(1/3)}) + 3/2*(-9)^{(1/3)}*\log(3*x + (-9)^{(2/3)}) - 1/12*\log(x^2 - x + 1) + 1/6*\log(x + 1)$

giac [A] time = 0.37, size = 96, normalized size = 0.77

$$\frac{1}{5}x^5 - 2x^2 + \frac{3}{4} \cdot 3^{\frac{2}{3}} \log(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}) - \frac{3}{2} \cdot 3^{\frac{2}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{9}{2} \cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}(2x-3^{\frac{1}{3}})\right) - \frac{1}{12}\log(x^2 - x + 1) + \frac{1}{6}\log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="giac")

[Out] $1/5*x^5 - 2*x^2 + 3/4*3^{(2/3)}*\log(x^2 - 3^{(1/3)}*x + 3^{(2/3)}) - 3/2*3^{(2/3)}*\log(\text{abs}(x + 3^{(1/3)})) - 1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + 9/2*3^{(1/6)}*\arctan(1/3*3^{(1/6)}*(2*x - 3^{(1/3)})) - 1/12*\log(x^2 - x + 1) + 1/6*\log(\text{abs}(x + 1))$

maple [A] time = 0.01, size = 94, normalized size = 0.76

$$\frac{x^5}{5} - 2x^2 + \frac{9 \cdot 3^{\frac{1}{6}} \arctan\left(\frac{\sqrt{3}\left(\frac{2 \cdot 3^{\frac{2}{3}}x}{3} - 1\right)}{3}\right)}{2} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x+1)}{6} - \frac{3 \cdot 3^{\frac{2}{3}} \ln\left(x + 3^{\frac{1}{3}}\right)}{2} + \frac{3 \cdot 3^{\frac{2}{3}} \ln\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right)}{4} - \frac{\ln(x^2 - x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10/(x^6+4*x^3+3),x)

[Out] $1/5*x^5 - 2*x^2 + 1/6*\ln(x+1) - 3/2*3^{(2/3)}*\ln(3^{(1/3)}+x) + 3/4*3^{(2/3)}*\ln(3^{(2/3)} - 3^{(1/3)}*x+x^2) + 9/2*3^{(1/6)}*\arctan(1/3*3^{(1/2)}*(2/3*3^{(2/3)}*x-1)) - 1/12*\ln(x^2 - x + 1) - 1/6*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})$

maxima [A] time = 1.23, size = 94, normalized size = 0.76

$$\frac{1}{5}x^5 - 2x^2 + \frac{3}{4} \cdot 3^{\frac{2}{3}} \log(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}) - \frac{3}{2} \cdot 3^{\frac{2}{3}} \log(x + 3^{\frac{1}{3}}) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{9}{2} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}(2x-3^{\frac{1}{3}})\right) - \frac{1}{12} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*log(x + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)

mupad [B] time = 0.24, size = 124, normalized size = 1.00

$$\frac{\ln(x+1)}{6} - \frac{33^{2/3} \ln(x+3^{1/3})}{2} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - 2x^2 + \frac{x^5}{5} - \frac{3(-1)^{1/3} \ln\left(x - \frac{(-1)^{1/3}3^{1/3}}{2} - \frac{(-1)^{1/6}3^{5/6}}{2} + \frac{3^{1/3}}{2}\right) (3^{2/3} + 3^{1/6}3i)}{4} + \frac{3(-1)^{1/3}3^{2/3} \ln(x + (-1)^{2/3}3^{1/3})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10/(4*x^3 + x^6 + 3),x)

[Out] log(x + 1)/6 - (3*3^(2/3)*log(x + 3^(1/3)))/2 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) - 2*x^2 + x^5/5 - (3*(-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/4 + (3*(-1)^(1/3)*3^(2/3)*log(x + (-1)^(2/3)*3^(1/3)))/2

sympy [C] time = 0.62, size = 144, normalized size = 1.16

$$\frac{x^5}{5} - 2x^2 + \frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{3872\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{3281} + \frac{3188648\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{88587}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{3188648\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{88587} + \frac{3872\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{3281}\right) + \text{RootSum}\left(8t^3 + 243, \left(t \mapsto t \log\left(\frac{3872t^5}{3281} + \frac{3188648t^2}{88587} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**10/(x**6+4*x**3+3),x)

[Out] x**5/5 - 2*x**2 + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 3872*(-1/12 - sqrt(3)*I/12)**5/3281 + 3188648*(-1/12 - sqrt(3)*I/12)**2/88587) + (-1/12 + sqrt(3)*I/12)*log(x + 3188648*(-1/12 + sqrt(3)*I/12)**2/88587 + 3872*(-1/12 + sqrt(3)*I/12)**5/3281) + RootSum(8*_t**3 + 243, Lambda(_t, _t*log(3872*_t**5/3281 + 3188648*_t**2/88587 + x)))

$$3.146 \quad \int \frac{x^9}{3+4x^3+x^6} dx$$

Optimal. Leaf size=122

$$\frac{x^4}{4} + \frac{1}{12} \log(x^2 - x + 1) - \frac{3}{4} \sqrt[3]{3} \log(x^2 - \sqrt[3]{3}x + 3^{2/3}) - 4x - \frac{1}{6} \log(x+1) + \frac{3}{2} \sqrt[3]{3} \log(x + \sqrt[3]{3}) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{3}{2} 3^{5/6}$$

Rubi [A] time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1367, 1502, 1422, 200, 31, 634, 618, 204, 628, 617}

$$\frac{x^4}{4} + \frac{1}{12} \log(x^2 - x + 1) - \frac{3}{4} \sqrt[3]{3} \log(x^2 - \sqrt[3]{3}x + 3^{2/3}) - 4x - \frac{1}{6} \log(x+1) + \frac{3}{2} \sqrt[3]{3} \log(x + \sqrt[3]{3}) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{3}{2} 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^9/(3 + 4*x^3 + x^6), x]

[Out] -4*x + x^4/4 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3*3^(1/3)*Log[3^(1/3) + x])/2 + Log[1 - x + x^2]/12 - (3*3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1367

Int[((d_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]

Rule 1422

Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])

Rule 1502

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^p, x_Symbol] := Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a

+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^9}{3 + 4x^3 + x^6} dx &= \frac{x^4}{4} - \frac{1}{4} \int \frac{x^3(12 + 16x^3)}{3 + 4x^3 + x^6} dx \\
 &= -4x + \frac{x^4}{4} + \frac{1}{4} \int \frac{48 + 52x^3}{3 + 4x^3 + x^6} dx \\
 &= -4x + \frac{x^4}{4} - \frac{1}{2} \int \frac{1}{1 + x^3} dx + \frac{27}{2} \int \frac{1}{3 + x^3} dx \\
 &= -4x + \frac{x^4}{4} - \frac{1}{6} \int \frac{1}{1 + x} dx - \frac{1}{6} \int \frac{2 - x}{1 - x + x^2} dx + \frac{1}{2} (3\sqrt[3]{3}) \int \frac{1}{\sqrt[3]{3} + x} dx + \frac{1}{2} (3\sqrt[3]{3}) \int \frac{1}{32} dx \\
 &= -4x + \frac{x^4}{4} - \frac{1}{6} \log(1 + x) + \frac{3}{2} \sqrt[3]{3} \log(\sqrt[3]{3} + x) + \frac{1}{12} \int \frac{-1 + 2x}{1 - x + x^2} dx - \frac{1}{4} \int \frac{1}{1 - x + x^2} dx \\
 &= -4x + \frac{x^4}{4} - \frac{1}{6} \log(1 + x) + \frac{3}{2} \sqrt[3]{3} \log(\sqrt[3]{3} + x) + \frac{1}{12} \log(1 - x + x^2) - \frac{3}{4} \sqrt[3]{3} \log(3^{2/3} - \sqrt[3]{3}) \\
 &= -4x + \frac{x^4}{4} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{3}{2} 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right) - \frac{1}{6} \log(1 + x) + \frac{3}{2} \sqrt[3]{3} \log(\sqrt[3]{3} + x) +
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 114, normalized size = 0.93

$$\frac{1}{12} \left(3x^4 + \log(x^2 - x + 1) - 9\sqrt[3]{3} \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) - 48x - 2\log(x + 1) + 18\sqrt[3]{3} \log(3^{2/3}x + 3) - 18 \cdot 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x - 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(3 + 4*x^3 + x^6), x]

[Out] (-48*x + 3*x^4 - 18*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 18*3^(1/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2] - 9*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{3 + 4x^3 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/(3 + 4*x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^9/(3 + 4*x^3 + x^6), x]

fricas [A] time = 1.12, size = 90, normalized size = 0.74

$$\frac{1}{4}x^4 + \frac{3}{2} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{2}{3} \cdot 3^{\frac{1}{6}}x - \frac{1}{3}\sqrt{3}\right) - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2} \cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) - 4x + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^6+4*x^3+3), x, algorithm="fricas")

[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)

giac [A] time = 0.43, size = 94, normalized size = 0.77

$$\frac{1}{4}x^4 + \frac{3}{2} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}(2x - 3^{\frac{1}{3}})\right) - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2} \cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) - 4x + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^6+4*x^3+3), x, algorithm="giac")

[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3/2*3^(1/3)*log(abs(x + 3^(1/3))) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))

maple [A] time = 0.01, size = 92, normalized size = 0.75

$$\frac{x^4}{4} - 4x + \frac{33^{\frac{5}{6}} \arctan\left(\frac{\sqrt{3}\left(\frac{23^{\frac{2}{3}}x-1}{3}\right)}{3}\right)}{2} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{33^{\frac{1}{3}} \ln\left(x + 3^{\frac{1}{3}}\right)}{2} - \frac{33^{\frac{1}{3}} \ln\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right)}{4} + \frac{\ln(x^2 - x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^6+4*x^3+3), x)

[Out] 1/4*x^4-4*x-1/6*ln(x+1)+3/2*3^(1/3)*ln(x+3^(1/3))-3/4*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2/3))+3/2*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [A] time = 1.33, size = 92, normalized size = 0.75

$$\frac{1}{4}x^4 + \frac{3}{2} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}(2x - 3^{\frac{1}{3}})\right) - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{4} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{3}{2} \cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) - 4x + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $\frac{1}{4}x^4 + \frac{3}{2}3^{(5/6)}*\arctan(1/3*3^{(1/6)}*(2*x - 3^{(1/3)})) - 1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 3/4*3^{(1/3)}*\log(x^2 - 3^{(1/3)}*x + 3^{(2/3)}) + 3/2*3^{(1/3)}*\log(x + 3^{(1/3)}) - 4*x + 1/12*\log(x^2 - x + 1) - 1/6*\log(x + 1)$

mupad [B] time = 1.42, size = 119, normalized size = 0.98

$$\frac{3^{3/3} \ln(x+3^{1/3})}{2} - \frac{\ln(x+1)}{6} - 4x + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) + \frac{x^4}{4} - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}i}{2}\right)\left(\frac{3^{1/3}}{4} + \frac{3^{5/6}i}{4}\right) + 3^{1/3} \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}i}{2}\right)\left(-\frac{3}{4} + \frac{\sqrt{3}i}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(4*x^3 + x^6 + 3),x)

[Out] $(3*3^{(1/3)}*\log(x + 3^{(1/3)}))/2 - \log(x + 1)/6 - 4*x + \log(x - (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/12 + 1/12) - \log(x + (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/12 - 1/12) + x^4/4 - \log(x - 3^{(1/3)}/2 - (3^{(5/6)}*1i)/2)*((3*3^{(1/3)})/4 + (3^{(5/6)}*3i)/4) + 3^{(1/3)}*\log(x - 3^{(1/3)}/2 + (3^{(5/6)}*1i)/2)*((3^{(1/2)}*3i)/4 - 3/4)$

sympy [C] time = 0.61, size = 129, normalized size = 1.06

$$\frac{x^4}{4} - 4x - \frac{\log(x+1)}{6} + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} - \frac{9841\sqrt{3}i}{19692} + \frac{360\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{547}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{9841}{19692} + \frac{360\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{547} + \frac{9841\sqrt{3}i}{19692}\right) + \text{RootSum}\left(8t^3 - 81, \left(t \mapsto t \log\left(\frac{360t^4}{547} - \frac{9841t}{1641} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(x**6+4*x**3+3),x)

[Out] $x^{**4}/4 - 4*x - \log(x + 1)/6 + (1/12 + \sqrt{3}*I/12)*\log(x - 9841/19692 - 9841*\sqrt{3}*I/19692 + 360*(1/12 + \sqrt{3}*I/12)**4/547) + (1/12 - \sqrt{3}*I/12)*\log(x - 9841/19692 + 360*(1/12 - \sqrt{3}*I/12)**4/547 + 9841*\sqrt{3}*I/19692) + \text{RootSum}(8*_t**3 - 81, \text{Lambda}(_t, _t*\log(360*_t**4/547 - 9841*_t/1641 + x)))$

$$3.147 \quad \int \frac{x^7}{3+4x^3+x^6} dx$$

Optimal. Leaf size=119

$$\frac{x^2}{2} + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{4} 3^{2/3} \log(x^2 - \sqrt[3]{3}x + 3^{2/3}) - \frac{1}{6} \log(x+1) + \frac{1}{2} 3^{2/3} \log(x + \sqrt[3]{3}) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{3}{2} \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1367, 1510, 292, 31, 634, 618, 204, 628, 617}

$$\frac{x^2}{2} + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{4} 3^{2/3} \log(x^2 - \sqrt[3]{3}x + 3^{2/3}) - \frac{1}{6} \log(x+1) + \frac{1}{2} 3^{2/3} \log(x + \sqrt[3]{3}) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{3}{2} \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^7/(3 + 4*x^3 + x^6), x]

[Out] x^2/2 - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3^(2/3)*Log[3^(1/3) + x])/2 + Log[1 - x + x^2]/12 - (3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1367

Int[((d_.)*(x_)^(m_.))*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]

Rule 1510

Int[(((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_.)))/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{3+4x^3+x^6} dx &= \frac{x^2}{2} - \frac{1}{2} \int \frac{x(6+8x^3)}{3+4x^3+x^6} dx \\
&= \frac{x^2}{2} + \frac{1}{2} \int \frac{x}{1+x^3} dx - \frac{9}{2} \int \frac{x}{3+x^3} dx \\
&= \frac{x^2}{2} - \frac{1}{6} \int \frac{1}{1+x} dx + \frac{1}{6} \int \frac{1+x}{1-x+x^2} dx + \frac{1}{2} 3^{2/3} \int \frac{1}{\sqrt[3]{3}+x} dx - \frac{1}{2} 3^{2/3} \int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx \\
&= \frac{x^2}{2} - \frac{1}{6} \log(1+x) + \frac{1}{2} 3^{2/3} \log(\sqrt[3]{3}+x) + \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1-x+x^2} dx - \frac{9}{4} \int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2} dx \\
&= \frac{x^2}{2} - \frac{1}{6} \log(1+x) + \frac{1}{2} 3^{2/3} \log(\sqrt[3]{3}+x) + \frac{1}{12} \log(1-x+x^2) - \frac{1}{4} 3^{2/3} \log(3^{2/3}-\sqrt[3]{3}x+x^2) \\
&= \frac{x^2}{2} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{3}{2} \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - \frac{1}{6} \log(1+x) + \frac{1}{2} 3^{2/3} \log(\sqrt[3]{3}+x) + \frac{1}{12} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 111, normalized size = 0.93

$$\frac{1}{12} \left(6x^2 + \log(x^2 - x + 1) - 3 \cdot 3^{2/3} \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) - 2 \log(x + 1) + 6 \cdot 3^{2/3} \log(3^{2/3}x + 3) + 18 \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(3 + 4*x^3 + x^6), x]

[Out] (6*x^2 + 18*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 6*3^(2/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2] - 3*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{3+4x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(3 + 4*x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^7/(3 + 4*x^3 + x^6), x]

fricas [A] time = 1.18, size = 99, normalized size = 0.83

$$\frac{1}{2} x^2 - \frac{1}{2} \cdot 9^{1/3} \sqrt{3} \arctan\left(\frac{2}{9} \cdot 9^{1/3} \sqrt{3} x - \frac{1}{3} \sqrt{3}\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) - \frac{1}{4} \cdot 9^{1/3} \log(3x^2 - 9^{2/3}x + 3 \cdot 9^{1/3}) + \frac{1}{2} \cdot 9^{1/3} \log(3x + 9^{2/3}) + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $\frac{1}{2}x^2 - \frac{1}{2}9^{(1/3)}\sqrt{3}\arctan\left(\frac{2}{9}9^{(1/3)}\sqrt{3}x - \frac{1}{3}\sqrt{3}\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) - \frac{1}{4}9^{(1/3)}\log(3x^2 - 9^{(2/3)}x + 3 \cdot 9^{(1/3)}) + \frac{1}{2}9^{(1/3)}\log(3x + 9^{(2/3)}) + \frac{1}{12}\log(x^2 - x + 1) - \frac{1}{6}\log(x + 1)$

giac [A] time = 0.34, size = 91, normalized size = 0.76

$$\frac{1}{2}x^2 - \frac{1}{4} \cdot 3^{2/3} \log(x^2 - 3^{1/3}x + 3^{2/3}) + \frac{1}{2} \cdot 3^{2/3} \log\left(x + 3^{1/3}\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) - \frac{3}{2} \cdot 3^{1/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6}(2x - 3^{1/3})\right) + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="giac")

[Out] $\frac{1}{2}x^2 - \frac{1}{4}3^{(2/3)}\log(x^2 - 3^{(1/3)}x + 3^{(2/3)}) + \frac{1}{2}3^{(2/3)}\log(\text{abs}(x + 3^{(1/3)})) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) - \frac{3}{2}3^{(1/6)}\arctan\left(\frac{1}{3}3^{(1/6)}(2x - 3^{(1/3)})\right) + \frac{1}{12}\log(x^2 - x + 1) - \frac{1}{6}\log(\text{abs}(x + 1))$

maple [A] time = 0.01, size = 89, normalized size = 0.75

$$\frac{x^2}{2} - \frac{3^{1/6} \arctan\left(\frac{\sqrt{3} \left(\frac{2 \cdot 3^{2/3} x - 1}{3}\right)}{3}\right)}{2} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{3^{2/3} \ln(x+3^{1/3})}{2} - \frac{3^{2/3} \ln(x^2 - 3^{1/3}x + 3^{2/3})}{4} + \frac{\ln(x^2 - x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^6+4*x^3+3),x)

[Out] $\frac{1}{2}x^2 - \frac{1}{6}\ln(x+1) + \frac{1}{2}3^{(2/3)}\ln(x+3^{(1/3)}) - \frac{1}{4}3^{(2/3)}\ln(x^2 - 3^{(1/3)}x + 3^{(2/3)}) - \frac{3}{2}3^{(1/6)}\arctan\left(\frac{1}{3}3^{(1/6)}(2x - 3^{(1/3)})\right) + \frac{1}{12}\ln(x^2 - x + 1) + \frac{1}{6}3^{(1/2)}\arctan\left(\frac{1}{3}(2x - 1)3^{(1/2)}\right)$

maxima [A] time = 1.31, size = 89, normalized size = 0.75

$$\frac{1}{2}x^2 - \frac{1}{4} \cdot 3^{2/3} \log(x^2 - 3^{1/3}x + 3^{2/3}) + \frac{1}{2} \cdot 3^{2/3} \log(x + 3^{1/3}) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) - \frac{3}{2} \cdot 3^{1/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6}(2x - 3^{1/3})\right) + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $\frac{1}{2}x^2 - \frac{1}{4}3^{(2/3)}\log(x^2 - 3^{(1/3)}x + 3^{(2/3)}) + \frac{1}{2}3^{(2/3)}\log(x + 3^{(1/3)}) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) - \frac{3}{2}3^{(1/6)}\arctan\left(\frac{1}{3}3^{(1/6)}(2x - 3^{(1/3)})\right) + \frac{1}{12}\log(x^2 - x + 1) - \frac{1}{6}\log(x + 1)$

mupad [B] time = 0.19, size = 118, normalized size = 0.99

$$\frac{3^{2/3} \ln(x + 3^{1/3})}{2} - \frac{\ln(x+1)}{6} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) + \frac{x^2}{2} - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}i}{2}\right) \left(\frac{3^{2/3}}{4} - \frac{3^{1/6}i}{4}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}i}{2}\right) \left(\frac{3^{2/3}}{4} + \frac{3^{1/6}i}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(4*x^3 + x^6 + 3), x)

[Out] (3^(2/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2) * ((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2) * ((3^(1/2)*1i)/12 + 1/12) + x^2/2 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2) * (3^(2/3)/4 - (3^(1/6)*3i)/4) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2) * (3^(2/3)/4 + (3^(1/6)*3i)/4)

sympy [C] time = 0.61, size = 134, normalized size = 1.13

$$\frac{x^2}{2} - \frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{6562\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{183} - \frac{1872\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{61}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x - \frac{1872\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{61} + \frac{6562\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{183}\right) + \text{RootSum}\left(8t^3 - 9, \left(t \mapsto t \log\left(-\frac{1872t^5}{61} + \frac{6562t^2}{183} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(x**6+4*x**3+3), x)

[Out] x**2/2 - log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 6562*(1/12 - sqrt(3)*I/12)**2/183 - 1872*(1/12 - sqrt(3)*I/12)**5/61) + (1/12 + sqrt(3)*I/12)*log(x - 1872*(1/12 + sqrt(3)*I/12)**5/61 + 6562*(1/12 + sqrt(3)*I/12)**2/183) + RootSum(8*_t**3 - 9, Lambda(_t, _t*log(-1872*_t**5/61 + 6562*_t**2/183 + x)))

$$3.148 \quad \int \frac{x^6}{3+4x^3+x^6} dx$$

Optimal. Leaf size=113

$$-\frac{1}{12} \log(x^2 - x + 1) + \frac{1}{4} \sqrt[3]{3} \log(x^2 - \sqrt[3]{3}x + 3^{2/3}) + x + \frac{1}{6} \log(x+1) - \frac{1}{2} \sqrt[3]{3} \log(x + \sqrt[3]{3}) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{2} 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1367, 1422, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12} \log(x^2 - x + 1) + \frac{1}{4} \sqrt[3]{3} \log(x^2 - \sqrt[3]{3}x + 3^{2/3}) + x + \frac{1}{6} \log(x+1) - \frac{1}{2} \sqrt[3]{3} \log(x + \sqrt[3]{3}) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{2} 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^6/(3 + 4*x^3 + x^6), x]

[Out] x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3^(1/3)*Log[3^(1/3) + x])/2 - Log[1 - x + x^2]/12 + (3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{3+4x^3+x^6} dx &= x - \int \frac{3+4x^3}{3+4x^3+x^6} dx \\
&= x + \frac{1}{2} \int \frac{1}{1+x^3} dx - \frac{9}{2} \int \frac{1}{3+x^3} dx \\
&= x + \frac{1}{6} \int \frac{1}{1+x} dx + \frac{1}{6} \int \frac{2-x}{1-x+x^2} dx - \frac{1}{2} \sqrt[3]{3} \int \frac{1}{\sqrt[3]{3}+x} dx - \frac{1}{2} \sqrt[3]{3} \int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx \\
&= x + \frac{1}{6} \log(1+x) - \frac{1}{2} \sqrt[3]{3} \log(\sqrt[3]{3}+x) - \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1-x+x^2} dx + \frac{1}{4} \sqrt[3]{3} \int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2} dx \\
&= x + \frac{1}{6} \log(1+x) - \frac{1}{2} \sqrt[3]{3} \log(\sqrt[3]{3}+x) - \frac{1}{12} \log(1-x+x^2) + \frac{1}{4} \sqrt[3]{3} \log(3^{2/3}-\sqrt[3]{3}x+x^2) \\
&= x - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{2} 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + \frac{1}{6} \log(1+x) - \frac{1}{2} \sqrt[3]{3} \log(\sqrt[3]{3}+x) - \frac{1}{12} \log(1-x+x^2) + \frac{1}{4} \sqrt[3]{3} \log(3^{2/3}-\sqrt[3]{3}x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 111, normalized size = 0.98

$$\frac{1}{12} \left(-\log(x^2-x+1) + 3\sqrt[3]{3} \log(\sqrt[3]{3}x^2-3^{2/3}x+3) + 12x + 2\log(x+1) - 6\sqrt[3]{3} \log(3^{2/3}x+3) + 6 \cdot 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(3 + 4*x^3 + x^6), x]

[Out] (12*x + 6*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - 6*3^(1/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x^2] + 3*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{3+4x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(3 + 4*x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^6/(3 + 4*x^3 + x^6), x]

fricas [A] time = 1.51, size = 88, normalized size = 0.78

$$\frac{1}{2} \sqrt{3} (-3)^{\frac{1}{3}} \arctan\left(\frac{1}{9} \sqrt{3} (2 (-3)^{\frac{2}{3}} x - 3)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{1}{4} (-3)^{\frac{1}{3}} \log(x^2 + (-3)^{\frac{1}{3}} x + (-3)^{\frac{2}{3}}) + \frac{1}{2} (-3)^{\frac{1}{3}} \log(x - (-3)^{\frac{1}{3}}) + x - \frac{1}{12} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $\frac{1}{2}\sqrt{3}\cdot(-3)^{(1/3)}\cdot\arctan\left(\frac{1}{9}\sqrt{3}\cdot(2\cdot(-3)^{(2/3)}\cdot x - 3)\right) + \frac{1}{6}\sqrt{3}\cdot\arctan\left(\frac{1}{3}\sqrt{3}\cdot(2\cdot x - 1)\right) - \frac{1}{4}\cdot(-3)^{(1/3)}\cdot\log(x^2 + (-3)^{(1/3)}\cdot x + (-3)^{(2/3)}) + \frac{1}{2}\cdot(-3)^{(1/3)}\cdot\log(x - (-3)^{(1/3)}) + x - \frac{1}{12}\cdot\log(x^2 - x + 1) + \frac{1}{6}\cdot\log(x + 1)$

giac [A] time = 0.45, size = 87, normalized size = 0.77

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}(2x-3^{\frac{1}{3}})\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + x - \frac{1}{12}\log(x^2-x+1) + \frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="giac")

[Out] $-\frac{1}{2}\cdot 3^{(5/6)}\cdot\arctan\left(\frac{1}{3}\cdot 3^{(1/6)}\cdot(2\cdot x - 3^{(1/3)})\right) + \frac{1}{6}\sqrt{3}\cdot\arctan\left(\frac{1}{3}\sqrt{3}\cdot(2\cdot x - 1)\right) + \frac{1}{4}\cdot 3^{(1/3)}\cdot\log(x^2 - 3^{(1/3)}\cdot x + 3^{(2/3)}) - \frac{1}{2}\cdot 3^{(1/3)}\cdot\log(\text{abs}(x + 3^{(1/3)})) + x - \frac{1}{12}\cdot\log(x^2 - x + 1) + \frac{1}{6}\cdot\log(\text{abs}(x + 1))$

maple [A] time = 0.01, size = 85, normalized size = 0.75

$$x - \frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{3}x}{3}-1\right)}{3}\right)}{2} + \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x+1)}{6} - \frac{3^{\frac{1}{3}}\ln(x+3^{\frac{1}{3}})}{2} + \frac{3^{\frac{1}{3}}\ln(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}})}{4} - \frac{\ln(x^2-x+1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^6+4*x^3+3),x)

[Out] $x + \frac{1}{6}\ln(x+1) - \frac{1}{2}\cdot 3^{(1/3)}\cdot\ln(x+3^{(1/3)}) + \frac{1}{4}\cdot 3^{(1/3)}\cdot\ln(x^2-3^{(1/3)}\cdot x+3^{(2/3)}) - \frac{1}{2}\cdot 3^{(5/6)}\cdot\arctan\left(\frac{1}{3}\cdot 3^{(1/6)}\cdot(2\cdot x - 3^{(1/3)})\right) - \frac{1}{12}\cdot\ln(x^2-x+1) + \frac{1}{6}\cdot 3^{(1/2)}\cdot\arctan\left(\frac{1}{3}\cdot(2\cdot x - 1)\cdot 3^{(1/2)}\right)$

maxima [A] time = 1.23, size = 85, normalized size = 0.75

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}(2x-3^{\frac{1}{3}})\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right) - \frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right) + x - \frac{1}{12}\log(x^2-x+1) + \frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $-\frac{1}{2}\cdot 3^{(5/6)}\cdot\arctan\left(\frac{1}{3}\cdot 3^{(1/6)}\cdot(2\cdot x - 3^{(1/3)})\right) + \frac{1}{6}\sqrt{3}\cdot\arctan\left(\frac{1}{3}\sqrt{3}\cdot(2\cdot x - 1)\right) + \frac{1}{4}\cdot 3^{(1/3)}\cdot\log(x^2 - 3^{(1/3)}\cdot x + 3^{(2/3)}) - \frac{1}{2}\cdot 3^{(1/3)}\cdot\log(x + 3^{(1/3)}) + x - \frac{1}{12}\cdot\log(x^2 - x + 1) + \frac{1}{6}\cdot\log(x + 1)$

mupad [B] time = 0.16, size = 104, normalized size = 0.92

$$x + \frac{\ln(x+1)}{6} - \frac{3^{1/3} \ln(x+3^{1/3})}{2} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) + \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}i}{2}\right) \left(\frac{3^{1/3}}{4} - \frac{3^{5/6}i}{4}\right) + \frac{(-1)^{1/3} 3^{1/3} \ln(x - (-1)^{1/3} 3^{1/3})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(4*x^3 + x^6 + 3), x)

[Out] x + log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/2 - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 - 1/12) + log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/4 - (3^(5/6)*1i)/4) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/2

sympy [C] time = 0.61, size = 126, normalized size = 1.12

$$x + \frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x - \frac{121}{246} - \frac{121\sqrt{3}i}{246} + \frac{864\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{41}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x - \frac{121}{246} + \frac{864\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{41} + \frac{121\sqrt{3}i}{246}\right) + \text{RootSum}\left(8t^3 + 3, \left(t \mapsto t \log\left(\frac{864t^4}{41} + \frac{242t}{41} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(x**6+4*x**3+3), x)

[Out] x + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 121/246 - 121*sqrt(3)*I/246 + 864*(-1/12 - sqrt(3)*I/12)**4/41) + (-1/12 + sqrt(3)*I/12)*log(x - 121/246 + 864*(-1/12 + sqrt(3)*I/12)**4/41 + 121*sqrt(3)*I/246) + RootSum(8*_t**3 + 3, Lambda(_t, _t*log(864*_t**4/41 + 242*_t/41 + x)))

$$3.149 \quad \int \frac{x^4}{3+4x^3+x^6} dx$$

Optimal. Leaf size=112

$$-\frac{1}{12} \log(x^2 - x + 1) + \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{4\sqrt[3]{3}} + \frac{1}{6} \log(x+1) - \frac{\log(x + \sqrt[3]{3})}{2\sqrt[3]{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2} \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1374, 292, 31, 634, 617, 204, 628, 618}

$$-\frac{1}{12} \log(x^2 - x + 1) + \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{4\sqrt[3]{3}} + \frac{1}{6} \log(x+1) - \frac{\log(x + \sqrt[3]{3})}{2\sqrt[3]{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2} \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^4/(3 + 4*x^3 + x^6), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - Log[3^(1/3) + x]/(2*3^(1/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_.)*(x_)^(m_))/((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{3+4x^3+x^6} dx &= -\left(\frac{1}{2} \int \frac{x}{1+x^3} dx\right) + \frac{3}{2} \int \frac{x}{3+x^3} dx \\
&= \frac{1}{6} \int \frac{1}{1+x} dx - \frac{1}{6} \int \frac{1+x}{1-x+x^2} dx - \frac{\int \frac{1}{\sqrt[3]{3}+x} dx}{2\sqrt[3]{3}} + \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{2\sqrt[3]{3}} \\
&= \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{2\sqrt[3]{3}} - \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{4} \int \frac{1}{1-x+x^2} dx + \frac{3}{4} \int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2} dx \\
&= \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{2\sqrt[3]{3}} - \frac{1}{12} \log(1-x+x^2) + \frac{\log(3^{2/3}-\sqrt[3]{3}x+x^2)}{4\sqrt[3]{3}} + \frac{1}{2} \text{Subst}\left(\frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2}, \sqrt[3]{3}+x\right) \\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2} \sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{2\sqrt[3]{3}} - \frac{1}{12} \log(1-x+x^2) + \frac{3}{4} \int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2} dx
\end{aligned}$$

Mathematica [A] time = 0.02, size = 107, normalized size = 0.96

$$\frac{1}{12} \left(-\log(x^2-x+1) + 3^{2/3} \log(\sqrt[3]{3}x^2-3^{2/3}x+3) + 2\log(x+1) - 2 \cdot 3^{2/3} \log(3^{2/3}x+3) - 6\sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(3 + 4*x^3 + x^6), x]

[Out] (-6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - 2*3^(2/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x^2] + 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{3+4x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(3 + 4*x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^4/(3 + 4*x^3 + x^6), x]

fricas [A] time = 1.47, size = 106, normalized size = 0.95

$$-\frac{1}{12} \cdot 3^{\frac{2}{3}} (-1)^{\frac{1}{3}} \log\left(-3^{\frac{1}{3}} (-1)^{\frac{2}{3}} x + x^2 - 3^{\frac{2}{3}} (-1)^{\frac{1}{3}}\right) + \frac{1}{6} \cdot 3^{\frac{2}{3}} (-1)^{\frac{1}{3}} \log\left(3^{\frac{1}{3}} (-1)^{\frac{2}{3}} + x\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + \frac{1}{2} \cdot 3^{\frac{1}{6}} (-1)^{\frac{1}{3}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} (2(-1)^{\frac{1}{3}} x + 3^{\frac{1}{3}})\right) - \frac{1}{12} \log(x^2-x+1) + \frac{1}{6} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $-1/12 \cdot 3^{2/3} \cdot (-1)^{1/3} \cdot \log(-3^{1/3} \cdot (-1)^{2/3} \cdot x + x^2 - 3^{2/3} \cdot (-1)^{1/3}) + 1/6 \cdot 3^{2/3} \cdot (-1)^{1/3} \cdot \log(3^{1/3} \cdot (-1)^{2/3} + x) - 1/6 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2x - 1)) + 1/2 \cdot 3^{1/6} \cdot (-1)^{1/3} \cdot \arctan(1/3 \cdot 3^{1/6} \cdot (2 \cdot (-1)^{1/3} \cdot x + 3^{1/3})) - 1/12 \cdot \log(x^2 - x + 1) + 1/6 \cdot \log(x + 1)$

giac [A] time = 0.30, size = 86, normalized size = 0.77

$$\frac{1}{12} \cdot 3^{2/3} \log(x^2 - 3^{1/3}x + 3^{2/3}) - \frac{1}{6} \cdot 3^{2/3} \log\left(x + 3^{1/3}\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{2} \cdot 3^{1/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6} (2x - 3^{1/3})\right) - \frac{1}{12} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="giac")

[Out] $1/12 \cdot 3^{2/3} \cdot \log(x^2 - 3^{1/3} \cdot x + 3^{2/3}) - 1/6 \cdot 3^{2/3} \cdot \log(\text{abs}(x + 3^{1/3})) - 1/6 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2x - 1)) + 1/2 \cdot 3^{1/6} \cdot \arctan(1/3 \cdot 3^{1/6} \cdot (2x - 3^{1/3})) - 1/12 \cdot \log(x^2 - x + 1) + 1/6 \cdot \log(\text{abs}(x + 1))$

maple [A] time = 0.01, size = 84, normalized size = 0.75

$$\frac{\frac{1}{3^6} \arctan\left(\frac{\sqrt{3} \left(\frac{2 \cdot 3^{2/3} x}{3} - 1\right)}{3}\right)}{2} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x+1)}{6} - \frac{3^{2/3} \ln(x+3^{1/3})}{6} + \frac{3^{2/3} \ln(x^2 - 3^{1/3}x + 3^{2/3})}{12} - \frac{\ln(x^2 - x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6+4*x^3+3),x)

[Out] $1/6 \cdot \ln(x+1) - 1/6 \cdot 3^{2/3} \cdot \ln(x + 3^{1/3}) + 1/12 \cdot 3^{2/3} \cdot \ln(x^2 - 3^{1/3} \cdot x + 3^{2/3}) + 1/2 \cdot 3^{1/6} \cdot \arctan(1/3 \cdot 3^{1/6} \cdot (2/3 \cdot 3^{2/3} \cdot x - 1)) - 1/12 \cdot \ln(x^2 - x + 1) - 1/6 \cdot 3^{1/2} \cdot \arctan(1/3 \cdot (2x - 1) \cdot 3^{1/2})$

maxima [A] time = 1.61, size = 84, normalized size = 0.75

$$\frac{1}{12} \cdot 3^{2/3} \log(x^2 - 3^{1/3}x + 3^{2/3}) - \frac{1}{6} \cdot 3^{2/3} \log(x + 3^{1/3}) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{2} \cdot 3^{1/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6} (2x - 3^{1/3})\right) - \frac{1}{12} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $1/12 \cdot 3^{2/3} \cdot \log(x^2 - 3^{1/3} \cdot x + 3^{2/3}) - 1/6 \cdot 3^{2/3} \cdot \log(x + 3^{1/3}) - 1/6 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2x - 1)) + 1/2 \cdot 3^{1/6} \cdot \arctan(1/3 \cdot 3^{1/6} \cdot (2x - 3^{1/3})) - 1/12 \cdot \log(x^2 - x + 1) + 1/6 \cdot \log(x + 1)$

mupad [B] time = 1.37, size = 114, normalized size = 1.02

$$\frac{\ln(x+1)}{6} - \frac{3^{2/3} \ln(x+3^{1/3})}{6} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \frac{(-1)^{1/3} \ln\left(x - \frac{(-1)^{1/3} 3^{1/3}}{2} - \frac{(-1)^{1/6} 3^{5/6}}{2} + \frac{3^{1/3}}{2}\right) (3^{2/3} + 3^{1/6} 3i)}{12} + \frac{(-1)^{1/3} 3^{2/3} \ln(x + (-1)^{2/3} 3^{1/3})}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(4*x^3 + x^6 + 3), x)`

[Out] $\log(x + 1)/6 - (3^{(2/3)} * \log(x + 3^{(1/3)}))/6 + \log(x - (3^{(1/2)} * 1i)/2 - 1/2) * ((3^{(1/2)} * 1i)/12 - 1/12) - \log(x + (3^{(1/2)} * 1i)/2 - 1/2) * ((3^{(1/2)} * 1i)/12 + 1/12) - ((-1)^{(1/3)} * \log(x - ((-1)^{(1/3)} * 3^{(1/3)})/2 - ((-1)^{(1/6)} * 3^{(5/6)})/2 + 3^{(1/3)}/2) * (3^{(2/3)} + 3^{(1/6)} * 3i))/12 + ((-1)^{(1/3)} * 3^{(2/3)} * \log(x + (-1)^{(2/3)} * 3^{(1/3)}))/6$

sympy [C] time = 0.60, size = 134, normalized size = 1.20

$$\frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{2592\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{5} + \frac{168\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{5}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{168\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{5} + \frac{2592\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{5}\right) + \text{RootSum}\left(24t^3 + 1, \left(t \mapsto t \log\left(\frac{2592t^5}{5} + \frac{168t^2}{5} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(x**6+4*x**3+3), x)`

[Out] $\log(x + 1)/6 + (-1/12 - \text{sqrt}(3)*I/12) * \log(x + 2592 * (-1/12 - \text{sqrt}(3)*I/12) ** 5/5 + 168 * (-1/12 - \text{sqrt}(3)*I/12) ** 2/5) + (-1/12 + \text{sqrt}(3)*I/12) * \log(x + 168 * (-1/12 + \text{sqrt}(3)*I/12) ** 2/5 + 2592 * (-1/12 + \text{sqrt}(3)*I/12) ** 5/5) + \text{RootSum}(24*_t**3 + 1, \text{Lambda}(_t, _t * \log(2592*_t**5/5 + 168*_t**2/5 + x)))$

$$3.150 \quad \int \frac{x^3}{3+4x^3+x^6} dx$$

Optimal. Leaf size=112

$$\frac{1}{12} \log(x^2 - x + 1) - \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{4 \cdot 3^{2/3}} - \frac{1}{6} \log(x+1) + \frac{\log(x + \sqrt[3]{3})}{2 \cdot 3^{2/3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1374, 200, 31, 634, 617, 204, 628, 618}

$$\frac{1}{12} \log(x^2 - x + 1) - \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{4 \cdot 3^{2/3}} - \frac{1}{6} \log(x+1) + \frac{\log(x + \sqrt[3]{3})}{2 \cdot 3^{2/3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(3 + 4*x^3 + x^6), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(2*3^(2/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_)*(x_)^m)/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symbo
l] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{3+4x^3+x^6} dx &= -\left(\frac{1}{2} \int \frac{1}{1+x^3} dx\right) + \frac{3}{2} \int \frac{1}{3+x^3} dx \\
&= -\left(\frac{1}{6} \int \frac{1}{1+x} dx\right) - \frac{1}{6} \int \frac{2-x}{1-x+x^2} dx + \frac{\int \frac{1}{\sqrt[3]{3}+x} dx}{2 \cdot 3^{2/3}} + \frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{2 \cdot 3^{2/3}} \\
&= -\frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{2 \cdot 3^{2/3}} + \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{4} \int \frac{1}{1-x+x^2} dx - \frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{4 \cdot 3^{2/3}} \\
&= -\frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{2 \cdot 3^{2/3}} + \frac{1}{12} \log(1-x+x^2) - \frac{\log(3^{2/3}-\sqrt[3]{3}x+x^2)}{4 \cdot 3^{2/3}} + \frac{1}{2} \text{Subst}\left(\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt{3}} - \frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{2 \cdot 3^{2/3}} + \frac{1}{12} \log(1-x+x^2) - \frac{\log(3^{2/3}-\sqrt[3]{3}x+x^2)}{4 \cdot 3^{2/3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 106, normalized size = 0.95

$$\frac{1}{12} \left(\log(x^2 - x + 1) - \sqrt[3]{3} \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) - 2 \log(x + 1) + 2\sqrt[3]{3} \log(3^{2/3}x + 3) - 2 \cdot 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x - 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(3 + 4*x^3 + x^6), x]

[Out] (-2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{3+4x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(3 + 4*x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^3/(3 + 4*x^3 + x^6), x]

fricas [A] time = 1.31, size = 102, normalized size = 0.91

$$\frac{1}{6} \cdot 9^{\frac{1}{6}} \sqrt{3} \arctan\left(\frac{1}{27} \cdot 9^{\frac{1}{6}} (2 \cdot 9^{\frac{2}{3}} \sqrt{3} x - 3 \cdot 9^{\frac{1}{3}} \sqrt{3})\right) - \frac{1}{36} \cdot 9^{\frac{2}{3}} \log(3x^2 - 9^{\frac{2}{3}}x + 3 \cdot 9^{\frac{1}{3}}) + \frac{1}{18} \cdot 9^{\frac{2}{3}} \log(3x + 9^{\frac{2}{3}}) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $\frac{1}{6} \cdot 9^{(1/6)} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{27} \cdot 9^{(1/6)} \cdot (2 \cdot 9^{(2/3)} \cdot \sqrt{3} \cdot x - 3 \cdot 9^{(1/3)} \cdot \sqrt{3})\right) - \frac{1}{36} \cdot 9^{(2/3)} \cdot \log(3 \cdot x^2 - 9^{(2/3)} \cdot x + 3 \cdot 9^{(1/3)}) + \frac{1}{18} \cdot 9^{(2/3)} \cdot \log(3 \cdot x + 9^{(2/3)}) - \frac{1}{6} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot (2x - 1)\right) + \frac{1}{12} \cdot \log(x^2 - x + 1) - \frac{1}{6} \cdot \log(x + 1)$

giac [A] time = 0.34, size = 86, normalized size = 0.77

$$\frac{1}{6} \cdot 3^{5/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6} (2x - 3^{1/3})\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{1}{12} \cdot 3^{1/3} \log\left(x^2 - 3^{1/3}x + 3^{2/3}\right) + \frac{1}{6} \cdot 3^{1/3} \log\left(x + 3^{1/3}\right) + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="giac")

[Out] $\frac{1}{6} \cdot 3^{(5/6)} \cdot \arctan\left(\frac{1}{3} \cdot 3^{(1/6)} \cdot (2x - 3^{(1/3)})\right) - \frac{1}{6} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot (2x - 1)\right) - \frac{1}{12} \cdot 3^{(1/3)} \cdot \log(x^2 - 3^{(1/3)} \cdot x + 3^{(2/3)}) + \frac{1}{6} \cdot 3^{(1/3)} \cdot \log(\text{abs}(x + 3^{(1/3)})) + \frac{1}{12} \cdot \log(x^2 - x + 1) - \frac{1}{6} \cdot \log(\text{abs}(x + 1))$

maple [A] time = 0.01, size = 84, normalized size = 0.75

$$\frac{3^{5/6} \arctan\left(\frac{\sqrt{3} \left(\frac{2 \cdot 3^{2/3} x - 1}{3}\right)}{3}\right)}{6} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{3^{1/3} \ln(x+3^{1/3})}{6} - \frac{3^{1/3} \ln(x^2 - 3^{1/3}x + 3^{2/3})}{12} + \frac{\ln(x^2 - x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^6+4*x^3+3),x)

[Out] $-1/6 \cdot \ln(x+1) + 1/6 \cdot 3^{(1/3)} \cdot \ln(x+3^{(1/3)}) - 1/12 \cdot 3^{(1/3)} \cdot \ln(x^2 - 3^{(1/3)} \cdot x + 3^{(2/3)}) + 1/6 \cdot 3^{(5/6)} \cdot \arctan\left(\frac{1}{3} \cdot 3^{(1/6)} \cdot (2/3 \cdot 3^{(2/3)} \cdot x - 1)\right) + 1/12 \cdot \ln(x^2 - x + 1) - 1/6 \cdot 3^{(1/2)} \cdot \arctan\left(\frac{1}{3} \cdot (2x - 1) \cdot 3^{(1/2)}\right)$

maxima [A] time = 1.71, size = 84, normalized size = 0.75

$$\frac{1}{6} \cdot 3^{5/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6} (2x - 3^{1/3})\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{1}{12} \cdot 3^{1/3} \log\left(x^2 - 3^{1/3}x + 3^{2/3}\right) + \frac{1}{6} \cdot 3^{1/3} \log\left(x + 3^{1/3}\right) + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $\frac{1}{6} \cdot 3^{(5/6)} \cdot \arctan\left(\frac{1}{3} \cdot 3^{(1/6)} \cdot (2x - 3^{(1/3)})\right) - \frac{1}{6} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot (2x - 1)\right) - \frac{1}{12} \cdot 3^{(1/3)} \cdot \log(x^2 - 3^{(1/3)} \cdot x + 3^{(2/3)}) + \frac{1}{6} \cdot 3^{(1/3)} \cdot \log(x + 3^{(1/3)}) + \frac{1}{12} \cdot \log(x^2 - x + 1) - \frac{1}{6} \cdot \log(x + 1)$

mupad [B] time = 1.36, size = 113, normalized size = 1.01

$$\frac{3^{1/3} \ln(x+3^{1/3})}{6} - \frac{\ln(x+1)}{6} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}i}{2}\right) \left(\frac{3^{1/3}}{12} + \frac{3^{5/6}i}{12}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}i}{2}\right) \left(\frac{3^{1/3}}{12} - \frac{3^{5/6}i}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(4*x^3 + x^6 + 3), x)`

[Out] $(3^{(1/3)} \log(x + 3^{(1/3)}))/6 - \log(x + 1)/6 + \log(x - (3^{(1/2)}*1i)/2 - 1/2) * ((3^{(1/2)}*1i)/12 + 1/12) - \log(x + (3^{(1/2)}*1i)/2 - 1/2) * ((3^{(1/2)}*1i)/12 - 1/12) - \log(x - 3^{(1/3)}/2 - (3^{(5/6)}*1i)/2) * (3^{(1/3)}/12 + (3^{(5/6)}*1i)/12) - \log(x - 3^{(1/3)}/2 + (3^{(5/6)}*1i)/2) * (3^{(1/3)}/12 - (3^{(5/6)}*1i)/12)$

sympy [C] time = 0.59, size = 110, normalized size = 0.98

$$-\frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x - \frac{1}{4} + 648\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4 + \frac{\sqrt{3}i}{4}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x - \frac{1}{4} + 648\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4 - \frac{\sqrt{3}i}{4}\right) + \text{RootSum}(72t^3 - 1, (t \mapsto t \log(648t^4 - 3t + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(x**6+4*x**3+3), x)`

[Out] $-\log(x + 1)/6 + (1/12 - \text{sqrt}(3)*I/12)*\log(x - 1/4 + 648*(1/12 - \text{sqrt}(3)*I/12)**4 + \text{sqrt}(3)*I/4) + (1/12 + \text{sqrt}(3)*I/12)*\log(x - 1/4 + 648*(1/12 + \text{sqrt}(3)*I/12)**4 - \text{sqrt}(3)*I/4) + \text{RootSum}(72*_t**3 - 1, \text{Lambda}(_t, _t*\log(648*_t**4 - 3*_t + x)))$

$$3.151 \quad \int \frac{x}{3+4x^3+x^6} dx$$

Optimal. Leaf size=112

$$\frac{1}{12} \log(x^2 - x + 1) - \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{12\sqrt[3]{3}} - \frac{1}{6} \log(x+1) + \frac{\log(x + \sqrt[3]{3})}{6\sqrt[3]{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2 \cdot 3^{5/6}}$$

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1375, 292, 31, 634, 618, 204, 628, 617}

$$\frac{1}{12} \log(x^2 - x + 1) - \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{12\sqrt[3]{3}} - \frac{1}{6} \log(x+1) + \frac{\log(x + \sqrt[3]{3})}{6\sqrt[3]{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2 \cdot 3^{5/6}}$$

Antiderivative was successfully verified.

[In] Int[x/(3 + 4*x^3 + x^6), x]

[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2 * 3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(6*3^(1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_)^(m_.))/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{3+4x^3+x^6} dx &= \frac{1}{2} \int \frac{x}{1+x^3} dx - \frac{1}{2} \int \frac{x}{3+x^3} dx \\
&= -\left(\frac{1}{6} \int \frac{1}{1+x} dx\right) + \frac{1}{6} \int \frac{1+x}{1-x+x^2} dx + \frac{\int \frac{1}{\sqrt[3]{3}+x} dx}{6\sqrt[3]{3}} - \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{6\sqrt[3]{3}} \\
&= -\frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{6\sqrt[3]{3}} + \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1-x+x^2} dx - \frac{1}{4} \int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2} dx \\
&= -\frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{6\sqrt[3]{3}} + \frac{1}{12} \log(1-x+x^2) - \frac{\log(3^{2/3}-\sqrt[3]{3}x+x^2)}{12\sqrt[3]{3}} - \frac{1}{2} \operatorname{Subst} \int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2} dx \\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2 \cdot 3^{5/6}} - \frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{6\sqrt[3]{3}} + \frac{1}{12} \log(1-x+x^2) - \frac{1}{2} \operatorname{Subst} \int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2} dx
\end{aligned}$$

Mathematica [A] time = 0.03, size = 108, normalized size = 0.96

$$\frac{1}{36} \left(3 \log(x^2 - x + 1) - 3^{2/3} \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) - 6 \log(x + 1) + 2 \cdot 3^{2/3} \log(3^{2/3}x + 3) + 6\sqrt[6]{3} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right) + 6\sqrt{3} \tan^{-1}\left(\frac{2x - 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(3 + 4*x^3 + x^6), x]

[Out] (6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 6*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] + 3*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{3+4x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(3 + 4*x^3 + x^6), x]

[Out] IntegrateAlgebraic[x/(3 + 4*x^3 + x^6), x]

fricas [A] time = 1.02, size = 84, normalized size = 0.75

$$-\frac{1}{36} \cdot 3^{2/3} \log(x^2 - 3^{1/3}x + 3^{2/3}) + \frac{1}{18} \cdot 3^{2/3} \log(x + 3^{1/3}) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) + \frac{1}{6} \cdot 3^{1/6} \arctan\left(-\frac{1}{3} \cdot 3^{1/6}(2x - 3^{1/3})\right) + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $-1/36 \cdot 3^{2/3} \cdot \log(x^2 - 3^{1/3} \cdot x + 3^{2/3}) + 1/18 \cdot 3^{2/3} \cdot \log(x + 3^{1/3}) + 1/6 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2x - 1)) + 1/6 \cdot 3^{1/6} \cdot \arctan(-1/3 \cdot 3^{1/6} \cdot (2x - 3^{1/3})) + 1/12 \cdot \log(x^2 - x + 1) - 1/6 \cdot \log(x + 1)$

giac [A] time = 0.36, size = 86, normalized size = 0.77

$$-\frac{1}{36} \cdot 3^{2/3} \log(x^2 - 3^{1/3}x + 3^{2/3}) + \frac{1}{18} \cdot 3^{2/3} \log\left(x + 3^{1/3}\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) - \frac{1}{6} \cdot 3^{1/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6}(2x-3^{1/3})\right) + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6+4*x^3+3),x, algorithm="giac")

[Out] $-1/36 \cdot 3^{2/3} \cdot \log(x^2 - 3^{1/3} \cdot x + 3^{2/3}) + 1/18 \cdot 3^{2/3} \cdot \log(\text{abs}(x + 3^{1/3})) + 1/6 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2x - 1)) - 1/6 \cdot 3^{1/6} \cdot \arctan(1/3 \cdot 3^{1/6} \cdot (2x - 3^{1/3})) + 1/12 \cdot \log(x^2 - x + 1) - 1/6 \cdot \log(\text{abs}(x + 1))$

maple [A] time = 0.00, size = 84, normalized size = 0.75

$$-\frac{1}{6} \arctan\left(\frac{\sqrt{3} \left(\frac{23^{2/3}x-1}{3}\right)}{3}\right) + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{3^{2/3} \ln(x+3^{1/3})}{18} - \frac{3^{2/3} \ln(x^2-3^{1/3}x+3^{2/3})}{36} + \frac{\ln(x^2-x+1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^6+4*x^3+3),x)

[Out] $-1/6 \cdot \ln(x+1) + 1/18 \cdot 3^{2/3} \cdot \ln(x+3^{1/3}) - 1/36 \cdot 3^{2/3} \cdot \ln(x^2 - 3^{1/3} \cdot x + 3^{2/3}) - 1/6 \cdot 3^{1/6} \cdot \arctan(1/3 \cdot 3^{1/6} \cdot (2x - 3^{1/3})) + 1/12 \cdot \ln(x^2 - x + 1) + 1/6 \cdot 3^{1/6} \cdot \arctan(1/3 \cdot (2x - 1) \cdot 3^{1/6})$

maxima [A] time = 1.60, size = 84, normalized size = 0.75

$$-\frac{1}{36} \cdot 3^{2/3} \log(x^2 - 3^{1/3}x + 3^{2/3}) + \frac{1}{18} \cdot 3^{2/3} \log(x + 3^{1/3}) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) - \frac{1}{6} \cdot 3^{1/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6}(2x-3^{1/3})\right) + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $-1/36 \cdot 3^{2/3} \cdot \log(x^2 - 3^{1/3} \cdot x + 3^{2/3}) + 1/18 \cdot 3^{2/3} \cdot \log(x + 3^{1/3}) + 1/6 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2x - 1)) - 1/6 \cdot 3^{1/6} \cdot \arctan(1/3 \cdot 3^{1/6} \cdot (2x - 3^{1/3})) + 1/12 \cdot \log(x^2 - x + 1) - 1/6 \cdot \log(x + 1)$

mupad [B] time = 1.36, size = 113, normalized size = 1.01

$$\frac{3^{2/3} \ln(x + 3^{1/3})}{18} - \frac{\ln(x + 1)}{6} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}i}{2}\right)\left(\frac{3^{2/3}}{36} - \frac{3^{1/6}i}{12}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}i}{2}\right)\left(\frac{3^{2/3}}{36} + \frac{3^{1/6}i}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(4*x^3 + x^6 + 3), x)

[Out] $(3^{2/3} \log(x + 3^{1/3}))/18 - \log(x + 1)/6 - \log(x - (3^{1/2} * 1i)/2 - 1/2) * ((3^{1/2} * 1i)/12 - 1/12) + \log(x + (3^{1/2} * 1i)/2 - 1/2) * ((3^{1/2} * 1i)/12 + 1/12) - \log(x - 3^{1/3}/2 - (3^{5/6} * 1i)/2) * (3^{2/3}/36 - (3^{1/6} * 1i)/12) - \log(x - 3^{1/3}/2 + (3^{5/6} * 1i)/2) * (3^{2/3}/36 + (3^{1/6} * 1i)/12)$

sympy [C] time = 1.86, size = 119, normalized size = 1.06

$$-\frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x + 90\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2 + 11664\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x + 11664\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5 + 90\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2\right) + \text{RootSum}(648t^3 - 1, (t \mapsto t \log(11664t^5 + 90t^2 + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**6+4*x**3+3), x)

[Out] $-\log(x + 1)/6 + (1/12 - \text{sqrt}(3)*I/12) * \log(x + 90*(1/12 - \text{sqrt}(3)*I/12)**2 + 11664*(1/12 - \text{sqrt}(3)*I/12)**5) + (1/12 + \text{sqrt}(3)*I/12) * \log(x + 11664*(1/12 + \text{sqrt}(3)*I/12)**5 + 90*(1/12 + \text{sqrt}(3)*I/12)**2) + \text{RootSum}(648*_t**3 - 1, \text{Lambda}(_t, _t * \log(11664*_t**5 + 90*_t**2 + x)))$

$$3.152 \quad \int \frac{1}{3+4x^3+x^6} dx$$

Optimal. Leaf size=112

$$-\frac{1}{12} \log(x^2 - x + 1) + \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{12 \cdot 3^{2/3}} + \frac{1}{6} \log(x+1) - \frac{\log(x + \sqrt[3]{3})}{6 \cdot 3^{2/3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1347, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12} \log(x^2 - x + 1) + \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{12 \cdot 3^{2/3}} + \frac{1}{6} \log(x+1) - \frac{\log(x + \sqrt[3]{3})}{6 \cdot 3^{2/3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x^3 + x^6)^(-1), x]

[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(6*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(6*3^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b


```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{3+4x^3+x^6} dx &= \frac{1}{2} \int \frac{1}{1+x^3} dx - \frac{1}{2} \int \frac{1}{3+x^3} dx \\
&= \frac{1}{6} \int \frac{1}{1+x} dx + \frac{1}{6} \int \frac{2-x}{1-x+x^2} dx - \frac{\int \frac{1}{\sqrt[3]{3}+x} dx}{6 \cdot 3^{2/3}} - \frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{6 \cdot 3^{2/3}} \\
&= \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{6 \cdot 3^{2/3}} - \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1-x+x^2} dx + \frac{\int \frac{-\sqrt[3]{3}+2x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{12 \cdot 3^{2/3}} \\
&= \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{6 \cdot 3^{2/3}} - \frac{1}{12} \log(1-x+x^2) + \frac{\log(3^{2/3}-\sqrt[3]{3}x+x^2)}{12 \cdot 3^{2/3}} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx \right) \\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}} + \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{6 \cdot 3^{2/3}} - \frac{1}{12} \log(1-x+x^2) + \frac{\log(3^{2/3}-\sqrt[3]{3}x+x^2)}{12 \cdot 3^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 107, normalized size = 0.96

$$\frac{1}{36} \left(-3 \log(x^2 - x + 1) + \sqrt[3]{3} \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) + 6 \log(x + 1) - 2\sqrt[3]{3} \log(3^{2/3}x + 3) + 2 \cdot 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3} - 2x}{3^{5/6}}\right) + 6\sqrt{3} \tan^{-1}\left(\frac{2x - 1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x^3 + x^6)^(-1), x]

[Out] (2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Log[1 + x] - 2*3^(1/3)*Log[3 + 3^(2/3)*x] - 3*Log[1 - x + x^2] + 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3+4x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 + 4*x^3 + x^6)^(-1), x]

[Out] IntegrateAlgebraic[(3 + 4*x^3 + x^6)^(-1), x]

fricas [A] time = 1.42, size = 124, normalized size = 1.11

$$\frac{1}{18} \cdot 9^{1/2} \sqrt{3} (-1)^{1/3} \arctan\left(\frac{1}{27} \cdot 9^{1/2} (2 \cdot 9^{1/2} \sqrt{3} (-1)^{2/3} x - 3 \cdot 9^{1/2} \sqrt{3})\right) - \frac{1}{108} \cdot 9^{1/2} (-1)^{1/3} \log\left(9^{1/2} (-1)^{1/3} x + 3x^2 + 3 \cdot 9^{1/2} (-1)^{2/3}\right) + \frac{1}{54} \cdot 9^{1/2} (-1)^{1/3} \log\left(-9^{1/2} (-1)^{1/3} + 3x\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{1}{12} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $\frac{1}{18} \cdot 9^{1/6} \cdot \sqrt{3} \cdot (-1)^{1/3} \cdot \arctan\left(\frac{1}{27} \cdot 9^{1/6} \cdot (2 \cdot 9^{2/3} \cdot \sqrt{3}) \cdot (-1)^{2/3} \cdot x - 3 \cdot 9^{1/3} \cdot \sqrt{3}\right) - \frac{1}{108} \cdot 9^{2/3} \cdot (-1)^{1/3} \cdot \log\left(9^{2/3} \cdot (-1)^{1/3} \cdot x + 3 \cdot x^2 + 3 \cdot 9^{1/3} \cdot (-1)^{2/3}\right) + \frac{1}{54} \cdot 9^{2/3} \cdot (-1)^{1/3} \cdot \log\left(-9^{2/3} \cdot (-1)^{1/3} + 3 \cdot x\right) + \frac{1}{6} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot (2 \cdot x - 1)\right) - \frac{1}{12} \cdot \log(x^2 - x + 1) + \frac{1}{6} \cdot \log(x + 1)$

giac [A] time = 0.34, size = 86, normalized size = 0.77

$$-\frac{1}{18} \cdot 3^{5/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6} (2x - 3^{1/3})\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{36} \cdot 3^{1/3} \log\left(x^2 - 3^{1/3}x + 3^{2/3}\right) - \frac{1}{18} \cdot 3^{1/3} \log\left(|x + 3^{1/3}|\right) - \frac{1}{12} \log(x^2 - x + 1) + \frac{1}{6} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+4*x^3+3),x, algorithm="giac")

[Out] $-\frac{1}{18} \cdot 3^{5/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6} \cdot (2 \cdot x - 3^{1/3})\right) + \frac{1}{6} \cdot \sqrt{3} \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot (2 \cdot x - 1)\right) + \frac{1}{36} \cdot 3^{1/3} \cdot \log(x^2 - 3^{1/3} \cdot x + 3^{2/3}) - \frac{1}{18} \cdot 3^{1/3} \cdot \log(\text{abs}(x + 3^{1/3})) - \frac{1}{12} \cdot \log(x^2 - x + 1) + \frac{1}{6} \cdot \log(\text{abs}(x + 1))$

maple [A] time = 0.01, size = 84, normalized size = 0.75

$$-\frac{3^{5/6} \arctan\left(\frac{\sqrt{3} \left(\frac{2 \cdot 3^{2/3} x - 1}{3}\right)}{3}\right)}{18} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x+1)}{6} - \frac{3^{1/3} \ln\left(x + 3^{1/3}\right)}{18} + \frac{3^{1/3} \ln\left(x^2 - 3^{1/3}x + 3^{2/3}\right)}{36} - \frac{\ln(x^2 - x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6+4*x^3+3),x)

[Out] $\frac{1}{6} \cdot \ln(x+1) - \frac{1}{18} \cdot 3^{1/3} \cdot \ln(x + 3^{1/3}) + \frac{1}{36} \cdot 3^{1/3} \cdot \ln(x^2 - 3^{1/3} \cdot x + 3^{2/3}) - \frac{1}{18} \cdot 3^{5/6} \cdot \arctan\left(\frac{1}{3} \cdot 3^{1/6} \cdot (2 \cdot 3^{2/3} \cdot x - 1)\right) - \frac{1}{12} \cdot \ln(x^2 - x + 1) + \frac{1}{6} \cdot 3^{1/2} \cdot \arctan\left(\frac{1}{3} \cdot (2 \cdot x - 1) \cdot 3^{1/2}\right)$

maxima [A] time = 1.28, size = 84, normalized size = 0.75

$$-\frac{1}{18} \cdot 3^{5/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6} (2x - 3^{1/3})\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{36} \cdot 3^{1/3} \log\left(x^2 - 3^{1/3}x + 3^{2/3}\right) - \frac{1}{18} \cdot 3^{1/3} \log\left(x + 3^{1/3}\right) - \frac{1}{12} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $-\frac{1}{18} \cdot 3^{5/6} \arctan\left(\frac{1}{3} \cdot 3^{1/6} \cdot (2 \cdot x - 3^{1/3})\right) + \frac{1}{6} \cdot \sqrt{3} \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot (2 \cdot x - 1)\right) + \frac{1}{36} \cdot 3^{1/3} \cdot \log(x^2 - 3^{1/3} \cdot x + 3^{2/3}) - \frac{1}{18} \cdot 3^{1/3} \cdot \log(x + 3^{1/3}) - \frac{1}{12} \cdot \log(x^2 - x + 1) + \frac{1}{6} \cdot \log(x + 1)$

mupad [B] time = 0.23, size = 110, normalized size = 0.98

$$\frac{\ln(x+1)}{6} - \frac{3^{1/3} \ln(x+3^{1/3})}{18} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) + \frac{(-1)^{1/3} 3^{1/3} \ln(x - (-1)^{1/3} 3^{1/3})}{18} - \frac{(-1)^{1/3} \ln\left(x + \frac{(-1)^{1/3} 3^{1/3}}{2} + \frac{(-1)^{1/3} 3^{5/6} i}{2}\right) (3^{1/3} + 3^{5/6} i)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*x^3 + x^6 + 3), x)`

[Out] `log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/18 - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 - 1/12) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/18 - ((-1)^(1/3)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3^(5/6)*1i)/2)*(3^(1/3) + 3^(5/6)*1i))/36`

sympy [C] time = 1.82, size = 124, normalized size = 1.11

$$\frac{\log(x+1)}{6} + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{13}{10} - \frac{13\sqrt{3}i}{10} + \frac{23328\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{5}\right) + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{13}{10} + \frac{23328\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{5} + \frac{13\sqrt{3}i}{10}\right) + \text{RootSum}\left(1944t^3 + 1, \left(t \mapsto t \log\left(\frac{23328t^4}{5} - \frac{78t}{5} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**6+4*x**3+3), x)`

[Out] `log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 13/10 - 13*sqrt(3)*I/10 + 23328*(-1/12 + sqrt(3)*I/12)**4/5) + (-1/12 - sqrt(3)*I/12)*log(x + 13/10 + 23328*(-1/12 - sqrt(3)*I/12)**4/5 + 13*sqrt(3)*I/10) + RootSum(1944*_t**3 + 1, Lambda(_t, _t*log(23328*_t**4/5 - 78*_t/5 + x)))`

$$3.153 \quad \int \frac{1}{x^2(3+4x^3+x^6)} dx$$

Optimal. Leaf size=119

$$-\frac{1}{12} \log(x^2 - x + 1) + \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{36\sqrt[3]{3}} - \frac{1}{3x} + \frac{1}{6} \log(x+1) - \frac{\log(x + \sqrt[3]{3})}{18\sqrt[3]{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6 \cdot 3^{5/6}}$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1368, 1510, 292, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12} \log(x^2 - x + 1) + \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{36\sqrt[3]{3}} - \frac{1}{3x} + \frac{1}{6} \log(x+1) - \frac{\log(x + \sqrt[3]{3})}{18\sqrt[3]{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6 \cdot 3^{5/6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(3 + 4*x^3 + x^6)),x]

[Out] -1/(3*x) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(6*3^(5/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(18*3^(1/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_
Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
c*(m + 2*n*(p + 1) + 1)*x^n*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) +
(c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(3+4x^3+x^6)} dx &= -\frac{1}{3x} + \frac{1}{3} \int \frac{x(-4-x^3)}{3+4x^3+x^6} dx \\
&= -\frac{1}{3x} + \frac{1}{6} \int \frac{x}{3+x^3} dx - \frac{1}{2} \int \frac{x}{1+x^3} dx \\
&= -\frac{1}{3x} + \frac{1}{6} \int \frac{1}{1+x} dx - \frac{1}{6} \int \frac{1+x}{1-x+x^2} dx - \frac{\int \frac{1}{\sqrt[3]{3}+x} dx}{18\sqrt[3]{3}} + \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{18\sqrt[3]{3}} \\
&= -\frac{1}{3x} + \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{18\sqrt[3]{3}} - \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{12} \int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2} dx \\
&= -\frac{1}{3x} + \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{18\sqrt[3]{3}} - \frac{1}{12} \log(1-x+x^2) + \frac{\log(3^{2/3}-\sqrt[3]{3}x+x^2)}{36\sqrt[3]{3}} \\
&= -\frac{1}{3x} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6 \cdot 3^{5/6}} + \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{18\sqrt[3]{3}} - \frac{1}{12} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 118, normalized size = 0.99

$$\frac{9x \log(x^2 - x + 1) - 3^{2/3}x \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) - 18x \log(x + 1) + 2 \cdot 3^{2/3}x \log(3^{2/3}x + 3) + 6\sqrt[3]{3}x \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 18\sqrt{3}x \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 36}{108x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(3 + 4*x^3 + x^6)),x]

[Out] -1/108*(36 + 6*3^(1/6)*x*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 18*Sqrt[3]*x*ArcTan[(-1 + 2*x)/Sqrt[3]] - 18*x*Log[1 + x] + 2*3^(2/3)*x*Log[3 + 3^(2/3)*x] + 9*x*Log[1 - x + x^2] - 3^(2/3)*x*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/x

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(3+4x^3+x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(3 + 4*x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x^2*(3 + 4*x^3 + x^6)), x]

fricas [A] time = 1.18, size = 117, normalized size = 0.98

$$\frac{3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x\log\left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+x^2-3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right)-2\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x\log\left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}}+x\right)+18\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6\cdot 3^{\frac{1}{6}}(-1)^{\frac{1}{3}}x\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}(2(-1)^{\frac{1}{3}}x+3^{\frac{1}{3}})\right)+9x\log(x^2-x+1)-18x\log(x+1)+36}{108x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $-1/108\cdot(3^{2/3})\cdot(-1)^{1/3}\cdot x\cdot\log(-3^{1/3}\cdot(-1)^{2/3}\cdot x + x^2 - 3^{2/3}\cdot(-1)^{1/3}) - 2\cdot 3^{2/3}\cdot(-1)^{1/3}\cdot x\cdot\log(3^{1/3}\cdot(-1)^{2/3} + x) + 18\cdot\sqrt{3}\cdot x\cdot\arctan(1/3\cdot\sqrt{3}\cdot(2x - 1)) - 6\cdot 3^{1/6}\cdot(-1)^{1/3}\cdot x\cdot\arctan(1/3\cdot 3^{1/6}\cdot(2\cdot(-1)^{1/3}\cdot x + 3^{1/3})) + 9\cdot x\cdot\log(x^2 - x + 1) - 18\cdot x\cdot\log(x + 1) + 36/x$

giac [A] time = 0.41, size = 91, normalized size = 0.76

$$\frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}(2x-3^{\frac{1}{3}})\right)-\frac{1}{3x}-\frac{1}{12}\log(x^2-x+1)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="giac")

[Out] $1/108\cdot 3^{2/3}\cdot\log(x^2 - 3^{1/3}\cdot x + 3^{2/3}) - 1/54\cdot 3^{2/3}\cdot\log(\text{abs}(x + 3^{1/3})) - 1/6\cdot\sqrt{3}\cdot\arctan(1/3\cdot\sqrt{3}\cdot(2x - 1)) + 1/18\cdot 3^{1/6}\cdot\arctan(1/3\cdot 3^{1/6}\cdot(2x - 3^{1/3})) - 1/3/x - 1/12\cdot\log(x^2 - x + 1) + 1/6\cdot\log(\text{abs}(x + 1))$

maple [A] time = 0.01, size = 89, normalized size = 0.75

$$\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{3}x-1}{3}\right)}{3}\right)}{18} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x+1)}{6} - \frac{3^{\frac{2}{3}}\ln\left(x+3^{\frac{1}{3}}\right)}{54} + \frac{3^{\frac{2}{3}}\ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{108} - \frac{\ln(x^2-x+1)}{12} - \frac{1}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(x^6+4*x^3+3),x)

[Out] $1/6\cdot\ln(x+1)-1/3/x-1/54\cdot 3^{2/3}\cdot\ln(x+3^{1/3})+1/108\cdot 3^{2/3}\cdot\ln(x^2-3^{1/3}\cdot x+3^{2/3})+1/18\cdot 3^{1/6}\cdot\arctan(1/3\cdot 3^{1/6}\cdot(2/3\cdot 3^{2/3}\cdot x-1))-1/12\cdot\ln(x^2-x+1)-1/6\cdot 3^{1/2}\cdot\arctan(1/3\cdot(2x-1)\cdot 3^{1/2})$

maxima [A] time = 1.09, size = 89, normalized size = 0.75

$$\frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}(2x-3^{\frac{1}{3}})\right)-\frac{1}{3x}-\frac{1}{12}\log(x^2-x+1)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $\frac{1}{108}3^{2/3}\log(x^2 - 3^{1/3}x + 3^{2/3}) - \frac{1}{54}3^{2/3}\log(x + 3^{1/3}) - \frac{1}{6}\sqrt{3}\arctan(1/3\sqrt{3}(2x - 1)) + \frac{1}{18}3^{1/6}\arctan(1/3*3^{1/6}(2x - 3^{1/3})) - \frac{1}{3x} - \frac{1}{12}\log(x^2 - x + 1) + \frac{1}{6}\log(x + 1)$

mupad [B] time = 1.38, size = 119, normalized size = 1.00

$$\frac{\ln(x+1)}{6} - \frac{3^{2/3}\ln(x+3^{1/3})}{54} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \frac{1}{3x} - \frac{(-1)^{1/3}\ln\left(x - \frac{(-1)^{1/3}3^{1/3}}{2} - \frac{(-1)^{1/6}3^{5/6}}{2} + \frac{3^{1/3}}{2}\right)(3^{2/3} + 3^{1/6}3i)}{108} + \frac{(-1)^{1/3}3^{2/3}\ln(x + (-1)^{2/3}3^{1/3})}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(4*x^3 + x^6 + 3)),x)

[Out] $\log(x + 1)/6 - (3^{2/3}\log(x + 3^{1/3}))/54 + \log(x - (3^{1/2}*1i)/2) - 1/2 * ((3^{1/2}*1i)/12 - 1/12) - \log(x + (3^{1/2}*1i)/2 - 1/2) * ((3^{1/2}*1i)/12 + 1/12) - 1/(3*x) - ((-1)^{1/3}\log(x - ((-1)^{1/3}*3^{1/3}))/2 - ((-1)^{1/6}*3^{5/6}))/2 + 3^{1/3}/2 * (3^{2/3} + 3^{1/6}*3i))/108 + ((-1)^{1/3}*3^{2/3})\log(x + (-1)^{2/3}*3^{1/3}))/54$

sympy [C] time = 1.83, size = 139, normalized size = 1.17

$$\frac{\log(x+1)}{6} + \left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)\log\left(x - \frac{8188128\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{41} + \frac{39384\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{41}\right) + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)\log\left(x + \frac{39384\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{41} - \frac{8188128\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{41}\right) + \text{RootSum}\left(17496t^3 + 1, \left(t \mapsto t\log\left(\frac{8188128t^5}{41} + \frac{39384t^2}{41} + x\right)\right) - \frac{1}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(x**6+4*x**3+3),x)

[Out] $\log(x + 1)/6 + (-1/12 - \sqrt{3}*I/12)\log(x - 8188128*(-1/12 - \sqrt{3}*I/12)**5/41 + 39384*(-1/12 - \sqrt{3}*I/12)**2/41) + (-1/12 + \sqrt{3}*I/12)\log(x + 39384*(-1/12 + \sqrt{3}*I/12)**2/41 - 8188128*(-1/12 + \sqrt{3}*I/12)**5/41) + \text{RootSum}(17496*_t**3 + 1, \text{Lambda}(_t, _t*\log(-8188128*_t**5/41 + 39384*_t**2/41 + x))) - 1/(3*x)$

$$3.154 \quad \int \frac{1}{x^3(3+4x^3+x^6)} dx$$

Optimal. Leaf size=119

$$-\frac{1}{6x^2} + \frac{1}{12} \log(x^2 - x + 1) - \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{36 \cdot 3^{2/3}} - \frac{1}{6} \log(x+1) + \frac{\log(x + \sqrt[3]{3})}{18 \cdot 3^{2/3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1368, 1422, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{6x^2} + \frac{1}{12} \log(x^2 - x + 1) - \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{36 \cdot 3^{2/3}} - \frac{1}{6} \log(x+1) + \frac{\log(x + \sqrt[3]{3})}{18 \cdot 3^{2/3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(3 + 4*x^3 + x^6)),x]

[Out] -1/(6*x^2) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(18*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(18*3^(2/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)^(p_), x_
Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
c*(m + 2*n*(p + 1) + 1)*x^n*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(3+4x^3+x^6)} dx &= -\frac{1}{6x^2} + \frac{1}{6} \int \frac{-8-2x^3}{3+4x^3+x^6} dx \\
&= -\frac{1}{6x^2} + \frac{1}{6} \int \frac{1}{3+x^3} dx - \frac{1}{2} \int \frac{1}{1+x^3} dx \\
&= -\frac{1}{6x^2} - \frac{1}{6} \int \frac{1}{1+x} dx - \frac{1}{6} \int \frac{2-x}{1-x+x^2} dx + \frac{\int \frac{1}{\sqrt[3]{3+x}} dx}{18 \cdot 3^{2/3}} + \frac{\int \frac{2\sqrt[3]{3-x}}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{18 \cdot 3^{2/3}} \\
&= -\frac{1}{6x^2} - \frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{18 \cdot 3^{2/3}} + \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{4} \int \frac{1}{1-x+x^2} dx - \int \frac{1}{1-x+x^2} dx \\
&= -\frac{1}{6x^2} - \frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{18 \cdot 3^{2/3}} + \frac{1}{12} \log(1-x+x^2) - \frac{\log(3^{2/3}-\sqrt[3]{3}x+x^2)}{36 \cdot 3^{2/3}} + \int \frac{1}{1-x+x^2} dx \\
&= -\frac{1}{6x^2} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}} - \frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{18 \cdot 3^{2/3}} + \frac{1}{12} \log(1-x+x^2) + \int \frac{1}{1-x+x^2} dx
\end{aligned}$$

Mathematica [A] time = 0.05, size = 113, normalized size = 0.95

$$\frac{1}{108} \left(-\frac{18}{x^2} + 9 \log(x^2 - x + 1) - \sqrt[3]{3} \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) - 18 \log(x + 1) + 2\sqrt[3]{3} \log(3^{2/3}x + 3) - 2 \cdot 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - 18\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(3 + 4*x^3 + x^6)),x]

[Out] (-18/x^2 - 2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 18*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 18*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + 9*Log[1 - x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/108

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(3+4x^3+x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(3 + 4*x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x^3*(3 + 4*x^3 + x^6)), x]

fricas [A] time = 1.16, size = 126, normalized size = 1.06

$$\frac{6 \cdot 9^{\frac{1}{6}} \sqrt{3} x^2 \arctan\left(\frac{1}{27} \cdot 9^{\frac{1}{6}} (2 \cdot 9^{\frac{2}{3}} \sqrt{3} x - 3 \cdot 9^{\frac{1}{3}} \sqrt{3})\right) - 9^{\frac{2}{3}} x^2 \log(3x^2 - 9^{\frac{2}{3}} x + 3 \cdot 9^{\frac{1}{3}}) + 2 \cdot 9^{\frac{2}{3}} x^2 \log(3x + 9^{\frac{2}{3}}) - 54 \sqrt{3} x^2 \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + 27 x^2 \log(x^2 - x + 1) - 54 x^2 \log(x + 1) - 54}{324 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] 1/324*(6*9^(1/6)*sqrt(3)*x^2*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9^(1/3)*sqrt(3))) - 9^(2/3)*x^2*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 2*9^(2/3)*x^2*log(3*x + 9^(2/3)) - 54*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x - 1)) + 27*x^2*log(x^2 - x + 1) - 54*x^2*log(x + 1) - 54)/x^2

giac [A] time = 0.41, size = 91, normalized size = 0.76

$$\frac{1}{54} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} (2x - 3^{\frac{1}{3}})\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{1}{108} \cdot 3^{\frac{1}{3}} \log(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}) + \frac{1}{54} \cdot 3^{\frac{1}{3}} \log(x + 3^{\frac{1}{3}}) - \frac{1}{6x^2} + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="giac")

[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(1/3)*log(abs(x + 3^(1/3))) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))

maple [A] time = 0.01, size = 89, normalized size = 0.75

$$\frac{3^{\frac{5}{6}} \arctan\left(\frac{\sqrt{3} \left(\frac{2 \cdot 3^{\frac{2}{3}} x - 1}{3}\right)}{3}\right)}{54} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{3^{\frac{1}{3}} \ln(x+3^{\frac{1}{3}})}{54} - \frac{3^{\frac{1}{3}} \ln(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}})}{108} + \frac{\ln(x^2 - x + 1)}{12} - \frac{1}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^6+4*x^3+3),x)

[Out] -1/6*ln(x+1)+1/54*3^(1/3)*ln(x+3^(1/3))-1/108*3^(1/3)*ln(x^2-3^(1/3)*x+3^(2/3))+1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2/3*3^(2/3)*x-1))-1/6/x^2+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [A] time = 1.27, size = 89, normalized size = 0.75

$$\frac{1}{54} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} (2x - 3^{\frac{1}{3}})\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{1}{108} \cdot 3^{\frac{1}{3}} \log(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}) + \frac{1}{54} \cdot 3^{\frac{1}{3}} \log(x + 3^{\frac{1}{3}}) - \frac{1}{6x^2} + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $\frac{1}{54} \cdot 3^{5/6} \cdot \arctan\left(\frac{1}{3} \cdot 3^{1/6} \cdot (2x - 3^{1/3})\right) - \frac{1}{6} \cdot \sqrt{3} \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot \sqrt[3]{x^2 - 3^{1/3} \cdot x + 3^{2/3}}\right) + \frac{1}{54} \cdot 3^{1/3} \cdot \log(x + 3^{1/3}) - \frac{1}{6} \cdot x^{-2} + \frac{1}{12} \cdot \log(x^2 - x + 1) - \frac{1}{6} \cdot \log(x + 1)$

mupad [B] time = 1.36, size = 118, normalized size = 0.99

$$\frac{3^{1/3} \ln(x+3^{1/3})}{54} - \frac{\ln(x+1)}{6} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) - \frac{1}{6x^2} - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}i}{2}\right) \left(\frac{3^{1/3}}{108} + \frac{3^{5/6}i}{108}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}i}{2}\right) \left(\frac{3^{1/3}}{108} - \frac{3^{5/6}i}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(4*x^3 + x^6 + 3)),x)

[Out] $\frac{3^{1/3} \cdot \log(x + 3^{1/3})}{54} - \frac{\log(x + 1)}{6} + \frac{\log(x - (3^{1/2} \cdot i))}{2} - \frac{1}{2} \cdot \left(\frac{3^{1/2} \cdot i}{12} + \frac{1}{12}\right) - \log(x + (3^{1/2} \cdot i)/2 - 1/2) \cdot \left(\frac{3^{1/2} \cdot i}{12} - \frac{1}{12}\right) - \frac{1}{6 \cdot x^2} - \log(x - 3^{1/3}/2 - (3^{5/6} \cdot i)/2) \cdot (3^{1/3}/108 + (3^{5/6} \cdot i)/108) - \log(x - 3^{1/3}/2 + (3^{5/6} \cdot i)/2) \cdot (3^{1/3}/108 - (3^{5/6} \cdot i)/108)$

sympy [C] time = 1.74, size = 128, normalized size = 1.08

$$-\frac{\log(x+1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{1093}{244} - \frac{1093\sqrt{3}i}{244} + \frac{787320\left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{61}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{1093}{244} + \frac{787320\left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{61} + \frac{1093\sqrt{3}i}{244}\right) + \text{RootSum}\left(52488t^3 - 1, \left(t \mapsto t \log\left(\frac{787320t^4}{61} + \frac{3279t}{61} + x\right)\right)\right) - \frac{1}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(x**6+4*x**3+3),x)

[Out] $-\log(x + 1)/6 + (1/12 - \sqrt{3} \cdot I/12) \cdot \log(x + 1093/244 - 1093 \cdot \sqrt{3} \cdot I/244 + 787320 \cdot (1/12 - \sqrt{3} \cdot I/12) \cdot I/61) + (1/12 + \sqrt{3} \cdot I/12) \cdot \log(x + 1093/244 + 787320 \cdot (1/12 + \sqrt{3} \cdot I/12) \cdot I/61 + 1093 \cdot \sqrt{3} \cdot I/244) + \text{RootSum}(52488 \cdot t^3 - 1, \text{Lambda}(t, t \cdot \log(787320 \cdot t^4/61 + 3279 \cdot t/61 + x))) - 1/(6 \cdot x^2)$

$$3.155 \quad \int \frac{1}{x^5(3+4x^3+x^6)} dx$$

Optimal. Leaf size=126

$$-\frac{1}{12x^4} + \frac{1}{12} \log(x^2 - x + 1) - \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{108\sqrt[3]{3}} + \frac{4}{9x} - \frac{1}{6} \log(x+1) + \frac{\log(x + \sqrt[3]{3})}{54\sqrt[3]{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18 \cdot 3^{5/6}}$$

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1368, 1504, 1510, 292, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{12x^4} + \frac{1}{12} \log(x^2 - x + 1) - \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{108\sqrt[3]{3}} + \frac{4}{9x} - \frac{1}{6} \log(x+1) + \frac{\log(x + \sqrt[3]{3})}{54\sqrt[3]{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18 \cdot 3^{5/6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(3 + 4*x^3 + x^6)),x]

[Out] -1/(12*x^4) + 4/(9*x) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(18*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(54*3^(1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1368

Int[((d_.)*(x_)^m)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1504

Int[((f_.)*(x_)^m)*((d_) + (e_.)*(x_)^(n_))*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m + n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1510

Int((((f_.)*(x_)^m)*((d_) + (e_.)*(x_)^(n_)))/((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +

$(2*c*d - b*e)/(2*q)$, Int $[(f*x)^m/(b/2 - q/2 + c*x^n)$, x], x] + Dist $[e/2 - (2*c*d - b*e)/(2*q)$, Int $[(f*x)^m/(b/2 + q/2 + c*x^n)$, x], x]] /; FreeQ $\{a, b, c, d, e, f, m\}$, x] && EqQ $[n2, 2*n]$ && NeQ $[b^2 - 4*a*c, 0]$ && IGtQ $[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x^5(3+4x^3+x^6)} dx &= -\frac{1}{12x^4} + \frac{1}{12} \int \frac{-16-4x^3}{x^2(3+4x^3+x^6)} dx \\ &= -\frac{1}{12x^4} + \frac{4}{9x} - \frac{1}{36} \int \frac{x(-52-16x^3)}{3+4x^3+x^6} dx \\ &= -\frac{1}{12x^4} + \frac{4}{9x} - \frac{1}{18} \int \frac{x}{3+x^3} dx + \frac{1}{2} \int \frac{x}{1+x^3} dx \\ &= -\frac{1}{12x^4} + \frac{4}{9x} - \frac{1}{6} \int \frac{1}{1+x} dx + \frac{1}{6} \int \frac{1+x}{1-x+x^2} dx + \frac{\int \frac{1}{\sqrt[3]{3}+x} dx}{54\sqrt[3]{3}} - \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{54\sqrt[3]{3}} \\ &= -\frac{1}{12x^4} + \frac{4}{9x} - \frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{54\sqrt[3]{3}} - \frac{1}{36} \int \frac{1}{3^{2/3}-\sqrt[3]{3}x+x^2} dx + \frac{1}{12} \int \frac{1}{1-x+x^2} dx \\ &= -\frac{1}{12x^4} + \frac{4}{9x} - \frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{54\sqrt[3]{3}} + \frac{1}{12} \log(1-x+x^2) - \frac{\log(3^{2/3}-\sqrt[3]{3}x)}{108\sqrt[3]{3}} \\ &= -\frac{1}{12x^4} + \frac{4}{9x} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18 \cdot 3^{5/6}} - \frac{1}{6} \log(1+x) + \frac{\log(\sqrt[3]{3}+x)}{54\sqrt[3]{3}} + \frac{1}{12} \log(1-x+x^2) \end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.94

$$\frac{1}{324} \left(-\frac{27}{x^4} + 27 \log(x^2 - x + 1) - 3^{2/3} \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) + \frac{144}{x} - 54 \log(x + 1) + 2 \cdot 3^{2/3} \log(3^{2/3}x + 3) + 6\sqrt[3]{3} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 54\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate $[1/(x^5*(3 + 4*x^3 + x^6)), x]$

[Out] $(-27/x^4 + 144/x + 6*3^{(1/6)}*ArcTan[(3^{(1/3)} - 2*x)/3^{(5/6)}] + 54*sqrt[3]*ArcTan[(-1 + 2*x)/sqrt[3]] - 54*Log[1 + x] + 2*3^{(2/3)}*Log[3 + 3^{(2/3)}*x] + 27*Log[1 - x + x^2] - 3^{(2/3)}*Log[3 - 3^{(2/3)}*x + 3^{(1/3)}*x^2])/324$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(3+4x^3+x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(3 + 4*x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x^5*(3 + 4*x^3 + x^6)), x]

fricas [A] time = 1.16, size = 112, normalized size = 0.89

$$\frac{-\frac{2}{324}x^4 \log(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}) - 2 \cdot 3^{\frac{2}{3}}x^4 \log(x + 3^{\frac{1}{3}}) - 54\sqrt{3}x^4 \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - 6 \cdot 3^{\frac{1}{3}}x^4 \arctan\left(-\frac{1}{3} \cdot 3^{\frac{1}{6}}(2x-3^{\frac{1}{3}})\right) - 27x^4 \log(x^2 - x + 1) + 54x^4 \log(x+1) - 144x^3 + 27}{324x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] -1/324*(3^(2/3)*x^4*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 2*3^(2/3)*x^4*log(x + 3^(1/3)) - 54*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*x^4*arctan(-1/3*3^(1/6)*(2*x - 3^(1/3))) - 27*x^4*log(x^2 - x + 1) + 54*x^4*log(x + 1) - 144*x^3 + 27)/x^4

giac [A] time = 0.36, size = 98, normalized size = 0.78

$$-\frac{1}{324} \cdot 3^{\frac{2}{3}} \log(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}) + \frac{1}{162} \cdot 3^{\frac{2}{3}} \log\left(x + 3^{\frac{1}{3}}\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{54} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}(2x-3^{\frac{1}{3}})\right) + \frac{16x^3-3}{36x^4} + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="giac")

[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(abs(x + 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))

maple [A] time = 0.01, size = 94, normalized size = 0.75

$$-\frac{3^{\frac{1}{6}} \arctan\left(\frac{\sqrt{3}\left(\frac{233x}{3}-1\right)}{3}\right)}{54} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x+1)}{6} + \frac{3^{\frac{2}{3}} \ln\left(x+3^{\frac{1}{3}}\right)}{162} - \frac{3^{\frac{2}{3}} \ln\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)}{324} + \frac{\ln(x^2-x+1)}{12} + \frac{4}{9x} - \frac{1}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(x^6+4*x^3+3),x)

[Out] -1/12/x^4+4/9/x-1/6*ln(x+1)+1/162*3^(2/3)*ln(x+3^(1/3))-1/324*3^(2/3)*ln(x^2-3^(1/3)*x+3^(2/3))-1/54*3^(1/6)*arctan(1/3*3^(1/6)*(2/3*3^(2/3)*x-1))+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [A] time = 1.20, size = 96, normalized size = 0.76

$$-\frac{1}{324} \cdot 3^{\frac{2}{3}} \log(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}) + \frac{1}{162} \cdot 3^{\frac{2}{3}} \log(x + 3^{\frac{1}{3}}) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) - \frac{1}{54} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}(2x - 3^{\frac{1}{3}})\right) + \frac{16x^3 - 3}{36x^4} + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] $-\frac{1}{324} \cdot 3^{\frac{2}{3}} \log(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}) + \frac{1}{162} \cdot 3^{\frac{2}{3}} \log(x + 3^{\frac{1}{3}}) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) - \frac{1}{54} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}(2x - 3^{\frac{1}{3}})\right) + \frac{16x^3 - 3}{36x^4} + \frac{1}{12} \log(x^2 - x + 1) - \frac{1}{6} \log(x + 1)$

mupad [B] time = 0.19, size = 124, normalized size = 0.98

$$\frac{3^{2/3} \ln(x + 3^{1/3})}{162} - \frac{\ln(x + 1)}{6} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3} i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3} i}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3} i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3} i}{12}\right) + \frac{4x^3 - 1}{9x^4} - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6} i}{2}\right) \left(\frac{3^{2/3}}{324} - \frac{3^{1/6} i}{108}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6} i}{2}\right) \left(\frac{3^{2/3}}{324} + \frac{3^{1/6} i}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(4*x^3 + x^6 + 3)),x)

[Out] $\frac{3^{\frac{2}{3}} \log(x + 3^{\frac{1}{3}})}{162} - \frac{\log(x + 1)}{6} - \frac{\log(x - (3^{\frac{1}{2}} * i))}{2} - \frac{1}{2} * \left(\frac{3^{\frac{1}{2}} * i}{12} - \frac{1}{12}\right) + \frac{\log(x + (3^{\frac{1}{2}} * i))}{2} - \frac{1}{2} * \left(\frac{3^{\frac{1}{2}} * i}{12} + \frac{1}{12}\right) + \frac{(4 * x^3)}{9} - \frac{1}{12} / x^4 - \frac{\log(x - 3^{\frac{1}{3}} / 2 - (3^{\frac{5}{6}} * i) / 2)}{2} * \left(\frac{3^{\frac{2}{3}}}{324} - \frac{(3^{\frac{1}{6}} * i)}{108}\right) - \frac{\log(x - 3^{\frac{1}{3}} / 2 + (3^{\frac{5}{6}} * i) / 2)}{2} * \left(\frac{3^{\frac{2}{3}}}{324} + \frac{(3^{\frac{1}{6}} * i)}{108}\right)$

sympy [C] time = 1.84, size = 141, normalized size = 1.12

$$-\frac{\log(x + 1)}{6} + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{4782978 \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^2}{547} + \frac{1028869776 \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^5}{547}\right) + \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{1028869776 \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^5}{547} + \frac{4782978 \left(\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^2}{547}\right) + \text{RootSum}\left(472392t^3 - 1, \left(t \mapsto t \log\left(\frac{1028869776t^5}{547} + \frac{4782978t^2}{547} + x\right)\right)\right) + \frac{16x^3 - 3}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(x**6+4*x**3+3),x)

[Out] $-\frac{\log(x + 1)}{6} + \frac{(1/12 - \sqrt{3}i/12) \log(x + 4782978(1/12 - \sqrt{3}i/12)^2/547 + 1028869776(1/12 - \sqrt{3}i/12)^5/547) + (1/12 + \sqrt{3}i/12) \log(x + 1028869776(1/12 + \sqrt{3}i/12)^5/547 + 4782978(1/12 + \sqrt{3}i/12)^2/547) + \text{RootSum}(472392 * t^3 - 1, \text{Lambda}(t, t * \log(1028869776 * t^5/547 + 4782978 * t^2/547 + x))) + (16 * x^3 - 3)/(36 * x^4)}$

$$3.156 \quad \int \frac{1}{x^6(3+4x^3+x^6)} dx$$

Optimal. Leaf size=126

$$-\frac{1}{15x^5} + \frac{2}{9x^2} - \frac{1}{12} \log(x^2 - x + 1) + \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{108 \cdot 3^{2/3}} + \frac{1}{6} \log(x+1) - \frac{\log(x + \sqrt[3]{3})}{54 \cdot 3^{2/3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}$$

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1368, 1504, 1422, 200, 31, 634, 618, 204, 628, 617}

$$\frac{2}{9x^2} - \frac{1}{15x^5} - \frac{1}{12} \log(x^2 - x + 1) + \frac{\log(x^2 - \sqrt[3]{3}x + 3^{2/3})}{108 \cdot 3^{2/3}} + \frac{1}{6} \log(x+1) - \frac{\log(x + \sqrt[3]{3})}{54 \cdot 3^{2/3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(3 + 4*x^3 + x^6)),x]

[Out] -1/(15*x^5) + 2/(9*x^2) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(54*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(54*3^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)^(p_), x_
Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
c*(m + 2*n*(p + 1) + 1)*x^n*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rule 1504

```

Int[((f_)*(x_))^(m_)*((d_)+(e_)*(x_)^n)*((a_)+(b_)*(x_)^n)+(
c_)*(x_)^n)^(p_), x_Symbol] := Simp[(d*(f*x)^(m+1)*(a+b*x^n+c*x^
(2*n))^(p+1))/(a*f*(m+1)), x] + Dist[1/(a*f^n*(m+1)), Int[(f*x)^(m+
n)*(a+b*x^n+c*x^(2*n))^p*Simp[a*e*(m+1)-b*d*(m+n*(p+1)+1)-c
*d*(m+2*n*(p+1)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
&& EqQ[n2, 2*n] && NeQ[b^2-4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6(3+4x^3+x^6)} dx &= -\frac{1}{15x^5} + \frac{1}{15} \int \frac{-20-5x^3}{x^3(3+4x^3+x^6)} dx \\
&= -\frac{1}{15x^5} + \frac{2}{9x^2} - \frac{1}{90} \int \frac{-130-40x^3}{3+4x^3+x^6} dx \\
&= -\frac{1}{15x^5} + \frac{2}{9x^2} - \frac{1}{18} \int \frac{1}{3+x^3} dx + \frac{1}{2} \int \frac{1}{1+x^3} dx \\
&= -\frac{1}{15x^5} + \frac{2}{9x^2} + \frac{1}{6} \int \frac{1}{1+x} dx + \frac{1}{6} \int \frac{2-x}{1-x+x^2} dx - \frac{\int \frac{1}{\sqrt[3]{3+x}} dx}{54 \cdot 3^{2/3}} - \frac{\int \frac{2\sqrt[3]{3-x}}{3^{2/3}-\sqrt[3]{3}x+x^2} dx}{54 \cdot 3^{2/3}} \\
&= -\frac{1}{15x^5} + \frac{2}{9x^2} + \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{54 \cdot 3^{2/3}} - \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1-x+x^2} \\
&= -\frac{1}{15x^5} + \frac{2}{9x^2} + \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{54 \cdot 3^{2/3}} - \frac{1}{12} \log(1-x+x^2) + \frac{\log(3^{2/3}-\sqrt[3]{3}x)}{108 \cdot 3^{2/3}} \\
&= -\frac{1}{15x^5} + \frac{2}{9x^2} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}} + \frac{1}{6} \log(1+x) - \frac{\log(\sqrt[3]{3}+x)}{54 \cdot 3^{2/3}} - \frac{1}{12} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.94

$$\frac{-\frac{108}{x^5} + \frac{360}{x^2} - 135 \log(x^2 - x + 1) + 5\sqrt[3]{3} \log(\sqrt[3]{3}x^2 - 3^{2/3}x + 3) + 270 \log(x + 1) - 10\sqrt[3]{3} \log(3^{2/3}x + 3) + 10 \cdot 3^{5/6} \tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 270\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{1620}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(3+4*x^3+x^6)),x]

[Out] (-108/x^5 + 360/x^2 + 10*3^(5/6)*ArcTan[(3^(1/3)-2*x)/3^(5/6)] + 270*Sqrt[3]*ArcTan[(-1+2*x)/Sqrt[3]] + 270*Log[1+x] - 10*3^(1/3)*Log[3+3^(2/3)

)*x] - 135*Log[1 - x + x^2] + 5*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/1
620

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6(3 + 4x^3 + x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^6*(3 + 4*x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x^6*(3 + 4*x^3 + x^6)), x]

fricas [A] time = 1.15, size = 153, normalized size = 1.21

$$\frac{30 \cdot 9^{\frac{1}{2}} \sqrt{5} (-1)^{\frac{1}{2}} x^5 \arctan\left(\frac{1}{27} \cdot 9^{\frac{1}{2}} (2 \cdot 9^{\frac{1}{2}} \sqrt{5} (-1)^{\frac{1}{2}} x - 3 \cdot 9^{\frac{1}{2}} \sqrt{5})\right) - 5 \cdot 9^{\frac{1}{2}} (-1)^{\frac{1}{2}} x^5 \log\left(9^{\frac{1}{2}} (-1)^{\frac{1}{2}} x + 3x^2 + 3 \cdot 9^{\frac{1}{2}} (-1)^{\frac{1}{2}}\right) + 10 \cdot 9^{\frac{1}{2}} (-1)^{\frac{1}{2}} x^5 \log\left(-9^{\frac{1}{2}} (-1)^{\frac{1}{2}} + 3x\right) + 810 \sqrt{5} x^5 \arctan\left(\frac{1}{3} \sqrt{5} (2x - 1)\right) - 405 x^5 \log(x^2 - x + 1) + 810 x^5 \log(x + 1) + 1080 x^5 - 324}{4860 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] $\frac{1}{4860} \cdot (30 \cdot 9^{(1/6)} \cdot \text{sqrt}(3) \cdot (-1)^{(1/3)} \cdot x^5 \cdot \arctan(1/27 \cdot 9^{(1/6)} \cdot (2 \cdot 9^{(2/3)} \cdot \text{sqrt}(3) \cdot (-1)^{(2/3)} \cdot x - 3 \cdot 9^{(1/3)} \cdot \text{sqrt}(3))) - 5 \cdot 9^{(2/3)} \cdot (-1)^{(1/3)} \cdot x^5 \cdot \log(9^{(2/3)} \cdot (-1)^{(1/3)} \cdot x + 3 \cdot x^2 + 3 \cdot 9^{(1/3)} \cdot (-1)^{(2/3)}) + 10 \cdot 9^{(2/3)} \cdot (-1)^{(1/3)} \cdot x^5 \cdot \log(-9^{(2/3)} \cdot (-1)^{(1/3)} + 3 \cdot x) + 810 \cdot \text{sqrt}(3) \cdot x^5 \cdot \arctan(1/3 \cdot \text{sqrt}(3) \cdot (2 \cdot x - 1)) - 405 \cdot x^5 \cdot \log(x^2 - x + 1) + 810 \cdot x^5 \cdot \log(x + 1) + 1080 \cdot x^3 - 324) / x^5$

giac [A] time = 0.36, size = 98, normalized size = 0.78

$$-\frac{1}{162} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} (2x - 3^{\frac{1}{3}})\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{324} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{162} \cdot 3^{\frac{1}{3}} \log\left(|x + 3^{\frac{1}{3}}|\right) + \frac{10x^3 - 3}{45x^5} - \frac{1}{12} \log(x^2 - x + 1) + \frac{1}{6} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="giac")

[Out] $-1/162 \cdot 3^{(5/6)} \cdot \arctan(1/3 \cdot 3^{(1/6)} \cdot (2 \cdot x - 3^{(1/3)})) + 1/6 \cdot \text{sqrt}(3) \cdot \arctan(1/3 \cdot \text{sqrt}(3) \cdot (2 \cdot x - 1)) + 1/324 \cdot 3^{(1/3)} \cdot \log(x^2 - 3^{(1/3)} \cdot x + 3^{(2/3)}) - 1/162 \cdot 3^{(1/3)} \cdot \log(\text{abs}(x + 3^{(1/3)})) + 1/45 \cdot (10 \cdot x^3 - 3) / x^5 - 1/12 \cdot \log(x^2 - x + 1) + 1/6 \cdot \log(\text{abs}(x + 1))$

maple [A] time = 0.01, size = 94, normalized size = 0.75

$$-\frac{3^{\frac{5}{6}} \arctan\left(\frac{\sqrt{3} \left(\frac{23^{\frac{2}{3}} x - 1}{3}\right)}{3}\right)}{162} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x+1)}{6} - \frac{3^{\frac{1}{3}} \ln\left(x + 3^{\frac{1}{3}}\right)}{162} + \frac{3^{\frac{1}{3}} \ln\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right)}{324} - \frac{\ln(x^2 - x + 1)}{12} + \frac{2}{9x^2} - \frac{1}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/x^6/(x^6+4*x^3+3), x)$

[Out] $-1/15/x^5+2/9/x^2+1/6*\ln(x+1)-1/162*3^{(1/3)}*\ln(x+3^{(1/3)})+1/324*3^{(1/3)}*\ln(x^2-3^{(1/3)}*x+3^{(2/3)})-1/162*3^{(5/6)}*\arctan(1/3*3^{(1/2)}*(2/3*3^{(2/3)}*x-1))-1/12*\ln(x^2-x+1)+1/6*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})$

maxima [A] time = 1.66, size = 96, normalized size = 0.76

$-\frac{1}{162} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}(2x - 3^{\frac{1}{6}})\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x - 1)\right) + \frac{1}{324} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) - \frac{1}{162} \cdot 3^{\frac{1}{3}} \log\left(x + 3^{\frac{1}{3}}\right) + \frac{10x^3 - 3}{45x^5} - \frac{1}{12} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/x^6/(x^6+4*x^3+3), x, \text{algorithm}="maxima")$

[Out] $-1/162*3^{(5/6)}*\arctan(1/3*3^{(1/6)}*(2*x - 3^{(1/3)})) + 1/6*\text{sqrt}(3)*\arctan(1/3*\text{sqrt}(3)*(2*x - 1)) + 1/324*3^{(1/3)}*\log(x^2 - 3^{(1/3)}*x + 3^{(2/3)}) - 1/162*3^{(1/3)}*\log(x + 3^{(1/3)}) + 1/45*(10*x^3 - 3)/x^5 - 1/12*\log(x^2 - x + 1) + 1/6*\log(x + 1)$

mupad [B] time = 1.40, size = 121, normalized size = 0.96

$\frac{\ln(x+1)}{6} - \frac{3^{1/3} \ln(x+3^{1/3})}{162} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3} i}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3} i}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3} i}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3} i}{12}\right) + \frac{2x^3 - \frac{1}{15}}{x^5} + \frac{(-1)^{1/3} \ln(x - (-1)^{1/3} 3^{1/3})}{162} - \frac{(-1)^{1/3} \ln\left(x + \frac{(-1)^{1/3} 3^{1/3}}{2} + \frac{(-1)^{1/3} 3^{5/6} i}{2}\right)}{324} (3^{1/3} + 3^{5/6} i)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^6*(4*x^3 + x^6 + 3)), x)$

[Out] $\log(x + 1)/6 - (3^{(1/3)}*\log(x + 3^{(1/3)}))/162 - \log(x - (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/12 + 1/12) + \log(x + (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/12 - 1/12) + ((2*x^3)/9 - 1/15)/x^5 + ((-1)^{(1/3)}*3^{(1/3)}*\log(x - (-1)^{(1/3)}*3^{(1/3)}))/162 - ((-1)^{(1/3)}*\log(x + ((-1)^{(1/3)}*3^{(1/3)})/2 + ((-1)^{(1/3)}*3^{(5/6)}*1i)/2)*(3^{(1/3)} + 3^{(5/6)}*1i))/324$

sympy [C] time = 1.78, size = 136, normalized size = 1.08

$\frac{\log(x+1)}{6} + \left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{88573}{6562} - \frac{88573\sqrt{3}i}{6562} + \frac{119042784\left(-\frac{1}{12} + \frac{\sqrt{3}i}{12}\right)^4}{3281}\right) + \left(\frac{1}{12} - \frac{\sqrt{3}i}{12}\right) \log\left(x + \frac{88573}{6562} + \frac{119042784\left(-\frac{1}{12} - \frac{\sqrt{3}i}{12}\right)^4}{3281} + \frac{88573\sqrt{3}i}{6562}\right) + \text{RootSum}\left(1417176t^3 + 1, \left(t \mapsto t \log\left(\frac{119042784t^4}{3281} - \frac{531438t}{3281} + x\right)\right) + \frac{10x^3 - 3}{45x^5}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/x**6/(x**6+4*x**3+3), x)$

[Out] $\log(x + 1)/6 + (-1/12 + \text{sqrt}(3)*I/12)*\log(x + 88573/6562 - 88573*\text{sqrt}(3)*I/6562 + 119042784*(-1/12 + \text{sqrt}(3)*I/12)**4/3281) + (-1/12 - \text{sqrt}(3)*I/12)*\log(x + 88573/6562 + 119042784*(-1/12 - \text{sqrt}(3)*I/12)**4/3281 + 88573*\text{sqrt}(3)*I/6562) + \text{RootSum}(1417176*_t**3 + 1, \text{Lambda}(_t, _t*\log(119042784*_t**4/3281 - 531438*_t/3281 + x))) + (10*x**3 - 3)/(45*x**5)$

$$3.157 \quad \int \frac{x^6}{1-x^3+x^6} dx$$

Optimal. Leaf size=412

$$\frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

Rubi [A] time = 0.43, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + x + \frac{(3-i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(-\sqrt{3}+i) \tan^{-1}\left(\frac{1+\sqrt[3]{2(1-i\sqrt{3})}}{\sqrt[3]{2(1-i\sqrt{3})}}\right)}{3\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(\sqrt{3}+i) \tan^{-1}\left(\frac{1+\sqrt[3]{2(1+i\sqrt{3})}}{\sqrt[3]{2(1+i\sqrt{3})}}\right)}{3\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(1 - x^3 + x^6), x]

[Out] x + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1367

```
Int[((d_.)*(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_), x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1-x^3+x^6} dx &= x - \int \frac{1-x^3}{1-x^3+x^6} dx \\
&= x - \frac{1}{6}(-3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2} + \frac{i\sqrt{3}}{2} + x^3} dx + \frac{1}{6}(3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^3} dx \\
&= x + \frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt{\frac{1}{2}(1-i\sqrt{3})} + x} dx}{9\sqrt[3]{2} (1-i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \int \frac{-2^{2/3} \sqrt[3]{1-i\sqrt{3}} - x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt{\frac{1}{2}(1-i\sqrt{3})} x + x^2} dx}{9\sqrt[3]{2} (1-i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \int \frac{1}{-\sqrt{\frac{1}{2}(1+i\sqrt{3})} + x} dx}{9\sqrt[3]{2} (1+i\sqrt{3})^{2/3}} \\
&= x + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2} (1-i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2} (1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2} (1-i\sqrt{3})^{2/3}} \\
&= x + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2} (1-i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2} (1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2} (1-i\sqrt{3})^{2/3}} \\
&= x + \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2} (1-i\sqrt{3})^{2/3}} - \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2} (1+i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2} (1-i\sqrt{3})^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.14

$$\frac{1}{3} \text{RootSum}\left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^3 \log(x - \#1) - \log(x - \#1)}{2\#1^5 - \#1^2} \&\right] + x$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(1 - x^3 + x^6), x]

[Out] x + RootSum[1 - #1^3 + #1^6 &, (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 + 2*#1^5) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1-x^3+x^6} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(1 - x^3 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^6/(1 - x^3 + x^6), x]
```

fricas [B] time = 1.38, size = 1028, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan
(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) + 2/27*18^(2/3)*12
^(1/6)*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
- 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3)
- 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arct
an(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)
*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) -
2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arctan
(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) +
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2))
- 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
```

$$\begin{aligned}
 &)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \\
 &\cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2 + 1/108 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \\
 &\cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} \\
 &- 2))) \cdot \log(-2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \\
 &\cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin \\
 &(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2) + x
 \end{aligned}$$

giac [B] time = 0.52, size = 638, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^6-x^3+1),x, algorithm="giac")

[Out]
$$\begin{aligned}
 &-1/9 \cdot (\sqrt{3} \cdot \cos(4/9 \cdot \pi))^4 - 6 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi)^2 \cdot \sin(4/9 \cdot \pi)^2 + \sqrt{3} \cdot \\
 &\sin(4/9 \cdot \pi)^4 + 4 \cdot \cos(4/9 \cdot \pi)^3 \cdot \sin(4/9 \cdot \pi) - 4 \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi)^3 \\
 &+ 2 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi) + 2 \cdot \sin(4/9 \cdot \pi)) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(4/9 \\
 &\cdot \pi) - 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(4/9 \cdot \pi))) - 1/9 \cdot (\sqrt{3} \cdot \cos(2/9 \cdot \pi))^4 - 6 \\
 &\cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi)^2 \cdot \sin(2/9 \cdot \pi)^2 + \sqrt{3} \cdot \sin(2/9 \cdot \pi)^4 + 4 \cdot \cos(2/9 \cdot \pi) \\
 &)^3 \cdot \sin(2/9 \cdot \pi) - 4 \cdot \cos(2/9 \cdot \pi) \cdot \sin(2/9 \cdot \pi)^3 + 2 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi) + 2 \cdot \sin \\
 &(2/9 \cdot \pi)) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(2/9 \cdot \pi) - 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin \\
 &(2/9 \cdot \pi))) - 1/9 \cdot (\sqrt{3} \cdot \cos(1/9 \cdot \pi))^4 - 6 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi)^2 \cdot \sin(1/9 \cdot \pi) \\
 &\pi)^2 + \sqrt{3} \cdot \sin(1/9 \cdot \pi)^4 - 4 \cdot \cos(1/9 \cdot \pi)^3 \cdot \sin(1/9 \cdot \pi) + 4 \cdot \cos(1/9 \cdot \pi) \\
 &\cdot \sin(1/9 \cdot \pi)^3 - 2 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi) + 2 \cdot \sin(1/9 \cdot \pi)) \cdot \arctan(((\sqrt{3} \cdot i \\
 &+ 1) \cdot \cos(1/9 \cdot \pi) + 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(1/9 \cdot \pi))) - 1/18 \cdot (4 \cdot \sqrt{3} \cdot \cos \\
 &(4/9 \cdot \pi)^3 \cdot \sin(4/9 \cdot \pi) - 4 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi)^3 - \cos(4/9 \cdot \pi) \\
 &\pi)^4 + 6 \cdot \cos(4/9 \cdot \pi)^2 \cdot \sin(4/9 \cdot \pi)^2 - \sin(4/9 \cdot \pi)^4 + 2 \cdot \sqrt{3} \cdot \sin(4/9 \cdot \pi) \\
 &- 2 \cdot \cos(4/9 \cdot \pi)) \cdot \log(-(\sqrt{3} \cdot i \cdot \cos(4/9 \cdot \pi) + \cos(4/9 \cdot \pi)) \cdot x + x^2 + 1) - \\
 &1/18 \cdot (4 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi)^3 \cdot \sin(2/9 \cdot \pi) - 4 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi) \cdot \sin(2/9 \cdot \pi) \\
 &\pi)^3 - \cos(2/9 \cdot \pi)^4 + 6 \cdot \cos(2/9 \cdot \pi)^2 \cdot \sin(2/9 \cdot \pi)^2 - \sin(2/9 \cdot \pi)^4 + 2 \cdot \sqrt{3} \\
 &\cdot \sin(2/9 \cdot \pi) - 2 \cdot \cos(2/9 \cdot \pi)) \cdot \log(-(\sqrt{3} \cdot i \cdot \cos(2/9 \cdot \pi) + \cos(2/9 \cdot \pi) \\
 &\pi)) \cdot x + x^2 + 1) + 1/18 \cdot (4 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi)^3 \cdot \sin(1/9 \cdot \pi) - 4 \cdot \sqrt{3} \cdot \cos \\
 &(1/9 \cdot \pi) \cdot \sin(1/9 \cdot \pi)^3 + \cos(1/9 \cdot \pi)^4 - 6 \cdot \cos(1/9 \cdot \pi)^2 \cdot \sin(1/9 \cdot \pi)^2 + \sin \\
 &(1/9 \cdot \pi)^4 - 2 \cdot \sqrt{3} \cdot \sin(1/9 \cdot \pi) - 2 \cdot \cos(1/9 \cdot \pi)) \cdot \log((\sqrt{3} \cdot i \cdot \cos(1/9 \\
 &\pi) + \cos(1/9 \cdot \pi)) \cdot x + x^2 + 1) + x
 \end{aligned}$$

maple [C] time = 0.01, size = 44, normalized size = 0.11

$$x + \frac{\left(\text{RootOf}\left(-Z^6 - Z^3 + 1\right)^3 - 1\right) \ln\left(-\text{RootOf}\left(-Z^6 - Z^3 + 1\right) + x\right)}{6 \text{RootOf}\left(-Z^6 - Z^3 + 1\right)^5 - 3 \text{RootOf}\left(-Z^6 - Z^3 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^6-x^3+1),x)

[Out] $x + \frac{1}{3} \sum \left(\frac{R^3 - 1}{2R^5 - R^2} \ln(-R + x) \right), R = \text{RootOf}(-Z^6 - Z^3 + 1)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x + \int \frac{x^3 - 1}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(x^6-x^3+1),x, algorithm="maxima")`

[Out] `x + integrate((x^3 - 1)/(x^6 - x^3 + 1), x)`

mupad [B] time = 1.82, size = 320, normalized size = 0.78

$$\frac{\ln\left(x + \frac{\sqrt[3]{3} + \sqrt[3]{9}i}{18}\right) (36 + \sqrt{12})^{1/3}}{18} + \frac{\ln\left(x - \frac{\sqrt[3]{3} + \sqrt[3]{9}i}{18}\right) (36 - \sqrt{12})^{1/3}}{18} + \frac{2^{2/3} \ln\left(x - \frac{2^{1/3}(\sqrt{3} + \sqrt[3]{9}i)^{1/3} + 2^{1/3}(\sqrt{3} - \sqrt[3]{9}i)^{1/3}}{36}\right) (36 - \sqrt{12})^{1/3}}{36} + \frac{2^{2/3} \ln\left(x + \frac{2^{1/3}(\sqrt{3} + \sqrt[3]{9}i)^{1/3} + 2^{1/3}(\sqrt{3} - \sqrt[3]{9}i)^{1/3}}{36}\right) (36 + \sqrt{12})^{1/3}}{36} + \frac{2^{2/3} \ln\left(x + \frac{2^{1/3}(\sqrt{3} + \sqrt[3]{9}i)^{1/3} - 2^{1/3}(\sqrt{3} - \sqrt[3]{9}i)^{1/3}}{36}\right) (36 - \sqrt{12})^{1/3}}{36} + \frac{2^{2/3} \ln\left(x - \frac{2^{1/3}(\sqrt{3} + \sqrt[3]{9}i)^{1/3} - 2^{1/3}(\sqrt{3} - \sqrt[3]{9}i)^{1/3}}{36}\right) (36 + \sqrt{12})^{1/3}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(x^6 - x^3 + 1), x)`

[Out] $x + \frac{(\log(x + ((3^{1/2} * 9i)/2 - 27/2) * (3^{1/2} * 12i + 36)^{1/3}) / 54) * (3^{1/2} * 12i + 36)^{1/3}}{18} + \frac{(\log(x - ((3^{1/2} * 9i)/2 + 27/2) * (36 - 3^{1/2} * 12i)^{1/3}) / 54) * (36 - 3^{1/2} * 12i)^{1/3}}{18} - \frac{(2^{2/3} * \log(x - (2^{2/3} * (3 - 3^{1/2} * 1i)^{1/3} * (3^{1/3} - 3^{5/6} * 1i) * ((3 * (3^{1/2} * 1i - 3) * (3^{1/3} - 3^{5/6} * 1i)^3) / 16 - 27)) / 108) * (3 - 3^{1/2} * 1i)^{1/3} * (3^{1/3} - 3^{5/6} * 1i))}{36} - \frac{(2^{2/3} * \log(x + (2^{2/3} * (3^{1/2} * 1i + 3)^{1/3} * (3^{1/3} + 3^{5/6} * 1i) * ((3 * (3^{1/2} * 1i + 3) * (3^{1/3} + 3^{5/6} * 1i)^3) / 16 + 27)) / 108) * (3^{1/2} * 1i + 3)^{1/3} * (3^{1/3} + 3^{5/6} * 1i))}{36} - \frac{(2^{2/3} * \log(x + (2^{2/3} * 3^{5/6} * (3 - 3^{1/2} * 1i)^{1/3} * 1i) / 6) * (3 - 3^{1/2} * 1i)^{1/3} * (3^{1/3} + 3^{5/6} * 1i))}{36} - \frac{(2^{2/3} * \log(x - (2^{2/3} * 3^{5/6} * (3^{1/2} * 1i + 3)^{1/3} * 1i) / 6) * (3^{1/2} * 1i + 3)^{1/3} * (3^{1/3} - 3^{5/6} * 1i))}{36}$

sympy [A] time = 0.18, size = 26, normalized size = 0.06

$$x + \text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log(729t^4 - 9t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(x**6-x**3+1),x)`

[Out] `x + RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t + x)))`

$$3.158 \quad \int \frac{x^5}{1-x^3+x^6} dx$$

Optimal. Leaf size=39

$$\frac{1}{6} \log(x^6 - x^3 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{6} \log(x^6 - x^3 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(1 - x^3 + x^6), x]

[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1-x^3+x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{1-x+x^2} dx, x, x^3 \right) \\
&= \frac{1}{6} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) + \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3 \right) \\
&= \frac{1}{6} \log(1-x^3+x^6) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\
&= -\frac{\tan^{-1} \left(\frac{1-2x^3}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{1}{6} \log(1-x^3+x^6)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{2x^3-1}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{1}{6} \log(x^6 - x^3 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(1 - x^3 + x^6), x]

[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1-x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(1 - x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^5/(1 - x^3 + x^6), x]

fricas [A] time = 1.18, size = 32, normalized size = 0.82

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) + \frac{1}{6} \log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6-x^3+1),x, algorithm="fricas")

[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)

giac [A] time = 0.43, size = 32, normalized size = 0.82

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) + \frac{1}{6} \log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6-x^3+1),x, algorithm="giac")

[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)

maple [A] time = 0.00, size = 33, normalized size = 0.85

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9} + \frac{\ln(x^6 - x^3 + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^6-x^3+1),x)

[Out] 1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))

maxima [A] time = 1.04, size = 32, normalized size = 0.82

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) + \frac{1}{6} \log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6-x^3+1),x, algorithm="maxima")

[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)

mupad [B] time = 1.21, size = 34, normalized size = 0.87

$$\frac{\ln(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(x^6 - x^3 + 1),x)`

[Out] `log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9`

sympy [A] time = 0.13, size = 37, normalized size = 0.95

$$\frac{\log(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(x**6-x**3+1),x)`

[Out] `log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9`

$$3.159 \quad \int \frac{x^4}{1-x^3+x^6} dx$$

Optimal. Leaf size=411

$$\frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}}$$

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1374, 292, 31, 634, 617, 204, 628}

$$\frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \frac{(3+i\sqrt{3}) \log\left(-\sqrt{2}x + \sqrt[3]{1-i\sqrt{3}}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(-\sqrt{2}x + \sqrt[3]{1+i\sqrt{3}}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \frac{(\sqrt{3}+i) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(-\sqrt{3}+i) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(1 - x^3 + x^6), x]

[Out] ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_)*(x_)^(m_))/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{1-x^3+x^6} dx &= -\left(\frac{1}{6}(-3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx\right) + \frac{1}{6}(3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3} dx \\
&= -\left(\frac{(-3-i\sqrt{3}) \int \frac{-\sqrt{\frac{1}{2}(1-i\sqrt{3})+x}}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt{\frac{1}{2}(1-i\sqrt{3})}x+x^2} dx}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}}\right) + \frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt{\frac{1}{2}(1+i\sqrt{3})+x}} dx}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{(-3-i\sqrt{3}) \int \frac{1}{\sqrt{\frac{1}{2}(1-i\sqrt{3})+x}} dx}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} \\
&= \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{(-3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} \\
&= \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} \\
&= \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.10

$$\frac{1}{3} \text{RootSum}\left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^2 \log(x - \#1)}{2\#1^3 - 1} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 - x^3 + x^6), x]

[Out] RootSum[1 - #1^3 + #1^6 &, (Log[x - #1]*#1^2)/(-1 + 2*#1^3) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1-x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(1 - x^3 + x^6),x]

[Out] IntegrateAlgebraic[x^4/(1 - x^3 + x^6), x]

fricas [B] time = 1.73, size = 1583, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6-x^3+1),x, algorithm="fricas")

[Out]
$$\frac{1}{54} \cdot 18^{2/3} \cdot 12^{1/6} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) \cdot \log\left(18^{2/3} \cdot 12^{2/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 2 \cdot \left(18^{2/3} \cdot 12^{2/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 36 \cdot x^2 + \frac{2}{27} \cdot 18^{2/3} \cdot 12^{1/6} \cdot \arctan\left(\frac{1}{108} \cdot \left(6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 108 \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 + 108 \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 + 864 \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^3 - 6 \cdot \left(18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x - 36 \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 - 12 \cdot \left(18^{2/3} \cdot 12^{2/3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 72 \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^3 \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - \sqrt{18^{2/3} \cdot 12^{2/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 2 \cdot \left(18^{2/3} \cdot 12^{2/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 36 \cdot x^2 \cdot \left(18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 - 2 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)\right) / \left(3 \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 - 10 \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 3 \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - \frac{1}{27} \cdot \left(18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - 18^{2/3} \cdot 12^{1/6} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)\right) \cdot \arctan\left(\frac{1}{108} \cdot \left(6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 108 \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 + 108 \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 - 864 \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^3 - 6 \cdot \left(18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x - 36 \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 12 \cdot \left(18^{2/3} \cdot 12^{2/3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 72 \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^3 \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - \sqrt{18^{2/3} \cdot 12^{2/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^4 + 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + \right.$$

$$\begin{aligned}
& 2*(18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 - 3*18^{(1/3)}*12^{(1/3)}* \\
& x*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 36*x^2)*(18^{(2/3)}*12^{(2/3)}*\sqrt{3}*\cos(\\
& 2/3*\arctan(\sqrt{3} + 2))^2 - 18^{(2/3)}*12^{(2/3)}*\sqrt{3}*\sin(2/3*\arctan(\sqrt{3} \\
& 3) + 2))^2 + 2*18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))*\sin(2/3*\arcta \\
& n(\sqrt{3} + 2))))/(3*\cos(2/3*\arctan(\sqrt{3} + 2))^4 - 10*\cos(2/3*\arctan(\sqrt{3} \\
& 2) + 2))^2*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 3*\sin(2/3*\arctan(\sqrt{3} + 2) \\
&)^4) - 1/27*(18^{(2/3)}*12^{(1/6)}*\sqrt{3}*\cos(2/3*\arctan(\sqrt{3} + 2)) + 18^{(\\
& 2/3)}*12^{(1/6)}*\sin(2/3*\arctan(\sqrt{3} + 2)))*\arctan(-1/432*(6*18^{(2/3)}*12^{(2 \\
& /3)}*x - 216*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 216*\sin(2/3*\arctan(\sqrt{3} + 2) \\
&))^2 - 18^{(2/3)}*12^{(2/3)}*\sqrt{18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2) \\
&)^4 + 18^{(2/3)}*12^{(2/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^4 - 12*18^{(1/3)}*12^{(1/ \\
& 3)}*x*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 2*(18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(s \\
& \sqrt{3} + 2))^2 + 6*18^{(1/3)}*12^{(1/3)}*x)*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 36 \\
& *x^2))/(\cos(2/3*\arctan(\sqrt{3} + 2))*\sin(2/3*\arctan(\sqrt{3} + 2)))) - 1/108 \\
& *(18^{(2/3)}*12^{(1/6)}*\sqrt{3}*\sin(2/3*\arctan(\sqrt{3} + 2)) + 18^{(2/3)}*12^{(1/6) \\
& }*\cos(2/3*\arctan(\sqrt{3} + 2)))*\log(18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} \\
&) + 2))^4 + 18^{(2/3)}*12^{(2/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^4 + 12*18^{(1/3)}* \\
& 12^{(1/3)}*\sqrt{3}*x*\cos(2/3*\arctan(\sqrt{3} + 2))*\sin(2/3*\arctan(\sqrt{3} + 2) \\
&) + 6*18^{(1/3)}*12^{(1/3)}*x*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 2*(18^{(2/3)}*12^{(\\
& 2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 - 3*18^{(1/3)}*12^{(1/3)}*x)*\sin(2/3*\arctan \\
& (\sqrt{3} + 2))^2 + 36*x^2) + 1/108*(18^{(2/3)}*12^{(1/6)}*\sqrt{3}*\sin(2/3*\arcta \\
& n(\sqrt{3} + 2)) - 18^{(2/3)}*12^{(1/6)}*\cos(2/3*\arctan(\sqrt{3} + 2)))*\log(18^{(2 \\
& /3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^4 + 18^{(2/3)}*12^{(2/3)}*\sin(2/3*\arc \\
& tan(\sqrt{3} + 2))^4 - 12*18^{(1/3)}*12^{(1/3)}*x*\cos(2/3*\arctan(\sqrt{3} + 2))^2 \\
& + 2*(18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 6*18^{(1/3)}*12^{(1/3) \\
& }*x)*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 36*x^2)
\end{aligned}$$

giac [B] time = 0.69, size = 824, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6-x^3+1),x, algorithm="giac")

[Out] $-1/9*(2*\sqrt{3}*\cos(4/9*\pi)^5 - 20*\sqrt{3}*\cos(4/9*\pi)^3*\sin(4/9*\pi)^2 + 10$
 $*\sqrt{3}*\cos(4/9*\pi)*\sin(4/9*\pi)^4 - 10*\cos(4/9*\pi)^4*\sin(4/9*\pi) + 20*\cos($
 $4/9*\pi)^2*\sin(4/9*\pi)^3 - 2*\sin(4/9*\pi)^5 + \sqrt{3}*\cos(4/9*\pi)^2 - \sqrt{3}$
 $*\sin(4/9*\pi)^2 - 2*\cos(4/9*\pi)*\sin(4/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(4/$
 $9*\pi) - 2*x)/((\sqrt{3}*i + 1)*\sin(4/9*\pi))) - 1/9*(2*\sqrt{3}*\cos(2/9*\pi)^5$
 $- 20*\sqrt{3}*\cos(2/9*\pi)^3*\sin(2/9*\pi)^2 + 10*\sqrt{3}*\cos(2/9*\pi)*\sin(2/9*\pi)$
 $i)^4 - 10*\cos(2/9*\pi)^4*\sin(2/9*\pi) + 20*\cos(2/9*\pi)^2*\sin(2/9*\pi)^3 - 2*\sin$
 $(2/9*\pi)^5 + \sqrt{3}*\cos(2/9*\pi)^2 - \sqrt{3}*\sin(2/9*\pi)^2 - 2*\cos(2/9*\pi)$
 $*\sin(2/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(2/9*\pi) - 2*x)/((\sqrt{3}*i + 1)*$
 $\sin(2/9*\pi))) + 1/9*(2*\sqrt{3}*\cos(1/9*\pi)^5 - 20*\sqrt{3}*\cos(1/9*\pi)^3*\sin$
 $(1/9*\pi)^2 + 10*\sqrt{3}*\cos(1/9*\pi)*\sin(1/9*\pi)^4 + 10*\cos(1/9*\pi)^4*\sin(1/$

$9\pi) - 20\cos(1/9\pi)^2\sin(1/9\pi)^3 + 2\sin(1/9\pi)^5 - \sqrt{3}\cos(1/9\pi)^2 + \sqrt{3}\sin(1/9\pi)^2 - 2\cos(1/9\pi)\sin(1/9\pi))\arctan(((\sqrt{3}) * i + 1)\cos(1/9\pi) + 2x)/((\sqrt{3}) * i + 1)\sin(1/9\pi))) - 1/18*(10\sqrt{3}) * \cos(4/9\pi)^4\sin(4/9\pi) - 20\sqrt{3}\cos(4/9\pi)^2\sin(4/9\pi)^3 + 2\sqrt{3}\sin(4/9\pi)^5 + 2\cos(4/9\pi)^5 - 20\cos(4/9\pi)^3\sin(4/9\pi)^2 + 10\cos(4/9\pi)\sin(4/9\pi)^4 + 2\sqrt{3}\cos(4/9\pi)\sin(4/9\pi) + \cos(4/9\pi)^2 - \sin(4/9\pi)^2)\log(-(\sqrt{3}) * i\cos(4/9\pi) + \cos(4/9\pi)) * x + x^2 + 1) - 1/18*(10\sqrt{3}) * \cos(2/9\pi)^4\sin(2/9\pi) - 20\sqrt{3}\cos(2/9\pi)^2\sin(2/9\pi)^3 + 2\sqrt{3}\sin(2/9\pi)^5 + 2\cos(2/9\pi)^5 - 20\cos(2/9\pi)^3\sin(2/9\pi)^2 + 10\cos(2/9\pi)\sin(2/9\pi)^4 + 2\sqrt{3}\cos(2/9\pi)\sin(2/9\pi) + \cos(2/9\pi)^2 - \sin(2/9\pi)^2)\log(-(\sqrt{3}) * i\cos(2/9\pi) + \cos(2/9\pi)) * x + x^2 + 1) - 1/18*(10\sqrt{3}) * \cos(1/9\pi)^4\sin(1/9\pi) - 20\sqrt{3}\cos(1/9\pi)^2\sin(1/9\pi)^3 + 2\sqrt{3}\sin(1/9\pi)^5 - 2\cos(1/9\pi)^5 + 20\cos(1/9\pi)^3\sin(1/9\pi)^2 - 10\cos(1/9\pi)\sin(1/9\pi)^4 - 2\sqrt{3}\cos(1/9\pi)\sin(1/9\pi) + \cos(1/9\pi)^2 - \sin(1/9\pi)^2)\log((\sqrt{3}) * i\cos(1/9\pi) + \cos(1/9\pi)) * x + x^2 + 1)$

maple [C] time = 0.01, size = 40, normalized size = 0.10

$$\frac{\text{RootOf}(-Z^6 - Z^3 + 1)^4 \ln(-\text{RootOf}(-Z^6 - Z^3 + 1) + x)}{6 \text{RootOf}(-Z^6 - Z^3 + 1)^5 - 3 \text{RootOf}(-Z^6 - Z^3 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6-x^3+1), x)

[Out] 1/3*sum(_R^4/(2*_R^5-_R^2)*ln(-_R+x), _R=RootOf(-Z^6-Z^3+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6-x^3+1), x, algorithm="maxima")

[Out] integrate(x^4/(x^6 - x^3 + 1), x)

mupad [B] time = 1.72, size = 304, normalized size = 0.74

$$\frac{\ln\left(x \left(122 + \frac{2\sqrt{3}\sqrt{13}}{13}\right) \left(\frac{1}{122} + \frac{\sqrt{13}}{122}\right)\right) (-36 + \sqrt{13})^{1/3}}{3} - \frac{\ln\left(x \left(122 + \frac{2\sqrt{3}\sqrt{13}}{13}\right) \left(\frac{1}{122} + \frac{\sqrt{13}}{122}\right)\right) (-36 - \sqrt{13})^{1/3}}{3} + \frac{20 \ln\left(x \left(\frac{2\sqrt{3}\sqrt{13}\sqrt{13}}{13} + \frac{2\sqrt{3}\sqrt{13}\sqrt{13}}{13}\right) (-3 - \sqrt{13})^{1/3}\right) (10 + 3\sqrt{13})}{36} + \frac{20 \ln\left(x \left(\frac{2\sqrt{3}\sqrt{13}\sqrt{13}}{13} + \frac{2\sqrt{3}\sqrt{13}\sqrt{13}}{13}\right) (-3 + \sqrt{13})^{1/3}\right) (10 - 3\sqrt{13})}{36} + \frac{20 \ln\left(x \left(\frac{2\sqrt{3}\sqrt{13}\sqrt{13}}{13} - \frac{2\sqrt{3}\sqrt{13}\sqrt{13}}{13}\right) (-3 - \sqrt{13})^{1/3}\right) (10 + 3\sqrt{13})}{36} + \frac{20 \ln\left(x \left(\frac{2\sqrt{3}\sqrt{13}\sqrt{13}}{13} - \frac{2\sqrt{3}\sqrt{13}\sqrt{13}}{13}\right) (-3 + \sqrt{13})^{1/3}\right) (10 + 3\sqrt{13})}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6 - x^3 + 1), x)


```
[Out] (log(x + (162*x + (27*(3^(1/2)*12i - 36)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/16
2))*3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (162*x + (27*(- 3^(1/2)*12i - 3
6)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(- 3^(1/2)*12i - 36)^(1/3))/18 - (
2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(- 3^(1/2)*1i - 3)^(2/3))/12 + (2^(1/3)*3^
(1/6)*(- 3^(1/2)*1i - 3)^(2/3)*1i)/4)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3
^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/
12 - (2^(1/3)*3^(1/6)*(3^(1/2)*1i - 3)^(2/3)*1i)/4)*(3^(1/2)*1i - 3)^(1/3)*
(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(- 3^(1/2)*1
i - 3)^(2/3))/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(
2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*1i - 3)^(
1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [A] time = 0.18, size = 26, normalized size = 0.06

$$\text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log(6561t^5 + 54t^2 + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t**
2 + x)))
```

$$3.160 \quad \int \frac{x^3}{1-x^3+x^6} dx$$

Optimal. Leaf size=411

$$\frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$\frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(\sqrt{3}+i) \tan^{-1}\left(\frac{1+\sqrt[3]{\frac{2}{3}(1-i\sqrt{3})}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(-\sqrt{3}+i) \tan^{-1}\left(\frac{1+\sqrt[3]{\frac{2}{3}(1+i\sqrt{3})}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 - x^3 + x^6), x]

[Out] $-\left(\left(I + \text{Sqrt}[3]\right) \text{ArcTan}\left[\frac{1 + (2x)}{\left(1 - I \text{Sqrt}[3]\right)^{1/3}}\right] / \text{Sqrt}[3]\right) / \left(3 \cdot 2^{1/3} \cdot \left(1 - I \text{Sqrt}[3]\right)^{2/3}\right) + \left(\left(I - \text{Sqrt}[3]\right) \text{ArcTan}\left[\frac{1 + (2x)}{\left(1 + I \text{Sqrt}[3]\right)^{1/3}}\right] / \text{Sqrt}[3]\right) / \left(3 \cdot 2^{1/3} \cdot \left(1 + I \text{Sqrt}[3]\right)^{2/3}\right) + \left(\left(3 + I \text{Sqrt}[3]\right) \text{Log}\left[\frac{\left(1 - I \text{Sqrt}[3]\right)^{1/3} - 2^{1/3}x}{\left(9 \cdot 2^{1/3} \cdot \left(1 - I \text{Sqrt}[3]\right)^{2/3}\right)}\right] + \left(\left(3 - I \text{Sqrt}[3]\right) \text{Log}\left[\frac{\left(1 + I \text{Sqrt}[3]\right)^{1/3} - 2^{1/3}x}{\left(9 \cdot 2^{1/3} \cdot \left(1 + I \text{Sqrt}[3]\right)^{2/3}\right)}\right] - \left(\left(3 + I \text{Sqrt}[3]\right) \text{Log}\left[\frac{\left(1 - I \text{Sqrt}[3]\right)^{2/3} + \left(2 \cdot \left(1 - I \text{Sqrt}[3]\right)^{1/3}x + 2^{2/3}x^2\right)}{\left(18 \cdot 2^{1/3} \cdot \left(1 - I \text{Sqrt}[3]\right)^{2/3}\right)}\right] - \left(\left(3 - I \text{Sqrt}[3]\right) \text{Log}\left[\frac{\left(1 + I \text{Sqrt}[3]\right)^{2/3} + \left(2 \cdot \left(1 + I \text{Sqrt}[3]\right)^{1/3}x + 2^{2/3}x^2\right)}{\left(18 \cdot 2^{1/3} \cdot \left(1 + I \text{Sqrt}[3]\right)^{2/3}\right)}\right]\right)$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)⁽⁻¹⁾, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_)*(x_)^m)/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1-x^3+x^6} dx &= -\left(\frac{1}{6}(-3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx\right) + \frac{1}{6}(3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3} dx \\
&= \frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}+x} dx}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}x+x^2} dx}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \int \frac{1}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x} dx}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} \\
&= \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \int \frac{1}{\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}x+x^2} dx}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} \\
&= \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} \\
&= -\frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&, \frac{\#1 \log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(1-x^3+x^6),x]

[Out] RootSum[1-#1^3+#1^6 & , (Log[x-#1]*#1)/(-1+2*#1^3) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1-x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(1 - x^3 + x^6),x]

[Out] IntegrateAlgebraic[x^3/(1 - x^3 + x^6), x]

fricas [B] time = 1.43, size = 1031, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6-x^3+1),x, algorithm="fricas")

[Out]
$$\frac{1}{54} \cdot 18^{2/3} \cdot 12^{1/6} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) \cdot \log\left(2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 18 \cdot x^2 + \frac{2}{27} \cdot 18^{2/3} \cdot 12^{1/6} \cdot \arctan\left(\frac{1}{216} \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3}) \cdot \sqrt{2} \cdot \sqrt{2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 18 \cdot x^2} - 6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x - 216 \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)\right) / \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - \frac{1}{27} \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - 18^{2/3} \cdot 12^{1/6} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)) \cdot \arctan\left(-\frac{1}{108} \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3}) \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 108 \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 108 \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 - 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x + 24 \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - \sqrt{-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 18 \cdot x^2} \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3}) \cdot \sqrt{2} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)\right) / (\cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 - 3 \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2) - \frac{1}{27} \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 18^{2/3} \cdot 12^{1/6} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)) \cdot \arctan\left(\frac{1}{108} \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3}) \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - 108 \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 - 108 \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x - 24 \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)) \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - \sqrt{-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 18 \cdot x^2} \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3}) \cdot \sqrt{2} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)\right) / (\cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 - 3 \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2) - \frac{1}{108} \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 18^{2/3} \cdot 12^{1/6} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)) \cdot \log\left(-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 18 \cdot x^2 + \frac{1}{108} \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - 18^{2/3} \cdot 12^{1/6} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)) \cdot \log\left(-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)^2 + 18 \cdot x^2\right) - 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right) - 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan(\sqrt{3} + 2)\right)$$

/6)*cos(2/3*arctan(sqrt(3) + 2))*log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)

giac [B] time = 0.57, size = 637, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6-x^3+1),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/9*(2*\sqrt{3}*\cos(4/9*\pi)^4 - 12*\sqrt{3}*\cos(4/9*\pi)^2*\sin(4/9*\pi)^2 + 2* \\ & \sqrt{3}*\sin(4/9*\pi)^4 + 8*\cos(4/9*\pi)^3*\sin(4/9*\pi) - 8*\cos(4/9*\pi)*\sin(4/9 \\ & *\pi)^3 + \sqrt{3}*\cos(4/9*\pi) + \sin(4/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(4/ \\ & 9*\pi) - 2*x)/((\sqrt{3}*i + 1)*\sin(4/9*\pi))) - 1/9*(2*\sqrt{3}*\cos(2/9*\pi)^4 \\ & - 12*\sqrt{3}*\cos(2/9*\pi)^2*\sin(2/9*\pi)^2 + 2*\sqrt{3}*\sin(2/9*\pi)^4 + 8*\cos(\\ & 2/9*\pi)^3*\sin(2/9*\pi) - 8*\cos(2/9*\pi)*\sin(2/9*\pi)^3 + \sqrt{3}*\cos(2/9*\pi) + \\ & \sin(2/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(2/9*\pi) - 2*x)/((\sqrt{3}*i + 1)* \\ & \sin(2/9*\pi))) - 1/9*(2*\sqrt{3}*\cos(1/9*\pi)^4 - 12*\sqrt{3}*\cos(1/9*\pi)^2*\sin \\ & (1/9*\pi)^2 + 2*\sqrt{3}*\sin(1/9*\pi)^4 - 8*\cos(1/9*\pi)^3*\sin(1/9*\pi) + 8*\cos(\\ & 1/9*\pi)*\sin(1/9*\pi)^3 - \sqrt{3}*\cos(1/9*\pi) + \sin(1/9*\pi))*\arctan(((\sqrt{3} \\ & *i + 1)*\cos(1/9*\pi) + 2*x)/((\sqrt{3}*i + 1)*\sin(1/9*\pi))) - 1/18*(8*\sqrt{3} \\ & *\cos(4/9*\pi)^3*\sin(4/9*\pi) - 8*\sqrt{3}*\cos(4/9*\pi)*\sin(4/9*\pi)^3 - 2*\cos(4/ \\ & 9*\pi)^4 + 12*\cos(4/9*\pi)^2*\sin(4/9*\pi)^2 - 2*\sin(4/9*\pi)^4 + \sqrt{3}*\sin(4/ \\ & 9*\pi) - \cos(4/9*\pi))*\log(-(\sqrt{3}*i*\cos(4/9*\pi) + \cos(4/9*\pi))*x + x^2 + 1 \\ &) - 1/18*(8*\sqrt{3}*\cos(2/9*\pi)^3*\sin(2/9*\pi) - 8*\sqrt{3}*\cos(2/9*\pi)*\sin(2 \\ & /9*\pi)^3 - 2*\cos(2/9*\pi)^4 + 12*\cos(2/9*\pi)^2*\sin(2/9*\pi)^2 - 2*\sin(2/9*\pi) \\ & ^4 + \sqrt{3}*\sin(2/9*\pi) - \cos(2/9*\pi))*\log(-(\sqrt{3}*i*\cos(2/9*\pi) + \cos(2 \\ & /9*\pi))*x + x^2 + 1) + 1/18*(8*\sqrt{3}*\cos(1/9*\pi)^3*\sin(1/9*\pi) - 8*\sqrt{3} \\ &)*\cos(1/9*\pi)*\sin(1/9*\pi)^3 + 2*\cos(1/9*\pi)^4 - 12*\cos(1/9*\pi)^2*\sin(1/9*\pi) \\ & ^2 + 2*\sin(1/9*\pi)^4 - \sqrt{3}*\sin(1/9*\pi) - \cos(1/9*\pi))*\log((\sqrt{3}*i*c \\ & \cos(1/9*\pi) + \cos(1/9*\pi))*x + x^2 + 1) \end{aligned}$$

maple [C] time = 0.01, size = 40, normalized size = 0.10

$$\frac{\text{RootOf}(-Z^6 - Z^3 + 1)^3 \ln(-\text{RootOf}(-Z^6 - Z^3 + 1) + x)}{6 \text{RootOf}(-Z^6 - Z^3 + 1)^5 - 3 \text{RootOf}(-Z^6 - Z^3 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^6-x^3+1),x)

[Out] $1/3*\sum(-R^3/(2*_R^5 - _R^2)*\ln(-_R+x), _R=\text{RootOf}(-Z^6 - Z^3 + 1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6-x^3+1),x, algorithm="maxima")

[Out] integrate(x^3/(x^6 - x^3 + 1), x)

mupad [B] time = 1.84, size = 327, normalized size = 0.80

$$\frac{\ln\left(\frac{e^{\frac{2\sqrt[3]{3}i-1}{3}} + \sqrt[3]{3}i}{e^{-\frac{2\sqrt[3]{3}i-1}{3}} - \sqrt[3]{3}i}\right)}{18} + \frac{\ln\left(\frac{e^{\frac{2\sqrt[3]{3}i-1}{3}} + \sqrt[3]{3}i}{e^{-\frac{2\sqrt[3]{3}i-1}{3}} - \sqrt[3]{3}i}\right)}{18} + \frac{2^{2/3} \ln\left(\frac{e^{\frac{2\sqrt[3]{3}i-1}{3}} + \sqrt[3]{3}i}{e^{-\frac{2\sqrt[3]{3}i-1}{3}} - \sqrt[3]{3}i}\right)}{36} + \frac{2^{2/3} \ln\left(\frac{e^{\frac{2\sqrt[3]{3}i-1}{3}} + \sqrt[3]{3}i}{e^{-\frac{2\sqrt[3]{3}i-1}{3}} - \sqrt[3]{3}i}\right)}{36} + \frac{2^{2/3} \ln\left(\frac{e^{\frac{2\sqrt[3]{3}i-1}{3}} + \sqrt[3]{3}i}{e^{-\frac{2\sqrt[3]{3}i-1}{3}} - \sqrt[3]{3}i}\right)}{36} + \frac{2^{2/3} \ln\left(\frac{e^{\frac{2\sqrt[3]{3}i-1}{3}} + \sqrt[3]{3}i}{e^{-\frac{2\sqrt[3]{3}i-1}{3}} - \sqrt[3]{3}i}\right)}{36} + \frac{2^{2/3} \ln\left(\frac{e^{\frac{2\sqrt[3]{3}i-1}{3}} + \sqrt[3]{3}i}{e^{-\frac{2\sqrt[3]{3}i-1}{3}} - \sqrt[3]{3}i}\right)}{36} + \frac{2^{2/3} \ln\left(\frac{e^{\frac{2\sqrt[3]{3}i-1}{3}} + \sqrt[3]{3}i}{e^{-\frac{2\sqrt[3]{3}i-1}{3}} - \sqrt[3]{3}i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^6 - x^3 + 1),x)

[Out] $(\log(x + (2^{2/3})3^{5/6})(-3^{1/2}i - 3)^{1/3}i)/6)(-3^{1/2}i - 3)^{1/3}i - 36)^{1/3})/18 + (\log(x - (2^{2/3})3^{5/6})(3^{1/2}i - 3)^{1/3}i)/6)(3^{1/2}i - 3)^{1/3}i - 36)^{1/3})/18 - (2^{2/3})\log(x + (2^{2/3})3^{1/3})(-3^{1/2}i - 3)^{1/3})/2 + (2^{2/3})3^{1/3}(-3^{1/2}i - 3)^{4/3})/12)(-3^{1/2}i - 3)^{1/3}(3^{1/3} + 3^{5/6}i)/36 - (2^{2/3})\log(x + (2^{2/3})3^{1/3})(-3^{1/2}i - 3)^{1/3})/2 + (2^{2/3})3^{1/3}(3^{1/2}i - 3)^{4/3})/12)(3^{1/2}i - 3)^{1/3}(3^{1/3} - 3^{5/6}i)/36 - (2^{2/3})\log(x - (2^{2/3})3^{1/3})(-3^{1/2}i - 3)^{1/3})/4 - (2^{2/3})3^{5/6}(-3^{1/2}i - 3)^{1/3}i)/12)(-3^{1/2}i - 3)^{1/3}(3^{1/3} - 3^{5/6}i)/36 - (2^{2/3})\log(x - (2^{2/3})3^{1/3})(3^{1/2}i - 3)^{1/3})/4 + (2^{2/3})3^{5/6}(3^{1/2}i - 3)^{1/3}i)/12)(3^{1/2}i - 3)^{1/3}(3^{1/3} + 3^{5/6}i)/36$

sympy [A] time = 0.18, size = 24, normalized size = 0.06

$$\text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log(-1458t^4 - 9t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**6-x**3+1),x)

[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t + x)))

$$3.161 \quad \int \frac{x^2}{1-x^3+x^6} dx$$

Optimal. Leaf size=23

$$-\frac{2 \tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1352, 618, 204}

$$-\frac{2 \tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 - x^3 + x^6),x]

[Out] (-2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{1-x^3+x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) \\ &= - \left(\frac{2}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \right) \\ &= - \frac{2 \tan^{-1} \left(\frac{1-2x^3}{\sqrt{3}} \right)}{3\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{2x^3-1}{\sqrt{3}} \right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 - x^3 + x^6), x]

[Out] (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1-x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1 - x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^2/(1 - x^3 + x^6), x]

fricas [A] time = 1.13, size = 18, normalized size = 0.78

$$\frac{2}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^3 - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6-x^3+1), x, algorithm="fricas")

[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))

giac [A] time = 0.49, size = 18, normalized size = 0.78

$$\frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6-x^3+1),x, algorithm="giac")

[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{2\sqrt{3} \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^6-x^3+1),x)

[Out] 2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))

maxima [A] time = 1.14, size = 18, normalized size = 0.78

$$\frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6-x^3+1),x, algorithm="maxima")

[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))

mupad [B] time = 1.22, size = 20, normalized size = 0.87

$$\frac{2\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^6 - x^3 + 1),x)

[Out] -(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9

sympy [A] time = 0.12, size = 27, normalized size = 1.17

$$\frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**6-x**3+1),x)

[Out] 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9

$$3.162 \quad \int \frac{x}{1-x^3+x^6} dx$$

Optimal. Leaf size=375

$$\frac{i \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt[3]{1-i\sqrt{3}}} + \frac{i \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt[3]{1+i\sqrt{3}}} + \frac{i \log\left(-\sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt[3]{\frac{1}{2}}}$$

Rubi [A] time = 0.25, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1375, 292, 31, 634, 617, 204, 628}

$$\frac{i \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt[3]{1-i\sqrt{3}}} + \frac{i \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt[3]{1+i\sqrt{3}}} + \frac{i \log\left(-\sqrt[3]{2}x + \sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \log\left(-\sqrt[3]{2}x + \sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}} + \frac{i \tan^{-1}\left(\frac{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \tan^{-1}\left(\frac{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}$$

Antiderivative was successfully verified.

[In] Int[x/(1 - x^3 + x^6), x]

[Out] ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 - I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_)*(x_)^(m_))/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symb
ol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1-x^3+x^6} dx &= -\frac{i \int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx}{\sqrt{3}} + \frac{i \int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3} dx}{\sqrt{3}} \\
&= \frac{i \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x} dx}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \int \frac{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x+x^2} dx}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}+x} dx}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}} + \frac{i \int \frac{1}{\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}x+x^2} dx}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}} \\
&= \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}} + \frac{i \int \frac{1}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x+x^2} dx}{2\sqrt{3}} \\
&= \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}} - \frac{i \log\left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2(1-i\sqrt{3})}\right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt[3]{1-i\sqrt{3}}} \\
&= \frac{i \tan^{-1}\left(\frac{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} + \frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \tan^{-1}\left(\frac{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} + \frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}} + \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.11

$$\frac{1}{3} \text{RootSum}\left[\#1^6 - \#1^3 + 1 \&, \frac{\log(x - \#1)}{2\#1^4 - \#1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 - x^3 + x^6), x]

[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1-x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(1 - x^3 + x^6),x]

[Out] IntegrateAlgebraic[x/(1 - x^3 + x^6), x]

fricas [B] time = 1.50, size = 1583, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6-x^3+1),x, algorithm="fricas")

[Out]
$$\frac{1}{54} \cdot 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \log(18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) - 2/27 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \arctan(1/108 \cdot (6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 864 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^3 - 6 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x - 36 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 12 \cdot (18^{2/3} \cdot 12^{2/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 72 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - \sqrt{18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) / (3 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 10 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 1/27 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 18^{2/3} \cdot 12^{1/6} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \arctan(1/108 \cdot (6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 864 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^3 - 6 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x - 36 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 12 \cdot (18^{2/3} \cdot 12^{2/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 72 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - \sqrt{18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \cos($$

$$\begin{aligned} & \frac{2}{3} \arctan(\sqrt{3} - 2)^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \sin(2/3 \arctan(\sqrt{3} - 2))^2 - 2 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \arctan(\sqrt{3} - 2)) / (3 \cos(2/3 \arctan(\sqrt{3} - 2))^4 - 10 \cos(2/3 \arctan(\sqrt{3} - 2))^2 \sin(2/3 \arctan(\sqrt{3} - 2))^2 + 3 \sin(2/3 \arctan(\sqrt{3} - 2))^4) \\ & + 1/27 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos(2/3 \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot \sin(2/3 \arctan(\sqrt{3} - 2))) \cdot \arctan(-1/432 \cdot (6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot x - 216 \cos(2/3 \arctan(\sqrt{3} - 2))^2 + 216 \sin(2/3 \arctan(\sqrt{3} - 2))^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \cos(2/3 \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \arctan(\sqrt{3} - 2))^2 + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) / (\cos(2/3 \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \arctan(\sqrt{3} - 2)))) + 1/108 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin(2/3 \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \arctan(\sqrt{3} - 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \arctan(\sqrt{3} - 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) - 1/108 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin(2/3 \arctan(\sqrt{3} - 2)) + 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \arctan(\sqrt{3} - 2))^2 + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) \end{aligned}$$

giac [B] time = 0.57, size = 812, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6-x^3+1),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/9 \cdot (\sqrt{3} \cdot \cos(4/9 \pi))^5 - 10 \sqrt{3} \cdot \cos(4/9 \pi)^3 \cdot \sin(4/9 \pi)^2 + 5 \sqrt{3} \cdot \cos(4/9 \pi) \cdot \sin(4/9 \pi)^4 - 5 \cos(4/9 \pi)^4 \cdot \sin(4/9 \pi) + 10 \cos(4/9 \pi)^2 \cdot \sin(4/9 \pi)^3 - \sin(4/9 \pi)^5 - \sqrt{3} \cdot \cos(4/9 \pi)^2 + \sqrt{3} \cdot \sin(4/9 \pi)^2 + 2 \cos(4/9 \pi) \cdot \sin(4/9 \pi) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(4/9 \pi) - 2x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(4/9 \pi))) - 1/9 \cdot (\sqrt{3} \cdot \cos(2/9 \pi))^5 - 10 \sqrt{3} \cdot \cos(2/9 \pi)^3 \cdot \sin(2/9 \pi)^2 + 5 \sqrt{3} \cdot \cos(2/9 \pi) \cdot \sin(2/9 \pi)^4 - 5 \cos(2/9 \pi)^4 \cdot \sin(2/9 \pi) + 10 \cos(2/9 \pi)^2 \cdot \sin(2/9 \pi)^3 - \sin(2/9 \pi)^5 - \sqrt{3} \cdot \cos(2/9 \pi)^2 + \sqrt{3} \cdot \sin(2/9 \pi)^2 + 2 \cos(2/9 \pi) \cdot \sin(2/9 \pi) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(2/9 \pi) - 2x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(2/9 \pi))) + 1/9 \cdot (\sqrt{3} \cdot \cos(1/9 \pi))^5 - 10 \sqrt{3} \cdot \cos(1/9 \pi)^3 \cdot \sin(1/9 \pi)^2 + 5 \sqrt{3} \cdot \cos(1/9 \pi) \cdot \sin(1/9 \pi)^4 + 5 \cos(1/9 \pi)^4 \cdot \sin(1/9 \pi) - 10 \cos(1/9 \pi)^2 \cdot \sin(1/9 \pi)^3 + \sin(1/9 \pi)^5 + \sqrt{3} \cdot \cos(1/9 \pi)^2 - \sqrt{3} \cdot \sin(1/9 \pi)^2 + 2 \cos(1/9 \pi) \cdot \sin(1/9 \pi) \cdot \arctan(((\sqrt{3} \cdot i + 1) \cdot \cos(1/9 \pi) \end{aligned}$$

) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) - cos(2/9*pi)^2 + sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) - cos(1/9*pi)^2 + sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)

maple [C] time = 0.01, size = 38, normalized size = 0.10

$$\frac{\text{RootOf}(-Z^6 - Z^3 + 1) \ln(-\text{RootOf}(-Z^6 - Z^3 + 1) + x)}{6 \text{RootOf}(-Z^6 - Z^3 + 1)^5 - 3 \text{RootOf}(-Z^6 - Z^3 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^6-x^3+1),x)

[Out] 1/3*sum(_R/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(-Z^6-Z^3+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6-x^3+1),x, algorithm="maxima")

[Out] integrate(x/(x^6 - x^3 + 1), x)

mupad [B] time = 0.45, size = 304, normalized size = 0.81

$$\frac{\ln\left(x + \frac{2(36 - \sqrt{12})}{4}\right) \left(\frac{1}{36} + \frac{\sqrt{12}}{36}\right) (36 + \sqrt{12})^{1/3}}{18} + \frac{\ln\left(x - \frac{2(36 - \sqrt{12})}{4}\right) \left(\frac{1}{36} + \frac{\sqrt{12}}{36}\right) (36 + \sqrt{12})^{1/3}}{18} + \frac{2^{2/3} \ln\left(x + \frac{2^{2/3}(36 - \sqrt{12})}{4}\right) \left(\frac{1}{36} + \frac{\sqrt{12}}{36}\right) (36 + \sqrt{12})^{1/3}}{36} + \frac{2^{2/3} \ln\left(x - \frac{2^{2/3}(36 - \sqrt{12})}{4}\right) \left(\frac{1}{36} + \frac{\sqrt{12}}{36}\right) (36 + \sqrt{12})^{1/3}}{36} + \frac{2^{2/3} \ln\left(x + \frac{2^{2/3}(36 + \sqrt{12})}{4}\right) \left(\frac{1}{36} + \frac{\sqrt{12}}{36}\right) (36 + \sqrt{12})^{1/3}}{36} + \frac{2^{2/3} \ln\left(x - \frac{2^{2/3}(36 + \sqrt{12})}{4}\right) \left(\frac{1}{36} + \frac{\sqrt{12}}{36}\right) (36 + \sqrt{12})^{1/3}}{36} + \frac{2^{2/3} \ln\left(x + \frac{2^{2/3}(36 + \sqrt{12})}{4}\right) \left(\frac{1}{36} + \frac{\sqrt{12}}{36}\right) (36 + \sqrt{12})^{1/3}}{36} + \frac{2^{2/3} \ln\left(x - \frac{2^{2/3}(36 + \sqrt{12})}{4}\right) \left(\frac{1}{36} + \frac{\sqrt{12}}{36}\right) (36 + \sqrt{12})^{1/3}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^6 - x^3 + 1),x)

[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i) + 36)^(

$$\begin{aligned} & \frac{2/3}{4}) * ((3^{1/2} * 1i) / 486 + 1/162)) * (3^{1/2} * 12i + 36)^{1/3} / 18 - (2^{2/3} \\ &) * \log(x + (2^{1/3} * 3^{2/3} * (3 - 3^{1/2} * 1i)^{2/3})) / 12 + (2^{1/3} * 3^{1/6} * (3 \\ & - 3^{1/2} * 1i)^{2/3} * 1i) / 4 * (3 - 3^{1/2} * 1i)^{1/3} * (3^{1/3} + 3^{5/6} * 1i)) / \\ & 36 - (2^{2/3} * \log(x + (2^{1/3} * 3^{2/3} * (3^{1/2} * 1i + 3)^{2/3})) / 12 - (2^{1/3} \\ &) * 3^{1/6} * (3^{1/2} * 1i + 3)^{2/3} * 1i) / 4 * (3^{1/2} * 1i + 3)^{1/3} * (3^{1/3} - 3 \\ & ^{5/6} * 1i)) / 36 - (2^{2/3} * \log(x - (2^{1/3} * 3^{2/3} * (3 - 3^{1/2} * 1i)^{2/3})) / \\ & 6) * (3 - 3^{1/2} * 1i)^{1/3} * (3^{1/3} - 3^{5/6} * 1i)) / 36 - (2^{2/3} * \log(x - (2^{1/3} * 3^{2/3} * \\ & (3^{1/2} * 1i + 3)^{2/3})) / 6) * (3^{1/2} * 1i + 3)^{1/3} * (3^{1/3} + \\ & 3^{5/6} * 1i)) / 36 \end{aligned}$$

sympy [A] time = 0.18, size = 26, normalized size = 0.07

$$\text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log(6561t^5 - 27t^2 + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**6-x**3+1),x)

[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 - 27*_t**2 + x)))

$$3.163 \quad \int \frac{1}{1-x^3+x^6} dx$$

Optimal. Leaf size=186

$$\frac{(-1)^{5/18} (3 \log(\sqrt[9]{-1} - x) + \log(2))}{9\sqrt{3}} + \frac{(-1)^{13/18} \log(-\sqrt[3]{2} (x + (-1)^{8/9}))}{3\sqrt{3}} - \frac{(-1)^{13/18} \log(-2^{2/3} ((-1)^{8/9} - x) x + \dots)}{6\sqrt{3}}$$

Rubi [C] time = 0.24, antiderivative size = 375, normalized size of antiderivative = 2.02, number of steps used = 13, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$\frac{i \log(2^{2/3} x^2 + \sqrt[3]{2(1-i\sqrt{3})} x + (1-i\sqrt{3})^{2/3})}{3\sqrt[3]{2}\sqrt{3}(1-i\sqrt{3})^{2/3}} + \frac{i \log(2^{2/3} x^2 + \sqrt[3]{2(1+i\sqrt{3})} x + (1+i\sqrt{3})^{2/3})}{3\sqrt[3]{2}\sqrt{3}(1+i\sqrt{3})^{2/3}} + \frac{i \log(-\sqrt[3]{2} x + \sqrt[3]{1-i\sqrt{3}})}{3\sqrt{3}(\frac{1}{2}(1-i\sqrt{3}))^{2/3}} - \frac{i \log(-\sqrt[3]{2} x + \sqrt[3]{1+i\sqrt{3}})}{3\sqrt{3}(\frac{1}{2}(1+i\sqrt{3}))^{2/3}} - \frac{i \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3(\frac{1}{2}(1-i\sqrt{3}))^{2/3}} + \frac{i \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3(\frac{1}{2}(1+i\sqrt{3}))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3 + x^6)^(-1), x]

[Out] ((-I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/((Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 - I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))(-1), x_Symbol] := With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-x^3+x^6} dx &= \frac{i \int \frac{1}{-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^3} dx}{\sqrt{3}} + \frac{i \int \frac{1}{-\frac{1}{2} + \frac{i\sqrt{3}}{2} + x^3} dx}{\sqrt{3}} \\
&= \frac{i \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} + x} dx}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} + \frac{i \int \frac{-2^{2/3} \sqrt[3]{1-i\sqrt{3}} - x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} x + x^2} dx}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} + x} dx}{3\sqrt{3} \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} - \frac{i \int \frac{1}{\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} x + x^2} dx}{3\sqrt{3} \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} \\
&= \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} - \frac{i \int \frac{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} + 2x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} x + x^2} dx}{3\sqrt[3]{2} \sqrt{3} (1-i\sqrt{3})^{2/3}} \\
&= \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} - \frac{i \log\left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2}(1-i\sqrt{3})\right)}{3\sqrt[3]{2} \sqrt{3} (1-i\sqrt{3})^{2/3}} \\
&= \frac{i \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3 \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} + \frac{i \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3 \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} + \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.23

$$\frac{1}{3} \text{RootSum}\left[\#1^6 - \#1^3 + 1 \&, \frac{\log(x - \#1)}{2\#1^5 - \#1^2} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3 + x^6)^(-1), x]

[Out] RootSum[1 - #1^3 + #1^6 &, Log[x - #1]/(-#1^2 + 2*#1^5) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^3 + x^6)^(-1),x]

[Out] IntegrateAlgebraic[(1 - x^3 + x^6)^(-1), x]

fricas [B] time = 1.29, size = 1027, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-x^3+1),x, algorithm="fricas")

[Out]
$$\frac{1}{54} \cdot 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \log(18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2 - 2/27 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \arctan(1/108 \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3}) \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) + 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x + 24 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - \sqrt{18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))} + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2 \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)))) / (\cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 1/27 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 18^{2/3} \cdot 12^{1/6} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \arctan(-1/108 \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3}) \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) - 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x - 24 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - \sqrt{18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))} - 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2 \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) - 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)))) / (\cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2) - 1/27 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \arctan(1/216 \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3}) \cdot \sqrt{2} \cdot \sqrt{-2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))} + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2) - 6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x + 216 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) / \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 1/108 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2 - 1/108 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3}$$

$(3)\sin(2/3\arctan(\sqrt{3} - 2)) + 18^{(2/3)}12^{(1/6)}\cos(2/3\arctan(\sqrt{3} - 2))\log(-2\cdot 18^{(2/3)}12^{(1/6)}\sqrt{3}\cdot x\sin(2/3\arctan(\sqrt{3} - 2)) + 3\cdot 18^{(1/3)}12^{(1/3)}\cos(2/3\arctan(\sqrt{3} - 2))^2 + 3\cdot 18^{(1/3)}12^{(1/3)}\sin(2/3\arctan(\sqrt{3} - 2))^2 + 18x^2)$

giac [B] time = 0.50, size = 629, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-x^3+1),x, algorithm="giac")

[Out] $-1/9(\sqrt{3}\cos(4/9\pi)^4 - 6\sqrt{3}\cos(4/9\pi)^2\sin(4/9\pi)^2 + \sqrt{3}\sin(4/9\pi)^4 + 4\cos(4/9\pi)^3\sin(4/9\pi) - 4\cos(4/9\pi)\sin(4/9\pi)^3 - \sqrt{3}\cos(4/9\pi) - \sin(4/9\pi))\arctan(-((\sqrt{3}i + 1)\cos(4/9\pi) - 2x)/((\sqrt{3}i + 1)\sin(4/9\pi))) - 1/9(\sqrt{3}\cos(2/9\pi)^4 - 6\sqrt{3}\cos(2/9\pi)^2\sin(2/9\pi)^2 + \sqrt{3}\sin(2/9\pi)^4 + 4\cos(2/9\pi)^3\sin(2/9\pi) - 4\cos(2/9\pi)\sin(2/9\pi)^3 - \sqrt{3}\cos(2/9\pi) - \sin(2/9\pi))\arctan(-((\sqrt{3}i + 1)\cos(2/9\pi) - 2x)/((\sqrt{3}i + 1)\sin(2/9\pi))) - 1/9(\sqrt{3}\cos(1/9\pi)^4 - 6\sqrt{3}\cos(1/9\pi)^2\sin(1/9\pi)^2 + \sqrt{3}\sin(1/9\pi)^4 - 4\cos(1/9\pi)^3\sin(1/9\pi) + 4\cos(1/9\pi)\sin(1/9\pi)^3 + \sqrt{3}\cos(1/9\pi) - \sin(1/9\pi))\arctan((\sqrt{3}i + 1)\cos(1/9\pi) + 2x)/((\sqrt{3}i + 1)\sin(1/9\pi))) - 1/18(4\sqrt{3}\cos(4/9\pi)^3\sin(4/9\pi) - 4\sqrt{3}\cos(4/9\pi)\sin(4/9\pi)^3 - \cos(4/9\pi)^4 + 6\cos(4/9\pi)^2\sin(4/9\pi)^2 - \sin(4/9\pi)^4 - \sqrt{3}\sin(4/9\pi) + \cos(4/9\pi))\log(-(\sqrt{3}i\cos(4/9\pi) + \cos(4/9\pi))x + x^2 + 1) - 1/18(4\sqrt{3}\cos(2/9\pi)^3\sin(2/9\pi) - 4\sqrt{3}\cos(2/9\pi)\sin(2/9\pi)^3 - \cos(2/9\pi)^4 + 6\cos(2/9\pi)^2\sin(2/9\pi)^2 - \sin(2/9\pi)^4 - \sqrt{3}\sin(2/9\pi) + \cos(2/9\pi))\log(-(\sqrt{3}i\cos(2/9\pi) + \cos(2/9\pi))x + x^2 + 1) + 1/18(4\sqrt{3}\cos(1/9\pi)^3\sin(1/9\pi) - 4\sqrt{3}\cos(1/9\pi)\sin(1/9\pi)^3 + \cos(1/9\pi)^4 - 6\cos(1/9\pi)^2\sin(1/9\pi)^2 + \sin(1/9\pi)^4 + \sqrt{3}\sin(1/9\pi) + \cos(1/9\pi))\log((\sqrt{3}i\cos(1/9\pi) + \cos(1/9\pi))x + x^2 + 1)$

maple [C] time = 0.01, size = 37, normalized size = 0.20

$$\frac{\ln(-\text{RootOf}(_Z^6 - _Z^3 + 1) + x)}{6\text{RootOf}(_Z^6 - _Z^3 + 1)^5 - 3\text{RootOf}(_Z^6 - _Z^3 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6-x^3+1),x)

[Out] $1/3\sum(1/(2\cdot R^5 - R^2)\ln(-R+x), R=\text{RootOf}(_Z^6 - _Z^3 + 1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-x^3+1),x, algorithm="maxima")

[Out] integrate(1/(x^6 - x^3 + 1), x)

mupad [B] time = 1.79, size = 327, normalized size = 1.76

$$\frac{\ln\left(x + \frac{2^{2/3} \sqrt{3} \sqrt{3x^2 - 2x + 1}}{12}\right) (36 - \sqrt{12})^{1/3}}{18} + \frac{\ln\left(x + \frac{2^{2/3} \sqrt{3} \sqrt{3x^2 - 2x + 1}}{12}\right) (36 + \sqrt{12})^{1/3}}{18} - \frac{2^{2/3} \ln\left(x + \frac{2^{2/3} \sqrt{3} \sqrt{3x^2 - 2x + 1}}{12}\right) (3 - \sqrt{3})^{1/3} (3^{1/3} + 3^{5/6})}{36} - \frac{2^{2/3} \ln\left(x + \frac{2^{2/3} \sqrt{3} \sqrt{3x^2 - 2x + 1}}{12}\right) (3 + \sqrt{3})^{1/3} (3^{1/3} - 3^{5/6})}{36} - \frac{2^{2/3} \ln\left(x + \frac{2^{2/3} \sqrt{3} \sqrt{3x^2 - 2x + 1}}{12}\right) (3 - \sqrt{3})^{1/3} (3^{1/3} - 3^{5/6})}{36} - \frac{2^{2/3} \ln\left(x + \frac{2^{2/3} \sqrt{3} \sqrt{3x^2 - 2x + 1}}{12}\right) (3 + \sqrt{3})^{1/3} (3^{1/3} + 3^{5/6})}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6 - x^3 + 1),x)

[Out] $(\log(x + (2^{(2/3)}*3^{(1/3)}*(3 - 3^{(1/2)}*1i)^{(1/3)}))/4 - (2^{(2/3)}*3^{(5/6)}*(3 - 3^{(1/2)}*1i)^{(1/3)}*1i)/12)*(36 - 3^{(1/2)}*12i)^{(1/3)}/18 + (\log(x + (2^{(2/3)}*3^{(1/3)}*(3^{(1/2)}*1i + 3)^{(1/3)}))/4 + (2^{(2/3)}*3^{(5/6)}*(3^{(1/2)}*1i + 3)^{(1/3)}*1i)/12)*(3^{(1/2)}*12i + 36)^{(1/3)}/18 - (2^{(2/3)}*\log(x - (2^{(2/3)}*3^{(1/3)}*(3 - 3^{(1/2)}*1i)^{(1/3)}))/2 + (2^{(2/3)}*3^{(1/3)}*(3 - 3^{(1/2)}*1i)^{(4/3)}))/12)*(3 - 3^{(1/2)}*1i)^{(1/3)}*(3^{(1/3)} + 3^{(5/6)}*1i))/36 - (2^{(2/3)}*\log(x - (2^{(2/3)}*3^{(1/3)}*(3^{(1/2)}*1i + 3)^{(1/3)}))/2 + (2^{(2/3)}*3^{(1/3)}*(3^{(1/2)}*1i + 3)^{(4/3)}))/12)*(3^{(1/2)}*1i + 3)^{(1/3)}*(3^{(1/3)} - 3^{(5/6)}*1i))/36 - (2^{(2/3)}*\log(x + (2^{(2/3)}*3^{(5/6)}*(3 - 3^{(1/2)}*1i)^{(1/3)}*1i)/6)*(3 - 3^{(1/2)}*1i)^{(1/3)}*(3^{(1/3)} - 3^{(5/6)}*1i))/36 - (2^{(2/3)}*\log(x - (2^{(2/3)}*3^{(5/6)}*(3^{(1/2)}*1i + 3)^{(1/3)}*1i)/6)*(3^{(1/2)}*1i + 3)^{(1/3)}*(3^{(1/3)} + 3^{(5/6)}*1i))/36$

sympy [A] time = 0.18, size = 20, normalized size = 0.11

$$\text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log(729t^4 + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6-x**3+1),x)

[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)))

$$3.164 \quad \int \frac{1}{x(1-x^3+x^6)} dx$$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(x)$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{6} \log(x^6 - x^3 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^3 + x^6)),x]

[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(1-x^3+x^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(1-x+x^2)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x} dx, x, x^3 \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1-x}{1-x+x^2} dx, x, x^3 \right) \\
 &= \log(x) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) - \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3 \right) \\
 &= \log(x) - \frac{1}{6} \log(1-x^3+x^6) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\
 &= -\frac{\tan^{-1} \left(\frac{1-2x^3}{\sqrt{3}} \right)}{3\sqrt{3}} + \log(x) - \frac{1}{6} \log(1-x^3+x^6)
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.34

$$\log(x) - \frac{1}{3} \text{RootSum} \left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^3 \log(x - \#1) - \log(x - \#1)}{2\#1^3 - 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - x^3 + x^6)),x]

[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-1 + 2*#1^3) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1 - x^3 + x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(1 - x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x*(1 - x^3 + x^6)), x]

fricas [A] time = 1.23, size = 34, normalized size = 0.83

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6-x^3+1),x, algorithm="fricas")

[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(x)

giac [A] time = 0.35, size = 35, normalized size = 0.85

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6-x^3+1),x, algorithm="giac")

[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(abs(x))

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9} + \ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(x^6-x^3+1),x)`

[Out] $-1/6*\ln(x^6-x^3+1)+1/9*3^{(1/2)}*\arctan(1/3*(2*x^3-1)*3^{(1/2)})+\ln(x)$

maxima [A] time = 1.45, size = 38, normalized size = 0.93

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{6}\log(x^6-x^3+1)+\frac{1}{3}\log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x^6-x^3+1),x, algorithm="maxima")`

[Out] $1/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^3-1))-1/6*\log(x^6-x^3+1)+1/3*\log(x^3)$

mupad [B] time = 1.23, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x^6-x^3+1)),x)`

[Out] $\log(x)-\log(x^6-x^3+1)/6-(3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3-(2*3^{(1/2)}*x^3)/3))/9$

sympy [A] time = 0.15, size = 41, normalized size = 1.00

$$\log(x)-\frac{\log(x^6-x^3+1)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x**6-x**3+1),x)`

[Out] $\log(x)-\log(x**6-x**3+1)/6+\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x**3/3-\sqrt{3}/3)/9$

$$3.165 \quad \int \frac{1}{x^2(1-x^3+x^6)} dx$$

Optimal. Leaf size=416

$$\frac{(3 - i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1 - i\sqrt{3})}x + (1 - i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1 - i\sqrt{3}}} + \frac{(3 + i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1 + i\sqrt{3})}x + (1 + i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1 + i\sqrt{3}}}$$

Rubi [A] time = 0.30, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1368, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{(3 - i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1 - i\sqrt{3})}x + (1 - i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1 - i\sqrt{3}}} + \frac{(3 + i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1 + i\sqrt{3})}x + (1 + i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1 + i\sqrt{3}}} - \frac{1}{x} - \frac{(3 - i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1 - i\sqrt{3}}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1 - i\sqrt{3}}} - \frac{(3 + i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1 + i\sqrt{3}}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1 + i\sqrt{3}}} + \frac{(-\sqrt{3} + i) \tan^{-1}\left(\frac{1 + \sqrt[3]{2(1 - i\sqrt{3})}}{\sqrt[3]{1 - i\sqrt{3}}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1 - i\sqrt{3}}} - \frac{(\sqrt{3} + i) \tan^{-1}\left(\frac{1 + \sqrt[3]{2(1 + i\sqrt{3})}}{\sqrt[3]{1 + i\sqrt{3}}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1 + i\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 - x^3 + x^6)), x]

[Out] $-x^{-1} + \frac{(1 - \sqrt{3}) \operatorname{ArcTan}\left[\frac{1 + (2x)}{\left(\frac{1 - \sqrt{3}}{2}\right)^{1/3}}\right]}{\sqrt{3}} \frac{1}{(3 \cdot 2^{2/3} (1 - \sqrt{3}))^{1/3}} - \frac{(1 + \sqrt{3}) \operatorname{ArcTan}\left[\frac{1 + (2x)}{\left(\frac{1 + \sqrt{3}}{2}\right)^{1/3}}\right]}{\sqrt{3}} \frac{1}{(3 \cdot 2^{2/3} (1 + \sqrt{3}))^{1/3}} - \frac{(3 - \sqrt{3}) \operatorname{Log}\left[\left(\frac{1 - \sqrt{3}}{2}\right)^{1/3} - 2^{1/3}x\right]}{(9 \cdot 2^{2/3} (1 - \sqrt{3}))^{1/3}} - \frac{(3 + \sqrt{3}) \operatorname{Log}\left[\left(\frac{1 + \sqrt{3}}{2}\right)^{1/3} - 2^{1/3}x\right]}{(9 \cdot 2^{2/3} (1 + \sqrt{3}))^{1/3}} + \frac{(3 - \sqrt{3}) \operatorname{Log}\left[\left(\frac{1 - \sqrt{3}}{2}\right)^{2/3} + (2(1 - \sqrt{3}))^{1/3}x + 2^{2/3}x^2\right]}{(18 \cdot 2^{2/3} (1 - \sqrt{3}))^{1/3}} + \frac{(3 + \sqrt{3}) \operatorname{Log}\left[\left(\frac{1 + \sqrt{3}}{2}\right)^{2/3} + (2(1 + \sqrt{3}))^{1/3}x + 2^{2/3}x^2\right]}{(18 \cdot 2^{2/3} (1 + \sqrt{3}))^{1/3}}$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[((d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(1-x^3+x^6)} dx &= -\frac{1}{x} + \int \frac{x(1-x^3)}{1-x^3+x^6} dx \\
&= -\frac{1}{x} + \frac{1}{6}(-3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2} + \frac{i\sqrt{3}}{2} + x^3} dx - \frac{1}{6}(3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^3} dx \\
&= -\frac{1}{x} - \frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} + x} dx}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \int \frac{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} + x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} x + x^2} dx}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3}) \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} + x} dx}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \frac{(3+i\sqrt{3}) \int \frac{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} + x}{\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} x + x^2} dx}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} \\
&= -\frac{1}{x} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(\frac{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} + x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} x + x^2}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3}) \log\left(\frac{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} + x}{\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} x + x^2}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} \\
&= -\frac{1}{x} + \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \log\left(\frac{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} + x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} x + x^2}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3+i\sqrt{3}) \log\left(\frac{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} + x}{\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})} x + x^2}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.15

$$-\frac{1}{3} \text{RootSum}\left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^3 \log(x - \#1) - \log(x - \#1)}{2\#1^4 - \#1} \&\right] - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(1 - x^3 + x^6)),x]

[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1 + 2*#1^4) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1-x^3+x^6)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(1 - x^3 + x^6)), x]
```

fricas [B] time = 1.65, size = 1598, normalized size = 3.84
result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6)*x*arctan(-1/432*(6*18^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))*sin(2/3*arctan(sqrt(3) + 2)) + 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4))
```


$$\begin{aligned} & \tan(\sqrt{3} + 2)^3 - 6*(18^{(2/3)}*12^{(2/3)}*\sqrt{3}*x - 36*\sqrt{3}*\cos(2/3*\arctan(\sqrt{3} + 2))^2*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 12*(18^{(2/3)}*12^{(2/3)}*x*\cos(2/3*\arctan(\sqrt{3} + 2)) + 72*\cos(2/3*\arctan(\sqrt{3} + 2))^3*\sin(2/3*\arctan(\sqrt{3} + 2)) - \sqrt{18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^4 + 18^{(2/3)}*12^{(2/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^4 + 12*18^{(1/3)}*12^{(1/3)}*\sqrt{3}*x*\cos(2/3*\arctan(\sqrt{3} + 2))*\sin(2/3*\arctan(\sqrt{3} + 2)) + 6*18^{(1/3)}*12^{(1/3)}*x*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 2*(18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 - 3*18^{(1/3)}*12^{(1/3)}*x)*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 36*x^2)*(18^{(2/3)}*12^{(2/3)}*\sqrt{3}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 - 18^{(2/3)}*12^{(2/3)}*\sqrt{3}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 2*18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))*\sin(2/3*\arctan(\sqrt{3} + 2))))/(3*\cos(2/3*\arctan(\sqrt{3} + 2))^4 - 10*\cos(2/3*\arctan(\sqrt{3} + 2))^2*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 3*\sin(2/3*\arctan(\sqrt{3} + 2))^4) + (18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x*\sin(2/3*\arctan(\sqrt{3} + 2)) - 18^{(2/3)}*12^{(1/6)}*x*\cos(2/3*\arctan(\sqrt{3} + 2)))*\log(18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^4 + 18^{(2/3)}*12^{(2/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^4 + 12*18^{(1/3)}*12^{(1/3)}*\sqrt{3}*x*\cos(2/3*\arctan(\sqrt{3} + 2))*\sin(2/3*\arctan(\sqrt{3} + 2)) + 6*18^{(1/3)}*12^{(1/3)}*x*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 2*(18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 - 3*18^{(1/3)}*12^{(1/3)}*x)*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 36*x^2) - (18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x*\sin(2/3*\arctan(\sqrt{3} + 2)) + 18^{(2/3)}*12^{(1/6)}*x*\cos(2/3*\arctan(\sqrt{3} + 2)))*\log(18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^4 + 18^{(2/3)}*12^{(2/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^4 - 12*18^{(1/3)}*12^{(1/3)}*\sqrt{3}*x*\cos(2/3*\arctan(\sqrt{3} + 2))*\sin(2/3*\arctan(\sqrt{3} + 2)) + 6*18^{(1/3)}*12^{(1/3)}*x*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 2*(18^{(2/3)}*12^{(2/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 - 3*18^{(1/3)}*12^{(1/3)}*x)*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 36*x^2) - 108)/x \end{aligned}$$

giac [B] time = 0.55, size = 826, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="giac")

[Out] $\frac{1}{9}*(\sqrt{3}*\cos(4/9*\pi))^5 - 10*\sqrt{3}*\cos(4/9*\pi)^3*\sin(4/9*\pi)^2 + 5*\sqrt{3}*\cos(4/9*\pi)*\sin(4/9*\pi)^4 - 5*\cos(4/9*\pi)^4*\sin(4/9*\pi) + 10*\cos(4/9*\pi)^2*\sin(4/9*\pi)^3 - \sin(4/9*\pi)^5 + 2*\sqrt{3}*\cos(4/9*\pi)^2 - 2*\sqrt{3}*\sin(4/9*\pi)^2 - 4*\cos(4/9*\pi)*\sin(4/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(4/9*\pi) - 2*x)/((\sqrt{3}*i + 1)*\sin(4/9*\pi))) + \frac{1}{9}*(\sqrt{3}*\cos(2/9*\pi))^5 - 10*\sqrt{3}*\cos(2/9*\pi)^3*\sin(2/9*\pi)^2 + 5*\sqrt{3}*\cos(2/9*\pi)*\sin(2/9*\pi)^4 - 5*\cos(2/9*\pi)^4*\sin(2/9*\pi) + 10*\cos(2/9*\pi)^2*\sin(2/9*\pi)^3 - \sin(2/9*\pi)^5 + 2*\sqrt{3}*\cos(2/9*\pi)^2 - 2*\sqrt{3}*\sin(2/9*\pi)^2 - 4*\cos(2/9*\pi)*\sin(2/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(2/9*\pi) - 2*x)/((\sqrt{3}*i + 1)*\sin(2/9*\pi))) - \frac{1}{9}*(\sqrt{3}*\cos(1/9*\pi))^5 - 10*\sqrt{3}*\cos(1/9*\pi)^3*\sin(1/9*\pi)^2 + 5*\sqrt{3}*\cos(1/9*\pi)*\sin(1/9*\pi)^4 + 5*\cos(1/9*\pi)^4*\sin(1/9*\pi) - 1$

$0 \cdot \cos(1/9\pi)^2 \sin(1/9\pi)^3 + \sin(1/9\pi)^5 - 2\sqrt{3} \cos(1/9\pi)^2 + 2\sqrt{3} \sin(1/9\pi)^2 - 4\cos(1/9\pi) \sin(1/9\pi) \arctan\left(\frac{(\sqrt{3}i + 1) \cos(1/9\pi) + 2x}{(\sqrt{3}i + 1) \sin(1/9\pi)}\right) + 1/18(5\sqrt{3} \cos(4/9\pi)^4 \sin(4/9\pi) - 10\sqrt{3} \cos(4/9\pi)^2 \sin(4/9\pi)^3 + \sqrt{3} \sin(4/9\pi)^5 + \cos(4/9\pi)^5 - 10\cos(4/9\pi)^3 \sin(4/9\pi)^2 + 5\cos(4/9\pi) \sin(4/9\pi)^4 + 4\sqrt{3} \cos(4/9\pi) \sin(4/9\pi) + 2\cos(4/9\pi)^2 - 2\sin(4/9\pi)^2) \log(-(\sqrt{3}i \cos(4/9\pi) + \cos(4/9\pi))x + x^2 + 1) + 1/18(5\sqrt{3} \cos(2/9\pi)^4 \sin(2/9\pi) - 10\sqrt{3} \cos(2/9\pi)^2 \sin(2/9\pi)^3 + \sqrt{3} \sin(2/9\pi)^5 + \cos(2/9\pi)^5 - 10\cos(2/9\pi)^3 \sin(2/9\pi)^2 + 5\cos(2/9\pi) \sin(2/9\pi)^4 + 4\sqrt{3} \cos(2/9\pi) \sin(2/9\pi) + 2\cos(2/9\pi)^2 - 2\sin(2/9\pi)^2) \log(-(\sqrt{3}i \cos(2/9\pi) + \cos(2/9\pi))x + x^2 + 1) + 1/18(5\sqrt{3} \cos(1/9\pi)^4 \sin(1/9\pi) - 10\sqrt{3} \cos(1/9\pi)^2 \sin(1/9\pi)^3 + \sqrt{3} \sin(1/9\pi)^5 - \cos(1/9\pi)^5 + 10\cos(1/9\pi)^3 \sin(1/9\pi)^2 - 5\cos(1/9\pi) \sin(1/9\pi)^4 - 4\sqrt{3} \cos(1/9\pi) \sin(1/9\pi) + 2\cos(1/9\pi)^2 - 2\sin(1/9\pi)^2) \log((\sqrt{3}i \cos(1/9\pi) + \cos(1/9\pi))x + x^2 + 1) - 1/x$

maple [C] time = 0.01, size = 50, normalized size = 0.12

$$\frac{\left(\text{RootOf}\left(-Z^6 - Z^3 + 1\right)^4 - \text{RootOf}\left(-Z^6 - Z^3 + 1\right)\right) \ln\left(-\text{RootOf}\left(-Z^6 - Z^3 + 1\right) + x\right) - \frac{1}{x}}{3\left(2\text{RootOf}\left(-Z^6 - Z^3 + 1\right)^5 - \text{RootOf}\left(-Z^6 - Z^3 + 1\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(x^6-x^3+1), x)

[Out] -1/3*sum((R^4-R)/(2*R^5-R^2)*ln(-R+x), R=RootOf(-Z^6-Z^3+1))-1/x

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x} - \int \frac{x^4 - x}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^6-x^3+1), x, algorithm="maxima")

[Out] -1/x - integrate((x^4 - x)/(x^6 - x^3 + 1), x)

mupad [B] time = 1.66, size = 286, normalized size = 0.69

$$\frac{\ln\left(\frac{2^{10} 3^{10} (-3 + \sqrt{5})^{10}}{18}\right) (-3 + \sqrt{5})^{10}}{18} - \frac{1}{x} - \frac{\ln\left(\frac{2^{10} (-3 + \sqrt{5})^{10}}{18}\right) (-3 + \sqrt{5})^{10}}{18} - \frac{2^{10} \ln\left(\frac{2^{10} (-3 + \sqrt{5})^{10}}{24}\right) (-3 + \sqrt{5})^{10}}{36} (3^{10} - 3^{10} 11) - \frac{2^{10} \ln\left(\frac{2^{10} (-3 + \sqrt{5})^{10}}{24}\right) (-3 + \sqrt{5})^{10}}{36} (3^{10} + 3^{10} 11) - \frac{2^{10} \ln\left(\frac{2^{10} (-3 + \sqrt{5})^{10}}{24}\right) (-3 + \sqrt{5})^{10}}{36} (3^{10} - 3^{10} 11) - \frac{2^{10} \ln\left(\frac{2^{10} (-3 + \sqrt{5})^{10}}{24}\right) (-3 + \sqrt{5})^{10}}{36} (3^{10} + 3^{10} 11)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(x^6 - x^3 + 1)), x)

```
[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/3))/18 - 1/x + (log(x - (-3^(1/2)*12i - 36)^(2/3)/12)*(-3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(1/3)*(-3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(-3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(-3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(-3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

sympy [A] time = 0.20, size = 24, normalized size = 0.06

$$\text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log(-27t^2 + x)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x))) - 1/x
```

$$3.166 \quad \int \frac{1}{x^3(1-x^3+x^6)} dx$$

Optimal. Leaf size=418

$$\frac{1}{2x^2} + \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

Rubi [A] time = 0.34, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1368, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{1}{2x^2} + \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(-\sqrt{3}+i) \tan^{-1}\left(\frac{1+\sqrt[3]{2(1-i\sqrt{3})}}{\sqrt[3]{2(1-i\sqrt{3})}}\right)}{3\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(\sqrt{3}+i) \tan^{-1}\left(\frac{1+\sqrt[3]{2(1+i\sqrt{3})}}{\sqrt[3]{2(1+i\sqrt{3})}}\right)}{3\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1 - x^3 + x^6)),x]

[Out] $-\frac{1}{2x^2} - \frac{((I - \text{Sqrt}[3]) \text{ArcTan}[(1 + (2x)/((1 - I\text{Sqrt}[3])/2)^{1/3}])/ \text{Sqrt}[3])}{(3 \cdot 2^{1/3} (1 - I\text{Sqrt}[3])^{2/3})} + \frac{((I + \text{Sqrt}[3]) \text{ArcTan}[(1 + (2x)/((1 + I\text{Sqrt}[3])/2)^{1/3}])/ \text{Sqrt}[3])}{(3 \cdot 2^{1/3} (1 + I\text{Sqrt}[3])^{2/3})} - \frac{((3 - I\text{Sqrt}[3]) \text{Log}[(1 - I\text{Sqrt}[3])^{1/3} - 2^{1/3}x])}{(9 \cdot 2^{1/3} (1 - I\text{Sqrt}[3])^{2/3})} - \frac{((3 + I\text{Sqrt}[3]) \text{Log}[(1 + I\text{Sqrt}[3])^{1/3} - 2^{1/3}x])}{(9 \cdot 2^{1/3} (1 + I\text{Sqrt}[3])^{2/3})} + \frac{((3 - I\text{Sqrt}[3]) \text{Log}[(1 - I\text{Sqrt}[3])^{2/3} + (2(1 - I\text{Sqrt}[3]))^{1/3}x + 2^{2/3}x^2])}{(18 \cdot 2^{1/3} (1 - I\text{Sqrt}[3])^{2/3})} + \frac{((3 + I\text{Sqrt}[3]) \text{Log}[(1 + I\text{Sqrt}[3])^{2/3} + (2(1 + I\text{Sqrt}[3]))^{1/3}x + 2^{2/3}x^2])}{(18 \cdot 2^{1/3} (1 + I\text{Sqrt}[3])^{2/3})}$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)⁽⁻¹⁾, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(1-x^3+x^6)} dx &= -\frac{1}{2x^2} + \frac{1}{2} \int \frac{2-2x^3}{1-x^3+x^6} dx \\
&= -\frac{1}{2x^2} + \frac{1}{6}(-3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2} + \frac{i\sqrt{3}}{2} + x^3} dx - \frac{1}{6}(3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^3} dx \\
&= -\frac{1}{2x^2} - \frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})} + x} dx}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}} - x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x + x^2} dx}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} \\
&= -\frac{1}{2x^2} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \dots \\
&= -\frac{1}{2x^2} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \dots \\
&= -\frac{1}{2x^2} - \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log}{9\sqrt[3]{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 65, normalized size = 0.16

$$-\frac{1}{3}\text{RootSum}\left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^3 \log(x - \#1) - \log(x - \#1)}{2\#1^5 - \#1^2} \&\right] - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(1 - x^3 + x^6)),x]

[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 + 2*#1^5) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(1-x^3+x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(1 - x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x^3*(1 - x^3 + x^6)), x]

fricas [B] time = 1.45, size = 1066, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="fricas")

[Out]
$$\frac{1}{108} \cdot (2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \log(-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2) - 8 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \arctan(-1/108 \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x + 24 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - \sqrt{-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2) \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) - 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)))) / (\cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 4 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^2 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) - 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \arctan(1/108 \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) - 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x - 24 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - \sqrt{-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2) \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)))) / (\cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2) - 4 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^2 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \arctan(1/216 \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \sqrt{2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2) - 6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x - 216 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) / \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) + (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \log(2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos$$

$$\begin{aligned} & (2/3*\arctan(\sqrt{3} + 2))^2 + 3*18^{(1/3)}*12^{(1/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 18*x^2 - (18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x^2*\sin(2/3*\arctan(\sqrt{3} + 2))) + 18^{(2/3)}*12^{(1/6)}*x^2*\cos(2/3*\arctan(\sqrt{3} + 2))*\log(-18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x*\sin(2/3*\arctan(\sqrt{3} + 2)) - 3*18^{(2/3)}*12^{(1/6)}*x*\cos(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(1/3)}*12^{(1/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 3*18^{(1/3)}*12^{(1/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 18*x^2 - 54)/x^2 \end{aligned}$$

giac [B] time = 0.55, size = 642, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="giac")

[Out] $\frac{1}{9}*(\sqrt{3}*\cos(4/9*\pi)^4 - 6*\sqrt{3}*\cos(4/9*\pi)^2*\sin(4/9*\pi)^2 + \sqrt{3}*\sin(4/9*\pi)^4 + 4*\cos(4/9*\pi)^3*\sin(4/9*\pi) - 4*\cos(4/9*\pi)*\sin(4/9*\pi)^3 + 2*\sqrt{3}*\cos(4/9*\pi) + 2*\sin(4/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(4/9*\pi) - 2*x)/((\sqrt{3}*i + 1)*\sin(4/9*\pi))) + \frac{1}{9}*(\sqrt{3}*\cos(2/9*\pi)^4 - 6*\sqrt{3}*\cos(2/9*\pi)^2*\sin(2/9*\pi)^2 + \sqrt{3}*\sin(2/9*\pi)^4 + 4*\cos(2/9*\pi)^3*\sin(2/9*\pi) - 4*\cos(2/9*\pi)*\sin(2/9*\pi)^3 + 2*\sqrt{3}*\cos(2/9*\pi) + 2*\sin(2/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(2/9*\pi) - 2*x)/((\sqrt{3}*i + 1)*\sin(2/9*\pi))) + \frac{1}{9}*(\sqrt{3}*\cos(1/9*\pi)^4 - 6*\sqrt{3}*\cos(1/9*\pi)^2*\sin(1/9*\pi)^2 + \sqrt{3}*\sin(1/9*\pi)^4 - 4*\cos(1/9*\pi)^3*\sin(1/9*\pi) + 4*\cos(1/9*\pi)*\sin(1/9*\pi)^3 - 2*\sqrt{3}*\cos(1/9*\pi) + 2*\sin(1/9*\pi))*\arctan(((\sqrt{3}*i + 1)*\cos(1/9*\pi) + 2*x)/((\sqrt{3}*i + 1)*\sin(1/9*\pi))) + \frac{1}{18}*(4*\sqrt{3}*\cos(4/9*\pi)^3*\sin(4/9*\pi) - 4*\sqrt{3}*\cos(4/9*\pi)*\sin(4/9*\pi)^3 - \cos(4/9*\pi)^4 + 6*\cos(4/9*\pi)^2*\sin(4/9*\pi)^2 - \sin(4/9*\pi)^4 + 2*\sqrt{3}*\sin(4/9*\pi) - 2*\cos(4/9*\pi))*\log(-(\sqrt{3}*i*\cos(4/9*\pi) + \cos(4/9*\pi))*x + x^2 + 1) + \frac{1}{18}*(4*\sqrt{3}*\cos(2/9*\pi)^3*\sin(2/9*\pi) - 4*\sqrt{3}*\cos(2/9*\pi)*\sin(2/9*\pi)^3 - \cos(2/9*\pi)^4 + 6*\cos(2/9*\pi)^2*\sin(2/9*\pi)^2 - \sin(2/9*\pi)^4 + 2*\sqrt{3}*\sin(2/9*\pi) - 2*\cos(2/9*\pi))*\log(-(\sqrt{3}*i*\cos(2/9*\pi) + \cos(2/9*\pi))*x + x^2 + 1) - \frac{1}{18}*(4*\sqrt{3}*\cos(1/9*\pi)^3*\sin(1/9*\pi) - 4*\sqrt{3}*\cos(1/9*\pi)*\sin(1/9*\pi)^3 + \cos(1/9*\pi)^4 - 6*\cos(1/9*\pi)^2*\sin(1/9*\pi)^2 + \sin(1/9*\pi)^4 - 2*\sqrt{3}*\sin(1/9*\pi) - 2*\cos(1/9*\pi))*\log((\sqrt{3}*i*\cos(1/9*\pi) + \cos(1/9*\pi))*x + x^2 + 1) - 1/2/x^2$

maple [C] time = 0.01, size = 50, normalized size = 0.12

$$\frac{(-\text{RootOf}(-Z^6 - Z^3 + 1)^3 + 1) \ln(-\text{RootOf}(-Z^6 - Z^3 + 1) + x)}{6 \text{RootOf}(-Z^6 - Z^3 + 1)^5 - 3 \text{RootOf}(-Z^6 - Z^3 + 1)^2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^6-x^3+1),x)

[Out] $1/3*\text{sum}((-R^3+1)/(2*R^5-R^2)*\ln(-R+x), R=\text{RootOf}(Z^6-Z^3+1))-1/2/x^2$
maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2x^2} - \int \frac{x^3 - 1}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(x^6-x^3+1),x, algorithm="maxima")`

[Out] $-1/2/x^2 - \text{integrate}((x^3 - 1)/(x^6 - x^3 + 1), x)$

mupad [B] time = 1.72, size = 324, normalized size = 0.78

$$\frac{\ln\left(x \cdot \frac{\left(\frac{\sqrt{3}-\sqrt{12}i}{3}\right)^{1/3} - \sqrt{-36-\sqrt{12}i}}{3}\right)}{18} + \frac{\ln\left(x \cdot \frac{\left(\frac{\sqrt{3}+\sqrt{12}i}{3}\right)^{1/3} - \sqrt{-36+\sqrt{12}i}}{3}\right)}{18} - \frac{1}{2x^2} - \frac{\ln\left(x \cdot \frac{\left(\frac{\sqrt{3}-\sqrt{12}i\right)^{1/3} \left(\frac{\sqrt{3}+\sqrt{12}i}{3}\right)^{1/3} - \sqrt{-36-\sqrt{12}i}}{3}\right)}{36} - \frac{\ln\left(x \cdot \frac{\left(\frac{\sqrt{3}+\sqrt{12}i\right)^{1/3} \left(\frac{\sqrt{3}-\sqrt{12}i}{3}\right)^{1/3} - \sqrt{-36+\sqrt{12}i}}{3}\right)}{36} - \frac{\ln\left(x \cdot \frac{\left(\frac{\sqrt{3}-\sqrt{12}i\right)^{1/3} \left(\frac{\sqrt{3}+\sqrt{12}i}{3}\right)^{1/3} - \sqrt{-36-\sqrt{12}i}}{3}\right)}{36} - \frac{\ln\left(x \cdot \frac{\left(\frac{\sqrt{3}+\sqrt{12}i\right)^{1/3} \left(\frac{\sqrt{3}-\sqrt{12}i}{3}\right)^{1/3} - \sqrt{-36+\sqrt{12}i}}{3}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(x^6 - x^3 + 1)),x)`

[Out] $(\log(x - ((3^{(1/2)}*9i)/2 - 27/2)*(-3^{(1/2)}*12i - 36)^{(1/3)})/54)*(-3^{(1/2)}*12i - 36)^{(1/3)}/18 + (\log(x + ((3^{(1/2)}*9i)/2 + 27/2)*(3^{(1/2)}*12i - 36)^{(1/3)})/54)*(3^{(1/2)}*12i - 36)^{(1/3)}/18 - 1/(2*x^2) - (2^{(2/3)}*\log(x - (2^{(2/3)}*(-3^{(1/2)}*1i - 3)^{(1/3)}*(3^{(1/3)} + 3^{(5/6)}*1i))*((3*(3^{(1/2)}*1i + 3)*(3^{(1/3)} + 3^{(5/6)}*1i)^3)/16 + 27))/108)*(-3^{(1/2)}*1i - 3)^{(1/3)}*(3^{(1/3)} + 3^{(5/6)}*1i))/36 - (2^{(2/3)}*\log(x + (2^{(2/3)}*(3^{(1/2)}*1i - 3)^{(1/3)}*(3^{(1/3)} - 3^{(5/6)}*1i))*((3*(3^{(1/2)}*1i - 3)*(3^{(1/3)} - 3^{(5/6)}*1i)^3)/16 - 27))/108)*(3^{(1/2)}*1i - 3)^{(1/3)}*(3^{(1/3)} - 3^{(5/6)}*1i))/36 - (2^{(2/3)}*\log(x + (2^{(2/3)}*3^{(5/6)}*(-3^{(1/2)}*1i - 3)^{(1/3)}*1i)/6)*(-3^{(1/2)}*1i - 3)^{(1/3)}*(3^{(1/3)} - 3^{(5/6)}*1i))/36 - (2^{(2/3)}*\log(x - (2^{(2/3)}*3^{(5/6)}*(3^{(1/2)}*1i - 3)^{(1/3)}*1i)/6)*(3^{(1/2)}*1i - 3)^{(1/3)}*(3^{(1/3)} + 3^{(5/6)}*1i))/36$

sympy [A] time = 0.21, size = 31, normalized size = 0.07

$$\text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log(729t^4 + 9t + x)\right)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(x**6-x**3+1),x)`

[Out] $\text{RootSum}(19683*_t**6 + 243*_t**3 + 1, \text{Lambda}(_t, _t*\log(729*_t**4 + 9*_t + x))) - 1/(2*x**2)$

$$3.167 \quad \int \frac{1}{x^4(1-x^3+x^6)} dx$$

Optimal. Leaf size=48

$$-\frac{1}{3x^3} + \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(x)$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{3x^3} - \frac{1}{6} \log(x^6 - x^3 + 1) + \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 - x^3 + x^6)),x]

[Out] -1/(3*x^3) + ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)(a + bx + cx^2)}{(a + bx + cx^2)^2} dx$; FreeQ[{a, b, c, d, e}, x] && NeQ[2cd - b^2 - 4ac, 0] && NeQ[b^2 - 4ac, 0] && !NiceSqrtQ[b^2 - 4ac]

Rule 709

$\int \frac{(d + ex)^m}{(a + bx + cx^2)} dx$; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4ac, 0] && NeQ[cd - b^2 - 4ac, 0] && NeQ[2cd - b^2 - 4ac, 0] && LtQ[m, -1]

Rule 800

$\int \frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)} dx$; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4ac, 0] && NeQ[cd - b^2 - 4ac, 0] && IntegerQ[m]

Rule 1357

$\int (a + bx + cx^2)^p dx$; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n, 2m] && NeQ[b^2 - 4ac, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(1-x^3+x^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2(1-x+x^2)} dx, x, x^3 \right) \\
&= -\frac{1}{3x^3} + \frac{1}{3} \text{Subst} \left(\int \frac{1-x}{x(1-x+x^2)} dx, x, x^3 \right) \\
&= -\frac{1}{3x^3} + \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{x} - \frac{x}{1-x+x^2} \right) dx, x, x^3 \right) \\
&= -\frac{1}{3x^3} + \log(x) - \frac{1}{3} \text{Subst} \left(\int \frac{x}{1-x+x^2} dx, x, x^3 \right) \\
&= -\frac{1}{3x^3} + \log(x) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) - \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3 \right) \\
&= -\frac{1}{3x^3} + \log(x) - \frac{1}{6} \log(1-x^3+x^6) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\
&= -\frac{1}{3x^3} + \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{6} \log(1-x^3+x^6)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 1.06

$$-\frac{1}{3} \text{RootSum} \left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^3 \log(x - \#1)}{2\#1^3 - 1} \& \right] - \frac{1}{3x^3} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(1 - x^3 + x^6)),x]

[Out] -1/3*1/x^3 + Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^3) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(1-x^3+x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(1 - x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x^4*(1 - x^3 + x^6)), x]

fricas [A] time = 1.17, size = 51, normalized size = 1.06

$$\frac{2\sqrt{3}x^3 \arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) + 3x^3 \log(x^6 - x^3 + 1) - 18x^3 \log(x) + 6}{18x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="fricas")

[Out] -1/18*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3*x^3*log(x^6 - x^3 + 1) - 18*x^3*log(x) + 6)/x^3

giac [A] time = 0.42, size = 45, normalized size = 0.94

$$-\frac{1}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{x^3+1}{3x^3} - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="giac")

[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3*(x^3 + 1)/x^3 - 1/6*log(x^6 - x^3 + 1) + log(abs(x))

maple [A] time = 0.01, size = 40, normalized size = 0.83

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9} + \ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(x^6-x^3+1),x)

[Out] -1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3+ln(x)

maxima [A] time = 1.13, size = 43, normalized size = 0.90

$$-\frac{1}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{3x^3} - \frac{1}{6} \log(x^6 - x^3 + 1) + \frac{1}{3} \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="maxima")

[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3 - 1/6*log(x^6 - x^3 + 1) + 1/3*log(x^3)

mupad [B] time = 0.06, size = 41, normalized size = 0.85

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{9} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(x^6 - x^3 + 1)),x)`

[Out] `log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - 1/(3*x^3)`

sympy [A] time = 0.17, size = 48, normalized size = 1.00

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(x**6-x**3+1),x)`

[Out] `log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)`

$$3.168 \quad \int \frac{1}{x^5(1-x^3+x^6)} dx$$

Optimal. Leaf size=423

$$\frac{1}{4x^4} + \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}}$$

Rubi [A] time = 0.37, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 16, number of rules / integrand size = 0.625, Rules used = {1368, 1504, 12, 1374, 292, 31, 634, 617, 204, 628}

$$\frac{1}{4x^4} + \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{1}{x} - \frac{(3+i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1-i\sqrt{3}}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \log\left(-\sqrt[3]{2}x + \sqrt[3]{1+i\sqrt{3}}\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{(\sqrt{3}+i) \tan^{-1}\left(\frac{1+\sqrt[3]{2(1-i\sqrt{3})}}{\sqrt[3]{1-i\sqrt{3}}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(-\sqrt{3}+i) \tan^{-1}\left(\frac{1+\sqrt[3]{2(1+i\sqrt{3})}}{\sqrt[3]{1+i\sqrt{3}}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(1 - x^3 + x^6)),x]

[Out] $-1/(4*x^4) - x^{(-1)} - ((I + \text{Sqrt}[3]) * \text{ArcTan}[(1 + (2*x)/((1 - I*\text{Sqrt}[3]))/2)^{(1/3)}/\text{Sqrt}[3]])/(3*2^{(2/3)}*(1 - I*\text{Sqrt}[3])^{(1/3)}) + ((I - \text{Sqrt}[3]) * \text{ArcTan}[(1 + (2*x)/((1 + I*\text{Sqrt}[3]))/2)^{(1/3)}/\text{Sqrt}[3]])/(3*2^{(2/3)}*(1 + I*\text{Sqrt}[3])^{(1/3)}) - ((3 + I*\text{Sqrt}[3]) * \text{Log}[(1 - I*\text{Sqrt}[3])^{(1/3)} - 2^{(1/3)}*x])/(9*2^{(2/3)}*(1 - I*\text{Sqrt}[3])^{(1/3)}) - ((3 - I*\text{Sqrt}[3]) * \text{Log}[(1 + I*\text{Sqrt}[3])^{(1/3)} - 2^{(1/3)}*x])/(9*2^{(2/3)}*(1 + I*\text{Sqrt}[3])^{(1/3)}) + ((3 + I*\text{Sqrt}[3]) * \text{Log}[(1 - I*\text{Sqrt}[3])^{(2/3)} + (2*(1 - I*\text{Sqrt}[3]))^{(1/3)}*x + 2^{(2/3)}*x^2])/(18*2^{(2/3)}*(1 - I*\text{Sqrt}[3])^{(1/3)}) + ((3 - I*\text{Sqrt}[3]) * \text{Log}[(1 + I*\text{Sqrt}[3])^{(2/3)} + (2*(1 + I*\text{Sqrt}[3]))^{(1/3)}*x + 2^{(2/3)}*x^2])/(18*2^{(2/3)}*(1 + I*\text{Sqrt}[3])^{(1/3)})$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 31

Int[((a_) + (b_.)*(x_))^{(-1)}, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^{(-1)}, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Rule 1374

```
Int[((d_)*(x_)^(m_))/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
```


NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]

Rule 1504

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m + n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(1-x^3+x^6)} dx &= -\frac{1}{4x^4} + \frac{1}{4} \int \frac{4-4x^3}{x^2(1-x^3+x^6)} dx \\
&= -\frac{1}{4x^4} - \frac{1}{x} - \frac{1}{4} \int \frac{4x^4}{1-x^3+x^6} dx \\
&= -\frac{1}{4x^4} - \frac{1}{x} - \int \frac{x^4}{1-x^3+x^6} dx \\
&= -\frac{1}{4x^4} - \frac{1}{x} + \frac{1}{6}(-3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx - \frac{1}{6}(3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3} dx \\
&= -\frac{1}{4x^4} - \frac{1}{x} + \frac{(-3-i\sqrt{3}) \int \frac{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})+x}}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x+x^2} dx}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})+x}} dx}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} \\
&= -\frac{1}{4x^4} - \frac{1}{x} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \\
&= -\frac{1}{4x^4} - \frac{1}{x} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \\
&= -\frac{1}{4x^4} - \frac{1}{x} - \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{(3+i\sqrt{3})}{3}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.13

$$-\frac{1}{3} \text{RootSum}\left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^2 \log(x - \#1)}{2\#1^3 - 1} \&\right] - \frac{1}{4x^4} - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(1 - x^3 + x^6)),x]

[Out] -1/4*1/x^4 - x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(1-x^3+x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(1 - x^3 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x^5*(1 - x^3 + x^6)), x]

fricas [B] time = 1.62, size = 1623, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="fricas")

[Out]
$$\frac{1}{108} \cdot (2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x^4 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \log(18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) + 8 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x^4 \cdot \arctan(1/108 \cdot (6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 864 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^3 - 6 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x - 36 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 12 \cdot (18^{2/3} \cdot 12^{2/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 72 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^3) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - \sqrt{18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 2 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)))) / (3 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 10 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 108 \cdot x^3 - 4 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^4 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot x^4 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \arctan(1/108 \cdot (6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 864 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^3 - 6 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x - 36 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))$$

$$\begin{aligned} & * \arctan(\sqrt{3} - 2))^2 + 12 \cdot (18^{2/3} \cdot 12^{2/3}) \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 72 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - \sqrt{3} \\ & \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3}) \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) / (3 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 10 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4) - 4 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^4 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 18^{2/3} \cdot 12^{1/6} \cdot x^4 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \arctan(-1/432 \cdot (6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot x - 216 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 216 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3}) \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2)) / (\cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) - (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^4 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 18^{2/3} \cdot 12^{1/6} \cdot x^4 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3}) \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) + (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^4 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot x^4 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3}) \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) - 27) / x^4 \end{aligned}$$

giac [B] time = 0.55, size = 836, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="giac")

[Out] $1/9 \cdot (2 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi))^5 - 20 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi)^3 \cdot \sin(4/9 \cdot \pi)^2 + 10 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi)^4 - 10 \cdot \cos(4/9 \cdot \pi)^4 \cdot \sin(4/9 \cdot \pi) + 20 \cdot \cos(4/9 \cdot \pi)^2 \cdot \sin(4/9 \cdot \pi)^3 - 2 \cdot \sin(4/9 \cdot \pi)^5 + \sqrt{3} \cdot \cos(4/9 \cdot \pi)^2 - \sqrt{3} \cdot \sin(4/9 \cdot \pi)^2 - 2 \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(4/9 \cdot \pi) - 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(4/9 \cdot \pi))) + 1/9 \cdot (2 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi))^5 -$

$20\sqrt{3}\cos(2/9\pi)^3\sin(2/9\pi)^2 + 10\sqrt{3}\cos(2/9\pi)\sin(2/9\pi)^4 - 10\cos(2/9\pi)^4\sin(2/9\pi) + 20\cos(2/9\pi)^2\sin(2/9\pi)^3 - 2\sin(2/9\pi)^5 + \sqrt{3}\cos(2/9\pi)^2 - \sqrt{3}\sin(2/9\pi)^2 - 2\cos(2/9\pi)\sin(2/9\pi)\arctan(-(\sqrt{3}i + 1)\cos(2/9\pi) - 2x)/((\sqrt{3}i + 1)\sin(2/9\pi)) - 1/9(2\sqrt{3}\cos(1/9\pi)^5 - 20\sqrt{3}\cos(1/9\pi)^3\sin(1/9\pi)^2 + 10\sqrt{3}\cos(1/9\pi)\sin(1/9\pi)^4 + 10\cos(1/9\pi)^4\sin(1/9\pi) - 20\cos(1/9\pi)^2\sin(1/9\pi)^3 + 2\sin(1/9\pi)^5 - \sqrt{3}\cos(1/9\pi)^2 + \sqrt{3}\sin(1/9\pi)^2 - 2\cos(1/9\pi)\sin(1/9\pi)\arctan(((\sqrt{3}i + 1)\cos(1/9\pi) + 2x)/((\sqrt{3}i + 1)\sin(1/9\pi))) + 1/18(10\sqrt{3}\cos(4/9\pi)^4\sin(4/9\pi) - 20\sqrt{3}\cos(4/9\pi)^2\sin(4/9\pi)^3 + 2\sqrt{3}\sin(4/9\pi)^5 + 2\cos(4/9\pi)^5 - 20\cos(4/9\pi)^3\sin(4/9\pi)^2 + 10\cos(4/9\pi)\sin(4/9\pi)^4 + 2\sqrt{3}\cos(4/9\pi)\sin(4/9\pi) + \cos(4/9\pi)^2 - \sin(4/9\pi)^2)\log(-(\sqrt{3}i\cos(4/9\pi) + \cos(4/9\pi))x + x^2 + 1) + 1/18(10\sqrt{3}\cos(2/9\pi)^4\sin(2/9\pi) - 20\sqrt{3}\cos(2/9\pi)^2\sin(2/9\pi)^3 + 2\sqrt{3}\sin(2/9\pi)^5 + 2\cos(2/9\pi)^5 - 20\cos(2/9\pi)^3\sin(2/9\pi)^2 + 10\cos(2/9\pi)\sin(2/9\pi)^4 + 2\sqrt{3}\cos(2/9\pi)\sin(2/9\pi) + \cos(2/9\pi)^2 - \sin(2/9\pi)^2)\log(-(\sqrt{3}i\cos(2/9\pi) + \cos(2/9\pi))x + x^2 + 1) + 1/18(10\sqrt{3}\cos(1/9\pi)^4\sin(1/9\pi) - 20\sqrt{3}\cos(1/9\pi)^2\sin(1/9\pi)^3 + 2\sqrt{3}\sin(1/9\pi)^5 - 2\cos(1/9\pi)^5 + 20\cos(1/9\pi)^3\sin(1/9\pi)^2 - 10\cos(1/9\pi)\sin(1/9\pi)^4 - 2\sqrt{3}\cos(1/9\pi)\sin(1/9\pi) + \cos(1/9\pi)^2 - \sin(1/9\pi)^2)\log((\sqrt{3}i\cos(1/9\pi) + \cos(1/9\pi))x + x^2 + 1) - 1/4(4x^3 + 1)/x^4$

maple [C] time = 0.01, size = 51, normalized size = 0.12

$$-\frac{\text{RootOf}(-Z^6 - Z^3 + 1)^4 \ln(-\text{RootOf}(-Z^6 - Z^3 + 1) + x)}{3\left(2\text{RootOf}(-Z^6 - Z^3 + 1)^5 - \text{RootOf}(-Z^6 - Z^3 + 1)^2\right)} - \frac{1}{x} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(x^6-x^3+1),x)

[Out] -1/3*sum(1/(2*_R^5-_R^2)*_R^4*ln(-_R+x),_R=RootOf(-Z^6-_Z^3+1))-1/4/x^4-1/x

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4x^3 + 1}{4x^4} - \int \frac{x^4}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="maxima")

[Out] -1/4*(4*x^3 + 1)/x^4 - integrate(x^4/(x^6 - x^3 + 1), x)

mupad [B] time = 1.59, size = 318, normalized size = 0.75

$$\frac{\ln\left(-x + \sqrt{162x + \frac{27(36 - \sqrt{12})}{4}}\right)\left(\frac{1}{18} + \frac{\sqrt{12}}{36}\right) + \ln\left(-x - \sqrt{162x + \frac{27(36 - \sqrt{12})}{4}}\right)\left(\frac{1}{18} - \frac{\sqrt{12}}{36}\right)}{x^4} + \frac{2^{2/3} \ln\left(x + \frac{2^{2/3} 3^{1/6} \sqrt{3} \sqrt{36 - 3^{1/2} 12i}}{4}\right) + 2^{2/3} 3^{1/6} \sqrt{3} \sqrt{36 - 3^{1/2} 12i}}{36} (0 - \sqrt{3} i)^{1/3} (3^{1/2} - 3^{5/6} i)}{x^4} + \frac{2^{2/3} \ln\left(x + \frac{2^{2/3} 3^{1/6} \sqrt{3} \sqrt{36 - 3^{1/2} 12i}}{4}\right) + 2^{2/3} 3^{1/6} \sqrt{3} \sqrt{36 - 3^{1/2} 12i}}{36} (0 + \sqrt{3} i)^{1/3} (3^{1/2} + 3^{5/6} i)}{x^4} + \frac{2^{2/3} \ln\left(x + \frac{2^{2/3} 3^{1/6} \sqrt{3} \sqrt{36 - 3^{1/2} 12i}}{4}\right) + 2^{2/3} 3^{1/6} \sqrt{3} \sqrt{36 - 3^{1/2} 12i}}{36} (0 - \sqrt{3} i)^{1/3} (3^{1/2} + 3^{5/6} i)}{x^4} + \frac{2^{2/3} \ln\left(x + \frac{2^{2/3} 3^{1/6} \sqrt{3} \sqrt{36 - 3^{1/2} 12i}}{4}\right) + 2^{2/3} 3^{1/6} \sqrt{3} \sqrt{36 - 3^{1/2} 12i}}{36} (0 + \sqrt{3} i)^{1/3} (3^{1/2} - 3^{5/6} i)}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(x^6 - x^3 + 1)),x)

[Out] (log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162) - x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(-x - (162*x + (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 - (x^3 + 1/4)/x^4 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 - (2^(1/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36

sympy [A] time = 0.22, size = 39, normalized size = 0.09

$$\text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log\left(-6561t^5 + 54t^2 + x\right)\right)\right) + \frac{-4x^3 - 1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(x**6-x**3+1),x)

[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**5 + 54*_t**2 + x))) + (-4*x**3 - 1)/(4*x**4)

$$3.169 \quad \int \frac{1}{2+x^3+x^6} dx$$

Optimal. Leaf size=381

$$\frac{i \log\left(2^{2/3}x^2 - \sqrt[3]{2(1-i\sqrt{7})}x + (1-i\sqrt{7})^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}(1-i\sqrt{7})^{2/3}} - \frac{i \log\left(2^{2/3}x^2 - \sqrt[3]{2(1+i\sqrt{7})}x + (1+i\sqrt{7})^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}(1+i\sqrt{7})^{2/3}} - \frac{i \log\left(\sqrt[3]{2}x + \sqrt[3]{\frac{1-i\sqrt{7}}{2}}\right)}{3\sqrt{7}\left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}}$$

Rubi [A] time = 0.40, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1347, 200, 31, 634, 617, 204, 628}

$$\frac{i \log\left(2^{2/3}x^2 - \sqrt[3]{2(1-i\sqrt{7})}x + (1-i\sqrt{7})^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}(1-i\sqrt{7})^{2/3}} - \frac{i \log\left(2^{2/3}x^2 - \sqrt[3]{2(1+i\sqrt{7})}x + (1+i\sqrt{7})^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}(1+i\sqrt{7})^{2/3}} - \frac{i \log\left(\sqrt[3]{2}x + \sqrt[3]{\frac{1-i\sqrt{7}}{2}}\right)}{3\sqrt{7}\left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}} + \frac{i \log\left(\sqrt[3]{2}x + \sqrt[3]{\frac{1+i\sqrt{7}}{2}}\right)}{3\sqrt{7}\left(\frac{1}{2}(1+i\sqrt{7})\right)^{2/3}} + \frac{i \tan^{-1}\left(\frac{1 - \sqrt[3]{\frac{2x}{1-i\sqrt{7}}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}} - \frac{i \tan^{-1}\left(\frac{1 - \sqrt[3]{\frac{2x}{1+i\sqrt{7}}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}(1+i\sqrt{7})\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x^3 + x^6)^(-1), x]

[Out] (I*ArcTan[(1 - (2*x))/((1 - I*Sqrt[7])/2)^(1/3)]/Sqrt[3])/(Sqrt[21]*((1 - I*Sqrt[7])/2)^(2/3)) - (I*ArcTan[(1 - (2*x))/((1 + I*Sqrt[7])/2)^(1/3)]/Sqrt[3])/(Sqrt[21]*((1 + I*Sqrt[7])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 - I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 + I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 - I*Sqrt[7])^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 + I*Sqrt[7])^(2/3))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2+x^3+x^6} dx &= -\frac{i \int \frac{1}{\frac{1-i\sqrt{7}}{2}+x^3} dx}{\sqrt{7}} + \frac{i \int \frac{1}{\frac{1+i\sqrt{7}}{2}+x^3} dx}{\sqrt{7}} \\
&= -\frac{i \int \frac{1}{\sqrt{\frac{1}{2}(1-i\sqrt{7})+x}} dx}{3\sqrt{7} \left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}} - \frac{i \int \frac{2^{2/3} \sqrt[3]{1-i\sqrt{7}-x}}{\left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3} - \sqrt{\frac{1}{2}(1-i\sqrt{7})+x^2}} dx}{3\sqrt{7} \left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}} + \frac{i \int \frac{1}{\sqrt{\frac{1}{2}(1+i\sqrt{7})+x}} dx}{3\sqrt{7} \left(\frac{1}{2}(1+i\sqrt{7})\right)^{2/3}} + \dots \\
&= -\frac{i \log\left(\sqrt[3]{1-i\sqrt{7}} + \sqrt[3]{2}x\right)}{3\sqrt{7} \left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}} + \frac{i \log\left(\sqrt[3]{1+i\sqrt{7}} + \sqrt[3]{2}x\right)}{3\sqrt{7} \left(\frac{1}{2}(1+i\sqrt{7})\right)^{2/3}} + \frac{i \int \frac{-\sqrt{\frac{1}{2}(1-i\sqrt{7})+2x}}{\left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3} - \sqrt{\frac{1}{2}(1-i\sqrt{7})+x^2}} dx}{3\sqrt[3]{2}\sqrt{7}(1-i\sqrt{7})^{2/3}} \\
&= -\frac{i \log\left(\sqrt[3]{1-i\sqrt{7}} + \sqrt[3]{2}x\right)}{3\sqrt{7} \left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}} + \frac{i \log\left(\sqrt[3]{1+i\sqrt{7}} + \sqrt[3]{2}x\right)}{3\sqrt{7} \left(\frac{1}{2}(1+i\sqrt{7})\right)^{2/3}} + \frac{i \log\left(\left(1-i\sqrt{7}\right)^{2/3} - \sqrt[3]{2}(1-i\sqrt{7})\right)}{3\sqrt[3]{2}\sqrt{7}(1-i\sqrt{7})^{2/3}} \\
&= \frac{i \tan^{-1}\left(\frac{\sqrt[3]{\frac{1-i\sqrt{7}}{2}}}{\sqrt{3}}\right)}{\sqrt{21} \left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}} - \frac{i \tan^{-1}\left(\frac{\sqrt[3]{\frac{1+i\sqrt{7}}{2}}}{\sqrt{3}}\right)}{\sqrt{21} \left(\frac{1}{2}(1+i\sqrt{7})\right)^{2/3}} - \frac{i \log\left(\sqrt[3]{1-i\sqrt{7}} + \sqrt[3]{2}x\right)}{3\sqrt{7} \left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3}} + \frac{i \log\left(\sqrt[3]{1+i\sqrt{7}} + \sqrt[3]{2}x\right)}{3\sqrt{7} \left(\frac{1}{2}(1+i\sqrt{7})\right)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.10

$$\frac{1}{3} \text{RootSum}\left[\#1^6 + \#1^3 + 2\&, \frac{\log(x - \#1)}{2\#1^5 + \#1^2} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^3 + x^6)^(-1), x]

[Out] RootSum[2 + #1^3 + #1^6 &, Log[x - #1]/(#1^2 + 2*#1^5) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x^3 + x^6)^(-1),x]

[Out] IntegrateAlgebraic[(2 + x^3 + x^6)^(-1), x]

fricas [B] time = 3.21, size = 1996, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+x^3+2),x, algorithm="fricas")

[Out]
$$\frac{1}{294} \cdot 112^{1/6} \cdot 49^{2/3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) \cdot \log\left(112^{1/6} \cdot 49^{2/3} \cdot \sqrt{7} \cdot x \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) + 7 \cdot 112^{1/6} \cdot 49^{2/3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) + 14 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 + 14 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 + 98 \cdot x^2\right) - \frac{2}{147} \cdot 112^{1/6} \cdot 49^{2/3} \cdot \arctan\left(\frac{1}{2744} \cdot (14 \cdot 112^{5/6} \cdot 49^{1/3} \cdot \sqrt{7}) \cdot x \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) + 2744 \cdot \sqrt{7} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 + 2744 \cdot \sqrt{7} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 + 98 \cdot (112^{5/6} \cdot 49^{1/3} \cdot x + 224 \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) - \sqrt{112^{1/6} \cdot 49^{2/3} \cdot \sqrt{7} \cdot x \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) + 7 \cdot 112^{1/6} \cdot 49^{2/3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) + 14 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 + 14 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 + 98 \cdot x^2\right) \cdot (112^{5/6} \cdot 49^{1/3} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) + 7 \cdot 112^{5/6} \cdot 49^{1/3} \cdot \sqrt{2} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)) / (\cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 - 7 \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) + \frac{1}{147} \cdot (112^{1/6} \cdot 49^{2/3} \cdot \sqrt{3}) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) + 112^{1/6} \cdot 49^{2/3} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) \cdot \arctan\left(\frac{1}{5488} \cdot (70 \cdot 112^{5/6} \cdot 49^{1/3} \cdot (\sqrt{7} \cdot x + 7 \cdot \sqrt{3}) \cdot x) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^3 - 27440 \cdot (\sqrt{7} + 2 \cdot \sqrt{3}) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^4 - 5488 \cdot (\sqrt{7} - 2 \cdot \sqrt{3}) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^4 - 14 \cdot (112^{5/6} \cdot 49^{1/3} \cdot (\sqrt{7} \cdot \sqrt{3}) \cdot x - 7 \cdot x) - 1568 \cdot (\sqrt{7} \cdot \sqrt{3} - 5) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^3 + 14 \cdot (112^{5/6} \cdot 49^{1/3} \cdot (13 \cdot \sqrt{7}) \cdot x - 21 \cdot \sqrt{3}) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) - 784 \cdot (3 \cdot \sqrt{7} + 4 \cdot \sqrt{3}) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 - 14 \cdot (112^{5/6} \cdot 49^{1/3} \cdot (9 \cdot \sqrt{7}) \cdot \sqrt{3} \cdot x + 49 \cdot x) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 - 1568 \cdot (\sqrt{7} \cdot \sqrt{3} + 11) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^3 \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) - (5 \cdot 112^{5/6} \cdot 49^{1/3} \cdot (\sqrt{7} + 7 \cdot \sqrt{3})) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^3 - 112^{5/6} \cdot 49^{1/3} \cdot (9 \cdot \sqrt{7}) \cdot \sqrt{3} \cdot (x + 49) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) + 112^{5/6} \cdot 49^{1/3} \cdot (13 \cdot \sqrt{7} - 21 \cdot \sqrt{3}) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^2 - 112^{5/6} \cdot 49^{1/3} \cdot (\sqrt{7} \cdot \sqrt{3} - 7) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right)^3 \cdot \sqrt{-112^{1/6} \cdot 49^{2/3}} \cdot (\sqrt{7} \cdot \sqrt{3}) \cdot x + 7 \cdot x) \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) - 112^{1/6} \cdot 49^{2/3} \cdot (\sqrt{7} \cdot x - 7 \cdot \sqrt{3}) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{1}{3} \sqrt{7} + \frac{4}{3}\right)\right) +$$

$$28 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 28 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 196 \cdot x^2) / (25 \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^4 - 38 \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^4) + 1/147 \cdot (112^{1/6} \cdot 49^{2/3} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) - 112^{1/6} \cdot 49^{2/3} \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))) \cdot \arctan(-1/5488 \cdot (70 \cdot 112^{5/6} \cdot 49^{1/3} \cdot (\sqrt{7} \cdot x - 7 \cdot \sqrt{3} \cdot x) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^3 - 274 \cdot 40 \cdot (\sqrt{7} - 2 \cdot \sqrt{3})) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^4 - 5488 \cdot (\sqrt{7} + 2 \cdot \sqrt{3})) \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^4 + 14 \cdot (112^{5/6} \cdot 49^{1/3} \cdot (\sqrt{7} \cdot \sqrt{3} \cdot x + 7 \cdot x) - 1568 \cdot (\sqrt{7} \cdot \sqrt{3} + 5) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))) \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^3 + 14 \cdot (112^{5/6} \cdot 49^{1/3} \cdot (13 \cdot \sqrt{7} \cdot x + 21 \cdot \sqrt{3} \cdot x) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) - 78 \cdot 4 \cdot (3 \cdot \sqrt{7} - 4 \cdot \sqrt{3})) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 14 \cdot (112^{5/6} \cdot 49^{1/3} \cdot (9 \cdot \sqrt{7} \cdot \sqrt{3} \cdot x - 4 \cdot 9 \cdot x) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 - 1568 \cdot (\sqrt{7} \cdot \sqrt{3} - 11) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^3 \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) - (5 \cdot 112^{5/6} \cdot 49^{1/3} \cdot (\sqrt{7} - 7 \cdot \sqrt{3})) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^3 + 112^{5/6} \cdot 49^{1/3} \cdot (9 \cdot \sqrt{7} \cdot \sqrt{3} - 49) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) + 112^{5/6} \cdot 49^{1/3} \cdot (13 \cdot \sqrt{7} \cdot x + 21 \cdot \sqrt{3} \cdot x) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 112^{5/6} \cdot 49^{1/3} \cdot (\sqrt{7} \cdot \sqrt{3} + 7) \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^3 \cdot \sqrt{112^{1/6} \cdot 49^{2/3} \cdot (\sqrt{7} \cdot \sqrt{3} \cdot x - 7 \cdot x) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) - 112^{1/6} \cdot 49^{2/3} \cdot (\sqrt{7} \cdot x + 7 \cdot \sqrt{3} \cdot x) \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) + 28 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 28 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 196 \cdot x^2) / (25 \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^4 - 38 \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^4) + 1/588 \cdot (112^{1/6} \cdot 49^{2/3} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) - 112^{1/6} \cdot 49^{2/3} \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))) \cdot \log(-112^{1/6} \cdot 49^{2/3} \cdot (\sqrt{7} \cdot \sqrt{3} \cdot x + 7 \cdot x) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) - 112^{1/6} \cdot 49^{2/3} \cdot (\sqrt{7} \cdot x - 7 \cdot \sqrt{3} \cdot x) \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) + 28 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 28 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 196 \cdot x^2) - 1/588 \cdot (112^{1/6} \cdot 49^{2/3} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) + 112^{1/6} \cdot 49^{2/3} \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))) \cdot \log(112^{1/6} \cdot 49^{2/3} \cdot (\sqrt{7} \cdot \sqrt{3} \cdot x - 7 \cdot x) \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) - 112^{1/6} \cdot 49^{2/3} \cdot (\sqrt{7} \cdot x + 7 \cdot \sqrt{3} \cdot x) \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3)) + 28 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \cos(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 28 \cdot 49^{1/3} \cdot 14^{1/3} \cdot \sin(2/3 \cdot \arctan(1/3 \cdot \sqrt{7} + 4/3))^2 + 196 \cdot x^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + x^3 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+x^3+2),x, algorithm="giac")

[Out] integrate(1/(x^6 + x^3 + 2), x)

maple [C] time = 0.01, size = 33, normalized size = 0.09

$$\frac{\ln\left(-\operatorname{RootOf}\left(-Z^6 + Z^3 + 2\right) + x\right)}{6 \operatorname{RootOf}\left(-Z^6 + Z^3 + 2\right)^5 + 3 \operatorname{RootOf}\left(-Z^6 + Z^3 + 2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6+x^3+2),x)

[Out] 1/3*sum(1/(2*_R^5+_R^2)*ln(-_R+x),_R=RootOf(-Z^6+Z^3+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

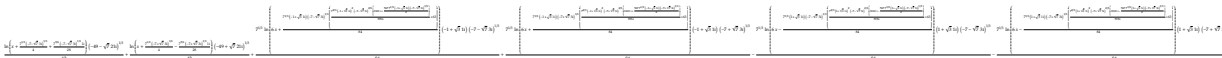
$$\int \frac{1}{x^6 + x^3 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+x^3+2),x, algorithm="maxima")

[Out] integrate(1/(x^6 + x^3 + 2), x)

mupad [B] time = 2.61, size = 513, normalized size = 1.35



Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3 + x^6 + 2),x)

[Out] $(\log(x + (7^{1/3}) * (-7^{1/2}) * 3i - 7)^{1/3}) / 4 + (7^{5/6}) * (-7^{1/2}) * 3i - 7)^{1/3} * 1i) / 28) * (-7^{1/2}) * 21i - 49)^{1/3} / 42 + (\log(x + (7^{1/3}) * (7^{1/2}) * 3i - 7)^{1/3}) / 4 - (7^{5/6}) * (7^{1/2}) * 3i - 7)^{1/3} * 1i) / 28) * (7^{1/2}) * 21i - 49)^{1/3} / 42 + (7^{1/3}) * \log(6 * x + (7^{1/3}) * (3^{1/2}) * 1i - 1) * (-7^{1/2}) * 3i - 7)^{1/3} * ((7^{2/3}) * (3^{1/2}) * 1i - 1)^2 * (-7^{1/2}) * 3i - 7)^{2/3} * (3969 * x + (567 * 7^{1/3}) * (3^{1/2}) * 1i - 1) * (-7^{1/2}) * 3i - 7)^{1/3}) / 2) / 7056 + 63) / 84) * (3^{1/2}) * 1i - 1) * (-7^{1/2}) * 3i - 7)^{1/3} / 84 + (7^{1/3}) * \log(6 * x + (7^{1/3}) * (3^{1/2}) * 1i - 1) * (7^{1/2}) * 3i - 7)^{1/3} * ((7^{2/3}) * (3^{1/2}) * 1i - 1)^2 * (7^{1/2}) * 3i - 7)^{2/3} * (3969 * x + (567 * 7^{1/3}) * (3^{1/2}) * 1i - 1) * (7^{1/2}) * 3i - 7)^{1/3}) / 2) / 7056 + 63) / 84) * (3^{1/2}) * 1i - 1) * (7^{1/2}) * 3i - 7)^{1/3} / 84 - (7^{1/3}) * \log(6 * x - (7^{1/3}) * (3^{1/2}) * 1i + 1) * (-7^{1/2}) * 3i - 7)^{1/3} * ((7^{2/3}) * (3^{1/2}) * 1i + 1)^2 * (-7^{1/2}) * 3i - 7)^{2/3} * (3969 * x - (567 * 7^{1/3}) * (3^{1/2}) * 1i + 1) * (-7^{1/2}) * 3i - 7)^{1/3}) / 2) / 7056 + 63) / 84) * (3^{1/2}) * 1i + 1) * (-7^{1/2}) * 3i - 7)^{1/3} / 84 - (7^{1/3}) * \log(6 * x - (7^{1/3}) * (3^{1/2}) * 1i + 1) * (-7^{1/2}) * 3i - 7)^{1/3} * ((7^{2/3}) * (3^{1/2}) * 1i + 1)^2 * (-7^{1/2}) * 3i - 7)^{2/3} * (3969 * x - (567 * 7^{1/3}) * (3^{1/2}) * 1i + 1) * (-7^{1/2}) * 3i - 7)^{1/3}) / 2) / 7056 + 63) / 84) * (3^{1/2}) * 1i + 1) * (-7^{1/2}) * 3i - 7)^{1/3} / 84$

$$\begin{aligned} & /2) * 1i + 1) * (-7^{(1/2)} * 3i - 7)^{(1/3)} / 2) / 7056 + 63) / 84) * (3^{(1/2)} * 1i + 1) * \\ & (-7^{(1/2)} * 3i - 7)^{(1/3)} / 84 - (7^{(1/3)} * \log(6 * x - (7^{(1/3)} * (3^{(1/2)} * 1i + 1) \\ & * (7^{(1/2)} * 3i - 7)^{(1/3)} * ((7^{(2/3)} * (3^{(1/2)} * 1i + 1)^2 * (7^{(1/2)} * 3i - 7)^{(2/3)} \\ & * (3969 * x - (567 * 7^{(1/3)} * (3^{(1/2)} * 1i + 1) * (7^{(1/2)} * 3i - 7)^{(1/3)})) / 2) / 7056 + \\ & 63) / 84) * (3^{(1/2)} * 1i + 1) * (7^{(1/2)} * 3i - 7)^{(1/3)} / 84 \end{aligned}$$

sympy [A] time = 0.15, size = 24, normalized size = 0.06

$$\text{RootSum}\left(1000188t^6 + 1323t^3 + 1, \left(t \mapsto t \log(-5292t^4 + 7t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6+x**3+2),x)

[Out] RootSum(1000188*_t**6 + 1323*_t**3 + 1, Lambda(_t, _t*log(-5292*_t**4 + 7*_t + x)))

$$3.170 \quad \int \frac{x^2}{2+x^3+x^6} dx$$

Optimal. Leaf size=23

$$\frac{2 \tan^{-1} \left(\frac{2x^3+1}{\sqrt{7}} \right)}{3\sqrt{7}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1352, 618, 204}

$$\frac{2 \tan^{-1} \left(\frac{2x^3+1}{\sqrt{7}} \right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 + x^3 + x^6),x]

[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{2+x^3+x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{2+x+x^2} dx, x, x^3 \right) \\ &= -\left(\frac{2}{3} \text{Subst} \left(\int \frac{1}{-7-x^2} dx, x, 1+2x^3 \right) \right) \\ &= \frac{2 \tan^{-1} \left(\frac{1+2x^3}{\sqrt{7}} \right)}{3\sqrt{7}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{2x^3+1}{\sqrt{7}} \right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2 + x^3 + x^6), x]

[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2+x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(2 + x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^2/(2 + x^3 + x^6), x]

fricas [A] time = 1.19, size = 18, normalized size = 0.78

$$\frac{2}{21} \sqrt{7} \arctan \left(\frac{1}{7} \sqrt{7} (2x^3 + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+x^3+2), x, algorithm="fricas")

[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))

giac [A] time = 0.50, size = 18, normalized size = 0.78

$$\frac{2}{21} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x^3 + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+x^3+2),x, algorithm="giac")

[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{2\sqrt{7} \arctan\left(\frac{(2x^3+1)\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^6+x^3+2),x)

[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)

maxima [A] time = 1.36, size = 18, normalized size = 0.78

$$\frac{2}{21} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x^3 + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+x^3+2),x, algorithm="maxima")

[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))

mupad [B] time = 0.05, size = 20, normalized size = 0.87

$$\frac{2\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x^3}{7} + \frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^3 + x^6 + 2),x)

[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^3)/7))/21

sympy [A] time = 0.11, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x^3}{7} + \frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**6+x**3+2),x)

[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**3/7 + sqrt(7)/7)/21

$$3.171 \quad \int \frac{x^3}{2+x^3+x^6} dx$$

Optimal. Leaf size=399

$$\frac{(7+i\sqrt{7}) \log\left(2^{2/3}x^2 - \sqrt[3]{2(1-i\sqrt{7})}x + (1-i\sqrt{7})^{2/3}\right)}{42\sqrt[3]{2}(1-i\sqrt{7})^{2/3}} - \frac{(7-i\sqrt{7}) \log\left(2^{2/3}x^2 - \sqrt[3]{2(1+i\sqrt{7})}x + (1+i\sqrt{7})^{2/3}\right)}{42\sqrt[3]{2}(1+i\sqrt{7})^{2/3}}$$

Rubi [A] time = 0.31, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1374, 200, 31, 634, 617, 204, 628}

$$\frac{(7+i\sqrt{7}) \log\left(2^{2/3}x^2 - \sqrt[3]{2(1-i\sqrt{7})}x + (1-i\sqrt{7})^{2/3}\right)}{42\sqrt[3]{2}(1-i\sqrt{7})^{2/3}} - \frac{(7-i\sqrt{7}) \log\left(2^{2/3}x^2 - \sqrt[3]{2(1+i\sqrt{7})}x + (1+i\sqrt{7})^{2/3}\right)}{42\sqrt[3]{2}(1+i\sqrt{7})^{2/3}} + \frac{(7+i\sqrt{7}) \log\left(\sqrt[3]{2}x + \sqrt[3]{1-i\sqrt{7}}\right)}{21\sqrt[3]{2}(1-i\sqrt{7})^{2/3}} + \frac{(7-i\sqrt{7}) \log\left(\sqrt[3]{2}x + \sqrt[3]{1+i\sqrt{7}}\right)}{21\sqrt[3]{2}(1+i\sqrt{7})^{2/3}} - \frac{i\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})} \tan^{-1}\left(\frac{\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{i\sqrt[3]{\frac{1}{2}(1+i\sqrt{7})} \tan^{-1}\left(\frac{\sqrt[3]{\frac{1}{2}(1+i\sqrt{7})}}{\sqrt{3}}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(2 + x^3 + x^6), x]

[Out] ((-I)*((1 - I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + (I*((1 + I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) + ((7 - I*Sqrt[7])*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 + I*Sqrt[7])^(2/3)) - ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(42*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) - ((7 - I*Sqrt[7])*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(42*2^(1/3)*(1 + I*Sqrt[7])^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1374

```
Int[((d_)*(x_)^m)/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{2+x^3+x^6} dx &= \frac{1}{14} (7-i\sqrt{7}) \int \frac{1}{\frac{1}{2} + \frac{i\sqrt{7}}{2} + x^3} dx + \frac{1}{14} (7+i\sqrt{7}) \int \frac{1}{\frac{1}{2} - \frac{i\sqrt{7}}{2} + x^3} dx \\
&= \frac{(7-i\sqrt{7}) \int \frac{1}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{7})} + x} dx}{21\sqrt[3]{2} (1+i\sqrt{7})^{2/3}} + \frac{(7-i\sqrt{7}) \int \frac{2^{2/3} \sqrt[3]{1+i\sqrt{7}} - x}{\left(\frac{1}{2}(1+i\sqrt{7})\right)^{2/3} - \sqrt[3]{\frac{1}{2}(1+i\sqrt{7})} x + x^2} dx}{21\sqrt[3]{2} (1+i\sqrt{7})^{2/3}} + \frac{(7+i\sqrt{7}) \int \frac{1}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})} + x} dx}{21\sqrt[3]{2} (1-i\sqrt{7})^{2/3}} \\
&= \frac{(7+i\sqrt{7}) \log\left(\sqrt[3]{1-i\sqrt{7}} + \sqrt[3]{2}x\right)}{21\sqrt[3]{2} (1-i\sqrt{7})^{2/3}} + \frac{(7-i\sqrt{7}) \log\left(\sqrt[3]{1+i\sqrt{7}} + \sqrt[3]{2}x\right)}{21\sqrt[3]{2} (1+i\sqrt{7})^{2/3}} - \frac{(7-i\sqrt{7}) \int \frac{1}{\left(\frac{1}{2}(1-i\sqrt{7})\right)^{2/3} - \sqrt[3]{\frac{1}{2}(1-i\sqrt{7})} x + x^2} dx}{42\sqrt[3]{2} (1-i\sqrt{7})^{2/3}} \\
&= \frac{(7+i\sqrt{7}) \log\left(\sqrt[3]{1-i\sqrt{7}} + \sqrt[3]{2}x\right)}{21\sqrt[3]{2} (1-i\sqrt{7})^{2/3}} + \frac{(7-i\sqrt{7}) \log\left(\sqrt[3]{1+i\sqrt{7}} + \sqrt[3]{2}x\right)}{21\sqrt[3]{2} (1+i\sqrt{7})^{2/3}} - \frac{(7+i\sqrt{7}) \log\left(\sqrt[3]{1-i\sqrt{7}} + \sqrt[3]{2}x\right)}{21\sqrt[3]{2} (1-i\sqrt{7})^{2/3}} \\
&= \frac{i\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})} \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{7})}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{i\sqrt[3]{\frac{1}{2}(1+i\sqrt{7})} \tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{7})}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{(7+i\sqrt{7}) \log\left(\sqrt[3]{1-i\sqrt{7}} + \sqrt[3]{2}x\right)}{21\sqrt[3]{2} (1-i\sqrt{7})^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.09

$$\frac{1}{3} \text{RootSum}\left[\#1^6 + \#1^3 + 2\&, \frac{\#1 \log(x - \#1)}{2\#1^3 + 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(2 + x^3 + x^6), x]

[Out] RootSum[2 + #1^3 + #1^6 &, (Log[x - #1]*#1)/(1 + 2*#1^3) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{2+x^3+x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(2 + x^3 + x^6), x]

[Out] IntegrateAlgebraic[x^3/(2 + x^3 + x^6), x]

fricas [B] time = 1.51, size = 1435, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+x^3+2),x, algorithm="fricas")

[Out]
$$\frac{1}{294} \cdot 98^{2/3} \cdot 56^{1/6} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) \cdot \log\left(\frac{-2 \cdot 98^{2/3} \cdot 56^{1/6} \cdot \sqrt{7} \cdot x \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + \sqrt{7}}{2 + 14 \cdot 98^{1/3} \cdot 7^{1/3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 14 \cdot 98^{1/3} \cdot 7^{1/3} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 98 \cdot x^2} - \frac{2}{147} \cdot 98^{2/3} \cdot 56^{1/6} \cdot \arctan\left(\frac{1}{5488} \cdot 98^{1/3} \cdot 56^{5/6} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \sqrt{-2 \cdot 98^{2/3} \cdot 56^{1/6} \cdot \sqrt{7} \cdot x \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + \sqrt{7}}\right) + 14 \cdot 98^{1/3} \cdot 7^{1/3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 14 \cdot 98^{1/3} \cdot 7^{1/3} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 98 \cdot x^2} - 14 \cdot 98^{1/3} \cdot 56^{5/6} \cdot \sqrt{7} \cdot x + 5488 \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) / \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)\right) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + \frac{1}{147} \cdot 98^{2/3} \cdot 56^{1/6} \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + 98^{2/3} \cdot 56^{1/6} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) \cdot \arctan\left(\frac{1}{2744} \cdot \left(14 \cdot 98^{1/3} \cdot 56^{5/6} \cdot \sqrt{7} \cdot x \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + \sqrt{7}\right)\right) + 2744 \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 2744 \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 14 \cdot \left(98^{1/3} \cdot 56^{5/6} \cdot \sqrt{7} \cdot \sqrt{3} \cdot x + 784 \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + \sqrt{7}\right) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) - \sqrt{98^{2/3} \cdot 56^{1/6} \cdot \sqrt{7} \cdot \sqrt{3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + 98^{2/3} \cdot 56^{1/6} \cdot \sqrt{7} \cdot x \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + 14 \cdot 98^{1/3} \cdot 7^{1/3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 14 \cdot 98^{1/3} \cdot 7^{1/3} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 98 \cdot x^2} \cdot \left(98^{1/3} \cdot 56^{5/6} \cdot \sqrt{7} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + \sqrt{7}\right) + 98^{1/3} \cdot 56^{5/6} \cdot \sqrt{7} \cdot \sqrt{2} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) \cdot \sqrt{7}\right) / \left(\cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)\right)^2 - 3 \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2\right) + \frac{1}{147} \cdot 98^{2/3} \cdot 56^{1/6} \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) - 98^{2/3} \cdot 56^{1/6} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) \cdot \arctan\left(-\frac{1}{2744} \cdot \left(14 \cdot 98^{1/3} \cdot 56^{5/6} \cdot \sqrt{7} \cdot x \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + \sqrt{7}\right)\right) - 2744 \cdot \sqrt{3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 - 2744 \cdot \sqrt{3} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 - 14 \cdot \left(98^{1/3} \cdot 56^{5/6} \cdot \sqrt{7} \cdot \sqrt{3} \cdot x - 784 \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + \sqrt{7}\right) \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + \sqrt{-98^{2/3} \cdot 56^{1/6} \cdot \sqrt{7} \cdot \sqrt{3} \cdot x \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + 98^{2/3} \cdot 56^{1/6} \cdot \sqrt{7} \cdot x \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right) + 14 \cdot 98^{1/3} \cdot 7^{1/3} \cdot \cos\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 14 \cdot 98^{1/3} \cdot 7^{1/3} \cdot \sin\left(\frac{2}{3} \arctan\left(\frac{2}{7} \sqrt{14} \sqrt{7} + \sqrt{7}\right)\right)^2 + 9$$

$8*x^2)*(98^{(1/3)}*56^{(5/6)}*\sqrt{7}*\sqrt{3}*\sqrt{2}*\sin(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})) - 98^{(1/3)}*56^{(5/6)}*\sqrt{7}*\sqrt{2}*\cos(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})))/(\cos(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7}))^2 - 3*\sin(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7}))^2) + 1/588*(98^{(2/3)}*56^{(1/6)}*\sqrt{3}*\sin(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})) - 98^{(2/3)}*56^{(1/6)}*\cos(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})))*\log(98^{(2/3)}*56^{(1/6)}*\sqrt{7}*\sqrt{3}*x*\cos(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})) + 98^{(2/3)}*56^{(1/6)}*\sqrt{7}*x*\sin(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})) + 14*98^{(1/3)}*7^{(1/3)}*\cos(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7}))^2 + 14*98^{(1/3)}*7^{(1/3)}*\sin(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7}))^2 + 98*x^2 - 1/588*(98^{(2/3)}*56^{(1/6)}*\sqrt{3}*\sin(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})) + 98^{(2/3)}*56^{(1/6)}*\cos(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})))*\log(-98^{(2/3)}*56^{(1/6)}*\sqrt{7}*\sqrt{3}*x*\cos(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})) + 98^{(2/3)}*56^{(1/6)}*\sqrt{7}*x*\sin(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7})) + 14*98^{(1/3)}*7^{(1/3)}*\cos(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7}))^2 + 14*98^{(1/3)}*7^{(1/3)}*\sin(2/3*\arctan(2/7*\sqrt{14})*\sqrt{7} + \sqrt{7}))^2 + 98*x^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + x^3 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+x^3+2),x, algorithm="giac")

[Out] integrate(x^3/(x^6 + x^3 + 2), x)

maple [C] time = 0.01, size = 36, normalized size = 0.09

$$\frac{\text{RootOf}(-Z^6 + Z^3 + 2)^3 \ln(-\text{RootOf}(-Z^6 + Z^3 + 2) + x)}{6 \text{RootOf}(-Z^6 + Z^3 + 2)^5 + 3 \text{RootOf}(-Z^6 + Z^3 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^6+x^3+2),x)

[Out] 1/3*sum(_R^3/(2*_R^5+_R^2)*ln(-_R+x),_R=RootOf(-Z^6+_Z^3+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + x^3 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+x^3+2),x, algorithm="maxima")

[Out] integrate(x^3/(x^6 + x^3 + 2), x)

mupad [B] time = 2.61, size = 351, normalized size = 0.88

$$\frac{\frac{1}{42} \left(x - \frac{2^{2/3} 7^{5/6}}{7} \right) \log(x - \frac{2^{2/3} 7^{5/6}}{7})}{42} + \frac{2^{2/3} 7^{1/3} \log(x + \frac{2^{2/3} 7^{5/6}}{7})}{42} + \frac{2^{2/3} 7^{1/3} \log(x + \frac{2^{2/3} 7^{5/6}}{7})}{42} + \frac{2^{2/3} 7^{1/3} \log(x + \frac{2^{2/3} 7^{5/6}}{7})}{42} + \frac{2^{2/3} 7^{1/3} \log(x + \frac{2^{2/3} 7^{5/6}}{7})}{42} + \frac{2^{2/3} 7^{1/3} \log(x + \frac{2^{2/3} 7^{5/6}}{7})}{42} + \frac{2^{2/3} 7^{1/3} \log(x + \frac{2^{2/3} 7^{5/6}}{7})}{42} + \frac{2^{2/3} 7^{1/3} \log(x + \frac{2^{2/3} 7^{5/6}}{7})}{42} + \frac{2^{2/3} 7^{1/3} \log(x + \frac{2^{2/3} 7^{5/6}}{7})}{42} + \frac{2^{2/3} 7^{1/3} \log(x + \frac{2^{2/3} 7^{5/6}}{7})}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^3 + x^6 + 2),x)

[Out] $(\log(x - (2^{2/3} 7^{5/6}) * (-7^{1/2} * i - 7)^{1/3} * i) / 14) * (-7^{1/2} * 28i - 196)^{1/3} / 42 + (2^{2/3} 7^{1/3} * \log(x + (2^{2/3} 7^{5/6}) * (7^{1/2} * i - 7)^{1/3} * i) / 14) * (7^{1/2} * i - 7)^{1/3} / 42 - (2^{2/3} 7^{1/3} * \log(x + (2^{2/3} 7^{5/6}) * (-7^{1/2} * i - 7)^{1/3} * i) / 28 - (2^{2/3} 3^{1/2} * 7^{5/6}) * (-7^{1/2} * i - 7)^{1/3} / 28) * (3^{1/2} * i + 1) * (-7^{1/2} * i - 7)^{1/3} / 84 + (2^{2/3} 7^{1/3} * \log(x + (2^{2/3} 7^{5/6}) * (-7^{1/2} * i - 7)^{1/3} * i) / 28 + (2^{2/3} 3^{1/2} * 7^{5/6}) * (-7^{1/2} * i - 7)^{1/3} / 28) * (3^{1/2} * i - 1) * (-7^{1/2} * i - 7)^{1/3} / 84 + (2^{2/3} 7^{1/3} * \log(x - (2^{2/3} 7^{5/6}) * (7^{1/2} * i - 7)^{1/3} * i) / 28 - (2^{2/3} 3^{1/2} * 7^{5/6}) * (7^{1/2} * i - 7)^{1/3} / 28) * (3^{1/2} * i - 1) * (7^{1/2} * i - 7)^{1/3} / 84 - (2^{2/3} 7^{1/3} * \log(x - (2^{2/3} 7^{5/6}) * (7^{1/2} * i - 7)^{1/3} * i) / 28 + (2^{2/3} 3^{1/2} * 7^{5/6}) * (7^{1/2} * i - 7)^{1/3} / 28) * (3^{1/2} * i + 1) * (7^{1/2} * i - 7)^{1/3} / 84$

sympy [A] time = 0.15, size = 24, normalized size = 0.06

$$\text{RootSum}\left(250047t^6 + 1323t^3 + 2, \left(t \mapsto t \log\left(7938t^4 + 21t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**6+x**3+2),x)

[Out] RootSum(250047*_t**6 + 1323*_t**3 + 2, Lambda(_t, _t*log(7938*_t**4 + 21*_t + x)))

$$3.172 \quad \int x^{14} \sqrt{a + bx^3 + cx^6} dx$$

Optimal. Leaf size=231

$$\frac{(b^2 - 4ac)(16a^2c^2 - 56ab^2c + 21b^4) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}} + \frac{(16a^2c^2 - 56ab^2c + 21b^4)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{1536c^5}$$

Rubi [A] time = 0.30, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 742, 832, 779, 612, 621, 206}

$$\frac{(16a^2c^2 - 56ab^2c + 21b^4)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{1536c^5} - \frac{(b^2 - 4ac)(16a^2c^2 - 56ab^2c + 21b^4) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}} - \frac{(7b(15b^2 - 28ac) - 6cx^3(21b^2 - 20ac))(a + bx^3 + cx^6)^{3/2}}{2880c^4} - \frac{bx^6(a + bx^3 + cx^6)^{3/2}}{20c^2} + \frac{x^9(a + bx^3 + cx^6)^{3/2}}{18c}$$

Antiderivative was successfully verified.

[In] Int[x¹⁴*Sqrt[a + b*x³ + c*x⁶],x]

[Out] ((21*b⁴ - 56*a*b²*c + 16*a²*c²)*(b + 2*c*x³)*Sqrt[a + b*x³ + c*x⁶])/ (1536*c⁵) - (b*x⁶*(a + b*x³ + c*x⁶)^(3/2))/(20*c²) + (x⁹*(a + b*x³ + c*x⁶)^(3/2))/(18*c) - ((7*b*(15*b² - 28*a*c) - 6*c*(21*b² - 20*a*c)*x³)*(a + b*x³ + c*x⁶)^(3/2))/(2880*c⁴) - ((b² - 4*a*c)*(21*b⁴ - 56*a*b²*c + 16*a²*c²)*ArcTanh[(b + 2*c*x³)/(2*Sqrt[c]*Sqrt[a + b*x³ + c*x⁶])])/(3072*c^(11/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 742


```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegerQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{14}\sqrt{a+bx^3+cx^6} dx &= \frac{1}{3} \text{Subst}\left(\int x^4\sqrt{a+bx+cx^2} dx, x, x^3\right) \\
&= \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} + \frac{\text{Subst}\left(\int x^2\left(-3a-\frac{9bx}{2}\right)\sqrt{a+bx+cx^2} dx, x, x^3\right)}{18c} \\
&= -\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} + \frac{\text{Subst}\left(\int x\left(9ab+\frac{3}{4}(21b^2-20ac)\right)\sqrt{a+bx+cx^2} dx, x, x^3\right)}{90c^2} \\
&= -\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} - \frac{(7b(15b^2-28ac)-6c(21b^2-20ac))\sqrt{a+bx+cx^2}}{2880c^4} \\
&= \frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} \\
&= \frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c} \\
&= \frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5} - \frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2} + \frac{x^9(a+bx^3+cx^6)^{3/2}}{18c}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 208, normalized size = 0.90

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}(16bc^2(113a^2-34acx^6+8c^2x^{12})+160c^3x^3(-3a^2+2acx^6+8c^2x^{12})+168b^3c(cx^6-10a)+16b^2c^2x^3(56a-9cx^6)+315b^5-210b^4cx^3)-15(-64a^3c^3+240a^2b^2c^2-140ab^4c+21b^6)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{46080c^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^14*Sqrt[a + b*x^3 + c*x^6], x]

[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 210*b^4*c*x^3 + 16*b^2*c^2*x^3*(56*a - 9*c*x^6) + 168*b^3*c*(-10*a + c*x^6) + 16*b*c^2*(113*a^2 - 34*a*c*x^6 + 8*c^2*x^12) + 160*c^3*x^3*(-3*a^2 + 2*a*c*x^6 + 8*c^2*x^12)) - 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(46080*c^(11/2))

IntegrateAlgebraic [A] time = 0.80, size = 209, normalized size = 0.90

$$\frac{\sqrt{a+bx^3+cx^6}(1808a^2bc^2-480a^2c^3x^3-1680ab^3c+896ab^2c^2x^3-544abc^3x^6+320ac^4x^9+315b^5-210b^4cx^3+168b^3c^2x^6-144b^2c^3x^9+128bc^4x^{12}+1280c^5x^{15})}{23040c^5} + \frac{(-64a^3c^3+240a^2b^2c^2-140ab^4c+21b^6)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6}+b+2cx^3\right)}{3072c^{11/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14*Sqrt[a + b*x^3 + c*x^6], x]

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 1680*a*b^3*c + 1808*a^2*b*c^2 - 210*b^4*c*x^3 + 896*a*b^2*c^2*x^3 - 480*a^2*c^3*x^3 + 168*b^3*c^2*x^6 - 544*a*b*c^3*x^6 - 144*b^2*c^3*x^9 + 320*a*c^4*x^9 + 128*b*c^4*x^12 + 1280*c^5*x^15))/(23040*c^5) + ((21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(3072*c^(11/2))
```

fricas [A] time = 1.35, size = 451, normalized size = 1.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c) *log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b) *sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*(9*b^2*c^4 - 20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6, 1/46080*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*(9*b^2*c^4 - 20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a} x^{14} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^14, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a} x^{14} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴*(c*x⁶+b*x³+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b²>0)', see `assume?` for more details)Is 4*a*c-b² positive, negative or zero?

mupad [B] time = 2.94, size = 543, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁴*(a + b*x³ + c*x⁶)^(1/2),x)

[Out] (x⁹*(a + b*x³ + c*x⁶)^(3/2))/(18*c) - (b*((x⁶*(a + b*x³ + c*x⁶)^(3/2))/(5*c) + (7*b*((a*((b/(4*c) + x^{3/2})*(a + b*x³ + c*x⁶)^(1/2) + (log((a + b*x³ + c*x⁶)^(1/2) + (b/2 + c*x³)/c^{(1/2)))*(a*c - b²/4))/(2*c^{(3/2)))))/(4*c) - (x³*(a + b*x³ + c*x⁶)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x⁶) - 3*b² + 2*b*c*x³)*(a + b*x³ + c*x⁶)^{(1/2)))/(24*c²) + (log(2*(a + b*x³ + c*x⁶)^(1/2) + (b + 2*c*x³)/c^{(1/2)))*(b³ - 4*a*b*c))/(16*c^{(5/2)))))/(8*c) - (2*a*(((8*c*(a + c*x⁶) - 3*b² + 2*b*c*x³)*(a + b*x³ + c*x⁶)^{(1/2)))/(24*c²) + (log(2*(a + b*x³ + c*x⁶)^(1/2) + (b + 2*c*x³)/c^{(1/2)))*(b³ - 4*a*b*c))/(16*c^{(5/2)))))/(5*c)))/(4*c) + (a*((a*((b/(4*c) + x^{3/2})*(a + b*x³ + c*x⁶)^(1/2) + (log((a + b*x³ + c*x⁶)^(1/2) + (b/2 + c*x³)/c^{(1/2)))*(a*c - b²/4))/(2*c^{(3/2)))))/(4*c) - (x³*(a + b*x³ + c*x⁶)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x⁶) - 3*b² + 2*b*c*x³)*(a + b*x³ + c*x⁶)^{(1/2)))/(24*c²) + (log(2*(a + b*x³ + c*x⁶)^(1/2) + (b + 2*c*x³)/c^{(1/2)))*(b³ - 4*a*b*c))/(16*c^{(5/2)))))/(8*c)))/(6*c)}}}}}}}}}}}}}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14} \sqrt{a + bx^3 + cx^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(x**14*sqrt(a + b*x**3 + c*x**6), x)

$$3.173 \quad \int x^{11} \sqrt{a + bx^3 + cx^6} dx$$

Optimal. Leaf size=171

$$\frac{b(7b^2 - 12ac)(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}} - \frac{b(7b^2 - 12ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{384c^4} + \frac{(-32ac + 35b^2 - 42bcx^3)(a + bx^3 + cx^6)^{3/2}}{720c^3} - \frac{b(7b^2 - 12ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{384c^4} + \frac{b(7b^2 - 12ac)(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}} + \frac{x^6(a + bx^3 + cx^6)^{3/2}}{15c}$$

Rubi [A] time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 742, 779, 612, 621, 206}

$$\frac{(-32ac + 35b^2 - 42bcx^3)(a + bx^3 + cx^6)^{3/2}}{720c^3} - \frac{b(7b^2 - 12ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{384c^4} + \frac{b(7b^2 - 12ac)(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}} + \frac{x^6(a + bx^3 + cx^6)^{3/2}}{15c}$$

Antiderivative was successfully verified.

[In] Int[x^11*Sqrt[a + b*x^3 + c*x^6], x]

[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*c^4) + (x^6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + ((35*b^2 - 32*a*c - 42*b*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(720*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{11} \sqrt{a + bx^3 + cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int x^3 \sqrt{a + bx + cx^2} dx, x, x^3 \right) \\
&= \frac{x^6 (a + bx^3 + cx^6)^{3/2}}{15c} + \frac{\text{Subst} \left(\int x \left(-2a - \frac{7bx}{2} \right) \sqrt{a + bx + cx^2} dx, x, x^3 \right)}{15c} \\
&= \frac{x^6 (a + bx^3 + cx^6)^{3/2}}{15c} + \frac{(35b^2 - 32ac - 42bcx^3) (a + bx^3 + cx^6)^{3/2}}{720c^3} - \frac{(b(7b^2 - 12ac))}{15c} \\
&= -\frac{b(7b^2 - 12ac)(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{384c^4} + \frac{x^6 (a + bx^3 + cx^6)^{3/2}}{15c} + \frac{(35b^2 - 32ac)}{15c} \\
&= -\frac{b(7b^2 - 12ac)(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{384c^4} + \frac{x^6 (a + bx^3 + cx^6)^{3/2}}{15c} + \frac{(35b^2 - 32ac)}{15c} \\
&= -\frac{b(7b^2 - 12ac)(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{384c^4} + \frac{x^6 (a + bx^3 + cx^6)^{3/2}}{15c} + \frac{(35b^2 - 32ac)}{15c}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 164, normalized size = 0.96

$$\frac{\frac{(32ac - 35b^2 + 42bcx^3)(a + bx^3 + cx^6)^{3/2}}{48c^2} + \frac{5(12abc - 7b^3) \left(2\sqrt{c} (b + 2cx^3) \sqrt{a + bx^3 + cx^6} - (b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right) \right)}{256c^{7/2}}}{15c} + x^6 (a + bx^3 + cx^6)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x¹¹*Sqrt[a + b*x³ + c*x⁶], x]

[Out] (x⁶*(a + b*x³ + c*x⁶)^(3/2) - (((-35*b² + 32*a*c + 42*b*c*x³)*(a + b*x³ + c*x⁶)^(3/2))/(48*c²) + (5*(-7*b³ + 12*a*b*c)*(2*Sqrt[c]*(b + 2*c*x³))*Sqrt[a + b*x³ + c*x⁶] - (b² - 4*a*c)*ArcTanh[(b + 2*c*x³)/(2*Sqrt[c]*Sqrt[a + b*x³ + c*x⁶])]))/(256*c^(7/2)))/(15*c)

IntegrateAlgebraic [A] time = 0.58, size = 170, normalized size = 0.99

$$\frac{(-48a^2bc^2 + 40ab^3c - 7b^5) \log \left(\frac{-2c^{9/2} \sqrt{a + bx^3 + cx^6} + bc^4 + 2c^5x^3}{768c^{9/2}} \right) + \frac{\sqrt{a + bx^3 + cx^6} (-256a^2c^2 + 460ab^2c - 232abc^2x^3 + 128ac^3x^6 - 105b^4 + 70b^3cx^3 - 56b^2c^2x^6 + 48bc^3x^9 + 384c^4x^{12})}{5760c^4}}{15c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x¹¹*Sqrt[a + b*x³ + c*x⁶], x]

[Out] (Sqrt[a + b*x³ + c*x⁶]*(-105*b⁴ + 460*a*b²*c - 256*a²*c² + 70*b³*c*x³ - 232*a*b*c²*x³ - 56*b²*c²*x⁶ + 128*a*c³*x⁶ + 48*b*c³*x⁹ + 384*

$c^4 x^{12}) / (5760 c^4) + ((-7 b^5 + 40 a b^3 c - 48 a^2 b c^2) \operatorname{Log}[b c^4 + 2 c^5 x^3 - 2 c^{(9/2)} \sqrt{a + b x^3 + c x^6}]) / (768 c^{(9/2)})$

fricas [A] time = 1.75, size = 367, normalized size = 2.15

$$\frac{15(7b^5 - 40ab^3c + 48a^2bc^2)\sqrt{c} \log\left(\frac{-8c^2x^6 - 8b^2c^2x^3 - b^2 - 4\sqrt{c}\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4ac}{2304c^4}\right) + 4(384c^5x^{12} + 48b^2c^4x^9 - 8(7b^2c^3 - 16ac^4)x^6 - 105b^4c^4 + 460ab^2c^2 - 256a^2c^3 + 2(35b^3c^2 - 116abc^3)x^3)\sqrt{cx^6 + bx^3 + a}}{11520c^5} - \frac{15(7b^5 - 40ab^3c + 48a^2bc^2)\sqrt{c} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}}{\sqrt{c}\sqrt{cx^6 + bx^3 + a}}\right) - 2(384c^5x^{12} + 48b^2c^4x^9 - 8(7b^2c^3 - 16ac^4)x^6 - 105b^4c^4 + 460ab^2c^2 - 256a^2c^3 + 2(35b^3c^2 - 116abc^3)x^3)\sqrt{cx^6 + bx^3 + a}}{11520c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(c*x⁶+b*x³+a)^(1/2),x, algorithm="fricas")

[Out] [1/23040*(15*(7*b⁵ - 40*a*b³*c + 48*a²*b*c²)*sqrt(c)*log(-8*c²*x⁶ - 8*b*c*x³ - b² - 4*sqrt(c*x⁶ + b*x³ + a)*(2*c*x³ + b)*sqrt(c) - 4*a*c) + 4*(384*c⁵*x¹² + 48*b*c⁴*x⁹ - 8*(7*b²*c³ - 16*a*c⁴)*x⁶ - 105*b⁴*c + 460*a*b²*c² - 256*a²*c³ + 2*(35*b³*c² - 116*a*b*c³)*x³)*sqrt(c*x⁶ + b*x³ + a))/c⁵, -1/11520*(15*(7*b⁵ - 40*a*b³*c + 48*a²*b*c²)*sqrt(-c)*arctan(1/2*sqrt(c*x⁶ + b*x³ + a)*(2*c*x³ + b)*sqrt(-c)/(c²*x⁶ + b*c*x³ + a*c)) - 2*(384*c⁵*x¹² + 48*b*c⁴*x⁹ - 8*(7*b²*c³ - 16*a*c⁴)*x⁶ - 105*b⁴*c + 460*a*b²*c² - 256*a²*c³ + 2*(35*b³*c² - 116*a*b*c³)*x³)*sqrt(c*x⁶ + b*x³ + a))/c⁵]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a} x^{11} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(c*x⁶+b*x³+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x⁶ + b*x³ + a)*x¹¹, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a} x^{11} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹*(c*x⁶+b*x³+a)^(1/2),x)

[Out] int(x¹¹*(c*x⁶+b*x³+a)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹*(c*x⁶+b*x³+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b²>0)', see `assume?` for more details)Is 4*a*c-b² positive, negative or zero?

mupad [B] time = 1.87, size = 315, normalized size = 1.84

$$\frac{x^6(c x^6 + b x^3 + a)^{3/2}}{15c} + \frac{7b \left(\frac{\left(\frac{b}{4c} + \frac{x^3}{2} \right) \sqrt{c x^6 + b x^3 + a} + \frac{\ln \left(\frac{\sqrt{c x^6 + b x^3 + a} + \frac{c x^3 + b}{\sqrt{c}} \right) \left(c - \frac{b^2}{4} \right)}{2c^{3/2}}}{4c} - \frac{x^3(c x^6 + b x^3 + a)^{3/2}}{4c} + \frac{5b \left(\frac{(8c(c x^6 + a) - 3b^2 + 2bcx^3) \sqrt{c x^6 + b x^3 + a} + \ln \left(\frac{2 \sqrt{c x^6 + b x^3 + a} + \frac{2cx^3 + b}{\sqrt{c}} \right) (b^2 - 4abc)}{24c^2} \right)}{8c} \right)}{30c} - \frac{2a \left(\frac{(8c(c x^6 + a) - 3b^2 + 2bcx^3) \sqrt{c x^6 + b x^3 + a} + \ln \left(\frac{2 \sqrt{c x^6 + b x^3 + a} + \frac{2cx^3 + b}{\sqrt{c}} \right) (b^2 - 4abc)}{24c^2} \right)}{15c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹*(a + b*x³ + c*x⁶)^(1/2),x)

[Out] (x⁶*(a + b*x³ + c*x⁶)^(3/2))/(15*c) + (7*b*((a*((b/(4*c) + x³/2)*(a + b*x³ + c*x⁶)^(1/2) + (log((a + b*x³ + c*x⁶)^(1/2) + (b/2 + c*x³)/c^{(1/2)))*(a*c - b²/4))/(2*c^(3/2))))/(4*c) - (x³*(a + b*x³ + c*x⁶)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x⁶) - 3*b² + 2*b*c*x³)*(a + b*x³ + c*x⁶)^(1/2)))/(24*c²) + (log(2*(a + b*x³ + c*x⁶)^(1/2) + (b + 2*c*x³)/c^(1/2))*(b³ - 4*a*b*c))/(16*c^(5/2)))/(8*c))/(30*c) - (2*a*(((8*c*(a + c*x⁶) - 3*b² + 2*b*c*x³)*(a + b*x³ + c*x⁶)^(1/2))/(24*c²) + (log(2*(a + b*x³ + c*x⁶)^(1/2) + (b + 2*c*x³)/c^(1/2))*(b³ - 4*a*b*c))/(16*c^(5/2))))/(15*c)}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11} \sqrt{a + bx^3 + cx^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(x**11*sqrt(a + b*x**3 + c*x**6), x)

3.174 $\int x^8 \sqrt{a + bx^3 + cx^6} dx$

Optimal. Leaf size=153

$$-\frac{(b^2 - 4ac)(5b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}} + \frac{(5b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{192c^3} - \frac{5b(a + bx^3 + cx^6)^{3/2}}{72c^2} +$$

Rubi [A] time = 0.14, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 742, 640, 612, 621, 206}

$$\frac{(5b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{192c^3} - \frac{(b^2 - 4ac)(5b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}} - \frac{5b(a + bx^3 + cx^6)^{3/2}}{72c^2} + \frac{x^3(a + bx^3 + cx^6)^{3/2}}{12c}$$

Antiderivative was successfully verified.

[In] Int[x^8*Sqrt[a + b*x^3 + c*x^6], x]

[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*c^3) - (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72*c^2) + (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int x^8 \sqrt{a + bx^3 + cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int x^2 \sqrt{a + bx + cx^2} dx, x, x^3 \right) \\
 &= \frac{x^3 (a + bx^3 + cx^6)^{3/2}}{12c} + \frac{\text{Subst} \left(\int \left(-a - \frac{5bx}{2} \right) \sqrt{a + bx + cx^2} dx, x, x^3 \right)}{12c} \\
 &= -\frac{5b (a + bx^3 + cx^6)^{3/2}}{72c^2} + \frac{x^3 (a + bx^3 + cx^6)^{3/2}}{12c} + \frac{(5b^2 - 4ac) \text{Subst} \left(\int \sqrt{a + bx + cx^2} dx, x, x^3 \right)}{48c^2} \\
 &= \frac{(5b^2 - 4ac) (b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{192c^3} - \frac{5b (a + bx^3 + cx^6)^{3/2}}{72c^2} + \frac{x^3 (a + bx^3 + cx^6)^{3/2}}{12c} \\
 &= \frac{(5b^2 - 4ac) (b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{192c^3} - \frac{5b (a + bx^3 + cx^6)^{3/2}}{72c^2} + \frac{x^3 (a + bx^3 + cx^6)^{3/2}}{12c} \\
 &= \frac{(5b^2 - 4ac) (b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{192c^3} - \frac{5b (a + bx^3 + cx^6)^{3/2}}{72c^2} + \frac{x^3 (a + bx^3 + cx^6)^{3/2}}{12c}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 136, normalized size = 0.89

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(b(8c^2x^6-52ac)+24c^2x^3(a+2cx^6)+15b^3-10b^2cx^3\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{1152c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*Sqrt[a + b*x^3 + c*x^6], x]

[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 10*b^2*c*x^3 + 24*c^2*x^3*(a + 2*c*x^6) + b*(-52*a*c + 8*c^2*x^6)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(1152*c^(7/2))

IntegrateAlgebraic [A] time = 0.42, size = 132, normalized size = 0.86

$$\frac{(16a^2c^2 - 24ab^2c + 5b^4)\log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6} + b + 2cx^3\right)}{384c^{7/2}} + \frac{\sqrt{a+bx^3+cx^6}\left(-52abc + 24ac^2x^3 + 15b^3 - 10b^2cx^3 + 8bc^2x^6 + 48c^3x^9\right)}{576c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8*Sqrt[a + b*x^3 + c*x^6], x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^3 + 24*a*c^2*x^3 + 8*b*c^2*x^6 + 48*c^3*x^9))/(576*c^3) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(7/2))

fricas [A] time = 1.22, size = 303, normalized size = 1.98

$$\frac{3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{c}\log\left(-8c^2x^6 - 8bcx^3 - b^2 + 4\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4ac\right) + 4(48c^4x^9 + 8b^3c^3x^6 + 15b^3c^2x^3 - 52abc^2x^3 - 2(5b^2c^2 - 12ac^3)x^3)\sqrt{cx^6 + bx^3 + a} - 3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}\sqrt{c}}{2(b^2 + bcx^3 + ac)}\right) + 2(48c^4x^9 + 8b^3c^3x^6 + 15b^3c^2x^3 - 52abc^2 - 2(5b^2c^2 - 12ac^3)x^3)\sqrt{cx^6 + bx^3 + a}}{2304c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c^2*x^3 - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/1152*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c^2*x^3 - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a} x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^8, x)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a} x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 1.59, size = 193, normalized size = 1.26

$$\frac{x^3 (cx^6 + bx^3 + a)^{3/2}}{12c} - \frac{a \left(\left(\frac{b}{4c} + \frac{x^3}{2} \right) \sqrt{cx^6 + bx^3 + a} + \frac{\ln \left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + \frac{b}{2}}{\sqrt{c}} \right) \left(ac - \frac{b^2}{4} \right)}{2c^{3/2}} \right)}{12c} - \frac{5b \left(\frac{(8c(cx^6+a)-3b^2+2bcx^3)\sqrt{cx^6+bx^3+a}}{24c^2} + \frac{\ln \left(2\sqrt{cx^6+bx^3+a} + \frac{2cx^3+b}{\sqrt{c}} \right) (b^3-4abc)}{16c^{5/2}} \right)}{24c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(a + b*x^3 + c*x^6)^(1/2),x)

[Out] (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c) - (a*((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2)))*(a*c - b^2/4))/(2*c^(3/2)))/(12*c) - (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(24*c)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \sqrt{a + bx^3 + cx^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8*sqrt(a + b*x**3 + c*x**6), x)
```

3.175 $\int x^5 \sqrt{a + bx^3 + cx^6} dx$

Optimal. Leaf size=108

$$\frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}} - \frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{(a+bx^3+cx^6)^{3/2}}{9c}$$

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 640, 612, 621, 206}

$$\frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}} - \frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{(a+bx^3+cx^6)^{3/2}}{9c}$$

Antiderivative was successfully verified.

[In] Int[x^5*Sqrt[a + b*x^3 + c*x^6],x]

[Out] -(b*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c^2) + (a + b*x^3 + c*x^6)^(3/2)/(9*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

```
Int[((d._) + (e._)*(x._))*((a._) + (b._)*(x._) + (c._)*(x._)^2)^(p._), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m._)*((a_) + (c._)*(x_)^(n2._) + (b._)*(x_)^(n._))^(p._), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt{a + bx^3 + cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int x \sqrt{a + bx + cx^2} dx, x, x^3 \right) \\
&= \frac{(a + bx^3 + cx^6)^{3/2}}{9c} - \frac{b \text{Subst} \left(\int \sqrt{a + bx + cx^2} dx, x, x^3 \right)}{6c} \\
&= -\frac{b(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{24c^2} + \frac{(a + bx^3 + cx^6)^{3/2}}{9c} + \frac{(b(b^2 - 4ac)) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{48c^2} \\
&= -\frac{b(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{24c^2} + \frac{(a + bx^3 + cx^6)^{3/2}}{9c} + \frac{(b(b^2 - 4ac)) \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, x^3 \right)}{24c^2} \\
&= -\frac{b(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{24c^2} + \frac{(a + bx^3 + cx^6)^{3/2}}{9c} + \frac{b(b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{48c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.92

$$\frac{(b^3 - 4abc) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{48c^{5/2}} + \frac{\sqrt{a + bx^3 + cx^6} (8c(a + cx^6) - 3b^2 + 2bcx^3)}{72c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3 + c*x^6], x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 2*b*c*x^3 + 8*c*(a + c*x^6)))/(72*c^2) +
((b^3 - 4*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])
]/(48*c^(5/2)))
```


IntegrateAlgebraic [A] time = 0.32, size = 107, normalized size = 0.99

$$\frac{(4abc - b^3) \log\left(-2c^{5/2} \sqrt{a + bx^3 + cx^6} + bc^2 + 2c^3x^3\right)}{48c^{5/2}} + \frac{\sqrt{a + bx^3 + cx^6} (8ac - 3b^2 + 2bcx^3 + 8c^2x^6)}{72c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*Sqrt[a + b*x^3 + c*x^6], x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*a*c + 2*b*c*x^3 + 8*c^2*x^6))/(72*c^2) + ((-b^3 + 4*a*b*c)*Log[b*c^2 + 2*c^3*x^3 - 2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(48*c^(5/2))

fricas [A] time = 1.22, size = 237, normalized size = 2.19

$$\frac{3(b^3 - 4abc)\sqrt{c} \log\left(\frac{-8c^2x^6 - 8b^2cx^3 - b^2 + 4\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4(8c^3x^6 + 2bc^2x^3 - 3b^2c + 8ac^2)\sqrt{cx^6 + bx^3 + a}}{288c^3}\right) - 2(8c^3x^6 + 2bc^2x^3 - 3b^2c + 8ac^2)\sqrt{cx^6 + bx^3 + a}}{144c^3} - \frac{3(b^3 - 4abc)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{-c}}{2(c^2x^6 + bc^2x^3 + ac)}\right)}{144c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b^2*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 + 2*b*c^2*x^3 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/144*(3*(b^3 - 4*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(8*c^3*x^6 + 2*b*c^2*x^3 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3]

giac [A] time = 0.49, size = 98, normalized size = 0.91

$$\frac{1}{72} \sqrt{cx^6 + bx^3 + a} \left(2 \left(4x^3 + \frac{b}{c} \right) x^3 - \frac{3b^2 - 8ac}{c^2} \right) - \frac{(b^3 - 4abc) \log\left(\left| -2 \left(\sqrt{c} x^3 - \sqrt{cx^6 + bx^3 + a} \right) \sqrt{c} - b \right|\right)}{48c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2), x, algorithm="giac")

[Out] 1/72*sqrt(c*x^6 + b*x^3 + a)*(2*(4*x^3 + b/c)*x^3 - (3*b^2 - 8*a*c)/c^2) - 1/48*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(5/2)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a} x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(c*x^6+b*x^3+a)^(1/2),x)`

[Out] `int(x^5*(c*x^6+b*x^3+a)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 1.39, size = 87, normalized size = 0.81

$$\frac{(8c(c x^6 + a) - 3b^2 + 2bcx^3) \sqrt{c x^6 + b x^3 + a}}{72c^2} + \frac{\ln\left(2\sqrt{c x^6 + b x^3 + a} + \frac{2cx^3+b}{\sqrt{c}}\right) (b^3 - 4abc)}{48c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a + b*x^3 + c*x^6)^(1/2),x)`

[Out] `((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(72*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(48*c^(5/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \sqrt{a + bx^3 + cx^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)`

[Out] `Integral(x**5*sqrt(a + b*x**3 + c*x**6), x)`

$$3.176 \quad \int x^2 \sqrt{a + bx^3 + cx^6} dx$$

Optimal. Leaf size=83

$$\frac{(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{12c} - \frac{(b^2 - 4ac) \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c} \sqrt{a+bx^3+cx^6}} \right)}{24c^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1352, 612, 621, 206}

$$\frac{(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{12c} - \frac{(b^2 - 4ac) \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c} \sqrt{a+bx^3+cx^6}} \right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a + b*x^3 + c*x^6],x]

[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,

b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
 \int x^2 \sqrt{a + bx^3 + cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int \sqrt{a + bx + cx^2} dx, x, x^3 \right) \\
 &= \frac{(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{12c} - \frac{(b^2 - 4ac) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{24c} \\
 &= \frac{(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{12c} - \frac{(b^2 - 4ac) \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx^3}{\sqrt{a + bx^3 + cx^6}} \right)}{12c} \\
 &= \frac{(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{12c} - \frac{(b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{24c^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 87, normalized size = 1.05

$$\frac{1}{3} \left(\frac{(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{4c} - \frac{(b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{8c^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a + b*x^3 + c*x^6],x]

[Out] (((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(3/2)))/3

IntegrateAlgebraic [A] time = 0.25, size = 85, normalized size = 1.02

$$\frac{(b^2 - 4ac) \log \left(-2c^{3/2} \sqrt{a + bx^3 + cx^6} + bc + 2c^2 x^3 \right)}{24c^{3/2}} + \frac{(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{12c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*Sqrt[a + b*x^3 + c*x^6],x]

[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) + ((b^2 - 4*a*c)*Log[b*c + 2*c^2*x^3 - 2*c^(3/2)*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(3/2))

fricas [A] time = 1.34, size = 197, normalized size = 2.37

$$\left[\frac{(b^2 - 4ac)\sqrt{c} \log\left(-8c^2x^6 - 8bcx^3 - b^2 - 4\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4\sqrt{cx^6 + bx^3 + a}(2c^2x^3 + bc)\right)}{48c^2}, \frac{(b^2 - 4ac)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{-c}}{2(-2x^6 + bcx^3 + ac)}\right) + 2\sqrt{cx^6 + bx^3 + a}(2c^2x^3 + bc)}{24c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [-1/48*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2, 1/24*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2]

giac [A] time = 0.56, size = 76, normalized size = 0.92

$$\frac{1}{12} \sqrt{cx^6 + bx^3 + a} \left(2x^3 + \frac{b}{c}\right) + \frac{(b^2 - 4ac) \log\left(\left|-2\left(\sqrt{c}x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{24c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] 1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3/2)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{cx^6 + bx^3 + a} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 1.39, size = 72, normalized size = 0.87

$$\frac{\left(\frac{b}{4c} + \frac{x^3}{2}\right) \sqrt{cx^6 + bx^3 + a}}{3} + \frac{\ln\left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + \frac{b}{2}}{\sqrt{c}}\right) \left(ac - \frac{b^2}{4}\right)}{6c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*x^3 + c*x^6)^(1/2),x)

[Out] ((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2))/3 + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(6*c^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + bx^3 + cx^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(x**2*sqrt(a + b*x**3 + c*x**6), x)

$$3.177 \quad \int \frac{\sqrt{a+bx^3+cx^6}}{x} dx$$

Optimal. Leaf size=109

$$\frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}\sqrt{a} \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Rubi [A] time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 734, 843, 621, 206, 724}

$$\frac{1}{3}\sqrt{a+bx^3+cx^6} - \frac{1}{3}\sqrt{a} \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + \frac{b \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^3 + c*x^6]/x,x]

[Out] Sqrt[a + b*x^3 + c*x^6]/3 - (Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/3 + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(6*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

```

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
&& GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0]
&& IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1357

```

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + bx^3 + cx^6}}{x} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a + bx + cx^2}}{x} dx, x, x^3 \right) \\
&= \frac{1}{3} \sqrt{a + bx^3 + cx^6} - \frac{1}{6} \text{Subst} \left(\int \frac{-2a - bx}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= \frac{1}{3} \sqrt{a + bx^3 + cx^6} + \frac{1}{3} a \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right) + \frac{1}{6} b \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= \frac{1}{3} \sqrt{a + bx^3 + cx^6} - \frac{1}{3} (2a) \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx^3}{\sqrt{a + bx^3 + cx^6}} \right) + \frac{1}{3} b \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{2a + bx^3}{\sqrt{a + bx^3 + cx^6}} \right) \\
&= \frac{1}{3} \sqrt{a + bx^3 + cx^6} - \frac{1}{3} \sqrt{a} \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right) + \frac{b \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{6\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 106, normalized size = 0.97

$$\frac{1}{6} \left(2\sqrt{a + bx^3 + cx^6} - 2\sqrt{a} \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right) + \frac{b \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{\sqrt{c}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x,x]

[Out] (2*Sqrt[a + b*x^3 + c*x^6] - 2*Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]]) + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])))/Sqrt[c])/6

IntegrateAlgebraic [A] time = 0.29, size = 111, normalized size = 1.02

$$\frac{1}{3} \sqrt{a + bx^3 + cx^6} - \frac{b \log \left(-2\sqrt{c} \sqrt{a + bx^3 + cx^6} + b + 2cx^3 \right)}{6\sqrt{c}} + \frac{2}{3} \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{c} x^3}{\sqrt{a}} - \frac{\sqrt{a + bx^3 + cx^6}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x,x]

[Out] Sqrt[a + b*x^3 + c*x^6]/3 + (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a]])/3 - (b*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(6*Sqrt[c])

fricas [A] time = 1.37, size = 566, normalized size = 5.19

$$\frac{1}{12} \sqrt{c} \log(-8c^2x^6 - 8bcx^3 - b^2 - 4\sqrt{c}\sqrt{a + bx^3 + cx^6} + b^2 + 4ac) + 2\sqrt{a}c \log(-((b^2 + 4ac)x^6 + 8abx^3 - 4\sqrt{c}\sqrt{a + bx^3 + a}(bx^3 + 2a))\sqrt{a} + 8a^2)/x^6) + 4\sqrt{c}\sqrt{a + bx^3 + a}c/c, -1/6(b\sqrt{-c})\arctan(1/2\sqrt{c}\sqrt{a + bx^3 + a})/(c^2x^6 + bcx^3 + ac) - \sqrt{a}c \log(-((b^2 + 4ac)x^6 + 8abx^3 - 4\sqrt{c}\sqrt{a + bx^3 + a}(bx^3 + 2a))\sqrt{a} + 8a^2)/x^6) - 2\sqrt{c}\sqrt{a + bx^3 + a}c/c, 1/12(4\sqrt{-a}c \arctan(1/2\sqrt{c}\sqrt{a + bx^3 + a}(bx^3 + 2a))\sqrt{-a}/(acx^6 + abx^3 + a^2)) + b\sqrt{c} \log(-8c^2x^6 - 8bcx^3 - b^2 - 4\sqrt{c}\sqrt{a + bx^3 + a}(bx^3 + 2a))\sqrt{c} - 4ac) + 4\sqrt{c}\sqrt{a + bx^3 + a}c/c, 1/6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="fricas")

[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c)*sqrt(a + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c)*sqrt(a + b*x^3 + a)*(b*x^3 + 2*a))*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c)*sqrt(a + b*x^3 + a)*c/c, -1/6*(b*sqrt(-c))*arctan(1/2*sqrt(c)*sqrt(a + b*x^3 + a))/(c^2*x^6 + b*c*x^3 + a*c) - sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c)*sqrt(a + b*x^3 + a)*(b*x^3 + 2*a))*sqrt(a) + 8*a^2)/x^6) - 2*sqrt(c)*sqrt(a + b*x^3 + a)*c/c, 1/12*(4*sqrt(-a)*c*arctan(1/2*sqrt(c)*sqrt(a + b*x^3 + a)*(b*x^3 + 2*a))*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c)*sqrt(a + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c)*sqrt(a + b*x^3 + a)*c/c, 1/6

```
*(2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a
*c*x^6 + a*b*x^3 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a
*c)/c]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.36, size = 88, normalized size = 0.81

$$\frac{\sqrt{cx^6 + bx^3 + a}}{3} - \frac{\sqrt{a} \ln\left(\frac{b}{2} + \frac{a}{x^3} + \frac{\sqrt{a} \sqrt{cx^6 + bx^3 + a}}{x^3}\right)}{3} + \frac{b \ln\left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + \frac{b}{2}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3 + c*x^6)^(1/2)/x,x)`

[Out] $(a + b*x^3 + c*x^6)^{1/2}/3 - (a^{1/2}*\log(b/2 + a/x^3 + (a^{1/2}*(a + b*x^3 + c*x^6)^{1/2})/x^3))/3 + (b*\log((a + b*x^3 + c*x^6)^{1/2} + (b/2 + c*x^3)/c^{1/2}))/6*c^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**6+b*x**3+a)**(1/2)/x,x)`

[Out] `Integral(sqrt(a + b*x**3 + c*x**6)/x, x)`

$$3.178 \quad \int \frac{\sqrt{a+bx^3+cx^6}}{x^4} dx$$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c} \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 732, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c} \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^4,x]

[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*x^3) - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 732

```

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1357

```

Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + bx^3 + cx^6}}{x^4} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a + bx + cx^2}}{x^2} dx, x, x^3 \right) \\
&= -\frac{\sqrt{a + bx^3 + cx^6}}{3x^3} + \frac{1}{6} \text{Subst} \left(\int \frac{b + 2cx}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{a + bx^3 + cx^6}}{3x^3} + \frac{1}{6}b \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right) + \frac{1}{3}c \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{a + bx^3 + cx^6}}{3x^3} - \frac{1}{3}b \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx^3}{\sqrt{a + bx^3 + cx^6}} \right) + \frac{1}{3}(2c) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{a + bx^3 + cx^6}}{3x^3} - \frac{b \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}} \right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c} \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c}\sqrt{a + bx^3 + cx^6}} \right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 112, normalized size = 1.00

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c} \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^4,x]

[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/x^3 - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3

IntegrateAlgebraic [A] time = 0.30, size = 114, normalized size = 1.02

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{1}{3}\sqrt{c} \log\left(-2\sqrt{c}\sqrt{a+bx^3+cx^6} + b + 2cx^3\right) + \frac{b \tanh^{-1}\left(\frac{\sqrt{c}x^3}{\sqrt{a}} - \frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^4,x]

[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/x^3 + (b*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a]])/(3*Sqrt[a]) - (Sqrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/3

fricas [A] time = 1.57, size = 601, normalized size = 5.37

$$\frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right) - \frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right) - \frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right) - \frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right) - \frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right) - \frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right) - \frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right) - \frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right) - \frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right) - \frac{1}{12} \sqrt{c} \log\left(\frac{b^2 + 4ac}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")

[Out] [1/12*(2*a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), -1/12*(4*a*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) -

$2\sqrt{c*x^6 + b*x^3 + a}*a)/(a*x^3), 1/6*(\sqrt{-a}*b*x^3*\arctan(1/2*\sqrt{c*x^6 + b*x^3 + a}*(b*x^3 + 2*a)*\sqrt{-a}/(a*c*x^6 + a*b*x^3 + a^2)) - 2*a*\sqrt{-c}*x^3*\arctan(1/2*\sqrt{c*x^6 + b*x^3 + a}*(2*c*x^3 + b)*\sqrt{-c}/(c^2*x^6 + b*c*x^3 + a*c)) - 2*\sqrt{c*x^6 + b*x^3 + a}*a)/(a*x^3)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^4, x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)

[Out] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 1.55, size = 91, normalized size = 0.81

$$\frac{\sqrt{c} \ln\left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + \frac{b}{2}}{\sqrt{c}}\right)}{3} - \frac{\sqrt{cx^6 + bx^3 + a}}{3x^3} - \frac{b \ln\left(\frac{b}{2} + \frac{a}{x^3} + \frac{\sqrt{a} \sqrt{cx^6 + bx^3 + a}}{x^3}\right)}{6\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3 + c*x^6)^(1/2)/x^4,x)`

[Out] $(c^{1/2} \log((a + b x^3 + c x^6)^{1/2} + (b/2 + c x^3)/c^{1/2}))/3 - (a + b x^3 + c x^6)^{1/2}/(3 x^3) - (b \log(b/2 + a/x^3 + (a^{1/2})(a + b x^3 + c x^6)^{1/2})/x^3)/(6 a^{1/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)`

[Out] `Integral(sqrt(a + b*x**3 + c*x**6)/x**4, x)`

$$3.179 \quad \int \frac{\sqrt{a+bx^3+cx^6}}{x^7} dx$$

Optimal. Leaf size=88

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}} - \frac{(2a + bx^3) \sqrt{a + bx^3 + cx^6}}{12ax^6}$$

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1357, 720, 724, 206}

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}} - \frac{(2a + bx^3) \sqrt{a + bx^3 + cx^6}}{12ax^6}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^7,x]

[Out] -((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((b^2 - 4*a*c)*ArcTan h[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + bx^3 + cx^6}}{x^7} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a + bx + cx^2}}{x^3} dx, x, x^3 \right) \\ &= -\frac{(2a + bx^3) \sqrt{a + bx^3 + cx^6}}{12ax^6} - \frac{(b^2 - 4ac) \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{24a} \\ &= -\frac{(2a + bx^3) \sqrt{a + bx^3 + cx^6}}{12ax^6} + \frac{(b^2 - 4ac) \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx^3}{\sqrt{a + bx^3 + cx^6}} \right)}{12a} \\ &= -\frac{(2a + bx^3) \sqrt{a + bx^3 + cx^6}}{12ax^6} + \frac{(b^2 - 4ac) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right)}{24a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 1.01

$$\frac{(b^2 - 4ac) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right) - \frac{2\sqrt{a}(2a + bx^3) \sqrt{a + bx^3 + cx^6}}{x^6}}{24a^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^7, x]

[Out] ((-2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))

IntegrateAlgebraic [A] time = 0.38, size = 91, normalized size = 1.03

$$\frac{(4ac - b^2) \tanh^{-1} \left(\frac{\sqrt{c} x^3 - \sqrt{a + bx^3 + cx^6}}{\sqrt{a}} \right)}{12a^{3/2}} + \frac{(-2a - bx^3) \sqrt{a + bx^3 + cx^6}}{12ax^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^7,x]

[Out] $((-2*a - b*x^3)*\text{Sqrt}[a + b*x^3 + c*x^6])/(12*a*x^6) + ((-b^2 + 4*a*c)*\text{ArcTan}[\text{Sqrt}[c]*x^3 - \text{Sqrt}[a + b*x^3 + c*x^6])/\text{Sqrt}[a]]/(12*a^{(3/2)})$

fricas [A] time = 1.37, size = 215, normalized size = 2.44

$$\left[\frac{(b^2 - 4ac)\sqrt{a}x^6 \log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a+8a^2}}{x^6}\right) + 4\sqrt{cx^6+bx^3+a}(abx^3+2a^2)}{48a^2x^6}, \frac{(b^2-4ac)\sqrt{-a}x^6 \arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right) + 2\sqrt{cx^6+bx^3+a}(abx^3+2a^2)}{24a^2x^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")

[Out] $[-1/48*((b^2 - 4*a*c)*\text{sqrt}(a)*x^6*\log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a))*\text{sqrt}(a) + 8*a^2)/x^6) + 4*\text{sqrt}(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6), -1/24*((b^2 - 4*a*c)*\text{sqrt}(-a)*x^6*\arctan(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a))*\text{sqrt}(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*\text{sqrt}(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^7, x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)

[Out] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^(1/2)/x^7,x)

[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^7, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)

[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**7, x)

$$3.180 \quad \int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}} dx$$

Optimal. Leaf size=116

$$-\frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}} + \frac{b(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{24a^2x^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9ax^9}$$

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 730, 720, 724, 206}

$$-\frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}} + \frac{b(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{24a^2x^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^10,x]

[Out] (b*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/((24*a^2*x^6) - (a + b*x^3 + c*x^6)^(3/2)/(9*a*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(5/2)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 730

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a + bx^3 + cx^6}}{x^{10}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a + bx + cx^2}}{x^4} dx, x, x^3 \right) \\
 &= -\frac{(a + bx^3 + cx^6)^{3/2}}{9ax^9} - \frac{b \text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{x^3} dx, x, x^3 \right)}{6a} \\
 &= \frac{b(2a + bx^3) \sqrt{a + bx^3 + cx^6}}{24a^2x^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9ax^9} + \frac{(b(b^2 - 4ac)) \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x \right)}{48a^2} \\
 &= \frac{b(2a + bx^3) \sqrt{a + bx^3 + cx^6}}{24a^2x^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9ax^9} - \frac{(b(b^2 - 4ac)) \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x \right)}{24a^2} \\
 &= \frac{b(2a + bx^3) \sqrt{a + bx^3 + cx^6}}{24a^2x^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9ax^9} - \frac{b(b^2 - 4ac) \tanh^{-1} \left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}} \right)}{48a^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 108, normalized size = 0.93

$$-\frac{b(b^2 - 4ac) \tanh^{-1} \left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}} \right)}{48a^{5/2}} - \frac{\sqrt{a + bx^3 + cx^6} (8a^2 + 2ax^3(b + 4cx^3) - 3b^2x^6)}{72a^2x^9}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^10,x]

[Out]
$$\frac{-1/72*(\text{Sqrt}[a + b*x^3 + c*x^6]*(8*a^2 - 3*b^2*x^6 + 2*a*x^3*(b + 4*c*x^3)))/(a^2*x^9) - (b*(b^2 - 4*a*c)*\text{ArcTanh}[(2*a + b*x^3)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x^3 + c*x^6]))/(48*a^{5/2})}{24a^{5/2}} + \frac{\sqrt{a + bx^3 + cx^6} (-8a^2 - 2abx^3 - 8acx^6 + 3b^2x^6)}{72a^2x^9}$$

IntegrateAlgebraic [A] time = 0.65, size = 108, normalized size = 0.93

$$\frac{(b^3 - 4abc) \tanh^{-1}\left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{5/2}} + \frac{\sqrt{a + bx^3 + cx^6} (-8a^2 - 2abx^3 - 8acx^6 + 3b^2x^6)}{72a^2x^9}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^10,x]

[Out]
$$\frac{(\text{Sqrt}[a + b*x^3 + c*x^6]*(-8*a^2 - 2*a*b*x^3 + 3*b^2*x^6 - 8*a*c*x^6))/(72*a^2*x^9) + ((b^3 - 4*a*b*c)*\text{ArcTanh}[(\text{Sqrt}[c]*x^3 - \text{Sqrt}[a + b*x^3 + c*x^6])/\text{Sqrt}[a]])/(24*a^{5/2})}{288a^3x^9} + \frac{3(b^3 - 4abc)\sqrt{a}x^9 \log\left(\frac{(b^2+4ac)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a+8a^2}}{x^6}\right) - 4((3ab^2-8a^2c)x^6-2a^2bx^3-8a^3)\sqrt{cx^6+bx^3+a}}{288a^3x^9} - \frac{3(b^3-4abc)\sqrt{-a}x^9 \arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right) + 2((3ab^2-8a^2c)x^6-2a^2bx^3-8a^3)\sqrt{cx^6+bx^3+a}}{144a^3x^9}}$$

fricas [A] time = 1.36, size = 259, normalized size = 2.23

$$\frac{3(b^3 - 4abc)\sqrt{a}x^9 \log\left(\frac{(b^2+4ac)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a+8a^2}}{x^6}\right) - 4((3ab^2-8a^2c)x^6-2a^2bx^3-8a^3)\sqrt{cx^6+bx^3+a}}{288a^3x^9} - \frac{3(b^3-4abc)\sqrt{-a}x^9 \arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right) + 2((3ab^2-8a^2c)x^6-2a^2bx^3-8a^3)\sqrt{cx^6+bx^3+a}}{144a^3x^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/288*(3*(b^3 - 4*a*b*c)*\text{sqrt}(a)*x^9*\log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a))*\text{sqrt}(a) + 8*a^2)/x^6) - 4*((3*a*b^2 - 8*a^2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/(a^3*x^9) \\ & , 1/144*(3*(b^3 - 4*a*b*c)*\text{sqrt}(-a)*x^9*\arctan(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*\text{sqrt}(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 - 8*a^2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/(a^3*x^9)] \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^10, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)

[Out] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^(1/2)/x^10,x)

[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^10, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)

[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**10, x)

$$3.181 \quad \int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}} dx$$

Optimal. Leaf size=161

$$\frac{(b^2 - 4ac)(5b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}} - \frac{(5b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{192a^3x^6} + \frac{5b(a + bx^3 + cx^6)^{3/2}}{72a^2x^9}$$

Rubi [A] time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 744, 806, 720, 724, 206}

$$-\frac{(5b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{192a^3x^6} + \frac{(b^2 - 4ac)(5b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}} + \frac{5b(a + bx^3 + cx^6)^{3/2}}{72a^2x^9} - \frac{(a + bx^3 + cx^6)^{3/2}}{12ax^{12}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^13, x]

[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/((192*a^3*x^6) - (a + b*x^3 + c*x^6)^(3/2)/(12*a*x^12) + (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72*a^2*x^9) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(7/2)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 744

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{x^5} dx, x, x^3 \right) \\
&= \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} - \frac{\text{Subst} \left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4} dx, x, x^3 \right)}{12a} \\
&= -\frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} + \frac{(5b^2-4ac) \text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{x^3} dx, x, x^3 \right)}{48a^2} \\
&= -\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} \\
&= -\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} \\
&= -\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 139, normalized size = 0.86

$$\frac{(16a^2c^2 - 24ab^2c + 5b^4) \tanh^{-1} \left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}} \right) - \sqrt{a+bx^3+cx^6} (48a^3 + 8a^2x^3(b+3cx^3) - 2abx^6(5b+26cx^3) + 15b^3x^9)}{384a^{7/2} 576a^3x^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^13,x]

[Out] -1/576*(Sqrt[a + b*x^3 + c*x^6]*(48*a^3 + 15*b^3*x^9 + 8*a^2*x^3*(b + 3*c*x^3) - 2*a*b*x^6*(5*b + 26*c*x^3)))/(a^3*x^12) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(7/2))

IntegrateAlgebraic [A] time = 0.84, size = 141, normalized size = 0.88

$$\frac{(-16a^2c^2 + 24ab^2c - 5b^4) \tanh^{-1} \left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{\sqrt{a}} \right) + \sqrt{a+bx^3+cx^6} (-48a^3 - 8a^2bx^3 - 24a^2cx^6 + 10ab^2x^6 + 52abcx^9 - 15b^3x^9)}{192a^{7/2} 576a^3x^{12}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^13,x]

[Out] $(\sqrt{a + bx^3 + cx^6}) \cdot (-48a^3 - 8a^2bx^3 + 10ab^2x^6 - 24a^2cx^9 - 15b^3x^9 + 52abcx^9) / (576a^3x^{12}) + ((-5b^4 + 24ab^2c - 16a^2c^2) \cdot \text{ArcTanh}[(\sqrt{c}x^3 - \sqrt{a + bx^3 + cx^6}) / \sqrt{a}]) / (192a^{7/2})$

fricas [A] time = 1.40, size = 325, normalized size = 2.02

$$\frac{3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{a}x^{12} \log\left(-\frac{(b^2+4a)^2 + 8ab^3 + 4\sqrt{c^2(b^2+2a)^2 + 4a^2}}{2304a^4x^{12}}\right) - 4((15ab^3 - 52a^2bc)x^9 + 8a^3bx^3 - 2(5a^2b^2 - 12a^2c)x^6 + 48a^4)\sqrt{cx^6 + bx^3 + a}}{1152a^4x^{12}} + \frac{3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{-a}x^{12} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}}{2(\sqrt{a+bx^3+a})}\right) + 2((15ab^3 - 52a^2bc)x^9 + 8a^3bx^3 - 2(5a^2b^2 - 12a^2c)x^6 + 48a^4)\sqrt{cx^6 + bx^3 + a}}{1152a^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="fricas")

[Out] $[1/2304*(3*(5b^4 - 24ab^2c + 16a^2c^2)*\sqrt{a})x^{12}*\log(-((b^2 + 4a*c)x^6 + 8a*b*x^3 + 4*\sqrt{c*x^6 + b*x^3 + a})*(b*x^3 + 2*a)*\sqrt{a}) + 8*a^2)/x^6) - 4*((15*a*b^3 - 52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^6 + 48*a^4)*\sqrt{c*x^6 + b*x^3 + a})/(a^4*x^{12}), -1/1152*(3*(5b^4 - 24ab^2c + 16a^2c^2)*\sqrt{-a})x^{12}*\arctan(1/2*\sqrt{c*x^6 + b*x^3 + a})*(b*x^3 + 2*a)*\sqrt{-a}/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3 - 52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^6 + 48*a^4)*\sqrt{c*x^6 + b*x^3 + a})/(a^4*x^{12})]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^13, x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)

[Out] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^(1/2)/x^13,x)

[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^13, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)

[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**13, x)

$$3.182 \quad \int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}} dx$$

Optimal. Leaf size=199

$$\frac{b(7b^2 - 12ac)(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}} + \frac{b(7b^2 - 12ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{384a^4x^6} - \frac{(35b^2 - 32ac)(a + bx^3 + cx^6)^{3/2}}{720a^3x^9}$$

Rubi [A] time = 0.23, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 744, 834, 806, 720, 724, 206}

$$\frac{(35b^2 - 32ac)(a + bx^3 + cx^6)^{3/2}}{720a^3x^9} + \frac{b(7b^2 - 12ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{384a^4x^6} - \frac{b(7b^2 - 12ac)(b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}} + \frac{7b(a + bx^3 + cx^6)^{3/2}}{120a^2x^{12}} - \frac{(a + bx^3 + cx^6)^{3/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^16,x]

[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*a^4*x^6) - (a + b*x^3 + c*x^6)^(3/2)/(15*a*x^15) + (7*b*(a + b*x^3 + c*x^6)^(3/2))/(120*a^2*x^12) - ((35*b^2 - 32*a*c)*(a + b*x^3 + c*x^6)^(3/2))/(720*a^3*x^9) - (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(768*a^(9/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x], (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 744

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 834

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{x^6} dx, x, x^3 \right) \\
&= -\frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} - \frac{\text{Subst} \left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5} dx, x, x^3 \right)}{15a} \\
&= -\frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}} + \frac{\text{Subst} \left(\int \frac{\left(\frac{1}{4}(35b^2-32ac)+\frac{7bcx}{2}\right)\sqrt{a+bx+cx^2}}{x^4} dx, x, x^3 \right)}{60a^2} \\
&= -\frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}} - \frac{(35b^2-32ac)(a+bx^3+cx^6)^{3/2}}{720a^3x^9} - \frac{b(7b^2-12ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{384a^4x^6} \\
&= \frac{b(7b^2-12ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}} \\
&= \frac{b(7b^2-12ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}} \\
&= \frac{b(7b^2-12ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+cx^6)^{3/2}}{120a^2x^{12}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 173, normalized size = 0.87

$$-\frac{b(48a^2c^2-40ab^2c+7b^4)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}} - \frac{\sqrt{a+bx^3+cx^6}(384a^4+16a^3(3bx^3+8cx^6)-8a^2x^6(7b^2+29bcx^3+32c^2x^6)+10ab^2x^9(7b+46cx^3)-105b^4x^{12})}{5760a^4x^{15}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^16,x]

[Out] $-\frac{1}{5760}(\text{Sqrt}[a + b*x^3 + c*x^6]*(384*a^4 - 105*b^4*x^{12} + 10*a*b^2*x^9*(7*b + 46*c*x^3) + 16*a^3*(3*b*x^3 + 8*c*x^6) - 8*a^2*x^6*(7*b^2 + 29*b*c*x^3 + 32*c^2*x^6)))/(a^4*x^{15}) - (b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*\text{ArcTanh}[(2*a + b*x^3)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x^3 + c*x^6]))/(768*a^{(9/2)})$

IntegrateAlgebraic [A] time = 1.20, size = 176, normalized size = 0.88

$$\frac{(48a^2bc^2-40ab^3c+7b^5)\tanh^{-1}\left(\frac{\sqrt{cx^3-\sqrt{a+bx^3+cx^6}}}{\sqrt{a}}\right)}{384a^{9/2}} + \frac{\sqrt{a+bx^3+cx^6}(-384a^4-48a^3bx^3-128a^3cx^6+56a^2b^2x^6+232a^2bcx^9+256a^2c^2x^{12}-70ab^3x^9-460ab^2cx^{12}+105b^4x^{12})}{5760a^4x^{15}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x^3 + c*x^6]/x^16,x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-384*a^4 - 48*a^3*b*x^3 + 56*a^2*b^2*x^6 - 128*a^3*c*x^6 - 70*a*b^3*x^9 + 232*a^2*b*c*x^9 + 105*b^4*x^12 - 460*a*b^2*c*x^12 + 256*a^2*c^2*x^12))/(5760*a^4*x^15) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(384*a^(9/2))

fricas [A] time = 1.52, size = 389, normalized size = 1.95

$$\frac{15(7b^5 - 40ab^3c + 48a^2b^2c^2)\sqrt{c} \operatorname{arctan}\left(\frac{\sqrt{cx^6 + bx^3 + a} - \sqrt{a}}{\sqrt{c}x^3}\right) + ((105ab^4 - 460a^2b^2c + 256a^3c^2)x^{12} - 2(35a^2b^3 - 116a^3bc)x^9 - 48a^4b^2x^6 + 8(7a^3b^2 - 16a^4c)x^3 - 384a^5)\sqrt{cx^6 + bx^3 + a}}{11520a^4x^{15}} + \frac{(7b^5 - 40ab^3c + 48a^2b^2c^2)\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{cx^6 + bx^3 + a} - \sqrt{a}}{\sqrt{cx^6 + bx^3 + a}}\right)}{384a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="fricas")

[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^12 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a^3*b^2 - 16*a^4*c)*x^6 - 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15), 1/11520*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^12 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a^3*b^2 - 16*a^4*c)*x^6 - 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="giac")

[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^16, x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)

[Out] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^6 + bx^3 + a}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^(1/2)/x^16,x)

[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^16, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)

[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**16, x)

$$3.183 \quad \int x^{14} (a + bx^3 + cx^6)^{3/2} dx$$

Optimal. Leaf size=293

$$\frac{(b^2 - 4ac)^2 (16a^2c^2 - 72ab^2c + 33b^4) \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c} \sqrt{a+bx^3+cx^6}} \right)}{32768c^{13/2}} - \frac{(b^2 - 4ac) (16a^2c^2 - 72ab^2c + 33b^4) (b + 2cx^3)}{16384c^6}$$

Rubi [A] time = 0.40, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 742, 832, 779, 612, 621, 206}

$$\frac{(16a^2c^2 - 72ab^2c + 33b^4)(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{6144c^5} - \frac{(b^2 - 4ac)(16a^2c^2 - 72ab^2c + 33b^4)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{16384c^6} + \frac{(b^2 - 4ac)^2(16a^2c^2 - 72ab^2c + 33b^4) \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c} \sqrt{a+bx^3+cx^6}} \right)}{32768c^{13/2}} - \frac{(3b(77b^2 - 124ac) - 10c^3(33b^2 - 28ac))(a + bx^3 + cx^6)^{5/2}}{13440c^4} - \frac{11bx^6(a + bx^3 + cx^6)^{5/2}}{336c^2} + \frac{x^9(a + bx^3 + cx^6)^{5/2}}{24c}$$

Antiderivative was successfully verified.

[In] Int[x^14*(a + b*x^3 + c*x^6)^(3/2), x]

[Out] -((b^2 - 4*a*c)*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(16384*c^6) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(6144*c^5) - (11*b*x^6*(a + b*x^3 + c*x^6)^(5/2))/(336*c^2) + (x^9*(a + b*x^3 + c*x^6)^(5/2))/(24*c) - ((3*b*(77*b^2 - 124*a*c) - 10*c*(33*b^2 - 28*a*c)*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(13440*c^4) + ((b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(32768*c^(13/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{14} (a + bx^3 + cx^6)^{3/2} dx &= \frac{1}{3} \text{Subst} \left(\int x^4 (a + bx + cx^2)^{3/2} dx, x, x^3 \right) \\
&= \frac{x^9 (a + bx^3 + cx^6)^{5/2}}{24c} + \frac{\text{Subst} \left(\int x^2 \left(-3a - \frac{11bx}{2} \right) (a + bx + cx^2)^{3/2} dx, x, x^3 \right)}{24c} \\
&= -\frac{11bx^6 (a + bx^3 + cx^6)^{5/2}}{336c^2} + \frac{x^9 (a + bx^3 + cx^6)^{5/2}}{24c} + \frac{\text{Subst} \left(\int x \left(11ab + \frac{3}{4} (33b^2 \right. \right. \right. \\
&= -\frac{11bx^6 (a + bx^3 + cx^6)^{5/2}}{336c^2} + \frac{x^9 (a + bx^3 + cx^6)^{5/2}}{24c} - \frac{(3b(77b^2 - 124ac) - 10c(33b^2 - 124ac) + 10c^2)}{6144c^5} \\
&= \frac{(33b^4 - 72ab^2c + 16a^2c^2)(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{6144c^5} - \frac{11bx^6 (a + bx^3 + cx^6)^{5/2}}{336c^2} \\
&= -\frac{(b^2 - 4ac)(33b^4 - 72ab^2c + 16a^2c^2)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{16384c^6} + \frac{(33b^4 - 72ab^2c + 16a^2c^2)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{16384c^6} \\
&= -\frac{(b^2 - 4ac)(33b^4 - 72ab^2c + 16a^2c^2)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{16384c^6} + \frac{(33b^4 - 72ab^2c + 16a^2c^2)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{16384c^6} \\
&= -\frac{(b^2 - 4ac)(33b^4 - 72ab^2c + 16a^2c^2)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{16384c^6} + \frac{(33b^4 - 72ab^2c + 16a^2c^2)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{16384c^6}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 241, normalized size = 0.82

$$\frac{(16a^2c^2 - 72ab^2c + 33b^4) \left(2\sqrt{c(b+2cx^3)} \sqrt{a+bx^3+cx^6} (4c(5a+2cx^6) - 3b^2 + 8bcx^3) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}} \right) \right)}{4096c^{11/2}} + \frac{(372abc - 280a^2c^2 - 231b^2 + 330b^2cx^3)(a+bx^3+cx^6)^{5/2}}{560c^3} - \frac{11bx^6(a+bx^3+cx^6)^{5/2}}{14c} + x^9(a+bx^3+cx^6)^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^14*(a + b*x^3 + c*x^6)^(3/2), x]

[Out] ((-11*b*x^6*(a + b*x^3 + c*x^6)^(5/2))/(14*c) + x^9*(a + b*x^3 + c*x^6)^(5/2) + ((-231*b^3 + 372*a*b*c + 330*b^2*c*x^3 - 280*a*c^2*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(560*c^3) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(2*sqrt[c]*(b + 2*c*x^3)*sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*sqrt[c]*sqrt[a + b*x^3 + c*x^6])]))/(4096*c^(11/2)))/(24*c)

IntegrateAlgebraic [A] time = 1.40, size = 314, normalized size = 1.07

$$\frac{\sqrt{a+bx^3+cx^6} (5803a^3b^3c^2 - 13440a^2b^4c^2 - 81648a^2b^3c^2 + 37792a^2b^2c^2 - 19328a^2b^2c^2 + 8960a^2b^2c^2 + 30660a^2c^2 - 17976a^2c^2 + 12480ab^2c^2 - 9888ab^2c^2 + 6656ab^2c^2 + 10752ab^2c^2 - 34657 + 2310b^2c^2 - 1848b^2c^2 + 1584b^2c^2 - 1408b^2c^2 + 1280b^2c^2 + 87040c^2 + 71680c^2)}{173252a^3} \log \left(\frac{-256a^4 + 1280a^3b^2 - 1120a^2b^2c^2 + 336a^4c^2 - 33b^4}{-2\sqrt{c}\sqrt{a+bx^3+cx^6}} \right) \sqrt{a+bx^3+cx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3465*b^7 + 30660*a*b^5*c - 81648*a^2*b^3*c^2 + 5
8816*a^3*b*c^3 + 2310*b^6*c*x^3 - 17976*a*b^4*c^2*x^3 + 37792*a^2*b^2*c^3*x
^3 - 13440*a^3*c^4*x^3 - 1848*b^5*c^2*x^6 + 12480*a*b^3*c^3*x^6 - 19328*a^2
*b*c^4*x^6 + 1584*b^4*c^3*x^9 - 9088*a*b^2*c^4*x^9 + 8960*a^2*c^5*x^9 - 140
8*b^3*c^4*x^12 + 6656*a*b*c^5*x^12 + 1280*b^2*c^5*x^15 + 107520*a*c^6*x^15
+ 87040*b*c^6*x^18 + 71680*c^7*x^21))/(1720320*c^6) + ((-33*b^8 + 336*a*b^6
*c - 1120*a^2*b^4*c^2 + 1280*a^3*b^2*c^3 - 256*a^4*c^4)*Log[b + 2*c*x^3 - 2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(32768*c^(13/2))
```

```
fricas [A] time = 1.56, size = 641, normalized size = 2.19
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6881280*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
+ 256*a^4*c^4)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(71680*c^8*x^21 + 87040*b*c^7*x
^18 + 1280*(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12
+ 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*
b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3
*c^4 + 2416*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b
^2*c^4 - 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7, -1/3440640*(105*(
33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*s
qrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6
+ b*c*x^3 + a*c)) - 2*(71680*c^8*x^21 + 87040*b*c^7*x^18 + 1280*(b^2*c^6 +
84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12 + 16*(99*b^4*c^4 - 568*
a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3
*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*
x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7]
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} x^{14} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} x^{14} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{14} (cx^6 + bx^3 + a)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14*(a + b*x^3 + c*x^6)^(3/2),x)

[Out] int(x^14*(a + b*x^3 + c*x^6)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14} (a + bx^3 + cx^6)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral(x**14*(a + b*x**3 + c*x**6)**(3/2), x)

$$3.184 \quad \int x^{11} (a + bx^3 + cx^6)^{3/2} dx$$

Optimal. Leaf size=223

$$\frac{b(b^2 - 4ac)^2 (3b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}} + \frac{b(b^2 - 4ac)(3b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{1024c^5} - \frac{b(3b^2 - 4ac)}{21c}$$

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 742, 779, 612, 621, 206}

$$\frac{(-16ac + 21b^2 - 30bcx^3)(a + bx^3 + cx^6)^{5/2}}{840c^3} - \frac{b(3b^2 - 4ac)(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{384c^4} + \frac{b(b^2 - 4ac)(3b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{1024c^5} - \frac{b(b^2 - 4ac)^2(3b^2 - 4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}} + \frac{x^6(a + bx^3 + cx^6)^{5/2}}{21c}$$

Antiderivative was successfully verified.

[In] Int[x^11*(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1024*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(384*c^4) + (x^6*(a + b*x^3 + c*x^6)^(5/2))/(21*c) + ((21*b^2 - 16*a*c - 30*b*c*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(840*c^3) - (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2048*c^(11/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{11} (a + bx^3 + cx^6)^{3/2} dx &= \frac{1}{3} \text{Subst} \left(\int x^3 (a + bx + cx^2)^{3/2} dx, x, x^3 \right) \\
&= \frac{x^6 (a + bx^3 + cx^6)^{5/2}}{21c} + \frac{\text{Subst} \left(\int x \left(-2a - \frac{9bx}{2} \right) (a + bx + cx^2)^{3/2} dx, x, x^3 \right)}{21c} \\
&= \frac{x^6 (a + bx^3 + cx^6)^{5/2}}{21c} + \frac{(21b^2 - 16ac - 30bcx^3) (a + bx^3 + cx^6)^{5/2}}{840c^3} - \frac{(b(3b^2 - 4ac))}{384c^4} \\
&= -\frac{b(3b^2 - 4ac)(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{384c^4} + \frac{x^6 (a + bx^3 + cx^6)^{5/2}}{21c} + \frac{(21b^2 - 16ac)}{384c^4} \\
&= \frac{b(b^2 - 4ac)(3b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{1024c^5} - \frac{b(3b^2 - 4ac)(b + 2cx^3)(a + bx^3 + cx^6)^{5/2}}{384c^4} \\
&= \frac{b(b^2 - 4ac)(3b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{1024c^5} - \frac{b(3b^2 - 4ac)(b + 2cx^3)(a + bx^3 + cx^6)^{5/2}}{384c^4} \\
&= \frac{b(b^2 - 4ac)(3b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{1024c^5} - \frac{b(3b^2 - 4ac)(b + 2cx^3)(a + bx^3 + cx^6)^{5/2}}{384c^4}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 192, normalized size = 0.86

$$\frac{-(16ac - 21b^2 + 30bcx^3)(a + bx^3 + cx^6)^{5/2}}{40c^2} + \frac{7(4abc - 3b^3) \left(2\sqrt{c} (b + 2cx^3) \sqrt{a + bx^3 + cx^6} (4c(5a + 2cx^6) - 3b^2 + 8bcx^3) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right) \right)}{2048c^{9/2}} + x^6 (a + bx^3 + cx^6)^{5/2}}{21c}$$

Antiderivative was successfully verified.

[In] Integrate[x^11*(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (x^6*(a + b*x^3 + c*x^6)^(5/2) - ((-21*b^2 + 16*a*c + 30*b*c*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(2048*c^(9/2)))/(21*c)

IntegrateAlgebraic [A] time = 1.13, size = 255, normalized size = 1.14

$$\frac{(-64a^2bc^3 + 80a^2b^3c^2 - 28a^2b^2c + 3b^3) \log \left(-2\sqrt{c} \sqrt{a + bx^3 + cx^6} + b + 2cx^3 \right) + \sqrt{a + bx^3 + cx^6} (-2048a^3c^3 + 5488a^2b^2c^2 - 2336a^2bc^3 + 1024a^2c^4 + 2520ab^2c - 1456a^2b^2c^2 - 992a^2b^2c^3 + 704abc^4 + 8192a^2c^4 + 3150c^5 - 210b^3c^3 + 1680b^2c^2 + 144b^3c^2 + 128b^2c^3 + 6400bc^4 + 5120c^5)}{2048c^{11/2}}}{107520c^9}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11*(a + b*x^3 + c*x^6)^(3/2),x]

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*
a^3*c^3 - 210*b^5*c*x^3 + 1456*a*b^3*c^2*x^3 - 2336*a^2*b*c^3*x^3 + 168*b^4
*c^2*x^6 - 992*a*b^2*c^3*x^6 + 1024*a^2*c^4*x^6 - 144*b^3*c^3*x^9 + 704*a*b
*c^4*x^9 + 128*b^2*c^4*x^12 + 8192*a*c^5*x^12 + 6400*b*c^5*x^15 + 5120*c^6*
x^18))/(107520*c^5) + ((3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)
*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(2048*c^(11/2))
```

fricas [A] time = 1.24, size = 535, normalized size = 2.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c)
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128*(b^2*c^5 + 64
*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c - 2520*a*b^4*c^2
+ 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^
2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^3)*sqrt(c*x
^6 + b*x^3 + a))/c^6, 1/215040*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 -
64*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sq
rt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128
*(b^2*c^5 + 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c -
2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^
2*c^4 + 128*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} x^{11} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} x^{11} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(c*x^6+b*x^3+a)^(3/2),x)`

[Out] `int(x^11*(c*x^6+b*x^3+a)^(3/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{11} (cx^6 + bx^3 + a)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(a + b*x^3 + c*x^6)^(3/2),x)`

[Out] `int(x^11*(a + b*x^3 + c*x^6)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11} (a + bx^3 + cx^6)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)`

[Out] `Integral(x**11*(a + b*x**3 + c*x**6)**(3/2), x)`

$$3.185 \quad \int x^8 (a + bx^3 + cx^6)^{3/2} dx$$

Optimal. Leaf size=204

$$\frac{(b^2 - 4ac)^2 (7b^2 - 4ac) \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c} \sqrt{a+bx^3+cx^6}} \right)}{3072c^{9/2}} - \frac{(b^2 - 4ac)(7b^2 - 4ac)(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{1536c^4} + \frac{(7b^2 - 4ac)}{180c^2} - \frac{x^3 (a + bx^3 + cx^6)^{5/2}}{18c}$$

Rubi [A] time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 742, 640, 612, 621, 206}

$$\frac{(7b^2 - 4ac)(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{576c^3} - \frac{(b^2 - 4ac)(7b^2 - 4ac)(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{1536c^4} + \frac{(b^2 - 4ac)^2 (7b^2 - 4ac) \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c} \sqrt{a+bx^3+cx^6}} \right)}{3072c^{9/2}} - \frac{7b(a + bx^3 + cx^6)^{5/2}}{180c^2} + \frac{x^3 (a + bx^3 + cx^6)^{5/2}}{18c}$$

Antiderivative was successfully verified.

[In] Int[x^8*(a + b*x^3 + c*x^6)^(3/2),x]

[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1536*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*c^3) - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*c^2) + (x^3*(a + b*x^3 + c*x^6)^(5/2))/(18*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3072*c^(9/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^8 (a + bx^3 + cx^6)^{3/2} dx &= \frac{1}{3} \text{Subst} \left(\int x^2 (a + bx + cx^2)^{3/2} dx, x, x^3 \right) \\
&= \frac{x^3 (a + bx^3 + cx^6)^{5/2}}{18c} + \frac{\text{Subst} \left(\int \left(-a - \frac{7bx}{2} \right) (a + bx + cx^2)^{3/2} dx, x, x^3 \right)}{18c} \\
&= -\frac{7b (a + bx^3 + cx^6)^{5/2}}{180c^2} + \frac{x^3 (a + bx^3 + cx^6)^{5/2}}{18c} + \frac{(7b^2 - 4ac) \text{Subst} \left(\int (a + bx + cx^2)^{3/2} dx, x, x^3 \right)}{72c^2} \\
&= \frac{(7b^2 - 4ac) (b + 2cx^3) (a + bx^3 + cx^6)^{3/2}}{576c^3} - \frac{7b (a + bx^3 + cx^6)^{5/2}}{180c^2} + \frac{x^3 (a + bx^3 + cx^6)^{5/2}}{18c} \\
&= -\frac{(b^2 - 4ac) (7b^2 - 4ac) (b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{1536c^4} + \frac{(7b^2 - 4ac) (b + 2cx^3) (a + bx^3 + cx^6)^{5/2}}{576c^3} \\
&= -\frac{(b^2 - 4ac) (7b^2 - 4ac) (b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{1536c^4} + \frac{(7b^2 - 4ac) (b + 2cx^3) (a + bx^3 + cx^6)^{5/2}}{576c^3} \\
&= -\frac{(b^2 - 4ac) (7b^2 - 4ac) (b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{1536c^4} + \frac{(7b^2 - 4ac) (b + 2cx^3) (a + bx^3 + cx^6)^{5/2}}{576c^3}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 175, normalized size = 0.86

$$\frac{(7b^2 - 4ac) \left(2\sqrt{c} (b + 2cx^3) \sqrt{a + bx^3 + cx^6} (4c(5a + 2cx^6) - 3b^2 + 8bcx^3) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right) \right)}{512c^{7/2}} + \frac{x^3 (a + bx^3 + cx^6)^{5/2}}{18c} - \frac{7b(a + bx^3 + cx^6)^{5/2}}{10c}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*(a + b*x^3 + c*x^6)^(3/2), x]

[Out] ((-7*b*(a + b*x^3 + c*x^6)^(5/2))/(10*c) + x^3*(a + b*x^3 + c*x^6)^(5/2) + ((7*b^2 - 4*a*c)*(2*sqrt[c]*(b + 2*c*x^3)*sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 8*b*c*x^3 + 4*c*(5*a + 2*c*x^6)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*sqrt[c]*sqrt[a + b*x^3 + c*x^6])]))/(512*c^(7/2)))/(18*c)

IntegrateAlgebraic [A] time = 0.88, size = 209, normalized size = 1.02

$$\frac{\sqrt{a + bx^3 + cx^6} (-1296a^2bc^2 + 480a^2c^3x^3 + 760ab^3c - 432a^2c^2x^3 + 288abc^3x^6 + 2240ac^4x^9 - 105b^5 + 70b^4cx^3 - 56b^3c^2x^6 + 48b^2c^3x^9 + 1664bc^4x^{12} + 1280c^5x^{15})}{23040c^4} + \frac{(64a^3c^3 - 144a^2b^2c^2 + 60ab^4c - 7b^6) \log(-2\sqrt{c} \sqrt{a + bx^3 + cx^6} + b + 2cx^3)}{3072c^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8*(a + b*x^3 + c*x^6)^(3/2), x]

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-105*b^5 + 760*a*b^3*c - 1296*a^2*b*c^2 + 70*b^4*
c*x^3 - 432*a*b^2*c^2*x^3 + 480*a^2*c^3*x^3 - 56*b^3*c^2*x^6 + 288*a*b*c^3*
x^6 + 48*b^2*c^3*x^9 + 2240*a*c^4*x^9 + 1664*b*c^4*x^12 + 1280*c^5*x^15))/
(23040*c^4) + ((-7*b^6 + 60*a*b^4*c - 144*a^2*b^2*c^2 + 64*a^3*c^3)*Log[b +
2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(3072*c^(9/2))
```

fricas [A] time = 1.30, size = 451, normalized size = 2.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*1
og(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*b^2*c^4 + 140*
a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1
296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 6
4*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-
c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*
b^2*c^4 + 140*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760
*a*b^3*c^2 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```


maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^8 (cx^6 + bx^3 + a)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(a + b*x^3 + c*x^6)^(3/2),x)`

[Out] `int(x^8*(a + b*x^3 + c*x^6)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 (a + bx^3 + cx^6)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)`

[Out] `Integral(x**8*(a + b*x**3 + c*x**6)**(3/2), x)`

$$3.186 \quad \int x^5 (a + bx^3 + cx^6)^{3/2} dx$$

Optimal. Leaf size=150

$$\frac{b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}} + \frac{b(b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{128c^3} - \frac{b(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{48c^2}$$

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 640, 612, 621, 206}

$$\frac{b(b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{128c^3} - \frac{b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}} - \frac{b(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{48c^2} + \frac{(a + bx^3 + cx^6)^{5/2}}{15c}$$

Antiderivative was successfully verified.

[In] Int[x^5*(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (b*(b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*c^3) - (b*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*x^3 + c*x^6)^(5/2)/(15*c) - (b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(256*c^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int x^5 (a + bx^3 + cx^6)^{3/2} dx &= \frac{1}{3} \text{Subst} \left(\int x (a + bx + cx^2)^{3/2} dx, x, x^3 \right) \\
 &= \frac{(a + bx^3 + cx^6)^{5/2}}{15c} - \frac{b \text{Subst} \left(\int (a + bx + cx^2)^{3/2} dx, x, x^3 \right)}{6c} \\
 &= -\frac{b(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{48c^2} + \frac{(a + bx^3 + cx^6)^{5/2}}{15c} + \frac{(b(b^2 - 4ac)) \text{Subst} \left(\int \sqrt{a + bx + cx^2} dx, x, x^3 \right)}{32c^2} \\
 &= \frac{b(b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{128c^3} - \frac{b(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{48c^2} + \frac{(a + bx^3 + cx^6)^{5/2}}{15c} \\
 &= \frac{b(b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{128c^3} - \frac{b(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{48c^2} + \frac{(a + bx^3 + cx^6)^{5/2}}{15c} \\
 &= \frac{b(b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{128c^3} - \frac{b(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{48c^2} + \frac{(a + bx^3 + cx^6)^{5/2}}{15c}
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 149, normalized size = 0.99

$$-\frac{b(b^2 - 4ac) \left((b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c}\sqrt{a + bx^3 + cx^6}} \right) - 2\sqrt{c}(b + 2cx^3)\sqrt{a + bx^3 + cx^6} \right)}{256c^{7/2}} - \frac{b(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{48c^2} + \frac{(a + bx^3 + cx^6)^{5/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^(3/2), x]
```

```
[Out] -1/48*(b*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/c^2 + (a + b*x^3 + c*x^6)
^(5/2)/(15*c) - (b*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 +
```

$$c*x^6] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*sqrt[c]*sqrt[a + b*x^3 + c*x^6])])]/(256*c^(7/2))$$

IntegrateAlgebraic [A] time = 0.66, size = 162, normalized size = 1.08

$$\frac{(16a^2bc^2 - 8ab^3c + b^5) \log\left(-2\sqrt{c}\sqrt{a + bx^3 + cx^6} + b + 2cx^3\right)}{256c^{7/2}} + \frac{\sqrt{a + bx^3 + cx^6} (128a^2c^2 - 100ab^2c + 56abc^2x^3 + 256ac^3x^6 + 15b^4 - 10b^3cx^3 + 8b^2c^2x^6 + 176bc^3x^9 + 128c^4x^{12})}{1920c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (sqrt[a + b*x^3 + c*x^6]*(15*b^4 - 100*a*b^2*c + 128*a^2*c^2 - 10*b^3*c*c*x^3 + 56*a*b*c^2*x^3 + 8*b^2*c^2*x^6 + 256*a*c^3*x^6 + 176*b*c^3*x^9 + 128*c^4*x^12))/(1920*c^3) + ((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*Log[b + 2*c*x^3 - 2*sqrt[c]*sqrt[a + b*x^3 + c*x^6]])/(256*c^(7/2))

fricas [A] time = 1.21, size = 361, normalized size = 2.41

$$\frac{15(b^5 - 8ab^3c + 16a^2bc^2)\sqrt{c} \log\left(-8c^2x^6 - 8b^3cx^3 - b^2 + 4\sqrt{c}\sqrt{a + bx^3 + cx^6} + (2cx^3 + b)\sqrt{c} - 4a\right) + 4(128c^5x^{12} + 176b^3c^4x^9 + 8(b^2c^3 + 32a^2c^4)x^6 + 15b^4c - 100ab^2c^2 + 128a^2c^3 - 2(5b^3c^2 - 28abc^3)x^3)\sqrt{c}\sqrt{a + bx^3 + cx^6} + 2(128c^5x^{12} + 176b^3c^4x^9 + 8(b^2c^3 + 32a^2c^4)x^6 + 15b^4c - 100ab^2c^2 + 128a^2c^3 - 2(5b^3c^2 - 28abc^3)x^3)\sqrt{c}\sqrt{a + bx^3 + cx^6}}{7680c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b^3*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(128*c^5*x^12 + 176*b^3*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(128*c^5*x^12 + 176*b^3*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4]

giac [A] time = 0.56, size = 172, normalized size = 1.15

$$\frac{1}{1920} \sqrt{cx^6 + bx^3 + a} \left(2 \left(4 \left(2(8cx^3 + 11b)x^3 + \frac{b^2c^3 + 32ac^4}{c^4} \right) x^3 - \frac{5b^3c^2 - 28abc^3}{c^4} \right) x^3 + \frac{15b^4c - 100ab^2c^2 + 128a^2c^3}{c^4} \right) + \frac{(b^5 - 8ab^3c + 16a^2bc^2) \log\left(-2\left(\sqrt{cx^3 - \sqrt{cx^6 + bx^3 + a}}\right)\sqrt{c} - b\right)}{256c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2), x, algorithm="giac")

[Out] 1/1920*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*(8*c*x^3 + 11*b)*x^3 + (b^2*c^3 + 32*a*c^4)/c^4)*x^3 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^3 + (15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3)/c^4) + 1/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(7/2)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(c*x^6+b*x^3+a)^(3/2),x)`

[Out] `int(x^5*(c*x^6+b*x^3+a)^(3/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 1.58, size = 223, normalized size = 1.49

$$\frac{(cx^6 + bx^3 + a)^{5/2}}{15c} - \frac{b \left(\frac{3a \left(\ln \left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + b}{\sqrt{c}} \right) \left(\frac{a}{2\sqrt{c}} - \frac{b^2}{8c^{3/2}} \right) + \frac{(2cx^3 + b)\sqrt{cx^6 + bx^3 + a}}{4c} \right)}{4} + \frac{x^3 (cx^6 + bx^3 + a)^{3/2}}{4} - \frac{3b^2 \left(\ln \left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + b}{\sqrt{c}} \right) \left(\frac{a}{2\sqrt{c}} - \frac{b^2}{8c^{3/2}} \right) + \frac{(2cx^3 + b)\sqrt{cx^6 + bx^3 + a}}{4c} \right)}{16c} + \frac{b(cx^6 + bx^3 + a)^{3/2}}{8c} \right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a + b*x^3 + c*x^6)^(3/2),x)`

[Out] $(a + bx^3 + cx^6)^{5/2}/(15c) - (b*((3*a*(\log((a + bx^3 + cx^6)^{1/2}) + (b/2 + cx^3)/c^{1/2})*(a/(2*c^{1/2}) - b^2/(8*c^{3/2}))) + ((b + 2*cx^3) * (a + bx^3 + cx^6)^{1/2})/(4*c)))/4 + (x^3*(a + bx^3 + cx^6)^{3/2})/4 - (3*b^2*(\log((a + bx^3 + cx^6)^{1/2}) + (b/2 + cx^3)/c^{1/2})*(a/(2*c^{1/2}) - b^2/(8*c^{3/2}))) + ((b + 2*cx^3)*(a + bx^3 + cx^6)^{1/2})/(4*c))/ (16*c) + (b*(a + bx^3 + cx^6)^{3/2})/(8*c))/ (6*c)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + bx^3 + cx^6)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)`

[Out] `Integral(x**5*(a + b*x**3 + c*x**6)**(3/2), x)`

$$3.187 \quad \int x^2 (a + bx^3 + cx^6)^{3/2} dx$$

Optimal. Leaf size=124

$$\frac{(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}} - \frac{(b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{64c^2} + \frac{(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{24c}$$

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1352, 612, 621, 206}

$$-\frac{(b^2 - 4ac)(b + 2cx^3)\sqrt{a + bx^3 + cx^6}}{64c^2} + \frac{(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}} + \frac{(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{24c}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^3 + c*x^6)^(3/2), x]

[Out] -((b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*c^2) + ((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*c) + ((b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(128*c^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol
] :=> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^2 (a + bx^3 + cx^6)^{3/2} dx &= \frac{1}{3} \text{Subst} \left(\int (a + bx + cx^2)^{3/2} dx, x, x^3 \right) \\ &= \frac{(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{24c} - \frac{(b^2 - 4ac) \text{Subst} \left(\int \sqrt{a + bx + cx^2} dx, x, x^3 \right)}{16c} \\ &= -\frac{(b^2 - 4ac)(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{64c^2} + \frac{(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{24c} + \frac{(b^2 - 4ac)(a + bx^3 + cx^6)^{3/2}}{16c} \\ &= -\frac{(b^2 - 4ac)(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{64c^2} + \frac{(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{24c} + \frac{(b^2 - 4ac)(a + bx^3 + cx^6)^{3/2}}{16c} \\ &= -\frac{(b^2 - 4ac)(b + 2cx^3) \sqrt{a + bx^3 + cx^6}}{64c^2} + \frac{(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{24c} + \frac{(b^2 - 4ac)(a + bx^3 + cx^6)^{3/2}}{16c} \end{aligned}$$

Mathematica [A] time = 0.09, size = 126, normalized size = 1.02

$$\frac{3(b^2 - 4ac) \left((b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right) - 2\sqrt{c} (b + 2cx^3) \sqrt{a + bx^3 + cx^6} \right)}{8c^{3/2}} + 2(b + 2cx^3)(a + bx^3 + cx^6)^{3/2}}{48c}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2) + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]))/(8*c^(3/2))/(48*c)

IntegrateAlgebraic [A] time = 0.50, size = 132, normalized size = 1.06

$$\frac{(-16a^2c^2 + 8ab^2c - b^4) \log \left(-2\sqrt{c} \sqrt{a + bx^3 + cx^6} + b + 2cx^3 \right)}{128c^{5/2}} + \frac{\sqrt{a + bx^3 + cx^6} (20abc + 40ac^2x^3 - 3b^3 + 2b^2cx^3 + 24bc^2x^6 + 16c^3x^9)}{192c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(a + b*x^3 + c*x^6)^(3/2), x]

[Out] $(\sqrt{a + bx^3 + cx^6}) \cdot (-3b^3 + 20abc + 2b^2cx^3 + 40a^2cx^3 + 24b^2c^2x^6 + 16c^3x^9) / (192c^2) + ((-b^4 + 8a^2b^2c - 16a^2c^2) \cdot \text{Log}[b + 2cx^3 - 2\sqrt{c} \cdot \sqrt{a + bx^3 + cx^6}]) / (128c^{5/2})$

fricas [A] time = 1.38, size = 297, normalized size = 2.40

$$\frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{c} \log\left(\frac{-8c^2x^6 - 8b^2cx^3 - b^2 - 4\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4ac}{768c^3}\right) + 4(16c^4x^9 + 24b^2c^3x^6 - 3b^3c + 20abc^2 + 2(b^2c^2 + 20ac^2)x^3)\sqrt{cx^6 + bx^3 + a} - 3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}\sqrt{-c}}{2(b^2cx^3 + b)\sqrt{-c}}\right) - 2(16c^4x^9 + 24b^2c^3x^6 - 3b^3c + 20abc^2 + 2(b^2c^2 + 20ac^2)x^3)\sqrt{cx^6 + bx^3 + a}}{384c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")`

[Out] $[1/768*(3*(b^4 - 8a^2b^2c + 16a^2c^2)*\sqrt{c}*\log(-8c^2x^6 - 8b^2cx^3 - b^2 - 4*\sqrt{cx^6 + bx^3 + a}*(2cx^3 + b)*\sqrt{c} - 4*a*c) + 4*(16c^4x^9 + 24b^2c^3x^6 - 3b^3c + 20abc^2 + 2*(b^2c^2 + 20a^2c^3)*x^3)*\sqrt{cx^6 + bx^3 + a})/c^3, -1/384*(3*(b^4 - 8a^2b^2c + 16a^2c^2)*\sqrt{c}*\arctan(1/2*\sqrt{cx^6 + bx^3 + a}*(2cx^3 + b)*\sqrt{-c}/(c^2x^6 + b^2cx^3 + a*c)) - 2*(16c^4x^9 + 24b^2c^3x^6 - 3b^3c + 20abc^2 + 2*(b^2c^2 + 20a^2c^3)*x^3)*\sqrt{cx^6 + bx^3 + a})/c^3]$

giac [A] time = 0.61, size = 135, normalized size = 1.09

$$\frac{1}{192} \sqrt{cx^6 + bx^3 + a} \left(2 \left(4(2cx^3 + 3b)x^3 + \frac{b^2c^2 + 20ac^3}{c^3} \right) x^3 - \frac{3b^3c - 20abc^2}{c^3} \right) - \frac{(b^4 - 8ab^2c + 16a^2c^2) \log\left(\left| -2 \left(\sqrt{cx^3 - \sqrt{cx^6 + bx^3 + a}} \right) \sqrt{c} - b \right|\right)}{128c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")`

[Out] $1/192*\sqrt{cx^6 + bx^3 + a}*(2*(4*(2cx^3 + 3b)*x^3 + (b^2c^2 + 20a^2c^3)/c^3)*x^3 - (3b^3c - 20a^2bc^2)/c^3) - 1/128*(b^4 - 8a^2b^2c + 16a^2c^2)*\log(\text{abs}(-2*(\sqrt{c})*x^3 - \sqrt{cx^6 + bx^3 + a})*\sqrt{c} - b))/c^{5/2}$

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(c*x^6+b*x^3+a)^(3/2),x)`

[Out] `int(x^2*(c*x^6+b*x^3+a)^(3/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 1.44, size = 115, normalized size = 0.93

$$\frac{\left(cx^3 + \frac{b}{2}\right) (cx^6 + bx^3 + a)^{3/2}}{12c} + \frac{\left(3ac - \frac{3b^2}{4}\right) \left(\frac{b}{4c} + \frac{x^3}{2}\right) \sqrt{cx^6 + bx^3 + a} + \frac{\ln\left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + \frac{b}{2}}{\sqrt{c}}\right) \left(ac - \frac{b^2}{4}\right)}{2c^{3/2}}}{12c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*x^3 + c*x^6)^(3/2),x)`

[Out] $\left(\frac{b}{2} + cx^3\right) (a + bx^3 + cx^6)^{3/2} / (12c) + \left(\frac{3ac - (3b^2)/4}{4c} + x^{3/2}\right) (a + bx^3 + cx^6)^{1/2} + \left(\log\left((a + bx^3 + cx^6)^{1/2} + \frac{b}{2} + cx^3/c^{1/2}\right) * (ac - b^2/4)\right) / (2c^{3/2}) / (12c)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + bx^3 + cx^6)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)`

[Out] `Integral(x**2*(a + b*x**3 + c*x**6)**(3/2), x)`

$$3.188 \quad \int \frac{(a+bx^3+cx^6)^{3/2}}{x} dx$$

Optimal. Leaf size=155

$$-\frac{1}{3}a^{3/2} \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - \frac{b(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}} + \frac{(8ac+b^2+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c}$$

Rubi [A] time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 734, 814, 843, 621, 206, 724}

$$-\frac{1}{3}a^{3/2} \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - \frac{b(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}} + \frac{(8ac+b^2+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}(a+bx^3+cx^6)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x, x]

[Out] ((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c) + (a + b*x^3 + c*x^6)^(3/2)/9 - (a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/3 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
&& GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0]
&& IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3 + cx^6)^{3/2}}{x} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x} dx, x, x^3 \right) \\
&= \frac{1}{9} (a + bx^3 + cx^6)^{3/2} - \frac{1}{6} \text{Subst} \left(\int \frac{(-2a - bx)\sqrt{a + bx + cx^2}}{x} dx, x, x^3 \right) \\
&= \frac{(b^2 + 8ac + 2bcx^3)\sqrt{a + bx^3 + cx^6}}{24c} + \frac{1}{9} (a + bx^3 + cx^6)^{3/2} + \frac{\text{Subst} \left(\int \frac{8a^2c - \frac{1}{2}b(b^2 - 12ac)}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{24c} \\
&= \frac{(b^2 + 8ac + 2bcx^3)\sqrt{a + bx^3 + cx^6}}{24c} + \frac{1}{9} (a + bx^3 + cx^6)^{3/2} + \frac{1}{3} a^2 \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= \frac{(b^2 + 8ac + 2bcx^3)\sqrt{a + bx^3 + cx^6}}{24c} + \frac{1}{9} (a + bx^3 + cx^6)^{3/2} - \frac{1}{3} (2a^2) \text{Subst} \left(\int \frac{1}{4a - x} dx, x, x^3 \right) \\
&= \frac{(b^2 + 8ac + 2bcx^3)\sqrt{a + bx^3 + cx^6}}{24c} + \frac{1}{9} (a + bx^3 + cx^6)^{3/2} - \frac{1}{3} a^{3/2} \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right)
\end{aligned}$$

Mathematica [A] time = 0.14, size = 143, normalized size = 0.92

$$\frac{1}{144} \left(-48a^{3/2} \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right) - \frac{3b(b^2 - 12ac) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{c^{3/2}} + \frac{2\sqrt{a + bx^3 + cx^6} (8c(4a + cx^6) + 3b^2 + 14bcx^3)}{c} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x,x]

[Out] ((2*sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 14*b*c*x^3 + 8*c*(4*a + c*x^6)))/c - 4*8*a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*sqrt[a]*sqrt[a + b*x^3 + c*x^6])] - (3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*sqrt[c]*sqrt[a + b*x^3 + c*x^6])])/c^(3/2))/144

IntegrateAlgebraic [A] time = 0.70, size = 151, normalized size = 0.97

$$\frac{2}{3} a^{3/2} \tanh^{-1} \left(\frac{\sqrt{c} x^3}{\sqrt{a}} - \frac{\sqrt{a + bx^3 + cx^6}}{\sqrt{a}} \right) + \frac{(b^3 - 12abc) \log(-2c^{3/2} \sqrt{a + bx^3 + cx^6} + bc + 2c^2 x^3)}{48c^{3/2}} + \frac{\sqrt{a + bx^3 + cx^6} (32ac + 3b^2 + 14bcx^3 + 8c^2 x^6)}{72c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x,x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 32*a*c + 14*b*c*x^3 + 8*c^2*x^6))/(72*c) + (2*a^(3/2)*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a]])/3 + ((b^3 - 12*a*b*c)*Log[b*c + 2*c^2*x^3 - 2*c^(3/2)*Sqrt[a + b*x^3 + c*x^6]])/(48*c^(3/2))

fricas [A] time = 1.82, size = 727, normalized size = 4.69

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="fricas")

[Out] [1/288*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/144*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/288*(96*sqrt(-a)*a*c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/144*(48*sqrt(-a)*a*c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x, x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^6+b*x^3+a)^(3/2)/x,x)`

[Out] `int((c*x^6+b*x^3+a)^(3/2)/x,x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^6 + bx^3 + a)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3 + c*x^6)^(3/2)/x,x)`

[Out] `int((a + b*x^3 + c*x^6)^(3/2)/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**6+b*x**3+a)**(3/2)/x,x)`

[Out] `Integral((a + b*x**3 + c*x**6)**(3/2)/x, x)`

$$3.189 \quad \int \frac{(a+bx^3+cx^6)^{3/2}}{x^4} dx$$

Optimal. Leaf size=150

$$\frac{(4ac + b^2) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}} - \frac{(a + bx^3 + cx^6)^{3/2}}{3x^3} + \frac{1}{4}(3b + 2cx^3)\sqrt{a + bx^3 + cx^6} - \frac{1}{2}\sqrt{a}b \tanh^{-1}\left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}}\right)$$

Rubi [A] time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 732, 814, 843, 621, 206, 724}

$$\frac{(4ac + b^2) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}} - \frac{(a + bx^3 + cx^6)^{3/2}}{3x^3} + \frac{1}{4}(3b + 2cx^3)\sqrt{a + bx^3 + cx^6} - \frac{1}{2}\sqrt{a}b \tanh^{-1}\left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]

[Out] ((3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/4 - (a + b*x^3 + c*x^6)^(3/2)/(3*x^3) - (Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/2 + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^4} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^2} dx, x, x^3 \right) \\
&= -\frac{(a + bx^3 + cx^6)^{3/2}}{3x^3} + \frac{1}{2} \text{Subst} \left(\int \frac{(b + 2cx)\sqrt{a + bx + cx^2}}{x} dx, x, x^3 \right) \\
&= \frac{1}{4} (3b + 2cx^3) \sqrt{a + bx^3 + cx^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{3x^3} - \frac{\text{Subst} \left(\int \frac{-4abc - c(b^2 + 4ac)x}{x\sqrt{a + bx + cx^2}} dx, x, \right)}{8c} \\
&= \frac{1}{4} (3b + 2cx^3) \sqrt{a + bx^3 + cx^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{3x^3} + \frac{1}{2}(ab) \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, \right) \\
&= \frac{1}{4} (3b + 2cx^3) \sqrt{a + bx^3 + cx^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{3x^3} - (ab) \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \right) \\
&= \frac{1}{4} (3b + 2cx^3) \sqrt{a + bx^3 + cx^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{3x^3} - \frac{1}{2} \sqrt{a} b \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right)
\end{aligned}$$

Mathematica [A] time = 0.11, size = 134, normalized size = 0.89

$$\frac{1}{24} \left(\frac{3(4ac + b^2) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{\sqrt{c}} + \frac{2\sqrt{a + bx^3 + cx^6} (-4a + 5bx^3 + 2cx^6)}{x^3} - 12\sqrt{a} b \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^4, x]

[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/x^3 - 12*Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])] + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[c])/24

IntegrateAlgebraic [A] time = 0.73, size = 137, normalized size = 0.91

$$\frac{(-4ac - b^2) \log \left(-2\sqrt{c} \sqrt{a + bx^3 + cx^6} + b + 2cx^3 \right)}{8\sqrt{c}} + \frac{\sqrt{a + bx^3 + cx^6} (-4a + 5bx^3 + 2cx^6)}{12x^3} + \sqrt{a} b \tanh^{-1} \left(\frac{\sqrt{c} x^3}{\sqrt{a}} - \frac{\sqrt{a + bx^3 + cx^6}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^4, x]

[Out] $(\text{Sqrt}[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/(12*x^3) + \text{Sqrt}[a]*b*\text{ArcTanh}[(\text{Sqrt}[c]*x^3)/\text{Sqrt}[a] - \text{Sqrt}[a + b*x^3 + c*x^6]/\text{Sqrt}[a]] + ((-b^2 - 4*a*c)*\text{Log}[b + 2*c*x^3 - 2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x^3 + c*x^6]])/(8*\text{Sqrt}[c])$

fricas [A] time = 1.55, size = 713, normalized size = 4.75

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")`

[Out] $[1/48*(12*\text{sqrt}(a)*b*c*x^3*\text{log}(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*\text{sqrt}(c*x^6 + b*x^3 + a))*(b*x^3 + 2*a)*\text{sqrt}(a) + 8*a^2)/x^6) + 3*(b^2 + 4*a*c)*\text{sqrt}(c)*x^3*\text{log}(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*\text{sqrt}(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*\text{sqrt}(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*\text{sqrt}(c*x^6 + b*x^3 + a)/(c*x^3), 1/24*(6*\text{sqrt}(a)*b*c*x^3*\text{log}(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*\text{sqrt}(c*x^6 + b*x^3 + a))*(b*x^3 + 2*a)*\text{sqrt}(a) + 8*a^2)/x^6) - 3*(b^2 + 4*a*c)*\text{sqrt}(-c)*x^3*\text{arctan}(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*\text{sqrt}(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*\text{sqrt}(c*x^6 + b*x^3 + a)/(c*x^3), 1/48*(24*\text{sqrt}(-a)*b*c*x^3*\text{arctan}(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*\text{sqrt}(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 3*(b^2 + 4*a*c)*\text{sqrt}(c)*x^3*\text{log}(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*\text{sqrt}(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*\text{sqrt}(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*\text{sqrt}(c*x^6 + b*x^3 + a)/(c*x^3), 1/24*(12*\text{sqrt}(-a)*b*c*x^3*\text{arctan}(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*\text{sqrt}(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 3*(b^2 + 4*a*c)*\text{sqrt}(-c)*x^3*\text{arctan}(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*\text{sqrt}(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*\text{sqrt}(c*x^6 + b*x^3 + a)/(c*x^3)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="giac")`

[Out] `integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4, x)`

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^6+b*x^3+a)^(3/2)/x^4,x)`

[Out] `int((c*x^6+b*x^3+a)^(3/2)/x^4,x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^6 + bx^3 + a)^{3/2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3 + c*x^6)^(3/2)/x^4,x)`

[Out] `int((a + b*x^3 + c*x^6)^(3/2)/x^4, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)`

[Out] `Integral((a + b*x**3 + c*x**6)**(3/2)/x**4, x)`

$$3.190 \quad \int \frac{(a+bx^3+cx^6)^{3/2}}{x^7} dx$$

Optimal. Leaf size=151

$$\frac{(4ac + b^2) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}} - \frac{(a + bx^3 + cx^6)^{3/2}}{6x^6} - \frac{(b - 2cx^3)\sqrt{a + bx^3 + cx^6}}{4x^3} + \frac{1}{2}b\sqrt{c} \tanh^{-1}\left(\frac{b + 2cx^3}{2\sqrt{c}\sqrt{a + bx^3 + cx^6}}\right)$$

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 732, 812, 843, 621, 206, 724}

$$\frac{(4ac + b^2) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}} - \frac{(a + bx^3 + cx^6)^{3/2}}{6x^6} - \frac{(b - 2cx^3)\sqrt{a + bx^3 + cx^6}}{4x^3} + \frac{1}{2}b\sqrt{c} \tanh^{-1}\left(\frac{b + 2cx^3}{2\sqrt{c}\sqrt{a + bx^3 + cx^6}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^7, x]

[Out] -((b - 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*x^3) - (a + b*x^3 + c*x^6)^(3/2)/(6*x^6) - ((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*Sqrt[a]) + (b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^7} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^3} dx, x, x^3 \right) \\
&= -\frac{(a + bx^3 + cx^6)^{3/2}}{6x^6} + \frac{1}{4} \text{Subst} \left(\int \frac{(b + 2cx)\sqrt{a + bx + cx^2}}{x^2} dx, x, x^3 \right) \\
&= -\frac{(b - 2cx^3)\sqrt{a + bx^3 + cx^6}}{4x^3} - \frac{(a + bx^3 + cx^6)^{3/2}}{6x^6} - \frac{1}{8} \text{Subst} \left(\int \frac{-b^2 - 4ac - 4bcx}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= -\frac{(b - 2cx^3)\sqrt{a + bx^3 + cx^6}}{4x^3} - \frac{(a + bx^3 + cx^6)^{3/2}}{6x^6} + \frac{1}{2}(bc) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= -\frac{(b - 2cx^3)\sqrt{a + bx^3 + cx^6}}{4x^3} - \frac{(a + bx^3 + cx^6)^{3/2}}{6x^6} + (bc) \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b}{\sqrt{a + bx + cx^2}} \right) \\
&= -\frac{(b - 2cx^3)\sqrt{a + bx^3 + cx^6}}{4x^3} - \frac{(a + bx^3 + cx^6)^{3/2}}{6x^6} - \frac{(b^2 + 4ac) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}} \right)}{8\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 134, normalized size = 0.89

$$\frac{1}{24} \left(-\frac{3(4ac + b^2) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}} \right)}{\sqrt{a}} - \frac{2\sqrt{a + bx^3 + cx^6} (2a + 5bx^3 - 4cx^6)}{x^6} + 12b\sqrt{c} \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c}\sqrt{a + bx^3 + cx^6}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^7, x]

[Out] ((-2*(2*a + 5*b*x^3 - 4*c*x^6)*Sqrt[a + b*x^3 + c*x^6])/x^6 - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/Sqrt[a] + 12*b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/24

IntegrateAlgebraic [A] time = 0.80, size = 134, normalized size = 0.89

$$\frac{(4ac + b^2) \tanh^{-1} \left(\frac{\sqrt{c}x^3 - \sqrt{a + bx^3 + cx^6}}{\sqrt{a}} \right)}{4\sqrt{a}} + \frac{\sqrt{a + bx^3 + cx^6} (-2a - 5bx^3 + 4cx^6)}{12x^6} - \frac{1}{2}b\sqrt{c} \log \left(-2\sqrt{c}\sqrt{a + bx^3 + cx^6} + b + 2cx^3 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^7, x]

[Out] $(\sqrt{a + bx^3 + cx^6}) * (-2a - 5bx^3 + 4cx^6) / (12x^6) + ((b^2 + 4ac) * \text{ArcTanh}[\sqrt{c}x^3 - \sqrt{a + bx^3 + cx^6}] / \sqrt{a}) / (4\sqrt{a}) - (b\sqrt{c} * \text{Log}[b + 2cx^3 - 2\sqrt{c}\sqrt{a + bx^3 + cx^6}]) / 2$

fricas [A] time = 1.46, size = 713, normalized size = 4.72

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")`

[Out] $[1/48*(12ab\sqrt{c}x^6\log(-8c^2x^6 - 8b^2cx^3 - b^2 - 4\sqrt{c}x^6 + bx^3 + a)(2cx^3 + b)\sqrt{c} - 4a^2c) + 3(b^2 + 4ac)\sqrt{a}x^6\log(-((b^2 + 4ac)x^6 + 8abx^3 - 4\sqrt{c}x^6 + bx^3 + a)(bx^3 + 2a)\sqrt{a} + 8a^2)/x^6) + 4(4acx^6 - 5abx^3 - 2a^2)\sqrt{c}x^6 + bx^3 + a)/(ax^6), -1/48*(24ab\sqrt{-c}x^6\arctan(1/2\sqrt{c}x^6 + bx^3 + a)(2cx^3 + b)\sqrt{-c}/(c^2x^6 + b^2cx^3 + a^2c)) - 3(b^2 + 4ac)\sqrt{a}x^6\log(-((b^2 + 4ac)x^6 + 8abx^3 - 4\sqrt{c}x^6 + bx^3 + a)(bx^3 + 2a)\sqrt{a} + 8a^2)/x^6) - 4(4acx^6 - 5abx^3 - 2a^2)\sqrt{c}x^6 + bx^3 + a)/(ax^6), 1/24*(6ab\sqrt{c}x^6\log(-8c^2x^6 - 8b^2cx^3 - b^2 - 4\sqrt{c}x^6 + bx^3 + a)(2cx^3 + b)\sqrt{c} - 4a^2c) + 3(b^2 + 4ac)\sqrt{-a}x^6\arctan(1/2\sqrt{c}x^6 + bx^3 + a)(bx^3 + 2a)\sqrt{-a}/(acx^6 + abx^3 + a^2)) + 2(4acx^6 - 5abx^3 - 2a^2)\sqrt{c}x^6 + bx^3 + a)/(ax^6), -1/24*(12ab\sqrt{-c}x^6\arctan(1/2\sqrt{c}x^6 + bx^3 + a)(2cx^3 + b)\sqrt{-c}/(c^2x^6 + b^2cx^3 + a^2c)) - 3(b^2 + 4ac)\sqrt{-a}x^6\arctan(1/2\sqrt{c}x^6 + bx^3 + a)(bx^3 + 2a)\sqrt{-a}/(acx^6 + abx^3 + a^2)) - 2(4acx^6 - 5abx^3 - 2a^2)\sqrt{c}x^6 + bx^3 + a)/(ax^6)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="giac")`

[Out] `integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7, x)`

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^6+b*x^3+a)^(3/2)/x^7,x)`

[Out] `int((c*x^6+b*x^3+a)^(3/2)/x^7,x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^6 + bx^3 + a)^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3 + c*x^6)^(3/2)/x^7,x)`

[Out] `int((a + b*x^3 + c*x^6)^(3/2)/x^7, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)`

[Out] `Integral((a + b*x**3 + c*x**6)**(3/2)/x**7, x)`

$$3.191 \quad \int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}} dx$$

Optimal. Leaf size=163

$$\frac{b(b^2 - 12ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - (x^3(8ac+b^2)+2ab)\sqrt{a+bx^3+cx^6}}{48a^{3/2} \cdot 24ax^6} + \frac{1}{3}c^{3/2} \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Rubi [A] time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 732, 810, 843, 621, 206, 724}

$$\frac{b(b^2 - 12ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) - (x^3(8ac+b^2)+2ab)\sqrt{a+bx^3+cx^6}}{48a^{3/2} \cdot 24ax^6} + \frac{1}{3}c^{3/2} \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right) - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]

[Out] -((2*a*b + (b^2 + 8*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a*x^6) - (a + b*x^3 + c*x^6)^(3/2)/(9*x^9) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x)/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m - 2*p) + e*f*(m + 2*p + 2)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{10}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^4} dx, x, x^3 \right) \\
&= -\frac{(a + bx^3 + cx^6)^{3/2}}{9x^9} + \frac{1}{6} \text{Subst} \left(\int \frac{(b + 2cx)\sqrt{a + bx + cx^2}}{x^3} dx, x, x^3 \right) \\
&= -\frac{(2ab + (b^2 + 8ac)x^3)\sqrt{a + bx^3 + cx^6}}{24ax^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9x^9} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}b(b^2 - 12ac) - cx^2}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{24a} \\
&= -\frac{(2ab + (b^2 + 8ac)x^3)\sqrt{a + bx^3 + cx^6}}{24ax^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9x^9} + \frac{1}{3}c^2 \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= -\frac{(2ab + (b^2 + 8ac)x^3)\sqrt{a + bx^3 + cx^6}}{24ax^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9x^9} + \frac{1}{3}(2c^2) \text{Subst} \left(\int \frac{1}{4c\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\
&= -\frac{(2ab + (b^2 + 8ac)x^3)\sqrt{a + bx^3 + cx^6}}{24ax^6} - \frac{(a + bx^3 + cx^6)^{3/2}}{9x^9} + \frac{b(b^2 - 12ac) \tanh^{-1} \left(\frac{2\sqrt{a + bx + cx^2}}{2\sqrt{c}\sqrt{a + bx + cx^2}} \right)}{48a^3}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 149, normalized size = 0.91

$$\frac{1}{144} \left(\frac{3b(b^2 - 12ac) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}} \right)}{a^{3/2}} - \frac{2\sqrt{a + bx^3 + cx^6} (8a^2 + 14abx^3 + 32acx^6 + 3b^2x^6)}{ax^9} + 48c^{3/2} \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c}\sqrt{a + bx^3 + cx^6}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^10, x]

[Out] ((-2*Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 14*a*b*x^3 + 3*b^2*x^6 + 32*a*c*x^6))/(a*x^9) + (3*b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/a^(3/2) + 48*c^(3/2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/144

IntegrateAlgebraic [A] time = 0.99, size = 148, normalized size = 0.91

$$\frac{(b^3 - 12abc) \tanh^{-1} \left(\frac{\sqrt{a + bx^3 + cx^6} - \sqrt{cx^3}}{\sqrt{a}} \right)}{24a^{3/2}} + \frac{\sqrt{a + bx^3 + cx^6} (-8a^2 - 14abx^3 - 32acx^6 - 3b^2x^6)}{72ax^9} - \frac{1}{3}c^{3/2} \log \left(-2\sqrt{c}\sqrt{a + bx^3 + cx^6} + b + 2cx^3 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 14*a*b*x^3 - 3*b^2*x^6 - 32*a*c*x^6))/(7*2*a*x^9) + ((b^3 - 12*a*b*c)*ArcTanh[(-(Sqrt[c]*x^3) + Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(24*a^(3/2)) - (c^(3/2)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/3

fricas [A] time = 1.60, size = 771, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="fricas")

[Out] [1/288*(48*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/288*(96*a^2*sqrt(-c)*c*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), 1/144*(24*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/144*(48*a^2*sqrt(-c)*c*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10, x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)

[Out] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^(3/2)/x^10,x)

[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^10, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)

[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**10, x)

$$3.192 \quad \int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}} dx$$

Optimal. Leaf size=133

$$\frac{(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}} + \frac{(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{64a^2x^6} - \frac{(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{24ax^{12}}$$

Rubi [A] time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1357, 720, 724, 206}

$$\frac{(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{64a^2x^6} - \frac{(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}} - \frac{(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{24ax^{12}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]

[Out] ((b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*a^2*x^6) - ((2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*a*x^12) - ((b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(128*a^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{13}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^5} dx, x, x^3 \right) \\ &= -\frac{(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2 - 4ac) \text{Subst} \left(\int \frac{\sqrt{a+bx+cx^2}}{x^3} dx, x, x^3 \right)}{16a} \\ &= \frac{(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{64a^2x^6} - \frac{(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{24ax^{12}} + \frac{(b^2 - 4ac)^2}{(b^2 - 4ac)^2} \\ &= \frac{(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{64a^2x^6} - \frac{(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2 - 4ac)^2}{(b^2 - 4ac)^2} \\ &= \frac{(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{64a^2x^6} - \frac{(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2 - 4ac)^2}{(b^2 - 4ac)^2} \end{aligned}$$

Mathematica [A] time = 0.16, size = 138, normalized size = 1.04

$$\frac{3(b^2-4ac)\left(x^6(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-2\sqrt{a}(2a+bx^3)\sqrt{a+bx^3+cx^6}\right)}{8a^{3/2}x^6} + \frac{2(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{x^{12}}$$

48a

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^13, x]

[Out] -1/48*((2*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/x^12 + (3*(b^2 - 4*a*c)*(-2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6] + (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]))/(8*a^(3/2)*x^6)/a

IntegrateAlgebraic [A] time = 1.11, size = 139, normalized size = 1.05

$$\frac{(16a^2c^2 - 8ab^2c + b^4) \tanh^{-1}\left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{64a^{5/2}} + \frac{\sqrt{a+bx^3+cx^6}(-16a^3 - 24a^2bx^3 - 40a^2cx^6 - 2ab^2x^6 - 20abcx^9 + 3b^3x^9)}{192a^2x^{12}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-16*a^3 - 24*a^2*b*x^3 - 2*a*b^2*x^6 - 40*a^2*c*x^6 + 3*b^3*x^9 - 20*a*b*c*x^9))/(192*a^2*x^12) + ((b^4 - 8*a*b^2*c + 16*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(64*a^(5/2))

fricas [A] time = 1.40, size = 319, normalized size = 2.40

$$\frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{a}x^{12}\log\left(\frac{(b^2+4a)^2+8ab^2c-4\sqrt{a^2+bx^3+cx^6}\sqrt{a^2+bx^3+cx^6}}{x^6}\right)+4((3ab^3-20a^2bc)^2-24a^3bx^3-2(a^2b^2+20a^2c)^2-16a^4)\sqrt{a^2+bx^3+cx^6}}{768a^3x^{12}} + \frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-a}x^{12}\arctan\left(\frac{\sqrt{a^2+bx^3+cx^6}\sqrt{-a}}{2(a^2+ab^2+ap^2)}\right)+2((3ab^3-20a^2bc)^2-24a^3bx^3-2(a^2b^2+20a^2c)^2-16a^4)\sqrt{a^2+bx^3+cx^6}}{384a^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="fricas")

[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-(b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((3*a*b^3 - 20*a^2*b*c)*x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^12), 1/384*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^12)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^6+b*x^3+a)^(3/2)/x^13,x)`

[Out] `int((c*x^6+b*x^3+a)^(3/2)/x^13,x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^6 + bx^3 + a)^{3/2}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3 + c*x^6)^(3/2)/x^13,x)`

[Out] `int((a + b*x^3 + c*x^6)^(3/2)/x^13, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)`

[Out] `Integral((a + b*x**3 + c*x**6)**(3/2)/x**13, x)`

$$3.193 \quad \int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}} dx$$

Optimal. Leaf size=162

$$\frac{b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}} - \frac{b(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{128a^3x^6} + \frac{b(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{48a^2x^{12}}$$

Rubi [A] time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 730, 720, 724, 206}

$$-\frac{b(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{128a^3x^6} + \frac{b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}} + \frac{b(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a + bx^3 + cx^6)^{5/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^16, x]

[Out] -(b*(b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*a^3*x^6) + (b*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*a^2*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(15*a*x^15) + (b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(256*a^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_)*)Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x], (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

Rule 730

$\text{Int}[(d + e*x)^m * (a + b*x + c*x^2)^p, x] \text{Symbol} \rightarrow \text{Simp}[(e*(d + e*x)^{m+1} * (a + b*x + c*x^2)^{p+1}) / ((m+1)*(c*d^2 - b*d*e + a*e^2)), x] + \text{Dist}[(2*c*d - b*e) / (2*(c*d^2 - b*d*e + a*e^2)), \text{Int}[(d + e*x)^{m+1} * (a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{EqQ}[m + 2*p + 3, 0]$

Rule 1357

$\text{Int}[x^m * (a + c*x^n + b*x^n)^p, x] \text{Symbol} \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1) * (a + b*x + c*x^2)^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[n^2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{16}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^6} dx, x, x^3 \right) \\ &= -\frac{(a + bx^3 + cx^6)^{5/2}}{15ax^{15}} - \frac{b \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^5} dx, x, x^3 \right)}{6a} \\ &= \frac{b(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a + bx^3 + cx^6)^{5/2}}{15ax^{15}} + \frac{(b(b^2 - 4ac)) \text{Subst} \left(\int \frac{\sqrt{a + bx + cx^2}}{x} dx, x, x^3 \right)}{32a^2} \\ &= -\frac{b(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{128a^3x^6} + \frac{b(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a + b)}{1} \\ &= -\frac{b(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{128a^3x^6} + \frac{b(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a + b)}{1} \\ &= -\frac{b(b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{128a^3x^6} + \frac{b(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{48a^2x^{12}} - \frac{(a + b)}{1} \end{aligned}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 1.03

$$\frac{b \left(16a^{3/2} (2a + bx^3) (a + bx^3 + cx^6)^{3/2} - 3x^6 (b^2 - 4ac) \left(2\sqrt{a} (2a + bx^3) \sqrt{a + bx^3 + cx^6} - x^6 (b^2 - 4ac) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right) \right) \right)}{768a^{7/2}x^{12}} - \frac{(a + bx^3 + cx^6)^{5/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]

[Out] -1/15*(a + b*x^3 + c*x^6)^(5/2)/(a*x^15) + (b*(16*a^(3/2)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2) - 3*(b^2 - 4*a*c)*x^6*(2*sqrt[a]*(2*a + b*x^3)*sqrt[a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*sqrt[a]*sqrt[a + b*x^3 + c*x^6])])))/(768*a^(7/2)*x^12)

IntegrateAlgebraic [A] time = 1.55, size = 176, normalized size = 1.09

$$\frac{(-16a^2bc^2 + 8ab^3c - b^5) \tanh^{-1} \left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{\sqrt{a}} \right)}{128a^{7/2}} + \frac{\sqrt{a + bx^3 + cx^6} (-128a^4 - 176a^3bx^3 - 256a^3cx^6 - 8a^2b^2x^6 - 56a^2bcx^9 - 128a^2c^2x^{12} + 10ab^3x^9 + 100ab^2cx^{12} - 15b^4x^{12})}{1920a^3x^{15}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]

[Out] (sqrt[a + b*x^3 + c*x^6]*(-128*a^4 - 176*a^3*b*x^3 - 8*a^2*b^2*x^6 - 256*a^3*c*x^6 + 10*a*b^3*x^9 - 56*a^2*b*c*x^9 - 15*b^4*x^12 + 100*a*b^2*c*x^12 - 128*a^2*c^2*x^12))/(1920*a^3*x^15) + ((-b^5 + 8*a*b^3*c - 16*a^2*b*c^2)*ArcTanh[(sqrt[c]*x^3 - sqrt[a + b*x^3 + c*x^6])/sqrt[a]])/(128*a^(7/2))

fricas [A] time = 1.67, size = 383, normalized size = 2.36

$$\frac{15(b^5 - 8ab^3c + 16a^2b^2c^2)\sqrt{a} \log\left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right) - 4((15a^4 - 100a^2b^2c + 128a^3c^2)x^{12} - 2(5a^2b^3 - 28a^3bc)x^9 + 176a^4b^2x^3 + 8(a^3b^2 + 32a^4c)x^6 + 128a^5)\sqrt{c}x^3 + a}{3840a^3x^{15}} + \frac{15(b^5 - 8ab^3c + 16a^2b^2c^2)\sqrt{a} \arctan\left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right) + 2((15a^4 - 100a^2b^2c + 128a^3c^2)x^{12} - 2(5a^2b^3 - 28a^3bc)x^9 + 176a^4b^2x^3 + 8(a^3b^2 + 32a^4c)x^6 + 128a^5)\sqrt{c}x^3 + a}{3840a^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="fricas")

[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b^2*c^2)*sqrt(a)*x^15*log(-(b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^12 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^6 + 128*a^5)*sqrt(c*x^6 + b*x^3 + a)/(a^4*x^15), -1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^12 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^6 + 128*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16, x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)

[Out] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^(3/2)/x^16,x)

[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^16, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^{16}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)

[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**16, x)

$$3.194 \quad \int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}} dx$$

Optimal. Leaf size=216

$$\frac{(b^2 - 4ac)^2 (7b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}} + \frac{(b^2 - 4ac)(7b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1536a^4x^6} - \frac{(7b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{576a^3x^{12}} + \frac{(b^2 - 4ac)(7b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1536a^4x^6} - \frac{(b^2 - 4ac)^2 (7b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}} + \frac{7b(a + bx^3 + cx^6)^{5/2}}{180a^2x^{15}} - \frac{(a + bx^3 + cx^6)^{5/2}}{18ax^{18}}$$

Rubi [A] time = 0.21, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 744, 806, 720, 724, 206}

$$\frac{(7b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{576a^3x^{12}} + \frac{(b^2 - 4ac)(7b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1536a^4x^6} - \frac{(b^2 - 4ac)^2 (7b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}} + \frac{7b(a + bx^3 + cx^6)^{5/2}}{180a^2x^{15}} - \frac{(a + bx^3 + cx^6)^{5/2}}{18ax^{18}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^19, x]

[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1536*a^4*x^6) - ((7*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*a^3*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(18*a*x^18) + (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*a^2*x^15) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(3072*a^(9/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2

$$\frac{a^2e - b^2d - (2cd - b^2e)x}{\sqrt{ax^2 + bx + c}}$$
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4ac, 0] && NeQ[2cd - b^2e, 0]

Rule 744

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4ac, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2cd - b^2e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])

Rule 806

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4ac, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 1357

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4ac, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{19}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^7} dx, x, x^3 \right) \\
&= -\frac{(a + bx^3 + cx^6)^{5/2}}{18ax^{18}} - \frac{\text{Subst} \left(\int \frac{\left(\frac{7b}{2} + cx\right)(a + bx + cx^2)^{3/2}}{x^6} dx, x, x^3 \right)}{18a} \\
&= -\frac{(a + bx^3 + cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a + bx^3 + cx^6)^{5/2}}{180a^2x^{15}} + \frac{(7b^2 - 4ac) \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^5} dx, x, x^3 \right)}{72a^2} \\
&= -\frac{(7b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{576a^3x^{12}} - \frac{(a + bx^3 + cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a + bx^3 + cx^6)^{5/2}}{180a^2x^{15}} \\
&= \frac{(b^2 - 4ac)(7b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1536a^4x^6} - \frac{(7b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{5/2}}{576a^3x^{12}} \\
&= \frac{(b^2 - 4ac)(7b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1536a^4x^6} - \frac{(7b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{5/2}}{576a^3x^{12}} \\
&= \frac{(b^2 - 4ac)(7b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1536a^4x^6} - \frac{(7b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{5/2}}{576a^3x^{12}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 206, normalized size = 0.95

$$\frac{\left(\frac{7b^2}{2} - 2ac\right) \left(16a^{3/2}(2a + bx^3)(a + bx^3 + cx^6)^{3/2} - 3x^6(b^2 - 4ac) \left(2\sqrt{a}(2a + bx^3)\sqrt{a + bx^3 + cx^6} - x^6(b^2 - 4ac) \tanh^{-1}\left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}}\right)\right)\right)}{256a^{7/2}x^{12}} + \frac{(a + bx^3 + cx^6)^{5/2}}{x^{18}} - \frac{7b(a + bx^3 + cx^6)^{5/2}}{10ax^{15}}$$

18a

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^19, x]

[Out] -1/18*((a + b*x^3 + c*x^6)^(5/2)/x^18 - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(10*a*x^15) + (((7*b^2)/2 - 2*a*c)*(16*a^(3/2)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2) - 3*(b^2 - 4*a*c)*x^6*(2*sqrt[a]*(2*a + b*x^3)*sqrt[a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*sqrt[a]*sqrt[a + b*x^3 + c*x^6])])))/(256*a^(7/2)*x^12))/a

IntegrateAlgebraic [A] time = 2.27, size = 221, normalized size = 1.02

$$\frac{(-64a^3c^3 + 144a^2b^2c^2 - 60ab^4c + 7b^6) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a + bx^3 + cx^6}}{\sqrt{a}}\right) + \sqrt{a + bx^3 + cx^6}(-1280a^5 - 1664a^4bx^3 - 2240a^4cx^6 - 48a^3b^2x^6 - 288a^3bcx^9 - 480a^3c^2x^{12} + 56a^2b^3x^9 + 432a^2b^2cx^{12} + 1296a^2bc^2x^{15} - 70ab^4x^{12} - 760ab^3cx^{15} + 105b^5x^{15})}{1536a^{9/2}} + \frac{7b(a + bx^3 + cx^6)^{5/2}}{23040a^4x^{18}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-1280*a^5 - 1664*a^4*b*x^3 - 48*a^3*b^2*x^6 - 2240*a^4*c*x^6 + 56*a^2*b^3*x^9 - 288*a^3*b*c*x^9 - 70*a*b^4*x^12 + 432*a^2*b^2*c*x^12 - 480*a^3*c^2*x^12 + 105*b^5*x^15 - 760*a*b^3*c*x^15 + 1296*a^2*b*c^2*x^15))/(23040*a^4*x^18) + ((7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(1536*a^(9/2))

fricas [A] time = 2.09, size = 473, normalized size = 2.19

151297 - 48480*a^4 - 144297*c - 44271*c^2)*sqrt(a + b*x^3 + c*x^6)^(3/2)/x^19 + ((105*a^2*b^3*c^2 - 216*a^3*b^2*c^2 + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a)/(a^5*x^18), 1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^18*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18)]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="fricas")

[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x^18*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18), 1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^18*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{19}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19, x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{19}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^6+b*x^3+a)^(3/2)/x^19,x)`

[Out] `int((c*x^6+b*x^3+a)^(3/2)/x^19,x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^6 + bx^3 + a)^{3/2}}{x^{19}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3 + c*x^6)^(3/2)/x^19,x)`

[Out] `int((a + b*x^3 + c*x^6)^(3/2)/x^19, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{19}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)`

[Out] `Integral((a + b*x**3 + c*x**6)**(3/2)/x**19, x)`

$$3.195 \quad \int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}} dx$$

Optimal. Leaf size=255

$$\frac{b(b^2 - 4ac)^2 (3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}} - \frac{b(b^2 - 4ac)(3b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1024a^5x^6} + \frac{b(3b^2 - 4ac)}{21ax^{21}}$$

Rubi [A] time = 0.31, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1357, 744, 834, 806, 720, 724, 206}

$$\frac{(21b^2 - 16ac)(a + bx^3 + cx^6)^{5/2}}{840a^3x^{15}} + \frac{b(3b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{384a^4x^{12}} - \frac{b(b^2 - 4ac)(3b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1024a^5x^6} + \frac{b(b^2 - 4ac)^2(3b^2 - 4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}} + \frac{b(a + bx^3 + cx^6)^{5/2}}{28a^2x^{18}} - \frac{(a + bx^3 + cx^6)^{5/2}}{21ax^{21}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]

[Out] -(b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1024*a^5*x^6) + (b*(3*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(384*a^4*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(21*a*x^21) + (b*(a + b*x^3 + c*x^6)^(5/2))/(28*a^2*x^18) - ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(840*a^3*x^15) + (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2048*a^(11/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_)^2)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x], (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 744

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])

Rule 806

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 834

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3 + cx^6)^{3/2}}{x^{22}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^8} dx, x, x^3 \right) \\
&= -\frac{(a + bx^3 + cx^6)^{5/2}}{21ax^{21}} - \frac{\text{Subst} \left(\int \frac{\left(\frac{9b}{2} + 2cx\right)(a + bx + cx^2)^{3/2}}{x^7} dx, x, x^3 \right)}{21a} \\
&= -\frac{(a + bx^3 + cx^6)^{5/2}}{21ax^{21}} + \frac{b(a + bx^3 + cx^6)^{5/2}}{28a^2x^{18}} + \frac{\text{Subst} \left(\int \frac{\left(\frac{3}{4}(21b^2 - 16ac) + \frac{9bcx}{2}\right)(a + bx + cx^2)^{3/2}}{x^6} dx, x, x^3 \right)}{126a^2} \\
&= -\frac{(a + bx^3 + cx^6)^{5/2}}{21ax^{21}} + \frac{b(a + bx^3 + cx^6)^{5/2}}{28a^2x^{18}} - \frac{(21b^2 - 16ac)(a + bx^3 + cx^6)^{5/2}}{840a^3x^{15}} - \frac{b(3b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{5/2}}{384a^4x^{12}} \\
&= \frac{b(3b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{3/2}}{384a^4x^{12}} - \frac{(a + bx^3 + cx^6)^{5/2}}{21ax^{21}} + \frac{b(a + bx^3 + cx^6)^{5/2}}{28a^2x^{18}} \\
&= -\frac{b(b^2 - 4ac)(3b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1024a^5x^6} + \frac{b(3b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{5/2}}{384a^4x^{12}} \\
&= -\frac{b(b^2 - 4ac)(3b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1024a^5x^6} + \frac{b(3b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{5/2}}{384a^4x^{12}} \\
&= -\frac{b(b^2 - 4ac)(3b^2 - 4ac)(2a + bx^3)\sqrt{a + bx^3 + cx^6}}{1024a^5x^6} + \frac{b(3b^2 - 4ac)(2a + bx^3)(a + bx^3 + cx^6)^{5/2}}{384a^4x^{12}}
\end{aligned}$$

Mathematica [A] time = 0.49, size = 243, normalized size = 0.95

$$\frac{\left(\frac{21abc - 63b^3}{4}\right)\left(16a^{3/2}(2a + bx^3)(a + bx^3 + cx^6)^{3/2} - 3x^6(b^2 - 4ac)\left(2\sqrt{a}(2a + bx^3)\sqrt{a + bx^3 + cx^6} - x^6(b^2 - 4ac)\tanh^{-1}\left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}}\right)\right)\right)}{1536a^{9/2}x^{12}} + \frac{(21b^2 - 16ac)(a + bx^3 + cx^6)^{5/2}}{40a^2x^{15}} + \frac{(a + bx^3 + cx^6)^{5/2}}{x^{21}} - \frac{3b(a + bx^3 + cx^6)^{5/2}}{4ax^{18}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]

[Out] -1/21*((a + b*x^3 + c*x^6)^(5/2)/x^21 - (3*b*(a + b*x^3 + c*x^6)^(5/2))/(4*a*x^18) + ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))/(40*a^2*x^15) + (((-63*b^3)/4 + 21*a*b*c)*(16*a^(3/2)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2) - 3*(b^2 - 4*a*c)*x^6*(2*Sqrt[a]*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*x^6*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]])))/(1536*a^(9/2)*x^12))/a

IntegrateAlgebraic [A] time = 2.48, size = 270, normalized size = 1.06

$$\frac{(64a^2bc^3 - 80a^2b^2c^2 + 28a^2c^3 - 3b^7) \operatorname{tanh}^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right) + \sqrt{a+bx^3+cx^6}(-5120a^6 - 6400a^5bx^3 - 128a^4b^2cx^6 - 8192a^3b^3x^9 - 704a^2b^4cx^{12} - 1024a^2c^2x^{15} + 144a^2b^5x^9 + 992a^2b^2cx^{12} + 2336a^2b^3cx^{15} + 2048a^2b^3x^{18} - 168a^2b^4cx^{12} - 1456a^2b^5cx^{15} - 5488a^2b^2cx^{18} + 210a^2b^5x^{15} + 2520a^2b^4cx^{18} - 315a^2x^{18})}{1024a^{11/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-5120*a^6 - 6400*a^5*b*x^3 - 128*a^4*b^2*x^6 - 8192*a^3*b^3*x^9 + 144*a^2*b^4*c*x^12 + 992*a^2*b^2*c*x^12 - 1024*a^4*c^2*x^12 + 210*a*b^5*x^15 - 1456*a^2*b^3*c*x^15 + 2336*a^3*b*c^2*x^15 - 315*b^6*x^18 + 2520*a*b^4*c*x^18 - 5488*a^2*b^2*c^2*x^18 + 2048*a^3*c^3*x^18))/(107520*a^5*x^21) + ((-3*b^7 + 28*a*b^5*c - 80*a^2*b^3*c^2 + 64*a^3*b*c^3)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(1024*a^(11/2))

fricas [A] time = 2.63, size = 557, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="fricas")

[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(a)*x^21*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a))*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((315*a*b^6 - 2520*a^2*b^4*c + 5488*a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5 - 728*a^3*b^3*c + 1168*a^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 128*a^5*c^2)*x^12 + 6400*a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a)/(a^6*x^21), -1/215040*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-a)*x^21*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((315*a*b^6 - 2520*a^2*b^4*c + 5488*a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5 - 728*a^3*b^3*c + 1168*a^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 128*a^5*c^2)*x^12 + 6400*a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a)/(a^6*x^21)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{22}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22, x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{22}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)

[Out] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^6 + bx^3 + a)^{\frac{3}{2}}}{x^{22}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^(3/2)/x^22,x)

[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^22, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^{22}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)

[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**22, x)

$$3.196 \quad \int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=171

$$\frac{(48a^2c^2 - 120ab^2c + 35b^4) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}} - \frac{(5b(21b^2 - 44ac) - 2cx^3(35b^2 - 36ac))\sqrt{a+bx^3+cx^6}}{576c^4}$$

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 742, 832, 779, 621, 206}

$$\frac{(48a^2c^2 - 120ab^2c + 35b^4) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}} - \frac{(5b(21b^2 - 44ac) - 2cx^3(35b^2 - 36ac))\sqrt{a+bx^3+cx^6}}{576c^4} - \frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c}$$

Antiderivative was successfully verified.

[In] Int[x^14/Sqrt[a + b*x^3 + c*x^6], x]

[Out] (-7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(72*c^2) + (x^9*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(576*c^4) + ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(9/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[In t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 742

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad

raticQ[a, b, c, d, e, m, p, x]

Rule 779

```
Int[((d_.) + (e_.)*(x_.))*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_.))^(m_.))*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^4}{\sqrt{a+bx+cx^2}} dx, x, x^3 \right) \\
&= \frac{x^9 \sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst} \left(\int \frac{x^2 \left(-3a - \frac{7bx}{2} \right)}{\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{12c} \\
&= -\frac{7bx^6 \sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9 \sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst} \left(\int \frac{x \left(7ab + \frac{1}{4}(35b^2 - 36ac)x \right)}{\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{36c^2} \\
&= -\frac{7bx^6 \sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9 \sqrt{a+bx^3+cx^6}}{12c} - \frac{(5b(21b^2 - 44ac) - 2c(35b^2 - 36ac))x^3}{576c^4} \\
&= -\frac{7bx^6 \sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9 \sqrt{a+bx^3+cx^6}}{12c} - \frac{(5b(21b^2 - 44ac) - 2c(35b^2 - 36ac))x^3}{576c^4} \\
&= -\frac{7bx^6 \sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9 \sqrt{a+bx^3+cx^6}}{12c} - \frac{(5b(21b^2 - 44ac) - 2c(35b^2 - 36ac))x^3}{576c^4}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 137, normalized size = 0.80

$$\frac{3(48a^2c^2 - 120ab^2c + 35b^4) \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}} \right) + 2\sqrt{c}\sqrt{a+bx^3+cx^6} (4bc(55a - 14cx^6) + 24c^2x^3(2cx^6 - 3a) - 105b^3 + 70b^2cx^3)}{1152c^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^14/Sqrt[a + b*x^3 + c*x^6], x]

[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 70*b^2*c*x^3 + 4*b*c*(55*a - 14*c*x^6) + 24*c^2*x^3*(-3*a + 2*c*x^6)) + 3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(1152*c^(9/2))

IntegrateAlgebraic [A] time = 0.42, size = 138, normalized size = 0.81

$$\frac{(-48a^2c^2 + 120ab^2c - 35b^4) \log \left(-2c^{9/2}\sqrt{a+bx^3+cx^6} + bc^4 + 2c^5x^3 \right)}{384c^{9/2}} + \frac{\sqrt{a+bx^3+cx^6} (220abc - 72ac^2x^3 - 105b^3 + 70b^2cx^3 - 56bc^2x^6 + 48c^3x^9)}{576c^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14/Sqrt[a + b*x^3 + c*x^6], x]

[Out] $(\sqrt{a + b*x^3 + c*x^6}*(-105*b^3 + 220*a*b*c + 70*b^2*c*x^3 - 72*a*c^2*x^3 - 56*b*c^2*x^6 + 48*c^3*x^9))/(576*c^4) + ((-35*b^4 + 120*a*b^2*c - 48*a^2*c^2)*\text{Log}[b*c^4 + 2*c^5*x^3 - 2*c^{(9/2)}*\sqrt{a + b*x^3 + c*x^6}])/(384*c^{(9/2)})$

fricas [A] time = 0.64, size = 303, normalized size = 1.77

$$\frac{3(35b^4 - 120ab^2c + 48a^2c^2)\sqrt{c} \log\left(\frac{-8c^2x^6 - 8bc^3x^6 - 4\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4ac}{2304c^5}\right) + 4(48c^4x^9 - 56b^2c^3x^6 - 105b^3c^2 + 220ab^2c + 2(35b^2c^2 - 36ac^3)x^3)\sqrt{cx^6 + bx^3 + a}}{1152c^5} - \frac{3(35b^4 - 120ab^2c + 48a^2c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}\sqrt{-c}}{2(c^2x^6 + bcx^3 + ac)}\right) - 2(48c^4x^9 - 56b^2c^3x^6 - 105b^3c^2 + 220ab^2c + 2(35b^2c^2 - 36ac^3)x^3)\sqrt{cx^6 + bx^3 + a}}{1152c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")`

[Out] $[1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*\text{sqrt}(c)*\text{log}(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*\text{sqrt}(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*\text{sqrt}(c) - 4*a*c) + 4*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*a*c^3)*x^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/c^5, -1/1152*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*\text{sqrt}(-c)*\text{arctan}(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*\text{sqrt}(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*a*c^3)*x^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/c^5]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")`

[Out] `integrate(x^14/sqrt(c*x^6 + b*x^3 + a), x)`

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14/(c*x^6+b*x^3+a)^(1/2),x)`

[Out] `int(x^14/(c*x^6+b*x^3+a)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14/(a + b*x^3 + c*x^6)^(1/2),x)`

[Out] `int(x^14/(a + b*x^3 + c*x^6)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{a + bx^3 + cx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)`

[Out] `Integral(x**14/sqrt(a + b*x**3 + c*x**6), x)`

$$3.197 \quad \int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=121

$$-\frac{b(5b^2 - 12ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}} + \frac{(-16ac + 15b^2 - 10bcx^3) \sqrt{a+bx^3+cx^6}}{72c^3} + \frac{x^6 \sqrt{a+bx^3+cx^6}}{9c}$$

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 742, 779, 621, 206}

$$\frac{(-16ac + 15b^2 - 10bcx^3) \sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b(5b^2 - 12ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}} + \frac{x^6 \sqrt{a+bx^3+cx^6}}{9c}$$

Antiderivative was successfully verified.

[In] Int[x^11/Sqrt[a + b*x^3 + c*x^6], x]

[Out] (x^6*Sqrt[a + b*x^3 + c*x^6])/(9*c) + ((15*b^2 - 16*a*c - 10*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(72*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 742

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad

raticQ[a, b, c, d, e, m, p, x]

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) -
2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{\sqrt{a+bx+cx^2}} dx, x, x^3 \right) \\ &= \frac{x^6 \sqrt{a+bx^3+cx^6}}{9c} + \frac{\text{Subst} \left(\int \frac{x^{(-2a-\frac{5bx}{2})}}{\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{9c} \\ &= \frac{x^6 \sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2 - 16ac - 10bcx^3) \sqrt{a+bx^3+cx^6}}{72c^3} - \frac{(b(5b^2 - 12ac)) \text{Subst}}{48c^3} \\ &= \frac{x^6 \sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2 - 16ac - 10bcx^3) \sqrt{a+bx^3+cx^6}}{72c^3} - \frac{(b(5b^2 - 12ac)) \text{Subst}}{48c^3} \\ &= \frac{x^6 \sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2 - 16ac - 10bcx^3) \sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b(5b^2 - 12ac) \tanh^{-1}}{48c^{7/2}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 0.86

$$\frac{(36abc - 15b^3) \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c} \sqrt{a+bx^3+cx^6}} \right) + 2\sqrt{c} \sqrt{a+bx^3+cx^6} (8c(cx^6 - 2a) + 15b^2 - 10bcx^3)}{144c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/Sqrt[a + b*x^3 + c*x^6],x]

[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 10*b*c*x^3 + 8*c*(-2*a + c*x^6)) + (-15*b^3 + 36*a*b*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(144*c^(7/2))

IntegrateAlgebraic [A] time = 0.32, size = 101, normalized size = 0.83

$$\frac{(5b^3 - 12abc) \log\left(-2\sqrt{c} \sqrt{a + bx^3 + cx^6} + b + 2cx^3\right)}{48c^{7/2}} + \frac{\sqrt{a + bx^3 + cx^6} (-16ac + 15b^2 - 10bcx^3 + 8c^2x^6)}{72c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/Sqrt[a + b*x^3 + c*x^6],x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 16*a*c - 10*b*c*x^3 + 8*c^2*x^6))/(72*c^3) + ((5*b^3 - 12*a*b*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(48*c^(7/2))

fricas [A] time = 1.36, size = 241, normalized size = 1.99

$$\left[\frac{3(5b^3 - 12abc)\sqrt{c} \log\left(-8c^2x^6 - 8b^2cx^3 - b^2 - 4\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4ac\right) - 4(8c^3x^6 - 10bc^2x^3 + 15b^2c - 16ac^2)\sqrt{cx^6 + bx^3 + a}}{288c^4}, \frac{3(5b^3 - 12abc)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c}}{2(\sqrt{c^2 + bcx^3 + ac})}\right) + 2(8c^3x^6 - 10bc^2x^3 + 15b^2c - 16ac^2)\sqrt{cx^6 + bx^3 + a}}{144c^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b^2*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 - 10*b*c^2*x^3 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/144*(3*(5*b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 - 10*b*c^2*x^3 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(x^11/sqrt(c*x^6 + b*x^3 + a), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(c*x^6+b*x^3+a)^(1/2),x)`

[Out] `int(x^11/(c*x^6+b*x^3+a)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(a + b*x^3 + c*x^6)^(1/2),x)`

[Out] `int(x^11/(a + b*x^3 + c*x^6)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{a + bx^3 + cx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)`

[Out] `Integral(x**11/sqrt(a + b*x**3 + c*x**6), x)`

$$3.198 \quad \int \frac{x^8}{\sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=104

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}} - \frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 742, 640, 621, 206}

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}} - \frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

Antiderivative was successfully verified.

[In] Int[x^8/Sqrt[a + b*x^3 + c*x^6], x]

[Out] -(b*Sqrt[a + b*x^3 + c*x^6])/(4*c^2) + (x^3*Sqrt[a + b*x^3 + c*x^6])/(6*c) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 742

```
Int[((d._) + (e._)*(x_))^(m_)*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^8}{\sqrt{a + bx^3 + cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\ &= \frac{x^3 \sqrt{a + bx^3 + cx^6}}{6c} + \frac{\text{Subst} \left(\int \frac{-a - \frac{3bx}{2}}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{6c} \\ &= -\frac{b\sqrt{a + bx^3 + cx^6}}{4c^2} + \frac{x^3 \sqrt{a + bx^3 + cx^6}}{6c} + \frac{(3b^2 - 4ac) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{24c^2} \\ &= -\frac{b\sqrt{a + bx^3 + cx^6}}{4c^2} + \frac{x^3 \sqrt{a + bx^3 + cx^6}}{6c} + \frac{(3b^2 - 4ac) \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx^3}{\sqrt{a + bx^3 + cx^6}} \right)}{12c^2} \\ &= -\frac{b\sqrt{a + bx^3 + cx^6}}{4c^2} + \frac{x^3 \sqrt{a + bx^3 + cx^6}}{6c} + \frac{(3b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{24c^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 88, normalized size = 0.85

$$\frac{(3b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right) + 2\sqrt{c} (2cx^3 - 3b) \sqrt{a + bx^3 + cx^6}}{24c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/Sqrt[a + b*x^3 + c*x^6],x]

[Out] (2*Sqrt[c]*(-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6] + (3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(5/2))

IntegrateAlgebraic [A] time = 0.29, size = 91, normalized size = 0.88

$$\frac{(4ac - 3b^2) \log\left(-2c^{5/2} \sqrt{a + bx^3 + cx^6} + bc^2 + 2c^3x^3\right)}{24c^{5/2}} + \frac{(2cx^3 - 3b) \sqrt{a + bx^3 + cx^6}}{12c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/Sqrt[a + b*x^3 + c*x^6],x]

[Out] ((-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^2) + ((-3*b^2 + 4*a*c)*Log[b*c^2 + 2*c^3*x^3 - 2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(5/2))

fricas [A] time = 1.46, size = 203, normalized size = 1.95

$$\left[\frac{(3b^2 - 4ac)\sqrt{c} \log\left(-8c^2x^6 - 8b*c*x^3 - b^2 + 4\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4\sqrt{cx^6 + bx^3 + a}(2c^2x^3 - 3bc)\right)}{48c^3}, -\frac{(3b^2 - 4ac)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{-c}}{2(2c^2x^3 - 3bc)\sqrt{-c}}\right) - 2\sqrt{cx^6 + bx^3 + a}(2c^2x^3 - 3bc)}{24c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 - 3*b*c))/c^3, -1/24*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*sqrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 - 3*b*c))/c^3]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(x^8/sqrt(c*x^6 + b*x^3 + a), x)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(c*x^6+b*x^3+a)^(1/2),x)`

[Out] `int(x^8/(c*x^6+b*x^3+a)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(a + b*x^3 + c*x^6)^(1/2),x)`

[Out] `int(x^8/(a + b*x^3 + c*x^6)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{a + bx^3 + cx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)`

[Out] `Integral(x**8/sqrt(a + b*x**3 + c*x**6), x)`

$$3.199 \quad \int \frac{x^5}{\sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1357, 640, 621, 206}

$$\frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sqrt[a + b*x^3 + c*x^6], x]

[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(6*c^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{x^5}{\sqrt{a + bx^3 + cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\ &= \frac{\sqrt{a + bx^3 + cx^6}}{3c} - \frac{b \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{6c} \\ &= \frac{\sqrt{a + bx^3 + cx^6}}{3c} - \frac{b \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx^3}{\sqrt{a + bx^3 + cx^6}} \right)}{3c} \\ &= \frac{\sqrt{a + bx^3 + cx^6}}{3c} - \frac{b \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{6c^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.00

$$\frac{\sqrt{a + bx^3 + cx^6}}{3c} - \frac{b \tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c} \sqrt{a + bx^3 + cx^6}} \right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[a + b*x^3 + c*x^6], x]

[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(6*c^(3/2))

IntegrateAlgebraic [A] time = 0.19, size = 70, normalized size = 1.03

$$\frac{b \log \left(-2c^{3/2} \sqrt{a + bx^3 + cx^6} + bc + 2c^2 x^3 \right)}{6c^{3/2}} + \frac{\sqrt{a + bx^3 + cx^6}}{3c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/Sqrt[a + b*x^3 + c*x^6], x]

[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) + (b*Log[b*c + 2*c^2*x^3 - 2*c^(3/2)*Sqrt[a + b*x^3 + c*x^6])]/(6*c^(3/2))

fricas [A] time = 1.35, size = 161, normalized size = 2.37

$$\left[\frac{b\sqrt{c} \log\left(-8c^2x^6 - 8bcx^3 - b^2 + 4\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4ac\right) + 4\sqrt{cx^6 + bx^3 + a}c}{12c^2}, \frac{b\sqrt{-c} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{-c}}{2(c^2x^6 + bcx^3 + ac)}\right) + 2\sqrt{cx^6 + bx^3 + a}c}{6c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c^2, 1/6*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*c)/c^2]

giac [A] time = 0.67, size = 61, normalized size = 0.90

$$\frac{b \log\left(\left|-2\left(\sqrt{c}x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{6c^{\frac{3}{2}}} + \frac{\sqrt{cx^6 + bx^3 + a}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] 1/6*b*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3/2) + 1/3*sqrt(c*x^6 + b*x^3 + a)/c

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 zero or nonzero?

mupad [B] time = 1.49, size = 55, normalized size = 0.81

$$\frac{\sqrt{cx^6 + bx^3 + a}}{3c} - \frac{b \ln\left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + \frac{b}{2}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a + b*x^3 + c*x^6)^(1/2), x)

[Out] (a + b*x^3 + c*x^6)^(1/2)/(3*c) - (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2)))/(6*c^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a + bx^3 + cx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**6+b*x**3+a)**(1/2), x)

[Out] Integral(x**5/sqrt(a + b*x**3 + c*x**6), x)

$$3.200 \quad \int \frac{x^2}{\sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1352, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a + b*x^3 + c*x^6], x]

[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+bx^3+cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^3 \right) \\ &= \frac{2}{3} \text{Subst} \left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}} \right) \\ &= \frac{\tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}} \right)}{3\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}} \right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a + b*x^3 + c*x^6], x]

[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])

IntegrateAlgebraic [A] time = 0.14, size = 41, normalized size = 0.95

$$\frac{\log \left(-2\sqrt{c}\sqrt{a+bx^3+cx^6} + b + 2cx^3 \right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/Sqrt[a + b*x^3 + c*x^6], x]

[Out] -1/3*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]]/Sqrt[c]

fricas [A] time = 1.21, size = 118, normalized size = 2.74

$$\left[\frac{\log \left(-8c^2x^6 - 8bcx^3 - b^2 - 4\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{c} - 4ac \right)}{6\sqrt{c}}, -\frac{\sqrt{-c} \arctan \left(\frac{\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{-c}}{2(c^2x^6+bcx^3+ac)} \right)}{3c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] $\left[\frac{1}{6} \log(-8c^2x^6 - 8bcx^3 - b^2 - 4\sqrt{cx^6 + bx^3 + a})(2cx^3 + b)\sqrt{c} - 4ac\right] / \sqrt{c}, -\frac{1}{3}\sqrt{-c} \arctan\left(\frac{1}{2}\sqrt{cx^6 + bx^3 + a}\right) / \sqrt{-c} \left[\frac{1}{6} \log(-8c^2x^6 - 8bcx^3 - b^2 - 4\sqrt{cx^6 + bx^3 + a})(2cx^3 + b)\sqrt{c} - 4ac\right] / \sqrt{c}$

giac [A] time = 0.68, size = 40, normalized size = 0.93

$$\frac{\log\left(\left|-2\left(\sqrt{c}x^3 - \sqrt{cx^6 + bx^3 + a}\right)\sqrt{c} - b\right|\right)}{3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")`

[Out] $-1/3 \log(\text{abs}(-2(\sqrt{c})x^3 - \sqrt{cx^6 + bx^3 + a})\sqrt{c} - b) / \sqrt{c}$

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(c*x^6+b*x^3+a)^(1/2),x)`

[Out] `int(x^2/(c*x^6+b*x^3+a)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [B] time = 1.58, size = 34, normalized size = 0.79

$$\frac{\ln\left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + \frac{b}{2}}{\sqrt{c}}\right)}{3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*x^3 + c*x^6)^(1/2), x)`

[Out] `log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + bx^3 + cx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**6+b*x**3+a)**(1/2), x)`

[Out] `Integral(x**2/sqrt(a + b*x**3 + c*x**6), x)`

$$3.201 \quad \int \frac{1}{x\sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=44

$$\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1357, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] -ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a+bx^3+cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^3 \right) \\ &= -\left(\frac{2}{3} \text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}} \right) \right) \\ &= -\frac{\tanh^{-1} \left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}} \right)}{3\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{\tanh^{-1} \left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}} \right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x^3 + c*x^6]), x]

[Out] -1/3*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/Sqrt[a]

IntegrateAlgebraic [A] time = 0.13, size = 48, normalized size = 1.09

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{c}x^3}{\sqrt{a}} - \frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}} \right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^3 + c*x^6]), x]

[Out] (2*ArcTanh[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a]])/(3*Sqrt[a])

fricas [A] time = 1.10, size = 124, normalized size = 2.82

$$\left[\frac{\log \left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6} \right)}{6\sqrt{a}}, \frac{\sqrt{-a} \arctan \left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)} \right)}{3a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6)/sqrt(a), 1/3*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2))/a]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x), x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 1.57, size = 36, normalized size = 0.82

$$\frac{\ln\left(\frac{b}{2} + \frac{a}{x^3} + \frac{\sqrt{a}\sqrt{cx^6+bx^3+a}}{x^3}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*x^3 + c*x^6)^(1/2)),x)`

[Out] `-log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))/x^3)/(3*a^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + bx^3 + cx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(a + b*x**3 + c*x**6)), x)`

$$3.202 \quad \int \frac{1}{x^4 \sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=72

$$\frac{b \tanh^{-1} \left(\frac{2a+bx^3}{2\sqrt{a} \sqrt{a+bx^3+cx^6}} \right)}{6a^{3/2}} - \frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1357, 730, 724, 206}

$$\frac{b \tanh^{-1} \left(\frac{2a+bx^3}{2\sqrt{a} \sqrt{a+bx^3+cx^6}} \right)}{6a^{3/2}} - \frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] -Sqrt[a + b*x^3 + c*x^6]/(3*a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(6*a^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 730

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]

Rule 1357

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 \sqrt{a + bx^3 + cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 \sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\ &= -\frac{\sqrt{a + bx^3 + cx^6}}{3ax^3} - \frac{b \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{6a} \\ &= -\frac{\sqrt{a + bx^3 + cx^6}}{3ax^3} + \frac{b \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx^3}{\sqrt{a + bx^3 + cx^6}} \right)}{3a} \\ &= -\frac{\sqrt{a + bx^3 + cx^6}}{3ax^3} + \frac{b \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right)}{6a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.00

$$\frac{b \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right)}{6a^{3/2}} - \frac{\sqrt{a + bx^3 + cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
 Sqrt[a + b*x^3 + c*x^6]))/(6*a^(3/2))

IntegrateAlgebraic [A] time = 0.21, size = 76, normalized size = 1.06

$$-\frac{b \tanh^{-1} \left(\frac{\sqrt{c} x^3}{\sqrt{a}} - \frac{\sqrt{a + bx^3 + cx^6}}{\sqrt{a}} \right)}{3a^{3/2}} - \frac{\sqrt{a + bx^3 + cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] $-1/3*\text{Sqrt}[a + b*x^3 + c*x^6]/(a*x^3) - (b*\text{ArcTanh}[(\text{Sqrt}[c]*x^3)/\text{Sqrt}[a] - \text{Sqrt}[a + b*x^3 + c*x^6]/\text{Sqrt}[a]])/(3*a^{(3/2)})$

fricas [A] time = 1.36, size = 179, normalized size = 2.49

$$\left[\frac{\sqrt{a} b x^3 \log\left(-\frac{(b^2+4ac)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a+8a^2}}{x^6}\right) - 4\sqrt{cx^6+bx^3+a} a}{12a^2x^3}, -\frac{\sqrt{-a} b x^3 \arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right) + 2\sqrt{cx^6+bx^3+a} a}{6a^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")`

[Out] $[1/12*(\text{sqrt}(a)*b*x^3*\log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a))*\text{sqrt}(a) + 8*a^2)/x^6) - 4*\text{sqrt}(c*x^6 + b*x^3 + a)*a)/(a^2*x^3), -1/6*(\text{sqrt}(-a)*b*x^3*\arctan(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a))*\text{sqrt}(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*\text{sqrt}(c*x^6 + b*x^3 + a)*a)/(a^2*x^3)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4), x)`

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)`

[Out] `int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is $4ac - b^2$ positive, negative or zero?

mupad [B] time = 1.56, size = 56, normalized size = 0.78

$$\frac{b \operatorname{atanh}\left(\frac{\frac{bx^3}{2} + a}{\sqrt{a} \sqrt{cx^6 + bx^3 + a}}\right)}{6a^{3/2}} - \frac{\sqrt{cx^6 + bx^3 + a}}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(a + b*x^3 + c*x^6)^(1/2)),x)`

[Out] $(b \operatorname{atanh}((a + (bx^3)/2)/(a^{1/2}(a + bx^3 + cx^6)^{1/2}))) / (6a^{3/2}) - (a + bx^3 + cx^6)^{1/2} / (3ax^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt{a + bx^3 + cx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)`

[Out] `Integral(1/(x**4*sqrt(a + b*x**3 + c*x**6)), x)`

$$3.203 \quad \int \frac{1}{x^7 \sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=108

$$-\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 744, 806, 724, 206}

$$-\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}} + \frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3} - \frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] -Sqrt[a + b*x^3 + c*x^6]/(6*a*x^6) + (b*Sqrt[a + b*x^3 + c*x^6])/(4*a^2*x^3) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 744

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && NeQ[m + 1, 0]

$Q[m, -1] \&\& ((LtQ[m, -1] \&\& IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] \&\& IntegerQ[p])) || ILtQ[Simplify[m + 2*p + 3], 0]$

Rule 806

$Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] \rightarrow -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& NeQ[c*d^2 - b*d*e + a*e^2, 0] \&\& EqQ[Simplify[m + 2*p + 3], 0]$

Rule 1357

$Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] \rightarrow Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] \&\& EqQ[n2, 2*n] \&\& NeQ[b^2 - 4*a*c, 0] \&\& IntegerQ[Simplify[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x^7 \sqrt{a + bx^3 + cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^3 \sqrt{a + bx + cx^2}} dx, x, x^3 \right) \\ &= -\frac{\sqrt{a + bx^3 + cx^6}}{6ax^6} - \frac{\text{Subst} \left(\int \frac{\frac{3b}{2} + cx}{x^2 \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{6a} \\ &= -\frac{\sqrt{a + bx^3 + cx^6}}{6ax^6} + \frac{b\sqrt{a + bx^3 + cx^6}}{4a^2x^3} + \frac{(3b^2 - 4ac) \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{24a^2} \\ &= -\frac{\sqrt{a + bx^3 + cx^6}}{6ax^6} + \frac{b\sqrt{a + bx^3 + cx^6}}{4a^2x^3} - \frac{(3b^2 - 4ac) \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx^3}{\sqrt{a + bx^3 + cx^6}} \right)}{12a^2} \\ &= -\frac{\sqrt{a + bx^3 + cx^6}}{6ax^6} + \frac{b\sqrt{a + bx^3 + cx^6}}{4a^2x^3} - \frac{(3b^2 - 4ac) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right)}{24a^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 92, normalized size = 0.85

$$\frac{(4ac - 3b^2) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right) + \frac{2\sqrt{a}(3bx^3 - 2a)\sqrt{a + bx^3 + cx^6}}{x^6}}{24a^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] ((2*Sqrt[a]*(-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^6 + (-3*b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(5/2))

IntegrateAlgebraic [A] time = 0.34, size = 91, normalized size = 0.84

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{5/2}} + \frac{(3bx^3 - 2a)\sqrt{a+bx^3+cx^6}}{12a^2x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] ((-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a^2*x^6) + ((3*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(5/2))

fricas [A] time = 1.41, size = 221, normalized size = 2.05

$$\left[\frac{(3b^2 - 4ac)\sqrt{a}x^6 \log\left(-\frac{(b^2+4ac)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right) - 4\sqrt{cx^6+bx^3+a}(3abx^3-2a^2)(3b^2-4ac)\sqrt{-a}x^6 \arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right) + 2\sqrt{cx^6+bx^3+a}(3abx^3-2a^2)}{48a^3x^6}, \frac{(3b^2-4ac)\sqrt{-a}x^6 \arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right) + 2\sqrt{cx^6+bx^3+a}(3abx^3-2a^2)}{24a^3x^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6), 1/24*((3*b^2 - 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a} x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)`

[Out] `int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7 \sqrt{cx^6 + bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)),x)`

[Out] `int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \sqrt{a + bx^3 + cx^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)`

[Out] `Integral(1/(x**7*sqrt(a + b*x**3 + c*x**6)), x)`

$$3.204 \quad \int \frac{1}{x^{10} \sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=145

$$\frac{b(5b^2 - 12ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}} - \frac{(15b^2 - 16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

Rubi [A] time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 744, 834, 806, 724, 206}

$$-\frac{(15b^2 - 16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{b(5b^2 - 12ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

Antiderivative was successfully verified.

[In] Int[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] -Sqrt[a + b*x^3 + c*x^6]/(9*a*x^9) + (5*b*Sqrt[a + b*x^3 + c*x^6])/(36*a^2*x^6) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(72*a^3*x^3) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 744

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0]

$2 - 4ac, 0]$ && $\text{NeQ}[c^2d - bde + ae^2, 0]$ && $\text{NeQ}[2cd - be, 0]$ && $\text{NeQ}[m, -1]$ && $(\text{LtQ}[m, -1] \&\& \text{IntQuadraticQ}[a, b, c, d, e, m, p, x]) \mid\mid (\text{SumSimplerQ}[m, 1] \&\& \text{IntegerQ}[p]) \mid\mid \text{ILtQ}[\text{Simplify}[m + 2p + 3], 0]$

Rule 806

$\text{Int}[(d_.) + (e_.)x^{(m_)} * ((f_.) + (g_.)x) * ((a_.) + (b_.)x + (c_.)x^2)^{(p_.)}, x_Symbol] \rightarrow -\text{Simp}[(ef - dg)(d + ex)^{(m+1)}(a + bx + cx^2)^{(p+1)} / (2(p+1)(c^2d - bde + ae^2)), x] - \text{Dist}[(b(ef + dg) - 2(cdf + aeg)) / (2(c^2d - bde + ae^2)), \text{Int}[(d + ex)^{(m+1)}(a + bx + cx^2)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x]$ && $\text{NeQ}[b^2 - 4ac, 0]$ && $\text{NeQ}[c^2d - bde + ae^2, 0]$ && $\text{EqQ}[\text{Simplify}[m + 2p + 3], 0]$

Rule 834

$\text{Int}[(d_.) + (e_.)x^{(m_)} * ((f_.) + (g_.)x) * ((a_.) + (b_.)x + (c_.)x^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(ef - dg)(d + ex)^{(m+1)}(a + bx + cx^2)^{(p+1)} / ((m+1)(c^2d - bde + ae^2)), x] + \text{Dist}[1 / ((m+1)(c^2d - bde + ae^2)), \text{Int}[(d + ex)^{(m+1)}(a + bx + cx^2)^p \text{Simp}[(cdf - fbe + aeg)(m+1) + b(dg - ef)(p+1) - c(ef - dg)(m + 2p + 3)x, x], x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, p\}, x]$ && $\text{NeQ}[b^2 - 4ac, 0]$ && $\text{NeQ}[c^2d - bde + ae^2, 0]$ && $\text{LtQ}[m, -1]$ && $(\text{IntegerQ}[m] \mid\mid \text{IntegerQ}[p] \mid\mid \text{IntegersQ}[2m, 2p])$

Rule 1357

$\text{Int}[x^{(m_)} * ((a_.) + (c_.)x^{(n2_)} + (b_.)x^{(n_)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)}(a + bx + cx^2)^p, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, m, n, p\}, x]$ && $\text{EqQ}[n2, 2*n]$ && $\text{NeQ}[b^2 - 4ac, 0]$ && $\text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} - \frac{\text{Subst} \left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{9a} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} + \frac{\text{Subst} \left(\int \frac{\frac{1}{4}(15b^2-16ac)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{18a^2} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} - \frac{b(5b^2-16ac)}{72a^3x^3} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{b(5b^2-16ac)}{72a^3x^3} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{b(5b^2-16ac)}{72a^3x^3}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.77

$$\frac{b(5b^2-12ac) \tanh^{-1} \left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}} \right)}{48a^{7/2}} + \frac{\sqrt{a+bx^3+cx^6} (-8a^2+2a(5bx^3+8cx^6)-15b^2x^6)}{72a^3x^9}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 15*b^2*x^6 + 2*a*(5*b*x^3 + 8*c*x^6)))/(72*a^3*x^9) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(7/2))

IntegrateAlgebraic [A] time = 0.53, size = 110, normalized size = 0.76

$$\frac{(12abc-5b^3) \tanh^{-1} \left(\frac{\sqrt{c}x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}} \right)}{24a^{7/2}} + \frac{\sqrt{a+bx^3+cx^6} (-8a^2+10abx^3+16acx^6-15b^2x^6)}{72a^3x^9}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]

[Out] $(\sqrt{a + b*x^3 + c*x^6}*(-8*a^2 + 10*a*b*x^3 - 15*b^2*x^6 + 16*a*c*x^6))/((72*a^3*x^9) + ((-5*b^3 + 12*a*b*c)*\text{ArcTanh}[(\sqrt{c}*x^3 - \sqrt{a + b*x^3 + c*x^6})/\sqrt{a}])/(24*a^{(7/2)}))$

fricas [A] time = 1.28, size = 263, normalized size = 1.81

$$\frac{3(5b^3 - 12abc)\sqrt{a}x^9 \log\left(\frac{-(b^2+4ac)x^6 + 8abx^3 - 4\sqrt{c^2+b^2+a}(bx^3+2a)\sqrt{a^2+a^2}}{x^6}\right) + 4((15ab^2 - 16a^2c)x^6 - 10a^2bx^3 + 8a^3)\sqrt{cx^6 + bx^3 + a}}{288a^4x^9} - \frac{3(5b^3 - 12abc)\sqrt{-a}x^9 \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}(bx^3+2a)\sqrt{-a}}{2(acx^6 + abx^3 + a^2)}\right) + 2((15ab^2 - 16a^2c)x^6 - 10a^2bx^3 + 8a^3)\sqrt{cx^6 + bx^3 + a}}{144a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")`

[Out] $[-1/288*(3*(5*b^3 - 12*a*b*c)*\text{sqrt}(a)*x^9*\log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*\text{sqrt}(a) + 8*a^2)/x^6) + 4*((15*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/(a^4*x^9), -1/144*(3*(5*b^3 - 12*a*b*c)*\text{sqrt}(-a)*x^9*\arctan(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*\text{sqrt}(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/(a^4*x^9)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10), x)`

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a}x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)`

[Out] `int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10} \sqrt{c x^6 + b x^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)),x)

[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10} \sqrt{a + b x^3 + c x^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)

[Out] Integral(1/(x**10*sqrt(a + b*x**3 + c*x**6)), x)

$$3.205 \quad \int \frac{1}{x^{13} \sqrt{a+bx^3+cx^6}} dx$$

Optimal. Leaf size=192

$$\frac{5b(21b^2 - 44ac) \sqrt{a+bx^3+cx^6}}{576a^4x^3} - \frac{(35b^2 - 36ac) \sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(48a^2c^2 - 120ab^2c + 35b^4)}{384a^{9/2}}$$

Rubi [A] time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 744, 834, 806, 724, 206}

$$-\frac{(48a^2c^2 - 120ab^2c + 35b^4) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}} + \frac{5b(21b^2 - 44ac) \sqrt{a+bx^3+cx^6}}{576a^4x^3} - \frac{(35b^2 - 36ac) \sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^13*sqrt[a + b*x^3 + c*x^6]),x]

[Out] -sqrt[a + b*x^3 + c*x^6]/(12*a*x^12) + (7*b*sqrt[a + b*x^3 + c*x^6])/(72*a^2*x^9) - ((35*b^2 - 36*a*c)*sqrt[a + b*x^3 + c*x^6])/(288*a^3*x^6) + (5*b*(21*b^2 - 44*a*c)*sqrt[a + b*x^3 + c*x^6])/(576*a^4*x^3) - ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*sqrt[a]*sqrt[a + b*x^3 + c*x^6])])/(384*a^(9/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 744

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0]

$2 - 4ac, 0] \&\& \text{NeQ}[c^2d - bde + ae^2, 0] \&\& \text{NeQ}[2cd - be, 0] \&\& \text{NeQ}[m, -1] \&\& ((\text{LtQ}[m, -1] \&\& \text{IntQuadraticQ}[a, b, c, d, e, m, p, x]) \mid\mid (\text{SumSimplerQ}[m, 1] \&\& \text{IntegerQ}[p]) \mid\mid \text{ILtQ}[\text{Simplify}[m + 2p + 3], 0])$

Rule 806

$\text{Int}[(d + e x)^m ((f + g x)(a + b x + c x^2)^p), x_Symbol] \rightarrow -\text{Simp}[(ef - dg)(d + ex)^{m+1}(a + bx + cx^2)^{p+1} / (2(p+1)(c^2d - bde + ae^2)), x] - \text{Dist}[(b(ef + dg) - 2(cdf + aeg)) / (2(c^2d - bde + ae^2)), \text{Int}[(d + ex)^{m+1}(a + bx + cx^2)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[c^2d - bde + ae^2, 0] \&\& \text{EqQ}[\text{Simplify}[m + 2p + 3], 0]$

Rule 834

$\text{Int}[(d + e x)^m ((f + g x)(a + b x + c x^2)^p), x_Symbol] \rightarrow \text{Simp}[(ef - dg)(d + ex)^{m+1}(a + bx + cx^2)^{p+1} / ((m+1)(c^2d - bde + ae^2)), x] + \text{Dist}[1 / ((m+1)(c^2d - bde + ae^2)), \text{Int}[(d + ex)^{m+1}(a + bx + cx^2)^p \text{Simp}[(cdf - fbe + aeg)(m+1) + b(dg - ef)(p+1) - c(ef - dg)(m + 2p + 3)x], x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, p\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[c^2d - bde + ae^2, 0] \&\& \text{LtQ}[m, -1] \&\& (\text{IntegerQ}[m] \mid\mid \text{IntegerQ}[p] \mid\mid \text{IntegersQ}[2m, 2p])$

Rule 1357

$\text{Int}[x^m ((a + c x)^{n2} + b x^n)^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)(a + bx + cx^2)^p}, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[n2, 2n] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^5\sqrt{a+bx+cx^2}} dx, x, x^3 \right) \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} - \frac{\text{Subst} \left(\int \frac{\frac{7b}{2}+3cx}{x^4\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{12a} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} + \frac{\text{Subst} \left(\int \frac{\frac{1}{4}(35b^2-36ac)+7bcx}{x^3\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{36a^2} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} - \frac{\text{Subst} \left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{36a} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-36ac)\sqrt{a+bx^3+cx^6}}{384a^4x^3} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-36ac)\sqrt{a+bx^3+cx^6}}{384a^4x^3} \\
&= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} + \frac{5b(21b^2-36ac)\sqrt{a+bx^3+cx^6}}{384a^4x^3}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 141, normalized size = 0.73

$$\frac{\sqrt{a+bx^3+cx^6} \left(-48a^3 + 8a^2(7bx^3+9cx^6) - 10abx^6(7b+22cx^3) + 105b^3x^9 \right)}{576a^4x^{12}} - \frac{(48a^2c^2 - 120ab^2c + 35b^4) \tanh^{-1} \left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}} \right)}{384a^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^13*Sqrt[a + b*x^3 + c*x^6]), x]

[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 105*b^3*x^9 - 10*a*b*x^6*(7*b + 22*c*x^3) + 8*a^2*(7*b*x^3 + 9*c*x^6)))/(576*a^4*x^12) - ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(9/2))

IntegrateAlgebraic [A] time = 0.78, size = 141, normalized size = 0.73

$$\frac{(48a^2c^2 - 120ab^2c + 35b^4) \tanh^{-1} \left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{\sqrt{a}} \right)}{192a^{9/2}} + \frac{\sqrt{a+bx^3+cx^6} \left(-48a^3 + 56a^2bx^3 + 72a^2cx^6 - 70ab^2x^6 - 220abcx^9 + 105b^3x^9 \right)}{576a^4x^{12}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^13*sqrt[a + b*x^3 + c*x^6]),x]

[Out] (sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 56*a^2*b*x^3 - 70*a*b^2*x^6 + 72*a^2*c*x^6 + 105*b^3*x^9 - 220*a*b*c*x^9))/(576*a^4*x^12) + ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(sqrt[c]*x^3 - sqrt[a + b*x^3 + c*x^6])/sqrt[a]])/(192*a^(9/2))

fricas [A] time = 1.78, size = 327, normalized size = 1.70

$$\frac{3(35b^4 - 120ab^2c + 48a^2c^2)\sqrt{a} \log\left(\frac{(b^2+4a)^2c^2+4\sqrt{c}\sqrt{a+b^2x^3+cx^6}\sqrt{a^2+bx^3+cx^6}}{2304a^3x^{12}}\right) + 4(5(21ab^3 - 44a^2bc)^2 + 56a^2bx^3 - 2(35a^2b^2 - 36a^2c)x^6 - 48a^4)\sqrt{cx^3 + bx^3 + a}}{1152a^3x^{12}} + 2(5(21ab^3 - 44a^2bc)^2 + 56a^2bx^3 - 2(35a^2b^2 - 36a^2c)x^6 - 48a^4)\sqrt{cx^3 + bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*(5*(21*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2 - 36*a^3*c)*x^6 - 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12), 1/1152*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(5*(21*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2 - 36*a^3*c)*x^6 - 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a} x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^6 + bx^3 + a} x^{13}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)

[Out] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{13} \sqrt{c x^6 + b x^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)),x)`

[Out] `int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13} \sqrt{a + b x^3 + c x^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)`

[Out] `Integral(1/(x**13*sqrt(a + b*x**3 + c*x**6)), x)`

$$3.206 \quad \int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}} dx$$

Optimal. Leaf size=195

$$\frac{(5b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}} - \frac{(b(15b^2 - 52ac) - 2cx^3(5b^2 - 12ac))\sqrt{a+bx^3+cx^6}}{12c^3(b^2 - 4ac)} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2 - 4ac)}$$

Rubi [A] time = 0.23, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 738, 832, 779, 621, 206}

$$-\frac{(b(15b^2 - 52ac) - 2cx^3(5b^2 - 12ac))\sqrt{a+bx^3+cx^6}}{12c^3(b^2 - 4ac)} + \frac{(5b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}} + \frac{2x^9(2a+bx^3)}{3(b^2 - 4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Int[x^14/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (2*x^9*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*x^6*Sqrt[a + b*x^3 + c*x^6]/(3*c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*(5*b^2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6]/(12*c^3*(b^2 - 4*a*c)) + ((5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(7/2)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 738

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^4}{(a+bx+cx^2)^{3/2}} dx, x, x^3 \right) \\
&= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2 \text{Subst} \left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{3(b^2-4ac)} \\
&= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{2 \text{Subst} \left(\int \frac{x(-6ab-\frac{3}{2}(5b^2-12ac)x)}{\sqrt{a+bx+cx^2}} dx, x, x^3 \right)}{9c(b^2-4ac)} \\
&= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-12ac))\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)} \\
&= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-12ac))\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)} \\
&= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-12ac))\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 181, normalized size = 0.93

$$\frac{2\sqrt{c}(4a^2c(6cx^3-13b)+a(15b^3-62b^2cx^3-20bc^2x^6+8c^3x^9))+b^2x^3(15b^2+5bcx^3-2c^2x^6)}{\sqrt{a+bx^3+cx^6}} - 3(16a^2c^2-24ab^2c+5b^4)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

$$24c^{7/2}(4ac-b^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^14/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] ((2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x^3) + b^2*x^3*(15*b^2 + 5*b*c*x^3 - 2*c^2*x^6) + a*(15*b^3 - 62*b^2*c*x^3 - 20*b*c^2*x^6 + 8*c^3*x^9)))/Sqrt[a + b*x^3 + c*x^6] - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(7/2)*(-b^2 + 4*a*c))

IntegrateAlgebraic [A] time = 0.96, size = 175, normalized size = 0.90

$$\frac{-52a^2bc + 24a^2c^2x^3 + 15ab^3 - 62ab^2cx^3 - 20abc^2x^6 + 8ac^3x^9 + 15b^4x^3 + 5b^3cx^6 - 2b^2c^2x^9}{12c^3(4ac-b^2)\sqrt{a+bx^3+cx^6}} + \frac{(4ac-5b^2)\log\left(-2c^{7/2}\sqrt{a+bx^3+cx^6} + bc^3 + 2c^4x^3\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^14/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] $(15*a*b^3 - 52*a^2*b*c + 15*b^4*x^3 - 62*a*b^2*c*x^3 + 24*a^2*c^2*x^3 + 5*b^3*c*x^6 - 20*a*b*c^2*x^6 - 2*b^2*c^2*x^9 + 8*a*c^3*x^9)/(12*c^3*(-b^2 + 4*a*c)*\text{Sqrt}[a + b*x^3 + c*x^6]) + ((-5*b^2 + 4*a*c)*\text{Log}[b*c^3 + 2*c^4*x^3 - 2*c^{7/2}*\text{Sqrt}[a + b*x^3 + c*x^6]])/(8*c^{7/2})$

fricas [A] time = 1.91, size = 591, normalized size = 3.03

$\frac{3(15b^3c - 52a^2bc + 15b^4x^3 - 62ab^2cx^3 + 24a^2c^2x^3 + 5b^3cx^6 - 20abc^2x^6 - 2b^2c^2x^9 + 8ac^3x^9)}{12c^3(-b^2 + 4ac)\sqrt{ax^3 + bx^3 + cx^6}} + \frac{(-5b^2 + 4ac)\log(bc^3 + 2c^4x^3 - 2c^{7/2}\sqrt{ax^3 + bx^3 + cx^6})}{8c^{7/2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] $[-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*\text{sqrt}(c)*\log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*\text{sqrt}(c*x^6 + b*x^3 + a))*(2*c*x^3 + b)*\text{sqrt}(c) - 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*b*c^3)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^6 + (b^3*c^4 - 4*a*b*c^5)*x^3), -1/24*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*\text{sqrt}(-c)*\text{arctan}(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*\text{sqrt}(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*b*c^3)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^6 + (b^3*c^4 - 4*a*b*c^5)*x^3)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2), x, algorithm="giac")

[Out] integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2), x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14/(c*x^6+b*x^3+a)^(3/2),x)`

[Out] `int(x^14/(c*x^6+b*x^3+a)^(3/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{(cx^6 + bx^3 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14/(a + b*x^3 + c*x^6)^(3/2),x)`

[Out] `int(x^14/(a + b*x^3 + c*x^6)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(a + bx^3 + cx^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)`

[Out] `Integral(x**14/(a + b*x**3 + c*x**6)**(3/2), x)`

$$3.207 \quad \int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}} dx$$

Optimal. Leaf size=137

$$\frac{(-8ac + 3b^2 - 2bcx^3) \sqrt{a + bx^3 + cx^6}}{3c^2 (b^2 - 4ac)} + \frac{2x^6 (2a + bx^3)}{3 (b^2 - 4ac) \sqrt{a + bx^3 + cx^6}} - \frac{b \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c} \sqrt{a+bx^3+cx^6}} \right)}{2c^{5/2}}$$

Rubi [A] time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 738, 779, 621, 206}

$$\frac{(-8ac + 3b^2 - 2bcx^3) \sqrt{a + bx^3 + cx^6}}{3c^2 (b^2 - 4ac)} + \frac{2x^6 (2a + bx^3)}{3 (b^2 - 4ac) \sqrt{a + bx^3 + cx^6}} - \frac{b \tanh^{-1} \left(\frac{b+2cx^3}{2\sqrt{c} \sqrt{a+bx^3+cx^6}} \right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (2*x^6*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + ((3*b^2 - 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(3*c^2*(b^2 - 4*a*c)) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2*c^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 738

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x]]

1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 779

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{(a + bx^3 + cx^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^3}{(a + bx + cx^2)^{3/2}} dx, x, x^3 \right) \\
 &= \frac{2x^6(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{2 \text{Subst} \left(\int \frac{x(4a + 2bx)}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3(b^2 - 4ac)} \\
 &= \frac{2x^6(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} + \frac{(3b^2 - 8ac - 2bcx^3)\sqrt{a + bx^3 + cx^6}}{3c^2(b^2 - 4ac)} - \frac{b \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{2} \\
 &= \frac{2x^6(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} + \frac{(3b^2 - 8ac - 2bcx^3)\sqrt{a + bx^3 + cx^6}}{3c^2(b^2 - 4ac)} - \frac{b \text{Subst} \left(\int \frac{1}{4c - x} dx, x, x^3 \right)}{2} \\
 &= \frac{2x^6(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} + \frac{(3b^2 - 8ac - 2bcx^3)\sqrt{a + bx^3 + cx^6}}{3c^2(b^2 - 4ac)} - \frac{b \tanh^{-1} \left(\frac{b}{2\sqrt{c}\sqrt{a + bx + cx^2}} \right)}{2c^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.12, size = 137, normalized size = 1.00

$$\frac{2\sqrt{c}(8a^2c+a(-3b^2+10bcx^3+4c^2x^6)-b^2x^3(3b+cx^3))}{\sqrt{a+bx^3+cx^6}} + 3b(b^2-4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{5/2}(4ac-b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] ((2*sqrt[c]*(8*a^2*c - b^2*x^3*(3*b + c*x^3) + a*(-3*b^2 + 10*b*c*x^3 + 4*c^2*x^6)))/sqrt[a + b*x^3 + c*x^6] + 3*b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*sqrt[c]*sqrt[a + b*x^3 + c*x^6])])/(6*c^(5/2)*(-b^2 + 4*a*c))

IntegrateAlgebraic [A] time = 0.67, size = 131, normalized size = 0.96

$$\frac{8a^2c - 3ab^2 + 10abcx^3 + 4ac^2x^6 - 3b^3x^3 - b^2cx^6}{3c^2(4ac - b^2)\sqrt{a + bx^3 + cx^6}} + \frac{b \log\left(-2c^{5/2}\sqrt{a + bx^3 + cx^6} + bc^2 + 2c^3x^3\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^11/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (-3*a*b^2 + 8*a^2*c - 3*b^3*x^3 + 10*a*b*c*x^3 - b^2*c*x^6 + 4*a*c^2*x^6)/(3*c^2*(-b^2 + 4*a*c)*sqrt[a + b*x^3 + c*x^6]) + (b*Log[b*c^2 + 2*c^3*x^3 - 2*c^(5/2)*sqrt[a + b*x^3 + c*x^6]])/(2*c^(5/2))

fricas [A] time = 1.37, size = 459, normalized size = 3.35

$$\frac{3((b^3c - 4abc^2)c^2 + ab^3c + (b^4 - 4ab^2c^2)\sqrt{c} \log(-8c^2x^6 - 8b^3cx^3 - b^2 + 4\sqrt{c}\sqrt{cx^6 + bx^3 + a}) + 4((b^3c - 4abc^2)c^2 + 3ab^3c - 8a^2c^2 + (3b^3c - 10abc^2)\sqrt{cx^6 + bx^3 + a})}{12((b^3c - 4abc^2)c^2 + ab^3c - 4a^2c^2 + (b^3c - 4abc^2)\sqrt{c})} + \frac{3((b^3c - 4abc^2)c^2 + ab^3c + (b^4 - 4ab^2c^2)\sqrt{c} \arctan\left(\frac{\sqrt{c}\sqrt{cx^6 + bx^3 + a}}{2\sqrt{a+bx^3+cx^6}}\right) + 2((b^3c - 4abc^2)c^2 + 3ab^3c - 8a^2c^2 + (3b^3c - 10abc^2)\sqrt{cx^6 + bx^3 + a})}{6((b^3c - 4abc^2)c^2 + ab^3c - 4a^2c^2 + (b^3c - 4abc^2)\sqrt{c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*sqrt(c)*log(-8*c^2*x^6 - 8*b^3*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3), 1/6*(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2), x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(a + b*x^3 + c*x^6)^(3/2),x)

[Out] int(x^11/(a + b*x^3 + c*x^6)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a + bx^3 + cx^6)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(c*x**6+b*x**3+a)**(3/2), x)

[Out] Integral(x**11/(a + b*x**3 + c*x**6)**(3/2), x)

$$3.208 \quad \int \frac{x^8}{(a+bx^3+cx^6)^{3/2}} dx$$

Optimal. Leaf size=120

$$\frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 738, 640, 621, 206}

$$\frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (2*x^3*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a + bx^3 + cx^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{(a + bx + cx^2)^{3/2}} dx, x, x^3 \right) \\
&= \frac{2x^3(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{2 \text{Subst} \left(\int \frac{2a + bx}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3(b^2 - 4ac)} \\
&= \frac{2x^3(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{2b\sqrt{a + bx^3 + cx^6}}{3c(b^2 - 4ac)} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3c} \\
&= \frac{2x^3(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{2b\sqrt{a + bx^3 + cx^6}}{3c(b^2 - 4ac)} + \frac{2 \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx^3}{\sqrt{a + bx^3 + cx^6}} \right)}{3c} \\
&= \frac{2x^3(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{2b\sqrt{a + bx^3 + cx^6}}{3c(b^2 - 4ac)} + \frac{\tanh^{-1} \left(\frac{b + 2cx^3}{2\sqrt{c}\sqrt{a + bx^3 + cx^6}} \right)}{3c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 107, normalized size = 0.89

$$\frac{2\sqrt{c}\left(\frac{a(b-2cx^3)+b^2x^3}{\sqrt{a+bx^3+cx^6}}\right) - (b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] ((2*Sqrt[c]*(b^2*x^3 + a*(b - 2*c*x^3)))/Sqrt[a + b*x^3 + c*x^6] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(3*c^(3/2)*(-b^2 + 4*a*c))

IntegrateAlgebraic [A] time = 0.52, size = 99, normalized size = 0.82

$$\frac{2(ab - 2acx^3 + b^2x^3)}{3c(4ac - b^2)\sqrt{a + bx^3 + cx^6}} - \frac{\log\left(-2c^{3/2}\sqrt{a + bx^3 + cx^6} + bc + 2c^2x^3\right)}{3c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (2*(a*b + b^2*x^3 - 2*a*c*x^3))/(3*c*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - Log[b*c + 2*c^2*x^3 - 2*c^(3/2)*Sqrt[a + b*x^3 + c*x^6]]/(3*c^(3/2))

fricas [A] time = 1.34, size = 387, normalized size = 3.22

$$\frac{\left(\frac{((b^2c - 4ac^2)x^6 + (b^3 - 4a^2bc)x^3 + a^2b^2 - 4a^2c^2)\sqrt{c} \log\left(\frac{-8c^2x^6 - 8b^2cx^3 - b^2 - 4\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c} - 4\sqrt{cx^6 + bx^3 + a}((b^2c - 2ac^2)x^3 + abc)}{6((b^2c^3 - 4ac^2)x^6 + ab^2c^2 - 4a^2c^3 + (b^2c - 4abc^2)x^3)}\right)}{6((b^2c^3 - 4ac^2)x^6 + ab^2c^2 - 4a^2c^3 + (b^2c - 4abc^2)x^3)}\right) - \frac{((b^2c - 4ac^2)x^6 + (b^3 - 4a^2bc)x^3 + a^2b^2 - 4a^2c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)\sqrt{c}}{2(b^2c^3 - 4ac^2)x^3 + abc}\right) + 2\sqrt{cx^6 + bx^3 + a}((b^2c - 2ac^2)x^3 + abc)}{3((b^2c^3 - 4ac^2)x^6 + ab^2c^2 - 4a^2c^3 + (b^2c - 4abc^2)x^3)}}{3((b^2c^3 - 4ac^2)x^6 + ab^2c^2 - 4a^2c^3 + (b^2c - 4abc^2)x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3 + a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a*b*c^3)*x^3), -1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3 + a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a*b*c^3)*x^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2), x)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(cx^6 + bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [B] time = 1.66, size = 84, normalized size = 0.70

$$\frac{\ln\left(\sqrt{cx^6 + bx^3 + a} + \frac{cx^3 + \frac{b}{2}}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{\frac{ab}{2} - x^3\left(ac - \frac{b^2}{2}\right)}{3c\left(ac - \frac{b^2}{4}\right)\sqrt{cx^6 + bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(a + b*x^3 + c*x^6)^(3/2),x)

[Out] $\log((a + b*x^3 + c*x^6)^{(1/2)} + (b/2 + c*x^3)/c^{(1/2)})/(3*c^{(3/2)}) + ((a*b)/2 - x^3*(a*c - b^2/2))/(3*c*(a*c - b^2/4)*(a + b*x^3 + c*x^6)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a + bx^3 + cx^6)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral(x**8/(a + b*x**3 + c*x**6)**(3/2), x)

$$3.209 \quad \int \frac{x^5}{(a+bx^3+cx^6)^{3/2}} dx$$

Optimal. Leaf size=39

$$\frac{2(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1357, 636}

$$\frac{2(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(a + bx^3 + cx^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{(a + bx + cx^2)^{3/2}} dx, x, x^3 \right) \\ &= \frac{2(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 41, normalized size = 1.05

$$\frac{2(2a + bx^3)}{3(4ac - b^2)\sqrt{a + bx^3 + cx^6}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (-2*(2*a + b*x^3))/(3*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])

IntegrateAlgebraic [A] time = 0.39, size = 39, normalized size = 1.00

$$\frac{2(2a + bx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])

fricas [A] time = 0.95, size = 68, normalized size = 1.74

$$\frac{2\sqrt{cx^6 + bx^3 + a}(bx^3 + 2a)}{3((b^2c - 4ac^2)x^6 + (b^3 - 4abc)x^3 + ab^2 - 4a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] 2/3*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)/((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)

giac [A] time = 1.32, size = 45, normalized size = 1.15

$$\frac{2\left(\frac{bx^3}{b^2-4ac} + \frac{2a}{b^2-4ac}\right)}{3\sqrt{cx^6 + bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] 2/3*(b*x^3/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)

maple [A] time = 0.01, size = 38, normalized size = 0.97

$$\frac{2(bx^3 + 2a)}{3\sqrt{cx^6 + bx^3 + a}(4ac - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(c*x^6+b*x^3+a)^(3/2),x)`

[Out] `-2/3/(c*x^6+b*x^3+a)^(1/2)*(b*x^3+2*a)/(4*a*c-b^2)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [B] time = 1.43, size = 38, normalized size = 0.97

$$\frac{2bx^3 + 4a}{(12ac - 3b^2)\sqrt{cx^6 + bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(a + b*x^3 + c*x^6)^(3/2),x)`

[Out] `-(4*a + 2*b*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^3 + cx^6)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)`

[Out] `Integral(x**5/(a + b*x**3 + c*x**6)**(3/2), x)`

$$3.210 \quad \int \frac{x^2}{(a+bx^3+cx^6)^{3/2}} dx$$

Optimal. Leaf size=38

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1352, 613}

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a+bx^3+cx^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{(a+bx+cx^2)^{3/2}} dx, x, x^3 \right) \\ &= -\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$-\frac{2(b + 2cx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])

IntegrateAlgebraic [A] time = 0.32, size = 38, normalized size = 1.00

$$-\frac{2(b + 2cx^3)}{3(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(a + b*x^3 + c*x^6)^(3/2), x]

[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])

fricas [A] time = 1.20, size = 67, normalized size = 1.76

$$\frac{2\sqrt{cx^6 + bx^3 + a}(2cx^3 + b)}{3((b^2c - 4ac^2)x^6 + (b^3 - 4abc)x^3 + ab^2 - 4a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2), x, algorithm="fricas")

[Out] -2/3*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)/((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)

giac [A] time = 1.41, size = 45, normalized size = 1.18

$$-\frac{2\left(\frac{2cx^3}{b^2-4ac} + \frac{b}{b^2-4ac}\right)}{3\sqrt{cx^6 + bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2), x, algorithm="giac")

[Out] -2/3*(2*c*x^3/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)

maple [A] time = 0.01, size = 37, normalized size = 0.97

$$\frac{\frac{4cx^3}{3} + \frac{2b}{3}}{\sqrt{cx^6 + bx^3 + a} (4ac - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(c*x^6+b*x^3+a)^(3/2),x)`

[Out] `2/3/(c*x^6+b*x^3+a)^(1/2)*(2*c*x^3+b)/(4*a*c-b^2)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?

mupad [B] time = 1.37, size = 37, normalized size = 0.97

$$\frac{4cx^3 + 2b}{(12ac - 3b^2) \sqrt{cx^6 + bx^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*x^3 + c*x^6)^(3/2),x)`

[Out] `(2*b + 4*c*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + bx^3 + cx^6)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)`

[Out] `Integral(x**2/(a + b*x**3 + c*x**6)**(3/2), x)`

$$3.211 \quad \int \frac{1}{x(a+bx^3+cx^6)^{3/2}} dx$$

Optimal. Leaf size=92

$$\frac{2(-2ac + b^2 + bcx^3)}{3a(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 740, 12, 724, 206}

$$\frac{2(-2ac + b^2 + bcx^3)}{3a(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*a^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

```

Int[((d._) + (e._)*(x._))^(m._)*((a._) + (b._)*(x._) + (c._)*(x._)^2)^(p._), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 1357

```

Int[(x._)^(m._)*((a._) + (c._)*(x._)^(n2._) + (b._)*(x._)^(n._))^(p._), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a + bx^3 + cx^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(a + bx + cx^2)^{3/2}} dx, x, x^3 \right) \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{2 \text{Subst} \left(\int \frac{-\frac{b^2}{2} + 2ac}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3a} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{2 \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + bx^3}{\sqrt{a + bx^3 + cx^6}} \right)}{3a} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{\tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}} \right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 92, normalized size = 1.00

$$\frac{1}{3} \left(\frac{2(-2ac + b^2 + bcx^3)}{a(b^2 - 4ac)\sqrt{a + bx^3 + cx^6}} - \frac{\tanh^{-1}\left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}}\right)}{a^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] ((2*(b^2 - 2*a*c + b*c*x^3))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - ArcTanH[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/a^(3/2))/3

IntegrateAlgebraic [A] time = 0.54, size = 101, normalized size = 1.10

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c}x^3}{\sqrt{a}} - \frac{\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3a^{3/2}} + \frac{2(2ac - b^2 - bcx^3)}{3a(4ac - b^2)\sqrt{a + bx^3 + cx^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] (2*(-b^2 + 2*a*c - b*c*x^3))/(3*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + (2*ArcTanH[(Sqrt[c]*x^3)/Sqrt[a] - Sqrt[a + b*x^3 + c*x^6]/Sqrt[a]])/(3*a^(3/2))

fricas [B] time = 1.67, size = 389, normalized size = 4.23

$$\frac{\left(\frac{((b^2c - 4a^2c)x^6 + (b^3 - 4abc)x^3 + ab^2 - 4a^2c)\sqrt{a} \log\left(\frac{-(b^2+4a)x^4 + 8abx^3 - 4\sqrt{a+bx^3+cx^6}(bx^3+2a)\sqrt{a+8a^2}}{x^6}\right) + 4\sqrt{cx^6+bx^3+a}(abcx^3+ab^2-2a^2c)}{6((a^2bc-4a^3c^2)x^6+a^3b^2-4a^4c+(a^2b^3-4a^2bc)x^3)}\right)}{3((a^2bc-4a^3c^2)x^6+a^3b^2-4a^4c+(a^2b^3-4a^2bc)x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 - 4*a^3*b*c)*x^3), 1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 - 4*a^3*b*c)*x^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x), x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x(c x^6 + b x^3 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)),x)

[Out] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^3 + cx^6)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**6+b*x**3+a)**(3/2), x)

[Out] Integral(1/(x*(a + b*x**3 + c*x**6)**(3/2)), x)

$$3.212 \quad \int \frac{1}{x^4(a+bx^3+cx^6)^{3/2}} dx$$

Optimal. Leaf size=142

$$\frac{b \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}} - \frac{(3b^2 - 8ac)\sqrt{a+bx^3+cx^6}}{3a^2x^3(b^2 - 4ac)} + \frac{2(-2ac + b^2 + bcx^3)}{3ax^3(b^2 - 4ac)\sqrt{a+bx^3+cx^6}}$$

Rubi [A] time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 740, 806, 724, 206}

$$-\frac{(3b^2 - 8ac)\sqrt{a+bx^3+cx^6}}{3a^2x^3(b^2 - 4ac)} + \frac{b \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}} + \frac{2(-2ac + b^2 + bcx^3)}{3ax^3(b^2 - 4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(3*a^2*(b^2 - 4*a*c)*x^3) + (b *ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +

```
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a + bx^3 + cx^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 (a + bx + cx^2)^{3/2}} dx, x, x^3 \right) \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^3 \sqrt{a + bx^3 + cx^6}} - \frac{2 \text{Subst} \left(\int \frac{\frac{1}{2}(-3b^2 + 8ac) - bcx}{x^2 \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^3 \sqrt{a + bx^3 + cx^6}} - \frac{(3b^2 - 8ac) \sqrt{a + bx^3 + cx^6}}{3a^2(b^2 - 4ac)x^3} - \frac{b \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{2a^2} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^3 \sqrt{a + bx^3 + cx^6}} - \frac{(3b^2 - 8ac) \sqrt{a + bx^3 + cx^6}}{3a^2(b^2 - 4ac)x^3} + \frac{b \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, x^3 \right)}{a} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^3 \sqrt{a + bx^3 + cx^6}} - \frac{(3b^2 - 8ac) \sqrt{a + bx^3 + cx^6}}{3a^2(b^2 - 4ac)x^3} + \frac{b \tanh^{-1} \left(\frac{2a + \sqrt{a + bx^3 + cx^6}}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 0.96

$$\frac{\frac{2\sqrt{a}(-4a^2c + a(b^2 - 10bcx^3 - 8c^2x^6) + 3b^2x^3(b + cx^3))}{x^3 \sqrt{a + bx^3 + cx^6}} - 3b(b^2 - 4ac) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right)}{6a^{5/2}(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] ((2*sqrt[a]*(-4*a^2*c + 3*b^2*x^3*(b + c*x^3) + a*(b^2 - 10*b*c*x^3 - 8*c^2*x^6)))/(x^3*sqrt[a + b*x^3 + c*x^6]) - 3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*sqrt[a]*sqrt[a + b*x^3 + c*x^6])])/(6*a^(5/2)*(-b^2 + 4*a*c))

IntegrateAlgebraic [A] time = 0.68, size = 132, normalized size = 0.93

$$\frac{-4a^2c + ab^2 - 10abcx^3 - 8ac^2x^6 + 3b^3x^3 + 3b^2cx^6}{3a^2x^3(4ac - b^2)\sqrt{a + bx^3 + cx^6}} - \frac{b \tanh^{-1} \left(\frac{\sqrt{c}x^3}{\sqrt{a}} - \frac{\sqrt{a + bx^3 + cx^6}}{\sqrt{a}} \right)}{a^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] $(a*b^2 - 4*a^2*c + 3*b^3*x^3 - 10*a*b*c*x^3 + 3*b^2*c*x^6 - 8*a*c^2*x^6)/(3*a^2*(-b^2 + 4*a*c)*x^3*\text{Sqrt}[a + b*x^3 + c*x^6]) - (b*\text{ArcTanh}[(\text{Sqrt}[c]*x^3)/\text{Sqrt}[a] - \text{Sqrt}[a + b*x^3 + c*x^6]/\text{Sqrt}[a]])/a^{(5/2)}$

fricas [A] time = 1.53, size = 485, normalized size = 3.42

$$\frac{3((b^2c - 4ab^2)x^3 + (b^3 - 4a^2c)x^6 + (ab^2 - 4a^2bc)x^9)\sqrt{a}\log\left(\frac{(b^2c - 4ab^2)x^3 + (b^3 - 4a^2c)x^6 + (ab^2 - 4a^2bc)x^9}{a}\right) - 4((3ab^2c - 8a^2b^2c^2)x^6 + a^2b^2c + (3ab^3 - 10a^2bc^2)x^3)\sqrt{a} + b^2c^2}{12((b^2c - 4a^2b^2)x^3 + (b^3 - 4a^2c)x^6 + (ab^2 - 4a^2bc)x^9)} - \frac{3((b^2c - 4ab^2)x^3 + (b^3 - 4a^2c)x^6 + (ab^2 - 4a^2bc)x^9)\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}x^3}{2(a^2 + abx^3 + ax^6)}\right) + 2((3ab^2c - 8a^2b^2c^2)x^6 + a^2b^2c + (3ab^3 - 10a^2bc^2)x^3)\sqrt{a} + b^2c^2}{6((b^2c - 4a^2b^2)x^3 + (b^3 - 4a^2c)x^6 + (ab^2 - 4a^2bc)x^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")`

[Out] $[1/12*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2*b*c)*x^3)*\text{sqrt}(a)*\log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*\text{sqrt}(c*x^6 + b*x^3 + a))*(b*x^3 + 2*a)*\text{sqrt}(a) + 8*a^2)/x^6) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^6 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)*x^3), -1/6*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2*b*c)*x^3)*\text{sqrt}(-a)*\arctan(1/2*\text{sqrt}(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*\text{sqrt}(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^6 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*\text{sqrt}(c*x^6 + b*x^3 + a))/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)*x^3)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")`

[Out] `integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4), x)`

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)`

[Out] `int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 (c x^6 + b x^3 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x)

[Out] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b x^3 + c x^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral(1/(x**4*(a + b*x**3 + c*x**6)**(3/2)), x)

$$3.213 \quad \int \frac{1}{x^7(a+bx^3+cx^6)^{3/2}} dx$$

Optimal. Leaf size=198

$$\frac{(5b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}} + \frac{b(15b^2 - 52ac)\sqrt{a+bx^3+cx^6}}{12a^3x^3(b^2 - 4ac)} - \frac{(5b^2 - 12ac)\sqrt{a+bx^3+cx^6}}{6a^2x^6(b^2 - 4ac)} + \frac{2}{3ax^6(b^2 - 4ac)}$$

Rubi [A] time = 0.20, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 740, 834, 806, 724, 206}

$$\frac{b(15b^2 - 52ac)\sqrt{a+bx^3+cx^6}}{12a^3x^3(b^2 - 4ac)} - \frac{(5b^2 - 12ac)\sqrt{a+bx^3+cx^6}}{6a^2x^6(b^2 - 4ac)} - \frac{(5b^2 - 4ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}} + \frac{2(-2ac + b^2 + bcx^3)}{3ax^6(b^2 - 4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)), x]

[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3 + c*x^6]) - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(6*a^2*(b^2 - 4*a*c)*x^6) + (b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/(12*a^3*(b^2 - 4*a*c)*x^3) - ((5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(8*a^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]

```

^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 806

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]

```

Rule 834

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 1357

```

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^3 (a + bx + cx^2)^{3/2}} dx, x, x^3 \right) \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^6 \sqrt{a + bx^3 + cx^6}} - \frac{2 \text{Subst} \left(\int \frac{\frac{1}{2}(-5b^2 + 12ac) - 2bcx}{x^3 \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^6 \sqrt{a + bx^3 + cx^6}} - \frac{(5b^2 - 12ac) \sqrt{a + bx^3 + cx^6}}{6a^2(b^2 - 4ac)x^6} + \frac{\text{Subst} \left(\int \frac{-\frac{1}{4}b(1}{x^3 \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^6 \sqrt{a + bx^3 + cx^6}} - \frac{(5b^2 - 12ac) \sqrt{a + bx^3 + cx^6}}{6a^2(b^2 - 4ac)x^6} + \frac{b(15b^2 - 52ac)}{12a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^6 \sqrt{a + bx^3 + cx^6}} - \frac{(5b^2 - 12ac) \sqrt{a + bx^3 + cx^6}}{6a^2(b^2 - 4ac)x^6} + \frac{b(15b^2 - 52ac)}{12a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^6 \sqrt{a + bx^3 + cx^6}} - \frac{(5b^2 - 12ac) \sqrt{a + bx^3 + cx^6}}{6a^2(b^2 - 4ac)x^6} + \frac{b(15b^2 - 52ac)}{12a^3(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 179, normalized size = 0.90

$$\frac{3(16a^2c^2 - 24ab^2c + 5b^4) \tanh^{-1} \left(\frac{2a + bx^3}{2\sqrt{a} \sqrt{a + bx^3 + cx^6}} \right) + \frac{2\sqrt{a}(-8a^3c + 2a^2(b^2 + 10bcx^3 - 12c^2x^6) + abx^3(-5b^2 + 62bcx^3 + 52c^2x^6) - 15b^3x^6(b + cx^3))}{x^6 \sqrt{a + bx^3 + cx^6}}}{24a^{7/2}(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] ((2*sqrt[a]*(-8*a^3*c - 15*b^3*x^6*(b + c*x^3) + 2*a^2*(b^2 + 10*b*c*x^3 - 12*c^2*x^6) + a*b*x^3*(-5*b^2 + 62*b*c*x^3 + 52*c^2*x^6)))/(x^6*sqrt[a + b*x^3 + c*x^6]) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*sqrt[a]*sqrt[a + b*x^3 + c*x^6])])/(24*a^(7/2)*(-b^2 + 4*a*c))

IntegrateAlgebraic [A] time = 1.01, size = 175, normalized size = 0.88

$$\frac{(5b^2 - 4ac) \tanh^{-1} \left(\frac{\sqrt{c}x^3 - \sqrt{a + bx^3 + cx^6}}{\sqrt{a}} \right)}{4a^{7/2}} + \frac{-8a^3c + 2a^2b^2 + 20a^2bcx^3 - 24a^2c^2x^6 - 5ab^3x^3 + 62ab^2cx^6 + 52abc^2x^9 - 15b^4x^6 - 15b^3cx^9}{12a^3x^6(4ac - b^2)\sqrt{a + bx^3 + cx^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] $(2a^2b^2 - 8a^3c - 5ab^3x^3 + 20a^2bcx^3 - 15b^4x^6 + 62ab^2cx^6 - 24a^2c^2x^6 - 15b^3cx^9 + 52ab^2cx^9)/(12a^3(-b^2 + 4ac)x^6\sqrt{a + bx^3 + cx^6}) + ((5b^2 - 4ac)\operatorname{ArcTanh}[\frac{\sqrt{c}x^3 - \sqrt{a + bx^3 + cx^6}}{\sqrt{a}}])/4a^{7/2})$

fricas [A] time = 1.85, size = 615, normalized size = 3.11

$\frac{3((15c^2 - 24ab^2 + 16a^2b^2)^2 - (3c^2 - 24ab^2 + 16a^2b^2)^2 + (15c^2 - 24ab^2 + 16a^2b^2)^2)\sqrt{a}}{48((15c^2 - 24ab^2 + 16a^2b^2)^2 - (3c^2 - 24ab^2 + 16a^2b^2)^2 + (15c^2 - 24ab^2 + 16a^2b^2)^2)\sqrt{a}} - 4((15c^2 - 24ab^2 + 16a^2b^2)^2 - (3c^2 - 24ab^2 + 16a^2b^2)^2 + (15c^2 - 24ab^2 + 16a^2b^2)^2)\sqrt{a}}{24((15c^2 - 24ab^2 + 16a^2b^2)^2 - (3c^2 - 24ab^2 + 16a^2b^2)^2 + (15c^2 - 24ab^2 + 16a^2b^2)^2)\sqrt{a}} + 2((15c^2 - 24ab^2 + 16a^2b^2)^2 - (3c^2 - 24ab^2 + 16a^2b^2)^2 + (15c^2 - 24ab^2 + 16a^2b^2)^2)\sqrt{a}}{24((15c^2 - 24ab^2 + 16a^2b^2)^2 - (3c^2 - 24ab^2 + 16a^2b^2)^2 + (15c^2 - 24ab^2 + 16a^2b^2)^2)\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] $[-1/48*(3*((5b^4c - 24ab^2c^2 + 16a^2c^3)x^{12} + (5b^5 - 24ab^3c + 16a^2bc^2)x^9 + (5ab^4 - 24a^2b^2c + 16a^3c^2)x^6)*\sqrt{a})\log(-((b^2 + 4ac)x^6 + 8abx^3 + 4\sqrt{c(x^6 + bx^3 + a)}(bx^3 + 2a))\sqrt{a} + 8a^2)/x^6 - 4*((15ab^3c - 52a^2bc^2)x^9 + (15ab^4 - 62a^2b^2c + 24a^3c^2)x^6 - 2a^3b^2 + 8a^4c + 5(a^2b^3 - 4a^3bc)*x^3)*\sqrt{c(x^6 + bx^3 + a)})/(a^4b^2c - 4a^5c^2)x^{12} + (a^4b^3 - 4a^5bc)x^9 + (a^5b^2 - 4a^6c)x^6, 1/24*(3*((5b^4c - 24ab^2c^2 + 16a^2c^3)x^{12} + (5b^5 - 24ab^3c + 16a^2bc^2)x^9 + (5ab^4 - 24a^2b^2c + 16a^3c^2)x^6)*\sqrt{-a})\arctan(1/2*\sqrt{c(x^6 + bx^3 + a)}(bx^3 + 2a)\sqrt{-a}/(acx^6 + abx^3 + a^2)) + 2*((15ab^3c - 52a^2bc^2)x^9 + (15ab^4 - 62a^2b^2c + 24a^3c^2)x^6 - 2a^3b^2 + 8a^4c + 5(a^2b^3 - 4a^3bc)*x^3)*\sqrt{c(x^6 + bx^3 + a)})/(a^4b^2c - 4a^5c^2)x^{12} + (a^4b^3 - 4a^5bc)x^9 + (a^5b^2 - 4a^6c)x^6]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7), x)

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)`

[Out] `int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7 (cx^6 + bx^3 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x)`

[Out] `int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (a + bx^3 + cx^6)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)`

[Out] `Integral(1/(x**7*(a + b*x**3 + c*x**6)**(3/2)), x)`

$$3.214 \quad \int \frac{1}{x^{10}(a+bx^3+cx^6)^{3/2}} dx$$

Optimal. Leaf size=256

$$\frac{5b(7b^2 - 12ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}} + \frac{b(35b^2 - 116ac)\sqrt{a+bx^3+cx^6}}{36a^3x^6(b^2 - 4ac)} - \frac{(7b^2 - 16ac)\sqrt{a+bx^3+cx^6}}{9a^2x^9(b^2 - 4ac)} \quad (256)$$

Rubi [A] time = 0.29, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 740, 834, 806, 724, 206}

$$-\frac{(256a^2c^2 - 460ab^2c + 105b^4)\sqrt{a+bx^3+cx^6}}{72a^4x^3(b^2 - 4ac)} + \frac{b(35b^2 - 116ac)\sqrt{a+bx^3+cx^6}}{36a^3x^6(b^2 - 4ac)} - \frac{(7b^2 - 16ac)\sqrt{a+bx^3+cx^6}}{9a^2x^9(b^2 - 4ac)} + \frac{5b(7b^2 - 12ac) \tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}} + \frac{2(-2ac + b^2 + bcx^3)}{3ax^9(b^2 - 4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)), x]

[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^9*sqrt[a + b*x^3 + c*x^6]) - ((7*b^2 - 16*a*c)*sqrt[a + b*x^3 + c*x^6])/(9*a^2*(b^2 - 4*a*c)*x^9) + (b*(35*b^2 - 116*a*c)*sqrt[a + b*x^3 + c*x^6])/(36*a^3*(b^2 - 4*a*c)*x^6) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*sqrt[a + b*x^3 + c*x^6])/(72*a^4*(b^2 - 4*a*c)*x^3) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*sqrt[a]*sqrt[a + b*x^3 + c*x^6])])/(48*a^(9/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e


```

^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 806

```

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]

```

Rule 834

```

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(m + 1)
*(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 1357

```

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{10} (a + bx^3 + cx^6)^{3/2}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^4 (a + bx + cx^2)^{3/2}} dx, x, x^3 \right) \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^9 \sqrt{a + bx^3 + cx^6}} - \frac{2 \text{Subst} \left(\int \frac{\frac{1}{2}(-7b^2 + 16ac) - 3bcx}{x^4 \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^9 \sqrt{a + bx^3 + cx^6}} - \frac{(7b^2 - 16ac) \sqrt{a + bx^3 + cx^6}}{9a^2(b^2 - 4ac)x^9} + \frac{2 \text{Subst} \left(\int \frac{-\frac{1}{4}b^2}{x^4 \sqrt{a + bx + cx^2}} dx, x, x^3 \right)}{3a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^9 \sqrt{a + bx^3 + cx^6}} - \frac{(7b^2 - 16ac) \sqrt{a + bx^3 + cx^6}}{9a^2(b^2 - 4ac)x^9} + \frac{b(35b^2 - 116ac)}{36a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^9 \sqrt{a + bx^3 + cx^6}} - \frac{(7b^2 - 16ac) \sqrt{a + bx^3 + cx^6}}{9a^2(b^2 - 4ac)x^9} + \frac{b(35b^2 - 116ac)}{36a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^9 \sqrt{a + bx^3 + cx^6}} - \frac{(7b^2 - 16ac) \sqrt{a + bx^3 + cx^6}}{9a^2(b^2 - 4ac)x^9} + \frac{b(35b^2 - 116ac)}{36a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^9 \sqrt{a + bx^3 + cx^6}} - \frac{(7b^2 - 16ac) \sqrt{a + bx^3 + cx^6}}{9a^2(b^2 - 4ac)x^9} + \frac{b(35b^2 - 116ac)}{36a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx^3)}{3a(b^2 - 4ac)x^9 \sqrt{a + bx^3 + cx^6}} - \frac{(7b^2 - 16ac) \sqrt{a + bx^3 + cx^6}}{9a^2(b^2 - 4ac)x^9} + \frac{b(35b^2 - 116ac)}{36a^3(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 223, normalized size = 0.87

$$\frac{2\sqrt{a}(-32a^4c + 8a^3(b^2 + 7bcx^3 + 16c^2x^6) + 2a^2x^3(-7b^3 - 86b^2cx^3 + 244bc^2x^6 + 128c^3x^9) + 5ab^2x^6(7b^2 - 106bcx^3 - 92c^2x^6) + 105b^4x^9(b + cx^3)) - 15b(48a^2c^2 - 40ab^2c + 7b^4) \tanh^{-1}\left(\frac{2a + bx^3}{2\sqrt{a}\sqrt{a + bx^3 + cx^6}}\right)}{x^9 \sqrt{a + bx^3 + cx^6}} \frac{144a^{9/2}(4ac - b^2)}{144a^{9/2}(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] ((2*sqrt[a]*(-32*a^4*c + 105*b^4*x^9*(b + c*x^3) + 5*a*b^2*x^6*(7*b^2 - 106*b*c*x^3 - 92*c^2*x^6) + 8*a^3*(b^2 + 7*b*c*x^3 + 16*c^2*x^6) + 2*a^2*x^3*(-7*b^3 - 86*b^2*c*x^3 + 244*b*c^2*x^6 + 128*c^3*x^9)))/(x^9*sqrt[a + b*x^3 + c*x^6]) - 15*b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*sqrt[a]*sqrt[a + b*x^3 + c*x^6])])/(144*a^(9/2)*(-b^2 + 4*a*c))

IntegrateAlgebraic [A] time = 1.35, size = 224, normalized size = 0.88

$$\frac{5(12abc - 7b^3) \tanh^{-1}\left(\frac{\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{9/2}} + \frac{-32a^4c + 8a^3b^2 + 56a^3bcx^3 + 128a^3c^2x^6 - 14a^2b^3x^3 - 172a^2b^2cx^6 + 488a^2bc^2x^9 + 256a^2c^3x^{12} + 35ab^4x^6 - 530ab^3cx^9 - 460ab^2c^2x^{12} + 105b^5x^9 + 105b^4cx^{12}}{72a^4x^9(4ac - b^2)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]

[Out] $(8a^3b^2 - 32a^4c - 14a^2b^3x^3 + 56a^3b^2cx^6 + 35a^2b^4x^6 - 172a^2b^3cx^9 + 256a^2c^3x^{12} + 35ab^4x^6 - 530ab^3cx^9 + 488a^2b^2cx^9 + 105b^5x^9 - 460ab^2c^2x^{12} + 105b^4cx^{12}) / (72a^4(-b^2 + 4ac)x^9 \sqrt{a+bx^3+cx^6}) + (5(-7b^3 + 12ab^2c) \operatorname{ArcTanh}[(\sqrt{c}x^3 - \sqrt{a+bx^3+cx^6})/\sqrt{a}]) / (24a^{9/2})$

fricas [A] time = 2.10, size = 705, normalized size = 2.75

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] $[-1/288(15((7b^5c - 40ab^3c^2 + 48a^2b^2c^3)x^{15} + (7b^6 - 40ab^4c + 48a^2b^2c^2)x^{12} + (7ab^5 - 40a^2b^3c + 48a^3b^2c^2)x^9) \sqrt{a} \log(-((b^2 + 4ac)x^6 + 8abx^3 - 4\sqrt{c(x^6 + bx^3 + a)})(bx^3 + 2a)\sqrt{a} + 8a^2)/x^6) + 4((105ab^4c - 460a^2b^2c^2 + 256a^3c^3)x^{12} + (105ab^5 - 530a^2b^3c + 488a^3b^2c^2)x^9 + (35a^2b^4 - 172a^3b^2c + 128a^4c^2)x^6 + 8a^4b^2 - 32a^5c - 14(a^3b^3 - 4a^4b^2c)x^3) \sqrt{c(x^6 + bx^3 + a)} / ((a^5b^2c - 4a^6c^2)x^{15} + (a^5b^3 - 4a^6b^2c)x^{12} + (a^6b^2 - 4a^7c)x^9), -1/144(15((7b^5c - 40ab^3c^2 + 48a^2b^2c^3)x^{15} + (7b^6 - 40ab^4c + 48a^2b^2c^2)x^{12} + (7ab^5 - 40a^2b^3c + 48a^3b^2c^2)x^9) \sqrt{-a} \arctan(1/2 \sqrt{c(x^6 + bx^3 + a)}(bx^3 + 2a)\sqrt{-a} / (acx^6 + abx^3 + a^2)) + 2((105ab^4c - 460a^2b^2c^2 + 256a^3c^3)x^{12} + (105ab^5 - 530a^2b^3c + 488a^3b^2c^2)x^9 + (35a^2b^4 - 172a^3b^2c + 128a^4c^2)x^6 + 8a^4b^2 - 32a^5c - 14(a^3b^3 - 4a^4b^2c)x^3) \sqrt{c(x^6 + bx^3 + a)} / ((a^5b^2c - 4a^6c^2)x^{15} + (a^5b^3 - 4a^6b^2c)x^{12} + (a^6b^2 - 4a^7c)x^9)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10), x)

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^6 + bx^3 + a)^{\frac{3}{2}} x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)

[Out] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{10} (cx^6 + bx^3 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x)

[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10} (a + bx^3 + cx^6)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral(1/(x**10*(a + b*x**3 + c*x**6)**(3/2)), x)

$$3.215 \quad \int (dx)^m (a + bx^3 + cx^6)^2 dx$$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{m+1}}{d(m+1)} + \frac{(2ac + b^2)(dx)^{m+7}}{d^7(m+7)} + \frac{2ab(dx)^{m+4}}{d^4(m+4)} + \frac{2bc(dx)^{m+10}}{d^{10}(m+10)} + \frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1353}

$$\frac{a^2(dx)^{m+1}}{d(m+1)} + \frac{(2ac + b^2)(dx)^{m+7}}{d^7(m+7)} + \frac{2ab(dx)^{m+4}}{d^4(m+4)} + \frac{2bc(dx)^{m+10}}{d^{10}(m+10)} + \frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]

[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 + m)) + ((b^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)^(10 + m))/(d^10*(10 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))

Rule 1353

Int[((d_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int (dx)^m (a + bx^3 + cx^6)^2 dx &= \int \left(a^2(dx)^m + \frac{2ab(dx)^{3+m}}{d^3} + \frac{(b^2 + 2ac)(dx)^{6+m}}{d^6} + \frac{2bc(dx)^{9+m}}{d^9} + \frac{c^2(dx)^{12+m}}{d^{12}} \right) dx \\ &= \frac{a^2(dx)^{1+m}}{d(1+m)} + \frac{2ab(dx)^{4+m}}{d^4(4+m)} + \frac{(b^2 + 2ac)(dx)^{7+m}}{d^7(7+m)} + \frac{2bc(dx)^{10+m}}{d^{10}(10+m)} + \frac{c^2(dx)^{13+m}}{d^{13}(13+m)} \end{aligned}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 0.69

$$x(dx)^m \left(\frac{a^2}{m+1} + \frac{x^6(2ac + b^2)}{m+7} + \frac{2abx^3}{m+4} + \frac{2bcx^9}{m+10} + \frac{c^2x^{12}}{m+13} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]

[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + ((b^2 + 2*a*c)*x^6)/(7 + m) + (2*b*c*x^9)/(10 + m) + (c^2*x^12)/(13 + m))

IntegrateAlgebraic [F] time = 1.17, size = 0, normalized size = 0.00

$$\int (dx)^m (a + bx^3 + cx^6)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(a + b*x^3 + c*x^6)^2, x]

fricas [B] time = 1.08, size = 241, normalized size = 2.39

$$\frac{((c^2m^4 + 22c^2m^3 + 159c^2m^2 + 418c^2m + 280c^2)x^{13} + 2(bc^2m^4 + 25b^2c^2m^3 + 195b^2c^2m^2 + 535b^2c^2m + 364b^2c^2)x^{10} + ((b^2 + 2ac)m^4 + 28(b^2 + 2ac)m^3 + 249(b^2 + 2ac)m^2 + 520b^2 + 1040ac + 742(b^2 + 2ac)m)x^7 + 2(abm^4 + 31abm^3 + 321abm^2 + 1201abm + 910ab)x^4 + (a^2m^4 + 34a^2m^3 + 411a^2m^2 + 2074a^2m + 3640a^2)x)(dx)^m}{m^5 + 35m^4 + 445m^3 + 2485m^2 + 5714m + 3640}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="fricas")

[Out] ((c^2*m^4 + 22*c^2*m^3 + 159*c^2*m^2 + 418*c^2*m + 280*c^2)*x^13 + 2*(b*c*m^4 + 25*b^2*c*m^3 + 195*b^2*c*m^2 + 535*b^2*c*m + 364*b^2*c)*x^10 + ((b^2 + 2*a*c)*m^4 + 28*(b^2 + 2*a*c)*m^3 + 249*(b^2 + 2*a*c)*m^2 + 520*b^2 + 1040*a*c + 742*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 31*a*b*m^3 + 321*a*b*m^2 + 1201*a*b*m + 910*a*b)*x^4 + (a^2*m^4 + 34*a^2*m^3 + 411*a^2*m^2 + 2074*a^2*m + 3640*a^2)*x)*(d*x)^m/(m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)

giac [B] time = 0.47, size = 449, normalized size = 4.45

$$\frac{(d^4m^4x^{13} + 22d^3m^3x^{13} + 159d^2m^2x^{13} + 418d^2m^2x^{13} + 280d^2m^2x^{13} + 2d^4m^4x^{10} + 25d^3m^3x^{10} + 195d^2m^2x^{10} + 535d^2m^2x^{10} + 364d^2m^2x^{10} + (b^2 + 2ac)d^4m^4x^7 + 28(b^2 + 2ac)d^3m^3x^7 + 249(b^2 + 2ac)d^2m^2x^7 + 520d^2m^2x^7 + 1040ad^2m^2x^7 + 742(b^2 + 2ac)d^2m^2x^7 + 2(abd^4m^4x^4 + 31abd^3m^3x^4 + 321abd^2m^2x^4 + 1201abd^2m^2x^4 + 910abd^2m^2x^4) + (a^2d^4m^4x^4 + 34a^2d^3m^3x^4 + 411a^2d^2m^2x^4 + 2074a^2d^2m^2x^4 + 3640a^2d^2m^2x^4)dx)^m}{m^5 + 35m^4 + 445m^3 + 2485m^2 + 5714m + 3640}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="giac")

[Out] ((d*x)^m*c^2*m^4*x^13 + 22*(d*x)^m*c^2*m^3*x^13 + 159*(d*x)^m*c^2*m^2*x^13 + 2*(d*x)^m*b^2*c*m^4*x^10 + 418*(d*x)^m*c^2*m^2*x^13 + 50*(d*x)^m*b^2*c*m^3*x^10 + 280*(d*x)^m*c^2*x^13 + 390*(d*x)^m*b^2*c*m^2*x^10 + (d*x)^m*b^2*m^4*x^7 + 2*(d*x)^m*a*c*m^4*x^7 + 1070*(d*x)^m*b^2*c*m^3*x^10 + 28*(d*x)^m*b^2*m^3*x^7 + 56*(d*x)^m*a*c*m^3*x^7 + 728*(d*x)^m*b^2*c*x^10 + 249*(d*x)^m*b^2*m^2*x^7 + 498*(d*x)^m*a*c*m^2*x^7 + 2*(d*x)^m*a*b*m^4*x^4 + 742*(d*x)^m*b^2*m*x^7 + 1484*(d*x)^m*a*c*m*x^7 + 62*(d*x)^m*a*b*m^3*x^4 + 520*(d*x)^m*b^2*x^7 + 1040*

$$(d*x)^m * a * c * x^7 + 642 * (d*x)^m * a * b * m^2 * x^4 + (d*x)^m * a^2 * m^4 * x + 2402 * (d*x)^m * a * b * m * x^4 + 34 * (d*x)^m * a^2 * m^3 * x + 1820 * (d*x)^m * a * b * x^4 + 411 * (d*x)^m * a^2 * m^2 * x + 2074 * (d*x)^m * a^2 * m * x + 3640 * (d*x)^m * a^2 * x / (m^5 + 35 * m^4 + 445 * m^3 + 2485 * m^2 + 5714 * m + 3640)$$

maple [B] time = 0.01, size = 301, normalized size = 2.98

$$\frac{(c^2 m^4 x^{12} + 22 a^2 m^3 x^{12} + 159 b^2 m^2 x^{12} + 2 c^2 m^3 x^9 + 418 b^2 m^2 x^9 + 50 b c m^2 x^9 + 280 c^2 m^2 x^9 + 360 b c m^2 x^9 + 2 a c m^3 x^6 + 728 b^2 m^3 x^6 + 1070 b c m^3 x^6 + 56 a c m^3 x^6 + 280 b^2 m^3 x^6 + 728 b c m^3 x^6 + 2402 a^2 m^3 x^6 + 2 a b m^4 x^6 + 1484 a c m^3 x^6 + 742 b^2 m^3 x^6 + 62 a b m^3 x^6 + 1040 c^2 m^3 x^6 + 520 b^2 m^3 x^6 + 642 a b m^3 x^6 + a^2 m^4 + 2402 a b m^3 x^3 + 542 b^2 m^3 x^3 + 1820 a b m^3 x^3 + 411 a^2 m^3 x^3 + 2074 a^2 m^3 x^3 + 3640 a^2 m^3 x^3) x^{d m}}{(m+13)(m+10)(m+7)(m+4)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(c*x^6+b*x^3+a)^2,x)

$$[Out] x * (c^2 * m^4 * x^{12} + 22 * c^2 * m^3 * x^{12} + 159 * c^2 * m^2 * x^{12} + 2 * b * c * m^4 * x^9 + 418 * c^2 * m * x^{12} + 50 * b * c * m^3 * x^9 + 280 * c^2 * x^{12} + 390 * b * c * m^2 * x^9 + 2 * a * c * m^4 * x^6 + b^2 * m^4 * x^6 + 1070 * b * c * m * x^9 + 56 * a * c * m^3 * x^6 + 28 * b^2 * m^3 * x^6 + 728 * b * c * x^9 + 498 * a * c * m^2 * x^6 + 249 * b^2 * m^2 * x^6 + 2 * a * b * m^4 * x^3 + 1484 * a * c * m * x^6 + 742 * b^2 * m * x^6 + 62 * a * b * m^3 * x^3 + 1040 * a * c * x^6 + 520 * b^2 * x^6 + 642 * a * b * m^2 * x^3 + a^2 * m^4 + 2402 * a * b * m * x^3 + 34 * a^2 * m^3 + 1820 * a * b * x^3 + 411 * a^2 * m^2 + 2074 * a^2 * m + 3640 * a^2) * (d*x)^m / (m+13) / (m+10) / (m+7) / (m+4) / (m+1)$$

maxima [A] time = 1.10, size = 110, normalized size = 1.09

$$\frac{c^2 d^m x^{13} x^m}{m+13} + \frac{2 b c d^m x^{10} x^m}{m+10} + \frac{b^2 d^m x^7 x^m}{m+7} + \frac{2 a c d^m x^7 x^m}{m+7} + \frac{2 a b d^m x^4 x^m}{m+4} + \frac{(d x)^{m+1} a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="maxima")

$$[Out] c^2 * d^m * x^{13} * x^m / (m + 13) + 2 * b * c * d^m * x^{10} * x^m / (m + 10) + b^2 * d^m * x^7 * x^m / (m + 7) + 2 * a * c * d^m * x^7 * x^m / (m + 7) + 2 * a * b * d^m * x^4 * x^m / (m + 4) + (d*x)^{(m+1)} * a^2 / (d * (m+1))$$

mupad [B] time = 1.52, size = 260, normalized size = 2.57

$$(d x)^m \left(\frac{c^2 x^{13} (m^4 + 22 m^3 + 159 m^2 + 418 m + 280)}{m^5 + 35 m^4 + 445 m^3 + 2485 m^2 + 5714 m + 3640} + \frac{x^7 (b^2 + 2 a c) (m^4 + 28 m^3 + 249 m^2 + 742 m + 520)}{m^5 + 35 m^4 + 445 m^3 + 2485 m^2 + 5714 m + 3640} + \frac{a^2 x (m^4 + 34 m^3 + 411 m^2 + 2074 m + 3640)}{m^5 + 35 m^4 + 445 m^3 + 2485 m^2 + 5714 m + 3640} + \frac{2 a b x^4 (m^4 + 31 m^3 + 321 m^2 + 1201 m + 910)}{m^5 + 35 m^4 + 445 m^3 + 2485 m^2 + 5714 m + 3640} + \frac{2 b c x^{10} (m^4 + 25 m^3 + 195 m^2 + 535 m + 364)}{m^5 + 35 m^4 + 445 m^3 + 2485 m^2 + 5714 m + 3640} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a + b*x^3 + c*x^6)^2,x)

$$[Out] (d*x)^m * ((c^2 * x^{13} * (418 * m + 159 * m^2 + 22 * m^3 + m^4 + 280)) / (5714 * m + 2485 * m^2 + 445 * m^3 + 35 * m^4 + m^5 + 3640) + (x^7 * (2 * a * c + b^2) * (742 * m + 249 * m^2 + 28 * m^3 + m^4 + 520)) / (5714 * m + 2485 * m^2 + 445 * m^3 + 35 * m^4 + m^5 + 3640) + (a^2 * x * (2074 * m + 411 * m^2 + 34 * m^3 + m^4 + 3640)) / (5714 * m + 2485 * m^2 + 445 * m^3 + 35 * m^4 + m^5 + 3640) + (2 * a * b * x^4 * (1201 * m + 321 * m^2 + 31 * m^3 + m^4 +$$

910))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (2*b*c*x^10*(53
5*m + 195*m^2 + 25*m^3 + m^4 + 364))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4
+ m^5 + 3640))

sympy [A] time = 5.82, size = 1510, normalized size = 14.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)

[Out] Piecewise(((-a**2/(12*x**12) - 2*a*b/(9*x**9) - a*c/(3*x**6) - b**2/(6*x**6)
) - 2*b*c/(3*x**3) + c**2*log(x))/d**13, Eq(m, -13)), ((-a**2/(9*x**9) - a
b/(3*x**6) - 2*a*c/(3*x**3) - b**2/(3*x**3) + 2*b*c*log(x) + c**2*x**3/3)/d
10, Eq(m, -10)), ((-a2/(6*x**6) - 2*a*b/(3*x**3) + 2*a*c*log(x) + b**2*
log(x) + 2*b*c*x**3/3 + c**2*x**6/6)/d**7, Eq(m, -7)), ((-a**2/(3*x**3) + 2
*a*b*log(x) + 2*a*c*x**3/3 + b**2*x**3/3 + b*c*x**6/3 + c**2*x**9/9)/d**4,
Eq(m, -4)), ((a**2*log(x) + 2*a*b*x**3/3 + a*c*x**6/3 + b**2*x**6/6 + 2*b*c
*x**9/9 + c**2*x**12/12)/d, Eq(m, -1)), (a**2*d**m**m**4*x*x**m/(m**5 + 35*m
4 + 445*m3 + 2485*m**2 + 5714*m + 3640) + 34*a**2*d**m**m**3*x*x**m/(m**
5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 411*a**2*d**m**m**2*x*x
m/(m5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2074*a**2*d*
mx*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 3640*
a**2*d**m**x*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) +
2*a*b*d**m**m**4*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
3640) + 62*a*b*d**m**m**3*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2
+ 5714*m + 3640) + 642*a*b*d**m**m**2*x**4*x**m/(m**5 + 35*m**4 + 445*m**3 +
2485*m**2 + 5714*m + 3640) + 2402*a*b*d**m**m**x**4*x**m/(m**5 + 35*m**4 + 4
45*m**3 + 2485*m**2 + 5714*m + 3640) + 1820*a*b*d**m**x**4*x**m/(m**5 + 35*m
4 + 445*m3 + 2485*m**2 + 5714*m + 3640) + 2*a*c*d**m**m**4*x**7*x**m/(m*
*5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 56*a*c*d**m**m**3*x**
7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 498*a*c*d*
mm**2*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) +
1484*a*c*d**m**m**x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m
+ 3640) + 1040*a*c*d**m**x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 +
5714*m + 3640) + b**2*d**m**m**4*x**7*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485
*m**2 + 5714*m + 3640) + 28*b**2*d**m**m**3*x**7*x**m/(m**5 + 35*m**4 + 445*
m**3 + 2485*m**2 + 5714*m + 3640) + 249*b**2*d**m**m**2*x**7*x**m/(m**5 + 35
*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 742*b**2*d**m**m**x**7*x**m/(
m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 520*b**2*d**m**x**7
*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*b*c*d**m*
m**4*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 5
0*b*c*d**m**m**3*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m
+ 3640) + 390*b*c*d**m**m**2*x**10*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m*
*2 + 5714*m + 3640) + 1070*b*c*d**m**m**x**10*x**m/(m**5 + 35*m**4 + 445*m**3


```

+ 2485*m**2 + 5714*m + 3640) + 728*b*c*d**m*x**10*x**m/(m**5 + 35*m**4 + 4
45*m**3 + 2485*m**2 + 5714*m + 3640) + c**2*d**m*m**4*x**13*x**m/(m**5 + 35
*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 22*c**2*d**m*m**3*x**13*x**
m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 159*c**2*d**m*m
**2*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 41
8*c**2*d**m*m*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
3640) + 280*c**2*d**m*x**13*x**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5
714*m + 3640), True))

```

3.216 $\int (dx)^m (a + bx^3 + cx^6) dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{m+1}}{d(m+1)} + \frac{b(dx)^{m+4}}{d^4(m+4)} + \frac{c(dx)^{m+7}}{d^7(m+7)}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{a(dx)^{m+1}}{d(m+1)} + \frac{b(dx)^{m+4}}{d^4(m+4)} + \frac{c(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a + b*x^3 + c*x^6), x]

[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) + (c*(d*x)^(7 + m))/(d^7*(7 + m))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rubi steps

$$\begin{aligned} \int (dx)^m (a + bx^3 + cx^6) dx &= \int \left(a(dx)^m + \frac{b(dx)^{3+m}}{d^3} + \frac{c(dx)^{6+m}}{d^6} \right) dx \\ &= \frac{a(dx)^{1+m}}{d(1+m)} + \frac{b(dx)^{4+m}}{d^4(4+m)} + \frac{c(dx)^{7+m}}{d^7(7+m)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.67

$$x(dx)^m \left(\frac{a}{m+1} + \frac{bx^3}{m+4} + \frac{cx^6}{m+7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6), x]

[Out] $x*(d*x)^m*(a/(1+m) + (b*x^3)/(4+m) + (c*x^6)/(7+m))$

IntegrateAlgebraic [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (dx)^m (a + bx^3 + cx^6) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^3 + c*x^6), x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(a + b*x^3 + c*x^6), x]

fricas [A] time = 1.31, size = 71, normalized size = 1.37

$$\frac{((cm^2 + 5cm + 4c)x^7 + (bm^2 + 8bm + 7b)x^4 + (am^2 + 11am + 28a)x)(dx)^m}{m^3 + 12m^2 + 39m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] $((cm^2 + 5cm + 4c)*x^7 + (bm^2 + 8bm + 7b)*x^4 + (am^2 + 11am + 28a)*x)*(d*x)^m/(m^3 + 12m^2 + 39m + 28)$

giac [B] time = 0.35, size = 119, normalized size = 2.29

$$\frac{(dx)^m cm^2x^7 + 5(dx)^m cmx^7 + 4(dx)^m cx^7 + (dx)^m bm^2x^4 + 8(dx)^m bmx^4 + 7(dx)^m bx^4 + (dx)^m am^2x + 11(dx)^m amx + 28(dx)^m ax}{m^3 + 12m^2 + 39m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(c*x^6+b*x^3+a), x, algorithm="giac")

[Out] $((d*x)^m*c*m^2*x^7 + 5*(d*x)^m*c*m*x^7 + 4*(d*x)^m*c*x^7 + (d*x)^m*b*m^2*x^4 + 8*(d*x)^m*b*m*x^4 + 7*(d*x)^m*b*x^4 + (d*x)^m*a*m^2*x + 11*(d*x)^m*a*m*x + 28*(d*x)^m*a*x)/(m^3 + 12m^2 + 39m + 28)$

maple [A] time = 0.00, size = 78, normalized size = 1.50

$$\frac{(cm^2x^6 + 5cmx^6 + 4cx^6 + bm^2x^3 + 8bmx^3 + 7bx^3 + am^2 + 11am + 28a)x(dx)^m}{(m+7)(m+4)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(c*x^6+b*x^3+a), x)

[Out] $x*(cm^2*x^6+5*c*m*x^6+4*c*x^6+b*m^2*x^3+8*b*m*x^3+7*b*x^3+a*m^2+11*a*m+28*a)*(d*x)^m/(m+7)/(m+4)/(m+1)$

maxima [A] time = 1.16, size = 50, normalized size = 0.96

$$\frac{cd^m x^7 x^m}{m+7} + \frac{bd^m x^4 x^m}{m+4} + \frac{(dx)^{m+1} a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] c*d^m*x^7*x^m/(m+7) + b*d^m*x^4*x^m/(m+4) + (d*x)^(m+1)*a/(d*(m+1))

mupad [B] time = 1.36, size = 89, normalized size = 1.71

$$(dx)^m \left(\frac{bx^4(m^2+8m+7)}{m^3+12m^2+39m+28} + \frac{cx^7(m^2+5m+4)}{m^3+12m^2+39m+28} + \frac{ax(m^2+11m+28)}{m^3+12m^2+39m+28} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a + b*x^3 + c*x^6),x)

[Out] (d*x)^m*((b*x^4*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (c*x^7*(5*m + m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (a*x*(11*m + m^2 + 28))/(39*m + 12*m^2 + m^3 + 28))

sympy [A] time = 1.47, size = 314, normalized size = 6.04

$$\begin{cases} \frac{-\frac{a}{6c^6} - \frac{b}{3c^3} + c \log(x)}{d^7} & \text{for } m = -7 \\ \frac{-\frac{a}{3c^3} + b \log(x) + \frac{c^3}{3}}{d^4} & \text{for } m = -4 \\ \frac{a \log(x) + \frac{bc^3}{3} + \frac{c^6}{6}}{d} & \text{for } m = -1 \\ \frac{ad^m n^2 x^m}{m^3+12m^2+39m+28} + \frac{11ad^m m x^m}{m^3+12m^2+39m+28} + \frac{28ad^m x x^m}{m^3+12m^2+39m+28} + \frac{bd^m n^2 x^4 x^m}{m^3+12m^2+39m+28} + \frac{8bd^m m x^4 x^m}{m^3+12m^2+39m+28} + \frac{7bd^m x^4 x^m}{m^3+12m^2+39m+28} + \frac{cd^m m^2 x^7 x^m}{m^3+12m^2+39m+28} + \frac{5cd^m m x^7 x^m}{m^3+12m^2+39m+28} + \frac{4cd^m x^7 x^m}{m^3+12m^2+39m+28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(c*x**6+b*x**3+a),x)

[Out] Piecewise(((-a/(6*x**6) - b/(3*x**3) + c*log(x))/d**7, Eq(m, -7)), ((-a/(3*x**3) + b*log(x) + c*x**3/3)/d**4, Eq(m, -4)), ((a*log(x) + b*x**3/3 + c*x**6/6)/d, Eq(m, -1)), (a*d**m*m**2*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 11*a*d**m*m*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a*d**m*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + b*d**m*m**2*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 8*b*d**m*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*b*d**m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + c*d**m*m**2*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 5*c*d**m*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*c*d**m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))

$$3.217 \quad \int \frac{x^9}{1+2x^4+x^8} dx$$

Optimal. Leaf size=30

$$\frac{3x^2}{4} - \frac{3}{4} \tan^{-1}(x^2) - \frac{x^6}{4(x^4+1)}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 275, 288, 321, 203}

$$-\frac{x^6}{4(x^4+1)} + \frac{3x^2}{4} - \frac{3}{4} \tan^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[x^9/(1 + 2*x^4 + x^8), x]

[Out] (3*x^2)/4 - x^6/(4*(1 + x^4)) - (3*ArcTan[x^2])/4

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :=
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))]^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 288

Int[((c_.)*(x_)]^(m_.)*((a_) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^(n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1 + 2x^4 + x^8} dx &= \int \frac{x^9}{(1 + x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(1 + x^2)^2} dx, x, x^2 \right) \\
&= -\frac{x^6}{4(1 + x^4)} + \frac{3}{4} \text{Subst} \left(\int \frac{x^2}{1 + x^2} dx, x, x^2 \right) \\
&= \frac{3x^2}{4} - \frac{x^6}{4(1 + x^4)} - \frac{3}{4} \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, x^2 \right) \\
&= \frac{3x^2}{4} - \frac{x^6}{4(1 + x^4)} - \frac{3}{4} \tan^{-1}(x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.80

$$\frac{1}{4} \left(x^2 \left(\frac{1}{x^4 + 1} + 2 \right) - 3 \tan^{-1}(x^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(1 + 2*x^4 + x^8), x]

[Out] (x^2*(2 + (1 + x^4)^(-1)) - 3*ArcTan[x^2])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1 + 2x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/(1 + 2*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^9/(1 + 2*x^4 + x^8), x]

fricas [A] time = 0.93, size = 31, normalized size = 1.03

$$\frac{2x^6 + 3x^2 - 3(x^4 + 1)\arctan(x^2)}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] 1/4*(2*x^6 + 3*x^2 - 3*(x^4 + 1)*arctan(x^2))/(x^4 + 1)

giac [A] time = 0.34, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2 + \frac{x^2}{4(x^4 + 1)} - \frac{3}{4}\arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="giac")

[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{x^2}{2} + \frac{x^2}{4x^4 + 4} - \frac{3\arctan(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^8+2*x^4+1),x)

[Out] 1/2*x^2+1/4*x^2/(x^4+1)-3/4*arctan(x^2)

maxima [A] time = 2.16, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2 + \frac{x^2}{4(x^4 + 1)} - \frac{3}{4}\arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)

mupad [B] time = 0.05, size = 25, normalized size = 0.83

$$\frac{x^2}{4(x^4 + 1)} - \frac{3 \operatorname{atan}(x^2)}{4} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(2*x^4 + x^8 + 1), x)`

[Out] `x^2/(4*(x^4 + 1)) - (3*atan(x^2))/4 + x^2/2`

sympy [A] time = 0.13, size = 22, normalized size = 0.73

$$\frac{x^2}{2} + \frac{x^2}{4x^4 + 4} - \frac{3 \operatorname{atan}(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(x**8+2*x**4+1), x)`

[Out] `x**2/2 + x**2/(4*x**4 + 4) - 3*atan(x**2)/4`

$$3.218 \quad \int \frac{x^7}{1+2x^4+x^8} dx$$

Optimal. Leaf size=22

$$\frac{1}{4(x^4+1)} + \frac{1}{4} \log(x^4+1)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 266, 43}

$$\frac{1}{4(x^4+1)} + \frac{1}{4} \log(x^4+1)$$

Antiderivative was successfully verified.

[In] Int[x^7/(1 + 2*x^4 + x^8), x]

[Out] 1/(4*(1 + x^4)) + Log[1 + x^4]/4

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :=
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+2x^4+x^8} dx &= \int \frac{x^7}{(1+x^4)^2} dx \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{x}{(1+x)^2} dx, x, x^4 \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \left(-\frac{1}{(1+x)^2} + \frac{1}{1+x} \right) dx, x, x^4 \right) \\
&= \frac{1}{4(1+x^4)} + \frac{1}{4} \log(1+x^4)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.82

$$\frac{1}{4} \left(\frac{1}{x^4+1} + \log(x^4+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(1+2*x^4+x^8),x]

[Out] ((1+x^4)^(-1)+Log[1+x^4])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1+2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(1+2*x^4+x^8),x]

[Out] IntegrateAlgebraic[x^7/(1+2*x^4+x^8),x]

fricas [A] time = 0.94, size = 23, normalized size = 1.05

$$\frac{(x^4+1)\log(x^4+1)+1}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] 1/4*((x^4+1)*log(x^4+1)+1)/(x^4+1)

giac [A] time = 0.28, size = 18, normalized size = 0.82

$$\frac{1}{4(x^4 + 1)} + \frac{1}{4} \log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="giac")

[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$\frac{\ln(x^4 + 1)}{4} + \frac{1}{4x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^8+2*x^4+1),x)

[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)

maxima [A] time = 0.98, size = 18, normalized size = 0.82

$$\frac{1}{4(x^4 + 1)} + \frac{1}{4} \log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)

mupad [B] time = 1.32, size = 18, normalized size = 0.82

$$\frac{\ln(x^4 + 1)}{4} + \frac{1}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(2*x^4 + x^8 + 1),x)

[Out] log(x^4 + 1)/4 + 1/(4*(x^4 + 1))

sympy [A] time = 0.11, size = 15, normalized size = 0.68

$$\frac{\log(x^4 + 1)}{4} + \frac{1}{4x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+2*x**4+1),x)
```

```
[Out] log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

$$3.219 \quad \int \frac{x^5}{1+2x^4+x^8} dx$$

Optimal. Leaf size=23

$$\frac{1}{4} \tan^{-1}(x^2) - \frac{x^2}{4(x^4+1)}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {28, 275, 288, 203}

$$\frac{1}{4} \tan^{-1}(x^2) - \frac{x^2}{4(x^4+1)}$$

Antiderivative was successfully verified.

[In] Int[x^5/(1 + 2*x^4 + x^8), x]

[Out] -x^2/(4*(1 + x^4)) + ArcTan[x^2]/4

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :=
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^(n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1+2x^4+x^8} dx &= \int \frac{x^5}{(1+x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(1+x^2)^2} dx, x, x^2 \right) \\
&= -\frac{x^2}{4(1+x^4)} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, x^2 \right) \\
&= -\frac{x^2}{4(1+x^4)} + \frac{1}{4} \tan^{-1}(x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{1}{4} \tan^{-1}(x^2) - \frac{x^2}{4(x^4+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(1+2*x^4+x^8),x]

[Out] -1/4*x^2/(1+x^4) + ArcTan[x^2]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1+2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(1+2*x^4+x^8),x]

[Out] IntegrateAlgebraic[x^5/(1+2*x^4+x^8),x]

fricas [A] time = 0.94, size = 24, normalized size = 1.04

$$-\frac{x^2 - (x^4 + 1) \arctan(x^2)}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] -1/4*(x^2 - (x^4 + 1)*arctan(x^2))/(x^4 + 1)

giac [A] time = 0.37, size = 19, normalized size = 0.83

$$-\frac{x^2}{4(x^4 + 1)} + \frac{1}{4} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="giac")

[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$-\frac{x^2}{4(x^4 + 1)} + \frac{\arctan(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^8+2*x^4+1),x)

[Out] -1/4/(x^4+1)*x^2+1/4*arctan(x^2)

maxima [A] time = 2.02, size = 19, normalized size = 0.83

$$-\frac{x^2}{4(x^4 + 1)} + \frac{1}{4} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)

mupad [B] time = 1.37, size = 21, normalized size = 0.91

$$\frac{\operatorname{atan}(x^2)}{4} - \frac{x^2}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(2*x^4 + x^8 + 1),x)

[Out] $\text{atan}(x^2)/4 - x^2/(4*(x^4 + 1))$

sympy [A] time = 0.12, size = 15, normalized size = 0.65

$$-\frac{x^2}{4x^4 + 4} + \frac{\text{atan}(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(x**8+2*x**4+1), x)`

[Out] $-x^{**2}/(4*x^{**4} + 4) + \text{atan}(x^{**2})/4$

$$3.220 \quad \int \frac{x^3}{1+2x^4+x^8} dx$$

Optimal. Leaf size=11

$$-\frac{1}{4(x^4+1)}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 261}

$$-\frac{1}{4(x^4+1)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 + 2*x^4 + x^8),x]

[Out] -1/(4*(1 + x^4))

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p+1)/(b*n*(p+1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n-1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{1+2x^4+x^8} dx &= \int \frac{x^3}{(1+x^4)^2} dx \\ &= -\frac{1}{4(1+x^4)} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{4(x^4+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(1 + 2*x^4 + x^8),x]

[Out] -1/4*1/(1 + x^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1 + 2x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(1 + 2*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^3/(1 + 2*x^4 + x^8), x]

fricas [A] time = 0.86, size = 9, normalized size = 0.82

$$-\frac{1}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] -1/4/(x^4 + 1)

giac [A] time = 0.36, size = 9, normalized size = 0.82

$$-\frac{1}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="giac")

[Out] -1/4/(x^4 + 1)

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{1}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^8+2*x^4+1),x)

[Out] $-1/4/(x^4+1)$

maxima [A] time = 0.89, size = 9, normalized size = 0.82

$$-\frac{1}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(x^8+2*x^4+1),x, algorithm="maxima")`

[Out] $-1/4/(x^4 + 1)$

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(2*x^4 + x^8 + 1),x)`

[Out] $-1/(4*(x^4 + 1))$

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\frac{1}{4x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(x**8+2*x**4+1),x)`

[Out] $-1/(4*x**4 + 4)$

$$3.221 \quad \int \frac{x}{1+2x^4+x^8} dx$$

Optimal. Leaf size=23

$$\frac{1}{4} \tan^{-1}(x^2) + \frac{x^2}{4(x^4 + 1)}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {28, 275, 199, 203}

$$\frac{x^2}{4(x^4 + 1)} + \frac{1}{4} \tan^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[x/(1 + 2*x^4 + x^8), x]

[Out] x^2/(4*(1 + x^4)) + ArcTan[x^2]/4

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+2x^4+x^8} dx &= \int \frac{x}{(1+x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x^2)^2} dx, x, x^2 \right) \\
&= \frac{x^2}{4(1+x^4)} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, x^2 \right) \\
&= \frac{x^2}{4(1+x^4)} + \frac{1}{4} \tan^{-1}(x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{4} \left(\tan^{-1}(x^2) + \frac{x^2}{x^4+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + 2*x^4 + x^8),x]

[Out] (x^2/(1 + x^4) + ArcTan[x^2])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1+2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(1 + 2*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x/(1 + 2*x^4 + x^8), x]

fricas [A] time = 1.12, size = 23, normalized size = 1.00

$$\frac{x^2 + (x^4 + 1) \arctan(x^2)}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] $1/4*(x^2 + (x^4 + 1)*\arctan(x^2))/(x^4 + 1)$

giac [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{x^2}{4(x^4 + 1)} + \frac{1}{4} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^8+2*x^4+1),x, algorithm="giac")`

[Out] $1/4*x^2/(x^4 + 1) + 1/4*\arctan(x^2)$

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{x^2}{4x^4 + 4} + \frac{\arctan(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^8+2*x^4+1),x)`

[Out] $1/4/(x^4+1)*x^2+1/4*\arctan(x^2)$

maxima [A] time = 2.17, size = 19, normalized size = 0.83

$$\frac{x^2}{4(x^4 + 1)} + \frac{1}{4} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^8+2*x^4+1),x, algorithm="maxima")`

[Out] $1/4*x^2/(x^4 + 1) + 1/4*\arctan(x^2)$

mupad [B] time = 0.03, size = 20, normalized size = 0.87

$$\frac{\operatorname{atan}(x^2)}{4} + \frac{x^2}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(2*x^4 + x^8 + 1),x)`

[Out] $\operatorname{atan}(x^2)/4 + x^2/(4*(x^4 + 1))$

sympy [A] time = 0.12, size = 15, normalized size = 0.65

$$\frac{x^2}{4x^4 + 4} + \frac{\operatorname{atan}(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**8+2*x**4+1),x)

[Out] x**2/(4*x**4 + 4) + atan(x**2)/4

$$3.222 \quad \int \frac{1}{x(1+2x^4+x^8)} dx$$

Optimal. Leaf size=24

$$\frac{1}{4(x^4+1)} - \frac{1}{4} \log(x^4+1) + \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 266, 44}

$$\frac{1}{4(x^4+1)} - \frac{1}{4} \log(x^4+1) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + 2*x^4 + x^8)),x]

[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1+2x^4+x^8)} dx &= \int \frac{1}{x(1+x^4)^2} dx \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(1+x)^2} dx, x, x^4 \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{-1-x} + \frac{1}{x} - \frac{1}{(1+x)^2} \right) dx, x, x^4 \right) \\
&= \frac{1}{4(1+x^4)} + \log(x) - \frac{1}{4} \log(1+x^4)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{4(x^4+1)} - \frac{1}{4} \log(x^4+1) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 + 2*x^4 + x^8)), x]

[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1+2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(1 + 2*x^4 + x^8)), x]

[Out] IntegrateAlgebraic[1/(x*(1 + 2*x^4 + x^8)), x]

fricas [A] time = 0.78, size = 32, normalized size = 1.33

$$\frac{(x^4+1) \log(x^4+1) - 4(x^4+1) \log(x) - 1}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8+2*x^4+1), x, algorithm="fricas")

[Out] -1/4*((x^4 + 1)*log(x^4 + 1) - 4*(x^4 + 1)*log(x) - 1)/(x^4 + 1)

giac [A] time = 0.36, size = 29, normalized size = 1.21

$$\frac{x^4 + 2}{4(x^4 + 1)} - \frac{1}{4} \log(x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="giac")

[Out] 1/4*(x^4 + 2)/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\ln(x) - \frac{\ln(x^4 + 1)}{4} + \frac{1}{4x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^8+2*x^4+1),x)

[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)

maxima [A] time = 0.97, size = 24, normalized size = 1.00

$$\frac{1}{4(x^4 + 1)} - \frac{1}{4} \log(x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] 1/4/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$\ln(x) - \frac{\ln(x^4 + 1)}{4} + \frac{1}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(2*x^4 + x^8 + 1)),x)

[Out] log(x) - log(x^4 + 1)/4 + 1/(4*(x^4 + 1))

sympy [A] time = 0.13, size = 19, normalized size = 0.79

$$\log(x) - \frac{\log(x^4 + 1)}{4} + \frac{1}{4x^4 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+2*x**4+1),x)
```

```
[Out] log(x) - log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

$$3.223 \quad \int \frac{1}{x^3(1+2x^4+x^8)} dx$$

Optimal. Leaf size=30

$$-\frac{3}{4x^2} - \frac{3}{4} \tan^{-1}(x^2) + \frac{1}{4x^2(x^4+1)}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 275, 290, 325, 203}

$$\frac{1}{4x^2(x^4+1)} - \frac{3}{4x^2} - \frac{3}{4} \tan^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1 + 2*x^4 + x^8)),x]

[Out] -3/(4*x^2) + 1/(4*x^2*(1 + x^4)) - (3*ArcTan[x^2])/4

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3(1+2x^4+x^8)} dx &= \int \frac{1}{x^3(1+x^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(1+x^2)^2} dx, x, x^2 \right) \\
 &= \frac{1}{4x^2(1+x^4)} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{x^2(1+x^2)} dx, x, x^2 \right) \\
 &= -\frac{3}{4x^2} + \frac{1}{4x^2(1+x^4)} - \frac{3}{4} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, x^2 \right) \\
 &= -\frac{3}{4x^2} + \frac{1}{4x^2(1+x^4)} - \frac{3}{4} \tan^{-1}(x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{1}{2x^2} + \frac{3}{4} \tan^{-1}\left(\frac{1}{x^2}\right) - \frac{x^2}{4(x^4+1)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(1 + 2*x^4 + x^8)), x]

[Out] -1/2*1/x^2 - x^2/(4*(1 + x^4)) + (3*ArcTan[x^(-2)])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(1+2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(1 + 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^3*(1 + 2*x^4 + x^8)), x]

fricas [A] time = 1.02, size = 31, normalized size = 1.03

$$\frac{3x^4 + 3(x^6 + x^2)\arctan(x^2) + 2}{4(x^6 + x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] -1/4*(3*x^4 + 3*(x^6 + x^2)*arctan(x^2) + 2)/(x^6 + x^2)

giac [A] time = 0.34, size = 25, normalized size = 0.83

$$-\frac{3x^4 + 2}{4(x^6 + x^2)} - \frac{3}{4}\arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="giac")

[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{x^2}{4(x^4 + 1)} - \frac{3\arctan(x^2)}{4} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^8+2*x^4+1),x)

[Out] -1/2/x^2-1/4/(x^4+1)*x^2-3/4*arctan(x^2)

maxima [A] time = 1.98, size = 25, normalized size = 0.83

$$-\frac{3x^4 + 2}{4(x^6 + x^2)} - \frac{3}{4}\arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)

mupad [B] time = 0.04, size = 25, normalized size = 0.83

$$-\frac{3 \operatorname{atan}\left(x^2\right)}{4} - \frac{\frac{3x^4}{4} + \frac{1}{2}}{x^6 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(2*x^4 + x^8 + 1)),x)`

[Out] `-(3*atan(x^2))/4 - ((3*x^4)/4 + 1/2)/(x^2 + x^6)`

sympy [A] time = 0.15, size = 26, normalized size = 0.87

$$\frac{-3x^4 - 2}{4x^6 + 4x^2} - \frac{3 \operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(x**8+2*x**4+1),x)`

[Out] `(-3*x**4 - 2)/(4*x**6 + 4*x**2) - 3*atan(x**2)/4`

$$3.224 \quad \int \frac{1}{x^5(1+2x^4+x^8)} dx$$

Optimal. Leaf size=33

$$-\frac{1}{4(x^4+1)} - \frac{1}{4x^4} + \frac{1}{2} \log(x^4+1) - 2\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 266, 44}

$$-\frac{1}{4(x^4+1)} - \frac{1}{4x^4} + \frac{1}{2} \log(x^4+1) - 2\log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(1 + 2*x^4 + x^8)),x]

[Out] -1/(4*x^4) - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(1+2x^4+x^8)} dx &= \int \frac{1}{x^5(1+x^4)^2} dx \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2(1+x)^2} dx, x, x^4 \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{x^2} - \frac{2}{x} + \frac{1}{(1+x)^2} + \frac{2}{1+x} \right) dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} - \frac{1}{4(1+x^4)} - 2\log(x) + \frac{1}{2}\log(1+x^4)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{4(x^4+1)} - \frac{1}{4x^4} + \frac{1}{2}\log(x^4+1) - 2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(1+2*x^4+x^8)),x]

[Out] -1/4*1/x^4 - 1/(4*(1+x^4)) - 2*Log[x] + Log[1+x^4]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(1+2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(1+2*x^4+x^8)),x]

[Out] IntegrateAlgebraic[1/(x^5*(1+2*x^4+x^8)), x]

fricas [A] time = 1.13, size = 44, normalized size = 1.33

$$\frac{2x^4 - 2(x^8 + x^4)\log(x^4 + 1) + 8(x^8 + x^4)\log(x) + 1}{4(x^8 + x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] $-1/4*(2*x^4 - 2*(x^8 + x^4)*\log(x^4 + 1) + 8*(x^8 + x^4)*\log(x) + 1)/(x^8 + x^4)$

giac [A] time = 0.35, size = 33, normalized size = 1.00

$$-\frac{2x^4 + 1}{4(x^8 + x^4)} + \frac{1}{2} \log(x^4 + 1) - \frac{1}{2} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="giac")`

[Out] $-1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*\log(x^4 + 1) - 1/2*\log(x^4)$

maple [A] time = 0.02, size = 28, normalized size = 0.85

$$-2 \ln(x) + \frac{\ln(x^4 + 1)}{2} - \frac{1}{4x^4} - \frac{1}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(x^8+2*x^4+1),x)`

[Out] $-1/4/x^4-1/4/(x^4+1)-2*\ln(x)+1/2*\ln(x^4+1)$

maxima [A] time = 0.88, size = 33, normalized size = 1.00

$$-\frac{2x^4 + 1}{4(x^8 + x^4)} + \frac{1}{2} \log(x^4 + 1) - \frac{1}{2} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="maxima")`

[Out] $-1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*\log(x^4 + 1) - 1/2*\log(x^4)$

mupad [B] time = 0.05, size = 31, normalized size = 0.94

$$\frac{\ln(x^4 + 1)}{2} - 2 \ln(x) - \frac{\frac{x^4}{2} + \frac{1}{4}}{x^8 + x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(2*x^4 + x^8 + 1)),x)`

[Out] $\log(x^4 + 1)/2 - 2*\log(x) - (x^4/2 + 1/4)/(x^4 + x^8)$

sympy [A] time = 0.15, size = 31, normalized size = 0.94

$$\frac{-2x^4 - 1}{4x^8 + 4x^4} - 2 \log(x) + \frac{\log(x^4 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(x**8+2*x**4+1),x)

[Out] (-2*x**4 - 1)/(4*x**8 + 4*x**4) - 2*log(x) + log(x**4 + 1)/2

$$3.225 \quad \int \frac{1}{x^7(1+2x^4+x^8)} dx$$

Optimal. Leaf size=37

$$-\frac{5}{12x^6} + \frac{5}{4x^2} + \frac{5}{4} \tan^{-1}(x^2) + \frac{1}{4x^6(x^4+1)}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 275, 290, 325, 203}

$$\frac{1}{4x^6(x^4+1)} + \frac{5}{4x^2} - \frac{5}{12x^6} + \frac{5}{4} \tan^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(1 + 2*x^4 + x^8)),x]

[Out] -5/(12*x^6) + 5/(4*x^2) + 1/(4*x^6*(1 + x^4)) + (5*ArcTan[x^2])/4

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7(1+2x^4+x^8)} dx &= \int \frac{1}{x^7(1+x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4(1+x^2)^2} dx, x, x^2 \right) \\
&= \frac{1}{4x^6(1+x^4)} + \frac{5}{4} \text{Subst} \left(\int \frac{1}{x^4(1+x^2)} dx, x, x^2 \right) \\
&= -\frac{5}{12x^6} + \frac{1}{4x^6(1+x^4)} - \frac{5}{4} \text{Subst} \left(\int \frac{1}{x^2(1+x^2)} dx, x, x^2 \right) \\
&= -\frac{5}{12x^6} + \frac{5}{4x^2} + \frac{1}{4x^6(1+x^4)} + \frac{5}{4} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, x^2 \right) \\
&= -\frac{5}{12x^6} + \frac{5}{4x^2} + \frac{1}{4x^6(1+x^4)} + \frac{5}{4} \tan^{-1}(x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.89

$$-\frac{1}{6x^6} + \frac{1}{x^2} - \frac{5}{4} \tan^{-1}\left(\frac{1}{x^2}\right) + \frac{x^2}{4(x^4+1)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(1 + 2*x^4 + x^8)), x]

[Out] -1/6*1/x^6 + x^(-2) + x^2/(4*(1 + x^4)) - (5*ArcTan[x^(-2)])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7(1+2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^7*(1+2*x^4+x^8)),x]

[Out] IntegrateAlgebraic[1/(x^7*(1+2*x^4+x^8)), x]

fricas [A] time = 1.10, size = 36, normalized size = 0.97

$$\frac{15x^8 + 10x^4 + 15(x^{10} + x^6) \arctan(x^2) - 2}{12(x^{10} + x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] 1/12*(15*x^8 + 10*x^4 + 15*(x^10 + x^6)*arctan(x^2) - 2)/(x^10 + x^6)

giac [A] time = 0.30, size = 31, normalized size = 0.84

$$\frac{x^2}{4(x^4+1)} + \frac{6x^4-1}{6x^6} + \frac{5}{4} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="giac")

[Out] 1/4*x^2/(x^4+1) + 1/6*(6*x^4-1)/x^6 + 5/4*arctan(x^2)

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$\frac{x^2}{4x^4+4} + \frac{5 \arctan(x^2)}{4} + \frac{1}{x^2} - \frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(x^8+2*x^4+1),x)

[Out] -1/6/x^6+1/x^2+1/4/(x^4+1)*x^2+5/4*arctan(x^2)

maxima [A] time = 2.49, size = 30, normalized size = 0.81

$$\frac{15x^8 + 10x^4 - 2}{12(x^{10} + x^6)} + \frac{5}{4} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="maxima")`

[Out] $1/12*(15*x^8 + 10*x^4 - 2)/(x^{10} + x^6) + 5/4*\arctan(x^2)$

mupad [B] time = 0.05, size = 30, normalized size = 0.81

$$\frac{5 \operatorname{atan}(x^2)}{4} + \frac{\frac{5x^8}{4} + \frac{5x^4}{6} - \frac{1}{6}}{x^6(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(2*x^4 + x^8 + 1)),x)`

[Out] $(5*\operatorname{atan}(x^2))/4 + ((5*x^4)/6 + (5*x^8)/4 - 1/6)/(x^6*(x^4 + 1))$

sympy [A] time = 0.17, size = 29, normalized size = 0.78

$$\frac{5 \operatorname{atan}(x^2)}{4} + \frac{15x^8 + 10x^4 - 2}{12x^{10} + 12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(x**8+2*x**4+1),x)`

[Out] $5*\operatorname{atan}(x**2)/4 + (15*x**8 + 10*x**4 - 2)/(12*x**10 + 12*x**6)$

$$3.226 \quad \int \frac{x^8}{1+2x^4+x^8} dx$$

Optimal. Leaf size=104

$$\frac{5 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{5 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{x^5}{4(x^4 + 1)} + \frac{5x}{4} + \frac{5 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} - \frac{5 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 288, 321, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x^5}{4(x^4 + 1)} + \frac{5 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{5 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{5x}{4} + \frac{5 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} - \frac{5 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^8/(1 + 2*x^4 + x^8),x]

[Out] (5*x)/4 - x^5/(4*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_.) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 288


```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1+2x^4+x^8} dx &= \int \frac{x^8}{(1+x^4)^2} dx \\
&= -\frac{x^5}{4(1+x^4)} + \frac{5}{4} \int \frac{x^4}{1+x^4} dx \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} - \frac{5}{4} \int \frac{1}{1+x^4} dx \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} - \frac{5}{8} \int \frac{1-x^2}{1+x^4} dx - \frac{5}{8} \int \frac{1+x^2}{1+x^4} dx \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} - \frac{5}{16} \int \frac{1}{1-\sqrt{2}x+x^2} dx - \frac{5}{16} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{5 \int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{16\sqrt{2}} + \dots \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} + \frac{5 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{5 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{5 \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1\right)}{8\sqrt{2}} \\
&= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} + \frac{5 \tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} - \frac{5 \tan^{-1}(1+\sqrt{2}x)}{8\sqrt{2}} + \frac{5 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{5 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.90

$$\frac{1}{32} \left(\frac{8x}{x^4+1} + 5\sqrt{2} \log(x^2 - \sqrt{2}x + 1) - 5\sqrt{2} \log(x^2 + \sqrt{2}x + 1) + 32x + 10\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) - 10\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(1 + 2*x^4 + x^8), x]

[Out] (32*x + (8*x)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 5*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/32

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1+2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(1 + 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^8/(1 + 2*x^4 + x^8), x]

fricas [A] time = 1.02, size = 132, normalized size = 1.27

$$\frac{32x^5 + 20\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1\right) + 20\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1\right) - 5\sqrt{2}(x^4+1)\log(x^2 + \sqrt{2}x + 1) + 5\sqrt{2}(x^4+1)\log(x^2 - \sqrt{2}x + 1) + 40x}{32(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] 1/32*(32*x^5 + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + 5*sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) + 40*x)/(x^4 + 1)

giac [A] time = 0.34, size = 83, normalized size = 0.80

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{5}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{5}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) + x + \frac{x}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="giac")

[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)

maple [A] time = 0.01, size = 69, normalized size = 0.66

$$x + \frac{x}{4x^4 + 4} - \frac{5\sqrt{2}\arctan(\sqrt{2}x - 1)}{16} - \frac{5\sqrt{2}\arctan(\sqrt{2}x + 1)}{16} - \frac{5\sqrt{2}\ln\left(\frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^8+2*x^4+1),x)

[Out] x+1/4*x/(x^4+1)-5/32*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))-5/16*arctan(-1+2^(1/2)*x)*2^(1/2)-5/16*arctan(1+2^(1/2)*x)*2^(1/2)

maxima [A] time = 2.02, size = 83, normalized size = 0.80

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{5}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{5}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) + x + \frac{x}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] $-5/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})) - 5/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})) - 5/32*\sqrt{2}*\log(x^2 + \sqrt{2}*x + 1) + 5/32*\sqrt{2}*\log(x^2 - \sqrt{2}*x + 1) + x + 1/4*x/(x^4 + 1)$

mupad [B] time = 1.37, size = 45, normalized size = 0.43

$$x + \frac{x}{4(x^4 + 1)} + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} - \frac{1}{2}i\right)\right) \left(-\frac{5}{16} - \frac{5}{16}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(-\frac{5}{16} + \frac{5}{16}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(x^8/(2*x^4 + x^8 + 1), x)$

[Out] $x - 2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 - 1i/2))*(5/16 + 5i/16) - 2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 + 1i/2))*(5/16 - 5i/16) + x/(4*(x^4 + 1))$

sympy [A] time = 0.18, size = 90, normalized size = 0.87

$$x + \frac{x}{4x^4 + 4} + \frac{5\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{32} - \frac{5\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{32} - \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{16} - \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(x^{**8}/(x^{**8}+2*x^{**4}+1), x)$

[Out] $x + x/(4*x^{**4} + 4) + 5*\sqrt{2}*\log(x^{**2} - \sqrt{2}*x + 1)/32 - 5*\sqrt{2}*\log(x^{**2} + \sqrt{2}*x + 1)/32 - 5*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x - 1)/16 - 5*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x + 1)/16$

$$3.227 \quad \int \frac{x^6}{1+2x^4+x^8} dx$$

Optimal. Leaf size=99

$$\frac{3 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{3 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{x^3}{4(x^4 + 1)} - \frac{3 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {28, 288, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^3}{4(x^4 + 1)} + \frac{3 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{3 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{3 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(1 + 2*x^4 + x^8), x]

[Out] -x^3/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (3*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && ! LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1+2x^4+x^8} dx &= \int \frac{x^6}{(1+x^4)^2} dx \\
&= -\frac{x^3}{4(1+x^4)} + \frac{3}{4} \int \frac{x^2}{1+x^4} dx \\
&= -\frac{x^3}{4(1+x^4)} - \frac{3}{8} \int \frac{1-x^2}{1+x^4} dx + \frac{3}{8} \int \frac{1+x^2}{1+x^4} dx \\
&= -\frac{x^3}{4(1+x^4)} + \frac{3}{16} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{3}{16} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{3}{16\sqrt{2}} \int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx + \frac{3}{16\sqrt{2}} \int \frac{\sqrt{2}-2x}{-1-\sqrt{2}x-x^2} dx \\
&= -\frac{x^3}{4(1+x^4)} + \frac{3 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{3 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} - \frac{3 \operatorname{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt{2}x\right)}{8\sqrt{2}} \\
&= -\frac{x^3}{4(1+x^4)} - \frac{3 \tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} + \frac{3 \tan^{-1}(1+\sqrt{2}x)}{8\sqrt{2}} + \frac{3 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{3 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.94

$$\frac{1}{32} \left(3\sqrt{2} \log(x^2 - \sqrt{2}x + 1) - 3\sqrt{2} \log(x^2 + \sqrt{2}x + 1) - \frac{8x^3}{x^4 + 1} - 6\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) + 6\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(1 + 2*x^4 + x^8),x]

[Out] ((-8*x^3)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 3*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/32

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1+2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(1 + 2*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^6/(1 + 2*x^4 + x^8), x]

fricas [A] time = 1.37, size = 129, normalized size = 1.30

$$\frac{8x^3 + 12\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1\right) + 12\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1\right) + 3\sqrt{2}(x^4+1)\log(x^2 + \sqrt{2}x + 1) - 3\sqrt{2}(x^4+1)\log(x^2 - \sqrt{2}x + 1)}{32(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] $-1/32*(8*x^3 + 12*\sqrt{2}*(x^4 + 1)*\arctan(-\sqrt{2}*x + \sqrt{2}*\sqrt{x^2 + \sqrt{2}*x + 1} - 1) + 12*\sqrt{2}*(x^4 + 1)*\arctan(-\sqrt{2}*x + \sqrt{2}*\sqrt{x^2 - \sqrt{2}*x + 1} + 1) + 3*\sqrt{2}*(x^4 + 1)*\log(x^2 + \sqrt{2}*x + 1) - 3*\sqrt{2}*(x^4 + 1)*\log(x^2 - \sqrt{2}*x + 1))/(x^4 + 1)$

giac [A] time = 0.29, size = 84, normalized size = 0.85

$$-\frac{x^3}{4(x^4+1)} + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{3}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{3}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="giac")

[Out] $-1/4*x^3/(x^4 + 1) + 3/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})) + 3/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})) - 3/32*\sqrt{2}*\log(x^2 + \sqrt{2}*x + 1) + 3/32*\sqrt{2}*\log(x^2 - \sqrt{2}*x + 1)$

maple [A] time = 0.01, size = 70, normalized size = 0.71

$$-\frac{x^3}{4(x^4+1)} + \frac{3\sqrt{2}\arctan(\sqrt{2}x-1)}{16} + \frac{3\sqrt{2}\arctan(\sqrt{2}x+1)}{16} + \frac{3\sqrt{2}\ln\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^8+2*x^4+1),x)

[Out] $-1/4*x^3/(x^4+1) + 3/16*2^{(1/2)}*\arctan(2^{(1/2)}*x-1) + 3/32*2^{(1/2)}*\ln((x^2-2^{(1/2)}*x+1)/(x^2+2^{(1/2)}*x+1)) + 3/16*2^{(1/2)}*\arctan(2^{(1/2)}*x+1)$

maxima [A] time = 2.07, size = 84, normalized size = 0.85

$$-\frac{x^3}{4(x^4+1)} + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{3}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{3}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] $-1/4*x^3/(x^4 + 1) + 3/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})) + 3/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})) - 3/32*\sqrt{2}*\log(x^2 + \sqrt{2}*x + 1) + 3/32*\sqrt{2}*\log(x^2 - \sqrt{2}*x + 1)$

mupad [B] time = 1.33, size = 47, normalized size = 0.47

$$-\frac{x^3}{4(x^4 + 1)} + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} - \frac{1}{2}i\right)\right) \left(\frac{3}{16} - \frac{3}{16}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(\frac{3}{16} + \frac{3}{16}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(2*x^4 + x^8 + 1),x)`

[Out] $2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 - 1i/2))*(3/16 - 3i/16) + 2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 + 1i/2))*(3/16 + 3i/16) - x^3/(4*(x^4 + 1))$

sympy [A] time = 0.18, size = 90, normalized size = 0.91

$$-\frac{x^3}{4x^4 + 4} + \frac{3\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{32} - \frac{3\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{32} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{16} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(x**8+2*x**4+1),x)`

[Out] $-x**3/(4*x**4 + 4) + 3*\sqrt{2}*\log(x**2 - \sqrt{2}*x + 1)/32 - 3*\sqrt{2}*\log(x**2 + \sqrt{2}*x + 1)/32 + 3*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x - 1)/16 + 3*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x + 1)/16$

$$3.228 \quad \int \frac{x^4}{1+2x^4+x^8} dx$$

Optimal. Leaf size=97

$$\frac{x}{4(x^4+1)} - \frac{\log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{\log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {28, 288, 211, 1165, 628, 1162, 617, 204}

$$\frac{x}{4(x^4+1)} - \frac{\log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{\log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(1 + 2*x^4 + x^8),x]

[Out] -x/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(16*Sqrt[2])

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_.) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{1+2x^4+x^8} dx &= \int \frac{x^4}{(1+x^4)^2} dx \\
&= -\frac{x}{4(1+x^4)} + \frac{1}{4} \int \frac{1}{1+x^4} dx \\
&= -\frac{x}{4(1+x^4)} + \frac{1}{8} \int \frac{1-x^2}{1+x^4} dx + \frac{1}{8} \int \frac{1+x^2}{1+x^4} dx \\
&= -\frac{x}{4(1+x^4)} + \frac{1}{16} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{16} \int \frac{1}{1+\sqrt{2}x+x^2} dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{16\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{16\sqrt{2}} \\
&= -\frac{x}{4(1+x^4)} - \frac{\log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{\log(1+\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} \\
&= -\frac{x}{4(1+x^4)} - \frac{\tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{8\sqrt{2}} - \frac{\log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{\log(1+\sqrt{2}x+x^2)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 90, normalized size = 0.93

$$\frac{1}{32} \left(-\frac{8x}{x^4+1} - \sqrt{2} \log(x^2 - \sqrt{2}x + 1) + \sqrt{2} \log(x^2 + \sqrt{2}x + 1) - 2\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) + 2\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 + 2*x^4 + x^8), x]

[Out] ((-8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/32

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1+2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(1 + 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^4/(1 + 2*x^4 + x^8), x]

fricas [A] time = 1.30, size = 126, normalized size = 1.30

$$\frac{4\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+4\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\sqrt{2}(x^4+1)\log(x^2+\sqrt{2}x+1)+\sqrt{2}(x^4+1)\log(x^2-\sqrt{2}x+1)+8x}{32(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] $-1/32*(4*\sqrt{2}*(x^4+1)*\arctan(-\sqrt{2}*x+\sqrt{2}*\sqrt{x^2+\sqrt{2}*x+1})-1)+4*\sqrt{2}*(x^4+1)*\arctan(-\sqrt{2}*x+\sqrt{2}*\sqrt{x^2-\sqrt{2}*x+1})-\sqrt{2}*(x^4+1)*\log(x^2+\sqrt{2}*x+1)+\sqrt{2}*(x^4+1)*\log(x^2-\sqrt{2}*x+1)+8*x)/(x^4+1)$

giac [A] time = 0.36, size = 82, normalized size = 0.85

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)+\frac{1}{32}\sqrt{2}\log(x^2+\sqrt{2}x+1)-\frac{1}{32}\sqrt{2}\log(x^2-\sqrt{2}x+1)-\frac{x}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="giac")

[Out] $1/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x+\sqrt{2})) + 1/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x-\sqrt{2})) + 1/32*\sqrt{2}*\log(x^2+\sqrt{2}*x+1) - 1/32*\sqrt{2}*\log(x^2-\sqrt{2}*x+1) - 1/4*x/(x^4+1)$

maple [A] time = 0.01, size = 68, normalized size = 0.70

$$-\frac{x}{4(x^4+1)} + \frac{\sqrt{2}\arctan(\sqrt{2}x-1)}{16} + \frac{\sqrt{2}\arctan(\sqrt{2}x+1)}{16} + \frac{\sqrt{2}\ln\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^8+2*x^4+1),x)

[Out] $-1/4/(x^4+1)*x+1/32*2^{(1/2)}*\ln((x^2+2^{(1/2)}*x+1)/(x^2-2^{(1/2)}*x+1))+1/16*2^{(1/2)}*\arctan(2^{(1/2)}*x-1)+1/16*2^{(1/2)}*\arctan(2^{(1/2)}*x+1)$

maxima [A] time = 1.99, size = 82, normalized size = 0.85

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)+\frac{1}{32}\sqrt{2}\log(x^2+\sqrt{2}x+1)-\frac{1}{32}\sqrt{2}\log(x^2-\sqrt{2}x+1)-\frac{x}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] $1/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})) + 1/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})) + 1/32*\sqrt{2}*\log(x^2 + \sqrt{2}*x + 1) - 1/32*\sqrt{2}*\log(x^2 - \sqrt{2}*x + 1) - 1/4*x/(x^4 + 1)$

mupad [B] time = 0.08, size = 45, normalized size = 0.46

$$-\frac{x}{4(x^4+1)} + \sqrt{2} \operatorname{atan}\left(\sqrt{2}x\left(\frac{1}{2} - \frac{1}{2}i\right)\right)\left(\frac{1}{16} + \frac{1}{16}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2}x\left(\frac{1}{2} + \frac{1}{2}i\right)\right)\left(\frac{1}{16} - \frac{1}{16}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(2*x^4 + x^8 + 1),x)`

[Out] $2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 - 1i/2))*(1/16 + 1i/16) + 2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 + 1i/2))*(1/16 - 1i/16) - x/(4*(x^4 + 1))$

sympy [A] time = 0.17, size = 82, normalized size = 0.85

$$-\frac{x}{4x^4+4} - \frac{\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{32} + \frac{\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{32} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{16} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(x**8+2*x**4+1),x)`

[Out] $-x/(4*x**4 + 4) - \sqrt{2}*\log(x**2 - \sqrt{2}*x + 1)/32 + \sqrt{2}*\log(x**2 + \sqrt{2}*x + 1)/32 + \sqrt{2}*\operatorname{atan}(\sqrt{2}*x - 1)/16 + \sqrt{2}*\operatorname{atan}(\sqrt{2}*x + 1)/16$

$$3.229 \quad \int \frac{x^2}{1+2x^4+x^8} dx$$

Optimal. Leaf size=99

$$\frac{\log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{\log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{x^3}{4(x^4 + 1)} - \frac{\tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {28, 290, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^3}{4(x^4 + 1)} + \frac{\log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{\log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + 2*x^4 + x^8),x]

[Out] x^3/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(8*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(16*Sqrt[2])

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+2x^4+x^8} dx &= \int \frac{x^2}{(1+x^4)^2} dx \\
&= \frac{x^3}{4(1+x^4)} + \frac{1}{4} \int \frac{x^2}{1+x^4} dx \\
&= \frac{x^3}{4(1+x^4)} - \frac{1}{8} \int \frac{1-x^2}{1+x^4} dx + \frac{1}{8} \int \frac{1+x^2}{1+x^4} dx \\
&= \frac{x^3}{4(1+x^4)} + \frac{1}{16} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{16} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{16\sqrt{2}} + \frac{\int \frac{\sqrt{2}-2x}{-1+x^2} dx}{16\sqrt{2}} \\
&= \frac{x^3}{4(1+x^4)} + \frac{\log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} \\
&= \frac{x^3}{4(1+x^4)} - \frac{\tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{8\sqrt{2}} + \frac{\log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 92, normalized size = 0.93

$$\frac{1}{32} \left(\sqrt{2} \log(x^2 - \sqrt{2}x + 1) - \sqrt{2} \log(x^2 + \sqrt{2}x + 1) + \frac{8x^3}{x^4 + 1} - 2\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) + 2\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + 2*x^4 + x^8), x]

[Out] ((8*x^3)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/32

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1+2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1 + 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^2/(1 + 2*x^4 + x^8), x]

fricas [A] time = 1.11, size = 128, normalized size = 1.29

$$\frac{8x^3 - 4\sqrt{2}(x^4 + 1)\arctan(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1) - 4\sqrt{2}(x^4 + 1)\arctan(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1) - \sqrt{2}(x^4 + 1)\log(x^2 + \sqrt{2}x + 1) + \sqrt{2}(x^4 + 1)\log(x^2 - \sqrt{2}x + 1)}{32(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] 1/32*(8*x^3 - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) + sqrt(2)*(x^4 + 1)*log(x^2 - sqrt(2)*x + 1))/(x^4 + 1)

giac [A] time = 0.35, size = 84, normalized size = 0.85

$$\frac{x^3}{4(x^4 + 1)} + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{1}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{1}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="giac")

[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)

maple [A] time = 0.01, size = 70, normalized size = 0.71

$$\frac{x^3}{4x^4 + 4} + \frac{\sqrt{2}\arctan(\sqrt{2}x - 1)}{16} + \frac{\sqrt{2}\arctan(\sqrt{2}x + 1)}{16} + \frac{\sqrt{2}\ln\left(\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^8+2*x^4+1),x)

[Out] 1/4/(x^4+1)*x^3+1/16*2^(1/2)*arctan(2^(1/2)*x-1)+1/32*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)*x+1))+1/16*2^(1/2)*arctan(2^(1/2)*x+1)

maxima [A] time = 2.06, size = 84, normalized size = 0.85

$$\frac{x^3}{4(x^4 + 1)} + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{1}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{1}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] $\frac{1}{4}x^3/(x^4 + 1) + \frac{1}{16}\sqrt{2}\arctan(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})) + \frac{1}{16}\sqrt{2}\arctan(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})) - \frac{1}{32}\sqrt{2}\log(x^2 + \sqrt{2}(2)x + 1) + \frac{1}{32}\sqrt{2}\log(x^2 - \sqrt{2}(2)x + 1)$

mupad [B] time = 0.05, size = 46, normalized size = 0.46

$$\frac{x^3}{4(x^4 + 1)} + \sqrt{2} \operatorname{atan}\left(\sqrt{2}x\left(\frac{1}{2} - \frac{1}{2}i\right)\right)\left(\frac{1}{16} - \frac{1}{16}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2}x\left(\frac{1}{2} + \frac{1}{2}i\right)\right)\left(\frac{1}{16} + \frac{1}{16}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(2*x^4 + x^8 + 1),x)`

[Out] $2^{(1/2)}\operatorname{atan}(2^{(1/2)}x(1/2 - 1i/2))*(1/16 - 1i/16) + 2^{(1/2)}\operatorname{atan}(2^{(1/2)}x(1/2 + 1i/2))*(1/16 + 1i/16) + x^3/(4*(x^4 + 1))$

sympy [A] time = 0.17, size = 83, normalized size = 0.84

$$\frac{x^3}{4x^4 + 4} + \frac{\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{32} - \frac{\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{32} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{16} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(x**8+2*x**4+1),x)`

[Out] $x**3/(4*x**4 + 4) + \sqrt{2}\log(x**2 - \sqrt{2}*x + 1)/32 - \sqrt{2}\log(x**2 + \sqrt{2}*x + 1)/32 + \sqrt{2}\operatorname{atan}(\sqrt{2}*x - 1)/16 + \sqrt{2}\operatorname{atan}(\sqrt{2}*x + 1)/16$

$$3.230 \quad \int \frac{1}{1+2x^4+x^8} dx$$

Optimal. Leaf size=97

$$\frac{x}{4(x^4+1)} - \frac{3 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{3 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{3 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {28, 199, 211, 1165, 628, 1162, 617, 204}

$$\frac{x}{4(x^4+1)} - \frac{3 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{3 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{3 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^4 + x^8)^(-1), x]

[Out] x/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (3*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 199

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+2x^4+x^8} dx &= \int \frac{1}{(1+x^4)^2} dx \\
&= \frac{x}{4(1+x^4)} + \frac{3}{4} \int \frac{1}{1+x^4} dx \\
&= \frac{x}{4(1+x^4)} + \frac{3}{8} \int \frac{1-x^2}{1+x^4} dx + \frac{3}{8} \int \frac{1+x^2}{1+x^4} dx \\
&= \frac{x}{4(1+x^4)} + \frac{3}{16} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{3}{16} \int \frac{1}{1+\sqrt{2}x+x^2} dx - \frac{3 \int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{16\sqrt{2}} - \frac{3 \int \frac{1}{-1-x^2} dx}{16\sqrt{2}} \\
&= \frac{x}{4(1+x^4)} - \frac{3 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{3 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{3 \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}x\right)}{8\sqrt{2}} \\
&= \frac{x}{4(1+x^4)} - \frac{3 \tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} + \frac{3 \tan^{-1}(1+\sqrt{2}x)}{8\sqrt{2}} - \frac{3 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{3 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.94

$$\frac{1}{32} \left(\frac{8x}{x^4+1} - 3\sqrt{2} \log(x^2 - \sqrt{2}x + 1) + 3\sqrt{2} \log(x^2 + \sqrt{2}x + 1) - 6\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) + 6\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^4 + x^8)^(-1), x]

[Out] ((8*x)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/32

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2*x^4 + x^8)^(-1), x]

[Out] IntegrateAlgebraic[(1 + 2*x^4 + x^8)^(-1), x]

fricas [A] time = 1.13, size = 127, normalized size = 1.31

$$\frac{12\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+12\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}\right)-3\sqrt{2}(x^4+1)\log(x^2+\sqrt{2}x+1)+3\sqrt{2}(x^4+1)\log(x^2-\sqrt{2}x+1)-8x}{32(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] $-1/32*(12*\sqrt{2}*(x^4+1)*\arctan(-\sqrt{2}*x+\sqrt{2}*\sqrt{x^2+\sqrt{2}*x+1})-1)+12*\sqrt{2}*(x^4+1)*\arctan(-\sqrt{2}*x+\sqrt{2}*\sqrt{x^2-\sqrt{2}*x+1})-3*\sqrt{2}*(x^4+1)*\log(x^2+\sqrt{2}*x+1)+3*\sqrt{2}*(x^4+1)*\log(x^2-\sqrt{2}*x+1)-8*x)/(x^4+1)$

giac [A] time = 0.34, size = 82, normalized size = 0.85

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)+\frac{3}{32}\sqrt{2}\log(x^2+\sqrt{2}x+1)-\frac{3}{32}\sqrt{2}\log(x^2-\sqrt{2}x+1)+\frac{x}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+2*x^4+1),x, algorithm="giac")

[Out] $3/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x+\sqrt{2})))+3/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x-\sqrt{2})))+3/32*\sqrt{2}*\log(x^2+\sqrt{2}*x+1)-3/32*\sqrt{2}*\log(x^2-\sqrt{2}*x+1)+1/4*x/(x^4+1)$

maple [A] time = 0.00, size = 68, normalized size = 0.70

$$\frac{x}{4x^4+4}+\frac{3\sqrt{2}\arctan(\sqrt{2}x-1)}{16}+\frac{3\sqrt{2}\arctan(\sqrt{2}x+1)}{16}+\frac{3\sqrt{2}\ln\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8+2*x^4+1),x)

[Out] $1/4/(x^4+1)*x+3/16*2^{(1/2)}*\arctan(2^{(1/2)}*x-1)+3/32*2^{(1/2)}*\ln((x^2+2^{(1/2)}*x+1)/(x^2-2^{(1/2)}*x+1))+3/16*2^{(1/2)}*\arctan(2^{(1/2)}*x+1)$

maxima [A] time = 2.00, size = 82, normalized size = 0.85

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)+\frac{3}{32}\sqrt{2}\log(x^2+\sqrt{2}x+1)-\frac{3}{32}\sqrt{2}\log(x^2-\sqrt{2}x+1)+\frac{x}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] $3/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})) + 3/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})) + 3/32*\sqrt{2}*\log(x^2 + \sqrt{2}*x + 1) - 3/32*\sqrt{2}*\log(x^2 - \sqrt{2}*x + 1) + 1/4*x/(x^4 + 1)$

mupad [B] time = 1.31, size = 44, normalized size = 0.45

$$\frac{x}{4(x^4 + 1)} + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} - \frac{1}{2}i\right)\right) \left(\frac{3}{16} + \frac{3}{16}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(\frac{3}{16} - \frac{3}{16}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^4 + x^8 + 1),x)`

[Out] $2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 - 1i/2))*(3/16 + 3i/16) + 2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 + 1i/2))*(3/16 - 3i/16) + x/(4*(x^4 + 1))$

sympy [A] time = 0.20, size = 88, normalized size = 0.91

$$\frac{x}{4x^4 + 4} - \frac{3\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{32} + \frac{3\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{32} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{16} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**8+2*x**4+1),x)`

[Out] $x/(4*x**4 + 4) - 3*\sqrt{2}*\log(x**2 - \sqrt{2}*x + 1)/32 + 3*\sqrt{2}*\log(x**2 + \sqrt{2}*x + 1)/32 + 3*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x - 1)/16 + 3*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x + 1)/16$

$$3.231 \quad \int \frac{1}{x^2(1+2x^4+x^8)} dx$$

Optimal. Leaf size=106

$$\frac{1}{4x(x^4+1)} - \frac{5 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{5 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{5}{4x} + \frac{5 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} - \frac{5 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{4x(x^4+1)} - \frac{5 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{5 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{5}{4x} + \frac{5 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} - \frac{5 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 + 2*x^4 + x^8)),x]

[Out] -5/(4*x) + 1/(4*x*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> -Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 297

```
Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(1+2x^4+x^8)} dx &= \int \frac{1}{x^2(1+x^4)^2} dx \\
&= \frac{1}{4x(1+x^4)} + \frac{5}{4} \int \frac{1}{x^2(1+x^4)} dx \\
&= -\frac{5}{4x} + \frac{1}{4x(1+x^4)} - \frac{5}{4} \int \frac{x^2}{1+x^4} dx \\
&= -\frac{5}{4x} + \frac{1}{4x(1+x^4)} + \frac{5}{8} \int \frac{1-x^2}{1+x^4} dx - \frac{5}{8} \int \frac{1+x^2}{1+x^4} dx \\
&= -\frac{5}{4x} + \frac{1}{4x(1+x^4)} - \frac{5}{16} \int \frac{1}{1-\sqrt{2}x+x^2} dx - \frac{5}{16} \int \frac{1}{1+\sqrt{2}x+x^2} dx - \frac{5 \int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x}}{16\sqrt{2}} \\
&= -\frac{5}{4x} + \frac{1}{4x(1+x^4)} - \frac{5 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{5 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{5 \text{Subst}\left(\int \frac{1}{-1-\sqrt{2}x}\right)}{8} \\
&= -\frac{5}{4x} + \frac{1}{4x(1+x^4)} + \frac{5 \tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} - \frac{5 \tan^{-1}(1+\sqrt{2}x)}{8\sqrt{2}} - \frac{5 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 98, normalized size = 0.92

$$\frac{1}{32} \left(-5\sqrt{2} \log(x^2 - \sqrt{2}x + 1) + 5\sqrt{2} \log(x^2 + \sqrt{2}x + 1) - \frac{8x^3}{x^4 + 1} - \frac{32}{x} + 10\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) - 10\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(1 + 2*x^4 + x^8)),x]

[Out] (-32/x - (8*x^3)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 5*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/32

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1+2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(1 + 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^2*(1 + 2*x^4 + x^8)), x]

fricas [A] time = 1.28, size = 130, normalized size = 1.23

$$\frac{40x^4 - 20\sqrt{2}(x^5 + x)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1\right) - 20\sqrt{2}(x^5 + x)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1\right) - 5\sqrt{2}(x^5 + x)\log(x^2 + \sqrt{2}x + 1) + 5\sqrt{2}(x^5 + x)\log(x^2 - \sqrt{2}x + 1) + 32}{32(x^5 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] -1/32*(40*x^4 - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^5 + x)*log(x^2 + sqrt(2)*x + 1) + 5*sqrt(2)*(x^5 + x)*log(x^2 - sqrt(2)*x + 1) + 32)/(x^5 + x)

giac [A] time = 0.36, size = 88, normalized size = 0.83

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) + \frac{5}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) - \frac{5}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) - \frac{5x^4 + 4}{4(x^5 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="giac")

[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)

maple [A] time = 0.01, size = 75, normalized size = 0.71

$$\frac{x^3}{4(x^4 + 1)} - \frac{5\sqrt{2}\arctan(\sqrt{2}x - 1)}{16} - \frac{5\sqrt{2}\arctan(\sqrt{2}x + 1)}{16} - \frac{5\sqrt{2}\ln\left(\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right)}{32} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(x^8+2*x^4+1),x)

[Out] -1/x - 1/4/(x^4+1)*x^3 - 5/16*2^(1/2)*arctan(2^(1/2)*x-1) - 5/32*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)*x+1)) - 5/16*2^(1/2)*arctan(2^(1/2)*x+1)

maxima [A] time = 2.03, size = 88, normalized size = 0.83

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) - \frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) + \frac{5}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) - \frac{5}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) - \frac{5x^4 + 4}{4(x^5 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] $-5/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})) - 5/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})) + 5/32*\sqrt{2}*\log(x^2 + \sqrt{2}*x + 1) - 5/32*\sqrt{2}*\log(x^2 - \sqrt{2}*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)$

mupad [B] time = 1.31, size = 49, normalized size = 0.46

$$-\frac{\frac{5x^4}{4} + 1}{x^5 + x} + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} - \frac{1}{2}i\right)\right) \left(-\frac{5}{16} + \frac{5}{16}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(-\frac{5}{16} - \frac{5}{16}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(2*x^4 + x^8 + 1)),x)

[Out] $-((5*x^4)/4 + 1)/(x + x^5) - 2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 - 1i/2))*(5/16 - 5i/16) - 2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 + 1i/2))*(5/16 + 5i/16)$

sympy [A] time = 0.21, size = 97, normalized size = 0.92

$$\frac{-5x^4 - 4}{4x^5 + 4x} - \frac{5\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{32} + \frac{5\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{32} - \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{16} - \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(x**8+2*x**4+1),x)

[Out] $(-5*x**4 - 4)/(4*x**5 + 4*x) - 5*\sqrt{2}*\log(x**2 - \sqrt{2}*x + 1)/32 + 5*\sqrt{2}*\log(x**2 + \sqrt{2}*x + 1)/32 - 5*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x - 1)/16 - 5*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x + 1)/16$

$$3.232 \quad \int \frac{1}{x^4(1+2x^4+x^8)} dx$$

Optimal. Leaf size=106

$$-\frac{7}{12x^3} + \frac{7 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{7 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{1}{4x^3(x^4 + 1)} + \frac{7 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} - \frac{7 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 290, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4x^3(x^4 + 1)} - \frac{7}{12x^3} + \frac{7 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{7 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{7 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} - \frac{7 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 + 2*x^4 + x^8)),x]

[Out] -7/(12*x^3) + 1/(4*x^3*(1 + x^4)) + (7*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (7*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (7*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(1+2x^4+x^8)} dx &= \int \frac{1}{x^4(1+x^4)^2} dx \\
&= \frac{1}{4x^3(1+x^4)} + \frac{7}{4} \int \frac{1}{x^4(1+x^4)} dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} - \frac{7}{4} \int \frac{1}{1+x^4} dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} - \frac{7}{8} \int \frac{1-x^2}{1+x^4} dx - \frac{7}{8} \int \frac{1+x^2}{1+x^4} dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} - \frac{7}{16} \int \frac{1}{1-\sqrt{2}x+x^2} dx - \frac{7}{16} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{7}{16\sqrt{2}} \int \frac{\sqrt{2}x+1}{-1-\sqrt{2}x+x^2} dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} + \frac{7 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{7 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{7 \operatorname{Subst}\left(\int \frac{\sqrt{2}x+1}{-1-\sqrt{2}x+x^2} dx\right)}{16\sqrt{2}} \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} + \frac{7 \tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} - \frac{7 \tan^{-1}(1+\sqrt{2}x)}{8\sqrt{2}} + \frac{7 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 96, normalized size = 0.91

$$\frac{1}{96} \left(-\frac{24x}{x^4+1} - \frac{32}{x^3} + 21\sqrt{2} \log(x^2 - \sqrt{2}x + 1) - 21\sqrt{2} \log(x^2 + \sqrt{2}x + 1) + 42\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) - 42\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(1 + 2*x^4 + x^8)),x]

[Out] (-32/x^3 - (24*x)/(1 + x^4) + 42*sqrt(2)*ArcTan[1 - sqrt(2)*x] - 42*sqrt(2)*ArcTan[1 + sqrt(2)*x] + 21*sqrt(2)*Log[1 - sqrt(2)*x + x^2] - 21*sqrt(2)*Log[1 + sqrt(2)*x + x^2])/96

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(1+2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(1 + 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^4*(1 + 2*x^4 + x^8)), x]

fricas [A] time = 1.31, size = 140, normalized size = 1.32

$$\frac{56x^4 - 84\sqrt{2}(x^7 + x^3)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1\right) - 84\sqrt{2}(x^7 + x^3)\arctan\left(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1\right) + 21\sqrt{2}(x^7 + x^3)\log(x^2 + \sqrt{2}x + 1) - 21\sqrt{2}(x^7 + x^3)\log(x^2 - \sqrt{2}x + 1) + 32}{96(x^7 + x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] $-1/96*(56*x^4 - 84*\sqrt{2}*(x^7 + x^3)*\arctan(-\sqrt{2}*x + \sqrt{2}*\sqrt{x^2 + \sqrt{2}*x + 1} - 1) - 84*\sqrt{2}*(x^7 + x^3)*\arctan(-\sqrt{2}*x + \sqrt{2}*\sqrt{x^2 - \sqrt{2}*x + 1} + 1) + 21*\sqrt{2}*(x^7 + x^3)*\log(x^2 + \sqrt{2}*x + 1) - 21*\sqrt{2}*(x^7 + x^3)*\log(x^2 - \sqrt{2}*x + 1) + 32)/(x^7 + x^3)$

giac [A] time = 0.44, size = 87, normalized size = 0.82

$$-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) - \frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{7}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{7}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) - \frac{x}{4(x^4 + 1)} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="giac")

[Out] $-7/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})) - 7/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})) - 7/32*\sqrt{2}*\log(x^2 + \sqrt{2}*x + 1) + 7/32*\sqrt{2}*\log(x^2 - \sqrt{2}*x + 1) - 1/4*x/(x^4 + 1) - 1/3/x^3$

maple [A] time = 0.01, size = 73, normalized size = 0.69

$$-\frac{x}{4(x^4 + 1)} - \frac{7\sqrt{2}\arctan(\sqrt{2}x - 1)}{16} - \frac{7\sqrt{2}\arctan(\sqrt{2}x + 1)}{16} - \frac{7\sqrt{2}\ln\left(\frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1}\right)}{32} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(x^8+2*x^4+1),x)

[Out] $-1/3/x^3 - 1/4/(x^4 + 1) * x - 7/32 * 2^{(1/2)} * \ln((x^2 + 2^{(1/2)} * x + 1)/(x^2 - 2^{(1/2)} * x + 1)) - 7/16 * 2^{(1/2)} * \arctan(2^{(1/2)} * x + 1) - 7/16 * 2^{(1/2)} * \arctan(2^{(1/2)} * x - 1)$

maxima [A] time = 2.20, size = 90, normalized size = 0.85

$$-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) - \frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{7}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{7}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) - \frac{7x^4 + 4}{12(x^7 + x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] $-7/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2})) - 7/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2})) - 7/32*\sqrt{2}*\log(x^2 + \sqrt{2}*x + 1) + 7/32*\sqrt{2}*\log(x^2 - \sqrt{2}*x + 1) - 1/12*(7*x^4 + 4)/(x^7 + x^3)$

mupad [B] time = 1.36, size = 51, normalized size = 0.48

$$-\frac{\frac{7x^4}{12} + \frac{1}{3}}{x^7 + x^3} + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} - \frac{1}{2}i\right)\right) \left(-\frac{7}{16} - \frac{7}{16}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(-\frac{7}{16} + \frac{7}{16}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(2*x^4 + x^8 + 1)),x)

[Out] $-2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 - 1i/2))*(7/16 + 7i/16) - 2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*x*(1/2 + 1i/2))*(7/16 - 7i/16) - ((7*x^4)/12 + 1/3)/(x^3 + x^7)$

sympy [A] time = 0.22, size = 99, normalized size = 0.93

$$\frac{-7x^4 - 4}{12x^7 + 12x^3} + \frac{7\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{32} - \frac{7\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{32} - \frac{7\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{16} - \frac{7\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(x**8+2*x**4+1),x)

[Out] $(-7*x**4 - 4)/(12*x**7 + 12*x**3) + 7*\sqrt{2}*\log(x**2 - \sqrt{2}*x + 1)/32 - 7*\sqrt{2}*\log(x**2 + \sqrt{2}*x + 1)/32 - 7*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x - 1)/16 - 7*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x + 1)/16$

$$3.233 \quad \int \frac{1}{x^6(1+2x^4+x^8)} dx$$

Optimal. Leaf size=113

$$-\frac{9}{20x^5} + \frac{9 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{9 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{1}{4x^5(x^4 + 1)} + \frac{9}{4x} - \frac{9 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{9 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{4x^5(x^4 + 1)} - \frac{9}{20x^5} + \frac{9 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{9 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{9}{4x} - \frac{9 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{9 \tan^{-1}(\sqrt{2}x + 1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(1 + 2*x^4 + x^8)),x]

[Out] -9/(20*x^5) + 9/(4*x) + 1/(4*x^5*(1 + x^4)) - (9*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (9*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (9*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> -Simp[(((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 297

```
Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6(1+2x^4+x^8)} dx &= \int \frac{1}{x^6(1+x^4)^2} dx \\
&= \frac{1}{4x^5(1+x^4)} + \frac{9}{4} \int \frac{1}{x^6(1+x^4)} dx \\
&= -\frac{9}{20x^5} + \frac{1}{4x^5(1+x^4)} - \frac{9}{4} \int \frac{1}{x^2(1+x^4)} dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} + \frac{9}{4} \int \frac{x^2}{1+x^4} dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} - \frac{9}{8} \int \frac{1-x^2}{1+x^4} dx + \frac{9}{8} \int \frac{1+x^2}{1+x^4} dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} + \frac{9}{16} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{9}{16} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{9}{16} \int \frac{1}{1+x^2} dx \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} + \frac{9 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} - \frac{9 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{9 \operatorname{Subst}(\log(x^2+1), x, \sqrt{2}x)}{16\sqrt{2}} \\
&= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5(1+x^4)} - \frac{9 \tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} + \frac{9 \tan^{-1}(1+\sqrt{2}x)}{8\sqrt{2}} + \frac{9 \log(1+x^2)}{16\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 103, normalized size = 0.91

$$\frac{1}{160} \left(-\frac{32}{x^5} + 45\sqrt{2} \log(x^2 - \sqrt{2}x + 1) - 45\sqrt{2} \log(x^2 + \sqrt{2}x + 1) + \frac{40x^3}{x^4+1} + \frac{320}{x} - 90\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) + 90\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(1 + 2*x^4 + x^8)),x]

[Out] (-32/x^5 + 320/x + (40*x^3)/(1 + x^4) - 90*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 90*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 45*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 45*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/160

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6(1+2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^6*(1 + 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^6*(1 + 2*x^4 + x^8)), x]

fricas [A] time = 1.28, size = 145, normalized size = 1.28

$$\frac{360x^8 + 288x^4 - 180\sqrt{2}(x^9 + x^5)\arctan(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1) - 180\sqrt{2}(x^9 + x^5)\arctan(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1) - 45\sqrt{2}(x^9 + x^5)\log(x^2 + \sqrt{2}x + 1) + 45\sqrt{2}(x^9 + x^5)\log(x^2 - \sqrt{2}x + 1) - 32}{160(x^9 + x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] 1/160*(360*x^8 + 288*x^4 - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 45*sqrt(2)*(x^9 + x^5)*log(x^2 + sqrt(2)*x + 1) + 45*sqrt(2)*(x^9 + x^5)*log(x^2 - sqrt(2)*x + 1) - 32)/(x^9 + x^5)

giac [A] time = 0.30, size = 96, normalized size = 0.85

$$\frac{x^3}{4(x^4 + 1)} + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{9}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{9}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) + \frac{10x^4 - 1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="giac")

[Out] 1/4*x^3/(x^4 + 1) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 9/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/5*(10*x^4 - 1)/x^5

maple [A] time = 0.01, size = 80, normalized size = 0.71

$$\frac{x^3}{4x^4 + 4} + \frac{9\sqrt{2}\arctan(\sqrt{2}x - 1)}{16} + \frac{9\sqrt{2}\arctan(\sqrt{2}x + 1)}{16} + \frac{9\sqrt{2}\ln\left(\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right)}{32} + \frac{2}{x} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(x^8+2*x^4+1),x)

[Out] -1/5/x^5+2/x+1/4/(x^4+1)*x^3+9/32*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)*x+1))+9/16*2^(1/2)*arctan(2^(1/2)*x+1)+9/16*2^(1/2)*arctan(2^(1/2)*x-1)

maxima [A] time = 2.01, size = 95, normalized size = 0.84

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{9}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{9}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) + \frac{45x^8 + 36x^4 - 4}{20(x^9 + x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] $\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{9}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{9}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) + \frac{1}{20}(45x^8 + 36x^4 - 4)/(x^9 + x^5)$

mupad [B] time = 0.09, size = 55, normalized size = 0.49

$$\frac{\frac{9x^8}{4} + \frac{9x^4}{5} - \frac{1}{5}}{x^9 + x^5} + \sqrt{2} \operatorname{atan}\left(\sqrt{2}x\left(\frac{1}{2} - \frac{1}{2}i\right)\right)\left(\frac{9}{16} - \frac{9}{16}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2}x\left(\frac{1}{2} + \frac{1}{2}i\right)\right)\left(\frac{9}{16} + \frac{9}{16}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(2*x^4 + x^8 + 1)),x)

[Out] $2^{1/2}\operatorname{atan}(2^{1/2}x(1/2 - 1i/2))*(9/16 - 9i/16) + 2^{1/2}\operatorname{atan}(2^{1/2}x(1/2 + 1i/2))*(9/16 + 9i/16) + ((9x^4)/5 + (9x^8)/4 - 1/5)/(x^5 + x^9)$

sympy [A] time = 0.23, size = 102, normalized size = 0.90

$$\frac{9\sqrt{2}\log(x^2 - \sqrt{2}x + 1)}{32} - \frac{9\sqrt{2}\log(x^2 + \sqrt{2}x + 1)}{32} + \frac{9\sqrt{2}\operatorname{atan}(\sqrt{2}x - 1)}{16} + \frac{9\sqrt{2}\operatorname{atan}(\sqrt{2}x + 1)}{16} + \frac{45x^8 + 36x^4 - 4}{20x^9 + 20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**6/(x**8+2*x**4+1),x)

[Out] $9\sqrt{2}\log(x^2 - \sqrt{2}x + 1)/32 - 9\sqrt{2}\log(x^2 + \sqrt{2}x + 1)/32 + 9\sqrt{2}\operatorname{atan}(\sqrt{2}x - 1)/16 + 9\sqrt{2}\operatorname{atan}(\sqrt{2}x + 1)/16 + (45x^8 + 36x^4 - 4)/(20x^9 + 20x^5)$

$$3.234 \quad \int \frac{1}{x^8(1+2x^4+x^8)} dx$$

Optimal. Leaf size=113

$$-\frac{11}{28x^7} + \frac{11}{12x^3} - \frac{11 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{11 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{1}{4x^7(x^4 + 1)} - \frac{11 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{11 \tan^{-1}(\sqrt{2}x)}{8\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 290, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4x^7(x^4 + 1)} + \frac{11}{12x^3} - \frac{11}{28x^7} - \frac{11 \log(x^2 - \sqrt{2}x + 1)}{16\sqrt{2}} + \frac{11 \log(x^2 + \sqrt{2}x + 1)}{16\sqrt{2}} - \frac{11 \tan^{-1}(1 - \sqrt{2}x)}{8\sqrt{2}} + \frac{11 \tan^{-1}(\sqrt{2}x)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^8*(1 + 2*x^4 + x^8)),x]

[Out] -11/(28*x^7) + 11/(12*x^3) + 1/(4*x^7*(1 + x^4)) - (11*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (11*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (11*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (11*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_.) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 290


```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8(1+2x^4+x^8)} dx &= \int \frac{1}{x^8(1+x^4)^2} dx \\
&= \frac{1}{4x^7(1+x^4)} + \frac{11}{4} \int \frac{1}{x^8(1+x^4)} dx \\
&= -\frac{11}{28x^7} + \frac{1}{4x^7(1+x^4)} - \frac{11}{4} \int \frac{1}{x^4(1+x^4)} dx \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7(1+x^4)} + \frac{11}{4} \int \frac{1}{1+x^4} dx \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7(1+x^4)} + \frac{11}{8} \int \frac{1-x^2}{1+x^4} dx + \frac{11}{8} \int \frac{1+x^2}{1+x^4} dx \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7(1+x^4)} + \frac{11}{16} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{11}{16} \int \frac{1}{1+\sqrt{2}x+x^2} dx - \frac{11}{8} \int \frac{1}{1+x^2} dx \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7(1+x^4)} - \frac{11 \log(1-\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{11 \log(1+\sqrt{2}x+x^2)}{16\sqrt{2}} + \frac{11}{8} \arctan(x) \\
&= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7(1+x^4)} - \frac{11 \tan^{-1}(1-\sqrt{2}x)}{8\sqrt{2}} + \frac{11 \tan^{-1}(1+\sqrt{2}x)}{8\sqrt{2}} - \frac{11 \log(1+x^2)}{8}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 101, normalized size = 0.89

$$\frac{1}{672} \left(-\frac{96}{x^7} + \frac{168x}{x^4+1} + \frac{448}{x^3} - 231\sqrt{2} \log(x^2 - \sqrt{2}x + 1) + 231\sqrt{2} \log(x^2 + \sqrt{2}x + 1) - 462\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) + 462\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^8*(1 + 2*x^4 + x^8)),x]

[Out] (-96/x^7 + 448/x^3 + (168*x)/(1 + x^4) - 462*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 462*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 231*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 231*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/672

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8(1+2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^8*(1 + 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^8*(1 + 2*x^4 + x^8)), x]

fricas [A] time = 1.23, size = 145, normalized size = 1.28

$$\frac{616x^8 + 352x^4 - 924\sqrt{2}(x^{11} + x^7)\arctan(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1) - 924\sqrt{2}(x^{11} + x^7)\arctan(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1) + 231\sqrt{2}(x^{11} + x^7)\log(x^2 + \sqrt{2}x + 1) - 231\sqrt{2}(x^{11} + x^7)\log(x^2 - \sqrt{2}x + 1) - 96}{672(x^{11} + x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{672} \cdot (616x^8 + 352x^4 - 924\sqrt{2}(x^{11} + x^7)\arctan(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 + \sqrt{2}x + 1} - 1) - 924\sqrt{2}(x^{11} + x^7)\arctan(-\sqrt{2}x + \sqrt{2}\sqrt{x^2 - \sqrt{2}x + 1} + 1) + 231\sqrt{2}(x^{11} + x^7)\log(x^2 + \sqrt{2}x + 1) - 231\sqrt{2}(x^{11} + x^7)\log(x^2 - \sqrt{2}x + 1) - 96) / (x^{11} + x^7)$

giac [A] time = 0.36, size = 94, normalized size = 0.83

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) + \frac{11}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) - \frac{11}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) + \frac{x}{4(x^4 + 1)} + \frac{14x^4 - 3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="giac")

[Out] $\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) + \frac{11}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) - \frac{11}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) + \frac{1}{4}\frac{x}{x^4 + 1} + \frac{1}{21}\frac{(14x^4 - 3)}{x^7}$

maple [A] time = 0.01, size = 78, normalized size = 0.69

$$\frac{x}{4x^4 + 4} + \frac{11\sqrt{2}\arctan(\sqrt{2}x - 1)}{16} + \frac{11\sqrt{2}\arctan(\sqrt{2}x + 1)}{16} + \frac{11\sqrt{2}\ln\left(\frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1}\right)}{32} + \frac{2}{3x^3} - \frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^8/(x^8+2*x^4+1),x)

[Out] $-\frac{1}{7x^7} + \frac{2}{3x^3} + \frac{1}{4(x^4 + 1)}x + \frac{11}{16}2^{(1/2)}\arctan(2^{(1/2)}x - 1) + \frac{11}{32}2^{(1/2)}\ln\left(\frac{x^2 + 2^{(1/2)}x + 1}{x^2 - 2^{(1/2)}x + 1}\right) + \frac{11}{16}2^{(1/2)}\arctan(2^{(1/2)}x + 1)$

maxima [A] time = 1.93, size = 95, normalized size = 0.84

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) + \frac{11}{32}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) - \frac{11}{32}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) + \frac{77x^8 + 44x^4 - 12}{84(x^{11} + x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="maxima")

[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/84*(77*x^8 + 44*x^4 - 12)/(x^11 + x^7)

mupad [B] time = 0.10, size = 55, normalized size = 0.49

$$\frac{11x^8}{12} + \frac{11x^4}{21} - \frac{1}{7} + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} - \frac{1}{2}i\right)\right) \left(\frac{11}{16} + \frac{11}{16}i\right) + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(\frac{11}{16} - \frac{11}{16}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8*(2*x^4 + x^8 + 1)),x)

[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(11/16 + 11i/16) + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(11/16 - 11i/16) + ((11*x^4)/21 + (11*x^8)/12 - 1/7)/(x^7 + x^11)

sympy [A] time = 0.23, size = 102, normalized size = 0.90

$$-\frac{11\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{32} + \frac{11\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{32} + \frac{11\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{16} + \frac{11\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{16} + \frac{77x^8 + 44x^4 - 12}{84x^{11} + 84x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**8/(x**8+2*x**4+1),x)

[Out] -11*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 11*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 + 11*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 11*sqrt(2)*atan(sqrt(2)*x + 1)/16 + (77*x**8 + 44*x**4 - 12)/(84*x**11 + 84*x**7)

$$3.235 \quad \int \frac{x^9}{1-2x^4+x^8} dx$$

Optimal. Leaf size=32

$$\frac{3x^2}{4} - \frac{3}{4} \tanh^{-1}(x^2) + \frac{x^6}{4(1-x^4)}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 275, 288, 321, 207}

$$\frac{x^6}{4(1-x^4)} + \frac{3x^2}{4} - \frac{3}{4} \tanh^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[x^9/(1 - 2*x^4 + x^8), x]

[Out] (3*x^2)/4 + x^6/(4*(1 - x^4)) - (3*ArcTanh[x^2])/4

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 207

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))]^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 288

Int[((c_.)*(x_)]^(m_.)*((a_.) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^(n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1 - 2x^4 + x^8} dx &= \int \frac{x^9}{(-1 + x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(-1 + x^2)^2} dx, x, x^2 \right) \\
&= \frac{x^6}{4(1 - x^4)} + \frac{3}{4} \text{Subst} \left(\int \frac{x^2}{-1 + x^2} dx, x, x^2 \right) \\
&= \frac{3x^2}{4} + \frac{x^6}{4(1 - x^4)} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{-1 + x^2} dx, x, x^2 \right) \\
&= \frac{3x^2}{4} + \frac{x^6}{4(1 - x^4)} - \frac{3}{4} \tanh^{-1}(x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.22

$$\frac{1}{8} \left(3 \log(1 - x^2) - 3 \log(x^2 + 1) + 2 \left(\frac{1}{1 - x^4} + 2 \right) x^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(1 - 2*x^4 + x^8), x]

[Out] (2*x^2*(2 + (1 - x^4)^(-1)) + 3*Log[1 - x^2] - 3*Log[1 + x^2])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1 - 2x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/(1 - 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^9/(1 - 2*x^4 + x^8), x]

fricas [A] time = 1.14, size = 46, normalized size = 1.44

$$\frac{4x^6 - 6x^2 - 3(x^4 - 1)\log(x^2 + 1) + 3(x^4 - 1)\log(x^2 - 1)}{8(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8-2*x^4+1), x, algorithm="fricas")

[Out] 1/8*(4*x^6 - 6*x^2 - 3*(x^4 - 1)*log(x^2 + 1) + 3*(x^4 - 1)*log(x^2 - 1))/(x^4 - 1)

giac [A] time = 0.33, size = 35, normalized size = 1.09

$$\frac{1}{2}x^2 - \frac{x^2}{4(x^4 - 1)} - \frac{3}{8}\log(x^2 + 1) + \frac{3}{8}\log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8-2*x^4+1), x, algorithm="giac")

[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(abs(x^2 - 1))

maple [A] time = 0.01, size = 41, normalized size = 1.28

$$\frac{x^2}{2} + \frac{3\ln(x^2 - 1)}{8} - \frac{3\ln(x^2 + 1)}{8} - \frac{1}{8(x^2 + 1)} - \frac{1}{8(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^8-2*x^4+1), x)

[Out] 1/2*x^2-1/8/(x^2+1)-3/8*ln(x^2+1)-1/8/(x^2-1)+3/8*ln(x^2-1)

maxima [A] time = 1.03, size = 34, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{x^2}{4(x^4 - 1)} - \frac{3}{8}\log(x^2 + 1) + \frac{3}{8}\log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8-2*x^4+1), x, algorithm="maxima")

[Out] $\frac{1}{2}x^2 - \frac{1}{4}x^2/(x^4 - 1) - \frac{3}{8}\log(x^2 + 1) + \frac{3}{8}\log(x^2 - 1)$

mupad [B] time = 0.05, size = 26, normalized size = 0.81

$$\frac{x^2}{2} - \frac{x^2}{4(x^4 - 1)} - \frac{3 \operatorname{atanh}(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(x^8 - 2*x^4 + 1), x)`

[Out] $x^2/2 - x^2/(4*(x^4 - 1)) - (3*\operatorname{atanh}(x^2))/4$

sympy [A] time = 0.12, size = 34, normalized size = 1.06

$$\frac{x^2}{2} - \frac{x^2}{4x^4 - 4} + \frac{3 \log(x^2 - 1)}{8} - \frac{3 \log(x^2 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(x**8-2*x**4+1), x)`

[Out] $x**2/2 - x**2/(4*x**4 - 4) + 3*\log(x**2 - 1)/8 - 3*\log(x**2 + 1)/8$

$$3.236 \quad \int \frac{x^7}{1-2x^4+x^8} dx$$

Optimal. Leaf size=26

$$\frac{1}{4(1-x^4)} + \frac{1}{4} \log(1-x^4)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 266, 43}

$$\frac{1}{4(1-x^4)} + \frac{1}{4} \log(1-x^4)$$

Antiderivative was successfully verified.

[In] Int[x^7/(1 - 2*x^4 + x^8), x]

[Out] 1/(4*(1 - x^4)) + Log[1 - x^4]/4

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :=
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1-2x^4+x^8} dx &= \int \frac{x^7}{(-1+x^4)^2} dx \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{x}{(-1+x)^2} dx, x, x^4 \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{(-1+x)^2} + \frac{1}{-1+x} \right) dx, x, x^4 \right) \\
&= \frac{1}{4(1-x^4)} + \frac{1}{4} \log(1-x^4)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.85

$$\frac{1}{4} \log(x^4 - 1) - \frac{1}{4(x^4 - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(1 - 2*x^4 + x^8), x]

[Out] -1/4*1/(-1 + x^4) + Log[-1 + x^4]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1-2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(1 - 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^7/(1 - 2*x^4 + x^8), x]

fricas [A] time = 1.27, size = 23, normalized size = 0.88

$$\frac{(x^4 - 1) \log(x^4 - 1) - 1}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8-2*x^4+1), x, algorithm="fricas")

[Out] 1/4*((x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)

giac [A] time = 0.41, size = 19, normalized size = 0.73

$$-\frac{1}{4(x^4-1)} + \frac{1}{4} \log(|x^4-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4/(x^4 - 1) + 1/4*log(abs(x^4 - 1))

maple [A] time = 0.01, size = 19, normalized size = 0.73

$$\frac{\ln(x^4-1)}{4} - \frac{1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^8-2*x^4+1),x)

[Out] -1/4/(x^4-1)+1/4*ln(x^4-1)

maxima [A] time = 1.13, size = 18, normalized size = 0.69

$$-\frac{1}{4(x^4-1)} + \frac{1}{4} \log(x^4-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] -1/4/(x^4 - 1) + 1/4*log(x^4 - 1)

mupad [B] time = 0.05, size = 20, normalized size = 0.77

$$\frac{\ln(x^4-1)}{4} - \frac{1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^8 - 2*x^4 + 1),x)

[Out] log(x^4 - 1)/4 - 1/(4*(x^4 - 1))

sympy [A] time = 0.10, size = 15, normalized size = 0.58

$$\frac{\log(x^4 - 1)}{4} - \frac{1}{4x^4 - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(x**8-2*x**4+1),x)

[Out] log(x**4 - 1)/4 - 1/(4*x**4 - 4)

$$3.237 \quad \int \frac{x^5}{1-2x^4+x^8} dx$$

Optimal. Leaf size=25

$$\frac{x^2}{4(1-x^4)} - \frac{1}{4} \tanh^{-1}(x^2)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {28, 275, 288, 207}

$$\frac{x^2}{4(1-x^4)} - \frac{1}{4} \tanh^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[x^5/(1 - 2*x^4 + x^8), x]

[Out] x^2/(4*(1 - x^4)) - ArcTanh[x^2]/4

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :=
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 207

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^(n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1-2x^4+x^8} dx &= \int \frac{x^5}{(-1+x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(-1+x^2)^2} dx, x, x^2 \right) \\
&= \frac{x^2}{4(1-x^4)} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, x^2 \right) \\
&= \frac{x^2}{4(1-x^4)} - \frac{1}{4} \tanh^{-1}(x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.32

$$\frac{1}{8} \left(\log(1-x^2) - \log(x^2+1) - \frac{2x^2}{x^4-1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(1 - 2*x^4 + x^8), x]

[Out] ((-2*x^2)/(-1 + x^4) + Log[1 - x^2] - Log[1 + x^2])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1-2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(1 - 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^5/(1 - 2*x^4 + x^8), x]

fricas [B] time = 1.17, size = 40, normalized size = 1.60

$$-\frac{2x^2 + (x^4 - 1)\log(x^2 + 1) - (x^4 - 1)\log(x^2 - 1)}{8(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] $-1/8*(2*x^2 + (x^4 - 1)*\log(x^2 + 1) - (x^4 - 1)*\log(x^2 - 1))/(x^4 - 1)$

giac [A] time = 0.50, size = 30, normalized size = 1.20

$$-\frac{x^2}{4(x^4 - 1)} - \frac{1}{8} \log(x^2 + 1) + \frac{1}{8} \log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="giac")

[Out] $-1/4*x^2/(x^4 - 1) - 1/8*\log(x^2 + 1) + 1/8*\log(\text{abs}(x^2 - 1))$

maple [A] time = 0.01, size = 36, normalized size = 1.44

$$\frac{\ln(x^2 - 1)}{8} - \frac{\ln(x^2 + 1)}{8} - \frac{1}{8(x^2 + 1)} - \frac{1}{8(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^8-2*x^4+1),x)

[Out] $-1/8/(x^2+1)-1/8*\ln(x^2+1)-1/8/(x^2-1)+1/8*\ln(x^2-1)$

maxima [A] time = 0.91, size = 29, normalized size = 1.16

$$-\frac{x^2}{4(x^4 - 1)} - \frac{1}{8} \log(x^2 + 1) + \frac{1}{8} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] $-1/4*x^2/(x^4 - 1) - 1/8*\log(x^2 + 1) + 1/8*\log(x^2 - 1)$

mupad [B] time = 1.27, size = 21, normalized size = 0.84

$$-\frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^8 - 2*x^4 + 1),x)

[Out] - atanh(x²)/4 - x²/(4*(x⁴ - 1))

sympy [A] time = 0.12, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4 - 4} + \frac{\log(x^2 - 1)}{8} - \frac{\log(x^2 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(x**8-2*x**4+1), x)

[Out] -x**2/(4*x**4 - 4) + log(x**2 - 1)/8 - log(x**2 + 1)/8

$$3.238 \quad \int \frac{x^3}{1-2x^4+x^8} dx$$

Optimal. Leaf size=13

$$\frac{1}{4(1-x^4)}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 261}

$$\frac{1}{4(1-x^4)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 - 2*x^4 + x^8),x]

[Out] 1/(4*(1 - x^4))

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{1-2x^4+x^8} dx &= \int \frac{x^3}{(-1+x^4)^2} dx \\ &= \frac{1}{4(1-x^4)} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4-1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(1 - 2*x^4 + x^8),x]

[Out] -1/4*1/(-1 + x^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1 - 2x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(1 - 2*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^3/(1 - 2*x^4 + x^8), x]

fricas [A] time = 1.15, size = 9, normalized size = 0.69

$$-\frac{1}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] -1/4/(x^4 - 1)

giac [A] time = 0.50, size = 9, normalized size = 0.69

$$-\frac{1}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4/(x^4 - 1)

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^8-2*x^4+1),x)

[Out] $-1/4/(x^4-1)$

maxima [A] time = 0.88, size = 9, normalized size = 0.69

$$-\frac{1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(x^8-2*x^4+1),x, algorithm="maxima")`

[Out] $-1/4/(x^4 - 1)$

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(x^8 - 2*x^4 + 1),x)`

[Out] $-1/(4*(x^4 - 1))$

sympy [A] time = 0.10, size = 8, normalized size = 0.62

$$-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(x**8-2*x**4+1),x)`

[Out] $-1/(4*x**4 - 4)$

$$3.239 \quad \int \frac{x}{1-2x^4+x^8} dx$$

Optimal. Leaf size=25

$$\frac{1}{4} \tanh^{-1}(x^2) + \frac{x^2}{4(1-x^4)}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {28, 275, 199, 207}

$$\frac{x^2}{4(1-x^4)} + \frac{1}{4} \tanh^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[x/(1 - 2*x^4 + x^8), x]

[Out] x^2/(4*(1 - x^4)) + ArcTanh[x^2]/4

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x}{1-2x^4+x^8} dx &= \int \frac{x}{(-1+x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(-1+x^2)^2} dx, x, x^2 \right) \\
&= \frac{x^2}{4(1-x^4)} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, x^2 \right) \\
&= \frac{x^2}{4(1-x^4)} + \frac{1}{4} \tanh^{-1}(x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.32

$$\frac{1}{8} \left(-\log(1-x^2) + \log(x^2+1) - \frac{2x^2}{x^4-1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 - 2*x^4 + x^8),x]

[Out] ((-2*x^2)/(-1 + x^4) - Log[1 - x^2] + Log[1 + x^2])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1-2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(1 - 2*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x/(1 - 2*x^4 + x^8), x]

fricas [B] time = 1.26, size = 40, normalized size = 1.60

$$-\frac{2x^2 - (x^4 - 1)\log(x^2 + 1) + (x^4 - 1)\log(x^2 - 1)}{8(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] $-1/8*(2*x^2 - (x^4 - 1)*\log(x^2 + 1) + (x^4 - 1)*\log(x^2 - 1))/(x^4 - 1)$

giac [A] time = 0.47, size = 30, normalized size = 1.20

$$-\frac{x^2}{4(x^4 - 1)} + \frac{1}{8} \log(x^2 + 1) - \frac{1}{8} \log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^8-2*x^4+1),x, algorithm="giac")`

[Out] $-1/4*x^2/(x^4 - 1) + 1/8*\log(x^2 + 1) - 1/8*\log(\text{abs}(x^2 - 1))$

maple [A] time = 0.01, size = 36, normalized size = 1.44

$$-\frac{\ln(x^2 - 1)}{8} + \frac{\ln(x^2 + 1)}{8} - \frac{1}{8(x^2 + 1)} - \frac{1}{8(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^8-2*x^4+1),x)`

[Out] $-1/8/(x^2+1)+1/8*\ln(x^2+1)-1/8/(x^2-1)-1/8*\ln(x^2-1)$

maxima [A] time = 0.92, size = 29, normalized size = 1.16

$$-\frac{x^2}{4(x^4 - 1)} + \frac{1}{8} \log(x^2 + 1) - \frac{1}{8} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^8-2*x^4+1),x, algorithm="maxima")`

[Out] $-1/4*x^2/(x^4 - 1) + 1/8*\log(x^2 + 1) - 1/8*\log(x^2 - 1)$

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$\frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^8 - 2*x^4 + 1),x)`

[Out] $\operatorname{atanh}(x^2)/4 - x^2/(4*(x^4 - 1))$

sympy [A] time = 0.12, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4 - 4} - \frac{\log(x^2 - 1)}{8} + \frac{\log(x^2 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**8-2*x**4+1),x)

[Out] -x**2/(4*x**4 - 4) - log(x**2 - 1)/8 + log(x**2 + 1)/8

$$3.240 \quad \int \frac{1}{x(1-2x^4+x^8)} dx$$

Optimal. Leaf size=28

$$\frac{1}{4(1-x^4)} - \frac{1}{4} \log(1-x^4) + \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 266, 44}

$$\frac{1}{4(1-x^4)} - \frac{1}{4} \log(1-x^4) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - 2*x^4 + x^8)),x]

[Out] 1/(4*(1 - x^4)) + Log[x] - Log[1 - x^4]/4

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-2x^4+x^8)} dx &= \int \frac{1}{x(-1+x^4)^2} dx \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(-1+x)^2 x} dx, x, x^4 \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{1-x} + \frac{1}{(-1+x)^2} + \frac{1}{x} \right) dx, x, x^4 \right) \\
&= \frac{1}{4(1-x^4)} + \log(x) - \frac{1}{4} \log(1-x^4)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{1}{4(x^4-1)} - \frac{1}{4} \log(1-x^4) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - 2*x^4 + x^8)), x]

[Out] -1/4*1/(-1 + x^4) + Log[x] - Log[1 - x^4]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(1 - 2*x^4 + x^8)), x]

[Out] IntegrateAlgebraic[1/(x*(1 - 2*x^4 + x^8)), x]

fricas [A] time = 0.98, size = 32, normalized size = 1.14

$$\frac{(x^4-1) \log(x^4-1) - 4(x^4-1) \log(x) + 1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8-2*x^4+1), x, algorithm="fricas")

[Out] -1/4*((x^4 - 1)*log(x^4 - 1) - 4*(x^4 - 1)*log(x) + 1)/(x^4 - 1)

giac [A] time = 0.38, size = 30, normalized size = 1.07

$$\frac{x^4 - 2}{4(x^4 - 1)} + \frac{1}{4} \log(x^4) - \frac{1}{4} \log(|x^4 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="giac")

[Out] 1/4*(x^4 - 2)/(x^4 - 1) + 1/4*log(x^4) - 1/4*log(abs(x^4 - 1))

maple [A] time = 0.02, size = 47, normalized size = 1.68

$$\ln(x) - \frac{\ln(x-1)}{4} - \frac{\ln(x+1)}{4} - \frac{\ln(x^2+1)}{4} + \frac{1}{16x+16} + \frac{1}{8x^2+8} - \frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^8-2*x^4+1),x)

[Out] ln(x)+1/16/(x+1)-1/4*ln(x+1)-1/4*ln(x^2+1)+1/8/(x^2+1)-1/16/(x-1)-1/4*ln(x-1)

maxima [A] time = 0.94, size = 24, normalized size = 0.86

$$-\frac{1}{4(x^4 - 1)} - \frac{1}{4} \log(x^4 - 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] -1/4/(x^4 - 1) - 1/4*log(x^4 - 1) + 1/4*log(x^4)

mupad [B] time = 0.06, size = 22, normalized size = 0.79

$$\ln(x) - \frac{\ln(x^4 - 1)}{4} - \frac{1}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(x^8 - 2*x^4 + 1)),x)

[Out] log(x) - log(x^4 - 1)/4 - 1/(4*(x^4 - 1))

sympy [A] time = 0.13, size = 19, normalized size = 0.68

$$\log(x) - \frac{\log(x^4 - 1)}{4} - \frac{1}{4x^4 - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x**8-2*x**4+1),x)

[Out] log(x) - log(x**4 - 1)/4 - 1/(4*x**4 - 4)

$$3.241 \quad \int \frac{1}{x^3(1-2x^4+x^8)} dx$$

Optimal. Leaf size=32

$$-\frac{3}{4x^2} + \frac{3}{4} \tanh^{-1}(x^2) + \frac{1}{4x^2(1-x^4)}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 275, 290, 325, 207}

$$\frac{1}{4x^2(1-x^4)} - \frac{3}{4x^2} + \frac{3}{4} \tanh^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1 - 2*x^4 + x^8)),x]

[Out] -3/(4*x^2) + 1/(4*x^2*(1 - x^4)) + (3*ArcTanh[x^2])/4

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3(1-2x^4+x^8)} dx &= \int \frac{1}{x^3(-1+x^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(-1+x^2)^2} dx, x, x^2 \right) \\
 &= \frac{1}{4x^2(1-x^4)} - \frac{3}{4} \text{Subst} \left(\int \frac{1}{x^2(-1+x^2)} dx, x, x^2 \right) \\
 &= -\frac{3}{4x^2} + \frac{1}{4x^2(1-x^4)} - \frac{3}{4} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, x^2 \right) \\
 &= -\frac{3}{4x^2} + \frac{1}{4x^2(1-x^4)} + \frac{3}{4} \tanh^{-1}(x^2)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.28

$$\frac{1}{8} \left(-3 \log(1-x^2) + 3 \log(x^2+1) + \frac{4-6x^4}{x^2(x^4-1)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(1 - 2*x^4 + x^8)), x]

[Out] ((4 - 6*x^4)/(x^2*(-1 + x^4)) - 3*Log[1 - x^2] + 3*Log[1 + x^2])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(1-2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(1 - 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^3*(1 - 2*x^4 + x^8)), x]

fricas [B] time = 1.16, size = 54, normalized size = 1.69

$$\frac{6x^4 - 3(x^6 - x^2)\log(x^2 + 1) + 3(x^6 - x^2)\log(x^2 - 1) - 4}{8(x^6 - x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] -1/8*(6*x^4 - 3*(x^6 - x^2)*log(x^2 + 1) + 3*(x^6 - x^2)*log(x^2 - 1) - 4)/(x^6 - x^2)

giac [A] time = 0.41, size = 38, normalized size = 1.19

$$-\frac{3x^4 - 2}{4(x^6 - x^2)} + \frac{3}{8}\log(x^2 + 1) - \frac{3}{8}\log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(abs(x^2 - 1))

maple [A] time = 0.02, size = 50, normalized size = 1.56

$$-\frac{3\ln(x-1)}{8} - \frac{3\ln(x+1)}{8} + \frac{3\ln(x^2+1)}{8} - \frac{1}{2x^2} + \frac{1}{16x+16} - \frac{1}{8(x^2+1)} - \frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^8-2*x^4+1),x)

[Out] -1/2/x^2+1/16/(x+1)-3/8*ln(x+1)+3/8*ln(x^2+1)-1/8/(x^2+1)-1/16/(x-1)-3/8*ln(x-1)

maxima [A] time = 0.95, size = 37, normalized size = 1.16

$$-\frac{3x^4 - 2}{4(x^6 - x^2)} + \frac{3}{8}\log(x^2 + 1) - \frac{3}{8}\log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(x^2 - 1)

mupad [B] time = 0.04, size = 26, normalized size = 0.81

$$\frac{3 \operatorname{atanh}(x^2)}{4} + \frac{\frac{3x^4}{4} - \frac{1}{2}}{x^2 - x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(x^8 - 2*x^4 + 1)),x)

[Out] (3*atanh(x^2))/4 + ((3*x^4)/4 - 1/2)/(x^2 - x^6)

sympy [A] time = 0.15, size = 36, normalized size = 1.12

$$\frac{2 - 3x^4}{4x^6 - 4x^2} - \frac{3 \log(x^2 - 1)}{8} + \frac{3 \log(x^2 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(x**8-2*x**4+1),x)

[Out] (2 - 3*x**4)/(4*x**6 - 4*x**2) - 3*log(x**2 - 1)/8 + 3*log(x**2 + 1)/8

$$3.242 \quad \int \frac{1}{x^5(1-2x^4+x^8)} dx$$

Optimal. Leaf size=37

$$\frac{1}{4(1-x^4)} - \frac{1}{4x^4} - \frac{1}{2} \log(1-x^4) + 2 \log(x)$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 266, 44}

$$\frac{1}{4(1-x^4)} - \frac{1}{4x^4} - \frac{1}{2} \log(1-x^4) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(1 - 2*x^4 + x^8)),x]

[Out] -1/(4*x^4) + 1/(4*(1 - x^4)) + 2*Log[x] - Log[1 - x^4]/2

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(1-2x^4+x^8)} dx &= \int \frac{1}{x^5(-1+x^4)^2} dx \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{(-1+x)^2 x^2} dx, x, x^4 \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{(-1+x)^2} - \frac{2}{-1+x} + \frac{1}{x^2} + \frac{2}{x} \right) dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{1}{4(1-x^4)} + 2 \log(x) - \frac{1}{2} \log(1-x^4)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.95

$$-\frac{1}{4(x^4-1)} - \frac{1}{4x^4} - \frac{1}{2} \log(1-x^4) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(1 - 2*x^4 + x^8)), x]

[Out] -1/4*1/x^4 - 1/(4*(-1 + x^4)) + 2*Log[x] - Log[1 - x^4]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(1-2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(1 - 2*x^4 + x^8)), x]

[Out] IntegrateAlgebraic[1/(x^5*(1 - 2*x^4 + x^8)), x]

fricas [A] time = 0.86, size = 50, normalized size = 1.35

$$\frac{2x^4 + 2(x^8 - x^4) \log(x^4 - 1) - 8(x^8 - x^4) \log(x) - 1}{4(x^8 - x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8-2*x^4+1), x, algorithm="fricas")

[Out] $-1/4*(2*x^4 + 2*(x^8 - x^4)*\log(x^4 - 1) - 8*(x^8 - x^4)*\log(x) - 1)/(x^8 - x^4)$

giac [A] time = 0.45, size = 36, normalized size = 0.97

$$-\frac{2x^4 - 1}{4(x^8 - x^4)} + \frac{1}{2} \log(x^4) - \frac{1}{2} \log(|x^4 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="giac")`

[Out] $-1/4*(2*x^4 - 1)/(x^8 - x^4) + 1/2*\log(x^4) - 1/2*\log(\text{abs}(x^4 - 1))$

maple [A] time = 0.02, size = 54, normalized size = 1.46

$$2 \ln(x) - \frac{\ln(x-1)}{2} - \frac{\ln(x+1)}{2} - \frac{\ln(x^2+1)}{2} - \frac{1}{4x^4} + \frac{1}{16x+16} + \frac{1}{8x^2+8} - \frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(x^8-2*x^4+1),x)`

[Out] $-1/4/x^4+2*\ln(x)+1/16/(x+1)-1/2*\ln(x+1)-1/2*\ln(x^2+1)+1/8/(x^2+1)-1/16/(x-1)-1/2*\ln(x-1)$

maxima [A] time = 0.83, size = 35, normalized size = 0.95

$$-\frac{2x^4 - 1}{4(x^8 - x^4)} - \frac{1}{2} \log(x^4 - 1) + \frac{1}{2} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="maxima")`

[Out] $-1/4*(2*x^4 - 1)/(x^8 - x^4) - 1/2*\log(x^4 - 1) + 1/2*\log(x^4)$

mupad [B] time = 0.05, size = 32, normalized size = 0.86

$$2 \ln(x) - \frac{\ln(x^4 - 1)}{2} + \frac{\frac{x^4}{2} - \frac{1}{4}}{x^4 - x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(x^8 - 2*x^4 + 1)),x)`

[Out] $2*\log(x) - \log(x^4 - 1)/2 + (x^4/2 - 1/4)/(x^4 - x^8)$

sympy [A] time = 0.15, size = 29, normalized size = 0.78

$$\frac{1 - 2x^4}{4x^8 - 4x^4} + 2 \log(x) - \frac{\log(x^4 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(x**8-2*x**4+1),x)

[Out] (1 - 2*x**4)/(4*x**8 - 4*x**4) + 2*log(x) - log(x**4 - 1)/2

$$3.243 \quad \int \frac{1}{x^7(1-2x^4+x^8)} dx$$

Optimal. Leaf size=39

$$-\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{5}{4} \tanh^{-1}(x^2) + \frac{1}{4x^6(1-x^4)}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 275, 290, 325, 207}

$$\frac{1}{4x^6(1-x^4)} - \frac{5}{4x^2} - \frac{5}{12x^6} + \frac{5}{4} \tanh^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(1 - 2*x^4 + x^8)),x]

[Out] -5/(12*x^6) - 5/(4*x^2) + 1/(4*x^6*(1 - x^4)) + (5*ArcTanh[x^2])/4

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

]

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7(1-2x^4+x^8)} dx &= \int \frac{1}{x^7(-1+x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4(-1+x^2)^2} dx, x, x^2 \right) \\
&= \frac{1}{4x^6(1-x^4)} - \frac{5}{4} \text{Subst} \left(\int \frac{1}{x^4(-1+x^2)} dx, x, x^2 \right) \\
&= -\frac{5}{12x^6} + \frac{1}{4x^6(1-x^4)} - \frac{5}{4} \text{Subst} \left(\int \frac{1}{x^2(-1+x^2)} dx, x, x^2 \right) \\
&= -\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6(1-x^4)} - \frac{5}{4} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, x^2 \right) \\
&= -\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6(1-x^4)} + \frac{5}{4} \tanh^{-1}(x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.26

$$-\frac{1}{6x^6} - \frac{1}{x^2} - \frac{5}{8} \log(1-x^2) + \frac{5}{8} \log(x^2+1) - \frac{x^2}{4(x^4-1)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(1 - 2*x^4 + x^8)), x]

[Out] -1/6*1/x^6 - x^(-2) - x^2/(4*(-1 + x^4)) - (5*Log[1 - x^2])/8 + (5*Log[1 + x^2])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7(1-2x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^7*(1-2*x^4+x^8)),x]

[Out] IntegrateAlgebraic[1/(x^7*(1-2*x^4+x^8)),x]

fricas [B] time = 1.23, size = 59, normalized size = 1.51

$$-\frac{30x^8 - 20x^4 - 15(x^{10} - x^6)\log(x^2 + 1) + 15(x^{10} - x^6)\log(x^2 - 1) - 4}{24(x^{10} - x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] -1/24*(30*x^8 - 20*x^4 - 15*(x^10 - x^6)*log(x^2 + 1) + 15*(x^10 - x^6)*log(x^2 - 1) - 4)/(x^10 - x^6)

giac [A] time = 0.33, size = 42, normalized size = 1.08

$$-\frac{x^2}{4(x^4-1)} - \frac{6x^4+1}{6x^6} + \frac{5}{8}\log(x^2+1) - \frac{5}{8}\log(|x^2-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4*x^2/(x^4-1) - 1/6*(6*x^4+1)/x^6 + 5/8*log(x^2+1) - 5/8*log(abs(x^2-1))

maple [A] time = 0.02, size = 55, normalized size = 1.41

$$-\frac{5\ln(x-1)}{8} - \frac{5\ln(x+1)}{8} + \frac{5\ln(x^2+1)}{8} - \frac{1}{x^2} - \frac{1}{6x^6} + \frac{1}{16x+16} - \frac{1}{8(x^2+1)} - \frac{1}{16(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(x^8-2*x^4+1),x)

[Out] -1/6/x^6-1/x^2+1/16/(x+1)-5/8*ln(x+1)+5/8*ln(x^2+1)-1/8/(x^2+1)-1/16/(x-1)-5/8*ln(x-1)

maxima [A] time = 0.94, size = 42, normalized size = 1.08

$$-\frac{15x^8 - 10x^4 - 2}{12(x^{10} - x^6)} + \frac{5}{8} \log(x^2 + 1) - \frac{5}{8} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] -1/12*(15*x^8 - 10*x^4 - 2)/(x^10 - x^6) + 5/8*log(x^2 + 1) - 5/8*log(x^2 - 1)

mupad [B] time = 0.05, size = 32, normalized size = 0.82

$$\frac{5 \operatorname{atanh}(x^2)}{4} - \frac{-\frac{5x^8}{4} + \frac{5x^4}{6} + \frac{1}{6}}{x^6 - x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(x^8 - 2*x^4 + 1)),x)

[Out] (5*atanh(x^2))/4 - ((5*x^4)/6 - (5*x^8)/4 + 1/6)/(x^6 - x^10)

sympy [A] time = 0.18, size = 41, normalized size = 1.05

$$-\frac{5 \log(x^2 - 1)}{8} + \frac{5 \log(x^2 + 1)}{8} + \frac{-15x^8 + 10x^4 + 2}{12x^{10} - 12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(x**8-2*x**4+1),x)

[Out] -5*log(x**2 - 1)/8 + 5*log(x**2 + 1)/8 + (-15*x**8 + 10*x**4 + 2)/(12*x**10 - 12*x**6)

$$3.244 \quad \int \frac{x^8}{1-2x^4+x^8} dx$$

Optimal. Leaf size=34

$$\frac{x^5}{4(1-x^4)} + \frac{5x}{4} - \frac{5}{8} \tan^{-1}(x) - \frac{5}{8} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {28, 288, 321, 212, 206, 203}

$$\frac{x^5}{4(1-x^4)} + \frac{5x}{4} - \frac{5}{8} \tan^{-1}(x) - \frac{5}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^8/(1 - 2*x^4 + x^8), x]

[Out] (5*x)/4 + x^5/(4*(1 - x^4)) - (5*ArcTan[x])/8 - (5*ArcTanh[x])/8

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1 - 2x^4 + x^8} dx &= \int \frac{x^8}{(-1 + x^4)^2} dx \\
&= \frac{x^5}{4(1 - x^4)} + \frac{5}{4} \int \frac{x^4}{-1 + x^4} dx \\
&= \frac{5x}{4} + \frac{x^5}{4(1 - x^4)} + \frac{5}{4} \int \frac{1}{-1 + x^4} dx \\
&= \frac{5x}{4} + \frac{x^5}{4(1 - x^4)} - \frac{5}{8} \int \frac{1}{1 - x^2} dx - \frac{5}{8} \int \frac{1}{1 + x^2} dx \\
&= \frac{5x}{4} + \frac{x^5}{4(1 - x^4)} - \frac{5}{8} \tan^{-1}(x) - \frac{5}{8} \tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.12

$$-\frac{x}{4(x^4 - 1)} + x + \frac{5}{16} \log(1 - x) - \frac{5}{16} \log(x + 1) - \frac{5}{8} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(1 - 2*x^4 + x^8),x]

[Out] x - x/(4*(-1 + x^4)) - (5*ArcTan[x])/8 + (5*Log[1 - x])/16 - (5*Log[1 + x])/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1 - 2x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(1 - 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^8/(1 - 2*x^4 + x^8), x]

fricas [B] time = 1.19, size = 49, normalized size = 1.44

$$\frac{16x^5 - 10(x^4 - 1)\arctan(x) - 5(x^4 - 1)\log(x + 1) + 5(x^4 - 1)\log(x - 1) - 20x}{16(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8-2*x^4+1), x, algorithm="fricas")

[Out] 1/16*(16*x^5 - 10*(x^4 - 1)*arctan(x) - 5*(x^4 - 1)*log(x + 1) + 5*(x^4 - 1)*log(x - 1) - 20*x)/(x^4 - 1)

giac [A] time = 0.28, size = 30, normalized size = 0.88

$$x - \frac{x}{4(x^4 - 1)} - \frac{5}{8} \arctan(x) - \frac{5}{16} \log(|x + 1|) + \frac{5}{16} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8-2*x^4+1), x, algorithm="giac")

[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(abs(x + 1)) + 5/16*log(abs(x - 1))

maple [A] time = 0.01, size = 43, normalized size = 1.26

$$x + \frac{x}{8x^2 + 8} - \frac{5 \arctan(x)}{8} + \frac{5 \ln(x - 1)}{16} - \frac{5 \ln(x + 1)}{16} - \frac{1}{16(x + 1)} - \frac{1}{16(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^8-2*x^4+1), x)

[Out] x-1/16/(x+1)-5/16*ln(x+1)+1/8/(x^2+1)*x-5/8*arctan(x)-1/16/(x-1)+5/16*ln(x-1)

maxima [A] time = 2.13, size = 28, normalized size = 0.82

$$x - \frac{x}{4(x^4 - 1)} - \frac{5}{8} \arctan(x) - \frac{5}{16} \log(x + 1) + \frac{5}{16} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(x + 1) + 5/16*log(x - 1)

mupad [B] time = 1.29, size = 26, normalized size = 0.76

$$x - \frac{5 \operatorname{atan}(x)}{8} - \frac{x}{4(x^4 - 1)} + \frac{\operatorname{atan}(x) 5i}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^8 - 2*x^4 + 1),x)

[Out] x + (atan(x*i)*5i)/8 - (5*atan(x))/8 - x/(4*(x^4 - 1))

sympy [A] time = 0.15, size = 32, normalized size = 0.94

$$x - \frac{x}{4x^4 - 4} + \frac{5 \log(x - 1)}{16} - \frac{5 \log(x + 1)}{16} - \frac{5 \operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(x**8-2*x**4+1),x)

[Out] x - x/(4*x**4 - 4) + 5*log(x - 1)/16 - 5*log(x + 1)/16 - 5*atan(x)/8

$$3.245 \quad \int \frac{x^6}{1-2x^4+x^8} dx$$

Optimal. Leaf size=29

$$\frac{x^3}{4(1-x^4)} + \frac{3}{8} \tan^{-1}(x) - \frac{3}{8} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 288, 298, 203, 206}

$$\frac{x^3}{4(1-x^4)} + \frac{3}{8} \tan^{-1}(x) - \frac{3}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^6/(1 - 2*x^4 + x^8), x]

[Out] x^3/(4*(1 - x^4)) + (3*ArcTan[x])/8 - (3*ArcTanh[x])/8

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !I

LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^6}{1 - 2x^4 + x^8} dx &= \int \frac{x^6}{(-1 + x^4)^2} dx \\ &= \frac{x^3}{4(1 - x^4)} + \frac{3}{4} \int \frac{x^2}{-1 + x^4} dx \\ &= \frac{x^3}{4(1 - x^4)} - \frac{3}{8} \int \frac{1}{1 - x^2} dx + \frac{3}{8} \int \frac{1}{1 + x^2} dx \\ &= \frac{x^3}{4(1 - x^4)} + \frac{3}{8} \tan^{-1}(x) - \frac{3}{8} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.21

$$\frac{1}{16} \left(-\frac{4x^3}{x^4 - 1} + 3 \log(1 - x) - 3 \log(x + 1) + 6 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(1 - 2*x^4 + x^8), x]

[Out] ((-4*x^3)/(-1 + x^4) + 6*ArcTan[x] + 3*Log[1 - x] - 3*Log[1 + x])/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1 - 2x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(1 - 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^6/(1 - 2*x^4 + x^8), x]

fricas [B] time = 1.23, size = 46, normalized size = 1.59

$$\frac{4x^3 - 6(x^4 - 1)\arctan(x) + 3(x^4 - 1)\log(x + 1) - 3(x^4 - 1)\log(x - 1)}{16(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] -1/16*(4*x^3 - 6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)*log(x - 1))/(x^4 - 1)

giac [A] time = 0.40, size = 31, normalized size = 1.07

$$-\frac{x^3}{4(x^4 - 1)} + \frac{3}{8}\arctan(x) - \frac{3}{16}\log(|x + 1|) + \frac{3}{16}\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(abs(x + 1)) + 3/16*log(abs(x - 1))

maple [A] time = 0.02, size = 42, normalized size = 1.45

$$-\frac{x}{8(x^2 + 1)} + \frac{3\arctan(x)}{8} + \frac{3\ln(x - 1)}{16} - \frac{3\ln(x + 1)}{16} - \frac{1}{16(x + 1)} - \frac{1}{16(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^8-2*x^4+1),x)

[Out] -1/16/(x+1)-3/16*ln(x+1)-1/8/(x^2+1)*x+3/8*arctan(x)-1/16/(x-1)+3/16*ln(x-1)

maxima [A] time = 2.03, size = 29, normalized size = 1.00

$$-\frac{x^3}{4(x^4 - 1)} + \frac{3}{8}\arctan(x) - \frac{3}{16}\log(x + 1) + \frac{3}{16}\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] $-1/4*x^3/(x^4 - 1) + 3/8*\arctan(x) - 3/16*\log(x + 1) + 3/16*\log(x - 1)$

mupad [B] time = 0.03, size = 23, normalized size = 0.79

$$\frac{3 \operatorname{atan}(x)}{8} - \frac{3 \operatorname{atanh}(x)}{8} - \frac{x^3}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(x^6/(x^8 - 2*x^4 + 1), x)$

[Out] $(3*\operatorname{atan}(x))/8 - (3*\operatorname{atanh}(x))/8 - x^3/(4*(x^4 - 1))$

sympy [A] time = 0.15, size = 32, normalized size = 1.10

$$-\frac{x^3}{4x^4 - 4} + \frac{3 \log(x - 1)}{16} - \frac{3 \log(x + 1)}{16} + \frac{3 \operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(x^{**6}/(x^{**8}-2*x^{**4}+1), x)$

[Out] $-x^{**3}/(4*x^{**4} - 4) + 3*\log(x - 1)/16 - 3*\log(x + 1)/16 + 3*\operatorname{atan}(x)/8$

$$3.246 \quad \int \frac{x^4}{1-2x^4+x^8} dx$$

Optimal. Leaf size=27

$$\frac{x}{4(1-x^4)} - \frac{1}{8} \tan^{-1}(x) - \frac{1}{8} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 288, 212, 206, 203}

$$\frac{x}{4(1-x^4)} - \frac{1}{8} \tan^{-1}(x) - \frac{1}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^4/(1 - 2*x^4 + x^8), x]

[Out] x/(4*(1 - x^4)) - ArcTan[x]/8 - ArcTanh[x]/8

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{1 - 2x^4 + x^8} dx &= \int \frac{x^4}{(-1 + x^4)^2} dx \\ &= \frac{x}{4(1 - x^4)} + \frac{1}{4} \int \frac{1}{-1 + x^4} dx \\ &= \frac{x}{4(1 - x^4)} - \frac{1}{8} \int \frac{1}{1 - x^2} dx - \frac{1}{8} \int \frac{1}{1 + x^2} dx \\ &= \frac{x}{4(1 - x^4)} - \frac{1}{8} \tan^{-1}(x) - \frac{1}{8} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{16} \left(-\frac{4x}{x^4 - 1} + \log(1 - x) - \log(x + 1) - 2 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 - 2*x^4 + x^8), x]

[Out] ((-4*x)/(-1 + x^4) - 2*ArcTan[x] + Log[1 - x] - Log[1 + x])/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1 - 2x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(1 - 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^4/(1 - 2*x^4 + x^8), x]

fricas [B] time = 1.27, size = 43, normalized size = 1.59

$$\frac{2(x^4 - 1) \arctan(x) + (x^4 - 1) \log(x + 1) - (x^4 - 1) \log(x - 1) + 4x}{16(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] -1/16*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1) + 4*x)/(x^4 - 1)

giac [A] time = 0.51, size = 29, normalized size = 1.07

$$-\frac{x}{4(x^4 - 1)} - \frac{1}{8} \arctan(x) - \frac{1}{16} \log(|x + 1|) + \frac{1}{16} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(abs(x + 1)) + 1/16*log(abs(x - 1))

maple [A] time = 0.01, size = 42, normalized size = 1.56

$$\frac{x}{8x^2 + 8} - \frac{\arctan(x)}{8} + \frac{\ln(x - 1)}{16} - \frac{\ln(x + 1)}{16} - \frac{1}{16(x + 1)} - \frac{1}{16(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^8-2*x^4+1),x)

[Out] -1/16/(x+1)-1/16*ln(x+1)+1/8/(x^2+1)*x-1/8*arctan(x)-1/16/(x-1)+1/16*ln(x-1)

maxima [A] time = 1.92, size = 27, normalized size = 1.00

$$-\frac{x}{4(x^4 - 1)} - \frac{1}{8} \arctan(x) - \frac{1}{16} \log(x + 1) + \frac{1}{16} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(x + 1) + 1/16*log(x - 1)

mupad [B] time = 0.03, size = 21, normalized size = 0.78

$$-\frac{\operatorname{atan}(x)}{8} - \frac{\operatorname{atanh}(x)}{8} - \frac{x}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(x^8 - 2*x^4 + 1), x)`

[Out] `- atan(x)/8 - atanh(x)/8 - x/(4*(x^4 - 1))`

sympy [A] time = 0.15, size = 26, normalized size = 0.96

$$-\frac{x}{4x^4 - 4} + \frac{\log(x - 1)}{16} - \frac{\log(x + 1)}{16} - \frac{\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(x**8-2*x**4+1), x)`

[Out] `-x/(4*x**4 - 4) + log(x - 1)/16 - log(x + 1)/16 - atan(x)/8`

$$3.247 \quad \int \frac{x^2}{1-2x^4+x^8} dx$$

Optimal. Leaf size=29

$$\frac{x^3}{4(1-x^4)} - \frac{1}{8} \tan^{-1}(x) + \frac{1}{8} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 290, 298, 203, 206}

$$\frac{x^3}{4(1-x^4)} - \frac{1}{8} \tan^{-1}(x) + \frac{1}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 - 2*x^4 + x^8), x]

[Out] x^3/(4*(1 - x^4)) - ArcTan[x]/8 + ArcTanh[x]/8

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m + n*(p+1) + 1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

]

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{1-2x^4+x^8} dx &= \int \frac{x^2}{(-1+x^4)^2} dx \\
 &= \frac{x^3}{4(1-x^4)} - \frac{1}{4} \int \frac{x^2}{-1+x^4} dx \\
 &= \frac{x^3}{4(1-x^4)} + \frac{1}{8} \int \frac{1}{1-x^2} dx - \frac{1}{8} \int \frac{1}{1+x^2} dx \\
 &= \frac{x^3}{4(1-x^4)} - \frac{1}{8} \tan^{-1}(x) + \frac{1}{8} \tanh^{-1}(x)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.14

$$\frac{1}{16} \left(-\frac{4x^3}{x^4-1} - \log(1-x) + \log(x+1) - 2 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 - 2*x^4 + x^8), x]

[Out] ((-4*x^3)/(-1 + x^4) - 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1-2x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1 - 2*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^2/(1 - 2*x^4 + x^8), x]

fricas [B] time = 1.22, size = 45, normalized size = 1.55

$$\frac{4x^3 + 2(x^4 - 1)\arctan(x) - (x^4 - 1)\log(x + 1) + (x^4 - 1)\log(x - 1)}{16(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] -1/16*(4*x^3 + 2*(x^4 - 1)*arctan(x) - (x^4 - 1)*log(x + 1) + (x^4 - 1)*log(x - 1))/(x^4 - 1)

giac [A] time = 0.34, size = 31, normalized size = 1.07

$$-\frac{x^3}{4(x^4 - 1)} - \frac{1}{8}\arctan(x) + \frac{1}{16}\log(|x + 1|) - \frac{1}{16}\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(abs(x + 1)) - 1/16*log(abs(x - 1))

maple [A] time = 0.01, size = 42, normalized size = 1.45

$$-\frac{x}{8(x^2 + 1)} - \frac{\arctan(x)}{8} - \frac{\ln(x - 1)}{16} + \frac{\ln(x + 1)}{16} - \frac{1}{16(x + 1)} - \frac{1}{16(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^8-2*x^4+1),x)

[Out] -1/16/(x+1)+1/16*ln(x+1)-1/8/(x^2+1)*x-1/8*arctan(x)-1/16/(x-1)-1/16*ln(x-1)

maxima [A] time = 1.71, size = 29, normalized size = 1.00

$$-\frac{x^3}{4(x^4 - 1)} - \frac{1}{8}\arctan(x) + \frac{1}{16}\log(x + 1) - \frac{1}{16}\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] $-1/4*x^3/(x^4 - 1) - 1/8*\arctan(x) + 1/16*\log(x + 1) - 1/16*\log(x - 1)$

mupad [B] time = 0.03, size = 23, normalized size = 0.79

$$\frac{\operatorname{atanh}(x)}{8} - \frac{\operatorname{atan}(x)}{8} - \frac{x^3}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x^8 - 2*x^4 + 1),x)`

[Out] $\operatorname{atanh}(x)/8 - \operatorname{atan}(x)/8 - x^3/(4*(x^4 - 1))$

sympy [A] time = 0.15, size = 27, normalized size = 0.93

$$-\frac{x^3}{4x^4 - 4} - \frac{\log(x - 1)}{16} + \frac{\log(x + 1)}{16} - \frac{\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(x**8-2*x**4+1),x)`

[Out] $-x**3/(4*x**4 - 4) - \log(x - 1)/16 + \log(x + 1)/16 - \operatorname{atan}(x)/8$

$$3.248 \quad \int \frac{1}{1-2x^4+x^8} dx$$

Optimal. Leaf size=27

$$\frac{x}{4(1-x^4)} + \frac{3}{8} \tan^{-1}(x) + \frac{3}{8} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {28, 199, 212, 206, 203}

$$\frac{x}{4(1-x^4)} + \frac{3}{8} \tan^{-1}(x) + \frac{3}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x^4 + x^8)^(-1), x]

[Out] x/(4*(1 - x^4)) + (3*ArcTan[x])/8 + (3*ArcTanh[x])/8

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 199

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{1 - 2x^4 + x^8} dx &= \int \frac{1}{(-1 + x^4)^2} dx \\ &= \frac{x}{4(1 - x^4)} - \frac{3}{4} \int \frac{1}{-1 + x^4} dx \\ &= \frac{x}{4(1 - x^4)} + \frac{3}{8} \int \frac{1}{1 - x^2} dx + \frac{3}{8} \int \frac{1}{1 + x^2} dx \\ &= \frac{x}{4(1 - x^4)} + \frac{3}{8} \tan^{-1}(x) + \frac{3}{8} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.22

$$\frac{1}{16} \left(-\frac{4x}{x^4 - 1} - 3 \log(1 - x) + 3 \log(x + 1) + 6 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x^4 + x^8)^(-1), x]

[Out] ((-4*x)/(-1 + x^4) + 6*ArcTan[x] - 3*Log[1 - x] + 3*Log[1 + x])/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1 - 2x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 2*x^4 + x^8)^(-1), x]

[Out] IntegrateAlgebraic[(1 - 2*x^4 + x^8)^(-1), x]

fricas [B] time = 1.30, size = 44, normalized size = 1.63

$$\frac{6(x^4 - 1) \arctan(x) + 3(x^4 - 1) \log(x + 1) - 3(x^4 - 1) \log(x - 1) - 4x}{16(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] 1/16*(6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)*log(x - 1) - 4*x)/(x^4 - 1)

giac [A] time = 0.37, size = 29, normalized size = 1.07

$$-\frac{x}{4(x^4 - 1)} + \frac{3}{8} \arctan(x) + \frac{3}{16} \log(|x + 1|) - \frac{3}{16} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(abs(x + 1)) - 3/16*log(abs(x - 1))

maple [A] time = 0.01, size = 42, normalized size = 1.56

$$\frac{x}{8x^2 + 8} + \frac{3 \arctan(x)}{8} - \frac{3 \ln(x - 1)}{16} + \frac{3 \ln(x + 1)}{16} - \frac{1}{16(x + 1)} - \frac{1}{16(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8-2*x^4+1),x)

[Out] -1/16/(x+1)+3/16*ln(x+1)+1/8/(x^2+1)*x+3/8*arctan(x)-1/16/(x-1)-3/16*ln(x-1)

maxima [A] time = 2.13, size = 27, normalized size = 1.00

$$-\frac{x}{4(x^4 - 1)} + \frac{3}{8} \arctan(x) + \frac{3}{16} \log(x + 1) - \frac{3}{16} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(x + 1) - 3/16*log(x - 1)

mupad [B] time = 0.03, size = 21, normalized size = 0.78

$$\frac{3 \operatorname{atan}(x)}{8} + \frac{3 \operatorname{atanh}(x)}{8} - \frac{x}{4(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^8 - 2*x^4 + 1),x)`

[Out] `(3*atan(x))/8 + (3*atanh(x))/8 - x/(4*(x^4 - 1))`

sympy [A] time = 0.16, size = 31, normalized size = 1.15

$$-\frac{x}{4x^4 - 4} - \frac{3 \log(x - 1)}{16} + \frac{3 \log(x + 1)}{16} + \frac{3 \operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**8-2*x**4+1),x)`

[Out] `-x/(4*x**4 - 4) - 3*log(x - 1)/16 + 3*log(x + 1)/16 + 3*atan(x)/8`

$$3.249 \quad \int \frac{1}{x^2(1-2x^4+x^8)} dx$$

Optimal. Leaf size=36

$$\frac{1}{4x(1-x^4)} - \frac{5}{4x} - \frac{5}{8} \tan^{-1}(x) + \frac{5}{8} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {28, 290, 325, 298, 203, 206}

$$\frac{1}{4x(1-x^4)} - \frac{5}{4x} - \frac{5}{8} \tan^{-1}(x) + \frac{5}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 - 2*x^4 + x^8)),x]

[Out] -5/(4*x) + 1/(4*x*(1 - x^4)) - (5*ArcTan[x])/8 + (5*ArcTanh[x])/8

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m + n*(p+1) + 1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

]

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2(1-2x^4+x^8)} dx &= \int \frac{1}{x^2(-1+x^4)^2} dx \\
 &= \frac{1}{4x(1-x^4)} - \frac{5}{4} \int \frac{1}{x^2(-1+x^4)} dx \\
 &= -\frac{5}{4x} + \frac{1}{4x(1-x^4)} - \frac{5}{4} \int \frac{x^2}{-1+x^4} dx \\
 &= -\frac{5}{4x} + \frac{1}{4x(1-x^4)} + \frac{5}{8} \int \frac{1}{1-x^2} dx - \frac{5}{8} \int \frac{1}{1+x^2} dx \\
 &= -\frac{5}{4x} + \frac{1}{4x(1-x^4)} - \frac{5}{8} \tan^{-1}(x) + \frac{5}{8} \tanh^{-1}(x)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.11

$$\frac{1}{16} \left(-\frac{4x^3}{x^4-1} - \frac{16}{x} - 5 \log(1-x) + 5 \log(x+1) - 10 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(1 - 2*x^4 + x^8)),x]

[Out] $(-16/x - (4*x^3)/(-1 + x^4) - 10*\text{ArcTan}[x] - 5*\text{Log}[1 - x] + 5*\text{Log}[1 + x])/16$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1 - 2x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(1 - 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^2*(1 - 2*x^4 + x^8)), x]

fricas [B] time = 1.26, size = 55, normalized size = 1.53

$$\frac{20x^4 + 10(x^5 - x)\arctan(x) - 5(x^5 - x)\log(x + 1) + 5(x^5 - x)\log(x - 1) - 16}{16(x^5 - x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] $-1/16*(20*x^4 + 10*(x^5 - x)*\arctan(x) - 5*(x^5 - x)*\log(x + 1) + 5*(x^5 - x)*\log(x - 1) - 16)/(x^5 - x)$

giac [A] time = 0.37, size = 37, normalized size = 1.03

$$-\frac{5x^4 - 4}{4(x^5 - x)} - \frac{5}{8}\arctan(x) + \frac{5}{16}\log(|x + 1|) - \frac{5}{16}\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="giac")

[Out] $-1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*\arctan(x) + 5/16*\log(\text{abs}(x + 1)) - 5/16*\log(\text{abs}(x - 1))$

maple [A] time = 0.01, size = 47, normalized size = 1.31

$$-\frac{x}{8(x^2 + 1)} - \frac{5\arctan(x)}{8} - \frac{5\ln(x - 1)}{16} + \frac{5\ln(x + 1)}{16} - \frac{1}{x} - \frac{1}{16(x + 1)} - \frac{1}{16(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(x^8-2*x^4+1),x)

[Out] $-1/x - 1/16/(x+1) + 5/16*\ln(x+1) - 1/8/(x^2+1)*x - 5/8*\arctan(x) - 1/16/(x-1) - 5/16*\ln(x-1)$

maxima [A] time = 2.09, size = 35, normalized size = 0.97

$$-\frac{5x^4 - 4}{4(x^5 - x)} - \frac{5}{8} \arctan(x) + \frac{5}{16} \log(x + 1) - \frac{5}{16} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="maxima")`

[Out] $-1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*\arctan(x) + 5/16*\log(x + 1) - 5/16*\log(x - 1)$

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{5 \operatorname{atanh}(x)}{8} - \frac{5 \operatorname{atan}(x)}{8} + \frac{\frac{5x^4}{4} - 1}{x - x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(x^8 - 2*x^4 + 1)),x)`

[Out] $(5*\operatorname{atanh}(x))/8 - (5*\operatorname{atan}(x))/8 + ((5*x^4)/4 - 1)/(x - x^5)$

sympy [A] time = 0.18, size = 37, normalized size = 1.03

$$\frac{4 - 5x^4}{4x^5 - 4x} - \frac{5 \log(x - 1)}{16} + \frac{5 \log(x + 1)}{16} - \frac{5 \operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(x**8-2*x**4+1),x)`

[Out] $(4 - 5*x**4)/(4*x**5 - 4*x) - 5*\log(x - 1)/16 + 5*\log(x + 1)/16 - 5*\operatorname{atan}(x)/8$

$$3.250 \quad \int \frac{1}{x^4(1-2x^4+x^8)} dx$$

Optimal. Leaf size=36

$$-\frac{7}{12x^3} + \frac{1}{4x^3(1-x^4)} + \frac{7}{8} \tan^{-1}(x) + \frac{7}{8} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {28, 290, 325, 212, 206, 203}

$$\frac{1}{4x^3(1-x^4)} - \frac{7}{12x^3} + \frac{7}{8} \tan^{-1}(x) + \frac{7}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 - 2*x^4 + x^8)),x]

[Out] -7/(12*x^3) + 1/(4*x^3*(1 - x^4)) + (7*ArcTan[x])/8 + (7*ArcTanh[x])/8

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(1-2x^4+x^8)} dx &= \int \frac{1}{x^4(-1+x^4)^2} dx \\
&= \frac{1}{4x^3(1-x^4)} - \frac{7}{4} \int \frac{1}{x^4(-1+x^4)} dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1-x^4)} - \frac{7}{4} \int \frac{1}{-1+x^4} dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1-x^4)} + \frac{7}{8} \int \frac{1}{1-x^2} dx + \frac{7}{8} \int \frac{1}{1+x^2} dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1-x^4)} + \frac{7}{8} \tan^{-1}(x) + \frac{7}{8} \tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.06

$$\frac{1}{48} \left(-\frac{12x}{x^4-1} - \frac{16}{x^3} - 21 \log(1-x) + 21 \log(x+1) + 42 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x^3 - (12*x)/(-1 + x^4) + 42*ArcTan[x] - 21*Log[1 - x] + 21*Log[1 + x])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(1 - 2x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(1 - 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^4*(1 - 2*x^4 + x^8)), x]

fricas [B] time = 1.22, size = 63, normalized size = 1.75

$$\frac{28x^4 - 42(x^7 - x^3)\arctan(x) - 21(x^7 - x^3)\log(x + 1) + 21(x^7 - x^3)\log(x - 1) - 16}{48(x^7 - x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] -1/48*(28*x^4 - 42*(x^7 - x^3)*arctan(x) - 21*(x^7 - x^3)*log(x + 1) + 21*(x^7 - x^3)*log(x - 1) - 16)/(x^7 - x^3)

giac [A] time = 0.46, size = 34, normalized size = 0.94

$$-\frac{x}{4(x^4 - 1)} - \frac{1}{3x^3} + \frac{7}{8}\arctan(x) + \frac{7}{16}\log(|x + 1|) - \frac{7}{16}\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4*x/(x^4 - 1) - 1/3/x^3 + 7/8*arctan(x) + 7/16*log(abs(x + 1)) - 7/16*log(abs(x - 1))

maple [A] time = 0.01, size = 47, normalized size = 1.31

$$\frac{x}{8x^2 + 8} + \frac{7\arctan(x)}{8} - \frac{7\ln(x - 1)}{16} + \frac{7\ln(x + 1)}{16} - \frac{1}{3x^3} - \frac{1}{16(x + 1)} - \frac{1}{16(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(x^8-2*x^4+1),x)

[Out] -1/3/x^3-1/16/(x+1)+7/16*ln(x+1)+1/8/(x^2+1)*x+7/8*arctan(x)-1/16/(x-1)-7/16*ln(x-1)

maxima [A] time = 1.98, size = 37, normalized size = 1.03

$$-\frac{7x^4 - 4}{12(x^7 - x^3)} + \frac{7}{8} \arctan(x) + \frac{7}{16} \log(x + 1) - \frac{7}{16} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="maxima")

[Out] -1/12*(7*x^4 - 4)/(x^7 - x^3) + 7/8*arctan(x) + 7/16*log(x + 1) - 7/16*log(x - 1)

mupad [B] time = 1.29, size = 28, normalized size = 0.78

$$\frac{7 \operatorname{atan}(x)}{8} + \frac{7 \operatorname{atanh}(x)}{8} + \frac{\frac{7x^4}{12} - \frac{1}{3}}{x^3 - x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(x^8 - 2*x^4 + 1)),x)

[Out] (7*atan(x))/8 + (7*atanh(x))/8 + ((7*x^4)/12 - 1/3)/(x^3 - x^7)

sympy [A] time = 0.18, size = 39, normalized size = 1.08

$$\frac{4 - 7x^4}{12x^7 - 12x^3} - \frac{7 \log(x - 1)}{16} + \frac{7 \log(x + 1)}{16} + \frac{7 \operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(x**8-2*x**4+1),x)

[Out] (4 - 7*x**4)/(12*x**7 - 12*x**3) - 7*log(x - 1)/16 + 7*log(x + 1)/16 + 7*atan(x)/8

$$3.251 \quad \int \frac{1}{x^6(1-2x^4+x^8)} dx$$

Optimal. Leaf size=43

$$-\frac{9}{20x^5} + \frac{1}{4x^5(1-x^4)} - \frac{9}{4x} - \frac{9}{8} \tan^{-1}(x) + \frac{9}{8} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {28, 290, 325, 298, 203, 206}

$$\frac{1}{4x^5(1-x^4)} - \frac{9}{20x^5} - \frac{9}{4x} - \frac{9}{8} \tan^{-1}(x) + \frac{9}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(1 - 2*x^4 + x^8)),x]

[Out] -9/(20*x^5) - 9/(4*x) + 1/(4*x^5*(1 - x^4)) - (9*ArcTan[x])/8 + (9*ArcTanh[x])/8

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^ (p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 290

Int[((c_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_.))^ (p_.), x_Symbol] :> -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b}

, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^6(1-2x^4+x^8)} dx &= \int \frac{1}{x^6(-1+x^4)^2} dx \\
 &= \frac{1}{4x^5(1-x^4)} - \frac{9}{4} \int \frac{1}{x^6(-1+x^4)} dx \\
 &= -\frac{9}{20x^5} + \frac{1}{4x^5(1-x^4)} - \frac{9}{4} \int \frac{1}{x^2(-1+x^4)} dx \\
 &= -\frac{9}{20x^5} - \frac{9}{4x} + \frac{1}{4x^5(1-x^4)} - \frac{9}{4} \int \frac{x^2}{-1+x^4} dx \\
 &= -\frac{9}{20x^5} - \frac{9}{4x} + \frac{1}{4x^5(1-x^4)} + \frac{9}{8} \int \frac{1}{1-x^2} dx - \frac{9}{8} \int \frac{1}{1+x^2} dx \\
 &= -\frac{9}{20x^5} - \frac{9}{4x} + \frac{1}{4x^5(1-x^4)} - \frac{9}{8} \tan^{-1}(x) + \frac{9}{8} \tanh^{-1}(x)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.19

$$-\frac{1}{5x^5} - \frac{x^3}{4(x^4-1)} - \frac{2}{x} - \frac{9}{16} \log(1-x) + \frac{9}{16} \log(x+1) - \frac{9}{8} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(1 - 2*x^4 + x^8)),x]

[Out] -1/5*1/x^5 - 2/x - x^3/(4*(-1 + x^4)) - (9*ArcTan[x])/8 - (9*Log[1 - x])/16 + (9*Log[1 + x])/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6(1 - 2x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^6*(1 - 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^6*(1 - 2*x^4 + x^8)), x]

fricas [B] time = 1.36, size = 68, normalized size = 1.58

$$\frac{180x^8 - 144x^4 + 90(x^9 - x^5) \arctan(x) - 45(x^9 - x^5) \log(x+1) + 45(x^9 - x^5) \log(x-1) - 16}{80(x^9 - x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] -1/80*(180*x^8 - 144*x^4 + 90*(x^9 - x^5)*arctan(x) - 45*(x^9 - x^5)*log(x + 1) + 45*(x^9 - x^5)*log(x - 1) - 16)/(x^9 - x^5)

giac [A] time = 0.45, size = 43, normalized size = 1.00

$$-\frac{x^3}{4(x^4 - 1)} - \frac{10x^4 + 1}{5x^5} - \frac{9}{8} \arctan(x) + \frac{9}{16} \log(|x + 1|) - \frac{9}{16} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4*x^3/(x^4 - 1) - 1/5*(10*x^4 + 1)/x^5 - 9/8*arctan(x) + 9/16*log(abs(x + 1)) - 9/16*log(abs(x - 1))

maple [A] time = 0.02, size = 52, normalized size = 1.21

$$-\frac{x}{8(x^2 + 1)} - \frac{9 \arctan(x)}{8} - \frac{9 \ln(x - 1)}{16} + \frac{9 \ln(x + 1)}{16} - \frac{2}{x} - \frac{1}{5x^5} - \frac{1}{16(x + 1)} - \frac{1}{16(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(x^8-2*x^4+1),x)`

[Out] $-1/5/x^5-2/x-1/16/(x+1)+9/16*\ln(x+1)-1/8/(x^2+1)*x-9/8*\arctan(x)-1/16/(x-1)-9/16*\ln(x-1)$

maxima [A] time = 2.43, size = 42, normalized size = 0.98

$$-\frac{45x^8 - 36x^4 - 4}{20(x^9 - x^5)} - \frac{9}{8} \arctan(x) + \frac{9}{16} \log(x + 1) - \frac{9}{16} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="maxima")`

[Out] $-1/20*(45*x^8 - 36*x^4 - 4)/(x^9 - x^5) - 9/8*\arctan(x) + 9/16*\log(x + 1) - 9/16*\log(x - 1)$

mupad [B] time = 0.04, size = 34, normalized size = 0.79

$$\frac{9 \operatorname{atanh}(x)}{8} - \frac{9 \operatorname{atan}(x)}{8} - \frac{-\frac{9x^8}{4} + \frac{9x^4}{5} + \frac{1}{5}}{x^5 - x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^6*(x^8 - 2*x^4 + 1)),x)`

[Out] $(9*\operatorname{atanh}(x))/8 - (9*\operatorname{atan}(x))/8 - ((9*x^4)/5 - (9*x^8)/4 + 1/5)/(x^5 - x^9)$

sympy [A] time = 0.20, size = 44, normalized size = 1.02

$$-\frac{9 \log(x - 1)}{16} + \frac{9 \log(x + 1)}{16} - \frac{9 \operatorname{atan}(x)}{8} + \frac{-45x^8 + 36x^4 + 4}{20x^9 - 20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(x**8-2*x**4+1),x)`

[Out] $-9*\log(x - 1)/16 + 9*\log(x + 1)/16 - 9*\operatorname{atan}(x)/8 + (-45*x**8 + 36*x**4 + 4)/(20*x**9 - 20*x**5)$

$$3.252 \quad \int \frac{1}{x^8(1-2x^4+x^8)} dx$$

Optimal. Leaf size=43

$$-\frac{11}{28x^7} - \frac{11}{12x^3} + \frac{1}{4x^7(1-x^4)} + \frac{11}{8} \tan^{-1}(x) + \frac{11}{8} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {28, 290, 325, 212, 206, 203}

$$\frac{1}{4x^7(1-x^4)} - \frac{11}{12x^3} - \frac{11}{28x^7} + \frac{11}{8} \tan^{-1}(x) + \frac{11}{8} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^8*(1 - 2*x^4 + x^8)),x]

[Out] -11/(28*x^7) - 11/(12*x^3) + 1/(4*x^7*(1 - x^4)) + (11*ArcTan[x])/8 + (11*ArcTanh[x])/8

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ

[a/b, 0]

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8(1-2x^4+x^8)} dx &= \int \frac{1}{x^8(-1+x^4)^2} dx \\
&= \frac{1}{4x^7(1-x^4)} - \frac{11}{4} \int \frac{1}{x^8(-1+x^4)} dx \\
&= -\frac{11}{28x^7} + \frac{1}{4x^7(1-x^4)} - \frac{11}{4} \int \frac{1}{x^4(-1+x^4)} dx \\
&= -\frac{11}{28x^7} - \frac{11}{12x^3} + \frac{1}{4x^7(1-x^4)} - \frac{11}{4} \int \frac{1}{-1+x^4} dx \\
&= -\frac{11}{28x^7} - \frac{11}{12x^3} + \frac{1}{4x^7(1-x^4)} + \frac{11}{8} \int \frac{1}{1-x^2} dx + \frac{11}{8} \int \frac{1}{1+x^2} dx \\
&= -\frac{11}{28x^7} - \frac{11}{12x^3} + \frac{1}{4x^7(1-x^4)} + \frac{11}{8} \tan^{-1}(x) + \frac{11}{8} \tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{1}{336} \left(-\frac{48}{x^7} - \frac{84x}{x^4-1} - \frac{224}{x^3} - 231 \log(1-x) + 231 \log(x+1) + 462 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^8*(1 - 2*x^4 + x^8)),x]

[Out] (-48/x^7 - 224/x^3 - (84*x)/(-1 + x^4) + 462*ArcTan[x] - 231*Log[1 - x] + 231*Log[1 + x])/336

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8(1 - 2x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^8*(1 - 2*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^8*(1 - 2*x^4 + x^8)), x]

fricas [B] time = 1.18, size = 68, normalized size = 1.58

$$\frac{308x^8 - 176x^4 - 462(x^{11} - x^7) \arctan(x) - 231(x^{11} - x^7) \log(x+1) + 231(x^{11} - x^7) \log(x-1) - 48}{336(x^{11} - x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="fricas")

[Out] -1/336*(308*x^8 - 176*x^4 - 462*(x^11 - x^7)*arctan(x) - 231*(x^11 - x^7)*log(x + 1) + 231*(x^11 - x^7)*log(x - 1) - 48)/(x^11 - x^7)

giac [A] time = 0.45, size = 41, normalized size = 0.95

$$-\frac{x}{4(x^4 - 1)} - \frac{14x^4 + 3}{21x^7} + \frac{11}{8} \arctan(x) + \frac{11}{16} \log(|x + 1|) - \frac{11}{16} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="giac")

[Out] -1/4*x/(x^4 - 1) - 1/21*(14*x^4 + 3)/x^7 + 11/8*arctan(x) + 11/16*log(abs(x + 1)) - 11/16*log(abs(x - 1))

maple [A] time = 0.02, size = 52, normalized size = 1.21

$$\frac{x}{8x^2 + 8} + \frac{11 \arctan(x)}{8} - \frac{11 \ln(x - 1)}{16} + \frac{11 \ln(x + 1)}{16} - \frac{2}{3x^3} - \frac{1}{7x^7} - \frac{1}{16(x + 1)} - \frac{1}{16(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^8/(x^8-2*x^4+1),x)`

[Out] $-1/7/x^7-2/3/x^3-1/16/(x+1)+11/16*\ln(x+1)+1/8/(x^2+1)*x+11/8*\arctan(x)-1/16/(x-1)-11/16*\ln(x-1)$

maxima [A] time = 1.94, size = 42, normalized size = 0.98

$$-\frac{77x^8 - 44x^4 - 12}{84(x^{11} - x^7)} + \frac{11}{8} \arctan(x) + \frac{11}{16} \log(x+1) - \frac{11}{16} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="maxima")`

[Out] $-1/84*(77*x^8 - 44*x^4 - 12)/(x^{11} - x^7) + 11/8*\arctan(x) + 11/16*\log(x + 1) - 11/16*\log(x - 1)$

mupad [B] time = 0.05, size = 34, normalized size = 0.79

$$\frac{11 \operatorname{atan}(x)}{8} + \frac{11 \operatorname{atanh}(x)}{8} - \frac{-\frac{11x^8}{12} + \frac{11x^4}{21} + \frac{1}{7}}{x^7 - x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^8*(x^8 - 2*x^4 + 1)),x)`

[Out] $(11*\operatorname{atan}(x))/8 + (11*\operatorname{atanh}(x))/8 - ((11*x^4)/21 - (11*x^8)/12 + 1/7)/(x^7 - x^{11})$

sympy [A] time = 0.21, size = 44, normalized size = 1.02

$$-\frac{11 \log(x-1)}{16} + \frac{11 \log(x+1)}{16} + \frac{11 \operatorname{atan}(x)}{8} + \frac{-77x^8 + 44x^4 + 12}{84x^{11} - 84x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**8/(x**8-2*x**4+1),x)`

[Out] $-11*\log(x - 1)/16 + 11*\log(x + 1)/16 + 11*\operatorname{atan}(x)/8 + (-77*x**8 + 44*x**4 + 12)/(84*x**11 - 84*x**7)$

$$3.253 \quad \int \frac{x^{11}}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=81

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx^4 + cx^8)}{8c^2} + \frac{x^4}{4c}$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx^4 + cx^8)}{8c^2} + \frac{x^4}{4c}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a + b*x^4 + c*x^8), x]

[Out] x^4/(4*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c^2 * Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(8*c^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4ac]$

Rule 703

$\text{Int}[(d + e x)^m / (a + b x + c x^2), x_Symbol] \rightarrow \text{Simp}[e(d + e x)^{m-1} / (c(m-1)), x] + \text{Dist}[1/c, \text{Int}[(d + e x)^{m-2} \text{Simp}[c d^2 - a e^2 + e(2cd - b e)x, x] / (a + b x + c x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[c d^2 - b d e + a e^2, 0] \ \&\& \ \text{NeQ}[2cd - b e, 0] \ \&\& \ \text{GtQ}[m, 1]$

Rule 1357

$\text{Int}[x^m ((a + c x^{n_2}) + (b x^{n_1}))^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{\text{Simplify}[(m+1)/n] - 1} (a + b x + c x^2)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, m, n, p, x\} \ \&\& \ \text{EqQ}[n^2, 2n] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^{11}}{a + bx^4 + cx^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{a + bx + cx^2} dx, x, x^4 \right) \\ &= \frac{x^4}{4c} + \frac{\text{Subst} \left(\int \frac{-a - bx}{a + bx + cx^2} dx, x, x^4 \right)}{4c} \\ &= \frac{x^4}{4c} - \frac{b \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^4 \right)}{8c^2} + \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^4 \right)}{8c^2} \\ &= \frac{x^4}{4c} - \frac{b \log(a + bx^4 + cx^8)}{8c^2} - \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^4 \right)}{4c^2} \\ &= \frac{x^4}{4c} - \frac{(b^2 - 2ac) \tanh^{-1} \left(\frac{b + 2cx^4}{\sqrt{b^2 - 4ac}} \right)}{4c^2 \sqrt{b^2 - 4ac}} - \frac{b \log(a + bx^4 + cx^8)}{8c^2} \end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.96

$$\frac{2(b^2 - 2ac) \tan^{-1} \left(\frac{b + 2cx^4}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}} - \frac{b \log(a + bx^4 + cx^8) + 2cx^4}{8c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x¹¹/(a + b*x⁴ + c*x⁸),x]

[Out] (2*c*x⁴ + (2*(b² - 2*a*c)*ArcTan[(b + 2*c*x⁴)/Sqrt[-b² + 4*a*c]])/Sqrt[-b² + 4*a*c] - b*Log[a + b*x⁴ + c*x⁸]/(8*c²)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x¹¹/(a + b*x⁴ + c*x⁸),x]

[Out] IntegrateAlgebraic[x¹¹/(a + b*x⁴ + c*x⁸), x]

fricas [A] time = 1.37, size = 254, normalized size = 3.14

$$\left[\frac{2(b^2c - 4ac^2)x^4 - (b^2 - 2ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^8 + 2bcx^4 + b^2 - 2ac + (2cx^4 + b)\sqrt{b^2 - 4ac}}{cx^8 + bx^4 + a}\right) - (b^3 - 4abc) \log(cx^8 + bx^4 + a)}{8(b^2c^2 - 4ac^3)}, \frac{2(b^2c - 4ac^2)x^4 - 2(b^2 - 2ac)\sqrt{-b^2 + 4ac} \arctan\left(\frac{(2cx^4 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) - (b^3 - 4abc) \log(cx^8 + bx^4 + a)}{8(b^2c^2 - 4ac^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(c*x⁸+b*x⁴+a),x, algorithm="fricas")

[Out] [1/8*(2*(b²*c - 4*a*c²)*x⁴ - (b² - 2*a*c)*sqrt(b² - 4*a*c)*log((2*c²*x⁸ + 2*b*c*x⁴ + b² - 2*a*c + (2*c*x⁴ + b)*sqrt(b² - 4*a*c))/(c*x⁸ + b*x⁴ + a) - (b³ - 4*a*b*c)*log(c*x⁸ + b*x⁴ + a))/(b²*c² - 4*a*c³), 1/8*(2*(b²*c - 4*a*c²)*x⁴ - 2*(b² - 2*a*c)*sqrt(-b² + 4*a*c)*arctan(-(2*c*x⁴ + b)*sqrt(-b² + 4*a*c)/(b² - 4*a*c)) - (b³ - 4*a*b*c)*log(c*x⁸ + b*x⁴ + a))/(b²*c² - 4*a*c³)]

giac [A] time = 17.07, size = 75, normalized size = 0.93

$$\frac{x^4}{4c} - \frac{b \log(cx^8 + bx^4 + a)}{8c^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^4 + b}{\sqrt{-b^2 + 4ac}}\right)}{4\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(c*x⁸+b*x⁴+a),x, algorithm="giac")

[Out] 1/4*x⁴/c - 1/8*b*log(c*x⁸ + b*x⁴ + a)/c² + 1/4*(b² - 2*a*c)*arctan((2*c*x⁴ + b)/sqrt(-b² + 4*a*c))/(sqrt(-b² + 4*a*c)*c²)

maple [A] time = 0.01, size = 111, normalized size = 1.37

$$\frac{x^4}{4c} - \frac{a \arctan\left(\frac{2cx^4 + b}{\sqrt{4ac - b^2}}\right)}{2\sqrt{4ac - b^2}c} + \frac{b^2 \arctan\left(\frac{2cx^4 + b}{\sqrt{4ac - b^2}}\right)}{4\sqrt{4ac - b^2}c^2} - \frac{b \ln(cx^8 + bx^4 + a)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{11}/(c*x^8+b*x^4+a), x)$

[Out] $\frac{1}{4}x^4/c - \frac{1}{8}b*\ln(c*x^8+b*x^4+a)/c^2 - \frac{1}{2}c/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^4+b)/(4*a*c-b^2)^{(1/2)})*a + \frac{1}{4}c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^4+b)/(4*a*c-b^2)^{(1/2)})*b^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{11}/(c*x^8+b*x^4+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.69, size = 3916, normalized size = 48.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{11}/(a + b*x^4 + c*x^8), x)$

[Out] $x^4/(4*c) + (\log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))/(2*(64*a*c^3 - 16*b^2*c^2)) - (\text{atan}((8*c^4*x^4*((a*c - b^2)*(((2*a*c - b^2)*(((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))* (2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^{(1/2)} + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^{(1/2)}*(64*a*c^3 - 16*b^2*c^2)))))/(8*c^2*(4*a*c - b^2)^{(1/2)} + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^{(1/2)})) - ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))* (2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^{(1/2)} + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^{(1/2)}*(64*a*c^3 - 16*b^2*c^2)))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)))/ (8*c^2*(4*a*c - b^2)^{(1/2)))/ (2*(64*a*c^3 - 16*b^2*c^2)) + (((8*a^3*c^5 - 20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)))/ (2*(64*a*c^3 - 16*b^2*c^2)))* (2*$

$$\begin{aligned}
& a*c - b^2)) / (8*c^2*(4*a*c - b^2)^{(1/2)}) + (b^3*(4*b^3 - 16*a*b*c)*(2*a*c - \\
& b^2)^3) / (2*c^2*(4*a*c - b^2)^{(3/2)}*(64*a*c^3 - 16*b^2*c^2))) / (8*a^3*c^2) - \\
& ((b^3 - 3*a*b*c)*(((4*b^3 - 16*a*b*c)*((8*a^3*c^5 - 20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c)))/(64*a*c^3 - 16*b^2*c^2)))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))) / (2*(64*a*c^3 - 16*b^2*c^2)) - (b^7 - a^3*b*c^3 + 3*a^2*b^3*c^2 - 3*a*b^5*c)/c^4 + ((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*(((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c)))/(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2)))/(8*c^2*(4*a*c - b^2)^{(1/2)}) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^{(1/2)}*(64*a*c^3 - 16*b^2*c^2))) / (8*c^2*(4*a*c - b^2)^{(1/2)}) + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2))) / (2*(64*a*c^3 - 16*b^2*c^2)) + (((4*b^3 - 16*a*b*c)*(((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c)))/(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2)))/(8*c^2*(4*a*c - b^2)^{(1/2)}) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^{(1/2)}*(64*a*c^3 - 16*b^2*c^2))) / (2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c)))/(64*a*c^3 - 16*b^2*c^2)))) / (2*(64*a*c^3 - 16*b^2*c^2))) / (8*c^2*(4*a*c - b^2)^{(1/2)))*(2*a*c - b^2)) / (8*c^2*(4*a*c - b^2)^{(1/2)}) - (b^3*(2*a*c - b^2)^4)/(8*c^4*(4*a*c - b^2)^2))) / (8*a^3*c^2*(4*a*c - b^2)^{(1/2)))*(4*a*c - b^2)^2)/(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b^6*c) - (c^2*(a*c - b^2)*(4*a*c - b^2)^2*((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c)))/(64*a*c^3 - 16*b^2*c^2)))))/(8*c^2*(4*a*c - b^2)^{(1/2)}) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^{(1/2)}*(64*a*c^3 - 16*b^2*c^2)))) / (2*(64*a*c^3 - 16*b^2*c^2)) + (((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c)))/(64*a*c^3 - 16*b^2*c^2)))) / (2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2)) / (8*c^2*(4*a*c - b^2)^{(1/2))))) / (2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((24*a*b^5*c^2 + 16*a^3*b*c^4 - 40*a^2*b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c)))/(64*a*c^3 - 16*b^2*c^2)))) / (2*(64*a*c^3 - 16*b^2*c^2)))) / (2*(64*a*c^3 - 16*b^2*c^2))) / (8*c^2*(4*a*c - b^2)^{(1/2)}) - ((2*a*c - b^2)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c)))/(64*a*c^3 - 16*b^2*c^2)))) / (8*c^2*(4*a*c - b^2)^{(1/2)}) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^{(1/2)}*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2)) / (8*c^2*(4*a*c - b^2)^{(1/2)}) + (8*a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2))) / (8*c^2*(4*a*c - b^2)^{(1/2)}) - (a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(c^2*(4*a*c - b^2)^{(3/2)}*(64*a*c^3 - 16*b^2*c^2))) / (a^3*(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b^6*c)) + (c^2*(4*a*c - b^2)^{(3/2)}*(b^3 - 3*a*b*c)*((a*b^6 - 2*a^2*b^4*c + a^3
\end{aligned}$$

$$\begin{aligned}
& *b^2*c^2)/c^4 + ((4*b^3 - 16*a*b*c)*((24*a*b^5*c^2 + 16*a^3*b*c^4 - 40*a^2* \\
& b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b \\
& ^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (5 \\
& 12*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 1 \\
& 6*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)) - (\\
& (4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + \\
& (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c \\
& - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - \\
& b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2)/(8*c^2*(4*a*c - b^2)^(1 \\
& /2)) + (8*a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^ \\
& 3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)) - ((2*a*c - b^2)*((4*b^3 - \\
& 16*a*b*c)*((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2 \\
& *c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/ \\
& 2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)* \\
& (64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((64*a^3*c^6 + 20 \\
& 8*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - \\
& 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2* \\
& c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/ \\
& 2)))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*b^2*(2*a*c - b^2)^4)/(4*c^4*(4*a*c - \\
& b^2)^2))/(a^3*(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b \\
& ^6*c)))*(2*a*c - b^2)/(4*c^2*(4*a*c - b^2)^(1/2))
\end{aligned}$$

sympy [B] time = 4.11, size = 316, normalized size = 3.90

$$\left(\frac{-b}{8c^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{8c^2(4ac - b^2)} \right) \log \left(x^4 + \frac{-ab - 16ac^2 \left(\frac{-b}{8c^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{8c^2(4ac - b^2)} \right) + 4b^2c \left(\frac{-b}{8c^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{8c^2(4ac - b^2)} \right)}{2ac - b^2} \right) + \left(\frac{-b}{8c^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{8c^2(4ac - b^2)} \right) \log \left(x^4 + \frac{-ab - 16ac^2 \left(\frac{-b}{8c^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{8c^2(4ac - b^2)} \right) + 4b^2c \left(\frac{-b}{8c^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{8c^2(4ac - b^2)} \right)}{2ac - b^2} \right) + \frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(c*x**8+b*x**4+a),x)

[Out] $(-b/(8*c**2) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))* \log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(8*c**2) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))* \log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**4/(4*c)$

$$3.254 \quad \int \frac{x^9}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=192

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x^2}{2c}$$

Rubi [A] time = 0.34, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1359, 1122, 1166, 205}

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a + b*x^4 + c*x^8), x]

[Out] x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1122

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m+4*p+1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x^9}{a + bx^4 + cx^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{a + bx^2 + cx^4} dx, x, x^2 \right) \\ &= \frac{x^2}{2c} - \frac{\text{Subst} \left(\int \frac{a+bx^2}{a+bx^2+cx^4} dx, x, x^2 \right)}{2c} \\ &= \frac{x^2}{2c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{Subst} \left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx, x, x^2 \right)}{4c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{Subst} \left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx, x, x^2 \right)}{4c} \\ &= \frac{x^2}{2c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}} \right)}{2\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 210, normalized size = 1.09

$$\frac{\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{4c^{3/2}} + 2\sqrt{c}x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^4 + c*x^8), x]
```

```
[Out] (2*Sqrt[c]*x^2 - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
```

$\text{Sqrt}[b^2 - 4*a*c]] - (\text{Sqrt}[2]*(b^2 - 2*a*c + b*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x^2)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]]) / (\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]) / (4*c^{(3/2)})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/(a + b*x^4 + c*x^8), x]

[Out] IntegrateAlgebraic[x^9/(a + b*x^4 + c*x^8), x]

fricas [B] time = 1.44, size = 1071, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(\text{sqrt}(1/2)*c*\text{sqrt}(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))))/(b^2*c^3 - 4*a*c^4))*\log(-(a*b^2 - a^2*c)*x^2 + 1/2*\text{sqrt}(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*\text{sqrt}(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))))/(b^2*c^3 - 4*a*c^4)) - \text{sqrt}(1/2)*c*\text{sqrt}(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))))/(b^2*c^3 - 4*a*c^4))*\log(-(a*b^2 - a^2*c)*x^2 - 1/2*\text{sqrt}(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*\text{sqrt}(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))))/(b^2*c^3 - 4*a*c^4)) + \text{sqrt}(1/2)*c*\text{sqrt}(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))))/(b^2*c^3 - 4*a*c^4))*\log(-(a*b^2 - a^2*c)*x^2 + 1/2*\text{sqrt}(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*\text{sqrt}(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))))/(b^2*c^3 - 4*a*c^4)) - \text{sqrt}(1/2)*c*\text{sqrt}(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))))/(b^2*c^3 - 4*a*c^4))*\log(-(a*b^2 - a^2*c)*x^2 - 1/2*\text{sqrt}(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*\text{sqrt}(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))))/(b^2*c^3 - 4*a*c^4)) - 2*x^2)/c \end{aligned}$$

giac [B] time = 18.17, size = 2043, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c + 1/8*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt
(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c))*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c))*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c))*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c))*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^3*c -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c))*a*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^3*c^2 - 16*a*b
^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b*c^3 + 32*a^2*b*c^3 -
2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 -
8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^
4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^2
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^3*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^2*c^3 - 2*(b^2 -
4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^4 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c))*a*b^3*c + 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c))*a^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*b*c^2 + sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c^2 - 16*a^2*b^2*c^2 - 4*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*c^3 + 32*a^3*c^3 - 2*(b^2 - 4*a*c)*a*b^
2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*abs(c))*arctan(2*sqrt(1/2)*x^2/sqrt((b*c + s
qrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*
a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*c^2) + 1/8*(2*a*b^3*c^3 - 8*
a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3
*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b*c^2
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^3 - 2*(b^2 -
4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^5 - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c))*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*b
*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^2 + sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c))*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c))*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 -
4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c))*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c))*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c))*b^2*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c))*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
```

) * c) * a^2 * b^2 * c - 2 * sqrt(2) * sqrt(b * c + sqrt(b^2 - 4 * a * c)) * c) * a * b^3 * c - 2 * a * b^4 * c + 16 * sqrt(2) * sqrt(b * c + sqrt(b^2 - 4 * a * c)) * c) * a^3 * c^2 + 8 * sqrt(2) * sqrt(b * c + sqrt(b^2 - 4 * a * c)) * c) * a^2 * b * c^2 + sqrt(2) * sqrt(b * c + sqrt(b^2 - 4 * a * c)) * c) * a * b^2 * c^2 + 16 * a^2 * b^2 * c^2 - 4 * sqrt(2) * sqrt(b * c + sqrt(b^2 - 4 * a * c)) * c) * a^2 * c^3 - 32 * a^3 * c^3 + 2 * (b^2 - 4 * a * c) * a * b^2 * c - 8 * (b^2 - 4 * a * c) * a^2 * c^2) * abs(c) * arctan(2 * sqrt(1/2) * x^2 / sqrt((b * c - sqrt(b^2 * c^2 - 4 * a * c^3)) / c^2)) / ((a * b^4 * c - 8 * a^2 * b^2 * c^2 - 2 * a * b^3 * c^2 + 16 * a^3 * c^3 + 8 * a^2 * b * c^3 + a * b^2 * c^3 - 4 * a^2 * c^4) * c^2)

maple [B] time = 0.03, size = 360, normalized size = 1.88

$$\frac{\sqrt{2} a \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2} a \operatorname{arctan}\left(\frac{\sqrt{2} c x^2}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} b^2 \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{4\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c}c} - \frac{\sqrt{2} b^2 \operatorname{arctan}\left(\frac{\sqrt{2} c x^2}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{4\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}c} + \frac{\sqrt{2} b \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{4\sqrt{(-b+\sqrt{-4ac+b^2})c}c} - \frac{\sqrt{2} b \operatorname{arctan}\left(\frac{\sqrt{2} c x^2}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{4\sqrt{(b+\sqrt{-4ac+b^2})c}c} + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(c*x^8+b*x^4+a), x)

[Out] 1/2/c*x^2+1/4/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/4/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-1/4/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/4/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(c*x^8+b*x^4+a), x, algorithm="maxima")

[Out] 1/2*x^2/c - integrate((b*x^4 + a)*x/(c*x^8 + b*x^4 + a), x)/c

mupad [B] time = 3.07, size = 5659, normalized size = 29.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(a + b*x^4 + c*x^8), x)

[Out] atan((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3

$$\begin{aligned}
 &(- (4*a*c - b^2)^3)^{(1/2)}) / (2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)) * (- \\
 &(b^5 + b^2*(- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(- (4*a \\
 &*c - b^2)^3)^{(1/2)}) / (32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (4*x \\
 &^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2) * (- (b^5 + b^2*(- (4 \\
 &*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(- (4*a*c - b^2)^3)^{(1 \\
 &/2)}) / (32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}) * (- (b^5 + b^2*(- (4*a* \\
 &c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(- (4*a*c - b^2)^3)^{(1/2)} \\
 &)/ (32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (4*x^2*(a^2*b^6 - 2*a^ \\
 &5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2) * (- (b^5 + b^2*(- (4*a*c - b^2)^3)^{(\\
 &1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (32*(16*a^ \\
 &2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}) * (- (b^5 + b^2*(- (4*a*c - b^2)^3)^{(1 \\
 &/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (32*(16*a^2* \\
 &c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}) * 2i + atan(((((16*(a*b^8 + 4*a^5*c^4 - \\
 &8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + ((((16*(32*a*b^7*c^3 - \\
 &256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 \\
 &- 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(- (4*a*c - b^2)^3)^{(1/2)} \\
 &+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(- (4*a*c - b^2)^3)^{(1/2)})) / (2*c^2*(16*a^2* \\
 &c^5 + b^4*c^3 - 8*a*b^2*c^4))) * (- (b^5 - b^2*(- (4*a*c - b^2)^3)^{(1/2)} + 12*a \\
 &^2*b*c^2 - 7*a*b^3*c + a*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (32*(16*a^2*c^5 + b^4* \\
 &c^3 - 8*a*b^2*c^4))^{(1/2)} - (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3 \\
 &*b^3*c^4))/c^2) * (- (b^5 - b^2*(- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a* \\
 &b^3*c + a*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^ \\
 &4))^{(1/2)}) * (- (b^5 - b^2*(- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3 \\
 &*c + a*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4) \\
 &))^{(1/2)} - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2) \\
 &* (- (b^5 - b^2*(- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(- (\\
 &4*a*c - b^2)^3)^{(1/2)}) / (32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}) * 1i \\
 &- ((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3) \\
 &)/c^2 + ((((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3 \\
 &*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - \\
 &b^2*(- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(- (4*a*c - b^ \\
 &2)^3)^{(1/2)})) / (2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))) * (- (b^5 - b^2*(- \\
 &(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(- (4*a*c - b^2)^3)^{(\\
 &1/2)}) / (32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (4*x^2*(64*a^4*b* \\
 &c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2) * (- (b^5 - b^2*(- (4*a*c - b^2)^3) \\
 &)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (32*(16* \\
 &a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}) * (- (b^5 - b^2*(- (4*a*c - b^2)^3)^{(\\
 &1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (32*(16*a^2 \\
 &*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)} + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3 \\
 &*b^4*c + 6*a^4*b^2*c^2))/c^2) * (- (b^5 - b^2*(- (4*a*c - b^2)^3)^{(1/2)} + 12*a^ \\
 &2*b*c^2 - 7*a*b^3*c + a*c*(- (4*a*c - b^2)^3)^{(1/2)}) / (32*(16*a^2*c^5 + b^4*c \\
 &^3 - 8*a*b^2*c^4))^{(1/2)}) * 1i) / ((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20* \\
 &a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + ((((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - \\
 &256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^ \\
 &7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 -
 \end{aligned}$$

$$\begin{aligned}
& 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8 \\
& *a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} - (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(- \\
& (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)})*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} - (4*x^2 \\
& *(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} + (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}))/(2*c^2 * (16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} + (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)})*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} + (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)})))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)})*2i + x^2/(2*c)
\end{aligned}$$

sympy [A] time = 4.42, size = 134, normalized size = 0.70

$$\text{RootSum}\left(t^4(4096a^2c^5 - 2048ab^2c^4 + 256b^4c^3) + t^2(192a^2bc^2 - 112ab^3c + 16b^5) + a^3, \left(t \mapsto t \log\left(x^2 + \frac{256t^3abc^4 - 64t^3b^3c^3 - 8ta^2c^2 + 16tab^2c - 4tb^4}{a^2c - ab^2}\right)\right)\right) + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(c*x**8+b*x**4+a), x)

[Out] RootSum(_t**4*(4096*a**2*c**5 - 2048*a*b**2*c**4 + 256*b**4*c**3) + _t**2*(192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + a**3, Lambda(_t, _t*log(x**2 + (256*_t**3*a*b*c**4 - 64*_t**3*b**3*c**3 - 8*_t*a**2*c**2 + 16*_t*a*b**2*c - 4*_t*b**4)/(a**2*c - a*b**2)))) + x**2/(2*c)

$$3.255 \quad \int \frac{x^7}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log(a+bx^4+cx^8)}{8c}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log(a+bx^4+cx^8)}{8c}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b*x^4 + c*x^8), x]

[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x^4 + c*x^8]/(8*c)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1357

$\text{Int}[(x_)^{(m_.)*((a_) + (c_.)*(x_)^{(n2_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^7}{a + bx^4 + cx^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{a + bx + cx^2} dx, x, x^4 \right) \\ &= \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^4 \right)}{8c} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^4 \right)}{8c} \\ &= \frac{\log(a + bx^4 + cx^8)}{8c} + \frac{b \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^4 \right)}{4c} \\ &= \frac{b \tanh^{-1} \left(\frac{b+2cx^4}{\sqrt{b^2 - 4ac}} \right)}{4c\sqrt{b^2 - 4ac}} + \frac{\log(a + bx^4 + cx^8)}{8c} \end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$\frac{\log(a + bx^4 + cx^8) - \frac{2b \tan^{-1} \left(\frac{b+2cx^4}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}}}{8c}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b*x^4 + c*x^8),x]

[Out] ((-2*b*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a + b*x^4 + c*x^8])/(8*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(a + b*x^4 + c*x^8),x]

[Out] IntegrateAlgebraic[x^7/(a + b*x^4 + c*x^8), x]

fricas [A] time = 1.42, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^8 + 2bcx^4 + b^2 - 2ac + (2cx^4 + b)\sqrt{b^2 - 4ac}}{cx^8 + bx^4 + a}\right) + (b^2 - 4ac) \log(cx^8 + bx^4 + a)}{8(b^2c - 4ac^2)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(-\frac{(2cx^4 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) + (b^2 - 4ac) \log(cx^8 + bx^4 + a)}{8(b^2c - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + (b^2 - 4*a*c)*log(c*x^8 + b*x^4 + a))/(b^2*c - 4*a*c^2), 1/8*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^8 + b*x^4 + a))/(b^2*c - 4*a*c^2)]

giac [A] time = 17.12, size = 59, normalized size = 0.94

$$-\frac{b \arctan\left(\frac{2cx^4+b}{\sqrt{-b^2+4ac}}\right)}{4\sqrt{-b^2+4ac}c} + \frac{\log(cx^8+bx^4+a)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/8*log(c*x^8 + b*x^4 + a)/c

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b \arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}c} + \frac{\ln(cx^8+bx^4+a)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(c*x^8+b*x^4+a),x)

[Out] 1/8*ln(c*x^8+b*x^4+a)/c-1/4*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.61, size = 2654, normalized size = 42.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^4 + c*x^8),x)
```

```
[Out] (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*at
an((8*x^4*(((a*c - b^2)*(((16*a*c - 4*b^2)*((b*(448*b^3*c^3 - (256*b^3*c^
4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32
*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2)))))/(2
*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(1
6*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*
b^2*c)))))/(8*c*(4*a*c - b^2)^(1/2))*((16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b
^2*c)) - (b*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2
- 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((6
4*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2)))))/(8*c*(4*a*c - b^2)^(1/2)) - (4*b
^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)))))/(8*c*(4*a*
c - b^2)^(1/2)) + (b*(20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c
^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2
- 16*b^2*c)))*((16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))))/(8*c*(4*a*c - b
^2)^(1/2)) + (b^6*c*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)*(4*a*c - b^2
)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^7/(8*(4*a*c - b^2)^2) + b^3 -
((20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))
/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*((16*a*
c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*((16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*
b^2*c)) + ((16*a*c - 4*b^2)*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4
*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*
a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2)))))/(8*c*(4*a*c - b
^2)^(1/2)) - (4*b^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b
^2)))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*((16*a*c - 4*b^2)*((b*(448*b^3*c^3 -
(256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^
(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)
^(1/2)))))/(2*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (25
6*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64
*a*c^2 - 16*b^2*c)))/((8*c*(4*a*c - b^2)^(1/2)))))/(8*c*(4*a*c - b^2)^(1/2))
)/((8*a^3*c^2*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/b^4 + ((4*a*c - b^2)^(
```

$$\begin{aligned} & \frac{3}{2} * (b^3 - 3*a*b*c) * (a*b^2 + (((b*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2)) / (64*a*c^2 - 16*b^2*c))) / (8*c*(4*a*c - b^2)^{(1/2)}) - (64*a*b^3*c^3*(16*a*c - 4*b^2)) / ((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^{(1/2)}))) / (8*c*(4*a*c - b^2)^{(1/2)}) - (8*a*b^4*c^2*(16*a*c - 4*b^2)) / ((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))) * (16*a*c - 4*b^2) / (2*(64*a*c^2 - 16*b^2*c)) + (a*b^6) / (4*(4*a*c - b^2)^2) - ((16*a*c - 4*b^2)*(((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2)) / (64*a*c^2 - 16*b^2*c)) * (16*a*c - 4*b^2)) / (2*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2)) / (2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c)) / (2*(64*a*c^2 - 16*b^2*c)) + (b*((((16*a*c - 4*b^2)*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2)) / (64*a*c^2 - 16*b^2*c))) / (8*c*(4*a*c - b^2)^{(1/2)}) - (64*a*b^3*c^3*(16*a*c - 4*b^2)) / ((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^{(1/2)}))) / (2*(64*a*c^2 - 16*b^2*c)) + (b*((((768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2)) / (64*a*c^2 - 16*b^2*c)) * (16*a*c - 4*b^2)) / (2*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2)) / (8*c*(4*a*c - b^2)^{(1/2)}))) / (8*c*(4*a*c - b^2)^{(1/2)}))) / (a^3*b^4*c^2) + ((a*c - b^2)*(4*a*c - b^2)^2 * (((((16*a*c - 4*b^2)*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2)) / (64*a*c^2 - 16*b^2*c))) / (8*c*(4*a*c - b^2)^{(1/2)}) - (64*a*b^3*c^3*(16*a*c - 4*b^2)) / ((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^{(1/2)}))) / (2*(64*a*c^2 - 16*b^2*c)) + (b*((((768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2)) / (64*a*c^2 - 16*b^2*c)) * (16*a*c - 4*b^2)) / (2*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2)) / (8*c*(4*a*c - b^2)^{(1/2)}))) * (16*a*c - 4*b^2) / (2*(64*a*c^2 - 16*b^2*c)) - (b*((b*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2)) / (64*a*c^2 - 16*b^2*c))) / (8*c*(4*a*c - b^2)^{(1/2)}) - (64*a*b^3*c^3*(16*a*c - 4*b^2)) / ((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^{(1/2)}))) / (8*c*(4*a*c - b^2)^{(1/2)}) - (8*a*b^4*c^2*(16*a*c - 4*b^2)) / ((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)))) / (8*c*(4*a*c - b^2)^{(1/2)}) + (b*((((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2)) / (64*a*c^2 - 16*b^2*c)) * (16*a*c - 4*b^2)) / (2*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2)) / (2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c)) / (8*c*(4*a*c - b^2)^{(1/2)}) + (a*b^5*c*(16*a*c - 4*b^2)) / ((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^{(3/2)})) / (a^3*b^4*c^2))) / (4*c*(4*a*c - b^2)^{(1/2)}) \end{aligned}$$

sympy [B] time = 2.18, size = 223, normalized size = 3.54

$$\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)} + \frac{1}{8c} \right) \log \left(x^4 + \frac{-16ac \left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)} + \frac{1}{8c} \right) + 2a + 4b^2 \left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)} + \frac{1}{8c} \right)}{b} \right) + \left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)} + \frac{1}{8c} \right) \log \left(x^4 + \frac{-16ac \left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)} + \frac{1}{8c} \right) + 2a + 4b^2 \left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)} + \frac{1}{8c} \right)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(c*x**8+b*x**4+a), x)

[Out] $(-b*\sqrt{-4*a*c + b**2}) / (8*c*(4*a*c - b**2)) + 1 / (8*c)) * \log(x**4 + (-16*a*c * (-b*\sqrt{-4*a*c + b**2}) / (8*c*(4*a*c - b**2)) + 1 / (8*c)) + 2*a + 4*b**2 * (-b*\sqrt{-4*a*c + b**2}) / (8*c*(4*a*c - b**2)) + 1 / (8*c))) / b + (b*\sqrt{-4*a*c + b**2}) / (8*c*(4*a*c - b**2)) + 1 / (8*c)) * \log(x**4 + (-16*a*c * (b*\sqrt{-4*a*c + b**2}) / (8*c*(4*a*c - b**2)) + 1 / (8*c)) + 2*a + 4*b**2 * (b*\sqrt{-4*a*c + b**2}) / (8*c*(4*a*c - b**2)) + 1 / (8*c))) / b$

$$3.256 \quad \int \frac{x^5}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=159

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1359, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b*x^4 + c*x^8),x]

[Out] -(Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1130

Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 1359

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, c, p}

} , x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{a + bx^4 + cx^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{a + bx^2 + cx^4} dx, x, x^2 \right) \\ &= \frac{1}{4} \left(1 - \frac{b}{\sqrt{b^2 - 4ac}} \right) \text{Subst} \left(\int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx, x, x^2 \right) + \frac{1}{4} \left(1 + \frac{b}{\sqrt{b^2 - 4ac}} \right) \text{Subst} \left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx, x, x^2 \right) \\ &= -\frac{\sqrt{b - \sqrt{b^2 - 4ac}} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} \sqrt{c} \sqrt{b^2 - 4ac}} + \frac{\sqrt{b + \sqrt{b^2 - 4ac}} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} \sqrt{c} \sqrt{b^2 - 4ac}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 171, normalized size = 1.08

$$\frac{\left(\sqrt{b^2 - 4ac} - b \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{\sqrt{b^2 - 4ac} + b} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{2\sqrt{2} \sqrt{c} \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b*x^4 + c*x^8), x]

[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(a + b*x^4 + c*x^8), x]

[Out] IntegrateAlgebraic[x^5/(a + b*x^4 + c*x^8), x]

fricas [B] time = 1.25, size = 567, normalized size = 3.57

$$\frac{1}{4} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}} \log \left(x^2 + \frac{\sqrt{\frac{b^2 - 4ac}{b^2 - 4ac^2}} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}}}{\sqrt{b^2 - 4ac^2}} \right) - \frac{1}{4} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}} \log \left(x^2 - \frac{\sqrt{\frac{b^2 - 4ac}{b^2 - 4ac^2}} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}}}{\sqrt{b^2 - 4ac^2}} \right) - \frac{1}{4} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}} \log \left(x^2 + \frac{\sqrt{\frac{b^2 - 4ac}{b^2 - 4ac^2}} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}}}{\sqrt{b^2 - 4ac^2}} \right) + \frac{1}{4} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}} \log \left(x^2 - \frac{\sqrt{\frac{b^2 - 4ac}{b^2 - 4ac^2}} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}}}{\sqrt{b^2 - 4ac^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) +
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3))
```

giac [B] time = 18.74, size = 1036, normalized size = 6.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
- 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4
- 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^
3)*abs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
```

$$b^2 - 4ac) * c) * b * c^2 - 2 * (b^2 - 4ac) * b^2 * c + 8 * (b^2 - 4ac) * a * c^2 + 2 * (b^2 - 4ac) * b * c^2) * x^4 * \arctan(2 * \sqrt{1/2} * x^2 / \sqrt{(b - \sqrt{b^2 - 4ac}) / c}) / ((a * b^4 - 8 * a^2 * b^2 * c - 2 * a * b^3 * c + 16 * a^3 * c^2 + 8 * a^2 * b * c^2 + a * b^2 * c^2 - 4 * a^2 * c^3) * \text{abs}(c))$$

maple [A] time = 0.02, size = 216, normalized size = 1.36

$$\frac{\sqrt{2} b \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{4\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} b \operatorname{arctan}\left(\frac{\sqrt{2} c x^2}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{4\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{4\sqrt{(-b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2} c x^2}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{4\sqrt{(b + \sqrt{-4ac + b^2})c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(c*x^8+b*x^4+a),x)`

[Out]
$$-1/4 * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x^2) + 1/4 / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x^2) * b + 1/4 * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x^2) + 1/4 / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x^2) * b$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

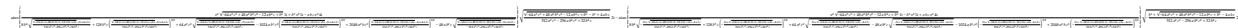
$$\int \frac{x^5}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")`

[Out] `integrate(x^5/(c*x^8 + b*x^4 + a), x)`

mupad [B] time = 2.81, size = 1220, normalized size = 7.67



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(a + b*x^4 + c*x^8),x)`

[Out]
$$\operatorname{atan}\left(\frac{x^2 * (b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} * i + b^3 * x^2 * i - a * b * c * x^2 * 4i}{(8 * b^4 * ((b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - b^3 + 4 * a * b * c) / (32 * b^4 * c + 512 * a^2 * c^3 - 256 * a * b^2 * c^2))^{(1/2)} + 128 * b^5 * c * ((b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - b^3 + 4 * a * b * c) / (32 * b^4 * c + 512 * a^2 * c^3 - 256 * a * b^2 * c^2))^{(3/2)} + 64 * a^2 * c^2 * ((b^6 - 64 * a^3 * c^3 + 48 * a^2 * b^2 * c^2 - 12 * a * b^4 * c)^{(1/2)} - b^3 + 4 * a * b * c) / (32 * b^4 * c + 512 * a^2 * c^3 - 256 * a * b^2 * c^2)}\right)$$

$$\begin{aligned}
& (b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{(1/2)} - b^3 + 4abc) / (32 \\
& *b^4c + 512a^2c^3 - 256ab^2c^2)^{(1/2)} - 1024ab^3c^2 * ((b^6 - 64a \\
& ^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{(1/2)} - b^3 + 4abc) / (32b^4c + 51 \\
& 2a^2c^3 - 256ab^2c^2)^{(3/2)} + 2048a^2b^3c^3 * ((b^6 - 64a^3c^3 + 48 \\
& *a^2b^2c^2 - 12ab^4c)^{(1/2)} - b^3 + 4abc) / (32b^4c + 512a^2c^3 - \\
& 256ab^2c^2)^{(3/2)} - 48ab^2c * ((b^6 - 64a^3c^3 + 48a^2b^2c^2 - \\
& 12ab^4c)^{(1/2)} - b^3 + 4abc) / (32b^4c + 512a^2c^3 - 256ab^2c^2) \\
&)^{(1/2)} * (((b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{(1/2)} - b^3 + \\
& 4abc) / (32b^4c + 512a^2c^3 - 256ab^2c^2)^{(1/2)} * 2i - \operatorname{atan}((x^2 * (b^6 \\
& - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{(1/2)} * i - b^3 * x^2 * i + abc \\
& * x^2 * 4i) / (8b^4 * (-b^3 + (b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{(1/2)} - \\
& 4abc) / (32b^4c + 512a^2c^3 - 256ab^2c^2)^{(1/2)} + 128b^5c \\
& * (-b^3 + (b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{(1/2)} - 4abc) \\
& / (32b^4c + 512a^2c^3 - 256ab^2c^2)^{(3/2)} + 64a^2c^2 * (-b^3 + (b^6 \\
& - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{(1/2)} - 4abc) / (32b^4c + 5 \\
& 12a^2c^3 - 256ab^2c^2)^{(1/2)} - 48ab^2c * (-b^3 + (b^6 - 64a^3c^3 \\
& + 48a^2b^2c^2 - 12ab^4c)^{(1/2)} - 4abc) / (32b^4c + 512a^2c^3 - 2 \\
& 56ab^2c^2)^{(1/2)} - 1024ab^3c^2 * (-b^3 + (b^6 - 64a^3c^3 + 48a^2b \\
& ^2c^2 - 12ab^4c)^{(1/2)} - 4abc) / (32b^4c + 512a^2c^3 - 256ab^2c \\
& ^2)^{(3/2)} + 2048a^2b^3c^3 * (-b^3 + (b^6 - 64a^3c^3 + 48a^2b^2c^2 - 1 \\
& 2ab^4c)^{(1/2)} - 4abc) / (32b^4c + 512a^2c^3 - 256ab^2c^2)^{(3/2)} \\
&)) * (-b^3 + (b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)^{(1/2)} - 4abc \\
& c) / (32b^4c + 512a^2c^3 - 256ab^2c^2)^{(1/2)} * 2i
\end{aligned}$$

sympy [A] time = 2.51, size = 76, normalized size = 0.48

$$\operatorname{RootSum}\left(t^4(4096a^2c^3 - 2048ab^2c^2 + 256b^4c) + t^2(-64abc + 16b^3) + a, (t \mapsto t \log(512t^3ac^2 - 128t^3b^2c - 4tb + x^2))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**8+b*x**4+a),x)

[Out] RootSum(_t**4*(4096*a**2*c**3 - 2048*a*b**2*c**2 + 256*b**4*c) + _t**2*(-64*a*b*c + 16*b**3) + a, Lambda(_t, _t*log(512*_t**3*a*c**2 - 128*_t**3*b**2*c - 4*_t*b + x**2)))

$$3.257 \quad \int \frac{x^3}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*x^4 + c*x^8), x]

[Out] -ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/(2*Sqrt[b^2 - 4*a*c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{a + bx^4 + cx^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^4 \right) \\ &= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^4 \right) \right) \\ &= - \frac{\tanh^{-1} \left(\frac{b+2cx^4}{\sqrt{b^2-4ac}} \right)}{2\sqrt{b^2-4ac}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.11

$$\frac{\tan^{-1} \left(\frac{b+2cx^4}{\sqrt{4ac-b^2}} \right)}{2\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*x^4 + c*x^8), x]

[Out] ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]/(2*Sqrt[-b^2 + 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(a + b*x^4 + c*x^8), x]

[Out] IntegrateAlgebraic[x^3/(a + b*x^4 + c*x^8), x]

fricas [A] time = 1.29, size = 129, normalized size = 3.39

$$\left[\frac{\log \left(\frac{2c^2x^8 + 2bcx^4 + b^2 - 2ac - (2cx^4 + b)\sqrt{b^2 - 4ac}}{cx^8 + bx^4 + a} \right)}{4\sqrt{b^2 - 4ac}}, - \frac{\sqrt{-b^2 + 4ac} \arctan \left(- \frac{(2cx^4 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac} \right)}{2(b^2 - 4ac)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^8+b*x^4+a), x, algorithm="fricas")

[Out] $\left[\frac{1}{4} \log\left(\frac{(2c^2x^8 + 2b^2cx^4 + b^2 - 2ac - (2cx^4 + b)\sqrt{b^2 - 4ac})}{(cx^8 + bx^4 + a)}\right) / \sqrt{b^2 - 4ac}, -\frac{1}{2}\sqrt{-b^2 + 4ac} \arctan\left(\frac{-(2cx^4 + b)\sqrt{-b^2 + 4ac}}{(b^2 - 4ac)}\right) / (b^2 - 4ac) \right]$

giac [A] time = 17.34, size = 36, normalized size = 0.95

$$\frac{\arctan\left(\frac{2cx^4+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="giac")`

[Out] $\frac{1}{2} \arctan\left(\frac{2cx^4 + b}{\sqrt{-b^2 + 4ac}}\right) / \sqrt{-b^2 + 4ac}$

maple [A] time = 0.00, size = 37, normalized size = 0.97

$$\frac{\arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(c*x^8+b*x^4+a),x)`

[Out] $\frac{1}{2} / (4ac - b^2)^{(1/2)} \arctan\left(\frac{2cx^4 + b}{(4ac - b^2)^{(1/2)}\right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.37, size = 260, normalized size = 6.84

$$\text{atan} \left(\frac{\left((4ac-b^2)^2 \left(\frac{\frac{4ac^4}{4ac-b^2} - \frac{4ab^2c^4}{(4ac-b^2)^2} \right) (b^3-3abc) \right) - x^4 \left(\frac{\frac{2c^4}{\sqrt{4ac-b^2}} - \frac{6b^2c^4}{(4ac-b^2)^{3/2}} \right) (ac-b^2) \left(\frac{b^3-3abc}{8a^3c^2\sqrt{4ac-b^2}} \right) \left(\frac{6bc^4}{4ac-b^2} - \frac{2b^3c^4}{(4ac-b^2)^2} \right) \right) + \frac{bc^2(ac-b^2)}{a^2(4ac-b^2)^{3/2}}}{2c^4} \right)$$

$$2\sqrt{4ac-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a + b*x^4 + c*x^8), x)`

[Out]
$$-\text{atan}\left(\frac{(4ac-b^2)^2 \left(\frac{4ac^4}{4ac-b^2} - \frac{4ab^2c^4}{(4ac-b^2)^2} \right) (b^3-3abc) - x^4 \left(\frac{2c^4}{\sqrt{4ac-b^2}} - \frac{6b^2c^4}{(4ac-b^2)^{3/2}} \right) (ac-b^2) \left(\frac{b^3-3abc}{8a^3c^2\sqrt{4ac-b^2}} \right) \left(\frac{6bc^4}{4ac-b^2} - \frac{2b^3c^4}{(4ac-b^2)^2} \right) + \frac{bc^2(ac-b^2)}{a^2(4ac-b^2)^{3/2}}}{2c^4} \right)$$

sympy [B] time = 0.77, size = 131, normalized size = 3.45

$$\frac{\sqrt{-\frac{1}{4ac-b^2}} \log \left(x^4 + \frac{-4ac\sqrt{-\frac{1}{4ac-b^2}} + b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c} \right)}{4} + \frac{\sqrt{-\frac{1}{4ac-b^2}} \log \left(x^4 + \frac{4ac\sqrt{-\frac{1}{4ac-b^2}} - b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c} \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(c*x**8+b*x**4+a), x)`

[Out]
$$-\sqrt{-1/(4ac-b^2)} \log(x^4 + (-4ac\sqrt{-1/(4ac-b^2)} + b^2\sqrt{-1/(4ac-b^2)} + b)/(2c))/4 + \sqrt{-1/(4ac-b^2)} \log(x^4 + (4ac\sqrt{-1/(4ac-b^2)} - b^2\sqrt{-1/(4ac-b^2)} + b)/(2c))/4$$

$$3.258 \quad \int \frac{x}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=154

$$\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.09, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1359, 1093, 205}

$$\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^4 + c*x^8), x]

[Out] (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1359

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}

} , x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{a + bx^4 + cx^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{a + bx^2 + cx^4} dx, x, x^2 \right) \\ &= \frac{c \text{Subst} \left(\int \frac{1}{\frac{b}{2} - \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2} dx, x, x^2 \right)}{2\sqrt{b^2 - 4ac}} - \frac{c \text{Subst} \left(\int \frac{1}{\frac{b}{2} + \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2} dx, x, x^2 \right)}{2\sqrt{b^2 - 4ac}} \\ &= \frac{\sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 133, normalized size = 0.86

$$\frac{\sqrt{c} \left(\frac{\tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{2} \sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^4 + c*x^8), x]

[Out] (Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a + b*x^4 + c*x^8), x]

[Out] IntegrateAlgebraic[x/(a + b*x^4 + c*x^8), x]

fricas [B] time = 1.25, size = 619, normalized size = 4.02

$$\frac{1}{4}\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{ab^2-4a^2c}}}{ab^2-4a^2c}} \log\left(cx^2 + \frac{1}{2}\sqrt{\frac{b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{ab^2-4a^2c}}}{\sqrt{ab^2-4a^2c}}}\right) \sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{ab^2-4a^2c}}}{ab^2-4a^2c}} - \frac{1}{4}\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{ab^2-4a^2c}}}{ab^2-4a^2c}} \log\left(cx^2 - \frac{1}{2}\sqrt{\frac{b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{ab^2-4a^2c}}}{\sqrt{ab^2-4a^2c}}}\right) \sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{ab^2-4a^2c}}}{ab^2-4a^2c}} - \frac{1}{4}\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{ab^2-4a^2c}}}{ab^2-4a^2c}} \log\left(cx^2 + \frac{1}{2}\sqrt{\frac{b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{ab^2-4a^2c}}}{\sqrt{ab^2-4a^2c}}}\right) \sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{ab^2-4a^2c}}}{ab^2-4a^2c}} + \frac{1}{4}\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{ab^2-4a^2c}}}{ab^2-4a^2c}} \log\left(cx^2 - \frac{1}{2}\sqrt{\frac{b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{ab^2-4a^2c}}}{\sqrt{ab^2-4a^2c}}}\right) \sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{ab^2-4a^2c}}}{ab^2-4a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*\sqrt{1/2}*\sqrt{-(b+(a*b^2-4*a^2*c)/\sqrt{a^2*b^2-4*a^3*c})}/(a*b^2-4*a^2*c)) * \log(c*x^2 + 1/2*\sqrt{1/2}*(b^2-4*a*c - (a*b^3-4*a^2*b*c)/\sqrt{a^2*b^2-4*a^3*c}))/\sqrt{a^2*b^2-4*a^3*c} \\ & + 1/4*\sqrt{1/2}*\sqrt{-(b+(a*b^2-4*a^2*c)/\sqrt{a^2*b^2-4*a^3*c})}/(a*b^2-4*a^2*c)) * \log(c*x^2 - 1/2*\sqrt{1/2}*(b^2-4*a*c - (a*b^3-4*a^2*b*c)/\sqrt{a^2*b^2-4*a^3*c}))/\sqrt{a^2*b^2-4*a^3*c} \\ & - 1/4*\sqrt{1/2}*\sqrt{-(b-(a*b^2-4*a^2*c)/\sqrt{a^2*b^2-4*a^3*c})}/(a*b^2-4*a^2*c)) * \log(c*x^2 + 1/2*\sqrt{1/2}*(b^2-4*a*c + (a*b^3-4*a^2*b*c)/\sqrt{a^2*b^2-4*a^3*c}))/\sqrt{a^2*b^2-4*a^3*c} \\ & + 1/4*\sqrt{1/2}*\sqrt{-(b-(a*b^2-4*a^2*c)/\sqrt{a^2*b^2-4*a^3*c})}/(a*b^2-4*a^2*c)) * \log(c*x^2 - 1/2*\sqrt{1/2}*(b^2-4*a*c + (a*b^3-4*a^2*b*c)/\sqrt{a^2*b^2-4*a^3*c}))/\sqrt{a^2*b^2-4*a^3*c} \end{aligned}$$

giac [B] time = 19.50, size = 1030, normalized size = 6.69

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/8*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*b^4 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*a*b^2*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*b^3*c \\ & - 2*b^4*c + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*a^2*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*a*b*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*b^2*c^2 \\ & + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2-4*a*c}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*b^3 \\ & + 4*\sqrt{2}*\sqrt{b^2-4*a*c}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*a*b*c + 2*\sqrt{2}*\sqrt{b^2-4*a*c}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*b^2*c - \sqrt{2}*\sqrt{b^2-4*a*c}*\sqrt{b*c + \sqrt{b^2-4*a*c}}*c)*b*c^2 \\ & + 2*(b^2-4*a*c)*b^2*c - 8*(b^2-4*a*c)*a*c^2 - 2*(b^2-4*a*c)*b*c^2)*\arctan(2*\sqrt{1/2}*x^2/\sqrt{(b+\sqrt{b^2-4*a*c})/c})/((a*b^4-8*a^2*b^2*c-2*a*b^3*c+16*a^3*c^2+8*a^2*b*c^2+a*b^2*c^2-4*a^2*c^3)*b*c) \\ & + 1/8*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2-4*a*c}}*c)*b^4 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2-4*a*c}}*c)*a*b^2*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2-4*a*c}}*c)*b^3*c \\ & + 2*b^4*c + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2-4*a*c}}*c)*a^2*c^2 + \end{aligned}$$

$8\sqrt{2}\sqrt{b^2c - \sqrt{b^2 - 4ac}}c^2 + \sqrt{2}\sqrt{b^2c - \sqrt{b^2 - 4ac}}b^2c^2 - 16ab^2c^2 - 2b^3c^2 - 4\sqrt{2}\sqrt{b^2c - \sqrt{b^2 - 4ac}}ac^3 + 32a^2c^3 + 8ab^3c^3 + \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2c - \sqrt{b^2 - 4ac}}b^3 - 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2c - \sqrt{b^2 - 4ac}}ab^2c + \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b^2c - \sqrt{b^2 - 4ac}}b^2c^2 - 2(b^2 - 4ac)b^2c + 8(b^2 - 4ac)ac^2 + 2(b^2 - 4ac)b^2c^2) \arctan(2\sqrt{1/2}x^2/\sqrt{(b - \sqrt{b^2 - 4ac})/c}) / ((ab^4 - 8a^2b^2c - 2ab^3c + 16a^3c^2 + 8a^2b^2c^2 + ab^2c^2 - 4a^2c^3) \operatorname{abs}(c))$

maple [A] time = 0.01, size = 120, normalized size = 0.78

$$\frac{\sqrt{2} c \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c}} - \frac{\sqrt{2} c \operatorname{arctan}\left(\frac{\sqrt{2} c x^2}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(c*x^8+b*x^4+a),x)`

[Out] $-1/2c/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctanh}(2^{(1/2)}/((-b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c*x^2) - 1/2c/(-4ac+b^2)^{(1/2)}2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}\operatorname{arctan}(2^{(1/2)}/((b+(-4ac+b^2)^{(1/2)})c)^{(1/2)}c*x^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^8+b*x^4+a),x, algorithm="maxima")`

[Out] `integrate(x/(c*x^8 + b*x^4 + a), x)`

mupad [B] time = 2.27, size = 1105, normalized size = 7.18

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*x^4 + c*x^8),x)`

```
[Out] atan((b^4*x^2*1i + b*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^2*6i)/(128*a^2*b^5*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) + 16*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) - 1024*a^3*b^3*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) + 2048*a^4*b*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2)))*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i + atan((b^4*x^2*1i - b*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^2*6i)/(128*a^2*b^5*((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*c^2*((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) + 16*a^2*b^2*c*((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) - 1024*a^3*b^3*c*((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) + 2048*a^4*b*c^2*((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i
```

sympy [A] time = 3.36, size = 88, normalized size = 0.57

$$\text{RootSum}\left(t^4(4096a^3c^2 - 2048a^2b^2c + 256ab^4) + t^2(-64abc + 16b^3) + c, \left(t \mapsto t \log\left(x^2 + \frac{256t^3a^2bc - 64t^3ab^3 + 8tac - 4tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**3*c**2 - 2048*a**2*b**2*c + 256*a*b**4) + _t**2*(-64*a*b*c + 16*b**3) + c, Lambda(_t, _t*log(x**2 + (256*_t**3*a**2*b*c - 64*_t**3*a*b**3 + 8*_t*a*c - 4*_t*b**2)/c)))
```

$$3.259 \quad \int \frac{1}{x(a+bx^4+cx^8)} dx$$

Optimal. Leaf size=69

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} - \frac{\log(a+bx^4+cx^8)}{8a} + \frac{\log(x)}{a}$$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} - \frac{\log(a+bx^4+cx^8)}{8a} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^4 + c*x^8)),x]

[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/(4*a*Sqrt[b^2 - 4*a*c])) + Log[x]/a - Log[a + b*x^4 + c*x^8]/(8*a)

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a + bx^4 + cx^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(a + bx + cx^2)} dx, x, x^4 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^4 \right)}{4a} + \frac{\text{Subst} \left(\int \frac{-b-cx}{a+bx+cx^2} dx, x, x^4 \right)}{4a} \\
&= \frac{\log(x)}{a} - \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^4 \right)}{8a} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^4 \right)}{8a} \\
&= \frac{\log(x)}{a} - \frac{\log(a + bx^4 + cx^8)}{8a} + \frac{b \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^4 \right)}{4a} \\
&= \frac{b \tanh^{-1} \left(\frac{b+2cx^4}{\sqrt{b^2-4ac}} \right)}{4a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a + bx^4 + cx^8)}{8a}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a} - \frac{\text{RootSum}\left[\#1^8 c + \#1^4 b + a \&, \frac{\#1^4 c \log(x-\#1) + b \log(x-\#1)}{2\#1^4 c + b} \&\right]}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^4 + c*x^8)),x]

[Out] Log[x]/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) &]/(4*a)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^4 + cx^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a + b*x^4 + c*x^8)),x]

[Out] IntegrateAlgebraic[1/(x*(a + b*x^4 + c*x^8)), x]

fricas [A] time = 1.24, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^8 + 2bcx^4 + b^2 - 2ac + (2cx^4 + b)\sqrt{b^2 - 4ac}}{cx^8 + bx^4 + a}\right) - (b^2 - 4ac) \log(cx^8 + bx^4 + a) + 8(b^2 - 4ac) \log(x)}{8(ab^2 - 4a^2c)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(\frac{-2cx^4 + b}{b^2 - 4ac}\right) - (b^2 - 4ac) \log(cx^8 + bx^4 + a) + 8(b^2 - 4ac) \log(x)}{8(ab^2 - 4a^2c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^2 - 4*a*c)*log(c*x^8 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/8*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^8 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]

giac [A] time = 16.28, size = 68, normalized size = 0.99

$$\frac{b \arctan\left(\frac{2cx^4 + b}{\sqrt{-b^2 + 4ac}}\right)}{4\sqrt{-b^2 + 4ac}a} - \frac{\log(cx^8 + bx^4 + a)}{8a} + \frac{\log(x^4)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] $-1/4*b*\arctan((2*c*x^4 + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c}*a) - 1/8*\log(c*x^8 + b*x^4 + a)/a + 1/4*\log(x^4)/a$

maple [A] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2cx^4+b}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}a} + \frac{\ln(x)}{a} - \frac{\ln(cx^8+bx^4+a)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(c*x^8+b*x^4+a),x)`

[Out] $1/a*\ln(x)-1/8*\ln(c*x^8+b*x^4+a)/a-1/4/a*b/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^4+b)/(4*a*c-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.18, size = 1690, normalized size = 24.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*x^4 + c*x^8)),x)`

[Out] $\log(x)/a + (\log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)) - (b*\operatorname{atan}((4*(4*a*c - b^2)^2*(5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*a*b^4*c)*(b^9*c^4)/(128*a^4*(4*a*c - b^2)^{(5/2)}) + (2*b^5*c^4*(16*a*c - 4*b^2)^4)/((16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^{(1/2)}) - (b*(16*a*c - 4*b^2)^3*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c))))/(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^{(1/2)}) + (b^3*(16*a*c - 4*b^2)*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(256*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^{(3/2)}) - (3*b^7*c^4*(16*a*c - 4*b^2)^2)/(4*a^2*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2)^{(3/2)))/(b^4*c^8*(81*a*c - 20*b^2)) + (128*a^5*x^4*((5*b^5 + 23*a^2*b*c^2 - 24*a*b^3*c)*((576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 -$

$$\begin{aligned}
& 64a^2c)) \cdot (16ac - 4b^2)^4 / ((16(16ab^2 - 64a^2c)^4) + (b^4(576b^3c^5 - ((1280b^5c^4 - 4608ab^3c^5)(16ac - 4b^2)) / (2(16ab^2 - 64a^2c)))) / (4096a^4(4ac - b^2)^2) + (b^2(1280b^5c^4 - 4608ab^3c^5)(16ac - 4b^2)^3) / (128a^2(16ab^2 - 64a^2c)^3(4ac - b^2)) - (3b^2(576b^3c^5 - ((1280b^5c^4 - 4608ab^3c^5)(16ac - 4b^2)) / (2(16ab^2 - 64a^2c)))) \cdot (16ac - 4b^2)^2) / (128a^2(16ab^2 - 64a^2c)^2(4ac - b^2)) - (b^4(1280b^5c^4 - 4608ab^3c^5)(16ac - 4b^2)) / (2048a^4(16ab^2 - 64a^2c)(4ac - b^2)^2)) / (32a^5c^4(81ac - 20b^2)) + ((5b^6 - 18a^3c^3 + 61a^2b^2c^2 - 34ab^4c) \cdot ((b^5(1280b^5c^4 - 4608ab^3c^5)) / (32768a^5(4ac - b^2)^{5/2})) - (3b^3(1280b^5c^4 - 4608ab^3c^5)(16ac - 4b^2)^2) / (1024a^3(16ab^2 - 64a^2c)^2(4ac - b^2)^{3/2})) + (b(1280b^5c^4 - 4608ab^3c^5)(16ac - 4b^2)^4) / (128a(16ab^2 - 64a^2c)^4(4ac - b^2)^{1/2}) - (b(576b^3c^5 - ((1280b^5c^4 - 4608ab^3c^5)(16ac - 4b^2)) / (2(16ab^2 - 64a^2c)))) \cdot (16ac - 4b^2)^3) / (16a(16ab^2 - 64a^2c)^3(4ac - b^2)^{1/2})) + (b^3(576b^3c^5 - ((1280b^5c^4 - 4608ab^3c^5)(16ac - 4b^2)) / (2(16ab^2 - 64a^2c)))) \cdot (16ac - 4b^2) / (256a^3(16ab^2 - 64a^2c)(4ac - b^2)^{3/2})) / (32a^5c^4(4ac - b^2)^{1/2}(81ac - 20b^2))) \cdot (4ac - b^2)^{5/2} / (b^4c^4) + (4(4ac - b^2)^{5/2}(5b^5 + 23a^2b^2c^2 - 24ab^3c) \cdot (((16ac - 4b^2)^4(256b^4c^4 - (128ab^4c^4(16ac - 4b^2)) / (16ab^2 - 64a^2c)))) / (16(16ab^2 - 64a^2c)^4) + (b^4(256b^4c^4 - (128ab^4c^4(16ac - 4b^2)) / (16ab^2 - 64a^2c))) / (4096a^4(4ac - b^2)^2) - (b^8c^4(16ac - 4b^2)) / (8a^3(16ab^2 - 64a^2c)(4ac - b^2)^2) + (2b^6c^4(16ac - 4b^2)^3) / (a(16ab^2 - 64a^2c)^3(4ac - b^2)) - (3b^2(16ac - 4b^2)^2(256b^4c^4 - (128ab^4c^4(16ac - 4b^2)) / (16ab^2 - 64a^2c))) / (128a^2(16ab^2 - 64a^2c)^2(4ac - b^2))) / (b^4c^8(81ac - 20b^2))) / (4a(4ac - b^2)^{1/2})
\end{aligned}$$

sympy [B] time = 14.47, size = 253, normalized size = 3.67

$$\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a} \right) \log \left(x^4 + \frac{-16a^2c \left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a} \right) + 4ab^2 \left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a} \right) - 2ac + b^2}{bc} \right) + \left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a} \right) \log \left(x^4 + \frac{-16a^2c \left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a} \right) + 4ab^2 \left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)} - \frac{1}{8a} \right) - 2ac + b^2}{bc} \right) + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**8+b*x**4+a), x)

[Out] $(-b\sqrt{-4ac+b^2})/(8a(4ac-b^2)) - 1/(8a)) \cdot \log(x^4 + (-16a^2c(-b\sqrt{-4ac+b^2})/(8a(4ac-b^2)) - 1/(8a)) + 4ab^2(-b\sqrt{-4ac+b^2})/(8a(4ac-b^2)) - 1/(8a)) - 2ac + b^2)/(bc)) + (b\sqrt{-4ac+b^2})/(8a(4ac-b^2)) - 1/(8a)) \cdot \log(x^4 + (-16a^2c(b\sqrt{-4ac+b^2})/(8a(4ac-b^2)) - 1/(8a)) + 4ab^2(b\sqrt{-4ac+b^2})/(8a(4ac-b^2)) - 1/(8a)) - 2ac + b^2)/(bc)) + \log(x)/a$

$$3.260 \quad \int \frac{1}{x^3(a+bx^4+cx^8)} dx$$

Optimal. Leaf size=184

$$\frac{\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{2\sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{2ax^2}$$

Rubi [A] time = 0.22, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1359, 1123, 1166, 205}

$$\frac{\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{2\sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*x^4 + c*x^8)),x]

[Out] -1/(2*a*x^2) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1123

Int[((d_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[((d*x)^(m+1)*(a + b*x^2 + c*x^4)^(p+1))/(a*d*(m+1)), x] - Dist[1/(a*d^2*(m+1)), Int[(d*x)^(m+2)*(b*(m+2*p+3) + c*(m+4*p+5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2

- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1359

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 (a + bx^4 + cx^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (a + bx^2 + cx^4)} dx, x, x^2 \right) \\ &= -\frac{1}{2ax^2} + \frac{\text{Subst} \left(\int \frac{-b-cx^2}{a+bx^2+cx^4} dx, x, x^2 \right)}{2a} \\ &= -\frac{1}{2ax^2} - \frac{\left(c \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx, x, x^2 \right)}{4a} - \frac{\left(c \left(1 + \frac{b}{\sqrt{b^2-4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx, x, x^2 \right)}{4a} \\ &= -\frac{1}{2ax^2} - \frac{\sqrt{c} \left(1 + \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b - \sqrt{b^2-4ac}}} \right)}{2\sqrt{2} a \sqrt{b - \sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b + \sqrt{b^2-4ac}}} \right)}{2\sqrt{2} a \sqrt{b + \sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.41

$$-\frac{\text{RootSum} \left[\#1^8 c + \#1^4 b + a \&, \frac{\#1^4 c \log(x-\#1) + b \log(x-\#1)}{2\#1^6 c + \#1^2 b} \& \right]}{4a} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*x^4 + c*x^8)),x]

[Out] -1/2*1/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 &, (b*Log[x - #1] + c*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) &]/(4*a)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + bx^4 + cx^8)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^4 + c*x^8)), x]
```

fricas [B] time = 1.40, size = 1134, normalized size = 6.16



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x^2)
```

giac [B] time = 15.89, size = 2055, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
```

$$\begin{aligned}
& - 4*a*c)*c)*a^2*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c \\
& *a*b^3*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c \\
&)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + (\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c) \\
& *b^4*c - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - 2*\sqrt{2} \\
& *\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c^2 - 2*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}} \\
& *c)*a^2*c^3 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}} \\
& *c)*b^2*c^3 + 16*a*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 - \\
& 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2* \\
& b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + \\
& 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + 2* \\
& \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{2} \\
& *\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 2*(b^2 - 4*a \\
& *c)*a*b*c^3)*x^4 + (\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5 - 8*\sqrt{2} \\
& *\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}} \\
& *c)*b^4*c - 2*b^5*c + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2* \\
& b*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}} \\
& *c)*b^3*c^2 + 16*a*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c) \\
& *b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*abs(a))*arctan(2*\sqrt{1/2}*x^2/\sqrt{(a*b + \sqrt{a^2*b^2 - 4*a^3*c})}) \\
& /((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) + 1/8*(2*a*b^4*c^2 - 8*a \\
& ^2*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4 \\
& + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 - (\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 8*\sqrt{2} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *b^3*c^2 + 2*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& *c)*a*b*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^3 - 16*a*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& *c)*a*c^4 + 32*a^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *c)*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*x^4 - (\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}} \\
& *c)*a*b^3*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c + 2*b^5*c + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *a^2*b*c^2 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) \\
& *b^3*c^2 - 16*a*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c) \\
& *b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(a))*arctan(2*\sqrt{1/2}*x^2/\sqrt{(a*b - \sqrt{a^2*b^2 - 4*a^3*c})}) \\
& /((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2
\end{aligned}$$

- 4*a^3*c^3)*abs(a)*abs(c)) - 1/2/(a*x^2)

maple [A] time = 0.02, size = 240, normalized size = 1.30

$$\frac{\sqrt{2} bc \operatorname{arctanh}\left(\frac{\sqrt{2} cx^2}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{4\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c}a} + \frac{\sqrt{2} bc \operatorname{arctan}\left(\frac{\sqrt{2} cx^2}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{4\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}a} + \frac{\sqrt{2} c \operatorname{arctanh}\left(\frac{\sqrt{2} cx^2}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{4\sqrt{(-b+\sqrt{-4ac+b^2})c}a} - \frac{\sqrt{2} c \operatorname{arctan}\left(\frac{\sqrt{2} cx^2}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{4\sqrt{(b+\sqrt{-4ac+b^2})c}a} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(c*x^8+b*x^4+a), x)

[Out] 1/4/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b-1/4/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)+1/4/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x^2)*b-1/2/a/x^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^8+b*x^4+a), x, algorithm="maxima")

[Out] -integrate((c*x^4 + b)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2/(a*x^2)

mupad [B] time = 2.42, size = 5451, normalized size = 29.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x^4 + c*x^8)), x)

[Out] - atan((((64*a^10*c^8 + ((-b^5 + b^2*(-4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((b^5 + b^2*(-4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*((-b^5 + b^2*(-4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*((-b^5 + b^2*(-4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))

$$\begin{aligned}
& *c - b^2)^3)^{(1/2)} / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + 16*a \\
& ^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b \\
& *c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)})*i / (32*(a^3*b^4 + 16*a^5* \\
& c^2 - 8*a^4*b^2*c)) - ((64*a^10*c^8 + ((-b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (32*(a^3*b^4 + \\
& 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * (((-b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (32*(a^3*b^4 + 1 \\
& 6*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * (4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + \\
& 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11 \\
& *b^5*c^5 - 24576*a^12*b^3*c^6)) * (-b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12* \\
& a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (32*(a^3*b^4 + 16*a^5 \\
& *c^2 - 8*a^4*b^2*c))^{(1/2)} + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^1 \\
& 1*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^1 \\
& 0*b^2*c^7)) * (-b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3* \\
& c - a*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) \\
&)^{(1/2)} + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)})*i / (32*(a^3 \\
& *b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) / (((64*a^10*c^8 + ((-b^5 + b^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)}) / \\
& (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * (((-b^5 + b^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (3 \\
& 2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * (4096*a^12*b^6*c^4 - 32768*a \\
& ^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c \\
& ^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6)) * (-b^5 + b^2*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (32*(a^ \\
& 3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + 4096*a^12*b*c^7 + 512*a^10*b^5* \\
& c^5 - 3072*a^11*b^3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4 \\
& *c^6 - 896*a^10*b^2*c^7)) * (-b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b* \\
& c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (32*(a^3*b^4 + 16*a^5*c^2 - \\
& 8*a^4*b^2*c))^{(1/2)} + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)} \\
&)) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + ((64*a^10*c^8 + ((-b^5 + b^ \\
& 2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2) \\
& ^3)^{(1/2)}) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * (((-b^5 + b^2* \\
& (-4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3 \\
&)^{(1/2)}) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * (4096*a^12*b^6*c^ \\
& 4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024 \\
& *a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6)) * (-b^5 + b^2*(-(4* \\
& a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)}) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + 4096*a^12*b*c^7 + 51 \\
& 2*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + \\
& 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7)) * (-b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)}) / (32*(a^3*b^4 + 1 \\
& 6*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + \\
& b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
\end{aligned}$$

$$\begin{aligned}
& 2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))) * (- (b^5 + b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} * i - \operatorname{atan}(\frac{((64*a^{10}*c^8 + ((- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} * ((- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} * (4096*a^{12}*b^6*c^4 - 32768*a^{13}*b^4*c^5 + 65536*a^{14}*b^2*c^6) + x^2*(16384*a^{13}*b*c^7 - 1024*a^{10}*b^7*c^4 + 9216*a^{11}*b^5*c^5 - 24576*a^{12}*b^3*c^6)) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} + 4096*a^{12}*b*c^7 + 512*a^{10}*b^5*c^5 - 3072*a^{11}*b^3*c^6) + x^2*(512*a^{11}*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^{10}*b^2*c^7)) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7) * (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2))} * i) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) - ((64*a^{10}*c^8 + ((- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} * (((- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} * (4096*a^{12}*b^6*c^4 - 32768*a^{13}*b^4*c^5 + 65536*a^{14}*b^2*c^6) - x^2*(16384*a^{13}*b*c^7 - 1024*a^{10}*b^7*c^4 + 9216*a^{11}*b^5*c^5 - 24576*a^{12}*b^3*c^6)) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} + 4096*a^{12}*b*c^7 + 512*a^{10}*b^5*c^5 - 3072*a^{11}*b^3*c^6) - x^2*(512*a^{11}*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^{10}*b^2*c^7)) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7) * (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2))} * i) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) / (((64*a^{10}*c^8 + ((- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} * (((- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} * (4096*a^{12}*b^6*c^4 - 32768*a^{13}*b^4*c^5 + 65536*a^{14}*b^2*c^6) + x^2*(16384*a^{13}*b*c^7 - 1024*a^{10}*b^7*c^4 + 9216*a^{11}*b^5*c^5 - 24576*a^{12}*b^3*c^6)) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} + 4096*a^{12}*b*c^7 + 512*a^{10}*b^5*c^5 - 3072*a^{11}*b^3*c^6) + x^2*(512*a^{11}*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^{10}*b^2*c^7)) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^{(1/2)} + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7) * (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2))} / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
\end{aligned}$$

$$\begin{aligned}
& 16a^5c^2 - 8a^4b^2c) + ((64a^{10}c^8 + ((-(b^5 - b^2(-(4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + ac(-(4ac - b^2)^3)^{1/2}))/((32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{1/2} * (((-(b^5 - b^2(-(4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + ac(-(4ac - b^2)^3)^{1/2}))/((32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{1/2} * (4096a^{12}b^6c^4 - 32768a^{13}b^4c^5 + 65536a^{14}b^2c^6) - x^2(16384a^{13}b^7c^7 - 1024a^{10}b^7c^4 + 9216a^{11}b^5c^5 - 24576a^{12}b^3c^6)) * (-(b^5 - b^2(-(4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + ac(-(4ac - b^2)^3)^{1/2}))/((32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{1/2} + 4096a^{12}b^7c^7 + 512a^{10}b^5c^5 - 3072a^{11}b^3c^6) - x^2(512a^{11}c^8 - 64a^8b^6c^5 + 448a^9b^4c^6 - 896a^{10}b^2c^7)) * (-(b^5 - b^2(-(4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + ac(-(4ac - b^2)^3)^{1/2}))/((32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{1/2} + 16a^8b^4c^6 - 64a^9b^2c^7) * (b^5 - b^2(-(4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + ac(-(4ac - b^2)^3)^{1/2}))/((32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{1/2} * (-(b^5 - b^2(-(4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + ac(-(4ac - b^2)^3)^{1/2}))/((32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{1/2} * 2i - 1/(2ax^2)
\end{aligned}$$

sympy [A] time = 15.34, size = 153, normalized size = 0.83

$$\text{RootSum}\left(t^4(4096a^5c^2 - 2048a^4b^2c + 256a^3b^4) + t^2(192a^2bc^2 - 112ab^3c + 16b^5) + c^3, \left(t \mapsto t \log\left(x^2 + \frac{-512t^3a^5c^2 + 384t^3a^4b^2c - 64t^3a^3b^4 - 20ta^2bc^2 + 20tab^3c - 4tb^5}{ac^3 - b^2c^2}\right)\right)\right) - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(c*x**8+b*x**4+a),x)

[Out] RootSum(_t**4*(4096*a**5*c**2 - 2048*a**4*b**2*c + 256*a**3*b**4) + _t**2*(192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + c**3, Lambda(_t, _t*log(x**2 + (-512*_t**3*a**5*c**2 + 384*_t**3*a**4*b**2*c - 64*_t**3*a**3*b**4 - 20*_t*a**2*b*c**2 + 20*_t*a*b**3*c - 4*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(2*a*x**2)

$$3.261 \quad \int \frac{1}{x^5(a+bx^4+cx^8)} dx$$

Optimal. Leaf size=89

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx^4 + cx^8)}{8a^2} - \frac{b \log(x)}{a^2} - \frac{1}{4ax^4}$$

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx^4 + cx^8)}{8a^2} - \frac{b \log(x)}{a^2} - \frac{1}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b*x^4 + c*x^8)),x]

[Out] -1/(4*a*x^4) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^4 + c*x^8])/(8*a^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)(a + bx + cx^2)}{(a + bx + cx^2)^2}, x \int ; \text{FreeQ}\{a, b, c, d, e, x\} \&\& \text{NeQ}[2cd - be, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 709

$\text{Int}[\frac{(d + ex)^m}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Simp}[e(d + ex)^{m+1}/((m+1)(cd^2 - bde + ae^2)), x] + \text{Dist}[1/(cd^2 - bde + ae^2), \text{Int}[\frac{(d + ex)^{m+1} \text{Simp}[cd - be - cex, x]}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, x\} \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[cd^2 - bde + ae^2, 0] \&\& \text{NeQ}[2cd - be, 0] \&\& \text{LtQ}[m, -1]$

Rule 800

$\text{Int}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, x\} \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[cd^2 - bde + ae^2, 0] \&\& \text{IntegerQ}[m]$

Rule 1357

$\text{Int}[(x^m)((a + cx^{n2}) + (b + cx^n)^p), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)(a + bx + cx^2)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, m, n, p, x\} \&\& \text{EqQ}[n2, 2n] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(a+bx^4+cx^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2(a+bx+cx^2)} dx, x, x^4 \right) \\
&= -\frac{1}{4ax^4} + \frac{\text{Subst} \left(\int \frac{-b-cx}{x(a+bx+cx^2)} dx, x, x^4 \right)}{4a} \\
&= -\frac{1}{4ax^4} + \frac{\text{Subst} \left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)} \right) dx, x, x^4 \right)}{4a} \\
&= -\frac{1}{4ax^4} - \frac{b \log(x)}{a^2} + \frac{\text{Subst} \left(\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx, x, x^4 \right)}{4a^2} \\
&= -\frac{1}{4ax^4} - \frac{b \log(x)}{a^2} + \frac{b \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^4 \right)}{8a^2} + \frac{(b^2-2ac) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^4 \right)}{8a^2} \\
&= -\frac{1}{4ax^4} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx^4+cx^8)}{8a^2} - \frac{(b^2-2ac) \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2cx^4 \right)}{4a^2} \\
&= -\frac{1}{4ax^4} - \frac{(b^2-2ac) \tanh^{-1} \left(\frac{b+2cx^4}{\sqrt{b^2-4ac}} \right)}{4a^2 \sqrt{b^2-4ac}} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx^4+cx^8)}{8a^2}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 92, normalized size = 1.03

$$\frac{\text{RootSum} \left[\#1^8 c + \#1^4 b + a \&, \frac{\#1^4 bc \log(x-\#1) - ac \log(x-\#1) + b^2 \log(x-\#1)}{2\#1^4 c + b} \& \right]}{4a^2} - \frac{b \log(x)}{a^2} - \frac{1}{4ax^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(a + b*x^4 + c*x^8)),x]

[Out] -1/4*1/(a*x^4) - (b*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8 &, (b^2*Log[x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) &]/(4*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(a+bx^4+cx^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(a + b*x^4 + c*x^8)),x]

[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^4 + c*x^8)), x]

fricas [A] time = 1.88, size = 293, normalized size = 3.29

$$\frac{\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac}x^4 \log\left(\frac{2cx^8 + 2bx^4 + b^2 - 2ac + (2cx^4 + b)\sqrt{b^2 - 4ac}}{cx^8 + bx^4 + a}\right) - (b^3 - 4abc)x^4 \log(cx^8 + bx^4 + a) + 8(b^3 - 4abc)x^4 \log(x) + 2ab^2 - 8a^2c}{8(a^2b^2 - 4a^2c)x^4} - \frac{2(b^2 - 2ac)\sqrt{-b^2 + 4ac}x^4 \arctan\left(\frac{(2cx^4 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) - (b^3 - 4abc)x^4 \log(cx^8 + bx^4 + a) + 8(b^3 - 4abc)x^4 \log(x) + 2ab^2 - 8a^2c}{8(a^2b^2 - 4a^2c)x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] [-1/8*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^3 - 4*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4), -1/8*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4)]

giac [A] time = 14.50, size = 94, normalized size = 1.06

$$\frac{b \log(cx^8 + bx^4 + a)}{8a^2} - \frac{b \log(x^4)}{4a^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^4 + b}{\sqrt{-b^2 + 4ac}}\right)}{4\sqrt{-b^2 + 4ac}a^2} + \frac{bx^4 - a}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] 1/8*b*log(c*x^8 + b*x^4 + a)/a^2 - 1/4*b*log(x^4)/a^2 + 1/4*(b^2 - 2*a*c)*a*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/4*(b*x^4 - a)/(a^2*x^4)

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{c \arctan\left(\frac{2cx^4 + b}{\sqrt{4ac - b^2}}\right)}{2\sqrt{4ac - b^2}a} + \frac{b^2 \arctan\left(\frac{2cx^4 + b}{\sqrt{4ac - b^2}}\right)}{4\sqrt{4ac - b^2}a^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln(cx^8 + bx^4 + a)}{8a^2} - \frac{1}{4ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(c*x^8+b*x^4+a),x)

[Out] -1/4/x^4/a - 1/a^2*b*ln(x) + 1/8*b*ln(c*x^8 + b*x^4 + a)/a^2 - 1/2/a/(4*a*c - b^2)^(1/2)*arctan((2*c*x^4 + b)/(4*a*c - b^2)^(1/2))*c + 1/4/a^2/(4*a*c - b^2)^(1/2)*arctan((2*c*x^4 + b)/(4*a*c - b^2)^(1/2))*b^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.79, size = 8817, normalized size = 99.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + b*x^4 + c*x^8)),x)

[Out]
$$\begin{aligned} & \left(\operatorname{atan}\left(\frac{(4ac - b^2)^2(5b^7 - 23a^3bc^3 + 66a^2b^3c^2 - 35ab^5c)}{(4b^3 - 16abc) \left(\frac{(256a^4b^5c^4 - 256a^5b^3c^5)/a^5 - (128ab^4c^4(4b^3 - 16abc))/(64a^3c - 16a^2b^2)}{(2ac - b^2)} \right)}{(8a^2(4ac - b^2)^{1/2}) - (16b^4c^4(4b^3 - 16abc)(2ac - b^2)) / (a(4ac - b^2)^{1/2}(64a^3c - 16a^2b^2))} \right) \right) \\ & - \frac{(2b^4c^4(4b^3 - 16abc)(2ac - b^2)^2)}{(a^3(4ac - b^2)(64a^3c - 16a^2b^2))} \frac{(2ac - b^2)}{(8a^2(4ac - b^2)^{1/2})} - \frac{(b^4c^4(4b^3 - 16abc)(2ac - b^2)^3)}{(4a^5(4ac - b^2)^{3/2}(64a^3c - 16a^2b^2))} \frac{1}{(2(64a^3c - 16a^2b^2))} \\ & - \frac{((4b^3 - 16abc) \left(\frac{(4b^3 - 16abc) \left(\frac{(256a^4b^5c^4 - 256a^5b^3c^5)/a^5 - (128ab^4c^4(4b^3 - 16abc))/(64a^3c - 16a^2b^2)}{(2ac - b^2)} \right)}{(8a^2(4ac - b^2)^{1/2})} - (16b^4c^4(4b^3 - 16abc)(2ac - b^2)) / (a(4ac - b^2)^{1/2}(64a^3c - 16a^2b^2)) \right)}{(2(64a^3c - 16a^2b^2))} \\ & + \left(\frac{(256a^3b^4c^5 - 96a^4b^2c^6)/a^5 + ((4b^3 - 16abc) \left(\frac{(256a^4b^5c^4 - 256a^5b^3c^5)/a^5 - (128ab^4c^4(4b^3 - 16abc))/(64a^3c - 16a^2b^2)}{(2ac - b^2)} \right)}{(8a^2(4ac - b^2)^{1/2})} \right)}{(2(64a^3c - 16a^2b^2))} \\ & - \frac{((16a^3b^3c^7 - 96a^2b^3c^6)/a^5 - ((4b^3 - 16abc) \left(\frac{(256a^3b^4c^5 - 96a^4b^2c^6)/a^5 + ((4b^3 - 16abc) \left(\frac{(256a^4b^5c^4 - 256a^5b^3c^5)/a^5 - (128ab^4c^4(4b^3 - 16abc))/(64a^3c - 16a^2b^2)}{(2ac - b^2)} \right)}{(8a^2(4ac - b^2)^{1/2})} \right)}{(2(64a^3c - 16a^2b^2))} \right)}{(2(64a^3c - 16a^2b^2))} \\ & + \left(\frac{(256a^3b^4c^5 - 96a^4b^2c^6)/a^5 + ((4b^3 - 16abc) \left(\frac{(256a^4b^5c^4 - 256a^5b^3c^5)/a^5 - (128ab^4c^4(4b^3 - 16abc))/(64a^3c - 16a^2b^2)}{(2ac - b^2)} \right)}{(8a^2(4ac - b^2)^{1/2})} \right)}{(2(64a^3c - 16a^2b^2))} \\ & - \frac{(16a^3b^3c^7 - 96a^2b^3c^6)/a^5 - ((4b^3 - 16abc) \left(\frac{(256a^3b^4c^5 - 96a^4b^2c^6)/a^5 + ((4b^3 - 16abc) \left(\frac{(256a^4b^5c^4 - 256a^5b^3c^5)/a^5 - (128ab^4c^4(4b^3 - 16abc))/(64a^3c - 16a^2b^2)}{(2ac - b^2)} \right)}{(8a^2(4ac - b^2)^{1/2})} \right)}{(2(64a^3c - 16a^2b^2))} \right)}{(8a^2(4ac - b^2)^{1/2})} \\ & - \frac{(16b^4c^4(4b^3 - 16abc)(2ac - b^2))}{(a(4ac - b^2)^{1/2}(64a^3c - 16a^2b^2))} \frac{(2ac - b^2)}{(8a^2(4ac - b^2)^{1/2})} \end{aligned}$$

$$\begin{aligned}
&^{(1/2)} - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2) \\
&*(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)) + (((4*b^3 - 16*a* \\
&b*c)*(((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 1 \\
&6*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)} \\
&)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^{(1/2)}*(\\
&64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 \\
&- 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3 \\
&*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2 \\
&*(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)))*(2*a \\
&*c - b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)))/(8*a^2*(4*a*c - b^2)^{(1/2)} + (((a \\
&^2*c^8 - 16*a*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((16*a^3*b*c^7 - 96*a^2*b^ \\
&3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + \\
&((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c \\
&^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2) \\
&)))/(2*(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2 \\
&))/(8*a^2*(4*a*c - b^2)^{(1/2)} + (b^4*c^4*(2*a*c - b^2)^5)/(128*a^9*(4*a*c \\
&- b^2)^{(5/2)))/(c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)*(16*a^4*c^8 + b^8*c^4 \\
&- 8*a*b^6*c^5 + 24*a^2*b^4*c^6 - 32*a^3*b^2*c^7)) - (128*a^10*x^4*((5*b^6 \\
&- a^3*c^3 + 26*a^2*b^2*c^2 - 25*a*b^4*c)*(c^9/a^5 + ((4*b^3 - 16*a*b*c)*((2 \\
&0*b*c^8)/a^4 + ((4*b^3 - 16*a*b*c)*((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((\\
&4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b \\
&*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a \\
&^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))/((2*(64*a^ \\
&3*c - 16*a^2*b^2)))/((2*(64*a^3*c - 16*a^2*b^2)))/((2*(64*a^3*c - 16*a^2*b^ \\
&2)))/((2*(64*a^3*c - 16*a^2*b^2)) + ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c \\
&)*(((2*a*c - b^2)*(((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 1 \\
&6*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b \\
&^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b^3 - 16*a*b*c)*(2*a \\
&*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^{(1/2} \\
&)*(64*a^3*c - 16*a^2*b^2)))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b^3 - 16*a*b \\
&*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c \\
&- b^2)*(64*a^3*c - 16*a^2*b^2)))/((2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b \\
&^2)*(((4*b^3 - 16*a*b*c)*(((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4* \\
&b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 1 \\
&6*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b^3 - 16*a*b* \\
&c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^ \\
&2)^{(1/2)}*(64*a^3*c - 16*a^2*b^2)))/((2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^ \\
&4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 34 \\
&56*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3* \\
&c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))/((2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c \\
&- b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)))/(8*a^2*(4*a*c - b^2)^{(1/2)))/(2*(64 \\
&a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a \\
&*b*c)*(((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(12 \\
&80*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c \\
&- b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1
\end{aligned}$$

$$\begin{aligned}
& (280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5)/(16*a^7*(4*a*c - b^2)^{(1/2)}*(64*a^3*c \\
& - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b \\
& ^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 \\
& + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^ \\
& 3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))/(2*a*c - b^2))/(8*a^2*(4* \\
& a*c - b^2)^{(1/2)))/(2*(64*a^3*c - 16*a^2*b^2)) - (((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - \\
& ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 1 \\
& 6*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b \\
& ^2)))))/(2*(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - \\
& b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)))/(8*a^2*(4*a*c - b^2)^{(1/2)} - (((2*a*c \\
& - b^2)*((2*a*c - b^2)*(((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b \\
& ^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16 \\
& *a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b^3 - 16*a*b*c \\
&)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2 \\
&)^{(1/2)}*(64*a^3*c - 16*a^2*b^2)))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b^3 - \\
& 16*a*b*c)*(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(\\
& 4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2)))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b \\
& ^3 - 16*a*b*c)*(2*a*c - b^2)^3*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(1024 \\
& *a^11*(4*a*c - b^2)^{(3/2)}*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(\\
& 4*a*c - b^2)^{(1/2)} - ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)^4*(1280*a^5*b^5*c^4 \\
& - 4608*a^6*b^3*c^5))/(8192*a^13*(4*a*c - b^2)^2*(64*a^3*c - 16*a^2*b^2)))/ \\
& /((32*a^5*c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)) + ((5*b^7 - 23*a^3*b*c^3 + 66 \\
& *a^2*b^3*c^2 - 35*a*b^5*c)*((4*b^3 - 16*a*b*c)*((2*a*c - b^2)*((2*a*c - \\
& b^2)*(((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(128 \\
& 0*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c \\
& - b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(12 \\
& 80*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^{(1/2)}*(64*a^3*c - \\
& 16*a^2*b^2)))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b^3 - 16*a*b*c)*(2*a*c - \\
& b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c - b^2)*(64*a^ \\
& 3*c - 16*a^2*b^2)))/(8*a^2*(4*a*c - b^2)^{(1/2)} + ((4*b^3 - 16*a*b*c)*(2*a \\
& *c - b^2)^3*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(1024*a^11*(4*a*c - b^2 \\
&)^{(3/2)}*(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4*b^3 - 1 \\
& 6*a*b*c)*(((4*b^3 - 16*a*b*c)*((4*b^3 - 16*a*b*c)*(((448*a^4*b^4*c^5 - 34 \\
& 56*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3* \\
& c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^ \\
& (1/2)) + ((4*b^3 - 16*a*b*c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3 \\
& *c^5))/(16*a^7*(4*a*c - b^2)^{(1/2)}*(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c \\
& - 16*a^2*b^2)) - (((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b \\
& *c)*((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a \\
& ^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^ \\
& 3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^{(1/2)))/(2*(64*a^3 \\
& *c - 16*a^2*b^2)) - (((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b \\
& *c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4* \\
& b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4
\end{aligned}$$

$$\begin{aligned}
& a^2 b^3 c^6 / a^5 - ((4b^3 - 16a*b*c) * ((256a^3 b^4 c^5 - 96a^4 b^2 c^6) / \\
& a^5 + ((4b^3 - 16a*b*c) * ((256a^4 b^5 c^4 - 256a^5 b^3 c^5) / a^5 - (128a \\
& * b^4 c^4 * (4b^3 - 16a*b*c)) / (64a^3 c - 16a^2 b^2))) / (2 * (64a^3 c - 16a^2 \\
& b^2))) / (2 * (64a^3 c - 16a^2 b^2))) / (2 * (64a^3 c - 16a^2 b^2))) / (2 * (6 \\
& 4a^3 c - 16a^2 b^2)) + (((4b^3 - 16a*b*c) * ((4b^3 - 16a*b*c) * (((256 \\
& * a^4 b^5 c^4 - 256a^5 b^3 c^5) / a^5 - (128a*b^4 c^4 * (4b^3 - 16a*b*c)) / (6 \\
& 4a^3 c - 16a^2 b^2)) * (2a*c - b^2)) / (8a^2 * (4a*c - b^2)^(1/2)) - (16b^4 \\
& * c^4 * (4b^3 - 16a*b*c) * (2a*c - b^2)) / (a * (4a*c - b^2)^(1/2) * (64a^3 c - 1 \\
& 6a^2 b^2)))) / (2 * (64a^3 c - 16a^2 b^2)) + (((256a^3 b^4 c^5 - 96a^4 b^2 \\
& * c^6) / a^5 + ((4b^3 - 16a*b*c) * ((256a^4 b^5 c^4 - 256a^5 b^3 c^5) / a^5 - \\
& (128a*b^4 c^4 * (4b^3 - 16a*b*c)) / (64a^3 c - 16a^2 b^2))) / (2 * (64a^3 c - \\
& 16a^2 b^2))) * (2a*c - b^2)) / (8a^2 * (4a*c - b^2)^(1/2))) / (2 * (64a^3 c - \\
& 16a^2 b^2)) - (((16a^3 b*c^7 - 96a^2 b^3 c^6) / a^5 - ((4b^3 - 16a*b*c) * \\
& ((256a^3 b^4 c^5 - 96a^4 b^2 c^6) / a^5 + ((4b^3 - 16a*b*c) * ((256a^4 b^5 \\
& c^4 - 256a^5 b^3 c^5) / a^5 - (128a*b^4 c^4 * (4b^3 - 16a*b*c)) / (64a^3 c \\
& - 16a^2 b^2))) / (2 * (64a^3 c - 16a^2 b^2)))) / (2 * (64a^3 c - 16a^2 b^2))) * \\
& (2a*c - b^2)) / (8a^2 * (4a*c - b^2)^(1/2))) * (2a*c - b^2)) / (8a^2 * (4a*c - \\
& b^2)^(1/2)) - ((((((((((256a^4 b^5 c^4 - 256a^5 b^3 c^5) / a^5 - (128a*b^4 \\
& c^4 * (4b^3 - 16a*b*c)) / (64a^3 c - 16a^2 b^2)) * (2a*c - b^2)) / (8a^2 * (4a \\
& * c - b^2)^(1/2)) - (16b^4 c^4 * (4b^3 - 16a*b*c) * (2a*c - b^2)) / (a * (4a*c \\
& - b^2)^(1/2) * (64a^3 c - 16a^2 b^2))) * (2a*c - b^2)) / (8a^2 * (4a*c - b^2)^(\\
& 1/2)) - (2b^4 c^4 * (4b^3 - 16a*b*c) * (2a*c - b^2)^2) / (a^3 * (4a*c - b^2) * \\
& (64a^3 c - 16a^2 b^2))) * (2a*c - b^2)) / (8a^2 * (4a*c - b^2)^(1/2)) - (b^4 \\
& * c^4 * (4b^3 - 16a*b*c) * (2a*c - b^2)^3) / (4a^5 * (4a*c - b^2)^(3/2) * (64a^3 \\
& c - 16a^2 b^2))) * (2a*c - b^2)) / (8a^2 * (4a*c - b^2)^(1/2)) + (b^4 c^4 * (4 \\
& * b^3 - 16a*b*c) * (2a*c - b^2)^4) / (32a^7 * (4a*c - b^2)^2 * (64a^3 c - 16a^2 \\
& b^2)))) / (c^4 * (a^2 c^2 - 20b^4 + 80a*b^2 c) * (16a^4 c^8 + b^8 c^4 - 8a * \\
& b^6 c^5 + 24a^2 b^4 c^6 - 32a^3 b^2 c^7))) * (2a*c - b^2)) / (4a^2 * (4a*c - \\
& b^2)^(1/2)) - (b*log(x)) / a^2 - (log(a + b*x^4 + c*x^8) * (4b^3 - 16a*b*c)) \\
& / (2 * (64a^3 c - 16a^2 b^2)) - 1 / (4a*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.262 \quad \int \frac{x^{10}}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=381

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) - \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) + \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}} c^{7/4} \sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) + \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}} c^{7/4} \sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}} c^{7/4} \sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Rubi [A] time = 0.65, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1367, 1510, 298, 205, 208}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) - \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) + \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) - \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}} c^{7/4} \sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) + \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) + \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) - \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}} c^{7/4} \sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{x^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[x^10/(a + b*x^4 + c*x^8), x]

[Out] $x^3/(3*c) - ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*c^{(1/4)}*x)/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(3/4)}*c^{(7/4)}*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}) - ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*c^{(1/4)}*x)/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(3/4)}*c^{(7/4)}*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}) + ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{(1/4)}*c^{(1/4)}*x)/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(3/4)}*c^{(7/4)}*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}) + ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{(1/4)}*c^{(1/4)}*x)/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(3/4)}*c^{(7/4)}*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)})$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 298

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x

], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1367

Int[((d_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]

Rule 1510

Int[(((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^(n_)))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^{10}}{a + bx^4 + cx^8} dx &= \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^4)}{a+bx^4+cx^8} dx}{3c} \\ &= \frac{x^3}{3c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{x^2}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{x^2}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2c} \\ &= \frac{x^3}{3c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b + \sqrt{b^2-4ac} - \sqrt{2}} \sqrt{cx^2}} dx}{2\sqrt{2}c^{3/2}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b + \sqrt{b^2-4ac} + \sqrt{2}} \sqrt{cx^2}} dx}{2\sqrt{2}c^{3/2}} \\ &= \frac{x^3}{3c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b - \sqrt{b^2-4ac}}}\right)}{2 \cdot 2^{3/4} c^{7/4} \sqrt[4]{-b - \sqrt{b^2-4ac}}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b + \sqrt{b^2-4ac}}}\right)}{2 \cdot 2^{3/4} c^{7/4} \sqrt[4]{-b + \sqrt{b^2-4ac}}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b + \sqrt{b^2-4ac}}}\right)}{2 \cdot 2^{3/4} c^{7/4} \sqrt[4]{-b + \sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.18

$$\frac{4x^3 - 3\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4b \log(x-\#1) + a \log(x-\#1)}{2\#1^5c + \#1b}\right]}{12c}$$

Antiderivative was successfully verified.

[In] Integrate[x¹⁰/(a + b*x⁴ + c*x⁸),x]

[Out] (4*x³ - 3*RootSum[a + b*#1⁴ + c*#1⁸ & , (a*Log[x - #1] + b*Log[x - #1]*#1⁴)/(b*#1 + 2*c*#1⁵) &])/(12*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x¹⁰/(a + b*x⁴ + c*x⁸),x]

[Out] IntegrateAlgebraic[x¹⁰/(a + b*x⁴ + c*x⁸), x]

fricas [B] time = 5.66, size = 6296, normalized size = 16.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁰/(c*x⁸+b*x⁴+a),x, algorithm="fricas")

[Out] 1/12*(4*x³ + 12*c*sqrt(sqrt(1/2)*sqrt(-(b⁷ - 7*a*b⁵*c + 14*a²*b³*c² - 7*a³*b*c³ + (b⁴*c⁷ - 8*a*b²*c⁸ + 16*a²*c⁹)*sqrt((b¹² - 10*a*b¹⁰*c + 37*a²*b⁸*c² - 62*a³*b⁶*c³ + 46*a⁴*b⁴*c⁴ - 12*a⁵*b²*c⁵ + a⁶*c⁶)/(b⁶*c¹⁴ - 12*a*b⁴*c¹⁵ + 48*a²*b²*c¹⁶ - 64*a³*c¹⁷)))/(b⁴*c⁷ - 8*a*b²*c⁸ + 16*a²*c⁹))*arctan(-1/2*((b⁶*c⁷ - 10*a*b⁴*c⁸ + 32*a²*b²*c⁹ - 32*a³*c¹⁰)*x*sqrt((b¹² - 10*a*b¹⁰*c + 37*a²*b⁸*c² - 62*a³*b⁶*c³ + 46*a⁴*b⁴*c⁴ - 12*a⁵*b²*c⁵ + a⁶*c⁶)/(b⁶*c¹⁴ - 12*a*b⁴*c¹⁵ + 48*a²*b²*c¹⁶ - 64*a³*c¹⁷)) - (b⁹ - 9*a*b⁷*c + 26*a²*b⁵*c² - 25*a³*b³*c³ + 4*a⁴*b*c⁴)*x + sqrt(1/2)*(b⁹ - 9*a*b⁷*c + 26*a²*b⁵*c² - 25*a³*b³*c³ + 4*a⁴*b*c⁴ - (b⁶*c⁷ - 10*a*b⁴*c⁸ + 32*a²*b²*c⁹ - 32*a³*c¹⁰)*sqrt((b¹² - 10*a*b¹⁰*c + 37*a²*b⁸*c² - 62*a³*b⁶*c³ + 46*a⁴*b⁴*c⁴ - 12*a⁵*b²*c⁵ + a⁶*c⁶)/(b⁶*c¹⁴ - 12*a*b⁴*c¹⁵ + 48*a²*b²*c¹⁶ - 64*a³*c¹⁷))*sqrt((2*(a³*b⁶ - 5*a⁴*b⁴*c + 6*a⁵*b²*c² - a⁶*c³)*x² - sqrt(1/2)*(b¹¹ - 12*a*b⁹*c + 53*a²*b⁷*c² - 103*a³*b⁵*c³ + 79*a⁴*b³*c⁴ - 12*a⁵*b*c⁵ - (b⁸*c⁷ - 13*a*b⁶*c⁸ + 60*a²*b⁴*c⁹ - 112*a³*b²*c¹⁰ + 64*a⁴*c¹¹)*sqrt((b¹² - 10*a*b¹⁰*c + 37*a²*b⁸*c² - 62*a³*b⁶*c³ + 46*a⁴*b⁴*c⁴ - 12*a⁵*b²*c⁵ + a⁶*c⁶)/(b⁶*c¹⁴ - 12*a*b⁴*c¹⁵ + 48*a²*b²*c¹⁶ - 64*a³*c¹⁷))*sqrt(-(b⁷ - 7*a*b⁵*c + 14*a²*b³*c² - 7*a³*b*c³ + (b⁴*c⁷ - 8*a*b²*c⁸ + 16*a²*c⁹)*sqrt((b¹² - 10*a*b¹⁰*c + 37*a²*b⁸*c² - 62*a³*b⁶*c³ + 46*a⁴*b⁴*c⁴ - 12*a⁵*b²*c⁵ + a⁶*c⁶)/(b⁶*c¹⁴ - 12*a*b⁴*c¹⁵ + 48*a²*b²*c¹⁶ - 64*a³*c¹⁷)))/(b⁴*c⁷ - 8*a*b²*c⁸ + 16*a²*c⁹))/(a³*b⁶ - 5*a⁴

$$\begin{aligned}
& 4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)) * \text{sqrt}(\text{sqrt}(1/2) * \text{sqrt}(-(b^7 - 7*a*b^5*c \\
& + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9) * \text{sqrt}(\\
& (b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12 \\
& *a^5*b^2*c^5 + a^6*c^6)/(b^6*c^{14} - 12*a*b^4*c^{15} + 48*a^2*b^2*c^{16} - 64*a^3*c^{17}))) \\
&)/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^2*b^6 - 5*a^3*b^4*c + 6 \\
& *a^4*b^2*c^2 - a^5*c^3) - 12*c * \text{sqrt}(\text{sqrt}(1/2) * \text{sqrt}(-(b^7 - 7*a*b^5*c + 14* \\
& a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9) * \text{sqrt}((b^{12} \\
& - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5* \\
& b^2*c^5 + a^6*c^6)/(b^6*c^{14} - 12*a*b^4*c^{15} + 48*a^2*b^2*c^{16} - 64*a^3*c^{17}))) \\
&)/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) * \arctan(1/2 * (\text{sqrt}(1/2) * (b^9 - 9* \\
& a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (b^6*c^7 - 10*a*b^4*c^8 + \\
& 32*a^2*b^2*c^9 - 32*a^3*c^{10}) * \text{sqrt}((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - \\
& 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^{14} - 12*a*b^4*c^{15} + \\
& 48*a^2*b^2*c^{16} - 64*a^3*c^{17}))) * \text{sqrt}(\text{sqrt}(1/2) * \text{sqrt}(- \\
& (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + \\
& 16*a^2*c^9) * \text{sqrt}((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46 \\
& *a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^{14} - 12*a*b^4*c^{15} + 48*a^2 \\
& *b^2*c^{16} - 64*a^3*c^{17}))))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) * \text{sqrt}((2*(\\
& a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3) * x^2 - \text{sqrt}(1/2) * (b^{11} - 12 \\
& *a*b^9*c + 53*a^2*b^7*c^2 - 103*a^3*b^5*c^3 + 79*a^4*b^3*c^4 - 12*a^5*b*c^5 \\
& + (b^8*c^7 - 13*a*b^6*c^8 + 60*a^2*b^4*c^9 - 112*a^3*b^2*c^{10} + 64*a^4*c^{11}) * \\
& \text{sqrt}((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - \\
& 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^{14} - 12*a*b^4*c^{15} + 48*a^2*b^2*c^{16} - \\
& 64*a^3*c^{17}))) * \text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (\\
& b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9) * \text{sqrt}((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - \\
& 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^{14} - 12*a*b^4*c^{15} + \\
& 48*a^2*b^2*c^{16} - 64*a^3*c^{17}))))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3) - ((b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^{10}) * x * \text{sqrt}((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^{14} - 12*a*b^4*c^{15} + 48*a^2*b^2*c^{16} - 64*a^3*c^{17}))) + (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4) * x) * \text{sqrt}(\text{sqrt}(1/2) * \text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9) * \text{sqrt}((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^{14} - 12*a*b^4*c^{15} + 48*a^2*b^2*c^{16} - 64*a^3*c^{17}))))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^2*b^6 - 5*a^3*b^4*c + 6*a^4*b^2*c^2 - a^5*c^3) - 3*c * \text{sqrt}(\text{sqrt}(1/2) * \text{sqrt}(- \\
& (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9) * \text{sqrt}((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^{14} - 12*a*b^4*c^{15} + 48*a^2*b^2*c^{16} - 64*a^3*c^{17}))))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) * \log(1/2 * \text{sqrt}(1/2) * (b^{14} - 16*a*b^{12}*c + 102*a^2*b^{10}*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^{11}*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^{10} + 560*a^4*b^3*c^{11} - 320*a^5*b*c^{12}) * \text{sqrt}((b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
\end{aligned}$$

$$\begin{aligned}
& *a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2* \\
& *b^2*c^16 - 64*a^3*c^17)))*\text{sqrt}(\text{sqrt}(1/2)*\text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a^2*b \\
& ^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*\text{sqrt}((b^12 - 10 \\
& *a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c \\
& ^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/ \\
& (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*\text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a^2*b^3* \\
& c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*\text{sqrt}((b^12 - 10*a* \\
& b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 \\
& + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^ \\
& 4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 \\
& - a^8*c^3)*x) + 3*c*\text{sqrt}(\text{sqrt}(1/2)*\text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 \\
& - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*\text{sqrt}((b^12 - 10*a*b^1 \\
& 0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a \\
& ^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c \\
& ^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*\log(-1/2*\text{sqrt}(1/2)*(b^14 - 16*a*b^12*c + 1 \\
& 02*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152 \\
& *a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 36 \\
& 4*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*\text{sqrt}((b^12 - 10*a*b^10* \\
& c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6 \\
& *c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*\text{sqrt}(\text{sqr} \\
& t(1/2)*\text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8 \\
& *a*b^2*c^8 + 16*a^2*c^9)*\text{sqrt}((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3 \\
& *b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4* \\
& c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9 \\
&)))*\text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a* \\
& b^2*c^8 + 16*a^2*c^9)*\text{sqrt}((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^ \\
& 6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^1 \\
& 5 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) \\
& - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*x) - 3*c*\text{sqrt}(\text{sqrt}(1/2) \\
& *\text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2 \\
& *c^8 + 16*a^2*c^9)*\text{sqrt}((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c \\
& ^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + \\
& 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*\text{lo} \\
& g(1/2*\text{sqrt}(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + \\
& 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^ \\
& 7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - \\
& 320*a^5*b*c^12)*\text{sqrt}((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 \\
& + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48 \\
& *a^2*b^2*c^16 - 64*a^3*c^17)))*\text{sqrt}(\text{sqrt}(1/2)*\text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a \\
& ^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*\text{sqrt}((b^12 \\
& - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b \\
& ^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17 \\
&)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*\text{sqrt}(-(b^7 - 7*a*b^5*c + 14*a^2* \\
& b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*\text{sqrt}((b^12 - 1 \\
& 0*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*
\end{aligned}$$

$$\frac{c^5 + a^6 c^6}{(b^6 c^{14} - 12 a b^4 c^{15} + 48 a^2 b^2 c^{16} - 64 a^3 c^{17})} \left(\frac{1}{(b^4 c^7 - 8 a b^2 c^8 + 16 a^2 c^9)} - (a^5 b^6 - 5 a^6 b^4 c + 6 a^7 b^2 c^2 - a^8 c^3) x \right) + 3 c \sqrt{\sqrt{\frac{1}{2}} \sqrt{-(b^7 - 7 a b^5 c + 14 a^2 b^3 c^2 - 7 a^3 b c^3 - (b^4 c^7 - 8 a b^2 c^8 + 16 a^2 c^9) \sqrt{(b^{12} - 10 a b^{10} c + 37 a^2 b^8 c^2 - 62 a^3 b^6 c^3 + 46 a^4 b^4 c^4 - 12 a^5 b^2 c^5 + a^6 c^6) / (b^6 c^{14} - 12 a b^4 c^{15} + 48 a^2 b^2 c^{16} - 64 a^3 c^{17}))} / (b^4 c^7 - 8 a b^2 c^8 + 16 a^2 c^9))} \log(-1/2 \sqrt{\frac{1}{2}} (b^{14} - 16 a b^{12} c + 102 a^2 b^{10} c^2 - 328 a^3 b^8 c^3 + 553 a^4 b^6 c^4 - 457 a^5 b^4 c^5 + 152 a^6 b^2 c^6 - 16 a^7 c^7 + (b^{11} c^7 - 17 a b^9 c^8 + 113 a^2 b^7 c^9 - 364 a^3 b^5 c^{10} + 560 a^4 b^3 c^{11} - 320 a^5 b c^{12}) \sqrt{(b^{12} - 10 a b^{10} c + 37 a^2 b^8 c^2 - 62 a^3 b^6 c^3 + 46 a^4 b^4 c^4 - 12 a^5 b^2 c^5 + a^6 c^6) / (b^6 c^{14} - 12 a b^4 c^{15} + 48 a^2 b^2 c^{16} - 64 a^3 c^{17}))} \sqrt{\sqrt{\frac{1}{2}} \sqrt{-(b^7 - 7 a b^5 c + 14 a^2 b^3 c^2 - 7 a^3 b c^3 - (b^4 c^7 - 8 a b^2 c^8 + 16 a^2 c^9) \sqrt{(b^{12} - 10 a b^{10} c + 37 a^2 b^8 c^2 - 62 a^3 b^6 c^3 + 46 a^4 b^4 c^4 - 12 a^5 b^2 c^5 + a^6 c^6) / (b^6 c^{14} - 12 a b^4 c^{15} + 48 a^2 b^2 c^{16} - 64 a^3 c^{17}))} / (b^4 c^7 - 8 a b^2 c^8 + 16 a^2 c^9))} \sqrt{-(b^7 - 7 a b^5 c + 14 a^2 b^3 c^2 - 7 a^3 b c^3 - (b^4 c^7 - 8 a b^2 c^8 + 16 a^2 c^9) \sqrt{(b^{12} - 10 a b^{10} c + 37 a^2 b^8 c^2 - 62 a^3 b^6 c^3 + 46 a^4 b^4 c^4 - 12 a^5 b^2 c^5 + a^6 c^6) / (b^6 c^{14} - 12 a b^4 c^{15} + 48 a^2 b^2 c^{16} - 64 a^3 c^{17}))} / (b^4 c^7 - 8 a b^2 c^8 + 16 a^2 c^9))} - (a^5 b^6 - 5 a^6 b^4 c + 6 a^7 b^2 c^2 - a^8 c^3) x) / c$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁰/(c*x⁸+b*x⁴+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Evaluation time: 19.53Unable to convert to real 1/4 Error: Bad Argument Value

maple [C] time = 0.04, size = 63, normalized size = 0.17

$$\frac{x^3}{3c} \frac{\left(\text{RootOf}(-Z^8 c + b Z^4 + a)^6 b + \text{RootOf}(-Z^8 c + b Z^4 + a)^2 a \right) \ln(-\text{RootOf}(-Z^8 c + b Z^4 + a) + x)}{4c \left(2 \text{RootOf}(-Z^8 c + b Z^4 + a)^7 c + \text{RootOf}(-Z^8 c + b Z^4 + a)^3 b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁰/(c*x⁸+b*x⁴+a),x)

[Out] 1/3/c*x³-1/4/c*sum((R⁶*b+R²*a)/(2*R⁷*c+R³*b)*ln(-R+x),R=RootOf(Z⁸*c+Z⁴*b+a))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁰/(c*x⁸+b*x⁴+a),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 3.49, size = 12709, normalized size = 33.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁰/(a + b*x⁴ + c*x⁸),x)

[Out] atan((((8192*a⁶*b*c⁶ - 256*a³*b⁷*c³ + 2560*a⁴*b⁵*c⁴ - 8192*a⁵*b³*c⁵)/c³ - (4*x*(-(b¹¹ + b⁶*(-(4*a*c - b²)⁵)^(1/2) - 112*a⁵*b*c⁵ + 86*a²*b⁷*c² - 231*a³*b⁵*c³ + 280*a⁴*b³*c⁴ - a³*c³*(-(4*a*c - b²)⁵)^(1/2) - 15*a*b⁹*c + 6*a²*b²*c²*(-(4*a*c - b²)⁵)^(1/2) - 5*a*b⁴*c*(-(4*a*c - b²)⁵)^(1/2))/(512*(256*a⁴*c¹¹ + b⁸*c⁷ - 16*a*b⁶*c⁸ + 96*a²*b⁴*c⁹ - 256*a³*b²*c¹⁰))^(1/4)*(8192*a⁶*c⁸ - 256*a³*b⁶*c⁵ + 2560*a⁴*b⁴*c⁶ - 8192*a⁵*b²*c⁷)/c³)*(-(b¹¹ + b⁶*(-(4*a*c - b²)⁵)^(1/2) - 112*a⁵*b*c⁵ + 86*a²*b⁷*c² - 231*a³*b⁵*c³ + 280*a⁴*b³*c⁴ - a³*c³*(-(4*a*c - b²)⁵)^(1/2) - 15*a*b⁹*c + 6*a²*b²*c²*(-(4*a*c - b²)⁵)^(1/2) - 5*a*b⁴*c*(-(4*a*c - b²)⁵)^(1/2))/(512*(256*a⁴*c¹¹ + b⁸*c⁷ - 16*a*b⁶*c⁸ + 96*a²*b⁴*c⁹ - 256*a³*b²*c¹⁰))^(3/4) + (4*x*(a⁵*b⁵ - 5*a⁶*b³*c + 5*a⁷*b*c²)/c³)*(-(b¹¹ + b⁶*(-(4*a*c - b²)⁵)^(1/2) - 112*a⁵*b*c⁵ + 86*a²*b⁷*c² - 231*a³*b⁵*c³ + 280*a⁴*b³*c⁴ - a³*c³*(-(4*a*c - b²)⁵)^(1/2) - 15*a*b⁹*c + 6*a²*b²*c²*(-(4*a*c - b²)⁵)^(1/2) - 5*a*b⁴*c*(-(4*a*c - b²)⁵)^(1/2))/(512*(256*a⁴*c¹¹ + b⁸*c⁷ - 16*a*b⁶*c⁸ + 96*a²*b⁴*c⁹ - 256*a³*b²*c¹⁰))^(1/4)*1i - (((8192*a⁶*b*c⁶ - 256*a³*b⁷*c³ + 2560*a⁴*b⁵*c⁴ - 8192*a⁵*b³*c⁵)/c³ + (4*x*(-(b¹¹ + b⁶*(-(4*a*c - b²)⁵)^(1/2) - 112*a⁵*b*c⁵ + 86*a²*b⁷*c² - 231*a³*b⁵*c³ + 280*a⁴*b³*c⁴ - a³*c³*(-(4*a*c - b²)⁵)^(1/2) - 15*a*b⁹*c + 6*a²*b²*c²*(-(4*a*c - b²)⁵)^(1/2) - 5*a*b⁴*c*(-(4*a*c - b²)⁵)^(1/2))/(512*(256*a⁴*c¹¹ + b⁸*c⁷ - 16*a*b⁶*c⁸ + 96*a²*b⁴*c⁹ - 256*a³*b²*c¹⁰))^(1/4)*(8192*a⁶*c⁸ - 256*a³*b⁶*c⁵ + 2560*a⁴*b⁴*c⁶ - 8192*a⁵*b²*c⁷)/c³)*(-(b¹¹ + b⁶*(-(4*a*c - b²)⁵)^(1/2) - 112*a⁵*b*c⁵ + 86*a²*b⁷*c² - 231*a³*b⁵*c³ + 280*a⁴*b³*c⁴ - a³*c³*(-(4*a*c - b²)⁵)^(1/2) - 15*a*b⁹*c + 6*a²*b²*c²*(-(4*a*c - b²)⁵)^(1/2) - 5*a*b⁴*c*(-(4*a*c - b²)⁵)^(1/2))/(512*(256*a⁴*c¹¹ + b⁸*c⁷ - 16*a*b⁶*c⁸ + 96*a²*b⁴*c⁹ - 256*a³*b²*c¹⁰))^(3/4) - (4*x*(a⁵*b⁵ - 5*a⁶*b³*c + 5*a⁷*b*c²)/c³)*(-(b¹¹ + b⁶*(-(4*a*c - b²)⁵)^(1/2) - 112*a⁵*b*c⁵ + 86*a²*b⁷*c² - 231*a³*b⁵*c³ + 280*a⁴*b³*c⁴ - a³*c³

$$\begin{aligned}
& 3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 \\
& + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)}*i)/((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 \\
& + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
&)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)} \\
& *(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
&)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)} \\
& + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
&)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)} \\
& + (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 + b^6 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 \\
& - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/c^3 + (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
&)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)} \\
& - (2*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 \\
& - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/c^3 + (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
&)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)} \\
& *2i + \operatorname{atan}((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(-(b^11 \\
& - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3 \\
& *c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
&)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)}*(8192*a^6*c^8
\end{aligned}$$

$$\begin{aligned}
& - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)/c^3)*(-(b^{11} - b \\
& ^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5* \\
& c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a \\
& ^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(\\
& 512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c \\
& ^{10}))^{(3/4)} + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^{11} - b \\
& ^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5* \\
& c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a \\
& ^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(\\
& 512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c \\
& ^{10}))^{(1/4)}*i - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - \\
& 8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a \\
& ^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a* \\
& b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)}*(8192*a^6*c^8 - 256*a^ \\
& 3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^{11} - b^6*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280 \\
& *a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^ \\
& ^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256* \\
& a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(3 \\
& /4)} - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^{11} - b^6*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280 \\
& *a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^ \\
& ^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256* \\
& a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1 \\
& /4)}*i)/((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5* \\
& b^3*c^5)/c^3 - (4*x*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 \\
& + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b \\
& ^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^ \\
& 4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + \\
& 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)}*(8192*a^6*c^8 - 256*a^3*b^6*c^5 \\
& + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^{11} - b^6*(-(4*a*c - b^2) \\
& ^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3* \\
& c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a* \\
& c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^{11} \\
& + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(3/4)} + (4* \\
& x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^{11} - b^6*(-(4*a*c - b^2) \\
& ^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3* \\
& c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a* \\
& c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^{11} \\
& + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)} + (((\\
& 8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 \\
& + (4*x*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7 \\
& *c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c \\
& - b^2)^5)^{(1/2)}/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 \\
& - 256*a^3*b^2*c^10)))^{(1/4)}*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 \\
& - 8192*a^5*b^2*c^7)/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3 \\
& 3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^11 + b^8*c^7 - \\
& 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^{(3/4)} - (4*x*(a^5*b^5 - \\
& 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3 \\
& 3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^11 + b^8*c^7 - \\
& 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^{(1/4)} - (2*(a^8*c - a^7 \\
& *b^2))/c^3))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2 \\
& *b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4 \\
& *a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2 \\
& *b^4*c^9 - 256*a^3*b^2*c^10)))^{(1/4)}*2i + 2*atan((((8192*a^6*b*c^6 - 256*a^3 \\
& *b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^11 + b^6*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + \\
& 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2 \\
& *c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(\\
& 256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10))) \\
&)^{(1/4)}*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7) \\
& *4i)/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2 \\
& *b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4 \\
& *a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2 \\
& *b^4*c^9 - 256*a^3*b^2*c^10)))^{(3/4)}*1i - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7 \\
& *b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86 \\
& *a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4 \\
& *a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2 \\
& *b^4*c^9 - 256*a^3*b^2*c^10)))^{(1/4)} - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 \\
& + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(-(b^11 + b^6*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4 \\
& *b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4 \\
& *a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^11 \\
& + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^{(1/4)}* \\
& (8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/ \\
& c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 \\
& - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c \\
& - b^2)^5)^{(1/2)}/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 \\
& - 256*a^3*b^2*c^10)))^{(3/4)}*1i + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2)
\end{aligned}$$

$$\begin{aligned}
&))/c^3)*(-(b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4))}/(((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 256 \\
& 0*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (x*(-(b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 \\
& - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^{11} + b \\
& ^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)}*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(\\
& b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231* \\
& a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9 \\
& *c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a \\
& ^3*b^2*c^{10}))^{(3/4)}*1i - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)* \\
& (-(b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 2 \\
& 31*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a* \\
& b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^ \\
& 5)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 25 \\
& 6*a^3*b^2*c^{10}))^{(1/4)}*1i + (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4 \\
& *b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(-(b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} \\
&) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^ \\
& 3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2) \\
& ^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^{11} + b^8*c^ \\
& 7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)}*(8192*a^6*c^8 \\
& - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3)*(-(b^{11} \\
& + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b \\
& ^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + \\
& 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)} \\
&)/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^ \\
& 2*c^{10}))^{(3/4)}*1i + (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^ \\
& 11 + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^ \\
& 3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c \\
& + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1 \\
& /2)))/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3 \\
& *b^2*c^{10}))^{(1/4)}*1i + (2*(a^8*c - a^7*b^2))/c^3))*(-(b^{11} + b^6*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^ \\
& 4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(\\
& -(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4 \\
& *c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)} \\
& + 2*atan((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^ \\
& 5*b^3*c^5)/c^3 - (x*(-(b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 \\
& + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b \\
& ^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^
\end{aligned}$$

$$\begin{aligned}
& 4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + \\
& 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)}*(8192*a^6*c^8 - 256*a^3*b^6*c^5 \\
& + 2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i/c^3*(-(b^{11} - b^6*(-(4*a*c - b \\
& ^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b \\
& ^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4 \\
& *a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^{11} \\
& + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(3/4)}*1i \\
& - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3*(-(b^{11} - b^6*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4 \\
& *b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4 \\
& *a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4 \\
& *c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)} \\
& - (((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + \\
& (x*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2 \\
& *b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4 \\
& *a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2* \\
& b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)}*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560* \\
& a^4*b^4*c^6 - 8192*a^5*b^2*c^7)*4i)/c^3*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + \\
& a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b \\
& ^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^{11} + b^8 \\
& *c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(3/4)}*1i + (4*x* \\
& (a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3*(-(b^{11} - b^6*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 \\
& + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^{11} + \\
& b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)})/(((81 \\
& 92*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - \\
& (x*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 \\
& - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 1 \\
& 5*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b \\
& ^2)^5)^{(1/2)})/(512*(256*a^4*c^{11} + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 \\
& - 256*a^3*b^2*c^{10}))^{(1/4)}*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4* \\
& c^6 - 8192*a^5*b^2*c^7)*4i)/c^3*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 1 \\
& 12*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3 \\
& *(- (4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^{11} + b^8*c^7 - 1 \\
& 6*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(3/4)}*1i - (4*x*(a^5*b^5 \\
& - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3* \\
& c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^{11} + b^8*c^7 \\
& - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^{10}))^{(1/4)}*1i + (((8192*a^6 \\
& *b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (x*(
\end{aligned}$$

$$\begin{aligned}
& -(b^{11} - b^6(-4ac - b^2)^5)^{1/2} - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 + a^3c^3(-4ac - b^2)^5)^{1/2} - 15ab^9c - 6a^2b^2c^2(-4ac - b^2)^5)^{1/2} + 5ab^4c(-4ac - b^2)^5)^{1/2}) / (512(256a^4c^{11} + b^8c^7 - 16ab^6c^8 + 96a^2b^4c^9 - 256a^3b^2c^{10}))^{1/4} * (8192a^6c^8 - 256a^3b^6c^5 + 2560a^4b^4c^6 - 8192a^5b^2c^7) * 4i) / c^3 * (-b^{11} - b^6(-4ac - b^2)^5)^{1/2} - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 + a^3c^3(-4ac - b^2)^5)^{1/2} - 15ab^9c - 6a^2b^2c^2(-4ac - b^2)^5)^{1/2} + 5ab^4c(-4ac - b^2)^5)^{1/2}) / (512(256a^4c^{11} + b^8c^7 - 16ab^6c^8 + 96a^2b^4c^9 - 256a^3b^2c^{10}))^{3/4} * i + (4x*(a^5b^5 - 5a^6b^3c + 5a^7b^2c^2)) / c^3 * (-b^{11} - b^6(-4ac - b^2)^5)^{1/2} - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 + a^3c^3(-4ac - b^2)^5)^{1/2} - 15ab^9c - 6a^2b^2c^2(-4ac - b^2)^5)^{1/2} + 5ab^4c(-4ac - b^2)^5)^{1/2}) / (512(256a^4c^{11} + b^8c^7 - 16ab^6c^8 + 96a^2b^4c^9 - 256a^3b^2c^{10}))^{1/4} * i + (2*(a^8c - a^7b^2)) / c^3 * (-b^{11} - b^6(-4ac - b^2)^5)^{1/2} - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 + a^3c^3(-4ac - b^2)^5)^{1/2} - 15ab^9c - 6a^2b^2c^2(-4ac - b^2)^5)^{1/2} + 5ab^4c(-4ac - b^2)^5)^{1/2}) / (512(256a^4c^{11} + b^8c^7 - 16ab^6c^8 + 96a^2b^4c^9 - 256a^3b^2c^{10}))^{1/4} + x^3 / (3c)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**10/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.263 \quad \int \frac{x^8}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \dots$$

Rubi [A] time = 0.57, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a + b*x^4 + c*x^8), x]

[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x]/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x]/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x]/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x]/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],

$x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /; \text{FreeQ}\{a, b\}, x\} \&\amp; \text{!GtQ}[a/b, 0]$

Rule 1367

$\text{Int}[\{(d_)*(x_)\}^{(m_)}*\{(a_)+(c_)*(x_)\}^{(n2_)}+(b_)*(x_)\}^{(n_)}\}^{(p_)}, x_Symbol] :> \text{Simp}[(d^{(2*n-1)}*(d*x)^{(m-2*n+1)}*(a+b*x^n+c*x^{(2*n)})^{(p+1)})/(c*(m+2*n*p+1)), x] - \text{Dist}[d^{(2*n)}/(c*(m+2*n*p+1)), \text{Int}[(d*x)^{(m-2*n)}*\text{Simp}[a*(m-2*n+1)+b*(m+n*(p-1)+1)*x^n, x]*(a+b*x^n+c*x^{(2*n)})^p, x], x] /; \text{FreeQ}\{a, b, c, d, p\}, x\} \&\amp; \text{EqQ}[n2, 2*n] \&\amp; \text{NeQ}[b^2-4*a*c, 0] \&\amp; \text{IGtQ}[n, 0] \&\amp; \text{GtQ}[m, 2*n-1] \&\amp; \text{NeQ}[m+2*n*p+1, 0] \&\amp; \text{IntegerQ}[p]$

Rule 1422

$\text{Int}[\{(d_)+(e_)*(x_)\}^{(n_)}\}/\{(a_)+(b_)*(x_)\}^{(n_)}+(c_)*(x_)\}^{(n2_)}\}, x_Symbol] :> \text{With}\{q = \text{Rt}[b^2-4*a*c, 2]\}, \text{Dist}[e/2+(2*c*d-b*e)/(2*q), \text{Int}[1/(b/2-q/2+c*x^n), x], x] + \text{Dist}[e/2-(2*c*d-b*e)/(2*q), \text{Int}[1/(b/2+q/2+c*x^n), x], x]] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\amp; \text{EqQ}[n2, 2*n] \&\amp; \text{NeQ}[b^2-4*a*c, 0] \&\amp; \text{NeQ}[c*d^2-b*d*e+a*e^2, 0] \&\amp; (\text{PosQ}[b^2-4*a*c] || \text{!IGtQ}[n/2, 0])$

Rubi steps

$$\begin{aligned} \int \frac{x^8}{a+bx^4+cx^8} dx &= \frac{x}{c} - \frac{\int \frac{a+bx^4}{a+bx^4+cx^8} dx}{c} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4} dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4} dx}{2c} \\ &= \frac{x}{c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{cx^2}} dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{cx^2}} dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} \\ &= \frac{x}{c} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}c^{5/4}} \end{aligned}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.19

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^8 c + \#1^4 b + a \&, \frac{\#1^4 b \log(x-\#1) + a \log(x-\#1)}{2\#1^7 c + \#1^3 b} \&\right]}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a + b*x^4 + c*x^8),x]

[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) &]/(4*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(a + b*x^4 + c*x^8),x]

[Out] IntegrateAlgebraic[x^8/(a + b*x^4 + c*x^8), x]

fricas [B] time = 2.61, size = 5082, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out]
$$-1/4*(4*c*\text{sqrt}(\text{sqrt}(1/2)*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\text{arctan}(1/4*(2*\text{sqrt}(1/2)*((b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*x*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))) + (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*x)*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^10)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - ($$

$$\begin{aligned}
& b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{(2(a^2b^4 - 3a^3b^2c + a^4c^2)x^2 + \sqrt{1/2}(b^8 - 9ab^6c + 27a^2b^4c^2 - 30a^3b^2c^3 + 8a^4c^4 + (b^7c^5 - 12ab^5c^6 + 48a^2b^3c^7 - 64a^3b^2c^8) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 - (b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^4c^5 - 8ab^2c^6 + 16a^2c^7))} / (a^2b^4 - 3a^3b^2c + a^4c^2)) \sqrt{\sqrt{1/2} \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 - (b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^4c^5 - 8ab^2c^6 + 16a^2c^7))} / (a^4b^4 - 3a^5b^2c + a^6c^2) - 4c \sqrt{\sqrt{1/2} \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^4c^5 - 8ab^2c^6 + 16a^2c^7))} \arctan(1/4(2\sqrt{1/2}((b^{10}c^5 - 16ab^8c^6 + 98a^2b^6c^7 - 280a^3b^4c^8 + 352a^4b^2c^9 - 128a^5c^{10})x \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} - (b^{11} - 13ab^9c + 63a^2b^7c^2 - 138a^3b^5c^3 + 128a^4b^3c^4 - 32a^5b^2c^5)x) \sqrt{\sqrt{1/2} \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^4c^5 - 8ab^2c^6 + 16a^2c^7))} \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^4c^5 - 8ab^2c^6 + 16a^2c^7))} + (b^{11} - 13ab^9c + 63a^2b^7c^2 - 138a^3b^5c^3 + 128a^4b^3c^4 - 32a^5b^2c^5 - (b^{10}c^5 - 16ab^8c^6 + 98a^2b^6c^7 - 280a^3b^4c^8 + 352a^4b^2c^9 - 128a^5c^{10}) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} \sqrt{\sqrt{1/2} \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^4c^5 - 8ab^2c^6 + 16a^2c^7))} \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^4c^5 - 8ab^2c^6 + 16a^2c^7))} \sqrt{(2(a^2b^4 - 3a^3b^2c + a^4c^2)x^2 + \sqrt{1/2}(b^8 - 9ab^6c + 27a^2b^4c^2 - 30a^3b^2c^3 + 8a^4c^4 - (b^7c^5 - 12ab^5c^6 + 48a^2b^3c^7 - 64a^3b^2c^8) \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (b^4c^5 - 8ab^2c^6 + 16a^2c^7) \sqrt{
\end{aligned}$$

$$\begin{aligned}
& t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - \\
& 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))/((b^4*c^5 - 8*a*b^2*c^6 + 1 \\
& 6*a^2*c^7)))/((a^2*b^4 - 3*a^3*b^2*c + a^4*c^2))/((a^4*b^4 - 3*a^5*b^2*c + a \\
& ^6*c^2)) - c*\sqrt{\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 \\
& - 8*a*b^2*c^6 + 16*a^2*c^7)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3 \\
& *b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13} \\
&))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*\log((a*b^4 - 3*a^2*b^2*c + a^3 \\
& *c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8* \\
& a*b^3*c^6 + 16*a^2*b*c^7)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2 \\
& *c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13} \\
&))*\sqrt{\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2 \\
& *c^6 + 16*a^2*c^7)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + \\
& a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))/} \\
& (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))))) + c*\sqrt{\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3 \\
& *c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*\sqrt{(b^8 - 6*a*b^6 \\
& *c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + \\
& 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))/} (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*\log \\
& ((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 \\
& - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*\sqrt{(b^8 - 6*a*b^6*c \\
& + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48* \\
& a^2*b^2*c^{12} - 64*a^3*c^{13}))*\sqrt{\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2 \\
& *b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^ \\
& 2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2 \\
& *c^{12} - 64*a^3*c^{13}))/} (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))))) - c*\sqrt{\sqrt{ \\
& t(1/2)*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a \\
& ^2*c^7)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(\\
& b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))/} (b^4*c^5 - 8*a* \\
& b^2*c^6 + 16*a^2*c^7)))*\log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 - \\
& 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b* \\
& c^7)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6 \\
& *c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))*\sqrt{\sqrt{1/2}*\sqrt{ \\
& t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*\sqrt{ \\
& t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - \\
& 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13}))/} (b^4*c^5 - 8*a*b^2*c^6 + \\
& 16*a^2*c^7))))) + c*\sqrt{\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b \\
& ^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - \\
& 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64* \\
& a^3*c^{13}))/} (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*\log((a*b^4 - 3*a^2*b^2*c \\
& + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 \\
& - 8*a*b^3*c^6 + 16*a^2*b*c^7)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6* \\
& a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3 \\
& *c^{13}))*\sqrt{\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8 \\
& *a*b^2*c^6 + 16*a^2*c^7)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2 \\
& *c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13} \\
&))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))))) - 4*x)/c
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 14.73Unable to convert to r
eal 1/4 Error: Bad Argument Value

maple [C] time = 0.01, size = 59, normalized size = 0.16

$$\frac{x}{c} + \frac{\left(-\text{RootOf}\left(-Z^8c + b_Z^4 + a\right)^4 b - a\right) \ln\left(-\text{RootOf}\left(-Z^8c + b_Z^4 + a\right) + x\right)}{4c\left(2\text{RootOf}\left(-Z^8c + b_Z^4 + a\right)^7 c + \text{RootOf}\left(-Z^8c + b_Z^4 + a\right)^3 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(c*x^8+b*x^4+a),x)

[Out] 1/c*x+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(-Z^8*c+_Z^4*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c} - \frac{\int \frac{bx^4+a}{cx^8+bx^4+a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c

mupad [B] time = 3.97, size = 10382, normalized size = 27.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(a + b*x^4 + c*x^8),x)

[Out] atan((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*
(-b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-

$$\begin{aligned}
& ((4*a*c - b^2)^5)^{1/2}) / (512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2* \\
& *b^4*c^7 - 256*a^3*b^2*c^8)))^{3/4}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 204 \\
& 8*a^4*b^3*c^5)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{1/2}) + 80*a^4*b*c^4 + 6 \\
& 1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{1/2} - 13*a*b \\
& ^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{1/2}) / (512*(256*a^4*c^9 + b^8*c^5 - 16 \\
& *a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{1/4} - (4*x*(a^4*b^4 + 2* \\
& a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{1/2}) + 80*a^4*b \\
& *c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{1/2} \\
& - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{1/2}) / (512*(256*a^4*c^9 + b^8* \\
& c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{1/4} * 1i - (((16*(\\
& a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4* \\
& -(4*a*c - b^2)^5)^{1/2}) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 \\
& + a^2*c^2*(-(4*a*c - b^2)^5)^{1/2} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2) \\
& ^5)^{1/2}) / (512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25 \\
& 6*a^3*b^2*c^8)))^{3/4}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5 \\
&))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{1/2}) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 \\
& - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{1/2} - 13*a*b^7*c - 3*a*b^ \\
& 2*c*(-(4*a*c - b^2)^5)^{1/2}) / (512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + \\
& 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{1/4} + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a \\
& ^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{1/2}) + 80*a^4*b*c^4 + 61*a^2 \\
& *b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{1/2} - 13*a*b^7*c \\
& - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{1/2}) / (512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^ \\
& 6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{1/4} * 1i / (((16*(a^3*b^6 - 4*a \\
& ^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^ \\
& 2)^5)^{1/2}) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(\\
& 4*a*c - b^2)^5)^{1/2} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{1/2}) / (5 \\
& 12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8 \\
&)))^{3/4}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 \\
& + b^4*(-(4*a*c - b^2)^5)^{1/2}) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^ \\
& 3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{1/2} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c \\
& - b^2)^5)^{1/2}) / (512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^ \\
& 7 - 256*a^3*b^2*c^8)))^{1/4} - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c) \\
& *(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{1/2}) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12 \\
& 0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{1/2} - 13*a*b^7*c - 3*a*b^2*c*(\\
& -(4*a*c - b^2)^5)^{1/2}) / (512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^ \\
& 2*b^4*c^7 - 256*a^3*b^2*c^8)))^{1/4} + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b \\
& ^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{1/2}) + 80 \\
& *a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^ \\
& ^{1/2} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{1/2}) / (512*(256*a^4*c^9 \\
& + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{3/4}*(4096* \\
& a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - \\
& b^2)^5)^{1/2}) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2* \\
& (- (4*a*c - b^2)^5)^{1/2} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{1/2}) \\
& / (512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2* \\
& c^8)))^{1/4} + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(
\end{aligned}$$

$$\begin{aligned}
& (256*a^3*b^2*c^8))^{(1/4)} + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 \\
& + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - \\
& 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3/4)}*(4096*a^5*b*c^6 + \\
& 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} \\
&) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} \\
&))^{(1/4)}))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + \\
& 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*2i - 2*atan((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)} \\
&))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)* \\
& (- (b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120 \\
& *a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4 \\
& *a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2 \\
& *b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c^ \\
&))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3 \\
& *a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} - (((16*(a^3*b^6 - 4*a^6*c^3 \\
& - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3/4)} \\
&)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 \\
& + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2 \\
&)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2 \\
& 56*a^3*b^2*c^8)))^{(1/4)}*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)* \\
& (- (b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120 \\
& *a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4 \\
& *a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2 \\
& *b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)})/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4 \\
& *c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4 \\
& *b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
&) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)}*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)}*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)}*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)}*1i)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)} - 2*atan((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)}*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)} - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 -
\end{aligned}$$

$$\begin{aligned}
& b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)}*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)})/(((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i)) + x/c
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.264 \quad \int \frac{x^6}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=325

$$\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}}c^{3/4}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.31, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1374, 298, 205, 208}

$$\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}}c^{3/4}\sqrt{b^2-4ac}} - \frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{2^{3/4}}c^{3/4}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b*x^4 + c*x^8), x]

[Out] $-\left(-b - \sqrt{b^2 - 4ac}\right)^{3/4} \text{ArcTan}\left[\frac{2^{1/4}c^{1/4}x}{-b - \sqrt{b^2 - 4ac}}\right] / \left(2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}\right) + \left(-b + \sqrt{b^2 - 4ac}\right)^{3/4} \text{ArcTan}\left[\frac{2^{1/4}c^{1/4}x}{-b + \sqrt{b^2 - 4ac}}\right] / \left(2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}\right) + \left(-b - \sqrt{b^2 - 4ac}\right)^{3/4} \text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}x}{-b - \sqrt{b^2 - 4ac}}\right] / \left(2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}\right) - \left(-b + \sqrt{b^2 - 4ac}\right)^{3/4} \text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}x}{-b + \sqrt{b^2 - 4ac}}\right] / \left(2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}\right)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1374

Int[((d_.)*(x_)^(m_))/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{x^6}{a + bx^4 + cx^8} dx &= -\left(\frac{1}{2}\left(-1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{x^2}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^4} dx\right) + \frac{1}{2}\left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{x^2}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^4} dx \\ &= -\frac{\left(1 - \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\sqrt{-b + \sqrt{b^2 - 4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{2}\sqrt{c}} + \frac{\left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\sqrt{-b + \sqrt{b^2 - 4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{2}\sqrt{c}} \\ &= -\frac{\left(-b - \sqrt{b^2 - 4ac}\right)^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b - \sqrt{b^2 - 4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}} + \frac{\left(-b + \sqrt{b^2 - 4ac}\right)^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b + \sqrt{b^2 - 4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^3 \log(x - \#1)}{2\#1^4c + b}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b*x^4 + c*x^8), x]

[Out] RootSum[a + b*#1^4 + c*#1^8 &, (Log[x - #1]*#1^3)/(b + 2*c*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(a + b*x^4 + c*x^8), x]

[Out] IntegrateAlgebraic[x^6/(a + b*x^4 + c*x^8), x]

fricas [B] time = 1.93, size = 3912, normalized size = 12.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -\sqrt{\sqrt{1/2}*\sqrt{-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))}} \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & / (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) * \arctan(1/2*((b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5) \\ & * x * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x - \sqrt{1/2}*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5) \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & * \sqrt{((2*(a*b^2 - a^2*c)*x^2 - \sqrt{1/2}*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6) \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & * \sqrt{-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))} \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & / (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))) / (a*b^2 - a^2*c)) \\ & * \sqrt{\sqrt{1/2}*\sqrt{-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))}} \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & / (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) * \arctan(1/2*(\sqrt{1/2} \\ & *(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5) * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & * \sqrt{\sqrt{1/2}*\sqrt{-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))}} \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & / (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) * \sqrt{((2*(a*b^2 - a^2*c)*x^2 - \sqrt{1/2}*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6) \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & * \sqrt{-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))} \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & / (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))) / (a*b^2 - a^2*c)) + ((b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5) \\ & * x * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x) * \sqrt{\sqrt{1/2}*\sqrt{-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))}} \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & / (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) + 1/4*\sqrt{\sqrt{1/2}*\sqrt{-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))}} \\ & * \sqrt{((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9))} \\ & / (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) * \log(1/2*\sqrt{1/2}*(b^7 - 9*a*b^5*c \end{aligned}$$

$$\begin{aligned}
& + 24a^2b^3c^2 - 16a^3b^2c^3 - (b^8c^3 - 14ab^6c^4 + 72a^2b^4c^5 \\
& - 160a^3b^2c^6 + 128a^4c^7) \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} \sqrt{\sqrt{1/2} \sqrt{-(b^3 - 3ab^2c + (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) \sqrt{-(b^3 - 3ab^2c + (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) - (a^2b^2 - a^3c) * x) - 1/4 \sqrt{\sqrt{1/2} \sqrt{-(b^3 - 3ab^2c + (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) * \log(-1/2 \sqrt{1/2} * (b^7 - 9ab^5c + 24a^2b^3c^2 - 16a^3b^2c^3 - (b^8c^3 - 14ab^6c^4 + 72a^2b^4c^5 - 160a^3b^2c^6 + 128a^4c^7) \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} \sqrt{\sqrt{1/2} \sqrt{-(b^3 - 3ab^2c + (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) \sqrt{-(b^3 - 3ab^2c + (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) - (a^2b^2 - a^3c) * x) + 1/4 \sqrt{\sqrt{1/2} \sqrt{-(b^3 - 3ab^2c - (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) * \log(1/2 \sqrt{1/2} * (b^7 - 9ab^5c + 24a^2b^3c^2 - 16a^3b^2c^3 + (b^8c^3 - 14ab^6c^4 + 72a^2b^4c^5 - 160a^3b^2c^6 + 128a^4c^7) \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} \sqrt{\sqrt{1/2} \sqrt{-(b^3 - 3ab^2c - (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) \sqrt{-(b^3 - 3ab^2c - (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) - (a^2b^2 - a^3c) * x) - 1/4 \sqrt{\sqrt{1/2} \sqrt{-(b^3 - 3ab^2c - (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) * \log(-1/2 \sqrt{1/2} * (b^7 - 9ab^5c + 24a^2b^3c^2 - 16a^3b^2c^3 + (b^8c^3 - 14ab^6c^4 + 72a^2b^4c^5 - 160a^3b^2c^6 + 128a^4c^7) \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} \sqrt{\sqrt{1/2} \sqrt{-(b^3 - 3ab^2c - (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) \sqrt{-(b^3 - 3ab^2c - (b^4c^3 - 8ab^2c^4 + 16a^2c^5))} \sqrt{(b^4 - 2ab^2c + a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))} / (b^4c^3 - 8ab^2c^4 + 16a^2c^5) - (a^2b^2 - a^3c) * x)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)

maple [C] time = 0.00, size = 43, normalized size = 0.13

$$\frac{\text{RootOf}(-Z^8c + b_Z^4 + a)^6 \ln(-\text{RootOf}(-Z^8c + b_Z^4 + a) + x)}{8 \text{RootOf}(-Z^8c + b_Z^4 + a)^7 c + 4 \text{RootOf}(-Z^8c + b_Z^4 + a)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(c*x^8+b*x^4+a),x)

[Out] 1/4*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(-Z^8*c+_Z^4*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)

mupad [B] time = 3.51, size = 8033, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(a + b*x^4 + c*x^8),x)

[Out] atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) + x

$$\begin{aligned}
& * (4a^3b^3c - 12a^4b^2c^2) * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)) \\
&)^{1/4} * i - ((- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{3/4} * (4096a^5c^5 + 256a^3b^4c^3 - 2048a^4b^2c^4 - x * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4} * (32768a^5c^6 + 2048a^3b^4c^4 - 16384a^4b^2c^5) - x * (4a^3b^3c - 12a^4b^2c^2) * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4} * i) / (((- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{3/4} * (4096a^5c^5 + 256a^3b^4c^3 - 2048a^4b^2c^4 + x * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4} * (32768a^5c^6 + 2048a^3b^4c^4 - 16384a^4b^2c^5) + x * (4a^3b^3c - 12a^4b^2c^2) * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4} + (((- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{3/4} * (4096a^5c^5 + 256a^3b^4c^3 - 2048a^4b^2c^4 - x * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4} * (32768a^5c^6 + 2048a^3b^4c^4 - 16384a^4b^2c^5) - x * (4a^3b^3c - 12a^4b^2c^2) * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4} - 2a^4b^2c) * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4} * 2i + \operatorname{atan}((((- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c + a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{3/4} * (4096a^5c^5 + 256a^3b^4c^3 - 2048a^4b^2c^4 + x * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c + a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4} * (32768a^5c^6 + 2048a^3b^4c^4 - 16384a^4b^2c^5) + x * (4a^3b^3c - 12a^4b^2c^2) * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c + a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4} + x * (4a^3b^3c - 12a^4b^2c^2) * (- (b^7 + b^2 * (- (4ac - b^2)^5)^{1/2}) - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c + a * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^7 + b^8c^3 - 16ab^6c^4 + 96a^2b^4c^5 - 256a^3b^2c^6)))^{1/4}
\end{aligned}$$

$$\begin{aligned}
 & - b^2)^5)^{(1/2)})/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)}*1i - (((-b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(3/4)}*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)}*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)}*1i) / (((-b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(3/4)}*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)}*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)} + (((-b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(3/4)}*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)}*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)} - 2*a^4*b*c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)}*2i - 2*atan((((-b^7 + b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(3/4)}*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)}*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^{(1/4)} - (((-b^7 + b^2
 \end{aligned}$$

$$\begin{aligned}
& *(- (4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c \\
& *(- (4*a*c - b^2)^5)^{(1/2)} / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96 \\
& *a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(3/4)} * (4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2 \\
& 048*a^4*b^2*c^4 + x*(- (b^7 + b^2*(- (4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + \\
& 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(- (4*a*c - b^2)^5)^{(1/2)}) / (512*(256*a^4*c \\
& ^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(1/4)} * (32 \\
& 768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c \\
& - 12*a^4*b*c^2) * (- (b^7 + b^2*(- (4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40 \\
& *a^2*b^3*c^2 - 11*a*b^5*c - a*c*(- (4*a*c - b^2)^5)^{(1/2)}) / (512*(256*a^4*c^7 \\
& + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(1/4)} / (((- \\
& (b^7 + b^2*(- (4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a* \\
& b^5*c - a*c*(- (4*a*c - b^2)^5)^{(1/2)}) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^ \\
& 6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(3/4)} * (4096*a^5*c^5 + 256*a^3*b \\
& ^4*c^3 - 2048*a^4*b^2*c^4 - x*(- (b^7 + b^2*(- (4*a*c - b^2)^5)^{(1/2)} - 48*a^ \\
& 3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(- (4*a*c - b^2)^5)^{(1/2)}) / (512* \\
& (256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)) \\
& ^{(1/4)} * (32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4 \\
& *a^3*b^3*c - 12*a^4*b*c^2) * (- (b^7 + b^2*(- (4*a*c - b^2)^5)^{(1/2)} - 48*a^3* \\
& b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(- (4*a*c - b^2)^5)^{(1/2)}) / (512*(2 \\
& 56*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(\\
& 1/4)} * 1i + (((- (b^7 + b^2*(- (4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^ \\
& 3*c^2 - 11*a*b^5*c - a*c*(- (4*a*c - b^2)^5)^{(1/2)}) / (512*(256*a^4*c^7 + b^8* \\
& c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(3/4)} * (4096*a^5*c^ \\
& 5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(- (b^7 + b^2*(- (4*a*c - b^2)^5)^ \\
& ^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(- (4*a*c - b^2)^5) \\
& ^{(1/2)}) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a \\
& ^3*b^2*c^6))^{(1/4)} * (32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)* \\
& 1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2) * (- (b^7 + b^2*(- (4*a*c - b^2)^5)^{(1 \\
& /2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(- (4*a*c - b^2)^5)^{(\\
& 1/2)}) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3 \\
& *b^2*c^6))^{(1/4)} * 1i + 2*a^4*b*c) * (- (b^7 + b^2*(- (4*a*c - b^2)^5)^{(1/2)} - \\
& 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(- (4*a*c - b^2)^5)^{(1/2)}) / \\
& (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c \\
& ^6))^{(1/4)} - 2*atan((((- (b^7 - b^2*(- (4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 \\
& + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(- (4*a*c - b^2)^5)^{(1/2)}) / (512*(256*a^ \\
& 4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(3/4)} * \\
& (4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(- (b^7 - b^2*(- (4*a* \\
& c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(- (4*a \\
& *c - b^2)^5)^{(1/2)}) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4 \\
& *c^5 - 256*a^3*b^2*c^6))^{(1/4)} * (32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a \\
& ^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2) * (- (b^7 - b^2*(- (4*a*c \\
& - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(- (4*a*c \\
& - b^2)^5)^{(1/2)}) / (512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c \\
& ^5 - 256*a^3*b^2*c^6))^{(1/4)} - (((- (b^7 - b^2*(- (4*a*c - b^2)^5)^{(1/2)} - 48 \\
& *a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(- (4*a*c - b^2)^5)^{(1/2)}) / (5
\end{aligned}$$

$$\begin{aligned}
& 12*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6) \\
&))^{(3/4)}*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b \\
& ^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + \\
& a*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + \\
& 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(1/4)}*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 \\
& - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a* \\
& c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96 \\
& *a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(1/4)})/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^ \\
& (1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5) \\
& ^{(1/2)))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a \\
& ^3*b^2*c^6))^{(3/4)}*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x* \\
& (-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11* \\
& a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a* \\
& b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(1/4)}*(32768*a^5*c^6 + 2048*a \\
& ^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(- \\
& (b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a* \\
& b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^ \\
& 6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(1/4)}*1i + (((-(b^7 - b^2*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4* \\
& a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^ \\
& 4*c^5 - 256*a^3*b^2*c^6))^{(3/4)}*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4 \\
& *b^2*c^4 + x*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2* \\
& b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^7 + b^ \\
& 8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(1/4)}*(32768*a^5 \\
& *c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)*1i)*1i - x*(4*a^3*b^3*c - 12*a \\
& ^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^ \\
& 3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^7 + b^8* \\
& c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(1/4)}*1i + 2*a^4*b \\
& *c))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 \\
& - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^7 + b^8*c^3 - \\
& 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))^{(1/4)}
\end{aligned}$$

sympy [A] time = 61.58, size = 230, normalized size = 0.71

$\text{RootSum}\left(t^8(16777216a^4c^7 - 16777216a^3b^2c^6 + 6291456a^2b^4c^5 - 1048576ab^6c^4 + 65536b^8c^3) + t^4(-12288a^3b^3c^3 + 10240a^2b^3c^2 - 2816ab^5c + 256b^7) + a^3, \left(1 + t \log\left(x + \frac{2097152t^7a^4c^7 - 2621440t^7a^3b^2c^6 + 1179648t^7a^2b^4c^5 - 229376t^7ab^6c^4 + 16384t^7b^8c^3 - 1280t^3a^4b^2c^4 - 1600t^3a^3b^4c^3 - 576t^3ab^5c + 64t^3b^7}{a^4c - a^2b^3}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(c*x**8+b*x**4+a), x)

[Out] RootSum(_t**8*(16777216*a**4*c**7 - 16777216*a**3*b**2*c**6 + 6291456*a**2*
b**4*c**5 - 1048576*a*b**6*c**4 + 65536*b**8*c**3) + _t**4*(-12288*a**3*b*c
3 + 10240*a2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + a**3, Lambda(_t, _
t*log(x + (2097152*_t**7*a**4*c**7 - 2621440*_t**7*a**3*b**2*c**6 + 1179648
*_t**7*a**2*b**4*c**5 - 229376*_t**7*a*b**6*c**4 + 16384*_t**7*b**8*c**3 -

$$\frac{1280*t^3*a^3*b*c^3 + 1600*t^3*a^2*b^3*c^2 - 576*t^3*a*b^5*c + 64*t^3*b^7}{(a^3*c - a^2*b^2))}$$

$$3.265 \quad \int \frac{x^4}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=325

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{\sqrt{b^2-4ac}-b} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-\sqrt{b^2-4ac}-b} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.30, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1374, 212, 208, 205}

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{\sqrt{b^2-4ac}-b} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-\sqrt{b^2-4ac}-b} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{\sqrt{b^2-4ac}-b} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b*x^4 + c*x^8),x]

[Out] $((-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)} * \text{ArcTan}[(2^{(1/4)} * c^{(1/4)} * x) / (-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}]) / (2 * 2^{(1/4)} * c^{(1/4)} * \text{Sqrt}[b^2 - 4*a*c]) - ((-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)} * \text{ArcTan}[(2^{(1/4)} * c^{(1/4)} * x) / (-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}]) / (2 * 2^{(1/4)} * c^{(1/4)} * \text{Sqrt}[b^2 - 4*a*c]) + ((-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)} * \text{ArcTanh}[(2^{(1/4)} * c^{(1/4)} * x) / (-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}]) / (2 * 2^{(1/4)} * c^{(1/4)} * \text{Sqrt}[b^2 - 4*a*c]) - ((-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)} * \text{ArcTanh}[(2^{(1/4)} * c^{(1/4)} * x) / (-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}]) / (2 * 2^{(1/4)} * c^{(1/4)} * \text{Sqrt}[b^2 - 4*a*c])$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ

[a/b, 0]

Rule 1374

Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{a + bx^4 + cx^8} dx &= -\left(\frac{1}{2}\left(-1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^4} dx\right) + \frac{1}{2}\left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^4} dx \\ &= \frac{\sqrt{-b - \sqrt{b^2 - 4ac}} \int \frac{1}{\sqrt{-b - \sqrt{b^2 - 4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{b^2 - 4ac}} + \frac{\sqrt{-b - \sqrt{b^2 - 4ac}} \int \frac{1}{\sqrt{-b - \sqrt{b^2 - 4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{b^2 - 4ac}} \\ &= \frac{\sqrt[4]{-b - \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2 - 4ac}} - \frac{\sqrt[4]{-b + \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2 - 4ac}} + \dots \end{aligned}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.13

$$\frac{1}{4} \text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1 \log(x - \#1)}{2\#1^4c + b}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b*x^4 + c*x^8), x]

[Out] RootSum[a + b*#1^4 + c*#1^8 &, (Log[x - #1]*#1)/(b + 2*c*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(a + b*x^4 + c*x^8), x]

[Out] IntegrateAlgebraic[x^4/(a + b*x^4 + c*x^8), x]

fricas [B] time = 1.53, size = 2479, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out]
$$-\sqrt{\sqrt{1/2} \sqrt{-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})}/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}} \arctan\left(\frac{1/2 \left(\sqrt{1/2} (b^4 - 8*a*b^2*c + 16*a^2*c^2 - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})\right) \sqrt{x^2 + \sqrt{1/2} (b^2 - 4*a*c)} \sqrt{-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})}}{(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)}\right) \sqrt{-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})} / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}} - \sqrt{1/2} \left((b^4 - 8*a*b^2*c + 16*a^2*c^2) * x - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4) * x / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5} \right) \sqrt{-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})} / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}} \sqrt{\sqrt{1/2} \sqrt{-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})}/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}}) / a + \sqrt{\sqrt{1/2} \sqrt{-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})}/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}}) \arctan\left(\frac{-1/2 \left(\sqrt{1/2} (b^4 - 8*a*b^2*c + 16*a^2*c^2 + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})\right) \sqrt{x^2 + \sqrt{1/2} (b^2 - 4*a*c)} \sqrt{-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})}}{(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)}\right) \sqrt{-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})} / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}} - \sqrt{1/2} \left((b^4 - 8*a*b^2*c + 16*a^2*c^2) * x + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4) * x / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5} \right) \sqrt{-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})} / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}} \sqrt{\sqrt{1/2} \sqrt{-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})}/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}}) / a + 1/4 \sqrt{\sqrt{1/2} \sqrt{-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})}/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}}) \log\left(x + \frac{(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3) \sqrt{\sqrt{1/2} \sqrt{-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})}}}{\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}}}\right) / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}} - 1/4 \sqrt{\sqrt{1/2} \sqrt{-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/\sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5})}} / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}}$$

$$\frac{64a^3c^5}{(b^4c - 8ab^2c^2 + 16a^2c^3)} \log(x - (b^4c - 8ab^2c^2 + 16a^2c^3) \sqrt{\sqrt{1/2} \sqrt{-(b + (b^4c - 8ab^2c^2 + 16a^2c^3) / \sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5}) / (b^4c - 8ab^2c^2 + 16a^2c^3)}}) / \sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5}) - 1/4 \sqrt{\sqrt{1/2} \sqrt{-(b - (b^4c - 8ab^2c^2 + 16a^2c^3) / \sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5}) / (b^4c - 8ab^2c^2 + 16a^2c^3)}}) / \sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5}) + 1/4 \sqrt{\sqrt{1/2} \sqrt{-(b - (b^4c - 8ab^2c^2 + 16a^2c^3) / \sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5}) / (b^4c - 8ab^2c^2 + 16a^2c^3)}}) / \sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5}) \log(x - (b^4c - 8ab^2c^2 + 16a^2c^3) \sqrt{\sqrt{1/2} \sqrt{-(b - (b^4c - 8ab^2c^2 + 16a^2c^3) / \sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5}) / (b^4c - 8ab^2c^2 + 16a^2c^3)}}) / \sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5}) / (b^4c - 8ab^2c^2 + 16a^2c^3)) / \sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5})$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)

maple [C] time = 0.00, size = 43, normalized size = 0.13

$$\frac{\text{RootOf}(-Z^8c + b_Z^4 + a)^4 \ln(-\text{RootOf}(-Z^8c + b_Z^4 + a) + x)}{8 \text{RootOf}(-Z^8c + b_Z^4 + a)^7 c + 4 \text{RootOf}(-Z^8c + b_Z^4 + a)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^8+b*x^4+a),x)

[Out] 1/4*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(-Z^8*c+_Z^4*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& (8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))^{(1/4)} \\
&)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6 \\
& ^6) - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (\\
& -(4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 \\
& ^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(3/4)} + 64*a^3*b*c^4 \\
& - 16*a^2*b^3*c^3) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-4*a*c - b^ \\
& 2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6 \\
& ^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(1/4)}))*(-(b^5 + (-4*a*c - b^2 \\
&)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6 \\
& *c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(1/4)}*2i - 2*atan((((-(b^5 + (- \\
& 4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 \\
& - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(1/4)}*(((-(b^5 + (-4 \\
& *a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 \\
& - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(1/4)}*(262144*a^5*c^7 \\
& - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i + x*(16384* \\
& a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-4*a*c - b^2)^ \\
& 5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^ \\
& ^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(3/4)}*1i - 64*a^3*b*c^4 + 16*a^2*b \\
& ^3*c^3)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-4*a*c - b^2)^5)^{(1/ \\
& 2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 9 \\
& 6*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(1/4)} - (((-(b^5 + (-4*a*c - b^2)^5)^{(1/ \\
& 2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 9 \\
& 6*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(1/4)}*(((-(b^5 + (-4*a*c - b^2)^5)^{(1/2) \\
&) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96 \\
& *a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(1/4)}*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 \\
& + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^ \\
& 2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b \\
& *c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 \\
& - 256*a^3*b^2*c^4)))^{(3/4)}*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i - x*(8*a \\
& ^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - \\
& 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256 \\
& *a^3*b^2*c^4)))^{(1/4)})/((((-(b^5 + (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - \\
& 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256 \\
& *a^3*b^2*c^4)))^{(1/4)}*(((-(b^5 + (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - \\
& 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256* \\
& a^3*b^2*c^4)))^{(1/4)}*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 \\
& - 196608*a^4*b^2*c^6)*1i + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^ \\
& 3*b^3*c^5))*(-(b^5 + (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(\\
& 512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4) \\
&))^{(3/4)}*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i + x*(8*a^3*c^4 - 4*a^2*b^2* \\
& c^3))*(-(b^5 + (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b \\
& ^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(1/ \\
& 4)}*1i + (((-(b^5 + (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512 \\
& *(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^{(\\
& 1/4)}*(((-(b^5 + (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*
\end{aligned}$$

$$\begin{aligned}
& (16384a^4b^5c^6 + 1024a^2b^5c^4 - 8192a^3b^3c^5) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{3/4} + 64a^3b^5c^4 - 16a^2b^3c^3) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4} + x(8a^3c^4 - 4a^2b^2c^3) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4} * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4} * 2i - 2 \operatorname{atan}(\frac{(-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c}{512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4}}) * (262144a^5c^7 - 4096a^2b^6c^4 + 49152a^3b^4c^5 - 196608a^4b^2c^6) * i + x(16384a^4b^5c^6 + 1024a^2b^5c^4 - 8192a^3b^3c^5) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{3/4} * i - 64a^3b^5c^4 + 16a^2b^3c^3) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4} * i + x(8a^3c^4 - 4a^2b^2c^3) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4} - (((-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4}) * (262144a^5c^7 - 4096a^2b^6c^4 + 49152a^3b^4c^5 - 196608a^4b^2c^6) * i - x(16384a^4b^5c^6 + 1024a^2b^5c^4 - 8192a^3b^3c^5) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{3/4} * i - 64a^3b^5c^4 + 16a^2b^3c^3) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4} * i - x(8a^3c^4 - 4a^2b^2c^3) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4} / (((-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4}) * (262144a^5c^7 - 4096a^2b^6c^4 + 49152a^3b^4c^5 - 196608a^4b^2c^6) * i + x(16384a^4b^5c^6 + 1024a^2b^5c^4 - 8192a^3b^3c^5) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{3/4} * i - 64a^3b^5c^4 + 16a^2b^3c^3) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4} * i + x(8a^3c^4 - 4a^2b^2c^3) * (-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4} * i + (((-b^5 - (-4ac - b^2)^5)^{1/2} + 16a^2b^5c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4))^{1/4}) * (262144a^5c^7 - 4096a^2b^6c^4 + 49152a^3b^4c^5 - 196608a^4b^2c^6) * i
\end{aligned}$$

$$i - x \cdot (16384a^4b^2c^6 + 1024a^2b^5c^4 - 8192a^3b^3c^5) \cdot (-(b^5 - (-(4ac - b^2)^5)^{1/2} + 16a^2b^2c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4)))^{3/4} \cdot 1i - 64a^3b^2c^4 + 16a^2b^3c^3) \cdot (-(b^5 - (-(4ac - b^2)^5)^{1/2} + 16a^2b^2c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4)))^{1/4} \cdot 1i - x \cdot (8a^3c^4 - 4a^2b^2c^3) \cdot (-(b^5 - (-(4ac - b^2)^5)^{1/2} + 16a^2b^2c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4)))^{1/4} \cdot 1i) \cdot (-(b^5 - (-(4ac - b^2)^5)^{1/2} + 16a^2b^2c^2 - 8ab^3c) / (512(b^8c + 256a^4c^5 - 16ab^6c^2 + 96a^2b^4c^3 - 256a^3b^2c^4)))^{1/4}$$

sympy [A] time = 2.92, size = 126, normalized size = 0.39

RootSum($t^8(16777216a^4c^5 - 16777216a^3b^2c^4 + 6291456a^2b^4c^3 - 1048576ab^6c^2 + 65536b^8c) + t^4(4096a^2bc^2 - 2048ab^3c + 256b^5) + a, (t \mapsto t \log(-32768t^5a^2c^3 + 16384t^5ab^2c^2 - 2048t^5b^4c - 4tb + x))$)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**8+b*x**4+a), x)

[Out] RootSum(_t**8*(16777216*a**4*c**5 - 16777216*a**3*b**2*c**4 + 6291456*a**2*b**4*c**3 - 1048576*a*b**6*c**2 + 65536*b**8*c) + _t**4*(4096*a**2*b*c**2 - 2048*a*b**3*c + 256*b**5) + a, Lambda(_t, _t*log(-32768*_t**5*a**2*c**3 + 16384*_t**5*a*b**2*c**2 - 2048*_t**5*b**4*c - 4*_t*b + x)))

$$3.266 \quad \int \frac{x^2}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=315

$$\frac{\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4} \sqrt{b^2-4ac} \sqrt[4]{-\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4} \sqrt{b^2-4ac} \sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4} \sqrt{b^2-4ac} \sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\sqrt[4]{c}}{2^{3/4} \sqrt{b^2-4ac}}$$

Rubi [A] time = 0.29, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1375, 298, 205, 208}

$$\frac{\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4} \sqrt{b^2-4ac} \sqrt[4]{-\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4} \sqrt{b^2-4ac} \sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4} \sqrt{b^2-4ac} \sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\sqrt[4]{c} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4} \sqrt{b^2-4ac} \sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*x^4 + c*x^8), x]

[Out] $-\left(\frac{c^{1/4} \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} x}{-b - \sqrt{b^2 - 4ac}}\right]^{1/4}}{2^{3/4} \sqrt{b^2 - 4ac} (-b - \sqrt{b^2 - 4ac})^{1/4}}\right) + \left(\frac{c^{1/4} \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} x}{-b + \sqrt{b^2 - 4ac}}\right]^{1/4}}{2^{3/4} \sqrt{b^2 - 4ac} (-b + \sqrt{b^2 - 4ac})^{1/4}}\right) + \left(\frac{c^{1/4} \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} x}{-b - \sqrt{b^2 - 4ac}}\right]^{1/4}}{2^{3/4} \sqrt{b^2 - 4ac} (-b - \sqrt{b^2 - 4ac})^{1/4}}\right) - \left(\frac{c^{1/4} \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} x}{-b + \sqrt{b^2 - 4ac}}\right]^{1/4}}{2^{3/4} \sqrt{b^2 - 4ac} (-b + \sqrt{b^2 - 4ac})^{1/4}}\right)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G

tQ[a/b, 0]

Rule 1375

Int[((d_.)*(x_)^(m_.)/((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{a + bx^4 + cx^8} dx &= \frac{c \int \frac{x^2}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{\sqrt{b^2-4ac}} - \frac{c \int \frac{x^2}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{\sqrt{b^2-4ac}} \\ &= \frac{\sqrt{c} \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c} \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c} \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{\sqrt{2}\sqrt{b^2-4ac}} \\ &= -\frac{\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.14

$$\frac{1}{4} \text{RootSum}\left[\#1^8 c + \#1^4 b + a \&, \frac{\log(x - \#1)}{2\#1^5 c + \#1 b} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*x^4 + c*x^8),x]

[Out] RootSum[a + b*#1^4 + c*#1^8 &, Log[x - #1]/(b*#1 + 2*c*#1^5) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a + b*x^4 + c*x^8),x]

[Out] IntegrateAlgebraic[x^2/(a + b*x^4 + c*x^8), x]

fricas [B] time = 1.66, size = 2746, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out]
$$\begin{aligned} & \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \arctan\left(-\frac{(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*x*\sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}}{(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)}\right) \\ & * \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & + \sqrt{\frac{1}{2}}*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*\sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & / \sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & - \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \arctan\left(-\frac{(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*x/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)}}{\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)}}\right) \\ & * \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \log\left(\frac{1}{2}\sqrt{\frac{1}{2}}*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3))\right) \\ & / \sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & + c*x + \frac{1}{4}\sqrt{\sqrt{\frac{1}{2}}}\sqrt{-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))}/\sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \\ & * \log\left(-\frac{1}{2}\sqrt{\frac{1}{2}}*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3))\right) \\ & / \sqrt{(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)} \end{aligned}$$

```

*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt
(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)) + c*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2
*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt(sqrt(1/2)*
sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/
2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*
b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a
^3*c^2)) + c*x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)

maple [C] time = 0.00, size = 43, normalized size = 0.14

$$\frac{\text{RootOf}(-Z^8c + b_Z^4 + a)^2 \ln(-\text{RootOf}(-Z^8c + b_Z^4 + a) + x)}{8\text{RootOf}(-Z^8c + b_Z^4 + a)^7 c + 4\text{RootOf}(-Z^8c + b_Z^4 + a)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^8+b*x^4+a),x)

[Out] 1/4*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(-Z^8*c+_Z^4*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)

mupad [B] time = 2.34, size = 6067, normalized size = 19.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*x^4 + c*x^8),x)

[Out]
$$2*\operatorname{atan}\left(\frac{(-(b^5 - (-4*a*c - b^2)^5)^{1/2} + 16*a^2*b*c^2 - 8*a*b^3*c)}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{3/4}*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-4*a*c - b^2)^5)^{1/2} + 16*a^2*b*c^2 - 8*a*b^3*c)}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{1/4}*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*i - 2048*a^2*b^3*c^5)*i - 4*a*b*c^5*x}\right) - \frac{(-(b^5 - (-4*a*c - b^2)^5)^{1/2} + 16*a^2*b*c^2 - 8*a*b^3*c)}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{1/4}} - \frac{(-(b^5 - (-4*a*c - b^2)^5)^{1/2} + 16*a^2*b*c^2 - 8*a*b^3*c)}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{3/4}}*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-4*a*c - b^2)^5)^{1/2} + 16*a^2*b*c^2 - 8*a*b^3*c)}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{1/4}}*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*i - 2048*a^2*b^3*c^5)*i + 4*a*b*c^5*x)\left(-\frac{b^5 - (-4*a*c - b^2)^5}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{1/4}}\right) - \frac{(-(b^5 - (-4*a*c - b^2)^5)^{1/2} + 16*a^2*b*c^2 - 8*a*b^3*c)}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{3/4}}*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-4*a*c - b^2)^5)^{1/2} + 16*a^2*b*c^2 - 8*a*b^3*c)}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{1/4}}*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*i - 2048*a^2*b^3*c^5)*i - 4*a*b*c^5*x)\left(-\frac{b^5 - (-4*a*c - b^2)^5}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{1/4}}\right) + \frac{(-(b^5 - (-4*a*c - b^2)^5)^{1/2} + 16*a^2*b*c^2 - 8*a*b^3*c)}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{3/4}}*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-4*a*c - b^2)^5)^{1/2} + 16*a^2*b*c^2 - 8*a*b^3*c)}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{1/4}}*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*i - 2048*a^2*b^3*c^5)*i + 4*a*b*c^5*x)\left(-\frac{b^5 - (-4*a*c - b^2)^5}{(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3))^{1/4}}\right)$$

$$\begin{aligned}
& 56*a^4*b^2*c^3))^{(1/4)}*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 \\
& - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 - (- \\
& 4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 \\
& - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(1/4)}*1i - 2*a*c^5))* \\
& (-(b^5 - (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + \\
& 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(1/4)} - a \\
& \tan((((-(b^5 - (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a \\
& *b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(3/ \\
& 4)}*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-4*a*c - b^2)^5)^{(1/2)} + \\
& 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3 \\
& *b^4*c^2 - 256*a^4*b^2*c^3)))^{(1/4)}*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240 \\
& *a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 - \\
& (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a \\
& ^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(1/4)}*1i - ((- \\
& b^5 - (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25 \\
& 6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(3/4)}*(256*a \\
& *b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b* \\
& c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 \\
& - 256*a^4*b^2*c^3)))^{(1/4)}*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4* \\
& c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-4*a \\
& *c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - \\
& 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(1/4)}*1i)/((((-(b^5 - (- \\
& 4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 \\
& - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(3/4)}*(256*a*b^5*c^4 \\
& + 4096*a^3*b*c^6 + x*(-(b^5 - (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a \\
& *b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4 \\
& *b^2*c^3)))^{(1/4)}*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 327 \\
& 68*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 - (-4*a*c - b^2) \\
& ^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^ \\
& 6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(1/4)} + (((-(b^5 - (-4*a*c - b^2) \\
& ^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^ \\
& 6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(3/4)}*(256*a*b^5*c^4 + 4096*a^3*b \\
& *c^6 - x*(-(b^5 - (-4*a*c - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512 \\
& *(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{ \\
& (1/4)}*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c \\
& ^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-4*a*c - b^2)^5)^{(1/2)} + \\
& 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3 \\
& *b^4*c^2 - 256*a^4*b^2*c^3)))^{(1/4)} + 2*a*c^5))*(-(b^5 - (-4*a*c - b^2)^5) \\
& ^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c \\
& + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(1/4)}*2i - \operatorname{atan}((((-(b^5 + (-4*a*c \\
& - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16* \\
& a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(3/4)}*(x*(-(b^5 + (-4*a*c \\
& - b^2)^5)^{(1/2)} + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16* \\
& a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^{(1/4)}*(32768*a^4*c^7 - 1024 \\
& *a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 256*a*b^5*c^4 - 4096*
\end{aligned}$$

$$\begin{aligned}
& a^3 b^3 c^6 + 2048 a^2 b^3 c^5 - 4 a^2 b^3 c^5 x) * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) \\
& / 2) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + \\
& 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} * i + ((- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) \\
& ^{1/2}) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c \\
& + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{3/4} * (x * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) \\
& ^{1/2}) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c \\
& + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} * (32768 a^4 c^7 - 1024 a^2 b^6 c^4 \\
& + 10240 a^2 b^4 c^5 - 32768 a^3 b^2 c^6) + 256 a^2 b^5 c^4 + 4096 a^3 b^3 c^6 \\
& - 2048 a^2 b^3 c^5) - 4 a^2 b^3 c^5 x) * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 \\
& - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} * i) / (((- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + \\
& 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{3/4} * (x * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + \\
& 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} * (32768 a^4 c^7 - 1024 a^2 b^6 c^4 + 10240 \\
& a^2 b^4 c^5 - 32768 a^3 b^2 c^6) + 256 a^2 b^5 c^4 + 4096 a^3 b^3 c^6 - 2048 a^2 b^3 c^5) - 4 a^2 b^3 c^5 x) * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 \\
& - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} - ((- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 \\
& - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{3/4} * (x * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 \\
& - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} * (32768 a^4 c^7 - 1024 a^2 b^6 c^4 + 10240 a^2 b^4 c^5 \\
& - 32768 a^3 b^2 c^6) - 256 a^2 b^5 c^4 - 4096 a^3 b^3 c^6 + 2048 a^2 b^3 c^5) - 4 a^2 b^3 c^5 x) * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c \\
&) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} + 2 a^2 c^5) * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 - \\
& 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} * 2i + 2 * \operatorname{atan}((((- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{3/4} * (x * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} * (32768 a^4 c^7 - 1024 a^2 b^6 c^4 + 10240 a^2 b^4 c^5 - 32768 a^3 b^2 c^6) * i - 256 a^2 b^5 c^4 - 4096 a^3 b^3 c^6 + 2048 a^2 b^3 c^5) * i + 4 a^2 b^3 c^5 x) * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} + ((- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{3/4} * (x * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4} * (32768 a^4 c^7 - 1024 a^2 b^6 c^4 + 10240 a^2 b^4 c^5 - 32768 a^3 b^2 c^6) * i + 256 a^2 b^5 c^4 + 4096 a^3 b^3 c^6 - 2048 a^2 b^3 c^5) * i + 4 a^2 b^3 c^5 x) * (- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 - 8 a^2 b^3 c) / (512 (a^2 b^8 + 256 a^5 c^4 - 16 a^2 b^6 c + 96 a^3 b^4 c^2 - 256 a^4 b^2 c^3))^{1/4}))) / (((- (b^5 + (- (4 a^2 c - b^2)^5)^{1/2}) + 16 a^2 b^3 c^2 - 8
\end{aligned}$$

$$\frac{a^3 b^3 c}{(512(a^8 b^8 + 256a^5 c^4 - 16a^2 b^6 c + 96a^3 b^4 c^2 - 256a^4 b^2 c^3))^{3/4}} \left(x \sqrt{-b^5 + (-4ac - b^2)^5} + 16a^2 b^2 c^2 - 8a^3 b^3 c \right) / (512(a^8 b^8 + 256a^5 c^4 - 16a^2 b^6 c + 96a^3 b^4 c^2 - 256a^4 b^2 c^3))^{1/4} \left(32768a^4 c^7 - 1024a^3 b^6 c^4 + 10240a^2 b^4 c^5 - 32768a^3 b^2 c^6 \right) i - 256a^5 b^5 c^4 - 4096a^3 b^6 c^6 + 2048a^2 b^3 c^5 \left(i + 4a^3 b^5 c^5 x \right) \sqrt{-b^5 + (-4ac - b^2)^5} + 16a^2 b^2 c^2 - 8a^3 b^3 c / (512(a^8 b^8 + 256a^5 c^4 - 16a^2 b^6 c + 96a^3 b^4 c^2 - 256a^4 b^2 c^3))^{1/4} i - \left(\sqrt{-b^5 + (-4ac - b^2)^5} + 16a^2 b^2 c^2 - 8a^3 b^3 c \right) / (512(a^8 b^8 + 256a^5 c^4 - 16a^2 b^6 c + 96a^3 b^4 c^2 - 256a^4 b^2 c^3))^{3/4} \left(x \sqrt{-b^5 + (-4ac - b^2)^5} + 16a^2 b^2 c^2 - 8a^3 b^3 c \right) / (512(a^8 b^8 + 256a^5 c^4 - 16a^2 b^6 c + 96a^3 b^4 c^2 - 256a^4 b^2 c^3))^{1/4} \left(32768a^4 c^7 - 1024a^3 b^6 c^4 + 10240a^2 b^4 c^5 - 32768a^3 b^2 c^6 \right) i + 256a^5 b^5 c^4 + 4096a^3 b^6 c^6 - 2048a^2 b^3 c^5 \left(i + 4a^3 b^5 c^5 x \right) \sqrt{-b^5 + (-4ac - b^2)^5} + 16a^2 b^2 c^2 - 8a^3 b^3 c / (512(a^8 b^8 + 256a^5 c^4 - 16a^2 b^6 c + 96a^3 b^4 c^2 - 256a^4 b^2 c^3))^{1/4} i + 2a^3 c^5 \left(\sqrt{-b^5 + (-4ac - b^2)^5} + 16a^2 b^2 c^2 - 8a^3 b^3 c \right) / (512(a^8 b^8 + 256a^5 c^4 - 16a^2 b^6 c + 96a^3 b^4 c^2 - 256a^4 b^2 c^3))^{1/4}$$

sympy [A] time = 4.10, size = 172, normalized size = 0.55

$$\text{RootSum}\left(t^8(16777216a^5c^4 - 16777216a^4b^2c^3 + 6291456a^3b^4c^2 - 1048576a^2b^6c + 65536ab^8) + t^4(4096a^2b^2c^2 - 2048ab^5c + 256b^8) + c, \left(t \mapsto t \log\left(x + \frac{1048576t^7a^4b^3c - 786432t^7a^3b^5c^2 + 196608t^7a^2b^7c^3 - 16384t^7ab^9 - 512t^3a^2c^2 + 384t^3ab^2c - 64t^3b^4}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**8+b*x**4+a),x)

[Out] RootSum(_t**8*(16777216*a**5*c**4 - 16777216*a**4*b**2*c**3 + 6291456*a**3*b**4*c**2 - 1048576*a**2*b**6*c + 65536*a*b**8) + _t**4*(4096*a**2*b*c**2 - 2048*a*b**3*c + 256*b**5) + c, Lambda(_t, _t*log(x + (1048576*_t**7*a**4*b*c**3 - 786432*_t**7*a**3*b**3*c**2 + 196608*_t**7*a**2*b**5*c - 16384*_t**7*a*b**7 - 512*_t**3*a**2*c**2 + 384*_t**3*a*b**2*c - 64*_t**3*b**4)/c)))

$$3.267 \quad \int \frac{1}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=315

$$\frac{c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2} \sqrt{b^2-4ac} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2} \sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2} \sqrt{b^2-4ac} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2} \sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Rubi [A] time = 0.30, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1347, 212, 208, 205}

$$\frac{c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2} \sqrt{b^2-4ac} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2} \sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2} \sqrt{b^2-4ac} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2} \sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^4 + c*x^8)^(-1), x]

[Out] (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ

[a/b, 0]

Rule 1347

Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(n_+1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a + bx^4 + cx^8} dx &= \frac{c \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{\sqrt{b^2-4ac}} - \frac{c \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{\sqrt{b^2-4ac}} \\ &= \frac{c \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{c \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} - \frac{c \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}} \\ &= \frac{c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.14

$$\frac{1}{4} \text{RootSum}\left[\#1^8 c + \#1^4 b + a \&, \frac{\log(x - \#1)}{2\#1^7 c + \#1^3 b} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^4 + c*x^8)^(-1), x]

[Out] RootSum[a + b*#1^4 + c*#1^8 &, Log[x - #1]/(b*#1^3 + 2*c*#1^7) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + bx^4 + cx^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^4 + c*x^8)^(-1), x]

$$\begin{aligned}
& t(\sqrt{1/2})\sqrt{-(b^3 - 3ab^2c - (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{t((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))\sqrt{-(b^3 - 3ab^2c - (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))\sqrt{(2(b^2c^2 - ac^3) x^2 + \sqrt{1/2}(b^6 - 7a^2b^4c + 14a^2b^2c^2 - 8a^3c^3 + (a^3b^7 - 12a^4b^5c + 48a^5b^3c^2 - 64a^6b^2c^3))\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}\sqrt{-(b^3 - 3ab^2c - (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2)))/(b^2c^2 - ac^3)) + 1/4\sqrt{(\sqrt{1/2})\sqrt{-(b^3 - 3ab^2c + (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))}\log(-(b^2c - ac^2)x + 1/2(b^4 - 5ab^2c + 4a^2c^2 - (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2))\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))})\sqrt{(\sqrt{1/2})\sqrt{-(b^3 - 3ab^2c + (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))}) - 1/4\sqrt{(\sqrt{1/2})\sqrt{-(b^3 - 3ab^2c + (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))}\log(-(b^2c - ac^2)x - 1/2(b^4 - 5ab^2c + 4a^2c^2 - (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2))\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))})\sqrt{(\sqrt{1/2})\sqrt{-(b^3 - 3ab^2c + (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))}) + 1/4\sqrt{(\sqrt{1/2})\sqrt{-(b^3 - 3ab^2c + (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))}\log(-(b^2c - ac^2)x + 1/2(b^4 - 5ab^2c + 4a^2c^2 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2))\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))})\sqrt{(\sqrt{1/2})\sqrt{-(b^3 - 3ab^2c - (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))}) - 1/4\sqrt{(\sqrt{1/2})\sqrt{-(b^3 - 3ab^2c - (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))}\log(-(b^2c - ac^2)x - 1/2(b^4 - 5ab^2c + 4a^2c^2 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2))\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))})\sqrt{(\sqrt{1/2})\sqrt{-(b^3 - 3ab^2c - (a^3b^4 - 8a^4b^2c + 16a^5c^2))}\sqrt{((b^4 - 2ab^2c + a^2c^2)/(a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3))}/(a^3b^4 - 8a^4b^2c + 16a^5c^2))})
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] integrate(1/(c*x^8 + b*x^4 + a), x)

maple [C] time = 0.00, size = 40, normalized size = 0.13

$$\frac{\ln\left(-\text{RootOf}\left(-Z^8c + b_Z^4 + a\right) + x\right)}{8\text{RootOf}\left(-Z^8c + b_Z^4 + a\right)^7c + 4\text{RootOf}\left(-Z^8c + b_Z^4 + a\right)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^8+b*x^4+a),x)

[Out] 1/4*sum(1/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(-Z^8*c+_Z^4*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] integrate(1/(c*x^8 + b*x^4 + a), x)

mupad [B] time = 3.42, size = 10337, normalized size = 32.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*x^4 + c*x^8),x)

[Out] - atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11

$$\begin{aligned}
& - a*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c \\
& + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))^{(1/4)})*(-(b^7 + b^2*(-(4*a*c - b^2) \\
& ^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2) \\
&)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 2 \\
& 56*a^6*b^2*c^3))^{(1/4)}*2i - \operatorname{atan}((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
& / (512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c \\
& ^3)))^{(1/4)}*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c \\
& ^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^3 \\
& *b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^{(1/ \\
& 4)}*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c \\
& ^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c \\
& ^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 \\
& - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^3*b^8 + 256*a^7*c^4 - \\
& 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^{(3/4)} - 16*b^2*c^6) + 8 \\
& *c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c \\
& ^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^3*b^8 + 256*a^7*c^4 \\
& - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^{(1/4)}*1i - ((-(b^7 - \\
& b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + \\
& a*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + \\
& 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^{(1/4)}*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4* \\
& a*c - b^2)^5)^{(1/2)})/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^ \\
& 4*c^2 - 256*a^6*b^2*c^3)))^{(1/4)}*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152 \\
& *a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49 \\
& 152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
& / (512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c \\
& ^3)))^{(3/4)} - 16*b^2*c^6) - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)} \\
&)/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^ \\
& 2*c^3)))^{(1/4)}*1i)/((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + \\
& 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^3*b^8 \\
& + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^{(1/4)}*(6 \\
& 4*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^ \\
& 3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^3*b^8 + 256*a^7* \\
& c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^{(1/4)}*(4096*a*b^7* \\
& c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024* \\
& b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - \\
& b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + \\
& a*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + \\
& 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^{(3/4)} - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 \\
& - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c \\
& + a*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c \\
& + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^{(1/4)} + (((-(b^7 - b^2*(-(4*a*c - b^2)
\end{aligned}$$

$$\begin{aligned}
&)^5)^{(1/2)} - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c + a * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3b^8 + 256a^7c^4 - 16a^4b^6c + 96a^5b^4c^2 - \\
& 256a^6b^2c^3))^{(1/4)} * (64a^7c^7 + ((-b^7 - b^2 * (-(4ac - b^2)^5)^{(1/2)} \\
& - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c + a * (-(4ac - b^2)^5)^{(1/2)} \\
&)) / (512(a^3b^8 + 256a^7c^4 - 16a^4b^6c + 96a^5b^4c^2 - 256a^6b^2c^3))^{(1/4)} * (4096a^7c^4 - 262144a^4b^7c^7 - 49152a^2b^5c^5 + 196 \\
& 608a^3b^3c^6) - x * (1024b^7c^4 - 11264ab^5c^5 - 49152a^3b^7c^7 + 40 \\
& 960a^2b^3c^6) * (-b^7 - b^2 * (-(4ac - b^2)^5)^{(1/2)} - 48a^3b^3c^3 + 40 \\
& a^2b^3c^2 - 11ab^5c + a * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3b^8 + 2 \\
& 56a^7c^4 - 16a^4b^6c + 96a^5b^4c^2 - 256a^6b^2c^3))^{(3/4)} - 16 \\
& b^2c^6) - 8c^7 * x * (-b^7 - b^2 * (-(4ac - b^2)^5)^{(1/2)} - 48a^3b^3c^3 + \\
& 40a^2b^3c^2 - 11ab^5c + a * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3b^8 + \\
& 256a^7c^4 - 16a^4b^6c + 96a^5b^4c^2 - 256a^6b^2c^3))^{(1/4)})) * (\\
& - (b^7 - b^2 * (-(4ac - b^2)^5)^{(1/2)} - 48a^3b^3c^3 + 40a^2b^3c^2 - 11a \\
& * b^5c + a * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3b^8 + 256a^7c^4 - 16a^4 \\
& * b^6c + 96a^5b^4c^2 - 256a^6b^2c^3))^{(1/4)} * 2i - 2 * \operatorname{atan}(((- (b^7 + b \\
& ^2 * (-(4ac - b^2)^5)^{(1/2)} - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - \\
& a * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3b^8 + 256a^7c^4 - 16a^4b^6c + \\
& 96a^5b^4c^2 - 256a^6b^2c^3))^{(1/4)} * (((- (b^7 + b^2 * (-(4ac - b^2)^5) \\
& ^{(1/2)} - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (-(4ac - b^2)^5 \\
&)^{(1/2)}) / (512(a^3b^8 + 256a^7c^4 - 16a^4b^6c + 96a^5b^4c^2 - 256a^6 \\
& b^2c^3))^{(1/4)} * (4096a^7c^4 - 262144a^4b^7c^7 - 49152a^2b^5c^5 \\
& + 196608a^3b^3c^6) * i + x * (1024b^7c^4 - 11264ab^5c^5 - 49152a^3b \\
& * c^7 + 40960a^2b^3c^6) * (-b^7 + b^2 * (-(4ac - b^2)^5)^{(1/2)} - 48a^3b \\
& * c^3 + 40a^2b^3c^2 - 11ab^5c - a * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^ \\
& 3b^8 + 256a^7c^4 - 16a^4b^6c + 96a^5b^4c^2 - 256a^6b^2c^3))^{(3 \\
& / 4)} * i - 64a^7c^7 + 16b^2c^6) * i - 8c^7 * x * (-b^7 + b^2 * (-(4ac - b^2)^ \\
& 5)^{(1/2)} - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * (-(4ac - b^2) \\
& ^5)^{(1/2)}) / (512(a^3b^8 + 256a^7c^4 - 16a^4b^6c + 96a^5b^4c^2 - 25 \\
& 6a^6b^2c^3))^{(1/4)} - ((- (b^7 + b^2 * (-(4ac - b^2)^5)^{(1/2)} - 48a^3b * \\
& c^3 + 40a^2b^3c^2 - 11ab^5c - a * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3 \\
& * b^8 + 256a^7c^4 - 16a^4b^6c + 96a^5b^4c^2 - 256a^6b^2c^3))^{(1/ \\
& 4)} * (((- (b^7 + b^2 * (-(4ac - b^2)^5)^{(1/2)} - 48a^3b^3c^3 + 40a^2b^3c^2 \\
& - 11ab^5c - a * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3b^8 + 256a^7c^4 - \\
& 16a^4b^6c + 96a^5b^4c^2 - 256a^6b^2c^3))^{(1/4)} * (4096a^7c^4 - \\
& 262144a^4b^7c^7 - 49152a^2b^5c^5 + 196608a^3b^3c^6) * i - x * (1024b^7 \\
& * c^4 - 11264ab^5c^5 - 49152a^3b^7c^7 + 40960a^2b^3c^6) * (-b^7 + b^2 \\
& * (-(4ac - b^2)^5)^{(1/2)} - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * \\
& c * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3b^8 + 256a^7c^4 - 16a^4b^6c + 96 \\
& a^5b^4c^2 - 256a^6b^2c^3))^{(3/4)} * i - 64a^7c^7 + 16b^2c^6) * i + 8 * \\
& c^7 * x * (-b^7 + b^2 * (-(4ac - b^2)^5)^{(1/2)} - 48a^3b^3c^3 + 40a^2b^3c^ \\
& 2 - 11ab^5c - a * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3b^8 + 256a^7c^4 \\
& - 16a^4b^6c + 96a^5b^4c^2 - 256a^6b^2c^3))^{(1/4)} / (((- (b^7 + b^2 * \\
& (-(4ac - b^2)^5)^{(1/2)} - 48a^3b^3c^3 + 40a^2b^3c^2 - 11ab^5c - a * c \\
& * (-(4ac - b^2)^5)^{(1/2)}) / (512(a^3b^8 + 256a^7c^4 - 16a^4b^6c + 96
\end{aligned}$$

$$\begin{aligned}
& a^5 b^4 c^2 - 256 a^6 b^2 c^3))^{1/4} * (((-(b^7 + b^2 * (-4 a^* c - b^2)^5)^{1/2}) - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c - a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{1/4} * (4096 a^* b^7 c^4 - 262144 a^4 b^* c^7 - 49152 a^2 b^5 c^5 + 196608 a^3 b^3 c^6) * 1i + x * (1024 b^7 c^4 - 11264 a^* b^5 c^5 - 49152 a^3 b^* c^7 + 40960 a^2 b^3 c^6) * (- (b^7 + b^2 * (-4 a^* c - b^2)^5)^{1/2} - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c - a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{3/4} * 1i - 64 a^* c^7 + 16 b^2 c^6) * 1i - 8 c^7 x * (- (b^7 + b^2 * (-4 a^* c - b^2)^5)^{1/2} - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c - a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{1/4} * 1i + (((-(b^7 + b^2 * (-4 a^* c - b^2)^5)^{1/2}) - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c - a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{1/4} * (4096 a^* b^7 c^4 - 262144 a^4 b^* c^7 - 49152 a^2 b^5 c^5 + 196608 a^3 b^3 c^6) * 1i - x * (1024 b^7 c^4 - 11264 a^* b^5 c^5 - 49152 a^3 b^* c^7 + 40960 a^2 b^3 c^6) * (- (b^7 + b^2 * (-4 a^* c - b^2)^5)^{1/2} - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c - a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{3/4} * 1i - 64 a^* c^7 + 16 b^2 c^6) * 1i + 8 c^7 x * (- (b^7 + b^2 * (-4 a^* c - b^2)^5)^{1/2} - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c - a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{1/4} * 1i)) * (- (b^7 + b^2 * (-4 a^* c - b^2)^5)^{1/2} - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c - a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{1/4} - 2 * atan((((-(b^7 - b^2 * (-4 a^* c - b^2)^5)^{1/2}) - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c + a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{1/4} * (((-(b^7 - b^2 * (-4 a^* c - b^2)^5)^{1/2}) - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c + a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{1/4} * (4096 a^* b^7 c^4 - 262144 a^4 b^* c^7 - 49152 a^2 b^5 c^5 + 196608 a^3 b^3 c^6) * 1i + x * (1024 b^7 c^4 - 11264 a^* b^5 c^5 - 49152 a^3 b^* c^7 + 40960 a^2 b^3 c^6) * (- (b^7 - b^2 * (-4 a^* c - b^2)^5)^{1/2} - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c + a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{3/4} * 1i - 64 a^* c^7 + 16 b^2 c^6) * 1i - 8 c^7 x * (- (b^7 - b^2 * (-4 a^* c - b^2)^5)^{1/2} - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c + a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{1/4} - (((-(b^7 - b^2 * (-4 a^* c - b^2)^5)^{1/2}) - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c + a^* c * (-4 a^* c - b^2)^5)^{1/2}) / (512 * (a^3 b^8 + 256 a^7 c^4 - 16 a^4 b^6 c + 96 a^5 b^4 c^2 - 256 a^6 b^2 c^3)))^{1/4} * (((-(b^7 - b^2 * (-4 a^* c - b^2)^5)^{1/2}) - 48 a^3 b^* c^3 + 40 a^2 b^3 c^2 - 11 a^* b^5 c
\end{aligned}$$

$$\begin{aligned}
& + a*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c \\
& + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))^{(1/4)}*(4096*a*b^7*c^4 - 262144*a^4*b* \\
& c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264* \\
& a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b \\
& ^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - \\
& b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 \\
& - 256*a^6*b^2*c^3))^{(3/4)}*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*c^7*x)*(-(b^7 \\
& - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c \\
& c + a*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c \\
& + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))^{(1/4)})/(((-(b^7 - b^2*(-(4*a*c - b^2 \\
& ^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b \\
& ^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - \\
& 256*a^6*b^2*c^3))^{(1/4)}*((-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3* \\
& b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a \\
& ^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^{(\\
& 1/4)}*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^ \\
& 3*c^6)*1i + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2 \\
& *b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^ \\
& 3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7* \\
& c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))^{(3/4)}*1i - 64*a*c^ \\
& 7 + 16*b^2*c^6)*1i - 8*c^7*x)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^ \\
& 3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512* \\
& (a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))) \\
& ^{(1/4)}*1i + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2* \\
& b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^ \\
& 7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))^{(1/4)}*((-(b^7 - \\
& b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c \\
& + a*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c \\
& + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))^{(1/4)}*(4096*a*b^7*c^4 - 262144*a^4*b* \\
& c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6)*1i - x*(1024*b^7*c^4 - 11264* \\
& a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b \\
& ^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - \\
& b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 \\
& - 256*a^6*b^2*c^3))^{(3/4)}*1i - 64*a*c^7 + 16*b^2*c^6)*1i + 8*c^7*x)*(-(b^7 \\
& - b^2*(-(4*a*c - b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c \\
& c + a*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c \\
& + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))^{(1/4)}*1i))*(-(b^7 - b^2*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - \\
& b^2)^5)^{(1/2)}/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 \\
& - 256*a^6*b^2*c^3))^{(1/4)}
\end{aligned}$$

sympy [A] time = 19.79, size = 177, normalized size = 0.56

$$\text{RootSum}\left(t^8(16777216a^7c^4 - 16777216a^6b^2c^3 + 6291456a^5b^4c^2 - 1048576a^4b^6c + 65536a^3b^8) + t^4(-12288a^3bc^3 + 10240a^2b^3c^2 - 2816ab^5c + 256b^7) + c^3, \left(t \mapsto t \log\left(x + \frac{16384t^5a^2bc^2 - 8192t^5a^4b^3c + 1024t^5a^6b^5 + 81a^2c^2 - 16tab^2c + 4tb^4}{ac^2 - b^2c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**8+b*x**4+a),x)

[Out] RootSum(_t**8*(16777216*a**7*c**4 - 16777216*a**6*b**2*c**3 + 6291456*a**5*b**4*c**2 - 1048576*a**4*b**6*c + 65536*a**3*b**8) + _t**4*(-12288*a**3*b*c**3 + 10240*a**2*b**3*c**2 - 2816*a*b**5*c + 256*b**7) + c**3, Lambda(_t, _t*log(x + (16384*_t**5*a**5*b*c**2 - 8192*_t**5*a**4*b**3*c + 1024*_t**5*a**3*b**5 + 8*_t*a**2*c**2 - 16*_t*a*b**2*c + 4*_t*b**4)/(a*c**2 - b**2*c))))

$$3.268 \quad \int \frac{1}{x^2(a+bx^4+cx^8)} dx$$

Optimal. Leaf size=363

$$\frac{\sqrt[4]{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) - \sqrt[4]{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) + \sqrt[4]{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2 \cdot 2^{3/4} a \sqrt[4]{-\sqrt{b^2-4ac}-b} - 2 \cdot 2^{3/4} a \sqrt[4]{\sqrt{b^2-4ac}-b} + 2 \cdot 2^{3/4} a \sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Rubi [A] time = 0.41, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1368, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) - \sqrt[4]{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) + \sqrt[4]{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) + \sqrt[4]{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2 \cdot 2^{3/4} a \sqrt[4]{-\sqrt{b^2-4ac}-b} - 2 \cdot 2^{3/4} a \sqrt[4]{\sqrt{b^2-4ac}-b} + 2 \cdot 2^{3/4} a \sqrt[4]{-\sqrt{b^2-4ac}-b} + 2 \cdot 2^{3/4} a \sqrt[4]{\sqrt{b^2-4ac}-b}} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^4 + c*x^8)),x]

[Out] -(1/(a*x)) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G

tQ[a/b, 0]

Rule 1368

Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n*(a + b*x^n + c*x^(2*n))^(p), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1510

Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2(a + bx^4 + cx^8)} dx &= -\frac{1}{ax} + \frac{\int \frac{x^2(-b-cx^4)}{a+bx^4+cx^8} dx}{a} \\
 &= -\frac{1}{ax} - \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{x^2}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2a} - \frac{\left(c\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{x^2}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2a} \\
 &= -\frac{1}{ax} + \frac{\left(\sqrt{c}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\sqrt{-b - \sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{2}a} - \frac{\left(\sqrt{c}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\sqrt{-b + \sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{2}a} \\
 &= -\frac{1}{ax} - \frac{\sqrt[4]{c}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b - \sqrt{b^2-4ac}}}\right)}{2 \cdot 2^{3/4} a \sqrt[4]{-b - \sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b + \sqrt{b^2-4ac}}}\right)}{2 \cdot 2^{3/4} a \sqrt[4]{-b + \sqrt{b^2-4ac}}}
 \end{aligned}$$

Mathematica [C] time = 0.04, size = 71, normalized size = 0.20

$$-\frac{\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4c \log(x-\#1) + b \log(x-\#1)}{2\#1^5c + \#1b}\& \right]}{4a} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*x^4 + c*x^8)),x]

[Out] -(1/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) &]/(4*a)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^4 + cx^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(a + b*x^4 + c*x^8)),x]

[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^4 + c*x^8)), x]

fricas [B] time = 3.47, size = 5125, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] -1/4*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*((a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*x - sqrt(1/2)*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x^2 - sqrt(1/2)*(b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 43*a^3*b^3*c^3 + 12*a^4*b*c^4 + (a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^4*c - 3*a*b^2*c^2 + a^2*c^3)) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-

$$\begin{aligned}
& (b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)))/((a^5b^4 - 8a^6b^2c + 16a^7c^2)) \cdot \arctan(-1/2(\sqrt{1/2}(b^6 - 7ab^4c + 13a^2b^2c^2 - 4a^3c^3 - (a^5b^5 - 8a^6b^3c + 16a^7b^2c^2))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))\sqrt{(1/2)\sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))})/((a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{(2(b^4c^3 - 3ab^2c^4 + a^2c^5))x^2 - \sqrt{1/2}(b^9 - 10ab^7c + 34a^2b^5c^2 - 43a^3b^3c^3 + 12a^4b^2c^4 - (a^5b^8 - 13a^6b^6c + 60a^7b^4c^2 - 112a^8b^2c^3 + 64a^9c^4))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))\sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))})/(a^5b^4 - 8a^6b^2c + 16a^7c^2)))/(b^4c^3 - 3ab^2c^4 + a^2c^5)) + ((a^5b^5 - 8a^6b^3c + 16a^7b^2c^2) \cdot x \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)) - (b^6 - 7ab^4c + 13a^2b^2c^2 - 4a^3c^3) \cdot x) \sqrt{(1/2)\sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))})/(b^4c^3 - 3ab^2c^4 + a^2c^5)) - a \cdot x \sqrt{(1/2)\sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))})/(a^5b^4 - 8a^6b^2c + 16a^7c^2))} \cdot \log(1/2\sqrt{1/2}(b^{11} - 13ab^9c + 63a^2b^7c^2 - 138a^3b^5c^3 + 128a^4b^3c^4 - 32a^5b^2c^5 - (a^5b^{10} - 16a^6b^8c + 98a^7b^6c^2 - 280a^8b^4c^3 + 352a^9b^2c^4 - 128a^{10}c^5))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))\sqrt{(1/2)\sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))})/(a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))})/(a^5b^4 - 8a^6b^2c + 16a^7c^2)) + (b^4c^4 - 3ab^2c^5 + a^2c^6) \cdot x) + a \cdot x \sqrt{(1/2)\sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2))\sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))})/(a^5b^4 - 8a^6b^2c + 16a^7c^2))} \cdot \log(-1/2\sqrt{1/2}(b^{11} - 13ab^9c + 63a^2b^7c^2 - 138a^3b^5c^3 + 128a^4b^3c^4 - 32a^5b^2c^5 - (a^5b^{10} - 16a^6b^8c + 98
\end{aligned}$$

$$\begin{aligned}
& *a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^{10}*c^5)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)))*\sqrt{(\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})}*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})}*(b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) - a*x*\sqrt{(\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})}*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})*\log(1/2*\sqrt{1/2}*(b^{11} - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^{10} - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^{10}*c^5)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})*\sqrt{(\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})}*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})}*(b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + a*x*\sqrt{(\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})}*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})*\log(-1/2*\sqrt{1/2}*(b^{11} - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^{10} - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^{10}*c^5)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})*\sqrt{(\sqrt{1/2}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})}*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))})}*(b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*x) + 4)/(a*x)
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 14.78Unable to convert to r
eal 1/4 Error: Bad Argument Value

maple [C] time = 0.01, size = 63, normalized size = 0.17

$$\frac{\left(\text{RootOf}\left(-Z^8c + b_Z^4 + a\right)^6 c + \text{RootOf}\left(-Z^8c + b_Z^4 + a\right)^2 b\right) \ln\left(-\text{RootOf}\left(-Z^8c + b_Z^4 + a\right) + x\right)}{4a\left(2\text{RootOf}\left(-Z^8c + b_Z^4 + a\right)^7 c + \text{RootOf}\left(-Z^8c + b_Z^4 + a\right)^3 b\right)} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^8+b*x^4+a),x)

[Out] -1/a/x-1/4/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(_Z^8*c+_Z^4*b+a))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 2.81, size = 10509, normalized size = 28.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*x^4 + c*x^8)),x)

[Out] 2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c^8*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)

$$\begin{aligned}
& 2*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + \\
& 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} - ((-(b^9 + b^4*(-(4*a*c - b^2)^5) \\
&)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a* \\
& c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(\\
& a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(\\
& (3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c \\
& ^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^ \\
& 4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*(32768*a^16*c^ \\
& 8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1 \\
& 5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - \\
& 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(\\
& 1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5 \\
& *b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/ \\
& 4)}/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 \\
& - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^ \\
& 2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + \\
& 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)}*(4096*a^15*c^8 - x*(-(b^9 + b^4*(\\
& -(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + \\
& a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^ \\
& 5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256 \\
& *a^8*b^2*c^3))^{(1/4)}*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6* \\
& c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816 \\
& *a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c^8*x \\
&)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - \\
& 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c \\
& *(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96* \\
& a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*1i + ((-(b^9 + b^4*(-(4*a*c - b^2)^5) \\
&)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a* \\
& c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(\\
& a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(\\
& (3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c \\
& ^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^ \\
& 4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*(32768*a^16*c^ \\
& 8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1 \\
& 5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - \\
& 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(\\
& 1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5 \\
& *b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/ \\
& 4)}*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c \\
& ^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a* \\
& b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
\end{aligned}$$

$$\begin{aligned}
& + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^{(1/4)} - \operatorname{atan}\left(\left(\left(-b^9 - b^4*(-4*a*c - b^2)^5\right)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2 * \right. \right. \\
& \left. \left. (-4*a*c - b^2)^5\right)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-4*a*c - b^2)^5\right)^{(1/2)} \left. \right) / (512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)} * (4096*a^{15}*c^8 + x*(-b^9 - b^4*(-4*a*c - b^2)^5)^{(1/2)} + 80 \\
& *a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-4*a*c - b^2)^5)^{(1/2)} \left. \right) / (512*(a^5*b^8 + 25 \\
& 6*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} * (32768 \\
& *a^{16}*c^8 + 1024*a^{12}*b^8*c^4 - 12288*a^{13}*b^6*c^5 + 51200*a^{14}*b^4*c^6 - 8 \\
& 1920*a^{15}*b^2*c^7) + 256*a^{11}*b^8*c^4 - 2816*a^{12}*b^6*c^5 + 10496*a^{13}*b^4* \\
& c^6 - 14336*a^{14}*b^2*c^7) + 4*a^{11}*b*c^8*x) * (-b^9 - b^4*(-4*a*c - b^2)^5) \\
& ^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-4*a*c \\
& - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-4*a*c - b^2)^5)^{(1/2)} \left. \right) / (512*(a \\
& ^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} * 1i - \left(\left(-b^9 - b^4*(-4*a*c - b^2)^5 \right)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^ \\
& 5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3 \\
& *a*b^2*c*(-4*a*c - b^2)^5)^{(1/2)} \left. \right) / (512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6 \\
& *c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)} * (4096*a^{15}*c^8 - x*(-b^9 - \\
& b^4*(-4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3* \\
& c^3 - a^2*c^2*(-4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-4*a*c - \\
& b^2)^5)^{(1/2)} \left. \right) / (512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 \\
& - 256*a^8*b^2*c^3))^{(1/4)} * (32768*a^{16}*c^8 + 1024*a^{12}*b^8*c^4 - 12288*a^{13} \\
& *b^6*c^5 + 51200*a^{14}*b^4*c^6 - 81920*a^{15}*b^2*c^7) + 256*a^{11}*b^8*c^4 - 28 \\
& 16*a^{12}*b^6*c^5 + 10496*a^{13}*b^4*c^6 - 14336*a^{14}*b^2*c^7) - 4*a^{11}*b*c^8*x \\
&) * (-b^9 - b^4*(-4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1 \\
& 20*a^3*b^3*c^3 - a^2*c^2*(-4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c* \\
& (-4*a*c - b^2)^5)^{(1/2)} \left. \right) / (512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a \\
& ^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} * 1i) / \left(\left(-b^9 - b^4*(-4*a*c - b^2)^5 \right) \right. \\
& ^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-4*a*c \\
& - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-4*a*c - b^2)^5)^{(1/2)} \left. \right) / (512*(a \\
& ^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)} * (4096*a^{15}*c^8 + x*(-b^9 - b^4*(-4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^ \\
& 4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-4*a*c - b^2)^5)^{(1/2)} - 1 \\
& 3*a*b^7*c + 3*a*b^2*c*(-4*a*c - b^2)^5)^{(1/2)} \left. \right) / (512*(a^5*b^8 + 256*a^9*c^4 \\
& - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} * (32768*a^{16}*c^8 \\
& + 1024*a^{12}*b^8*c^4 - 12288*a^{13}*b^6*c^5 + 51200*a^{14}*b^4*c^6 - 81920*a^{15} \\
& *b^2*c^7) + 256*a^{11}*b^8*c^4 - 2816*a^{12}*b^6*c^5 + 10496*a^{13}*b^4*c^6 - 143 \\
& 36*a^{14}*b^2*c^7) + 4*a^{11}*b*c^8*x) * (-b^9 - b^4*(-4*a*c - b^2)^5)^{(1/2)} + \\
& 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-4*a*c - b^2)^5 \\
&)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-4*a*c - b^2)^5)^{(1/2)} \left. \right) / (512*(a^5*b^8 + \\
& 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} + \left(\left(-b^9 - b^4*(-4*a*c - b^2)^5 \right)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120* \right. \\
& \left. a^3*b^3*c^3 - a^2*c^2*(-4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-4 \right. \\
& \left. 4*a*c - b^2)^5)^{(1/2)} \left. \right) / (512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7* \right. \\
& \left. b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)} * (4096*a^{15}*c^8 - x*(-b^9 - b^4*(-4*a*c
\end{aligned}$$

$$\begin{aligned}
& - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^8 \\
& 2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)} \\
&))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 5 \\
& 1200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x)*(-(b^9 - b \\
& ^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b \\
& ^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b \\
& c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*2i - \operatorname{atan}((\\
& ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12 \\
& 0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^ \\
& 7*b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)}*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a \\
& *c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1 \\
& /2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + \\
& 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) + 4*a^11*b*c^8*x)*(-(b^9 + \\
& b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - \\
& b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 \\
& - 256*a^8*b^2*c^3))^{(1/4)}*1i - ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 8 \\
& 0*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 2 \\
& 56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)}*(4096 \\
& *a^15*c^8 - x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2 \\
& *b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c \\
& - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*(32768*a^16*c^8 + 1024*a^ \\
& 12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7) \\
& + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7) - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*1i)/(((-(b^9 + \\
& b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3 \\
& *c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - \\
& b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 \\
& - 256*a^8*b^2*c^3))^{(3/4)}*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2) \\
& ^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*
\end{aligned}$$

$$\begin{aligned}
& a^5c - b^2)^5)^{(1/2)} - 13a^5b^7c - 3a^5b^2c^2 * (-4a^5c - b^2)^5)^{(1/2)) / (512 \\
& * (a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3)) \\
&)^{(1/4)} * (32768a^{16}c^8 + 1024a^{12}b^8c^4 - 12288a^{13}b^6c^5 + 51200a^{14}b^4c^6 \\
& - 81920a^{15}b^2c^7) + 256a^{11}b^8c^4 - 2816a^{12}b^6c^5 + 1 \\
& 0496a^{13}b^4c^6 - 14336a^{14}b^2c^7) + 4a^{11}b^8c^4 * (-b^9 + b^4 * (-4a^5c - b^2)^5)^{(1/2)} \\
& + 80a^4b^4c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (-4a^5c - b^2)^5)^{(1/2)} - 13a^5b^7c \\
& - 3a^5b^2c^2 * (-4a^5c - b^2)^5)^{(1/2)) / (512 * (a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 \\
& - 256a^8b^2c^3))^{(1/4)} + ((-b^9 + b^4 * (-4a^5c - b^2)^5)^{(1/2)} + 80a^4b^4c^4 \\
& + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (-4a^5c - b^2)^5)^{(1/2)} - 13a^5b^7c - 3a^5b^2c^2 * \\
& (-4a^5c - b^2)^5)^{(1/2)) / (512 * (a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(3/4)} \\
& * (4096a^{15}c^8 - x * (-b^9 + b^4 * (-4a^5c - b^2)^5)^{(1/2)} + 80a^4b^4c^4 + 61a^2b^5c^2 - 1 \\
& 20a^3b^3c^3 + a^2c^2 * (-4a^5c - b^2)^5)^{(1/2)} - 13a^5b^7c - 3a^5b^2c^2 * \\
& (-4a^5c - b^2)^5)^{(1/2)) / (512 * (a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)} \\
& * (32768a^{16}c^8 + 1024a^{12}b^8c^4 - 12288a^{13}b^6c^5 + 51200a^{14}b^4c^6 - 81920a^{15}b^2c^7) + 256a^{11}b^8c^4 \\
& - 2816a^{12}b^6c^5 + 10496a^{13}b^4c^6 - 14336a^{14}b^2c^7) - 4a^{11}b^8c^4 * (-b^9 + b^4 * (-4a^5c - b^2)^5)^{(1/2)} \\
& + 80a^4b^4c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (-4a^5c - b^2)^5)^{(1/2)} - 13a^5b^7c - \\
& 3a^5b^2c^2 * (-4a^5c - b^2)^5)^{(1/2)) / (512 * (a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)} \\
&)) * (-b^9 + b^4 * (-4a^5c - b^2)^5)^{(1/2)} + 80a^4b^4c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * \\
& (-4a^5c - b^2)^5)^{(1/2)} - 13a^5b^7c - 3a^5b^2c^2 * (-4a^5c - b^2)^5)^{(1/2)) / \\
& (512 * (a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)} * 2i + 2 * \operatorname{atan}(((-b^9 - b^4 * (-4a^5c - b^2)^5)^{(1/2)} \\
& + 80a^4b^4c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 - a^2c^2 * (-4a^5c - b^2)^5)^{(1/2)} - \\
& 13a^5b^7c + 3a^5b^2c^2 * (-4a^5c - b^2)^5)^{(1/2)) / (512 * (a^5b^8 + 256a^9c^4 \\
& - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(3/4)} * (4096a^{15}c^8 \\
& - x * (-b^9 - b^4 * (-4a^5c - b^2)^5)^{(1/2)} + 80a^4b^4c^4 + 61a^2b^5c^2 \\
& - 120a^3b^3c^3 - a^2c^2 * (-4a^5c - b^2)^5)^{(1/2)} - 13a^5b^7c + 3a^5b^2c^2 * \\
& (-4a^5c - b^2)^5)^{(1/2)) / (512 * (a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)} \\
& * (32768a^{16}c^8 + 1024a^{12}b^8c^4 - 12288a^{13}b^6c^5 + 51200a^{14}b^4c^6 - 81920a^{15}b^2c^7) * 1i + 256 \\
& a^{11}b^8c^4 - 2816a^{12}b^6c^5 + 10496a^{13}b^4c^6 - 14336a^{14}b^2c^7) * 1i + 4a^{11}b^8c^4 * (-b^9 - b^4 * (-4a^5c - b^2)^5)^{(1/2)} \\
& + 80a^4b^4c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 - a^2c^2 * (-4a^5c - b^2)^5)^{(1/2)} - 13 \\
& a^5b^7c + 3a^5b^2c^2 * (-4a^5c - b^2)^5)^{(1/2)) / (512 * (a^5b^8 + 256a^9c^4 \\
& - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)} - ((-b^9 - b^4 * (-4a^5c - b^2)^5)^{(1/2)} \\
& + 80a^4b^4c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 - a^2c^2 * (-4a^5c - b^2)^5)^{(1/2)} - 13a^5b^7c + 3a^5b^2c^2 * \\
& (-4a^5c - b^2)^5)^{(1/2)) / (512 * (a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 25 \\
& 6a^8b^2c^3))^{(3/4)} * (4096a^{15}c^8 + x * (-b^9 - b^4 * (-4a^5c - b^2)^5)^{(1/2)} \\
& + 80a^4b^4c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 - a^2c^2 * (-4a^5c - \\
& b^2)^5)^{(1/2)} - 13a^5b^7c + 3a^5b^2c^2 * (-4a^5c - b^2)^5)^{(1/2)) / (512 * (a^5
\end{aligned}$$

```

*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^
4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 104
96*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)
^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4))/(((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4
- 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 -
x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*
a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4
- 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^
11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1
i + 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 +
61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*
b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 1
6*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + (((-(b^9 - b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
- a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 25
6*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^
4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 104
96*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 - b^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a
^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)
^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4)*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - 1/(a*x)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.269 \quad \int \frac{1}{x^4(a+bx^4+cx^8)} dx$$

Optimal. Leaf size=365

$$\frac{c^{3/4} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} a \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} a \left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} a \left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Rubi [A] time = 0.40, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1368, 1422, 212, 208, 205}

$$\frac{c^{3/4} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} a \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} a \left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} a \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4} \left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} a \left(\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*x^4 + c*x^8)),x]

[Out] $-1/(3*a*x^3) + (c^{3/4}*(1 - b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{1/4}*c^{1/4}*x)/(-b - \text{Sqrt}[b^2 - 4*a*c])^{1/4}])/(2*2^{1/4}*a*(-b - \text{Sqrt}[b^2 - 4*a*c])^{3/4}) + (c^{3/4}*(1 + b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{1/4}*c^{1/4}*x)/(-b + \text{Sqrt}[b^2 - 4*a*c])^{1/4}])/(2*2^{1/4}*a*(-b + \text{Sqrt}[b^2 - 4*a*c])^{3/4}) + (c^{3/4}*(1 - b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{1/4}*c^{1/4}*x)/(-b - \text{Sqrt}[b^2 - 4*a*c])^{1/4}])/(2*2^{1/4}*a*(-b - \text{Sqrt}[b^2 - 4*a*c])^{3/4}) + (c^{3/4}*(1 + b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{1/4}*c^{1/4}*x)/(-b + \text{Sqrt}[b^2 - 4*a*c])^{1/4}])/(2*2^{1/4}*a*(-b + \text{Sqrt}[b^2 - 4*a*c])^{3/4})$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ

[a/b, 0]

Rule 1368

Int[((d_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[((d*x)^(m+1)*(a+b*x^n+c*x^(2*n))^(p+1))/(a*d*(m+1)), x] - Dist[1/(a*d^n*(m+1)), Int[(d*x)^(m+n)*(b*(m+n*(p+1)+1)+c*(m+2*n*(p+1)+1)*x^n*(a+b*x^n+c*x^(2*n))]^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2-4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1422

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2-4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2-4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2-4*a*c] || !IGtQ[n/2, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^4(a+bx^4+cx^8)} dx &= -\frac{1}{3ax^3} + \frac{\int \frac{-3b-3cx^4}{a+bx^4+cx^8} dx}{3a} \\
 &= -\frac{1}{3ax^3} - \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2a} - \frac{\left(c\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2a} \\
 &= -\frac{1}{3ax^3} + \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}} \\
 &= -\frac{1}{3ax^3} + \frac{c^{3/4}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}
 \end{aligned}$$

Mathematica [C] time = 0.04, size = 75, normalized size = 0.21

$$\frac{\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4c \log(x-\#1)+b \log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4a} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a + b*x^4 + c*x^8)),x]

[Out] -1/3*1/(a*x^3) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) &]/(4*a)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a + bx^4 + cx^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x^4 + c*x^8)),x]

[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^4 + c*x^8)), x]

fricas [B] time = 4.08, size = 6324, normalized size = 17.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] 1/12*(12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))*arctan(-1/4*(2*sqrt(1/2)*((a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)) + (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7)*x)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) - (b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))

$$\begin{aligned}
& 13a^9b^7c^2 - 364a^{10}b^5c^3 + 560a^{11}b^3c^4 - 320a^{12}b^1c^5) \sqrt{ \\
& ((b^{12} - 10a^5b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 1 \\
& 2a^5b^2c^5 + a^6c^6)/(a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a \\
& ^{17}c^3))} \sqrt{-(b^7 - 7a^5b^5c + 14a^2b^3c^2 - 7a^3b^1c^3 - (a^7b^4 \\
& - 8a^8b^2c + 16a^9c^2))} \sqrt{((b^{12} - 10a^5b^{10}c + 37a^2b^8c^2 - 62 \\
& a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6)/(a^{14}b^6 - 12a^{15}b^4c \\
& + 48a^{16}b^2c^2 - 64a^{17}c^3)))/(a^7b^4 - 8a^8b^2c + 16a^9 \\
& c^2)} \sqrt{((2(b^6c^4 - 5a^5b^4c^5 + 6a^2b^2c^6 - a^3c^7))x^2 + \sqrt{ \\
& (1/2)(b^{12} - 13a^5b^{10}c + 64a^2b^8c^2 - 147a^3b^6c^3 + 156a^4b^4c^4 \\
& c^4 - 66a^5b^2c^5 + 8a^6c^6 + (a^7b^9 - 14a^8b^7c + 72a^9b^5c^2 \\
& - 160a^{10}b^3c^3 + 128a^{11}b^1c^4))} \sqrt{((b^{12} - 10a^5b^{10}c + 37a^2b^8 \\
& c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6)/(a^{14}b^6 \\
& - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))} \sqrt{-(b^7 - 7a^5b^5c \\
& + 14a^2b^3c^2 - 7a^3b^1c^3 - (a^7b^4 - 8a^8b^2c + 16a^9c^2))} \sqrt{ \\
& ((b^{12} - 10a^5b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 1 \\
& 2a^5b^2c^5 + a^6c^6)/(a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a \\
& ^{17}c^3)))/(a^7b^4 - 8a^8b^2c + 16a^9c^2)))/(b^6c^4 - 5a^5b^4c^5 + \\
& 6a^2b^2c^6 - a^3c^7))} \sqrt{(\sqrt{1/2})\sqrt{-(b^7 - 7a^5b^5c + 14a^2b^ \\
& ^3c^2 - 7a^3b^1c^3 - (a^7b^4 - 8a^8b^2c + 16a^9c^2))} \sqrt{((b^{12} - 10 \\
& a^5b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^ \\
& ^5 + a^6c^6)/(a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3)))/ \\
& (a^7b^4 - 8a^8b^2c + 16a^9c^2)))/(b^6c^5 - 5a^5b^4c^6 + 6a^2b^2c^ \\
& ^7 - a^3c^8)) - 12ax^3\sqrt{(\sqrt{1/2})\sqrt{-(b^7 - 7a^5b^5c + 14a^2b^ \\
& ^3c^2 - 7a^3b^1c^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2))} \sqrt{((b^{12} - 10 \\
& a^5b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^ \\
& ^5 + a^6c^6)/(a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3)))/ \\
& (a^7b^4 - 8a^8b^2c + 16a^9c^2))} \arctan(-1/4(2\sqrt{1/2})((a^7b^{11} - \\
& 17a^8b^9c + 113a^9b^7c^2 - 364a^{10}b^5c^3 + 560a^{11}b^3c^4 - 320 \\
& a^{12}b^1c^5)x\sqrt{((b^{12} - 10a^5b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + \\
& 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6)/(a^{14}b^6 - 12a^{15}b^4c + 48 \\
& a^{16}b^2c^2 - 64a^{17}c^3)) - (b^{14} - 16a^5b^{12}c + 102a^2b^{10}c^2 - 328 \\
& a^3b^8c^3 + 553a^4b^6c^4 - 457a^5b^4c^5 + 152a^6b^2c^6 - 16a^7 \\
& c^7)x)} \sqrt{(\sqrt{1/2})\sqrt{-(b^7 - 7a^5b^5c + 14a^2b^3c^2 - 7a^3b^1c^ \\
& ^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2))} \sqrt{((b^{12} - 10a^5b^{10}c + 37a^2 \\
& b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6)/(a^{14} \\
& b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3)))/(a^7b^4 - 8a^8b^ \\
& ^2c + 16a^9c^2))} \sqrt{-(b^7 - 7a^5b^5c + 14a^2b^3c^2 - 7a^3b^1c^3 \\
& + (a^7b^4 - 8a^8b^2c + 16a^9c^2))} \sqrt{((b^{12} - 10a^5b^{10}c + 37a^2b^ \\
& 8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6)/(a^{14}b^ \\
& ^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3)))/(a^7b^4 - 8a^8b^2 \\
& c + 16a^9c^2)) + (b^{14} - 16a^5b^{12}c + 102a^2b^{10}c^2 - 328a^3b^8c^3 \\
& + 553a^4b^6c^4 - 457a^5b^4c^5 + 152a^6b^2c^6 - 16a^7c^7 - (a^7b^ \\
& ^{11} - 17a^8b^9c + 113a^9b^7c^2 - 364a^{10}b^5c^3 + 560a^{11}b^3c^4 \\
& - 320a^{12}b^1c^5)\sqrt{((b^{12} - 10a^5b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^ \\
& ^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6)/(a^{14}b^6 - 12a^{15}b^4c +
\end{aligned}$$

$$\begin{aligned}
& 48a^{16}b^2c^2 - 64a^{17}c^3)) * \sqrt{\sqrt{1/2} * \sqrt{-(b^7 - 7a^*b^5c + 14a^2b^3c^2 - 7a^3b^*c^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} / (a^7b^4 - 8a^8b^2c + 16a^9c^2)) * \sqrt{-(b^7 - 7a^*b^5c + 14a^2b^3c^2 - 7a^3b^*c^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} / (a^7b^4 - 8a^8b^2c + 16a^9c^2)) * \sqrt{((b^6c^4 - 5a^*b^4c^5 + 6a^2b^2c^6 - a^3c^7) * x^2 + \sqrt{1/2} * (b^{12} - 13a^*b^{10}c + 64a^2b^8c^2 - 147a^3b^6c^3 + 156a^4b^4c^4 - 66a^5b^2c^5 + 8a^6c^6 - (a^7b^9 - 14a^8b^7c + 72a^9b^5c^2 - 160a^{10}b^3c^3 + 128a^{11}b^*c^4) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} * \sqrt{-(b^7 - 7a^*b^5c + 14a^2b^3c^2 - 7a^3b^*c^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} / (a^7b^4 - 8a^8b^2c + 16a^9c^2)) / (b^6c^4 - 5a^*b^4c^5 + 6a^2b^2c^6 - a^3c^7)) / (b^6c^5 - 5a^*b^4c^6 + 6a^2b^2c^7 - a^3c^8)) - 3a^*x^3 * \sqrt{\sqrt{1/2} * \sqrt{-(b^7 - 7a^*b^5c + 14a^2b^3c^2 - 7a^3b^*c^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} / (a^7b^4 - 8a^8b^2c + 16a^9c^2)) * \log(-(b^6c^2 - 5a^*b^4c^3 + 6a^2b^2c^4 - a^3c^5) * x + 1/2 * (b^9 - 9a^*b^7c + 26a^2b^5c^2 - 25a^3b^3c^3 + 4a^4b^*c^4 - (a^7b^6 - 10a^8b^4c + 32a^9b^2c^2 - 32a^{10}c^3) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} * \sqrt{\sqrt{1/2} * \sqrt{-(b^7 - 7a^*b^5c + 14a^2b^3c^2 - 7a^3b^*c^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} / (a^7b^4 - 8a^8b^2c + 16a^9c^2))}} + 3a^*x^3 * \sqrt{\sqrt{1/2} * \sqrt{-(b^7 - 7a^*b^5c + 14a^2b^3c^2 - 7a^3b^*c^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} / (a^7b^4 - 8a^8b^2c + 16a^9c^2)) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} * \sqrt{\sqrt{1/2} * \sqrt{-(b^7 - 7a^*b^5c + 14a^2b^3c^2 - 7a^3b^*c^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} / (a^7b^4 - 8a^8b^2c + 16a^9c^2))}} * \log(-(b^6c^2 - 5a^*b^4c^3 + 6a^2b^2c^4 - a^3c^5) * x - 1/2 * (b^9 - 9a^*b^7c + 26a^2b^5c^2 - 25a^3b^3c^3 + 4a^4b^*c^4 - (a^7b^6 - 10a^8b^4c + 32a^9b^2c^2 - 32a^{10}c^3) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} * \sqrt{\sqrt{1/2} * \sqrt{-(b^7 - 7a^*b^5c + 14a^2b^3c^2 - 7a^3b^*c^3 + (a^7b^4 - 8a^8b^2c + 16a^9c^2) * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}} / (a^7b^4 - 8a^8b^2c + 16a^9c^2))}} * \sqrt{(b^{12} - 10a^*b^{10}c + 37a^2b^8c^2 - 62a^3b^6c^3 + 46a^4b^4c^4 - 12a^5b^2c^5 + a^6c^6) / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3))}}
\end{aligned}$$

```

^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 3*a*x^3*sqrt(
sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4
- 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*
a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^1
5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x + 1/2*(b^9
- 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10
*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sq
rt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48
*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*
a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
- (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b
^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*
c + 16*a^9*c^2)))*log(-(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*x
- 1/2*(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a
^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(s
qrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 -
8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c
^2)))) - 4)/(a*x^3)

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Evaluation time: 21.84Unable to convert to r
eal 1/4 Error: Bad Argument Value

maple [C] time = 0.01, size = 62, normalized size = 0.17

$$\frac{\left(-\operatorname{RootOf}\left(-Z^8c + b_Z^4 + a\right)^4 c - b\right) \ln\left(-\operatorname{RootOf}\left(-Z^8c + b_Z^4 + a\right) + x\right)}{4a\left(2\operatorname{RootOf}\left(-Z^8c + b_Z^4 + a\right)^7 c + \operatorname{RootOf}\left(-Z^8c + b_Z^4 + a\right)^3 b\right)} - \frac{1}{3ax^3}$$

$$\begin{aligned}
& 4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} - 15 a b^9 c + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2} \\
& / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} ((x (81920 a^{15} b^8 c^8 + 1024 a^{11} b^9 c^4 - 13312 a^{12} b^7 c^5 + 62464 a^{13} b^5 c^6 - 122880 a^{14} b^3 c^7) \\
& + (- (b^{11} + b^6 (- (4 a c - b^2)^5)^{1/2} - 112 a^5 b^5 c^5 + 86 a^2 b^7 c^2 - 231 a^3 b^5 c^3 + 280 a^4 b^3 c^4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} \\
& - 15 a b^9 c + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2})) / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 \\
& - 256 a^{10} b^2 c^3))^{1/4} (262144 a^{17} c^8 + 4096 a^{13} b^8 c^4 - 53248 a^{14} b^6 c^5 + 245760 a^{15} b^4 c^6 - 458752 a^{16} b^2 c^7) * i) \\
& * (- (b^{11} + b^6 (- (4 a c - b^2)^5)^{1/2} - 112 a^5 b^5 c^5 + 86 a^2 b^7 c^2 - 231 a^3 b^5 c^3 + 280 a^4 b^3 c^4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} \\
& - 15 a b^9 c + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2}) / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 \\
& - 256 a^{10} b^2 c^3))^{3/4} * i + 128 a^{11} b^9 c^9 + 16 a^9 b^5 c^7 - 96 a^{10} b^3 c^8) * i + x (8 a^{10} c^{10} - 4 a^9 b^2 c^9)) * (- (b^{11} + b^6 (- (4 a c - b^2)^5)^{1/2} \\
& - 112 a^5 b^5 c^5 + 86 a^2 b^7 c^2 - 231 a^3 b^5 c^3 + 280 a^4 b^3 c^4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} - 15 a b^9 c + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} \\
& - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2}) / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} \\
& / (((- (b^{11} + b^6 (- (4 a c - b^2)^5)^{1/2} - 112 a^5 b^5 c^5 + 86 a^2 b^7 c^2 - 231 a^3 b^5 c^3 + 280 a^4 b^3 c^4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} - 15 a b^9 c \\
& + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2}) / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} \\
& * ((x (81920 a^{15} b^8 c^8 + 1024 a^{11} b^9 c^4 - 13312 a^{12} b^7 c^5 + 62464 a^{13} b^5 c^6 - 122880 a^{14} b^3 c^7) - (- (b^{11} + b^6 (- (4 a c - b^2)^5)^{1/2} \\
& - 112 a^5 b^5 c^5 + 86 a^2 b^7 c^2 - 231 a^3 b^5 c^3 + 280 a^4 b^3 c^4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} - 15 a b^9 c + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} \\
& - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2}) / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} * ((x (81920 a^{15} b^8 c^8 + 1024 a^{11} b^9 c^4 \\
& - 13312 a^{12} b^7 c^5 + 62464 a^{13} b^5 c^6 - 122880 a^{14} b^3 c^7) - (- (b^{11} + b^6 (- (4 a c - b^2)^5)^{1/2} - 112 a^5 b^5 c^5 + 86 a^2 b^7 c^2 \\
& - 231 a^3 b^5 c^3 + 280 a^4 b^3 c^4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} - 15 a b^9 c + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2}) \\
& / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} * i) - 128 a^{11} b^9 c^9 - 16 a^9 b^5 c^7 + 96 a^{10} b^3 c^8) * i + x (8 a^{10} c^{10} - 4 a^9 b^2 c^9)) \\
& * (- (b^{11} + b^6 (- (4 a c - b^2)^5)^{1/2} - 112 a^5 b^5 c^5 + 86 a^2 b^7 c^2 - 231 a^3 b^5 c^3 + 280 a^4 b^3 c^4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} \\
& - 15 a b^9 c + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2}) / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 \\
& - 256 a^{10} b^2 c^3))^{1/4} * i) - ((- (b^{11} + b^6 (- (4 a c - b^2)^5)^{1/2} - 112 a^5 b^5 c^5 + 86 a^2 b^7 c^2 - 231 a^3 b^5 c^3 + 280 a^4 b^3 c^4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} \\
& - 15 a b^9 c + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2}) / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} \\
& * i) - 128 a^{11} b^9 c^9 - 16 a^9 b^5 c^7 + 96 a^{10} b^3 c^8) * i + x (8 a^{10} c^{10} - 4 a^9 b^2 c^9)) * (- (b^{11} + b^6 (- (4 a c - b^2)^5)^{1/2} - 112 a^5 b^5 c^5 + 86 a^2 b^7 c^2 \\
& - 231 a^3 b^5 c^3 + 280 a^4 b^3 c^4 - a^3 c^3 (- (4 a c - b^2)^5)^{1/2} - 15 a b^9 c + 6 a^2 b^2 c^2 (- (4 a c - b^2)^5)^{1/2} - 5 a b^4 c (- (4 a c - b^2)^5)^{1/2}) \\
& / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} * i)
\end{aligned}$$

$$\begin{aligned}
& c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{1/4} * ((x*(81920a^{15}b^8c^8 + 1024a^{11}b^9c^4 - 13312a^{12}b^7c^5 + 62464a^{13}b^5c^6 - \\
& 122880a^{14}b^3c^7) + (-(b^{11} + b^6*(-(4ac - b^2)^5)^{1/2}) - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 - a^3c^3*(-(4ac - \\
& b^2)^5)^{1/2} - 15ab^9c + 6a^2b^2c^2*(-(4ac - b^2)^5)^{1/2} - 5ab^4c*(-(4ac - b^2)^5)^{1/2}) / (512*(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 \\
& c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{1/4} * (262144a^{17}c^8 + 4096a^{13}b^8c^4 - 53248a^{14}b^6c^5 + 245760a^{15}b^4c^6 - 458752a^{16}b^2c^7) * \\
& 1i) * (-(b^{11} + b^6*(-(4ac - b^2)^5)^{1/2}) - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 - a^3c^3*(-(4ac - b^2)^5)^{1/2} - 1 \\
& 5ab^9c + 6a^2b^2c^2*(-(4ac - b^2)^5)^{1/2} - 5ab^4c*(-(4ac - b^2)^5)^{1/2}) / (512*(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 \\
& - 256a^{10}b^2c^3))^{3/4} * 1i + 128a^{11}b^9c^9 + 16a^9b^5c^7 - 96a^{10}b^3c^8) * 1i + x*(8a^{10}c^{10} - 4a^9b^2c^9) * (-(b^{11} + b^6*(-(4ac - b^2)^5)^{1/2}) \\
&)^{1/2} - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 - a^3c^3*(-(4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2b^2c^2*(-(4ac - b^2)^5)^{1/2} - 5ab^4c*(-(4ac - b^2)^5)^{1/2}) / (512*(a^7b^8 + 25 \\
& 6a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{1/4} * 1i) * (-(b^{11} + b^6*(-(4ac - b^2)^5)^{1/2}) - 112a^5b^5c^5 + 86a^2b^7c^2 - \\
& 231a^3b^5c^3 + 280a^4b^3c^4 - a^3c^3*(-(4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2b^2c^2*(-(4ac - b^2)^5)^{1/2} - 5ab^4c*(-(4ac - b^2)^5)^{1/2}) / (512*(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 2 \\
& 56a^{10}b^2c^3))^{1/4} - \operatorname{atan}(-((-(b^{11} - b^6*(-(4ac - b^2)^5)^{1/2}) - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 + a^3c^3*(-(4ac - b^2)^5)^{1/2} - 15ab^9c - 6a^2b^2c^2*(-(4ac - b^2)^5)^{1/2} \\
&)^{1/2} + 5ab^4c*(-(4ac - b^2)^5)^{1/2}) / (512*(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{1/4} * ((x*(81920a^{15}b^8c^8 + 1024a^{11}b^9c^4 - 13312a^{12}b^7c^5 + 62464a^{13}b^5c^6 - 12288 \\
& 0a^{14}b^3c^7) + (-(b^{11} - b^6*(-(4ac - b^2)^5)^{1/2}) - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 + a^3c^3*(-(4ac - b^2)^5)^{1/2} - 15ab^9c - 6a^2b^2c^2*(-(4ac - b^2)^5)^{1/2} + 5ab^4c*(-(4ac - b^2)^5)^{1/2}) / (512*(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 9 \\
& 6a^9b^4c^2 - 256a^{10}b^2c^3))^{1/4} * (262144a^{17}c^8 + 4096a^{13}b^8c^4 - 53248a^{14}b^6c^5 + 245760a^{15}b^4c^6 - 458752a^{16}b^2c^7) * (-(b^{11} - b^6*(-(4ac - b^2)^5)^{1/2}) - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 + a^3c^3*(-(4ac - b^2)^5)^{1/2} - 15ab^9c \\
& c - 6a^2b^2c^2*(-(4ac - b^2)^5)^{1/2} + 5ab^4c*(-(4ac - b^2)^5)^{1/2}) / (512*(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{3/4} - 128a^{11}b^9c^9 - 16a^9b^5c^7 + 96a^{10}b^3c^8) - \\
& x*(8a^{10}c^{10} - 4a^9b^2c^9) * (-(b^{11} - b^6*(-(4ac - b^2)^5)^{1/2}) - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3 + 280a^4b^3c^4 + a^3c^3*(-(4ac - b^2)^5)^{1/2} - 15ab^9c - 6a^2b^2c^2*(-(4ac - b^2)^5)^{1/2} \\
&)^{1/2} + 5ab^4c*(-(4ac - b^2)^5)^{1/2}) / (512*(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{1/4} * 1i + ((-(b^{11} - b^6*(-(4ac - b^2)^5)^{1/2}) - 112a^5b^5c^5 + 86a^2b^7c^2 - 231a^3b^5c^3
\end{aligned}$$

$$\begin{aligned}
&^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - ((b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(3/4)} + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*ii)/(((b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) + ((b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(3/4)} - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8) - x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)} - (((b^11 - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11
\end{aligned}$$

$$\begin{aligned}
& *c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*((x*(81920 \\
& *a^{15}*b*c^8 + 1024*a^{11}*b^9*c^4 - 13312*a^{12}*b^7*c^5 + 62464*a^{13}*b^5*c^6 - \\
& 122880*a^{14}*b^3*c^7) - (b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b* \\
& c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5* \\
& a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6 \\
& *c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*(262144*a^{17}*c^8 + 4096*a^1 \\
& 3*b^8*c^4 - 53248*a^{14}*b^6*c^5 + 245760*a^{15}*b^4*c^6 - 458752*a^{16}*b^2*c^7) \\
&)*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - \\
& 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15* \\
& a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2 \\
&)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - \\
& 256*a^{10}*b^2*c^3))^{(3/4)} + 128*a^{11}*b*c^9 + 16*a^9*b^5*c^7 - 96*a^{10}*b^3*c \\
& ^8) - x*(8*a^{10}*c^{10} - 4*a^9*b^2*c^9))*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/ \\
& 2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a \\
& ^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^{11}*c \\
& ^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}))*(-(b^{11} - \\
& b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5 \\
& *c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6* \\
& a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/ \\
& (512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2 \\
& *c^3))^{(1/4)}*2i - \operatorname{atan}(-(((b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5 \\
& *b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4* \\
& a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8* \\
& b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*((x*(81920*a^{15}*b*c^8 + \\
& 1024*a^{11}*b^9*c^4 - 13312*a^{12}*b^7*c^5 + 62464*a^{13}*b^5*c^6 - 122880*a^{14}*b \\
& ^3*c^7) + (b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b \\
& ^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/ \\
& 2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a \\
& *c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^ \\
& 4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*(262144*a^{17}*c^8 + 4096*a^{13}*b^8*c^4 - 53 \\
& 248*a^{14}*b^6*c^5 + 245760*a^{15}*b^4*c^6 - 458752*a^{16}*b^2*c^7))*(-(b^{11} + b^ \\
& 6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c \\
& ^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^ \\
& 2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(5 \\
& 12*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c \\
& ^3))^{(3/4)} - 128*a^{11}*b*c^9 - 16*a^9*b^5*c^7 + 96*a^{10}*b^3*c^8) - x*(8*a^1 \\
& 0*c^{10} - 4*a^9*b^2*c^9))*(-(b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b \\
& *c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a* \\
& c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5 \\
& *a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^ \\
& 6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*1i + (((b^{11} + b^6*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
\end{aligned}$$

$$\begin{aligned}
& a^4 b^3 c^4 - a^3 c^3 (-4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2 b^2 c^2 (-4ac - b^2)^5)^{1/2} - 5ab^4 c (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^{11} c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^{10} b^2 c^3))^{1/4} \\
& ((x(81920a^{15} b^3 c^8 + 1024a^{11} b^9 c^4 - 13312a^{12} b^7 c^5 + 62464a^{13} b^5 c^6 - 122880a^{14} b^3 c^7) - (b^{11} + b^6 (-4ac - b^2)^5)^{1/2} - 112a^5 b^3 c^5 + 86a^2 b^7 c^2 - 231a^3 b^5 c^3 + 280a^4 b^3 c^4 - a^3 c^3 (-4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2 b^2 c^2 (-4ac - b^2)^5)^{1/2} - 5ab^4 c (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^{11} c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^{10} b^2 c^3))^{1/4} \\
& (262144a^{17} c^8 + 4096a^{13} b^8 c^4 - 53248a^{14} b^6 c^5 + 245760a^{15} b^4 c^6 - 458752a^{16} b^2 c^7) * (-b^{11} + b^6 (-4ac - b^2)^5)^{1/2} - 112a^5 b^3 c^5 + 86a^2 b^7 c^2 - 231a^3 b^5 c^3 + 280a^4 b^3 c^4 - a^3 c^3 (-4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2 b^2 c^2 (-4ac - b^2)^5)^{1/2} - 5ab^4 c (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^{11} c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^{10} b^2 c^3))^{3/4} \\
& + 128a^{11} b^3 c^9 + 16a^9 b^5 c^7 - 96a^{10} b^3 c^8) - x(8a^{10} c^{10} - 4a^9 b^2 c^9) * (-b^{11} + b^6 (-4ac - b^2)^5)^{1/2} - 112a^5 b^3 c^5 + 86a^2 b^7 c^2 - 231a^3 b^5 c^3 + 280a^4 b^3 c^4 - a^3 c^3 (-4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2 b^2 c^2 (-4ac - b^2)^5)^{1/2} - 5ab^4 c (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^{11} c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^{10} b^2 c^3))^{1/4} \\
& * i) / (((-b^{11} + b^6 (-4ac - b^2)^5)^{1/2} - 112a^5 b^3 c^5 + 86a^2 b^7 c^2 - 231a^3 b^5 c^3 + 280a^4 b^3 c^4 - a^3 c^3 (-4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2 b^2 c^2 (-4ac - b^2)^5)^{1/2} - 5ab^4 c (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^{11} c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^{10} b^2 c^3))^{1/4} \\
& * ((x(81920a^{15} b^3 c^8 + 1024a^{11} b^9 c^4 - 13312a^{12} b^7 c^5 + 62464a^{13} b^5 c^6 - 122880a^{14} b^3 c^7) + (-b^{11} + b^6 (-4ac - b^2)^5)^{1/2} - 112a^5 b^3 c^5 + 86a^2 b^7 c^2 - 231a^3 b^5 c^3 + 280a^4 b^3 c^4 - a^3 c^3 (-4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2 b^2 c^2 (-4ac - b^2)^5)^{1/2} - 5ab^4 c (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^{11} c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^{10} b^2 c^3))^{1/4} \\
& * (262144a^{17} c^8 + 4096a^{13} b^8 c^4 - 53248a^{14} b^6 c^5 + 245760a^{15} b^4 c^6 - 458752a^{16} b^2 c^7) * (-b^{11} + b^6 (-4ac - b^2)^5)^{1/2} - 112a^5 b^3 c^5 + 86a^2 b^7 c^2 - 231a^3 b^5 c^3 + 280a^4 b^3 c^4 - a^3 c^3 (-4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2 b^2 c^2 (-4ac - b^2)^5)^{1/2} - 5ab^4 c (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^{11} c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^{10} b^2 c^3))^{3/4} \\
& - 128a^{11} b^3 c^9 - 16a^9 b^5 c^7 + 96a^{10} b^3 c^8) - x(8a^{10} c^{10} - 4a^9 b^2 c^9) * (-b^{11} + b^6 (-4ac - b^2)^5)^{1/2} - 112a^5 b^3 c^5 + 86a^2 b^7 c^2 - 231a^3 b^5 c^3 + 280a^4 b^3 c^4 - a^3 c^3 (-4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2 b^2 c^2 (-4ac - b^2)^5)^{1/2} - 5ab^4 c (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^{11} c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^{10} b^2 c^3))^{1/4} \\
& - ((-b^{11} + b^6 (-4ac - b^2)^5)^{1/2} - 112a^5 b^3 c^5 + 86a^2 b^7 c^2 - 231a^3 b^5 c^3 + 280a^4 b^3 c^4 - a^3 c^3 (-4ac - b^2)^5)^{1/2} - 15ab^9c + 6a^2 b^2 c^2 (-4ac - b^2)^5)^{1/2} - 5ab^4 c (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^{11} c^4 - 1
\end{aligned}$$

$$\begin{aligned}
& (6*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*((x*(81920*a^{15}*b^8*c^8 + 1024*a^{11}*b^9*c^4 - 13312*a^{12}*b^7*c^5 + 62464*a^{13}*b^5*c^6 - 122880*a^{14}*b^3*c^7) - ((b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*(262144*a^{17}*c^8 + 4096*a^{13}*b^8*c^4 - 53248*a^{14}*b^6*c^5 + 245760*a^{15}*b^4*c^6 - 458752*a^{16}*b^2*c^7))*(-(b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(3/4)} + 128*a^{11}*b*c^9 + 16*a^9*b^5*c^7 - 96*a^{10}*b^3*c^8) - x*(8*a^{10}*c^{10} - 4*a^9*b^2*c^9))*(-(b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)})))*(-(b^{11} + b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)})*2i + 2*atan(-(((b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*((x*(81920*a^{15}*b^8*c^8 + 1024*a^{11}*b^9*c^4 - 13312*a^{12}*b^7*c^5 + 62464*a^{13}*b^5*c^6 - 122880*a^{14}*b^3*c^7) - ((b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*(262144*a^{17}*c^8 + 4096*a^{13}*b^8*c^4 - 53248*a^{14}*b^6*c^5 + 245760*a^{15}*b^4*c^6 - 458752*a^{16}*b^2*c^7)*1i))*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)})*1i - 128*a^{11}*b*c^9 - 16*a^9*b^5*c^7 + 96*a^{10}*b^3*c^8)*1i + x*(8*a^{10}*c^{10} - 4*a^9*b^2*c^9))*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)} + (((b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280
\end{aligned}$$

$$\begin{aligned}
& + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}* \\
& ((x*(81920*a^{15}*b*c^8 + 1024*a^{11}*b^9*c^4 - 13312*a^{12}*b^7*c^5 + 62464*a^{13} \\
& *b^5*c^6 - 122880*a^{14}*b^3*c^7) + (-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^7*b^8 + 256*a^{11}*c^4 - \\
& 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)}*(262144*a^{17}*c^8 \\
& + 4096*a^{13}*b^8*c^4 - 53248*a^{14}*b^6*c^5 + 245760*a^{15}*b^4*c^6 - 458752*a^{16} \\
& *b^2*c^7)*1i)*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86* \\
& a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(\\
& -(4*a*c - b^2)^5)^{(1/2)))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a \\
& ^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(3/4)}*1i + 128*a^{11}*b*c^9 + 16*a^9*b^5*c^7 \\
& - 96*a^{10}*b^3*c^8)*1i + x*(8*a^{10}*c^{10} - 4*a^9*b^2*c^9))*(-(b^{11} - b^6*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + \\
& 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(1/2)} - 15*a*b^9*c - 6*a^2*b^2 \\
& *c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a \\
& ^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3)) \\
& ^{(1/4)}*1i))*(-(b^{11} - b^6*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5 + 86*a^2 \\
& *b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 + a^3*c^3*(-(4*a*c - b^2)^5)^{(\\
& 1/2)} - 15*a*b^9*c - 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*(\\
& -(4*a*c - b^2)^5)^{(1/2)))/(512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9* \\
& b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)} - 1/(3*a*x^3)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.270 \quad \int \frac{x^{11}}{1+x^4+x^8} dx$$

Optimal. Leaf size=44

$$\frac{x^4}{4} - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8} \log(x^8 + x^4 + 1)$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1357, 703, 634, 618, 204, 628}

$$\frac{x^4}{4} - \frac{1}{8} \log(x^8 + x^4 + 1) - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^11/(1 + x^4 + x^8), x]

[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(
m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
+ a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{1+x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{1+x+x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{4} \text{Subst} \left(\int \frac{-1-x}{1+x+x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^4 \right) - \frac{1}{8} \text{Subst} \left(\int \frac{1+2x}{1+x+x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} - \frac{1}{8} \log(1+x^4+x^8) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^4 \right) \\
 &= \frac{x^4}{4} - \frac{\tan^{-1} \left(\frac{1+2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} - \frac{1}{8} \log(1+x^4+x^8)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$\frac{x^4}{4} - \frac{\tan^{-1} \left(\frac{2x^4+1}{\sqrt{3}} \right)}{4\sqrt{3}} - \frac{1}{8} \log(x^8 + x^4 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(1 + x^4 + x^8), x]

[Out] $x^4/4 - \text{ArcTan}[(1 + 2x^4)/\text{Sqrt}[3]]/(4*\text{Sqrt}[3]) - \text{Log}[1 + x^4 + x^8]/8$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{1 + x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[x^11/(1 + x^4 + x^8), x]`

[Out] `IntegrateAlgebraic[x^11/(1 + x^4 + x^8), x]`

fricas [A] time = 1.42, size = 35, normalized size = 0.80

$$\frac{1}{4}x^4 - \frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^4 + 1)\right) - \frac{1}{8}\log(x^8 + x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(x^8+x^4+1), x, algorithm="fricas")`

[Out] $1/4*x^4 - 1/12*\text{sqrt}(3)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x^4 + 1)) - 1/8*\text{log}(x^8 + x^4 + 1)$

giac [A] time = 0.34, size = 35, normalized size = 0.80

$$\frac{1}{4}x^4 - \frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^4 + 1)\right) - \frac{1}{8}\log(x^8 + x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(x^8+x^4+1), x, algorithm="giac")`

[Out] $1/4*x^4 - 1/12*\text{sqrt}(3)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x^4 + 1)) - 1/8*\text{log}(x^8 + x^4 + 1)$

maple [A] time = 0.00, size = 36, normalized size = 0.82

$$\frac{x^4}{4} - \frac{\sqrt{3} \arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^8 + x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(x^8+x^4+1), x)`

[Out] $1/4*x^4 - 1/8*\ln(x^8+x^4+1) - 1/12*\text{arctan}(1/3*(2*x^4+1)*3^{(1/2)})*3^{(1/2)}$

maxima [A] time = 2.42, size = 35, normalized size = 0.80

$$\frac{1}{4}x^4 - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{8}\log(x^8+x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸+x⁴+1),x, algorithm="maxima")

[Out] 1/4*x⁴ - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x⁴ + 1)) - 1/8*log(x⁸ + x⁴ + 1)

mupad [B] time = 0.05, size = 37, normalized size = 0.84

$$\frac{x^4}{4} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^8+x^4+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(x⁴ + x⁸ + 1),x)

[Out] x⁴/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x⁴)/3))/12 - log(x⁴ + x⁸ + 1)/8

sympy [A] time = 0.14, size = 42, normalized size = 0.95

$$\frac{x^4}{4} - \frac{\log(x^8+x^4+1)}{8} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸+x⁴+1),x)

[Out] x⁴/4 - log(x⁸ + x⁴ + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x⁴/3 + sqrt(3)/3)/12

$$3.271 \quad \int \frac{x^9}{1+x^4+x^8} dx$$

Optimal. Leaf size=54

$$\frac{x^2}{2} + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1359, 1122, 1161, 618, 204}

$$\frac{x^2}{2} + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^9/(1 + x^4 + x^8),x]

[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1122

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)), x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m + 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1+x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{1+x^2+x^4} dx, x, x^2 \right) \\
&= \frac{x^2}{2} - \frac{1}{2} \text{Subst} \left(\int \frac{1+x^2}{1+x^2+x^4} dx, x, x^2 \right) \\
&= \frac{x^2}{2} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^2 \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^2 \right) \\
&= \frac{x^2}{2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^2 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^2 \right) \\
&= \frac{x^2}{2} + \frac{\tan^{-1} \left(\frac{1-2x^2}{\sqrt{3}} \right)}{2\sqrt{3}} - \frac{\tan^{-1} \left(\frac{1+2x^2}{\sqrt{3}} \right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.17, size = 98, normalized size = 1.81

$$\frac{x^2}{2} - \frac{(\sqrt{3} + i) \tan^{-1} \left(\frac{1}{2} (\sqrt{3} - i) x^2 \right)}{2\sqrt{6 + 6i\sqrt{3}}} - \frac{(\sqrt{3} - i) \tan^{-1} \left(\frac{1}{2} (\sqrt{3} + i) x^2 \right)}{2\sqrt{6 - 6i\sqrt{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/(1 + x^4 + x^8), x]
```

```
[Out] x^2/2 - ((I + Sqrt[3])*ArcTan[(-I + Sqrt[3])*x^2/2])/(2*Sqrt[6 + (6*I)*Sqr
t[3]]) - ((-I + Sqrt[3])*ArcTan[(I + Sqrt[3])*x^2/2])/(2*Sqrt[6 - (6*I)*
Sqrt[3]])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1+x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/(1 + x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^9/(1 + x^4 + x^8), x]

fricas [A] time = 1.23, size = 40, normalized size = 0.74

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}x^2\right) - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(x^6 + 2x^2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8+x^4+1), x, algorithm="fricas")

[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x^2) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^6 + 2*x^2))

giac [A] time = 0.32, size = 42, normalized size = 0.78

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^2 + 1)\right) - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^2 - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8+x^4+1), x, algorithm="giac")

[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))

maple [A] time = 0.01, size = 43, normalized size = 0.80

$$\frac{x^2}{2} - \frac{\sqrt{3} \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3} \arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^8+x^4+1), x)

[Out] 1/2*x^2-1/6*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))

maxima [A] time = 2.39, size = 42, normalized size = 0.78

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^2 + 1)\right) - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^2 - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8+x^4+1),x, algorithm="maxima")

[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))

mupad [B] time = 0.04, size = 43, normalized size = 0.80

$$\frac{x^2}{2} - \frac{\sqrt{3} \left(2 \operatorname{atan}\left(\frac{\sqrt{3}x^6}{3} + \frac{2\sqrt{3}x^2}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right) \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^4 + x^8 + 1),x)

[Out] x^2/2 - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(1/2)*x^2)/3)))/12

sympy [A] time = 0.14, size = 51, normalized size = 0.94

$$\frac{x^2}{2} + \frac{\sqrt{3} \left(-2 \operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right) - 2 \operatorname{atan}\left(\frac{\sqrt{3}x^6}{3} + \frac{2\sqrt{3}x^2}{3}\right) \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(x**8+x**4+1),x)

[Out] x**2/2 + sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(3)*x**2/3))/12

$$3.272 \quad \int \frac{x^7}{1+x^4+x^8} dx$$

Optimal. Leaf size=37

$$\frac{1}{8} \log(x^8 + x^4 + 1) - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{8} \log(x^8 + x^4 + 1) - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^7/(1 + x^4 + x^8),x]

[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 + x^4 + x^8]/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{1+x+x^2} dx, x, x^4 \right) \\
&= - \left(\frac{1}{8} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^4 \right) \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1+2x}{1+x+x^2} dx, x, x^4 \right) \\
&= \frac{1}{8} \log(1+x^4+x^8) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^4 \right) \\
&= -\frac{\tan^{-1} \left(\frac{1+2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} + \frac{1}{8} \log(1+x^4+x^8)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{8} \log(x^8 + x^4 + 1) - \frac{\tan^{-1} \left(\frac{2x^4+1}{\sqrt{3}} \right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(1 + x^4 + x^8), x]

[Out] -1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 + x^4 + x^8]/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1+x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(1 + x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^7/(1 + x^4 + x^8), x]

fricas [A] time = 1.21, size = 30, normalized size = 0.81

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 + 1)\right) + \frac{1}{8} \log(x^8 + x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8+x^4+1),x, algorithm="fricas")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)

giac [A] time = 0.39, size = 30, normalized size = 0.81

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 + 1)\right) + \frac{1}{8} \log(x^8 + x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8+x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)}{12} + \frac{\ln(x^8 + x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^8+x^4+1),x)

[Out] 1/8*ln(x^8+x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4+1)*3^(1/2))

maxima [A] time = 2.43, size = 30, normalized size = 0.81

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 + 1)\right) + \frac{1}{8} \log(x^8 + x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8+x^4+1),x, algorithm="maxima")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)

mupad [B] time = 0.04, size = 32, normalized size = 0.86

$$\frac{\ln(x^8 + x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(x^4 + x^8 + 1),x)`

[Out] `log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12`

sympy [A] time = 0.13, size = 37, normalized size = 1.00

$$\frac{\log(x^8 + x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(x**8+x**4+1),x)`

[Out] `log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/12`

$$3.273 \quad \int \frac{x^5}{1+x^4+x^8} dx$$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8} \log(x^4 - x^2 + 1) - \frac{1}{8} \log(x^4 + x^2 + 1)$$

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1359, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{8} \log(x^4 - x^2 + 1) - \frac{1}{8} \log(x^4 + x^2 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(1 + x^4 + x^8), x]

[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1127

Int[(x_)^2/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I

`Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4*a*c, 0] && PosQ[a*c]`

Rule 1161

`Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))`

Rule 1164

`Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]`

Rule 1359

`Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{1+x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{1+x^2+x^4} dx, x, x^2 \right) \\
 &= -\left(\frac{1}{4} \text{Subst} \left(\int \frac{1-x^2}{1+x^2+x^4} dx, x, x^2 \right) \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1+x^2}{1+x^2+x^4} dx, x, x^2 \right) \\
 &= \frac{1}{8} \text{Subst} \left(\int \frac{1+2x}{-1-x-x^2} dx, x, x^2 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1-2x}{-1+x-x^2} dx, x, x^2 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x} dx, x, x^2 \right) \\
 &= \frac{1}{8} \log(1-x^2+x^4) - \frac{1}{8} \log(1+x^2+x^4) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^2 \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-x} dx, x, x^2 \right) \\
 &= -\frac{\tan^{-1} \left(\frac{1-2x^2}{\sqrt{3}} \right)}{4\sqrt{3}} + \frac{\tan^{-1} \left(\frac{1+2x^2}{\sqrt{3}} \right)}{4\sqrt{3}} + \frac{1}{8} \log(1-x^2+x^4) - \frac{1}{8} \log(1+x^2+x^4) - \frac{1}{4} \log(1-x)
 \end{aligned}$$

Mathematica [C] time = 0.12, size = 94, normalized size = 1.25

$$\frac{\sqrt{1-i\sqrt{3}} (\sqrt{3}-i) \tan^{-1}\left(\frac{1}{2}(\sqrt{3}-i)x^2\right) + \sqrt{1+i\sqrt{3}} (\sqrt{3}+i) \tan^{-1}\left(\frac{1}{2}(\sqrt{3}+i)x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5/(1 + x^4 + x^8),x]

[Out] (Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2] + Sqrt[1 + I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(4*Sqrt[6])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1+x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(1 + x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^5/(1 + x^4 + x^8), x]

fricas [A] time = 0.97, size = 61, normalized size = 0.81

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) - \frac{1}{8} \log(x^4 + x^2 + 1) + \frac{1}{8} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+x^4+1),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)

giac [A] time = 0.39, size = 61, normalized size = 0.81

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) - \frac{1}{8} \log(x^4 + x^2 + 1) + \frac{1}{8} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+x^4+1),x, algorithm="giac")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)

maple [A] time = 0.00, size = 62, normalized size = 0.83

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{12} + \frac{\ln(x^4 - x^2 + 1)}{8} - \frac{\ln(x^4 + x^2 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^8+x^4+1),x)

[Out] -1/8*ln(x^4+x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2+1)*3^(1/2))+1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))

maxima [A] time = 2.49, size = 61, normalized size = 0.81

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) - \frac{1}{8} \log(x^4 + x^2 + 1) + \frac{1}{8} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+x^4+1),x, algorithm="maxima")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)

mupad [B] time = 0.09, size = 51, normalized size = 0.68

$$\operatorname{atanh}\left(\frac{2x^2}{-1 + \sqrt{3} 1i}\right) \left(\frac{1}{4} + \frac{\sqrt{3} 1i}{12}\right) + \operatorname{atanh}\left(\frac{2x^2}{1 + \sqrt{3} 1i}\right) \left(-\frac{1}{4} + \frac{\sqrt{3} 1i}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^4 + x^8 + 1),x)

[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)

sympy [A] time = 0.21, size = 76, normalized size = 1.01

$$\frac{\log(x^4 - x^2 + 1)}{8} - \frac{\log(x^4 + x^2 + 1)}{8} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(x**8+x**4+1),x)

[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12

$$3.274 \quad \int \frac{x^3}{1+x^4+x^8} dx$$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1352, 618, 204}

$$\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 + x^4 + x^8),x]

[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{1+x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^4 \right) \\
 &= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^4 \right) \right) \\
 &= \frac{\tan^{-1} \left(\frac{1+2x^4}{\sqrt{3}} \right)}{2\sqrt{3}}
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{2x^4+1}{\sqrt{3}} \right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(1 + x^4 + x^8), x]

[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1+x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(1 + x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^3/(1 + x^4 + x^8), x]

fricas [A] time = 1.14, size = 18, normalized size = 0.78

$$\frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^4 + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+x^4+1), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))

giac [A] time = 0.36, size = 18, normalized size = 0.78

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+x^4+1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^8+x^4+1),x)

[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4+1)*3^(1/2))

maxima [A] time = 2.47, size = 18, normalized size = 0.78

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+x^4+1),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))

mupad [B] time = 1.30, size = 17, normalized size = 0.74

$$\frac{\sqrt{3} \operatorname{atan}\left(\sqrt{3} \left(\frac{2x^4}{3} + \frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^4 + x^8 + 1),x)

[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 + 1/3)))/6

sympy [A] time = 0.12, size = 26, normalized size = 1.13

$$\frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**8+x**4+1),x)

[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/6

$$3.275 \quad \int \frac{x}{1+x^4+x^8} dx$$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8} \log(x^4 - x^2 + 1) + \frac{1}{8} \log(x^4 + x^2 + 1)$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1359, 1094, 634, 618, 204, 628}

$$-\frac{1}{8} \log(x^4 - x^2 + 1) + \frac{1}{8} \log(x^4 + x^2 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x^4 + x^8), x]

[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^2 + x^4]/8 + Log[1 + x^2 + x^4]/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:= With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x}{1+x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x^2+x^4} dx, x, x^2 \right) \\
 &= \frac{1}{4} \text{Subst} \left(\int \frac{1-x}{1-x+x^2} dx, x, x^2 \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1+x}{1+x+x^2} dx, x, x^2 \right) \\
 &= \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^2 \right) - \frac{1}{8} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^2 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^2 \right) \\
 &= -\frac{1}{8} \log(1-x^2+x^4) + \frac{1}{8} \log(1+x^2+x^4) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^2 \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^2 \right) \\
 &= -\frac{\tan^{-1} \left(\frac{1-2x^2}{\sqrt{3}} \right)}{4\sqrt{3}} + \frac{\tan^{-1} \left(\frac{1+2x^2}{\sqrt{3}} \right)}{4\sqrt{3}} - \frac{1}{8} \log(1-x^2+x^4) + \frac{1}{8} \log(1+x^2+x^4)
 \end{aligned}$$

Mathematica [C] time = 0.05, size = 79, normalized size = 1.05

$$\frac{i \left(\sqrt{1-i\sqrt{3}} \tan^{-1} \left(\frac{1}{2} (\sqrt{3}-i)x^2 \right) - \sqrt{1+i\sqrt{3}} \tan^{-1} \left(\frac{1}{2} (\sqrt{3}+i)x^2 \right) \right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(1 + x^4 + x^8), x]

[Out] $((I/2)*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x^2)/2] - Sqrt[1 + I*Sqrt[3]]*ArcTan[((I + Sqrt[3])*x^2)/2]))/Sqrt[6]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1 + x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(1 + x^4 + x^8), x]

[Out] IntegrateAlgebraic[x/(1 + x^4 + x^8), x]

fricas [A] time = 1.50, size = 61, normalized size = 0.81

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) + \frac{1}{8} \log(x^4 + x^2 + 1) - \frac{1}{8} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8+x^4+1),x, algorithm="fricas")

[Out] $1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^2 + 1)) + 1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^2 - 1)) + 1/8*\log(x^4 + x^2 + 1) - 1/8*\log(x^4 - x^2 + 1)$

giac [A] time = 0.31, size = 61, normalized size = 0.81

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) + \frac{1}{8} \log(x^4 + x^2 + 1) - \frac{1}{8} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8+x^4+1),x, algorithm="giac")

[Out] $1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^2 + 1)) + 1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^2 - 1)) + 1/8*\log(x^4 + x^2 + 1) - 1/8*\log(x^4 - x^2 + 1)$

maple [A] time = 0.00, size = 62, normalized size = 0.83

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^4 - x^2 + 1)}{8} + \frac{\ln(x^4 + x^2 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^8+x^4+1),x)

[Out] $\frac{1}{8} \ln(x^4 + x^2 + 1) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) - \frac{1}{8} \ln(x^4 - x^2 + 1) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right)$

maxima [A] time = 2.59, size = 61, normalized size = 0.81

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) + \frac{1}{8} \log(x^4 + x^2 + 1) - \frac{1}{8} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8+x^4+1),x, algorithm="maxima")

[Out] $\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 + 1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) + \frac{1}{8} \log(x^4 + x^2 + 1) - \frac{1}{8} \log(x^4 - x^2 + 1)$

mupad [B] time = 1.28, size = 51, normalized size = 0.68

$$\operatorname{atan}\left(\frac{\sqrt{3} x^2}{2} - \frac{x^2 1i}{2}\right) \left(\frac{\sqrt{3}}{12} + \frac{1}{4} i\right) + \operatorname{atan}\left(\frac{\sqrt{3} x^2}{2} + \frac{x^2 1i}{2}\right) \left(\frac{\sqrt{3}}{12} - \frac{1}{4} i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^4 + x^8 + 1),x)

[Out] $\operatorname{atan}\left(\frac{\sqrt{3} x^2}{2} - \frac{x^2 1i}{2}\right) \left(\frac{\sqrt{3}}{12} + \frac{1}{4} i\right) + \operatorname{atan}\left(\frac{\sqrt{3} x^2}{2} + \frac{x^2 1i}{2}\right) \left(\frac{\sqrt{3}}{12} - \frac{1}{4} i\right)$

sympy [A] time = 0.20, size = 76, normalized size = 1.01

$$-\frac{\log(x^4 - x^2 + 1)}{8} + \frac{\log(x^4 + x^2 + 1)}{8} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**8+x**4+1),x)

[Out] $-\log(x^4 - x^2 + 1)/8 + \log(x^4 + x^2 + 1)/8 + \sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)/12 + \sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)/12$

$$3.276 \quad \int \frac{1}{x(1+x^4+x^8)} dx$$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8} \log(x^8 + x^4 + 1) + \log(x)$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{8} \log(x^8 + x^4 + 1) - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + x^4 + x^8)),x]

[Out] -ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 + x^4 + x^8]/8

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d._) + (e._)*(x_))/((a._) + (b._)*(x_) + (c._)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d._) + (e._)*(x_))*((a._) + (b._)*(x_) + (c._)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(1+x^4+x^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(1+x+x^2)} dx, x, x^4 \right) \\
 &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x} dx, x, x^4 \right) + \frac{1}{4} \text{Subst} \left(\int \frac{-1-x}{1+x+x^2} dx, x, x^4 \right) \\
 &= \log(x) - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^4 \right) - \frac{1}{8} \text{Subst} \left(\int \frac{1+2x}{1+x+x^2} dx, x, x^4 \right) \\
 &= \log(x) - \frac{1}{8} \log(1+x^4+x^8) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^4 \right) \\
 &= -\frac{\tan^{-1} \left(\frac{1+2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} + \log(x) - \frac{1}{8} \log(1+x^4+x^8)
 \end{aligned}$$

Mathematica [C] time = 0.08, size = 138, normalized size = 3.54

$$\frac{1}{24} \left(-\sqrt{3}(\sqrt{3}-i) \log \left(x^2 - \frac{i\sqrt{3}}{2} - \frac{1}{2} \right) - \sqrt{3}(\sqrt{3}+i) \log \left(x^2 + \frac{1}{2}i(\sqrt{3}+i) \right) - 3 \log(x^2-x+1) - 3 \log(x^2+x+1) + 24 \log(x) + 2\sqrt{3} \tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right) - 2\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 + x^4 + x^8)),x]

[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 24*Log[x] - Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2] - Sqrt[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1 + x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(1 + x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x*(1 + x^4 + x^8)), x]

fricas [A] time = 1.26, size = 32, normalized size = 0.82

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 + 1)\right) - \frac{1}{8} \log(x^8 + x^4 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8+x^4+1),x, algorithm="fricas")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + log(x)

giac [A] time = 0.38, size = 36, normalized size = 0.92

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 + 1)\right) - \frac{1}{8} \log(x^8 + x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8+x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/4*log(x^4)

maple [B] time = 0.01, size = 87, normalized size = 2.23

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} - \frac{\sqrt{3} \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12} + \ln(x) - \frac{\ln(x^2-x+1)}{8} - \frac{\ln(x^2+x+1)}{8} - \frac{\ln(x^4-x^2+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(x^8+x^4+1),x)`

[Out] $\ln(x) - \frac{1}{8} \ln(x^2+x+1) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{8} \ln(x^4-x^2+1) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2-1)\right) - \frac{1}{8} \ln(x^2-x+1) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right)$

maxima [A] time = 3.05, size = 36, normalized size = 0.92

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 + 1)\right) - \frac{1}{8} \log(x^8 + x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x^8+x^4+1),x, algorithm="maxima")`

[Out] $-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 + 1)\right) - \frac{1}{8} \log(x^8 + x^4 + 1) + \frac{1}{4} \log(x^4)$

mupad [B] time = 1.30, size = 34, normalized size = 0.87

$$\ln(x) - \frac{\ln(x^8 + x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x^4 + x^8 + 1)),x)`

[Out] $\log(x) - \log(x^4 + x^8 + 1)/8 - (\sqrt{3} \operatorname{atan}(\sqrt{3}/3 + (2\sqrt{3}x^4)/3))/12$

sympy [A] time = 0.15, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^8 + x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x**8+x**4+1),x)`

[Out] $\log(x) - \log(x^8 + x^4 + 1)/8 - \sqrt{3} \operatorname{atan}(2\sqrt{3}x^4/3 + \sqrt{3}/3)/12$

$$3.277 \quad \int \frac{1}{x^3(1+x^4+x^8)} dx$$

Optimal. Leaf size=54

$$-\frac{1}{2x^2} + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1359, 1123, 1161, 618, 204}

$$-\frac{1}{2x^2} + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1 + x^4 + x^8)),x]

[Out] -1/(2*x^2) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1123

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*x^2 + c*x^4)^(p+1))/(a*d*(m+1)), x] - Dist[1/(a*d^2*(m+1)), Int[(d*x)^(m+2)*(b*(m+2*p+3) + c*(m+4*p+5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(1+x^4+x^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(1+x^2+x^4)} dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{-1-x^2}{1+x^2+x^4} dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^2 \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^2 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^2 \right) \\
&= -\frac{1}{2x^2} + \frac{\tan^{-1} \left(\frac{1-2x^2}{\sqrt{3}} \right)}{2\sqrt{3}} - \frac{\tan^{-1} \left(\frac{1+2x^2}{\sqrt{3}} \right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 100, normalized size = 1.85

$$\frac{1}{12} \left(-\frac{6}{x^2} + i\sqrt{3} \log(2x^2 - i\sqrt{3} - 1) - i\sqrt{3} \log(2x^2 + i\sqrt{3} - 1) - 2\sqrt{3} \tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right) + 2\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] + I*Sqrt[3]*Log[-1 - I*Sqrt[3] + 2*x^2] - I*Sqrt[3]*Log[-1 + I*Sq
rt[3] + 2*x^2])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(1+x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(1 + x^4 + x^8)), x]

[Out] IntegrateAlgebraic[1/(x^3*(1 + x^4 + x^8)), x]

fricas [A] time = 1.08, size = 45, normalized size = 0.83

$$\frac{\sqrt{3}x^2 \arctan\left(\frac{1}{3}\sqrt{3}x^2\right) + \sqrt{3}x^2 \arctan\left(\frac{1}{3}\sqrt{3}(x^6 + 2x^2)\right) + 3}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8+x^4+1), x, algorithm="fricas")

[Out] -1/6*(sqrt(3)*x^2*arctan(1/3*sqrt(3)*x^2) + sqrt(3)*x^2*arctan(1/3*sqrt(3)*(x^6 + 2*x^2)) + 3)/x^2

giac [A] time = 0.38, size = 42, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^2 + 1)\right) - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^2 - 1)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8+x^4+1), x, algorithm="giac")

[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/2/x^2

maple [A] time = 0.01, size = 57, normalized size = 1.06

$$\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3} \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^8+x^4+1), x)

[Out] 1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/2/x^2-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [A] time = 2.39, size = 42, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="maxima")

[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/2/x^2

mupad [B] time = 0.04, size = 43, normalized size = 0.80

$$\frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{\sqrt{3}x^6}{3}+\frac{2\sqrt{3}x^2}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right)\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(x^4 + x^8 + 1)),x)

[Out] -(3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(1/2)*x^2)/3)))/12 - 1/(2*x^2)

sympy [A] time = 0.16, size = 53, normalized size = 0.98

$$\frac{\sqrt{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{3}x^2}{3}\right)-2\operatorname{atan}\left(\frac{\sqrt{3}x^6}{3}+\frac{2\sqrt{3}x^2}{3}\right)\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(x**8+x**4+1),x)

[Out] sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(3)*x**2/3))/12 - 1/(2*x**2)

$$3.278 \quad \int \frac{1}{x^5(1+x^4+x^8)} dx$$

Optimal. Leaf size=48

$$-\frac{1}{4x^4} - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8} \log(x^8 + x^4 + 1) - \log(x)$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{4x^4} + \frac{1}{8} \log(x^8 + x^4 + 1) - \frac{\tan^{-1}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(1 + x^4 + x^8)),x]

[Out] -1/(4*x^4) - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[x] + Log[1 + x^4 + x^8]/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)/(a + bx + cx^2)}{x} dx$; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 709

$\int \frac{(d + e)x^m}{(a + bx + cx^2)} dx$, x_Symbol
] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

$\int \frac{(d + e)x^m * (f + gx)}{(a + bx + cx^2)} dx$, x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1357

$\int (x^m * (a + c*x^{n2}) + b*x^n)^p dx$, x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(1+x^4+x^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2(1+x+x^2)} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{1}{4} \text{Subst} \left(\int \frac{-1-x}{x(1+x+x^2)} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{1}{4} \text{Subst} \left(\int \left(-\frac{1}{x} + \frac{x}{1+x+x^2} \right) dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} - \log(x) + \frac{1}{4} \text{Subst} \left(\int \frac{x}{1+x+x^2} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} - \log(x) - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^4 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1+2x}{1+x+x^2} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} - \log(x) + \frac{1}{8} \log(1+x^4+x^8) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^4 \right) \\
&= -\frac{1}{4x^4} - \frac{\tan^{-1} \left(\frac{1+2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} - \log(x) + \frac{1}{8} \log(1+x^4+x^8)
\end{aligned}$$

Mathematica [C] time = 0.10, size = 141, normalized size = 2.94

$$\frac{1}{24} \left(-\frac{6}{x^4} + \sqrt{3}(\sqrt{3}+i) \log \left(x^2 - \frac{i\sqrt{3}}{2} - \frac{1}{2} \right) + \sqrt{3}(\sqrt{3}-i) \log \left(x^2 + \frac{1}{2}i(\sqrt{3}+i) \right) + 3\log(x^2-x+1) + 3\log(x^2+x+1) - 24\log(x) + 2\sqrt{3} \tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right) - 2\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(1 + x^4 + x^8)), x]

[Out] $(-6/x^4 + 2*\text{Sqrt}[3]*\text{ArcTan}[(-1 + 2*x)/\text{Sqrt}[3]] - 2*\text{Sqrt}[3]*\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]] - 24*\text{Log}[x] + \text{Sqrt}[3]*(I + \text{Sqrt}[3])*\text{Log}[-1/2 - (I/2)*\text{Sqrt}[3] + x^2] + \text{Sqrt}[3]*(-I + \text{Sqrt}[3])* \text{Log}[(I/2)*(I + \text{Sqrt}[3]) + x^2] + 3*\text{Log}[1 - x + x^2] + 3*\text{Log}[1 + x + x^2])/24$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(1+x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(1 + x^4 + x^8)), x]

[Out] IntegrateAlgebraic[1/(x^5*(1 + x^4 + x^8)), x]

fricas [A] time = 1.08, size = 49, normalized size = 1.02

$$\frac{2\sqrt{3}x^4 \arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - 3x^4 \log(x^8+x^4+1) + 24x^4 \log(x) + 6}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="fricas")

[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 3*x^4*log(x^8 + x^4 + 1) + 24*x^4*log(x) + 6)/x^4

giac [A] time = 0.31, size = 46, normalized size = 0.96

$$-\frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) + \frac{x^4-1}{4x^4} + \frac{1}{8} \log(x^8+x^4+1) - \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/4*(x^4 - 1)/x^4 + 1/8*log(x^8 + x^4 + 1) - 1/4*log(x^4)

maple [B] time = 0.01, size = 94, normalized size = 1.96

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} - \frac{\sqrt{3} \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12} - \ln(x) + \frac{\ln(x^2-x+1)}{8} + \frac{\ln(x^2+x+1)}{8} + \frac{\ln(x^4-x^2+1)}{8} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(x^8+x^4+1),x)

[Out] -1/4/x^4-ln(x)+1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/8*ln(x^4-x^2+1)-1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))+1/8*ln(x^2-x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [A] time = 2.79, size = 41, normalized size = 0.85

$$-\frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^4+1)\right) - \frac{1}{4x^4} + \frac{1}{8} \log(x^8+x^4+1) - \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="maxima")

[Out] $-1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^4 + 1)) - 1/4/x^4 + 1/8*\log(x^8 + x^4 + 1) - 1/4*\log(x^4)$

mupad [B] time = 0.06, size = 41, normalized size = 0.85

$$\frac{\ln(x^8 + x^4 + 1)}{8} - \ln(x) - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(x^4 + x^8 + 1)),x)`

[Out] $\log(x^4 + x^8 + 1)/8 - \log(x) - (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 + (2*3^{(1/2)}*x^4)/3))/12 - 1/(4*x^4)$

sympy [A] time = 0.18, size = 48, normalized size = 1.00

$$-\log(x) + \frac{\log(x^8 + x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(x**8+x**4+1),x)`

[Out] $-\log(x) + \log(x**8 + x**4 + 1)/8 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x**4/3 + \sqrt{3}/3)/12 - 1/(4*x**4)$

$$3.279 \quad \int \frac{1}{x^7(1+x^4+x^8)} dx$$

Optimal. Leaf size=89

$$-\frac{1}{6x^6} + \frac{1}{2x^2} - \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8} \log(x^4 - x^2 + 1) - \frac{1}{8} \log(x^4 + x^2 + 1)$$

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {1359, 1123, 1281, 12, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{2x^2} - \frac{1}{6x^6} + \frac{1}{8} \log(x^4 - x^2 + 1) - \frac{1}{8} \log(x^4 + x^2 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(1 + x^4 + x^8)),x]

[Out] -1/(6*x^6) + 1/(2*x^2) - ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1123

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1127

Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4*a*c, 0] && PosQ[a*c]

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1281

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1359

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7(1+x^4+x^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4(1+x^2+x^4)} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} + \frac{1}{6} \text{Subst} \left(\int \frac{-3-3x^2}{x^2(1+x^2+x^4)} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} + \frac{1}{2x^2} - \frac{1}{6} \text{Subst} \left(\int -\frac{3x^2}{1+x^2+x^4} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} + \frac{1}{2x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{1+x^2+x^4} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} + \frac{1}{2x^2} - \frac{1}{4} \text{Subst} \left(\int \frac{1-x^2}{1+x^2+x^4} dx, x, x^2 \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1+x^2}{1+x^2+x^4} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} + \frac{1}{2x^2} + \frac{1}{8} \text{Subst} \left(\int \frac{1+2x}{-1-x-x^2} dx, x, x^2 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1-2x}{-1+x-x^2} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} + \frac{1}{2x^2} + \frac{1}{8} \log(1-x^2+x^4) - \frac{1}{8} \log(1+x^2+x^4) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} + \frac{1}{2x^2} - \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8} \log(1-x^2+x^4) - \frac{1}{8} \log(1+x^2+x^4)
\end{aligned}$$

Mathematica [C] time = 0.11, size = 142, normalized size = 1.60

$$\frac{1}{24} \left(-\frac{4}{x^6} + \frac{12}{x^2} + \sqrt{3}(\sqrt{3}-i) \log\left(x^2 - \frac{i\sqrt{3}}{2} - \frac{1}{2}\right) + \sqrt{3}(\sqrt{3}+i) \log\left(x^2 + \frac{1}{2}i(\sqrt{3}+i)\right) - 3 \log(x^2-x+1) - 3 \log(x^2+x+1) + 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] (-4/x^6 + 12/x^2 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[
(1 + 2*x)/Sqrt[3]] + Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2]
+ Sqrt[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2]
] - 3*Log[1 + x + x^2])/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7(1+x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^7*(1 + x^4 + x^8)), x]

[Out] IntegrateAlgebraic[1/(x^7*(1 + x^4 + x^8)), x]

fricas [A] time = 1.46, size = 84, normalized size = 0.94

$$\frac{2\sqrt{3}x^6 \arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + 2\sqrt{3}x^6 \arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - 3x^6 \log(x^4+x^2+1) + 3x^6 \log(x^4-x^2+1) + 12x^4 - 4}{24x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8+x^4+1), x, algorithm="fricas")

[Out] 1/24*(2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 3*x^6*log(x^4 + x^2 + 1) + 3*x^6*log(x^4 - x^2 + 1) + 12*x^4 - 4)/x^6

giac [A] time = 0.27, size = 73, normalized size = 0.82

$$\frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{3x^4-1}{6x^6} - \frac{1}{8} \log(x^4+x^2+1) + \frac{1}{8} \log(x^4-x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8+x^4+1), x, algorithm="giac")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)

maple [A] time = 0.01, size = 95, normalized size = 1.07

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^2-x+1)}{8} - \frac{\ln(x^2+x+1)}{8} + \frac{\ln(x^4-x^2+1)}{8} + \frac{1}{2x^2} - \frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(x^8+x^4+1), x)

[Out] -1/6/x^6+1/2/x^2-1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^2-x+1)+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [A] time = 2.37, size = 73, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x^2 + 1)\right) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x^2 - 1)\right) + \frac{3x^4 - 1}{6x^6} - \frac{1}{8} \log(x^4 + x^2 + 1) + \frac{1}{8} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="maxima")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)

mupad [B] time = 0.04, size = 62, normalized size = 0.70

$$\operatorname{atanh}\left(\frac{2x^2}{-1 + \sqrt{3} 1i}\right) \left(\frac{1}{4} + \frac{\sqrt{3} 1i}{12}\right) + \operatorname{atanh}\left(\frac{2x^2}{1 + \sqrt{3} 1i}\right) \left(-\frac{1}{4} + \frac{\sqrt{3} 1i}{12}\right) + \frac{x^4 - \frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(x^4 + x^8 + 1)),x)

[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) + (x^4/2 - 1/6)/x^6

sympy [A] time = 0.25, size = 88, normalized size = 0.99

$$\frac{\log(x^4 - x^2 + 1)}{8} - \frac{\log(x^4 + x^2 + 1)}{8} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{12} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{12} + \frac{3x^4 - 1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(x**8+x**4+1),x)

[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12 + (3*x**4 - 1)/(6*x**6)

$$3.280 \quad \int \frac{x^8}{1+x^4+x^8} dx$$

Optimal. Leaf size=141

$$\frac{1}{8} \log(x^2 - x + 1) - \frac{1}{8} \log(x^2 + x + 1) + \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}\left(\sqrt{\frac{1-2x}{3}}\right)$$

Rubi [A] time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1367, 1419, 1094, 634, 618, 204, 628}

$$\frac{1}{8} \log(x^2 - x + 1) - \frac{1}{8} \log(x^2 + x + 1) + \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(2x + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[x^8/(1 + x^4 + x^8), x]

[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)/(a + bx + cx^2)}{x} dx$; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1094

$\int ((a_1) + (b_1)x^2 + (c_1)x^4)^{-1} dx$; With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]

Rule 1367

$\int ((d_1)x^{m_1})((a_1) + (c_1)x^{n_2}) + (b_1)x^{n_1})^{p_1} dx$; Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]

Rule 1419

$\int ((d_1) + (e_1)x^{n_1})/((a_1) + (b_1)x^{n_1} + (c_1)x^{n_2}) dx$; With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1+x^4+x^8} dx &= x - \int \frac{1+x^4}{1+x^4+x^8} dx \\
&= x - \frac{1}{2} \int \frac{1}{1-x^2+x^4} dx - \frac{1}{2} \int \frac{1}{1+x^2+x^4} dx \\
&= x - \frac{1}{4} \int \frac{1-x}{1-x+x^2} dx - \frac{1}{4} \int \frac{1+x}{1+x+x^2} dx - \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2} dx}{4\sqrt{3}} - \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2} dx}{4\sqrt{3}} \\
&= x - \frac{1}{8} \int \frac{1}{1-x+x^2} dx + \frac{1}{8} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{8} \int \frac{1}{1+x+x^2} dx - \frac{1}{8} \int \frac{1+2x}{1+x+x^2} dx - \frac{1}{8} \\
&= x + \frac{1}{8} \log(1-x+x^2) - \frac{1}{8} \log(1+x+x^2) + \frac{\log(1-\sqrt{3}x+x^2)}{8\sqrt{3}} - \frac{\log(1+\sqrt{3}x+x^2)}{8\sqrt{3}} + \\
&= x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3}-2x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3}+2x) + \frac{1}{8} \log(1-x+x^2)
\end{aligned}$$

Mathematica [C] time = 0.28, size = 139, normalized size = 0.99

$$\frac{1}{24} \left(3 \log(x^2 - x + 1) - 3 \log(x^2 + x + 1) + 24x - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) - \frac{i \tan^{-1}\left(\frac{1}{2}(1-i\sqrt{3})x\right)}{\sqrt{-6+6i\sqrt{3}}} + \frac{i \tan^{-1}\left(\frac{1}{2}(1+i\sqrt{3})x\right)}{\sqrt{-6-6i\sqrt{3}}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^8/(1 + x^4 + x^8), x]

[Out] ((-I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + (I*ArcTan[(1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]] + (24*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1+x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(1 + x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^8/(1 + x^4 + x^8), x]

fricas [A] time = 0.95, size = 212, normalized size = 1.50

$$\frac{1}{12}\sqrt{6}\sqrt{2}\arctan\left(\frac{1}{3}\sqrt{6}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2}x+2x^2+2}+\sqrt{3}\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log(\sqrt{6}\sqrt{2}x+2x^2+2)+\frac{1}{48}\sqrt{6}\sqrt{2}\log(-\sqrt{6}\sqrt{2}x+2x^2+2)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{8}\log(x^2+x+1)+\frac{1}{8}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8+x^4+1),x, algorithm="fricas")

[Out] 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sqrt(2)*log(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)

giac [A] time = 0.39, size = 109, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log(x^2+\sqrt{3}x+1)+\frac{1}{24}\sqrt{3}\log(x^2-\sqrt{3}x+1)+x-\frac{1}{4}\arctan(2x+\sqrt{3})-\frac{1}{4}\arctan(2x-\sqrt{3})-\frac{1}{8}\log(x^2+x+1)+\frac{1}{8}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8+x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^2 - sqrt(3)*x + 1) + x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)

maple [A] time = 0.04, size = 110, normalized size = 0.78

$$x-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\arctan(2x-\sqrt{3})}{4}-\frac{\arctan(2x+\sqrt{3})}{4}+\frac{\sqrt{3}\ln(x^2-\sqrt{3}x+1)}{24}-\frac{\sqrt{3}\ln(x^2+\sqrt{3}x+1)}{24}+\frac{\ln(x^2-x+1)}{8}-\frac{\ln(x^2+x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^8+x^4+1),x)

[Out] x-1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/4*arctan(2*x-3^(1/2))-1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{2}\int\frac{1}{x^4-x^2+1}dx-\frac{1}{8}\log(x^2+x+1)+\frac{1}{8}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8+x^4+1),x, algorithm="maxima")

[Out] $-1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + x - 1/2*\integrate(1/(x^4 - x^2 + 1), x) - 1/8*\log(x^2 + x + 1) + 1/8*\log(x^2 - x + 1)$

mupad [B] time = 0.10, size = 100, normalized size = 0.71

$$x - \operatorname{atan}\left(\frac{2x}{-1 + \sqrt{3}1i}\right)\left(-\frac{1}{4} + \frac{\sqrt{3}1i}{12}\right) - \operatorname{atan}\left(\frac{2x}{1 + \sqrt{3}1i}\right)\left(\frac{1}{4} + \frac{\sqrt{3}1i}{12}\right) - \operatorname{atan}\left(\frac{x2i}{-1 + \sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12} + \frac{1}{4}i\right) - \operatorname{atan}\left(\frac{x2i}{1 + \sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12} - \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^4 + x^8 + 1),x)

[Out] $x - \operatorname{atan}((2*x)/(3^{(1/2)*1i} - 1))*((3^{(1/2)*1i})/12 - 1/4) - \operatorname{atan}((2*x)/(3^{(1/2)*1i} + 1))*((3^{(1/2)*1i})/12 + 1/4) - \operatorname{atan}((x*2i)/(3^{(1/2)*1i} - 1))*(3^{(1/2)}/12 + 1i/4) - \operatorname{atan}((x*2i)/(3^{(1/2)*1i} + 1))*(3^{(1/2)}/12 - 1i/4)$

sympy [C] time = 0.71, size = 192, normalized size = 1.36

$$x + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x - 1 - \frac{\sqrt{3}i}{3} - 9216\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\right) + \left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x - 1 - 9216\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) + \frac{\sqrt{3}i}{3}\right) + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x + 1 - \frac{\sqrt{3}i}{3} - 9216\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\right) + \left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x + 1 - 9216\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) + \frac{\sqrt{3}i}{3}\right) + \operatorname{RootSum}(2304t^4 + 48t^2 + 1, (t \mapsto t \log(-9216t^5 - 8t + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(x**8+x**4+1),x)

[Out] $x + (1/8 + \sqrt{3}I/24)*\log(x - 1 - \sqrt{3}I/3 - 9216*(1/8 + \sqrt{3}I/24)**5) + (1/8 - \sqrt{3}I/24)*\log(x - 1 - 9216*(1/8 - \sqrt{3}I/24)**5 + \sqrt{3}I/3) + (-1/8 + \sqrt{3}I/24)*\log(x + 1 - \sqrt{3}I/3 - 9216*(-1/8 + \sqrt{3}I/24)**5) + (-1/8 - \sqrt{3}I/24)*\log(x + 1 - 9216*(-1/8 - \sqrt{3}I/24)**5 + \sqrt{3}I/3) + \operatorname{RootSum}(2304*_t**4 + 48*_t**2 + 1, \operatorname{Lambda}(_t, _t*\log(-9216*_t**5 - 8*_t + x)))$

$$3.281 \quad \int \frac{x^6}{1+x^4+x^8} dx$$

Optimal. Leaf size=88

$$\frac{\log(x^2 - \sqrt{3}x + 1)}{4\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{4\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1372, 1164, 628, 1161, 618, 204}

$$\frac{\log(x^2 - \sqrt{3}x + 1)}{4\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{4\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(1 + x^4 + x^8), x]

[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2

, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1372

Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n)), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{x^6}{1+x^4+x^8} dx &= -\left(\frac{1}{2} \int \frac{1-x^2}{1-x^2+x^4} dx\right) + \frac{1}{2} \int \frac{1+x^2}{1+x^2+x^4} dx \\ &= \frac{1}{4} \int \frac{1}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1+x+x^2} dx + \frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2} dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2} dx}{4\sqrt{3}} \\ &= \frac{\log(1-\sqrt{3}x+x^2)}{4\sqrt{3}} - \frac{\log(1+\sqrt{3}x+x^2)}{4\sqrt{3}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\log(1-\sqrt{3}x+x^2)}{4\sqrt{3}} - \frac{\log(1+\sqrt{3}x+x^2)}{4\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.77

$$\frac{\log(-x^2 + \sqrt{3}x - 1) - \log(x^2 + \sqrt{3}x + 1) + 2 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(1 + x^4 + x^8), x]

[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[-1 + Sqrt[3]*x - x^2] - Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1 + x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(1 + x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^6/(1 + x^4 + x^8), x]

fricas [A] time = 1.21, size = 70, normalized size = 0.80

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (x^3 + 2x)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} x\right) + \frac{1}{12} \sqrt{3} \log\left(\frac{x^4 + 5x^2 - 2\sqrt{3}(x^3 + x) + 1}{x^4 - x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8+x^4+1), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1))

giac [A] time = 0.33, size = 66, normalized size = 0.75

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{1}{12} \sqrt{3} \log(x^2 + \sqrt{3}x + 1) + \frac{1}{12} \sqrt{3} \log(x^2 - \sqrt{3}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8+x^4+1), x, algorithm="giac")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x + 1)

maple [A] time = 0.01, size = 67, normalized size = 0.76

$$\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} \ln(x^2 - \sqrt{3}x + 1)}{12} - \frac{\sqrt{3} \ln(x^2 + \sqrt{3}x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(x^8+x^4+1),x)`

[Out] $\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{12}\sqrt{3} \ln(x^2 - \sqrt{3}x + 1) - \frac{1}{12}\sqrt{3} \ln(x^2 + \sqrt{3}x + 1) + \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2} \int \frac{x^2-1}{x^4-x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(x^8+x^4+1),x, algorithm="maxima")`

[Out] $\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2} \int \frac{x^2-1}{x^4-x^2+1} dx$

mupad [B] time = 1.31, size = 38, normalized size = 0.43

$$\frac{\sqrt{3} \left(\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right) + \operatorname{atanh}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(x^4 + x^8 + 1),x)`

[Out] $-\frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right) + \sqrt{3} \operatorname{atanh}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)}{6}$

sympy [A] time = 0.18, size = 82, normalized size = 0.93

$$\frac{\sqrt{3} \left(2 \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right) \right)}{12} + \frac{\sqrt{3} \log(x^2 - \sqrt{3}x + 1)}{12} - \frac{\sqrt{3} \log(x^2 + \sqrt{3}x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(x**8+x**4+1),x)`

[Out] $\frac{\sqrt{3} \left(2 \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right) \right)}{12} + \frac{\sqrt{3} \log(x^2 - \sqrt{3}x + 1)}{12} - \frac{\sqrt{3} \log(x^2 + \sqrt{3}x + 1)}{12}$

2

$$3.282 \quad \int \frac{x^4}{1+x^4+x^8} dx$$

Optimal. Leaf size=140

$$-\frac{1}{8} \log(x^2 - x + 1) + \frac{1}{8} \log(x^2 + x + 1) + \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x) + \frac{1}{4} \tan^{-1}(2x + \sqrt{3})$$

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{8} \log(x^2 - x + 1) + \frac{1}{8} \log(x^2 + x + 1) + \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(2x + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[x^4/(1 + x^4 + x^8), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1127

Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I


```
Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n)/2] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{1+x^4+x^8} dx &= \frac{1}{2} \int \frac{x^2}{1-x^2+x^4} dx - \frac{1}{2} \int \frac{x^2}{1+x^2+x^4} dx \\
&= -\left(\frac{1}{4} \int \frac{1-x^2}{1-x^2+x^4} dx\right) + \frac{1}{4} \int \frac{1+x^2}{1-x^2+x^4} dx + \frac{1}{4} \int \frac{1-x^2}{1+x^2+x^4} dx - \frac{1}{4} \int \frac{1+x^2}{1+x^2+x^4} dx \\
&= -\left(\frac{1}{8} \int \frac{1+2x}{-1-x-x^2} dx\right) - \frac{1}{8} \int \frac{1-2x}{-1+x-x^2} dx - \frac{1}{8} \int \frac{1}{1-x+x^2} dx - \frac{1}{8} \int \frac{1}{1+x+x^2} dx + \dots \\
&= -\frac{1}{8} \log(1-x+x^2) + \frac{1}{8} \log(1+x+x^2) + \frac{\log(1-\sqrt{3}x+x^2)}{8\sqrt{3}} - \frac{\log(1+\sqrt{3}x+x^2)}{8\sqrt{3}} + \frac{1}{4} \operatorname{S} \\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3}-2x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3}+2x) - \frac{1}{8} \log(1-x+x^2)
\end{aligned}$$

Mathematica [C] time = 0.16, size = 135, normalized size = 0.96

$$\frac{1}{24} \left(-3 \log(x^2 - x + 1) + 3 \log(x^2 + x + 1) - 2i\sqrt{-6 + 6i\sqrt{3}} \tan^{-1}\left(\frac{1}{2}(1 - i\sqrt{3})x\right) + 2i\sqrt{-6 - 6i\sqrt{3}} \tan^{-1}\left(\frac{1}{2}(1 + i\sqrt{3})x\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/(1 + x^4 + x^8), x]

[Out] ((-2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] + (2*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x + x^2] + 3*Log[1 + x + x^2])/24

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1+x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(1 + x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^4/(1 + x^4 + x^8), x]

fricas [A] time = 1.53, size = 211, normalized size = 1.51

$$\frac{1}{12}\sqrt{6}\sqrt{2}\arctan\left(\frac{1}{3}\sqrt{6}\sqrt{2}x + \frac{1}{3}\sqrt{6}\sqrt{6\sqrt{2}x+2x^2+2-\sqrt{3}}\right) - \frac{1}{12}\sqrt{6}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{2}x + \frac{1}{3}\sqrt{6}\sqrt{6\sqrt{2}x+2x^2+2+\sqrt{3}}\right) - \frac{1}{24}\sqrt{6}\sqrt{2}\log(\sqrt{6}\sqrt{2}x+2x^2+2) + \frac{1}{24}\sqrt{6}\sqrt{2}\log(-\sqrt{6}\sqrt{2}x+2x^2+2) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{8}\log(x^2+x+1) - \frac{1}{8}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8+x^4+1),x, algorithm="fricas")

[Out] $-1/12\sqrt{6}\sqrt{3}\sqrt{2}\arctan(-1/3\sqrt{6}\sqrt{3}\sqrt{2}x + 1/3\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{\sqrt{6}\sqrt{2}x + 2x^2 + 2} - \sqrt{3}) - 1/12\sqrt{6}\sqrt{3}\sqrt{2}\arctan(-1/3\sqrt{6}\sqrt{3}\sqrt{2}x + 1/3\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-\sqrt{6}\sqrt{2}x + 2x^2 + 2} + \sqrt{3}) - 1/48\sqrt{6}\sqrt{2}\log(\sqrt{6}\sqrt{2}x + 2x^2 + 2) + 1/48\sqrt{6}\sqrt{2}\log(-\sqrt{6}\sqrt{2}x + 2x^2 + 2) - 1/12\sqrt{3}\arctan(1/3\sqrt{3}(2x + 1)) - 1/12\sqrt{3}\arctan(1/3\sqrt{3}(2x - 1)) + 1/8\log(x^2 + x + 1) - 1/8\log(x^2 - x + 1)$

giac [A] time = 0.40, size = 108, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{24}\sqrt{3}\log(x^2 + \sqrt{3}x + 1) + \frac{1}{24}\sqrt{3}\log(x^2 - \sqrt{3}x + 1) + \frac{1}{4}\arctan(2x + \sqrt{3}) + \frac{1}{4}\arctan(2x - \sqrt{3}) + \frac{1}{8}\log(x^2 + x + 1) - \frac{1}{8}\log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8+x^4+1),x, algorithm="giac")

[Out] $-1/12\sqrt{3}\arctan(1/3\sqrt{3}(2x + 1)) - 1/12\sqrt{3}\arctan(1/3\sqrt{3}(2x - 1)) - 1/24\sqrt{3}\log(x^2 + \sqrt{3}x + 1) + 1/24\sqrt{3}\log(x^2 - \sqrt{3}x + 1) + 1/4\arctan(2x + \sqrt{3}) + 1/4\arctan(2x - \sqrt{3}) + 1/8\log(x^2 + x + 1) - 1/8\log(x^2 - x + 1)$

maple [A] time = 0.02, size = 109, normalized size = 0.78

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12} - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} + \frac{\arctan(2x - \sqrt{3})}{4} + \frac{\arctan(2x + \sqrt{3})}{4} + \frac{\sqrt{3}\ln(x^2 - \sqrt{3}x + 1)}{24} - \frac{\sqrt{3}\ln(x^2 + \sqrt{3}x + 1)}{24} - \frac{\ln(x^2 - x + 1)}{8} + \frac{\ln(x^2 + x + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^8+x^4+1),x)

[Out] $1/8\ln(x^2+x+1) - 1/12\sqrt{3}\arctan(1/3(2x+1)\sqrt{3}) + 1/24\sqrt{3}\ln(x^2 - \sqrt{3}x + 1) + 1/4\arctan(2x - \sqrt{3}) - 1/24\sqrt{3}\ln(x^2 + \sqrt{3}x + 1) + 1/4\arctan(2x + \sqrt{3}) - 1/8\ln(x^2 - x + 1) - 1/12\sqrt{3}\arctan(1/3(2x-1)\sqrt{3})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\int\frac{x^2}{x^4-x^2+1}dx + \frac{1}{8}\log(x^2+x+1) - \frac{1}{8}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8+x^4+1),x, algorithm="maxima")

[Out] $-1/12\sqrt{3}\arctan(1/3\sqrt{3}(2x + 1)) - 1/12\sqrt{3}\arctan(1/3\sqrt{3}(2x - 1)) + 1/2\int(x^2/(x^4 - x^2 + 1), x) + 1/8\log(x^2 + x + 1) - 1/8\log(x^2 - x + 1)$

mupad [B] time = 0.07, size = 99, normalized size = 0.71

$$-\operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}1i}\right)\left(\frac{1}{4}+\frac{\sqrt{3}1i}{12}\right)-\operatorname{atan}\left(\frac{2x}{1+\sqrt{3}1i}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}1i}{12}\right)-\operatorname{atan}\left(\frac{x2i}{-1+\sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}i\right)-\operatorname{atan}\left(\frac{x2i}{1+\sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^4 + x^8 + 1), x)

[Out] - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4)

sympy [C] time = 0.72, size = 197, normalized size = 1.41

$$\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,t\mapsto t\log(-18432t^5-4t+x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**8+x**4+1), x)

[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - sqrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sqrt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-18432*_t**5 - 4*_t + x)))

$$3.283 \quad \int \frac{x^2}{1+x^4+x^8} dx$$

Optimal. Leaf size=140

$$\frac{1}{8} \log(x^2 - x + 1) - \frac{1}{8} \log(x^2 + x + 1) - \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x) + \frac{1}{4} \tan^{-1}(2x + \sqrt{3})$$

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1373, 1094, 634, 618, 204, 628}

$$\frac{1}{8} \log(x^2 - x + 1) - \frac{1}{8} \log(x^2 + x + 1) - \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(2x + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + x^4 + x^8), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1094

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]

Rule 1373

Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n)/2] && NegQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{1+x^4+x^8} dx &= \frac{1}{2} \int \frac{1}{1-x^2+x^4} dx - \frac{1}{2} \int \frac{1}{1+x^2+x^4} dx \\ &= -\left(\frac{1}{4} \int \frac{1-x}{1-x+x^2} dx\right) - \frac{1}{4} \int \frac{1+x}{1+x+x^2} dx + \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2} dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2} dx}{4\sqrt{3}} \\ &= -\left(\frac{1}{8} \int \frac{1}{1-x+x^2} dx\right) + \frac{1}{8} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{8} \int \frac{1}{1+x+x^2} dx - \frac{1}{8} \int \frac{1+2x}{1+x+x^2} dx + \frac{1}{8} \int \frac{1}{1+x+x^2} dx \\ &= \frac{1}{8} \log(1-x+x^2) - \frac{1}{8} \log(1+x+x^2) - \frac{\log(1-\sqrt{3}x+x^2)}{8\sqrt{3}} + \frac{\log(1+\sqrt{3}x+x^2)}{8\sqrt{3}} + \frac{1}{4} \operatorname{Su} \\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3}-2x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3}+2x) + \frac{1}{8} \log(1-x+x^2) \end{aligned}$$

Mathematica [C] time = 0.15, size = 135, normalized size = 0.96

$$\frac{1}{48} \left(6 \log(x^2 - x + 1) - 6 \log(x^2 + x + 1) + 4i\sqrt{-6 - 6i\sqrt{3}} \tan^{-1}\left(\frac{1}{2}(1 - i\sqrt{3})x\right) - 4i\sqrt{-6 + 6i\sqrt{3}} \tan^{-1}\left(\frac{1}{2}(1 + i\sqrt{3})x\right) - 4\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 4\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/(1 + x^4 + x^8),x]

[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 4*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 6*Log[1 - x + x^2] - 6*Log[1 + x + x^2])/48

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1 + x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1 + x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^2/(1 + x^4 + x^8), x]

fricas [A] time = 1.45, size = 211, normalized size = 1.51

$$\frac{1}{12} \sqrt{6} \sqrt{2} \arctan\left(\frac{1}{3} \sqrt{6} \sqrt{2} x + \frac{1}{3} \sqrt{6} \sqrt{2} \sqrt{\sqrt{6} \sqrt{2} x + 2x^2 + 2} - \sqrt{3}\right) - \frac{1}{12} \sqrt{6} \sqrt{2} \arctan\left(\frac{1}{3} \sqrt{6} \sqrt{2} \sqrt{-\sqrt{6} \sqrt{2} x + 2x^2 + 2} + \sqrt{3}\right) + \frac{1}{48} \sqrt{6} \sqrt{2} \log(\sqrt{6} \sqrt{2} x + 2x^2 + 2) - \frac{1}{48} \sqrt{6} \sqrt{2} \log(-\sqrt{6} \sqrt{2} x + 2x^2 + 2) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{1}{8} \log(x^2 + x + 1) + \frac{1}{8} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8+x^4+1),x, algorithm="fricas")

[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sqrt(2)*log(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/48*sqrt(6)*sqrt(2)*log(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)

giac [A] time = 0.34, size = 108, normalized size = 0.77

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{24} \sqrt{3} \log(x^2 + \sqrt{3}x + 1) - \frac{1}{24} \sqrt{3} \log(x^2 - \sqrt{3}x + 1) + \frac{1}{4} \arctan(2x + \sqrt{3}) + \frac{1}{4} \arctan(2x - \sqrt{3}) - \frac{1}{8} \log(x^2 + x + 1) + \frac{1}{8} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8+x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)

maple [A] time = 0.01, size = 109, normalized size = 0.78

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} + \frac{\arctan(2x-\sqrt{3})}{4} + \frac{\arctan(2x+\sqrt{3})}{4} - \frac{\sqrt{3} \ln(x^2-\sqrt{3}x+1)}{24} + \frac{\sqrt{3} \ln(x^2+\sqrt{3}x+1)}{24} + \frac{\ln(x^2-x+1)}{8} - \frac{\ln(x^2+x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^8+x^4+1), x)

[Out] $-1/8*\ln(x^2+x+1)-1/12*3^{(1/2)}*\arctan(1/3*(2*x+1)*3^{(1/2)})-1/24*3^{(1/2)}*\ln(x^2-3^{(1/2)}*x+1)+1/4*\arctan(2*x-3^{(1/2)})+1/24*3^{(1/2)}*\ln(x^2+3^{(1/2)}*x+1)+1/4*\arctan(2*x+3^{(1/2)})+1/8*\ln(x^2-x+1)-1/12*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2} \int \frac{1}{x^4-x^2+1} dx - \frac{1}{8} \log(x^2+x+1) + \frac{1}{8} \log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8+x^4+1), x, algorithm="maxima")

[Out] $-1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + 1/2*\integrate(1/(x^4 - x^2 + 1), x) - 1/8*\log(x^2 + x + 1) + 1/8*\log(x^2 - x + 1)$

mupad [B] time = 1.31, size = 97, normalized size = 0.69

$$\operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}1i}\right)\left(\frac{1}{4}+\frac{\sqrt{3}1i}{12}\right) + \operatorname{atan}\left(\frac{2x}{1+\sqrt{3}1i}\right)\left(\frac{1}{4}+\frac{\sqrt{3}1i}{12}\right) - \operatorname{atan}\left(\frac{x2i}{-1+\sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}i\right) - \operatorname{atan}\left(\frac{x2i}{1+\sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^4 + x^8 + 1), x)

[Out] $\operatorname{atan}((2*x)/(3^{(1/2)}*1i - 1))*((3^{(1/2)}*1i)/12 - 1/4) + \operatorname{atan}((2*x)/(3^{(1/2)}*1i + 1))*((3^{(1/2)}*1i)/12 + 1/4) - \operatorname{atan}((x*2i)/(3^{(1/2)}*1i - 1))*(3^{(1/2)}/12 + 1i/4) - \operatorname{atan}((x*2i)/(3^{(1/2)}*1i + 1))*(3^{(1/2)}/12 - 1i/4)$

sympy [C] time = 0.71, size = 214, normalized size = 1.53

$$\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x + 442368\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)^3 - 192\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)^2\right) + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x - 192\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)^3 + 442368\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)^2\right) - \left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x + 442368\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)^3 - 192\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)^2\right) + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x - 192\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)^3 + 442368\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)^2\right) + \operatorname{RootSum}(2304t^4 + 48t^2 + 1, (t \mapsto t \log(442368t^3 - 192t^2 + 1)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**8+x**4+1), x)


```
[Out] (-1/8 - sqrt(3)*I/24)*log(x + 442368*(-1/8 - sqrt(3)*I/24)**7 - 192*(-1/8 -
sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 192*(-1/8 + sqrt(3)*I/24
)**3 + 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x + 4423
68*(1/8 - sqrt(3)*I/24)**7 - 192*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 192*(1/8 + sqrt(3)*I/24)**3 + 442368*(1/8 + sqrt(3)*I/24)**7)
+ RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(442368*_t**7 - 192*_
_t**3 + x)))
```

$$3.284 \quad \int \frac{1}{1+x^4+x^8} dx$$

Optimal. Leaf size=88

$$-\frac{\log(x^2 - \sqrt{3}x + 1)}{4\sqrt{3}} + \frac{\log(x^2 + \sqrt{3}x + 1)}{4\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1346, 1164, 628, 1161, 618, 204}

$$-\frac{\log(x^2 - \sqrt{3}x + 1)}{4\sqrt{3}} + \frac{\log(x^2 + \sqrt{3}x + 1)}{4\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4 + x^8)^(-1), x]

[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2

, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1346

Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(n_ - 1), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+x^4+x^8} dx &= \frac{1}{2} \int \frac{1-x^2}{1-x^2+x^4} dx + \frac{1}{2} \int \frac{1+x^2}{1+x^2+x^4} dx \\ &= \frac{1}{4} \int \frac{1}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1+x+x^2} dx - \frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2} dx}{4\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2} dx}{4\sqrt{3}} \\ &= -\frac{\log(1-\sqrt{3}x+x^2)}{4\sqrt{3}} + \frac{\log(1+\sqrt{3}x+x^2)}{4\sqrt{3}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log(1-\sqrt{3}x+x^2)}{4\sqrt{3}} + \frac{\log(1+\sqrt{3}x+x^2)}{4\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.77

$$\frac{-\log(-x^2 + \sqrt{3}x - 1) + \log(x^2 + \sqrt{3}x + 1) + 2 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4 + x^8)^(-1), x]

[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[-1 + Sqrt[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1 + x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^4 + x^8)^(-1), x]

[Out] IntegrateAlgebraic[(1 + x^4 + x^8)^(-1), x]

fricas [A] time = 1.12, size = 70, normalized size = 0.80

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (x^3 + 2x)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} x\right) + \frac{1}{12} \sqrt{3} \log\left(\frac{x^4 + 5x^2 + 2\sqrt{3}(x^3 + x) + 1}{x^4 - x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+x^4+1), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1))

giac [A] time = 0.39, size = 66, normalized size = 0.75

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{12} \sqrt{3} \log(x^2 + \sqrt{3}x + 1) - \frac{1}{12} \sqrt{3} \log(x^2 - \sqrt{3}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+x^4+1), x, algorithm="giac")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x + 1)

maple [A] time = 0.01, size = 67, normalized size = 0.76

$$\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3} \ln(x^2 - \sqrt{3}x + 1)}{12} + \frac{\sqrt{3} \ln(x^2 + \sqrt{3}x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8+x^4+1),x)

[Out] $\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3} \ln(x^2 - \sqrt{3}x + 1) + \frac{1}{12}\sqrt{3} \ln(x^2 + \sqrt{3}x + 1) + \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{2} \int \frac{x^2-1}{x^4-x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+x^4+1),x, algorithm="maxima")

[Out] $\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{2} \int \frac{x^2-1}{x^4-x^2+1} dx$

mupad [B] time = 0.04, size = 40, normalized size = 0.45

$$\frac{\sqrt{3} \left(\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right) - \operatorname{atanh}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right) \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4 + x^8 + 1),x)

[Out] $-\frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right) - \operatorname{atanh}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)}{6}$

sympy [A] time = 0.18, size = 82, normalized size = 0.93

$$\frac{\sqrt{3} \left(2 \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right) \right)}{12} - \frac{\sqrt{3} \log(x^2 - \sqrt{3}x + 1)}{12} + \frac{\sqrt{3} \log(x^2 + \sqrt{3}x + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**8+x**4+1),x)

[Out] $\frac{\sqrt{3} \left(2 \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right) \right)}{12} - \frac{\sqrt{3} \log(x^2 - \sqrt{3}x + 1)}{12} + \frac{\sqrt{3} \log(x^2 + \sqrt{3}x + 1)}{12}$

2

$$3.285 \quad \int \frac{1}{x^2(1+x^4+x^8)} dx$$

Optimal. Leaf size=145

$$-\frac{1}{8} \log(x^2 - x + 1) + \frac{1}{8} \log(x^2 + x + 1) - \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{1}{x} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1368, 1506, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{8} \log(x^2 - x + 1) + \frac{1}{8} \log(x^2 + x + 1) - \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{1}{x} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(2x + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 + x^4 + x^8)),x]

[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1127

Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I

Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4*a*c, 0] && PosQ[a*c]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 1164

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1368

Int[((d_)*(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1506

Int[((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^(n_)))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q - b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r + (c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x] /; !LtQ[2*c*q - b*c, 0] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c, 0] && IntegerQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(1+x^4+x^8)} dx &= -\frac{1}{x} + \int \frac{x^2(-1-x^4)}{1+x^4+x^8} dx \\
&= -\frac{1}{x} - \frac{1}{2} \int \frac{x^2}{1-x^2+x^4} dx - \frac{1}{2} \int \frac{x^2}{1+x^2+x^4} dx \\
&= -\frac{1}{x} + \frac{1}{4} \int \frac{1-x^2}{1-x^2+x^4} dx - \frac{1}{4} \int \frac{1+x^2}{1-x^2+x^4} dx + \frac{1}{4} \int \frac{1-x^2}{1+x^2+x^4} dx - \frac{1}{4} \int \frac{1+x^2}{1+x^2+x^4} dx \\
&= -\frac{1}{x} - \frac{1}{8} \int \frac{1+2x}{-1-x-x^2} dx - \frac{1}{8} \int \frac{1-2x}{-1+x-x^2} dx - \frac{1}{8} \int \frac{1}{1-x+x^2} dx - \frac{1}{8} \int \frac{1}{1+x+x^2} dx \\
&= -\frac{1}{x} - \frac{1}{8} \log(1-x+x^2) + \frac{1}{8} \log(1+x+x^2) - \frac{\log(1-\sqrt{3}x+x^2)}{8\sqrt{3}} + \frac{\log(1+\sqrt{3}x+x^2)}{8\sqrt{3}} \\
&= -\frac{1}{x} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3}-2x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3}+2x) - \frac{1}{8} \log(1-x+x^2) + \frac{1}{8} \log(1+x+x^2)
\end{aligned}$$

Mathematica [C] time = 0.21, size = 140, normalized size = 0.97

$$\frac{1}{24} \left(-3 \log(x^2 - x + 1) + 3 \log(x^2 + x + 1) - \frac{24}{x} + 2i\sqrt{-6 + 6i\sqrt{3}} \tan^{-1}\left(\frac{1}{2}(1 - i\sqrt{3})x\right) - 2i\sqrt{-6 - 6i\sqrt{3}} \tan^{-1}\left(\frac{1}{2}(1 + i\sqrt{3})x\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^2*(1 + x^4 + x^8)),x]

[Out] $(-24/x + (2*I)*\text{Sqrt}[-6 + (6*I)*\text{Sqrt}[3]]*\text{ArcTan}[\frac{(1 - I*\text{Sqrt}[3])*x}{2}] - (2*I)*\text{Sqrt}[-6 - (6*I)*\text{Sqrt}[3]]*\text{ArcTan}[\frac{(1 + I*\text{Sqrt}[3])*x}{2}] - 2*\text{Sqrt}[3]*\text{ArcTan}[\frac{-1 + 2*x}{\text{Sqrt}[3]}] - 2*\text{Sqrt}[3]*\text{ArcTan}[\frac{1 + 2*x}{\text{Sqrt}[3]}] - 3*\text{Log}[1 - x + x^2] + 3*\text{Log}[1 + x + x^2])/24$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1+x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(1 + x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^2*(1 + x^4 + x^8)), x]

fricas [A] time = 1.32, size = 224, normalized size = 1.54

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{2x^2+2-\sqrt{3}}\right) + 4\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{2x^2+2+\sqrt{3}}\right) + \sqrt{6}\sqrt{2}\log(\sqrt{6}\sqrt{2}x+2x^2+2) - \sqrt{6}\sqrt{2}\log(-\sqrt{6}\sqrt{2}x+2x^2+2) - 4\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}(2x+1)\right) - 4\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}(2x-1)\right) + 6x\log(x^2+x+1) - 6x\log(x^2-x+1) - 48}{48x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{48} \cdot (4 \cdot \sqrt{6} \cdot \sqrt{3} \cdot \sqrt{2} \cdot x \cdot \arctan(-\frac{1}{3} \sqrt{6} \sqrt{3} \sqrt{2} \cdot x + \frac{1}{3} \sqrt{6} \sqrt{3} \sqrt{2} \cdot x + 2 \cdot x^2 + 2) - \sqrt{3}) + 4 \cdot \sqrt{6} \sqrt{3} \sqrt{2} \cdot x \cdot \arctan(-\frac{1}{3} \sqrt{6} \sqrt{3} \sqrt{2} \cdot x + \frac{1}{3} \sqrt{6} \sqrt{3} \sqrt{2} \cdot x + 2 \cdot x^2 + 2) + \sqrt{3}) + \sqrt{6} \sqrt{2} \cdot x \cdot \log(\sqrt{6} \sqrt{2} \cdot x + 2 \cdot x^2 + 2) - \sqrt{6} \sqrt{2} \cdot x \cdot \log(-\sqrt{6} \sqrt{2} \cdot x + 2 \cdot x^2 + 2) - 4 \cdot \sqrt{3} \cdot x \cdot \arctan(\frac{1}{3} \sqrt{3} \cdot (2 \cdot x + 1)) - 4 \cdot \sqrt{3} \cdot x \cdot \arctan(\frac{1}{3} \sqrt{3} \cdot (2 \cdot x - 1)) + 6 \cdot x \cdot \log(x^2 + x + 1) - 6 \cdot x \cdot \log(x^2 - x + 1) - 48) / x$

giac [A] time = 0.33, size = 113, normalized size = 0.78

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + \frac{1}{24} \sqrt{3} \log(x^2 + \sqrt{3}x + 1) - \frac{1}{24} \sqrt{3} \log(x^2 - \sqrt{3}x + 1) - \frac{1}{x} - \frac{1}{4} \arctan(2x + \sqrt{3}) - \frac{1}{4} \arctan(2x - \sqrt{3}) + \frac{1}{8} \log(x^2 + x + 1) - \frac{1}{8} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="giac")

[Out] $-1/12 \cdot \sqrt{3} \cdot \arctan(1/3 \sqrt{3} \cdot (2 \cdot x + 1)) - 1/12 \cdot \sqrt{3} \cdot \arctan(1/3 \sqrt{3} \cdot (2 \cdot x - 1)) + 1/24 \cdot \sqrt{3} \cdot \log(x^2 + \sqrt{3} \cdot x + 1) - 1/24 \cdot \sqrt{3} \cdot \log(x^2 - \sqrt{3} \cdot x + 1) - 1/x - 1/4 \cdot \arctan(2 \cdot x + \sqrt{3}) - 1/4 \cdot \arctan(2 \cdot x - \sqrt{3}) + 1/8 \cdot \log(x^2 + x + 1) - 1/8 \cdot \log(x^2 - x + 1)$

maple [A] time = 0.01, size = 114, normalized size = 0.79

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} - \frac{\arctan(2x - \sqrt{3})}{4} - \frac{\arctan(2x + \sqrt{3})}{4} - \frac{\sqrt{3} \ln(x^2 - \sqrt{3}x + 1)}{24} + \frac{\sqrt{3} \ln(x^2 + \sqrt{3}x + 1)}{24} - \frac{\ln(x^2 - x + 1)}{8} + \frac{\ln(x^2 + x + 1)}{8} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(x^8+x^4+1),x)

[Out] $\frac{1}{8} \ln(x^2+x+1) - \frac{1}{12} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot (2 \cdot x + 1) \cdot 3^{(1/2)}) - \frac{1}{x} - \frac{1}{24} \cdot 3^{(1/2)} \cdot \ln(x^2 - 3^{(1/2)} \cdot x + 1) - \frac{1}{4} \cdot \arctan(2 \cdot x - 3^{(1/2)}) + \frac{1}{24} \cdot 3^{(1/2)} \cdot \ln(x^2 + 3^{(1/2)} \cdot x + 1) - \frac{1}{4} \cdot \arctan(2 \cdot x + 3^{(1/2)}) - \frac{1}{8} \ln(x^2 - x + 1) - \frac{1}{12} \cdot 3^{(1/2)} \cdot \arctan(1/3 \cdot (2 \cdot x - 1) \cdot 3^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) - \frac{1}{x} - \frac{1}{2} \int \frac{x^2}{x^4 - x^2 + 1} dx + \frac{1}{8} \log(x^2 + x + 1) - \frac{1}{8} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="maxima")

[Out] $-1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 1/x - 1/2*\integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*\log(x^2 + x + 1) - 1/8*\log(x^2 - x + 1)$

mupad [B] time = 0.05, size = 102, normalized size = 0.70

$$\operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}1i}\right)\left(\frac{1}{4}+\frac{\sqrt{3}1i}{12}\right)+\operatorname{atan}\left(\frac{2x}{1+\sqrt{3}1i}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}1i}{12}\right)-\operatorname{atan}\left(\frac{x2i}{-1+\sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}i\right)-\operatorname{atan}\left(\frac{x2i}{1+\sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}i\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(1/(x^2*(x^4 + x^8 + 1)), x)$

[Out] $\operatorname{atan}((2*x)/(3^{(1/2)*1i} - 1))*((3^{(1/2)*1i})/12 + 1/4) + \operatorname{atan}((2*x)/(3^{(1/2)*1i} + 1))*((3^{(1/2)*1i})/12 - 1/4) - \operatorname{atan}((x*2i)/(3^{(1/2)*1i} - 1))*(3^{(1/2)}/12 - 1i/4) - \operatorname{atan}((x*2i)/(3^{(1/2)*1i} + 1))*(3^{(1/2)}/12 + 1i/4) - 1/x$

sympy [C] time = 0.74, size = 218, normalized size = 1.50

$$\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x - 442368\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) - 384\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)\right) + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x - 384\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) - 442368\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\right) + \left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x - 442368\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) - 384\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)\right) + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x - 384\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) - 442368\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right)\right) + \operatorname{RootSum}(2304*t^4 + 48*t^2 + 1, \operatorname{Lambda}(t, t*\log(-442368*t^7 - 384*t^3 + x))) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(1/x**2/(x**8+x**4+1), x)$

[Out] $(-1/8 - \sqrt{3}*I/24)*\log(x - 442368*(-1/8 - \sqrt{3}*I/24)**7 - 384*(-1/8 - \sqrt{3}*I/24)**3) + (-1/8 + \sqrt{3}*I/24)*\log(x - 384*(-1/8 + \sqrt{3}*I/24)**3 - 442368*(-1/8 + \sqrt{3}*I/24)**7) + (1/8 - \sqrt{3}*I/24)*\log(x - 442368*(1/8 - \sqrt{3}*I/24)**7 - 384*(1/8 - \sqrt{3}*I/24)**3) + (1/8 + \sqrt{3}*I/24)*\log(x - 384*(1/8 + \sqrt{3}*I/24)**3 - 442368*(1/8 + \sqrt{3}*I/24)**7) + \operatorname{RootSum}(2304*_t**4 + 48*_t**2 + 1, \operatorname{Lambda}(t, t*\log(-442368*_t**7 - 384*_t**3 + x))) - 1/x$

$$3.286 \quad \int \frac{1}{x^4(1+x^4+x^8)} dx$$

Optimal. Leaf size=147

$$-\frac{1}{3x^3} + \frac{1}{8} \log(x^2 - x + 1) - \frac{1}{8} \log(x^2 + x + 1) + \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)$$

Rubi [A] time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1368, 1419, 1094, 634, 618, 204, 628}

$$-\frac{1}{3x^3} + \frac{1}{8} \log(x^2 - x + 1) - \frac{1}{8} \log(x^2 + x + 1) + \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(2x + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 + x^4 + x^8)),x]

[Out] -1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)(a + bx + cx^2)}{(a + bx + cx^2)^2}, x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1094

$\text{Int}[(a_ + (b_ \cdot)(x_)^2 + (c_ \cdot)(x_)^4)^{-1}, x_Symbol] := \text{With}[\{q = \text{Rt}[a/c, 2]\}, \text{With}[\{r = \text{Rt}[2q - b/c, 2]\}, \text{Dist}[1/(2cq*r), \text{Int}[(r - x)/(q - r*x + x^2), x], x] + \text{Dist}[1/(2cq*r), \text{Int}[(r + x)/(q + r*x + x^2), x], x]]] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]

Rule 1368

$\text{Int}[(d_ \cdot)(x_)^m * ((a_ + (c_ \cdot)(x_)^{n2_}) + (b_ \cdot)(x_)^{n_})^{p_}, x_Symbol] := \text{Simp}[(d*x)^{m+1} * (a + b*x^n + c*x^{2n})^{p+1} / (a*d*(m+1)), x] - \text{Dist}[1/(a*d^n*(m+1)), \text{Int}[(d*x)^{m+n} * (b*(m+n*(p+1)+1) + c*(m+2*n*(p+1)+1)*x^n) * (a + b*x^n + c*x^{2n})^p, x], x] /;$ FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1419

$\text{Int}[(d_ + (e_ \cdot)(x_)^{n_}) / ((a_ + (b_ \cdot)(x_)^{n_}) + (c_ \cdot)(x_)^{n2_}), x_Symbol] := \text{With}[\{q = \text{Rt}[(2*d)/e - b/c, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x^{n/2} + x^n, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x^{n/2} + x^n, x], x], x]] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(1+x^4+x^8)} dx &= -\frac{1}{3x^3} + \frac{1}{3} \int \frac{-3-3x^4}{1+x^4+x^8} dx \\
&= -\frac{1}{3x^3} - \frac{1}{2} \int \frac{1}{1-x^2+x^4} dx - \frac{1}{2} \int \frac{1}{1+x^2+x^4} dx \\
&= -\frac{1}{3x^3} - \frac{1}{4} \int \frac{1-x}{1-x+x^2} dx - \frac{1}{4} \int \frac{1+x}{1+x+x^2} dx - \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2} dx}{4\sqrt{3}} - \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2} dx}{4\sqrt{3}} \\
&= -\frac{1}{3x^3} - \frac{1}{8} \int \frac{1}{1-x+x^2} dx + \frac{1}{8} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{8} \int \frac{1}{1+x+x^2} dx - \frac{1}{8} \int \frac{1+2x}{1+x+x^2} dx \\
&= -\frac{1}{3x^3} + \frac{1}{8} \log(1-x+x^2) - \frac{1}{8} \log(1+x+x^2) + \frac{\log(1-\sqrt{3}x+x^2)}{8\sqrt{3}} - \frac{\log(1+\sqrt{3}x+x^2)}{8\sqrt{3}} \\
&= -\frac{1}{3x^3} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3}-2x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3}+2x) + \frac{1}{8} \log\left(\frac{1-x+x^2}{1+x+x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.29, size = 148, normalized size = 1.01

$$\frac{1}{24} \left(-\frac{8}{x^3} + 3 \log(x^2-x+1) - 3 \log(x^2+x+1) - \frac{4i \tan^{-1}\left(\frac{1-i\sqrt{3}}{2}x\right)}{\sqrt{\frac{1}{6}i(\sqrt{3}+i)}} + \frac{4i \tan^{-1}\left(\frac{1+i\sqrt{3}}{2}x\right)}{\sqrt{-\frac{1}{6}i(\sqrt{3}-i)}} - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^4*(1 + x^4 + x^8)),x]

[Out] $(-8/x^3 - ((4*I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[(I/6)*(I + Sqrt[3])] + ((4*I)*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[(-1/6*I)*(-I + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(1+x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(1 + x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^4*(1 + x^4 + x^8)), x]

fricas [B] time = 1.23, size = 240, normalized size = 1.63

$$\frac{4\sqrt{6}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x+1}+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{6\sqrt{2x+2}-\sqrt{3}}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x+1}+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{-\sqrt{6}\sqrt{2x+2}+\sqrt{3}}\right)-\sqrt{6}\sqrt{3}\log(\sqrt{6}\sqrt{2x+2})+\sqrt{6}\sqrt{3}\log(-\sqrt{6}\sqrt{2x+2})-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6x^3\log(x^2+x+1)+6x^3\log(x^2-x+1)-16}{48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="fricas")

[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 4*sqrt(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)*sqrt(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(3)) - sqrt(6)*sqrt(2)*x^3*log(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) + sqrt(6)*sqrt(2)*x^3*log(-sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - 4*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x + 1)) - 4*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^3*log(x^2 + x + 1) + 6*x^3*log(x^2 - x + 1) - 16)/x^3

giac [A] time = 0.38, size = 113, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log(x^2+\sqrt{3}x+1)+\frac{1}{24}\sqrt{3}\log(x^2-\sqrt{3}x+1)-\frac{1}{3x^3}-\frac{1}{4}\arctan(2x+\sqrt{3})-\frac{1}{4}\arctan(2x-\sqrt{3})-\frac{1}{8}\log(x^2+x+1)+\frac{1}{8}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^2 - sqrt(3)*x + 1) - 1/3/x^3 - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)

maple [A] time = 0.01, size = 114, normalized size = 0.78

$$\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\arctan(2x-\sqrt{3})}{4}-\frac{\arctan(2x+\sqrt{3})}{4}+\frac{\sqrt{3}\ln(x^2-\sqrt{3}x+1)}{24}-\frac{\sqrt{3}\ln(x^2+\sqrt{3}x+1)}{24}+\frac{\ln(x^2-x+1)}{8}-\frac{\ln(x^2+x+1)}{8}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(x^8+x^4+1),x)

[Out] -1/8*ln(x^2+x+1)-1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/3/x^3+1/24*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/4*arctan(2*x*3^(1/2))-1/24*3^(1/2)*ln(x^2+3^(1/2)*x+1)-1/4*arctan(2*x*3^(1/2))+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{3x^3}-\frac{1}{2}\int\frac{1}{x^4-x^2+1}dx-\frac{1}{8}\log(x^2+x+1)+\frac{1}{8}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="maxima")

[Out] $-1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(3)*(2*x - 1)) - 1/3/x^3 - 1/2*\integrate(1/(x^4 - x^2 + 1), x) - 1/8*\log(x^2 + x + 1) + 1/8*\log(x^2 - x + 1)$

mupad [B] time = 0.03, size = 104, normalized size = 0.71

$$-\operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}1i}\right)\left(\frac{1}{4}+\frac{\sqrt{3}1i}{12}\right)-\operatorname{atan}\left(\frac{2x}{1+\sqrt{3}1i}\right)\left(\frac{1}{4}+\frac{\sqrt{3}1i}{12}\right)-\operatorname{atan}\left(\frac{x2i}{-1+\sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}i\right)-\operatorname{atan}\left(\frac{x2i}{1+\sqrt{3}1i}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}i\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(x^4 + x^8 + 1)),x)

[Out] $-\operatorname{atan}\left(\frac{2*x}{3^{1/2}*1i-1}\right)*\left(\frac{3^{1/2}*1i}{12}-\frac{1}{4}\right)-\operatorname{atan}\left(\frac{2*x}{3^{1/2}*1i+1}\right)*\left(\frac{3^{1/2}*1i}{12}+\frac{1}{4}\right)-\operatorname{atan}\left(\frac{x*2i}{3^{1/2}*1i-1}\right)*\left(\frac{3^{1/2}}{12}+\frac{1i}{4}\right)-\operatorname{atan}\left(\frac{x*2i}{3^{1/2}*1i+1}\right)*\left(\frac{3^{1/2}}{12}-\frac{1i}{4}\right)-\frac{1}{3*x^3}$

sympy [C] time = 0.75, size = 197, normalized size = 1.34

$$\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,(t\mapsto t\log(-9216t^5-8t+x))\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(x**8+x**4+1),x)

[Out] $(1/8 + \sqrt{3}*I/24)*\log(x - 1 - \sqrt{3}*I/3 - 9216*(1/8 + \sqrt{3}*I/24)**5) + (1/8 - \sqrt{3}*I/24)*\log(x - 1 - 9216*(1/8 - \sqrt{3}*I/24)**5 + \sqrt{3}*I/3) + (-1/8 + \sqrt{3}*I/24)*\log(x + 1 - \sqrt{3}*I/3 - 9216*(-1/8 + \sqrt{3}*I/24)**5) + (-1/8 - \sqrt{3}*I/24)*\log(x + 1 - 9216*(-1/8 - \sqrt{3}*I/24)**5 + \sqrt{3}*I/3) + \operatorname{RootSum}(2304*_t**4 + 48*_t**2 + 1, \operatorname{Lambda}(_t, _t*\log(-9216*_t**5 - 8*_t + x))) - 1/(3*x**3)$

$$3.287 \quad \int \frac{1}{x^6(1+x^4+x^8)} dx$$

Optimal. Leaf size=98

$$-\frac{1}{5x^5} + \frac{\log(x^2 - \sqrt{3}x + 1)}{4\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{4\sqrt{3}} + \frac{1}{x} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1368, 1504, 12, 1372, 1164, 628, 1161, 618, 204}

$$-\frac{1}{5x^5} + \frac{\log(x^2 - \sqrt{3}x + 1)}{4\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{4\sqrt{3}} + \frac{1}{x} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(1 + x^4 + x^8)),x]

[Out] -1/(5*x^5) + x^(-1) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 1164

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1368

Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n*(a + b*x^n + c*x^(2*n)))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1372

Int[(x_)^(m_)/((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)], x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]

Rule 1504

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_)) + (c_)*(x_)^(n2_)]^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m + n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte

gerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6(1+x^4+x^8)} dx &= -\frac{1}{5x^5} + \frac{1}{5} \int \frac{-5-5x^4}{x^2(1+x^4+x^8)} dx \\
&= -\frac{1}{5x^5} + \frac{1}{x} - \frac{1}{5} \int \frac{5x^6}{1+x^4+x^8} dx \\
&= -\frac{1}{5x^5} + \frac{1}{x} + \int \frac{x^6}{1+x^4+x^8} dx \\
&= -\frac{1}{5x^5} + \frac{1}{x} - \frac{1}{2} \int \frac{1-x^2}{1-x^2+x^4} dx + \frac{1}{2} \int \frac{1+x^2}{1+x^2+x^4} dx \\
&= -\frac{1}{5x^5} + \frac{1}{x} + \frac{1}{4} \int \frac{1}{1-x+x^2} dx + \frac{1}{4} \int \frac{1}{1+x+x^2} dx + \frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2} dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2} dx}{4\sqrt{3}} \\
&= -\frac{1}{5x^5} + \frac{1}{x} + \frac{\log(1-\sqrt{3}x+x^2)}{4\sqrt{3}} - \frac{\log(1+\sqrt{3}x+x^2)}{4\sqrt{3}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1\right) \\
&= -\frac{1}{5x^5} + \frac{1}{x} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\log(1-\sqrt{3}x+x^2)}{4\sqrt{3}} - \frac{\log(1+\sqrt{3}x+x^2)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 0.97

$$\frac{1}{60} \left(-\frac{12}{x^5} + 5\sqrt{3} \log(-x^2 + \sqrt{3}x - 1) - 5\sqrt{3} \log(x^2 + \sqrt{3}x + 1) + \frac{60}{x} + 10\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 10\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(1 + x^4 + x^8)),x]

```
[Out] (-12/x^5 + 60/x + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt[3]*ArcTan
[(1 + 2*x)/Sqrt[3]] + 5*Sqrt[3]*Log[-1 + Sqrt[3]*x - x^2] - 5*Sqrt[3]*Log[1
+ Sqrt[3]*x + x^2])/60
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6(1+x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^6*(1 + x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^6*(1 + x^4 + x^8)), x]

fricas [A] time = 1.20, size = 90, normalized size = 0.92

$$\frac{10\sqrt{3}x^5 \arctan\left(\frac{1}{3}\sqrt{3}(x^3+2x)\right) + 10\sqrt{3}x^5 \arctan\left(\frac{1}{3}\sqrt{3}x\right) + 5\sqrt{3}x^5 \log\left(\frac{x^4+5x^2-2\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right) + 60x^4 - 12}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="fricas")

[Out] 1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*x) + 5*sqrt(3)*x^5*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1)) + 60*x^4 - 12)/x^5

giac [A] time = 0.40, size = 100, normalized size = 1.02

$$\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{24}\sqrt{3} \log(x^2 + \sqrt{3}x+1) + \frac{1}{24}\sqrt{3} \log(x^2 - \sqrt{3}x+1) + \frac{5x^4-1}{5x^5} + \frac{1}{4} \arctan(2x + \sqrt{3}) + \frac{1}{4} \arctan(2x - \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^2 - sqrt(3)*x + 1) + 1/5*(5*x^4 - 1)/x^5 + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3))

maple [A] time = 0.01, size = 75, normalized size = 0.77

$$\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} \ln(x^2 - \sqrt{3}x + 1)}{12} - \frac{\sqrt{3} \ln(x^2 + \sqrt{3}x + 1)}{12} + \frac{1}{x} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(x^8+x^4+1),x)

[Out] -1/5/x^5+1/x+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/12*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{5x^4-1}{5x^5} + \frac{1}{2} \int \frac{x^2-1}{x^4-x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/5*(5*x^4 - 1)/x^5 + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)

mupad [B] time = 0.04, size = 52, normalized size = 0.53

$$\frac{x^4 - \frac{1}{5}}{x^5} - \frac{\sqrt{3} \operatorname{atanh}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3} + \frac{2}{3}\right)}\right)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3\left(\frac{2x^2}{3} - \frac{2}{3}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(x^4 + x^8 + 1)),x)

[Out] (x^4 - 1/5)/x^5 - (3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3)))/6 - (3^(1/2)*atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3)))/6

sympy [A] time = 0.22, size = 94, normalized size = 0.96

$$\frac{\sqrt{3} \left(2 \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right) \right)}{12} + \frac{\sqrt{3} \log(x^2 - \sqrt{3}x + 1)}{12} - \frac{\sqrt{3} \log(x^2 + \sqrt{3}x + 1)}{12} + \frac{5x^4 - 1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**6/(x**8+x**4+1),x)

[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 + sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/12 + (5*x**4 - 1)/(5*x**5)

$$3.288 \quad \int \frac{1}{x^8(1+x^4+x^8)} dx$$

Optimal. Leaf size=154

$$-\frac{1}{7x^7} + \frac{1}{3x^3} - \frac{1}{8} \log(x^2 - x + 1) + \frac{1}{8} \log(x^2 + x + 1) + \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}$$

Rubi [A] time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {1368, 1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{3x^3} - \frac{1}{7x^7} - \frac{1}{8} \log(x^2 - x + 1) + \frac{1}{8} \log(x^2 + x + 1) + \frac{\log(x^2 - \sqrt{3}x + 1)}{8\sqrt{3}} - \frac{\log(x^2 + \sqrt{3}x + 1)}{8\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(2x + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[1/(x^8*(1 + x^4 + x^8)),x]

[Out] -1/(7*x^7) + 1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1127

Int[(x_)^2/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4*a*c, 0] && PosQ[a*c]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 1164

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1368

Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1373

Int[(x_)^(m_)/((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n)/2] && NegQ[b^2 - 4*a*c]

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^(n_.))*((a_.) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
&& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8(1+x^4+x^8)} dx &= -\frac{1}{7x^7} + \frac{1}{7} \int \frac{-7-7x^4}{x^4(1+x^4+x^8)} dx \\
&= -\frac{1}{7x^7} + \frac{1}{3x^3} - \frac{1}{21} \int \frac{21x^4}{1+x^4+x^8} dx \\
&= -\frac{1}{7x^7} + \frac{1}{3x^3} + \int \frac{x^4}{1+x^4+x^8} dx \\
&= -\frac{1}{7x^7} + \frac{1}{3x^3} + \frac{1}{2} \int \frac{x^2}{1-x^2+x^4} dx - \frac{1}{2} \int \frac{x^2}{1+x^2+x^4} dx \\
&= -\frac{1}{7x^7} + \frac{1}{3x^3} - \frac{1}{4} \int \frac{1-x^2}{1-x^2+x^4} dx + \frac{1}{4} \int \frac{1+x^2}{1-x^2+x^4} dx + \frac{1}{4} \int \frac{1-x^2}{1+x^2+x^4} dx - \frac{1}{4} \int \frac{1+x^2}{1+x^2+x^4} dx \\
&= -\frac{1}{7x^7} + \frac{1}{3x^3} - \frac{1}{8} \int \frac{1+2x}{-1-x-x^2} dx - \frac{1}{8} \int \frac{1-2x}{-1+x-x^2} dx - \frac{1}{8} \int \frac{1}{1-x+x^2} dx - \frac{1}{8} \int \frac{1}{1+x+x^2} dx \\
&= -\frac{1}{7x^7} + \frac{1}{3x^3} - \frac{1}{8} \log(1-x+x^2) + \frac{1}{8} \log(1+x+x^2) + \frac{\log(1-\sqrt{3}x+x^2)}{8\sqrt{3}} - \frac{\log(1+\sqrt{3}x+x^2)}{8\sqrt{3}} \\
&= -\frac{1}{7x^7} + \frac{1}{3x^3} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3}-2x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3}+2x)
\end{aligned}$$

Mathematica [C] time = 0.34, size = 171, normalized size = 1.11

$$-\frac{1}{7x^7} + \frac{1}{3x^3} - \frac{1}{8} \log(x^2-x+1) + \frac{1}{8} \log(x^2+x+1) + \frac{(\sqrt{3}+i) \tan^{-1}\left(\frac{1}{2}(1-i\sqrt{3})x\right)}{2\sqrt{-6+6i\sqrt{3}}} + \frac{(\sqrt{3}-i) \tan^{-1}\left(\frac{1}{2}(1+i\sqrt{3})x\right)}{2\sqrt{-6-6i\sqrt{3}}} - \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^8*(1 + x^4 + x^8)),x]

[Out] $-1/7*1/x^7 + 1/(3*x^3) + ((I + \text{Sqrt}[3])*\text{ArcTan}[\frac{(1 - I*\text{Sqrt}[3])*x}{2}])/(2*\text{Sqrt}[-6 + (6*I)*\text{Sqrt}[3]]) + ((-I + \text{Sqrt}[3])*\text{ArcTan}[\frac{(1 + I*\text{Sqrt}[3])*x}{2}])/(2*\text{Sqrt}[-6 - (6*I)*\text{Sqrt}[3]]) - \text{ArcTan}[\frac{-1 + 2*x}{\text{Sqrt}[3]}]/(4*\text{Sqrt}[3]) - \text{ArcTan}[\frac{1 + 2*x}{\text{Sqrt}[3]}]/(4*\text{Sqrt}[3]) - \text{Log}[1 - x + x^2]/8 + \text{Log}[1 + x + x^2]/8$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8(1+x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^8*(1 + x^4 + x^8)), x]

[Out] IntegrateAlgebraic[1/(x^8*(1 + x^4 + x^8)), x]

fricas [B] time = 1.20, size = 246, normalized size = 1.60

$28\sqrt{6}\sqrt{3}\sqrt{2}x^7 \arctan\left(\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x + \frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{2x^2+2} - \sqrt{3}\right) + 28\sqrt{6}\sqrt{3}\sqrt{2}x^7 \arctan\left(\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x + \frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{2x^2+2} + \sqrt{3}\right) + 7\sqrt{6}\sqrt{2}x^7 \log(\sqrt{6}\sqrt{2x^2+2}) - 7\sqrt{6}\sqrt{2}x^7 \log(-\sqrt{6}\sqrt{2x^2+2}) + 28\sqrt{5}x^7 \arctan\left(\frac{1}{3}\sqrt{5}(2x+1)\right) + 28\sqrt{5}x^7 \arctan\left(\frac{1}{3}\sqrt{5}(2x-1)\right) - 42x^7 \log(x^2+x+1) + 42x^7 \log(x^2-x+1) - 112x^4 + 48$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8+x^4+1), x, algorithm="fricas")

[Out] $-1/336*(28*\text{sqrt}(6)*\text{sqrt}(3)*\text{sqrt}(2)*x^7*\text{arctan}(-1/3*\text{sqrt}(6)*\text{sqrt}(3)*\text{sqrt}(2)*x + 1/3*\text{sqrt}(6)*\text{sqrt}(3)*\text{sqrt}(\text{sqrt}(6)*\text{sqrt}(2)*x + 2*x^2 + 2) - \text{sqrt}(3)) + 28*\text{sqrt}(6)*\text{sqrt}(3)*\text{sqrt}(2)*x^7*\text{arctan}(-1/3*\text{sqrt}(6)*\text{sqrt}(3)*\text{sqrt}(2)*x + 1/3*\text{sqrt}(6)*\text{sqrt}(3)*\text{sqrt}(-\text{sqrt}(6)*\text{sqrt}(2)*x + 2*x^2 + 2) + \text{sqrt}(3)) + 7*\text{sqrt}(6)*\text{sqrt}(2)*x^7*\log(\text{sqrt}(6)*\text{sqrt}(2)*x + 2*x^2 + 2) - 7*\text{sqrt}(6)*\text{sqrt}(2)*x^7*\log(-\text{sqrt}(6)*\text{sqrt}(2)*x + 2*x^2 + 2) + 28*\text{sqrt}(3)*x^7*\text{arctan}(1/3*\text{sqrt}(3)*(2*x + 1)) + 28*\text{sqrt}(3)*x^7*\text{arctan}(1/3*\text{sqrt}(3)*(2*x - 1)) - 42*x^7*\log(x^2 + x + 1) + 42*x^7*\log(x^2 - x + 1) - 112*x^4 + 48)/x^7$

giac [A] time = 0.43, size = 120, normalized size = 0.78

$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{24}\sqrt{3}\log(x^2 + \sqrt{3}x + 1) + \frac{1}{24}\sqrt{3}\log(x^2 - \sqrt{3}x + 1) + \frac{7x^4 - 3}{21x^2} + \frac{1}{4}\arctan(2x + \sqrt{3}) + \frac{1}{4}\arctan(2x - \sqrt{3}) + \frac{1}{8}\log(x^2 + x + 1) - \frac{1}{8}\log(x^2 - x + 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8+x^4+1), x, algorithm="giac")

[Out] $-1/12*\text{sqrt}(3)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x + 1)) - 1/12*\text{sqrt}(3)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x - 1)) - 1/24*\text{sqrt}(3)*\log(x^2 + \text{sqrt}(3)*x + 1) + 1/24*\text{sqrt}(3)*\log(x^2 - \text{sqrt}(3)*x + 1) + 1/21*(7*x^4 - 3)/x^7 + 1/4*\text{arctan}(2*x + \text{sqrt}(3)) + 1/4*\text{arctan}(2*x - \text{sqrt}(3)) + 1/8*\log(x^2 + x + 1) - 1/8*\log(x^2 - x + 1)$

maple [A] time = 0.01, size = 119, normalized size = 0.77

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} + \frac{\arctan(2x - \sqrt{3})}{4} + \frac{\arctan(2x + \sqrt{3})}{4} + \frac{\sqrt{3} \ln(x^2 - \sqrt{3}x + 1)}{24} - \frac{\sqrt{3} \ln(x^2 + \sqrt{3}x + 1)}{24} - \frac{\ln(x^2 - x + 1)}{8} + \frac{\ln(x^2 + x + 1)}{8} + \frac{1}{3x^3} - \frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^8/(x^8+x^4+1),x)

[Out] $-1/7/x^7 + 1/3/x^3 + 1/8 \ln(x^2 + x + 1) - 1/12 \cdot 3^{1/2} \arctan(1/3 \cdot (2x + 1) \cdot 3^{1/2}) + 1/24 \cdot 3^{1/2} \ln(x^2 - 3^{1/2} \cdot x + 1) + 1/4 \arctan(2x - 3^{1/2}) - 1/24 \cdot 3^{1/2} \ln(x^2 + 3^{1/2} \cdot x + 1) + 1/4 \arctan(2x + 3^{1/2}) - 1/8 \ln(x^2 - x + 1) - 1/12 \cdot 3^{1/2} \arctan(1/3 \cdot (2x - 1) \cdot 3^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{7x^4 - 3}{21x^7} + \frac{1}{2} \int \frac{x^2}{x^4 - x^2 + 1} dx + \frac{1}{8} \log(x^2 + x + 1) - \frac{1}{8} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="maxima")

[Out] $-1/12 \cdot \text{sqrt}(3) \cdot \arctan(1/3 \cdot \text{sqrt}(3) \cdot (2x + 1)) - 1/12 \cdot \text{sqrt}(3) \cdot \arctan(1/3 \cdot \text{sqrt}(3) \cdot (2x - 1)) + 1/21 \cdot (7x^4 - 3)/x^7 + 1/2 \cdot \text{integrate}(x^2/(x^4 - x^2 + 1), x) + 1/8 \cdot \log(x^2 + x + 1) - 1/8 \cdot \log(x^2 - x + 1)$

mupad [B] time = 0.03, size = 110, normalized size = 0.71

$$\frac{x^4 - 1}{x^7} - \text{atan}\left(\frac{2x}{1 + \sqrt{3} \text{li}}\right) \left(\frac{1}{4} + \frac{\sqrt{3} \text{li}}{12}\right) - \text{atan}\left(\frac{x \cdot 2i}{-1 + \sqrt{3} \text{li}}\right) \left(\frac{\sqrt{3}}{12} - \frac{1}{4}i\right) - \text{atan}\left(\frac{x \cdot 2i}{1 + \sqrt{3} \text{li}}\right) \left(\frac{\sqrt{3}}{12} + \frac{1}{4}i\right) - \text{atan}\left(\frac{2x}{-1 + \sqrt{3} \text{li}}\right) \left(\frac{1}{4} + \frac{\sqrt{3} \text{li}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8*(x^4 + x^8 + 1)),x)

[Out] $(x^4/3 - 1/7)/x^7 - \text{atan}((2x)/(3^{1/2} \cdot 1i + 1)) \cdot ((3^{1/2} \cdot 1i)/12 - 1/4) - \text{atan}((x \cdot 2i)/(3^{1/2} \cdot 1i - 1)) \cdot (3^{1/2}/12 - 1i/4) - \text{atan}((x \cdot 2i)/(3^{1/2} \cdot 1i + 1)) \cdot (3^{1/2}/12 + 1i/4) - \text{atan}((2x)/(3^{1/2} \cdot 1i - 1)) \cdot ((3^{1/2} \cdot 1i)/12 + 1/4)$

sympy [C] time = 0.76, size = 209, normalized size = 1.36

$$\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x - \frac{1}{2} + \frac{\sqrt{3}i}{6} - 18432\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)\right) + \left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x - \frac{1}{2} - 18432\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) - \frac{\sqrt{3}i}{6}\right) + \left(-\frac{1}{8} - \frac{\sqrt{3}i}{24}\right) \log\left(x + \frac{1}{2} + \frac{\sqrt{3}i}{6} - 18432\left(\frac{1}{8} - \frac{\sqrt{3}i}{24}\right)\right) + \left(-\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) \log\left(x + \frac{1}{2} - 18432\left(\frac{1}{8} + \frac{\sqrt{3}i}{24}\right) - \frac{\sqrt{3}i}{6}\right) + \text{RootSum}(2304x^4 + 48x^2 + 1, (t \mapsto t \log(-18432t^5 - 4t + x))) + \frac{7x^4 - 3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**8/(x**8+x**4+1),x)

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
- sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x))) + (7*x**4 - 3)/(21*x**7)
```

$$3.289 \quad \int \frac{x^{11}}{1-x^4+x^8} dx$$

Optimal. Leaf size=46

$$\frac{x^4}{4} + \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{8} \log(x^8 - x^4 + 1)$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1357, 703, 634, 618, 204, 628}

$$\frac{x^4}{4} + \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^11/(1 - x^4 + x^8), x]

[Out] x^4/4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(
m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
+ a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{1-x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{1-x+x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{4} \text{Subst} \left(\int \frac{-1+x}{1-x+x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^4 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{8} \log(1-x^4+x^8) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4 \right) \\
 &= \frac{x^4}{4} + \frac{\tan^{-1} \left(\frac{1-2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} + \frac{1}{8} \log(1-x^4+x^8)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{x^4}{4} - \frac{\tan^{-1} \left(\frac{2x^4-1}{\sqrt{3}} \right)}{4\sqrt{3}} + \frac{1}{8} \log(x^8 - x^4 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(1 - x^4 + x^8), x]

[Out] $x^4/4 - \text{ArcTan}[(-1 + 2*x^4)/\text{Sqrt}[3]]/(4*\text{Sqrt}[3]) + \text{Log}[1 - x^4 + x^8]/8$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{1 - x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[x^11/(1 - x^4 + x^8), x]`

[Out] `IntegrateAlgebraic[x^11/(1 - x^4 + x^8), x]`

fricas [A] time = 1.14, size = 37, normalized size = 0.80

$$\frac{1}{4} x^4 - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) + \frac{1}{8} \log(x^8 - x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(x^8-x^4+1), x, algorithm="fricas")`

[Out] $1/4*x^4 - 1/12*\text{sqrt}(3)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x^4 - 1)) + 1/8*\text{log}(x^8 - x^4 + 1)$

giac [A] time = 0.34, size = 37, normalized size = 0.80

$$\frac{1}{4} x^4 - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) + \frac{1}{8} \log(x^8 - x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(x^8-x^4+1), x, algorithm="giac")`

[Out] $1/4*x^4 - 1/12*\text{sqrt}(3)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x^4 - 1)) + 1/8*\text{log}(x^8 - x^4 + 1)$

maple [A] time = 0.01, size = 38, normalized size = 0.83

$$\frac{x^4}{4} - \frac{\sqrt{3} \arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12} + \frac{\ln(x^8 - x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(x^8-x^4+1), x)`

[Out] $1/4*x^4+1/8*\ln(x^8-x^4+1)-1/12*3^{(1/2)}*\text{arctan}(1/3*(2*x^4-1)*3^{(1/2)})$

maxima [A] time = 2.11, size = 37, normalized size = 0.80

$$\frac{1}{4}x^4 - \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4 - 1)\right) + \frac{1}{8}\log(x^8 - x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸-x⁴+1),x, algorithm="maxima")

[Out] 1/4*x⁴ - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x⁴ - 1)) + 1/8*log(x⁸ - x⁴ + 1)

mupad [B] time = 0.05, size = 39, normalized size = 0.85

$$\frac{\ln(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^4}{3}\right)}{12} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(x⁸ - x⁴ + 1),x)

[Out] log(x⁸ - x⁴ + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x⁴)/3))/12 + x⁴/4

sympy [A] time = 0.14, size = 42, normalized size = 0.91

$$\frac{x^4}{4} + \frac{\log(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸-x⁴+1),x)

[Out] x⁴/4 + log(x⁸ - x⁴ + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x⁴/3 - sqrt(3)/3)/12

$$3.290 \quad \int \frac{x^9}{1-x^4+x^8} dx$$

Optimal. Leaf size=57

$$\frac{x^2}{2} + \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{4\sqrt{3}} - \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{4\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1359, 1122, 1164, 628}

$$\frac{x^2}{2} + \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{4\sqrt{3}} - \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^9/(1 - x^4 + x^8), x]

[Out] x^2/2 + Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(4*Sqrt[3])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1122

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1164

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_, x_Symbol]
  :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^9}{1-x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{1-x^2+x^4} dx, x, x^2 \right) \\
 &= \frac{x^2}{2} - \frac{1}{2} \text{Subst} \left(\int \frac{1-x^2}{1-x^2+x^4} dx, x, x^2 \right) \\
 &= \frac{x^2}{2} + \frac{\text{Subst} \left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2} dx, x, x^2 \right)}{4\sqrt{3}} + \frac{\text{Subst} \left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2} dx, x, x^2 \right)}{4\sqrt{3}} \\
 &= \frac{x^2}{2} + \frac{\log(1-\sqrt{3}x^2+x^4)}{4\sqrt{3}} - \frac{\log(1+\sqrt{3}x^2+x^4)}{4\sqrt{3}}
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{12} (6x^2 + \sqrt{3} \log(-x^4 + \sqrt{3}x^2 - 1) - \sqrt{3} \log(x^4 + \sqrt{3}x^2 + 1))$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(1 - x^4 + x^8), x]

[Out] (6*x^2 + Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] - Sqrt[3]*Log[1 + Sqrt[3]*x^2 + x^4])/12

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1-x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/(1 - x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^9/(1 - x^4 + x^8), x]

fricas [A] time = 1.17, size = 47, normalized size = 0.82

$$\frac{1}{2}x^2 + \frac{1}{12}\sqrt{3}\log\left(\frac{x^8 + 5x^4 - 2\sqrt{3}(x^6 + x^2) + 1}{x^8 - x^4 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8-x^4+1),x, algorithm="fricas")

[Out] 1/2*x^2 + 1/12*sqrt(3)*log((x^8 + 5*x^4 - 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1))

giac [B] time = 0.32, size = 99, normalized size = 1.74

$$\frac{1}{2}x^2 + \frac{1}{4}(x^4 - 1)\arctan(2x^2 + \sqrt{3}) + \frac{1}{4}(x^4 - 1)\arctan(2x^2 - \sqrt{3}) + \frac{1}{24}(\sqrt{3}x^4 - \sqrt{3})\log(x^4 + \sqrt{3}x^2 + 1) - \frac{1}{24}(\sqrt{3}x^4 - \sqrt{3})\log(x^4 - \sqrt{3}x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8-x^4+1),x, algorithm="giac")

[Out] 1/2*x^2 + 1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) + 1/4*(x^4 - 1)*arctan(2*x^2 - sqrt(3)) + 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1)

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$\frac{x^2}{2} + \frac{\sqrt{3}\ln(x^4 - \sqrt{3}x^2 + 1)}{12} - \frac{\sqrt{3}\ln(x^4 + \sqrt{3}x^2 + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^8-x^4+1),x)

[Out] 1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2 + \int \frac{(x^4 - 1)x}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8-x^4+1),x, algorithm="maxima")

[Out] 1/2*x^2 + integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)

mupad [B] time = 1.31, size = 29, normalized size = 0.51

$$\frac{x^2}{2} - \frac{\sqrt{3} \operatorname{atanh}\left(\frac{2\sqrt{3}x^2}{9\left(\frac{2x^4}{9} + \frac{2}{9}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(x^8 - x^4 + 1),x)`

[Out] $x^2/2 - (3^{(1/2)}*\operatorname{atanh}((2*3^{(1/2)}*x^2)/(9*((2*x^4)/9 + 2/9))))/6$

sympy [A] time = 0.13, size = 48, normalized size = 0.84

$$\frac{x^2}{2} + \frac{\sqrt{3} \log(x^4 - \sqrt{3}x^2 + 1)}{12} - \frac{\sqrt{3} \log(x^4 + \sqrt{3}x^2 + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(x**8-x**4+1),x)`

[Out] $x**2/2 + \operatorname{sqrt}(3)*\log(x**4 - \operatorname{sqrt}(3)*x**2 + 1)/12 - \operatorname{sqrt}(3)*\log(x**4 + \operatorname{sqrt}(3)*x**2 + 1)/12$

$$3.291 \quad \int \frac{x^7}{1-x^4+x^8} dx$$

Optimal. Leaf size=39

$$\frac{1}{8} \log(x^8 - x^4 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{8} \log(x^8 - x^4 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^7/(1 - x^4 + x^8),x]

[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1-x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{1-x+x^2} dx, x, x^4 \right) \\
&= \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^4 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^4 \right) \\
&= \frac{1}{8} \log(1-x^4+x^8) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4 \right) \\
&= -\frac{\tan^{-1} \left(\frac{1-2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} + \frac{1}{8} \log(1-x^4+x^8)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{2x^4-1}{\sqrt{3}} \right)}{4\sqrt{3}} + \frac{1}{8} \log(x^8 - x^4 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(1 - x^4 + x^8), x]

[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1-x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(1 - x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^7/(1 - x^4 + x^8), x]

fricas [A] time = 1.16, size = 32, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) + \frac{1}{8} \log(x^8 - x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8-x^4+1),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)

giac [A] time = 0.42, size = 32, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) + \frac{1}{8} \log(x^8 - x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8-x^4+1),x, algorithm="giac")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)

maple [A] time = 0.00, size = 33, normalized size = 0.85

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12} + \frac{\ln(x^8 - x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^8-x^4+1),x)

[Out] 1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))

maxima [A] time = 1.97, size = 32, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) + \frac{1}{8} \log(x^8 - x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8-x^4+1),x, algorithm="maxima")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)

mupad [B] time = 1.28, size = 34, normalized size = 0.87

$$\frac{\ln(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(x^8 - x^4 + 1),x)`

[Out] `log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12`

sympy [A] time = 0.14, size = 37, normalized size = 0.95

$$\frac{\log(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(x**8-x**4+1),x)`

[Out] `log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12`

$$3.292 \quad \int \frac{x^5}{1-x^4+x^8} dx$$

Optimal. Leaf size=82

$$-\frac{1}{4} \tan^{-1}(\sqrt{3} - 2x^2) + \frac{1}{4} \tan^{-1}(2x^2 + \sqrt{3}) + \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{8\sqrt{3}} - \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{8\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1359, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log(x^4 - \sqrt{3}x^2 + 1)}{8\sqrt{3}} - \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{8\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x^2) + \frac{1}{4} \tan^{-1}(2x^2 + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[x^5/(1 - x^4 + x^8), x]

[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 + Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1127

Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2

- 4*a*c, 0] && PosQ[a*c]

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1-x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{1-x^2+x^4} dx, x, x^2 \right) \\
&= -\left(\frac{1}{4} \text{Subst} \left(\int \frac{1-x^2}{1-x^2+x^4} dx, x, x^2 \right) \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1+x^2}{1-x^2+x^4} dx, x, x^2 \right) \\
&= \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-\sqrt{3}x+x^2} dx, x, x^2 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1}{1+\sqrt{3}x+x^2} dx, x, x^2 \right) + \frac{\text{Subst} \left(\int \frac{\sqrt{3}}{-1-x} dx, x, x^2 \right)}{8} \\
&= \frac{\log(1-\sqrt{3}x^2+x^4)}{8\sqrt{3}} - \frac{\log(1+\sqrt{3}x^2+x^4)}{8\sqrt{3}} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, -\sqrt{3}+2x^2 \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, \sqrt{3}+2x^2 \right) \\
&= -\frac{1}{4} \tan^{-1}(\sqrt{3}-2x^2) + \frac{1}{4} \tan^{-1}(\sqrt{3}+2x^2) + \frac{\log(1-\sqrt{3}x^2+x^4)}{8\sqrt{3}} - \frac{\log(1+\sqrt{3}x^2+x^4)}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 98, normalized size = 1.20

$$\frac{\sqrt{-1-i\sqrt{3}}(\sqrt{3}+i)\tan^{-1}\left(\frac{1}{2}(1-i\sqrt{3})x^2\right)+\sqrt{-1+i\sqrt{3}}(\sqrt{3}-i)\tan^{-1}\left(\frac{1}{2}(1+i\sqrt{3})x^2\right)}{4\sqrt{6}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5/(1 - x^4 + x^8),x]

[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^2)/2] + Sqrt[-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^2)/2])/(4*Sqrt[6])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1-x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(1 - x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^5/(1 - x^4 + x^8), x]

fricas [B] time = 1.17, size = 171, normalized size = 2.09

$$-\frac{1}{12}\sqrt{6}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^2+1}+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2x^2+2}-\sqrt{3}}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^2+1}+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4-\sqrt{6}\sqrt{2x^2+2}+\sqrt{3}}\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log(2x^4+\sqrt{6}\sqrt{2x^2+2})+\frac{1}{48}\sqrt{6}\sqrt{2}\log(2x^4-\sqrt{6}\sqrt{2x^2+2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8-x^4+1),x, algorithm="fricas")

[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) - 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sqrt(2)*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) + 1/48*sqrt(6)*sqrt(2)*log(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2)

giac [A] time = 0.35, size = 76, normalized size = 0.93

$$\frac{1}{24}\sqrt{3}x^4\log(x^4+\sqrt{3}x^2+1)-\frac{1}{24}\sqrt{3}x^4\log(x^4-\sqrt{3}x^2+1)+\frac{1}{4}x^4\arctan(2x^2+\sqrt{3})+\frac{1}{4}x^4\arctan(2x^2-\sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8-x^4+1),x, algorithm="giac")

[Out] $\frac{1}{24}\sqrt{3}x^4\log(x^4 + \sqrt{3}x^2 + 1) - \frac{1}{24}\sqrt{3}x^4\log(x^4 - \sqrt{3}x^2 + 1) + \frac{1}{4}x^4\arctan(2x^2 + \sqrt{3}) + \frac{1}{4}x^4\arctan(2x^2 - \sqrt{3})$

maple [A] time = 0.01, size = 65, normalized size = 0.79

$$\frac{\arctan(2x^2 - \sqrt{3})}{4} + \frac{\arctan(2x^2 + \sqrt{3})}{4} + \frac{\sqrt{3} \ln(x^4 - \sqrt{3}x^2 + 1)}{24} - \frac{\sqrt{3} \ln(x^4 + \sqrt{3}x^2 + 1)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5/(x^8-x^4+1), x)$

[Out] $\frac{1}{4}\arctan(2x^2-3^{1/2})+\frac{1}{4}\arctan(2x^2+3^{1/2})+\frac{1}{24}3^{1/2}\ln(x^4-3^{1/2}x^2+1)-\frac{1}{24}3^{1/2}\ln(x^4+3^{1/2}x^2+1)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^5/(x^8-x^4+1), x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(x^5/(x^8 - x^4 + 1), x)$

mupad [B] time = 0.05, size = 53, normalized size = 0.65

$$-\text{atan}\left(\frac{2x^2}{-1 + \sqrt{3}1i}\right)\left(\frac{1}{4} + \frac{\sqrt{3}1i}{12}\right) - \text{atan}\left(\frac{2x^2}{1 + \sqrt{3}1i}\right)\left(-\frac{1}{4} + \frac{\sqrt{3}1i}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5/(x^8 - x^4 + 1), x)$

[Out] $-\text{atan}((2x^2)/(3^{1/2}1i - 1))*((3^{1/2}1i)/12 + 1/4) - \text{atan}((2x^2)/(3^{1/2}1i + 1))*((3^{1/2}1i)/12 - 1/4)$

sympy [A] time = 0.21, size = 70, normalized size = 0.85

$$\frac{\sqrt{3} \log(x^4 - \sqrt{3}x^2 + 1)}{24} - \frac{\sqrt{3} \log(x^4 + \sqrt{3}x^2 + 1)}{24} + \frac{\text{atan}(2x^2 - \sqrt{3})}{4} + \frac{\text{atan}(2x^2 + \sqrt{3})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{**5}/(x^{**8}-x^{**4}+1), x)$

[Out] $\sqrt{3}\log(x^{**4} - \sqrt{3}x^{**2} + 1)/24 - \sqrt{3}\log(x^{**4} + \sqrt{3}x^{**2} + 1)/24 + \text{atan}(2x^{**2} - \sqrt{3})/4 + \text{atan}(2x^{**2} + \sqrt{3})/4$

$$3.293 \quad \int \frac{x^3}{1-x^4+x^8} dx$$

Optimal. Leaf size=23

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1352, 618, 204}

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 - x^4 + x^8),x]

[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(2*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{1-x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^4 \right) \\
 &= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4 \right) \right) \\
 &= - \frac{\tan^{-1} \left(\frac{1-2x^4}{\sqrt{3}} \right)}{2\sqrt{3}}
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{2x^4-1}{\sqrt{3}} \right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(1 - x^4 + x^8), x]

[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1-x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(1 - x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^3/(1 - x^4 + x^8), x]

fricas [A] time = 1.70, size = 18, normalized size = 0.78

$$\frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^4 - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8-x^4+1), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))

giac [A] time = 0.41, size = 18, normalized size = 0.78

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8-x^4+1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^8-x^4+1),x)

[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))

maxima [A] time = 1.94, size = 18, normalized size = 0.78

$$\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8-x^4+1),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))

mupad [B] time = 1.29, size = 17, normalized size = 0.74

$$\frac{\sqrt{3} \operatorname{atan}\left(\sqrt{3} \left(\frac{2x^4}{3} - \frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^8 - x^4 + 1),x)

[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 - 1/3)))/6

sympy [A] time = 0.12, size = 26, normalized size = 1.13

$$\frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**8-x**4+1),x)

[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/6

$$3.294 \quad \int \frac{x}{1-x^4+x^8} dx$$

Optimal. Leaf size=82

$$-\frac{1}{4} \tan^{-1}(\sqrt{3} - 2x^2) + \frac{1}{4} \tan^{-1}(2x^2 + \sqrt{3}) - \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{8\sqrt{3}} + \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{8\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1359, 1094, 634, 618, 204, 628}

$$-\frac{\log(x^4 - \sqrt{3}x^2 + 1)}{8\sqrt{3}} + \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{8\sqrt{3}} - \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x^2) + \frac{1}{4} \tan^{-1}(2x^2 + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[x/(1 - x^4 + x^8), x]

[Out] -ArcTan[Sqrt[3] - 2*x^2]/4 + ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1094

```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1359

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol]
:= With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x}{1-x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x^2+x^4} dx, x, x^2 \right) \\
 &= \frac{\text{Subst} \left(\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2} dx, x, x^2 \right)}{4\sqrt{3}} + \frac{\text{Subst} \left(\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2} dx, x, x^2 \right)}{4\sqrt{3}} \\
 &= \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-\sqrt{3}x+x^2} dx, x, x^2 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1}{1+\sqrt{3}x+x^2} dx, x, x^2 \right) - \frac{\text{Subst} \left(\int \frac{-\sqrt{3}}{1-\sqrt{3}x+x^2} dx, x, x^2 \right)}{8\sqrt{3}} \\
 &= -\frac{\log(1-\sqrt{3}x^2+x^4)}{8\sqrt{3}} + \frac{\log(1+\sqrt{3}x^2+x^4)}{8\sqrt{3}} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, -\sqrt{3}+2x^2 \right) - \frac{1}{4} \\
 &= -\frac{1}{4} \tan^{-1}(\sqrt{3}-2x^2) + \frac{1}{4} \tan^{-1}(\sqrt{3}+2x^2) - \frac{\log(1-\sqrt{3}x^2+x^4)}{8\sqrt{3}} + \frac{\log(1+\sqrt{3}x^2+x^4)}{8\sqrt{3}}
 \end{aligned}$$

Mathematica [C] time = 0.05, size = 83, normalized size = 1.01

$$\frac{i \left(\sqrt{-1-i\sqrt{3}} \tan^{-1} \left(\frac{1}{2} (1-i\sqrt{3}) x^2 \right) - \sqrt{-1+i\sqrt{3}} \tan^{-1} \left(\frac{1}{2} (1+i\sqrt{3}) x^2 \right) \right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/(1 - x^4 + x^8), x]

[Out] $((I/2)*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/2] - Sqrt[-1 + I*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2]))/Sqrt[6]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1 - x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(1 - x^4 + x^8), x]

[Out] IntegrateAlgebraic[x/(1 - x^4 + x^8), x]

fricas [B] time = 1.17, size = 171, normalized size = 2.09

$-\frac{1}{12}\sqrt{6}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^2+1}+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2x^2+2}-\sqrt{3}}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^2+1}+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4-\sqrt{6}\sqrt{2x^2+2}+\sqrt{3}}\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log(2x^4+\sqrt{6}\sqrt{2x^2+2})-\frac{1}{48}\sqrt{6}\sqrt{2}\log(2x^4-\sqrt{6}\sqrt{2x^2+2})$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8-x^4+1),x, algorithm="fricas")

[Out] $-1/12*\sqrt{6}*\sqrt{3}*\sqrt{2}*\arctan(-1/3*\sqrt{6}*\sqrt{3}*\sqrt{2}*x^2 + 1/3*\sqrt{6}*\sqrt{3}*\sqrt{2*x^4 + \sqrt{6}*\sqrt{2}*x^2 + 2} - \sqrt{3}) - 1/12*\sqrt{6}*\sqrt{3}*\sqrt{2}*\arctan(-1/3*\sqrt{6}*\sqrt{3}*\sqrt{2}*x^2 + 1/3*\sqrt{6}*\sqrt{3}*\sqrt{2*x^4 - \sqrt{6}*\sqrt{2}*x^2 + 2} + \sqrt{3}) + 1/48*\sqrt{6}*\sqrt{2}*\log(2*x^4 + \sqrt{6}*\sqrt{2}*x^2 + 2) - 1/48*\sqrt{6}*\sqrt{2}*\log(2*x^4 - \sqrt{6}*\sqrt{2}*x^2 + 2)$

giac [A] time = 0.41, size = 64, normalized size = 0.78

$$\frac{1}{24}\sqrt{3}\log(x^4 + \sqrt{3}x^2 + 1) - \frac{1}{24}\sqrt{3}\log(x^4 - \sqrt{3}x^2 + 1) + \frac{1}{4}\arctan(2x^2 + \sqrt{3}) + \frac{1}{4}\arctan(2x^2 - \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8-x^4+1),x, algorithm="giac")

[Out] $1/24*\sqrt{3}*\log(x^4 + \sqrt{3}*x^2 + 1) - 1/24*\sqrt{3}*\log(x^4 - \sqrt{3}*x^2 + 1) + 1/4*\arctan(2*x^2 + \sqrt{3}) + 1/4*\arctan(2*x^2 - \sqrt{3})$

maple [A] time = 0.01, size = 65, normalized size = 0.79

$$\frac{\arctan(2x^2 - \sqrt{3})}{4} + \frac{\arctan(2x^2 + \sqrt{3})}{4} - \frac{\sqrt{3}\ln(x^4 - \sqrt{3}x^2 + 1)}{24} + \frac{\sqrt{3}\ln(x^4 + \sqrt{3}x^2 + 1)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^8-x^4+1),x)

[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate(x/(x^8 - x^4 + 1), x)

mupad [B] time = 0.04, size = 53, normalized size = 0.65

$$-\operatorname{atan}\left(-\frac{x^2}{2} + \frac{\sqrt{3} x^2 1i}{2}\right)\left(\frac{1}{4} + \frac{\sqrt{3} 1i}{12}\right) - \operatorname{atan}\left(\frac{x^2}{2} + \frac{\sqrt{3} x^2 1i}{2}\right)\left(-\frac{1}{4} + \frac{\sqrt{3} 1i}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^8 - x^4 + 1),x)

[Out] - atan((3^(1/2)*x^2*1i)/2 - x^2/2)*((3^(1/2)*1i)/12 + 1/4) - atan((3^(1/2)*x^2*1i)/2 + x^2/2)*((3^(1/2)*1i)/12 - 1/4)

sympy [A] time = 0.21, size = 70, normalized size = 0.85

$$-\frac{\sqrt{3} \log(x^4 - \sqrt{3} x^2 + 1)}{24} + \frac{\sqrt{3} \log(x^4 + \sqrt{3} x^2 + 1)}{24} + \frac{\operatorname{atan}(2x^2 - \sqrt{3})}{4} + \frac{\operatorname{atan}(2x^2 + \sqrt{3})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**8-x**4+1),x)

[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2 + 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4

$$3.295 \quad \int \frac{1}{x(1-x^4+x^8)} dx$$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8} \log(x^8 - x^4 + 1) + \log(x)$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{8} \log(x^8 - x^4 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^4 + x^8)),x]

[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(1-x^4+x^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(1-x+x^2)} dx, x, x^4 \right) \\
 &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x} dx, x, x^4 \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1-x}{1-x+x^2} dx, x, x^4 \right) \\
 &= \log(x) + \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^4 \right) - \frac{1}{8} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^4 \right) \\
 &= \log(x) - \frac{1}{8} \log(1-x^4+x^8) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4 \right) \\
 &= -\frac{\tan^{-1} \left(\frac{1-2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} + \log(x) - \frac{1}{8} \log(1-x^4+x^8)
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.34

$$\log(x) - \frac{1}{4} \text{RootSum} \left[\#1^8 - \#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) - \log(x - \#1)}{2\#1^4 - 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 - x^4 + x^8)),x]

[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-1 + 2*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1 - x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(1 - x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x*(1 - x^4 + x^8)), x]

fricas [A] time = 1.12, size = 34, normalized size = 0.83

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8-x^4+1),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + log(x)

giac [A] time = 0.35, size = 38, normalized size = 0.93

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8-x^4+1),x, algorithm="giac")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4*log(x^4)

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12} + \ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(x^8-x^4+1),x)`

[Out] $\ln(x) - \frac{1}{8} \ln(x^8 - x^4 + 1) + \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{1}{4} \log(x^4)$

maxima [A] time = 1.95, size = 38, normalized size = 0.93

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x^8-x^4+1),x, algorithm="maxima")`

[Out] $\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{1}{4} \log(x^4)$

mupad [B] time = 1.29, size = 36, normalized size = 0.88

$$\ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x^8 - x^4 + 1)),x)`

[Out] $\log(x) - \log(x^8 - x^4 + 1)/8 - (3^{1/2} \operatorname{atan}(3^{1/2}/3 - (2 \cdot 3^{1/2} x^4)/3))/12$

sympy [A] time = 0.16, size = 41, normalized size = 1.00

$$\log(x) - \frac{\log(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x**8-x**4+1),x)`

[Out] $\log(x) - \log(x^8 - x^4 + 1)/8 + \sqrt{3} \operatorname{atan}(2\sqrt{3}x^4/3 - \sqrt{3}/3)/12$

$$3.296 \quad \int \frac{1}{x^3(1-x^4+x^8)} dx$$

Optimal. Leaf size=57

$$-\frac{1}{2x^2} - \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{4\sqrt{3}} + \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{4\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1359, 1123, 1164, 628}

$$-\frac{1}{2x^2} - \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{4\sqrt{3}} + \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1 - x^4 + x^8)),x]

[Out] -1/(2*x^2) - Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(4*Sqrt[3])

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1123

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(1-x^4+x^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(1-x^2+x^4)} dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1-x^2}{1-x^2+x^4} dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} - \frac{\text{Subst} \left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2} dx, x, x^2 \right)}{4\sqrt{3}} - \frac{\text{Subst} \left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2} dx, x, x^2 \right)}{4\sqrt{3}} \\ &= -\frac{1}{2x^2} - \frac{\log(1-\sqrt{3}x^2+x^4)}{4\sqrt{3}} + \frac{\log(1+\sqrt{3}x^2+x^4)}{4\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{12} \left(-\frac{6}{x^2} - \sqrt{3} \log(-x^4 + \sqrt{3}x^2 - 1) + \sqrt{3} \log(x^4 + \sqrt{3}x^2 + 1) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] + Sqrt[3]*Log[1 + Sqrt[3]*x^2 + x^4])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(1-x^4+x^8)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(1 - x^4 + x^8)), x]
```


fricas [A] time = 1.24, size = 50, normalized size = 0.88

$$\frac{\sqrt{3} x^2 \log\left(\frac{x^8 + 5x^4 + 2\sqrt{3}(x^6 + x^2) + 1}{x^8 - x^4 + 1}\right) - 6}{12 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="fricas")

[Out] 1/12*(sqrt(3)*x^2*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1)) - 6)/x^2

giac [B] time = 0.31, size = 99, normalized size = 1.74

$$-\frac{1}{4}(x^4 - 1) \arctan(2x^2 + \sqrt{3}) - \frac{1}{4}(x^4 - 1) \arctan(2x^2 - \sqrt{3}) - \frac{1}{24}(\sqrt{3}x^4 - \sqrt{3}) \log(x^4 + \sqrt{3}x^2 + 1) + \frac{1}{24}(\sqrt{3}x^4 - \sqrt{3}) \log(x^4 - \sqrt{3}x^2 + 1) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="giac")

[Out] -1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) - 1/4*(x^4 - 1)*arctan(2*x^2 - sqrt(3)) - 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1) - 1/2/x^2

maple [A] time = 0.01, size = 44, normalized size = 0.77

$$-\frac{\sqrt{3} \ln(x^4 - \sqrt{3} x^2 + 1)}{12} + \frac{\sqrt{3} \ln(x^4 + \sqrt{3} x^2 + 1)}{12} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^8-x^4+1),x)

[Out] -1/2/x^2-1/12*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/12*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2x^2} - \int \frac{(x^4 - 1)x}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="maxima")

[Out] -1/2/x^2 - integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)

mupad [B] time = 1.27, size = 29, normalized size = 0.51

$$\frac{\sqrt{3} \operatorname{atanh}\left(\frac{2\sqrt{3}x^2}{9\left(\frac{2x^4}{9} + \frac{2}{9}\right)}\right)}{6} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(x^8 - x^4 + 1)),x)`

[Out] $(3^{(1/2)}*\operatorname{atanh}((2*3^{(1/2)}*x^2)/(9*((2*x^4)/9 + 2/9))))/6 - 1/(2*x^2)$

sympy [A] time = 0.15, size = 49, normalized size = 0.86

$$-\frac{\sqrt{3} \log(x^4 - \sqrt{3}x^2 + 1)}{12} + \frac{\sqrt{3} \log(x^4 + \sqrt{3}x^2 + 1)}{12} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(x**8-x**4+1),x)`

[Out] $-\operatorname{sqrt}(3)*\log(x**4 - \operatorname{sqrt}(3)*x**2 + 1)/12 + \operatorname{sqrt}(3)*\log(x**4 + \operatorname{sqrt}(3)*x**2 + 1)/12 - 1/(2*x**2)$

$$3.297 \quad \int \frac{1}{x^5(1-x^4+x^8)} dx$$

Optimal. Leaf size=48

$$-\frac{1}{4x^4} + \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8} \log(x^8 - x^4 + 1) + \log(x)$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{4x^4} - \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(1 - x^4 + x^8)),x]

[Out] -1/(4*x^4) + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)/(a + bx + cx^2)}{x} dx$; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 709

$\int \frac{(d + e)x^m}{(a + bx + cx^2)} dx$; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

$\int \frac{(d + e)x^m (f + gx)}{(a + bx + cx^2)} dx$; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1357

$\int (x^m (a + c x^{n2}) + b x^n)^p dx$; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(1-x^4+x^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2(1-x+x^2)} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{1}{4} \text{Subst} \left(\int \frac{1-x}{x(1-x+x^2)} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{x} - \frac{x}{1-x+x^2} \right) dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \log(x) - \frac{1}{4} \text{Subst} \left(\int \frac{x}{1-x+x^2} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \log(x) - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^4 \right) - \frac{1}{8} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \log(x) - \frac{1}{8} \log(1-x^4+x^8) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{\tan^{-1} \left(\frac{1-2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} + \log(x) - \frac{1}{8} \log(1-x^4+x^8)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 1.06

$$-\frac{1}{4} \text{RootSum} \left[\#1^8 - \#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1)}{2\#1^4 - 1} \& \right] - \frac{1}{4x^4} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(1 - x^4 + x^8)),x]

[Out] -1/4*1/x^4 + Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(1-x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(1 - x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^5*(1 - x^4 + x^8)), x]

fricas [A] time = 1.22, size = 51, normalized size = 1.06

$$\frac{2\sqrt{3}x^4 \arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) + 3x^4 \log(x^8 - x^4 + 1) - 24x^4 \log(x) + 6}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="fricas")

[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 3*x^4*log(x^8 - x^4 + 1) - 24*x^4*log(x) + 6)/x^4

giac [A] time = 0.40, size = 48, normalized size = 1.00

$$-\frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) - \frac{x^4+1}{4x^4} - \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4*(x^4 + 1)/x^4 - 1/8*log(x^8 - x^4 + 1) + 1/4*log(x^4)

maple [A] time = 0.01, size = 40, normalized size = 0.83

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12} + \ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(x^8-x^4+1),x)

[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))

maxima [A] time = 1.97, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right) - \frac{1}{4x^4} - \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="maxima")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4/x^4 - 1/8*log(x^8 - x^4 + 1) + 1/4*log(x^4)

mupad [B] time = 0.07, size = 41, normalized size = 0.85

$$\ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^4}{3}\right)}{12} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(x^8 - x^4 + 1)),x)`

[Out] `log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12 - 1/(4*x^4)`

sympy [A] time = 0.19, size = 48, normalized size = 1.00

$$\log(x) - \frac{\log(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(x**8-x**4+1),x)`

[Out] `log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12 - 1/(4*x**4)`

$$3.298 \quad \int \frac{1}{x^7(1-x^4+x^8)} dx$$

Optimal. Leaf size=96

$$-\frac{1}{6x^6} - \frac{1}{2x^2} + \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x^2) - \frac{1}{4} \tan^{-1}(2x^2 + \sqrt{3}) - \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{8\sqrt{3}} + \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{8\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1359, 1123, 1281, 12, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{2x^2} - \frac{1}{6x^6} - \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{8\sqrt{3}} + \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{8\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x^2) - \frac{1}{4} \tan^{-1}(2x^2 + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(1 - x^4 + x^8)),x]

[Out] -1/(6*x^6) - 1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
- 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
```

$x^{(n/k)} + c*x^{((2*n)/k)} \wedge p, x], x, x^k], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{EqQ}[n^2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^7(1-x^4+x^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4(1-x^2+x^4)} dx, x, x^2 \right) \\
 &= -\frac{1}{6x^6} + \frac{1}{6} \text{Subst} \left(\int \frac{3-3x^2}{x^2(1-x^2+x^4)} dx, x, x^2 \right) \\
 &= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{6} \text{Subst} \left(\int \frac{3x^2}{1-x^2+x^4} dx, x, x^2 \right) \\
 &= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{1-x^2+x^4} dx, x, x^2 \right) \\
 &= -\frac{1}{6x^6} - \frac{1}{2x^2} + \frac{1}{4} \text{Subst} \left(\int \frac{1-x^2}{1-x^2+x^4} dx, x, x^2 \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1+x^2}{1-x^2+x^4} dx, x, x^2 \right) \\
 &= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-\sqrt{3}x+x^2} dx, x, x^2 \right) - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1+\sqrt{3}x+x^2} dx, x, x^2 \right) \\
 &= -\frac{1}{6x^6} - \frac{1}{2x^2} - \frac{\log(1-\sqrt{3}x^2+x^4)}{8\sqrt{3}} + \frac{\log(1+\sqrt{3}x^2+x^4)}{8\sqrt{3}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, x^2 \right) \\
 &= -\frac{1}{6x^6} - \frac{1}{2x^2} + \frac{1}{4} \tan^{-1}(\sqrt{3}-2x^2) - \frac{1}{4} \tan^{-1}(\sqrt{3}+2x^2) - \frac{\log(1-\sqrt{3}x^2+x^4)}{8\sqrt{3}} + \frac{\log(1+\sqrt{3}x^2+x^4)}{8\sqrt{3}}
 \end{aligned}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.58

$$-\frac{1}{4} \text{RootSum} \left[\#1^8 - \#1^4 + 1 \&, \frac{\#1^2 \log(x - \#1)}{2\#1^4 - 1} \& \right] - \frac{1}{6x^6} - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(1-x^4+x^8)),x]

[Out] -1/6*1/x^6 - 1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7(1-x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^7*(1 - x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^7*(1 - x^4 + x^8)), x]

fricas [B] time = 1.33, size = 193, normalized size = 2.01

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^6\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^6\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4-\sqrt{6}\sqrt{2}x^2+2}+\sqrt{3}\right)+\sqrt{6}\sqrt{2}x^6\log(2x^4+\sqrt{6}\sqrt{2}x^2+2)-\sqrt{6}\sqrt{2}x^6\log(2x^4-\sqrt{6}\sqrt{2}x^2+2)-24x^4-8}{48x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="fricas")

[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*sqrt(2)*x^6*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(6)*sqrt(2)*x^6*log(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) - 24*x^4 - 8)/x^6

giac [A] time = 0.42, size = 56, normalized size = 0.58

$$-\frac{1}{12}\sqrt{3}x^4\log(x^4+\sqrt{3}x^2+1)+\frac{1}{12}\sqrt{3}x^4\log(x^4-\sqrt{3}x^2+1)-\frac{3x^4+1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/12*sqrt(3)*x^4*log(x^4 - sqrt(3)*x^2 + 1) - 1/6*(3*x^4 + 1)/x^6

maple [A] time = 0.01, size = 75, normalized size = 0.78

$$\frac{\arctan(2x^2-\sqrt{3})}{4}-\frac{\arctan(2x^2+\sqrt{3})}{4}-\frac{\sqrt{3}\ln(x^4-\sqrt{3}x^2+1)}{24}+\frac{\sqrt{3}\ln(x^4+\sqrt{3}x^2+1)}{24}-\frac{1}{2x^2}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(x^8-x^4+1),x)

[Out] -1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3x^4+1}{6x^6}-\int\frac{x^5}{x^8-x^4+1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="maxima")

[Out] -1/6*(3*x^4 + 1)/x^6 - integrate(x^5/(x^8 - x^4 + 1), x)

mupad [B] time = 0.06, size = 63, normalized size = 0.66

$$\operatorname{atan}\left(\frac{2x^2}{-1 + \sqrt{3}1i}\right)\left(\frac{1}{4} + \frac{\sqrt{3}1i}{12}\right) + \operatorname{atan}\left(\frac{2x^2}{1 + \sqrt{3}1i}\right)\left(-\frac{1}{4} + \frac{\sqrt{3}1i}{12}\right) - \frac{x^4}{2} + \frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^7*(x^8 - x^4 + 1)),x)

[Out] atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - (x^4/2 + 1/6)/x^6

sympy [A] time = 0.26, size = 83, normalized size = 0.86

$$-\frac{\sqrt{3} \log(x^4 - \sqrt{3}x^2 + 1)}{24} + \frac{\sqrt{3} \log(x^4 + \sqrt{3}x^2 + 1)}{24} - \frac{\operatorname{atan}(2x^2 - \sqrt{3})}{4} - \frac{\operatorname{atan}(2x^2 + \sqrt{3})}{4} + \frac{-3x^4 - 1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**7/(x**8-x**4+1),x)

[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2 + 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 + (-3*x**4 - 1)/(6*x**6)

$$3.299 \quad \int \frac{x^8}{1-x^4+x^8} dx$$

Optimal. Leaf size=356

$$-\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) + \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) + \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log$$

Rubi [A] time = 0.33, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1367, 1421, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) + \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) + \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 - \sqrt{2+\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 + \sqrt{2+\sqrt{3}}x + 1\right) + x + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} - \frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} + \frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Int[x^8/(1 - x^4 + x^8), x]

[Out] x + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1367

Int[((d_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] :> Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]

Rule 1421

Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1-x^4+x^8} dx &= x - \int \frac{1-x^4}{1-x^4+x^8} dx \\
&= x + \frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4} dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4} dx}{2\sqrt{3}} \\
&= x - \frac{\int \frac{\sqrt{3(2-\sqrt{3})} - (-2+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{\sqrt{3(2-\sqrt{3})} + (-2+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{\sqrt{3(2+\sqrt{3})} - (-2+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} - \frac{\int \frac{\sqrt{3(2+\sqrt{3})} + (-2+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} \\
&= x + \frac{1}{8}\sqrt{\frac{1}{3}(7-4\sqrt{3})} \int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx + \frac{1}{8}\sqrt{\frac{1}{3}(7-4\sqrt{3})} \int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx \\
&= x - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}} \log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}} \log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right) \\
&= x + \frac{1}{4}\sqrt{\frac{1}{3}(2+\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}(2-\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}(2+\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}(2-\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) - \log(x - \#1)}{2\#1^7 - \#1^3} \&\right] + x$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(1 - x^4 + x^8),x]

[Out] x + RootSum[1 - #1^4 + #1^8 &, (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 + 2*#1^7) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1-x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(1 - x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^8/(1 - x^4 + x^8), x]

fricas [B] time = 1.43, size = 716, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8-x^4+1),x, algorithm="fricas")

[Out]
$$-1/48*\sqrt{6}*(\sqrt{3}*\sqrt{2} - 2*\sqrt{2})*\sqrt{\sqrt{3} + 2}*\log(12*x^2 + 2*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2})*\sqrt{\sqrt{3} + 2} + 12) + 1/48*\sqrt{6}*(\sqrt{3}*\sqrt{2} - 2*\sqrt{2})*\sqrt{\sqrt{3} + 2}*\log(12*x^2 - 2*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2})*\sqrt{\sqrt{3} + 2} + 12) - 1/96*\sqrt{6}*(\sqrt{3}*\sqrt{2} + 2*\sqrt{2})*\sqrt{-4*\sqrt{3} + 8}*\log(12*x^2 + \sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2})*\sqrt{-4*\sqrt{3} + 8} + 12) + 1/96*\sqrt{6}*(\sqrt{3}*\sqrt{2} + 2*\sqrt{2})*\sqrt{-4*\sqrt{3} + 8}*\log(12*x^2 - \sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2})*\sqrt{-4*\sqrt{3} + 8} + 12) - 1/12*\sqrt{6}*\sqrt{2}*\sqrt{\sqrt{3} + 2}*\arctan(1/6*\sqrt{6}*\sqrt{12*x^2 + 2*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2})*\sqrt{\sqrt{3} + 2} + 12}*(\sqrt{3}*\sqrt{2} - 2*\sqrt{2}))*\sqrt{\sqrt{3} + 2} + 1/3*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2})*\sqrt{\sqrt{3} + 2} - \sqrt{3} + 2) - 1/12*\sqrt{6}*\sqrt{2}*\sqrt{\sqrt{3} + 2}*\arctan(1/6*\sqrt{6}*\sqrt{12*x^2 - 2*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2})*\sqrt{\sqrt{3} + 2} + 12}*(\sqrt{3}*\sqrt{2} - 2*\sqrt{2}))*\sqrt{\sqrt{3} + 2} + 1/3*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2})*\sqrt{\sqrt{3} + 2} + \sqrt{3} - 2) - 1/24*\sqrt{6}*\sqrt{2}*\sqrt{-4*\sqrt{3} + 8}*\arctan(1/12*\sqrt{6}*\sqrt{12*x^2 + \sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2})*\sqrt{-4*\sqrt{3} + 8} + 12}*(\sqrt{3}*\sqrt{2} + 2*\sqrt{2}))*\sqrt{-4*\sqrt{3} + 8} - 1/6*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2})*\sqrt{-4*\sqrt{3} + 8} - \sqrt{3} - 2) - 1/24*\sqrt{6}*\sqrt{2}*\sqrt{-4*\sqrt{3} + 8}*\arctan(1/12*\sqrt{6}*\sqrt{12*x^2 - \sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2})*\sqrt{-4*\sqrt{3} + 8} + 12}*(\sqrt{3}*\sqrt{2} + 2*\sqrt{2}))*\sqrt{-4*\sqrt{3} + 8} - 1/6*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2})*\sqrt{-4*\sqrt{3} + 8} + \sqrt{3} + 2) + x$$

giac [A] time = 0.35, size = 254, normalized size = 0.71

$$\frac{1}{24}(\sqrt{6} + 3\sqrt{2})\arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) - \frac{1}{24}(\sqrt{6} + 3\sqrt{2})\arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) - \frac{1}{24}(\sqrt{6} - 3\sqrt{2})\arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) - \frac{1}{24}(\sqrt{6} - 3\sqrt{2})\arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) - \frac{1}{48}(\sqrt{6} + 3\sqrt{2})\log\left(x^2 + \frac{1}{2}(\sqrt{6} + \sqrt{2}) + 1\right) + \frac{1}{48}(\sqrt{6} + 3\sqrt{2})\log\left(x^2 - \frac{1}{2}(\sqrt{6} + \sqrt{2}) + 1\right) - \frac{1}{48}(\sqrt{6} - 3\sqrt{2})\log\left(x^2 + \frac{1}{2}(\sqrt{6} - \sqrt{2}) + 1\right) + \frac{1}{48}(\sqrt{6} - 3\sqrt{2})\log\left(x^2 - \frac{1}{2}(\sqrt{6} - \sqrt{2}) + 1\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8-x^4+1),x, algorithm="giac")

[Out]
$$-1/24*(\sqrt{6} + 3*\sqrt{2})*\arctan((4*x + \sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})) - 1/24*(\sqrt{6} + 3*\sqrt{2})*\arctan((4*x - \sqrt{6} + \sqrt{2})/(\sqrt{6} + \sqrt{2})) - 1/24*(\sqrt{6} - 3*\sqrt{2})*\arctan((4*x + \sqrt{6} + \sqrt{2})/(\sqrt{6} - \sqrt{2})) - 1/24*(\sqrt{6} - 3*\sqrt{2})*\arctan((4*x - \sqrt{6} - \sqrt{2})/(\sqrt{6} - \sqrt{2})) - 1/48*(\sqrt{6} + 3*\sqrt{2})*\log(x^2 + 1/2*x$$

(sqrt(6) + sqrt(2)) + 1) + 1/48(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) + x

maple [C] time = 0.02, size = 44, normalized size = 0.12

$$x + \frac{\left(\text{RootOf}(-Z^8 - Z^4 + 1)^4 - 1\right) \ln\left(-\text{RootOf}(-Z^8 - Z^4 + 1) + x\right)}{8 \text{RootOf}(-Z^8 - Z^4 + 1)^7 - 4 \text{RootOf}(-Z^8 - Z^4 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^8-x^4+1),x)

[Out] x+1/4*sum((R^4-1)/(2*R^7-R^3)*ln(-R+x),R=RootOf(-Z^8-Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x + \int \frac{x^4 - 1}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8-x^4+1),x, algorithm="maxima")

[Out] x + integrate((x^4 - 1)/(x^8 - x^4 + 1), x)

mupad [B] time = 0.16, size = 209, normalized size = 0.59

$$x + \frac{\sqrt{3} \operatorname{atan}\left(\frac{x}{(8-\sqrt{3}8i)^{1/4}} + \frac{\sqrt{3}x11}{(8-\sqrt{3}8i)^{1/4}}\right) (8-\sqrt{3}8i)^{1/4} 1i}{12} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{x11}{(8-\sqrt{3}8i)^{1/4}} - \frac{\sqrt{3}x}{(8-\sqrt{3}8i)^{1/4}}\right) (8-\sqrt{3}8i)^{1/4}}{12} - \frac{2^{3/4} \sqrt{3} \operatorname{atan}\left(\frac{2^{1/4}x}{2(1+\sqrt{3}11)^{1/4}} - \frac{2^{1/4}\sqrt{3}x11}{2(1+\sqrt{3}11)^{1/4}}\right) (1+\sqrt{3}11)^{1/4} 1i}{12} - \frac{2^{3/4} \sqrt{3} \operatorname{atan}\left(\frac{2^{1/4}x11}{2(1+\sqrt{3}11)^{1/4}} + \frac{2^{1/4}\sqrt{3}x}{2(1+\sqrt{3}11)^{1/4}}\right) (1+\sqrt{3}11)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^8 - x^4 + 1),x)

[Out] x + (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12

sympy [A] time = 3.20, size = 26, normalized size = 0.07

$$x + \text{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(9216t^5 - 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-x**4+1),x)
```

```
[Out] x + RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 -  
8*_t + x)))
```

$$3.300 \quad \int \frac{x^6}{1-x^4+x^8} dx$$

Optimal. Leaf size=275

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}}$$

Rubi [A] time = 0.24, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1372, 1169, 634, 618, 204, 628}

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2 - \sqrt{3}} - 2x}{\sqrt{2 + \sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2 + \sqrt{3}} - 2x}{\sqrt{2 - \sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{2x + \sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{2x + \sqrt{2 + \sqrt{3}}}{\sqrt{2 - \sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(1 - x^4 + x^8), x]

[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1-x^4+x^8} dx &= -\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= \frac{\int \frac{\sqrt{2-\sqrt{3}}-(1-\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+(1-\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{\sqrt{2+\sqrt{3}}-(1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} - \frac{\int \frac{\sqrt{2+\sqrt{3}}+(1+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} \\
&= \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} \\
&= \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\#1^3 \log(x - \#1)}{2\#1^4 - 1}\& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(1 - x^4 + x^8), x]

[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1-x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(1 - x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^6/(1 - x^4 + x^8), x]

fricas [A] time = 1.27, size = 215, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x-2)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}(x^3-x)-x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x+2)}{3x^2-2}\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log(x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1)+\frac{1}{24}\sqrt{3}\sqrt{2}\log(x^4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8-x^4+1),x, algorithm="fricas")

[Out]
$$-1/6*\sqrt{3}*\sqrt{2}*\arctan(-(\sqrt{3}*\sqrt{2}*(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}*(\sqrt{3}*\sqrt{2}*x-2))/(3*x^2-2)) - 1/6*\sqrt{3}*\sqrt{2}*\arctan(-(\sqrt{3}*\sqrt{2}*(x^3-x)-x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}*(\sqrt{3}*\sqrt{2}*x+2))/(3*x^2-2)) - 1/24*\sqrt{3}*\sqrt{2}*\log(x^4+\sqrt{3}*\sqrt{2}*(x^3+x)+3*x^2+1) + 1/24*\sqrt{3}*\sqrt{2}*\log(x^4-\sqrt{3}*\sqrt{2}*(x^3+x)+3*x^2+1)$$

giac [A] time = 0.40, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\sqrt{6}\log(x^2+\frac{1}{2}(\sqrt{6}+\sqrt{2})+1)+\frac{1}{24}\sqrt{6}\log(x^2-\frac{1}{2}(\sqrt{6}+\sqrt{2})+1)-\frac{1}{24}\sqrt{6}\log(x^2+\frac{1}{2}(\sqrt{6}-\sqrt{2})+1)+\frac{1}{24}\sqrt{6}\log(x^2-\frac{1}{2}(\sqrt{6}-\sqrt{2})+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8-x^4+1),x, algorithm="giac")

[Out]
$$1/12*\sqrt{6}*\arctan((4*x+\sqrt{6}-\sqrt{2})/(\sqrt{6}+\sqrt{2}))+1/12*\sqrt{6}*\arctan((4*x-\sqrt{6}+\sqrt{2})/(\sqrt{6}+\sqrt{2}))+1/12*\sqrt{6}*\arctan((4*x+\sqrt{6}+\sqrt{2})/(\sqrt{6}-\sqrt{2}))+1/12*\sqrt{6}*\arctan((4*x-\sqrt{6}-\sqrt{2})/(\sqrt{6}-\sqrt{2}))-1/24*\sqrt{6}*\log(x^2+1/2*x*(\sqrt{6}+\sqrt{2})+1)+1/24*\sqrt{6}*\log(x^2-1/2*x*(\sqrt{6}+\sqrt{2})+1)-1/24*\sqrt{6}*\log(x^2+1/2*x*(\sqrt{6}-\sqrt{2})+1)+1/24*\sqrt{6}*\log(x^2-1/2*x*(\sqrt{6}-\sqrt{2})+1)$$

maple [C] time = 0.01, size = 32, normalized size = 0.12

$$\frac{\text{RootOf}(9_Z^4+1)\ln\left(9\text{RootOf}(9_Z^4+1)^3x-3\text{RootOf}(9_Z^4+1)^2+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^8-x^4+1),x)

[Out]
$$1/4*\sum(_R*\ln(9*_R^3*x-3*_R^2*x^2),_R=\text{RootOf}(9*_Z^4+1))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{x^8-x^4+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate(x^6/(x^8 - x^4 + 1), x)

mupad [B] time = 0.10, size = 53, normalized size = 0.19

$$\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x \left(\frac{1}{3} + \frac{1}{3}i\right)}{\frac{2x^2}{3} - \frac{2}{3}i}\right) \left(-\frac{1}{12} + \frac{1}{12}i\right) + \sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x \left(\frac{1}{3} - \frac{1}{3}i\right)}{\frac{2x^2}{3} + \frac{2}{3}i}\right) \left(-\frac{1}{12} - \frac{1}{12}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^8 - x^4 + 1),x)

[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12)

sympy [A] time = 0.22, size = 165, normalized size = 0.60

$$\frac{\sqrt{6} \left(2 \operatorname{atan}\left(\frac{\sqrt{6}x}{3} - \frac{1}{3}\right) + 2 \operatorname{atan}\left(\sqrt{6}x^3 - 4x^2 + 2\sqrt{6}x - 3\right) \right)}{24} + \frac{\sqrt{6} \left(2 \operatorname{atan}\left(\frac{\sqrt{6}x}{3} + \frac{1}{3}\right) + 2 \operatorname{atan}\left(\sqrt{6}x^3 + 4x^2 + 2\sqrt{6}x + 3\right) \right)}{24} + \frac{\sqrt{6} \log(x^4 - \sqrt{6}x^3 + 3x^2 - \sqrt{6}x + 1)}{24} - \frac{\sqrt{6} \log(x^4 + \sqrt{6}x^3 + 3x^2 + \sqrt{6}x + 1)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(x**8-x**4+1),x)

[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 + sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 - sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24

$$3.301 \quad \int \frac{x^4}{1-x^4+x^8} dx$$

Optimal. Leaf size=347

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{8\sqrt{3}(2 - \sqrt{3})} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{8\sqrt{3}(2 - \sqrt{3})} + \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{8\sqrt{3}(2 + \sqrt{3})} - \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{8\sqrt{3}(2 + \sqrt{3})}$$

Rubi [A] time = 0.21, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1373, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{8\sqrt{3}(2 - \sqrt{3})} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{8\sqrt{3}(2 - \sqrt{3})} + \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{8\sqrt{3}(2 + \sqrt{3})} - \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{8\sqrt{3}(2 + \sqrt{3})} + \frac{\tan^{-1}\left(\frac{\sqrt{2 - \sqrt{3}} - 2x}{\sqrt{2 + \sqrt{3}}}\right)}{4\sqrt{3}(2 + \sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2 + \sqrt{3}} - 2x}{\sqrt{2 - \sqrt{3}}}\right)}{4\sqrt{3}(2 - \sqrt{3})} - \frac{\tan^{-1}\left(\frac{2x + \sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}}\right)}{4\sqrt{3}(2 + \sqrt{3})} + \frac{\tan^{-1}\left(\frac{2x + \sqrt{2 + \sqrt{3}}}{\sqrt{2 - \sqrt{3}}}\right)}{4\sqrt{3}(2 - \sqrt{3})}$$

Antiderivative was successfully verified.

[In] Int[x^4/(1 - x^4 + x^8), x]

[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1127

Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4*a*c, 0] && PosQ[a*c]

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1373

Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n)/2] && NegQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{1-x^4+x^8} dx &= \frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} - \frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= -\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} + \frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} + \frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} - \frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} \\
&= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{8\sqrt{3}} - \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} \\
&= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2+\sqrt{3})} - \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2+\sqrt{3})} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\#1 \log(x - \#1)}{2\#1^4 - 1}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 - x^4 + x^8), x]

[Out] RootSum[1 - #1^4 + #1^8 &, (Log[x - #1]*#1)/(-1 + 2*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1-x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(1 - x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^4/(1 - x^4 + x^8), x]

fricas [B] time = 1.32, size = 567, normalized size = 1.63

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8-x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{48}\sqrt{6}(\sqrt{3}\sqrt{2} - 2\sqrt{2})\sqrt{\sqrt{3} + 2}\log(2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3} + 2} + 12x^2 + 12) - \frac{1}{48}\sqrt{6}(\sqrt{3}\sqrt{2} - 2\sqrt{2})\sqrt{\sqrt{3} + 2}\log(-2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3} + 2} + 12x^2 + 12) + \frac{1}{96}\sqrt{6}(\sqrt{3}\sqrt{2} + 2\sqrt{2})\sqrt{-4\sqrt{3} + 8}\log(\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3} + 8} + 12x^2 + 12) - \frac{1}{96}\sqrt{6}(\sqrt{3}\sqrt{2} + 2\sqrt{2})\sqrt{-4\sqrt{3} + 8}\log(-\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3} + 8} + 12x^2 + 12) - \frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{3} + 2}\arctan(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3} + 2} + \frac{1}{6}\sqrt{6}\sqrt{2}\sqrt{2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3} + 2} + 12x^2 + 12}\sqrt{\sqrt{3} + 2} - \sqrt{3} - 2) - \frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{3} + 2}\arctan(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3} + 2} + \frac{1}{6}\sqrt{6}\sqrt{2}\sqrt{-2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3} + 2} + 12x^2 + 12}\sqrt{\sqrt{3} + 2} + \sqrt{3} + 2) + \frac{1}{24}\sqrt{6}\sqrt{2}\sqrt{-4\sqrt{3} + 8}\arctan(-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3} + 8} + \frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3} + 8} + 12x^2 + 12}\sqrt{-4\sqrt{3} + 8} + \sqrt{3} - 2) + \frac{1}{24}\sqrt{6}\sqrt{2}\sqrt{-4\sqrt{3} + 8}\arctan(-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3} + 8} + \frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{-\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3} + 8} + 12x^2 + 12}\sqrt{-4\sqrt{3} + 8} - \sqrt{3} + 2)$

giac [A] time = 0.48, size = 253, normalized size = 0.73

$\frac{1}{24}(\sqrt{6} - 3\sqrt{2})\arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{24}(\sqrt{6} - 3\sqrt{2})\arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{24}(\sqrt{6} + 3\sqrt{2})\arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{24}(\sqrt{6} + 3\sqrt{2})\arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{24}(\sqrt{6} - 3\sqrt{2})\log\left(x^2 + \frac{1}{2}(\sqrt{6} + \sqrt{2})x + 1\right) - \frac{1}{24}(\sqrt{6} - 3\sqrt{2})\log\left(x^2 - \frac{1}{2}(\sqrt{6} + \sqrt{2})x + 1\right) + \frac{1}{24}(\sqrt{6} + 3\sqrt{2})\log\left(x^2 + \frac{1}{2}(\sqrt{6} - \sqrt{2})x + 1\right) - \frac{1}{24}(\sqrt{6} + 3\sqrt{2})\log\left(x^2 - \frac{1}{2}(\sqrt{6} - \sqrt{2})x + 1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8-x^4+1),x, algorithm="giac")

[Out] $\frac{1}{24}(\sqrt{6} - 3\sqrt{2})\arctan((4x + \sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})) + \frac{1}{24}(\sqrt{6} - 3\sqrt{2})\arctan((4x - \sqrt{6} + \sqrt{2})/(\sqrt{6} + \sqrt{2})) + \frac{1}{24}(\sqrt{6} + 3\sqrt{2})\arctan((4x + \sqrt{6} + \sqrt{2})/(\sqrt{6} - \sqrt{2})) + \frac{1}{24}(\sqrt{6} + 3\sqrt{2})\arctan((4x - \sqrt{6} - \sqrt{2})/(\sqrt{6} - \sqrt{2})) + \frac{1}{48}(\sqrt{6} - 3\sqrt{2})\log(x^2 + \frac{1}{2}x(\sqrt{6} + \sqrt{2}) + 1) - \frac{1}{48}(\sqrt{6} - 3\sqrt{2})\log(x^2 - \frac{1}{2}x(\sqrt{6} + \sqrt{2}) + 1) + \frac{1}{48}(\sqrt{6} + 3\sqrt{2})\log(x^2 + \frac{1}{2}x(\sqrt{6} - \sqrt{2}) + 1) - \frac{1}{48}(\sqrt{6} + 3\sqrt{2})\log(x^2 - \frac{1}{2}x(\sqrt{6} - \sqrt{2}) + 1)$

maple [C] time = 0.01, size = 40, normalized size = 0.12

$$\frac{\text{RootOf}(-Z^8 - Z^4 + 1)^4 \ln(-\text{RootOf}(-Z^8 - Z^4 + 1) + x)}{8 \text{RootOf}(-Z^8 - Z^4 + 1)^7 - 4 \text{RootOf}(-Z^8 - Z^4 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^8-x^4+1),x)

[Out] 1/4*sum(_R^4/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(-_Z^8-_Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate(x^4/(x^8 - x^4 + 1), x)

mupad [B] time = 1.33, size = 474, normalized size = 1.37

$$\frac{\sqrt{3} \operatorname{atan}\left(\frac{x(\sqrt{3}n)^{14}}{\sqrt{\frac{25}{4}x^2 - \frac{25}{4}n^{14}}}\right) + \frac{\sqrt{3}x(\sqrt{3}n)^{14}}{\sqrt{\frac{25}{4}x^2 - \frac{25}{4}n^{14}}}}{12} + \frac{(8 - \sqrt{3}n)^{14}i}{\sqrt{3}} \operatorname{atan}\left(\frac{x(\sqrt{3}n)^{14}}{\sqrt{\frac{25}{4}x^2 - \frac{25}{4}n^{14}}}\right) - \frac{\sqrt{3}x(\sqrt{3}n)^{14}}{\sqrt{\frac{25}{4}x^2 - \frac{25}{4}n^{14}}}}{12} + \frac{(8 - \sqrt{3}n)^{14}}{2^{14}\sqrt{3}} \operatorname{atan}\left(\frac{2^{14}x(1+\sqrt{3}i)^{14}}{\sqrt{\frac{25}{4}x^2 - \frac{25}{4}(1+\sqrt{3}i)^{14}}}\right) - \frac{\sqrt{3}x(1+\sqrt{3}i)^{14}}{\sqrt{\frac{25}{4}x^2 - \frac{25}{4}(1+\sqrt{3}i)^{14}}}}{12} + \frac{(1+\sqrt{3}i)^{14}}{2^{14}\sqrt{3}} \operatorname{atan}\left(\frac{2^{14}x(1+\sqrt{3}i)^{14}}{\sqrt{\frac{25}{4}x^2 - \frac{25}{4}(1+\sqrt{3}i)^{14}}}\right) + \frac{\sqrt{3}x(1+\sqrt{3}i)^{14}}{\sqrt{\frac{25}{4}x^2 - \frac{25}{4}(1+\sqrt{3}i)^{14}}}}{12} + \frac{(1+\sqrt{3}i)^{14}}{2^{14}\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^8 - x^4 + 1),x)

[Out] (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)^(1/4))/12

sympy [A] time = 3.19, size = 24, normalized size = 0.07

$$\text{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log(-18432t^5 + 4t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4*_t + x)))
```

$$3.302 \quad \int \frac{x^2}{1-x^4+x^8} dx$$

Optimal. Leaf size=355

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}}x + 1\right)}{8\sqrt{3}(2 - \sqrt{3})} - \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}}x + 1\right)}{8\sqrt{3}(2 - \sqrt{3})} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}}x + 1\right)}{8\sqrt{3}(2 + \sqrt{3})} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}}x + 1\right)}{8\sqrt{3}(2 + \sqrt{3})} + \dots$$

Rubi [A] time = 0.20, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1373, 1094, 634, 618, 204, 628}

$$\frac{\log\left(\frac{x^2 - \sqrt{2 - \sqrt{3}}x + 1}{8\sqrt{3}(2 - \sqrt{3})}\right)}{8\sqrt{3}(2 - \sqrt{3})} - \frac{\log\left(\frac{x^2 + \sqrt{2 - \sqrt{3}}x + 1}{8\sqrt{3}(2 - \sqrt{3})}\right)}{8\sqrt{3}(2 - \sqrt{3})} - \frac{\log\left(\frac{x^2 - \sqrt{2 + \sqrt{3}}x + 1}{8\sqrt{3}(2 + \sqrt{3})}\right)}{8\sqrt{3}(2 + \sqrt{3})} + \frac{\log\left(\frac{x^2 + \sqrt{2 + \sqrt{3}}x + 1}{8\sqrt{3}(2 + \sqrt{3})}\right)}{8\sqrt{3}(2 + \sqrt{3})} + \frac{1}{4}\sqrt{\frac{1}{3}(2 - \sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2 - \sqrt{3}} - 2x}{\sqrt{2 + \sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}(2 + \sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2 + \sqrt{3}} - 2x}{\sqrt{2 - \sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}(2 - \sqrt{3})} \tan^{-1}\left(\frac{2x + \sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}(2 + \sqrt{3})} \tan^{-1}\left(\frac{2x + \sqrt{2 + \sqrt{3}}}{\sqrt{2 - \sqrt{3}}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 - x^4 + x^8), x]

[Out] (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]])/4 + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1094

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]

Rule 1373

Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n)/2] && NegQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1-x^4+x^8} dx &= \frac{\int \frac{1}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= -\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} \\
&= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} \\
&= \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2+\sqrt{3})} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2+\sqrt{3})} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\log(x - \#1)}{2\#1^5 - \#1}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 - x^4 + x^8),x]

[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1-x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1 - x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^2/(1 - x^4 + x^8), x]

fricas [B] time = 1.30, size = 567, normalized size = 1.60

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8-x^4+1),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/48*\sqrt{6}*(\sqrt{3}*\sqrt{2} - 2*\sqrt{2})*\sqrt{(\sqrt{3} + 2)*\log(2*\sqrt{6})} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(\sqrt{3} + 2) + 12*x^2 + 12} + 1/48*\sqrt{6}*(\sqrt{3} \\ & * \sqrt{2} - 2*\sqrt{2})*\sqrt{(\sqrt{3} + 2)*\log(-2*\sqrt{6})} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(\sqrt{3} + 2) + 12*x^2 + 12} - 1/96*\sqrt{6}*(\sqrt{3}*\sqrt{2} + 2*\sqrt{2}) \\ & * \sqrt{(-4*\sqrt{3} + 8)*\log(\sqrt{6})} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(-4*\sqrt{3} + 8) + 12*x^2 + 12} \\ & + 1/96*\sqrt{6}*(\sqrt{3}*\sqrt{2} + 2*\sqrt{2})*\sqrt{(-4*\sqrt{3} + 8)} \\ & * \log(-\sqrt{6}) \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(-4*\sqrt{3} + 8) + 12*x^2 + 12} - 1/ \\ & 12*\sqrt{6}*\sqrt{2}*\sqrt{(\sqrt{3} + 2)*\arctan(-1/3*\sqrt{6})} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(\sqrt{3} + 2) + 12*x^2 + 12} \\ & + 1/6*\sqrt{6}*\sqrt{2}*\sqrt{2*\sqrt{6}} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(\sqrt{3} + 2) + 12*x^2 + 12} \\ & * \sqrt{(\sqrt{3} + 2) - \sqrt{3} - 2} - 1/12*\sqrt{6} \\ & * \sqrt{2}*\sqrt{(\sqrt{3} + 2)*\arctan(-1/3*\sqrt{6})} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(\sqrt{3} + 2) + 12*x^2 + 12} \\ & * \sqrt{(\sqrt{3} + 2) + \sqrt{3} + 2} + 1/24*\sqrt{6} \\ & * \sqrt{2}*\sqrt{(-4*\sqrt{3} + 8)*\arctan(-1/6*\sqrt{6})} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(-4*\sqrt{3} + 8) + 12*x^2 + 12} \\ & * \sqrt{(\sqrt{6})} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(-4*\sqrt{3} + 8) + 12*x^2 + 12} \\ & * \sqrt{(-4*\sqrt{3} + 8) + \sqrt{3} - 2} + 1/24*\sqrt{6} \\ & * \sqrt{2}*\sqrt{(-4*\sqrt{3} + 8)*\arctan(-1/6*\sqrt{6})} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(-4*\sqrt{3} + 8) + 12*x^2 + 12} \\ & * \sqrt{(-\sqrt{6})} \\ & * \sqrt{3}*\sqrt{2}*x*\sqrt{(-4*\sqrt{3} + 8) + 12*x^2 + 12} \\ & * \sqrt{(-4*\sqrt{3} + 8) - \sqrt{3} + 2} \end{aligned}$$

giac [A] time = 0.46, size = 253, normalized size = 0.71

$$\frac{1}{24}(\sqrt{6}-3\sqrt{2})\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}(\sqrt{6}-3\sqrt{2})\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}(\sqrt{6}+3\sqrt{2})\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}(\sqrt{6}+3\sqrt{2})\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{48}(\sqrt{6}-3\sqrt{2})\log\left(x^2 + \frac{1}{2}(\sqrt{6}+\sqrt{2})+1\right) + \frac{1}{48}(\sqrt{6}-3\sqrt{2})\log\left(x^2 - \frac{1}{2}(\sqrt{6}+\sqrt{2})+1\right) - \frac{1}{48}(\sqrt{6}+3\sqrt{2})\log\left(x^2 + \frac{1}{2}(\sqrt{6}-\sqrt{2})+1\right) + \frac{1}{48}(\sqrt{6}+3\sqrt{2})\log\left(x^2 - \frac{1}{2}(\sqrt{6}-\sqrt{2})+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8-x^4+1),x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/24*(\sqrt{6} - 3*\sqrt{2})*\arctan((4*x + \sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})) \\ & + 1/24*(\sqrt{6} - 3*\sqrt{2})*\arctan((4*x - \sqrt{6} + \sqrt{2})/(\sqrt{6} \\ & + \sqrt{2})) + 1/24*(\sqrt{6} + 3*\sqrt{2})*\arctan((4*x + \sqrt{6} + \sqrt{2}) \\ & /(\sqrt{6} - \sqrt{2})) + 1/24*(\sqrt{6} + 3*\sqrt{2})*\arctan((4*x - \sqrt{6} - \\ & \sqrt{2})/(\sqrt{6} - \sqrt{2})) - 1/48*(\sqrt{6} - 3*\sqrt{2})*\log(x^2 + 1/2*x* \\ & (\sqrt{6} + \sqrt{2}) + 1) + 1/48*(\sqrt{6} - 3*\sqrt{2})*\log(x^2 - 1/2*x*(\sqrt{6} \\ & + \sqrt{2}) + 1) - 1/48*(\sqrt{6} + 3*\sqrt{2})*\log(x^2 + 1/2*x*(\sqrt{6} - \\ & \sqrt{2}) + 1) + 1/48*(\sqrt{6} + 3*\sqrt{2})*\log(x^2 - 1/2*x*(\sqrt{6} - \sqrt{2} \\ & + 1) \end{aligned}$$

maple [C] time = 0.01, size = 40, normalized size = 0.11

$$\frac{\text{RootOf}(-Z^8 - Z^4 + 1)^2 \ln(-\text{RootOf}(-Z^8 - Z^4 + 1) + x)}{8 \text{RootOf}(-Z^8 - Z^4 + 1)^7 - 4 \text{RootOf}(-Z^8 - Z^4 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^8-x^4+1),x)

[Out] 1/4*sum(_R^2/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(-Z^8-Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate(x^2/(x^8 - x^4 + 1), x)

mupad [B] time = 0.08, size = 286, normalized size = 0.81

$$\frac{\sqrt{3} \operatorname{atan}\left(\frac{x(8-\sqrt{3}8i)^{1/4} + \sqrt{3}x(8-\sqrt{3}8i)^{1/4}i}{2(1+\sqrt{3}i)}\right)(8-\sqrt{3}8i)^{1/4}i + \sqrt{3} \operatorname{atan}\left(\frac{x(8-\sqrt{3}8i)^{1/4}i - \sqrt{3}x(8-\sqrt{3}8i)^{1/4}}{2(1+\sqrt{3}i)}\right)(8-\sqrt{3}8i)^{1/4} - 2^{3/4}\sqrt{3} \operatorname{atan}\left(\frac{2^{3/4}x(1+\sqrt{3}i)^{1/4}}{2(-1+\sqrt{3}i)} - \frac{2^{3/4}\sqrt{3}x(1+\sqrt{3}i)^{1/4}i}{2(-1+\sqrt{3}i)}\right)(1+\sqrt{3}i)^{1/4}i + 2^{3/4}\sqrt{3} \operatorname{atan}\left(\frac{2^{3/4}x(1+\sqrt{3}i)^{1/4}i + 2^{3/4}\sqrt{3}x(1+\sqrt{3}i)^{1/4}}{2(-1+\sqrt{3}i)}\right)(1+\sqrt{3}i)^{1/4}}{12} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{x(8-\sqrt{3}8i)^{1/4}i - \sqrt{3}x(8-\sqrt{3}8i)^{1/4}}{2(1+\sqrt{3}i)}\right)(8-\sqrt{3}8i)^{1/4} - 2^{3/4}\sqrt{3} \operatorname{atan}\left(\frac{2^{3/4}x(1+\sqrt{3}i)^{1/4}}{2(-1+\sqrt{3}i)} - \frac{2^{3/4}\sqrt{3}x(1+\sqrt{3}i)^{1/4}i}{2(-1+\sqrt{3}i)}\right)(1+\sqrt{3}i)^{1/4}i + 2^{3/4}\sqrt{3} \operatorname{atan}\left(\frac{2^{3/4}x(1+\sqrt{3}i)^{1/4}i + 2^{3/4}\sqrt{3}x(1+\sqrt{3}i)^{1/4}}{2(-1+\sqrt{3}i)}\right)(1+\sqrt{3}i)^{1/4}}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^8 - x^4 + 1),x)

[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*(3^(1/2)*1i - 1)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4))/12

sympy [A] time = 3.28, size = 26, normalized size = 0.07

$$\text{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(-442368t^7 - 192t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 - 1  
92*_t**3 + x)))
```

$$3.303 \quad \int \frac{1}{1-x^4+x^8} dx$$

Optimal. Leaf size=275

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}}$$

Rubi [A] time = 0.21, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1346, 1169, 634, 618, 204, 628}

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2 - \sqrt{3}} - 2x}{\sqrt{2 + \sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2 + \sqrt{3}} - 2x}{\sqrt{2 - \sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{2x + \sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{2x + \sqrt{2 + \sqrt{3}}}{\sqrt{2 - \sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4 + x^8)^(-1), x]

[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6])

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(n_ - 1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-x^4+x^8} dx &= \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= \frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(1+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3(2-\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})}+(1+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3(2-\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3(2+\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3(2+\sqrt{3})}} \\
&= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} \\
&= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\log(x - \#1)}{2\#1^7 - \#1^3}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4 + x^8)^(-1), x]

[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^4 + x^8)^(-1), x]

[Out] IntegrateAlgebraic[(1 - x^4 + x^8)^(-1), x]

fricas [A] time = 1.46, size = 215, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{-\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x-2)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{-\sqrt{3}\sqrt{2}(x^3-x)-x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x+2)}{3x^2-2}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log(x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1)-\frac{1}{24}\sqrt{3}\sqrt{2}\log(x^4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")

[Out] $-1/6*\sqrt{3}*\sqrt{2}*\arctan(-(\sqrt{3}*\sqrt{2}*(x^3 - x) + x^2 - \sqrt{x^4 + \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1}*(\sqrt{3}*\sqrt{2}*x - 2)))/(3*x^2 - 2)$
 $) - 1/6*\sqrt{3}*\sqrt{2}*\arctan(-(\sqrt{3}*\sqrt{2}*(x^3 - x) - x^2 - \sqrt{x^4 - \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1}*(\sqrt{3}*\sqrt{2}*x + 2)))/(3*x^2 - 2)$
 $) + 1/24*\sqrt{3}*\sqrt{2}*\log(x^4 + \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1)$
 $) - 1/24*\sqrt{3}*\sqrt{2}*\log(x^4 - \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1)$

giac [A] time = 0.40, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x(\sqrt{6}+\sqrt{2})+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x(\sqrt{6}+\sqrt{2})+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x(\sqrt{6}-\sqrt{2})+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x(\sqrt{6}-\sqrt{2})+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-x^4+1),x, algorithm="giac")

[Out] $1/12*\sqrt{6}*\arctan((4*x + \sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})) + 1/12*\sqrt{6}*\arctan((4*x - \sqrt{6} + \sqrt{2})/(\sqrt{6} + \sqrt{2})) + 1/12*\sqrt{6}*\arctan((4*x + \sqrt{6} + \sqrt{2})/(\sqrt{6} - \sqrt{2})) + 1/12*\sqrt{6}*\arctan((4*x - \sqrt{6} - \sqrt{2})/(\sqrt{6} - \sqrt{2})) + 1/24*\sqrt{6}*\log(x^2 + 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) + 1/24*\sqrt{6}*\log(x^2 + 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} - \sqrt{2}) + 1)$

maple [C] time = 0.01, size = 30, normalized size = 0.11

$$\frac{\text{RootOf}(9_Z^4 + 1) \ln\left(3 \text{RootOf}(9_Z^4 + 1)^2 + 3 \text{RootOf}(9_Z^4 + 1)x + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8-x^4+1),x)

[Out] $1/4*\text{sum}(_R*\ln(3*_R^2+3*_R*x+x^2), _R=\text{RootOf}(9*_Z^4+1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")

[Out] integrate(1/(x^8 - x^4 + 1), x)

mupad [B] time = 0.04, size = 53, normalized size = 0.19

$$\sqrt{6} \operatorname{atan} \left(\frac{\sqrt{6} x \left(\frac{1}{3} + \frac{1}{3}i \right)}{\frac{2x^2}{3} - \frac{2}{3}i} \right) \left(-\frac{1}{12} - \frac{1}{12}i \right) + \sqrt{6} \operatorname{atan} \left(\frac{\sqrt{6} x \left(\frac{1}{3} - \frac{1}{3}i \right)}{\frac{2x^2}{3} + \frac{2}{3}i} \right) \left(-\frac{1}{12} + \frac{1}{12}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8 - x^4 + 1),x)

[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12)

sympy [A] time = 0.22, size = 165, normalized size = 0.60

$$\frac{\sqrt{6} \left(2 \operatorname{atan} \left(\frac{\sqrt{6}x}{3} - \frac{1}{3} \right) + 2 \operatorname{atan} \left(\sqrt{6}x^3 - 4x^2 + 2\sqrt{6}x - 3 \right) \right)}{24} + \frac{\sqrt{6} \left(2 \operatorname{atan} \left(\frac{\sqrt{6}x}{3} + \frac{1}{3} \right) + 2 \operatorname{atan} \left(\sqrt{6}x^3 + 4x^2 + 2\sqrt{6}x + 3 \right) \right)}{24} - \frac{\sqrt{6} \log(x^4 - \sqrt{6}x^3 + 3x^2 - \sqrt{6}x + 1)}{24} + \frac{\sqrt{6} \log(x^4 + \sqrt{6}x^3 + 3x^2 + \sqrt{6}x + 1)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**8-x**4+1),x)

[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24

$$3.304 \quad \int \frac{1}{x^2(1-x^4+x^8)} dx$$

Optimal. Leaf size=360

$$\frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 - \sqrt{2+\sqrt{3}}x + 1\right) + \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 + \sqrt{2+\sqrt{3}}x + 1\right) - \frac{1}{x} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} - \frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} + \frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})}$$

Rubi [A] time = 0.24, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1368, 1506, 1279, 1169, 634, 618, 204, 628}

$$\frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 - \sqrt{2+\sqrt{3}}x + 1\right) + \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 + \sqrt{2+\sqrt{3}}x + 1\right) - \frac{1}{x} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} - \frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} + \frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 - x^4 + x^8)),x]

[Out] $-x^{-1} + \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}} - 2x}{\sqrt{2+\sqrt{3}}}\right] / (4\sqrt{3}(2-\sqrt{3})) - \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}} - 2x}{\sqrt{2-\sqrt{3}}}\right] / (4\sqrt{3}(2+\sqrt{3})) - \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}} + 2x}{\sqrt{2+\sqrt{3}}}\right] / (4\sqrt{3}(2-\sqrt{3})) + \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}} + 2x}{\sqrt{2-\sqrt{3}}}\right] / (4\sqrt{3}(2+\sqrt{3})) + (\sqrt{3}(2-\sqrt{3})/3) \text{Log}[1 - \sqrt{2-\sqrt{3}}x + x^2] / 8 - (\sqrt{3}(2-\sqrt{3})/3) \text{Log}[1 + \sqrt{2-\sqrt{3}}x + x^2] / 8 - (\sqrt{3}(2+\sqrt{3})/3) \text{Log}[1 - \sqrt{2+\sqrt{3}}x + x^2] / 8 + (\sqrt{3}(2+\sqrt{3})/3) \text{Log}[1 + \sqrt{2+\sqrt{3}}x + x^2] / 8$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1279

Int[((f_.)*(x_)^m)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1368

Int[((d_.)*(x_)^m)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1506

Int((((f_.)*(x_)^m)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q - b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x] / (q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r + (c*d - e*q)*x^(n/2), x] / (q + r*x^(n/2) + c*x^n), x], x]]] /; !LtQ[2*c*q - b*c, 0] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c

, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2(1-x^4+x^8)} dx &= -\frac{1}{x} + \int \frac{x^2(1-x^4)}{1-x^4+x^8} dx \\
 &= -\frac{1}{x} + \frac{\int \frac{x^2(\sqrt{3}-2x^2)}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} + \frac{\int \frac{x^2(\sqrt{3}+2x^2)}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
 &= -\frac{1}{x} - \frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} - \frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
 &= -\frac{1}{x} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}-(2-\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}+(2-\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}-(-2-\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}+(-2+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} \\
 &= -\frac{1}{x} + \frac{1}{8}\sqrt{\frac{1}{3}(7-4\sqrt{3})} \int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx + \frac{1}{8}\sqrt{\frac{1}{3}(7-4\sqrt{3})} \int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx \\
 &= -\frac{1}{x} + \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}} \log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) \\
 &= -\frac{1}{x} + \frac{1}{4}\sqrt{\frac{1}{3}(2+\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}(2-\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)
 \end{aligned}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\#1^4 \log(x - \#1) - \log(x - \#1)}{2\#1^5 - \#1}\&\right] - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(1 - x^4 + x^8)),x]

[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 + 2*#1^5) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1-x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(1 - x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^2*(1 - x^4 + x^8)), x]

fricas [B] time = 1.56, size = 732, normalized size = 2.03

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/96*(8*\sqrt{6}*\sqrt{2}*x*\sqrt{\sqrt{3}+2}*\arctan(1/6*\sqrt{6}*\sqrt{12*x^2} \\ & + 2*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2}*x)*\sqrt{\sqrt{3}+2} + 12)*(s \\ & \sqrt{3}*\sqrt{2} - 2*\sqrt{2})*\sqrt{\sqrt{3}+2} + 1/3*\sqrt{6}*(2*\sqrt{3}*\sqrt{2} \\ & *x - 3*\sqrt{2}*x)*\sqrt{\sqrt{3}+2} - \sqrt{3} + 2) + 8*\sqrt{6}*\sqrt{2}*x \\ & *\sqrt{\sqrt{3}+2}*\arctan(1/6*\sqrt{6}*\sqrt{12*x^2 - 2*\sqrt{6}*(2*\sqrt{3}*\sqrt{2} \\ & *x - 3*\sqrt{2}*x)*\sqrt{\sqrt{3}+2} + 12)*(\sqrt{3}*\sqrt{2} - 2*\sqrt{2} \\ &)*\sqrt{\sqrt{3}+2} + 1/3*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2}*x)*\sqrt{ \\ & \sqrt{3}+2} + \sqrt{3} - 2) + 4*\sqrt{6}*\sqrt{2}*x*\sqrt{-4*\sqrt{3}+8}*\arct \\ & \arctan(1/12*\sqrt{6}*\sqrt{12*x^2 + \sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2}*x)*\sqrt{ \\ & -4*\sqrt{3}+8} + 12)*(\sqrt{3}*\sqrt{2} + 2*\sqrt{2})*\sqrt{-4*\sqrt{3}+8} \\ &) - 1/6*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2}*x)*\sqrt{-4*\sqrt{3}+8} - \\ & \sqrt{3} - 2) + 4*\sqrt{6}*\sqrt{2}*x*\sqrt{-4*\sqrt{3}+8}*\arctan(1/12*\sqrt{6} \\ & *\sqrt{12*x^2 - \sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2}*x)*\sqrt{-4*\sqrt{3} \\ & + 8} + 12)*(\sqrt{3}*\sqrt{2} + 2*\sqrt{2})*\sqrt{-4*\sqrt{3}+8} - 1/6*\sqrt{6} \\ & *(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2}*x)*\sqrt{-4*\sqrt{3}+8} + \sqrt{3} + 2) - \\ & 2*\sqrt{6}*(\sqrt{3}*\sqrt{2}*x - 2*\sqrt{2}*x)*\sqrt{\sqrt{3}+2}*\log(12*x^2 + \\ & 2*\sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2}*x)*\sqrt{\sqrt{3}+2} + 12) + 2*s \\ & \sqrt{6}*(\sqrt{3}*\sqrt{2}*x - 2*\sqrt{2}*x)*\sqrt{\sqrt{3}+2}*\log(12*x^2 - 2*s \\ & \sqrt{6}*(2*\sqrt{3}*\sqrt{2}*x - 3*\sqrt{2}*x)*\sqrt{\sqrt{3}+2} + 12) - \sqrt{6} \\ &)*(\sqrt{3}*\sqrt{2}*x + 2*\sqrt{2}*x)*\sqrt{-4*\sqrt{3}+8}*\log(12*x^2 + \sqrt{6} \\ & *(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2}*x)*\sqrt{-4*\sqrt{3}+8} + 12) + \sqrt{6} \\ & *(\sqrt{3}*\sqrt{2}*x + 2*\sqrt{2}*x)*\sqrt{-4*\sqrt{3}+8}*\log(12*x^2 - \sqrt{6} \\ & *(2*\sqrt{3}*\sqrt{2}*x + 3*\sqrt{2}*x)*\sqrt{-4*\sqrt{3}+8} + 12) + 96)/x \end{aligned}$$

giac [A] time = 0.46, size = 258, normalized size = 0.72

$$\frac{1}{24}(\sqrt{6+3\sqrt{2}})\arctan\left(\frac{43+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}(\sqrt{6+3\sqrt{2}})\arctan\left(\frac{43-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}(\sqrt{6-3\sqrt{2}})\arctan\left(\frac{43+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}(\sqrt{6-3\sqrt{2}})\arctan\left(\frac{43-\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}(\sqrt{6+3\sqrt{2}})\log\left(x+\frac{1}{2}(\sqrt{6}+\sqrt{2})+1\right) + \frac{1}{24}(\sqrt{6+3\sqrt{2}})\log\left(x-\frac{1}{2}(\sqrt{6}+\sqrt{2})+1\right) + \frac{1}{24}(\sqrt{6-3\sqrt{2}})\log\left(x+\frac{1}{2}(\sqrt{6}-\sqrt{2})+1\right) - \frac{1}{24}(\sqrt{6-3\sqrt{2}})\log\left(x-\frac{1}{2}(\sqrt{6}-\sqrt{2})+1\right) - \frac{1}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="giac")

[Out]
$$-1/24*(\sqrt{6} + 3*\sqrt{2})*\arctan((4*x + \sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})) - 1/24*(\sqrt{6} + 3*\sqrt{2})*\arctan((4*x - \sqrt{6} + \sqrt{2})/(\sqrt{6} + \sqrt{2})) - 1/24*(\sqrt{6} - 3*\sqrt{2})*\arctan((4*x + \sqrt{6} + \sqrt{2})/(\sqrt{6} - \sqrt{2})) - 1/24*(\sqrt{6} - 3*\sqrt{2})*\arctan((4*x - \sqrt{6} - \sqrt{2})/(\sqrt{6} - \sqrt{2})) + 1/48*(\sqrt{6} + 3*\sqrt{2})*\log(x^2 + 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) - 1/48*(\sqrt{6} + 3*\sqrt{2})*\log(x^2 - 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) + 1/48*(\sqrt{6} - 3*\sqrt{2})*\log(x^2 + 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - 1/48*(\sqrt{6} - 3*\sqrt{2})*\log(x^2 - 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - 1/x$$

maple [C] time = 0.02, size = 52, normalized size = 0.14

$$\frac{\left(\text{RootOf}\left(-Z^8 - Z^4 + 1\right)^6 - \text{RootOf}\left(-Z^8 - Z^4 + 1\right)^2\right) \ln\left(-\text{RootOf}\left(-Z^8 - Z^4 + 1\right) + x\right)}{4\left(2\text{RootOf}\left(-Z^8 - Z^4 + 1\right)^7 - \text{RootOf}\left(-Z^8 - Z^4 + 1\right)^3\right)} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(x^8-x^4+1),x)

[Out]
$$-1/x - 1/4*\sum((_R^6 - _R^2)/((2*_R^7 - _R^3)*\ln(-_R + x)), _R = \text{RootOf}(-Z^8 - Z^4 + 1))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x} - \int \frac{x^6 - x^2}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="maxima")

[Out]
$$-1/x - \text{integrate}((x^6 - x^2)/(x^8 - x^4 + 1), x)$$

mupad [B] time = 1.29, size = 253, normalized size = 0.70

$$-\frac{1}{x} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{x(8-\sqrt{3}8i)^{1/4}}{2(-1+\sqrt{3}i)} + \frac{\sqrt{3}x(8-\sqrt{3}8i)^{1/4}i}{2(-1+\sqrt{3}i)}\right)(8-\sqrt{3}8i)^{1/4}i + \sqrt{3} \operatorname{atan}\left(\frac{x(8-\sqrt{3}8i)^{1/4}i}{2(-1+\sqrt{3}i)} - \frac{\sqrt{3}x(8-\sqrt{3}8i)^{1/4}}{2(-1+\sqrt{3}i)}\right)(8-\sqrt{3}8i)^{1/4}}{12} + \frac{2^{3/4}\sqrt{3} \operatorname{atan}\left(\frac{2^{3/4}x}{2(1+\sqrt{3}i)^{3/4}} - \frac{2^{3/4}\sqrt{3}x1i}{2(1+\sqrt{3}i)^{3/4}}\right)(1+\sqrt{3}1i)^{1/4} + 2^{3/4}\sqrt{3} \operatorname{atan}\left(\frac{2^{3/4}x1i}{2(1+\sqrt{3}i)^{3/4}} + \frac{2^{3/4}\sqrt{3}x}{2(1+\sqrt{3}i)^{3/4}}\right)(1+\sqrt{3}1i)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(x^8 - x^4 + 1)),x)

[Out]
$$(3^{1/2}*\operatorname{atan}((x*(8 - 3^{1/2})*8i)^{1/4})/(2*(3^{1/2}*1i - 1)) + (3^{1/2})*x*(8 - 3^{1/2})*8i)^{1/4}*1i)/(2*(3^{1/2}*1i - 1))*(8 - 3^{1/2})*8i)^{1/4}*1i$$

$$\begin{aligned} & /12 - 1/x - (3^{(1/2)} * \operatorname{atan}((x * (8 - 3^{(1/2)} * 8i)^{(1/4)} * 1i) / (2 * (3^{(1/2)} * 1i - 1) \\ &) - (3^{(1/2)} * x * (8 - 3^{(1/2)} * 8i)^{(1/4)} / (2 * (3^{(1/2)} * 1i - 1))) * (8 - 3^{(1/2)} * 8 \\ & i)^{(1/4)}) / 12 + (2^{(3/4)} * 3^{(1/2)} * \operatorname{atan}((2^{(3/4)} * x) / (2 * (3^{(1/2)} * 1i + 1)^{(3/4)})) \\ & - (2^{(3/4)} * 3^{(1/2)} * x * 1i) / (2 * (3^{(1/2)} * 1i + 1)^{(3/4)})) * (3^{(1/2)} * 1i + 1)^{(1/4)} \\ &) * 1i) / 12 - (2^{(3/4)} * 3^{(1/2)} * \operatorname{atan}((2^{(3/4)} * x * 1i) / (2 * (3^{(1/2)} * 1i + 1)^{(3/4)})) \\ & + (2^{(3/4)} * 3^{(1/2)} * x) / (2 * (3^{(1/2)} * 1i + 1)^{(3/4)})) * (3^{(1/2)} * 1i + 1)^{(1/4)}) / 1 \\ & 2 \end{aligned}$$

sympy [A] time = 3.29, size = 29, normalized size = 0.08

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(-442368t^7 + 384t^3 + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(x**8-x**4+1),x)

[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 + 384*_t**3 + x))) - 1/x

$$3.305 \quad \int \frac{1}{x^4(1-x^4+x^8)} dx$$

Optimal. Leaf size=370

$$-\frac{1}{3x^3} + \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})}$$

Rubi [A] time = 0.24, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1368, 1421, 1169, 634, 618, 204, 628}

$$\frac{1}{3x^3} + \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log(x^2 - \sqrt{2-\sqrt{3}}x + 1) - \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log(x^2 + \sqrt{2-\sqrt{3}}x + 1) + \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log(x^2 - \sqrt{2+\sqrt{3}}x + 1) + \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log(x^2 + \sqrt{2+\sqrt{3}}x + 1) - \frac{1}{4} \sqrt{\frac{1}{3}(2+\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4} \sqrt{\frac{1}{3}(2-\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4} \sqrt{\frac{1}{3}(2+\sqrt{3})} \tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4} \sqrt{\frac{1}{3}(2-\sqrt{3})} \tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 - x^4 + x^8)),x]

[Out] -1/(3*x^3) - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]])/4 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_.)*(x_)^(m_))*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(1-x^4+x^8)} dx &= -\frac{1}{3x^3} + \frac{1}{3} \int \frac{3-3x^4}{1-x^4+x^8} dx \\
&= -\frac{1}{3x^3} - \frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4} dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4} dx}{2\sqrt{3}} \\
&= -\frac{1}{3x^3} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})} - (-2+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})} + (-2+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})} - (2+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} \\
&= -\frac{1}{3x^3} - \frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx - \frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx \\
&= -\frac{1}{3x^3} + \frac{1}{8}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{2}{3}} - \frac{1}{\sqrt{3}} \log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) \\
&= -\frac{1}{3x^3} - \frac{1}{4}\sqrt{\frac{1}{3}}(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 65, normalized size = 0.18

$$-\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\#1^4 \log(x - \#1) - \log(x - \#1)}{2\#1^7 - \#1^3}\&\right] - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(1 - x^4 + x^8)),x]

[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 + 2*#1^7) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(1-x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(1 - x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^4*(1 - x^4 + x^8)), x]

fricas [B] time = 0.96, size = 756, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{96} \cdot (8 \cdot \sqrt{6} \cdot \sqrt{2} \cdot x^3 \cdot \sqrt{\sqrt{3} + 2} \cdot \arctan\left(\frac{1}{6} \cdot \sqrt{6} \cdot \sqrt{2} \cdot \sqrt{12x^2 + 2\sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x - 3\sqrt{2}x)} \cdot \sqrt{\sqrt{3} + 2} + 12\right) \cdot (\sqrt{3}\sqrt{2} - 2\sqrt{2}) \cdot \sqrt{\sqrt{3} + 2} + \frac{1}{3} \cdot \sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x - 3\sqrt{2}x) \cdot \sqrt{\sqrt{3} + 2} - \sqrt{3} + 2) + 8 \cdot \sqrt{6} \cdot \sqrt{2} \cdot x^3 \cdot \sqrt{\sqrt{3} + 2} \cdot \arctan\left(\frac{1}{6} \cdot \sqrt{6} \cdot \sqrt{2} \cdot \sqrt{12x^2 - 2\sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x - 3\sqrt{2}x)} \cdot \sqrt{\sqrt{3} + 2} + 12\right) \cdot (\sqrt{3}\sqrt{2} - 2\sqrt{2}) \cdot \sqrt{\sqrt{3} + 2} + \frac{1}{3} \cdot \sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x - 3\sqrt{2}x) \cdot \sqrt{\sqrt{3} + 2} + \sqrt{3} - 2) + 4 \cdot \sqrt{6} \cdot \sqrt{2} \cdot x^3 \cdot \sqrt{-4\sqrt{3} + 8} \cdot \arctan\left(\frac{1}{12} \cdot \sqrt{6} \cdot \sqrt{2} \cdot \sqrt{12x^2 + \sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x + 3\sqrt{2}x)} \cdot \sqrt{-4\sqrt{3} + 8} + 12\right) \cdot (\sqrt{3}\sqrt{2} + 2\sqrt{2}) \cdot \sqrt{-4\sqrt{3} + 8} - \frac{1}{6} \cdot \sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x + 3\sqrt{2}x) \cdot \sqrt{-4\sqrt{3} + 8} - \sqrt{3} - 2) + 4 \cdot \sqrt{6} \cdot \sqrt{2} \cdot x^3 \cdot \sqrt{-4\sqrt{3} + 8} \cdot \arctan\left(\frac{1}{12} \cdot \sqrt{6} \cdot \sqrt{2} \cdot \sqrt{12x^2 - \sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x + 3\sqrt{2}x)} \cdot \sqrt{-4\sqrt{3} + 8} + 12\right) \cdot (\sqrt{3}\sqrt{2} + 2\sqrt{2}) \cdot \sqrt{-4\sqrt{3} + 8} - \frac{1}{6} \cdot \sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x + 3\sqrt{2}x) \cdot \sqrt{-4\sqrt{3} + 8} + \sqrt{3} + 2) + 2 \cdot \sqrt{6} \cdot (\sqrt{3}\sqrt{2}x^3 - 2\sqrt{2}x^3) \cdot \sqrt{\sqrt{3} + 2} \cdot \log(12x^2 + 2\sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x - 3\sqrt{2}x) \cdot \sqrt{\sqrt{3} + 2} + 12) - 2 \cdot \sqrt{6} \cdot (\sqrt{3}\sqrt{2}x^3 - 2\sqrt{2}x^3) \cdot \sqrt{\sqrt{3} + 2} \cdot \log(12x^2 - 2\sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x - 3\sqrt{2}x) \cdot \sqrt{\sqrt{3} + 2} + 12) + \sqrt{6} \cdot (\sqrt{3}\sqrt{2}x^3 + 2\sqrt{2}x^3) \cdot \sqrt{-4\sqrt{3} + 8} \cdot \log(12x^2 + \sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x + 3\sqrt{2}x) \cdot \sqrt{-4\sqrt{3} + 8} + 12) - \sqrt{6} \cdot (\sqrt{3}\sqrt{2}x^3 + 2\sqrt{2}x^3) \cdot \sqrt{-4\sqrt{3} + 8} \cdot \log(12x^2 - \sqrt{6} \cdot (2\sqrt{3}\sqrt{2}x + 3\sqrt{2}x) \cdot \sqrt{-4\sqrt{3} + 8} + 12) - 32) / x^3$

giac [A] time = 0.35, size = 258, normalized size = 0.70

$\frac{1}{24}(\sqrt{6} + 3\sqrt{2}) \arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{24}(\sqrt{6} + 3\sqrt{2}) \arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{24}(\sqrt{6} - 3\sqrt{2}) \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{24}(\sqrt{6} - 3\sqrt{2}) \arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{24}(\sqrt{6} + 3\sqrt{2}) \log\left(x + \frac{1}{2}(\sqrt{6} + \sqrt{2})\right) - \frac{1}{24}(\sqrt{6} + 3\sqrt{2}) \log\left(x - \frac{1}{2}(\sqrt{6} + \sqrt{2})\right) + \frac{1}{24}(\sqrt{6} - 3\sqrt{2}) \log\left(x + \frac{1}{2}(\sqrt{6} - \sqrt{2})\right) - \frac{1}{24}(\sqrt{6} - 3\sqrt{2}) \log\left(x - \frac{1}{2}(\sqrt{6} - \sqrt{2})\right) - \frac{1}{32}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="giac")

[Out] $\frac{1}{24} \cdot (\sqrt{6} + 3\sqrt{2}) \cdot \arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{24} \cdot (\sqrt{6} + 3\sqrt{2}) \cdot \arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{24} \cdot (\sqrt{6} - 3\sqrt{2}) \cdot \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) - \frac{1}{24} \cdot (\sqrt{6} - 3\sqrt{2}) \cdot \arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{24} \cdot (\sqrt{6} + 3\sqrt{2}) \cdot \log\left(x + \frac{1}{2}(\sqrt{6} + \sqrt{2})\right) - \frac{1}{24} \cdot (\sqrt{6} + 3\sqrt{2}) \cdot \log\left(x - \frac{1}{2}(\sqrt{6} + \sqrt{2})\right) + \frac{1}{24} \cdot (\sqrt{6} - 3\sqrt{2}) \cdot \log\left(x + \frac{1}{2}(\sqrt{6} - \sqrt{2})\right) - \frac{1}{24} \cdot (\sqrt{6} - 3\sqrt{2}) \cdot \log\left(x - \frac{1}{2}(\sqrt{6} - \sqrt{2})\right) - \frac{1}{32}$

) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/3/x^3

maple [C] time = 0.01, size = 50, normalized size = 0.14

$$\frac{\left(-\text{RootOf}\left(-Z^8 - Z^4 + 1\right)^4 + 1\right) \ln\left(-\text{RootOf}\left(-Z^8 - Z^4 + 1\right) + x\right)}{8 \text{RootOf}\left(-Z^8 - Z^4 + 1\right)^7 - 4 \text{RootOf}\left(-Z^8 - Z^4 + 1\right)^3} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(x^8-x^4+1),x)

[Out] -1/3/x^3+1/4*sum((-R^4+1)/(2*_R^7-_R^3)*ln(-R+x),_R=RootOf(-Z^8-Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3x^3} - \int \frac{x^4 - 1}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="maxima")

[Out] -1/3/x^3 - integrate((x^4 - 1)/(x^8 - x^4 + 1), x)

mupad [B] time = 1.29, size = 213, normalized size = 0.58

$$-\frac{1}{3x^3} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{x}{(8-\sqrt{3}8)^{1/4}} + \frac{\sqrt{3}x11}{(8-\sqrt{3}8)^{1/4}}\right) (8-\sqrt{3}8)^{1/4} \operatorname{li}\left(\frac{\sqrt{3}x}{(8-\sqrt{3}8)^{1/4}} - \frac{\sqrt{3}x}{(8-\sqrt{3}8)^{1/4}}\right) (8-\sqrt{3}8)^{1/4}}{12} + \frac{2^{3/4}\sqrt{3} \operatorname{atan}\left(\frac{2^{1/4}x}{2(1+\sqrt{3}11)^{1/4}} - \frac{2^{1/4}\sqrt{3}x11}{2(1+\sqrt{3}11)^{1/4}}\right) (1+\sqrt{3}11)^{1/4} \operatorname{li}\left(\frac{2^{1/4}x}{2(1+\sqrt{3}11)^{1/4}} + \frac{2^{1/4}\sqrt{3}x}{2(1+\sqrt{3}11)^{1/4}}\right) (1+\sqrt{3}11)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(x^8 - x^4 + 1)),x)

[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4)))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - 1/(3*x^3) + (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12

sympy [A] time = 3.25, size = 31, normalized size = 0.08

$$\text{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log(-9216t^5 + 8t + x)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(x**8-x**4+1),x)

[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-9216*_t**5 + 8*_t + x))) - 1/(3*x**3)

$$3.306 \quad \int \frac{1}{x^6(1-x^4+x^8)} dx$$

Optimal. Leaf size=287

$$\frac{1}{5x^5} - \frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}}x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}}x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}}x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}}x + 1\right)}{4\sqrt{6}}$$

Rubi [A] time = 0.24, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1368, 1504, 12, 1372, 1169, 634, 618, 204, 628}

$$\frac{1}{5x^5} - \frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}}x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}}x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}}x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}}x + 1\right)}{4\sqrt{6}} - \frac{1}{x} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(1 - x^4 + x^8)),x]

[Out] $-1/(5*x^5) - x^{(-1)} + \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[3]] - 2*x)/\text{Sqrt}[2 + \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) + \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[3]] - 2*x)/\text{Sqrt}[2 - \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) - \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[3]] + 2*x)/\text{Sqrt}[2 + \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) - \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[3]] + 2*x)/\text{Sqrt}[2 - \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) - \text{Log}[1 - \text{Sqrt}[2 - \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6]) + \text{Log}[1 + \text{Sqrt}[2 - \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6]) - \text{Log}[1 - \text{Sqrt}[2 + \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6]) + \text{Log}[1 + \text{Sqrt}[2 + \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6])$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[((d_)*(x_)^m)/((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^p, x_
Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m +
1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
c*(m + 2*n*(p + 1) + 1)*x^n*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1372

```
Int[(x_)^m/((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)), x_Symbol] := W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_)*(x_)^m)/((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_)) + (
c_)*(x_)^(n2_))^p, x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
```

```
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
&& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6(1-x^4+x^8)} dx &= -\frac{1}{5x^5} + \frac{1}{5} \int \frac{5-5x^4}{x^2(1-x^4+x^8)} dx \\
&= -\frac{1}{5x^5} - \frac{1}{x} - \frac{1}{5} \int \frac{5x^6}{1-x^4+x^8} dx \\
&= -\frac{1}{5x^5} - \frac{1}{x} - \int \frac{x^6}{1-x^4+x^8} dx \\
&= -\frac{1}{5x^5} - \frac{1}{x} + \frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} - \frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= -\frac{1}{5x^5} - \frac{1}{x} - \frac{\int \frac{\sqrt{2-\sqrt{3}}-(1-\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+(1-\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-(1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+(1+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} \\
&= -\frac{1}{5x^5} - \frac{1}{x} - \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} \\
&= -\frac{1}{5x^5} - \frac{1}{x} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} \\
&= -\frac{1}{5x^5} - \frac{1}{x} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.19

$$-\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\#1^3 \log(x - \#1)}{2\#1^4 - 1}\&\right] - \frac{1}{5x^5} - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(1 - x^4 + x^8)),x]

[Out] $-1/5*1/x^5 - x^{(-1)} - \text{RootSum}[1 - \#1^4 + \#1^8 \& , (\text{Log}[x - \#1]*\#1^3)/(-1 + 2*\#1^4) \&]/4$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6(1 - x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^6*(1 - x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^6*(1 - x^4 + x^8)), x]

fricas [A] time = 2.13, size = 238, normalized size = 0.83

$$\frac{20\sqrt{3}\sqrt{2}x^5 \arctan\left(\frac{-\sqrt{3}\sqrt{2}(x^3-x) + \sqrt{4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x-2)}{3x^2-2}\right) + 20\sqrt{3}\sqrt{2}x^5 \arctan\left(\frac{\sqrt{3}\sqrt{2}(x^3-x) - \sqrt{4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x+2)}{3x^2-2}\right) + 5\sqrt{3}\sqrt{2}x^5 \log(x^4 + \sqrt{3}\sqrt{2}(x^3+x) + 3x^2+1) - 5\sqrt{3}\sqrt{2}x^5 \log(x^4 - \sqrt{3}\sqrt{2}(x^3+x) + 3x^2+1) - 120x^4 - 24}{120x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="fricas")

[Out] $1/120*(20*\text{sqrt}(3)*\text{sqrt}(2)*x^5*\arctan(-(\text{sqrt}(3)*\text{sqrt}(2)*(x^3 - x) + x^2 - \text{sqrt}(x^4 + \text{sqrt}(3)*\text{sqrt}(2)*(x^3 + x) + 3*x^2 + 1))*(\text{sqrt}(3)*\text{sqrt}(2)*x - 2)))/(3*x^2 - 2)) + 20*\text{sqrt}(3)*\text{sqrt}(2)*x^5*\arctan(-(\text{sqrt}(3)*\text{sqrt}(2)*(x^3 - x) - x^2 - \text{sqrt}(x^4 - \text{sqrt}(3)*\text{sqrt}(2)*(x^3 + x) + 3*x^2 + 1))*(\text{sqrt}(3)*\text{sqrt}(2)*x + 2)))/(3*x^2 - 2)) + 5*\text{sqrt}(3)*\text{sqrt}(2)*x^5*\log(x^4 + \text{sqrt}(3)*\text{sqrt}(2)*(x^3 + x) + 3*x^2 + 1) - 5*\text{sqrt}(3)*\text{sqrt}(2)*x^5*\log(x^4 - \text{sqrt}(3)*\text{sqrt}(2)*(x^3 + x) + 3*x^2 + 1) - 120*x^4 - 24)/x^5$

giac [A] time = 0.38, size = 217, normalized size = 0.76

$$\frac{1}{12}\sqrt{6} \arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6} \arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\sqrt{6} \arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{12}\sqrt{6} \arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\sqrt{6} \log\left(x^2 + \frac{1}{2}(\sqrt{6}+\sqrt{2})+1\right) - \frac{1}{24}\sqrt{6} \log\left(x^2 - \frac{1}{2}(\sqrt{6}+\sqrt{2})+1\right) + \frac{1}{24}\sqrt{6} \log\left(x^2 + \frac{1}{2}(\sqrt{6}-\sqrt{2})+1\right) - \frac{1}{24}\sqrt{6} \log\left(x^2 - \frac{1}{2}(\sqrt{6}-\sqrt{2})+1\right) - \frac{5x^4+1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="giac")

[Out] $-1/12*\text{sqrt}(6)*\arctan((4*x + \text{sqrt}(6) - \text{sqrt}(2))/(\text{sqrt}(6) + \text{sqrt}(2))) - 1/12*\text{sqrt}(6)*\arctan((4*x - \text{sqrt}(6) + \text{sqrt}(2))/(\text{sqrt}(6) + \text{sqrt}(2))) - 1/12*\text{sqrt}(6)*\arctan((4*x + \text{sqrt}(6) + \text{sqrt}(2))/(\text{sqrt}(6) - \text{sqrt}(2))) - 1/12*\text{sqrt}(6)*\arctan((4*x - \text{sqrt}(6) - \text{sqrt}(2))/(\text{sqrt}(6) - \text{sqrt}(2))) + 1/24*\text{sqrt}(6)*\log(x^2 + 1/2*x*(\text{sqrt}(6) + \text{sqrt}(2)) + 1) - 1/24*\text{sqrt}(6)*\log(x^2 - 1/2*x*(\text{sqrt}(6) + \text{sqrt}(2)) + 1) + 1/24*\text{sqrt}(6)*\log(x^2 + 1/2*x*(\text{sqrt}(6) - \text{sqrt}(2)) + 1) - 1/24*\text{sqrt}(6)*\log(x^2 - 1/2*x*(\text{sqrt}(6) - \text{sqrt}(2)) + 1) - 1/5*(5*x^4 + 1)/x^5$

maple [C] time = 0.01, size = 43, normalized size = 0.15

$$\frac{\text{RootOf}(9_Z^4 + 1) \ln\left(9 \text{RootOf}(9_Z^4 + 1)^3 x - 3 \text{RootOf}(9_Z^4 + 1)^2 + x^2\right)}{4} - \frac{1}{x} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(x^8-x^4+1),x)

[Out] -1/5/x^5-1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{5x^4 + 1}{5x^5} - \int \frac{x^6}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="maxima")

[Out] -1/5*(5*x^4 + 1)/x^5 - integrate(x^6/(x^8 - x^4 + 1), x)

mupad [B] time = 1.30, size = 63, normalized size = 0.22

$$-\frac{x^4 + \frac{1}{5}}{x^5} + \sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x \left(\frac{1}{3} + \frac{1}{3}i\right)}{\frac{2x^2}{3} - \frac{2}{3}i}\right) \left(\frac{1}{12} - \frac{1}{12}i\right) + \sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x \left(\frac{1}{3} - \frac{1}{3}i\right)}{\frac{2x^2}{3} + \frac{2}{3}i}\right) \left(\frac{1}{12} + \frac{1}{12}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(x^8 - x^4 + 1)),x)

[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) + 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) - (x^4 + 1/5)/x^5

sympy [A] time = 0.27, size = 182, normalized size = 0.63

$$\frac{\sqrt{6} \left(-2 \operatorname{atan}\left(\frac{\sqrt{6}x}{3} - \frac{1}{3}\right) - 2 \operatorname{atan}\left(\sqrt{6}x^3 - 4x^2 + 2\sqrt{6}x - 3\right)\right)}{24} + \frac{\sqrt{6} \left(-2 \operatorname{atan}\left(\frac{\sqrt{6}x}{3} + \frac{1}{3}\right) - 2 \operatorname{atan}\left(\sqrt{6}x^3 + 4x^2 + 2\sqrt{6}x + 3\right)\right)}{24} - \frac{\sqrt{6} \log(x^4 - \sqrt{6}x^3 + 3x^2 - \sqrt{6}x + 1)}{24} + \frac{\sqrt{6} \log(x^4 + \sqrt{6}x^3 + 3x^2 + \sqrt{6}x + 1)}{24} + \frac{-5x^4 - 1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**6/(x**8-x**4+1),x)

[Out] sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(6)*x - 3))/24 + sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24 + (-5*x**4 - 1)/(5*x**5)

$$3.307 \quad \int \frac{1}{x^8(1-x^4+x^8)} dx$$

Optimal. Leaf size=377

$$-\frac{1}{7x^7} - \frac{1}{3x^3} + \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2+\sqrt{3}}x + 1\right) + \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2+\sqrt{3}}x + 1\right)$$

Rubi [A] time = 0.29, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1368, 1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$\frac{1}{30} \frac{1}{x^7} + \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log(x^2 - \sqrt{2-\sqrt{3}}x + 1) - \frac{1}{8} \sqrt{\frac{1}{3}(2+\sqrt{3})} \log(x^2 + \sqrt{2-\sqrt{3}}x + 1) - \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log(x^2 - \sqrt{2+\sqrt{3}}x + 1) + \frac{1}{8} \sqrt{\frac{1}{3}(2-\sqrt{3})} \log(x^2 + \sqrt{2+\sqrt{3}}x + 1) - \frac{1}{4} \sqrt{\frac{1}{3}(2-\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4} \sqrt{\frac{1}{3}(2+\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4} \sqrt{\frac{1}{3}(2-\sqrt{3})} \tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4} \sqrt{\frac{1}{3}(2+\sqrt{3})} \tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^8*(1 - x^4 + x^8)),x]

[Out] -1/(7*x^7) - 1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
- 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_
Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1373

```
Int[(x_)^(m_)/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symbol] := W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
- n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
```

$b^2 - 4ac, 0] \ \&\& \ \text{IGtQ}[n/2, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{GeQ}[m, n/2] \ \&\& \ \text{LtQ}[m, (3n)/2] \ \&\& \ \text{NegQ}[b^2 - 4ac]$

Rule 1504

$\text{Int}[(f(x))^m * (d + e(x)^n) * (a + b(x)^n + c(x)^{2n})^p, x_Symbol] \ :> \ \text{Simp}[(d * (f * x)^{m+1} * (a + b * x^n + c * x^{2n})^{p+1}) / (a * f * (m+1)), x] + \text{Dist}[1 / (a * f^n * (m+1)), \text{Int}[(f * x)^{m+n} * (a + b * x^n + c * x^{2n})^p * \text{Simp}[a * e * (m+1) - b * d * (m+n * (p+1) + 1) - c * d * (m+2 * n * (p+1) + 1) * x^n, x], x] \ /; \ \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[n^2, 2 * n] \ \&\& \ \text{NeQ}[b^2 - 4 * a * c, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^8(1-x^4+x^8)} dx &= -\frac{1}{7x^7} + \frac{1}{7} \int \frac{7-7x^4}{x^4(1-x^4+x^8)} dx \\
 &= -\frac{1}{7x^7} - \frac{1}{3x^3} - \frac{1}{21} \int \frac{21x^4}{1-x^4+x^8} dx \\
 &= -\frac{1}{7x^7} - \frac{1}{3x^3} - \int \frac{x^4}{1-x^4+x^8} dx \\
 &= -\frac{1}{7x^7} - \frac{1}{3x^3} - \frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} + \frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
 &= -\frac{1}{7x^7} - \frac{1}{3x^3} + \frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} - \frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} - \frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} + \frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} \\
 &= -\frac{1}{7x^7} - \frac{1}{3x^3} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} - \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{8\sqrt{3}} \\
 &= -\frac{1}{7x^7} - \frac{1}{3x^3} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2+\sqrt{3})} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2+\sqrt{3})} \\
 &= -\frac{1}{7x^7} - \frac{1}{3x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.14

$$-\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\#1 \log(x - \#1)}{2\#1^4 - 1}\&\right] - \frac{1}{7x^7} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^8*(1 - x^4 + x^8)),x]

[Out] -1/7*1/x^7 - 1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8(1 - x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^8*(1 - x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^8*(1 - x^4 + x^8)), x]

fricas [B] time = 1.73, size = 614, normalized size = 1.63

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="fricas")

[Out] 1/672*(56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) + 56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 28*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) - 28*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2) - 224*x^4 - 14*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 - 2*sqrt(2)*x^7)*sqrt(sqrt(3) + 2)*log(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) + 14*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 - 2*sqrt(2)*x^7)*sqrt(sqrt(3) + 2)*log

$(-2*\sqrt{6}*\sqrt{3}*\sqrt{2}*x*\sqrt{\sqrt{3} + 2} + 12*x^2 + 12) - 7*\sqrt{6}*(\sqrt{3}*\sqrt{2}*x^7 + 2*\sqrt{2}*x^7)*\sqrt{-4*\sqrt{3} + 8}*\log(\sqrt{6}*\sqrt{3}*\sqrt{2}*x*\sqrt{-4*\sqrt{3} + 8} + 12*x^2 + 12) + 7*\sqrt{6}*(\sqrt{3}*\sqrt{2}*x^7 + 2*\sqrt{2}*x^7)*\sqrt{-4*\sqrt{3} + 8}*\log(-\sqrt{6}*\sqrt{3}*\sqrt{2}*x*\sqrt{-4*\sqrt{3} + 8} + 12*x^2 + 12) - 96)/x^7$

giac [A] time = 0.38, size = 265, normalized size = 0.70

$\frac{1}{24}(\sqrt{6}-3\sqrt{2})\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}(\sqrt{6}-3\sqrt{2})\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{24}(\sqrt{6}+3\sqrt{2})\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{24}(\sqrt{6}+3\sqrt{2})\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{48}(\sqrt{6}-3\sqrt{2})\log\left(x^2+\frac{1}{2}(\sqrt{6}+\sqrt{2})+1\right) + \frac{1}{48}(\sqrt{6}-3\sqrt{2})\log\left(x^2-\frac{1}{2}(\sqrt{6}+\sqrt{2})+1\right) + \frac{1}{48}(\sqrt{6}+3\sqrt{2})\log\left(x^2+\frac{1}{2}(\sqrt{6}-\sqrt{2})+1\right) + \frac{1}{48}(\sqrt{6}+3\sqrt{2})\log\left(x^2-\frac{1}{2}(\sqrt{6}-\sqrt{2})+1\right) - \frac{7x^4+3}{21x^7}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="giac")

[Out] $-1/24*(\sqrt{6} - 3*\sqrt{2})*\arctan((4*x + \sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})) - 1/24*(\sqrt{6} - 3*\sqrt{2})*\arctan((4*x - \sqrt{6} + \sqrt{2})/(\sqrt{6} + \sqrt{2})) - 1/24*(\sqrt{6} + 3*\sqrt{2})*\arctan((4*x + \sqrt{6} + \sqrt{2})/(\sqrt{6} - \sqrt{2})) - 1/24*(\sqrt{6} + 3*\sqrt{2})*\arctan((4*x - \sqrt{6} - \sqrt{2})/(\sqrt{6} - \sqrt{2})) - 1/48*(\sqrt{6} - 3*\sqrt{2})*\log(x^2 + 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) + 1/48*(\sqrt{6} - 3*\sqrt{2})*\log(x^2 - 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) - 1/48*(\sqrt{6} + 3*\sqrt{2})*\log(x^2 + 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) + 1/48*(\sqrt{6} + 3*\sqrt{2})*\log(x^2 - 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - 1/21*(7*x^4 + 3)/x^7$

maple [C] time = 0.02, size = 51, normalized size = 0.14

$$-\frac{\text{RootOf}(-Z^8 - Z^4 + 1)^4 \ln(-\text{RootOf}(-Z^8 - Z^4 + 1) + x)}{4 \left(2 \text{RootOf}(-Z^8 - Z^4 + 1)^7 - \text{RootOf}(-Z^8 - Z^4 + 1)^3 \right)} - \frac{1}{3x^3} - \frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^8/(x^8-x^4+1),x)

[Out] $-1/7/x^7 - 1/3/x^3 - 1/4*\text{sum}(1/(2*_R^7 - _R^3)*_R^4*\ln(-_R+x), _R=\text{RootOf}(-Z^8 - Z^4 + 1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{7x^4 + 3}{21x^7} - \int \frac{x^4}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="maxima")

[Out] $-1/21*(7*x^4 + 3)/x^7 - \text{integrate}(x^4/(x^8 - x^4 + 1), x)$

mupad [B] time = 0.06, size = 486, normalized size = 1.29

$$\frac{\sqrt{3} \operatorname{atan}\left(\frac{(1-i\sqrt{3})^{14}}{\sqrt{\frac{3}{4}(1-i\sqrt{3})^{14} + \frac{3}{4}(1+i\sqrt{3})^{14}}}\right) + \sqrt{3} \operatorname{atan}\left(\frac{(1+i\sqrt{3})^{14}}{\sqrt{\frac{3}{4}(1+i\sqrt{3})^{14} + \frac{3}{4}(1-i\sqrt{3})^{14}}}\right)}{12} + \frac{(8-\sqrt{3}i)^{14}i}{\sqrt{3}} \operatorname{atan}\left(\frac{(1-i\sqrt{3})^{14}}{\sqrt{\frac{3}{4}(1-i\sqrt{3})^{14} + \frac{3}{4}(1+i\sqrt{3})^{14}}}\right) - \frac{(8-\sqrt{3}i)^{14}}{\sqrt{3}} \operatorname{atan}\left(\frac{(1+i\sqrt{3})^{14}}{\sqrt{\frac{3}{4}(1+i\sqrt{3})^{14} + \frac{3}{4}(1-i\sqrt{3})^{14}}}\right)}{12} + \frac{2^{14}\sqrt{3} \operatorname{atan}\left(\frac{2^{14}(1-i\sqrt{3})^{14}}{\sqrt{\frac{3}{4}(1-i\sqrt{3})^{14} + \frac{3}{4}(1+i\sqrt{3})^{14}}}\right) - 2^{14}\sqrt{3} \operatorname{atan}\left(\frac{2^{14}(1+i\sqrt{3})^{14}}{\sqrt{\frac{3}{4}(1+i\sqrt{3})^{14} + \frac{3}{4}(1-i\sqrt{3})^{14}}}\right)}{12} + (1+\sqrt{3}i)^{14}i \operatorname{atan}\left(\frac{2^{14}(1-i\sqrt{3})^{14}}{\sqrt{\frac{3}{4}(1-i\sqrt{3})^{14} + \frac{3}{4}(1+i\sqrt{3})^{14}}}\right) - (1+\sqrt{3}i)^{14} \operatorname{atan}\left(\frac{2^{14}(1+i\sqrt{3})^{14}}{\sqrt{\frac{3}{4}(1+i\sqrt{3})^{14} + \frac{3}{4}(1-i\sqrt{3})^{14}}}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^8*(x^8 - x^4 + 1)),x)`

[Out] $(3^{1/2} \operatorname{atan}((x(8 - 3^{1/2} * 8i)^{1/4}) / (2 * ((3^{1/2} * (8 - 3^{1/2} * 8i)^{1/2})^{1/2} * i) / 4 + (8 - 3^{1/2} * 8i)^{1/2} / 4)) + (3^{1/2} * x * (8 - 3^{1/2} * 8i)^{1/4} * i) / (2 * ((3^{1/2} * (8 - 3^{1/2} * 8i)^{1/2})^{1/2} * i) / 4 + (8 - 3^{1/2} * 8i)^{1/2} / 4))) * (8 - 3^{1/2} * 8i)^{1/4} * i) / 12 - (x^4 / 3 + 1/7) / x^7 + (3^{1/2} \operatorname{atan}((x(8 - 3^{1/2} * 8i)^{1/4} * i) / (2 * ((3^{1/2} * (8 - 3^{1/2} * 8i)^{1/2})^{1/2} * i) / 4 + (8 - 3^{1/2} * 8i)^{1/2} / 4)) - (3^{1/2} * x * (8 - 3^{1/2} * 8i)^{1/4}) / (2 * ((3^{1/2} * (8 - 3^{1/2} * 8i)^{1/2})^{1/2} * i) / 4 + (8 - 3^{1/2} * 8i)^{1/2} / 4))) * (8 - 3^{1/2} * 8i)^{1/4} / 12 - (2^{3/4} * 3^{1/2} \operatorname{atan}((2^{3/4} * x * (3^{1/2} * i + 1)^{1/4}) / (2 * ((2^{1/2} * (3^{1/2} * i + 1)^{1/2})^{1/2} / 2 - (2^{1/2} * 3^{1/2} * (3^{1/2} * i + 1)^{1/2} * i) / 2))) - (2^{3/4} * 3^{1/2} * x * (3^{1/2} * i + 1)^{1/4} * i) / (2 * ((2^{1/2} * (3^{1/2} * i + 1)^{1/2})^{1/2} / 2 - (2^{1/2} * 3^{1/2} * (3^{1/2} * i + 1)^{1/2} * i) / 2))) * (3^{1/2} * i + 1)^{1/4} * i) / 12 - (2^{3/4} * 3^{1/2} \operatorname{atan}((2^{3/4} * x * (3^{1/2} * i + 1)^{1/4}) * i) / (2 * ((2^{1/2} * (3^{1/2} * i + 1)^{1/2})^{1/2} / 2 - (2^{1/2} * 3^{1/2} * (3^{1/2} * i + 1)^{1/2} * i) / 2))) + (2^{3/4} * 3^{1/2} * x * (3^{1/2} * i + 1)^{1/4}) / (2 * ((2^{1/2} * (3^{1/2} * i + 1)^{1/2})^{1/2} / 2 - (2^{1/2} * 3^{1/2} * (3^{1/2} * i + 1)^{1/2} * i) / 2))) * (3^{1/2} * i + 1)^{1/4} / 12$

sympy [A] time = 3.37, size = 37, normalized size = 0.10

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(18432t^5 - 4t + x\right)\right)\right) + \frac{-7x^4 - 3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**8/(x**8-x**4+1),x)`

[Out] `RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(18432*_t**5 - 4*_t + x))) + (-7*x**4 - 3)/(21*x**7)`

$$3.308 \quad \int \frac{x^{11}}{1+3x^4+x^8} dx$$

Optimal. Leaf size=62

$$\frac{x^4}{4} - \frac{1}{40} (15 - 7\sqrt{5}) \log(2x^4 - \sqrt{5} + 3) - \frac{1}{40} (15 + 7\sqrt{5}) \log(2x^4 + \sqrt{5} + 3)$$

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4} - \frac{1}{40} (15 - 7\sqrt{5}) \log(2x^4 - \sqrt{5} + 3) - \frac{1}{40} (15 + 7\sqrt{5}) \log(2x^4 + \sqrt{5} + 3)$$

Antiderivative was successfully verified.

[In] Int[x^11/(1 + 3*x^4 + x^8), x]

[Out] x^4/4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((15 + 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x

], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n^2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{1 + 3x^4 + x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{1 + 3x + x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{4} \text{Subst} \left(\int \frac{-1 - 3x}{1 + 3x + x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{40} (-15 + 7\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) - \frac{1}{40} (15 + 7\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} - \frac{1}{40} (15 - 7\sqrt{5}) \log(3 - \sqrt{5} + 2x^4) - \frac{1}{40} (15 + 7\sqrt{5}) \log(3 + \sqrt{5} + 2x^4)
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.92

$$\frac{1}{40} (10x^4 + (7\sqrt{5} - 15) \log(-2x^4 + \sqrt{5} - 3) - (15 + 7\sqrt{5}) \log(2x^4 + \sqrt{5} + 3))$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(1 + 3*x^4 + x^8), x]

[Out] (10*x^4 + (-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] - (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{1 + 3x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^11/(1 + 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^11/(1 + 3*x^4 + x^8), x]

fricas [A] time = 1.48, size = 62, normalized size = 1.00

$$\frac{1}{4} x^4 + \frac{7}{40} \sqrt{5} \log \left(\frac{2x^8 + 6x^4 - \sqrt{5}(2x^4 + 3) + 7}{x^8 + 3x^4 + 1} \right) - \frac{3}{8} \log(x^8 + 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸+3*x⁴+1),x, algorithm="fricas")

[Out] 1/4*x⁴ + 7/40*sqrt(5)*log((2*x⁸ + 6*x⁴ - sqrt(5)*(2*x⁴ + 3) + 7)/(x⁸ + 3*x⁴ + 1)) - 3/8*log(x⁸ + 3*x⁴ + 1)

giac [A] time = 0.50, size = 50, normalized size = 0.81

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) - \frac{3}{8}\log(x^8 + 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸+3*x⁴+1),x, algorithm="giac")

[Out] 1/4*x⁴ + 7/40*sqrt(5)*log((2*x⁴ - sqrt(5) + 3)/(2*x⁴ + sqrt(5) + 3)) - 3/8*log(x⁸ + 3*x⁴ + 1)

maple [A] time = 0.00, size = 38, normalized size = 0.61

$$\frac{x^4}{4} - \frac{7\sqrt{5}\operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{20} - \frac{3\ln(x^8 + 3x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(x⁸+3*x⁴+1),x)

[Out] 1/4*x⁴-3/8*ln(x⁸+3*x⁴+1)-7/20*arctanh(1/5*(2*x⁴+3)*5^(1/2))*5^(1/2)

maxima [A] time = 1.28, size = 50, normalized size = 0.81

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) - \frac{3}{8}\log(x^8 + 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸+3*x⁴+1),x, algorithm="maxima")

[Out] 1/4*x⁴ + 7/40*sqrt(5)*log((2*x⁴ - sqrt(5) + 3)/(2*x⁴ + sqrt(5) + 3)) - 3/8*log(x⁸ + 3*x⁴ + 1)

mupad [B] time = 0.13, size = 64, normalized size = 1.03

$$\frac{7\sqrt{5}\ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{40} - \frac{3\ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{8} - \frac{3\ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{8} - \frac{7\sqrt{5}\ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{40} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(3*x^4 + x^8 + 1),x)`

[Out] $(7*5^{(1/2)}*\log(x^4 - 5^{(1/2)}/2 + 3/2))/40 - (3*\log(5^{(1/2)}/2 + x^4 + 3/2))/8 - (3*\log(x^4 - 5^{(1/2)}/2 + 3/2))/8 - (7*5^{(1/2)}*\log(5^{(1/2)}/2 + x^4 + 3/2))/40 + x^4/4$

sympy [A] time = 0.14, size = 60, normalized size = 0.97

$$\frac{x^4}{4} + \left(-\frac{3}{8} + \frac{7\sqrt{5}}{40}\right) \log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{7\sqrt{5}}{40} - \frac{3}{8}\right) \log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(x**8+3*x**4+1),x)`

[Out] $x**4/4 + (-3/8 + 7*\text{sqrt}(5)/40)*\log(x**4 - \text{sqrt}(5)/2 + 3/2) + (-7*\text{sqrt}(5)/40 - 3/8)*\log(x**4 + \text{sqrt}(5)/2 + 3/2)$

$$3.309 \quad \int \frac{x^9}{1+3x^4+x^8} dx$$

Optimal. Leaf size=90

$$\frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}(9+4\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{2}\sqrt{\frac{1}{5}(9-4\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right)$$

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1359, 1122, 1166, 203}

$$\frac{x^2}{2} - \frac{1}{2}\sqrt{\frac{1}{5}(9+4\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) + \frac{1}{2}\sqrt{\frac{1}{5}(9-4\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x^9/(1 + 3*x^4 + x^8),x]

[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1122

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m+4*p+1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_, x_Symbol]
  :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x^9}{1 + 3x^4 + x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{1 + 3x^2 + x^4} dx, x, x^2 \right) \\ &= \frac{x^2}{2} - \frac{1}{2} \text{Subst} \left(\int \frac{1 + 3x^2}{1 + 3x^2 + x^4} dx, x, x^2 \right) \\ &= \frac{x^2}{2} - \frac{1}{20} (15 - 7\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) - \frac{1}{20} (15 + 7\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2}} dx, x, x^2 \right) \\ &= \frac{x^2}{2} - \frac{1}{2} \sqrt{\frac{1}{5}} (9 + 4\sqrt{5}) \tan^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right) + \frac{1}{20} \sqrt{180 - 80\sqrt{5}} \tan^{-1} \left(\sqrt{\frac{1}{2}} (3 + \sqrt{5}) x^2 \right) \end{aligned}$$

Mathematica [A] time = 0.15, size = 97, normalized size = 1.08

$$\frac{1}{40} \left(20x^2 - \sqrt{6 - 2\sqrt{5}} (15 + 7\sqrt{5}) \tan^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right) + \sqrt{2(3 + \sqrt{5})} (7\sqrt{5} - 15) \tan^{-1} \left(\sqrt{\frac{1}{2}} (3 + \sqrt{5}) x^2 \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + 3*x^4 + x^8), x]
```

```
[Out] (20*x^2 - Sqrt[6 - 2*Sqrt[5]]*(15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]]*x^2) + Sqrt[2*(3 + Sqrt[5])]*(-15 + 7*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1 + 3x^4 + x^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(1 + 3*x^4 + x^8), x]
```

[Out] IntegrateAlgebraic[x^9/(1 + 3*x^4 + x^8), x]

fricas [B] time = 1.29, size = 154, normalized size = 1.71

$$\frac{1}{2}x^2 - \frac{1}{5}\sqrt{5}\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{2x^4-\sqrt{5}+3}(3\sqrt{5}\sqrt{2}+7\sqrt{2})\sqrt{-4\sqrt{5}+9} - \frac{1}{2}(3\sqrt{5}x^2+7x^2)\sqrt{-4\sqrt{5}+9}\right) - \frac{1}{5}\sqrt{5}\sqrt{4\sqrt{5}+9}\arctan\left(-\frac{1}{4}(6\sqrt{5}x^2-14x^2-\sqrt{2x^4+\sqrt{5}+3}(3\sqrt{5}\sqrt{2}-7\sqrt{2}))\sqrt{4\sqrt{5}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{2}x^2 - \frac{1}{5}\sqrt{5}\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{2x^4-\sqrt{5}+3}(3\sqrt{5}\sqrt{2}+7\sqrt{2})\sqrt{-4\sqrt{5}+9} - \frac{1}{2}(3\sqrt{5}x^2+7x^2)\sqrt{-4\sqrt{5}+9}\right) - \frac{1}{5}\sqrt{5}\sqrt{4\sqrt{5}+9}\arctan\left(-\frac{1}{4}(6\sqrt{5}x^2-14x^2-\sqrt{2x^4+\sqrt{5}+3}(3\sqrt{5}\sqrt{2}-7\sqrt{2}))\sqrt{4\sqrt{5}+9}\right)$

giac [A] time = 0.56, size = 66, normalized size = 0.73

$$\frac{1}{2}x^2 - \frac{1}{20}\left(3x^4(\sqrt{5}-5) + \sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right) - \frac{1}{20}\left(3x^4(\sqrt{5}+5) + \sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="giac")

[Out] $\frac{1}{2}x^2 - \frac{1}{20}(3x^4(\sqrt{5}-5) + \sqrt{5}-5)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right) - \frac{1}{20}(3x^4(\sqrt{5}+5) + \sqrt{5}+5)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$

maple [B] time = 0.05, size = 117, normalized size = 1.30

$$\frac{x^2}{2} + \frac{7\sqrt{5}\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5(-2+2\sqrt{5})} - \frac{3\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}} - \frac{7\sqrt{5}\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5(2+2\sqrt{5})} - \frac{3\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^8+3*x^4+1),x)

[Out] $\frac{1}{2}x^2 - \frac{7}{5}5^{(1/2)}/(2+2*5^{(1/2)})\arctan(4*x^2/(2+2*5^{(1/2)})) - \frac{3}{(2+2*5^{(1/2)})}\arctan(4*x^2/(2+2*5^{(1/2)})) + \frac{7}{5}5^{(1/2)}/(-2+2*5^{(1/2)})\arctan(4*x^2/(-2+2*5^{(1/2)})) - \frac{3}{(-2+2*5^{(1/2)})}\arctan(4*x^2/(-2+2*5^{(1/2)}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2 - \int \frac{(3x^4 + 1)x}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] 1/2*x^2 - integrate((3*x^4 + 1)*x/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 1.34, size = 130, normalized size = 1.44

$$2 \operatorname{atanh} \left(\frac{1280x^2 \sqrt{\frac{\sqrt{5}}{20} - \frac{9}{80}}}{64\sqrt{5} - 192} + \frac{768\sqrt{5}x^2 \sqrt{\frac{\sqrt{5}}{20} - \frac{9}{80}}}{64\sqrt{5} - 192} \right) \sqrt{\frac{\sqrt{5}}{20} - \frac{9}{80}} - 2 \operatorname{atanh} \left(\frac{1280x^2 \sqrt{-\frac{\sqrt{5}}{20} - \frac{9}{80}}}{64\sqrt{5} + 192} - \frac{768\sqrt{5}x^2 \sqrt{-\frac{\sqrt{5}}{20} - \frac{9}{80}}}{64\sqrt{5} + 192} \right) \sqrt{-\frac{\sqrt{5}}{20} - \frac{9}{80}} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(3*x^4 + x^8 + 1),x)

[Out] 2*atanh((1280*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192) + (768*5^(1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192))*(5^(1/2)/20 - 9/80)^(1/2) - 2*atanh((1280*x^2*(-5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192) - (768*5^(1/2)*x^2*(-5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192))*(-5^(1/2)/20 - 9/80)^(1/2) + x^2/2

sympy [A] time = 0.21, size = 54, normalized size = 0.60

$$\frac{x^2}{2} + 2 \left(\frac{1}{4} - \frac{\sqrt{5}}{10} \right) \operatorname{atan} \left(\frac{2x^2}{-1 + \sqrt{5}} \right) - 2 \left(\frac{\sqrt{5}}{10} + \frac{1}{4} \right) \operatorname{atan} \left(\frac{2x^2}{1 + \sqrt{5}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(x**8+3*x**4+1),x)

[Out] x**2/2 + 2*(1/4 - sqrt(5)/10)*atan(2*x**2/(-1 + sqrt(5))) - 2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(1 + sqrt(5)))

$$3.310 \quad \int \frac{x^7}{1+3x^4+x^8} dx$$

Optimal. Leaf size=55

$$\frac{1}{40} (5 - 3\sqrt{5}) \log(2x^4 - \sqrt{5} + 3) + \frac{1}{40} (5 + 3\sqrt{5}) \log(2x^4 + \sqrt{5} + 3)$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1357, 632, 31}

$$\frac{1}{40} (5 - 3\sqrt{5}) \log(2x^4 - \sqrt{5} + 3) + \frac{1}{40} (5 + 3\sqrt{5}) \log(2x^4 + \sqrt{5} + 3)$$

Antiderivative was successfully verified.

[In] Int[x^7/(1 + 3*x^4 + x^8),x]

[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+3x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{1+3x+x^2} dx, x, x^4 \right) \\
&= \frac{1}{40} (5-3\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x} dx, x, x^4 \right) + \frac{1}{40} (5+3\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x} dx, x, x^4 \right) \\
&= \frac{1}{40} (5-3\sqrt{5}) \log(3-\sqrt{5}+2x^4) + \frac{1}{40} (5+3\sqrt{5}) \log(3+\sqrt{5}+2x^4)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.96

$$\frac{1}{40} (5-3\sqrt{5}) \log(-2x^4 + \sqrt{5} - 3) + \frac{1}{40} (5+3\sqrt{5}) \log(2x^4 + \sqrt{5} + 3)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(1 + 3*x^4 + x^8), x]

[Out] ((5 - 3*sqrt(5))*Log[-3 + sqrt(5) - 2*x^4])/40 + ((5 + 3*sqrt(5))*Log[3 + sqrt(5) + 2*x^4])/40

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1+3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(1 + 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^7/(1 + 3*x^4 + x^8), x]

fricas [A] time = 1.40, size = 56, normalized size = 1.02

$$\frac{3}{40} \sqrt{5} \log\left(\frac{2x^8 + 6x^4 + \sqrt{5}(2x^4 + 3) + 7}{x^8 + 3x^4 + 1}\right) + \frac{1}{8} \log(x^8 + 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8+3*x^4+1), x, algorithm="fricas")

[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1)) + 1/8*log(x^8 + 3*x^4 + 1)

giac [A] time = 0.50, size = 45, normalized size = 0.82

$$-\frac{3}{40} \sqrt{5} \log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) + \frac{1}{8} \log(x^8 + 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="giac")

[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^8 + 3*x^4 + 1)

maple [A] time = 0.00, size = 33, normalized size = 0.60

$$\frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{20} + \frac{\ln(x^8 + 3x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^8+3*x^4+1),x)

[Out] 1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)

maxima [A] time = 1.17, size = 45, normalized size = 0.82

$$-\frac{3}{40} \sqrt{5} \log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) + \frac{1}{8} \log(x^8 + 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^8 + 3*x^4 + 1)

mupad [B] time = 1.36, size = 59, normalized size = 1.07

$$\frac{\ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{8} + \frac{\ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{8} - \frac{3\sqrt{5} \ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{40} + \frac{3\sqrt{5} \ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(3*x^4 + x^8 + 1),x)

[Out] log(x^4 - 5^(1/2)/2 + 3/2)/8 + log(5^(1/2)/2 + x^4 + 3/2)/8 - (3*5^(1/2)*log(x^4 - 5^(1/2)/2 + 3/2))/40 + (3*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2))/40

sympy [A] time = 0.14, size = 53, normalized size = 0.96

$$\left(\frac{1}{8} - \frac{3\sqrt{5}}{40}\right) \log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(\frac{1}{8} + \frac{3\sqrt{5}}{40}\right) \log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(x**8+3*x**4+1),x)

[Out] (1/8 - 3*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (1/8 + 3*sqrt(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)

$$3.311 \quad \int \frac{x^5}{1+3x^4+x^8} dx$$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}(3+\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) - \frac{1}{2}\sqrt{\frac{1}{10}(3-\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right)$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1359, 1130, 203}

$$\frac{1}{2}\sqrt{\frac{1}{10}(3+\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) - \frac{1}{2}\sqrt{\frac{1}{10}(3-\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x^5/(1 + 3*x^4 + x^8), x]

[Out] (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - (Sqrt[(3 - Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1130

Int[((d_.)*(x_)^(m_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 1359

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1+3x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{1+3x^2+x^4} dx, x, x^2 \right) \\
&= \frac{1}{20} (5-3\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) + \frac{1}{20} (5+3\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \sqrt{\frac{1}{10} (3+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right) - \frac{1}{2} \sqrt{\frac{1}{10} (3-\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{1}{2} (3+\sqrt{5})} x^2 \right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.93

$$\frac{2\sqrt{5} \tan^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right) + (5-3\sqrt{5}) \tan^{-1} \left(\sqrt{\frac{1}{2} (3+\sqrt{5})} x^2 \right)}{10\sqrt{6-2\sqrt{5}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(1 + 3*x^4 + x^8), x]

[Out] (2*Sqrt[5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]]*x^2) + (5 - 3*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/(10*Sqrt[6 - 2*Sqrt[5]])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1+3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(1 + 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^5/(1 + 3*x^4 + x^8), x]

fricas [B] time = 1.45, size = 165, normalized size = 2.04

$$-\frac{1}{10} \sqrt{10} \sqrt{\sqrt{5}+3} \arctan\left(\frac{1}{40} \sqrt{10} \sqrt{2x^4+\sqrt{5}+3} (3\sqrt{5}\sqrt{2}-5\sqrt{2}) \sqrt{\sqrt{5}+3}\right) - \frac{1}{20} \sqrt{10} (3\sqrt{5}x^2-5x^2) \sqrt{\sqrt{5}+3} + \frac{1}{10} \sqrt{10} \sqrt{-\sqrt{5}+3} \arctan\left(\frac{1}{40} \sqrt{10} \sqrt{2x^4-\sqrt{5}+3} (3\sqrt{5}\sqrt{2}+5\sqrt{2}) \sqrt{-\sqrt{5}+3}\right) - \frac{1}{20} \sqrt{10} (3\sqrt{5}x^2+5x^2) \sqrt{-\sqrt{5}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+3*x^4+1), x, algorithm="fricas")

[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(1/40*sqrt(10)*sqrt(2*x^4 + sqrt(5) + 3)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 3) - 1/20*sqrt(10)*(3*s

```

qrt(5)*x^2 - 5*x^2)*sqrt(sqrt(5) + 3)) + 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*a
rctan(1/40*sqrt(10)*sqrt(2*x^4 - sqrt(5) + 3)*(3*sqrt(5)*sqrt(2) + 5*sqrt(2
))*sqrt(-sqrt(5) + 3) - 1/20*sqrt(10)*(3*sqrt(5)*x^2 + 5*x^2)*sqrt(-sqrt(5)
+ 3))

```

giac [A] time = 0.55, size = 47, normalized size = 0.58

$$\frac{1}{20} x^4 (\sqrt{5} - 5) \arctan\left(\frac{2x^2}{\sqrt{5} + 1}\right) + \frac{1}{20} x^4 (\sqrt{5} + 5) \arctan\left(\frac{2x^2}{\sqrt{5} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="giac")

[Out] 1/20*x^4*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*x^4*(sqrt(5) + 5)*arctan(2*x^2/(sqrt(5) - 1))

maple [B] time = 0.02, size = 110, normalized size = 1.36

$$\frac{\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}} - \frac{3\sqrt{5} \arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5(-2+2\sqrt{5})} + \frac{\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}} + \frac{3\sqrt{5} \arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5(2+2\sqrt{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^8+3*x^4+1),x)

[Out] 1/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+3/5*5^(1/2)/(2+2*5^(1/2))*arctan(4/(2+2*5^(1/2))*x^2)+1/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)-3/5*5^(1/2)/(-2+2*5^(1/2))*arctan(4/(-2+2*5^(1/2))*x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] integrate(x^5/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 0.12, size = 117, normalized size = 1.44

$$2 \operatorname{atanh}\left(\frac{60x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3} + \frac{28\sqrt{5}x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}\right) \sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}} - 2 \operatorname{atanh}\left(\frac{60x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3} - \frac{28\sqrt{5}x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3}\right) \sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(3*x^4 + x^8 + 1),x)`

[Out] $2*\operatorname{atanh}\left(\frac{60*x^2*(5^{(1/2)}/160 - 3/160)^{(1/2)}}{5^{(1/2)} + 3} + (28*5^{(1/2)}*x^2*(5^{(1/2)}/160 - 3/160)^{(1/2)})/(5^{(1/2)} + 3)\right)*5^{(1/2)}/160 - 3/160)^{(1/2)} - 2*\operatorname{atanh}\left(\frac{60*x^2*(-5^{(1/2)}/160 - 3/160)^{(1/2)}}{5^{(1/2)} - 3} - (28*5^{(1/2)}*x^2*(-5^{(1/2)}/160 - 3/160)^{(1/2)})/(5^{(1/2)} - 3)\right)*(-5^{(1/2)}/160 - 3/160)^{(1/2)}$

sympy [A] time = 0.20, size = 49, normalized size = 0.60

$$-2\left(\frac{1}{8} - \frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{-1 + \sqrt{5}}\right) + 2\left(\frac{\sqrt{5}}{40} + \frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{1 + \sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(x**8+3*x**4+1),x)`

[Out] $-2*(1/8 - \operatorname{sqrt}(5)/40)*\operatorname{atan}(2*x**2/(-1 + \operatorname{sqrt}(5))) + 2*(\operatorname{sqrt}(5)/40 + 1/8)*\operatorname{atan}(2*x**2/(1 + \operatorname{sqrt}(5)))$

$$3.312 \quad \int \frac{x^3}{1+3x^4+x^8} dx$$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 + 3*x^4 + x^8),x]

[Out] -ArcTanh[(3 + 2*x^4)/Sqrt[5]]/(2*Sqrt[5])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{1+3x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{1+3x+x^2} dx, x, x^4 \right) \\ &= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{5-x^2} dx, x, 3+2x^4 \right) \right) \\ &= - \frac{\tanh^{-1} \left(\frac{3+2x^4}{\sqrt{5}} \right)}{2\sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.65

$$\frac{\log(-2x^4 + \sqrt{5} - 3) - \log(2x^4 + \sqrt{5} + 3)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(1 + 3*x^4 + x^8),x]

[Out] (Log[-3 + Sqrt[5] - 2*x^4] - Log[3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1+3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(1 + 3*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^3/(1 + 3*x^4 + x^8), x]

fricas [B] time = 1.59, size = 43, normalized size = 1.87

$$\frac{1}{20} \sqrt{5} \log \left(\frac{2x^8 + 6x^4 - \sqrt{5}(2x^4 + 3) + 7}{x^8 + 3x^4 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] 1/20*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1))

giac [A] time = 0.60, size = 31, normalized size = 1.35

$$\frac{1}{20} \sqrt{5} \log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="giac")

[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^8+3*x^4+1),x)

[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)

maxima [A] time = 1.48, size = 31, normalized size = 1.35

$$\frac{1}{20} \sqrt{5} \log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))

mupad [B] time = 1.33, size = 30, normalized size = 1.30

$$\frac{\sqrt{5} \operatorname{atanh}\left(\frac{8\sqrt{5}x^4+3\sqrt{5}}{18x^4+7}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(3*x^4 + x^8 + 1),x)

[Out] (5^(1/2)*atanh((3*5^(1/2) + 8*5^(1/2)*x^4)/(18*x^4 + 7)))/10

sympy [A] time = 0.12, size = 42, normalized size = 1.83

$$\frac{\sqrt{5} \log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{20} - \frac{\sqrt{5} \log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**8+3*x**4+1),x)

[Out] sqrt(5)*log(x**4 - sqrt(5)/2 + 3/2)/20 - sqrt(5)*log(x**4 + sqrt(5)/2 + 3/2)/20

$$3.313 \quad \int \frac{x}{1+3x^4+x^8} dx$$

Optimal. Leaf size=75

$$\frac{1}{2} \sqrt{\frac{1}{10} (3 + \sqrt{5})} \tan^{-1} \left(\sqrt{\frac{1}{2} (3 + \sqrt{5})} x^2 \right) - \frac{\tan^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right)}{\sqrt{10} (3 + \sqrt{5})}$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1359, 1093, 203}

$$\frac{1}{2} \sqrt{\frac{1}{10} (3 + \sqrt{5})} \tan^{-1} \left(\sqrt{\frac{1}{2} (3 + \sqrt{5})} x^2 \right) - \frac{\tan^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right)}{\sqrt{10} (3 + \sqrt{5})}$$

Antiderivative was successfully verified.

[In] Int[x/(1 + 3*x^4 + x^8),x]

[Out] -(ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1359

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+3x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+3x^2+x^4} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{\frac{3-\sqrt{5}}{2} + x^2} dx, x, x^2 \right)}{2\sqrt{5}} - \frac{\text{Subst} \left(\int \frac{1}{\frac{3+\sqrt{5}}{2} + x^2} dx, x, x^2 \right)}{2\sqrt{5}} \\
&= -\frac{\tan^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right)}{\sqrt{10(3+\sqrt{5})}} + \frac{1}{2} \sqrt{\frac{1}{10}(3+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x^2 \right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.99

$$\frac{\tan^{-1} \left(\sqrt{\frac{2}{3-\sqrt{5}}} x^2 \right)}{\sqrt{10(3-\sqrt{5})}} - \frac{\tan^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right)}{\sqrt{10(3+\sqrt{5})}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + 3*x^4 + x^8),x]

[Out] ArcTan[Sqrt[2/(3 - Sqrt[5])]*x^2]/Sqrt[10*(3 - Sqrt[5])] - ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1+3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(1 + 3*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x/(1 + 3*x^4 + x^8), x]

fricas [B] time = 1.36, size = 128, normalized size = 1.71

$$\frac{1}{10} \sqrt{10} \sqrt{-\sqrt{5}+3} \arctan \left(-\frac{1}{10} \sqrt{10} \sqrt{5} x^2 \sqrt{-\sqrt{5}+3} + \frac{1}{20} \sqrt{10} \sqrt{5} \sqrt{2x^4+\sqrt{5}+3} \sqrt{-\sqrt{5}+3} \right) - \frac{1}{10} \sqrt{10} \sqrt{\sqrt{5}+3} \arctan \left(-\frac{1}{20} \left(2 \sqrt{10} \sqrt{5} x^2 - \sqrt{10} \sqrt{5} \sqrt{2x^4-\sqrt{5}+3} \right) \sqrt{\sqrt{5}+3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{10}\sqrt{10}\sqrt{-\sqrt{5} + 3}\arctan\left(\frac{-1}{10}\sqrt{10}\sqrt{5}x^2\sqrt{-\sqrt{5} + 3}\right) + \frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^4 + \sqrt{5} + 3}\sqrt{-\sqrt{5} + 3} - \frac{1}{10}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^4 - \sqrt{5} + 3}\sqrt{\sqrt{5} + 3} - \frac{1}{10}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^4 + \sqrt{5} + 3}\sqrt{\sqrt{5} + 3}$

giac [A] time = 0.42, size = 41, normalized size = 0.55

$$\frac{1}{20}(\sqrt{5} - 5)\arctan\left(\frac{2x^2}{\sqrt{5} + 1}\right) + \frac{1}{20}(\sqrt{5} + 5)\arctan\left(\frac{2x^2}{\sqrt{5} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8+3*x^4+1),x, algorithm="giac")

[Out] $\frac{1}{20}(\sqrt{5} - 5)\arctan\left(\frac{2x^2}{\sqrt{5} + 1}\right) + \frac{1}{20}(\sqrt{5} + 5)\arctan\left(\frac{2x^2}{\sqrt{5} - 1}\right)$

maple [A] time = 0.02, size = 60, normalized size = 0.80

$$\frac{2\sqrt{5}\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5(-2+2\sqrt{5})} - \frac{2\sqrt{5}\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5(2+2\sqrt{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^8+3*x^4+1),x)

[Out] $-\frac{2}{5}5^{(1/2)}/(2+2*5^{(1/2)})\arctan(4/(2+2*5^{(1/2)})x^2)+\frac{2}{5}5^{(1/2)}/(-2+2*5^{(1/2)})\arctan(4/(-2+2*5^{(1/2)})x^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] integrate(x/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 0.05, size = 125, normalized size = 1.67

$$2\operatorname{atanh}\left(\frac{160x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\sqrt{5}-18}-\frac{72\sqrt{5}x^2\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\sqrt{5}-18}\right)\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}-2\operatorname{atanh}\left(\frac{160x^2\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\sqrt{5}+18}+\frac{72\sqrt{5}x^2\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\sqrt{5}+18}\right)\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(3*x^4 + x^8 + 1),x)`

[Out] $2*\operatorname{atanh}\left(\frac{160*x^2*(5^{1/2}/160 - 3/160)^{1/2}}{(8*5^{1/2} - 18) - (72*5^{1/2})*x^2*(5^{1/2}/160 - 3/160)^{1/2}}\right)/(8*5^{1/2} - 18) - (72*5^{1/2})*x^2*(5^{1/2}/160 - 3/160)^{1/2} - 2*\operatorname{atanh}\left(\frac{160*x^2*(-5^{1/2}/160 - 3/160)^{1/2}}{(8*5^{1/2} + 18) + (72*5^{1/2})*x^2*(-5^{1/2}/160 - 3/160)^{1/2}}\right)/(8*5^{1/2} + 18)*(-5^{1/2}/160 - 3/160)^{1/2}$

sympy [A] time = 0.20, size = 49, normalized size = 0.65

$$2\left(\frac{\sqrt{5}}{40} + \frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1 + \sqrt{5}}\right) - 2\left(\frac{1}{8} - \frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{1 + \sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**8+3*x**4+1),x)`

[Out] $2*(\operatorname{sqrt}(5)/40 + 1/8)*\operatorname{atan}(2*x**2/(-1 + \operatorname{sqrt}(5))) - 2*(1/8 - \operatorname{sqrt}(5)/40)*\operatorname{atan}(2*x**2/(1 + \operatorname{sqrt}(5)))$

$$3.314 \quad \int \frac{1}{x(1+3x^4+x^8)} dx$$

Optimal. Leaf size=57

$$-\frac{1}{40}(5+3\sqrt{5})\log(2x^4-\sqrt{5}+3)-\frac{1}{40}(5-3\sqrt{5})\log(2x^4+\sqrt{5}+3)+\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{40}(5+3\sqrt{5})\log(2x^4-\sqrt{5}+3)-\frac{1}{40}(5-3\sqrt{5})\log(2x^4+\sqrt{5}+3)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + 3*x^4 + x^8)), x]

[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((5 - 3*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(1+3x^4+x^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(1+3x+x^2)} dx, x, x^4 \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x} dx, x, x^4 \right) + \frac{1}{4} \text{Subst} \left(\int \frac{-3-x}{1+3x+x^2} dx, x, x^4 \right) \\ &= \log(x) + \frac{1}{40} (-5 + 3\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) - \frac{1}{40} (5 + 3\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) \\ &= \log(x) - \frac{1}{40} (5 + 3\sqrt{5}) \log(3 - \sqrt{5} + 2x^4) - \frac{1}{40} (5 - 3\sqrt{5}) \log(3 + \sqrt{5} + 2x^4) \end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.96

$$\frac{1}{40} (-5 - 3\sqrt{5}) \log(-2x^4 + \sqrt{5} - 3) + \frac{1}{40} (3\sqrt{5} - 5) \log(2x^4 + \sqrt{5} + 3) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + 3*x^4 + x^8)), x]
```

```
[Out] Log[x] + ((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((-5 + 3*Sqrt[5])
)*Log[3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1+3x^4+x^8)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + 3*x^4 + x^8)), x]
```

```
[Out] IntegrateAlgebraic[1/(x*(1 + 3*x^4 + x^8)), x]
```

fricas [A] time = 1.45, size = 58, normalized size = 1.02

$$\frac{3}{40} \sqrt{5} \log\left(\frac{2x^8 + 6x^4 + \sqrt{5}(2x^4 + 3) + 7}{x^8 + 3x^4 + 1}\right) - \frac{1}{8} \log(x^8 + 3x^4 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1)) - 1/8*log(x^8 + 3*x^4 + 1) + log(x)

giac [A] time = 0.48, size = 51, normalized size = 0.89

$$-\frac{3}{40} \sqrt{5} \log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) - \frac{1}{8} \log(x^8 + 3x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="giac")

[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^8 + 3*x^4 + 1) + 1/4*log(x^4)

maple [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{20} + \ln(x) - \frac{\ln(x^8 + 3x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^8+3*x^4+1),x)

[Out] ln(x)-1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)

maxima [A] time = 1.50, size = 51, normalized size = 0.89

$$-\frac{3}{40} \sqrt{5} \log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) - \frac{1}{8} \log(x^8 + 3x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^8 + 3*x^4 + 1) + 1/4*log(x^4)

mupad [B] time = 1.41, size = 42, normalized size = 0.74

$$\ln(x) - \ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40} + \frac{1}{8}\right) + \ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(3*x^4 + x^8 + 1)),x)`

[Out] `log(x) - log(x^4 - 5^(1/2)/2 + 3/2)*((3*5^(1/2))/40 + 1/8) + log(5^(1/2)/2 + x^4 + 3/2)*((3*5^(1/2))/40 - 1/8)`

sympy [A] time = 0.16, size = 58, normalized size = 1.02

$$\log(x) + \left(-\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(-\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x**8+3*x**4+1),x)`

[Out] `log(x) + (-3*sqrt(5)/40 - 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (-1/8 + 3*sqrt(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)`

$$3.315 \quad \int \frac{1}{x^3(1+3x^4+x^8)} dx$$

Optimal. Leaf size=89

$$-\frac{1}{2x^2} + \frac{1}{2}\sqrt{\frac{1}{5}(9-4\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) - \frac{(3+\sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right)}{4\sqrt{10}}$$

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1359, 1123, 1166, 203}

$$-\frac{1}{2x^2} + \frac{1}{2}\sqrt{\frac{1}{5}(9-4\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right) - \frac{(3+\sqrt{5})^{3/2} \tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1 + 3*x^4 + x^8)),x]

[Out] -1/(2*x^2) + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1123

Int[((d_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*x^2 + c*x^4)^(p+1))/(a*d*(m+1)), x] - Dist[1/(a*d^2*(m+1)), Int[(d*x)^(m+2)*(b*(m+2*p+3) + c*(m+4*p+5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1359

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))]^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(1+3x^4+x^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(1+3x^2+x^4)} dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{-3-x^2}{1+3x^2+x^4} dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} + \frac{1}{20}(-5+3\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) - \frac{1}{20}(5+3\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} + \frac{1}{10} \sqrt{45-20\sqrt{5}} \tan^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right) - \frac{(3+\sqrt{5})^{3/2} \tan^{-1} \left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x^2 \right)}{4\sqrt{10}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.73

$$-\frac{1}{4} \text{RootSum} \left[\#1^8 + 3\#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) + 3 \log(x - \#1)}{2\#1^6 + 3\#1^2} \& \right] - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(1 + 3*x^4 + x^8)), x]

[Out] -1/2*1/x^2 - RootSum[1 + 3*#1^4 + #1^8 &, (3*Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(1+3x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(1 + 3*x^4 + x^8)), x]

[Out] IntegrateAlgebraic[1/(x^3*(1 + 3*x^4 + x^8)), x]

fricas [B] time = 0.96, size = 158, normalized size = 1.78

$$\frac{2\sqrt{5}x^2\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{2x^4+\sqrt{5}+3}(\sqrt{5}\sqrt{2}+3\sqrt{2})\sqrt{-4\sqrt{5}+9}-\frac{1}{2}(\sqrt{5}x^2+3x^2)\sqrt{-4\sqrt{5}+9}\right)+2\sqrt{5}x^2\sqrt{4\sqrt{5}+9}\arctan\left(-\frac{1}{4}(2\sqrt{5}x^2-6x^2-\sqrt{2x^4-\sqrt{5}+3}(\sqrt{5}\sqrt{2}-3\sqrt{2}))\sqrt{4\sqrt{5}+9}\right)+5}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] $-\frac{1}{10}(2\sqrt{5}x^2\sqrt{-4\sqrt{5}+9}\arctan(1/4\sqrt{2x^4+\sqrt{5}+3}(\sqrt{5}\sqrt{2}+3\sqrt{2})\sqrt{-4\sqrt{5}+9}-1/2(\sqrt{5}x^2+3x^2)\sqrt{-4\sqrt{5}+9})+2\sqrt{5}x^2\sqrt{4\sqrt{5}+9}\arctan(-1/4(2\sqrt{5}x^2-6x^2-\sqrt{2x^4-\sqrt{5}+3}(\sqrt{5}\sqrt{2}-3\sqrt{2}))\sqrt{4\sqrt{5}+9}))+5)/x^2$

giac [A] time = 0.46, size = 68, normalized size = 0.76

$$-\frac{1}{20}(x^4(\sqrt{5}-5)+3\sqrt{5}-15)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)-\frac{1}{20}(x^4(\sqrt{5}+5)+3\sqrt{5}+15)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="giac")

[Out] $-\frac{1}{20}(x^4(\sqrt{5}-5)+3\sqrt{5}-15)\arctan(2x^2/(\sqrt{5}+1))-1/20(x^4(\sqrt{5}+5)+3\sqrt{5}+15)\arctan(2x^2/(\sqrt{5}-1))-1/2x^2$

maple [B] time = 0.03, size = 117, normalized size = 1.31

$$-\frac{\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}}-\frac{3\sqrt{5}\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5(-2+2\sqrt{5})}-\frac{\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}}+\frac{3\sqrt{5}\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5(2+2\sqrt{5})}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^8+3*x^4+1),x)

[Out] $-\frac{1}{(2+2*5^{1/2})}\arctan(4/(2+2*5^{1/2})*x^2)+\frac{3}{5*5^{1/2}}\arctan(4/(-2+2*5^{1/2})*x^2)-\frac{1}{(-2+2*5^{1/2})}\arctan(4/(-2+2*5^{1/2})*x^2)-\frac{3}{5*5^{1/2}}\arctan(4/(-2+2*5^{1/2})*x^2)-\frac{1}{2x^2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2x^2}-\int\frac{(x^4+3)x}{x^8+3x^4+1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] -1/2/x^2 - integrate((x^4 + 3)*x/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 1.30, size = 130, normalized size = 1.46

$$2 \operatorname{atanh} \left(\frac{26880 x^2 \sqrt{-\frac{\sqrt{5}}{20} - \frac{9}{80}}}{3520 \sqrt{5} + 7872} + \frac{12032 \sqrt{5} x^2 \sqrt{-\frac{\sqrt{5}}{20} - \frac{9}{80}}}{3520 \sqrt{5} + 7872} \right) \sqrt{-\frac{\sqrt{5}}{20} - \frac{9}{80}} - 2 \operatorname{atanh} \left(\frac{26880 x^2 \sqrt{\frac{\sqrt{5}}{20} - \frac{9}{80}}}{3520 \sqrt{5} - 7872} - \frac{12032 \sqrt{5} x^2 \sqrt{\frac{\sqrt{5}}{20} - \frac{9}{80}}}{3520 \sqrt{5} - 7872} \right) \sqrt{\frac{\sqrt{5}}{20} - \frac{9}{80}} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(3*x^4 + x^8 + 1)),x)

[Out] 2*atanh((26880*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872) + (12032*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872))*(- 5^(1/2)/20 - 9/80)^(1/2) - 2*atanh((26880*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) - 7872) - (12032*5^(1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) - 7872))*(5^(1/2)/20 - 9/80)^(1/2) - 1/(2*x^2)

sympy [A] time = 0.24, size = 56, normalized size = 0.63

$$-2 \left(\frac{\sqrt{5}}{10} + \frac{1}{4} \right) \operatorname{atan} \left(\frac{2x^2}{-1 + \sqrt{5}} \right) + 2 \left(\frac{1}{4} - \frac{\sqrt{5}}{10} \right) \operatorname{atan} \left(\frac{2x^2}{1 + \sqrt{5}} \right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(x**8+3*x**4+1),x)

[Out] -2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) + 2*(1/4 - sqrt(5)/10)*atan(2*x**2/(1 + sqrt(5))) - 1/(2*x**2)

$$3.316 \quad \int \frac{1}{x^5(1+3x^4+x^8)} dx$$

Optimal. Leaf size=66

$$-\frac{1}{4x^4} + \frac{1}{40} (15 + 7\sqrt{5}) \log(2x^4 - \sqrt{5} + 3) + \frac{1}{40} (15 - 7\sqrt{5}) \log(2x^4 + \sqrt{5} + 3) - 3 \log(x)$$

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1357, 709, 800, 632, 31}

$$-\frac{1}{4x^4} + \frac{1}{40} (15 + 7\sqrt{5}) \log(2x^4 - \sqrt{5} + 3) + \frac{1}{40} (15 - 7\sqrt{5}) \log(2x^4 + \sqrt{5} + 3) - 3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(1 + 3*x^4 + x^8)),x]

[Out] -1/(4*x^4) - 3*Log[x] + ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800


```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(1+3x^4+x^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2(1+3x+x^2)} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{1}{4} \text{Subst} \left(\int \frac{-3-x}{x(1+3x+x^2)} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{1}{4} \text{Subst} \left(\int \left(-\frac{3}{x} + \frac{8+3x}{1+3x+x^2} \right) dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} - 3 \log(x) + \frac{1}{4} \text{Subst} \left(\int \frac{8+3x}{1+3x+x^2} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} - 3 \log(x) + \frac{1}{40} (15 - 7\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) + \frac{1}{40} (15 + 7\sqrt{5}) \\
&= -\frac{1}{4x^4} - 3 \log(x) + \frac{1}{40} (15 + 7\sqrt{5}) \log(3 - \sqrt{5} + 2x^4) + \frac{1}{40} (15 - 7\sqrt{5}) \log(3 + \sqrt{5}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.91

$$\frac{1}{40} \left(-\frac{10}{x^4} + (15 + 7\sqrt{5}) \log(-2x^4 + \sqrt{5} - 3) + (15 - 7\sqrt{5}) \log(2x^4 + \sqrt{5} + 3) - 120 \log(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 - 120*Log[x] + (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] + (15 -
7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 (1 + 3x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(1 + 3*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^5*(1 + 3*x^4 + x^8)), x]

fricas [A] time = 1.18, size = 76, normalized size = 1.15

$$\frac{7\sqrt{5}x^4 \log\left(\frac{2x^8+6x^4-\sqrt{5}(2x^4+3)+7}{x^8+3x^4+1}\right) + 15x^4 \log(x^8 + 3x^4 + 1) - 120x^4 \log(x) - 10}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1)) + 15*x^4*log(x^8 + 3*x^4 + 1) - 120*x^4*log(x) - 10)/x^4

giac [A] time = 0.55, size = 63, normalized size = 0.95

$$\frac{7}{40} \sqrt{5} \log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) + \frac{3x^4 - 1}{4x^4} + \frac{3}{8} \log(x^8 + 3x^4 + 1) - \frac{3}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="giac")

[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/4*(3*x^4 - 1)/x^4 + 3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)

maple [A] time = 0.01, size = 42, normalized size = 0.64

$$-\frac{7\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)}{20} - 3 \ln(x) + \frac{3 \ln(x^8 + 3x^4 + 1)}{8} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(x^8+3*x^4+1),x)

[Out] $-1/4/x^4 - 3*\ln(x) + 3/8*\ln(x^8 + 3*x^4 + 1) - 7/20*\operatorname{arctanh}(1/5*(2*x^4 + 3)*5^{(1/2)})*5^{(1/2)}$

maxima [A] time = 1.29, size = 56, normalized size = 0.85

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} + 3}{2x^4 + \sqrt{5} + 3}\right) - \frac{1}{4x^4} + \frac{3}{8}\log(x^8 + 3x^4 + 1) - \frac{3}{4}\log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="maxima")`

[Out] $7/40*\sqrt{5}*\log((2*x^4 - \sqrt{5} + 3)/(2*x^4 + \sqrt{5} + 3)) - 1/4/x^4 + 3/8*\log(x^8 + 3*x^4 + 1) - 3/4*\log(x^4)$

mupad [B] time = 1.36, size = 49, normalized size = 0.74

$$\ln\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40} + \frac{3}{8}\right) - \frac{1}{4x^4} - 3\ln(x) - \ln\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40} - \frac{3}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(3*x^4 + x^8 + 1)),x)`

[Out] $\log(x^4 - 5^{(1/2)}/2 + 3/2)*((7*5^{(1/2)})/40 + 3/8) - 1/(4*x^4) - 3*\log(x) - \log(5^{(1/2)}/2 + x^4 + 3/2)*((7*5^{(1/2)})/40 - 3/8)$

sympy [A] time = 0.19, size = 65, normalized size = 0.98

$$-3\log(x) + \left(\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{\sqrt{5}}{2} + \frac{3}{2}\right) + \left(\frac{3}{8} - \frac{7\sqrt{5}}{40}\right)\log\left(x^4 + \frac{\sqrt{5}}{2} + \frac{3}{2}\right) - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(x**8+3*x**4+1),x)`

[Out] $-3*\log(x) + (3/8 + 7*\sqrt{5}/40)*\log(x**4 - \sqrt{5}/2 + 3/2) + (3/8 - 7*\sqrt{5}/40)*\log(x**4 + \sqrt{5}/2 + 3/2) - 1/(4*x**4)$

$$3.317 \quad \int \frac{1}{x^7(1+3x^4+x^8)} dx$$

Optimal. Leaf size=97

$$-\frac{1}{6x^6} + \frac{3}{2x^2} - \frac{1}{2} \sqrt{\frac{1}{10} (123 - 55\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right) + \frac{1}{2} \sqrt{\frac{1}{10} (123 + 55\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{1}{2} (3 + \sqrt{5})} x^2 \right)$$

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1359, 1123, 1281, 1166, 203}

$$\frac{3}{2x^2} - \frac{1}{6x^6} - \frac{1}{2} \sqrt{\frac{1}{10} (123 - 55\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right) + \frac{1}{2} \sqrt{\frac{1}{10} (123 + 55\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{1}{2} (3 + \sqrt{5})} x^2 \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(1 + 3*x^4 + x^8)),x]

[Out] -1/(6*x^6) + 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1123

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^7(1+3x^4+x^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4(1+3x^2+x^4)} dx, x, x^2 \right) \\ &= -\frac{1}{6x^6} + \frac{1}{6} \text{Subst} \left(\int \frac{-9-3x^2}{x^2(1+3x^2+x^4)} dx, x, x^2 \right) \\ &= -\frac{1}{6x^6} + \frac{3}{2x^2} - \frac{1}{6} \text{Subst} \left(\int \frac{-24-9x^2}{1+3x^2+x^4} dx, x, x^2 \right) \\ &= -\frac{1}{6x^6} + \frac{3}{2x^2} - \frac{1}{20} (-15+7\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) + \frac{1}{20} (15+7\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) \\ &= -\frac{1}{6x^6} + \frac{3}{2x^2} - \frac{1}{2} \sqrt{\frac{1}{10} (123-55\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right) + \frac{1}{20} \sqrt{1230+550\sqrt{5}} \tan^{-1} \left(\sqrt{\frac{2}{3-\sqrt{5}}} x^2 \right) \end{aligned}$$

Mathematica [C] time = 0.02, size = 73, normalized size = 0.75

$$\frac{1}{4} \text{RootSum} \left[\#1^8 + 3\#1^4 + 1 \&, \frac{3\#1^4 \log(x - \#1) + 8 \log(x - \#1)}{2\#1^6 + 3\#1^2} \& \right] - \frac{1}{6x^6} + \frac{3}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(1 + 3*x^4 + x^8)),x]

[Out] $-1/6*1/x^6 + 3/(2*x^2) + \text{RootSum}[1 + 3*#1^4 + #1^8 \& , (8*\text{Log}[x - #1] + 3*\text{Log}[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) \&]/4$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 (1 + 3x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^7*(1 + 3*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^7*(1 + 3*x^4 + x^8)), x]

fricas [B] time = 0.79, size = 180, normalized size = 1.86

$$\frac{3\sqrt{10}x^4\sqrt{-55\sqrt{5}+123}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{2x^4+\sqrt{5}+3(7\sqrt{5}\sqrt{2}+15\sqrt{2})}\sqrt{-55\sqrt{5}+123}-\frac{1}{20}\sqrt{10}(7\sqrt{5}x^2+15x^2)\sqrt{-55\sqrt{5}+123}\right)-3\sqrt{10}x^4\sqrt{55\sqrt{5}+123}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{2x^4-\sqrt{5}+3(7\sqrt{5}\sqrt{2}-15\sqrt{2})}\sqrt{55\sqrt{5}+123}\right)+45x^4-5}{30x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{30}*(3*\text{sqrt}(10)*x^6*\text{sqrt}(-55*\text{sqrt}(5) + 123)*\text{arctan}(1/40*\text{sqrt}(10)*\text{sqrt}(2*x^4 + \text{sqrt}(5) + 3)*(7*\text{sqrt}(5)*\text{sqrt}(2) + 15*\text{sqrt}(2))*\text{sqrt}(-55*\text{sqrt}(5) + 123) - 1/20*\text{sqrt}(10)*(7*\text{sqrt}(5)*x^2 + 15*x^2)*\text{sqrt}(-55*\text{sqrt}(5) + 123)) - 3*\text{sqrt}(10)*x^6*\text{sqrt}(55*\text{sqrt}(5) + 123)*\text{arctan}(1/40*(\text{sqrt}(10)*\text{sqrt}(2*x^4 - \text{sqrt}(5) + 3)*(7*\text{sqrt}(5)*\text{sqrt}(2) - 15*\text{sqrt}(2)) - 2*\text{sqrt}(10)*(7*\text{sqrt}(5)*x^2 - 15*x^2))*\text{sqrt}(55*\text{sqrt}(5) + 123)) + 45*x^4 - 5)/x^6$

giac [A] time = 0.51, size = 77, normalized size = 0.79

$$\frac{1}{20}(3x^4(\sqrt{5}-5)+8\sqrt{5}-40)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)+\frac{1}{20}(3x^4(\sqrt{5}+5)+8\sqrt{5}+40)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)+\frac{9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="giac")

[Out] $\frac{1}{20}*(3*x^4*(\text{sqrt}(5) - 5) + 8*\text{sqrt}(5) - 40)*\text{arctan}(2*x^2/(\text{sqrt}(5) + 1)) + \frac{1}{20}*(3*x^4*(\text{sqrt}(5) + 5) + 8*\text{sqrt}(5) + 40)*\text{arctan}(2*x^2/(\text{sqrt}(5) - 1)) + \frac{1}{6}*(9*x^4 - 1)/x^6$

maple [B] time = 0.03, size = 122, normalized size = 1.26

$$\frac{7\sqrt{5}\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{5(-2+2\sqrt{5})} + \frac{3\arctan\left(\frac{4x^2}{-2+2\sqrt{5}}\right)}{-2+2\sqrt{5}} - \frac{7\sqrt{5}\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{5(2+2\sqrt{5})} + \frac{3\arctan\left(\frac{4x^2}{2+2\sqrt{5}}\right)}{2+2\sqrt{5}} + \frac{3}{2x^2} - \frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(x^8+3*x^4+1),x)`

[Out]
$$-1/6/x^6+3/2/x^2-7/5*5^{(1/2)}/(2+2*5^{(1/2)})*\arctan(4/(2+2*5^{(1/2)})x^2)+3/(2+2*5^{(1/2)})*\arctan(4/(2+2*5^{(1/2)})x^2)+7/5*5^{(1/2)}/(-2+2*5^{(1/2)})*\arctan(4/(-2+2*5^{(1/2)})x^2)+3/(-2+2*5^{(1/2)})*\arctan(4/(-2+2*5^{(1/2)})x^2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9x^4 - 1}{6x^6} + \int \frac{(3x^4 + 8)x}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="maxima")`

[Out] $1/6*(9*x^4 - 1)/x^6 + \text{integrate}((3*x^4 + 8)*x/(x^8 + 3*x^4 + 1), x)$

mupad [B] time = 0.12, size = 136, normalized size = 1.40

$$2 \operatorname{atanh} \left(\frac{3327500x^2 \sqrt{\frac{11\sqrt{5}}{32} - \frac{123}{160}} - \frac{1488300\sqrt{5}x^2 \sqrt{\frac{11\sqrt{5}}{32} - \frac{123}{160}}}{1140425\sqrt{5} - 2550075}}{\sqrt{\frac{11\sqrt{5}}{32} - \frac{123}{160}}} \right) - 2 \operatorname{atanh} \left(\frac{3327500x^2 \sqrt{-\frac{11\sqrt{5}}{32} - \frac{123}{160}} + \frac{1488300\sqrt{5}x^2 \sqrt{-\frac{11\sqrt{5}}{32} - \frac{123}{160}}}{1140425\sqrt{5} + 2550075}}{\sqrt{-\frac{11\sqrt{5}}{32} - \frac{123}{160}}} \right) + \frac{3x^4 - 1}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(3*x^4 + x^8 + 1)),x)`

[Out]
$$2*\operatorname{atanh}((3327500*x^2*((11*5^{(1/2))}/32 - 123/160)^{(1/2)})/(1140425*5^{(1/2)} - 2550075) - (1488300*5^{(1/2)}*x^2*((11*5^{(1/2))}/32 - 123/160)^{(1/2)})/(1140425*5^{(1/2)} - 2550075))*((11*5^{(1/2))}/32 - 123/160)^{(1/2)} - 2*\operatorname{atanh}((3327500*x^2*(-(11*5^{(1/2))}/32 - 123/160)^{(1/2)})/(1140425*5^{(1/2)} + 2550075) + (1488300*5^{(1/2)}*x^2*(-(11*5^{(1/2))}/32 - 123/160)^{(1/2)})/(1140425*5^{(1/2)} + 2550075))*(-(11*5^{(1/2))}/32 - 123/160)^{(1/2)} + ((3*x^4)/2 - 1/6)/x^6$$

sympy [A] time = 0.27, size = 65, normalized size = 0.67

$$2 \left(\frac{11\sqrt{5}}{40} + \frac{5}{8} \right) \operatorname{atan} \left(\frac{2x^2}{-1 + \sqrt{5}} \right) - 2 \left(\frac{5}{8} - \frac{11\sqrt{5}}{40} \right) \operatorname{atan} \left(\frac{2x^2}{1 + \sqrt{5}} \right) + \frac{9x^4 - 1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(x**8+3*x**4+1),x)`

[Out] $2*(11*\operatorname{sqrt}(5)/40 + 5/8)*\operatorname{atan}(2*x**2/(-1 + \operatorname{sqrt}(5))) - 2*(5/8 - 11*\operatorname{sqrt}(5)/40)*\operatorname{atan}(2*x**2/(1 + \operatorname{sqrt}(5))) + (9*x**4 - 1)/(6*x**6)$

$$3.318 \quad \int \frac{x^8}{1+3x^4+x^8} dx$$

Optimal. Leaf size=460

$$\frac{\sqrt[4]{123-55\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{123-55\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.42, antiderivative size = 440, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 8, integrand size = 16, number of rules / integrand size = 0.500, Rules used = {1367, 1422, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{984-440\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{8\sqrt{10}} - \frac{\sqrt[4]{984-440\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{8\sqrt{10}} + \frac{\sqrt[4]{123+55\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{123+55\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{984-440\sqrt{5}} \tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt{3-\sqrt{5}}}\right)}{4\sqrt{10}} - \frac{\sqrt[4]{984-440\sqrt{5}} \tan^{-1}\left(\frac{2^{3/4}x}{\sqrt{3-\sqrt{5}}}\right)}{4\sqrt{10}} + \frac{\sqrt[4]{123+55\sqrt{5}} \tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt{3+\sqrt{5}}}\right)}{2 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{123+55\sqrt{5}} \tan^{-1}\left(\frac{2^{3/4}x}{\sqrt{3+\sqrt{5}}}\right)}{2 \cdot 2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x^8/(1 + 3*x^4 + x^8), x]

[Out] x - ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(4*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(4*Sqrt[10]) + ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2)/(8*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2)/(8*Sqrt[10]) + ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])]] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2)/(4*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])]] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2)/(4*2^(3/4)*Sqrt[5])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1367

```
Int[((d_)*(x_)^(m_))*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

*c] || !IGtQ[n/2, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{1+3x^4+x^8} dx &= x - \int \frac{1+3x^4}{1+3x^4+x^8} dx \\
 &= x - \frac{1}{10} (15-7\sqrt{5}) \int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx - \frac{1}{10} (15+7\sqrt{5}) \int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx \\
 &= x + \frac{1}{2} \sqrt{\frac{1}{10} (9-4\sqrt{5})} \int \frac{\sqrt{3-\sqrt{5}} - \sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx + \frac{1}{2} \sqrt{\frac{1}{10} (9-4\sqrt{5})} \int \frac{\sqrt{3-\sqrt{5}} + \sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx \\
 &= x + \frac{1}{4} \sqrt{\frac{1}{5} (9-4\sqrt{5})} \int \frac{1}{\sqrt{\frac{1}{2} (3-\sqrt{5})} - \sqrt[4]{2} (3-\sqrt{5}) x + x^2} dx + \frac{1}{4} \sqrt{\frac{1}{5} (9-4\sqrt{5})} \int \frac{1}{\sqrt{\frac{1}{2} (3-\sqrt{5})} + \sqrt[4]{2} (3-\sqrt{5}) x + x^2} dx \\
 &= x - \frac{1}{8} \sqrt[4]{\frac{246}{25} - \frac{22}{\sqrt{5}}} \log \left(\sqrt{2(3-\sqrt{5})} - 2\sqrt[4]{2(3-\sqrt{5})} x + 2x^2 \right) + \frac{1}{8} \sqrt[4]{\frac{246}{25} - \frac{22}{\sqrt{5}}} \log \left(\sqrt{2(3-\sqrt{5})} + 2\sqrt[4]{2(3-\sqrt{5})} x + 2x^2 \right) \\
 &= x - \frac{\sqrt[4]{123-55\sqrt{5}} \tan^{-1} \left(1 - \frac{2^{3/4} x}{\sqrt[4]{3-\sqrt{5}}} \right)}{2 \cdot 2^{3/4} \sqrt{5}} + \frac{\sqrt[4]{123-55\sqrt{5}} \tan^{-1} \left(1 + \frac{2^{3/4} x}{\sqrt[4]{3-\sqrt{5}}} \right)}{2 \cdot 2^{3/4} \sqrt{5}} + \frac{\sqrt[4]{123+55\sqrt{5}} \tan^{-1} \left(1 - \frac{2^{3/4} x}{\sqrt[4]{3-\sqrt{5}}} \right)}{2 \cdot 2^{3/4} \sqrt{5}} + \frac{\sqrt[4]{123+55\sqrt{5}} \tan^{-1} \left(1 + \frac{2^{3/4} x}{\sqrt[4]{3-\sqrt{5}}} \right)}{2 \cdot 2^{3/4} \sqrt{5}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 58, normalized size = 0.13

$$x - \frac{1}{4} \text{RootSum} \left[\#1^8 + 3\#1^4 + 1 \&, \frac{3\#1^4 \log(x - \#1) + \log(x - \#1)}{2\#1^7 + 3\#1^3} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(1+3*x^4+x^8),x]

[Out] x - RootSum[1 + 3*#1^4 + #1^8 &, (Log[x - #1] + 3*Log[x - #1]*#1^4)/(3*#1^3 + 2*#1^7) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1+3x^4+x^8} dx$$

2)*(-110*sqrt(5) + 246)^(1/4)*log(20*x^2 - sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(5) + 246)) + x

giac [A] time = 0.73, size = 240, normalized size = 0.52

$\frac{1}{80} \left(\pi + 4 \arctan(\sqrt{5}-1) \right) \sqrt{25\sqrt{5}-55} - \frac{1}{80} \left(\pi + 4 \arctan(\sqrt{5}+1) \right) \sqrt{25\sqrt{5}+55} - \frac{1}{80} \left(\pi + 4 \arctan(\sqrt{5}-1) \right) \sqrt{25\sqrt{5}-55} - \frac{1}{80} \left(\pi + 4 \arctan(\sqrt{5}+1) \right) \sqrt{25\sqrt{5}+55} - \frac{1}{40} \sqrt{25\sqrt{5}-55} \log(722500(x + \sqrt{5}+1)^2 + 722500x^2) - \frac{1}{40} \sqrt{25\sqrt{5}+55} \log(722500(x - \sqrt{5}+1)^2 + 722500x^2) + \frac{1}{40} \sqrt{25\sqrt{5}-55} \log(2992900(x + \sqrt{5}-1)^2 + 2992900x^2) - \frac{1}{40} \sqrt{25\sqrt{5}+55} \log(2992900(x - \sqrt{5}-1)^2 + 2992900x^2) + x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="giac")

[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*sqrt(5) + 55)*log(722500*(x + sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sqrt(25*sqrt(5) + 55)*log(722500*(x - sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x + sqrt(sqrt(5) - 1))^2 + 2992900*x^2) - 1/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x - sqrt(sqrt(5) - 1))^2 + 2992900*x^2) + x

maple [C] time = 0.01, size = 46, normalized size = 0.10

$$x + \frac{\left(-3 \operatorname{RootOf}\left(-Z^8 + 3Z^4 + 1\right)^4 - 1\right) \ln\left(-\operatorname{RootOf}\left(-Z^8 + 3Z^4 + 1\right) + x\right)}{8 \operatorname{RootOf}\left(-Z^8 + 3Z^4 + 1\right)^7 + 12 \operatorname{RootOf}\left(-Z^8 + 3Z^4 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^8+3*x^4+1),x)

[Out] x+1/4*sum((-3*_R^4-1)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(-_Z^8+3*_Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x - \int \frac{3x^4 + 1}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] x - integrate((3*x^4 + 1)/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 1.44, size = 216, normalized size = 0.47

$x - \frac{2^{3/4} \sqrt{5} \operatorname{atan}\left(\frac{32^{1/4} x}{2(-55 \sqrt{5}-123)^{1/4}} + \frac{2^{1/4} \sqrt{5} x}{2(-55 \sqrt{5}-123)^{1/4}}\right) (-55 \sqrt{5}-123)^{1/4}}{20} + \frac{2^{3/4} \sqrt{5} \operatorname{atan}\left(\frac{32^{1/4} x}{2(55 \sqrt{5}-123)^{1/4}} - \frac{2^{1/4} \sqrt{5} x}{2(55 \sqrt{5}-123)^{1/4}}\right) (55 \sqrt{5}-123)^{1/4}}{20} + \frac{2^{3/4} \sqrt{5} \operatorname{atan}\left(\frac{2^{1/4} x \sqrt{3i}}{2(-55 \sqrt{5}-123)^{1/4}} + \frac{2^{1/4} \sqrt{5} x \sqrt{3i}}{2(-55 \sqrt{5}-123)^{1/4}}\right) (-55 \sqrt{5}-123)^{1/4} \operatorname{li}}{20} - \frac{2^{3/4} \sqrt{5} \operatorname{atan}\left(\frac{2^{1/4} x \sqrt{3i}}{2(55 \sqrt{5}-123)^{1/4}} - \frac{2^{1/4} \sqrt{5} x \sqrt{3i}}{2(55 \sqrt{5}-123)^{1/4}}\right) (55 \sqrt{5}-123)^{1/4} \operatorname{li}}{20}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(3*x^4 + x^8 + 1),x)`

[Out] $x - (2^{3/4} \cdot 5^{1/2} \cdot \operatorname{atan}((3 \cdot 2^{1/4}) \cdot x) / (2 \cdot (-55 \cdot 5^{1/2} - 123)^{1/4})) + (2^{1/4} \cdot 5^{1/2} \cdot x) / (2 \cdot (-55 \cdot 5^{1/2} - 123)^{1/4})) \cdot (-55 \cdot 5^{1/2} - 123)^{1/4} / 20 + (2^{3/4} \cdot 5^{1/2} \cdot \operatorname{atan}((3 \cdot 2^{1/4}) \cdot x) / (2 \cdot (55 \cdot 5^{1/2} - 123)^{1/4})) - (2^{1/4} \cdot 5^{1/2} \cdot x) / (2 \cdot (55 \cdot 5^{1/2} - 123)^{1/4})) \cdot (55 \cdot 5^{1/2} - 123)^{1/4} / 20 + (2^{3/4} \cdot 5^{1/2} \cdot \operatorname{atan}((2^{1/4}) \cdot x \cdot 3i) / (2 \cdot (-55 \cdot 5^{1/2} - 123)^{1/4})) + (2^{1/4} \cdot 5^{1/2} \cdot x \cdot 1i) / (2 \cdot (-55 \cdot 5^{1/2} - 123)^{1/4})) \cdot (-55 \cdot 5^{1/2} - 123)^{1/4} \cdot 1i / 20 - (2^{3/4} \cdot 5^{1/2} \cdot \operatorname{atan}((2^{1/4}) \cdot x \cdot 3i) / (2 \cdot (55 \cdot 5^{1/2} - 123)^{1/4})) - (2^{1/4} \cdot 5^{1/2} \cdot x \cdot 1i) / (2 \cdot (55 \cdot 5^{1/2} - 123)^{1/4})) \cdot (55 \cdot 5^{1/2} - 123)^{1/4} \cdot 1i / 20$

sympy [A] time = 1.59, size = 29, normalized size = 0.06

$$x + \operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t \log\left(\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8/(x**8+3*x**4+1),x)`

[Out] `x + RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(15360*_t**5/11 + 1288*_t/55 + x)))`

$$3.319 \quad \int \frac{x^6}{1+3x^4+x^8} dx$$

Optimal. Leaf size=431

$$\frac{\sqrt[4]{9-4\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9-4\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4\sqrt{10}}$$

Rubi [A] time = 0.29, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1374, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{9-4\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4\sqrt{10}} - \frac{(3+\sqrt{5})^{3/4} \log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{8\sqrt{2}\sqrt{5}} - \frac{(3+\sqrt{5})^{3/4} \log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{8\sqrt{2}\sqrt{5}} + \frac{\sqrt[4]{9-4\sqrt{5}} \tan^{-1}\left(1 - \frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}}\right)}{2\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}} \tan^{-1}\left(\frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}} + 1\right)}{2\sqrt{10}} - \frac{(3+\sqrt{5})^{3/4} \tan^{-1}\left(1 - \frac{2x^2}{\sqrt[4]{2(3+\sqrt{5})}}\right)}{4\sqrt{2}\sqrt{5}} - \frac{(3+\sqrt{5})^{3/4} \tan^{-1}\left(\frac{2x^2}{\sqrt[4]{2(3+\sqrt{5})}} + 1\right)}{4\sqrt{2}\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(1 + 3*x^4 + x^8), x]

[Out] ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2*Sqrt[10]) - (9 - 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2*Sqrt[10]) - ((3 + Sqrt[5])^(3/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(4*2^(1/4)*Sqrt[5]) + ((3 + Sqrt[5])^(3/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(4*2^(1/4)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2)/(4*Sqrt[10]) + ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2)/(4*Sqrt[10]) + ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3 + Sqrt[5])]] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2)/(8*2^(1/4)*Sqrt[5]) - ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3 + Sqrt[5])]] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2)/(8*2^(1/4)*Sqrt[5])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1374

```
Int[((d_.)*(x_)^(m_))/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1+3x^4+x^8} dx &= -\left(\frac{1}{10}(-5+3\sqrt{5}) \int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx\right) + \frac{1}{10}(5+3\sqrt{5}) \int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx \\
&= \frac{(3-\sqrt{5}) \int \frac{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{4\sqrt{10}} - \frac{(3-\sqrt{5}) \int \frac{\sqrt{3-\sqrt{5}+\sqrt{2}x^2}}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{4\sqrt{10}} - \frac{(3+\sqrt{5}) \int \frac{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{4\sqrt{10}} \\
&= -\frac{\sqrt[4]{9-4\sqrt{5}} \int \frac{\sqrt[4]{2(3-\sqrt{5})+2x}}{-\sqrt{\frac{1}{2}(3-\sqrt{5})}-\sqrt[4]{2(3-\sqrt{5})}x-x^2} dx}{4\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}} \int \frac{\sqrt[4]{2(3-\sqrt{5})-2x}}{-\sqrt{\frac{1}{2}(3-\sqrt{5})}+\sqrt[4]{2(3-\sqrt{5})}x-x^2} dx}{4\sqrt{10}} + \\
&= -\frac{\sqrt[4]{9-4\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})}-2\sqrt[4]{2(3-\sqrt{5})}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9-4\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})}+\sqrt[4]{2(3-\sqrt{5})}x-x^2\right)}{4\sqrt{10}} \\
&= \frac{(3-\sqrt{5})^{3/4} \tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{36-16\sqrt{5}} \tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}} - \frac{(3+\sqrt{5})^{3/4} \tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.10

$$\frac{1}{4} \text{RootSum}\left[\#1^8 + 3\#1^4 + 1 \&, \frac{\#1^3 \log(x - \#1)}{2\#1^4 + 3} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(1+3*x^4+x^8),x]

[Out] RootSum[1+3*#1^4+#1^8 &, (Log[x-#1]*#1^3)/(3+2*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1+3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(1+3*x^4+x^8),x]

[Out] IntegrateAlgebraic[x^6/(1+3*x^4+x^8),x]

fricas [B] time = 1.58, size = 725, normalized size = 1.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{10}\sqrt{5}\sqrt{2}(4\sqrt{5}+9)^{1/4}\arctan\left(\frac{1}{2}\sqrt{2x^2+(3\sqrt{5}\sqrt{2}x-7\sqrt{2}x)(4\sqrt{5}+9)^{3/4}}-\sqrt{4\sqrt{5}+9}(\sqrt{5}-3)\right)(21\sqrt{5}-47)(4\sqrt{5}+9)^{5/4}-\frac{1}{2}(21\sqrt{5}\sqrt{2}x-47\sqrt{2}x)(4\sqrt{5}+9)^{5/4}-1+\frac{1}{10}\sqrt{5}\sqrt{2}(4\sqrt{5}+9)^{1/4}\arctan\left(\frac{1}{2}\sqrt{2x^2-(3\sqrt{5}\sqrt{2}x-7\sqrt{2}x)(4\sqrt{5}+9)^{3/4}}-\sqrt{4\sqrt{5}+9}(\sqrt{5}-3)\right)(21\sqrt{5}-47)(4\sqrt{5}+9)^{5/4}-\frac{1}{2}(21\sqrt{5}\sqrt{2}x-47\sqrt{2}x)(4\sqrt{5}+9)^{5/4}+1+\frac{1}{10}\sqrt{5}\sqrt{2}(-4\sqrt{5}+9)^{1/4}\arctan\left(\frac{1}{2}\sqrt{2x^2+(3\sqrt{5}\sqrt{2}x+7\sqrt{2}x)(-4\sqrt{5}+9)^{3/4}}+(\sqrt{5}+3)\sqrt{-4\sqrt{5}+9}\right)(21\sqrt{5}+47)(-4\sqrt{5}+9)^{5/4}-\frac{1}{2}(21\sqrt{5}\sqrt{2}x+47\sqrt{2}x)(-4\sqrt{5}+9)^{5/4}-1+\frac{1}{10}\sqrt{5}\sqrt{2}(-4\sqrt{5}+9)^{1/4}\arctan\left(\frac{1}{2}\sqrt{2x^2-(3\sqrt{5}\sqrt{2}x+7\sqrt{2}x)(-4\sqrt{5}+9)^{3/4}}+(\sqrt{5}+3)\sqrt{-4\sqrt{5}+9}\right)(21\sqrt{5}+47)(-4\sqrt{5}+9)^{5/4}-\frac{1}{2}(21\sqrt{5}\sqrt{2}x+47\sqrt{2}x)(-4\sqrt{5}+9)^{5/4}+1+\frac{1}{40}\sqrt{5}\sqrt{2}(4\sqrt{5}+9)^{1/4}\log(2x^2+(3\sqrt{5}\sqrt{2}x-7\sqrt{2}x)(4\sqrt{5}+9)^{3/4}-\sqrt{4\sqrt{5}+9}(\sqrt{5}-3))-1+\frac{1}{40}\sqrt{5}\sqrt{2}(4\sqrt{5}+9)^{1/4}\log(2x^2-(3\sqrt{5}\sqrt{2}x-7\sqrt{2}x)(4\sqrt{5}+9)^{3/4}-\sqrt{4\sqrt{5}+9}(\sqrt{5}-3))+1+\frac{1}{40}\sqrt{5}\sqrt{2}(-4\sqrt{5}+9)^{1/4}\log(2x^2+(3\sqrt{5}\sqrt{2}x+7\sqrt{2}x)(-4\sqrt{5}+9)^{3/4}+(\sqrt{5}+3)\sqrt{-4\sqrt{5}+9})-1+\frac{1}{40}\sqrt{5}\sqrt{2}(-4\sqrt{5}+9)^{1/4}\log(2x^2-(3\sqrt{5}\sqrt{2}x+7\sqrt{2}x)(-4\sqrt{5}+9)^{3/4}+(\sqrt{5}+3)\sqrt{-4\sqrt{5}+9})$

giac [A] time = 0.66, size = 239, normalized size = 0.55

$\frac{1}{80}(\pi+4\arctan(\sqrt{5}-1))\sqrt{10\sqrt{5}+20}-\frac{1}{80}(\pi+4\arctan(-\sqrt{5}-1))\sqrt{10\sqrt{5}+20}-\frac{1}{80}(\pi+4\arctan(\sqrt{5}+1))\sqrt{10\sqrt{5}-20}-\frac{1}{80}(\pi+4\arctan(-\sqrt{5}+1))\sqrt{10\sqrt{5}-20}-\frac{1}{40}\sqrt{10\sqrt{5}+20}\log(400(x+\sqrt{5}-1)^2+400x^2)-\frac{1}{40}\sqrt{10\sqrt{5}+20}\log(400(x-\sqrt{5}-1)^2+400x^2)-\frac{1}{40}\sqrt{10\sqrt{5}-20}\log(4000(x+\sqrt{5}-1)^2+10000x^2)-\frac{1}{40}\sqrt{10\sqrt{5}-20}\log(4000(x-\sqrt{5}-1)^2+10000x^2)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="giac")

[Out] $\frac{1}{80}(\pi+4\arctan(x\sqrt{\sqrt{5}-1}-1))\sqrt{10\sqrt{5}+20}-\frac{1}{80}(\pi+4\arctan(-x\sqrt{\sqrt{5}-1}-1))\sqrt{10\sqrt{5}+20}-\frac{1}{80}(\pi+4\arctan(x\sqrt{\sqrt{5}+1}+1))\sqrt{10\sqrt{5}-20}+\frac{1}{80}(\pi+4\arctan(-x\sqrt{\sqrt{5}+1}+1))\sqrt{10\sqrt{5}-20}-\frac{1}{40}\sqrt{10\sqrt{5}+20}\log(400(x+\sqrt{\sqrt{5}+1})^2+400x^2)+\frac{1}{40}\sqrt{10\sqrt{5}+20}\log(400(x-\sqrt{\sqrt{5}+1})^2+400x^2)+\frac{1}{40}\sqrt{10\sqrt{5}-20}\log(4000(x+\sqrt{5}-1)^2+10000x^2)-\frac{1}{40}\sqrt{10\sqrt{5}-20}\log(4000(x-\sqrt{5}-1)^2+10000x^2)$

- 20)*log(10000*(x + sqrt(sqrt(5) - 1))^2 + 10000*x^2) - 1/40*sqrt(10*sqrt(5) - 20)*log(10000*(x - sqrt(sqrt(5) - 1))^2 + 10000*x^2)

maple [C] time = 0.01, size = 40, normalized size = 0.09

$$\frac{\text{RootOf}(-Z^8 + 3Z^4 + 1)^6 \ln(-\text{RootOf}(-Z^8 + 3Z^4 + 1) + x)}{8 \text{RootOf}(-Z^8 + 3Z^4 + 1)^7 + 12 \text{RootOf}(-Z^8 + 3Z^4 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^8+3*x^4+1),x)

[Out] 1/4*sum(_R^6/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(-Z^8+3*_Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] integrate(x^6/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 1.46, size = 149, normalized size = 0.35

$$\frac{\sqrt{5} \operatorname{atan}\left(\frac{16x(4\sqrt{5}-9)^{1/4}}{8\sqrt{5}+24}\right)(-4\sqrt{5}-9)^{1/4}}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{16x(4\sqrt{5}-9)^{1/4}}{8\sqrt{5}-24}\right)(4\sqrt{5}-9)^{1/4}}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{x(-4\sqrt{5}-9)^{1/4}16i}{8\sqrt{5}+24}\right)(-4\sqrt{5}-9)^{1/4}1i}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{x(4\sqrt{5}-9)^{1/4}16i}{8\sqrt{5}-24}\right)(4\sqrt{5}-9)^{1/4}1i}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(3*x^4 + x^8 + 1),x)

[Out] (5^(1/2)*atan((16*x*(-4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) + 24))*(-4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((16*x*(4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) - 24))*(4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((x*(-4*5^(1/2) - 9)^(1/4)*16i)/(8*5^(1/2) + 24))*(-4*5^(1/2) - 9)^(1/4)*1i)/10 + (5^(1/2)*atan((x*(4*5^(1/2) - 9)^(1/4)*16i)/(8*5^(1/2) - 24))*(4*5^(1/2) - 9)^(1/4)*1i)/10

sympy [A] time = 1.53, size = 26, normalized size = 0.06

$$\text{RootSum}\left(40960000t^8 + 115200t^4 + 1, \left(t \mapsto t \log(-1792000t^7 - 4920t^3 + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(x**8+3*x**4+1),x)

[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-1792000*_t**7 - 4920*_t**3 + x)))

$$3.320 \quad \int \frac{x^4}{1+3x^4+x^8} dx$$

Optimal. Leaf size=451

$$\frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.28, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1374, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}} \tan^{-1}\left(1 - \frac{2x^2}{\sqrt[4]{3-\sqrt{5}}}\right)}{2 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}} \tan^{-1}\left(\frac{2x^2}{\sqrt[4]{3-\sqrt{5}}} + 1\right)}{2 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}} \tan^{-1}\left(1 - \frac{2x^2}{\sqrt[4]{3+\sqrt{5}}}\right)}{2 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}} \tan^{-1}\left(\frac{2x^2}{\sqrt[4]{3+\sqrt{5}}} + 1\right)}{2 \cdot 2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(1 + 3*x^4 + x^8), x]

[Out] ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]/(2*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2)/(4*2^(3/4)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2)/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])]] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2)/(4*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])]] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2)/(4*2^(3/4)*Sqrt[5])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1374

```
Int[((d_.)*(x_)^(m_))/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{1+3x^4+x^8} dx &= -\left(\frac{1}{10}(-5+3\sqrt{5}) \int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx\right) + \frac{1}{10}(5+3\sqrt{5}) \int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx \\
&= -\left(\frac{1}{4}\sqrt{\frac{1}{5}}(3-\sqrt{5}) \int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx\right) - \frac{1}{4}\sqrt{\frac{1}{5}}(3-\sqrt{5}) \int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx \\
&= -\left(\frac{1}{4}\sqrt{\frac{1}{10}}(3-\sqrt{5}) \int \frac{1}{\sqrt{\frac{1}{2}}(3-\sqrt{5})-\sqrt[4]{2}(3-\sqrt{5})x+x^2} dx\right) - \frac{1}{4}\sqrt{\frac{1}{10}}(3-\sqrt{5}) \int \frac{1}{\sqrt{\frac{1}{2}}(3-\sqrt{5})+\sqrt[4]{2}(3-\sqrt{5})x+x^2} dx \\
&= \frac{\sqrt[4]{3-\sqrt{5}} \log\left(\sqrt{2}(3-\sqrt{5})-2\sqrt[4]{2}(3-\sqrt{5})x+2x^2\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}} \log\left(\sqrt{2}(3-\sqrt{5})+2\sqrt[4]{2}(3-\sqrt{5})x+2x^2\right)}{4 \cdot 2^{3/4}\sqrt{5}} \\
&= \frac{\sqrt[4]{3-\sqrt{5}} \tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3-\sqrt{5}} \tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}} \tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{3+\sqrt{5}} \tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2 \cdot 2^{3/4}\sqrt{5}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&, \frac{\#1 \log(x-\#1)}{2\#1^4+3}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1+3*x^4+x^8),x]

[Out] RootSum[1+3*#1^4+#1^8 &, (Log[x-#1]*#1)/(3+2*#1^4) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1+3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(1+3*x^4+x^8),x]

[Out] IntegrateAlgebraic[x^4/(1+3*x^4+x^8),x]

fricas [B] time = 1.10, size = 843, normalized size = 1.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
-1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3)
+ 1/80*sqrt(sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*
sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) +
3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3
)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*ar
ctan(-1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt(5)
+ 3) + 1/80*sqrt(-sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^
2 + 5*sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sqrt(5) + 6)^(5/4)*sqrt(sqrt
(5) + 3) - 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(
5) + 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^
(3/4)*arctan(1/80*sqrt(sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) +
10*x^2 + 5*sqrt(-2*sqrt(5) + 6))*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sq
rt(5) + 6)^(5/4) - 1/80*(sqrt(10)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/
4) + 10*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) +
3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4
)*arctan(1/80*sqrt(-sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*
x^2 + 5*sqrt(-2*sqrt(5) + 6))*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sqrt
(5) + 6)^(5/4) - 1/80*(sqrt(10)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/4)
- 10*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)
) + 1/80*sqrt(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(sqrt(10)*sqrt(5)*sqrt(2
)*x*(2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)
*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(-sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) +
6)^(1/4) + 10*x^2 + 5*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sq
rt(5) + 6)^(1/4)*log(sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*
x^2 + 5*sqrt(-2*sqrt(5) + 6)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4
)*log(-sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 10*x^2 + 5*sqrt(
-2*sqrt(5) + 6))
```

giac [A] time = 0.77, size = 239, normalized size = 0.53

$$\frac{1}{80}(\pi + 4 \arctan(x\sqrt{\sqrt{5}-1} + 1))\sqrt{5\sqrt{5}+5} - \frac{1}{80}(\pi + 4 \arctan(-x\sqrt{\sqrt{5}-1} + 1))\sqrt{5\sqrt{5}+5} - \frac{1}{80}(\pi + 4 \arctan(x\sqrt{\sqrt{5}+1} - 1))\sqrt{5\sqrt{5}-5} + \frac{1}{80}(\pi + 4 \arctan(-x\sqrt{\sqrt{5}+1} - 1))\sqrt{5\sqrt{5}-5} + \frac{1}{80}(\pi + 4 \arctan(x\sqrt{\sqrt{5}-1} + 1))\sqrt{5\sqrt{5}+5} \log(\cos(x\sqrt{\sqrt{5}+1})^2 + \sin(x\sqrt{\sqrt{5}+1})^2) - \frac{1}{80}(\pi + 4 \arctan(-x\sqrt{\sqrt{5}-1} + 1))\sqrt{5\sqrt{5}+5} \log(\cos(x\sqrt{\sqrt{5}+1})^2 + \sin(x\sqrt{\sqrt{5}+1})^2) + \frac{1}{80}(\pi + 4 \arctan(x\sqrt{\sqrt{5}+1} - 1))\sqrt{5\sqrt{5}-5} \log(\cos(x\sqrt{\sqrt{5}+1})^2 + \sin(x\sqrt{\sqrt{5}+1})^2) - \frac{1}{80}(\pi + 4 \arctan(-x\sqrt{\sqrt{5}+1} - 1))\sqrt{5\sqrt{5}-5} \log(\cos(x\sqrt{\sqrt{5}+1})^2 + \sin(x\sqrt{\sqrt{5}+1})^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) + 5)*1
```

$\log(625*(x + \sqrt{\sqrt{5} + 1})^2 + 625*x^2) - 1/40*\sqrt{5*\sqrt{5} + 5}*\log(625*(x - \sqrt{\sqrt{5} + 1})^2 + 625*x^2) - 1/40*\sqrt{5*\sqrt{5} - 5}*\log(4225*(x + \sqrt{\sqrt{5} - 1})^2 + 4225*x^2) + 1/40*\sqrt{5*\sqrt{5} - 5}*\log(4225*(x - \sqrt{\sqrt{5} - 1})^2 + 4225*x^2)$

maple [C] time = 0.02, size = 40, normalized size = 0.09

$$\frac{\text{RootOf}(-Z^8 + 3Z^4 + 1)^4 \ln(-\text{RootOf}(-Z^8 + 3Z^4 + 1) + x)}{8 \text{RootOf}(-Z^8 + 3Z^4 + 1)^7 + 12 \text{RootOf}(-Z^8 + 3Z^4 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^8+3*x^4+1),x)

[Out] 1/4*sum(_R^4/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(-Z^8+3*_Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] integrate(x^4/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 0.20, size = 454, normalized size = 1.01

$$\frac{2^{14} \sqrt{5} \operatorname{atan}\left(\frac{32^{14} (\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} (\sqrt{5}-3)^{14}}\right) - 2^{14} \sqrt{5} \operatorname{atan}\left(\frac{32^{14} (\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} (\sqrt{5}-3)^{14}}\right) (-\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} \operatorname{atan}\left(\frac{32^{14} (\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} (\sqrt{5}-3)^{14}}\right) - 2^{14} \sqrt{5} \operatorname{atan}\left(\frac{32^{14} (\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} (\sqrt{5}-3)^{14}}\right) (-\sqrt{5}-3)^{14}} + 2^{14} \sqrt{5} \operatorname{atan}\left(\frac{32^{14} (\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} (\sqrt{5}-3)^{14}}\right) + 2^{14} \sqrt{5} \operatorname{atan}\left(\frac{32^{14} (\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} (\sqrt{5}-3)^{14}}\right) (\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} \operatorname{atan}\left(\frac{32^{14} (\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} (\sqrt{5}-3)^{14}}\right) + 2^{14} \sqrt{5} \operatorname{atan}\left(\frac{32^{14} (\sqrt{5}-3)^{14}}{2^{14} \sqrt{5} (\sqrt{5}-3)^{14}}\right) (\sqrt{5}-3)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(3*x^4 + x^8 + 1),x)

[Out] $(2^{(3/4)}*5^{(1/2)}*\operatorname{atan}((3*2^{(3/4)})*x*(-5^{(1/2)}-3)^{(1/4)})/(2*((3*2^{(1/2)})*(-5^{(1/2)}-3)^{(1/2)})/2 - (2^{(1/2)}*5^{(1/2)}*(-5^{(1/2)}-3)^{(1/2)})/2)) - (2^{(3/4)}*5^{(1/2)}*x*(-5^{(1/2)}-3)^{(1/4)})/(2*((3*2^{(1/2)})*(-5^{(1/2)}-3)^{(1/2)})/2 - (2^{(1/2)}*5^{(1/2)}*(-5^{(1/2)}-3)^{(1/2)})/2)))*(-5^{(1/2)}-3)^{(1/4)}/20 - (2^{(3/4)}*5^{(1/2)}*\operatorname{atan}((2^{(3/4)})*x*(-5^{(1/2)}-3)^{(1/4)}*3i)/(2*((3*2^{(1/2)})*(-5^{(1/2)}-3)^{(1/2)})/2 - (2^{(1/2)}*5^{(1/2)}*(-5^{(1/2)}-3)^{(1/2)})/2)) - (2^{(3/4)}*5^{(1/2)}*x*(-5^{(1/2)}-3)^{(1/4)}*1i)/(2*((3*2^{(1/2)})*(-5^{(1/2)}-3)^{(1/2)})/2 - (2^{(1/2)}*5^{(1/2)}*(-5^{(1/2)}-3)^{(1/2)})/2)))*(-5^{(1/2)}-3)^{(1/4)}*1i)/20 - (2^{(3/4)}*5^{(1/2)}*\operatorname{atan}((3*2^{(3/4)})*x*(5^{(1/2)}-3)^{(1/4)})/(2*((3*2^{(1/2)})*(5^{(1/2)}-3)^{(1/2)})/2 + (2^{(1/2)}*5^{(1/2)}*(5^{(1/2)}-3)^{(1/2)})/2))$

$$\begin{aligned}
& + (2^{3/4} \cdot 5^{1/2} \cdot x \cdot (5^{1/2} - 3)^{1/4}) / (2 \cdot ((3 \cdot 2^{1/2}) \cdot (5^{1/2} - 3)^{1/2}) / 2 + (2^{1/2} \cdot 5^{1/2} \cdot (5^{1/2} - 3)^{1/2}) / 2)) \cdot (5^{1/2} - 3)^{1/4} / 20 \\
& + (2^{3/4} \cdot 5^{1/2} \cdot \operatorname{atan}((2^{3/4} \cdot x \cdot (5^{1/2} - 3)^{1/4} \cdot 3i) / (2 \cdot ((3 \cdot 2^{1/2}) \cdot (5^{1/2} - 3)^{1/2}) / 2 + (2^{1/2} \cdot 5^{1/2} \cdot (5^{1/2} - 3)^{1/2}) / 2))) + (2^{3/4} \\
& \cdot 5^{1/2} \cdot x \cdot (5^{1/2} - 3)^{1/4} \cdot 1i) / (2 \cdot ((3 \cdot 2^{1/2}) \cdot (5^{1/2} - 3)^{1/2}) / 2 + (2^{1/2} \cdot 5^{1/2} \cdot (5^{1/2} - 3)^{1/2}) / 2)) \cdot (5^{1/2} - 3)^{1/4} \cdot 1i) / 20
\end{aligned}$$

sympy [A] time = 1.48, size = 24, normalized size = 0.05

$$\operatorname{RootSum}(40960000t^8 + 19200t^4 + 1, (t \mapsto t \log(-51200t^5 - 12t + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**8+3*x**4+1),x)

[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-51200*_t**5 - 12*_t + x)))

$$3.321 \quad \int \frac{x^2}{1+3x^4+x^8} dx$$

Optimal. Leaf size=427

$$\frac{\log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4\sqrt{5}\sqrt[4]{2(3-\sqrt{5})}} - \frac{\log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4\sqrt{5}\sqrt[4]{2(3-\sqrt{5})}} - \frac{\log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}} - \frac{\log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}}$$

Rubi [A] time = 0.26, antiderivative size = 431, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1375, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{3+\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4^{2/3}\sqrt{5}} - \frac{\sqrt{3+\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4^{2/3}\sqrt{5}} - \frac{\log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}} + \frac{\log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}} - \frac{\sqrt{3+\sqrt{5}} \tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt[4]{2(3-\sqrt{5})}}\right)}{2^{2/3}\sqrt{5}} + \frac{\sqrt{3+\sqrt{5}} \tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{2(3-\sqrt{5})}} + 1\right)}{2^{2/3}\sqrt{5}} + \frac{\tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt[4]{2(3+\sqrt{5})}}\right)}{2\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}} + \frac{\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt[4]{2(3+\sqrt{5})}} + 1\right)}{2\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + 3*x^4 + x^8), x]

[Out] $-\left((3 + \sqrt{5})^{1/4} \operatorname{ArcTan}\left[1 - (2^{3/4}x)/(3 - \sqrt{5})^{1/4}\right]\right)/(2 \cdot 2^{3/4} \sqrt{5}) + \left((3 + \sqrt{5})^{1/4} \operatorname{ArcTan}\left[1 + (2^{3/4}x)/(3 - \sqrt{5})^{1/4}\right]\right)/(2 \cdot 2^{3/4} \sqrt{5}) + \operatorname{ArcTan}\left[1 - (2^{3/4}x)/(3 + \sqrt{5})^{1/4}\right]/(2 \cdot \sqrt{5} \cdot (2 \cdot (3 + \sqrt{5}))^{1/4}) - \operatorname{ArcTan}\left[1 + (2^{3/4}x)/(3 + \sqrt{5})^{1/4}\right]/(2 \cdot \sqrt{5} \cdot (2 \cdot (3 + \sqrt{5}))^{1/4}) + \left((3 + \sqrt{5})^{1/4} \operatorname{Log}\left[\sqrt{5} \cdot (2 \cdot (3 - \sqrt{5}))^{1/4} - 2 \cdot (2 \cdot (3 - \sqrt{5}))^{1/4} \cdot x + 2 \cdot x^2\right]\right)/(4 \cdot 2^{3/4} \sqrt{5}) - \left((3 + \sqrt{5})^{1/4} \operatorname{Log}\left[\sqrt{5} \cdot (2 \cdot (3 - \sqrt{5}))^{1/4} + 2 \cdot (2 \cdot (3 - \sqrt{5}))^{1/4} \cdot x + 2 \cdot x^2\right]\right)/(4 \cdot 2^{3/4} \sqrt{5}) - \operatorname{Log}\left[\sqrt{5} \cdot (2 \cdot (3 + \sqrt{5}))^{1/4} - 2 \cdot (2 \cdot (3 + \sqrt{5}))^{1/4} \cdot x + 2 \cdot x^2\right]/(4 \cdot \sqrt{5} \cdot (2 \cdot (3 + \sqrt{5}))^{1/4}) + \operatorname{Log}\left[\sqrt{5} \cdot (2 \cdot (3 + \sqrt{5}))^{1/4} + 2 \cdot (2 \cdot (3 + \sqrt{5}))^{1/4} \cdot x + 2 \cdot x^2\right]/(4 \cdot \sqrt{5} \cdot (2 \cdot (3 + \sqrt{5}))^{1/4})$

Rule 204

Int[((a_) + (b_) * (x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

Int[(x_)^2/((a_) + (b_) * (x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] & AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1375

```
Int[((d_.)*(x_)^(m_.))/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+3x^4+x^8} dx &= \frac{\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{\sqrt{5}} - \frac{\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{\sqrt{5}} \\
&= -\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{10}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{10}} \\
&= \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})}-\sqrt[4]{2(3-\sqrt{5})}x+x^2} dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})}+\sqrt[4]{2(3-\sqrt{5})}x+x^2} dx}{4\sqrt{5}} - \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3+\sqrt{5})}-\sqrt[4]{2(3+\sqrt{5})}x+x^2} dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3+\sqrt{5})}+\sqrt[4]{2(3+\sqrt{5})}x+x^2} dx}{4\sqrt{5}} \\
&= \frac{\sqrt[4]{3+\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})}-2\sqrt[4]{2(3-\sqrt{5})}x+2x^2\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})}+2\sqrt[4]{2(3-\sqrt{5})}x+2x^2\right)}{4 \cdot 2^{3/4}\sqrt{5}} \\
&\quad + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2(3-\sqrt{5})}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2(3-\sqrt{5})}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2(3+\sqrt{5})}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&, \frac{\log(x-\#1)}{2\#1^5+3\#1}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1+3*x^4+x^8),x]

[Out] RootSum[1+3*#1^4+#1^8 &, Log[x-#1]/(3*#1+2*#1^5) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1+3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1+3*x^4+x^8),x]

[Out] IntegrateAlgebraic[x^2/(1 + 3*x^4 + x^8), x]

fricas [B] time = 1.86, size = 955, normalized size = 2.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{80}\sqrt{10}(2\sqrt{5}+6)^{3/4}\sqrt{\sqrt{5}+3}(\sqrt{5}-3)\arctan\left(\frac{-1/40\sqrt{10}(3\sqrt{5}x-5x)(2\sqrt{5}+6)^{3/4}\sqrt{\sqrt{5}+3}}{1/80\sqrt{10}(3\sqrt{5})\sqrt{2}x-5\sqrt{2}x(2\sqrt{5}+6)^{3/4}+40x^2-10\sqrt{2\sqrt{5}+6}(\sqrt{5}-3)(3\sqrt{5}-5)(2\sqrt{5}+6)^{3/4}\sqrt{\sqrt{5}+3}+1/8(\sqrt{5}\sqrt{2}-3\sqrt{2})\sqrt{2\sqrt{5}+6}\sqrt{\sqrt{5}+3}}\right)+\frac{1}{80}\sqrt{10}(2\sqrt{5}+6)^{3/4}\sqrt{\sqrt{5}+3}(\sqrt{5}-3)\arctan\left(\frac{-1/40\sqrt{10}(3\sqrt{5})\sqrt{2}x-5\sqrt{2}x(2\sqrt{5}+6)^{3/4}+40x^2-10\sqrt{2\sqrt{5}+6}(\sqrt{5}-3)(3\sqrt{5}-5)(2\sqrt{5}+6)^{3/4}\sqrt{\sqrt{5}+3}-1/8(\sqrt{5}\sqrt{2}-3\sqrt{2})\sqrt{2\sqrt{5}+6}\sqrt{\sqrt{5}+3}}{1/80\sqrt{10}(3\sqrt{5})\sqrt{2}x+5\sqrt{2}x(-2\sqrt{5}+6)^{3/4}+40x^2+10(\sqrt{5}+3)\sqrt{-2\sqrt{5}+6}(3\sqrt{5}+5)\sqrt{-\sqrt{5}+3}(-2\sqrt{5}+6)^{3/4}-1/40(\sqrt{10}(3\sqrt{5})x+5x)(-2\sqrt{5}+6)^{3/4}+5(\sqrt{5}\sqrt{2}+3\sqrt{2})\sqrt{-2\sqrt{5}+6}\sqrt{-\sqrt{5}+3}}\right)+\frac{1}{80}\sqrt{10}(3\sqrt{5})\sqrt{2}x+5\sqrt{2}x(-2\sqrt{5}+6)^{3/4}+40x^2+10(\sqrt{5}+3)\sqrt{-2\sqrt{5}+6}(3\sqrt{5}+5)\sqrt{-\sqrt{5}+3}(-2\sqrt{5}+6)^{3/4}-1/40(\sqrt{10}(3\sqrt{5})x+5x)(-2\sqrt{5}+6)^{3/4}-5(\sqrt{5}\sqrt{2}+3\sqrt{2})\sqrt{-2\sqrt{5}+6}\sqrt{-\sqrt{5}+3}}\right)-\frac{1}{80}\sqrt{10}\sqrt{2}(2\sqrt{5}+6)^{1/4}\log\left(\frac{\sqrt{10}(3\sqrt{5})\sqrt{2}x-5\sqrt{2}x(2\sqrt{5}+6)^{3/4}+40x^2-10\sqrt{2\sqrt{5}+6}(\sqrt{5}-3)}{\sqrt{10}(3\sqrt{5})\sqrt{2}x+5\sqrt{2}x(-2\sqrt{5}+6)^{3/4}+40x^2+10(\sqrt{5}+3)\sqrt{-2\sqrt{5}+6}(3\sqrt{5}+5)\sqrt{-\sqrt{5}+3}}\right)+\frac{1}{80}\sqrt{10}\sqrt{2}(2\sqrt{5}+6)^{1/4}\log\left(\frac{-\sqrt{10}(3\sqrt{5})\sqrt{2}x-5\sqrt{2}x(2\sqrt{5}+6)^{3/4}+40x^2-10\sqrt{2\sqrt{5}+6}(\sqrt{5}-3)}{\sqrt{10}(3\sqrt{5})\sqrt{2}x+5\sqrt{2}x(-2\sqrt{5}+6)^{3/4}+40x^2+10(\sqrt{5}+3)\sqrt{-2\sqrt{5}+6}(3\sqrt{5}+5)\sqrt{-\sqrt{5}+3}}\right)+40x^2+10(\sqrt{5}+3)\sqrt{-2\sqrt{5}+6}$

giac [A] time = 0.59, size = 239, normalized size = 0.56

$\frac{1}{80}(1+\arctan(\sqrt{5}-1))\sqrt{5\sqrt{5}+5}-\frac{1}{80}(1+\arctan(-\sqrt{5}-1))\sqrt{5\sqrt{5}+5}+\frac{1}{80}(1+\arctan(\sqrt{5}-1))\sqrt{5\sqrt{5}-5}+\frac{1}{80}(1+\arctan(-\sqrt{5}-1))\sqrt{5\sqrt{5}-5}+\frac{1}{80}\sqrt{5\sqrt{5}-5}\log\left(\frac{1000(1+\sqrt{5}+1)+10000x^2}{1000(1-\sqrt{5}+1)+10000x^2}\right)+\frac{1}{80}\sqrt{5\sqrt{5}+5}\log\left(\frac{2500(1+\sqrt{5}-1)+2500x^2}{2500(1-\sqrt{5}-1)+2500x^2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="giac")

[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) - 5)*log(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) + 5)*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) + 1/40*sqrt(5*sqrt(5) + 5)*log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)

maple [C] time = 0.01, size = 40, normalized size = 0.09

$$\frac{\text{RootOf}(-Z^8 + 3Z^4 + 1)^2 \ln(-\text{RootOf}(-Z^8 + 3Z^4 + 1) + x)}{8 \text{RootOf}(-Z^8 + 3Z^4 + 1)^7 + 12 \text{RootOf}(-Z^8 + 3Z^4 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^8+3*x^4+1),x)

[Out] 1/4*sum(_R^2/(2*_R^7+3*_R^3)*ln(-_R+x), _R=RootOf(-Z^8+3*_Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] integrate(x^2/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 0.09, size = 275, normalized size = 0.64

$$\frac{2^{3/4} \sqrt{5} \operatorname{atan}\left(\frac{72^{3/4} (\sqrt{5}-3)^{1/4} - 32^{3/4} \sqrt{5} (\sqrt{5}-3)^{1/4}}{2(\beta \sqrt{5}-7)}\right) (\sqrt{5}-3)^{1/4}}{20} + \frac{2^{3/4} \sqrt{5} \operatorname{atan}\left(\frac{2^{3/4} (\sqrt{5}-3)^{1/4} \eta - 2^{3/4} \sqrt{5} (\sqrt{5}-3)^{1/4} \eta}{2(\beta \sqrt{5}-7)}\right) (\sqrt{5}-3)^{1/4} i}{20} + \frac{2^{3/4} \sqrt{5} \operatorname{atan}\left(\frac{72^{3/4} (-\sqrt{5}-3)^{1/4} + 32^{3/4} \sqrt{5} (-\sqrt{5}-3)^{1/4}}{2(\beta \sqrt{5}+7)}\right) (-\sqrt{5}-3)^{1/4}}{20} + \frac{2^{3/4} \sqrt{5} \operatorname{atan}\left(\frac{2^{3/4} (-\sqrt{5}-3)^{1/4} \eta + 2^{3/4} \sqrt{5} (-\sqrt{5}-3)^{1/4} \eta}{2(\beta \sqrt{5}+7)}\right) (-\sqrt{5}-3)^{1/4} i}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(3*x^4 + x^8 + 1),x)

[Out] (2^(3/4)*5^(1/2)*atan((7*2^(3/4))*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)) - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)))*(5^(1/2) - 3)^(1/4)/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4))*x*(5^(1/2) - 3)^(1/4)*7i)/(2*(3*5^(1/2) - 7)) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/2) - 7)))*(5^(1/2) - 3)^(1/4)*i)/20 + (2^(3/4)*5^(1/2)*atan((7*2^(3/4))*x*(

$$\begin{aligned}
& - 5^{1/2} - 3^{1/4}) / (2 * (3 * 5^{1/2} + 7)) + (3 * 2^{3/4} * 5^{1/2} * x * (- 5^{1/2} \\
& - 3^{1/4}) / (2 * (3 * 5^{1/2} + 7))) * (- 5^{1/2} - 3^{1/4}) / 20 + (2^{3/4} * 5^{1/2} \\
& / 2) * \operatorname{atan}((2^{3/4} * x * (- 5^{1/2} - 3^{1/4}) * 7i) / (2 * (3 * 5^{1/2} + 7))) + (2^{3/4} \\
&) * 5^{1/2} * x * (- 5^{1/2} - 3^{1/4}) * 3i) / (2 * (3 * 5^{1/2} + 7))) * (- 5^{1/2} - 3^{1/4}) \\
& (1/4) * 1i) / 20
\end{aligned}$$

sympy [A] time = 1.51, size = 26, normalized size = 0.06

$$\operatorname{RootSum}(40960000t^8 + 19200t^4 + 1, (t \mapsto t \log(-6144000t^7 - 2240t^3 + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(x**8+3*x**4+1),x)`

[Out] `RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-6144000*_t**7 - 2240*_t**3 + x)))`

$$3.322 \quad \int \frac{1}{1+3x^4+x^8} dx$$

Optimal. Leaf size=414

$$\frac{\sqrt[4]{9+4\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4\sqrt{10}}$$

Rubi [A] time = 0.26, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {1347, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{9+4\sqrt{5}} \log(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})})}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}} \log(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})})}{4\sqrt{10}} + \frac{\log(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})})}{2\sqrt{5}(2(3+\sqrt{5}))^{3/4}} - \frac{\log(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})})}{2\sqrt{5}(2(3+\sqrt{5}))^{3/4}} - \frac{\sqrt[4]{9+4\sqrt{5}} \tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}} \tan^{-1}\left(\frac{2^{3/4}x}{\sqrt{3-\sqrt{5}}} + 1\right)}{2\sqrt{10}} + \frac{\tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{5}(2(3+\sqrt{5}))^{3/4}} - \frac{\tan^{-1}\left(\frac{2^{3/4}x}{\sqrt{3+\sqrt{5}}} + 1\right)}{\sqrt{5}(2(3+\sqrt{5}))^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3*x^4 + x^8)^(-1), x]

[Out] -((9 + 4*sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - sqrt[5])^(1/4)]/(2*sqrt[10]) + ((9 + 4*sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - sqrt[5])^(1/4)]/(2*sqrt[10]) + ArcTan[1 - (2^(3/4)*x)/(3 + sqrt[5])^(1/4)]/(sqrt[5]*(2*(3 + sqrt[5]))^(3/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + sqrt[5])^(1/4)]/(sqrt[5]*(2*(3 + sqrt[5]))^(3/4)) - ((9 + 4*sqrt[5])^(1/4)*Log[sqrt[2*(3 - sqrt[5])]] - 2*(2*(3 - sqrt[5]))^(1/4)*x + 2*x^2)/(4*sqrt[10]) + ((9 + 4*sqrt[5])^(1/4)*Log[sqrt[2*(3 - sqrt[5])]] + 2*(2*(3 - sqrt[5]))^(1/4)*x + 2*x^2)/(4*sqrt[10]) + Log[sqrt[2*(3 + sqrt[5])]] - 2*(2*(3 + sqrt[5]))^(1/4)*x + 2*x^2/(2*sqrt[5]*(2*(3 + sqrt[5]))^(3/4)) - Log[sqrt[2*(3 + sqrt[5])]] + 2*(2*(3 + sqrt[5]))^(1/4)*x + 2*x^2/(2*sqrt[5]*(2*(3 + sqrt[5]))^(3/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(n_ - 1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^n), x], x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+3x^4+x^8} dx &= \frac{\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{\sqrt{5}} - \frac{\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{\sqrt{5}} \\
&= \frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{5}(3-\sqrt{5})} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{5}(3-\sqrt{5})} - \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{5}(3+\sqrt{5})} - \frac{\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{2\sqrt{5}(3+\sqrt{5})} \\
&= \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})}-\sqrt[4]{2(3-\sqrt{5})}x+x^2} dx}{2\sqrt{10}(3-\sqrt{5})} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}(3-\sqrt{5})}+\sqrt[4]{2(3-\sqrt{5})}x+x^2} dx}{2\sqrt{10}(3-\sqrt{5})} + \frac{\int \frac{\sqrt[4]{2(3+\sqrt{5})}+2x}{-\sqrt{\frac{1}{2}(3+\sqrt{5})}-\sqrt[4]{2(3+\sqrt{5})}x+x^2} dx}{2\sqrt{5}(2(3+\sqrt{5}))^{3/4}} \\
&= -\frac{\sqrt[4]{9+4\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})}-2\sqrt[4]{2(3-\sqrt{5})}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}} \log\left(\sqrt{2(3-\sqrt{5})}+2\sqrt[4]{2(3-\sqrt{5})}x+2x^2\right)}{4\sqrt{10}} \\
&= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}(2(3-\sqrt{5}))^{3/4}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}(2(3-\sqrt{5}))^{3/4}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}(2(3+\sqrt{5}))^{3/4}} - \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}(2(3+\sqrt{5}))^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.10

$$\frac{1}{4}\text{RootSum}\left[\#1^8+3\#1^4+1\&, \frac{\log(x-\#1)}{2\#1^7+3\#1^3}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3*x^4 + x^8)^(-1), x]

[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1^3 + 2*#1^7) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 3*x^4 + x^8)^(-1), x]

[Out] IntegrateAlgebraic[(1 + 3*x^4 + x^8)^(-1), x]

fricas [B] time = 1.76, size = 733, normalized size = 1.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{10}\sqrt{5}\sqrt{2}(4\sqrt{5}+9)^{1/4}\arctan\left(\frac{1}{2}\sqrt{2}x^2-\sqrt{4\sqrt{5}+9}\right)(3\sqrt{5}-7)+(\sqrt{5}\sqrt{2}x-3\sqrt{2}x)(4\sqrt{5}+9)^{1/4}(4\sqrt{5}+9)^{3/4}(3\sqrt{5}-7)-\frac{1}{2}(3\sqrt{5}\sqrt{2}x-7\sqrt{2}x)(4\sqrt{5}+9)^{3/4}-1+\frac{1}{10}\sqrt{5}\sqrt{2}(4\sqrt{5}+9)^{1/4}\arctan\left(\frac{1}{2}\sqrt{2}x^2-\sqrt{4\sqrt{5}+9}\right)(3\sqrt{5}-7)-(\sqrt{5}\sqrt{2}x-3\sqrt{2}x)(4\sqrt{5}+9)^{1/4}(4\sqrt{5}+9)^{3/4}(3\sqrt{5}-7)-\frac{1}{2}(3\sqrt{5}\sqrt{2}x-7\sqrt{2}x)(4\sqrt{5}+9)^{3/4}+1+\frac{1}{10}\sqrt{5}\sqrt{2}(-4\sqrt{5}+9)^{1/4}\arctan\left(\frac{1}{2}\sqrt{2}x^2+(3\sqrt{5}+7)\sqrt{-4\sqrt{5}+9}\right)+(\sqrt{5}\sqrt{2}x+3\sqrt{2}x)(-4\sqrt{5}+9)^{1/4}(3\sqrt{5}+7)(-4\sqrt{5}+9)^{3/4}-\frac{1}{2}(3\sqrt{5}\sqrt{2}x+7\sqrt{2}x)(-4\sqrt{5}+9)^{3/4}-1+\frac{1}{10}\sqrt{5}\sqrt{2}(-4\sqrt{5}+9)^{1/4}\arctan\left(\frac{1}{2}\sqrt{2}x^2+(3\sqrt{5}+7)\sqrt{-4\sqrt{5}+9}\right)-(\sqrt{5}\sqrt{2}x+3\sqrt{2}x)(-4\sqrt{5}+9)^{1/4}(3\sqrt{5}+7)(-4\sqrt{5}+9)^{3/4}-\frac{1}{2}(3\sqrt{5}\sqrt{2}x+7\sqrt{2}x)(-4\sqrt{5}+9)^{3/4}+1-\frac{1}{40}\sqrt{5}\sqrt{2}(4\sqrt{5}+9)^{1/4}\log(2x^2-\sqrt{4\sqrt{5}+9})(3\sqrt{5}-7)+(\sqrt{5}\sqrt{2}x-3\sqrt{2}x)(4\sqrt{5}+9)^{1/4}+\frac{1}{40}\sqrt{5}\sqrt{2}(4\sqrt{5}+9)^{1/4}\log(2x^2-\sqrt{4\sqrt{5}+9})(3\sqrt{5}-7)-(\sqrt{5}\sqrt{2}x-3\sqrt{2}x)(4\sqrt{5}+9)^{1/4}-\frac{1}{40}\sqrt{5}\sqrt{2}(-4\sqrt{5}+9)^{1/4}\log(2x^2+(3\sqrt{5}+7)\sqrt{-4\sqrt{5}+9})+(\sqrt{5}\sqrt{2}x+3\sqrt{2}x)(-4\sqrt{5}+9)^{1/4}+\frac{1}{40}\sqrt{5}\sqrt{2}(-4\sqrt{5}+9)^{1/4}\log(2x^2+(3\sqrt{5}+7)\sqrt{-4\sqrt{5}+9})-(\sqrt{5}\sqrt{2}x+3\sqrt{2}x)(-4\sqrt{5}+9)^{1/4}$

giac [A] time = 0.53, size = 239, normalized size = 0.58

$\frac{1}{20}\left(\frac{1}{2}+4\arctan(\sqrt{5}+1)\right)\sqrt{10\sqrt{5}+20}-\frac{1}{20}\left(\frac{1}{2}+4\arctan(-\sqrt{5}+1)\right)\sqrt{10\sqrt{5}+20}-\frac{1}{20}\left(\frac{1}{2}+4\arctan(\sqrt{5}-1)\right)\sqrt{10\sqrt{5}-20}+\frac{1}{20}\left(\frac{1}{2}+4\arctan(-\sqrt{5}-1)\right)\sqrt{10\sqrt{5}-20}-\frac{1}{20}\sqrt{10\sqrt{5}-20}\log\left(\frac{(x+\sqrt{5}+1)^2+10000x^2}{(x-\sqrt{5}+1)^2+10000x^2}\right)+\frac{1}{20}\sqrt{10\sqrt{5}-20}\log\left(\frac{(x-\sqrt{5}+1)^2+10000x^2}{(x+\sqrt{5}-1)^2+10000x^2}\right)+\frac{1}{20}\sqrt{10\sqrt{5}+20}\log\left(\frac{(x+\sqrt{5}-1)^2+400x^2}{(x-\sqrt{5}-1)^2+400x^2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+3*x^4+1),x, algorithm="giac")

[Out] $\frac{1}{80}(\pi+4\arctan(x\sqrt{\sqrt{5}+1}+1))\sqrt{10\sqrt{5}+20}-\frac{1}{80}(\pi+4\arctan(-x\sqrt{\sqrt{5}+1}+1))\sqrt{10\sqrt{5}+20}-\frac{1}{80}(\pi+4\arctan(x\sqrt{\sqrt{5}-1}-1))\sqrt{10\sqrt{5}-20}+\frac{1}{80}(\pi+4\arctan(-x\sqrt{\sqrt{5}-1}-1))\sqrt{10\sqrt{5}-20}-\frac{1}{40}\sqrt{10\sqrt{5}-20}\log(10000(x+\sqrt{\sqrt{5}+1})^2+10000x^2)+\frac{1}{40}\sqrt{10\sqrt{5}-20}\log(10000(x-\sqrt{\sqrt{5}+1})^2+10000x^2)+\frac{1}{40}\sqrt{10\sqrt{5}+20}\log(400(x+\sqrt{\sqrt{5}-1})^2+400x^2)-\frac{1}{40}\sqrt{10\sqrt{5}+20}\log(400(x-\sqrt{\sqrt{5}-1})^2+400x^2)$

*sqrt(5) + 20)*log(400*(x + sqrt(sqrt(5) - 1))^2 + 400*x^2) - 1/40*sqrt(10*sqrt(5) + 20)*log(400*(x - sqrt(sqrt(5) - 1))^2 + 400*x^2)

maple [C] time = 0.01, size = 37, normalized size = 0.09

$$\frac{\ln\left(-\text{RootOf}\left(-Z^8 + 3_Z^4 + 1\right) + x\right)}{8 \text{RootOf}\left(-Z^8 + 3_Z^4 + 1\right)^7 + 12 \text{RootOf}\left(-Z^8 + 3_Z^4 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8+3*x^4+1),x)

[Out] 1/4*sum(1/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(-Z^8+3*_Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] integrate(1/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 0.08, size = 403, normalized size = 0.97

$$\frac{\sqrt{5} \operatorname{atan}\left(\frac{144(-4\sqrt{5}-9)^{14}}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}\sqrt{4\sqrt{5}-9}} + \frac{64\sqrt{5}(4\sqrt{5}-9)^{14}}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}\sqrt{4\sqrt{5}-9}}\right)(-4\sqrt{5}-9)^{14}}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{144(4\sqrt{5}-9)^{14}}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}\sqrt{4\sqrt{5}-9}} - \frac{64\sqrt{5}(4\sqrt{5}-9)^{14}}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}\sqrt{4\sqrt{5}-9}}\right)(4\sqrt{5}-9)^{14}}{10} - \frac{\sqrt{5} \operatorname{atan}\left(\frac{(-4\sqrt{5}-9)^{14}144}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}\sqrt{4\sqrt{5}-9}} + \frac{\sqrt{5}(4\sqrt{5}-9)^{14}64}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}\sqrt{4\sqrt{5}-9}}\right)(-4\sqrt{5}-9)^{14}}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{(4\sqrt{5}-9)^{14}144}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}\sqrt{4\sqrt{5}-9}} - \frac{\sqrt{5}(4\sqrt{5}-9)^{14}64}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}\sqrt{4\sqrt{5}-9}}\right)(4\sqrt{5}-9)^{14}}{10} i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^4 + x^8 + 1),x)

[Out] (5^(1/2)*atan((144*x*(-4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(-4*5^(1/2) - 9)^(1/2) + 56*(-4*5^(1/2) - 9)^(1/2)) + (64*5^(1/2)*x*(-4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(-4*5^(1/2) - 9)^(1/2) + 56*(-4*5^(1/2) - 9)^(1/2)))*(-4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((144*x*(4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (64*5^(1/2)*x*(4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)))*(-4*5^(1/2) - 9)^(1/4))/10 - (5^(1/2)*atan((x*(-4*5^(1/2) - 9)^(1/4))*144i)/(24*5^(1/2)*(-4*5^(1/2) - 9)^(1/2) + 56*(-4*5^(1/2) - 9)^(1/2)) + (5^(1/2)*x*(-4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*(-4*5^(1/2) - 9)^(1/2) + 56*(-4*5^(1/2) - 9)^(1/2)))*(-4*5^(1/2) - 9)^(1/4)*1i)/10 - (5^(1/2)*atan((x*(4*5^(1/2) - 9)^(1/4))*144i)/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (5^(1/2)*x*(4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)))*(-4*5^(1/2) - 9)^(1/4)*1i)/10

sympy [A] time = 1.52, size = 26, normalized size = 0.06

$$\text{RootSum}\left(40960000t^8 + 115200t^4 + 1, \left(t \mapsto t \log\left(-9600t^5 - \frac{47t}{2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**8+3*x**4+1),x)

[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-9600*_t**5 - 47*_t/2 + x)))

$$3.323 \quad \int \frac{1}{x^2(1+3x^4+x^8)} dx$$

Optimal. Leaf size=416

$$\frac{(3 + \sqrt{5})^{5/4} \log\left(2x^2 - 2\sqrt[4]{2(3 - \sqrt{5})}x + \sqrt{2(3 - \sqrt{5})}\right)}{8 \cdot 2^{3/4}\sqrt{5}} + \frac{(3 + \sqrt{5})^{5/4} \log\left(2x^2 + 2\sqrt[4]{2(3 - \sqrt{5})}x + \sqrt{2(3 - \sqrt{5})}\right)}{8 \cdot 2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.29, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1368, 1510, 297, 1162, 617, 204, 1165, 628}

$$\frac{(1+\sqrt{5})^{\frac{5}{4}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{8 \cdot 2^{3/4}\sqrt{5}} - \frac{(1+\sqrt{5})^{\frac{5}{4}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{8 \cdot 2^{3/4}\sqrt{5}} + \frac{1}{20} \sqrt[4]{6150 - 2750\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right) - \frac{1}{20} \sqrt[4]{6150 - 2750\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right) + \frac{(1+\sqrt{5})^{\frac{5}{4}} \operatorname{arctan}\left(\frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{(1+\sqrt{5})^{\frac{5}{4}} \operatorname{arctan}\left(\frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}} + 1\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{1}{20} \sqrt[4]{6150 - 2750\sqrt{5}} \operatorname{arctan}\left(\frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}}\right) + \frac{1}{20} \sqrt[4]{6150 - 2750\sqrt{5}} \operatorname{arctan}\left(\frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 + 3*x^4 + x^8)),x]

[Out] $-x^{-1} + \left(\frac{(3 + \sqrt{5})^{5/4} \operatorname{ArcTan}\left[1 - (2^{3/4}x)/(3 - \sqrt{5})^{1/4}\right]}{(4 \cdot 2^{3/4} \sqrt{5})} - \frac{(3 + \sqrt{5})^{5/4} \operatorname{ArcTan}\left[1 + (2^{3/4}x)/(3 - \sqrt{5})^{1/4}\right]}{(4 \cdot 2^{3/4} \sqrt{5})} - \frac{(6150 - 2750 \sqrt{5})^{1/4} \operatorname{ArcTan}\left[1 - (2^{3/4}x)/(3 + \sqrt{5})^{1/4}\right]}{20} + \frac{(6150 - 2750 \sqrt{5})^{1/4} \operatorname{ArcTan}\left[1 + (2^{3/4}x)/(3 + \sqrt{5})^{1/4}\right]}{20} - \frac{(3 + \sqrt{5})^{5/4} \operatorname{Log}\left[\sqrt{2} \cdot (3 - \sqrt{5})\right] - 2 \cdot (2 \cdot (3 - \sqrt{5}))^{1/4} x + 2x^2}}{(8 \cdot 2^{3/4} \sqrt{5})} + \frac{(3 + \sqrt{5})^{5/4} \operatorname{Log}\left[\sqrt{2} \cdot (3 - \sqrt{5})\right] + 2 \cdot (2 \cdot (3 - \sqrt{5}))^{1/4} x + 2x^2}}{(8 \cdot 2^{3/4} \sqrt{5})} + \frac{(6150 - 2750 \sqrt{5})^{1/4} \operatorname{Log}\left[\sqrt{2} \cdot (3 + \sqrt{5})\right] - 2 \cdot (2 \cdot (3 + \sqrt{5}))^{1/4} x + 2x^2}}{40} - \frac{(6150 - 2750 \sqrt{5})^{1/4} \operatorname{Log}\left[\sqrt{2} \cdot (3 + \sqrt{5})\right] + 2 \cdot (2 \cdot (3 + \sqrt{5}))^{1/4} x + 2x^2}}{40}\right)$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] & AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1368

```
Int[((d_.)*(x_)^(m_))*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(1+3x^4+x^8)} dx &= -\frac{1}{x} + \int \frac{x^2(-3-x^4)}{1+3x^4+x^8} dx \\
&= -\frac{1}{x} + \frac{1}{10}(-5+3\sqrt{5}) \int \frac{x^2}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx - \frac{1}{10}(5+3\sqrt{5}) \int \frac{x^2}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx \\
&= -\frac{1}{x} - \frac{(3-\sqrt{5}) \int \frac{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx}{4\sqrt{10}} + \frac{(3-\sqrt{5}) \int \frac{\sqrt{3+\sqrt{5}+\sqrt{2}x^2}}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx}{4\sqrt{10}} + \frac{(3+\sqrt{5}) \int \frac{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx}{4\sqrt{10}} \\
&= -\frac{1}{x} - \frac{(3+\sqrt{5})^{5/4} \int \frac{\sqrt[4]{2(3-\sqrt{5})+2x}}{-\sqrt{\frac{1}{2}(3-\sqrt{5})} - \sqrt[4]{2(3-\sqrt{5})}x-x^2} dx}{8 \cdot 2^{3/4}\sqrt{5}} - \frac{(3+\sqrt{5})^{5/4} \int \frac{\sqrt[4]{2(3-\sqrt{5})-2x}}{-\sqrt{\frac{1}{2}(3-\sqrt{5})} + \sqrt[4]{2(3-\sqrt{5})}x-x^2} dx}{8 \cdot 2^{3/4}\sqrt{5}} \\
&= -\frac{1}{x} - \frac{(3+\sqrt{5})^{5/4} \log\left(\sqrt{2(3-\sqrt{5})} - 2\sqrt[4]{2(3-\sqrt{5})}x + 2x^2\right)}{8 \cdot 2^{3/4}\sqrt{5}} + \frac{(3+\sqrt{5})^{5/4} \log\left(\sqrt{2(3-\sqrt{5})} + 2\sqrt[4]{2(3-\sqrt{5})}x + 2x^2\right)}{8 \cdot 2^{3/4}\sqrt{5}} \\
&= -\frac{1}{x} + \frac{\sqrt[4]{246+110\sqrt{5}} \tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}} - \frac{\sqrt[4]{246+110\sqrt{5}} \tan^{-1}\left(1 + \frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}} - \frac{1}{x}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[\#1^8 + 3\#1^4 + 1\&, \frac{\#1^4 \log(x - \#1) + 3 \log(x - \#1)}{2\#1^5 + 3\#1}\&\right] - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(1+3*x^4+x^8)),x]

[Out] -x^(-1) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)/(3*#1 + 2*#1^5) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1+3x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(1 + 3*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^2*(1 + 3*x^4 + x^8)), x]

fricas [B] time = 1.50, size = 1017, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/80*(\sqrt{10}*(55*\sqrt{5}*x - 123*x)*(110*\sqrt{5} + 246)^{3/4}*\sqrt{55*\sqrt{5} + 123}*\arctan(-1/20*\sqrt{10}*(161*\sqrt{5}*x - 360*x)*(110*\sqrt{5} + 246)^{3/4}*\sqrt{55*\sqrt{5} + 123} \\ & + 1/40*\sqrt{(\sqrt{10}*(47*\sqrt{5}*\sqrt{2}*x - 105*\sqrt{2}*x)*(110*\sqrt{5} + 246)^{3/4} + 40*x^2 - 20*\sqrt{110*\sqrt{5} + 246}*(4*\sqrt{5} - 9))} \\ & *(161*\sqrt{5} - 360)*(110*\sqrt{5} + 246)^{3/4}*\sqrt{55*\sqrt{5} + 123} + 1/8*(55*\sqrt{5}*\sqrt{2} - 123*\sqrt{2})*\sqrt{110*\sqrt{5} + 246} \\ & *\sqrt{55*\sqrt{5} + 123}) + \sqrt{10}*(55*\sqrt{5}*x - 123*x)*(110*\sqrt{5} + 246)^{3/4}*\sqrt{55*\sqrt{5} + 123}*\arctan(-1/20*\sqrt{10}*(161*\sqrt{5} \\ & *x - 360*x)*(110*\sqrt{5} + 246)^{3/4}*\sqrt{55*\sqrt{5} + 123} + 1/40*\sqrt{(-\sqrt{10}*(47*\sqrt{5}*\sqrt{2}*x - 105*\sqrt{2}*x)*(110*\sqrt{5} + 246)^{3/4} \\ & + 40*x^2 - 20*\sqrt{110*\sqrt{5} + 246}*(4*\sqrt{5} - 9))}*(161*\sqrt{5} - 360)*(110*\sqrt{5} + 246)^{3/4}*\sqrt{55*\sqrt{5} + 123} \\ & - 1/8*(55*\sqrt{5}*\sqrt{2} - 123*\sqrt{2})*\sqrt{110*\sqrt{5} + 246}*\sqrt{55*\sqrt{5} + 123}) + \sqrt{10}*(55*\sqrt{5}*x + 123*x) \\ & *\sqrt{-55*\sqrt{5} + 123}*(-110*\sqrt{5} + 246)^{3/4}*\arctan(1/40*\sqrt{(\sqrt{10}*(47*\sqrt{5}*\sqrt{2}*x + 105*\sqrt{2}*x)*(-110*\sqrt{5} + 246)^{3/4} \\ & + 40*x^2 + 20*(4*\sqrt{5} + 9)*\sqrt{-110*\sqrt{5} + 246})}*(161*\sqrt{5} + 360)*\sqrt{-55*\sqrt{5} + 123}*(-110*\sqrt{5} + 246)^{3/4} \\ & - 1/40*(2*\sqrt{10}*(161*\sqrt{5}*x + 360*x)*(-110*\sqrt{5} + 246)^{3/4} + 5*(55*\sqrt{5}*\sqrt{2} + 123*\sqrt{2})*\sqrt{-110*\sqrt{5} + 246})*\sqrt{-55*\sqrt{5} + 123}) \\ & + \sqrt{10}*(55*\sqrt{5}*x + 123*x)*\sqrt{-55*\sqrt{5} + 123}*(-110*\sqrt{5} + 246)^{3/4}*\arctan(1/40*\sqrt{(-\sqrt{10}*(47*\sqrt{5}*\sqrt{2}*x + 105*\sqrt{2}*x)* \\ & (-110*\sqrt{5} + 246)^{3/4} + 40*x^2 + 20*(4*\sqrt{5} + 9)*\sqrt{-110*\sqrt{5} + 246})}*(161*\sqrt{5} + 360)*\sqrt{-55*\sqrt{5} + 123}*(-110*\sqrt{5} + 246)^{3/4} \\ & - 1/40*(2*\sqrt{10}*(161*\sqrt{5}*x + 360*x)*(-110*\sqrt{5} + 246)^{3/4} - 5*(55*\sqrt{5}*\sqrt{2} + 123*\sqrt{2})*\sqrt{-110*\sqrt{5} + 246})*\sqrt{-55*\sqrt{5} + 123}) \\ & - \sqrt{10}*\sqrt{2}*x*(110*\sqrt{5} + 246)^{1/4}*\log(\sqrt{10}*(47*\sqrt{5}*\sqrt{2}*x - 105*\sqrt{2}*x)*(110*\sqrt{5} + 246)^{3/4} + 40*x^2 - 20*\sqrt{110*\sqrt{5} + 246} \\ & *(4*\sqrt{5} - 9)) + \sqrt{10}*\sqrt{2}*x*(110*\sqrt{5} + 246)^{1/4}*\log(-\sqrt{10}*(47*\sqrt{5}*\sqrt{2}*x - 105*\sqrt{2}*x)*(110*\sqrt{5} + 246)^{3/4} \\ & + 40*x^2 - 20*\sqrt{110*\sqrt{5} + 246}*(4*\sqrt{5} - 9)) + \sqrt{10}*\sqrt{2}*x*(-110*\sqrt{5} + 246)^{1/4}*\log(\sqrt{10}*(47*\sqrt{5}*\sqrt{2}*x + 105*\sqrt{2}*x)* \\ & (-110*\sqrt{5} + 246)^{3/4} + 40*x^2 + 20*(4*\sqrt{5} + 9)*\sqrt{-110*\sqrt{5} + 246}) - \sqrt{10}*\sqrt{2}*x*(-110*\sqrt{5} + 246)^{1/4} \\ & *\log(\sqrt{10}*(47*\sqrt{5}*\sqrt{2}*x + 105*\sqrt{2}*x)*(-110*\sqrt{5} + 246)^{3/4} + 40*x^2 + 20*(4*\sqrt{5} + 9)*\sqrt{-110*\sqrt{5} + 246}) \end{aligned}$$

$(1/4)*\log(-\sqrt{10}*(47*\sqrt{5}*\sqrt{2}*x + 105*\sqrt{2}*x)*(-110*\sqrt{5} + 246)^{(3/4)} + 40*x^2 + 20*(4*\sqrt{5} + 9)*\sqrt{-110*\sqrt{5} + 246}) + 80)/x$

giac [A] time = 0.60, size = 244, normalized size = 0.59

$\frac{1}{80} \left(\pi + 4 \arctan(\sqrt{5} + 1) \right) \sqrt{25\sqrt{5} + 55} + \frac{1}{80} \left(\pi + 4 \arctan(-\sqrt{5} + 1) \right) \sqrt{25\sqrt{5} - 55} + \frac{1}{40} \left(\pi + 4 \arctan(\sqrt{5} - 1) \right) \sqrt{25\sqrt{5} - 55} - \frac{1}{40} \left(\pi + 4 \arctan(-\sqrt{5} - 1) \right) \sqrt{25\sqrt{5} - 55} - \frac{1}{40} \sqrt{25\sqrt{5} - 55} \log(748225(x + \sqrt{5} + 1)^2 + 748225x^2) + \frac{1}{40} \sqrt{25\sqrt{5} - 55} \log(180625(x + \sqrt{5} - 1)^2 + 180625x^2) - \frac{1}{40} \sqrt{25\sqrt{5} + 55} \log(180625(x - \sqrt{5} + 1)^2 + 180625x^2) - \frac{1}{40} \sqrt{25\sqrt{5} + 55} \log(180625(x - \sqrt{5} - 1)^2 + 180625x^2) - \frac{1}{x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="giac")

[Out] $-1/80*(\pi + 4*\arctan(x*\sqrt{\sqrt{5} + 1} - 1))*\sqrt{25*\sqrt{5} + 55} + 1/80*(\pi + 4*\arctan(-x*\sqrt{\sqrt{5} + 1} - 1))*\sqrt{25*\sqrt{5} + 55} + 1/80*(\pi + 4*\arctan(x*\sqrt{\sqrt{5} - 1} + 1))*\sqrt{25*\sqrt{5} - 55} - 1/80*(\pi + 4*\arctan(-x*\sqrt{\sqrt{5} - 1} + 1))*\sqrt{25*\sqrt{5} - 55} - 1/40*\sqrt{25*\sqrt{5} - 55}*\log(748225*(x + \sqrt{\sqrt{5} + 1})^2 + 748225*x^2) + 1/40*\sqrt{25*\sqrt{5} - 55}*\log(748225*(x - \sqrt{\sqrt{5} + 1})^2 + 748225*x^2) + 1/40*\sqrt{25*\sqrt{5} + 55}*\log(180625*(x + \sqrt{\sqrt{5} - 1})^2 + 180625*x^2) - 1/40*\sqrt{25*\sqrt{5} + 55}*\log(180625*(x - \sqrt{\sqrt{5} - 1})^2 + 180625*x^2) - 1/x$

maple [C] time = 0.01, size = 52, normalized size = 0.12

$$\frac{\left(\text{RootOf}\left(-Z^8 + 3Z^4 + 1\right)^6 + 3\text{RootOf}\left(-Z^8 + 3Z^4 + 1\right)^2\right) \ln\left(-\text{RootOf}\left(-Z^8 + 3Z^4 + 1\right) + x\right)}{4\left(2\text{RootOf}\left(-Z^8 + 3Z^4 + 1\right)^7 + 3\text{RootOf}\left(-Z^8 + 3Z^4 + 1\right)^3\right)} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(x^8+3*x^4+1),x)

[Out] $-1/4*\sum\left(\left(-R^6 + 3R^2\right)/\left(2R^7 + 3R^3\right)*\ln\left(-R + x\right), R = \text{RootOf}\left(-Z^8 + 3Z^4 + 1\right)\right) - 1/x$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x} - \int \frac{x^6 + 3x^2}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] $-1/x - \text{integrate}\left(\left(x^6 + 3x^2\right)/\left(x^8 + 3x^4 + 1\right), x\right)$

mupad [B] time = 1.29, size = 292, normalized size = 0.70

$\frac{1}{4} \left(\frac{2^{14} \sqrt{5} \operatorname{atan}\left(\frac{2082^{14} (-55\sqrt{5}-123)^{14}}{2(1025\sqrt{5}-6763)}\right) - 1152^{14} \sqrt{5} (-55\sqrt{5}-123)^{14}}{2(1025\sqrt{5}-6763)} \right) (-55\sqrt{5}-123)^{14} - \frac{2^{14} \sqrt{5} \operatorname{atan}\left(\frac{2082^{14} (55\sqrt{5}-123)^{14}}{2(1025\sqrt{5}-6763)}\right) - 1152^{14} \sqrt{5} (55\sqrt{5}-123)^{14}}{2(1025\sqrt{5}-6763)} \right) (55\sqrt{5}-123)^{14} - \frac{2^{14} \sqrt{5} \operatorname{atan}\left(\frac{2^{14} (-55\sqrt{5}-123)^{14} 2080}{2(1025\sqrt{5}-6763)}\right) + \frac{2^{14} \sqrt{5} (-55\sqrt{5}-123)^{14} 1152}{2(1025\sqrt{5}-6763)}}{2(1025\sqrt{5}-6763)} \right) (-55\sqrt{5}-123)^{14} - \frac{2^{14} \sqrt{5} \operatorname{atan}\left(\frac{2^{14} (55\sqrt{5}-123)^{14} 2080}{2(1025\sqrt{5}-6763)}\right) + \frac{2^{14} \sqrt{5} (55\sqrt{5}-123)^{14} 1152}{2(1025\sqrt{5}-6763)}}{2(1025\sqrt{5}-6763)} \right) (55\sqrt{5}-123)^{14} - \frac{1}{x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(3*x^4 + x^8 + 1)),x)`

[Out]
$$- \frac{1}{x} - \frac{2^{3/4} 5^{1/2} \operatorname{atan}\left(\frac{2585 \cdot 2^{3/4} x (-55 \cdot 5^{1/2} - 123)^{1/4}}{2(3025 \cdot 5^{1/2} + 6765)} + \frac{1155 \cdot 2^{3/4} 5^{1/2} x (-55 \cdot 5^{1/2} - 123)^{1/4}}{2(3025 \cdot 5^{1/2} + 6765)}\right) (-55 \cdot 5^{1/2} - 123)^{1/4}}{20} - \frac{2^{3/4} 5^{1/2} \operatorname{atan}\left(\frac{2585 \cdot 2^{3/4} x (55 \cdot 5^{1/2} - 123)^{1/4}}{2(3025 \cdot 5^{1/2} - 6765)} - \frac{1155 \cdot 2^{3/4} 5^{1/2} x (55 \cdot 5^{1/2} - 123)^{1/4}}{2(3025 \cdot 5^{1/2} - 6765)}\right) (55 \cdot 5^{1/2} - 123)^{1/4}}{20} - \frac{2^{3/4} 5^{1/2} \operatorname{atan}\left(\frac{2^{3/4} x (-55 \cdot 5^{1/2} - 123)^{1/4} \cdot 2585i}{2(3025 \cdot 5^{1/2} + 6765)} + \frac{2^{3/4} 5^{1/2} x (-55 \cdot 5^{1/2} - 123)^{1/4} \cdot 1155i}{2(3025 \cdot 5^{1/2} + 6765)}\right) (-55 \cdot 5^{1/2} - 123)^{1/4} \cdot 1i}{20} - \frac{2^{3/4} 5^{1/2} \operatorname{atan}\left(\frac{2^{3/4} x (55 \cdot 5^{1/2} - 123)^{1/4} \cdot 2585i}{2(3025 \cdot 5^{1/2} - 6765)} - \frac{2^{3/4} 5^{1/2} x (55 \cdot 5^{1/2} - 123)^{1/4} \cdot 1155i}{2(3025 \cdot 5^{1/2} - 6765)}\right) (55 \cdot 5^{1/2} - 123)^{1/4} \cdot 1i}{20}$$

sympy [A] time = 1.61, size = 32, normalized size = 0.08

$$\operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t \log\left(\frac{19251200t^7}{11} + \frac{369792t^3}{11} + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(x**8+3*x**4+1),x)`

[Out] `RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(19251200*_t**7/11 + 369792*_t**3/11 + x))) - 1/x`

$$3.324 \quad \int \frac{1}{x^4(1+3x^4+x^8)} dx$$

Optimal. Leaf size=466

$$-\frac{1}{3x^3} + \frac{\sqrt[4]{843+377\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843+377\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.37, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1368, 1422, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{3x^3} \sqrt[4]{843+377\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3-\sqrt{5})}x + \sqrt{2(3-\sqrt{5})}\right) + \frac{\sqrt[4]{843+377\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3-\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}} \log\left(2x^2 - 2\sqrt[4]{2(3+\sqrt{5})}x + \sqrt{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}} \log\left(2x^2 + 2\sqrt[4]{2(3+\sqrt{5})}\right)}{4 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{843+377\sqrt{5}} \arctan\left(\frac{1 - \frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}}}{\frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}} + 1}\right)}{2 \cdot 2^{3/4}\sqrt{5}} + \frac{\sqrt[4]{843+377\sqrt{5}} \arctan\left(\frac{1 - \frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}}}{\frac{2x^2}{\sqrt[4]{2(3-\sqrt{5})}} - 1}\right)}{2 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}} \arctan\left(\frac{1 - \frac{2x^2}{\sqrt[4]{2(3+\sqrt{5})}}}{\frac{2x^2}{\sqrt[4]{2(3+\sqrt{5})}} + 1}\right)}{2 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}} \arctan\left(\frac{1 - \frac{2x^2}{\sqrt[4]{2(3+\sqrt{5})}}}{\frac{2x^2}{\sqrt[4]{2(3+\sqrt{5})}} - 1}\right)}{2 \cdot 2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 + 3*x^4 + x^8)),x]

[Out] $-1/(3x^3) + ((843 + 377\sqrt{5})^{1/4} \text{ArcTan}[1 - (2^{3/4}x)/(3 - \sqrt{5})]^{1/4}) / (2 \cdot 2^{3/4} \sqrt{5}) - ((843 + 377\sqrt{5})^{1/4} \text{ArcTan}[1 + (2^{3/4}x)/(3 - \sqrt{5})]^{1/4}) / (2 \cdot 2^{3/4} \sqrt{5}) - ((843 - 377\sqrt{5})^{1/4} \text{ArcTan}[1 - (2^{3/4}x)/(3 + \sqrt{5})]^{1/4}) / (2 \cdot 2^{3/4} \sqrt{5}) + ((843 - 377\sqrt{5})^{1/4} \text{ArcTan}[1 + (2^{3/4}x)/(3 + \sqrt{5})]^{1/4}) / (2 \cdot 2^{3/4} \sqrt{5}) + ((843 + 377\sqrt{5})^{1/4} \text{Log}[\text{Sqrt}[2(3 - \sqrt{5})]]) - 2 \cdot (2(3 - \sqrt{5}))^{1/4} x + 2x^2) / (4 \cdot 2^{3/4} \sqrt{5}) - ((843 + 377\sqrt{5})^{1/4} \text{Log}[\text{Sqrt}[2(3 - \sqrt{5})]]) + 2 \cdot (2(3 - \sqrt{5}))^{1/4} x + 2x^2) / (4 \cdot 2^{3/4} \sqrt{5}) - ((843 - 377\sqrt{5})^{1/4} \text{Log}[\text{Sqrt}[2(3 + \sqrt{5})]]) - 2 \cdot (2(3 + \sqrt{5}))^{1/4} x + 2x^2) / (4 \cdot 2^{3/4} \sqrt{5}) + ((843 - 377\sqrt{5})^{1/4} \text{Log}[\text{Sqrt}[2(3 + \sqrt{5})]]) + 2 \cdot (2(3 + \sqrt{5}))^{1/4} x + 2x^2) / (4 \cdot 2^{3/4} \sqrt{5})$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1368

```
Int[((d_.)*(x_)^(m_))*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
c*(m + 2*n*(p + 1) + 1)*x^n*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(1+3x^4+x^8)} dx &= -\frac{1}{3x^3} + \frac{1}{3} \int \frac{-9-3x^4}{1+3x^4+x^8} dx \\
&= -\frac{1}{3x^3} + \frac{1}{10}(-5+3\sqrt{5}) \int \frac{1}{\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx - \frac{1}{10}(5+3\sqrt{5}) \int \frac{1}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx \\
&= -\frac{1}{3x^3} - \frac{(3+\sqrt{5})^{3/2} \int \frac{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx}{8\sqrt{5}} - \frac{(3+\sqrt{5})^{3/2} \int \frac{\sqrt{3-\sqrt{5}+\sqrt{2}x^2}}{\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx}{8\sqrt{5}} + \dots \\
&= -\frac{1}{3x^3} - \frac{\sqrt[4]{843-377\sqrt{5}} \int \frac{\sqrt[4]{2(3+\sqrt{5})+2x}}{-\sqrt{\frac{1}{2}(3+\sqrt{5})} - \sqrt[4]{2(3+\sqrt{5})}x-x^2} dx}{4 \cdot 2^{3/4} \sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}} \int \frac{\dots}{-\sqrt{\frac{1}{2}(3+\sqrt{5})} - \sqrt[4]{2(3+\sqrt{5})}x-x^2} dx}{4 \cdot 2^{3/4} \sqrt{5}} \\
&= -\frac{1}{3x^3} + \frac{(3+\sqrt{5})^{7/4} \log\left(\sqrt{2(3-\sqrt{5})} - 2\sqrt[4]{2(3-\sqrt{5})}x + 2x^2\right)}{16\sqrt[4]{2}\sqrt{5}} - \frac{(3+\sqrt{5})^{7/4} \log\left(\dots\right)}{16\sqrt[4]{2}\sqrt{5}} \\
&= -\frac{1}{3x^3} + \frac{(3+\sqrt{5})^{7/4} \tan^{-1}\left(1 - \frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}} - \frac{(3+\sqrt{5})^{7/4} \tan^{-1}\left(1 + \frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{843-377\sqrt{5}}}{\dots}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.14

$$-\frac{1}{4} \text{RootSum}\left[\#1^8 + 3\#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1) + 3 \log(x - \#1)}{2\#1^7 + 3\#1^3} \&\right] - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(1 + 3*x^4 + x^8)), x]

[Out] -1/3*1/x^3 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)/(3*#1^3 + 2*#1^7) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(1+3x^4+x^8)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(1 + 3*x^4 + x^8)), x]
```

fricas [B] time = 1.79, size = 1057, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/240*(3*sqrt(10)*sqrt(2)*x^3*(754*sqrt(5) + 1686)^(1/4)*log(20*x^2 + sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47)) - 3*sqrt(10)*sqrt(2)*x^3*(754*sqrt(5) + 1686)^(1/4)*log(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47)) - 3*sqrt(10)*sqrt(2)*x^3*(-754*sqrt(5) + 1686)^(1/4)*log(20*x^2 + sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686)) + 3*sqrt(10)*sqrt(2)*x^3*(-754*sqrt(5) + 1686)^(1/4)*log(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686)) - 3*sqrt(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754*sqrt(5) + 1686)^(3/4)*sqrt(377*sqrt(5) + 843)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47))*(23184*sqrt(5) - 51841)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) + 843) + 1/40*sqrt(10)*(51841*sqrt(5)*x - 115920*x)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) + 843) - 1/8*(377*sqrt(5)*sqrt(2) - 843*sqrt(2))*sqrt(754*sqrt(5) + 1686)*sqrt(377*sqrt(5) + 843)) - 3*sqrt(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754*sqrt(5) + 1686)^(3/4)*sqrt(377*sqrt(5) + 843)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47))*(23184*sqrt(5) - 51841)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) + 843) + 1/40*sqrt(10)*(51841*sqrt(5)*x - 115920*x)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) + 843) + 1/8*(377*sqrt(5)*sqrt(2) - 843*sqrt(2))*sqrt(754*sqrt(5) + 1686)*sqrt(377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt(5)*x^3 + 843*x^3)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 + sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686))*(23184*sqrt(5) + 51841)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(5/4) - 1/40*(sqrt(10)*(51841*sqrt(5)*x + 115920*x)*(-754*sqrt(5) + 1686)^(5/4) + 5*(377*sqrt(5)*sqrt(2) + 843*sqrt(2))*sqrt(-754*sqrt(5) + 1686))*sqrt(-377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt(5)*x^3 + 843*x^3)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(3/4)*arctan(1/80*sqrt(10)*sqrt(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47))*sqrt(754*sqrt(5) + 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47))
```

) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686))*(23184*sqrt(5) + 51841)*
sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(5/4) - 1/40*(sqrt(10)*(5184
1*sqrt(5)*x + 115920*x)*(-754*sqrt(5) + 1686)^(5/4) - 5*(377*sqrt(5)*sqrt(2
) + 843*sqrt(2))*sqrt(-754*sqrt(5) + 1686))*sqrt(-377*sqrt(5) + 843)) + 80
/x^3

giac [A] time = 0.70, size = 244, normalized size = 0.52

$\frac{1}{8}(\dots + \arctan(\sqrt{5}+1))\sqrt{65\sqrt{5}+145} + \frac{1}{8}(\dots + \arctan(\sqrt{5}+1))\sqrt{65\sqrt{5}-145} + \frac{1}{8}(\dots + \arctan(\sqrt{5}-1))\sqrt{65\sqrt{5}+145} + \frac{1}{8}(\dots + \arctan(\sqrt{5}-1))\sqrt{65\sqrt{5}-145} + \frac{1}{40}\sqrt{65\sqrt{5}-145}\log(93122500(x+\sqrt{5}+1)^2+93122500x^2) - \frac{1}{40}\sqrt{65\sqrt{5}+145}\log(93122500(x-\sqrt{5}+1)^2+93122500x^2) - \frac{1}{40}\sqrt{65\sqrt{5}-145}\log(53728900(x+\sqrt{5}-1)^2+53728900x^2) + \frac{1}{40}\sqrt{65\sqrt{5}+145}\log(53728900(x-\sqrt{5}-1)^2+53728900x^2) - \frac{1}{3x^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="giac")

[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/8
0*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/80*(
pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) - 1/80*(pi +
4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) + 1/40*sqrt(65*
sqrt(5) - 145)*log(93122500*(x + sqrt(sqrt(5) + 1))^2 + 93122500*x^2) - 1/4
0*sqrt(65*sqrt(5) - 145)*log(93122500*(x - sqrt(sqrt(5) + 1))^2 + 93122500*
x^2) - 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x + sqrt(sqrt(5) - 1))^2 +
53728900*x^2) + 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x - sqrt(sqrt(5)
- 1))^2 + 53728900*x^2) - 1/3/x^3

maple [C] time = 0.01, size = 50, normalized size = 0.11

$$\frac{(-\text{RootOf}(-Z^8 + 3Z^4 + 1)^4 - 3) \ln(-\text{RootOf}(-Z^8 + 3Z^4 + 1) + x)}{8 \text{RootOf}(-Z^8 + 3Z^4 + 1)^7 + 12 \text{RootOf}(-Z^8 + 3Z^4 + 1)^3} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(x^8+3*x^4+1),x)

[Out] 1/4*sum((-_R^4-3)/(2*_R^7+3*_R^3)*ln(-_R+x),_R=RootOf(-Z^8+3*_Z^4+1))-1/3/x
^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3x^3} - \int \frac{x^4 + 3}{x^8 + 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="maxima")

[Out] -1/3/x^3 - integrate((x^4 + 3)/(x^8 + 3*x^4 + 1), x)

mupad [B] time = 0.18, size = 492, normalized size = 1.06

$$\frac{\sqrt[4]{2} \sqrt[4]{5} \operatorname{atan}\left(\frac{46371 \sqrt[4]{2} \sqrt[4]{5} x (377 \sqrt[4]{5} - 843)^{1/4}}{2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (377 \sqrt[4]{5} - 843)^{1/4})}\right) - (20735 \sqrt[4]{2} \sqrt[4]{5} x (377 \sqrt[4]{5} - 843)^{1/4}) / (2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (377 \sqrt[4]{5} - 843)^{1/4})) * (377 \sqrt[4]{5} - 843)^{1/4} / 20 - (2 \sqrt[4]{2} \sqrt[4]{5} \operatorname{atan}\left(\frac{46371 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4}}{2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4})}\right) + 1508 \sqrt[4]{2} \sqrt[4]{5} (-377 \sqrt[4]{5} - 843)^{1/4} / (2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4})) + (20735 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4}) / (2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4})) * (-377 \sqrt[4]{5} - 843)^{1/4} / 20 - 1 / (3 x^3) + (2 \sqrt[4]{2} \sqrt[4]{5} \operatorname{atan}\left(\frac{2 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4} * 46371 i}{2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4})}\right) + 1508 \sqrt[4]{2} \sqrt[4]{5} (-377 \sqrt[4]{5} - 843)^{1/4} / (2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4})) + (2 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4} * 20735 i) / (2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (-377 \sqrt[4]{5} - 843)^{1/4})) * (-377 \sqrt[4]{5} - 843)^{1/4} / 20 - (2 \sqrt[4]{2} \sqrt[4]{5} \operatorname{atan}\left(\frac{2 \sqrt[4]{2} \sqrt[4]{5} x (377 \sqrt[4]{5} - 843)^{1/4} * 46371 i}{2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (377 \sqrt[4]{5} - 843)^{1/4})}\right) - (2 \sqrt[4]{2} \sqrt[4]{5} x (377 \sqrt[4]{5} - 843)^{1/4} * 20735 i) / (2 (3393 \sqrt[4]{2} \sqrt[4]{5} - 1508 \sqrt[4]{2} \sqrt[4]{5} - 20735 \sqrt[4]{2} \sqrt[4]{5} x (377 \sqrt[4]{5} - 843)^{1/4})) * (377 \sqrt[4]{5} - 843)^{1/4} / 20$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(3*x^4 + x^8 + 1)),x)

[Out] $(2^{3/4} \cdot 5^{1/2} \cdot \operatorname{atan}\left(\frac{46371 \cdot 2^{3/4} \cdot x \cdot (377 \cdot 5^{1/2} - 843)^{1/4}}{2 \cdot (3393 \cdot 2^{1/2} \cdot (377 \cdot 5^{1/2} - 843)^{1/2} - 1508 \cdot 2^{1/2} \cdot 5^{1/2} \cdot (377 \cdot 5^{1/2} - 843)^{1/2})}\right) - (20735 \cdot 2^{3/4} \cdot 5^{1/2} \cdot x \cdot (377 \cdot 5^{1/2} - 843)^{1/4}) / (2 \cdot (3393 \cdot 2^{1/2} \cdot (377 \cdot 5^{1/2} - 843)^{1/2} - 1508 \cdot 2^{1/2} \cdot 5^{1/2} \cdot (377 \cdot 5^{1/2} - 843)^{1/2})) \cdot (377 \cdot 5^{1/2} - 843)^{1/4} / 20 - (2^{3/4} \cdot 5^{1/2} \cdot \operatorname{atan}\left(\frac{46371 \cdot 2^{3/4} \cdot x \cdot (-377 \cdot 5^{1/2} - 843)^{1/4}}{2 \cdot (3393 \cdot 2^{1/2} \cdot (-377 \cdot 5^{1/2} - 843)^{1/2} + 1508 \cdot 2^{1/2} \cdot 5^{1/2} \cdot (-377 \cdot 5^{1/2} - 843)^{1/2})}\right) + (20735 \cdot 2^{3/4} \cdot 5^{1/2} \cdot x \cdot (-377 \cdot 5^{1/2} - 843)^{1/4}) / (2 \cdot (3393 \cdot 2^{1/2} \cdot (-377 \cdot 5^{1/2} - 843)^{1/2} + 1508 \cdot 2^{1/2} \cdot 5^{1/2} \cdot (-377 \cdot 5^{1/2} - 843)^{1/2})) \cdot (-377 \cdot 5^{1/2} - 843)^{1/4} / 20 - 1 / (3 \cdot x^3) + (2^{3/4} \cdot 5^{1/2} \cdot \operatorname{atan}\left(\frac{2^{3/4} \cdot x \cdot (-377 \cdot 5^{1/2} - 843)^{1/4} \cdot 46371 i}{2 \cdot (3393 \cdot 2^{1/2} \cdot (-377 \cdot 5^{1/2} - 843)^{1/2} + 1508 \cdot 2^{1/2} \cdot 5^{1/2} \cdot (-377 \cdot 5^{1/2} - 843)^{1/2})}\right) + (2^{3/4} \cdot 5^{1/2} \cdot x \cdot (-377 \cdot 5^{1/2} - 843)^{1/4} \cdot 20735 i) / (2 \cdot (3393 \cdot 2^{1/2} \cdot (-377 \cdot 5^{1/2} - 843)^{1/2} + 1508 \cdot 2^{1/2} \cdot 5^{1/2} \cdot (-377 \cdot 5^{1/2} - 843)^{1/2})) \cdot (-377 \cdot 5^{1/2} - 843)^{1/4} \cdot i / 20 - (2^{3/4} \cdot 5^{1/2} \cdot \operatorname{atan}\left(\frac{2^{3/4} \cdot x \cdot (377 \cdot 5^{1/2} - 843)^{1/4} \cdot 46371 i}{2 \cdot (3393 \cdot 2^{1/2} \cdot (377 \cdot 5^{1/2} - 843)^{1/2} - 1508 \cdot 2^{1/2} \cdot 5^{1/2} \cdot (377 \cdot 5^{1/2} - 843)^{1/2})}\right) - (2^{3/4} \cdot 5^{1/2} \cdot x \cdot (377 \cdot 5^{1/2} - 843)^{1/4} \cdot 20735 i) / (2 \cdot (3393 \cdot 2^{1/2} \cdot (377 \cdot 5^{1/2} - 843)^{1/2} - 1508 \cdot 2^{1/2} \cdot 5^{1/2} \cdot (377 \cdot 5^{1/2} - 843)^{1/2})) \cdot (377 \cdot 5^{1/2} - 843)^{1/4} \cdot i / 20$

sympy [A] time = 1.62, size = 34, normalized size = 0.07

$$\operatorname{RootSum}\left(40960000t^8 + 5395200t^4 + 1, \left(t \mapsto t \log\left(\frac{179200t^5}{377} + \frac{23112t}{377} + x\right)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(x**8+3*x**4+1),x)

[Out] $\operatorname{RootSum}(40960000 \cdot t^8 + 5395200 \cdot t^4 + 1, \operatorname{Lambda}(t, t \cdot \log(179200 \cdot t^5 / 377 + 23112 \cdot t / 377 + x))) - 1 / (3 \cdot x^3)$

$$3.325 \quad \int \frac{x^{11}}{1-3x^4+x^8} dx$$

Optimal. Leaf size=62

$$\frac{x^4}{4} + \frac{1}{40} (15 - 7\sqrt{5}) \log(-2x^4 - \sqrt{5} + 3) + \frac{1}{40} (15 + 7\sqrt{5}) \log(-2x^4 + \sqrt{5} + 3)$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4} + \frac{1}{40} (15 - 7\sqrt{5}) \log(-2x^4 - \sqrt{5} + 3) + \frac{1}{40} (15 + 7\sqrt{5}) \log(-2x^4 + \sqrt{5} + 3)$$

Antiderivative was successfully verified.

[In] Int[x^11/(1 - 3*x^4 + x^8), x]

[Out] x^4/4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1357

Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x

], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{1-3x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{1-3x+x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{4} \text{Subst} \left(\int \frac{-1+3x}{1-3x+x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{40} (15-7\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} + \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) + \frac{1}{40} (15+7\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{40} (15-7\sqrt{5}) \log(3-\sqrt{5}-2x^4) + \frac{1}{40} (15+7\sqrt{5}) \log(3+\sqrt{5}-2x^4)
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.90

$$\frac{1}{40} (10x^4 + (15 + 7\sqrt{5}) \log(-2x^4 + \sqrt{5} + 3) + (15 - 7\sqrt{5}) \log(2x^4 + \sqrt{5} - 3))$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(1 - 3*x^4 + x^8), x]

[Out] (10*x^4 + (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] + (15 - 7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{1-3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^11/(1 - 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^11/(1 - 3*x^4 + x^8), x]

fricas [A] time = 0.68, size = 62, normalized size = 1.00

$$\frac{1}{4} x^4 + \frac{7}{40} \sqrt{5} \log \left(\frac{2x^8 - 6x^4 - \sqrt{5}(2x^4 - 3) + 7}{x^8 - 3x^4 + 1} \right) + \frac{3}{8} \log(x^8 - 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸-3*x⁴+1),x, algorithm="fricas")

[Out] 1/4*x⁴ + 7/40*sqrt(5)*log((2*x⁸ - 6*x⁴ - sqrt(5)*(2*x⁴ - 3) + 7)/(x⁸ - 3*x⁴ + 1)) + 3/8*log(x⁸ - 3*x⁴ + 1)

giac [A] time = 0.42, size = 53, normalized size = 0.85

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{|2x^4 - \sqrt{5} - 3|}{|2x^4 + \sqrt{5} - 3|}\right) + \frac{3}{8}\log(|x^8 - 3x^4 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸-3*x⁴+1),x, algorithm="giac")

[Out] 1/4*x⁴ + 7/40*sqrt(5)*log(abs(2*x⁴ - sqrt(5) - 3)/abs(2*x⁴ + sqrt(5) - 3)) + 3/8*log(abs(x⁸ - 3*x⁴ + 1))

maple [A] time = 0.00, size = 38, normalized size = 0.61

$$\frac{x^4}{4} - \frac{7\sqrt{5}\operatorname{arctanh}\left(\frac{(2x^4-3)\sqrt{5}}{5}\right)}{20} + \frac{3\ln(x^8 - 3x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(x⁸-3*x⁴+1),x)

[Out] 1/4*x⁴+3/8*ln(x⁸-3*x⁴+1)-7/20*5^(1/2)*arctanh(1/5*(2*x⁴-3)*5^(1/2))

maxima [A] time = 1.39, size = 50, normalized size = 0.81

$$\frac{1}{4}x^4 + \frac{7}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3}\right) + \frac{3}{8}\log(x^8 - 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸-3*x⁴+1),x, algorithm="maxima")

[Out] 1/4*x⁴ + 7/40*sqrt(5)*log((2*x⁴ - sqrt(5) - 3)/(2*x⁴ + sqrt(5) - 3)) + 3/8*log(x⁸ - 3*x⁴ + 1)

mupad [B] time = 0.12, size = 64, normalized size = 1.03

$$\frac{3\ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{8} + \frac{3\ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{8} + \frac{7\sqrt{5}\ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{40} - \frac{7\sqrt{5}\ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{40} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(x^8 - 3*x^4 + 1), x)`

[Out] $(3*\log(x^4 - 5^{(1/2)}/2 - 3/2))/8 + (3*\log(5^{(1/2)}/2 + x^4 - 3/2))/8 + (7*5^{(1/2)}*\log(x^4 - 5^{(1/2)}/2 - 3/2))/40 - (7*5^{(1/2)}*\log(5^{(1/2)}/2 + x^4 - 3/2))/40 + x^4/4$

sympy [A] time = 0.14, size = 58, normalized size = 0.94

$$\frac{x^4}{4} + \left(\frac{3}{8} + \frac{7\sqrt{5}}{40}\right) \log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(\frac{3}{8} - \frac{7\sqrt{5}}{40}\right) \log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(x**8-3*x**4+1), x)`

[Out] $x**4/4 + (3/8 + 7*\text{sqrt}(5)/40)*\log(x**4 - 3/2 - \text{sqrt}(5)/2) + (3/8 - 7*\text{sqrt}(5)/40)*\log(x**4 - 3/2 + \text{sqrt}(5)/2)$

$$3.326 \quad \int \frac{x^9}{1-3x^4+x^8} dx$$

Optimal. Leaf size=90

$$\frac{x^2}{2} - \frac{1}{2} \sqrt{\frac{1}{5}(9+4\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right) + \frac{1}{2} \sqrt{\frac{1}{5}(9-4\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x^2 \right)$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1359, 1122, 1166, 207}

$$\frac{x^2}{2} - \frac{1}{2} \sqrt{\frac{1}{5}(9+4\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right) + \frac{1}{2} \sqrt{\frac{1}{5}(9-4\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x^2 \right)$$

Antiderivative was successfully verified.

[In] Int[x^9/(1 - 3*x^4 + x^8), x]

[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1122

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m+4*p+1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x^9}{1-3x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{1-3x^2+x^4} dx, x, x^2 \right) \\ &= \frac{x^2}{2} - \frac{1}{2} \text{Subst} \left(\int \frac{1-3x^2}{1-3x^2+x^4} dx, x, x^2 \right) \\ &= \frac{x^2}{2} - \frac{1}{20} (-15 + 7\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) + \frac{1}{20} (15 + 7\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) \\ &= \frac{x^2}{2} - \frac{1}{2} \sqrt{\frac{1}{5}} (9 + 4\sqrt{5}) \tanh^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right) + \frac{1}{20} \sqrt{180 - 80\sqrt{5}} \tanh^{-1} \left(\sqrt{\frac{1}{2}} (3 + \sqrt{5}) x^2 \right) \end{aligned}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 1.14

$$\frac{1}{20} (10x^2 + (2\sqrt{5} - 5) \log(-2x^2 + \sqrt{5} - 1) + (5 + 2\sqrt{5}) \log(-2x^2 + \sqrt{5} + 1) + (5 - 2\sqrt{5}) \log(2x^2 + \sqrt{5} - 1) - (5 + 2\sqrt{5}) \log(2x^2 + \sqrt{5} + 1))$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(1 - 3*x^4 + x^8), x]

[Out] (10*x^2 + (-5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + 2*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{1-3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/(1 - 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^9/(1 - 3*x^4 + x^8), x]

fricas [B] time = 1.16, size = 114, normalized size = 1.27

$$\frac{1}{2}x^2 + \frac{1}{10}\sqrt{5}\log\left(\frac{2x^4 + 2x^2 - \sqrt{5}(2x^2 + 1) + 3}{x^4 + x^2 - 1}\right) + \frac{1}{10}\sqrt{5}\log\left(\frac{2x^4 - 2x^2 - \sqrt{5}(2x^2 - 1) + 3}{x^4 - x^2 - 1}\right) - \frac{1}{4}\log(x^4 + x^2 - 1) + \frac{1}{4}\log(x^4 - x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1)) + 1/10*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 - 1)) - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)

giac [A] time = 0.46, size = 97, normalized size = 1.08

$$\frac{1}{2}x^2 + \frac{1}{10}\sqrt{5}\log\left(\frac{|2x^2 - \sqrt{5} + 1|}{2x^2 + \sqrt{5} + 1}\right) + \frac{1}{10}\sqrt{5}\log\left(\frac{|2x^2 - \sqrt{5} - 1|}{2x^2 + \sqrt{5} - 1}\right) - \frac{1}{4}\log(|x^4 + x^2 - 1|) + \frac{1}{4}\log(|x^4 - x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="giac")

[Out] 1/2*x^2 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))

maple [A] time = 0.01, size = 67, normalized size = 0.74

$$\frac{x^2}{2} - \frac{\sqrt{5}\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5} - \frac{\sqrt{5}\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{5} + \frac{\ln(x^4 - x^2 - 1)}{4} - \frac{\ln(x^4 + x^2 - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^8-3*x^4+1),x)

[Out] 1/2*x^2-1/4*ln(x^4+x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/4*ln(x^4-x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))

maxima [A] time = 1.36, size = 92, normalized size = 1.02

$$\frac{1}{2}x^2 + \frac{1}{10}\sqrt{5}\log\left(\frac{2x^2 - \sqrt{5} + 1}{2x^2 + \sqrt{5} + 1}\right) + \frac{1}{10}\sqrt{5}\log\left(\frac{2x^2 - \sqrt{5} - 1}{2x^2 + \sqrt{5} - 1}\right) - \frac{1}{4}\log(x^4 + x^2 - 1) + \frac{1}{4}\log(x^4 - x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] $\frac{1}{2}x^2 + \frac{1}{10}\sqrt{5}\log((2x^2 - \sqrt{5} + 1)/(2x^2 + \sqrt{5} + 1)) + \frac{1}{10}\sqrt{5}\log((2x^2 - \sqrt{5} - 1)/(2x^2 + \sqrt{5} - 1)) - \frac{1}{4}\log(x^4 + x^2 - 1) + \frac{1}{4}\log(x^4 - x^2 - 1)$

mupad [B] time = 1.33, size = 90, normalized size = 1.00

$$\frac{x^2}{2} - \operatorname{atanh}\left(\frac{64x^2}{64\sqrt{5} + 192} + \frac{64\sqrt{5}x^2}{64\sqrt{5} + 192}\right)\left(\frac{\sqrt{5}}{5} + \frac{1}{2}\right) - \operatorname{atanh}\left(\frac{64x^2}{64\sqrt{5} - 192} - \frac{64\sqrt{5}x^2}{64\sqrt{5} - 192}\right)\left(\frac{\sqrt{5}}{5} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(x^8 - 3*x^4 + 1), x)`

[Out] $x^2/2 - \operatorname{atanh}((64x^2)/(64*5^{(1/2)} + 192) + (64*5^{(1/2)}*x^2)/(64*5^{(1/2)} + 192))*(5^{(1/2)}/5 + 1/2) - \operatorname{atanh}((64x^2)/(64*5^{(1/2)} - 192) - (64*5^{(1/2)}*x^2)/(64*5^{(1/2)} - 192))*(5^{(1/2)}/5 - 1/2)$

sympy [B] time = 0.38, size = 170, normalized size = 1.89

$$\frac{x^2}{2} + \left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{47}{8} - \frac{47\sqrt{5}}{20} - 120\left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)^3\right) + \left(\frac{1}{4} + \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{47}{8} - 120\left(\frac{1}{4} + \frac{\sqrt{5}}{10}\right)^3 + \frac{47\sqrt{5}}{20}\right) + \left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{47\sqrt{5}}{20} - 120\left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)^3 + \frac{47}{8}\right) + \left(\frac{\sqrt{5}}{10} + \frac{1}{4}\right)\log\left(x^2 - 120\left(\frac{\sqrt{5}}{10} + \frac{1}{4}\right)^3 + \frac{47\sqrt{5}}{20} + \frac{47}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(x**8-3*x**4+1), x)`

[Out] $x^{**2}/2 + (-1/4 - \sqrt{5}/10)*\log(x^{**2} - 47/8 - 47*\sqrt{5}/20 - 120*(-1/4 - \sqrt{5}/10)**3) + (-1/4 + \sqrt{5}/10)*\log(x^{**2} - 47/8 - 120*(-1/4 + \sqrt{5}/10)**3 + 47*\sqrt{5}/20) + (1/4 - \sqrt{5}/10)*\log(x^{**2} - 47*\sqrt{5}/20 - 120*(1/4 - \sqrt{5}/10)**3 + 47/8) + (\sqrt{5}/10 + 1/4)*\log(x^{**2} - 120*(\sqrt{5}/10 + 1/4)**3 + 47*\sqrt{5}/20 + 47/8)$

$$3.327 \quad \int \frac{x^7}{1-3x^4+x^8} dx$$

Optimal. Leaf size=55

$$\frac{1}{40} (5 - 3\sqrt{5}) \log(-2x^4 - \sqrt{5} + 3) + \frac{1}{40} (5 + 3\sqrt{5}) \log(-2x^4 + \sqrt{5} + 3)$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1357, 632, 31}

$$\frac{1}{40} (5 - 3\sqrt{5}) \log(-2x^4 - \sqrt{5} + 3) + \frac{1}{40} (5 + 3\sqrt{5}) \log(-2x^4 + \sqrt{5} + 3)$$

Antiderivative was successfully verified.

[In] Int[x^7/(1 - 3*x^4 + x^8),x]

[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1-3x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{1-3x+x^2} dx, x, x^4 \right) \\
&= \frac{1}{40} (5-3\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} + \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) + \frac{1}{40} (5+3\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) \\
&= \frac{1}{40} (5-3\sqrt{5}) \log(3-\sqrt{5}-2x^4) + \frac{1}{40} (5+3\sqrt{5}) \log(3+\sqrt{5}-2x^4)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.96

$$\frac{1}{40} (5+3\sqrt{5}) \log(-2x^4 + \sqrt{5} + 3) + \frac{1}{40} (5-3\sqrt{5}) \log(2x^4 + \sqrt{5} - 3)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(1 - 3*x^4 + x^8), x]

[Out] ((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{1-3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(1 - 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^7/(1 - 3*x^4 + x^8), x]

fricas [A] time = 1.04, size = 57, normalized size = 1.04

$$\frac{3}{40} \sqrt{5} \log\left(\frac{2x^8 - 6x^4 - \sqrt{5}(2x^4 - 3) + 7}{x^8 - 3x^4 + 1}\right) + \frac{1}{8} \log(x^8 - 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8-3*x^4+1), x, algorithm="fricas")

[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1)) + 1/8*log(x^8 - 3*x^4 + 1)

giac [A] time = 0.42, size = 48, normalized size = 0.87

$$\frac{3}{40} \sqrt{5} \log \left(\left| \frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3} \right| \right) + \frac{1}{8} \log(|x^8 - 3x^4 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="giac")

[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/8*log(abs(x^8 - 3*x^4 + 1))

maple [A] time = 0.00, size = 33, normalized size = 0.60

$$-\frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^4-3)\sqrt{5}}{5}\right)}{20} + \frac{\ln(x^8 - 3x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^8-3*x^4+1),x)

[Out] 1/8*ln(x^8-3*x^4+1)-3/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))

maxima [A] time = 1.48, size = 45, normalized size = 0.82

$$\frac{3}{40} \sqrt{5} \log \left(\frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3} \right) + \frac{1}{8} \log(x^8 - 3x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 1/8*log(x^8 - 3*x^4 + 1)

mupad [B] time = 0.10, size = 59, normalized size = 1.07

$$\frac{\ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{8} + \frac{\ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{8} + \frac{3\sqrt{5} \ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{40} - \frac{3\sqrt{5} \ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^8 - 3*x^4 + 1),x)

[Out] $\log(x^4 - 5^{(1/2)}/2 - 3/2)/8 + \log(5^{(1/2)}/2 + x^4 - 3/2)/8 + (3*5^{(1/2)}*\log(x^4 - 5^{(1/2)}/2 - 3/2))/40 - (3*5^{(1/2)}*\log(5^{(1/2)}/2 + x^4 - 3/2))/40$

sympy [A] time = 0.14, size = 53, normalized size = 0.96

$$\left(\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(\frac{1}{8} - \frac{3\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(x**8-3*x**4+1),x)`

[Out] $(1/8 + 3*\text{sqrt}(5)/40)*\log(x**4 - 3/2 - \text{sqrt}(5)/2) + (1/8 - 3*\text{sqrt}(5)/40)*\log(x**4 - 3/2 + \text{sqrt}(5)/2)$

$$3.328 \quad \int \frac{x^5}{1-3x^4+x^8} dx$$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}(3-\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right) - \frac{1}{2}\sqrt{\frac{1}{10}(3+\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1359, 1130, 207}

$$\frac{1}{2}\sqrt{\frac{1}{10}(3-\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x^2\right) - \frac{1}{2}\sqrt{\frac{1}{10}(3+\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x^5/(1 - 3*x^4 + x^8),x]

[Out] -(Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5]])*x^2])/2 + (Sqrt[(3 - Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1130

Int[((d_.)*(x_)^(m_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 1359

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1-3x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{1-3x^2+x^4} dx, x, x^2 \right) \\
&= \frac{1}{20} (5-3\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} + \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) + \frac{1}{20} (5+3\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x^2} dx, x, x^2 \right) \\
&= -\frac{1}{2} \sqrt{\frac{1}{10} (3+\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right) + \frac{1}{2} \sqrt{\frac{1}{10} (3-\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{1}{2} (3+\sqrt{5})} x^2 \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.12

$$\frac{1}{40} ((\sqrt{5}-5) \log(-2x^2+\sqrt{5}-1) + (5+\sqrt{5}) \log(-2x^2+\sqrt{5}+1) - (\sqrt{5}-5) \log(2x^2+\sqrt{5}-1) - (5+\sqrt{5}) \log(2x^2+\sqrt{5}+1))$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(1 - 3*x^4 + x^8), x]

[Out] ((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] - (-5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/40

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{1-3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(1 - 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^5/(1 - 3*x^4 + x^8), x]

fricas [B] time = 1.17, size = 109, normalized size = 1.35

$$\frac{1}{40} \sqrt{5} \log \left(\frac{2x^4 + 2x^2 - \sqrt{5}(2x^2 + 1) + 3}{x^4 + x^2 - 1} \right) + \frac{1}{40} \sqrt{5} \log \left(\frac{2x^4 - 2x^2 - \sqrt{5}(2x^2 - 1) + 3}{x^4 - x^2 - 1} \right) - \frac{1}{8} \log(x^4 + x^2 - 1) + \frac{1}{8} \log(x^4 - x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8-3*x^4+1), x, algorithm="fricas")

[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1)) + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 - 1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)

giac [B] time = 0.44, size = 92, normalized size = 1.14

$$\frac{1}{40} \sqrt{5} \log\left(\frac{|2x^2 - \sqrt{5} + 1|}{2x^2 + \sqrt{5} + 1}\right) + \frac{1}{40} \sqrt{5} \log\left(\frac{|2x^2 - \sqrt{5} - 1|}{2x^2 + \sqrt{5} - 1}\right) - \frac{1}{8} \log(|x^4 + x^2 - 1|) + \frac{1}{8} \log(|x^4 - x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="giac")

[Out] 1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))

maple [A] time = 0.00, size = 62, normalized size = 0.77

$$-\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20} - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{20} + \frac{\ln(x^4 - x^2 - 1)}{8} - \frac{\ln(x^4 + x^2 - 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^8-3*x^4+1),x)

[Out] -1/8*ln(x^4+x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))+1/8*ln(x^4-x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))

maxima [B] time = 1.92, size = 87, normalized size = 1.07

$$\frac{1}{40} \sqrt{5} \log\left(\frac{2x^2 - \sqrt{5} + 1}{2x^2 + \sqrt{5} + 1}\right) + \frac{1}{40} \sqrt{5} \log\left(\frac{2x^2 - \sqrt{5} - 1}{2x^2 + \sqrt{5} - 1}\right) - \frac{1}{8} \log(x^4 + x^2 - 1) + \frac{1}{8} \log(x^4 - x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] 1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)

mupad [B] time = 1.38, size = 77, normalized size = 0.95

$$-\operatorname{atanh}\left(\frac{4x^2}{\sqrt{5}-3} - \frac{2\sqrt{5}x^2}{\sqrt{5}-3}\right)\left(\frac{\sqrt{5}}{20} + \frac{1}{4}\right) - \operatorname{atanh}\left(\frac{4x^2}{\sqrt{5}+3} + \frac{2\sqrt{5}x^2}{\sqrt{5}+3}\right)\left(\frac{\sqrt{5}}{20} - \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(x^8 - 3*x^4 + 1), x)`

[Out] $-\operatorname{atanh}\left(\frac{4x^2}{5^{1/2} - 3} - \frac{(2\sqrt{5})x^2}{(5^{1/2} - 3)}\right) \frac{5^{1/2}}{20} + \frac{1}{4} - \operatorname{atanh}\left(\frac{4x^2}{5^{1/2} + 3} + \frac{(2\sqrt{5})x^2}{(5^{1/2} + 3)}\right) \frac{5^{1/2}}{20} - \frac{1}{4}$

sympy [B] time = 0.37, size = 165, normalized size = 2.04

$$\left(-\frac{1}{8} - \frac{\sqrt{5}}{40}\right) \log\left(x^2 - \frac{3}{2} - \frac{3\sqrt{5}}{10} - 640\left(-\frac{1}{8} - \frac{\sqrt{5}}{40}\right)^3\right) + \left(-\frac{1}{8} + \frac{\sqrt{5}}{40}\right) \log\left(x^2 - \frac{3}{2} - 640\left(-\frac{1}{8} + \frac{\sqrt{5}}{40}\right)^3 + \frac{3\sqrt{5}}{10}\right) + \left(\frac{1}{8} - \frac{\sqrt{5}}{40}\right) \log\left(x^2 - \frac{3\sqrt{5}}{10} - 640\left(\frac{1}{8} - \frac{\sqrt{5}}{40}\right)^3 + \frac{3}{2}\right) + \left(\frac{\sqrt{5}}{40} + \frac{1}{8}\right) \log\left(x^2 - 640\left(\frac{\sqrt{5}}{40} + \frac{1}{8}\right)^3 + \frac{3\sqrt{5}}{10} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(x**8-3*x**4+1), x)`

[Out] $(-1/8 - \sqrt{5}/40) \log(x^2 - 3/2 - 3\sqrt{5}/10 - 640(-1/8 - \sqrt{5}/40)^3) + (-1/8 + \sqrt{5}/40) \log(x^2 - 3/2 - 640(-1/8 + \sqrt{5}/40)^3 + 3\sqrt{5}/10) + (1/8 - \sqrt{5}/40) \log(x^2 - 3\sqrt{5}/10 - 640(1/8 - \sqrt{5}/40)^3 + 3/2) + (\sqrt{5}/40 + 1/8) \log(x^2 - 640(\sqrt{5}/40 + 1/8)^3 + 3\sqrt{5}/10 + 3/2)$

$$3.329 \quad \int \frac{x^3}{1-3x^4+x^8} dx$$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1352, 618, 206}

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 - 3*x^4 + x^8),x]

[Out] ArcTanh[(3 - 2*x^4)/Sqrt[5]]/(2*Sqrt[5])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{1-3x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-3x+x^2} dx, x, x^4 \right) \\ &= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{5-x^2} dx, x, -3+2x^4 \right) \right) \\ &= \frac{\tanh^{-1} \left(\frac{3-2x^4}{\sqrt{5}} \right)}{2\sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.65

$$\frac{\log(-2x^4 + \sqrt{5} + 3) - \log(2x^4 + \sqrt{5} - 3)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(1 - 3*x^4 + x^8),x]

[Out] (Log[3 + Sqrt[5] - 2*x^4] - Log[-3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{1-3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(1 - 3*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^3/(1 - 3*x^4 + x^8), x]

fricas [B] time = 1.28, size = 43, normalized size = 1.87

$$\frac{1}{20} \sqrt{5} \log \left(\frac{2x^8 - 6x^4 - \sqrt{5}(2x^4 - 3) + 7}{x^8 - 3x^4 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] 1/20*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1))

giac [A] time = 0.54, size = 33, normalized size = 1.43

$$\frac{1}{20} \sqrt{5} \log \left(\frac{|2x^4 - \sqrt{5} - 3|}{|2x^4 + \sqrt{5} - 3|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="giac")

[Out] 1/20*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3))

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$-\frac{\sqrt{5} \operatorname{arctanh} \left(\frac{(2x^4-3)\sqrt{5}}{5} \right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^8-3*x^4+1),x)

[Out] -1/10*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))

maxima [A] time = 1.41, size = 31, normalized size = 1.35

$$\frac{1}{20} \sqrt{5} \log \left(\frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3))

mupad [B] time = 1.57, size = 30, normalized size = 1.30

$$\frac{\sqrt{5} \operatorname{atanh} \left(\frac{3\sqrt{5}-8\sqrt{5}x^4}{18x^4-7} \right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^8 - 3*x^4 + 1),x)

[Out] (5^(1/2)*atanh((3*5^(1/2) - 8*5^(1/2)*x^4)/(18*x^4 - 7)))/10

sympy [A] time = 0.12, size = 42, normalized size = 1.83

$$\frac{\sqrt{5} \log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right)}{20} - \frac{\sqrt{5} \log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**8-3*x**4+1),x)

[Out] sqrt(5)*log(x**4 - 3/2 - sqrt(5)/2)/20 - sqrt(5)*log(x**4 - 3/2 + sqrt(5)/2)/20

$$3.330 \quad \int \frac{x}{1-3x^4+x^8} dx$$

Optimal. Leaf size=75

$$\frac{1}{2} \sqrt{\frac{1}{10} (3 + \sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{1}{2} (3 + \sqrt{5})} x^2 \right) - \frac{\tanh^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right)}{\sqrt{10} (3 + \sqrt{5})}$$

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1359, 1093, 207}

$$\frac{1}{2} \sqrt{\frac{1}{10} (3 + \sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{1}{2} (3 + \sqrt{5})} x^2 \right) - \frac{\tanh^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right)}{\sqrt{10} (3 + \sqrt{5})}$$

Antiderivative was successfully verified.

[In] Int[x/(1 - 3*x^4 + x^8),x]

[Out] -(ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1359

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x}{1-3x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-3x^2+x^4} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2} dx, x, x^2 \right)}{2\sqrt{5}} - \frac{\text{Subst} \left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2} dx, x, x^2 \right)}{2\sqrt{5}} \\
&= -\frac{\tanh^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right)}{\sqrt{10(3+\sqrt{5})}} + \frac{1}{2} \sqrt{\frac{1}{10} (3+\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{1}{2} (3+\sqrt{5})} x^2 \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 91, normalized size = 1.21

$$\frac{1}{40} \left(-((5+\sqrt{5}) \log(-2x^2+\sqrt{5}-1)) - (\sqrt{5}-5) \log(-2x^2+\sqrt{5}+1) + (5+\sqrt{5}) \log(2x^2+\sqrt{5}-1) + (\sqrt{5}-5) \log(2x^2+\sqrt{5}+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 - 3*x^4 + x^8), x]

[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/40

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{1-3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(1 - 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x/(1 - 3*x^4 + x^8), x]

fricas [B] time = 1.23, size = 107, normalized size = 1.43

$$\frac{1}{40} \sqrt{5} \log \left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1} \right) + \frac{1}{40} \sqrt{5} \log \left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1} \right) - \frac{1}{8} \log(x^4+x^2-1) + \frac{1}{8} \log(x^4-x^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^8-3*x^4+1), x, algorithm="fricas")

[Out] $\frac{1}{40}\sqrt{5}\log((2x^4 + 2x^2 + \sqrt{5})(2x^2 + 1) + 3)/(x^4 + x^2 - 1)) + \frac{1}{40}\sqrt{5}\log((2x^4 - 2x^2 + \sqrt{5})(2x^2 - 1) + 3)/(x^4 - x^2 - 1)) - \frac{1}{8}\log(x^4 + x^2 - 1) + \frac{1}{8}\log(x^4 - x^2 - 1)$

giac [B] time = 0.43, size = 92, normalized size = 1.23

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{|2x^2 - \sqrt{5} + 1|}{2x^2 + \sqrt{5} + 1}\right) - \frac{1}{40}\sqrt{5}\log\left(\frac{|2x^2 - \sqrt{5} - 1|}{2x^2 + \sqrt{5} - 1}\right) - \frac{1}{8}\log(|x^4 + x^2 - 1|) + \frac{1}{8}\log(|x^4 - x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^8-3*x^4+1),x, algorithm="giac")`

[Out] $-\frac{1}{40}\sqrt{5}\log(\text{abs}(2x^2 - \sqrt{5} + 1)/(2x^2 + \sqrt{5} + 1)) - \frac{1}{40}\sqrt{5}\log(\text{abs}(2x^2 - \sqrt{5} - 1)/\text{abs}(2x^2 + \sqrt{5} - 1)) - \frac{1}{8}\log(\text{abs}(x^4 + x^2 - 1)) + \frac{1}{8}\log(\text{abs}(x^4 - x^2 - 1))$

maple [A] time = 0.00, size = 62, normalized size = 0.83

$$\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{20} + \frac{\ln(x^4 - x^2 - 1)}{8} - \frac{\ln(x^4 + x^2 - 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^8-3*x^4+1),x)`

[Out] $-\frac{1}{8}\ln(x^4+x^2-1)+\frac{1}{20}5^{(1/2)}*\operatorname{arctanh}(1/5*(2*x^2+1)*5^{(1/2)})+\frac{1}{8}\ln(x^4-x^2-1)+\frac{1}{20}5^{(1/2)}*\operatorname{arctanh}(1/5*(2*x^2-1)*5^{(1/2)})$

maxima [B] time = 1.45, size = 87, normalized size = 1.16

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2 - \sqrt{5} + 1}{2x^2 + \sqrt{5} + 1}\right) - \frac{1}{40}\sqrt{5}\log\left(\frac{2x^2 - \sqrt{5} - 1}{2x^2 + \sqrt{5} - 1}\right) - \frac{1}{8}\log(x^4 + x^2 - 1) + \frac{1}{8}\log(x^4 - x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^8-3*x^4+1),x, algorithm="maxima")`

[Out] $-\frac{1}{40}\sqrt{5}\log((2x^2 - \sqrt{5} + 1)/(2x^2 + \sqrt{5} + 1)) - \frac{1}{40}\sqrt{5}\log((2x^2 - \sqrt{5} - 1)/(2x^2 + \sqrt{5} - 1)) - \frac{1}{8}\log(x^4 + x^2 - 1) + \frac{1}{8}\log(x^4 - x^2 - 1)$

mupad [B] time = 1.30, size = 83, normalized size = 1.11

$$\operatorname{atanh}\left(\frac{29x^2}{8\sqrt{5}-18} - \frac{13\sqrt{5}x^2}{8\sqrt{5}-18}\right)\left(\frac{\sqrt{5}}{20} - \frac{1}{4}\right) + \operatorname{atanh}\left(\frac{29x^2}{8\sqrt{5}+18} + \frac{13\sqrt{5}x^2}{8\sqrt{5}+18}\right)\left(\frac{\sqrt{5}}{20} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^8 - 3*x^4 + 1), x)`

[Out] `atanh((29*x^2)/(8*5^(1/2) - 18) - (13*5^(1/2)*x^2)/(8*5^(1/2) - 18))*(5^(1/2)/20 - 1/4) + atanh((29*x^2)/(8*5^(1/2) + 18) + (13*5^(1/2)*x^2)/(8*5^(1/2) + 18))*(5^(1/2)/20 + 1/4)`

sympy [B] time = 0.37, size = 165, normalized size = 2.20

$$\left(\frac{\sqrt{5}}{40} + \frac{1}{8}\right) \log\left(x^2 - \frac{7}{2} - \frac{7\sqrt{5}}{10} + 960\left(\frac{\sqrt{5}}{40} + \frac{1}{8}\right)^3\right) + \left(\frac{1}{8} - \frac{\sqrt{5}}{40}\right) \log\left(x^2 - \frac{7}{2} + 960\left(\frac{1}{8} - \frac{\sqrt{5}}{40}\right)^3 + \frac{7\sqrt{5}}{10}\right) + \left(-\frac{1}{8} + \frac{\sqrt{5}}{40}\right) \log\left(x^2 - \frac{7\sqrt{5}}{10} + 960\left(-\frac{1}{8} + \frac{\sqrt{5}}{40}\right)^3 + \frac{7}{2}\right) + \left(-\frac{1}{8} - \frac{\sqrt{5}}{40}\right) \log\left(x^2 + 960\left(-\frac{1}{8} - \frac{\sqrt{5}}{40}\right)^3 + \frac{7\sqrt{5}}{10} + \frac{7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**8-3*x**4+1), x)`

[Out] `(sqrt(5)/40 + 1/8)*log(x**2 - 7/2 - 7*sqrt(5)/10 + 960*(sqrt(5)/40 + 1/8)**3) + (1/8 - sqrt(5)/40)*log(x**2 - 7/2 + 960*(1/8 - sqrt(5)/40)**3 + 7*sqrt(5)/10) + (-1/8 + sqrt(5)/40)*log(x**2 - 7*sqrt(5)/10 + 960*(-1/8 + sqrt(5)/40)**3 + 7/2) + (-1/8 - sqrt(5)/40)*log(x**2 + 960*(-1/8 - sqrt(5)/40)**3 + 7*sqrt(5)/10 + 7/2)`

$$3.331 \quad \int \frac{1}{x(1-3x^4+x^8)} dx$$

Optimal. Leaf size=57

$$-\frac{1}{40}(5+3\sqrt{5})\log(-2x^4-\sqrt{5}+3)-\frac{1}{40}(5-3\sqrt{5})\log(-2x^4+\sqrt{5}+3)+\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1357, 705, 29, 632, 31}

$$-\frac{1}{40}(5+3\sqrt{5})\log(-2x^4-\sqrt{5}+3)-\frac{1}{40}(5-3\sqrt{5})\log(-2x^4+\sqrt{5}+3)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - 3*x^4 + x^8)),x]

[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((5 - 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-3x^4+x^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x(1-3x+x^2)} dx, x, x^4 \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x} dx, x, x^4 \right) + \frac{1}{4} \text{Subst} \left(\int \frac{3-x}{1-3x+x^2} dx, x, x^4 \right) \\
&= \log(x) + \frac{1}{40} (-5 + 3\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) - \frac{1}{40} (5 + 3\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} + \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) \\
&= \log(x) - \frac{1}{40} (5 + 3\sqrt{5}) \log(3 - \sqrt{5} - 2x^4) - \frac{1}{40} (5 - 3\sqrt{5}) \log(3 + \sqrt{5} - 2x^4)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.96

$$\frac{1}{40} (3\sqrt{5} - 5) \log(-2x^4 + \sqrt{5} + 3) + \frac{1}{40} (-5 - 3\sqrt{5}) \log(2x^4 + \sqrt{5} - 3) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((-5 - 3*Sqrt[5])
*Log[-3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1-3x^4+x^8)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(1 - 3*x^4 + x^8)), x]
```

fricas [A] time = 1.28, size = 59, normalized size = 1.04

$$\frac{3}{40} \sqrt{5} \log\left(\frac{2x^8 - 6x^4 - \sqrt{5}(2x^4 - 3) + 7}{x^8 - 3x^4 + 1}\right) - \frac{1}{8} \log(x^8 - 3x^4 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1)) - 1/8*log(x^8 - 3*x^4 + 1) + log(x)

giac [A] time = 0.48, size = 54, normalized size = 0.95

$$\frac{3}{40} \sqrt{5} \log\left(\frac{|2x^4 - \sqrt{5} - 3|}{|2x^4 + \sqrt{5} - 3|}\right) + \frac{1}{4} \log(x^4) - \frac{1}{8} \log(|x^8 - 3x^4 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="giac")

[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/4*log(x^4) - 1/8*log(abs(x^8 - 3*x^4 + 1))

maple [A] time = 0.01, size = 64, normalized size = 1.12

$$\frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20} + \frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{20} + \ln(x) - \frac{\ln(x^4 - x^2 - 1)}{8} - \frac{\ln(x^4 + x^2 - 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^8-3*x^4+1),x)

[Out] ln(x)-1/8*ln(x^4-x^2-1)-3/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln(x^4+x^2-1)+3/20*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))

maxima [A] time = 1.45, size = 51, normalized size = 0.89

$$\frac{3}{40} \sqrt{5} \log\left(\frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3}\right) - \frac{1}{8} \log(x^8 - 3x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] $\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4 - \sqrt{5} - 3}{2x^4 + \sqrt{5} - 3}\right) - \frac{1}{8}\log(x^8 - 3x^4 + 1) + \frac{1}{4}\log(x^4)$

mupad [B] time = 0.43, size = 42, normalized size = 0.74

$$\ln(x) + \ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40} - \frac{1}{8}\right) - \ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40} + \frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x^8 - 3*x^4 + 1)),x)`

[Out] $\log(x) + \log(x^4 - 5^{(1/2)}/2 - 3/2)*((3*5^{(1/2)})/40 - 1/8) - \log(5^{(1/2)}/2 + x^4 - 3/2)*((3*5^{(1/2)})/40 + 1/8)$

sympy [A] time = 0.16, size = 58, normalized size = 1.02

$$\log(x) + \left(-\frac{1}{8} + \frac{3\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(-\frac{3\sqrt{5}}{40} - \frac{1}{8}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x**8-3*x**4+1),x)`

[Out] $\log(x) + (-1/8 + 3*\sqrt{5}/40)*\log(x**4 - 3/2 - \sqrt{5}/2) + (-3*\sqrt{5}/40 - 1/8)*\log(x**4 - 3/2 + \sqrt{5}/2)$

$$3.332 \quad \int \frac{1}{x^3(1-3x^4+x^8)} dx$$

Optimal. Leaf size=89

$$-\frac{1}{2x^2} - \frac{1}{2} \sqrt{\frac{1}{5}(9-4\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2\right) + \frac{(3+\sqrt{5})^{3/2} \tanh^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x^2\right)}{4\sqrt{10}}$$

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1359, 1123, 1166, 207}

$$-\frac{1}{2x^2} - \frac{1}{2} \sqrt{\frac{1}{5}(9-4\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2\right) + \frac{(3+\sqrt{5})^{3/2} \tanh^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(1 - 3*x^4 + x^8)),x]

[Out] -1/(2*x^2) - (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + ((3 + Sqrt[5])^(3/2)*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1123

Int[((d_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m+1)*(a+b*x^2+c*x^4)^(p+1))/(a*d*(m+1)), x] - Dist[1/(a*d^2*(m+1)), Int[(d*x)^(m+2)*(b*(m+2*p+3)+c*(m+4*p+5)*x^2)*(a+b*x^2+c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2-4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(1-3x^4+x^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(1-3x^2+x^4)} dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{3-x^2}{1-3x^2+x^4} dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} + \frac{1}{20}(-5+3\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2} dx, x, x^2 \right) - \frac{1}{20}(5+3\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2} dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} - \frac{1}{10}\sqrt{45-20\sqrt{5}} \tanh^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right) + \frac{(3+\sqrt{5})^{3/2} \tanh^{-1} \left(\sqrt{\frac{1}{2}(3+\sqrt{5})} x^2 \right)}{4\sqrt{10}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 103, normalized size = 1.16

$$\frac{1}{20} \left(-\frac{10}{x^2} - ((5+2\sqrt{5}) \log(-2x^2+\sqrt{5}-1)) + (5-2\sqrt{5}) \log(-2x^2+\sqrt{5}+1) + (5+2\sqrt{5}) \log(2x^2+\sqrt{5}-1) + (2\sqrt{5}-5) \log(2x^2+\sqrt{5}+1) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^2 - (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + 2*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(1-3x^4+x^8)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

[Out] IntegrateAlgebraic[1/(x^3*(1 - 3*x^4 + x^8)), x]

fricas [B] time = 1.11, size = 125, normalized size = 1.40

$$\frac{2\sqrt{5}x^2 \log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right) + 2\sqrt{5}x^2 \log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right) - 5x^2 \log(x^4+x^2-1) + 5x^2 \log(x^4-x^2-1) - 10}{20x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] 1/20*(2*sqrt(5)*x^2*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1)) + 2*sqrt(5)*x^2*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 - 1)) - 5*x^2*log(x^4 + x^2 - 1) + 5*x^2*log(x^4 - x^2 - 1) - 10)/x^2

giac [A] time = 0.38, size = 97, normalized size = 1.09

$$-\frac{1}{10}\sqrt{5} \log\left(\frac{|2x^2 - \sqrt{5} + 1|}{2x^2 + \sqrt{5} + 1}\right) - \frac{1}{10}\sqrt{5} \log\left(\frac{|2x^2 - \sqrt{5} - 1|}{2x^2 + \sqrt{5} - 1}\right) - \frac{1}{2x^2} - \frac{1}{4} \log(|x^4 + x^2 - 1|) + \frac{1}{4} \log(|x^4 - x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="giac")

[Out] -1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))

maple [A] time = 0.01, size = 67, normalized size = 0.75

$$\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{5} + \frac{\ln(x^4 - x^2 - 1)}{4} - \frac{\ln(x^4 + x^2 - 1)}{4} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^8-3*x^4+1),x)

[Out] 1/4*ln(x^4-x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4*ln(x^4+x^2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2+1)*5^(1/2))-1/2/x^2

maxima [A] time = 1.46, size = 92, normalized size = 1.03

$$-\frac{1}{10}\sqrt{5} \log\left(\frac{2x^2 - \sqrt{5} + 1}{2x^2 + \sqrt{5} + 1}\right) - \frac{1}{10}\sqrt{5} \log\left(\frac{2x^2 - \sqrt{5} - 1}{2x^2 + \sqrt{5} - 1}\right) - \frac{1}{2x^2} - \frac{1}{4} \log(x^4 + x^2 - 1) + \frac{1}{4} \log(x^4 - x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] $-1/10*\sqrt{5}*\log((2*x^2 - \sqrt{5} + 1)/(2*x^2 + \sqrt{5} + 1)) - 1/10*\sqrt{5}*\log((2*x^2 - \sqrt{5} - 1)/(2*x^2 + \sqrt{5} - 1)) - 1/2/x^2 - 1/4*\log(x^4 + x^2 - 1) + 1/4*\log(x^4 - x^2 - 1)$

mupad [B] time = 0.06, size = 88, normalized size = 0.99

$$\operatorname{atanh}\left(\frac{12736x^2}{3520\sqrt{5}-7872} - \frac{5696\sqrt{5}x^2}{3520\sqrt{5}-7872}\right)\left(\frac{\sqrt{5}}{5} - \frac{1}{2}\right) + \operatorname{atanh}\left(\frac{12736x^2}{3520\sqrt{5}+7872} + \frac{5696\sqrt{5}x^2}{3520\sqrt{5}+7872}\right)\left(\frac{\sqrt{5}}{5} + \frac{1}{2}\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(x^8 - 3*x^4 + 1)),x)

[Out] $\operatorname{atanh}((12736*x^2)/(3520*5^{(1/2)} - 7872) - (5696*5^{(1/2)}*x^2)/(3520*5^{(1/2)} - 7872))*(5^{(1/2)}/5 - 1/2) + \operatorname{atanh}((12736*x^2)/(3520*5^{(1/2)} + 7872) + (5696*5^{(1/2)}*x^2)/(3520*5^{(1/2)} + 7872))*(5^{(1/2)}/5 + 1/2) - 1/(2*x^2)$

sympy [B] time = 0.41, size = 172, normalized size = 1.93

$$\left(\frac{\sqrt{5}}{10} + \frac{1}{4}\right)\log\left(x^2 - \frac{123}{8} - \frac{123\sqrt{5}}{20} + 280\left(\frac{\sqrt{5}}{10} + \frac{1}{4}\right)^3\right) + \left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{123}{8} + 280\left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)^3 + \frac{123\sqrt{5}}{20}\right) + \left(\frac{1}{4} + \frac{\sqrt{5}}{10}\right)\log\left(x^2 - \frac{123\sqrt{5}}{20} + 280\left(\frac{1}{4} + \frac{\sqrt{5}}{10}\right)^3 + \frac{123}{8}\right) + \left(-\frac{1}{4} - \frac{\sqrt{5}}{10}\right)\log\left(x^2 + 280\left(\frac{1}{4} - \frac{\sqrt{5}}{10}\right)^3 + \frac{123\sqrt{5}}{20} + \frac{123}{8}\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(x**8-3*x**4+1),x)

[Out] $(\sqrt{5}/10 + 1/4)*\log(x**2 - 123/8 - 123*\sqrt{5}/20 + 280*(\sqrt{5}/10 + 1/4)**3) + (1/4 - \sqrt{5}/10)*\log(x**2 - 123/8 + 280*(1/4 - \sqrt{5}/10)**3 + 123*\sqrt{5}/20) + (-1/4 + \sqrt{5}/10)*\log(x**2 - 123*\sqrt{5}/20 + 280*(-1/4 + \sqrt{5}/10)**3 + 123/8) + (-1/4 - \sqrt{5}/10)*\log(x**2 + 280*(-1/4 - \sqrt{5}/10)**3 + 123*\sqrt{5}/20 + 123/8) - 1/(2*x**2)$

$$3.333 \quad \int \frac{1}{x^5(1-3x^4+x^8)} dx$$

Optimal. Leaf size=66

$$-\frac{1}{4x^4} - \frac{1}{40} (15 + 7\sqrt{5}) \log(-2x^4 - \sqrt{5} + 3) - \frac{1}{40} (15 - 7\sqrt{5}) \log(-2x^4 + \sqrt{5} + 3) + 3 \log(x)$$

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1357, 709, 800, 632, 31}

$$-\frac{1}{4x^4} - \frac{1}{40} (15 + 7\sqrt{5}) \log(-2x^4 - \sqrt{5} + 3) - \frac{1}{40} (15 - 7\sqrt{5}) \log(-2x^4 + \sqrt{5} + 3) + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(1 - 3*x^4 + x^8)),x]

[Out] -1/(4*x^4) + 3*Log[x] - ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((15 - 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(1-3x^4+x^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{x^2(1-3x+x^2)} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{1}{4} \text{Subst} \left(\int \frac{3-x}{x(1-3x+x^2)} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + \frac{1}{4} \text{Subst} \left(\int \left(\frac{3}{x} + \frac{8-3x}{1-3x+x^2} \right) dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + 3 \log(x) + \frac{1}{4} \text{Subst} \left(\int \frac{8-3x}{1-3x+x^2} dx, x, x^4 \right) \\
&= -\frac{1}{4x^4} + 3 \log(x) + \frac{1}{40} (-15 + 7\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x} dx, x, x^4 \right) - \frac{1}{40} (15 + 7\sqrt{5}) \\
&= -\frac{1}{4x^4} + 3 \log(x) - \frac{1}{40} (15 + 7\sqrt{5}) \log(3 - \sqrt{5} - 2x^4) - \frac{1}{40} (15 - 7\sqrt{5}) \log(3 + \sqrt{5} - 2x^4)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.92

$$\frac{1}{40} \left(-\frac{10}{x^4} + (7\sqrt{5} - 15) \log(-2x^4 + \sqrt{5} + 3) - (15 + 7\sqrt{5}) \log(2x^4 + \sqrt{5} - 3) + 120 \log(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 + 120*Log[x] + (-15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] - (15 +
7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(1-3x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(1 - 3*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^5*(1 - 3*x^4 + x^8)), x]

fricas [A] time = 1.28, size = 76, normalized size = 1.15

$$\frac{7\sqrt{5}x^4 \log\left(\frac{2x^8-6x^4-\sqrt{5}(2x^4-3)+7}{x^8-3x^4+1}\right) - 15x^4 \log(x^8-3x^4+1) + 120x^4 \log(x) - 10}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1)) - 15*x^4*log(x^8 - 3*x^4 + 1) + 120*x^4*log(x) - 10)/x^4

giac [A] time = 0.51, size = 66, normalized size = 1.00

$$\frac{7}{40}\sqrt{5} \log\left(\frac{|2x^4 - \sqrt{5} - 3|}{|2x^4 + \sqrt{5} - 3|}\right) - \frac{3x^4 + 1}{4x^4} + \frac{3}{4} \log(x^4) - \frac{3}{8} \log(|x^8 - 3x^4 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="giac")

[Out] 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) - 1/4*(3*x^4 + 1)/x^4 + 3/4*log(x^4) - 3/8*log(abs(x^8 - 3*x^4 + 1))

maple [A] time = 0.01, size = 71, normalized size = 1.08

$$-\frac{7\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20} + \frac{7\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{20} + 3\ln(x) - \frac{3\ln(x^4-x^2-1)}{8} - \frac{3\ln(x^4+x^2-1)}{8} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(x^8-3*x^4+1),x)

[Out] $-1/4/x^4+3*\ln(x)-3/8*\ln(x^4-x^2-1)-7/20*5^{(1/2)}*\operatorname{arctanh}(1/5*(2*x^2-1)*5^{(1/2)})-3/8*\ln(x^4+x^2-1)+7/20*5^{(1/2)}*\operatorname{arctanh}(1/5*(2*x^2+1)*5^{(1/2)})$

maxima [A] time = 1.40, size = 56, normalized size = 0.85

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{4x^4}-\frac{3}{8}\log(x^8-3x^4+1)+\frac{3}{4}\log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="maxima")`

[Out] $7/40*\sqrt{5}*\log((2*x^4 - \sqrt{5} - 3)/(2*x^4 + \sqrt{5} - 3)) - 1/4/x^4 - 3/8*\log(x^8 - 3*x^4 + 1) + 3/4*\log(x^4)$

mupad [B] time = 1.35, size = 49, normalized size = 0.74

$$3\ln(x) - \frac{1}{4x^4} + \ln\left(x^4 - \frac{\sqrt{5}}{2} - \frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40} - \frac{3}{8}\right) - \ln\left(x^4 + \frac{\sqrt{5}}{2} - \frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40} + \frac{3}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(x^8 - 3*x^4 + 1)),x)`

[Out] $3*\log(x) - 1/(4*x^4) + \log(x^4 - 5^{(1/2)}/2 - 3/2)*((7*5^{(1/2)})/40 - 3/8) - \log(5^{(1/2)}/2 + x^4 - 3/2)*((7*5^{(1/2)})/40 + 3/8)$

sympy [A] time = 0.19, size = 66, normalized size = 1.00

$$3\log(x) + \left(-\frac{3}{8} + \frac{7\sqrt{5}}{40}\right)\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) + \left(-\frac{7\sqrt{5}}{40} - \frac{3}{8}\right)\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right) - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(x**8-3*x**4+1),x)`

[Out] $3*\log(x) + (-3/8 + 7*\sqrt{5}/40)*\log(x**4 - 3/2 - \sqrt{5}/2) + (-7*\sqrt{5}/40 - 3/8)*\log(x**4 - 3/2 + \sqrt{5}/2) - 1/(4*x**4)$

$$3.334 \quad \int \frac{1}{x^7(1-3x^4+x^8)} dx$$

Optimal. Leaf size=97

$$-\frac{1}{6x^6} - \frac{3}{2x^2} - \frac{1}{2} \sqrt{\frac{1}{10} (123 - 55\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right) + \frac{1}{2} \sqrt{\frac{1}{10} (123 + 55\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{1}{2} (3 + \sqrt{5})} x^2 \right)$$

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1359, 1123, 1281, 1166, 207}

$$-\frac{3}{2x^2} - \frac{1}{6x^6} - \frac{1}{2} \sqrt{\frac{1}{10} (123 - 55\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{3 + \sqrt{5}}} x^2 \right) + \frac{1}{2} \sqrt{\frac{1}{10} (123 + 55\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{1}{2} (3 + \sqrt{5})} x^2 \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^7*(1 - 3*x^4 + x^8)),x]

[Out] -1/(6*x^6) - 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1123

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1359

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7(1-3x^4+x^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^4(1-3x^2+x^4)} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} + \frac{1}{6} \text{Subst} \left(\int \frac{9-3x^2}{x^2(1-3x^2+x^4)} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} - \frac{3}{2x^2} - \frac{1}{6} \text{Subst} \left(\int \frac{-24+9x^2}{1-3x^2+x^4} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} - \frac{3}{2x^2} - \frac{1}{20} (15-7\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2} dx, x, x^2 \right) - \frac{1}{20} (15+7\sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2} dx, x, x^2 \right) \\
&= -\frac{1}{6x^6} - \frac{3}{2x^2} - \frac{1}{2} \sqrt{\frac{1}{10} (123-55\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right) + \frac{1}{20} \sqrt{1230+550\sqrt{5}} \tan^{-1} \left(\sqrt{\frac{2}{3+\sqrt{5}}} x^2 \right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 111, normalized size = 1.14

$$\frac{1}{120} \left(-\frac{20}{x^6} - \frac{180}{x^2} - 3(25+11\sqrt{5}) \log(-2x^2+\sqrt{5}-1) + 3(25-11\sqrt{5}) \log(-2x^2+\sqrt{5}+1) + 3(25+11\sqrt{5}) \log(2x^2+\sqrt{5}-1) + 3(11\sqrt{5}-25) \log(2x^2+\sqrt{5}+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7*(1 - 3*x^4 + x^8)), x]

[Out] $(-20/x^6 - 180/x^2 - 3*(25 + 11*\text{Sqrt}[5])*Log[-1 + \text{Sqrt}[5] - 2*x^2] + 3*(25 - 11*\text{Sqrt}[5])*Log[1 + \text{Sqrt}[5] - 2*x^2] + 3*(25 + 11*\text{Sqrt}[5])*Log[-1 + \text{Sqrt}[5] + 2*x^2] + 3*(-25 + 11*\text{Sqrt}[5])*Log[1 + \text{Sqrt}[5] + 2*x^2])/120$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7(1 - 3x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^7*(1 - 3*x^4 + x^8)), x]

[Out] IntegrateAlgebraic[1/(x^7*(1 - 3*x^4 + x^8)), x]

fricas [B] time = 1.20, size = 130, normalized size = 1.34

$$\frac{33\sqrt{5}x^6 \log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right) + 33\sqrt{5}x^6 \log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right) - 75x^6 \log(x^4+x^2-1) + 75x^6 \log(x^4-x^2-1) - 180x^4 - 20}{120x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8-3*x^4+1), x, algorithm="fricas")

[Out] $1/120*(33*\text{sqrt}(5)*x^6*\log((2*x^4 + 2*x^2 + \text{sqrt}(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1)) + 33*\text{sqrt}(5)*x^6*\log((2*x^4 - 2*x^2 + \text{sqrt}(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 - 1)) - 75*x^6*\log(x^4 + x^2 - 1) + 75*x^6*\log(x^4 - x^2 - 1) - 180*x^4 - 20)/x^6$

giac [A] time = 0.41, size = 104, normalized size = 1.07

$$-\frac{11}{40}\sqrt{5} \log\left(\frac{|2x^2 - \sqrt{5} + 1|}{2x^2 + \sqrt{5} + 1}\right) - \frac{11}{40}\sqrt{5} \log\left(\frac{|2x^2 - \sqrt{5} - 1|}{2x^2 + \sqrt{5} - 1}\right) - \frac{9x^4 + 1}{6x^6} - \frac{5}{8} \log(|x^4 + x^2 - 1|) + \frac{5}{8} \log(|x^4 - x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^7/(x^8-3*x^4+1), x, algorithm="giac")

[Out] $-11/40*\text{sqrt}(5)*\log(\text{abs}(2*x^2 - \text{sqrt}(5) + 1)/(2*x^2 + \text{sqrt}(5) + 1)) - 11/40*\text{sqrt}(5)*\log(\text{abs}(2*x^2 - \text{sqrt}(5) - 1)/\text{abs}(2*x^2 + \text{sqrt}(5) - 1)) - 1/6*(9*x^4 + 1)/x^6 - 5/8*\log(\text{abs}(x^4 + x^2 - 1)) + 5/8*\log(\text{abs}(x^4 - x^2 - 1))$

maple [A] time = 0.01, size = 72, normalized size = 0.74

$$\frac{11\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20} + \frac{11\sqrt{5} \operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)}{20} + \frac{5 \ln(x^4 - x^2 - 1)}{8} - \frac{5 \ln(x^4 + x^2 - 1)}{8} - \frac{3}{2x^2} - \frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(x^8-3*x^4+1),x)`

[Out] $-1/6/x^6-3/2/x^2+5/8*\ln(x^4-x^2-1)+11/20*5^{(1/2)}*\operatorname{arctanh}(1/5*(2*x^2-1)*5^{(1/2)})-5/8*\ln(x^4+x^2-1)+11/20*5^{(1/2)}*\operatorname{arctanh}(1/5*(2*x^2+1)*5^{(1/2)})$

maxima [A] time = 1.32, size = 99, normalized size = 1.02

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log(x^4+x^2-1)+\frac{5}{8}\log(x^4-x^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="maxima")`

[Out] $-11/40*\sqrt{5}*\log((2*x^2 - \sqrt{5} + 1)/(2*x^2 + \sqrt{5} + 1)) - 11/40*\sqrt{5}*\log((2*x^2 - \sqrt{5} - 1)/(2*x^2 + \sqrt{5} - 1)) - 1/6*(9*x^4 + 1)/x^6 - 5/8*\log(x^4 + x^2 - 1) + 5/8*\log(x^4 - x^2 - 1)$

mupad [B] time = 1.38, size = 95, normalized size = 0.98

$$\operatorname{atanh}\left(\frac{4126100x^2}{1140425\sqrt{5}-2550075}-\frac{1845250\sqrt{5}x^2}{1140425\sqrt{5}-2550075}\right)\left(\frac{11\sqrt{5}}{20}-\frac{5}{4}\right)+\operatorname{atanh}\left(\frac{4126100x^2}{1140425\sqrt{5}+2550075}+\frac{1845250\sqrt{5}x^2}{1140425\sqrt{5}+2550075}\right)\left(\frac{11\sqrt{5}}{20}+\frac{5}{4}\right)-\frac{3x^4}{x^6}+\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(x^8-3*x^4+1)),x)`

[Out] $\operatorname{atanh}\left(\frac{4126100*x^2}{1140425*5^{(1/2)}-2550075}-\frac{1845250*5^{(1/2)}*x^2}{1140425*5^{(1/2)}-2550075}\right)\left(\frac{11*5^{(1/2)}}{20}-\frac{5}{4}\right)+\operatorname{atanh}\left(\frac{4126100*x^2}{1140425*5^{(1/2)}+2550075}+\frac{1845250*5^{(1/2)}*x^2}{1140425*5^{(1/2)}+2550075}\right)\left(\frac{11*5^{(1/2)}}{20}+\frac{5}{4}\right)-\frac{(3*x^4)}{2}+\frac{1}{6}/x^6$

sympy [B] time = 0.44, size = 199, normalized size = 2.05

$$\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\log\left(x^2-\frac{2207}{22}-\frac{2207\sqrt{5}}{50}+\frac{1152\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)^3}{11}\right)+\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207}{22}+\frac{1152\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}\right)+\left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207\sqrt{5}}{50}-\frac{1152\left(\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207}{22}\right)+\left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2+\frac{1152\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}+\frac{2207}{22}\right)+\frac{-9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(x**8-3*x**4+1),x)`

[Out] $(11*\sqrt{5}/40+5/8)*\log(x**2-2207/22-2207*\sqrt{5}/50+1152*(11*\sqrt{5}/40+5/8)**3/11)+(5/8-11*\sqrt{5}/40)*\log(x**2-2207/22+1152*(5/8-11*\sqrt{5}/40)**3/11+2207*\sqrt{5}/50)+(-5/8+11*\sqrt{5}/40)*\log(x**2-2207*\sqrt{5}/50+1152*(-5/8+11*\sqrt{5}/40)**3/11+2207/22)+(-5/8-11*\sqrt{5}/40)*\log(x**2+1152*(-5/8-11*\sqrt{5}/40)**3/11+2207*\sqrt{5}/50+2207/22)+(-9*x**4-1)/(6*x**6)$

$$3.335 \quad \int \frac{x^8}{1-3x^4+x^8} dx$$

Optimal. Leaf size=170

$$x - \frac{\sqrt[4]{\frac{1}{2}(123 + 55\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{984 - 440\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3 + \sqrt{5})} x\right)}{4\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}(123 + 55\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}}$$

Rubi [A] time = 0.11, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1367, 1422, 212, 206, 203}

$$x - \frac{\sqrt[4]{\frac{1}{2}(123 + 55\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{984 - 440\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3 + \sqrt{5})} x\right)}{4\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}(123 + 55\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{984 - 440\sqrt{5}} \tanh^{-1}\left(\sqrt[4]{\frac{1}{2}(3 + \sqrt{5})} x\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x^8/(1 - 3*x^4 + x^8), x]

[Out] x - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5]) - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
&& IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^8}{1 - 3x^4 + x^8} dx &= x - \int \frac{1 - 3x^4}{1 - 3x^4 + x^8} dx \\
 &= x - \frac{1}{10} (-15 + 7\sqrt{5}) \int \frac{1}{-\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx + \frac{1}{10} (15 + 7\sqrt{5}) \int \frac{1}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx \\
 &= x + \sqrt{\frac{1}{10} (9 - 4\sqrt{5})} \int \frac{1}{\sqrt{3 - \sqrt{5}} - \sqrt{2} x^2} dx + \sqrt{\frac{1}{10} (9 - 4\sqrt{5})} \int \frac{1}{\sqrt{3 - \sqrt{5}} + \sqrt{2} x^2} dx \\
 &= x - \frac{\sqrt{\frac{1}{2} (123 + 55\sqrt{5})} \tan^{-1} \left(\sqrt[4]{\frac{2}{3 + \sqrt{5}}} x \right)}{2\sqrt{5}} + \frac{\sqrt{\frac{1}{2} (123 - 55\sqrt{5})} \tan^{-1} \left(\sqrt[4]{\frac{1}{2} (3 + \sqrt{5})} x \right)}{2\sqrt{5}} - \dots
 \end{aligned}$$

Mathematica [A] time = 0.27, size = 160, normalized size = 0.94

$$x + \frac{(\sqrt{5} - 2) \tan^{-1} \left(\sqrt{\frac{2}{\sqrt{5} - 1}} x \right)}{\sqrt{10(\sqrt{5} - 1)}} - \frac{(2 + \sqrt{5}) \tan^{-1} \left(\sqrt{\frac{2}{1 + \sqrt{5}}} x \right)}{\sqrt{10(1 + \sqrt{5})}} + \frac{(\sqrt{5} - 2) \tanh^{-1} \left(\sqrt{\frac{2}{\sqrt{5} - 1}} x \right)}{\sqrt{10(\sqrt{5} - 1)}} - \frac{(2 + \sqrt{5}) \tanh^{-1} \left(\sqrt{\frac{2}{1 + \sqrt{5}}} x \right)}{\sqrt{10(1 + \sqrt{5})}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(1 - 3*x^4 + x^8), x]

```
[Out] x + ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5]])*x])/Sqrt[10*(-1 + Sqrt[5]]
) - ((2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5]])*x])/Sqrt[10*(1 + Sqrt[5])]
+ ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5]])*x])/Sqrt[10*(-1 + Sqrt[5]]
) - ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5]])*x])/Sqrt[10*(1 + Sqrt[5]]
]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{1 - 3x^4 + x^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(1 - 3*x^4 + x^8), x]
```

```
[Out] IntegrateAlgebraic[x^8/(1 - 3*x^4 + x^8), x]
```

fricas [B] time = 1.31, size = 304, normalized size = 1.79

```
 $\frac{1}{20} \sqrt{5} \sqrt{5 + 11} \arctan\left(\frac{1}{20} \sqrt{10} \sqrt{2x^2 + \sqrt{5} + 1} \sqrt{5\sqrt{5} + 11}\right) - \frac{1}{20} \sqrt{5} \sqrt{5 - 11} \arctan\left(\frac{1}{20} \sqrt{10} \sqrt{2x^2 + \sqrt{5} - 1} \sqrt{5\sqrt{5} - 11}\right) - \frac{1}{40} \sqrt{5} \sqrt{5 + 11} \log\left(\sqrt{10} \sqrt{5\sqrt{5} + 11} (3\sqrt{5} + 5) + 20x\right) - \frac{1}{40} \sqrt{5} \sqrt{5 - 11} \log\left(\sqrt{10} \sqrt{5\sqrt{5} - 11} (3\sqrt{5} - 5) + 20x\right) + \frac{1}{40} \sqrt{5} \sqrt{5 + 11} \log\left(\sqrt{10} \sqrt{5\sqrt{5} + 11} (3\sqrt{5} - 5) + 20x\right) + \frac{1}{40} \sqrt{5} \sqrt{5 - 11} \log\left(\sqrt{10} \sqrt{5\sqrt{5} - 11} (3\sqrt{5} + 5) + 20x\right) + x$ 
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1), x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(5*sqrt(5) + 11)*arctan(1/20*(sqrt(10)*sqrt(2*x^2 + sqrt
(5) + 1)*(2*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(2*sqrt(5)*x - 5*x))*
sqrt(5*sqrt(5) + 11)) - 1/10*sqrt(10)*sqrt(5*sqrt(5) - 11)*arctan(1/20*(sqr
t(10)*sqrt(2*x^2 + sqrt(5) - 1)*(2*sqrt(5)*sqrt(2) + 5*sqrt(2)) - 2*sqrt(10
)*(2*sqrt(5)*x + 5*x))*sqrt(5*sqrt(5) - 11)) + 1/40*sqrt(10)*sqrt(5*sqrt(5)
- 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) + 5) + 20*x) - 1/40*sq
rt(10)*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) +
5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*sqrt(5*sqrt(5)
+ 11)*(3*sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)*log(-sq
rt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5) + 20*x) + x
```

giac [A] time = 0.68, size = 148, normalized size = 0.87

```
 $-\frac{1}{20} \sqrt{50\sqrt{5} + 110} \arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{20} \sqrt{50\sqrt{5} - 110} \arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5} - \frac{1}{2}}}\right) - \frac{1}{40} \sqrt{50\sqrt{5} + 110} \log\left(\left|x + \sqrt{\frac{1}{2}\sqrt{5} + \frac{1}{2}}\right|\right) + \frac{1}{40} \sqrt{50\sqrt{5} + 110} \log\left(\left|x - \sqrt{\frac{1}{2}\sqrt{5} + \frac{1}{2}}\right|\right) + \frac{1}{40} \sqrt{50\sqrt{5} - 110} \log\left(\left|x + \sqrt{\frac{1}{2}\sqrt{5} - \frac{1}{2}}\right|\right) - \frac{1}{40} \sqrt{50\sqrt{5} - 110} \log\left(\left|x - \sqrt{\frac{1}{2}\sqrt{5} - \frac{1}{2}}\right|\right) + x$ 
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1), x, algorithm="giac")
```

```
[Out] -1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(
50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5)
+ 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*
```

$\log(\text{abs}(x - \sqrt{1/2*\sqrt{5} + 1/2})) + 1/40*\sqrt{50*\sqrt{5} - 110}*\log(\text{abs}(x + \sqrt{1/2*\sqrt{5} - 1/2})) - 1/40*\sqrt{50*\sqrt{5} - 110}*\log(\text{abs}(x - \sqrt{1/2*\sqrt{5} - 1/2})) + x$

maple [A] time = 0.06, size = 205, normalized size = 1.21

$$x + \frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}} - \frac{2\sqrt{5} \operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}} - \frac{2\sqrt{5} \operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}} - \frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}} + \frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}} - \frac{2\sqrt{5} \operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}} - \frac{2\sqrt{5} \operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}} - \frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8/(x^8-3*x^4+1), x)$

[Out] $x - 2/5*5^{1/2}/(2+2*5^{1/2})^{1/2}*\operatorname{arctanh}(2*x/(2+2*5^{1/2})^{1/2}) - 1/(2+2*5^{1/2})^{1/2}*\operatorname{arctanh}(2*x/(2+2*5^{1/2})^{1/2}) + 1/(-2+2*5^{1/2})^{1/2}*\operatorname{arctan}(2*x/(-2+2*5^{1/2})^{1/2}) - 2/5*5^{1/2}/(-2+2*5^{1/2})^{1/2}*\operatorname{arctan}(2*x/(-2+2*5^{1/2})^{1/2}) + 1/(-2+2*5^{1/2})^{1/2}*\operatorname{arctanh}(2*x/(-2+2*5^{1/2})^{1/2}) - 2/5*5^{1/2}/(-2+2*5^{1/2})^{1/2}*\operatorname{arctanh}(2*x/(-2+2*5^{1/2})^{1/2}) - 2/5*5^{1/2}/(2+2*5^{1/2})^{1/2}*\operatorname{arctan}(2*x/(2+2*5^{1/2})^{1/2}) - 1/(2+2*5^{1/2})^{1/2}*\operatorname{arctan}(2*x/(2+2*5^{1/2})^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x + \frac{1}{2} \int \frac{2x^2 + 1}{x^4 - x^2 - 1} dx - \frac{1}{2} \int \frac{2x^2 - 1}{x^4 + x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^8/(x^8-3*x^4+1), x, \text{algorithm}="maxima")$

[Out] $x + 1/2*\text{integrate}((2*x^2 + 1)/(x^4 - x^2 - 1), x) - 1/2*\text{integrate}((2*x^2 - 1)/(x^4 + x^2 - 1), x)$

mupad [B] time = 1.44, size = 246, normalized size = 1.45

$$x - \frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110} + \sqrt{5}x\sqrt{-50\sqrt{5}-110}33i}{2(275\sqrt{5}+605)}\right)\sqrt{-50\sqrt{5}-110}i}{20} - \frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}} - \sqrt{5}x\sqrt{110-50\sqrt{5}}33i}{2(275\sqrt{5}-605)}\right)\sqrt{110-50\sqrt{5}}i}{20} + \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}-110} - \sqrt{5}x\sqrt{50\sqrt{5}-110}33i}{2(275\sqrt{5}-605)}\right)\sqrt{50\sqrt{5}-110}i}{20} + \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}+110} + \sqrt{5}x\sqrt{50\sqrt{5}+110}33i}{2(275\sqrt{5}+605)}\right)\sqrt{50\sqrt{5}+110}i}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8/(x^8 - 3*x^4 + 1), x)$

[Out] $x - (\operatorname{atan}((x*(-50*5^{1/2} - 110))^{1/2}*55i)/(2*(275*5^{1/2} + 605)) + (5^{1/2}*x*(-50*5^{1/2} - 110))^{1/2}*33i)/(2*(275*5^{1/2} + 605)))*(-50*5^{1/2} - 110)^{1/2}*i/20 - (\operatorname{atan}((x*(110 - 50*5^{1/2}))^{1/2}*55i)/(2*(275*5^{1/2} - 605)) - (5^{1/2}*x*(110 - 50*5^{1/2}))^{1/2}*33i)/(2*(275*5^{1/2} - 605)))*((110 - 50*5^{1/2}))^{1/2}*i/20 + (\operatorname{atan}((x*(50*5^{1/2} - 110))^{1/2}*55i)/(2*(275*5^{1/2} - 605)) - (5^{1/2}*x*(50*5^{1/2} - 110))^{1/2}*33i)/(2*($

$$\frac{275 \cdot 5^{1/2} - 605}{(50 \cdot 5^{1/2} - 110)^{1/2} \cdot i} \cdot \frac{1}{20} + \frac{\operatorname{atan}\left(\frac{x \cdot (50 \cdot 5^{1/2} + 110)^{1/2} \cdot 55i}{2 \cdot (275 \cdot 5^{1/2} + 605)}\right) + (5^{1/2} \cdot x \cdot (50 \cdot 5^{1/2} + 110)^{1/2} \cdot 33i)}{2 \cdot (275 \cdot 5^{1/2} + 605)} \cdot \frac{1}{20}$$

sympy [A] time = 1.24, size = 58, normalized size = 0.34

$$x + \operatorname{RootSum}\left(6400t^4 - 880t^2 - 1, \left(t \mapsto t \log\left(-\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 880t^2 - 1, \left(t \mapsto t \log\left(-\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(x**8-3*x**4+1),x)

[Out] x + RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(-15360*_t**5/11 + 1288*_t/55 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*log(-15360*_t**5/11 + 1288*_t/55 + x)))

$$3.336 \quad \int \frac{x^6}{1-3x^4+x^8} dx$$

Optimal. Leaf size=167

$$\frac{(3 + \sqrt{5})^{3/4} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2 \cdot 2^{3/4} \sqrt{5}} - \frac{\sqrt[4]{144 - 64\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}}(3 + \sqrt{5}) x\right)}{4\sqrt{5}} - \frac{(3 + \sqrt{5})^{3/4} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2 \cdot 2^{3/4} \sqrt{5}} + \frac{\sqrt[4]{144 - 64\sqrt{5}} \tanh^{-1}\left(\sqrt[4]{\frac{1}{2}}(3 + \sqrt{5}) x\right)}{4\sqrt{5}}$$

Rubi [A] time = 0.08, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1374, 298, 203, 206}

$$\frac{(3 + \sqrt{5})^{3/4} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2 \cdot 2^{3/4} \sqrt{5}} - \frac{\sqrt[4]{144 - 64\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}}(3 + \sqrt{5}) x\right)}{4\sqrt{5}} - \frac{(3 + \sqrt{5})^{3/4} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2 \cdot 2^{3/4} \sqrt{5}} + \frac{\sqrt[4]{144 - 64\sqrt{5}} \tanh^{-1}\left(\sqrt[4]{\frac{1}{2}}(3 + \sqrt{5}) x\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(1 - 3*x^4 + x^8), x]

[Out] ((3 + Sqrt[5])^(3/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt[5]) - ((144 - 64*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5]) - ((3 + Sqrt[5])^(3/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt[5]) + ((144 - 64*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1374

Int[((d_.)*(x_)^(m_))/((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{x^6}{1 - 3x^4 + x^8} dx &= \frac{1}{10} (5 - 3\sqrt{5}) \int \frac{x^2}{-\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx + \frac{1}{10} (5 + 3\sqrt{5}) \int \frac{x^2}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx \\ &= \frac{(3 - \sqrt{5}) \int \frac{1}{\sqrt{3 - \sqrt{5} - \sqrt{2}x^2}} dx}{2\sqrt{10}} - \frac{(3 - \sqrt{5}) \int \frac{1}{\sqrt{3 - \sqrt{5} + \sqrt{2}x^2}} dx}{2\sqrt{10}} - \frac{(3 + \sqrt{5}) \int \frac{1}{\sqrt{3 + \sqrt{5} - \sqrt{2}x^2}} dx}{2\sqrt{10}} \\ &= \frac{(3 + \sqrt{5})^{3/4} \tan^{-1}\left(\sqrt[4]{\frac{2}{3 + \sqrt{5}}} x\right)}{2 \cdot 2^{3/4} \sqrt{5}} - \frac{(3 - \sqrt{5})^{3/4} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3 + \sqrt{5})} x\right)}{2 \cdot 2^{3/4} \sqrt{5}} - \frac{(3 + \sqrt{5})^{3/4} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3 + \sqrt{5}}} x\right)}{2 \cdot 2^{3/4} \sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.15, size = 160, normalized size = 0.96

$$\frac{(\sqrt{5}-3) \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right)}{\sqrt{\sqrt{5}-1}} + \frac{(3+\sqrt{5}) \tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)}{\sqrt{1+\sqrt{5}}} - \frac{(\sqrt{5}-3) \tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right)}{\sqrt{\sqrt{5}-1}} - \frac{(3+\sqrt{5}) \tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)}{\sqrt{1+\sqrt{5}}}$$

$$2\sqrt{10}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(1 - 3*x^4 + x^8),x]

[Out] (((-3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5]])*x])/Sqrt[-1 + Sqrt[5]] + ((3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5]])*x])/Sqrt[1 + Sqrt[5]] - ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5]])*x])/Sqrt[-1 + Sqrt[5]] - ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5]])*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{1 - 3x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(1 - 3*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^6/(1 - 3*x^4 + x^8), x]

fricas [B] time = 1.27, size = 255, normalized size = 1.53

$$\frac{1}{5}\sqrt{5}\sqrt{5+2}\arctan\left(\frac{1}{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}-3}\sqrt{\sqrt{5}+2}\right) - \frac{1}{5}\sqrt{5}\sqrt{5-2}\arctan\left(\frac{1}{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+3}\sqrt{\sqrt{5}-2}\right) - \frac{1}{20}\sqrt{5}\sqrt{5+2}\log\left(\sqrt{5+2}\sqrt{\sqrt{5}-1}+2x\right) + \frac{1}{20}\sqrt{5}\sqrt{5+2}\log\left(-\sqrt{5+2}\sqrt{\sqrt{5}-1}+2x\right) + \frac{1}{20}\sqrt{5}\sqrt{5-2}\log\left(\sqrt{5+1}\sqrt{\sqrt{5}-2}+2x\right) - \frac{1}{20}\sqrt{5}\sqrt{5-2}\log\left(-\sqrt{5+1}\sqrt{\sqrt{5}-2}+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{5}\sqrt{5}\sqrt{5+2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}+2}\right) - \frac{3}{5}\sqrt{5}\sqrt{5+2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}+2}\right) - \frac{1}{2}\sqrt{5}\sqrt{5+2}\log\left(\sqrt{5+2}\sqrt{\sqrt{5}-1}+2x\right) + \frac{1}{5}\sqrt{5}\sqrt{5+2}\log\left(\sqrt{5+2}\sqrt{\sqrt{5}-1}+2x\right) - \frac{1}{2}\sqrt{5}\sqrt{5-2}\log\left(\sqrt{5+1}\sqrt{\sqrt{5}-2}+2x\right) + \frac{3}{5}\sqrt{5}\sqrt{5-2}\log\left(\sqrt{5+1}\sqrt{\sqrt{5}-2}+2x\right) - \frac{1}{20}\sqrt{5}\sqrt{5+2}\log\left(-\sqrt{5+2}\sqrt{\sqrt{5}-1}+2x\right) + \frac{1}{20}\sqrt{5}\sqrt{5+2}\log\left(-\sqrt{5+2}\sqrt{\sqrt{5}-1}+2x\right) + \frac{1}{20}\sqrt{5}\sqrt{5-2}\log\left(\sqrt{5+1}\sqrt{\sqrt{5}-2}+2x\right) - \frac{1}{20}\sqrt{5}\sqrt{5-2}\log\left(-\sqrt{5+1}\sqrt{\sqrt{5}-2}+2x\right)$

giac [A] time = 0.75, size = 147, normalized size = 0.88

$$\frac{1}{10}\sqrt{5}\sqrt{5+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+1}}\right) - \frac{1}{10}\sqrt{5}\sqrt{5-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-1}}\right) - \frac{1}{20}\sqrt{5}\sqrt{5+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+1}\right|\right) + \frac{1}{20}\sqrt{5}\sqrt{5+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+1}\right|\right) + \frac{1}{20}\sqrt{5}\sqrt{5-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-1}\right|\right) - \frac{1}{20}\sqrt{5}\sqrt{5-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="giac")

[Out] $\frac{1}{10}\sqrt{5}\sqrt{5+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+1}}\right) - \frac{1}{10}\sqrt{5}\sqrt{5-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-1}}\right) - \frac{1}{20}\sqrt{5}\sqrt{5+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+1}\right|\right) + \frac{1}{20}\sqrt{5}\sqrt{5+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+1}\right|\right) + \frac{1}{20}\sqrt{5}\sqrt{5-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-1}\right|\right) - \frac{1}{20}\sqrt{5}\sqrt{5-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-1}\right|\right)$

maple [A] time = 0.03, size = 206, normalized size = 1.23

$$-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}} + \frac{3\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}} - \frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}} - \frac{3\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}} + \frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}} - \frac{3\sqrt{5}\operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}} + \frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}} + \frac{3\sqrt{5}\operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^8-3*x^4+1),x)

[Out] $-\frac{1}{2}\sqrt{2+2\sqrt{5}}\operatorname{arctanh}\left(\frac{2}{\sqrt{2+2\sqrt{5}}}\sqrt{2+2\sqrt{5}}x\right) - \frac{3}{10}\sqrt{2+2\sqrt{5}}\operatorname{arctanh}\left(\frac{2}{\sqrt{2+2\sqrt{5}}}\sqrt{2+2\sqrt{5}}x\right) + \frac{1}{2}\sqrt{-2+2\sqrt{5}}\operatorname{arctanh}\left(\frac{2}{\sqrt{-2+2\sqrt{5}}}\sqrt{-2+2\sqrt{5}}x\right) - \frac{3}{10}\sqrt{-2+2\sqrt{5}}\operatorname{arctanh}\left(\frac{2}{\sqrt{-2+2\sqrt{5}}}\sqrt{-2+2\sqrt{5}}x\right)$

$$\begin{aligned} & \operatorname{rctan}\left(\frac{2}{(-2+2*5^{(1/2)})^{(1/2)}}*x\right)-\frac{3}{10*5^{(1/2)}}/(-2+2*5^{(1/2)})^{(1/2)}*\arctan\left(\frac{2}{(-2+2*5^{(1/2)})^{(1/2)}}*x\right)-\frac{1}{2}/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}\left(\frac{2}{(-2+2*5^{(1/2)})^{(1/2)}}*x\right) \\ & +\frac{3}{10*5^{(1/2)}}/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}\left(\frac{2}{(-2+2*5^{(1/2)})^{(1/2)}}*x\right) \\ & +\frac{1}{2}/(2+2*5^{(1/2)})^{(1/2)}*\arctan\left(\frac{2}{(2+2*5^{(1/2)})^{(1/2)}}*x\right)+\frac{3}{10*5^{(1/2)}}/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}\left(\frac{2}{(2+2*5^{(1/2)})^{(1/2)}}*x\right) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{x^8 - 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] integrate(x^6/(x^8 - 3*x^4 + 1), x)

mupad [B] time = 0.19, size = 147, normalized size = 0.88

$$\frac{\sqrt{5} \operatorname{atan}\left(\frac{16x\sqrt{2-\sqrt{5}}}{8\sqrt{5-24}}\right)\sqrt{\sqrt{5}-2} \operatorname{li}}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{16x\sqrt{-\sqrt{5}-2}}{8\sqrt{5+24}}\right)\sqrt{\sqrt{5}+2} \operatorname{li}}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{x\sqrt{2-\sqrt{5}}16i}{8\sqrt{5-24}}\right)\sqrt{2-\sqrt{5}} \operatorname{li}}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{x\sqrt{-\sqrt{5}-2}16i}{8\sqrt{5+24}}\right)\sqrt{-\sqrt{5}-2} \operatorname{li}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^8 - 3*x^4 + 1),x)

[Out] $(5^{(1/2)}*\operatorname{atan}((16*x*(2 - 5^{(1/2)})^{(1/2)})/(8*5^{(1/2)} - 24))*(5^{(1/2)} - 2)^{(1/2)}*1i)/10 + (5^{(1/2)}*\operatorname{atan}((16*x*(- 5^{(1/2)} - 2)^{(1/2)})/(8*5^{(1/2)} + 24))*(5^{(1/2)} + 2)^{(1/2)}*1i)/10 + (5^{(1/2)}*\operatorname{atan}((x*(2 - 5^{(1/2)})^{(1/2)}*16i)/(8*5^{(1/2)} - 24))*(2 - 5^{(1/2)})^{(1/2)}*1i)/10 + (5^{(1/2)}*\operatorname{atan}((x*(- 5^{(1/2)} - 2)^{(1/2)}*16i)/(8*5^{(1/2)} + 24))*(- 5^{(1/2)} - 2)^{(1/2)}*1i)/10$

sympy [A] time = 1.22, size = 53, normalized size = 0.32

RootSum(6400t⁴ - 320t² - 1, (t ↦ t log(-1792000t⁷ + 4920t³ + x))) + RootSum(6400t⁴ + 320t² - 1, (t ↦ t log(-1792000t⁷ + 4920t³ + x)))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(x**8-3*x**4+1),x)

[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t**7 + 4920*_t**3 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t**7 + 4920*_t**3 + x)))

$$3.337 \quad \int \frac{x^4}{1-3x^4+x^8} dx$$

Optimal. Leaf size=173

$$\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}(3-\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}(3-\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}}$$

Rubi [A] time = 0.08, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1374, 212, 206, 203}

$$\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}(3-\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}(3-\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(1 - 3*x^4 + x^8), x]

[Out] -(((3 + Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((3 - Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - ((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + ((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol
] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{1-3x^4+x^8} dx &= \frac{1}{10} (5-3\sqrt{5}) \int \frac{1}{-\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx + \frac{1}{10} (5+3\sqrt{5}) \int \frac{1}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx \\ &= \frac{1}{2} \sqrt{\frac{1}{5}} (3-\sqrt{5}) \int \frac{1}{\sqrt{3-\sqrt{5}} - \sqrt{2}x^2} dx + \frac{1}{2} \sqrt{\frac{1}{5}} (3-\sqrt{5}) \int \frac{1}{\sqrt{3-\sqrt{5}} + \sqrt{2}x^2} dx - \frac{1}{2} \\ &= -\frac{\sqrt[4]{\frac{1}{2}} (3+\sqrt{5}) \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}} (3-\sqrt{5}) \tan^{-1}\left(\sqrt[4]{\frac{1}{2}} (3+\sqrt{5}) x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}} (3+\sqrt{5})}{2\sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.19, size = 132, normalized size = 0.76

$$\frac{\sqrt{\sqrt{5}-1} \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right) - \sqrt{1+\sqrt{5}} \tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right) + \sqrt{\sqrt{5}-1} \tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right) - \sqrt{1+\sqrt{5}} \tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 - 3*x^4 + x^8), x]

[Out] (Sqrt[-1 + Sqrt[5]]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]]*x - Sqrt[1 + Sqrt[5]]*ArcTan[Sqrt[2/(1 + Sqrt[5])]]*x + Sqrt[-1 + Sqrt[5]]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]]*x - Sqrt[1 + Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]]*x)/(2*Sqrt[10])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{1-3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(1 - 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^4/(1 - 3*x^4 + x^8), x]

fricas [B] time = 1.42, size = 271, normalized size = 1.57

$$\frac{1}{20}\sqrt{10}\sqrt{5+1}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10}\sqrt{5-1}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10}\sqrt{5+10}\log\left(x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)+\frac{1}{40}\sqrt{10}\sqrt{5-10}\log\left(x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] $-1/10*\sqrt{10}*\sqrt{\sqrt{5}+1}*\arctan(1/40*\sqrt{10}*\sqrt{2*x^2+\sqrt{5}+1}*(\sqrt{5}*\sqrt{2}-5*\sqrt{2}))*\sqrt{\sqrt{5}+1}-1/20*\sqrt{10}*(\sqrt{5}*x-5*x)*\sqrt{\sqrt{5}+1})-1/10*\sqrt{10}*\sqrt{\sqrt{5}-1}*\arctan(1/40*\sqrt{10}*\sqrt{2*x^2+\sqrt{5}-1}*(\sqrt{5}*\sqrt{2}+5*\sqrt{2}))*\sqrt{\sqrt{5}-1}-1/20*\sqrt{10}*(\sqrt{5}*x+5*x)*\sqrt{\sqrt{5}-1})-1/40*\sqrt{10}*\sqrt{\sqrt{5}+1}*\log(\sqrt{10}*\sqrt{5}*\sqrt{\sqrt{5}+1}+10*x)+1/40*\sqrt{10}*\sqrt{\sqrt{5}+1}*\log(-\sqrt{10}*\sqrt{5}*\sqrt{\sqrt{5}+1}+10*x)+1/40*\sqrt{10}*\sqrt{\sqrt{5}-1}*\log(\sqrt{10}*\sqrt{5}*\sqrt{\sqrt{5}-1}+10*x)-1/40*\sqrt{10}*\sqrt{\sqrt{5}-1}*\log(-\sqrt{10}*\sqrt{5}*\sqrt{\sqrt{5}-1}+10*x)$

giac [A] time = 0.61, size = 147, normalized size = 0.85

$$\frac{1}{20}\sqrt{10}\sqrt{5+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10}\sqrt{5-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10}\sqrt{5+10}\log\left(x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)+\frac{1}{40}\sqrt{10}\sqrt{5-10}\log\left(x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="giac")

[Out] $-1/20*\sqrt{10}*\sqrt{5+10}*\arctan(x/\sqrt{1/2*\sqrt{5}+1/2})+1/20*\sqrt{10}*(10*\sqrt{5}-10)*\arctan(x/\sqrt{1/2*\sqrt{5}-1/2})-1/40*\sqrt{10}*(10*\sqrt{5}+10)*\log(\text{abs}(x+\sqrt{1/2*\sqrt{5}+1/2}))+1/40*\sqrt{10}*(10*\sqrt{5}-10)*\log(\text{abs}(x-\sqrt{1/2*\sqrt{5}-1/2}))$

maple [A] time = 0.04, size = 206, normalized size = 1.19

$$\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{\sqrt{5}\operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^8-3*x^4+1),x)

[Out] $-1/10*5^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)})^{(1/2)}*x)-1/2/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)})^{(1/2)}*x)+1/2/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(-2+2*5^{(1/2)})^{(1/2)}*x)-1/10*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/$

$$\begin{aligned} & (-2+2*5^{(1/2)})^{(1/2)*x}+1/2/(-2+2*5^{(1/2)})^{(1/2)*x}*\operatorname{arctanh}(2/(-2+2*5^{(1/2)})^{(1/2)*x}) \\ & -1/10*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)*x}*\operatorname{arctanh}(2/(-2+2*5^{(1/2)})^{(1/2)*x}) \\ & -1/10*5^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)*x}*\operatorname{arctan}(2/(2+2*5^{(1/2)})^{(1/2)*x})-1/2/(2+2* \\ & 5^{(1/2)})^{(1/2)*x}*\operatorname{arctan}(2/(2+2*5^{(1/2)})^{(1/2)*x}) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8 - 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] integrate(x^4/(x^8 - 3*x^4 + 1), x)

mupad [B] time = 1.47, size = 269, normalized size = 1.55

$$\frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10}x\sqrt{\sqrt{5}-1}i - \sqrt{5}\sqrt{10}x\sqrt{\sqrt{5}-1}i}{2(\sqrt{5}-1)}\right) \sqrt{-\sqrt{5}-1}i}{20} + \frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10}x\sqrt{1-\sqrt{5}}i + \sqrt{5}\sqrt{10}x\sqrt{1-\sqrt{5}}i}{2(\sqrt{5}+1)}\right) \sqrt{1-\sqrt{5}}i}{20} - \frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10}x\sqrt{\sqrt{5}+1}i - \sqrt{5}\sqrt{10}x\sqrt{\sqrt{5}+1}i}{2(\sqrt{5}-1)}\right) \sqrt{\sqrt{5}+1}i}{20} - \frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10}x\sqrt{\sqrt{5}-1}i + \sqrt{5}\sqrt{10}x\sqrt{\sqrt{5}-1}i}{2(\sqrt{5}+1)}\right) \sqrt{\sqrt{5}-1}i}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^8 - 3*x^4 + 1),x)

[Out] $(10^{(1/2)}*\operatorname{atan}((10^{(1/2)}*x*(-5^{(1/2)}-1)^{(1/2)}*1i)/(2*(5^{(1/2)}-1))) - (5^{(1/2)}*10^{(1/2)}*x*(-5^{(1/2)}-1)^{(1/2)}*3i)/(10*(5^{(1/2)}-1)))*(-5^{(1/2)}-1)^{(1/2)}*1i)/20 + (10^{(1/2)}*\operatorname{atan}((10^{(1/2)}*x*(1-5^{(1/2)})^{(1/2)}*1i)/(2*(5^{(1/2)}+1))) + (5^{(1/2)}*10^{(1/2)}*x*(1-5^{(1/2)})^{(1/2)}*3i)/(10*(5^{(1/2)}+1)))*(1-5^{(1/2)})^{(1/2)}*1i)/20 - (10^{(1/2)}*\operatorname{atan}((10^{(1/2)}*x*(5^{(1/2)}+1)^{(1/2)}*1i)/(2*(5^{(1/2)}-1))) - (5^{(1/2)}*10^{(1/2)}*x*(5^{(1/2)}+1)^{(1/2)}*3i)/(10*(5^{(1/2)}-1)))*(5^{(1/2)}+1)^{(1/2)}*1i)/20 - (10^{(1/2)}*\operatorname{atan}((10^{(1/2)}*x*(5^{(1/2)}-1)^{(1/2)}*1i)/(2*(5^{(1/2)}+1))) + (5^{(1/2)}*10^{(1/2)}*x*(5^{(1/2)}-1)^{(1/2)}*3i)/(10*(5^{(1/2)}+1)))*(5^{(1/2)}-1)^{(1/2)}*1i)/20$

sympy [A] time = 1.20, size = 49, normalized size = 0.28

RootSum(6400t^4 - 80t^2 - 1, (t ↦ t log(-51200t^5 + 12t + x))) + RootSum(6400t^4 + 80t^2 - 1, (t ↦ t log(-51200t^5 + 12t + x)))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**8-3*x**4+1),x)

[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 + 12*_t + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 + 12*_t + x)))

$$3.338 \quad \int \frac{x^2}{1-3x^4+x^8} dx$$

Optimal. Leaf size=145

$$\frac{1}{20} \sqrt{10\sqrt{5}-10} \tan^{-1}\left(\frac{1}{2} \sqrt{2\sqrt{5}-2} x\right) - \frac{1}{20} \sqrt{10+10\sqrt{5}} \tan^{-1}\left(\frac{1}{2} \sqrt{2+2\sqrt{5}} x\right) - \frac{1}{20} \sqrt{10\sqrt{5}-10} \tanh^{-1}\left(\frac{1}{2} \sqrt{2}\right)$$

Rubi [A] time = 0.06, antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1375, 298, 203, 206}

$$\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2^{3/4} \sqrt{5} \sqrt[4]{3+\sqrt{5}}} - \frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} x\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2^{3/4} \sqrt{5} \sqrt[4]{3+\sqrt{5}}} + \frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 - 3*x^4 + x^8), x]

[Out] ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) - ((3 + Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x]/(2*Sqrt[5]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) + ((3 + Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]/(2*Sqrt[5])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.)), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{1-3x^4+x^8} dx &= \frac{\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{\sqrt{5}} - \frac{\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{\sqrt{5}} \\ &= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}} dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}} dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3-\sqrt{5}+\sqrt{2}x^2}} dx}{\sqrt{10}} + \frac{\int \frac{1}{\sqrt{3+\sqrt{5}+\sqrt{2}x^2}} dx}{\sqrt{10}} \\ &= \frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} - \frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} + \frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{2\sqrt{5}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 131, normalized size = 0.90

$$-\frac{\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10}(\sqrt{5}-1)} + \frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10}(1+\sqrt{5})} + \frac{\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10}(\sqrt{5}-1)} - \frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10}(1+\sqrt{5})}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 - 3*x^4 + x^8),x]

[Out] -(ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])]) + ArcTan[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1-3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1 - 3*x^4 + x^8),x]

[Out] IntegrateAlgebraic[x^2/(1 - 3*x^4 + x^8), x]

fricas [B] time = 1.08, size = 255, normalized size = 1.76

$\frac{1}{10} \sqrt{10} \sqrt{\sqrt{5} + 1} \arctan\left(\frac{x}{\sqrt{2} \sqrt{\sqrt{5} + 1}}\right) - \frac{1}{10} \sqrt{10} \sqrt{\sqrt{5} - 1} \arctan\left(\frac{x}{\sqrt{2} \sqrt{\sqrt{5} - 1}}\right) - \frac{1}{40} \sqrt{10} \sqrt{\sqrt{5} - 1} \log\left(\frac{x + \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}{x - \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{40} \sqrt{10} \sqrt{\sqrt{5} - 1} \log\left(\frac{x + \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}{x - \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}\right) - \frac{1}{40} \sqrt{10} \sqrt{\sqrt{5} + 1} \log\left(\frac{x + \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}{x - \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{40} \sqrt{10} \sqrt{\sqrt{5} + 1} \log\left(\frac{x + \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}{x - \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{10} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} + 1} \arctan\left(\frac{1}{20} \sqrt{10} \sqrt{\sqrt{5} + 1} \sqrt{2} \sqrt{2x^2 + \sqrt{5} - 1}\right) - \frac{1}{10} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} - 1} \arctan\left(\frac{1}{20} \sqrt{10} \sqrt{\sqrt{5} - 1} \sqrt{2} \sqrt{2x^2 + \sqrt{5} + 1}\right) - \frac{1}{40} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} - 1} \log\left(\frac{\sqrt{10} \sqrt{\sqrt{5} + 1} \sqrt{2x^2 + \sqrt{5} - 1}}{\sqrt{10} \sqrt{\sqrt{5} + 1} \sqrt{2x^2 + \sqrt{5} + 1}}\right) + \frac{1}{40} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} + 1} \log\left(\frac{\sqrt{10} \sqrt{\sqrt{5} - 1} \sqrt{2x^2 + \sqrt{5} - 1}}{\sqrt{10} \sqrt{\sqrt{5} - 1} \sqrt{2x^2 + \sqrt{5} + 1}}\right) - \frac{1}{40} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} - 1} \log\left(\frac{\sqrt{10} \sqrt{\sqrt{5} + 1} \sqrt{2x^2 + \sqrt{5} - 1}}{\sqrt{10} \sqrt{\sqrt{5} + 1} \sqrt{2x^2 + \sqrt{5} + 1}}\right) + \frac{1}{40} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} + 1} \log\left(\frac{\sqrt{10} \sqrt{\sqrt{5} - 1} \sqrt{2x^2 + \sqrt{5} - 1}}{\sqrt{10} \sqrt{\sqrt{5} - 1} \sqrt{2x^2 + \sqrt{5} + 1}}\right)$

giac [A] time = 0.62, size = 147, normalized size = 1.01

$\frac{1}{20} \sqrt{10} \sqrt{\sqrt{5} - 10} \arctan\left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}\right) - \frac{1}{20} \sqrt{10} \sqrt{\sqrt{5} + 10} \arctan\left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}\right) - \frac{1}{40} \sqrt{10} \sqrt{\sqrt{5} - 10} \log\left(\frac{x + \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}{x - \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{40} \sqrt{10} \sqrt{\sqrt{5} - 10} \log\left(\frac{x - \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}{x + \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{40} \sqrt{10} \sqrt{\sqrt{5} + 10} \log\left(\frac{x + \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}{x - \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}\right) - \frac{1}{40} \sqrt{10} \sqrt{\sqrt{5} + 10} \log\left(\frac{x - \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}{x + \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="giac")

[Out] $\frac{1}{20} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} - 10} \arctan\left(\frac{x}{\sqrt{1/2 \sqrt{5} + 1/2}}\right) - \frac{1}{20} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} + 10} \arctan\left(\frac{x}{\sqrt{1/2 \sqrt{5} - 1/2}}\right) - \frac{1}{40} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} - 10} \log\left(\frac{\text{abs}(x + \sqrt{1/2 \sqrt{5} + 1/2})}{\text{abs}(x - \sqrt{1/2 \sqrt{5} + 1/2})}\right) + \frac{1}{40} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} - 10} \log\left(\frac{\text{abs}(x - \sqrt{1/2 \sqrt{5} + 1/2})}{\text{abs}(x + \sqrt{1/2 \sqrt{5} + 1/2})}\right) + \frac{1}{40} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} + 10} \log\left(\frac{\text{abs}(x + \sqrt{1/2 \sqrt{5} - 1/2})}{\text{abs}(x - \sqrt{1/2 \sqrt{5} - 1/2})}\right) - \frac{1}{40} \sqrt{10} \sqrt{10} \sqrt{\sqrt{5} + 10} \log\left(\frac{\text{abs}(x - \sqrt{1/2 \sqrt{5} - 1/2})}{\text{abs}(x + \sqrt{1/2 \sqrt{5} - 1/2})}\right)$

maple [A] time = 0.03, size = 110, normalized size = 0.76

$\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}} - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}} - \frac{\sqrt{5} \operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}} + \frac{\sqrt{5} \operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^8-3*x^4+1),x)

[Out] $-\frac{1}{5} 5^{1/2} / (2+2 \cdot 5^{1/2})^{1/2} \operatorname{arctanh}\left(\frac{2}{(2+2 \cdot 5^{1/2})^{1/2}} x\right) - \frac{1}{5} 5^{1/2} / (-2+2 \cdot 5^{1/2})^{1/2} \operatorname{arctan}\left(\frac{2}{(-2+2 \cdot 5^{1/2})^{1/2}} x\right) + \frac{1}{5} 5^{1/2} / (-2+2 \cdot 5^{1/2})^{1/2} \operatorname{arctanh}\left(\frac{2}{(-2+2 \cdot 5^{1/2})^{1/2}} x\right) - \frac{1}{5} 5^{1/2} / (2+2 \cdot 5^{1/2})^{1/2} \operatorname{arctan}\left(\frac{2}{(2+2 \cdot 5^{1/2})^{1/2}} x\right)$

$5^{(1/2)} \arctanh(2/(-2+2*5^{(1/2)})^{(1/2)} * x) + 1/5 * 5^{(1/2)} / (2+2*5^{(1/2)})^{(1/2)} * \arctan(2/(2+2*5^{(1/2)})^{(1/2)} * x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8 - 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] integrate(x^2/(x^8 - 3*x^4 + 1), x)

mupad [B] time = 0.08, size = 269, normalized size = 1.86

$$\frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10} \sqrt{\sqrt{5}-1} 3i}{2(3\sqrt{5}-7)} - \frac{\sqrt{5} \sqrt{10} \sqrt{\sqrt{5}-1} 7i}{10(3\sqrt{5}-7)}\right) \sqrt{\sqrt{5}-1} 1i}{20} - \frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10} \sqrt{\sqrt{5}+1} 3i}{2(3\sqrt{5}+7)} + \frac{\sqrt{5} \sqrt{10} \sqrt{\sqrt{5}+1} 7i}{10(3\sqrt{5}+7)}\right) \sqrt{\sqrt{5}+1} 1i}{20} + \frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10} \sqrt{1-\sqrt{5}} 3i}{2(3\sqrt{5}-7)} - \frac{\sqrt{5} \sqrt{10} \sqrt{1-\sqrt{5}} 7i}{10(3\sqrt{5}-7)}\right) \sqrt{1-\sqrt{5}} 1i}{20} - \frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10} \sqrt{-\sqrt{5}-1} 3i}{2(3\sqrt{5}+7)} + \frac{\sqrt{5} \sqrt{10} \sqrt{-\sqrt{5}-1} 7i}{10(3\sqrt{5}+7)}\right) \sqrt{-\sqrt{5}-1} 1i}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^8 - 3*x^4 + 1),x)

[Out] $(10^{(1/2)} * \operatorname{atan}((10^{(1/2)} * x * (5^{(1/2)} - 1)^{(1/2)} * 3i) / (2 * (3 * 5^{(1/2)} - 7))) - (5^{(1/2)} * 10^{(1/2)} * x * (5^{(1/2)} - 1)^{(1/2)} * 7i) / (10 * (3 * 5^{(1/2)} - 7))) * (5^{(1/2)} - 1)^{(1/2)} * 1i) / 20 - (10^{(1/2)} * \operatorname{atan}((10^{(1/2)} * x * (5^{(1/2)} + 1)^{(1/2)} * 3i) / (2 * (3 * 5^{(1/2)} + 7))) + (5^{(1/2)} * 10^{(1/2)} * x * (5^{(1/2)} + 1)^{(1/2)} * 7i) / (10 * (3 * 5^{(1/2)} + 7))) * (5^{(1/2)} + 1)^{(1/2)} * 1i) / 20 + (10^{(1/2)} * \operatorname{atan}((10^{(1/2)} * x * (1 - 5^{(1/2)})^{(1/2)} * 3i) / (2 * (3 * 5^{(1/2)} - 7))) - (5^{(1/2)} * 10^{(1/2)} * x * (1 - 5^{(1/2)})^{(1/2)} * 7i) / (10 * (3 * 5^{(1/2)} - 7))) * (1 - 5^{(1/2)})^{(1/2)} * 1i) / 20 - (10^{(1/2)} * \operatorname{atan}((10^{(1/2)} * x * (-5^{(1/2)} - 1)^{(1/2)} * 3i) / (2 * (3 * 5^{(1/2)} + 7))) + (5^{(1/2)} * 10^{(1/2)} * x * (-5^{(1/2)} - 1)^{(1/2)} * 7i) / (10 * (3 * 5^{(1/2)} + 7))) * (-5^{(1/2)} - 1)^{(1/2)} * 1i) / 20$

sympy [A] time = 1.19, size = 53, normalized size = 0.37

$\operatorname{RootSum}(6400t^4 - 80t^2 - 1, (t \mapsto t \log(6144000t^7 - 2240t^3 + x))) + \operatorname{RootSum}(6400t^4 + 80t^2 - 1, (t \mapsto t \log(6144000t^7 - 2240t^3 + x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**8-3*x**4+1),x)

[Out] $\operatorname{RootSum}(6400 * t^{**4} - 80 * t^{**2} - 1, \operatorname{Lambda}(t, t * \log(6144000 * t^{**7} - 2240 * t^{**3} + x))) + \operatorname{RootSum}(6400 * t^{**4} + 80 * t^{**2} - 1, \operatorname{Lambda}(t, t * \log(6144000 * t^{**7} - 2240 * t^{**3} + x)))$

$$3.339 \quad \int \frac{1}{1-3x^4+x^8} dx$$

Optimal. Leaf size=169

$$\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}(3+\sqrt{5})^{3/4}} + \frac{(3+\sqrt{5})^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{2\cdot 2^{3/4}\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}(3+\sqrt{5})^{3/4}} + \frac{(3+\sqrt{5})^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{2\cdot 2^{3/4}\sqrt{5}}$$

Rubi [A] time = 0.06, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1347, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}(3+\sqrt{5})^{3/4}} + \frac{(3+\sqrt{5})^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{2\cdot 2^{3/4}\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}(3+\sqrt{5})^{3/4}} + \frac{(3+\sqrt{5})^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{2\cdot 2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x^4 + x^8)^(-1), x]

[Out] -(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4))) + ((3 + Sqrt[5])^(3/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4)) + ((3 + Sqrt[5])^(3/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1347

Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(n_+1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{1-3x^4+x^8} dx &= \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx}{\sqrt{5}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx}{\sqrt{5}} \\ &= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}} dx}{\sqrt{5(3-\sqrt{5})}} + \frac{\int \frac{1}{\sqrt{3-\sqrt{5}+\sqrt{2}x^2}} dx}{\sqrt{5(3-\sqrt{5})}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}} dx}{\sqrt{5(3+\sqrt{5})}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}+\sqrt{2}x^2}} dx}{\sqrt{5(3+\sqrt{5})}} \\ &= -\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}(3+\sqrt{5})^{3/4}} + \frac{(3+\sqrt{5})^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{2^{2/4}\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}(3+\sqrt{5})^{3/4}} + \frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt[4]{2}\sqrt{5}(3+\sqrt{5})^{3/4}} \end{aligned}$$

Mathematica [A] time = 0.16, size = 160, normalized size = 0.95

$$\frac{(1+\sqrt{5})\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}} - \frac{(\sqrt{5}-1)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}} + \frac{(1+\sqrt{5})\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{\sqrt{5}-1}} - \frac{(\sqrt{5}-1)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{1+\sqrt{5}}}$$

$$2\sqrt{10}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x^4 + x^8)^(-1), x]

[Out] (((1 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5]])*x])/Sqrt[-1 + Sqrt[5]] - ((-1 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5]])*x])/Sqrt[1 + Sqrt[5]] + ((1 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5]])*x])/Sqrt[-1 + Sqrt[5]] - ((-1 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5]])*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-3x^4+x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 3*x^4 + x^8)^(-1),x]

[Out] IntegrateAlgebraic[(1 - 3*x^4 + x^8)^(-1), x]

fricas [B] time = 1.49, size = 251, normalized size = 1.49

$$\frac{1}{5}\sqrt{5+2}\arctan\left(\frac{1}{2}\sqrt{2x^2+1}(\sqrt{5}\sqrt{2}-\sqrt{5})\sqrt{\sqrt{5}+2}-\frac{1}{2}(\sqrt{5}x-1)\sqrt{\sqrt{5}+2}\right)+\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}-2}\arctan\left(\frac{1}{2}\sqrt{2x^2+1}(\sqrt{5}\sqrt{2}+\sqrt{5})\sqrt{\sqrt{5}-2}-\frac{1}{2}(\sqrt{5}x+1)\sqrt{\sqrt{5}-2}\right)-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\frac{(5+3)\sqrt{\sqrt{5}-2}+2}{(5+3)\sqrt{\sqrt{5}-2}+2}\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\frac{(5+3)\sqrt{\sqrt{5}-2}-2}{(5+3)\sqrt{\sqrt{5}-2}-2}\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\frac{\sqrt{5}+2(\sqrt{5}-3)+2}{\sqrt{5}+2(\sqrt{5}-3)+2}\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\frac{-\sqrt{5}+2(\sqrt{5}-3)+2}{-\sqrt{5}+2(\sqrt{5}-3)+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] $-\frac{1}{5}\sqrt{5}\sqrt{5}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}-1}\sqrt{5}\sqrt{2}\right)+\frac{1}{5}\sqrt{5}\sqrt{5}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2x^2+\sqrt{5}+1}\sqrt{5}\sqrt{2}\right)+\frac{1}{20}\sqrt{5}\sqrt{5}\sqrt{2}\log\left(\frac{(5+3)\sqrt{5}\sqrt{2}+2}{(5+3)\sqrt{5}\sqrt{2}+2}\right)+\frac{1}{20}\sqrt{5}\sqrt{5}\sqrt{2}\log\left(\frac{(5+3)\sqrt{5}\sqrt{2}-2}{(5+3)\sqrt{5}\sqrt{2}-2}\right)+\frac{1}{20}\sqrt{5}\sqrt{5}\sqrt{2}\log\left(\frac{\sqrt{5}+2(\sqrt{5}-3)+2}{\sqrt{5}+2(\sqrt{5}-3)+2}\right)+\frac{1}{20}\sqrt{5}\sqrt{5}\sqrt{2}\log\left(\frac{-\sqrt{5}+2(\sqrt{5}-3)+2}{-\sqrt{5}+2(\sqrt{5}-3)+2}\right)$

giac [A] time = 0.48, size = 147, normalized size = 0.87

$$\frac{1}{10}\sqrt{5}\sqrt{5-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{10}\sqrt{5}\sqrt{5+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5}\sqrt{5-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5}\sqrt{5-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5}\sqrt{5+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{5}\sqrt{5+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-3*x^4+1),x, algorithm="giac")

[Out] $-\frac{1}{10}\sqrt{5}\sqrt{5}\sqrt{5-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{10}\sqrt{5}\sqrt{5}\sqrt{5+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5}\sqrt{5}\sqrt{5-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5}\sqrt{5}\sqrt{5-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5}\sqrt{5}\sqrt{5+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{5}\sqrt{5}\sqrt{5+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$

maple [A] time = 0.03, size = 206, normalized size = 1.22

$$\frac{\sqrt{5}\arctanh\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\arctanh\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\arctanh\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\arctanh\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}+\frac{\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8-3*x^4+1),x)

[Out] $-\frac{1}{2}\sqrt{2+2\sqrt{5}}\arctanh\left(\frac{2}{(2+2\sqrt{5})^{\frac{1}{2}}}\sqrt{2+2\sqrt{5}}\right)+\frac{1}{10}\sqrt{2+2\sqrt{5}}\sqrt{2+2\sqrt{5}}\arctanh\left(\frac{2}{(2+2\sqrt{5})^{\frac{1}{2}}}\sqrt{2+2\sqrt{5}}\right)+\frac{1}{10}\sqrt{2+2\sqrt{5}}\sqrt{-2+2\sqrt{5}}\arctanh\left(\frac{2}{(-2+2\sqrt{5})^{\frac{1}{2}}}\sqrt{-2+2\sqrt{5}}\right)$

$$\begin{aligned} &)^{(1/2)} * \arctan(2/(-2+2*5^{(1/2)})^{(1/2)} * x) + 1/2/(-2+2*5^{(1/2)})^{(1/2)} * \arctan(2/ \\ &(-2+2*5^{(1/2)})^{(1/2)} * x) + 1/10*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)} * \operatorname{arctanh}(2/(-2+2*5 \\ &^{(1/2)})^{(1/2)} * x) + 1/2/(-2+2*5^{(1/2)})^{(1/2)} * \operatorname{arctanh}(2/(-2+2*5^{(1/2)})^{(1/2)} * x) \\ &- 1/2/(2+2*5^{(1/2)})^{(1/2)} * \arctan(2/(2+2*5^{(1/2)})^{(1/2)} * x) + 1/10*5^{(1/2)}/(2+2* \\ &5^{(1/2)})^{(1/2)} * \arctan(2/(2+2*5^{(1/2)})^{(1/2)} * x) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 - 3x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] integrate(1/(x^8 - 3*x^4 + 1), x)

mupad [B] time = 0.08, size = 245, normalized size = 1.45

$$\frac{\sqrt{5} \operatorname{atan}\left(\frac{x\sqrt{2-\sqrt{5}}-144i}{104\sqrt{5}-232} - \frac{\sqrt{5}x\sqrt{2-\sqrt{5}}-64i}{104\sqrt{5}-232}\right)\sqrt{2-\sqrt{5}}-1i}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{x\sqrt{-5-2\sqrt{5}}-144i}{104\sqrt{5}+232} + \frac{\sqrt{5}x\sqrt{-5-2\sqrt{5}}-64i}{104\sqrt{5}+232}\right)\sqrt{-5-2\sqrt{5}}-2i}{10} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{x\sqrt{5-2\sqrt{5}}-144i}{104\sqrt{5}-232} - \frac{\sqrt{5}x\sqrt{5-2\sqrt{5}}-64i}{104\sqrt{5}-232}\right)\sqrt{5-2\sqrt{5}}-2i}{10} - \frac{\sqrt{5} \operatorname{atan}\left(\frac{x\sqrt{5+2\sqrt{5}}-144i}{104\sqrt{5}+232} + \frac{\sqrt{5}x\sqrt{5+2\sqrt{5}}-64i}{104\sqrt{5}+232}\right)\sqrt{5+2\sqrt{5}}-2i}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^8 - 3*x^4 + 1),x)

[Out] $(5^{(1/2)} * \operatorname{atan}((x * (-5^{(1/2)} - 2)^{(1/2)} * 144i) / (104 * 5^{(1/2)} + 232) + (5^{(1/2)} * x * (-5^{(1/2)} - 2)^{(1/2)} * 64i) / (104 * 5^{(1/2)} + 232)) * (-5^{(1/2)} - 2)^{(1/2)} * 1i) / 10 - (5^{(1/2)} * \operatorname{atan}((x * (2 - 5^{(1/2)})^{(1/2)} * 144i) / (104 * 5^{(1/2)} - 232) - (5^{(1/2)} * x * (2 - 5^{(1/2)})^{(1/2)} * 64i) / (104 * 5^{(1/2)} - 232)) * (2 - 5^{(1/2)})^{(1/2)} * 1i) / 10 + (5^{(1/2)} * \operatorname{atan}((x * (5^{(1/2)} - 2)^{(1/2)} * 144i) / (104 * 5^{(1/2)} - 232) - (5^{(1/2)} * x * (5^{(1/2)} - 2)^{(1/2)} * 64i) / (104 * 5^{(1/2)} - 232)) * (5^{(1/2)} - 2)^{(1/2)} * 1i) / 10 - (5^{(1/2)} * \operatorname{atan}((x * (5^{(1/2)} + 2)^{(1/2)} * 144i) / (104 * 5^{(1/2)} + 232) + (5^{(1/2)} * x * (5^{(1/2)} + 2)^{(1/2)} * 64i) / (104 * 5^{(1/2)} + 232)) * (5^{(1/2)} + 2)^{(1/2)} * 1i) / 10$

sympy [A] time = 1.21, size = 53, normalized size = 0.31

$$\operatorname{RootSum}\left(6400t^4 - 320t^2 - 1, \left(t \mapsto t \log\left(9600t^5 - \frac{47t}{2} + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 320t^2 - 1, \left(t \mapsto t \log\left(9600t^5 - \frac{47t}{2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**8-3*x**4+1),x)

[Out] $\operatorname{RootSum}(6400*_t**4 - 320*_t**2 - 1, \operatorname{Lambda}(_t, _t * \log(9600*_t**5 - 47*_t/2 + x))) + \operatorname{RootSum}(6400*_t**4 + 320*_t**2 - 1, \operatorname{Lambda}(_t, _t * \log(9600*_t**5 - 47*_t/2 + x)))$

$$3.340 \quad \int \frac{1}{x^2(1-3x^4+x^8)} dx$$

Optimal. Leaf size=172

$$-\frac{1}{x} + \frac{\sqrt[4]{984-440\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{4\sqrt{5}} - \frac{(3+\sqrt{5})^{5/4} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} x\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{984-440\sqrt{5}} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{4\sqrt{5}}$$

Rubi [A] time = 0.09, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1368, 1510, 298, 203, 206}

$$-\frac{1}{x} + \frac{\sqrt[4]{984-440\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{4\sqrt{5}} - \frac{(3+\sqrt{5})^{5/4} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} x\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{984-440\sqrt{5}} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{4\sqrt{5}} + \frac{(3+\sqrt{5})^{5/4} \tanh^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} x\right)}{4\sqrt[4]{2}\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 - 3*x^4 + x^8)), x]

[Out] -x^(-1) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(4*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(4*Sqrt[5]) + ((3 + Sqrt[5])^(5/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/4)*Sqrt[5])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1368

Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*x^n + c*x^(2*n))^(p+1))/(a*d*(m+1)), x] - Dist[1/(a*d^n*(m+1)), Int[(d*x)^(m+n)*(b*(m+n*(p+1)+1) + c*(m+2*n*(p+1)+1)*x^n*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1510

Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2(1-3x^4+x^8)} dx &= -\frac{1}{x} + \int \frac{x^2(3-x^4)}{1-3x^4+x^8} dx \\
 &= -\frac{1}{x} + \frac{1}{10}(-5+3\sqrt{5}) \int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx - \frac{1}{10}(5+3\sqrt{5}) \int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx \\
 &= -\frac{1}{x} - \frac{(3-\sqrt{5}) \int \frac{1}{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}} dx}{2\sqrt{10}} + \frac{(3-\sqrt{5}) \int \frac{1}{\sqrt{3+\sqrt{5}+\sqrt{2}x^2}} dx}{2\sqrt{10}} + \frac{(3+\sqrt{5}) \int \frac{1}{\sqrt{3-\sqrt{5}-\sqrt{2}x^2}} dx}{2\sqrt{10}} \\
 &= -\frac{1}{x} + \frac{\sqrt[4]{984-440\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{4\sqrt{5}} - \frac{(3+\sqrt{5})^{5/4} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} x\right)}{4\sqrt[4]{2}\sqrt{5}} - \frac{\sqrt[4]{984+440\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{2}{3-\sqrt{5}}} x\right)}{4\sqrt{5}}
 \end{aligned}$$

Mathematica [A] time = 0.27, size = 174, normalized size = 1.01

$$-\frac{1}{x} - \frac{(3+\sqrt{5}) \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right)}{2\sqrt{10}(\sqrt{5}-1)} - \frac{(\sqrt{5}-3) \tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)}{2\sqrt{10}(1+\sqrt{5})} + \frac{(3+\sqrt{5}) \tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right)}{2\sqrt{10}(\sqrt{5}-1)} + \frac{(\sqrt{5}-3) \tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)}{2\sqrt{10}(1+\sqrt{5})}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(1 - 3*x^4 + x^8)),x]

```
[Out] -x^(-1) - ((3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5]])*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((-3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5]])*x])/(2*Sqrt[10*(1 + Sqrt[5])]) + ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5]])*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) + ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5]])*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1 - 3x^4 + x^8)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(1 - 3*x^4 + x^8)), x]
```

fricas [B] time = 1.31, size = 313, normalized size = 1.82

$\frac{\sqrt{10}\sqrt{5}\sqrt{11}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) + \sqrt{10}\sqrt{5}\sqrt{11}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \sqrt{10}\sqrt{5}\sqrt{11}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \sqrt{10}\sqrt{5}\sqrt{11}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+1}\sqrt{5}+5\right) + \sqrt{10}\sqrt{5}\sqrt{11}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-1}\sqrt{5}+5\right) - \sqrt{10}\sqrt{5}\sqrt{11}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+1}\sqrt{5}-5\right) + \sqrt{10}\sqrt{5}\sqrt{11}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-1}\sqrt{5}-5\right) + \sqrt{10}\sqrt{5}\sqrt{11}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+1}\sqrt{5}+5\right) - \sqrt{10}\sqrt{5}\sqrt{11}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-1}\sqrt{5}+5\right) - \sqrt{10}\sqrt{5}\sqrt{11}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+1}\sqrt{5}-5\right) + \sqrt{10}\sqrt{5}\sqrt{11}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-1}\sqrt{5}-5\right) - 40}{x}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(4*sqrt(10)*x*sqrt(5*sqrt(5) + 11)*arctan(1/40*(sqrt(10)*sqrt(2*x^2 + sqrt(5) - 1)*(3*sqrt(5)*sqrt(2) - 5*sqrt(2)) - 2*sqrt(10)*(3*sqrt(5)*x - 5*x))*sqrt(5*sqrt(5) + 11)) - 4*sqrt(10)*x*sqrt(5*sqrt(5) - 11)*arctan(1/40*(sqrt(10)*sqrt(2*x^2 + sqrt(5) + 1)*(3*sqrt(5)*sqrt(2) + 5*sqrt(2)) - 2*sqrt(10)*(3*sqrt(5)*x + 5*x))*sqrt(5*sqrt(5) - 11)) - sqrt(10)*x*sqrt(5*sqrt(5) - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5) + 10*x) + sqrt(10)*x*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5) + 10*x) - sqrt(10)*x*sqrt(5*sqrt(5) + 11)*log(sqrt(10)*sqrt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x) + sqrt(10)*x*sqrt(5*sqrt(5) + 11)*log(-sqrt(10)*sqrt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x) - 40)/x
```

giac [A] time = 0.54, size = 152, normalized size = 0.88

$\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*
```


$\log(\text{abs}(x - \sqrt{1/2*\sqrt{5} + 1/2})) + 1/40*\sqrt{50*\sqrt{5} + 110}*\log(\text{abs}(x + \sqrt{1/2*\sqrt{5} - 1/2})) - 1/40*\sqrt{50*\sqrt{5} + 110}*\log(\text{abs}(x - \sqrt{1/2*\sqrt{5} - 1/2})) - 1/x$

maple [A] time = 0.03, size = 211, normalized size = 1.23

$$\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}} + \frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}} + \frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}} - \frac{3\sqrt{5} \operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}} - \frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}} - \frac{3\sqrt{5} \operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}} - \frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}} + \frac{3\sqrt{5} \operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/x^2/(x^8-3*x^4+1), x)$

[Out] $1/2/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)})^{(1/2)}*x)-3/10*5^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)})^{(1/2)}*x)-1/2/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(-2+2*5^{(1/2)})^{(1/2)}*x)-3/10*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(-2+2*5^{(1/2)})^{(1/2)}*x)-1/x+1/2/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(-2+2*5^{(1/2)})^{(1/2)}*x)+3/10*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(-2+2*5^{(1/2)})^{(1/2)}*x)-1/2/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(2+2*5^{(1/2)})^{(1/2)}*x)+3/10*5^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(2+2*5^{(1/2)})^{(1/2)}*x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x} - \frac{1}{2} \int \frac{x^2 + 2}{x^4 + x^2 - 1} dx - \frac{1}{2} \int \frac{x^2 - 2}{x^4 - x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/x^2/(x^8-3*x^4+1), x, \text{algorithm}="maxima")$

[Out] $-1/x - 1/2*\text{integrate}((x^2 + 2)/(x^4 + x^2 - 1), x) - 1/2*\text{integrate}((x^2 - 2)/(x^4 - x^2 - 1), x)$

mupad [B] time = 1.34, size = 250, normalized size = 1.45

$$\frac{1}{x} - \frac{\operatorname{atan}\left(\frac{\pm\sqrt{-50\sqrt{5}-110}1155i + \sqrt{5}\sqrt{-50\sqrt{5}-110}517i}{2(3025\sqrt{5}+6765)}\right)\sqrt{-50\sqrt{5}-110}1i}{20} + \frac{\operatorname{atan}\left(\frac{\pm\sqrt{110-50\sqrt{5}}1155i - \sqrt{5}\sqrt{110-50\sqrt{5}}517i}{2(3025\sqrt{5}-6765)}\right)\sqrt{110-50\sqrt{5}}1i}{20} + \frac{\operatorname{atan}\left(\frac{\pm\sqrt{50\sqrt{5}-110}1155i - \sqrt{5}\sqrt{50\sqrt{5}-110}517i}{2(3025\sqrt{5}-6765)}\right)\sqrt{50\sqrt{5}-110}1i}{20} - \frac{\operatorname{atan}\left(\frac{\pm\sqrt{50\sqrt{5}+110}1155i + \sqrt{5}\sqrt{50\sqrt{5}+110}517i}{2(3025\sqrt{5}+6765)}\right)\sqrt{50\sqrt{5}+110}1i}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^2*(x^8 - 3*x^4 + 1)), x)$

[Out] $(\operatorname{atan}((x*(110 - 50*5^{(1/2)})^{(1/2)}*1155i)/(2*(3025*5^{(1/2)} - 6765))) - (5^{(1/2)}*x*(110 - 50*5^{(1/2)})^{(1/2)}*517i)/(2*(3025*5^{(1/2)} - 6765)))*(110 - 50*5^{(1/2)})^{(1/2)}*1i)/20 - (\operatorname{atan}((x*(-50*5^{(1/2)} - 110)^{(1/2)}*1155i)/(2*(3025*5^{(1/2)} + 6765))) + (5^{(1/2)}*x*(-50*5^{(1/2)} - 110)^{(1/2)}*517i)/(2*(3025*5^{(1/2)} + 6765)))*(-50*5^{(1/2)} - 110)^{(1/2)}*1i)/20 + (\operatorname{atan}((x*(50*5^{(1/2)} - 110)^{(1/2)}*1155i)/(2*(3025*5^{(1/2)} - 6765))) - (5^{(1/2)}*x*(50*5^{(1/2)} - 110)^{(1/2)}*517i)/(2*(3025*5^{(1/2)} - 6765)))*(-50*5^{(1/2)} - 110)^{(1/2)}*1i)/20 - (\operatorname{atan}((x*(50*5^{(1/2)} - 110)^{(1/2)}*1155i)/(2*(3025*5^{(1/2)} - 6765))) - (5^{(1/2)}*x*(50*5^{(1/2)} - 110)^{(1/2)}*517i)/(2*(3025*5^{(1/2)} - 6765)))*(50*5^{(1/2)} - 110)^{(1/2)}*1i)/20$

$$\frac{1}{2} * 517i) / (2 * (3025 * 5^{(1/2)} - 6765))) * (50 * 5^{(1/2)} - 110)^{(1/2)} * 1i) / 20 - (\text{atan}((x * (50 * 5^{(1/2)} + 110)^{(1/2)} * 1155i) / (2 * (3025 * 5^{(1/2)} + 6765))) + (5^{(1/2)} * x * (50 * 5^{(1/2)} + 110)^{(1/2)} * 517i) / (2 * (3025 * 5^{(1/2)} + 6765))) * (50 * 5^{(1/2)} + 110)^{(1/2)} * 1i) / 20 - 1/x$$

sympy [A] time = 1.25, size = 63, normalized size = 0.37

$$\text{RootSum}\left(6400t^4 - 880t^2 - 1, \left(t \mapsto t \log\left(\frac{19251200t^7}{11} - \frac{369792t^3}{11} + x\right)\right)\right) + \text{RootSum}\left(6400t^4 + 880t^2 - 1, \left(t \mapsto t \log\left(\frac{19251200t^7}{11} - \frac{369792t^3}{11} + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(x**8-3*x**4+1),x)

[Out] RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t**7/11 - 369792*_t**3/11 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t**7/11 - 369792*_t**3/11 + x))) - 1/x

$$3.341 \quad \int \frac{1}{x^4(1-3x^4+x^8)} dx$$

Optimal. Leaf size=182

$$\frac{1}{3x^3} - \frac{\sqrt[4]{\frac{1}{2}(843-377\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{(3+\sqrt{5})^{7/4} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}(843-377\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}}$$

Rubi [A] time = 0.12, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1368, 1422, 212, 206, 203}

$$\frac{1}{3x^3} - \frac{\sqrt[4]{\frac{1}{2}(843-377\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{(3+\sqrt{5})^{7/4} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{4 \cdot 2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}(843-377\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{(3+\sqrt{5})^{7/4} \tanh^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x\right)}{4 \cdot 2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 - 3*x^4 + x^8)),x]

[Out] $-1/(3*x^3) - (((843 - 377*\text{Sqrt}[5])/2)^{(1/4)}*\text{ArcTan}[(2/(3 + \text{Sqrt}[5]))^{(1/4)}*x])/(2*\text{Sqrt}[5]) + ((3 + \text{Sqrt}[5])^{(7/4)}*\text{ArcTan}[(3 + \text{Sqrt}[5])/2]^{(1/4)}*x)/(4*2^{(3/4)}*\text{Sqrt}[5]) - (((843 - 377*\text{Sqrt}[5])/2)^{(1/4)}*\text{ArcTanh}[(2/(3 + \text{Sqrt}[5]))^{(1/4)}*x])/(2*\text{Sqrt}[5]) + ((3 + \text{Sqrt}[5])^{(7/4)}*\text{ArcTanh}[(3 + \text{Sqrt}[5])/2]^{(1/4)}*x)/(4*2^{(3/4)}*\text{Sqrt}[5])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1368

```
Int[((d_.)*(x_)^(m_))*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
c*(m + 2*n*(p + 1) + 1)*x^n*(a + b*x^n + c*x^(2*n))^(p), x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(1-3x^4+x^8)} dx &= -\frac{1}{3x^3} + \frac{1}{3} \int \frac{9-3x^4}{1-3x^4+x^8} dx \\
&= -\frac{1}{3x^3} + \frac{1}{10}(-5+3\sqrt{5}) \int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx - \frac{1}{10}(5+3\sqrt{5}) \int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx \\
&= -\frac{1}{3x^3} + \frac{(5-3\sqrt{5}) \int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2} dx}{10\sqrt{3+\sqrt{5}}} + \frac{(5-3\sqrt{5}) \int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2} dx}{10\sqrt{3+\sqrt{5}}} + \frac{(3+\sqrt{5})^{3/2}}{4} \\
&= -\frac{1}{3x^3} - \frac{\sqrt[4]{\frac{1}{2}}(843-377\sqrt{5}) \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}}(843+377\sqrt{5}) \tan^{-1}\left(\sqrt[4]{\frac{1}{2}}(3+\sqrt{5})x\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 166, normalized size = 0.91

$$-\frac{1}{3x^3} + \frac{(2+\sqrt{5}) \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10(\sqrt{5}-1)}} - \frac{(\sqrt{5}-2) \tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}} + \frac{(2+\sqrt{5}) \tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{\sqrt{10(\sqrt{5}-1)}} - \frac{(\sqrt{5}-2) \tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{\sqrt{10(1+\sqrt{5})}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(1 - 3*x^4 + x^8)),x]

[Out] $-\frac{1}{3} \frac{1}{x^3} + \frac{((2 + \sqrt{5}) \operatorname{ArcTan}[\sqrt{2/(-1 + \sqrt{5})}] * x)) / \sqrt{10(-1 + \sqrt{5})} - ((-2 + \sqrt{5}) \operatorname{ArcTan}[\sqrt{2/(1 + \sqrt{5})}] * x)) / \sqrt{10(1 + \sqrt{5})} + ((2 + \sqrt{5}) \operatorname{ArcTanh}[\sqrt{2/(-1 + \sqrt{5})}] * x)) / \sqrt{10(-1 + \sqrt{5})} - ((-2 + \sqrt{5}) \operatorname{ArcTanh}[\sqrt{2/(1 + \sqrt{5})}] * x)) / \sqrt{10(1 + \sqrt{5})}}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(1 - 3x^4 + x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(1 - 3*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^4*(1 - 3*x^4 + x^8)), x]

fricas [B] time = 1.40, size = 327, normalized size = 1.80

$\frac{1}{120} \sqrt{10} \sqrt{13 \sqrt{5} + 29} \arctan\left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{20} \sqrt{10} \sqrt{13 \sqrt{5} + 29} \arctan\left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}\right) - \frac{1}{40} \sqrt{130 \sqrt{5} - 290} \log\left(x + \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}\right) + \frac{1}{40} \sqrt{130 \sqrt{5} - 290} \log\left(x - \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}\right) + \frac{1}{40} \sqrt{130 \sqrt{5} + 290} \log\left(x + \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}\right) - \frac{1}{40} \sqrt{130 \sqrt{5} + 290} \log\left(x - \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}\right) - \frac{1}{3x^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{120} (12 \sqrt{10} x^3 \sqrt{13 \sqrt{5} + 29} \arctan(1/20 (\sqrt{10} \sqrt{2x^2 + \sqrt{5}} - 1) (2 \sqrt{5} \sqrt{2} - 5 \sqrt{2})) - 2 \sqrt{10} (2 \sqrt{5} x - 5x) \sqrt{13 \sqrt{5} + 29}) + 12 \sqrt{10} x^3 \sqrt{13 \sqrt{5} - 29} \arctan(1/20 (\sqrt{10} \sqrt{2x^2 + \sqrt{5}} + 1) (2 \sqrt{5} \sqrt{2} + 5 \sqrt{2})) - 2 \sqrt{10} (2 \sqrt{5} x + 5x) \sqrt{13 \sqrt{5} - 29}) - 3 \sqrt{10} x^3 \sqrt{13 \sqrt{5} - 29} \log(\sqrt{10} \sqrt{13 \sqrt{5} - 29}) (7 \sqrt{5} + 15) + 20x) + 3 \sqrt{10} x^3 \sqrt{13 \sqrt{5} - 29} \log(-\sqrt{10} \sqrt{13 \sqrt{5} - 29}) (7 \sqrt{5} + 15) + 20x) + 3 \sqrt{10} x^3 \sqrt{13 \sqrt{5} + 29} \log(\sqrt{10} \sqrt{13 \sqrt{5} + 29}) (7 \sqrt{5} - 15) + 20x) - 3 \sqrt{10} x^3 \sqrt{13 \sqrt{5} + 29} \log(-\sqrt{10} \sqrt{13 \sqrt{5} + 29}) (7 \sqrt{5} - 15) + 20x) - 40/x^3$

giac [A] time = 0.67, size = 152, normalized size = 0.84

$-\frac{1}{20} \sqrt{130 \sqrt{5} - 290} \arctan\left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}}\right) + \frac{1}{20} \sqrt{130 \sqrt{5} + 290} \arctan\left(\frac{x}{\sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}}\right) - \frac{1}{40} \sqrt{130 \sqrt{5} - 290} \log\left(x + \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}\right) + \frac{1}{40} \sqrt{130 \sqrt{5} - 290} \log\left(x - \sqrt{\frac{1}{2} \sqrt{5} + \frac{1}{2}}\right) + \frac{1}{40} \sqrt{130 \sqrt{5} + 290} \log\left(x + \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}\right) - \frac{1}{40} \sqrt{130 \sqrt{5} + 290} \log\left(x - \sqrt{\frac{1}{2} \sqrt{5} - \frac{1}{2}}\right) - \frac{1}{3x^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="giac")

[Out] $-1/20*\sqrt{130*\sqrt{5} - 290}*\arctan(x/\sqrt{1/2*\sqrt{5} + 1/2}) + 1/20*\sqrt{130*\sqrt{5} + 290}*\arctan(x/\sqrt{1/2*\sqrt{5} - 1/2}) - 1/40*\sqrt{130*\sqrt{5} - 290}*\log(\text{abs}(x + \sqrt{1/2*\sqrt{5} + 1/2})) + 1/40*\sqrt{130*\sqrt{5} - 290}*\log(\text{abs}(x - \sqrt{1/2*\sqrt{5} + 1/2})) + 1/40*\sqrt{130*\sqrt{5} + 290}*\log(\text{abs}(x + \sqrt{1/2*\sqrt{5} - 1/2})) - 1/40*\sqrt{130*\sqrt{5} + 290}*\log(\text{abs}(x - \sqrt{1/2*\sqrt{5} - 1/2})) - 1/3/x^3$

maple [A] time = 0.03, size = 209, normalized size = 1.15

$$\frac{2\sqrt{5} \operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}} + \frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}} + \frac{2\sqrt{5} \operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}} - \frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}} + \frac{2\sqrt{5} \operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{5\sqrt{-2+2\sqrt{5}}} + \frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}} + \frac{2\sqrt{5} \operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{5\sqrt{2+2\sqrt{5}}} - \frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(x^8-3*x^4+1),x)`

[Out] $2/5*5^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)})^{(1/2)}*x) - 1/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)})^{(1/2)}*x) + 2/5*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(-2+2*5^{(1/2)})^{(1/2)}*x) + 1/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(-2+2*5^{(1/2)})^{(1/2)}*x) + 2/5*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(-2+2*5^{(1/2)})^{(1/2)}*x) + 1/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(-2+2*5^{(1/2)})^{(1/2)}*x) + 2/5*5^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(2+2*5^{(1/2)})^{(1/2)}*x) - 1/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(2+2*5^{(1/2)})^{(1/2)}*x) - 1/3/x^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3x^3} - \frac{1}{2} \int \frac{2x^2 + 3}{x^4 + x^2 - 1} dx + \frac{1}{2} \int \frac{2x^2 - 3}{x^4 - x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="maxima")`

[Out] $-1/3/x^3 - 1/2*\operatorname{integrate}((2*x^2 + 3)/(x^4 + x^2 - 1), x) + 1/2*\operatorname{integrate}((2*x^2 - 3)/(x^4 - x^2 - 1), x)$

mupad [B] time = 0.20, size = 268, normalized size = 1.47

$$\frac{\operatorname{atan}\left(\frac{\sqrt{-130\sqrt{5}-290} \cdot 20735i + \sqrt{5} \cdot \sqrt{-130\sqrt{5}-290} \cdot 46371i}{2(87841\sqrt{5}+196417)}\right) \sqrt{-130\sqrt{5}-290} \cdot 11}{20} + \frac{\operatorname{atan}\left(\frac{\sqrt{-290-130\sqrt{5}} \cdot 20735i - \sqrt{5} \cdot \sqrt{-290-130\sqrt{5}} \cdot 46371i}{2(87841\sqrt{5}-196417)}\right) \sqrt{-130\sqrt{5}-290} \cdot 11}{20} - \frac{1}{3x^3} - \frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10} \cdot \sqrt{13\sqrt{5}-29} \cdot 20735i - \sqrt{5} \cdot \sqrt{10} \cdot \sqrt{13\sqrt{5}-29} \cdot 46371i}{2(87841\sqrt{5}-196417)}\right) \sqrt{13\sqrt{5}-29} \cdot 11}{20} - \frac{\sqrt{10} \operatorname{atan}\left(\frac{\sqrt{10} \cdot \sqrt{13\sqrt{5}+29} \cdot 20735i + \sqrt{5} \cdot \sqrt{10} \cdot \sqrt{13\sqrt{5}+29} \cdot 46371i}{2(87841\sqrt{5}+196417)}\right) \sqrt{13\sqrt{5}+29} \cdot 11}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(x^8 - 3*x^4 + 1)),x)`

[Out] $(\operatorname{atan}((x*(-130*5^{(1/2)} - 290)^{(1/2)}*20735i)/(2*(87841*5^{(1/2)} + 196417))) + (5^{(1/2)}*x*(-130*5^{(1/2)} - 290)^{(1/2)}*46371i)/(10*(87841*5^{(1/2)} + 196417))) * (-130*5^{(1/2)} - 290)^{(1/2)}*1i)/20 + (\operatorname{atan}((x*(290 - 130*5^{(1/2)})^{(1/2)})$

$$\begin{aligned} & *20735i)/(2*(87841*5^{(1/2)} - 196417)) - (5^{(1/2)}*x*(290 - 130*5^{(1/2)})^{(1/2)} \\ &)*46371i)/(10*(87841*5^{(1/2)} - 196417)))*(290 - 130*5^{(1/2)})^{(1/2)}*1i)/20 - \\ & 1/(3*x^3) - (10^{(1/2)}*atan((10^{(1/2)}*x*(13*5^{(1/2)} - 29)^{(1/2)}*20735i)/(2* \\ & (87841*5^{(1/2)} - 196417)) - (5^{(1/2)}*10^{(1/2)}*x*(13*5^{(1/2)} - 29)^{(1/2)}*463 \\ & 71i)/(10*(87841*5^{(1/2)} - 196417)))*(13*5^{(1/2)} - 29)^{(1/2)}*1i)/20 - (10^{(1 \\ & /2)}*atan((10^{(1/2)}*x*(13*5^{(1/2)} + 29)^{(1/2)}*20735i)/(2*(87841*5^{(1/2)} + 19 \\ & 6417)) + (5^{(1/2)}*10^{(1/2)}*x*(13*5^{(1/2)} + 29)^{(1/2)}*46371i)/(10*(87841*5^{(\\ & 1/2)} + 196417)))*(13*5^{(1/2)} + 29)^{(1/2)}*1i)/20 \end{aligned}$$

sympy [A] time = 1.28, size = 63, normalized size = 0.35

$$\text{RootSum}\left(6400t^4 - 2320t^2 - 1, \left(t \mapsto t \log\left(\frac{179200t^5}{377} - \frac{23112t}{377} + x\right)\right)\right) + \text{RootSum}\left(6400t^4 + 2320t^2 - 1, \left(t \mapsto t \log\left(\frac{179200t^5}{377} - \frac{23112t}{377} + x\right)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(x**8-3*x**4+1),x)

[Out] RootSum(6400*_t**4 - 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t**5/377 - 23112*_t/377 + x))) + RootSum(6400*_t**4 + 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t**5/377 - 23112*_t/377 + x))) - 1/(3*x**3)

$$3.342 \quad \int \frac{1}{x^6(1-3x^4+x^8)} dx$$

Optimal. Leaf size=173

$$-\frac{1}{5x^5} - \frac{3}{x} + \frac{\sqrt[4]{2889-1292\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889+1292\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889-1292\sqrt{5}}}{2\sqrt{5}}$$

Rubi [A] time = 0.14, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1368, 1504, 1510, 298, 203, 206}

$$-\frac{1}{5x^5} - \frac{3}{x} + \frac{\sqrt[4]{2889-1292\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889+1292\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{2889-1292\sqrt{5}} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{2889+1292\sqrt{5}} \tanh^{-1}\left(\sqrt[4]{\frac{1}{2}(3+\sqrt{5})} x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(1 - 3*x^4 + x^8)),x]

[Out] -1/(5*x^5) - 3/x + ((2889 - 1292*sqrt[5])^(1/4)*ArcTan[(2/(3 + sqrt[5]))^(1/4)*x])/(2*sqrt[5]) - ((2889 + 1292*sqrt[5])^(1/4)*ArcTan[((3 + sqrt[5])/2)^(1/4)*x])/(2*sqrt[5]) - ((2889 - 1292*sqrt[5])^(1/4)*ArcTanh[(2/(3 + sqrt[5]))^(1/4)*x])/(2*sqrt[5]) + ((2889 + 1292*sqrt[5])^(1/4)*ArcTanh[((3 + sqrt[5])/2)^(1/4)*x])/(2*sqrt[5])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1368

Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) + c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1504

Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)]^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m + n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rule 1510

Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6(1-3x^4+x^8)} dx &= -\frac{1}{5x^5} + \frac{1}{5} \int \frac{15-5x^4}{x^2(1-3x^4+x^8)} dx \\
&= -\frac{1}{5x^5} - \frac{3}{x} - \frac{1}{5} \int \frac{x^2(-40+15x^4)}{1-3x^4+x^8} dx \\
&= -\frac{1}{5x^5} - \frac{3}{x} - \frac{1}{10} (15-7\sqrt{5}) \int \frac{x^2}{-\frac{3}{2} - \frac{\sqrt{5}}{2} + x^4} dx - \frac{1}{10} (15+7\sqrt{5}) \int \frac{x^2}{-\frac{3}{2} + \frac{\sqrt{5}}{2} + x^4} dx \\
&= -\frac{1}{5x^5} - \frac{3}{x} - \frac{(7-3\sqrt{5}) \int \frac{1}{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}} dx}{2\sqrt{10}} + \frac{(7-3\sqrt{5}) \int \frac{1}{\sqrt{3+\sqrt{5}+\sqrt{2}x^2}} dx}{2\sqrt{10}} + \frac{(7+3\sqrt{5}) \int \frac{1}{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}} dx}{2\sqrt{10}} \\
&= -\frac{1}{5x^5} - \frac{3}{x} + \frac{\sqrt[4]{46224-20672\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}} x\right)}{4\sqrt{5}} - \frac{\sqrt[4]{46224+20672\sqrt{5}} \tan^{-1}\left(\sqrt[4]{\frac{2}{1+\sqrt{5}}} x\right)}{4\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 189, normalized size = 1.09

$$-\frac{1}{5x^5} - \frac{3}{x} + \frac{(-7-3\sqrt{5}) \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right)}{2\sqrt{10}(\sqrt{5}-1)} + \frac{(7-3\sqrt{5}) \tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)}{2\sqrt{10}(1+\sqrt{5})} - \frac{(-7-3\sqrt{5}) \tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right)}{2\sqrt{10}(\sqrt{5}-1)} - \frac{(7-3\sqrt{5}) \tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right)}{2\sqrt{10}(1+\sqrt{5})}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(1 - 3*x^4 + x^8)),x]

[Out]
$$-\frac{1}{5} \frac{1}{x^5} - \frac{3}{x} + \frac{((-7 - 3\sqrt{5}) \text{ArcTan}[\sqrt{2/(-1 + \sqrt{5})}] x))}{(2\sqrt{10}(-1 + \sqrt{5}))} + \frac{((7 - 3\sqrt{5}) \text{ArcTan}[\sqrt{2/(1 + \sqrt{5})}] x))}{(2\sqrt{10}(1 + \sqrt{5}))} - \frac{((-7 - 3\sqrt{5}) \text{ArcTanh}[\sqrt{2/(-1 + \sqrt{5})}] x))}{(2\sqrt{10}(-1 + \sqrt{5}))} - \frac{((7 - 3\sqrt{5}) \text{ArcTanh}[\sqrt{2/(1 + \sqrt{5})}] x))}{(2\sqrt{10}(1 + \sqrt{5}))}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6(1-3x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^6*(1 - 3*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^6*(1 - 3*x^4 + x^8)), x]

fricas [B] time = 1.19, size = 300, normalized size = 1.73

$$\frac{4\sqrt{5}\sqrt{17\sqrt{5}-38}\arctan\left(\frac{x}{\sqrt{2x^2+\sqrt{5}}}\right)-4\sqrt{5}\sqrt{17\sqrt{5}-38}\arctan\left(\frac{x}{\sqrt{2x^2+\sqrt{5}}}\right)-4\sqrt{5}\sqrt{17\sqrt{5}-38}\arctan\left(\frac{x}{\sqrt{2x^2+\sqrt{5}}}\right)-4\sqrt{5}\sqrt{17\sqrt{5}-38}\arctan\left(\frac{x}{\sqrt{2x^2+\sqrt{5}}}\right)+\sqrt{5}\sqrt{17\sqrt{5}-38}\log\left(\sqrt{17\sqrt{5}-38}(5\sqrt{5}-11)+2x\right)-\sqrt{5}\sqrt{17\sqrt{5}-38}\log\left(-\sqrt{17\sqrt{5}-38}(5\sqrt{5}-11)+2x\right)+\sqrt{5}\sqrt{17\sqrt{5}-38}\log\left(\sqrt{17\sqrt{5}-38}(5\sqrt{5}-11)+2x\right)+\sqrt{5}\sqrt{17\sqrt{5}-38}\log\left(-\sqrt{17\sqrt{5}-38}(5\sqrt{5}-11)+2x\right)+60x^4+4}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] $-1/20*(4*\sqrt{5}*x^5*\sqrt{17*\sqrt{5}+38}*\arctan(1/4*(\sqrt{2*x^2+\sqrt{5}}-1)*(3*\sqrt{5}*\sqrt{2}-7*\sqrt{2}))-6*\sqrt{5}*x+14*x)*\sqrt{17*\sqrt{5}+38})+4*\sqrt{5}*x^5*\sqrt{17*\sqrt{5}-38}*\arctan(1/4*(\sqrt{2*x^2+\sqrt{5}}+1)*(3*\sqrt{5}*\sqrt{2}+7*\sqrt{2}))-6*\sqrt{5}*x-14*x)*\sqrt{17*\sqrt{5}-38})+\sqrt{5}*x^5*\sqrt{17*\sqrt{5}-38}*\log(\sqrt{17*\sqrt{5}-38}*(5*\sqrt{5}+11)+2*x)-\sqrt{5}*x^5*\sqrt{17*\sqrt{5}-38}*\log(-\sqrt{17*\sqrt{5}-38}*(5*\sqrt{5}+11)+2*x)-\sqrt{5}*x^5*\sqrt{17*\sqrt{5}+38}*\log(\sqrt{17*\sqrt{5}+38}*(5*\sqrt{5}-11)+2*x)+\sqrt{5}*x^5*\sqrt{17*\sqrt{5}+38}*\log(-\sqrt{17*\sqrt{5}+38}*(5*\sqrt{5}-11)+2*x)+60*x^4+4)/x^5$

giac [A] time = 0.54, size = 159, normalized size = 0.92

$$\frac{1}{10}\sqrt{85\sqrt{5}-190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{85\sqrt{5}+190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)+\frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)+\frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)-\frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)-\frac{15x^4+1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="giac")

[Out] $1/10*\sqrt{85*\sqrt{5}-190}*\arctan(x/\sqrt{1/2*\sqrt{5}+1/2})-1/10*\sqrt{85*\sqrt{5}+190}*\arctan(x/\sqrt{1/2*\sqrt{5}-1/2})-1/20*\sqrt{85*\sqrt{5}-190}*\log(\text{abs}(x+\sqrt{1/2*\sqrt{5}+1/2}))+1/20*\sqrt{85*\sqrt{5}-190}*\log(\text{abs}(x-\sqrt{1/2*\sqrt{5}+1/2}))+1/20*\sqrt{85*\sqrt{5}+190}*\log(\text{abs}(x+\sqrt{1/2*\sqrt{5}-1/2}))-1/20*\sqrt{85*\sqrt{5}+190}*\log(\text{abs}(x-\sqrt{1/2*\sqrt{5}-1/2}))-1/5*(15*x^4+1)/x^5$

maple [A] time = 0.04, size = 216, normalized size = 1.25

$$\frac{7\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}+\frac{3\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}-\frac{7\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}+\frac{3\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{7\sqrt{5}\operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{10\sqrt{-2+2\sqrt{5}}}-\frac{3\operatorname{arctan}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{2\sqrt{-2+2\sqrt{5}}}+\frac{7\sqrt{5}\operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{10\sqrt{2+2\sqrt{5}}}-\frac{3\operatorname{arctan}\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{2\sqrt{2+2\sqrt{5}}}-\frac{3}{x}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(x^8-3*x^4+1),x)

[Out] $-1/5/x^5-3/x-7/10*5^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)}))^{(1/2)}*x)+3/2/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)}))^{(1/2)}*x)-7/10*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(-2+2*5^{(1/2)}))^{(1/2)}*x)-3/2/(-2+2*5^{(1/2)})^{(1/2)}*2*\operatorname{arctan}(2/(-2+2*5^{(1/2)}))^{(1/2)}*x)+7/10*5^{(1/2)}/(-2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctan}(2/(-2+2*5^{(1/2)}))^{(1/2)}*x$

$\text{nh}(2/(-2+2*5^{(1/2)})^{(1/2)}*x)+3/2/(-2+2*5^{(1/2)})^{(1/2)}*\text{arctanh}(2/(-2+2*5^{(1/2)})^{(1/2)}*x)+7/10*5^{(1/2)}/(2+2*5^{(1/2)})^{(1/2)}*\text{arctan}(2/(2+2*5^{(1/2)})^{(1/2)}*x)-3/2/(2+2*5^{(1/2)})^{(1/2)}*\text{arctan}(2/(2+2*5^{(1/2)})^{(1/2)}*x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{15x^4+1}{5x^5} - \frac{1}{2} \int \frac{3x^2+5}{x^4+x^2-1} dx - \frac{1}{2} \int \frac{3x^2-5}{x^4-x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="maxima")

[Out] -1/5*(15*x^4 + 1)/x^5 - 1/2*integrate((3*x^2 + 5)/(x^4 + x^2 - 1), x) - 1/2*integrate((3*x^2 - 5)/(x^4 - x^2 - 1), x)

mupad [B] time = 1.49, size = 257, normalized size = 1.49

$$\frac{3x^4 + \frac{1}{x^5} + \frac{\text{atan}\left(\frac{x\sqrt{85\sqrt{5}-190}-372096i}{2550408\sqrt{5}-5702888} + \frac{\sqrt{5}x\sqrt{85\sqrt{5}-190}-832048i}{5(2550408\sqrt{5}-5702888)}\right)\sqrt{-85\sqrt{5}-190} + \text{atan}\left(\frac{x\sqrt{190-85\sqrt{5}}-372096i}{2550408\sqrt{5}-5702888} - \frac{\sqrt{5}x\sqrt{190-85\sqrt{5}}-832048i}{5(2550408\sqrt{5}-5702888)}\right)\sqrt{190-85\sqrt{5}} + \text{atan}\left(\frac{x\sqrt{85\sqrt{5}-190}-372096i}{2550408\sqrt{5}-5702888} - \frac{\sqrt{5}x\sqrt{85\sqrt{5}-190}-832048i}{5(2550408\sqrt{5}-5702888)}\right)\sqrt{85\sqrt{5}-190} + \text{atan}\left(\frac{x\sqrt{85\sqrt{5}+190}-372096i}{2550408\sqrt{5}+5702888} + \frac{\sqrt{5}x\sqrt{85\sqrt{5}+190}-832048i}{5(2550408\sqrt{5}+5702888)}\right)\sqrt{85\sqrt{5}+190}}{10}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(x^8 - 3*x^4 + 1)),x)

[Out] (atan((x*(190 - 85*5^(1/2))^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (5^(1/2)*x*(190 - 85*5^(1/2))^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888))) * (190 - 85*5^(1/2))^(1/2)*1i)/10 - (atan((x*(-85*5^(1/2) - 190)^(1/2)*372096i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(-85*5^(1/2) - 190)^(1/2)*832048i)/(5*(2550408*5^(1/2) + 5702888))) * (-85*5^(1/2) - 190)^(1/2)*1i)/10 + (atan((x*(85*5^(1/2) - 190)^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (5^(1/2)*x*(85*5^(1/2) - 190)^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888))) * (85*5^(1/2) - 190)^(1/2)*1i)/10 - (atan((x*(85*5^(1/2) + 190)^(1/2)*372096i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(85*5^(1/2) + 190)^(1/2)*832048i)/(5*(2550408*5^(1/2) + 5702888))) * (85*5^(1/2) + 190)^(1/2)*1i)/10 - (3*x^4 + 1/5)/x^5

sympy [A] time = 1.29, size = 73, normalized size = 0.42

$$\text{RootSum}\left(6400t^4 - 6080t^2 - 1, \left(t \mapsto t \log\left(\frac{215808000t^7}{323} - \frac{194833880t^3}{323} + x\right)\right)\right) + \text{RootSum}\left(6400t^4 + 6080t^2 - 1, \left(t \mapsto t \log\left(\frac{215808000t^7}{323} - \frac{194833880t^3}{323} + x\right)\right)\right) + \frac{-15x^4 - 1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**6/(x**8-3*x**4+1),x)

[Out] RootSum(6400*_t**4 - 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323 - 194833880*_t**3/323 + x))) + RootSum(6400*_t**4 + 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323 - 194833880*_t**3/323 + x))) + (-15*x**4 - 1)/(5*x**5)

$$3.343 \quad \int \frac{1}{x^8(1-3x^4+x^8)} dx$$

Optimal. Leaf size=189

$$\frac{1}{7x^7} - \frac{1}{x^3} - \frac{\sqrt[4]{\frac{1}{2}(39603 - 17711\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}(39603 + 17711\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3 + \sqrt{5})}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}}}{\sqrt{5}}$$

Rubi [A] time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1368, 1504, 1422, 212, 206, 203}

$$\frac{1}{x^3} - \frac{1}{7x^7} - \frac{\sqrt[4]{\frac{1}{2}(39603 - 17711\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}(39603 + 17711\sqrt{5})} \tan^{-1}\left(\sqrt[4]{\frac{1}{2}(3 + \sqrt{5})}x\right)}{2\sqrt{5}} - \frac{\sqrt[4]{\frac{1}{2}(39603 - 17711\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}(39603 + 17711\sqrt{5})} \tanh^{-1}\left(\sqrt[4]{\frac{1}{2}(3 + \sqrt{5})}x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^8*(1 - 3*x^4 + x^8)),x]

[Out] -1/(7*x^7) - x^(-3) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1368

```

Int[((d_)*(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_), x_
_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*d*(m + 1
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]

```

Rule 1422

```

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])

```

Rule 1504

```

Int[((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (
c_)*(x_)^(n2_))^(p_), x_Symbol] := Simp[((d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
&& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8(1-3x^4+x^8)} dx &= -\frac{1}{7x^7} + \frac{1}{7} \int \frac{21-7x^4}{x^4(1-3x^4+x^8)} dx \\
&= -\frac{1}{7x^7} - \frac{1}{x^3} - \frac{1}{21} \int \frac{-168+63x^4}{1-3x^4+x^8} dx \\
&= -\frac{1}{7x^7} - \frac{1}{x^3} - \frac{1}{10} (15-7\sqrt{5}) \int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4} dx - \frac{1}{10} (15+7\sqrt{5}) \int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4} dx \\
&= -\frac{1}{7x^7} - \frac{1}{x^3} - \frac{(-15+7\sqrt{5}) \int \frac{1}{\sqrt{3+\sqrt{5}-\sqrt{2}x^2}} dx}{10\sqrt{3+\sqrt{5}}} - \frac{(-15+7\sqrt{5}) \int \frac{1}{\sqrt{3+\sqrt{5}+\sqrt{2}x^2}} dx}{10\sqrt{3+\sqrt{5}}} + \frac{1}{2} \\
&= -\frac{1}{7x^7} - \frac{1}{x^3} - \frac{\sqrt{\frac{1}{2}}(39603-17711\sqrt{5}) \tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}} x\right)}{2\sqrt{5}} + \frac{\sqrt{\frac{1}{2}}(39603+17711\sqrt{5})}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 189, normalized size = 1.00

$$-\frac{1}{7x^7} - \frac{1}{x^3} + \frac{(11+5\sqrt{5})\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10}(\sqrt{5}-1)} + \frac{(11-5\sqrt{5})\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10}(1+\sqrt{5})} - \frac{(-11-5\sqrt{5})\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)}{2\sqrt{10}(\sqrt{5}-1)} - \frac{(5\sqrt{5}-11)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)}{2\sqrt{10}(1+\sqrt{5})}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^8*(1 - 3*x^4 + x^8)),x]

[Out] $-\frac{1}{7} \frac{1}{x^7} - x^{-3} + \frac{((11 + 5\sqrt{5}) \text{ArcTan}[\sqrt{2/(-1 + \sqrt{5})}] * x)) / (2\sqrt{10}(-1 + \sqrt{5})) + ((11 - 5\sqrt{5}) \text{ArcTan}[\sqrt{2/(1 + \sqrt{5})}] * x)) / (2\sqrt{10}(1 + \sqrt{5})) - ((-11 - 5\sqrt{5}) \text{ArcTanh}[\sqrt{2/(-1 + \sqrt{5})}] * x)) / (2\sqrt{10}(-1 + \sqrt{5})) - ((-11 + 5\sqrt{5}) \text{ArcTanh}[\sqrt{2/(1 + \sqrt{5})}] * x)) / (2\sqrt{10}(1 + \sqrt{5}))}{1}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8(1-3x^4+x^8)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^8*(1 - 3*x^4 + x^8)),x]

[Out] IntegrateAlgebraic[1/(x^8*(1 - 3*x^4 + x^8)), x]

fricas [B] time = 1.21, size = 332, normalized size = 1.76

$\frac{2x\sqrt{5}\sqrt{890\sqrt{5}-1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-2\sqrt{890\sqrt{5}-1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)+\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)+\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)+\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)-\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)-\frac{7x^4+1}{7x^7}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{280}*(28*\sqrt{10}*x^7*\sqrt{89*\sqrt{5}+199}*\arctan(1/40*(\sqrt{10}*\sqrt{2*x^2+\sqrt{5}}-1)*(11*\sqrt{5}*\sqrt{2}-25*\sqrt{2}))-2*\sqrt{10}*(11*\sqrt{5}*x-25*x))*\sqrt{89*\sqrt{5}+199}+28*\sqrt{10}*x^7*\sqrt{89*\sqrt{5}-199}*\arctan(1/40*(\sqrt{10}*\sqrt{2*x^2+\sqrt{5}}+1)*(11*\sqrt{5}*\sqrt{2}+25*\sqrt{2}))-2*\sqrt{10}*(11*\sqrt{5}*x+25*x))*\sqrt{89*\sqrt{5}-199}-7*\sqrt{10}*x^7*\sqrt{89*\sqrt{5}-199}*\log(\sqrt{10}*\sqrt{89*\sqrt{5}-199}*(9*\sqrt{5}+20)+10*x)+7*\sqrt{10}*x^7*\sqrt{89*\sqrt{5}-199}*\log(-\sqrt{10}*\sqrt{89*\sqrt{5}-199}*(9*\sqrt{5}+20)+10*x)+7*\sqrt{10}*x^7*\sqrt{89*\sqrt{5}+199}*\log(\sqrt{10}*\sqrt{89*\sqrt{5}+199}*(9*\sqrt{5}-20)+10*x)-7*\sqrt{10}*x^7*\sqrt{89*\sqrt{5}+199}*\log(-\sqrt{10}*\sqrt{89*\sqrt{5}+199}*(9*\sqrt{5}-20)+10*x)-280*x^4-40)/x^7$

giac [A] time = 0.56, size = 159, normalized size = 0.84

$\frac{1}{20}\sqrt{890\sqrt{5}-1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{890\sqrt{5}+1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)+\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)+\frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)-\frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)-\frac{7x^4+1}{7x^7}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="giac")

[Out] $-1/20*\sqrt{890*\sqrt{5}-1990}*\arctan(x/\sqrt{1/2*\sqrt{5}+1/2})+1/20*\sqrt{890*\sqrt{5}+1990}*\arctan(x/\sqrt{1/2*\sqrt{5}-1/2})-1/40*\sqrt{890*\sqrt{5}-1990}*\log(\text{abs}(x+\sqrt{1/2*\sqrt{5}+1/2}))+1/40*\sqrt{890*\sqrt{5}-1990}*\log(\text{abs}(x-\sqrt{1/2*\sqrt{5}+1/2}))+1/40*\sqrt{890*\sqrt{5}+1990}*\log(\text{abs}(x+\sqrt{1/2*\sqrt{5}-1/2}))-1/40*\sqrt{890*\sqrt{5}+1990}*\log(\text{abs}(x-\sqrt{1/2*\sqrt{5}-1/2}))-1/7*(7*x^4+1)/x^7$

maple [A] time = 0.04, size = 216, normalized size = 1.14

$\frac{11\sqrt{5}\arctanh\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)+5\arctanh\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)+11\sqrt{5}\arctanh\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)-5\arctanh\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)+11\sqrt{5}\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)+5\arctan\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)+11\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)-5\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{1}{x^3}-\frac{1}{7x^7}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^8/(x^8-3*x^4+1),x)

[Out] $-1/7/x^7-1/x^3+11/10*5^{(1/2)/(2+2*5^{(1/2)})}^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)})^{(1/2)}*x)-5/2/(2+2*5^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2/(2+2*5^{(1/2)})^{(1/2)}*x)+11/10*5^{(1/2)/(-2+2*5^{(1/2)})}^{(1/2)}*\operatorname{arctan}(2/(-2+2*5^{(1/2)})^{(1/2)}*x)+5/2/(-2+2*5^{(1/2)})^{(1/2)}$


```
[Out] RootSum(6400*_t**4 - 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711 - 2842588*_t/17711 + x))) + RootSum(6400*_t**4 + 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711 - 2842588*_t/17711 + x))) + (-7*x**4 - 1)/(7*x*7)
```

$$3.344 \quad \int \frac{x^3}{2+3x^4+x^8} dx$$

Optimal. Leaf size=21

$$\frac{1}{4} \log(x^4 + 1) - \frac{1}{4} \log(x^4 + 2)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1352, 616, 31}

$$\frac{1}{4} \log(x^4 + 1) - \frac{1}{4} \log(x^4 + 2)$$

Antiderivative was successfully verified.

[In] Int[x^3/(2 + 3*x^4 + x^8), x]

[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{2 + 3x^4 + x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{2 + 3x + x^2} dx, x, x^4 \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{1 + x} dx, x, x^4 \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{2 + x} dx, x, x^4 \right) \\
&= \frac{1}{4} \log(1 + x^4) - \frac{1}{4} \log(2 + x^4)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{4} \log(x^4 + 1) - \frac{1}{4} \log(x^4 + 2)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(2 + 3*x^4 + x^8), x]

[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{2 + 3x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(2 + 3*x^4 + x^8), x]

[Out] IntegrateAlgebraic[x^3/(2 + 3*x^4 + x^8), x]

fricas [A] time = 1.28, size = 17, normalized size = 0.81

$$-\frac{1}{4} \log(x^4 + 2) + \frac{1}{4} \log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+3*x^4+2), x, algorithm="fricas")

[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)

giac [A] time = 0.28, size = 17, normalized size = 0.81

$$-\frac{1}{4} \log(x^4 + 2) + \frac{1}{4} \log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="giac")

[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln(x^4 + 1)}{4} - \frac{\ln(x^4 + 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^8+3*x^4+2),x)

[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)

maxima [A] time = 0.59, size = 17, normalized size = 0.81

$$-\frac{1}{4} \log(x^4 + 2) + \frac{1}{4} \log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="maxima")

[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)

mupad [B] time = 0.06, size = 16, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{256}{9(144x^4+160)} - \frac{7}{9}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(3*x^4 + x^8 + 2),x)

[Out] -atanh(256/(9*(144*x^4 + 160)) - 7/9)/2

sympy [A] time = 0.12, size = 15, normalized size = 0.71

$$\frac{\log(x^4 + 1)}{4} - \frac{\log(x^4 + 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**8+3*x**4+2),x)

[Out] log(x**4 + 1)/4 - log(x**4 + 2)/4

$$3.345 \quad \int \frac{x^{11}}{2+3x^4+x^8} dx$$

Optimal. Leaf size=26

$$\frac{x^4}{4} + \frac{1}{4} \log(x^4 + 1) - \log(x^4 + 2)$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 703, 632, 31}

$$\frac{x^4}{4} + \frac{1}{4} \log(x^4 + 1) - \log(x^4 + 2)$$

Antiderivative was successfully verified.

[In] Int[x^11/(2 + 3*x^4 + x^8), x]

[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -

4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{11}}{2 + 3x^4 + x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x^2}{2 + 3x + x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{4} \text{Subst} \left(\int \frac{-2 - 3x}{2 + 3x + x^2} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1 + x} dx, x, x^4 \right) - \text{Subst} \left(\int \frac{1}{2 + x} dx, x, x^4 \right) \\
 &= \frac{x^4}{4} + \frac{1}{4} \log(1 + x^4) - \log(2 + x^4)
 \end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x^4}{4} + \frac{1}{4} \log(x^4 + 1) - \log(x^4 + 2)$$

Antiderivative was successfully verified.

[In] Integrate[x¹¹/(2 + 3*x⁴ + x⁸), x]

[Out] x⁴/4 + Log[1 + x⁴]/4 - Log[2 + x⁴]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{2 + 3x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x¹¹/(2 + 3*x⁴ + x⁸), x]

[Out] IntegrateAlgebraic[x¹¹/(2 + 3*x⁴ + x⁸), x]

fricas [A] time = 1.20, size = 22, normalized size = 0.85

$$\frac{1}{4} x^4 - \log(x^4 + 2) + \frac{1}{4} \log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(x⁸+3*x⁴+2), x, algorithm="fricas")

[Out] $\frac{1}{4}x^4 - \log(x^4 + 2) + \frac{1}{4}\log(x^4 + 1)$

giac [A] time = 0.36, size = 22, normalized size = 0.85

$$\frac{1}{4}x^4 - \log(x^4 + 2) + \frac{1}{4}\log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(x^8+3*x^4+2),x, algorithm="giac")`

[Out] $\frac{1}{4}x^4 - \log(x^4 + 2) + \frac{1}{4}\log(x^4 + 1)$

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{x^4}{4} + \frac{\ln(x^4 + 1)}{4} - \ln(x^4 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(x^8+3*x^4+2),x)`

[Out] $\frac{1}{4}x^4 + \frac{1}{4}\ln(x^4 + 1) - \ln(x^4 + 2)$

maxima [A] time = 0.86, size = 22, normalized size = 0.85

$$\frac{1}{4}x^4 - \log(x^4 + 2) + \frac{1}{4}\log(x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(x^8+3*x^4+2),x, algorithm="maxima")`

[Out] $\frac{1}{4}x^4 - \log(x^4 + 2) + \frac{1}{4}\log(x^4 + 1)$

mupad [B] time = 1.32, size = 22, normalized size = 0.85

$$\frac{\ln(x^4 + 1)}{4} - \ln(x^4 + 2) + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(3*x^4 + x^8 + 2),x)`

[Out] $\log(x^4 + 1)/4 - \log(x^4 + 2) + x^4/4$

sympy [A] time = 0.13, size = 19, normalized size = 0.73

$$\frac{x^4}{4} + \frac{\log(x^4 + 1)}{4} - \log(x^4 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+3*x**4+2),x)
```

```
[Out] x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

$$3.346 \quad \int \frac{x^9}{2+x^5+x^{10}} dx$$

Optimal. Leaf size=37

$$\frac{1}{10} \log(x^{10} + x^5 + 2) - \frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1357, 634, 618, 204, 628}

$$\frac{1}{10} \log(x^{10} + x^5 + 2) - \frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[x^9/(2 + x^5 + x^10),x]

[Out] -ArcTan[(1 + 2*x^5)/Sqrt[7]]/(5*Sqrt[7]) + Log[2 + x^5 + x^10]/10

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1357

$\text{Int}[(x_)^{(m_.)}*((a_) + (c_.)*(x_)^{(n2_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol]$
 $] \text{:> Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x}], x, x^n], x] \text{;/; FreeQ}\{a, b, c, m, n, p\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^9}{2 + x^5 + x^{10}} dx &= \frac{1}{5} \text{Subst} \left(\int \frac{x}{2 + x + x^2} dx, x, x^5 \right) \\ &= - \left(\frac{1}{10} \text{Subst} \left(\int \frac{1}{2 + x + x^2} dx, x, x^5 \right) \right) + \frac{1}{10} \text{Subst} \left(\int \frac{1 + 2x}{2 + x + x^2} dx, x, x^5 \right) \\ &= \frac{1}{10} \log(2 + x^5 + x^{10}) + \frac{1}{5} \text{Subst} \left(\int \frac{1}{-7 - x^2} dx, x, 1 + 2x^5 \right) \\ &= - \frac{\tan^{-1} \left(\frac{1 + 2x^5}{\sqrt{7}} \right)}{5\sqrt{7}} + \frac{1}{10} \log(2 + x^5 + x^{10}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{10} \log(x^{10} + x^5 + 2) - \frac{\tan^{-1} \left(\frac{2x^5 + 1}{\sqrt{7}} \right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(2 + x^5 + x^10),x]

[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7] + Log[2 + x^5 + x^10]/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{2 + x^5 + x^{10}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/(2 + x^5 + x^10),x]

[Out] IntegrateAlgebraic[x^9/(2 + x^5 + x^10), x]

fricas [A] time = 1.20, size = 30, normalized size = 0.81

$$-\frac{1}{35} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x^5 + 1)\right) + \frac{1}{10} \log(x^{10} + x^5 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^10+x^5+2),x, algorithm="fricas")

[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)

giac [A] time = 2.72, size = 30, normalized size = 0.81

$$-\frac{1}{35} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x^5 + 1)\right) + \frac{1}{10} \log(x^{10} + x^5 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^10+x^5+2),x, algorithm="giac")

[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{7} \arctan\left(\frac{(2x^5+1)\sqrt{7}}{7}\right)}{35} + \frac{\ln(x^{10} + x^5 + 2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^10+x^5+2),x)

[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)

maxima [A] time = 2.05, size = 30, normalized size = 0.81

$$-\frac{1}{35} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x^5 + 1)\right) + \frac{1}{10} \log(x^{10} + x^5 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^10+x^5+2),x, algorithm="maxima")

[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)

mupad [B] time = 1.35, size = 32, normalized size = 0.86

$$\frac{\ln(x^{10} + x^5 + 2)}{10} - \frac{\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7} + \frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(x^5 + x^10 + 2),x)`

[Out] `log(x^5 + x^10 + 2)/10 - (7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35`

sympy [A] time = 0.14, size = 37, normalized size = 1.00

$$\frac{\log(x^{10} + x^5 + 2)}{10} - \frac{\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7} + \frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(x**10+x**5+2),x)`

[Out] `log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35`

$$3.347 \quad \int \frac{x^4}{2+x^5+x^{10}} dx$$

Optimal. Leaf size=23

$$\frac{2 \tan^{-1} \left(\frac{2x^5+1}{\sqrt{7}} \right)}{5\sqrt{7}}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1352, 618, 204}

$$\frac{2 \tan^{-1} \left(\frac{2x^5+1}{\sqrt{7}} \right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(2 + x^5 + x^10),x]

[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{2+x^5+x^{10}} dx &= \frac{1}{5} \text{Subst} \left(\int \frac{1}{2+x+x^2} dx, x, x^5 \right) \\
&= - \left(\frac{2}{5} \text{Subst} \left(\int \frac{1}{-7-x^2} dx, x, 1+2x^5 \right) \right) \\
&= \frac{2 \tan^{-1} \left(\frac{1+2x^5}{\sqrt{7}} \right)}{5\sqrt{7}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{2x^5+1}{\sqrt{7}} \right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(2 + x^5 + x^10), x]

[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{2+x^5+x^{10}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(2 + x^5 + x^10), x]

[Out] IntegrateAlgebraic[x^4/(2 + x^5 + x^10), x]

fricas [A] time = 1.22, size = 18, normalized size = 0.78

$$\frac{2}{35} \sqrt{7} \arctan \left(\frac{1}{7} \sqrt{7} (2x^5 + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^10+x^5+2), x, algorithm="fricas")

[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))

giac [A] time = 2.96, size = 18, normalized size = 0.78

$$\frac{2}{35} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x^5 + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^10+x^5+2),x, algorithm="giac")

[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{2\sqrt{7} \arctan\left(\frac{(2x^5+1)\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^10+x^5+2),x)

[Out] 2/35*7^(1/2)*arctan(1/7*(2*x^5+1)*7^(1/2))

maxima [A] time = 2.09, size = 18, normalized size = 0.78

$$\frac{2}{35} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x^5 + 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^10+x^5+2),x, algorithm="maxima")

[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))

mupad [B] time = 1.33, size = 20, normalized size = 0.87

$$\frac{2\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7} + \frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^5 + x^10 + 2),x)

[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35

sympy [A] time = 0.13, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7} + \frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**10+x**5+2),x)

[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35

$$3.348 \quad \int \frac{1}{x(1+x^5+x^{10})} dx$$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} - \frac{1}{10} \log(x^{10} + x^5 + 1) + \log(x)$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{10} \log(x^{10} + x^5 + 1) - \frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + x^5 + x^10)), x]

[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(1+x^5+x^{10})} dx &= \frac{1}{5} \text{Subst} \left(\int \frac{1}{x(1+x+x^2)} dx, x, x^5 \right) \\
 &= \frac{1}{5} \text{Subst} \left(\int \frac{1}{x} dx, x, x^5 \right) + \frac{1}{5} \text{Subst} \left(\int \frac{-1-x}{1+x+x^2} dx, x, x^5 \right) \\
 &= \log(x) - \frac{1}{10} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^5 \right) - \frac{1}{10} \text{Subst} \left(\int \frac{1+2x}{1+x+x^2} dx, x, x^5 \right) \\
 &= \log(x) - \frac{1}{10} \log(1+x^5+x^{10}) + \frac{1}{5} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^5 \right) \\
 &= -\frac{\tan^{-1} \left(\frac{1+2x^5}{\sqrt{3}} \right)}{5\sqrt{3}} + \log(x) - \frac{1}{10} \log(1+x^5+x^{10})
 \end{aligned}$$

Mathematica [C] time = 0.04, size = 197, normalized size = 5.05

$$\frac{1}{5} \text{RootSum} \left[\#1^8 - \#1^7 + \#1^5 - \#1^4 + \#1^3 - \#1 + 1 \& \frac{4\#1^7 \log(x - \#1) - 3\#1^6 \log(x - \#1) - \#1^5 \log(x - \#1) + 3\#1^4 \log(x - \#1) - \#1^3 \log(x - \#1) + 2\#1^2 \log(x - \#1) - \#1 \log(x - \#1)}{8\#1^7 - 7\#1^6 + 5\#1^4 - 4\#1^3 + 3\#1^2 - 1} \& \right] - \frac{1}{10} \log(x^2 + x + 1) + \log(x) + \frac{\tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)}{5\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 + x^5 + x^10)),x]

[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-Log[x - #1]*#1) + 2*Log[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 - 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) &]/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1 + x^5 + x^{10})} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(1 + x^5 + x^10)),x]

[Out] IntegrateAlgebraic[1/(x*(1 + x^5 + x^10)), x]

fricas [A] time = 1.16, size = 32, normalized size = 0.82

$$-\frac{1}{15} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^5 + 1)\right) - \frac{1}{10} \log(x^{10} + x^5 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^10+x^5+1),x, algorithm="fricas")

[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) + log(x)

giac [A] time = 0.41, size = 33, normalized size = 0.85

$$-\frac{1}{15} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^5 + 1)\right) - \frac{1}{10} \log(x^{10} + x^5 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^10+x^5+1),x, algorithm="giac")

[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) + log(abs(x))

maple [B] time = 0.04, size = 66, normalized size = 1.69

$$-\frac{\sqrt{3} \arctan\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15} + \ln(x) - \frac{\ln(x^2 + x + 1)}{10} - \frac{\ln(4x^8 - 4x^7 + 4x^5 - 4x^4 + 4x^3 - 4x + 4)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(x^10+x^5+1),x)`

[Out] $-1/10*\ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)-1/15*3^{(1/2)}*\arctan(2/3*3^{(1/2)}*x^5+1/3*3^{(1/2)})+\ln(x)-1/10*\ln(x^2+x+1)$

maxima [A] time = 2.03, size = 36, normalized size = 0.92

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right)-\frac{1}{10}\log(x^{10}+x^5+1)+\frac{1}{5}\log(x^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x^10+x^5+1),x, algorithm="maxima")`

[Out] $-1/15*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^5+1))-1/10*\log(x^{10}+x^5+1)+1/5*\log(x^5)$

mupad [B] time = 0.06, size = 34, normalized size = 0.87

$$\ln(x) - \frac{\ln(x^{10} + x^5 + 1)}{10} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x^5+x^10+1)),x)`

[Out] $\log(x) - \log(x^5 + x^{10} + 1)/10 - (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 + (2*3^{(1/2)}*x^5)/3))/15$

sympy [A] time = 0.17, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^{10} + x^5 + 1)}{10} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x**10+x**5+1),x)`

[Out] $\log(x) - \log(x^{10} + x^5 + 1)/10 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x^{5/3} + \sqrt{3}/3)/15$

$$3.349 \quad \int \frac{1}{x^6(1+x^5+x^{10})} dx$$

Optimal. Leaf size=48

$$-\frac{1}{5x^5} - \frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} + \frac{1}{10} \log(x^{10} + x^5 + 1) - \log(x)$$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1357, 709, 800, 634, 618, 204, 628}

$$-\frac{1}{5x^5} + \frac{1}{10} \log(x^{10} + x^5 + 1) - \frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} - \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(1 + x^5 + x^10)),x]

[Out] -1/(5*x^5) - ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) - Log[x] + Log[1 + x^5 + x^10]/10

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)(a + bx + cx^2)}{(a + bx + cx^2)^2} dx$; FreeQ[{a, b, c, d, e}, x] && NeQ[2cd - b^2 - 4ac, 0] && NeQ[b^2 - 4ac, 0] && !NiceSqrtQ[b^2 - 4ac]

Rule 709

$\int \frac{(d + ex)^m}{(a + bx + cx^2)^2} dx$; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4ac, 0] && NeQ[cd - b^2 - 4ac, 0] && NeQ[2cd - b^2 - 4ac, 0] && LtQ[m, -1]

Rule 800

$\int \frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)^2} dx$; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4ac, 0] && NeQ[cd - b^2 - 4ac, 0] && IntegerQ[m]

Rule 1357

$\int (a + bx + cx^2)^p dx$; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n, 2m] && NeQ[b^2 - 4ac, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6(1+x^5+x^{10})} dx &= \frac{1}{5} \text{Subst} \left(\int \frac{1}{x^2(1+x+x^2)} dx, x, x^5 \right) \\
&= -\frac{1}{5x^5} + \frac{1}{5} \text{Subst} \left(\int \frac{-1-x}{x(1+x+x^2)} dx, x, x^5 \right) \\
&= -\frac{1}{5x^5} + \frac{1}{5} \text{Subst} \left(\int \left(-\frac{1}{x} + \frac{x}{1+x+x^2} \right) dx, x, x^5 \right) \\
&= -\frac{1}{5x^5} - \log(x) + \frac{1}{5} \text{Subst} \left(\int \frac{x}{1+x+x^2} dx, x, x^5 \right) \\
&= -\frac{1}{5x^5} - \log(x) - \frac{1}{10} \text{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, x^5 \right) + \frac{1}{10} \text{Subst} \left(\int \frac{1+2x}{1+x+x^2} dx, x, x^5 \right) \\
&= -\frac{1}{5x^5} - \log(x) + \frac{1}{10} \log(1+x^5+x^{10}) + \frac{1}{5} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x^5 \right) \\
&= -\frac{1}{5x^5} - \frac{\tan^{-1} \left(\frac{1+2x^5}{\sqrt{3}} \right)}{5\sqrt{3}} - \log(x) + \frac{1}{10} \log(1+x^5+x^{10})
\end{aligned}$$

Mathematica [C] time = 0.04, size = 208, normalized size = 4.33

$$\frac{1}{30} \left(6\text{RootSum} \left[\#1^8 - \#1^7 + \#1^5 - \#1^4 + \#1^3 - \#1 + 1 \&, \frac{4\#1^7 \log(x - \#1) - 4\#1^6 \log(x - \#1) + \#1^5 \log(x - \#1) + 2\#1^4 \log(x - \#1) - 3\#1^3 \log(x - \#1) + \#1^2 \log(x - \#1) + \#1 \log(x - \#1) - \log(x - \#1)}{8\#1^7 - 7\#1^6 + 5\#1^4 - 4\#1^3 + 3\#1^2 - 1} \right] - \frac{6}{x^5} + 3 \log(x^2 + x + 1) - 30 \log(x) + 2\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6*(1 + x^5 + x^10)),x]

[Out] (-6/x^5 + 2*sqrt[3]*ArcTan[(1 + 2*x)/sqrt[3]] - 30*Log[x] + 3*Log[1 + x + x^2] + 6*RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 &, (-Log[x - #1] + Log[x - #1]*#1 + Log[x - #1]*#1^2 - 3*Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4 + Log[x - #1]*#1^5 - 4*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) &])/30

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6(1+x^5+x^{10})} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^6*(1 + x^5 + x^10)),x]

[Out] IntegrateAlgebraic[1/(x^6*(1 + x^5 + x^10)), x]

fricas [A] time = 1.27, size = 49, normalized size = 1.02

$$\frac{2\sqrt{3}x^5 \arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - 3x^5 \log(x^{10} + x^5 + 1) + 30x^5 \log(x) + 6}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="fricas")

[Out] -1/30*(2*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 3*x^5*log(x^10 + x^5 + 1) + 30*x^5*log(x) + 6)/x^5

giac [A] time = 0.25, size = 45, normalized size = 0.94

$$-\frac{1}{15}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) + \frac{x^5-1}{5x^5} + \frac{1}{10} \log(x^{10} + x^5 + 1) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="giac")

[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) + 1/5*(x^5 - 1)/x^5 + 1/10*log(x^10 + x^5 + 1) - log(abs(x))

maple [A] time = 0.02, size = 73, normalized size = 1.52

$$-\frac{\sqrt{3} \arctan\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15} - \ln(x) + \frac{\ln(x^2 + x + 1)}{10} + \frac{\ln(4x^8 - 4x^7 + 4x^5 - 4x^4 + 4x^3 - 4x + 4)}{10} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(x^10+x^5+1),x)

[Out] 1/10*ln(4*x^8-4*x^7+4*x^5-4*x^4+4*x^3-4*x+4)-1/15*3^(1/2)*arctan(2/3*3^(1/2)*x^5+1/3*3^(1/2))-1/5/x^5-ln(x)+1/10*ln(x^2+x+1)

maxima [A] time = 2.07, size = 41, normalized size = 0.85

$$-\frac{1}{15}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^5+1)\right) - \frac{1}{5x^5} + \frac{1}{10} \log(x^{10} + x^5 + 1) - \frac{1}{5} \log(x^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="maxima")

[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/5/x^5 + 1/10*log(x^10 + x^5 + 1) - 1/5*log(x^5)

mupad [B] time = 1.37, size = 41, normalized size = 0.85

$$\frac{\ln(x^{10} + x^5 + 1)}{10} - \ln(x) - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^6*(x^5 + x^10 + 1)),x)`

[Out] `log(x^5 + x^10 + 1)/10 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)/3))/15 - 1/(5*x^5)`

sympy [A] time = 0.20, size = 48, normalized size = 1.00

$$-\log(x) + \frac{\log(x^{10} + x^5 + 1)}{10} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15} - \frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(x**10+x**5+1),x)`

[Out] `-log(x) + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)/3)/15 - 1/(5*x**5)`

$$3.350 \quad \int \frac{1}{x+x^6+x^{11}} dx$$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} - \frac{1}{10} \log(x^{10} + x^5 + 1) + \log(x)$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {1594, 1357, 705, 29, 634, 618, 204, 628}

$$-\frac{1}{10} \log(x^{10} + x^5 + 1) - \frac{\tan^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(x + x^6 + x^11)^(-1), x]

[Out] -ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x^5 + x^10]/10

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x + x^6 + x^{11}} dx &= \int \frac{1}{x(1 + x^5 + x^{10})} dx \\
&= \frac{1}{5} \text{Subst} \left(\int \frac{1}{x(1 + x + x^2)} dx, x, x^5 \right) \\
&= \frac{1}{5} \text{Subst} \left(\int \frac{1}{x} dx, x, x^5 \right) + \frac{1}{5} \text{Subst} \left(\int \frac{-1 - x}{1 + x + x^2} dx, x, x^5 \right) \\
&= \log(x) - \frac{1}{10} \text{Subst} \left(\int \frac{1}{1 + x + x^2} dx, x, x^5 \right) - \frac{1}{10} \text{Subst} \left(\int \frac{1 + 2x}{1 + x + x^2} dx, x, x^5 \right) \\
&= \log(x) - \frac{1}{10} \log(1 + x^5 + x^{10}) + \frac{1}{5} \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2x^5 \right) \\
&= -\frac{\tan^{-1} \left(\frac{1+2x^5}{\sqrt{3}} \right)}{5\sqrt{3}} + \log(x) - \frac{1}{10} \log(1 + x^5 + x^{10})
\end{aligned}$$

Mathematica [C] time = 0.02, size = 197, normalized size = 5.05

$$-\frac{1}{5} \text{RootSum} \left[\#1^5 - \#1^7 + \#1^5 - \#1^4 + \#1^3 - \#1 + 1 \&, \frac{4\#1^7 \log(x - \#1) - 3\#1^6 \log(x - \#1) - \#1^5 \log(x - \#1) + 3\#1^4 \log(x - \#1) - \#1^3 \log(x - \#1) + 2\#1^2 \log(x - \#1) - \#1 \log(x - \#1)}{8\#1^7 - 7\#1^6 + 5\#1^4 - 4\#1^3 + 3\#1^2 - 1} \& \right] - \frac{1}{10} \log(x^2 + x + 1) + \log(x) + \frac{\tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)}{5\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^6 + x^11)^(-1), x]

[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 &, (- (Log[x - #1]*#1) + 2*Log[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 - 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) &]/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + x^6 + x^{11}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x + x^6 + x^11)^(-1), x]

[Out] IntegrateAlgebraic[(x + x^6 + x^11)^(-1), x]

fricas [A] time = 1.06, size = 32, normalized size = 0.82

$$-\frac{1}{15} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^5 + 1) \right) - \frac{1}{10} \log(x^{10} + x^5 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x¹¹+x⁶+x),x, algorithm="fricas")

[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x⁵ + 1)) - 1/10*log(x¹⁰ + x⁵ + 1) + log(x)

giac [A] time = 0.35, size = 33, normalized size = 0.85

$$-\frac{1}{15} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^5 + 1)\right) - \frac{1}{10} \log(x^{10} + x^5 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x¹¹+x⁶+x),x, algorithm="giac")

[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x⁵ + 1)) - 1/10*log(x¹⁰ + x⁵ + 1) + log(abs(x))

maple [B] time = 0.02, size = 66, normalized size = 1.69

$$-\frac{\sqrt{3} \arctan\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15} + \ln(x) - \frac{\ln(x^2 + x + 1)}{10} - \frac{\ln(4x^8 - 4x^7 + 4x^5 - 4x^4 + 4x^3 - 4x + 4)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x¹¹+x⁶+x),x)

[Out] -1/15*3^(1/2)*arctan(2/3*3^(1/2)*x⁵+1/3*3^(1/2))+ln(x)-1/10*ln(x²+x+1)-1/10*ln(4*x⁸-4*x⁷+4*x⁵-4*x⁴+4*x³-4*x+4)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{5} \int \frac{4x^7 - 3x^6 - x^5 + 3x^4 - x^3 + 2x^2 - x}{x^8 - x^7 + x^5 - x^4 + x^3 - x + 1} dx - \frac{1}{10} \log(x^2 + x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x¹¹+x⁶+x),x, algorithm="maxima")

[Out] 1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/5*integrate((4*x⁷ - 3*x⁶ - x⁵ + 3*x⁴ - x³ + 2*x² - x)/(x⁸ - x⁷ + x⁵ - x⁴ + x³ - x + 1), x) - 1/10*log(x² + x + 1) + log(x)

mupad [B] time = 0.03, size = 34, normalized size = 0.87

$$\ln(x) - \frac{\ln(x^{10} + x^5 + 1)}{10} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x + x^6 + x^11),x)`

[Out] $\log(x) - \log(x^5 + x^{10} + 1)/10 - (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 + (2*3^{(1/2)}*x^5)/3))/15$

sympy [A] time = 0.17, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^{10} + x^5 + 1)}{10} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**11+x**6+x),x)`

[Out] $\log(x) - \log(x^{10} + x^5 + 1)/10 - \operatorname{sqrt}(3)*\operatorname{atan}(2*\operatorname{sqrt}(3)*x^{5/3} + \operatorname{sqrt}(3)/3)/15$

$$3.351 \quad \int \frac{x^3}{c + \frac{a}{x^2} + \frac{b}{x}} dx$$

Optimal. Leaf size=147

$$\frac{(a^2c^2 - 3ab^2c + b^4) \log(a + bx + cx^2)}{2c^5} + \frac{b(5a^2c^2 - 5ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}} - \frac{bx(b^2-2ac)}{c^4} + \frac{x^2(b^2-ac)}{2c^3} - \frac{bx^3}{3c^2} + \frac{x^4}{4c}$$

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1354, 701, 634, 618, 206, 628}

$$\frac{(a^2c^2 - 3ab^2c + b^4) \log(a + bx + cx^2)}{2c^5} + \frac{b(5a^2c^2 - 5ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}} + \frac{x^2(b^2-ac)}{2c^3} - \frac{bx(b^2-2ac)}{c^4} - \frac{bx^3}{3c^2} + \frac{x^4}{4c}$$

Antiderivative was successfully verified.

[In] Int[x^3/(c + a/x^2 + b/x), x]

[Out] -((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/(3*c^2) + x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + b*x + c*x^2])/(2*c^5)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634


```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:= Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{c + \frac{a}{x^2} + \frac{b}{x}} dx &= \int \frac{x^5}{a + bx + cx^2} dx \\ &= \int \left(-\frac{b(b^2 - 2ac)}{c^4} + \frac{(b^2 - ac)x}{c^3} - \frac{bx^2}{c^2} + \frac{x^3}{c} + \frac{ab(b^2 - 2ac) + (b^4 - 3ab^2c + a^2c^2)x}{c^4(a + bx + cx^2)} \right) dx \\ &= -\frac{b(b^2 - 2ac)x}{c^4} + \frac{(b^2 - ac)x^2}{2c^3} - \frac{bx^3}{3c^2} + \frac{x^4}{4c} + \frac{\int \frac{ab(b^2 - 2ac) + (b^4 - 3ab^2c + a^2c^2)x}{a + bx + cx^2} dx}{c^4} \\ &= -\frac{b(b^2 - 2ac)x}{c^4} + \frac{(b^2 - ac)x^2}{2c^3} - \frac{bx^3}{3c^2} + \frac{x^4}{4c} + \frac{(b^4 - 3ab^2c + a^2c^2) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2c^5} - \frac{b(b^4 - 5ab^2c + 5a^2c^2)}{2c^5} \\ &= -\frac{b(b^2 - 2ac)x}{c^4} + \frac{(b^2 - ac)x^2}{2c^3} - \frac{bx^3}{3c^2} + \frac{x^4}{4c} + \frac{(b^4 - 3ab^2c + a^2c^2) \log(a + bx + cx^2)}{2c^5} + \frac{b(b^4 - 5ab^2c + 5a^2c^2)}{2c^5} \\ &= -\frac{b(b^2 - 2ac)x}{c^4} + \frac{(b^2 - ac)x^2}{2c^3} - \frac{bx^3}{3c^2} + \frac{x^4}{4c} + \frac{b(b^4 - 5ab^2c + 5a^2c^2) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{c^5 \sqrt{b^2 - 4ac}} + \frac{b(b^4 - 5ab^2c + 5a^2c^2)}{2c^5} \end{aligned}$$

Mathematica [A] time = 0.12, size = 140, normalized size = 0.95

$$\frac{6(a^2c^2 - 3ab^2c + b^4) \log(a + x(b + cx)) - \frac{12b(5a^2c^2 - 5ab^2c + b^4) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} + cx(-4bc(cx^2 - 6a) + 3c^2x(cx^2 - 2a) - 12b^3 + 6b^2cx)}{12c^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(c + a/x^2 + b/x),x]

[Out] (c*x*(-12*b^3 + 6*b^2*c*x - 4*b*c*(-6*a + c*x^2) + 3*c^2*x*(-2*a + c*x^2)) - (12*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 6*(b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + x*(b + c*x)]/(12*c^5)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{c + \frac{a}{x^2} + \frac{b}{x}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(c + a/x^2 + b/x),x]

[Out] IntegrateAlgebraic[x^3/(c + a/x^2 + b/x), x]

fricas [A] time = 1.30, size = 466, normalized size = 3.17

$$\frac{3(b^5 - 4ab^4)c^5 - 4(b^5 - 4ab^4)c^4 + 6(b^5 - 5ab^4 + 4a^2b^3)c^3 + 6(b^5 - 5ab^4 + 4a^2b^3)c^2 + 6(b^5 - 5ab^4 + 4a^2b^3)c + 6(b^5 - 5ab^4 + 4a^2b^3)\sqrt{-b^2 + 4ac} \log\left(\frac{3(b^2 + 2cx + a)\sqrt{-b^2 + 4ac}}{12(c^2 - 4a^2)}\right) - 12(b^5 - 6ab^4 + 8a^2b^3)c^2 + 6(b^5 - 7ab^4 + 13a^2b^3)c + 6(b^5 - 7ab^4 + 13a^2b^3)\log(cx^2 + bx + a) + 3(b^5 - 4ab^4)c^5 - 4(b^5 - 4ab^4)c^4 + 6(b^5 - 5ab^4 + 4a^2b^3)c^3 + 12(b^5 - 5ab^4 + 5a^2b^3)\sqrt{-b^2 + 4ac} \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right) - 12(b^5 - 6ab^4 + 8a^2b^3)c^2 + 6(b^5 - 7ab^4 + 13a^2b^3)c + 6(b^5 - 7ab^4 + 13a^2b^3)\log(cx^2 + bx + a)}{12(c^2 - 4a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="fricas")

[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 6*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a)/(b^2*c^5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 12*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a)/(b^2*c^5 - 4*a*c^6)]

giac [A] time = 0.29, size = 145, normalized size = 0.99

$$\frac{3c^3x^4 - 4bc^2x^3 + 6b^2cx^2 - 6ac^2x^2 - 12b^3x + 24abcx}{12c^4} + \frac{(b^4 - 3ab^2c + a^2c^2)\log(cx^2 + bx + a)}{2c^5} - \frac{(b^5 - 5ab^3c + 5a^2bc^2)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="giac")

[Out] $1/12*(3*c^3*x^4 - 4*b*c^2*x^3 + 6*b^2*c*x^2 - 6*a*c^2*x^2 - 12*b^3*x + 24*a*b*c*x)/c^4 + 1/2*(b^4 - 3*a*b^2*c + a^2*c^2)*\log(c*x^2 + b*x + a)/c^5 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c})*c^5)$

maple [A] time = 0.01, size = 236, normalized size = 1.61

$$\frac{x^4}{4c} - \frac{bx^3}{3c^2} - \frac{5a^2b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^3} + \frac{5ab^3 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^4} - \frac{ax^2}{2c^2} - \frac{b^5 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^5} + \frac{b^2x^2}{2c^3} + \frac{a^2 \ln(cx^2 + bx + a)}{2c^3} - \frac{3ab^2 \ln(cx^2 + bx + a)}{2c^4} + \frac{2abx}{c^3} + \frac{b^4 \ln(cx^2 + bx + a)}{2c^5} - \frac{b^3x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(c+a/x^2+b/x), x)`

[Out] $1/4/c*x^4 - 1/3*b*x^3/c^2 - 1/2/c^2*x^2*a + 1/2/c^3*x^2*b^2 + 2/c^3*a*b*x - 1/c^4*b^3*x + 1/2/c^3*\ln(c*x^2 + b*x + a)*a^2 - 3/2/c^4*\ln(c*x^2 + b*x + a)*a*b^2 + 1/2/c^5*\ln(c*x^2 + b*x + a)*b^4 - 5/c^3/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x + b)/(4*a*c - b^2)^{(1/2)})*a^2*b + 5/c^4/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x + b)/(4*a*c - b^2)^{(1/2)})*a*b^3 - 1/c^5/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x + b)/(4*a*c - b^2)^{(1/2)})*b^5$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c+a/x^2+b/x), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.16, size = 183, normalized size = 1.24

$$x \left(\frac{b \left(\frac{a}{2c} - \frac{b^2}{c^3} \right) + \frac{ab}{c^3}}{c} \right) + \frac{x^4}{4c} - x^2 \left(\frac{a}{2c^2} - \frac{b^2}{2c^3} \right) - \frac{\ln(cx^2 + bx + a) (-4a^3c^3 + 13a^2b^2c^2 - 7ab^4c + b^6)}{2(4ac^6 - b^2c^5)} - \frac{bx^3}{3c^2} - \frac{b \operatorname{atan}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) (5a^2c^2 - 5ab^2c + b^4)}{c^5 \sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(c + a/x^2 + b/x), x)`

[Out] $x*((b*(a/c^2 - b^2/c^3))/c + (a*b)/c^3) + x^4/(4*c) - x^2*(a/(2*c^2) - b^2/(2*c^3)) - (\log(a + b*x + c*x^2)*(b^6 - 4*a^3*c^3 + 13*a^2*b^2*c^2 - 7*a*b^4*c))/(2*(4*a*c^6 - b^2*c^5)) - (b*x^3)/(3*c^2) - (b*\operatorname{atan}((b + 2*c*x)/(4*a*c - b^2)^{(1/2)}))*(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c^5*(4*a*c - b^2)^{(1/2)})$

sympy [B] time = 1.25, size = 605, normalized size = 4.12

$$\frac{b^2}{3c^2} + x^2 \left(\frac{a}{2c^2} + \frac{b^2}{3c^2} \right) + \left(\frac{2ab}{3c^2} \right) \left(\frac{b\sqrt{-4ac + b^2}}{2^2(4ac - b^2)} \right) + \left(\frac{b^2 - 3ab^2 + b^3}{2c^2} \right) \log \left(\frac{2a^2x^2 - 4a^2bx + ab^2 - 4ac}{2^2(4ac - b^2)} + \frac{b^2 \sqrt{-4ac + b^2}}{2^2(4ac - b^2)} \right) + \left(\frac{b^2 - 3ab^2 + b^3}{2c^2} \right) \log \left(\frac{2a^2x^2 - 4a^2bx + ab^2 - 4ac}{2^2(4ac - b^2)} + \frac{b^2 \sqrt{-4ac + b^2}}{2^2(4ac - b^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c+a/x**2+b/x),x)

[Out] $-b*x**3/(3*c**2) + x**2*(-a/(2*c**2) + b**2/(2*c**3)) + x*(2*a*b/c**3 - b**3/c**4) + (-b*\text{sqrt}(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*\log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(-b*\text{sqrt}(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c**4*(-b*\text{sqrt}(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + (b*\text{sqrt}(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*\log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(b*\text{sqrt}(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c**4*(b*\text{sqrt}(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + x**4/(4*c)$

$$3.352 \quad \int \frac{x^2}{c + \frac{a}{x^2} + \frac{b}{x}} dx$$

Optimal. Leaf size=118

$$\frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) - b(b^2 - 2ac) \log(a + bx + cx^2)}{c^4\sqrt{b^2 - 4ac}} + \frac{x(b^2 - ac)}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c}$$

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1354, 701, 634, 618, 206, 628}

$$\frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) - b(b^2 - 2ac) \log(a + bx + cx^2)}{c^4\sqrt{b^2 - 4ac}} + \frac{x(b^2 - ac)}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[x^2/(c + a/x^2 + b/x), x]

[Out] ((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_)^m)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^m*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{c + \frac{a}{x^2} + \frac{b}{x}} dx &= \int \frac{x^4}{a + bx + cx^2} dx \\
 &= \int \left(\frac{b^2 - ac}{c^3} - \frac{bx}{c^2} + \frac{x^2}{c} - \frac{a(b^2 - ac) + b(b^2 - 2ac)x}{c^3(a + bx + cx^2)} \right) dx \\
 &= \frac{(b^2 - ac)x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{\int \frac{a(b^2 - ac) + b(b^2 - 2ac)x}{a + bx + cx^2} dx}{c^3} \\
 &= \frac{(b^2 - ac)x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{(b(b^2 - 2ac)) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2c^4} + \frac{(b^4 - 4ab^2c + 2a^2c^2) \int \frac{1}{a + bx + cx^2} dx}{2c^4} \\
 &= \frac{(b^2 - ac)x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{b(b^2 - 2ac) \log(a + bx + cx^2)}{2c^4} - \frac{(b^4 - 4ab^2c + 2a^2c^2) \text{Subst}\left(\int \frac{1}{b^2 - 4ac}\right)}{c^4} \\
 &= \frac{(b^2 - ac)x}{c^3} - \frac{bx^2}{2c^2} + \frac{x^3}{3c} - \frac{(b^4 - 4ab^2c + 2a^2c^2) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{c^4 \sqrt{b^2 - 4ac}} - \frac{b(b^2 - 2ac) \log(a + bx + cx^2)}{2c^4}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.95

$$\frac{6(2a^2c^2 - 4ab^2c + b^4) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) - 3(b^3 - 2abc) \log(a + x(b + cx)) + cx(-6ac + 6b^2 - 3bcx + 2c^2x^2)}{\sqrt{4ac-b^2} 6c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(c + a/x^2 + b/x), x]

[Out] (c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)]/(6*c^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{c + \frac{a}{x^2} + \frac{b}{x}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(c + a/x^2 + b/x), x]

[Out] IntegrateAlgebraic[x^2/(c + a/x^2 + b/x), x]

fricas [A] time = 1.34, size = 383, normalized size = 3.25

$$\frac{2(b^2c^2 - 4ac^3) - 3(b^2c^2 - 4ab^2c^2) + 3(b^4 - 4ab^2c + 2a^2c^2)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac}{c^2x^2 + bx + a}\right) + 6(b^2c - 5ab^2c + 4a^2c^2)x - 3(b^3 - 6ab^2c + 8a^2bc^2)\log(cx^2 + bx + a) + 2(b^2c^2 - 4ac^3)^2 - 3(b^2c^2 - 4ab^2c)^2 - 6(b^4 - 4ab^2c + 2a^2c^2)\sqrt{b^2 - 4ac} \arctan\left(\frac{-\sqrt{b^2 - 4ac}cx}{2c^2x + b}\right) + 6(b^2c - 5ab^2c + 4a^2c^2)x - 3(b^3 - 6ab^2c + 8a^2bc^2)\log(cx^2 + bx + a)}{6(b^2c^2 - 4ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c+a/x^2+b/x), x, algorithm="fricas")

[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 + 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 - 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5)]

giac [A] time = 0.39, size = 113, normalized size = 0.96

$$\frac{2c^2x^3 - 3bcx^2 + 6b^2x - 6acx}{6c^3} - \frac{(b^3 - 2abc) \log(cx^2 + bx + a)}{2c^4} + \frac{(b^4 - 4ab^2c + 2a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="giac")

[Out] $\frac{1}{6}*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - \frac{1}{2}*(b^3 - 2*a*b*c)*\log(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c}*c^4)$

maple [A] time = 0.00, size = 190, normalized size = 1.61

$$\frac{x^3}{3c} + \frac{2a^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^2} - \frac{4ab^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^3} + \frac{b^4 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^4} - \frac{bx^2}{2c^2} + \frac{ab \ln(cx^2 + bx + a)}{c^3} - \frac{ax}{c^2} - \frac{b^3 \ln(cx^2 + bx + a)}{2c^4} + \frac{b^2x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c+a/x^2+b/x),x)

[Out] $\frac{1}{3}/c*x^3 - \frac{1}{2}*b*x^2/c^2 - \frac{1}{c^2}*a*x + \frac{1}{c^3}*b^2*x + \frac{1}{c^3}*\ln(c*x^2+b*x+a)*a*b - \frac{1}{2}/c^4*\ln(c*x^2+b*x+a)*b^3 + \frac{2}{c^2}/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a^2 - \frac{4}{c^3}/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^2 + \frac{1}{c^4}/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^4$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.40, size = 151, normalized size = 1.28

$$\frac{x^3}{3c} - x \left(\frac{a}{c^2} - \frac{b^2}{c^3} \right) - \frac{bx^2}{2c^2} + \frac{\ln(cx^2 + bx + a) (8a^2bc^2 - 6ab^3c + b^5)}{2(4ac^5 - b^2c^4)} + \frac{\operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right) (2a^2c^2 - 4ab^2c + b^4)}{c^4 \sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c + a/x^2 + b/x),x)

[Out] $x^3/(3*c) - x*(a/c^2 - b^2/c^3) - (b*x^2)/(2*c^2) + (\log(a + b*x + c*x^2))*(b^5 + 8*a^2*b*c^2 - 6*a*b^3*c)/(2*(4*a*c^5 - b^2*c^4)) + (\operatorname{atan}(b/(4*a*c - b^2)^{(1/2)} + (2*c*x)/(4*a*c - b^2)^{(1/2)}))*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)/(c^4*(4*a*c - b^2)^{(1/2)})$

sympy [B] time = 1.01, size = 498, normalized size = 4.22

$$\frac{bx^2}{2c} + \left(\frac{a}{2c} + \frac{b^2}{c^2}\right) \left(\frac{b(2ac - b^2)}{2c^4} - \frac{\sqrt{-4ac + b^2}(2a^2c^2 - 4ab^2c + b^4)}{2c^4(4ac - b^2)}\right) \log\left(x + \frac{-3a^2c + ab^3 + 4ac^2\left(\frac{b(2ac - b^2)}{2c^4} - \frac{\sqrt{-4ac + b^2}(2a^2c^2 - 4ab^2c + b^4)}{2c^4(4ac - b^2)}\right) - b^2c^2\left(\frac{b(2ac - b^2)}{2c^4} - \frac{\sqrt{-4ac + b^2}(2a^2c^2 - 4ab^2c + b^4)}{2c^4(4ac - b^2)}\right)}{2a^2c^2 - 4ab^2c + b^4}\right) + \left(\frac{b(2ac - b^2)}{2c^4} + \frac{\sqrt{-4ac + b^2}(2a^2c^2 - 4ab^2c + b^4)}{2c^4(4ac - b^2)}\right) \log\left(x + \frac{-3a^2c + ab^3 + 4ac^2\left(\frac{b(2ac - b^2)}{2c^4} + \frac{\sqrt{-4ac + b^2}(2a^2c^2 - 4ab^2c + b^4)}{2c^4(4ac - b^2)}\right) - b^2c^2\left(\frac{b(2ac - b^2)}{2c^4} + \frac{\sqrt{-4ac + b^2}(2a^2c^2 - 4ab^2c + b^4)}{2c^4(4ac - b^2)}\right)}{2a^2c^2 - 4ab^2c + b^4}\right) + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c+a/x**2+b/x),x)

[Out] $-b*x**2/(2*c**2) + x*(-a/c**2 + b**2/c**3) + (b*(2*a*c - b**2)/(2*c**4) - \text{sqrt}(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) * \log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) - \text{sqrt}(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) - \text{sqrt}(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4) + (b*(2*a*c - b**2)/(2*c**4) + \text{sqrt}(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) * \log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) + \text{sqrt}(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) + \text{sqrt}(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4) + x**3/(3*c)$

$$3.353 \quad \int \frac{x}{c + \frac{a}{x^2} + \frac{b}{x}} dx$$

Optimal. Leaf size=89

$$\frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} - \frac{bx}{c^2} + \frac{x^2}{2c}$$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1354, 701, 634, 618, 206, 628}

$$\frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} - \frac{bx}{c^2} + \frac{x^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[x/(c + a/x^2 + b/x), x]

[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x}{c + \frac{a}{x^2} + \frac{b}{x}} dx &= \int \frac{x^3}{a + bx + cx^2} dx \\
 &= \int \left(-\frac{b}{c^2} + \frac{x}{c} + \frac{ab + (b^2 - ac)x}{c^2(a + bx + cx^2)} \right) dx \\
 &= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{\int \frac{ab + (b^2 - ac)x}{a + bx + cx^2} dx}{c^2} \\
 &= -\frac{bx}{c^2} + \frac{x^2}{2c} - \frac{(b(b^2 - 3ac)) \int \frac{1}{a + bx + cx^2} dx}{2c^3} + \frac{(b^2 - ac) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2c^3} \\
 &= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3} + \frac{(b(b^2 - 3ac)) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{c^3} \\
 &= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{c^3 \sqrt{b^2 - 4ac}} + \frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.94

$$\frac{(b^2 - ac) \log(a + x(b + cx)) - \frac{2b(b^2 - 3ac) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} + cx(cx - 2b)}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x/(c + a/x^2 + b/x), x]

[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)]/(2*c^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{c + \frac{a}{x^2} + \frac{b}{x}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(c + a/x^2 + b/x), x]

[Out] IntegrateAlgebraic[x/(c + a/x^2 + b/x), x]

fricas [A] time = 1.41, size = 297, normalized size = 3.34

$$\frac{(b^2c^2 - 4ac^3)x^2 - (b^3 - 3abc)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{c^2 + bx + a}\right) - 2(b^3c - 4abc^2)x + (b^4 - 5ab^2c + 4a^2c^2) \log(cx^2 + bx + a)}{2(b^2c^3 - 4ac^4)} + \frac{(b^2c^2 - 4ac^3)x^2 + 2(b^3 - 3abc)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) - 2(b^3c - 4abc^2)x + (b^4 - 5ab^2c + 4a^2c^2) \log(cx^2 + bx + a)}{2(b^2c^3 - 4ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+a/x^2+b/x), x, algorithm="fricas")

[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]

giac [A] time = 0.26, size = 86, normalized size = 0.97

$$\frac{cx^2 - 2bx}{2c^2} + \frac{(b^2 - ac) \log(cx^2 + bx + a)}{2c^3} - \frac{(b^3 - 3abc) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac} c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+a/x^2+b/x),x, algorithm="giac")

[Out] $\frac{1}{2}*(c*x^2 - 2*b*x)/c^2 + \frac{1}{2}*(b^2 - a*c)*\log(c*x^2 + b*x + a)/c^3 - (b^3 - 3*a*b*c)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c}*c^3)$

maple [A] time = 0.00, size = 132, normalized size = 1.48

$$\frac{3ab \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^2} - \frac{b^3 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^3} + \frac{x^2}{2c} - \frac{a \ln(cx^2 + bx + a)}{2c^2} + \frac{b^2 \ln(cx^2 + bx + a)}{2c^3} - \frac{bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c+a/x^2+b/x),x)

[Out] $\frac{1}{2}/c*x^2-b*x/c^2-1/2/c^2*\ln(c*x^2+b*x+a)*a+1/2/c^3*\ln(c*x^2+b*x+a)*b^2+3/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b-1/c^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^3$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+a/x^2+b/x),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.13, size = 112, normalized size = 1.26

$$\frac{x^2}{2c} - \frac{\ln(cx^2 + bx + a) (4a^2c^2 - 5ab^2c + b^4)}{2(4ac^4 - b^2c^3)} - \frac{bx}{c^2} + \frac{b \operatorname{atan}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) (3ac - b^2)}{c^3 \sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c + a/x^2 + b/x),x)

[Out] $x^2/(2*c) - (\log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c^4 - b^2*c^3)) - (b*x)/c^2 + (b*\operatorname{atan}((b + 2*c*x)/(4*a*c - b^2)^{(1/2)})*(3*a*c - b^2))/(c^3*(4*a*c - b^2)^{(1/2)})$

sympy [B] time = 0.84, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2} + \left(-\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) \log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(-\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(-\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)}{3abc - b^3}\right) + \left(\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) \log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)}{3abc - b^3}\right) + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+a/x**2+b/x),x)

[Out]
$$\begin{aligned} & -b*x/c**2 + (-b*\sqrt{-4*a*c + b**2}*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) \\ & - (a*c - b**2)/(2*c**3))*\log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*\sqrt{-4* \\ & a*c + b**2}*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)) \\ & - b**2*c**2*(-b*\sqrt{-4*a*c + b**2}*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) \\ & - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3) + (b*\sqrt{-4*a*c + b**2}*(3*a* \\ & c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*\log(x + (2*a**2* \\ & c - a*b**2 + 4*a*c**3*(b*\sqrt{-4*a*c + b**2}*(3*a*c - b**2)/(2*c**3*(4*a*c \\ & - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*\sqrt{-4*a*c + b**2}*(3*a*c \\ & - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3) \\ &) + x**2/(2*c) \end{aligned}$$

$$3.354 \quad \int \frac{1}{c + \frac{a}{x^2} + \frac{b}{x}} dx$$

Optimal. Leaf size=70

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx + cx^2)}{2c^2} + \frac{x}{c}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1340, 703, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx + cx^2)}{2c^2} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[(c + a/x^2 + b/x)^(-1),x]

[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\text{t}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \} \&\& \text{NeQ}[2cd - be, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 703

$\text{Int}[(d + e*x)^m / (a + b*x + c*x^2), x_Symbol] \text{ :> } \text{Simp}[(e*(d + e*x)^{m-1}) / (c*(m-1)), x] + \text{Dist}[1/c, \text{Int}[(d + e*x)^{m-2} * \text{Simp}[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]] / (a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \} \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{GtQ}[m, 1]$

Rule 1340

$\text{Int}[(a + c*x)^{n2} + (b*x)^{n1}]^p, x_Symbol] \text{ :> } \text{Int}[x^{(2*n*p)*(c + b/x^n + a/x^{2*n})^p}, x] /; \text{FreeQ}\{a, b, c\}, x \} \&\& \text{EqQ}[n2, 2*n] \&\& \text{LtQ}[n, 0] \&\& \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \frac{1}{c + \frac{a}{x^2} + \frac{b}{x}} dx &= \int \frac{x^2}{a + bx + cx^2} dx \\ &= \frac{x}{c} + \frac{\int \frac{-a-bx}{a+bx+cx^2} dx}{c} \\ &= \frac{x}{c} - \frac{b \int \frac{b+2cx}{a+bx+cx^2} dx}{2c^2} + \frac{(b^2 - 2ac) \int \frac{1}{a+bx+cx^2} dx}{2c^2} \\ &= \frac{x}{c} - \frac{b \log(a + bx + cx^2)}{2c^2} - \frac{(b^2 - 2ac) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{c^2} \\ &= \frac{x}{c} - \frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2 \sqrt{b^2 - 4ac}} - \frac{b \log(a + bx + cx^2)}{2c^2} \end{aligned}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.04

$$\frac{(b^2 - 2ac) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2 \sqrt{4ac - b^2}} - \frac{b \log(a + bx + cx^2)}{2c^2} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(c + a/x^2 + b/x)^(-1),x]

[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c + \frac{a}{x^2} + \frac{b}{x}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-1),x]

[Out] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-1), x]

fricas [A] time = 1.43, size = 235, normalized size = 3.36

$$\left[\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) - 2(b^2c - 4ac^2)x + (b^3 - 4abc) \log(cx^2 + bx + a)}{2(b^2c^2 - 4ac^3)}, \frac{2(b^2 - 2ac)\sqrt{-b^2 + 4ac} \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) - 2(b^2c - 4ac^2)x + (b^3 - 4abc) \log(cx^2 + bx + a)}{2(b^2c^2 - 4ac^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x),x, algorithm="fricas")

[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]

giac [A] time = 0.27, size = 67, normalized size = 0.96

$$\frac{x}{c} - \frac{b \log(cx^2 + bx + a)}{2c^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x),x, algorithm="giac")

[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)

maple [A] time = 0.00, size = 101, normalized size = 1.44

$$-\frac{2a \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2} c} + \frac{b^2 \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2} c^2} - \frac{b \ln(cx^2 + bx + a)}{2c^2} + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c+a/x^2+b/x),x)`

[Out] $1/c*x-1/2*b*\ln(c*x^2+b*x+a)/c^2-2/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a+1/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x^2+b/x),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.42, size = 172, normalized size = 2.46

$$\frac{x}{c} + \frac{b^3 \ln(cx^2 + bx + a)}{2(4ac^3 - b^2c^2)} - \frac{2a \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}} + \frac{b^2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}} - \frac{2abc \ln(cx^2 + bx + a)}{4ac^3 - b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c + a/x^2 + b/x),x)`

[Out] $x/c + (b^3*\log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*\operatorname{atan}(b/(4*a*c - b^2)^{(1/2)} + (2*c*x)/(4*a*c - b^2)^{(1/2)}))/(c*(4*a*c - b^2)^{(1/2)}) + (b^2*\operatorname{atan}(b/(4*a*c - b^2)^{(1/2)} + (2*c*x)/(4*a*c - b^2)^{(1/2)}))/(c^2*(4*a*c - b^2)^{(1/2)}) - (2*a*b*c*\log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)$

sympy [B] time = 0.60, size = 306, normalized size = 4.37

$$\left(\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) \log\left(x + \frac{-ab - 4ac^2\left(\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) + b^2c\left(\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right)}{2ac - b^2}\right) + \left(\frac{b}{2c^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) \log\left(x + \frac{-ab - 4ac^2\left(\frac{b}{2c^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) + b^2c\left(\frac{b}{2c^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right)}{2ac - b^2}\right) + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x**2+b/x),x)`

[Out] $(-b/(2*c**2) - \operatorname{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*\log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - \operatorname{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - \operatorname{sqrt}(-4*a*c + b**2)*(2*a*c$

$$\begin{aligned}
& - b^2/(2c^2(4ac - b^2)))/(2ac - b^2) + (-b/(2c^2) + \sqrt{-4ac + b^2}) * (2ac - b^2)/(2c^2(4ac - b^2)) * \log(x + (-ab - 4ac^2 * (-b/(2c^2) + \sqrt{-4ac + b^2}) * (2ac - b^2)/(2c^2(4ac - b^2)))) + b^2c * (-b/(2c^2) + \sqrt{-4ac + b^2}) * (2ac - b^2)/(2c^2(4ac - b^2)))/(2ac - b^2) + x/c
\end{aligned}$$

$$3.355 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x} dx$$

Optimal. Leaf size=56

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a+bx+cx^2)}{2c}$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1354, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a+bx+cx^2)}{2c}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)*x),x]

[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/(c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x + c*x^2]/(2*c)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1354

`Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]`

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x} dx &= \int \frac{x}{a + bx + cx^2} dx \\ &= \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2c} - \frac{b \int \frac{1}{a+bx+cx^2} dx}{2c} \\ &= \frac{\log(a + bx + cx^2)}{2c} + \frac{b \operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx\right)}{c} \\ &= \frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a + bx + cx^2)}{2c} \end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.02

$$\frac{\log(a + x(b + cx)) - \frac{2b \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)*x), x]

[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a + x*(b + c*x)])/(2*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x), x]

fricas [A] time = 1.35, size = 185, normalized size = 3.30

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + (b^2 - 4ac) \log(cx^2 + bx + a)}{2(b^2c - 4ac^2)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + (b^2 - 4ac) \log(cx^2 + bx + a)}{2(b^2c - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="fricas")

[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a*c)*c*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c - 4*a*c^2)]

giac [A] time = 0.29, size = 55, normalized size = 0.98

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}c} + \frac{\log(cx^2 + bx + a)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="giac")

[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(c*x^2 + b*x + a)/c

maple [A] time = 0.00, size = 56, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} + \frac{\ln(cx^2 + bx + a)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)/x,x)

[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.17, size = 112, normalized size = 2.00

$$\frac{2ac \ln(cx^2 + bx + a)}{4ac^2 - b^2c} - \frac{b \operatorname{atan}\left(\frac{b}{\sqrt{4ac - b^2}} + \frac{2cx}{\sqrt{4ac - b^2}}\right)}{c\sqrt{4ac - b^2}} - \frac{b^2 \ln(cx^2 + bx + a)}{2(4ac^2 - b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(c + a/x^2 + b/x)),x)

[Out] (2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a + b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))

sympy [B] time = 0.32, size = 216, normalized size = 3.86

$$\left(\frac{b\sqrt{-4ac + b^2}}{2c(4ac - b^2)} + \frac{1}{2c}\right) \log\left(x + \frac{-4ac\left(-\frac{b\sqrt{-4ac + b^2}}{2c(4ac - b^2)} + \frac{1}{2c}\right) + 2a + b^2\left(-\frac{b\sqrt{-4ac + b^2}}{2c(4ac - b^2)} + \frac{1}{2c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac + b^2}}{2c(4ac - b^2)} + \frac{1}{2c}\right) \log\left(x + \frac{-4ac\left(\frac{b\sqrt{-4ac + b^2}}{2c(4ac - b^2)} + \frac{1}{2c}\right) + 2a + b^2\left(\frac{b\sqrt{-4ac + b^2}}{2c(4ac - b^2)} + \frac{1}{2c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)/x,x)

[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b)

$$3.356 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^2} dx$$

Optimal. Leaf size=36

$$\frac{2 \tanh^{-1}\left(\frac{\frac{2a}{x} + b}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{b^2 - 4ac}}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1352, 618, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\frac{2a}{x} + b}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)*x^2),x]

[Out] (2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^2} dx &= -\text{Subst}\left(\int \frac{1}{c + bx + ax^2} dx, x, \frac{1}{x}\right) \\
&= 2\text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + \frac{2a}{x}\right) \\
&= \frac{2 \tanh^{-1}\left(\frac{b + \frac{2a}{x}}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{b^2 - 4ac}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.06

$$\frac{2 \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)*x^2), x]

[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^2), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^2), x]

fricas [A] time = 1.24, size = 120, normalized size = 3.33

$$\left[\frac{\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4ac}}, -\frac{2\sqrt{-b^2+4ac} \arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)}{b^2-4ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="fricas")

[Out] $\left[\log\left(\frac{(2c^2x^2 + 2b*cx + b^2 - 2*a*c - \sqrt{b^2 - 4*a*c})*(2*cx + b)}{(c*x^2 + b*x + a)}\right)/\sqrt{b^2 - 4*a*c}, -2*\sqrt{-b^2 + 4*a*c}*\arctan\left(\frac{-\sqrt{-b^2 + 4*a*c}*(2*cx + b)}{(b^2 - 4*a*c)}\right)/\sqrt{-b^2 + 4*a*c}\right]$

giac [A] time = 0.35, size = 34, normalized size = 0.94

$$\frac{2 \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="giac")`

[Out] $2*\arctan\left(\frac{2*cx + b}{\sqrt{-b^2 + 4*a*c}}\right)/\sqrt{-b^2 + 4*a*c}$

maple [A] time = 0.00, size = 35, normalized size = 0.97

$$\frac{2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c+a/x^2+b/x)/x^2,x)`

[Out] $2/(4*a*c-b^2)^{(1/2)}*\arctan\left(\frac{2*cx+b}{(4*a*c-b^2)^{(1/2)}}\right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.05, size = 46, normalized size = 1.28

$$\frac{2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(c + a/x^2 + b/x)),x)`

[Out] $(2*\operatorname{atan}(b/(4*a*c - b^2)^{(1/2)} + (2*c*x)/(4*a*c - b^2)^{(1/2)}))/(4*a*c - b^2)^{(1/2)}$

sympy [B] time = 0.22, size = 124, normalized size = 3.44

$$-\sqrt{-\frac{1}{4ac - b^2}} \log\left(x + \frac{-4ac\sqrt{-\frac{1}{4ac - b^2}} + b^2\sqrt{-\frac{1}{4ac - b^2}} + b}{2c}\right) + \sqrt{-\frac{1}{4ac - b^2}} \log\left(x + \frac{4ac\sqrt{-\frac{1}{4ac - b^2}} - b^2\sqrt{-\frac{1}{4ac - b^2}} + b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x**2+b/x)/x**2,x)`

[Out] $-\operatorname{sqrt}(-1/(4*a*c - b**2))*\log(x + (-4*a*c*\operatorname{sqrt}(-1/(4*a*c - b**2)) + b**2*\operatorname{sqrt}(-1/(4*a*c - b**2)) + b)/(2*c)) + \operatorname{sqrt}(-1/(4*a*c - b**2))*\log(x + (4*a*c*\operatorname{sqrt}(-1/(4*a*c - b**2)) - b**2*\operatorname{sqrt}(-1/(4*a*c - b**2)) + b)/(2*c))$

$$3.357 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^3} dx$$

Optimal. Leaf size=62

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} - \frac{\log(a+bx+cx^2)}{2a} + \frac{\log(x)}{a}$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1354, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} - \frac{\log(a+bx+cx^2)}{2a} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)*x^3),x]

[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a - Log[a + b*x + c*x^2]/(2*a)

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^3} dx &= \int \frac{1}{x(a + bx + cx^2)} dx \\
 &= \frac{\int \frac{1}{x} dx}{a} + \frac{\int \frac{-b-cx}{a+bx+cx^2} dx}{a} \\
 &= \frac{\log(x)}{a} - \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2a} - \frac{b \int \frac{1}{a+bx+cx^2} dx}{2a} \\
 &= \frac{\log(x)}{a} - \frac{\log(a + bx + cx^2)}{2a} + \frac{b \operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx\right)}{a} \\
 &= \frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a + bx + cx^2)}{2a}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 0.98

$$\frac{2b \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \log(a + x(b + cx)) - 2 \log(x)$$

$$2a$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)*x^3),x]

[Out] -1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*Log[x] + Log[a + x*(b + c*x)])/a

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^3),x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^3), x]

fricas [A] time = 1.41, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) - (b^2 - 4ac) \log(cx^2 + bx + a) + 2(b^2 - 4ac) \log(x)}{2(ab^2 - 4a^2c)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) - (b^2 - 4ac) \log(cx^2 + bx + a) + 2(b^2 - 4ac) \log(x)}{2(ab^2 - 4a^2c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="fricas")

[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]

giac [A] time = 0.30, size = 62, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac} a} - \frac{\log(cx^2 + bx + a)}{2a} + \frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="giac")

[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(c*x^2 + b*x + a)/a + log(abs(x))/a

maple [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} a} + \frac{\ln(x)}{a} - \frac{\ln(cx^2+bx+a)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)/x^3,x)

[Out] 1/a*ln(x)-1/2*ln(c*x^2+b*x+a)/a-1/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.72, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{a} - \ln\left(bc - (x(6ac^2 - 2b^2c) - abc)\left(\frac{1}{2a} - \frac{b\sqrt{b^2 - 4ac}}{2(a^2b^2 - 4a^2c)}\right) + 3c^2x\right)\left(\frac{1}{2a} - \frac{b\sqrt{b^2 - 4ac}}{2(a^2b^2 - 4a^2c)}\right) - \ln\left(x(6ac^2 - 2b^2c) - abc\right)\left(\frac{1}{2a} + \frac{b\sqrt{b^2 - 4ac}}{2(a^2b^2 - 4a^2c)}\right) - bc - 3c^2x\right)\left(\frac{1}{2a} + \frac{b\sqrt{b^2 - 4ac}}{2(a^2b^2 - 4a^2c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(c + a/x^2 + b/x)),x)

[Out] log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))) + 3*c^2*x*(1/(2*a) - (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))) - b*c - 3*c^2*x*(1/(2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))))

sympy [B] time = 4.08, size = 564, normalized size = 9.10

$$\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) \ln\left(\frac{24a^2c^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right)^2 - 14a^2b^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) - 12a^2c^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) + 2a^2b^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) + 3a^2b^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) - 12a^2c^2 + 11a^2b^2 - 2b^4}{9ab^2 - 2b^3c}\right) + \left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) \ln\left(\frac{24a^2c^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right)^2 - 14a^2b^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) - 12a^2c^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) + 2a^2b^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) + 3a^2b^2\left(\frac{b\sqrt{4ac-b^2}}{2(4ac-b^2)}\right) - 12a^2c^2 + 11a^2b^2 - 2b^4}{9ab^2 - 2b^3c}\right) + \frac{\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)/x**3,x)

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```


$$3.358 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^4} dx$$

Optimal. Leaf size=81

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx + cx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{ax}$$

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx + cx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)*x^4),x]

[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)/(a + bx + cx^2)}{x} dx$; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 709

$\int \frac{(d + ex)^m}{(a + bx + cx^2)} dx$:> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

$\int \frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)} dx$:> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1354

$\int (x^m (a + cx^{2n}) + b x^n)^p dx$:> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^4} dx &= \int \frac{1}{x^2(a + bx + cx^2)} dx \\
&= -\frac{1}{ax} + \frac{\int \frac{-b-cx}{x(a+bx+cx^2)} dx}{a} \\
&= -\frac{1}{ax} + \frac{\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right) dx}{a} \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx}{a^2} \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \int \frac{b+2cx}{a+bx+cx^2} dx}{2a^2} + \frac{(b^2 - 2ac) \int \frac{1}{a+bx+cx^2} dx}{2a^2} \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx + cx^2)}{2a^2} - \frac{(b^2 - 2ac) \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx\right)}{a^2} \\
&= -\frac{1}{ax} - \frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2 \sqrt{b^2 - 4ac}} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx + cx^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.95

$$\frac{2(b^2-2ac) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{b \log(a + x(b + cx)) - \frac{2a}{x} - 2b \log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)*x^4), x]

[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)]/(2*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^4), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^4), x]

fricas [A] time = 0.89, size = 269, normalized size = 3.32

$$\left[\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac}x \log\left(\frac{2x^2 + 2bx + a^2 - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + 2ab^2 - 8a^2c - (b^3 - 4abc)x \log(cx^2 + bx + a) + 2(b^3 - 4abc)x \log(x)}{2(a^2b^2 - 4a^3c)x}, \frac{2(b^2 - 2ac)\sqrt{-b^2 + 4ac}x \arctan\left(\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + 2ab^2 - 8a^2c - (b^3 - 4abc)x \log(cx^2 + bx + a) + 2(b^3 - 4abc)x \log(x)}{2(a^2b^2 - 4a^3c)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="fricas")

[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*a^3*c)*x)]

giac [A] time = 0.39, size = 79, normalized size = 0.98

$$\frac{b \log(cx^2 + bx + a)}{2a^2} - \frac{b \log(|x|)}{a^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac} a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="giac")

[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)

maple [A] time = 0.01, size = 112, normalized size = 1.38

$$-\frac{2c \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2} a} + \frac{b^2 \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2} a^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln(cx^2 + bx + a)}{2a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)/x^4,x)

[Out] -1/a/x-1/a^2*b*ln(x)+1/2*b*ln(c*x^2+b*x+a)/a^2-2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c+1/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.81, size = 339, normalized size = 4.19

$$\frac{\ln(2ab^3+2b^3x-2ab^2\sqrt{b^2-4ac}+a^2c\sqrt{b^2-4ac}-2b^2x\sqrt{b^2-4ac}+2a^2x^2-7a^2bc-8ab^2cx+4ab^2cx\sqrt{b^2-4ac})\left(\frac{1}{2}+\frac{1}{2}\frac{\sqrt{b^2-4ac}}{b}\right)-\frac{1}{2x}\frac{\ln(2ab^3+2b^3x+2ab^2\sqrt{b^2-4ac}-a^2c\sqrt{b^2-4ac}+2b^2x\sqrt{b^2-4ac}+2a^2x^2-7a^2bc-8ab^2cx-4ab^2cx\sqrt{b^2-4ac})\left(\frac{1}{2}-\frac{1}{2}\frac{\sqrt{b^2-4ac}}{b}\right)+\frac{1}{2}\frac{\ln(c)}{b}}{4b^3c-a^2b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(c + a/x^2 + b/x)),x)

[Out] $(\log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^{(1/2)} + a^2*c*(b^2 - 4*a*c)^{(1/2)} - 2*b^3*x*(b^2 - 4*a*c)^{(1/2)} + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x + 4*a*b*c*x*(b^2 - 4*a*c)^{(1/2}))*a*(2*b*c - c*(b^2 - 4*a*c)^{(1/2})) - b^3/2 + (b^2*(b^2 - 4*a*c)^{(1/2}))/2)/(4*a^3*c - a^2*b^2) - 1/(a*x) - (\log(2*a*b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^{(1/2)} - a^2*c*(b^2 - 4*a*c)^{(1/2)} + 2*b^3*x*(b^2 - 4*a*c)^{(1/2)} + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x - 4*a*b*c*x*(b^2 - 4*a*c)^{(1/2}))*a*(2*b*c + c*(b^2 - 4*a*c)^{(1/2})) + (b^2*(b^2 - 4*a*c)^{(1/2}))/2)/(4*a^3*c - a^2*b^2) - (b*\log(x))/a^2$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)/x**4,x)

[Out] Timed out

$$3.359 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^5} dx$$

Optimal. Leaf size=104

$$-\frac{(b^2 - ac) \log(a + bx + cx^2)}{2a^3} + \frac{\log(x)(b^2 - ac)}{a^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Rubi [A] time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - ac) \log(a + bx + cx^2)}{2a^3} + \frac{\log(x)(b^2 - ac)}{a^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)*x^5), x]

[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^3)

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^5} dx &= \int \frac{1}{x^3(a + bx + cx^2)} dx \\
&= -\frac{1}{2ax^2} + \frac{\int \frac{-b-cx}{x^2(a+bx+cx^2)} dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right) dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2} dx}{a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac)) \int \frac{1}{a+bx+cx^2} dx}{2a^3} - \frac{(b^2-ac) \int \frac{b+2cx}{a+bx+cx^2} dx}{2a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} + \frac{(b(b^2-3ac)) \operatorname{Subst}\left(\int \frac{1}{u} du, a+bx+cx^2\right)}{2a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2} + 2\log(x)(b^2 - ac) + (ac - b^2)\log(a + x(b + cx)) - \frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + \frac{2ab}{x}}{2a^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)*x^5),x]

[Out] $(-a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*\operatorname{ArcTan}[(b + 2*c*x)/\operatorname{Sqrt}[-b^2 + 4*a*c]])/\operatorname{Sqrt}[-b^2 + 4*a*c] + 2*(b^2 - a*c)*\operatorname{Log}[x] + (-b^2 + a*c)*\operatorname{Log}[a + x*(b + c*x)]/(2*a^3)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^5), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^5), x]

fricas [A] time = 1.36, size = 358, normalized size = 3.44

$$\frac{(b^3 - 3abc)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2 + 2bax + 2\sqrt{b^2 - 4ac}cx}{c^2 + bx + a}\right) + a^2b^2 - 4a^2c + (b^4 - 5ab^2c + 4a^2c^2)^2 \log(cx^2 + bx + a) - 2(b^4 - 5ab^2c + 4a^2c^2)^2 \log(x) - 2(a^3 - 4a^2bc) \sqrt{b^2 - 4ac} \arctan\left(\frac{\sqrt{b^2 - 4ac}cx}{c^2 + bx + a}\right) - a^2b^2 + 4a^2c - (b^4 - 5ab^2c + 4a^2c^2)^2 \log(cx^2 + bx + a) + 2(b^4 - 5ab^2c + 4a^2c^2)^2 \log(x) + 2(a^3 - 4a^2bc) \sqrt{b^2 - 4ac} \arctan\left(\frac{\sqrt{b^2 - 4ac}cx}{c^2 + bx + a}\right)}{2(a^2b^2 - 4a^2c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="fricas")

[Out] $[-1/2*((b^3 - 3*a*b*c)*\text{sqrt}(b^2 - 4*a*c)*x^2*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - \text{sqrt}(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*\log(c*x^2 + b*x + a) - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*\log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*\text{sqrt}(-b^2 + 4*a*c)*x^2*\arctan(-\text{sqrt}(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*\log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*\log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]$

giac [A] time = 0.31, size = 105, normalized size = 1.01

$$-\frac{(b^2 - ac) \log(cx^2 + bx + a)}{2a^3} + \frac{(b^2 - ac) \log(|x|)}{a^3} - \frac{(b^3 - 3abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac} a^3} + \frac{2abx - a^2}{2a^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="giac")

[Out] $-1/2*(b^2 - a*c)*\log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*\log(\text{abs}(x))/a^3 - ((b^3 - 3*a*b*c)*\arctan((2*c*x + b)/\text{sqrt}(-b^2 + 4*a*c))/(\text{sqrt}(-b^2 + 4*a*c)*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2))$

maple [A] time = 0.01, size = 150, normalized size = 1.44

$$\frac{3bc \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} a^2} - \frac{b^3 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} a^3} - \frac{c \ln(x)}{a^2} + \frac{c \ln(cx^2 + bx + a)}{2a^2} + \frac{b^2 \ln(x)}{a^3} - \frac{b^2 \ln(cx^2 + bx + a)}{2a^3} + \frac{b}{a^2 x} - \frac{1}{2a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)/x^5,x)

[Out] $-1/2/a/x^2 - 1/a^2*\ln(x)*c + 1/a^3*\ln(x)*b^2 + b/a^2/x + 1/2/a^2*c*\ln(c*x^2 + b*x + a) - 1/2/a^3*\ln(c*x^2 + b*x + a)*b^2 + 3/a^2/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x + b)/(4*a*c - b^2)^{(1/2)})*b*c - 1/a^3/(4*a*c - b^2)^{(1/2)}*\arctan((2*c*x + b)/(4*a*c - b^2)^{(1/2)})*b^3$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.87, size = 447, normalized size = 4.30

$$\frac{\ln(2a^2 + 2b^2 + 4a^2\sqrt{c} + 2a^2\sqrt{c^2} + 2a^2\sqrt{c^3} - 9a^2c - 10a^2bc - 3a^2b^2\sqrt{c} + 3a^2b^2\sqrt{c^2} - 6a^2b^2\sqrt{c^3})}{4a^2c - 2a^2b^2} \left(\frac{1}{x} - \frac{10a^2bc}{4a^2c - 2a^2b^2} \right) - \frac{\ln(2a^2 + 2b^2 + 4a^2\sqrt{c} - 2a^2\sqrt{c^2} - 2a^2\sqrt{c^3} - 9a^2c - 10a^2bc + 3a^2b^2\sqrt{c} + 9a^2b^2\sqrt{c^2} + 6a^2b^2\sqrt{c^3})}{4a^2c - 2a^2b^2} \left(\frac{1}{x} - \frac{10a^2bc}{4a^2c - 2a^2b^2} \right) - \frac{1}{2a^2} \ln\left(\frac{c-x}{c+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(c + a/x^2 + b/x)),x)

[Out] $(\log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^{(1/2)} + 2*b^4*x*(b^2 - 4*a*c)^{(1/2)} - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c)^{(1/2)} + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^{(1/2)} - 6*a*b^2*c*x*(b^2 - 4*a*c)^{(1/2)})*(b^{4/2} - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^{(1/2)}/2) + (b^3*(b^2 - 4*a*c)^{(1/2)}/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (\log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^{(1/2)} - 2*b^4*x*(b^2 - 4*a*c)^{(1/2)} - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^{(1/2)} + 9*a^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^{(1/2)} + 6*a*b^2*c*x*(b^2 - 4*a*c)^{(1/2})))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^{(1/2)}/2) - b^{4/2} + (b^3*(b^2 - 4*a*c)^{(1/2)}/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (b*x)/a^2)/x^2 - (\log(x)*(a*c - b^2))/a^3$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)/x**5,x)

[Out] Timed out

$$3.360 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^6} dx$$

Optimal. Leaf size=137

$$\frac{b(b^2 - 2ac) \log(a + bx + cx^2)}{2a^4} - \frac{b \log(x)(b^2 - 2ac)}{a^4} - \frac{b^2 - ac}{a^3x} + \frac{b}{2a^2x^2} - \frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1354, 709, 800, 634, 618, 206, 628}

$$-\frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} + \frac{b(b^2 - 2ac) \log(a + bx + cx^2)}{2a^4} - \frac{b^2 - ac}{a^3x} - \frac{b \log(x)(b^2 - 2ac)}{a^4} + \frac{b}{2a^2x^2} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)*x^6), x]

[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_)^m)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^m*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^6} dx &= \int \frac{1}{x^4(a + bx + cx^2)} dx \\
&= -\frac{1}{3ax^3} + \frac{\int \frac{-b-cx}{x^3(a+bx+cx^2)} dx}{a} \\
&= -\frac{1}{3ax^3} + \frac{\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right) dx}{a} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2} dx}{a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{(b(b^2-2ac)) \int \frac{b+2cx}{a+bx+cx^2} dx}{2a^4} + \frac{(b^4-3ab^2c+a^2c^2)}{2a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4} - \frac{(b^4-3ab^2c+a^2c^2)}{2a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b(b^2-2ac)\log(a+bx+cx^2)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 131, normalized size = 0.96

$$\frac{-\frac{2a^3}{x^3} + \frac{6(2a^2c^2-4ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + \frac{3a^2b}{x^2} - 6\log(x)(b^3-2abc) + 3(b^3-2abc)\log(a+x(b+cx)) + \frac{6a(ac-b^2)}{x}}{6a^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)*x^6),x]

[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)]/(6*a^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^6),x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)*x^6), x]

fricas [A] time = 1.47, size = 445, normalized size = 3.25

$$\frac{3(b^4 - 4ab^2c + 2a^2c^2)\sqrt{-b^2 + 4ac} \log\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right) - 2a^3b^2c + 3(b^5 - 6ab^3c + 8a^2b^2c^2) \log(cx^2 + bx + a) - 6(b^5 - 6ab^3c + 8a^2b^2c^2) \log(x) - 6(a^2b^3c - 4a^3b^2c^2) \log(x) + 6(a^2b^3c - 4a^3b^2c^2) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right) + 2a^3b^2c - 8a^4c - 3(b^5 - 6ab^3c + 8a^2b^2c^2) \log(cx^2 + bx + a) + 6(b^5 - 6ab^3c + 8a^2b^2c^2) \log(x) + 6(a^2b^3c - 4a^3b^2c^2) \log(x) - 3(a^2b^3c - 4a^3b^2c^2) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{6(b^5 - 4a^2b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="fricas")

[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]

giac [A] time = 0.38, size = 136, normalized size = 0.99

$$\frac{(b^3 - 2abc) \log(cx^2 + bx + a)}{2a^4} - \frac{(b^3 - 2abc) \log(|x|)}{a^4} + \frac{(b^4 - 4ab^2c + 2a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}a^4} + \frac{3a^2bx - 2a^3 - 6(ab^2 - a^2c)x^2}{6a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="giac")

[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/ (sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/ (a^4*x^3)

maple [A] time = 0.01, size = 214, normalized size = 1.56

$$\frac{2c^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}a^2} - \frac{4b^2c \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}a^3} + \frac{b^4 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}a^4} + \frac{2bc \ln(x)}{a^3} - \frac{bc \ln(cx^2 + bx + a)}{a^3} - \frac{b^3 \ln(x)}{a^4} + \frac{b^3 \ln(cx^2 + bx + a)}{2a^4} + \frac{c}{a^2x} - \frac{b^2}{a^3x} + \frac{b}{2a^2x^2} - \frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)/x^6,x)

[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/2*b/a^2/x^2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3+2/a^2/(4*a*c-b^2)^2

$$\frac{1}{2} \arctan\left(\frac{2cx+b}{4ac-b^2}\right)^{1/2} \cdot c^2 - 4/a^3 / (4ac-b^2)^{1/2} \arctan\left(\frac{2cx+b}{4ac-b^2}\right)^{1/2} \cdot b^2c + 1/a^4 / (4ac-b^2)^{1/2} \arctan\left(\frac{2cx+b}{4ac-b^2}\right)^{1/2} \cdot b^4$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.92, size = 524, normalized size = 3.82

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(c + a/x^2 + b/x)),x)

[Out] $\log(2ab^4(b^2 - 4ac)^{1/2} - 2b^6x - 2ab^5 + 2b^5x(b^2 - 4ac)^{1/2} + 11a^2b^3c - 13a^3b^2c^2 + 2a^3c^3x + a^3c^2(b^2 - 4ac)^{1/2} - 17a^2b^2c^2x + 12ab^4cx - 5a^2b^2c(b^2 - 4ac)^{1/2} - 8ab^3cx(b^2 - 4ac)^{1/2} + 7a^2b^2cx(b^2 - 4ac)^{1/2})(b^3/(2a^4) - (b^2(b^2 - 4ac)^{1/2})/(2a^4) - (bc)/a^3 + (a^2c^2(b^2 - 4ac)^{1/2})/(4a^5c - a^4b^2)) + \log(2ab^5 + 2b^6x + 2ab^4(b^2 - 4ac)^{1/2} + 2b^5x(b^2 - 4ac)^{1/2} - 11a^2b^3c + 13a^3b^2c^2 - 2a^3c^3x + a^3c^2(b^2 - 4ac)^{1/2} + 17a^2b^2c^2x - 12ab^4cx - 5a^2b^2c(b^2 - 4ac)^{1/2} - 8ab^3cx(b^2 - 4ac)^{1/2} + 7a^2b^2cx(b^2 - 4ac)^{1/2})(b^3/(2a^4) + (b^2(b^2 - 4ac)^{1/2})/(2a^4) - (bc)/a^3 - (a^2c^2(b^2 - 4ac)^{1/2})/(4a^5c - a^4b^2)) + ((x^2(ac - b^2))/a^3 - 1/(3a) + (bx)/(2a^2))/x^3 + (b \log(x)(2ac - b^2))/a^4$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)/x**6,x)

[Out] Timed out

$$3.361 \quad \int \frac{x}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} dx$$

Optimal. Leaf size=196

$$\frac{b(30a^2c^2 - 20ab^2c + 3b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) + (3b^2 - 2ac) \log(a + bx + cx^2) - \frac{bx(3b^2 - 11ac)}{c^3(b^2 - 4ac)} + \frac{x^2(3b^2 - 8ac)}{2c^2(b^2 - 4ac)}}{c^4(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.20, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1354, 738, 800, 634, 618, 206, 628}

$$\frac{b(30a^2c^2 - 20ab^2c + 3b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) + \frac{x^2(3b^2 - 8ac)}{2c^2(b^2 - 4ac)} + \frac{(3b^2 - 2ac) \log(a + bx + cx^2)}{2c^4} - \frac{bx(3b^2 - 11ac)}{c^3(b^2 - 4ac)} + \frac{x^4(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{bx^3}{c(b^2 - 4ac)}}{c^4(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x/(c + a/x^2 + b/x)^2, x]

[Out] -((b*(3*b^2 - 11*a*c)*x)/(c^3*(b^2 - 4*a*c))) + ((3*b^2 - 8*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634


```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} dx &= \int \frac{x^5}{(a + bx + cx^2)^2} dx \\
&= \frac{x^4(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{x^3(8a+3bx)}{a+bx+cx^2} dx}{-b^2 + 4ac} \\
&= \frac{x^4(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \left(\frac{b(3b^2-11ac)}{c^3} - \frac{(3b^2-8ac)x}{c^2} + \frac{3bx^2}{c} - \frac{ab(3b^2-11ac)+(b^2-4ac)(3b^2-2ac)x}{c^3(a+bx+cx^2)} \right) dx}{-b^2 + 4ac} \\
&= -\frac{b(3b^2 - 11ac)x}{c^3(b^2 - 4ac)} + \frac{(3b^2 - 8ac)x^2}{2c^2(b^2 - 4ac)} - \frac{bx^3}{c(b^2 - 4ac)} + \frac{x^4(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{ab(3b^2-2ac)x}{c^3(a+bx+cx^2)} dx}{-b^2 + 4ac} \\
&= -\frac{b(3b^2 - 11ac)x}{c^3(b^2 - 4ac)} + \frac{(3b^2 - 8ac)x^2}{2c^2(b^2 - 4ac)} - \frac{bx^3}{c(b^2 - 4ac)} + \frac{x^4(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{(3b^2 - 2ac)x^4}{c^3(b^2 - 4ac)(a + bx + cx^2)} \\
&= -\frac{b(3b^2 - 11ac)x}{c^3(b^2 - 4ac)} + \frac{(3b^2 - 8ac)x^2}{2c^2(b^2 - 4ac)} - \frac{bx^3}{c(b^2 - 4ac)} + \frac{x^4(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{(3b^2 - 2ac)x^4}{c^3(b^2 - 4ac)(a + bx + cx^2)} \\
&= -\frac{b(3b^2 - 11ac)x}{c^3(b^2 - 4ac)} + \frac{(3b^2 - 8ac)x^2}{2c^2(b^2 - 4ac)} - \frac{bx^3}{c(b^2 - 4ac)} + \frac{x^4(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{b(3b^4 - 2ac^2)}{c^3(b^2 - 4ac)(a + bx + cx^2)}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 163, normalized size = 0.83

$$\frac{2b(30a^2c^2 - 20ab^2c + 3b^4) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + \frac{2(2a^3c^2 + a^2bc(5cx-4b) + ab^3(b-5cx) + b^5x)}{(b^2-4ac)(a+x(b+cx))} + (3b^2 - 2ac) \log(a + x(b + cx)) - 4bcx + c^2x^2}{2c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x/(c + a/x^2 + b/x)^2,x]

[Out] (-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x)))/(b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (3*b^2 - 2*a*c)*Log[a + x*(b + c*x)]/(2*c^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(c + a/x^2 + b/x)^2,x]

[Out] IntegrateAlgebraic[x/(c + a/x^2 + b/x)^2, x]

fricas [B] time = 1.33, size = 1029, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="fricas")

[Out] [1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 - (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*log(c*x^2 + b*x + a)/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x), 1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 + 2*(3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*log(c*x^2 + b*x + a)/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x)]

giac [A] time = 0.33, size = 188, normalized size = 0.96

$$-\frac{(3b^5 - 20ab^3c + 30a^2bc^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2c^4 - 4ac^5)\sqrt{-b^2+4ac}} + \frac{(3b^2 - 2ac) \log(cx^2 + bx + a)}{2c^4} + \frac{c^2x^2 - 4bcx}{2c^4} + \frac{ab^4 - 4a^2b^2c + 2a^3c^2 + (b^5 - 5ab^3c + 5a^2bc^2)x}{(cx^2 + bx + a)(b^2 - 4ac)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="giac")

[Out] $-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c}) / ((b^2*c^4 - 4*a*c^5)*\sqrt{-b^2 + 4*a*c}) + 1/2*(3*b^2 - 2*a*c)*\log(c*x^2 + b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 + (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^4)$

maple [B] time = 0.01, size = 434, normalized size = 2.21

$$\frac{5a^2bx}{(c^2+bx+a)(4ac-b^2)^2} + \frac{30a^2b\arctan\left(\frac{2bx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c^2} + \frac{5a^2b^2}{(c^2+bx+a)(4ac-b^2)^2} - \frac{20ab^3\arctan\left(\frac{2bx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c^2} - \frac{b^5}{(c^2+bx+a)(4ac-b^2)^2} + \frac{30b^5\arctan\left(\frac{2bx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c^2} - \frac{2a^2}{(c^2+bx+a)(4ac-b^2)^2} + \frac{4a^2b^2}{(c^2+bx+a)(4ac-b^2)^2} - \frac{4a^2\ln(c^2+bx+a)}{(4ac-b^2)^2c^2} - \frac{a^4}{(c^2+bx+a)(4ac-b^2)^2} + \frac{7a^2\ln(c^2+bx+a)}{(4ac-b^2)^2c^2} - \frac{30^4\ln(c^2+bx+a)}{2(4ac-b^2)^2c^4} + \frac{x^2}{2c^2} - \frac{2bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c+a/x^2+b/x)^2,x)

[Out] $1/2/c^2*x^2-2/c^3*b*x-5/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b*x*a^2+5/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^3*x*a-1/c^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^5*x-2/c^2/(c*x^2+b*x+a)*a^3/(4*a*c-b^2)+4/c^3/(c*x^2+b*x+a)*a^2/(4*a*c-b^2)*b^2-1/c^4/(c*x^2+b*x+a)*a/(4*a*c-b^2)*b^4-4/c^2/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*a^2+7/c^3/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*a*b^2-3/2/c^4/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^4+30/c^2/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b-20/c^3/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3+3/c^4/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.82, size = 382, normalized size = 1.95

$$\frac{x^2}{2c^2} - \frac{a(2a^2c^2-4ab^2c+b^4)}{c(4ac-b^2)} + \frac{bx(5a^2c^2-5ab^2c+b^4)}{c(4ac-b^2)} - \frac{\ln(cx^2+bx+a)}{2(64a^3c^2-48a^2b^2c^2+12ab^4c^2-b^6c^4)} - \frac{2bx}{c^3} + \frac{b \operatorname{atan}\left(\frac{c^4\left(\frac{2bx(30a^2c^2-20a^2c^2+3b^4)}{c^3(4ac-b^2)^2} - \frac{b(3c^2-4ab^2c^2)}{30a^2b^2-20a^2b^2c+3b^4}\right)\sqrt{4ac-b^2}}{30a^2b^2-20a^2b^2c+3b^4}\right)}{c^4(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(c + a/x^2 + b/x)^2,x)`

[Out] $x^2/(2c^2) - ((a(b^4 + 2a^2c^2 - 4ab^2c))/(c(4ac - b^2)) + (bx(b^4 + 5a^2c^2 - 5ab^2c))/(c(4ac - b^2)))/(ac^3 + c^4x^2 + bc^3x) - (\log(a + bx + cx^2)(3b^8 + 128a^4c^4 + 168a^2b^4c^2 - 288a^3b^2c^3 - 38ab^6c))/(2(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6)) - (2bx)/c^3 + (b \operatorname{atan}((c^4((2bx(3b^4 + 30a^2c^2 - 20ab^2c))/c^3(4ac - b^2)^3) - (b(b^3c^3 - 4abc^4)(3b^4 + 30a^2c^2 - 20ab^2c))/(c^7(4ac - b^2)^4))(4ac - b^2)^{5/2})/(3b^5 + 30a^2bc^2 - 20ab^3c))(3b^4 + 30a^2c^2 - 20ab^2c))/(c^4(4ac - b^2)^{3/2})$

sympy [B] time = 2.37, size = 1012, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c+a/x**2+b/x)**2,x)`

[Out] $-2bx/c^3 + (-b\sqrt{-(4ac - b^2)^3}(30a^2c^2 - 20ab^2c + 3b^4)/(2c^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (2ac - 3b^2)/(2c^4))\log(x + (16a^3c^2 - 17a^2b^2c + 16a^2c^5(-b\sqrt{-(4ac - b^2)^3}(30a^2c^2 - 20ab^2c + 3b^4)/(2c^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (2ac - 3b^2)/(2c^4)) + 3ab^4 - 8ab^2c^4(-b\sqrt{-(4ac - b^2)^3}(30a^2c^2 - 20ab^2c + 3b^4)/(2c^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (2ac - 3b^2)/(2c^4)) + b^4c^3(-b\sqrt{-(4ac - b^2)^3}(30a^2c^2 - 20ab^2c + 3b^4)/(2c^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (2ac - 3b^2)/(2c^4)))/(30a^2bc^2 - 20ab^3c + 3b^5)) + (b\sqrt{-(4ac - b^2)^3}(30a^2c^2 - 20ab^2c + 3b^4)/(2c^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (2ac - 3b^2)/(2c^4))\log(x + (16a^3c^2 - 17a^2b^2c + 16a^2c^5(b\sqrt{-(4ac - b^2)^3}(30a^2c^2 - 20ab^2c + 3b^4)/(2c^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (2ac - 3b^2)/(2c^4)) + 3ab^4 - 8ab^2c^4(b\sqrt{-(4ac - b^2)^3}(30a^2c^2 - 20ab^2c + 3b^4)/(2c^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (2ac - 3b^2)/(2c^4)) + b^4c^3(b\sqrt{-(4ac - b^2)^3}(30a^2c^2 - 20ab^2c + 3b^4)/(2c^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (2ac - 3b^2)/(2c^4)))/(30a^2bc^2 - 20ab^3c + 3b^5)) + (-2a^3c^2 + 4a^2b^2c - ab^4 + x(-5a^2bc^2 + 5ab^3c - b^5))/(4a^2c^5 - ab^2c^4 + x^2(4ac^6 - b^2c^5) + x(4ab^3c^5 - b^3c^4)) + x^2/(2c^2)$

$$3.362 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} dx$$

Optimal. Leaf size=150

$$-\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{2x(b^2-3ac)}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{b \log(a+bx+cx^2)}{c^3}$$

Rubi [A] time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1340, 738, 800, 634, 618, 206, 628}

$$-\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{2x(b^2-3ac)}{c^2(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{bx^2}{c(b^2-4ac)} - \frac{b \log(a+bx+cx^2)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(c + a/x^2 + b/x)^(-2), x]

[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x + c*x^2])/c^3

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} dx &= \int \frac{x^4}{(a + bx + cx^2)^2} dx \\
&= \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2} dx}{-b^2 + 4ac} \\
&= \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \left(-\frac{2(b^2-3ac)}{c^2} + \frac{2bx}{c} + \frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)} \right) dx}{-b^2 + 4ac} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2 \int \frac{a(b^2-3ac)+b(b^2-4ac)x}{a+bx+cx^2} dx}{c^2(b^2 - 4ac)} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{b \int \frac{b+2cx}{a+bx+cx^2} dx}{c^3} + \frac{(b^4 - 6ab^2c)}{c^3} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{b \log(a + bx + cx^2)}{c^3} - \frac{(2(b^4 - 6ab^2c)) \tanh^{-1}}{c^3(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 132, normalized size = 0.88

$$\frac{-\frac{2(6a^2c^2-6ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + \frac{a^2c(3b-2cx)-ab^2(b-4cx)+b^4(-x)}{(b^2-4ac)(a+x(b+cx))} - b \log(a + x(b + cx)) + cx}{c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + a/x^2 + b/x)^(-2), x]

[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)]/c^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-2), x]

[Out] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-2), x]

fricas [B] time = 1.62, size = 837, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} &[-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) \\ &)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6 \\ &*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2 \\ &*b*c^2)*x]*\sqrt{b^2 - 4*a*c}*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c} \\ &*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2 \\ &*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a* \\ &b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*\log(c*x \\ &^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2 \\ &*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5 \\ &- 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 - \\ &(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c \\ &^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2) \\ &)*x]*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*c*x + b)/(b^2 - 4*a*c \\ &)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3 \\ &*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b \\ &^4*c + 16*a^2*b^2*c^2)*x)*\log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 \\ &+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3 \\ &c^4 + 16*a^2*b*c^5)*x)] \end{aligned}$$

giac [A] time = 0.37, size = 161, normalized size = 1.07

$$\frac{2(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) + \frac{x}{c^2} - \frac{b \log(cx^2 + bx + a)}{c^3} - \frac{(b^4 - 4ab^2c + 2a^2c^2)x}{c} + \frac{ab^3 - 3a^2bc}{c}}{(b^2c^3 - 4ac^4)\sqrt{-b^2 + 4ac}} - \frac{(b^4 - 4ab^2c + 2a^2c^2)*x}{c} + \frac{(a*b^3 - 3*a^2*b*c)/c}{(c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} &2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2 \\ &*c^3 - 4*a*c^4)*\sqrt{-b^2 + 4*a*c}) + x/c^2 - b*\log(c*x^2 + b*x + a)/c^3 - \\ &((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x \\ &+ a)*(b^2 - 4*a*c)*c^2) \end{aligned}$$

maple [B] time = 0.01, size = 352, normalized size = 2.35

$$\frac{2a^2x}{(cx^2+bx+a)(4ac-b^2)c} - \frac{12a^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c} - \frac{4ab^2x}{(cx^2+bx+a)(4ac-b^2)c^2} + \frac{12ab^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c^2} + \frac{b^4x}{(cx^2+bx+a)(4ac-b^2)c^3} - \frac{2b^4 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c^3} - \frac{3a^2b}{(cx^2+bx+a)(4ac-b^2)c^2} + \frac{ab^3}{(cx^2+bx+a)(4ac-b^2)c^3} - \frac{4ab \ln(cx^2+bx+a)}{(4ac-b^2)c^2} + \frac{b^3 \ln(cx^2+bx+a)}{(4ac-b^2)c^3} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^2,x)

[Out] $x/c^2 + 2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2 - 4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b^2 + 1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4 - 3/c^2/(c*x^2+b*x+a)*a^2*b/(4*a*c-b^2) + 1/c^3/(c*x^2+b*x+a)*a*b^3/(4*a*c-b^2) - 4/c^2/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*a*b + 1/c^3/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^3 - 12/c/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a^2 + 12/c^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^2 - 2/c^3/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^4$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.80, size = 261, normalized size = 1.74

$$\frac{x}{c^2} + \frac{a(b^3-3abc) + x(2a^2c^2-4ab^2c+b^4)}{c^3x^2+b^2cx+ac^2} + \frac{\ln(cx^2+bx+a)(-128a^3bc^3+96a^2b^3c^2-24ab^5c+2b^7)}{2(64a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3)} - \frac{2 \operatorname{atan}\left(\frac{2cx}{\sqrt{4ac-b^2}} - \frac{b^3c^2-4abc^3}{c^2(4ac-b^2)^{3/2}}\right)(6a^2c^2-6ab^2c+b^4)}{c^3(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c + a/x^2 + b/x)^2,x)

[Out] $x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (\log(a + b*x + c*x^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*\operatorname{atan}((2*c*x)/(4*a*c - b^2)^{(1/2)} - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^{(3/2)})))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^2)^{(3/2)})$

sympy [B] time = 1.73, size = 842, normalized size = 5.61

$$\frac{\frac{1}{2} \sqrt{\frac{4ac-b^2}{(4ac-b^2)^2-4a^2c^2}} (4ac-b^2+a)}{\frac{1}{2} \sqrt{\frac{4ac-b^2}{(4ac-b^2)^2-4a^2c^2}}} \left(\frac{-128a^3bc^3-128a^2b^3c^2-24ab^5c+2b^7}{128a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3} + \frac{2 \operatorname{atan}\left(\frac{2cx}{\sqrt{4ac-b^2}} - \frac{b^3c^2-4abc^3}{c^2(4ac-b^2)^{3/2}}\right)(6a^2c^2-6ab^2c+b^4)}{c^3(4ac-b^2)^{3/2}} \right) + \frac{x}{c^2} + \frac{a(b^3-3abc) + x(2a^2c^2-4ab^2c+b^4)}{c^3x^2+b^2cx+ac^2} + \frac{\ln(cx^2+bx+a)(-128a^3bc^3+96a^2b^3c^2-24ab^5c+2b^7)}{2(64a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**2,x)

[Out]
$$\begin{aligned} & (-b/c^{**3} - \sqrt{-(4*a*c - b^{**2})}^{**3}) * (6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4}) / (c^{**3} \\ & * (64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) * \log(x + (-10*a^{**2}*b*c - 16*a^{**2}*c^{**4} \\ & * (-b/c^{**3} - \sqrt{-(4*a*c - b^{**2})}^{**3}) * (6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4}) / (c^{**3} * (64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) \\ & + 2*a*b^{**3} + 8*a*b^{**2}*c^{**3} * (-b/c^{**3} - \sqrt{-(4*a*c - b^{**2})}^{**3}) * (6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4}) / (c^{**3} * (64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) \\ & - b^{**4}*c^{**2} * (-b/c^{**3} - \sqrt{-(4*a*c - b^{**2})}^{**3}) * (6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4}) / (c^{**3} * (64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) \\ & / (12*a^{**2}*c^{**2} - 12*a*b^{**2}*c + 2*b^{**4}) + (-b/c^{**3} + \sqrt{-(4*a*c - b^{**2})}^{**3}) * (6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4}) / (c^{**3} * (64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) \\ & * \log(x + (-10*a^{**2}*b*c - 16*a^{**2}*c^{**4} * (-b/c^{**3} + \sqrt{-(4*a*c - b^{**2})}^{**3}) * (6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4}) / (c^{**3} * (64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) \\ & + 2*a*b^{**3} + 8*a*b^{**2}*c^{**3} * (-b/c^{**3} + \sqrt{-(4*a*c - b^{**2})}^{**3}) * (6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4}) / (c^{**3} * (64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) \\ & - b^{**4}*c^{**2} * (-b/c^{**3} + \sqrt{-(4*a*c - b^{**2})}^{**3}) * (6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4}) / (c^{**3} * (64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) \\ & / (12*a^{**2}*c^{**2} - 12*a*b^{**2}*c + 2*b^{**4}) + (-3*a^{**2}*b*c + a*b^{**3} + x*(2*a^{**2}*c^{**2} - 4*a*b^{**2}*c + b^{**4})) / (4*a^{**2}*c^{**4} - a*b^{**2}*c^{**3} + x**2*(4*a*c^{**5} - b^{**2}*c^{**4}) \\ & + x*(4*a*b*c^{**4} - b^{**3}*c^{**3})) + x/c^{**2} \end{aligned}$$

$$3.363 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x} dx$$

Optimal. Leaf size=114

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)^{3/2}} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{bx}{c(b^2 - 4ac)} + \frac{\log(a + bx + cx^2)}{2c^2}$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1354, 738, 773, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)^{3/2}} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{bx}{c(b^2 - 4ac)} + \frac{\log(a + bx + cx^2)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^2*x), x]

[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1354

```
Int[(x_)^(m_)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x} dx &= \int \frac{x^3}{(a + bx + cx^2)^2} dx \\
&= \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2} dx}{-b^2 + 4ac} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{\int \frac{-ab + (-b^2 + 4ac)x}{a + bx + cx^2} dx}{c(b^2 - 4ac)} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2c^2} - \frac{(b(b^2 - 6ac)) \int \frac{1}{a+bx+cx^2} dx}{2c^2(b^2 - 4ac)} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\log(a + bx + cx^2)}{2c^2} + \frac{(b(b^2 - 6ac)) \text{Subst}\left(\frac{1}{a+bx+cx^2}, x, \frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)^{3/2}} + \frac{\log(a + bx + cx^2)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 109, normalized size = 0.96

$$\frac{\frac{2(-2a^2c + ab(b-3cx) + b^3x)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + \log(a + x(b + cx))}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^2*x),x]

[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x), x]

fricas [B] time = 1.28, size = 635, normalized size = 5.57

$$\frac{2a^2 - 12a^2b + 18a^2c + (6a^2 - 6ab + (b^2 - 4ac)^2)\sqrt{-b^2 + 4ac} \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - 2(b^2 - 7ab + 12a^2c) \log\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - (a^2 - 6a^2b + 18a^2c + (b^2 - 4ac)^2)\sqrt{-b^2 + 4ac} \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) \log(cx^2 + bx + a) + 2a^2 - 12a^2b + 18a^2c - 2(b^2 - 7ab + 12a^2c) \log\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - (a^2 - 6a^2b + 18a^2c + (b^2 - 4ac)^2)\sqrt{-b^2 + 4ac} \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) \log(cx^2 + bx + a)}{2(a^2 - 6a^2b + 18a^2c + (b^2 - 4ac)^2)\sqrt{-b^2 + 4ac} \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - 2(b^2 - 7ab + 12a^2c) \log\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - (a^2 - 6a^2b + 18a^2c + (b^2 - 4ac)^2)\sqrt{-b^2 + 4ac} \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) \log(cx^2 + bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*\sqrt{b^2 - 4*a*c}*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c})*(2*c*x + b))/(c*x^2 + b*x + a) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(c*x^2 + b*x + a)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x), \\ & 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(c*x^2 + b*x + a)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)] \end{aligned}$$

giac [A] time = 0.36, size = 125, normalized size = 1.10

$$-\frac{(b^3 - 6abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2c^2 - 4ac^3)\sqrt{-b^2+4ac}} + \frac{\log(cx^2 + bx + a)}{2c^2} + \frac{ab^2 - 2a^2c + (b^3 - 3abc)x}{(cx^2 + bx + a)(b^2 - 4ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="giac")

[Out]
$$-(b^3 - 6*a*b*c)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2*c^2 - 4*a*c^3)*\sqrt{-b^2 + 4*a*c}) + 1/2*\log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)$$

maple [A] time = 0.01, size = 209, normalized size = 1.83

$$-\frac{6ab \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}c} + \frac{b^3 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}c^2} + \frac{2a \ln(cx^2 + bx + a)}{(4ac-b^2)c} - \frac{b^2 \ln(cx^2 + bx + a)}{2(4ac-b^2)c^2} + \frac{\frac{(3ac-b^2)bx}{(4ac-b^2)c^2} + \frac{(2ac-b^2)a}{(4ac-b^2)c^2}}{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c+a/x^2+b/x)^2/x,x)`

[Out]
$$\left(\frac{3ac-b^2}{4ac-b^2}\right)\frac{b}{c^2}x + \left(\frac{2ac-b^2}{4ac-b^2}\right)\frac{a}{c^2} + \frac{2c}{(4ac-b^2)}\ln(cx^2+bx+a) - \frac{1}{2c^2}\frac{1}{(4ac-b^2)}\ln(cx^2+bx+a) \cdot b^2 - \frac{6c}{(4ac-b^2)^{3/2}}\arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) \cdot \frac{a+b}{c^2} + \frac{1}{(4ac-b^2)^{3/2}}\arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) \cdot b^3$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.86, size = 279, normalized size = 2.45

$$\frac{a(2ac-b^2)}{c^2(4ac-b^2)} + \frac{bx(3ac-b^2)}{c^2(4ac-b^2)} - \frac{\ln(cx^2+bx+a)(-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6)}{2(64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2)} + \frac{b \operatorname{atan}\left(\frac{c^2(4ac-b^2)^{3/2}\left(\frac{2bx(6ac-b^2)}{c(4ac-b^2)^3} + \frac{b^2(4a^2c^2-b^2c)(6ac-b^2)}{c^3(4ac-b^2)^4}\right)}{b^3-6abc}\right)}{c^2(4ac-b^2)^{3/2}}(6ac-b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(c+a/x^2+b/x)^2),x)`

[Out]
$$\left(\frac{a(2ac-b^2)}{c^2(4ac-b^2)}\right) + \frac{b^2x(3ac-b^2)}{c^2(4ac-b^2)} - \frac{(\log(a+bx+cx^2)(b^6-64a^3c^3+48a^2b^2c^2-12ab^4c))/(2(64a^3c^5-b^6c^2+12ab^4c^3-48a^2b^2c^4)) + (b \operatorname{atan}((c^2(4ac-b^2)^{5/2}((2bx(6ac-b^2))/(c(4ac-b^2)^3) + (b^2(4a^2c^2-b^2c)(6ac-b^2))/(c^3(4ac-b^2)^4))))/(b^3-6abc)) \cdot (6ac-b^2)}{c^2(4ac-b^2)^{3/2}}$$

sympy [B] time = 1.32, size = 729, normalized size = 6.39

$$\left(\frac{a(2ac-b^2)}{c^2(4ac-b^2)} + \frac{bx(3ac-b^2)}{c^2(4ac-b^2)} - \frac{\ln(cx^2+bx+a)(-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6)}{2(64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2)} + \frac{b \operatorname{atan}\left(\frac{c^2(4ac-b^2)^{3/2}\left(\frac{2bx(6ac-b^2)}{c(4ac-b^2)^3} + \frac{b^2(4a^2c^2-b^2c)(6ac-b^2)}{c^3(4ac-b^2)^4}\right)}{b^3-6abc}\right)}{c^2(4ac-b^2)^{3/2}}(6ac-b^2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x**2+b/x)**2/x,x)`


```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2))) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6)) + 1/(2*c**2))) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

$$3.364 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^2} dx$$

Optimal. Leaf size=71

$$\frac{\frac{2a}{x} + b}{(b^2 - 4ac)\left(\frac{a}{x^2} + \frac{b}{x} + c\right)} - \frac{4a \tanh^{-1}\left(\frac{\frac{2a}{x} + b}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1352, 614, 618, 206}

$$\frac{\frac{2a}{x} + b}{(b^2 - 4ac)\left(\frac{a}{x^2} + \frac{b}{x} + c\right)} - \frac{4a \tanh^{-1}\left(\frac{\frac{2a}{x} + b}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^2*x^2),x]

[Out] (b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 1352

$\text{Int}[(x_)^{(m_.)}*((a_) + (c_.)*(x_)^{(n2_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol]$
 $]:> \text{Dist}[1/n, \text{Subst}[\text{Int}[(a + b*x + c*x^2)^p, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{EqQ}[\text{Simplify}[m - n + 1], 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^2} dx &= -\text{Subst}\left(\int \frac{1}{(c + bx + ax^2)^2} dx, x, \frac{1}{x}\right) \\ &= \frac{b + \frac{2a}{x}}{(b^2 - 4ac)\left(c + \frac{a}{x^2} + \frac{b}{x}\right)} + \frac{(2a) \text{Subst}\left(\int \frac{1}{c + bx + ax^2} dx, x, \frac{1}{x}\right)}{b^2 - 4ac} \\ &= \frac{b + \frac{2a}{x}}{(b^2 - 4ac)\left(c + \frac{a}{x^2} + \frac{b}{x}\right)} - \frac{(4a) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + \frac{2a}{x}\right)}{b^2 - 4ac} \\ &= \frac{b + \frac{2a}{x}}{(b^2 - 4ac)\left(c + \frac{a}{x^2} + \frac{b}{x}\right)} - \frac{4a \tanh^{-1}\left(\frac{b + \frac{2a}{x}}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 1.14

$$\frac{a(b - 2cx) + b^2x}{c(4ac - b^2)(a + x(b + cx))} + \frac{4a \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^2*x^2), x]

[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^2), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^2), x]

fricas [B] time = 1.20, size = 387, normalized size = 5.45

$$\left[\frac{ab^3 - 4a^2bc + 2(ac^2x^2 + abcx + a^2c)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + (b^4 - 6ab^2c + 8a^2c^2)x}{ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^2 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x}, -\frac{ab^3 - 4a^2bc - 4(ac^2x^2 + abcx + a^2c)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + (b^4 - 6ab^2c + 8a^2c^2)x}{ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^2 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="fricas")

[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c))*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]

giac [A] time = 0.38, size = 88, normalized size = 1.24

$$-\frac{4a \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2-4ac)\sqrt{-b^2+4ac}} - \frac{b^2x-2acx+ab}{(b^2c-4ac^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="giac")

[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))

maple [A] time = 0.01, size = 97, normalized size = 1.37

$$\frac{4a \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{\frac{ab}{(4ac-b^2)c} - \frac{(2ac-b^2)x}{(4ac-b^2)c}}{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^2/x^2,x)

[Out] $(-(2*a*c-b^2)/(4*a*c-b^2)/c*x+1/c*a*b/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.37, size = 135, normalized size = 1.90

$$\frac{\frac{x(2ac-b^2)}{c(4ac-b^2)} - \frac{ab}{c(4ac-b^2)}}{cx^2 + bx + a} - \frac{4a \operatorname{atan}\left(\frac{\left(\frac{2a(b^3-4abc)}{(4ac-b^2)^{5/2}} - \frac{4acx}{(4ac-b^2)^{3/2}}\right)(4ac-b^2)}{2a}\right)}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(c + a/x^2 + b/x)^2),x)

[Out] $-((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x + c*x^2) - (4*a*\operatorname{atan}(((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^{(5/2)} - (4*a*c*x)/(4*a*c - b^2)^{(3/2)})*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^{(3/2)}$

sympy [B] time = 0.60, size = 280, normalized size = 3.94

$$-2a \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{-32a^3c^2 \sqrt{\frac{1}{(4ac-b^2)}} + 16a^2b^2c \sqrt{\frac{1}{(4ac-b^2)}} - 2ab^4 \sqrt{\frac{1}{(4ac-b^2)}} + 2ab}{4ac}\right) + 2a \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{32a^3c^2 \sqrt{\frac{1}{(4ac-b^2)}} - 16a^2b^2c \sqrt{\frac{1}{(4ac-b^2)}} + 2ab^4 \sqrt{\frac{1}{(4ac-b^2)}} + 2ab}{4ac}\right) + \frac{ab + x(-2ac + b^2)}{4a^2c^2 - ab^2c + x^2(4ac^3 - b^2c^2) + x(4abc^2 - b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x**2+b/x)**2/x**2,x)`

[Out]
$$\begin{aligned} & -2*a*\sqrt{-1/(4*a*c - b**2)**3}*\log(x + (-32*a**3*c**2*\sqrt{-1/(4*a*c - b**2)**3} \\ & + 16*a**2*b**2*c*\sqrt{-1/(4*a*c - b**2)**3} - 2*a*b**4*\sqrt{-1/(4*a*c - b**2)**3} \\ & + 2*a*b)/(4*a*c)) + 2*a*\sqrt{-1/(4*a*c - b**2)**3}*\log(x + (3 \\ & 2*a**3*c**2*\sqrt{-1/(4*a*c - b**2)**3} - 16*a**2*b**2*c*\sqrt{-1/(4*a*c - b**2)**3} \\ & + 2*a*b**4*\sqrt{-1/(4*a*c - b**2)**3} + 2*a*b)/(4*a*c)) + (a*b + x \\ & (-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x \\ & (4*a*b*c**2 - b**3*c)) \end{aligned}$$

$$3.365 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^3} dx$$

Optimal. Leaf size=66

$$\frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1354, 638, 618, 206}

$$\frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^2*x^3),x]

[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^3} dx &= \int \frac{x}{(a + bx + cx^2)^2} dx \\
&= \frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{b \int \frac{1}{a + bx + cx^2} dx}{b^2 - 4ac} \\
&= \frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2b) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{b^2 - 4ac} \\
&= \frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2b \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 1.05

$$\frac{2a + bx}{(b^2 - 4ac)(a + x(b + cx))} - \frac{2b \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^2*x^3), x]

[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^3), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^3), x]

fricas [B] time = 1.17, size = 338, normalized size = 5.12

$$\left[\frac{2ab^2 - 8a^2c - (bcx^2 + b^2x + ab)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + (b^3 - 4abc)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x}, \frac{2ab^2 - 8a^2c - 2(bc^2x + b^2x + ab)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + (b^3 - 4abc)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="fricas")

[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 - 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)]

giac [A] time = 0.37, size = 76, normalized size = 1.15

$$\frac{2b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2-4ac)\sqrt{-b^2+4ac}} + \frac{bx+2a}{(cx^2+bx+a)(b^2-4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="giac")

[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))

maple [A] time = 0.00, size = 70, normalized size = 1.06

$$-\frac{2b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{-bx-2a}{(4ac-b^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^2/x^3,x)

[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.37, size = 110, normalized size = 1.67

$$-\frac{\frac{2a}{4ac-b^2} + \frac{bx}{4ac-b^2}}{cx^2 + bx + a} - \frac{2b \operatorname{atan}\left(\frac{\left(\frac{b^2}{(4ac-b^2)^{3/2}} + \frac{2bcx}{(4ac-b^2)^{3/2}}\right)(4ac-b^2)}{b}\right)}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(c + a/x^2 + b/x)^2),x)

[Out] - ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*atan((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/b))/(4*a*c - b^2)^(3/2)

sympy [B] time = 0.56, size = 253, normalized size = 3.83

$$b \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{-16a^2bc^2 \sqrt{\frac{1}{(4ac-b^2)^3}} + 8ab^3c \sqrt{\frac{1}{(4ac-b^2)^3}} - b^5 \sqrt{\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right) - b \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{16a^2bc^2 \sqrt{\frac{1}{(4ac-b^2)^3}} - 8ab^3c \sqrt{\frac{1}{(4ac-b^2)^3}} + b^5 \sqrt{\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right) + \frac{-2a - bx}{4a^2c - ab^2 + x^2(4ac^2 - b^2c) + x(4abc - b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**2/x**3,x)

[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))

$$3.366 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^4} dx$$

Optimal. Leaf size=66

$$\frac{4c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1354, 614, 618, 206}

$$\frac{4c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^2*x^4),x]

[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1354

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
:> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^4} dx &= \int \frac{1}{(a + bx + cx^2)^2} dx \\ &= -\frac{b + 2cx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2c) \int \frac{1}{a + bx + cx^2} dx}{b^2 - 4ac} \\ &= -\frac{b + 2cx}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{(4c) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{b^2 - 4ac} \\ &= -\frac{b + 2cx}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{4c \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 70, normalized size = 1.06

$$-\frac{\frac{4c \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} + \frac{b + 2cx}{a + x(b + cx)}}{b^2 - 4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^4), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^4), x]

fricas [B] time = 1.26, size = 341, normalized size = 5.17

$$\left[\frac{b^3 - 4abc + 2(c^2x^2 + bcx + ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + 2(b^2c - 4ac^2)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x}, -\frac{b^3 - 4abc - 4(c^2x^2 + bcx + ac)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + 2(b^2c - 4ac^2)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="fricas")

[Out] $[-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c))*\text{sqrt}(b^2 - 4*a*c)*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - \text{sqrt}(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 - 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c))*\text{sqrt}(-b^2 + 4*a*c)*\arctan(-\text{sqrt}(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)]$

giac [A] time = 0.39, size = 76, normalized size = 1.15

$$-\frac{4c \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2-4ac)\sqrt{-b^2+4ac}} - \frac{2cx+b}{(cx^2+bx+a)(b^2-4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="giac")

[Out] $-4*c*\arctan((2*c*x + b)/\text{sqrt}(-b^2 + 4*a*c))/((b^2 - 4*a*c)*\text{sqrt}(-b^2 + 4*a*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))$

maple [A] time = 0.00, size = 68, normalized size = 1.03

$$\frac{4c \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{2cx+b}{(4ac-b^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^2/x^4,x)

[Out] $(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.08, size = 119, normalized size = 1.80

$$\frac{b}{4ac-b^2} + \frac{2cx}{4ac-b^2} - \frac{4c \operatorname{atan}\left(\frac{\left(\frac{2c(b^3-4abc)}{(4ac-b^2)^{5/2}} - \frac{4c^2x}{(4ac-b^2)^{3/2}}\right)(4ac-b^2)}{2c}\right)}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(c + a/x^2 + b/x)^2),x)

[Out] (b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan((((2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))* (4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)

sympy [B] time = 0.58, size = 265, normalized size = 4.02

$$-2c \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{-32a^2c^3 \sqrt{-\frac{1}{(4ac-b^2)^3}} + 16ab^2c^2 \sqrt{-\frac{1}{(4ac-b^2)^3}} - 2b^4c \sqrt{-\frac{1}{(4ac-b^2)^3}} + 2bc}{4c^2}\right) + 2c \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{32a^2c^3 \sqrt{-\frac{1}{(4ac-b^2)^3}} - 16ab^2c^2 \sqrt{-\frac{1}{(4ac-b^2)^3}} + 2b^4c \sqrt{-\frac{1}{(4ac-b^2)^3}} + 2bc}{4c^2}\right) + \frac{b+2cx}{4a^2c-ab^2+x^2(4ac^2-b^2c)+x(4abc-b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**2/x**4,x)

[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))

$$3.367 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^5} dx$$

Optimal. Leaf size=108

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx + cx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx}{a(b^2 - 4ac)(a + bx + cx^2)}$$

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx + cx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx}{a(b^2 - 4ac)(a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^2*x^5), x]

[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^5} dx &= \int \frac{1}{x(a+bx+cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a+bx+cx^2)} - \frac{\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a+bx+cx^2)} - \frac{\int \left(\frac{-b^2+4ac}{ax} + \frac{b(b^2-5ac)+c(b^2-4ac)x}{a(a+bx+cx^2)}\right) dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a+bx+cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2} dx}{a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a+bx+cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2a^2} - \frac{(b(b^2 - 6ac)) \int \frac{1}{a+bx+cx^2} dx}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a+bx+cx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx+cx^2)}{2a^2} + \frac{(b(b^2 - 6ac)) \text{Subst}\left(\int \frac{1}{u} du, a+bx+cx^2\right)}{a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a+bx+cx^2)} + \frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2 - 4ac)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log(a+bx+cx^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 107, normalized size = 0.99

$$\frac{2a(-2ac+b^2+bcx)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} - \log(a+x(b+cx)) + 2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^2*x^5),x]

[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^5), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^5), x]

fricas [B] time = 1.49, size = 781, normalized size = 7.23

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2))*x^2 + (b^4 - 6*a*b^2*c)*x)*\sqrt{b^2 - 4*a*c}*\log((2*c*x^2 + 2*b*c*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c})*(2*c*x + b))/(c*x^2 + b*x + a) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2))*x^2 + (b^4 - 6*a*b^2*c)*x)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)] \end{aligned}$$

giac [A] time = 0.36, size = 126, normalized size = 1.17

$$\frac{(b^3 - 6abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(a^2b^2 - 4a^3c)\sqrt{-b^2+4ac}} - \frac{\log(cx^2 + bx + a)}{2a^2} + \frac{\log(|x|)}{a^2} + \frac{abcx + ab^2 - 2a^2c}{(cx^2 + bx + a)(b^2 - 4ac)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="giac")

[Out]
$$-(b^3 - 6*a*b*c)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((a^2*b^2 - 4*a^3*c)*\sqrt{-b^2 + 4*a*c}) - 1/2*\log(c*x^2 + b*x + a)/a^2 + \log(\text{abs}(x))/a^2 + (a*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)$$

maple [B] time = 0.01, size = 237, normalized size = 2.19

$$\frac{bcx}{(cx^2+bx+a)(4ac-b^2)a} - \frac{6bc \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a} + \frac{b^3 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a^2} - \frac{b^2}{(cx^2+bx+a)(4ac-b^2)a} - \frac{2c \ln(cx^2+bx+a)}{(4ac-b^2)a} + \frac{b^2 \ln(cx^2+bx+a)}{2(4ac-b^2)a^2} + \frac{2c}{(cx^2+bx+a)(4ac-b^2)} + \frac{\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^2/x^5,x)

[Out] $\ln(x)/a^2 - 1/a/(cx^2+bx+a)*b*c/(4*a*c-b^2)*x+2/(cx^2+bx+a)/(4*a*c-b^2)*c - 1/a/(cx^2+bx+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*\ln(cx^2+bx+a)+1/2/a^2/(4*a*c-b^2)*\ln(cx^2+bx+a)*b^2-6/a/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b*c+1/a^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^3$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.10, size = 620, normalized size = 5.74

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(c + a/x^2 + b/x)^2),x)

[Out] $\log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))/(a + b*x + c*x^2) + (\log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^{(1/2)} + 84*a^3*b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)} - 120*a^3*b*c^3*x - 12*a*b^2*c*x*(-(4*a*c - b^2)^3)^{(1/2)})*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3)^{(1/2)}) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*a^2*(4*a*c - b^2)^3) + (\log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^{(1/2)} - 84*a^3*b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^{(1/2)} + 120*a^3*b*c^3*x - 12*a*b^2*c*$

$$x*(-(4*a*c - b^2)^3)^{(1/2)}*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*a^2*(4*a*c - b^2)^3)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**2/x**5,x)

[Out] Timed out

$$3.368 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^6} dx$$

Optimal. Leaf size=148

$$\frac{b \log(a + bx + cx^2)}{a^3} - \frac{2b \log(x)}{a^3} - \frac{2(b^2 - 3ac)}{a^2 x (b^2 - 4ac)} - \frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3 (b^2 - 4ac)^{3/2}} + \frac{-2ac + b^2 + b}{ax (b^2 - 4ac) (a + b)}$$

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$-\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3 (b^2 - 4ac)^{3/2}} - \frac{2(b^2 - 3ac)}{a^2 x (b^2 - 4ac)} + \frac{b \log(a + bx + cx^2)}{a^3} - \frac{2b \log(x)}{a^3} + \frac{-2ac + b^2 + bcx}{ax (b^2 - 4ac) (a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^2*x^6), x]

[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x + c*x^2])/a^3

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^6} dx &= \int \frac{1}{x^2 (a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{\int \frac{-2(b^2 - 3ac) - 2bcx}{x^2(a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{\int \left(\frac{2(-b^2 + 3ac)}{ax^2} - \frac{2b(-b^2 + 4ac)}{a^2x} + \frac{2(-b^4 + 5ab^2c - 3a^2c^2 - bc(b^2 - 4ac)x)}{a^2(a + bx + cx^2)} \right) dx}{a(b^2 - 4ac)} \\
&= -\frac{2(b^2 - 3ac)}{a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{2b \log(x)}{a^3} - \frac{2 \int \frac{-b^4 + 5ab^2c - 3a^2c^2 - bc(b^2 - 4ac)x}{a + bx + cx^2}}{a^3(b^2 - 4ac)} \\
&= -\frac{2(b^2 - 3ac)}{a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{2b \log(x)}{a^3} + \frac{b \int \frac{b + 2cx}{a + bx + cx^2} dx}{a^3} + \frac{(b^4 - 6ab^2c + 6a^2c^2) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{a^3(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 131, normalized size = 0.89

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right) + \frac{a(-3abc - 2ac^2x + b^3 + b^2cx)}{(b^2 - 4ac)(a + x(b + cx))} - b \log(a + x(b + cx)) + \frac{a}{x} + 2b \log(x)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^2*x^6),x]

[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/(b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)]/a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^6), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^6), x]

fricas [B] time = 1.79, size = 975, normalized size = 6.59

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="fricas")

[Out]
$$\begin{aligned} &[-(a^2b^4 - 8a^3b^2c + 16a^4c^2 + 2(a^2b^4c - 7a^2b^2c^2 + 12a^3c^3))x^2 + ((b^4c - 6a^2b^2c^2 + 6a^2c^3)x^3 + (b^5 - 6a^2b^3c + 6a^2b^2c^2)x^2 + (a^2b^4 - 6a^2b^2c + 6a^3c^2)x) \sqrt{b^2 - 4ac} \log(\\ &(2c^2x^2 + 2b^2cx + b^2 - 2ac + \sqrt{b^2 - 4ac})(2cx + b)) / (cx^2 + bx + a) + (2a^2b^5 - 15a^2b^3c + 28a^3b^2c^2)x - ((b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)x^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)x^2 + (a^2b^5 - 8a^2b^3c + 16a^3b^2c^2)x) \log(cx^2 + bx + a) + 2((b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)x^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)x^2 + (a^2b^5 - 8a^2b^3c + 16a^3b^2c^2)x) \log(x) / ((a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)x^3 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)x^2 + (a^4b^4 - 8a^5b^2c + 16a^6c^2)x), \\ &-(a^2b^4 - 8a^3b^2c + 16a^4c^2 + 2(a^2b^4c - 7a^2b^2c^2 + 12a^3c^3))x^2 + 2((b^4c - 6a^2b^2c^2 + 6a^2c^3)x^3 + (b^5 - 6a^2b^3c + 6a^2b^2c^2)x^2 + (a^2b^4 - 6a^2b^2c + 6a^3c^2)x) \sqrt{-b^2 + 4ac} \arctan(-\sqrt{-b^2 + 4ac})(2cx + b) / (b^2 - 4ac) \\ &+ (2a^2b^5 - 15a^2b^3c + 28a^3b^2c^2)x - ((b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)x^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)x^2 + (a^2b^5 - 8a^2b^3c + 16a^3b^2c^2)x) \log(cx^2 + bx + a) + 2((b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)x^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)x^2 + (a^2b^5 - 8a^2b^3c + 16a^3b^2c^2)x) \log(x) / ((a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)x^3 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)x^2 + (a^4b^4 - 8a^5b^2c + 16a^6c^2)x] \end{aligned}$$

giac [A] time = 0.42, size = 171, normalized size = 1.16

$$\frac{2(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(a^3b^2 - 4a^4c)\sqrt{-b^2+4ac}} - \frac{2b^2cx^2 - 6ac^2x^2 + 2b^3x - 7abcx + ab^2 - 4a^2c}{(a^2b^2 - 4a^3c)(cx^3 + bx^2 + ax)} + \frac{b \log(cx^2 + bx + a)}{a^3} - \frac{2b \log(|x|)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="giac")

[Out] $2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((a^3*b^2 - 4*a^4*c)*\sqrt{-b^2 + 4*a*c}) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x)) + b*\log(c*x^2 + b*x + a)/a^3 - 2*b*\log(\text{abs}(x))/a^3$

maple [B] time = 0.01, size = 328, normalized size = 2.22

$$\frac{2c^2x}{(cx^2+bx+a)(4ac-b^2)a} - \frac{12c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a} + \frac{b^2cx}{(cx^2+bx+a)(4ac-b^2)a^2} + \frac{12b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a^2} - \frac{2b^4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a^3} - \frac{3bc}{(cx^2+bx+a)(4ac-b^2)a} + \frac{b^3}{(cx^2+bx+a)(4ac-b^2)a^2} + \frac{4bc\ln(cx^2+bx+a)}{(4ac-b^2)a^2} - \frac{b^3\ln(cx^2+bx+a)}{(4ac-b^2)a^3} - \frac{2b\ln(x)}{a^3} - \frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^2/x^6,x)

[Out] $-1/a^2/x - 2*b*\ln(x)/a^3 - 2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x + 1/a^2/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^2 - 3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c + 1/a^2/(c*x^2+b*x+a)*b^3/(4*a*c-b^2) + 4/a^2/(4*a*c-b^2)*c*\ln(c*x^2+b*x+a)*b - 1/a^3/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^3 - 12/a/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2}))*c^2 + 12/a^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2}))*b^2*c - 2/a^3/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2}))*b^4$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.13, size = 775, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(c + a/x^2 + b/x)^2),x)

[Out] $\log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 23*a^2*b^5*c - 108*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^{(1/2)} + 87*a^3*b^3$

$$\begin{aligned}
& *c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*c*x* \\
& (-(4*a*c - b^2)^3)^{(1/2)} + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^{(1/2)}*((b^4*(\\
& -(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*(- \\
& (4*a*c - b^2)^3)^{(1/2)})/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) \\
& + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c)))/(a^2*(4*a*c - b^2)) + (2*c*x^2* \\
& (3*a*c - b^2))/(a^2*(4*a*c - b^2))/(a*x + b*x^2 + c*x^3) - \log(2*a*b^4*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^4*b*c^3 - \\
& 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^{(1/2)} - 87*a^3*b^3*c^2 + 3*a^3*c^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} - 97*a^2* \\
& b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^{(1/2)}*((b^4*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 6*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*(-(4*a*c - b^2)^3 \\
&)^{(1/2)})/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - b/a^3) - \\
& (2*b*\log(x))/a^3
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**2/x**6,x)

[Out] Timed out

$$3.369 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^7} dx$$

Optimal. Leaf size=202

$$\frac{(3b^2 - 2ac) \log(a + bx + cx^2)}{2a^4} + \frac{\log(x)(3b^2 - 2ac)}{a^4} + \frac{b(3b^2 - 11ac)}{a^3x(b^2 - 4ac)} - \frac{3b^2 - 8ac}{2a^2x^2(b^2 - 4ac)} + \frac{b(30a^2c^2 - 20ab^2c + 3b^3)}{a^4(b^2 - 4ac)}$$

Rubi [A] time = 0.24, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1354, 740, 800, 634, 618, 206, 628}

$$\frac{b(30a^2c^2 - 20ab^2c + 3b^3) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} - \frac{3b^2-8ac}{2a^2x^2(b^2-4ac)} - \frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4} + \frac{b(3b^2-11ac)}{a^3x(b^2-4ac)} + \frac{\log(x)(3b^2-2ac)}{a^4} + \frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^2*x^7), x]

[Out] $-(3b^2 - 8ac)/(2a^2(b^2 - 4ac)x^2) + (b(3b^2 - 11ac))/(a^3(b^2 - 4ac)x) + (b^2 - 2ac + bcx)/(a(b^2 - 4ac)x^2(a + bx + cx^2)) + (b(3b^4 - 20ab^2c + 30a^2c^2) \operatorname{ArcTanh}[(b + 2cx)/\sqrt{b^2 - 4ac}])/(a^4(b^2 - 4ac)^{3/2}) + ((3b^2 - 2ac) \log(x))/a^4 - ((3b^2 - 2ac) \log[a + bx + cx^2])/(2a^4)$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4ac, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + bx + cx^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^m*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^p, x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^7} dx &= \int \frac{1}{x^3 (a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} - \frac{\int \frac{-3b^2 + 8ac - 3bcx}{x^3(a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} - \frac{\int \left(\frac{-3b^2 + 8ac}{ax^3} + \frac{3b^3 - 11abc}{a^2x^2} + \frac{(b^2 - 4ac)(-3b^2 + 2ac)}{a^3x} + \frac{b(3b^4 - 17ab^2 + 17a^2c)}{a^3} \right) dx}{a(b^2 - 4ac)} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{(3b^2 - 2ac) \log(a + bx + cx^2)}{a^4} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{(3b^2 - 2ac) \log(a + bx + cx^2)}{a^4} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{(3b^2 - 2ac) \log(a + bx + cx^2)}{a^4} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{b(3b^4 - 20ab^2 + 17a^2c) \log(a + bx + cx^2)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 175, normalized size = 0.87

$$\frac{2b(30a^2c^2 - 20ab^2c + 3b^4) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + \frac{2a(2a^2c^2 - 4ab^2c - 3abc^2x + b^4 + b^3cx)}{(b^2 - 4ac)(a + x(b + cx))} - \frac{a^2}{x^2} + 2 \log(x)(3b^2 - 2ac) + (2ac - 3b^2) \log(a + x(b + cx)) + \frac{4ab}{x}}{2a^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^2*x^7), x]

[Out] $(-a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)]/(2*a^4)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2 x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^7), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^2*x^7), x]

fricas [B] time = 2.00, size = 1226, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="fricas")

[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]

giac [A] time = 0.33, size = 229, normalized size = 1.13

$$\frac{(3b^5 - 20ab^3c + 30a^2bc^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - (3b^2 - 2ac) \log(cx^2 + bx + a) + \frac{(3b^2 - 2ac) \log(|x|)}{a^4} - \frac{a^3b^2 - 4a^4c - 2(3ab^2c - 11a^2bc^2)x^3 - (6ab^4 - 25a^2b^2c + 8a^3c^2)x^2 - 3(a^2b^3 - 4a^3bc)x}{2(cx^2 + bx + a)(b^2 - 4ac)a^4x^2}}{(a^2b^2 - 4a^5c)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="giac")

[Out] $-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c}) / ((a^4*b^2 - 4*a^5*c)*\sqrt{-b^2 + 4*a*c}) - 1/2*(3*b^2 - 2*a*c)*\log(c*x^2 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*\log(\text{abs}(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x) / ((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)$

maple [B] time = 0.02, size = 418, normalized size = 2.07

$$\frac{3b^2c}{(c^2 + bx + a)(4ac - b^2)^2} + \frac{30b^2c \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^2 a^2} - \frac{b^2cx}{(c^2 + bx + a)(4ac - b^2)^3} - \frac{20b^2c \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^2 a^2} + \frac{3b^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^2 a^4} - \frac{2c^2}{(c^2 + bx + a)(4ac - b^2)^2} - \frac{4b^2c}{(c^2 + bx + a)(4ac - b^2)^2} - \frac{4c^2 \ln(cx^2 + bx + a)}{(4ac - b^2)^2} - \frac{b^4}{(c^2 + bx + a)(4ac - b^2)^2} - \frac{7b^2c \ln(cx^2 + bx + a)}{(4ac - b^2)^2} + \frac{3b^4 \ln(cx^2 + bx + a)}{2(4ac - b^2)^2} - \frac{2b \ln(a)}{a^2} + \frac{3b^2 \ln(a)}{a^4} + \frac{2b}{a^5} - \frac{1}{2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^2/x^7,x)

[Out] $-1/2/a^2/x^2 - 2/a^3*\ln(x)*c + 3/a^4*\ln(x)*b^2 + 2/a^3*b/x + 3/a^2/(c*x^2 + b*x + a)*b*c^2/(4*a*c - b^2)*x - 1/a^3/(c*x^2 + b*x + a)*b^3*c/(4*a*c - b^2)*x - 2/a/(c*x^2 + b*x + a)/(4*a*c - b^2)*c^2 + 4/a^2/(c*x^2 + b*x + a)/(4*a*c - b^2)*b^2*c - 1/a^3/(c*x^2 + b*x + a)/(4*a*c - b^2)*b^4 + 4/a^2/(4*a*c - b^2)*c^2*\ln(c*x^2 + b*x + a) - 7/a^3/(4*a*c - b^2)*c*\ln(c*x^2 + b*x + a)*b^2 + 3/2/a^4/(4*a*c - b^2)*\ln(c*x^2 + b*x + a)*b^4 + 30/a^2/(4*a*c - b^2)^(3/2)*\arctan((2*c*x + b)/(4*a*c - b^2)^(1/2))*b*c^2 - 20/a^3/(4*a*c - b^2)^(3/2)*\arctan((2*c*x + b)/(4*a*c - b^2)^(1/2))*b^3*c + 3/a^4/(4*a*c - b^2)^(3/2)*\arctan((2*c*x + b)/(4*a*c - b^2)^(1/2))*b^5$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c - b^2 > 0)', see `assume?` for more details) Is 4*a*c - b^2 positive or negative?

mupad [B] time = 2.30, size = 914, normalized size = 4.52

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^7*(c + a/x^2 + b/x)^2),x)`

[Out] $(\log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 73*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^{(1/2)} + 307*a^3*b^4*c^2 - 492*a^4*b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a^3*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(4*a*c - b^2)^3)^{(1/2)} - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4*a*c - b^2)^3)^{(1/2)} - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^{(1/2)})*(3*b^8 + 128*a^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 20*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (\log(x)*(2*a*c - 3*b^2))/a^4 - (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(2*a^3*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2)))/(a*x^2 + b*x^3 + c*x^4) + (\log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^{(1/2)} + 307*a^3*b^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^3*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x - 24*a^3*c^3*x*(-(4*a*c - b^2)^3)^{(1/2)} - 76*a*b^7*c*x + 312*a^4*b*c^4*x - 40*a*b^4*c*x*(-(4*a*c - b^2)^3)^{(1/2)} + 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^{(1/2)})*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^{(1/2)} + 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a*b^3*c*(-(4*a*c - b^2)^3)^{(1/2)))/(2*a^4*(4*a*c - b^2)^3)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x**2+b/x)**2/x**7,x)`

[Out] Timed out

$$3.370 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3} dx$$

Optimal. Leaf size=238

$$\frac{3x(10a^2c^2 - 7ab^2c + b^4)}{c^3(b^2 - 4ac)^2} - \frac{3(-20a^3c^3 + 30a^2b^2c^2 - 10ab^4c + b^6) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4(b^2 - 4ac)^{5/2}} - \frac{3bx^2(b^2 - 6ac)}{2c^2(b^2 - 4ac)^2} + \frac{x^5(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^3(bx(b^2 - 7ac) + a(b^2 - 10ac))}{c(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{3b \log(a + bx + cx^2)}{2c^4}$$

Rubi [A] time = 0.29, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1340, 738, 818, 800, 634, 618, 206, 628}

$$\frac{3x(10a^2c^2 - 7ab^2c + b^4)}{c^3(b^2 - 4ac)^2} - \frac{3(30a^2b^2c^2 - 20a^3c^3 - 10ab^4c + b^6) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4(b^2 - 4ac)^{5/2}} - \frac{3bx^2(b^2 - 6ac)}{2c^2(b^2 - 4ac)^2} + \frac{x^5(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^3(bx(b^2 - 7ac) + a(b^2 - 10ac))}{c(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{3b \log(a + bx + cx^2)}{2c^4}$$

Antiderivative was successfully verified.

[In] Int[(c + a/x^2 + b/x)^(-3), x]

[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b*(b^2 - 6*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[a + b*x + c*x^2])/(2*c^4)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x))/((c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/((c*(p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))] + e*(b^2*e*g*(m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])
```

Rule 1340

```
Int[((a_.) + (c_.)*(x_)^n2) + (b_.)*(x_)^n)^p, x_Symbol] := Int[x^(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3} dx &= \int \frac{x^6}{(a + bx + cx^2)^3} dx \\
 &= \frac{x^5(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{\int \frac{x^4(10a+2bx)}{(a+bx+cx^2)^2} dx}{2(b^2 - 4ac)} \\
 &= \frac{x^5(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^3(a(b^2 - 10ac) + b(b^2 - 7ac)x)}{c(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{\int \frac{x^2(6a(b^2-10ac)+6b(b^2-4ac))}{a+bx+cx^2} dx}{2c(b^2 - 4ac)} \\
 &= \frac{x^5(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^3(a(b^2 - 10ac) + b(b^2 - 7ac)x)}{c(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{\int \left(-\frac{6(b^4-7ab^2c+10a^2c^2)}{c^2}\right) dx}{2c(b^2 - 4ac)} \\
 &= \frac{3(b^4 - 7ab^2c + 10a^2c^2)x}{c^3(b^2 - 4ac)^2} - \frac{3b(b^2 - 6ac)x^2}{2c^2(b^2 - 4ac)^2} + \frac{x^5(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^3(a(b^2 - 10ac) + b(b^2 - 7ac)x)}{c(b^2 - 4ac)^2(a + bx + cx^2)} \\
 &= \frac{3(b^4 - 7ab^2c + 10a^2c^2)x}{c^3(b^2 - 4ac)^2} - \frac{3b(b^2 - 6ac)x^2}{2c^2(b^2 - 4ac)^2} + \frac{x^5(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^3(a(b^2 - 10ac) + b(b^2 - 7ac)x)}{c(b^2 - 4ac)^2(a + bx + cx^2)} \\
 &= \frac{3(b^4 - 7ab^2c + 10a^2c^2)x}{c^3(b^2 - 4ac)^2} - \frac{3b(b^2 - 6ac)x^2}{2c^2(b^2 - 4ac)^2} + \frac{x^5(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^3(a(b^2 - 10ac) + b(b^2 - 7ac)x)}{c(b^2 - 4ac)^2(a + bx + cx^2)} \\
 &= \frac{3(b^4 - 7ab^2c + 10a^2c^2)x}{c^3(b^2 - 4ac)^2} - \frac{3b(b^2 - 6ac)x^2}{2c^2(b^2 - 4ac)^2} + \frac{x^5(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^3(a(b^2 - 10ac) + b(b^2 - 7ac)x)}{c(b^2 - 4ac)^2(a + bx + cx^2)}
 \end{aligned}$$

Mathematica [A] time = 0.37, size = 260, normalized size = 1.09

$$\frac{a^2c^2(2cx-5b)+a^2b^2c(5b-9cx)-ab^4(b-6cx)+b^6(-x)}{(b^2-4ac)(a+bx+cx)^2} + \frac{6c(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + \frac{-78a^3bc^3+36a^2c^4x+61a^2b^2c^2-102a^2b^2c^3x-14ab^5c+48ab^4c^2x+b^7-6b^6cx}{(b^2-4ac)^2(a+bx+cx)} - 3bc \log(a + x(b + cx)) + 2c^2x}{2c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + a/x^2 + b/x)^(-3), x]

```
[Out] (2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a + x*
(b + c*x))) + (-b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) + a^3
*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)]/(2*c^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-3), x]
```

```
[Out] IntegrateAlgebraic[(c + a/x^2 + b/x)^(-3), x]
```

fricas [B] time = 1.01, size = 1926, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*
c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b
^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 1
23*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c +
225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 - 10*a^
3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^
2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3
*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c
^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^4*c^2
- 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3
*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c
^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^3 +
(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^2 +
2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x^2 + b
*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 - 12*a*
b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^5 + 24
*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*a^2*b^
5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x), -1/2*(5*a^2*b^7 - 56*a^3*b^5*c +
```

$$202a^4b^3c^2 - 232a^5b^2c^3 - 2(b^6c^3 - 12a^2b^4c^4 + 48a^2b^2c^5 - 64a^3c^6)x^5 - 4(b^7c^2 - 12a^2b^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)x^4 + 2(2b^8c - 26a^2b^6c^2 + 123a^2b^4c^3 - 254a^3b^2c^4 + 200a^4c^5)x^3 + (5b^9 - 58a^2b^7c + 225a^2b^5c^2 - 314a^3b^3c^3 + 88a^4b^2c^4)x^2 + 6(a^2b^6 - 10a^3b^4c + 30a^4b^2c^2 - 20a^5c^3 + (b^6c^2 - 10a^2b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)x^4 + 2(b^7c - 10a^2b^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)x^3 + (b^8 - 8a^2b^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)x^2 + 2(a^2b^7 - 10a^2b^5c + 30a^3b^3c^2 - 20a^4b^2c^3)x) \sqrt{-b^2 + 4ac} \arctan(\sqrt{-b^2 + 4ac}) + 2(5a^2b^8 - 59a^2b^6c + 235a^3b^4c^2 - 346a^4b^2c^3 + 120a^5c^4)x + 3(a^2b^7 - 12a^3b^5c + 48a^4b^3c^2 - 64a^5b^2c^3 + (b^7c^2 - 12a^2b^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)x^4 + 2(b^8c - 12a^2b^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)x^3 + (b^9 - 10a^2b^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4)x^2 + 2(a^2b^8 - 12a^2b^6c + 48a^3b^4c^2 - 64a^4b^2c^3)x) \log(cx^2 + bx + a) / (a^2b^6c^4 - 12a^3b^4c^5 + 48a^4b^2c^6 - 64a^5c^7 + (b^6c^6 - 12a^2b^4c^7 + 48a^2b^2c^8 - 64a^3c^9)x^4 + 2(b^7c^5 - 12a^2b^5c^6 + 48a^2b^3c^7 - 64a^3b^2c^8)x^3 + (b^8c^4 - 10a^2b^6c^5 + 24a^2b^4c^6 + 32a^3b^2c^7 - 128a^4c^8)x^2 + 2(a^2b^7c^4 - 12a^2b^5c^5 + 48a^3b^3c^6 - 64a^4b^2c^7)x]$$

giac [A] time = 0.38, size = 282, normalized size = 1.18

$$\frac{3(b^6 - 10ab^4c + 30a^2b^2c^2 - 20a^3c^3) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) + \frac{x}{c^3} - \frac{3b \log(cx^2 + bx + a)}{2c^4} - \frac{5a^2b^5 - 36a^2b^3c + 58a^4bc^2 + 6(b^6c - 8ab^4c^2 + 17a^2b^2c^3 - 6a^3c^4)x^3 + (5b^7 - 34ab^5c + 41a^2b^3c^2 + 42a^3b^2c^3)x^2 + 2(5ab^6 - 38a^2b^4c + 71a^3b^2c^2 - 14a^4c^3)x}{2(cx^2 + bx + a)^2(b^2 - 4ac)^2c^4}}{(b^6c^4 - 8ab^4c^5 + 16a^2c^6)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="giac")

[Out] $3(b^6 - 10a^2b^4c + 30a^2b^2c^2 - 20a^3c^3) \arctan((2cx + b)/\sqrt{-b^2 + 4ac}) / ((b^4c^4 - 8a^2b^2c^5 + 16a^2c^6) \sqrt{-b^2 + 4ac}) + x/c^3 - 3/2b \log(cx^2 + bx + a)/c^4 - 1/2(5a^2b^5 - 36a^3b^3c + 58a^4b^2c^2 + 6(b^6c - 8a^2b^4c^2 + 17a^2b^2c^3 - 6a^3c^4)x^3 + (5b^7 - 34a^2b^5c + 41a^2b^3c^2 + 42a^3b^2c^3)x^2 + 2(5a^2b^6 - 38a^2b^4c + 71a^3b^2c^2 - 14a^4c^3)x) / ((cx^2 + bx + a)^2(b^2 - 4ac)^2c^4)$

maple [B] time = 0.02, size = 1040, normalized size = 4.37

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^3,x)

[Out] $x/c^3 + 18/(cx^2 + bx + a)^2 / (16a^2c^2 - 8a^2b^2c + b^4)x^3 + a^3 - 51/c / (cx^2 + bx + a)^2 / (16a^2c^2 - 8a^2b^2c + b^4)x^3 + a^2b^2 + 24/c^2 / (cx^2 + bx + a)^2 / (16a^2$

$$\begin{aligned}
& *c^2-8*a*b^2*c+b^4)*x^3*a*b^4-3/c^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4) \\
& *x^3*b^6-21/c/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^3-41/2/ \\
& c^2/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a^2+17/c^3/(c*x^2+b*x+a)^2*b^5/ \\
& (16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a-5/2/c^4/(c*x^2+b*x+a)^2*b^7/(16*a^2*c^2-8*a*b^2*c+b^4) \\
& *x^2+14/c/(c*x^2+b*x+a)^2*a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x-71/c^2/(c*x^2+b*x+a)^2*a^3/ \\
& (16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2+38/c^3/(c*x^2+b*x+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4) \\
& *x*b^4-5/c^4/(c*x^2+b*x+a)^2*a/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^6-29/c^2/(c*x^2+b*x+a)^2*b*a^4/ \\
& (16*a^2*c^2-8*a*b^2*c+b^4)+18/c^3/(c*x^2+b*x+a)^2*b^3*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)-5/2/c^4/ \\
& (c*x^2+b*x+a)^2*b^5*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)-24/c^2/(16*a^2*c^2-8*a*b^2*c+b^4) \\
& *ln(c*x^2+b*x+a)*a^2*b+12/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a*b^3-3/2/c^4/ \\
& (16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^5-60/c/(16*a^2*c^2-8*a*b^2*c+b^4) \\
& /((4*a*c-b^2)^(1/2))*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3+90/c^2/(16*a^2*c^2-8*a*b^2*c+b^4) \\
& /((4*a*c-b^2)^(1/2))*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2-30/c^3/(16*a^2*c^2-8*a*b^2*c+b^4) \\
& /((4*a*c-b^2)^(1/2))*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4+3/c^4/(16*a^2*c^2-8*a*b^2*c+b^4) \\
& /((4*a*c-b^2)^(1/2))*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.00, size = 705, normalized size = 2.96

$$\frac{\frac{3}{8} \frac{(a^2 b^2 x^2 + 2 a b^2 x + b^3) \sqrt{4 a c - b^2}}{(c x^2 + b x + a)^3} + \frac{3}{4} \frac{b^2 \sqrt{4 a c - b^2}}{(c x^2 + b x + a)^2} + \frac{3}{2} \frac{b \sqrt{4 a c - b^2}}{(c x^2 + b x + a)} + \frac{3}{2} \frac{\sqrt{4 a c - b^2}}{(c x^2 + b x + a)} \ln\left(\frac{2 c x + b}{\sqrt{4 a c - b^2}}\right) + \frac{3}{2} \frac{\sqrt{4 a c - b^2}}{(c x^2 + b x + a)} \arctan\left(\frac{2 c x + b}{\sqrt{4 a c - b^2}}\right)}{2 (1024 a^4 c^3 - 1280 a^3 b c^3 + 640 a^2 b^2 c^3 - 160 a b^3 c^3 + 20 a^4 b^3 - 8 b^4 c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c + a/x^2 + b/x)^3,x)

[Out] $x/c^3 - ((3*x^3*(b^6 - 6*a^3*c^3 + 17*a^2*b^2*c^2 - 8*a*b^4*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (x^2*(5*b^7 + 42*a^3*b*c^3 + 41*a^2*b^3*c^2 - 34*a*b^5*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(5*b^5 + 58*a^2*b*c^2 - 36*a*b^3*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(5*b^6 - 14*a^3*c^3 + 71*a^2*b^2*c^2 - 38*a*b^4*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(a^2*c^3 + c^5*x^4 + x^2*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^3 + 2*a*b*c^3*x) + \log(a + b*x + c*x^2)*(3*b^11 - 3072*a^5*b*c^5 + 480*a^2*b^7*c^2 - 1920*a^3*b^5*c$

$$\frac{c^3 + 3840a^4b^3c^4 - 60ab^9c)}{(2(1024a^5c^9 - b^{10}c^4 + 20ab^8c^5 - 160a^2b^6c^6 + 640a^3b^4c^7 - 1280a^4b^2c^8)) + (3\operatorname{atan}\left(\frac{(3x(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c))}{(c^3(4ac - b^2)^5)}\right) + (3(b^5c^3 - 8ab^3c^4 + 16a^2b^2c^5)(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c)) / (2c^7(4ac - b^2)^5(b^4 + 16a^2c^2 - 8ab^2c)) + (32a^2c^6(4ac - b^2)^{5/2} + 2b^4c^4(4ac - b^2)^{5/2} - 16ab^2c^5(4ac - b^2)^{5/2})) / (3b^6 - 60a^3c^3 + 90a^2b^2c^2 - 30ab^4c)) * (b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c)) / (c^4(4ac - b^2)^{5/2})$$

sympy [B] time = 4.20, size = 1714, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**3,x)

[Out]
$$\begin{aligned} & \left(-\frac{3b}{2c^4} - 3\sqrt{-(4ac - b^2)^5} \frac{(20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)}{(2c^4(1024a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 + 20ab^8c - b^{10}))} \right) \log(x + (-66a^3b^2c^2 - 64a^3c^6(-\frac{3b}{2c^4}) - 3\sqrt{-(4ac - b^2)^5}) \frac{(20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)}{(2c^4(1024a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 + 20ab^8c - b^{10}))} + 27a^2b^3c + 48a^2b^2c^5(-\frac{3b}{2c^4}) - 3\sqrt{-(4ac - b^2)^5}) \frac{(20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)}{(2c^4(1024a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 + 20ab^8c - b^{10}))} - 3ab^5 - 12ab^4c^4(-\frac{3b}{2c^4}) - 3\sqrt{-(4ac - b^2)^5}) \frac{(20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)}{(2c^4(1024a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 + 20ab^8c - b^{10}))} + b^6c^3(-\frac{3b}{2c^4}) - 3\sqrt{-(4ac - b^2)^5}) \frac{(20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)}{(2c^4(1024a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 + 20ab^8c - b^{10}))} \bigg) / (60a^3c^3 - 90a^2b^2c^2 + 30ab^4c - 3b^6) + (-\frac{3b}{2c^4} + 3\sqrt{-(4ac - b^2)^5}) \frac{(20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)}{(2c^4(1024a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 + 20ab^8c - b^{10}))} \log(x + (-66a^3b^2c^2 - 64a^3c^6(-\frac{3b}{2c^4}) + 3\sqrt{-(4ac - b^2)^5}) \frac{(20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)}{(2c^4(1024a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 + 20ab^8c - b^{10}))} + 27a^2b^3c + 48a^2b^2c^5(-\frac{3b}{2c^4}) + 3\sqrt{-(4ac - b^2)^5}) \frac{(20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)}{(2c^4(1024a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 + 20ab^8c - b^{10}))} - 3ab^5 - 12ab^4c^4(-\frac{3b}{2c^4}) + 3\sqrt{-(4ac - b^2)^5}) \frac{(20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)}{(2c^4(1024a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 + 20ab^8c - b^{10}))} \bigg) / (60a^3c^3 - 90a^2b^2c^2 + 30ab^4c - 3b^6) \end{aligned}$$

$$\begin{aligned}
& 4a^{5c^5} - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 \\
& + 20ab^8c - b^{10})) + b^6c^3(-3b/(2c^4) + 3\sqrt{-(4ac - b^2)^5} \\
& (20a^3c^3 - 30a^2b^2c^2 + 10ab^4c - b^6)/(2c^4(102 \\
& 4a^5c^5 - 1280a^4b^2c^4 + 640a^3b^4c^3 - 160a^2b^6c^2 \\
& + 20ab^8c - b^{10}))) / (60a^3c^3 - 90a^2b^2c^2 + 30ab^4c \\
& - 3b^6) + (-58a^4b^2c^2 + 36a^3b^3c - 5a^2b^5 + x^3(36a^3 \\
& c^4 - 102a^2b^2c^3 + 48ab^4c^2 - 6b^6c) + x^2(-42a^3b \\
& c^3 - 41a^2b^3c^2 + 34ab^5c - 5b^7) + x(28a^4c^3 - 142a \\
& ^3b^2c^2 + 76a^2b^4c - 10ab^6)) / (32a^4c^6 - 16a^3b^2c \\
& ^5 + 2a^2b^4c^4 + x^4(32a^2c^8 - 16ab^2c^7 + 2b^4c^6) \\
& + x^3(64a^2b^2c^7 - 32ab^3c^6 + 4b^5c^5) + x^2(64a^3c^7 \\
& - 12ab^4c^5 + 2b^6c^4) + x(64a^3b^2c^6 - 32a^2b^3c^5 + \\
& 4ab^5c^4)) + x/c^3
\end{aligned}$$

$$3.371 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x} dx$$

Optimal. Leaf size=190

$$\frac{b(30a^2c^2 - 10ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) - \frac{bx(b^2-7ac)}{c^2(b^2-4ac)^2} + \frac{x^2(bx(b^2-10ac) + a(b^2-16ac))}{2c(b^2-4ac)^2(a+bx+cx^2)} + \frac{x^4(2a}{2(b^2-4ac)(a$$

Rubi [A] time = 0.28, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1354, 738, 818, 773, 634, 618, 206, 628}

$$\frac{b(30a^2c^2 - 10ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) - \frac{bx(b^2-7ac)}{c^2(b^2-4ac)^2} + \frac{x^4(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)} + \frac{x^2(bx(b^2-10ac) + a(b^2-16ac))}{2c(b^2-4ac)^2(a+bx+cx^2)} + \frac{\log(a+bx+cx^2)}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^3*x), x]

[Out] -((b*(b^2 - 7*a*c)*x)/(c^2*(b^2 - 4*a*c)^2)) + (x^4*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (x^2*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*c)*x))/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + Log[a + b*x + c*x^2]/(2*c^3)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x))/((c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))] + e*(b^2*e*g*(m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])
```

Rule 1354

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x} dx &= \int \frac{x^5}{(a + bx + cx^2)^3} dx \\
&= \frac{x^4(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{\int \frac{x^3(8a+bx)}{(a+bx+cx^2)^2} dx}{2(b^2 - 4ac)} \\
&= \frac{x^4(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^2(a(b^2 - 16ac) + b(b^2 - 10ac)x)}{2c(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{\int \frac{x(2a(b^2 - 16ac) + 2b(b^2 - 10ac)x)}{a + bx + cx^2} dx}{2c(b^2 - 4ac)} \\
&= -\frac{b(b^2 - 7ac)x}{c^2(b^2 - 4ac)^2} + \frac{x^4(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^2(a(b^2 - 16ac) + b(b^2 - 10ac)x)}{2c(b^2 - 4ac)^2(a + bx + cx^2)} \\
&= -\frac{b(b^2 - 7ac)x}{c^2(b^2 - 4ac)^2} + \frac{x^4(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^2(a(b^2 - 16ac) + b(b^2 - 10ac)x)}{2c(b^2 - 4ac)^2(a + bx + cx^2)} + \\
&= -\frac{b(b^2 - 7ac)x}{c^2(b^2 - 4ac)^2} + \frac{x^4(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^2(a(b^2 - 16ac) + b(b^2 - 10ac)x)}{2c(b^2 - 4ac)^2(a + bx + cx^2)} + \\
&= -\frac{b(b^2 - 7ac)x}{c^2(b^2 - 4ac)^2} + \frac{x^4(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x^2(a(b^2 - 16ac) + b(b^2 - 10ac)x)}{2c(b^2 - 4ac)^2(a + bx + cx^2)} +
\end{aligned}$$

Mathematica [A] time = 0.31, size = 221, normalized size = 1.16

$$\frac{\frac{2bc(30a^2c^2 - 10ab^2c + b^4) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + \frac{2a^3c^2 + a^2bc(5cx-4b) + ab^3(b-5cx) + b^5x}{(b^2-4ac)(a+x(b+cx))^2} + \frac{32a^3c^3 - 39a^2b^2c^2 + 50a^2bc^3x + 11ab^4c - 30ab^3c^2x - b^6 + 4b^5cx}{(b^2-4ac)^2(a+x(b+cx))} + c \log(a + x(b + cx))}{2c^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^3*x), x]

[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 30*a*b^3*c^2*x + 50*a^2*b*c^3*x)/(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x))/(b^2 - 4*a*c)*(a + x*(b + c*x))

$+ c*x))^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + x*(b + c*x)]/(2*c^4)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x), x]

fricas [B] time = 1.68, size = 1603, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="fricas")

[Out] $[1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x + (a^2*b^$

$$6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3 + (b^6c^2 - 12a^2b^4c^3 + 48a^3b^2c^4 - 64a^3c^5)x^4 + 2(b^7c - 12a^2b^5c^2 + 48a^3b^3c^3 - 64a^3b^2c^4)x^3 + (b^8 - 10a^2b^6c + 24a^3b^4c^2 + 32a^3b^2c^3 - 128a^4c^4)x^2 + 2(a^2b^7 - 12a^2b^5c + 48a^3b^3c^2 - 64a^4b^2c^3)x \log(cx^2 + bx + a) / (a^2b^6c^3 - 12a^3b^4c^4 + 48a^4b^2c^5 - 64a^5c^6 + (b^6c^5 - 12a^2b^4c^6 + 48a^3b^2c^7 - 64a^3c^8)x^4 + 2(b^7c^4 - 12a^2b^5c^5 + 48a^3b^3c^6 - 64a^3b^2c^7)x^3 + (b^8c^3 - 10a^2b^6c^4 + 24a^3b^4c^5 + 32a^3b^2c^6 - 128a^4c^7)x^2 + 2(a^2b^7c^3 - 12a^2b^5c^4 + 48a^3b^3c^5 - 64a^4b^2c^6)x]$$

giac [A] time = 0.40, size = 245, normalized size = 1.29

$$\frac{(b^5 - 10ab^3c + 30a^2bc^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) + \log(cx^2 + bx + a)}{(b^4c^3 - 8ab^2c^4 + 16a^2c^5)\sqrt{-b^2 + 4ac}} + \frac{\log(cx^2 + bx + a)}{2c^3} + \frac{3a^2b^4 - 21a^3b^2c + 24a^4c^2 + 2(2b^5c - 15ab^3c^2 + 25a^2bc^3)x^3 + (3b^6 - 19ab^4c + 11a^2b^2c^2 + 32a^3c^3)x^2 + 2(3ab^5 - 22a^2b^3c + 31a^3bc^2)x}{2(cx^2 + bx + a)^2(b^2 - 4ac)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="giac")

$$[Out] -(b^5 - 10a^2b^3c + 30a^2b^2c^2) \arctan((2cx + b)/\sqrt{-b^2 + 4ac}) / ((b^4c^3 - 8a^2b^2c^4 + 16a^2c^5) \sqrt{-b^2 + 4ac}) + 1/2 \log(cx^2 + bx + a) / c^3 + 1/2 (3a^2b^4 - 21a^3b^2c + 24a^4c^2 + 2(2b^5c - 15a^2b^3c^2 + 25a^2b^2c^3)x^3 + (3b^6 - 19a^2b^4c + 11a^2b^2c^2 + 32a^3c^3)x^2 + 2(3a^2b^5 - 22a^2b^3c + 31a^3b^2c^2)x) / ((cx^2 + bx + a)^2(b^2 - 4ac)^2c^3)$$

maple [B] time = 0.02, size = 530, normalized size = 2.79

$$\frac{30a^2b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac - b^2}c} + \frac{10ab^3 \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac - b^2}c^2} + \frac{b^5 \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac - b^2}c^3} + \frac{8a^2 \ln(cx^2 + bx + a)}{(16a^2c^2 - 8ab^2c + b^4)c} + \frac{4ab^2 \ln(cx^2 + bx + a)}{(16a^2c^2 - 8ab^2c + b^4)c^2} + \frac{b^4 \ln(cx^2 + bx + a)}{2(16a^2c^2 - 8ab^2c + b^4)c^3} + \frac{(25a^2c^2 - 15a^2b^2c + 25a^2b^2c^2)}{(16a^2c^2 - 8ab^2c + b^4)c^3} + \frac{(31a^2c^2 - 22a^2b^2c + 31a^2b^2c^2)}{(16a^2c^2 - 8ab^2c + b^4)c^3} + \frac{3(9a^2c^2 - 7a^2b^2c + b^4)c^2}{2(16a^2c^2 - 8ab^2c + b^4)c^3} + \frac{(30a^2c^2 + 11a^2b^2c - 39a^2b^2c^2)}{2(16a^2c^2 - 8ab^2c + b^4)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^3/x,x)

$$[Out] (1/c^2b(25a^2c^2 - 15a^2b^2c + 2b^4) / (16a^2c^2 - 8a^2b^2c + b^4)x^3 + 1/2(32a^3c^3 + 11a^2b^2c^2 - 19a^2b^4c + 3b^6) / c^3 / (16a^2c^2 - 8a^2b^2c + b^4)x^2 + a^2b(31a^2c^2 - 22a^2b^2c + 3b^4) / (16a^2c^2 - 8a^2b^2c + b^4) / c^3 x + 3/2a^2(8a^2c^2 - 7a^2b^2c + b^4) / c^3 / (16a^2c^2 - 8a^2b^2c + b^4) / (cx^2 + bx + a)^2 + 8/c / (16a^2c^2 - 8a^2b^2c + b^4) * ln(cx^2 + bx + a) * a^2 - 4/c^2 / (16a^2c^2 - 8a^2b^2c + b^4) * ln(cx^2 + bx + a) * a^2b + 1/2/c^3 / (16a^2c^2 - 8a^2b^2c + b^4) * ln(cx^2 + bx + a) * b^4 - 30/c / (16a^2c^2 - 8a^2b^2c + b^4) / (4ac - b^2)^(1/2) * arctan((2cx + b) / (4ac - b^2)^(1/2)) * a^2b + 10/c^2 / (16a^2c^2 - 8a^2b^2c + b^4) / (4ac - b^2)^(1/2) * arctan((2cx + b) / (4ac - b^2)^(1/2)) * a^2b^3 - 1/c^3 / (16a^2c^2 - 8a^2b^2c + b^4) / (4ac - b^2)^(1/2) * arctan((2cx + b) / (4ac - b^2)^(1/2)) * b^5$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo re details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.20, size = 620, normalized size = 3.26

$$\frac{\frac{3a^2(b^2c-7ab^2c+3a^2)}{2c^2(16a^2c-8ab^2c+3a^2)} - \frac{a^2(12a^2c-11a^2b^2c-3a^2c+3a^2)}{2c^2(16a^2c-8ab^2c+3a^2)} + \frac{a^2(2a^2c-11a^2b^2c+2a^2)}{a^2(16a^2c-8ab^2c+3a^2)} + \frac{ab(11a^2c-22a^2b^2c+3a^2)}{a^2(16a^2c-8ab^2c+3a^2)} - \ln(c^2+bx+a) \left(\frac{-1024a^5c^5+1280a^4b^2c^4-640a^3b^4c^3+160a^2b^6c^2-20a^2b^8c+4a^{10}}{2(1024a^5c^5-1280a^4b^2c^4+640a^3b^4c^3-160a^2b^6c^2+20a^2b^8c-10a^{10})} \right) - \frac{b \operatorname{atan}\left(\frac{\frac{1+1024a^5c^5+1280a^4b^2c^4+640a^3b^4c^3+160a^2b^6c^2-20a^2b^8c+4a^{10}}{2(1024a^5c^5-1280a^4b^2c^4+640a^3b^4c^3-160a^2b^6c^2+20a^2b^8c-10a^{10})}}{2(4ac-b^2)^{5/2}}\right)}{2(4ac-b^2)^{5/2}}}{2(4ac-b^2)^{5/2}} \left(30a^2c^2-10ab^2c+a^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(c + a/x^2 + b/x)^3),x)

[Out] ((3*a^2*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(2*b^4 + 25*a^2*c^2 - 15*a*b^2*c))/(c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*b*x*(3*b^4 + 31*a^2*c^2 - 22*a*b^2*c))/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (log(a + b*x + c*x^2)*(b^10 - 1024*a^5*c^5 + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 20*a*b^8*c))/(2*(1024*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b^4*c^6 - 1280*a^4*b^2*c^7)) - (b*atan(((b*x*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^2*(4*a*c - b^2)^5) + (b^2*(16*a^2*c^4 + b^4*c^2 - 8*a*b^2*c^3)*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*c^5*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a*c - b^2)^(5/2)))/(b^5 + 30*a^2*b*c^2 - 10*a*b^3*c))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^3*(4*a*c - b^2)^(5/2))

sympy [B] time = 3.21, size = 1510, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**3/x,x)

[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)))/(b^5 + 30*a^2*b*c^2 - 10*a*b^3*c))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^3*(4*a*c - b^2)^(5/2))

$$\begin{aligned}
& **5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a* \\
& b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(-b*\sqrt{-(4*a*c - b**2)**5})*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c** \\
& *5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b \\
& **8*c - b**10)) + 1/(2*c**3)) + a*b**4 + b**6*c**2*(-b*\sqrt{-(4*a*c - b**2) \\
& **5})*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a** \\
& 4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10 \\
&)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (b*\sqrt{-(4*a*c \\
& - b**2)**5})*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1 \\
& 280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c \\
& - b**10)) + 1/(2*c**3))*\log(x + (-64*a**3*c**5*(b*\sqrt{-(4*a*c - b**2)**5})* \\
& (30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b** \\
& 2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + \\
& 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(b*\sqrt{-(4*a*c - b**2)**5})* \\
& (30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b** \\
& 2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + \\
& 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(b*\sqrt{-(4*a*c - b**2)**5})*(3 \\
& 0*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2* \\
& c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/ \\
& (2*c**3)) + a*b**4 + b**6*c**2*(b*\sqrt{-(4*a*c - b**2)**5})*(30*a**2*c**2 - \\
& 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a** \\
& 3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)))/(30 \\
& *a**2*b*c**2 - 10*a*b**3*c + b**5)) + (24*a**4*c**2 - 21*a**3*b**2*c + 3*a* \\
& *2*b**4 + x**3*(50*a**2*b*c**3 - 30*a*b**3*c**2 + 4*b**5*c) + x**2*(32*a**3 \\
& *c**3 + 11*a**2*b**2*c**2 - 19*a*b**4*c + 3*b**6) + x*(62*a**3*b*c**2 - 44* \\
& a**2*b**3*c + 6*a*b**5)))/(32*a**4*c**5 - 16*a**3*b**2*c**4 + 2*a**2*b**4*c* \\
& *3 + x**4*(32*a**2*c**7 - 16*a*b**2*c**6 + 2*b**4*c**5) + x**3*(64*a**2*b*c \\
& **6 - 32*a*b**3*c**5 + 4*b**5*c**4) + x**2*(64*a**3*c**6 - 12*a*b**4*c**4 + \\
& 2*b**6*c**3) + x*(64*a**3*b*c**5 - 32*a**2*b**3*c**4 + 4*a*b**5*c**3))
\end{aligned}$$

$$3.372 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^2} dx$$

Optimal. Leaf size=111

$$\frac{12a^2 \tanh^{-1}\left(\frac{\frac{2a}{x} + b}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{5/2}} - \frac{3a\left(\frac{2a}{x} + b\right)}{(b^2 - 4ac)^2 \left(\frac{a}{x^2} + \frac{b}{x} + c\right)} + \frac{\frac{2a}{x} + b}{2(b^2 - 4ac) \left(\frac{a}{x^2} + \frac{b}{x} + c\right)^2}$$

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1352, 614, 618, 206}

$$\frac{12a^2 \tanh^{-1}\left(\frac{\frac{2a}{x} + b}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{5/2}} - \frac{3a\left(\frac{2a}{x} + b\right)}{(b^2 - 4ac)^2 \left(\frac{a}{x^2} + \frac{b}{x} + c\right)} + \frac{\frac{2a}{x} + b}{2(b^2 - 4ac) \left(\frac{a}{x^2} + \frac{b}{x} + c\right)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^3*x^2), x]

[Out] (b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) - (3*a*(b + (2*a)/x))/((b^2 - 4*a*c)^2*(c + a/x^2 + b/x)) + (12*a^2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^2} dx &= -\text{Subst}\left(\int \frac{1}{(c + bx + ax^2)^3} dx, x, \frac{1}{x}\right) \\
 &= \frac{b + \frac{2a}{x}}{2(b^2 - 4ac)\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} + \frac{(3a)\text{Subst}\left(\int \frac{1}{(c + bx + ax^2)^2} dx, x, \frac{1}{x}\right)}{b^2 - 4ac} \\
 &= \frac{b + \frac{2a}{x}}{2(b^2 - 4ac)\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} - \frac{3a\left(b + \frac{2a}{x}\right)}{(b^2 - 4ac)^2\left(c + \frac{a}{x^2} + \frac{b}{x}\right)} - \frac{(6a^2)\text{Subst}\left(\int \frac{1}{c + bx + ax^2} dx, x, \frac{1}{x}\right)}{(b^2 - 4ac)^2} \\
 &= \frac{b + \frac{2a}{x}}{2(b^2 - 4ac)\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} - \frac{3a\left(b + \frac{2a}{x}\right)}{(b^2 - 4ac)^2\left(c + \frac{a}{x^2} + \frac{b}{x}\right)} + \frac{(12a^2)\text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, \frac{1}{x}\right)}{(b^2 - 4ac)^2} \\
 &= \frac{b + \frac{2a}{x}}{2(b^2 - 4ac)\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^2} - \frac{3a\left(b + \frac{2a}{x}\right)}{(b^2 - 4ac)^2\left(c + \frac{a}{x^2} + \frac{b}{x}\right)} + \frac{12a^2 \tanh^{-1}\left(\frac{b + \frac{2a}{x}}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.17, size = 174, normalized size = 1.57

$$\frac{1}{2} \left(\frac{24a^2 \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + \frac{a^2c(2cx-3b) + ab^2(b-4cx) + b^4x}{c^3(4ac-b^2)(a+x(b+cx))^2} + \frac{22a^2bc^2 - 20a^2c^3x - 8ab^3c + 16ab^2c^2x + b^5 - 2b^4cx}{c^3(b^2-4ac)^2(a+x(b+cx))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^3*x^2), x]

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 20*a^2*c^3*x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) + a^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (24*a^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

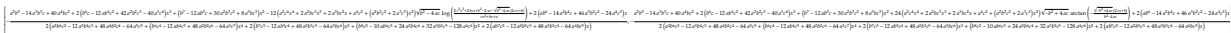
$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^2), x]
```

```
[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^2), x]
```

fricas [B] time = 0.76, size = 953, normalized size = 8.59



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^2 - 12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x), -1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^2 + 24*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x)]
```

giac [A] time = 0.42, size = 202, normalized size = 1.82

$$\frac{12a^2 \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4-8ab^2c+16a^2c^2)\sqrt{-b^2+4ac}} - \frac{2b^4cx^3-16ab^2c^2x^3+20a^2c^3x^3+b^5x^2-8ab^3cx^2-2a^2bc^2x^2+2ab^4x-20a^2b^2cx+12a^3c^2x+a^2b^3-10a^3bc}{2(b^4c^2-8ab^2c^3+16a^2c^4)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="giac")

[Out] $12a^2 \arctan((2cx+b)/\sqrt{-b^2+4ac})/((b^4-8a^2b^2c+16a^2c^2)\sqrt{-b^2+4ac}) - 1/2*(2b^4cx^3-16a^2b^2c^2x^3+20a^2c^3x^3+b^5x^2-8a^2b^3cx^2-2a^2b^2c^2x^2+2ab^4x-20a^2b^2cx+12a^3c^2x+a^2b^3-10a^3bc)/((b^4c^2-8a^2b^2c^3+16a^2c^4)(cx^2+bx+a)^2)$

maple [B] time = 0.01, size = 260, normalized size = 2.34

$$\frac{12a^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8ab^2c+b^4)\sqrt{4ac-b^2}} + \frac{-\frac{(10a^2c^2-8ab^2c+b^4)x^3}{(16a^2c^2-8ab^2c+b^4)c} + \frac{(10ac-b^2)a^2b}{2(16a^2c^2-8ab^2c+b^4)c^2} + \frac{(2a^2c^2+8ab^2c-b^4)bx^2}{2(16a^2c^2-8ab^2c+b^4)c^2} - \frac{(6a^2c^2-10ab^2c+b^4)ax}{(16a^2c^2-8ab^2c+b^4)c^2}}{(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^3/x^2,x)

[Out] $(-1/c*(10a^2c^2-8a^2b^2c+b^4)/(16a^2c^2-8a^2b^2c+b^4)*x^3+1/2*b*(2a^2c^2+8a^2b^2c-b^4)/c^2/(16a^2c^2-8a^2b^2c+b^4)*x^2-a*(6a^2c^2-10a^2b^2c+b^4)/(16a^2c^2-8a^2b^2c+b^4)/c^2*x+1/2*a^2*b*(10a^2c-b^2)/c^2/(16a^2c^2-8a^2b^2c+b^4))/(c*x^2+bx+a)^2+12a^2/(16a^2c^2-8a^2b^2c+b^4)/(4a^2c-b^2)^{(1/2)}*\arctan((2cx+b)/(4a^2c-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.19, size = 343, normalized size = 3.09

$$\frac{12a^2 \operatorname{atan}\left(\frac{\left(\frac{6a^2(16a^2b^2c^2-8ab^3c+b^5)}{(4ac-b^2)^{3/2}(16a^2c^2-8ab^2c+b^4)} + \frac{12a^2cx}{(4ac-b^2)^{3/2}}\right)(16a^2c^2-8ab^2c+b^4)}{6a^2}\right)}{(4ac-b^2)^{5/2}} - \frac{x^3(10a^2c^2-8ab^2c+b^4)}{c(16a^2c^2-8ab^2c+b^4)} + \frac{a^2(b^3-10abc)}{2c^2(16a^2c^2-8ab^2c+b^4)} - \frac{x^2(2a^2b^2c^2+8ab^3c-b^5)}{2c^2(16a^2c^2-8ab^2c+b^4)} + \frac{ax(6a^2c^2-10ab^2c+b^4)}{c^2(16a^2c^2-8ab^2c+b^4)}}{x^2(b^2+2ac)+a^2+c^2x^4+2abx+2bcx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(c + a/x^2 + b/x)^3),x)`

[Out] $(12a^2 \operatorname{atan}\left(\frac{(6a^2(b^5 + 16a^2bc^2 - 8ab^3c))}{(4ac - b^2)^{5/2}}\right) * (b^4 + 16a^2c^2 - 8ab^2c) + (12a^2cx)/(4ac - b^2)^{5/2}) * (b^4 + 16a^2c^2 - 8ab^2c) / (6a^2) / (4ac - b^2)^{5/2} - ((x^3(b^4 + 10a^2c^2 - 8ab^2c)) / (c(b^4 + 16a^2c^2 - 8ab^2c)) + (a^2(b^3 - 10ab^2c)) / (2c^2(b^4 + 16a^2c^2 - 8ab^2c)) - (x^2(2a^2bc^2 - b^5 + 8ab^3c)) / (2c^2(b^4 + 16a^2c^2 - 8ab^2c)) + (ax(b^4 + 6a^2c^2 - 10ab^2c)) / (c^2(b^4 + 16a^2c^2 - 8ab^2c))) / (x^2(2ac + b^2) + a^2 + c^2x^4 + 2abx + 2bcx^3)$

sympy [B] time = 1.41, size = 547, normalized size = 4.93

$$-a^2 \sqrt{\frac{1}{(4c-b^2)}} \operatorname{atan}\left(\frac{-384a^2c^2 \sqrt{\frac{1}{(4c-b^2)}} + 288a^2c^2 \sqrt{\frac{1}{(4c-b^2)}} - 72a^2b^4 \sqrt{\frac{1}{(4c-b^2)}} + 6a^2b^2 \sqrt{\frac{1}{(4c-b^2)}} + 6a^2b^2}{12bc}\right) + 6a^2 \sqrt{\frac{1}{(4c-b^2)}} \operatorname{atan}\left(\frac{384a^2c^2 \sqrt{\frac{1}{(4c-b^2)}} - 288a^2c^2 \sqrt{\frac{1}{(4c-b^2)}} + 72a^2b^4 \sqrt{\frac{1}{(4c-b^2)}} - 6a^2b^2 \sqrt{\frac{1}{(4c-b^2)}} + 6a^2b^2}{12bc}\right) + \frac{10a^2bc - a^2b^3 + x^3(-20a^2c^3 + 16a^2b^2c^2 - 2a^2b^2c + 8ab^3c - b^5) + x(-12a^2c^2 + 20a^2b^2c - 2a^2b^2)}{32a^4c^4 - 16a^3b^2c^2 + 2a^2b^4c^2 + c^4(32a^2c^2 - 16ab^2c^2 + 2b^4c^2) + a^2(64b^3c^2 - 32a^2b^3c^2 + 4b^5c^2) + c(64a^3b^3c^2 - 32a^2b^3c^2 + 4ab^5c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x**2+b/x)**3/x**2,x)`

[Out] $-6a^2 \operatorname{sqrt}(-1/(4ac - b^2))^{*5} \log(x + (-384a^2c^2 \operatorname{sqrt}(-1/(4ac - b^2))^{*5} + 288a^2c^2 b^2 \operatorname{sqrt}(-1/(4ac - b^2))^{*5} - 72a^2b^4 c \operatorname{sqrt}(-1/(4ac - b^2))^{*5} + 6a^2b^2 \operatorname{sqrt}(-1/(4ac - b^2))^{*5} + 6a^2b^2) / (12a^2c)) + 6a^2 \operatorname{sqrt}(-1/(4ac - b^2))^{*5} \log(x + (384a^2c^2 \operatorname{sqrt}(-1/(4ac - b^2))^{*5} - 288a^2c^2 b^2 \operatorname{sqrt}(-1/(4ac - b^2))^{*5} + 72a^2b^4 c \operatorname{sqrt}(-1/(4ac - b^2))^{*5} - 6a^2b^2 \operatorname{sqrt}(-1/(4ac - b^2))^{*5} + 6a^2b^2) / (12a^2c)) + (10a^2b^3c - a^2b^3 + x^3(-20a^2c^3 + 16a^2b^2c^2 - 2b^4c) + x^2(2a^2b^2c^2 + 8a^2b^3c - b^5) + x(-12a^2c^2 + 20a^2b^2c - 2a^2b^2)) / (32a^4c^4 - 16a^3b^2c^2 + 2a^2b^4c^2 + x^4(32a^2c^2 - 16ab^2c^2 + 2b^4c^2) + x^3(64a^2b^3c^2 - 32a^2b^3c^2 + 4b^5c^2) + x^2(64a^3b^3c^2 - 12a^2b^4c^2 + 2b^6c^2) + x(64a^3b^3c^2 - 32a^2b^3c^2 + 4a^2b^5c^2))$

$$3.373 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^3} dx$$

Optimal. Leaf size=107

$$\frac{3bx(2a + bx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{x^3(b + 2cx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{6ab \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1354, 728, 722, 618, 206}

$$-\frac{x^3(b + 2cx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3bx(2a + bx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{6ab \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^3*x^3),x]

[Out] -(x^3*(b + 2*c*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*x*(2*a + b*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*a*b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 722

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 - b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &

& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]

Rule 728

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[(m*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]

Rule 1354

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3} x^3 dx &= \int \frac{x^3}{(a + bx + cx^2)^3} dx \\
 &= -\frac{x^3(b + 2cx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{(3b) \int \frac{x^2}{(a + bx + cx^2)^2} dx}{2(b^2 - 4ac)} \\
 &= -\frac{x^3(b + 2cx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3bx(2a + bx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{(3ab) \int \frac{1}{a + bx + cx^2} dx}{(b^2 - 4ac)^2} \\
 &= -\frac{x^3(b + 2cx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3bx(2a + bx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{(6ab) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx\right)}{(b^2 - 4ac)} \\
 &= -\frac{x^3(b + 2cx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3bx(2a + bx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{6ab \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.19, size = 126, normalized size = 1.18

$$\frac{8a^3c + a^2(b^2 + 10bcx + 16c^2x^2) + abx(2b^2 + bcx + 6c^2x^2) + b^4x^2}{2c(b^2 - 4ac)^2(a + x(b + cx))^2} - \frac{6ab \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^3*x^3), x]

[Out] -1/2*(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^2 + 10*b*c*x + 16*c^2*x^2))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*a*b*ArcT an[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^3), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^3), x]

fricas [B] time = 1.11, size = 872, normalized size = 8.15

$$\frac{a^4 + 4ab^2c - 32a^2c^2 + 6(b^2 - 4ac)^2c^2 + (b^2 - 3ab^2 + 12a^2b^2 - 64a^2c^2)c^2 - 6(b^2c^2 + 2ab^2c^2 + 2a^2b^2c^2 + ab^2c + (ab^2 + 2a^2b^2c^2)\sqrt{b^2 - 4ac})\log\left(\frac{b^2 + 2cx + \sqrt{b^2 - 4ac}}{b^2 - 4ac}\right) - 2(ab^2 + ab^2c - 20a^2b^2c^2)}{2(b^2 - 12a^2b^2 + 48a^2b^2c - 64a^2c^2 + (b^2 - 12a^2b^2 + 48a^2b^2c - 64a^2c^2)c^2 + 2(b^2 - 12a^2b^2 + 48a^2b^2c - 64a^2c^2)c^2)} - \frac{a^4 + 4ab^2c - 32a^2c^2 + 6(b^2 - 4ac)^2c^2 + (b^2 - 3ab^2 + 12a^2b^2 - 64a^2c^2)c^2 - 12(ab^2c^2 + 2ab^2c^2 + 2a^2b^2c^2 + ab^2c + (ab^2 + 2a^2b^2c^2)\sqrt{b^2 - 4ac})\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{b^2 - 4ac}}\right) + 2(ab^2 + ab^2c - 20a^2b^2c^2)}{2(b^2 - 12a^2b^2 + 48a^2b^2c - 64a^2c^2 + (b^2 - 12a^2b^2 + 48a^2b^2c - 64a^2c^2)c^2 + 2(b^2 - 12a^2b^2 + 48a^2b^2c - 64a^2c^2)c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="fricas")

[Out] [-1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 6*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x), -1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 12*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c

$*c + 2*a^2*b*c^2)*x^2)*\text{sqrt}(-b^2 + 4*a*c)*\text{arctan}(-\text{sqrt}(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^3*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x]$

giac [A] time = 0.31, size = 163, normalized size = 1.52

$$\frac{6ab \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2+4ac}} - \frac{6abc^2x^3 + b^4x^2 + ab^2cx^2 + 16a^2c^2x^2 + 2ab^3x + 10a^2bcx + a^2b^2 + 8a^3c}{2(b^4c - 8ab^2c^2 + 16a^2c^3)(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="giac")

[Out] $-6*a*b*\text{arctan}((2*c*x + b)/\text{sqrt}(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*\text{sqrt}(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 + a*b^2*c*x^2 + 16*a^2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)$

maple [B] time = 0.01, size = 223, normalized size = 2.08

$$\frac{6ab \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac-b^2}} + \frac{\frac{3abcx^3}{16a^2c^2-8ab^2c+b^4} - \frac{(5ac+b^2)abx}{(16a^2c^2-8ab^2c+b^4)c} - \frac{(8ac+b^2)a^2}{2(16a^2c^2-8ab^2c+b^4)c} - \frac{(16a^2c^2+ab^2c+b^4)x^2}{2(16a^2c^2-8ab^2c+b^4)c}}{(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^3/x^3,x)

[Out] $(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2-6*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*\text{arctan}((2*c*x+b)/(4*a*c-b^2)^(1/2))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.43, size = 271, normalized size = 2.53

$$\frac{\frac{x^2(16a^2c^2+ab^2c+b^4)}{2c(16a^2c^2-8ab^2c+b^4)} + \frac{a^2(b^2+8ac)}{2c(16a^2c^2-8ab^2c+b^4)} + \frac{3abcx^3}{16a^2c^2-8ab^2c+b^4} + \frac{abx(b^2+5ac)}{c(16a^2c^2-8ab^2c+b^4)} - \frac{6ab \operatorname{atan}\left(\frac{\left(\frac{3ab^2}{(4ac-b^2)^{5/2}} + \frac{6abcx}{(4ac-b^2)^{5/2}}\right)(16a^2c^2-8ab^2c+b^4)}{3ab}\right)}{(4ac-b^2)^{5/2}}}{x^2(b^2+2ac) + a^2 + c^2x^4 + 2abx + 2bcx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(c + a/x^2 + b/x)^3),x)

[Out] $-\left(\frac{x^2(b^4 + 16a^2c^2 + ab^2c)}{2c(b^4 + 16a^2c^2 - 8ab^2c)} + \frac{a^2(8ac + b^2)}{2c(b^4 + 16a^2c^2 - 8ab^2c)} + \frac{3ab^2cx^3}{(b^4 + 16a^2c^2 - 8ab^2c)} + \frac{ab^2x(5ac + b^2)}{c(b^4 + 16a^2c^2 - 8ab^2c)}\right) / (x^2(2ac + b^2) + a^2 + c^2x^4 + 2abx + 2bcx^3) - \left(\frac{6ab^2 \operatorname{atan}\left(\frac{(3ab^2)/(4ac-b^2)^{5/2} + (6ab^2cx)/(4ac-b^2)^{5/2}}{(b^4 + 16a^2c^2 - 8ab^2c)}\right)}{(3ab)}\right) / (4ac-b^2)^{5/2}$

sympy [B] time = 1.21, size = 513, normalized size = 4.79

$$\frac{3ab \sqrt{\frac{1}{(4ac-b^2)^5}} \log\left(x + \frac{-192a^4b^3c^3 \sqrt{\frac{1}{(4ac-b^2)^5}} + 144a^3b^3c^2 \sqrt{\frac{1}{(4ac-b^2)^5}} - 36a^2b^3c \sqrt{\frac{1}{(4ac-b^2)^5}} + 3ab^3 \sqrt{\frac{1}{(4ac-b^2)^5}} + 3ab^3}{abc}\right) + 3ab \sqrt{\frac{1}{(4ac-b^2)^5}} \log\left(x + \frac{192a^4b^3c^3 \sqrt{\frac{1}{(4ac-b^2)^5}} - 144a^3b^3c^2 \sqrt{\frac{1}{(4ac-b^2)^5}} + 36a^2b^3c \sqrt{\frac{1}{(4ac-b^2)^5}} - 3ab^3 \sqrt{\frac{1}{(4ac-b^2)^5}} + 3ab^3}{abc}\right) - \frac{-8a^2c^2 - 6ab^2c^2 + x^2(-16a^2c^2 - ab^2c - b^4) + x(-10a^2c - 2ab^2)}{32a^3c^3 - 16a^2b^3c^2 + 2ab^3c^2 + x^2(32a^3c^3 - 16a^2b^3c^2 + 2ab^3c^2) + x^2(64a^3c^3 - 32a^2b^3c^2 + 4b^3c^2) + x(64a^3c^3 - 32a^2b^3c^2 + 4b^3c^2) + 3(64a^3c^3 - 32a^2b^3c^2 + 4b^3c^2)}{32a^3c^3 - 16a^2b^3c^2 + 2ab^3c^2 + x^2(32a^3c^3 - 16a^2b^3c^2 + 2ab^3c^2) + x^2(64a^3c^3 - 32a^2b^3c^2 + 4b^3c^2) + x(64a^3c^3 - 32a^2b^3c^2 + 4b^3c^2) + 3(64a^3c^3 - 32a^2b^3c^2 + 4b^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**3/x**3,x)

[Out] $3ab \sqrt{-1/(4ac-b^2)^5} \log(x + (-192a^4b^3c^3 \sqrt{-1/(4ac-b^2)^5} + 144a^3b^3c^2 \sqrt{-1/(4ac-b^2)^5} - 36a^2b^3c \sqrt{-1/(4ac-b^2)^5} + 3ab^3 \sqrt{-1/(4ac-b^2)^5} + 3ab^3)/(6ab^2c)) - 3ab \sqrt{-1/(4ac-b^2)^5} \log(x + (192a^4b^3c^3 \sqrt{-1/(4ac-b^2)^5} - 144a^3b^3c^2 \sqrt{-1/(4ac-b^2)^5} + 36a^2b^3c \sqrt{-1/(4ac-b^2)^5} - 3ab^3 \sqrt{-1/(4ac-b^2)^5} + 3ab^3)/(6ab^2c)) + (-8a^3c - a^2b^2 - 6ab^2c^2x^3 + x^2(-16a^2c^2 - ab^2c - b^4) + x(-10a^2b^2c - 2ab^3c^3))/(32a^4c^3 - 16a^3b^3c^2 + 2a^2b^4c + x^4(32a^2c^5 - 16ab^2c^4 + 2b^4c^3) + x^3(64a^2b^2c^4 - 32ab^3c^3 + 4b^5c^2) + x^2(64a^3c^4 - 12ab^4c^2 + 2b^6c) + x(64a^3b^3c^3 - 32a^2b^3c^2 + 4ab^5c^2) + 4ab^5c^2)$

$$3.374 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^4} dx$$

Optimal. Leaf size=115

$$\frac{x(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x(2ac + b^2) + 3ab}{(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{2(2ac + b^2) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Rubi [A] time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1354, 738, 638, 618, 206}

$$\frac{x(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{x(2ac + b^2) + 3ab}{(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{2(2ac + b^2) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^3*x^4),x]

[Out] (x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*a*b + (b^2 + 2*a*c)*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(b^2 + 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 738

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^4} dx &= \int \frac{x^2}{(a + bx + cx^2)^3} dx \\ &= \frac{x(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{\int \frac{2a - 2bx}{(a + bx + cx^2)^2} dx}{2(b^2 - 4ac)} \\ &= \frac{x(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3ab + (b^2 + 2ac)x}{(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{(b^2 + 2ac) \int \frac{1}{a + bx + cx^2} dx}{(b^2 - 4ac)^2} \\ &= \frac{x(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3ab + (b^2 + 2ac)x}{(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{(2(b^2 + 2ac)) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac + cx^2} dx\right)}{(b^2 - 4ac)^2} \\ &= \frac{x(2a + bx)}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3ab + (b^2 + 2ac)x}{(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{2(b^2 + 2ac) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.14, size = 131, normalized size = 1.14

$$\frac{1}{2} \left(\frac{(2ac + b^2)(b + 2cx)}{c(b^2 - 4ac)^2(a + x(b + cx))} + \frac{a(b - 2cx) + b^2x}{c(4ac - b^2)(a + x(b + cx))^2} + \frac{4(2ac + b^2) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^3*x^4),x]

[Out] ((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + ((b^2 + 2*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (4*(b^2 + 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^4),x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^4), x]

fricas [B] time = 1.10, size = 887, normalized size = 7.71

$$\frac{(b^2 - 2abx + 2a^2x^2 - a^2/x^2 + 2b^2 - 4abx^2 + 2a^2x^4 + 2b^2 - 2abx^2 + 2a^2x^4 + 2b^2 - 4abx^2 + 2a^2x^4 + 2b^2 - 4abx^2 + 2a^2x^4) \sqrt{b^2 - 4ac} \log\left(\frac{(2c^2x^2 + 2b^2cx + b^2 - 2ac - \sqrt{b^2 - 4ac})(2cx + b)}{(cx^2 + bx + a)}\right) + 2(5ab^4 - 22a^2b^2c + 8a^3c^2)x}{(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3 + (b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)x^4 + 2(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3b^2c^4)x^3 + (b^8 - 10ab^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4)x^2 + 2(ab^7 - 12a^2b^5c + 48a^3b^3c^2 - 64a^4b^2c^3)x)}, \frac{1}{2}(6a^2b^3 - 24a^3b^2c + 2(b^4c - 2ab^2c^2 - 8a^2c^3)x^3 + 3(b^5 - 2ab^3c - 8a^2b^2c^2)x^2 + 2((b^2c^2 + 2ac^3)x^4 + a^2b^2 + 2a^3c + 2(b^3c + 2ab^2c^2)x^3 + (b^4 + 4ab^2c + 4a^2c^2)x^2 + 2(ab^3 + 2a^2bc)x)x)\sqrt{b^2 - 4ac} + \arctan\left(\frac{-\sqrt{b^2 - 4ac}(2cx + b)}{(b^2 - 4ac)}\right) + 2(5ab^4 - 22a^2b^2c + 8a^3c^2)x}{(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3 + (b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)x^4 + 2(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3b^2c^4)x^3 + (b^8 - 10ab^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4)x^2 + 2(ab^7 - 12a^2b^5c + 48a^3b^3c^2 - 64a^4b^2c^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="fricas")

[Out] [1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b^2*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b^2*c^3)*x), 1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 - 4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b^2*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b^2*c^3)*x)]

giac [A] time = 0.44, size = 154, normalized size = 1.34

$$\frac{2(b^2 + 2ac) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2+4ac}} + \frac{2b^2cx^3 + 4ac^2x^3 + 3b^3x^2 + 6abcx^2 + 10ab^2x - 4a^2cx + 6a^2b}{2(b^4 - 8ab^2c + 16a^2c^2)(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="giac")

[Out] $2*(b^2 + 2*a*c)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*\sqrt{-b^2 + 4*a*c}) + 1/2*(2*b^2*c*x^3 + 4*a*c^2*x^3 + 3*b^3*x^2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*a^2*b)/(b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2$

maple [B] time = 0.01, size = 262, normalized size = 2.28

$$\frac{4ac \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac-b^2}} + \frac{2b^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac-b^2}} + \frac{\frac{(2ac+b^2)cx^3}{16a^2c^2-8ab^2c+b^4} + \frac{3a^2b}{16a^2c^2-8ab^2c+b^4} + \frac{3(2ac+b^2)bx^2}{2(16a^2c^2-8ab^2c+b^4)} - \frac{(2ac-5b^2)ax}{16a^2c^2-8ab^2c+b^4}}{(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^3/x^4,x)

[Out] $(c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*c+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.50, size = 313, normalized size = 2.72

$$\frac{\frac{3a^2b}{16a^2c^2-8ab^2c+b^4} - \frac{ax(2ac-5b^2)}{16a^2c^2-8ab^2c+b^4} + \frac{3bx^2(b^2+2ac)}{2(16a^2c^2-8ab^2c+b^4)} + \frac{cx^3(b^2+2ac)}{16a^2c^2-8ab^2c+b^4}}{x^2(b^2+2ac) + a^2 + c^2x^4 + 2abx + 2bcx^3} + \frac{2 \operatorname{atan}\left(\frac{\left(\frac{(b^2+2ac)(16a^2bc^2-8ab^3c+b^5)}{(4ac-b^2)^{5/2}(16a^2c^2-8ab^2c+b^4)} + \frac{2cx(b^2+2ac)}{(4ac-b^2)^{5/2}}\right)(16a^2c^2-8ab^2c+b^4)}{b^2+2ac}\right)}{(4ac-b^2)^{5/2}}}{(4ac-b^2)^{5/2}}(b^2+2ac)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(c + a/x^2 + b/x)^3),x)`

[Out]
$$\begin{aligned} & \left(\frac{3a^2b}{b^4 + 16a^2c^2 - 8ab^2c} - \frac{ax(2ac - 5b^2)}{(b^4 + 16a^2c^2 - 8ab^2c)} + \frac{3bx^2(2ac + b^2)}{2(b^4 + 16a^2c^2 - 8ab^2c)} \right) / (x^2(2ac + b^2) + a^2 + c^2x^4 + 2abx + 2bcx^3) \\ & + \frac{2 \operatorname{atan}\left(\frac{(2ac + b^2)(b^5 + 16a^2bc^2 - 8ab^3c)}{(4ac - b^2)^{5/2}(b^4 + 16a^2c^2 - 8ab^2c)}\right)}{(4ac - b^2)^{5/2}(b^4 + 16a^2c^2 - 8ab^2c)} \\ & + \frac{2cx(2ac + b^2)}{(4ac - b^2)^{5/2}(b^4 + 16a^2c^2 - 8ab^2c)} \end{aligned}$$

sympy [B] time = 1.28, size = 570, normalized size = 4.96

$$\frac{1}{\sqrt{(ac-b^2)} \log\left(\frac{-4ab^2\sqrt{(ac-b^2)} + 4b^3\sqrt{(ac-b^2)} + 12ab^2\sqrt{(ac-b^2)} + 2ab^2\sqrt{(ac-b^2)} + 2ab^2\sqrt{(ac-b^2)}}{4a^2 + 2b^2}\right)} + \frac{1}{\sqrt{(ac-b^2)} \log\left(\frac{4ab^2\sqrt{(ac-b^2)} - 4b^3\sqrt{(ac-b^2)} + 12ab^2\sqrt{(ac-b^2)} + 2ab^2\sqrt{(ac-b^2)} + 2ab^2\sqrt{(ac-b^2)}}{4a^2 + 2b^2}\right)} + \frac{4bx + c^2(4ac + 2b^2) - c^2(4ac + 2b^2) + c^2(4ac + 2b^2)}{12ab^2 - 12ab^2 - 2ab^2 + c^2(12ab^2 - 2ab^2) + c^2(12ab^2 - 2ab^2) + c^2(12ab^2 - 2ab^2) + c^2(12ab^2 - 2ab^2) + c^2(12ab^2 - 2ab^2) + c^2(12ab^2 - 2ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+a/x**2+b/x)**3/x**4,x)`

[Out]
$$\begin{aligned} & -\sqrt{-1/(4ac - b^2)^5} (2ac + b^2) \log(x + (-64a^3c^3 \sqrt{-1/(4ac - b^2)^5} (2ac + b^2) + 48a^2b^2c^2 \sqrt{-1/(4ac - b^2)^5} (2ac + b^2) - 12ab^4c \sqrt{-1/(4ac - b^2)^5} (2ac + b^2) \\ & + 2ab^2c + b^6 \sqrt{-1/(4ac - b^2)^5} (2ac + b^2) + b^3)/(4ac^2 + 2b^2c)) + \sqrt{-1/(4ac - b^2)^5} (2ac + b^2) \log(x + (64a^3c^3 \sqrt{-1/(4ac - b^2)^5} (2ac + b^2) - 48a^2b^2c^2 \sqrt{-1/(4ac - b^2)^5} (2ac + b^2) + 12ab^4c \sqrt{-1/(4ac - b^2)^5} (2ac + b^2) \\ & + 2ab^2c - b^6 \sqrt{-1/(4ac - b^2)^5} (2ac + b^2) + b^3)/(4ac^2 + 2b^2c)) + (6a^2b + x^3(4ac^2 + 2b^2c) + x^2(6ab^2c + 3b^3) + x(-4a^2c + 10ab^2))/(32a^4c^2 - 16a^3b^2c + 2a^2b^4 + x^4(32a^2c^4 - 16ab^2c^3 + 2b^4c^2) + x^3(64a^2b^3c - 32ab^3c^2 + 4b^5c) + x^2(64a^3c^3 - 12ab^4c + 2b^6) + x(64a^3b^2c^2 - 32a^2b^3c + 4ab^5)) \end{aligned}$$

$$3.375 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^5} dx$$

Optimal. Leaf size=103

$$\frac{2a + bx}{2(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{3b(b + 2cx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{6bc \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1354, 638, 614, 618, 206}

$$\frac{2a + bx}{2(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{3b(b + 2cx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{6bc \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^3*x^5), x]

[Out] (2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*b*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*b*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

```
Int[((d_.) + (e_.)*(x_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:= Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^(p), x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^5} dx &= \int \frac{x}{(a + bx + cx^2)^3} dx \\ &= \frac{2a + bx}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{(3b) \int \frac{1}{(a + bx + cx^2)^2} dx}{2(b^2 - 4ac)} \\ &= \frac{2a + bx}{2(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{3b(b + 2cx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{(3bc) \int \frac{1}{a + bx + cx^2} dx}{(b^2 - 4ac)^2} \\ &= \frac{2a + bx}{2(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{3b(b + 2cx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{(6bc) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx\right)}{(b^2 - 4ac)^2} \\ &= \frac{2a + bx}{2(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{3b(b + 2cx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{6bc \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 102, normalized size = 0.99

$$\frac{\frac{(b^2 - 4ac)(2a + bx)}{(a + x(b + cx))^2} - \frac{12bc \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} - \frac{3b(b + 2cx)}{a + x(b + cx)}}{2(b^2 - 4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^3*x^5),x]

[Out] ((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*x))/(a + x*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]/(2*(b^2 - 4*a*c)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^5),x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^5), x]

fricas [B] time = 1.19, size = 788, normalized size = 7.65

$$\frac{6bc \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2+4ac}} - \frac{6bc^2x^3 + 9b^2cx^2 + 2b^3x + 10abcx + ab^2 + 8a^2c}{2(b^4 - 8ab^2c + 16a^2c^2)(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="fricas")

[Out] [-1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3))*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^2 - 6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3))*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^2 - 12*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]

giac [A] time = 0.41, size = 135, normalized size = 1.31

$$\frac{6bc \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2+4ac}} - \frac{6bc^2x^3 + 9b^2cx^2 + 2b^3x + 10abcx + ab^2 + 8a^2c}{2(b^4 - 8ab^2c + 16a^2c^2)(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="giac")

[Out] $-6*b*c*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*\sqrt{-b^2 + 4*a*c}) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2 + 2*b^3*x + 10*a*b*c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)$

maple [A] time = 0.00, size = 130, normalized size = 1.26

$$-\frac{3bcx}{(4ac - b^2)^2 (cx^2 + bx + a)} - \frac{6bc \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{\frac{5}{2}}} - \frac{3b^2}{2(4ac - b^2)^2 (cx^2 + bx + a)} + \frac{-bx - 2a}{2(4ac - b^2)(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^3/x^5,x)

[Out] $1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3/(4*a*c-b^2)^2*b/(c*x^2+b*x+a)*c*x-3/2/(4*a*c-b^2)^2*b^2/(c*x^2+b*x+a)-6/(4*a*c-b^2)^{(5/2)}*b*c*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.43, size = 253, normalized size = 2.46

$$-\frac{\frac{8c^2+ab^2}{2(16a^2c^2-8ab^2c+b^4)} + \frac{9b^2cx^2}{2(16a^2c^2-8ab^2c+b^4)} + \frac{3b^2cx^3}{16a^2c^2-8ab^2c+b^4} + \frac{bx(b^2+5ac)}{16a^2c^2-8ab^2c+b^4}}{x^2(b^2+2ac) + a^2 + c^2x^4 + 2abx + 2bcx^3} - \frac{6bc \operatorname{atan}\left(\frac{\left(\frac{3b^2c}{(4ac-b^2)^{5/2}} + \frac{6b^2x}{(4ac-b^2)^{5/2}}\right)(16a^2c^2-8ab^2c+b^4)}{3bc}\right)}{(4ac-b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(c + a/x^2 + b/x)^3),x)

[Out] $-((a*b^2 + 8*a^2*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^2)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))$

$$*c) + (b*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (6*b*c*atan((((3*b^2*c)/(4*a*c - b^2))^(5/2) + (6*b*c^2*x)/(4*a*c - b^2))^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(3*b*c)))/(4*a*c - b^2)^(5/2)$$

sympy [B] time = 1.09, size = 481, normalized size = 4.67

$$3c \sqrt{\frac{1}{(4ac - b^2)}} \log \left(x + \frac{-192a^3b^4 \sqrt{\frac{1}{(4ac - b^2)}} + 144a^2b^3 \sqrt{\frac{1}{(4ac - b^2)}} - 36ab^2 \sqrt{\frac{1}{(4ac - b^2)}} + 3b \sqrt{\frac{1}{(4ac - b^2)}} + 3c^2}{6bc} \right) - 3c \sqrt{\frac{1}{(4ac - b^2)}} \log \left(x + \frac{192a^3b^4 \sqrt{\frac{1}{(4ac - b^2)}} - 144a^2b^3 \sqrt{\frac{1}{(4ac - b^2)}} + 36ab^2 \sqrt{\frac{1}{(4ac - b^2)}} - 3b \sqrt{\frac{1}{(4ac - b^2)}} + 3c^2}{6bc} \right) + \frac{-8b^2c - ab^3 - 9b^2c^2 - 6ab^2c + (-10ab - 2b^2)}{32a^2c^2 - 16a^2b^2 + 2a^2b^2 + 2a^2c^2 + 2b^2c^2 + 2c^2} + \frac{64b^2c^2 - 32a^2b^2c - 12a^2b^2c + 2b^2c^2 + 4b^2c^2}{(64a^3b^2 - 32a^2b^3c + 4a^2b^2c^2 + 4b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**3/x**5,x)

[Out] $3*b*c*\sqrt{-1/(4*a*c - b**2)**5}*\log(x + (-192*a**3*b*c**4*\sqrt{-1/(4*a*c - b**2)**5} + 144*a**2*b**3*c**3*\sqrt{-1/(4*a*c - b**2)**5} - 36*a*b**5*c**2*\sqrt{-1/(4*a*c - b**2)**5} + 3*b**7*c*\sqrt{-1/(4*a*c - b**2)**5} + 3*b**2*c)/(6*b*c**2)) - 3*b*c*\sqrt{-1/(4*a*c - b**2)**5}*\log(x + (192*a**3*b*c**4*\sqrt{-1/(4*a*c - b**2)**5} - 144*a**2*b**3*c**3*\sqrt{-1/(4*a*c - b**2)**5} + 36*a*b**5*c**2*\sqrt{-1/(4*a*c - b**2)**5} - 3*b**7*c*\sqrt{-1/(4*a*c - b**2)**5} + 3*b**2*c)/(6*b*c**2)) + (-8*a**2*c - a*b**2 - 9*b**2*c*x**2 - 6*b*c**2*x**3 + x*(-10*a*b*c - 2*b**3))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))$

$$3.376 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^6} dx$$

Optimal. Leaf size=103

$$-\frac{12c^2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}} + \frac{3c(b+2cx)}{(b^2-4ac)^2 (a+bx+cx^2)} + \frac{-b-2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1354, 614, 618, 206}

$$-\frac{12c^2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}} + \frac{3c(b+2cx)}{(b^2-4ac)^2 (a+bx+cx^2)} - \frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^3*x^6),x]

[Out] -(b + 2*c*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1354

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^6} dx &= \int \frac{1}{(a + bx + cx^2)^3} dx \\
 &= -\frac{b + 2cx}{2(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{(3c) \int \frac{1}{(a + bx + cx^2)^2} dx}{b^2 - 4ac} \\
 &= -\frac{b + 2cx}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3c(b + 2cx)}{(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{(6c^2) \int \frac{1}{a + bx + cx^2} dx}{(b^2 - 4ac)^2} \\
 &= -\frac{b + 2cx}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3c(b + 2cx)}{(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{(12c^2) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x} dx\right)}{(b^2 - 4ac)^2} \\
 &= -\frac{b + 2cx}{2(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{3c(b + 2cx)}{(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{12c^2 \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 97, normalized size = 0.94

$$\frac{\frac{24c^2 \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} - \frac{(b + 2cx)(-2c(5a + 3cx^2) + b^2 - 6bcx)}{(a + x(b + cx))^2}}{2(b^2 - 4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^3*x^6), x]

[Out] (-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2) + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^6), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^6), x]

fricas [B] time = 1.70, size = 785, normalized size = 7.62

$$\frac{\frac{12c^2 \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2+4ac}} + \frac{12c^3x^3 + 18bc^2x^2 + 4b^2cx + 20ac^2x - b^3 + 10abc}{2(b^4 - 8ab^2c + 16a^2c^2)(cx^2 + bx + a)^2}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="fricas")

[Out] [-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 + 24*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]

giac [A] time = 0.34, size = 136, normalized size = 1.32

$$\frac{12c^2 \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2+4ac}} + \frac{12c^3x^3 + 18bc^2x^2 + 4b^2cx + 20ac^2x - b^3 + 10abc}{2(b^4 - 8ab^2c + 16a^2c^2)(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="giac")

[Out] $12c^2 \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) / ((b^4 - 8ab^2c + 16a^2c^2) \sqrt{-b^2+4ac}) + \frac{1}{2} \frac{(12c^3x^3 + 18bc^2x^2 + 4b^2cx + 20a^2c^2x - b^3 + 10ab^2c)}{(b^4 - 8ab^2c + 16a^2c^2)(cx^2 + bx + a)^2}$

maple [A] time = 0.00, size = 129, normalized size = 1.25

$$\frac{6c^2x}{(4ac-b^2)^2(cx^2+bx+a)} + \frac{12c^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{5}{2}}} + \frac{3bc}{(4ac-b^2)^2(cx^2+bx+a)} + \frac{2cx+b}{2(4ac-b^2)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^3/x^6,x)

[Out] $\frac{1}{2} \frac{(2cx+b)}{(4ac-b^2)} / (cx^2+bx+a)^2 + 6c^2 / (4ac-b^2)^2 / (cx^2+bx+a) * x^3 + c / (4ac-b^2)^2 / (cx^2+bx+a) * b + 12c^2 / (4ac-b^2)^{5/2} * \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 1.42, size = 285, normalized size = 2.77

$$\frac{\frac{6c^3x^3}{16a^2c^2-8ab^2c+b^4} - \frac{b^3-10abc}{2(16a^2c^2-8ab^2c+b^4)} + \frac{9bc^2x^2}{16a^2c^2-8ab^2c+b^4} + \frac{2cx(b^2+5ac)}{16a^2c^2-8ab^2c+b^4}}{x^2(b^2+2ac)+a^2+c^2x^4+2abx+2bcx^3} + \frac{12c^2 \operatorname{atan}\left(\frac{\left(\frac{12c^3x}{(4ac-b^2)^{5/2}} + \frac{6c^2(16a^2b^2-8ab^3c+b^5)}{(4ac-b^2)^{5/2}(16a^2c^2-8ab^2c+b^4)}\right)(16a^2c^2-8ab^2c+b^4)}{6c^2}\right)}{(4ac-b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6*(c + a/x^2 + b/x)^3),x)

[Out] $\left(\frac{6c^3x^3}{b^4 + 16a^2c^2 - 8ab^2c} - \frac{b^3 - 10ab^2c}{2(b^4 + 16a^2c^2 - 8ab^2c)}\right) + \frac{9bc^2x^2}{b^4 + 16a^2c^2 - 8ab^2c} + \frac{2c}{2(b^4 + 16a^2c^2 - 8ab^2c)}$

$$c*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (12*c^2*atan((((12*c^3*x)/(4*a*c - b^2))^{5/2} + (6*c^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^{5/2}*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2)))/(4*a*c - b^2)^{5/2}$$

sympy [B] time = 1.11, size = 474, normalized size = 4.60

$$-c^2 \sqrt{\frac{1}{(4c-b^2)}} \log\left(x + \frac{-384a^2c^2 \sqrt{\frac{1}{(4c-b^2)}} + 288a^2b^2c^2 \sqrt{\frac{1}{(4c-b^2)}} - 72ab^4c^2 \sqrt{\frac{1}{(4c-b^2)}} + 6b^6c^2 \sqrt{\frac{1}{(4c-b^2)}} + 6bc^5}{12c}}\right) + c^2 \sqrt{\frac{1}{(4c-b^2)}} \log\left(x + \frac{384a^2c^2 \sqrt{\frac{1}{(4c-b^2)}} - 288a^2b^2c^2 \sqrt{\frac{1}{(4c-b^2)}} + 72ab^4c^2 \sqrt{\frac{1}{(4c-b^2)}} + 6b^6c^2 \sqrt{\frac{1}{(4c-b^2)}} + 6bc^5}{12c}}\right) \frac{30ab^2c - b^3 + 18b^2c^2 + 12c^3a^2 + c(20bc^2 + 4b^2c)}{52a^2c^2 - 16a^2b^2c + 2b^4c^2 + c^2(52a^2c^2 - 36ab^2c^2 + 2b^4c^2) + c^2(64a^2c^2 - 32ab^2c^2 + 4b^2c^2) + c^2(64a^2c^2 - 12ab^2c^2 + 2b^4c^2) + c^2(64a^2b^2c^2 - 32a^2b^2c^2 + 4ab^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**3/x**6,x)

[Out] $-6*c**2*\sqrt{-1/(4*a*c - b**2)**5}*\log(x + (-384*a**3*c**5*\sqrt{-1/(4*a*c - b**2)**5} + 288*a**2*b**2*c**4*\sqrt{-1/(4*a*c - b**2)**5} - 72*a*b**4*c**3*\sqrt{-1/(4*a*c - b**2)**5} + 6*b**6*c**2*\sqrt{-1/(4*a*c - b**2)**5} + 6*b*c**2)/(12*c**3)) + 6*c**2*\sqrt{-1/(4*a*c - b**2)**5}*\log(x + (384*a**3*c**5*\sqrt{-1/(4*a*c - b**2)**5} - 288*a**2*b**2*c**4*\sqrt{-1/(4*a*c - b**2)**5} + 72*a*b**4*c**3*\sqrt{-1/(4*a*c - b**2)**5} - 6*b**6*c**2*\sqrt{-1/(4*a*c - b**2)**5} + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**2 + 12*c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))$

$$3.377 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^7} dx$$

Optimal. Leaf size=185

$$-\frac{\log(a+bx+cx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{16a^2c^2 + 2bcx(b^2 - 7ac) - 15ab^2c + 2b^4}{2a^2(b^2 - 4ac)^2(a+bx+cx^2)} + \frac{b(30a^2c^2 - 10ab^2c + b^4) \tanh^{-1}\left(\frac{b+2c}{\sqrt{b^2-4ac}}\right)}{a^3(b^2 - 4ac)^{5/2}}$$

Rubi [A] time = 0.22, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1354, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2 + 2bcx(b^2 - 7ac) - 15ab^2c + 2b^4}{2a^2(b^2 - 4ac)^2(a+bx+cx^2)} + \frac{b(30a^2c^2 - 10ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2 - 4ac)^{5/2}} - \frac{\log(a+bx+cx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{-2ac + b^2 + bcx}{2a(b^2 - 4ac)(a+bx+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^3*x^7), x]

[Out] (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a + b*x + c*x^2]/(2*a^3)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1354

```
Int[(x_)^m*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^7} dx &= \int \frac{1}{x(a+bx+cx^2)^3} dx \\
&= \frac{b^2 - 2ac + bcx}{2a(b^2 - 4ac)(a+bx+cx^2)^2} - \frac{\int \frac{-2(b^2-4ac)-3bcx}{x(a+bx+cx^2)^2} dx}{2a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{2a(b^2 - 4ac)(a+bx+cx^2)^2} + \frac{2b^4 - 15ab^2c + 16a^2c^2 + 2bc(b^2 - 7ac)x}{2a^2(b^2 - 4ac)^2(a+bx+cx^2)} + \frac{\int \frac{2(b^2-4ac)}{x(a+bx+cx^2)} dx}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{2a(b^2 - 4ac)(a+bx+cx^2)^2} + \frac{2b^4 - 15ab^2c + 16a^2c^2 + 2bc(b^2 - 7ac)x}{2a^2(b^2 - 4ac)^2(a+bx+cx^2)} + \frac{\int \left(\frac{2(-b^2+4ac)}{ax}\right) dx}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{2a(b^2 - 4ac)(a+bx+cx^2)^2} + \frac{2b^4 - 15ab^2c + 16a^2c^2 + 2bc(b^2 - 7ac)x}{2a^2(b^2 - 4ac)^2(a+bx+cx^2)} + \frac{\log(x)}{a^3} + \frac{\int \frac{2(b^2-4ac)}{x(a+bx+cx^2)} dx}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{2a(b^2 - 4ac)(a+bx+cx^2)^2} + \frac{2b^4 - 15ab^2c + 16a^2c^2 + 2bc(b^2 - 7ac)x}{2a^2(b^2 - 4ac)^2(a+bx+cx^2)} + \frac{\log(x)}{a^3} - \frac{\int \frac{2(b^2-4ac)}{x(a+bx+cx^2)} dx}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{2a(b^2 - 4ac)(a+bx+cx^2)^2} + \frac{2b^4 - 15ab^2c + 16a^2c^2 + 2bc(b^2 - 7ac)x}{2a^2(b^2 - 4ac)^2(a+bx+cx^2)} + \frac{\log(x)}{a^3} - \frac{\int \frac{2(b^2-4ac)}{x(a+bx+cx^2)} dx}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{2a(b^2 - 4ac)(a+bx+cx^2)^2} + \frac{2b^4 - 15ab^2c + 16a^2c^2 + 2bc(b^2 - 7ac)x}{2a^2(b^2 - 4ac)^2(a+bx+cx^2)} + \frac{\log(x)}{a^3} - \frac{b(b^4 - 10ab^2c + 16a^2c^2)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 178, normalized size = 0.96

$$\frac{\frac{a^2(-2ac+b^2+bcx)}{(b^2-4ac)(a+x(b+cx))^2} - \frac{2b(30a^2c^2-10ab^2c+b^4) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + \frac{a(16a^2c^2-15ab^2c-14abc^2x+2b^4+2b^3cx)}{(b^2-4ac)^2(a+x(b+cx))} - \log(a+x(b+cx)) + 2\log(x)}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^3*x^7),x]

[Out] ((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + 2*Log[x] - Log[a + x*(b + c*x)])/ (2*a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^7} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^7),x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^7), x]

fricas [B] time = 3.24, size = 1985, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="fricas")

[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a) + 2*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(x)]/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^4 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 -

$128a^7c^4)x^2 + 2(a^4b^7 - 12a^5b^5c + 48a^6b^3c^2 - 64a^7b^1c^3)x$, $1/2(3a^2b^6 - 33a^3b^4c + 108a^4b^2c^2 - 96a^5c^3 + 2(a^5b^3c^2 - 11a^2b^3c^3 + 28a^3b^1c^4)x^3 + (4a^6b^2c - 45a^2b^4c^2 + 132a^3b^2c^3 - 64a^4c^4)x^2 + 2(a^2b^5 - 10a^3b^3c + 30a^4b^1c^2 + (b^5c^2 - 10a^2b^3c^3 + 30a^2b^1c^4)x^4 + 2(b^6c - 10a^2b^4c^2 + 30a^2b^2c^3)x^3 + (b^7 - 8a^2b^5c + 10a^2b^3c^2 + 60a^3b^1c^3)x^2 + 2(a^2b^6 - 10a^2b^4c + 30a^3b^2c^2)x) \sqrt{-b^2 + 4ac} \arctan(-\sqrt{-b^2 + 4ac})(2cx + b)/(b^2 - 4ac) + 2(a^2b^7 - 10a^2b^5c + 23a^3b^3c^2 + 4a^4b^1c^3)x - (a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3 + (b^6c^2 - 12a^2b^4c^3 + 48a^2b^2c^4 - 64a^3c^5)x^4 + 2(b^7c - 12a^2b^5c^2 + 48a^2b^3c^3 - 64a^3b^1c^4)x^3 + (b^8 - 10a^2b^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4)x^2 + 2(a^2b^7 - 12a^2b^5c + 48a^3b^3c^2 - 64a^4b^1c^3)x) \log(cx^2 + bx + a) + 2(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3 + (b^6c^2 - 12a^2b^4c^3 + 48a^2b^2c^4 - 64a^3c^5)x^4 + 2(b^7c - 12a^2b^5c^2 + 48a^2b^3c^3 - 64a^3b^1c^4)x^3 + (b^8 - 10a^2b^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4)x^2 + 2(a^2b^7 - 12a^2b^5c + 48a^3b^3c^2 - 64a^4b^1c^3)x) \log(x))/(a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3 + (a^3b^6c^2 - 12a^4b^4c^3 + 48a^5b^2c^4 - 64a^6c^5)x^4 + 2(a^3b^7c - 12a^4b^5c^2 + 48a^5b^3c^3 - 64a^6b^1c^4)x^3 + (a^3b^8 - 10a^4b^6c + 24a^5b^4c^2 + 32a^6b^2c^3 - 128a^7c^4)x^2 + 2(a^4b^7 - 12a^5b^5c + 48a^6b^3c^2 - 64a^7b^1c^3)x]$

giac [A] time = 0.32, size = 239, normalized size = 1.29

$$-\frac{(b^5 - 10ab^3c + 30a^2bc^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - \log(cx^2 + bx + a)}{(a^3b^4 - 8a^4b^2c + 16a^5c^2)\sqrt{-b^2+4ac}} + \frac{\log(|x|)}{a^3} + \frac{3a^2b^4 - 21a^3b^2c + 24a^4c^2 + 2(ab^3c^2 - 7a^2bc^3)x^3 + (4ab^4c - 29a^2b^2c^2 + 16a^3c^3)x^2 + 2(ab^5 - 6a^2b^3c - a^3bc^2)x}{2(cx^2 + bx + a)^2(b^2 - 4ac)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="giac")

[Out] $-(b^5 - 10a^2b^3c + 30a^2b^1c^2) \arctan((2cx + b)/\sqrt{-b^2 + 4ac}) / ((a^3b^4 - 8a^4b^2c + 16a^5c^2) \sqrt{-b^2 + 4ac}) - 1/2 \log(cx^2 + bx + a) / a^3 + \log(\text{abs}(x)) / a^3 + 1/2(3a^2b^4 - 21a^3b^2c + 24a^4c^2 + 2(a^2b^3c^2 - 7a^2b^1c^3)x^3 + (4a^2b^4c - 29a^2b^2c^2 + 16a^3c^3)x^2 + 2(a^2b^5 - 6a^2b^3c - a^3b^1c^2)x) / ((cx^2 + bx + a)^2(b^2 - 4ac)^2a^3)$

maple [B] time = 0.02, size = 781, normalized size = 4.22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^2+b/x)^3/x^7,x)

```
[Out] ln(x)/a^3-7/a/(c*x^2+b*x+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/a^2/(c*x^2+b*x+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+8/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-29/2/a/(c*x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2+2/a^2/(c*x^2+b*x+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4-1/(c*x^2+b*x+a)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-6/a/(c*x^2+b*x+a)^2*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c+1/a^2/(c*x^2+b*x+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x+12*a/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2-21/2/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c+3/2/a/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4-8/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^2+b*x+a)+4/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^2+b*x+a)*b^2-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^4-30/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2+10/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c-1/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.46, size = 1089, normalized size = 5.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + a/x^2 + b/x)^3),x)
```

```
[Out] log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*x*(a^2*c^2 - b^4 + 6*a*b^2*c))/(a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^3*(7*a*c - b^2))/(a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (log(1536*a^6*c^5 - 2*b^11*x - 2*a*b^10 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2) + 39*a^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2) - 303*a^3*b^6*c^2 + 1160*a^4*b^4*c^3 - 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) + 39*a^3*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 321*a^2*b^7*c^2*x + 1286*a^3*b^5*c^3*x - 2560*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^9*c*x + 2
```

$$\begin{aligned}
& 016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c - b^2)^5)^{(1/2)} + 63*a^2*b^2*c^2*x* \\
& (-(4*a*c - b^2)^5)^{(1/2)}*(1024*a^5*c^5 - b^{10} + b^5*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 160*a^2*b^6*c^2 + 640*a^3*b^4*c^3 - 1280*a^4*b^2*c^4 + 20*a*b^8*c + 30 \\
& *a^2*b*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 10*a*b^3*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
& / (2*a^3*(4*a*c - b^2)^5) + (\log(2*a*b^{10} + 2*b^{11}*x - 1536*a^6*c^5 + 2*a*b^5 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 39*a^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^{(1/2)} \\
&) + 303*a^3*b^6*c^2 - 1160*a^4*b^4*c^3 + 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(- \\
& (4*a*c - b^2)^5)^{(1/2)} + 39*a^3*b*c^2*(-(4*a*c - b^2)^5)^{(1/2)} + 321*a^2*b^7 \\
& *c^2*x - 1286*a^3*b^5*c^3*x + 2560*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 40*a*b^9*c*x - 2016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c - \\
& b^2)^5)^{(1/2)} + 63*a^2*b^2*c^2*x*(-(4*a*c - b^2)^5)^{(1/2)}*(b^{10} - 1024*a^5 \\
& *c^5 + b^5*(-(4*a*c - b^2)^5)^{(1/2)} + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1 \\
& 280*a^4*b^2*c^4 - 20*a*b^8*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 10*a \\
& *b^3*c*(-(4*a*c - b^2)^5)^{(1/2)}) / (2*a^3*(4*a*c - b^2)^5)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**3/x**7,x)

[Out] Timed out

$$3.378 \quad \int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^8} dx$$

Optimal. Leaf size=239

$$\frac{3b \log(a + bx + cx^2)}{2a^4} - \frac{3b \log(x)}{a^4} - \frac{3(b^2 - 5ac)(b^2 - 2ac)}{a^3 x (b^2 - 4ac)^2} + \frac{20a^2 c^2 + 3bcx(b^2 - 6ac) - 20ab^2 c + 3b^4}{2a^2 x (b^2 - 4ac)^2 (a + bx + cx^2)} - \frac{3(-20a^3 c^3}{$$

Rubi [A] time = 0.28, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1354, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2 c^2 + 3bcx(b^2 - 6ac) - 20ab^2 c + 3b^4}{2a^2 x (b^2 - 4ac)^2 (a + bx + cx^2)} - \frac{3(30a^2 b^2 c^2 - 20a^3 c^3 - 10ab^4 c + b^6) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4 (b^2 - 4ac)^{5/2}} - \frac{3(b^2 - 5ac)(b^2 - 2ac)}{a^3 x (b^2 - 4ac)^2} + \frac{3b \log(a + bx + cx^2)}{2a^4} - \frac{3b \log(x)}{a^4} + \frac{-2ac + b^2 + bcx}{2ax (b^2 - 4ac) (a + bx + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((c + a/x^2 + b/x)^3*x^8), x]

[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Log[a + b*x + c*x^2])/(2*a^4)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1354

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^8} dx &= \int \frac{1}{x^2 (a + bx + cx^2)^3} dx \\
&= \frac{b^2 - 2ac + bcx}{2a (b^2 - 4ac) x (a + bx + cx^2)^2} - \frac{\int \frac{-3b^2 + 10ac - 4bcx}{x^2 (a + bx + cx^2)^2} dx}{2a (b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{2a (b^2 - 4ac) x (a + bx + cx^2)^2} + \frac{3b^4 - 20ab^2c + 20a^2c^2 + 3bc (b^2 - 6ac) x}{2a^2 (b^2 - 4ac)^2 x (a + bx + cx^2)} + \frac{\int \frac{6(b^2 - 5ac)}{2} dx}{2} \\
&= \frac{b^2 - 2ac + bcx}{2a (b^2 - 4ac) x (a + bx + cx^2)^2} + \frac{3b^4 - 20ab^2c + 20a^2c^2 + 3bc (b^2 - 6ac) x}{2a^2 (b^2 - 4ac)^2 x (a + bx + cx^2)} + \frac{\int \frac{6(b^2 - 5ac)}{2} dx}{2} \\
&= -\frac{3 (b^2 - 5ac) (b^2 - 2ac)}{a^3 (b^2 - 4ac)^2 x} + \frac{b^2 - 2ac + bcx}{2a (b^2 - 4ac) x (a + bx + cx^2)^2} + \frac{3b^4 - 20ab^2c + 20a^2c^2 + 3bc (b^2 - 6ac) x}{2a^2 (b^2 - 4ac)^2 x (a + bx + cx^2)} \\
&= -\frac{3 (b^2 - 5ac) (b^2 - 2ac)}{a^3 (b^2 - 4ac)^2 x} + \frac{b^2 - 2ac + bcx}{2a (b^2 - 4ac) x (a + bx + cx^2)^2} + \frac{3b^4 - 20ab^2c + 20a^2c^2 + 3bc (b^2 - 6ac) x}{2a^2 (b^2 - 4ac)^2 x (a + bx + cx^2)} \\
&= -\frac{3 (b^2 - 5ac) (b^2 - 2ac)}{a^3 (b^2 - 4ac)^2 x} + \frac{b^2 - 2ac + bcx}{2a (b^2 - 4ac) x (a + bx + cx^2)^2} + \frac{3b^4 - 20ab^2c + 20a^2c^2 + 3bc (b^2 - 6ac) x}{2a^2 (b^2 - 4ac)^2 x (a + bx + cx^2)} \\
&= -\frac{3 (b^2 - 5ac) (b^2 - 2ac)}{a^3 (b^2 - 4ac)^2 x} + \frac{b^2 - 2ac + bcx}{2a (b^2 - 4ac) x (a + bx + cx^2)^2} + \frac{3b^4 - 20ab^2c + 20a^2c^2 + 3bc (b^2 - 6ac) x}{2a^2 (b^2 - 4ac)^2 x (a + bx + cx^2)}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 221, normalized size = 0.92

$$\frac{a^2(-3abc-2ac^2x+b^3+b^2cx)}{(4ac-b^2)(a+x(b+cx))^2} - \frac{a(46a^2bc^2+28a^2c^3x-29ab^3c-26ab^2c^2x+4b^5+4b^4cx)}{(b^2-4ac)^2(a+x(b+cx))} + \frac{6(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + 3b\log(a+x(b+cx)) - \frac{2a}{x} - 6b\log(x)$$

$2a^4$

Antiderivative was successfully verified.

[In] Integrate[1/((c + a/x^2 + b/x)^3*x^8),x]

[Out]
$$\frac{((-2a)/x + (a^2(b^3 - 3ab^2c + b^2cx - 2ac^2x)))/((-b^2 + 4ac)(a + x(b + cx))^2) - (a(4b^5 - 29ab^3c + 46a^2b^2c^2 + 4b^4cx - 26ab^2c^2x + 28a^2c^3x)))/((b^2 - 4ac)^2(a + x(b + cx))) + (6(b^6 - 10ab^4c + 30a^2b^2c^2 - 20a^3c^3) \operatorname{ArcTan}[(b + 2cx)/\sqrt{-b^2 + 4ac}]))/(-b^2 + 4ac)^{5/2} - 6b \operatorname{Log}[x] + 3b \operatorname{Log}[a + x(b + cx)])/(2a^4)}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c + \frac{a}{x^2} + \frac{b}{x}\right)^3 x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^8), x]

[Out] IntegrateAlgebraic[1/((c + a/x^2 + b/x)^3*x^8), x]

fricas [B] time = 3.19, size = 2280, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="fricas")

[Out]
$$\begin{aligned} &[-1/2*(2a^3b^6 - 24a^4b^4c + 96a^5b^2c^2 - 128a^6c^3 + 6(a^2b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*x^4 + 3(4ab^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)*x^3 + 2(3ab^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)*x^2 + 3((b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)*x^5 + 2(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)*x^4 + (b^8 - 8ab^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)*x^3 + 2(ab^7 - 10a^2b^5c + 30a^3b^3c^2 - 20a^4b^2c^3)*x^2 + (a^2b^6 - 10a^3b^4c + 30a^4b^2c^2 - 20a^5c^3)*x) * \sqrt{b^2 - 4ac} * \log((2c^2x^2 + 2b^2cx + b^2 - 2ac + \sqrt{b^2 - 4ac})*(2cx + b))/(cx^2 + bx + a)) + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3)*x - 3((b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)*x^5 + 2(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)*x^4 + (b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4)*x^3 + 2(ab^8 - 12a^2b^6c + 48a^3b^4c^2 - 64a^4b^2c^3)*x^2 + (a^2b^7 - 12a^3b^5c + 48a^4b^3c^2 - 64a^5b^2c^3)*x) * \log(cx^2 + bx + a) + 6((b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)*x^5 + 2(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)*x^4 + (b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4)*x^3 + 2(ab^8 - 12a^2b^6c + 48a^3b^4c^2 - 64a^4b^2c^3)*x^2 + (a^2b^7 - 12a^3b^5c + 48a^4b^3c^2 - 64a^5b^2c^3)*x) * \log(x)]/((a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5)*x^5 + 2(a^4b^7c - 12a^5b^5c^2 + 48a^6b^3c^3 - 64a^7c^4) * x^4 + (a^4b^8 - 12a^5b^6c + 48a^6b^4c^2 - 64a^7b^2c^3) * x^3 + (a^4b^9 - 12a^5b^7c + 48a^6b^5c^2 - 64a^7b^3c^3) * x^2 + (a^4b^{10} - 12a^5b^8c + 48a^6b^6c^2 - 64a^7b^4c^3) * x + (a^4b^{11} - 12a^5b^9c + 48a^6b^7c^2 - 64a^7b^5c^3) * x^0) \end{aligned}$$

$$\begin{aligned}
& *a^6*b^3*c^3 - 64*a^7*b*c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 \\
& + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x), \\
& -1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x)]
\end{aligned}$$

giac [A] time = 0.47, size = 309, normalized size = 1.29

$$\frac{3(b^6 - 10ab^5c + 30a^2b^4c^2 - 20a^3c^3) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) + 3b \log(cx^2 + bx + a) - \frac{3b \log(|x|)}{a^4} - \frac{2a^3b^4 - 16a^4b^2c + 32a^5c^2 + 6(ab^4c^2 - 7a^2b^3c^3 + 10a^3c^4)x^4 + 3(4ab^5c - 29a^2b^4c^2 + 46a^3bc^3)x^3 + 2(3ab^6 - 18a^2b^5c + 7a^3b^2c^2 + 50a^4c^3)x^2 + (9a^2b^5 - 68a^3b^3c + 122a^4bc^2)x}{(a^4b^4 - 8a^5b^2c + 16a^6c^2)\sqrt{-b^2+4ac}}}{2(\alpha^2 + bx + a)(b^2 - 4ac)^2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="giac")

[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) + 3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 - 16*a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)

maple [B] time = 0.02, size = 954, normalized size = 3.99

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(c+a/x^2+b/x)^3/x^8,x)$

[Out]
$$\begin{aligned} & -1/a^3/x-3*b*\ln(x)/a^4-14/a/(c*x^2+b*x+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)* \\ & x^3+13/a^2/(c*x^2+b*x+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2-2/a^3/(c* \\ & x^2+b*x+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-37/a/(c*x^2+b*x+a)^2*b* \\ & c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+55/2/a^2/(c*x^2+b*x+a)^2*b^3*c^2/(16*a^2 \\ & *c^2-8*a*b^2*c+b^4)*x^2-4/a^3/(c*x^2+b*x+a)^2*b^5*c/(16*a^2*c^2-8*a*b^2*c+b \\ & ^4)*x^2-18/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3-7/a/(c*x^2+b*x+ \\ & a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+12/a^2/(c*x^2+b*x+a)^2/(16*a^2*c^ \\ & 2-8*a*b^2*c+b^4)*x*b^4*c-2/a^3/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x \\ & *b^6-29/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2+18/a/(c*x^2+b*x+a) \\ & ^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c-5/2/a^2/(c*x^2+b*x+a)^2/(16*a^2*c^2-8*a \\ & *b^2*c+b^4)*b^5+24/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*\ln(c*x^2+b*x+a)*b-12/ \\ & a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*\ln(c*x^2+b*x+a)*b^3+3/2/a^4/(16*a^2*c^2-8* \\ & a*b^2*c+b^4)*\ln(c*x^2+b*x+a)*b^5-60/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2 \\ &)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*c^3+90/a^2/(16*a^2*c^2-8*a*b^2* \\ & c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2*c^2-30/a^3 \\ & /(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^ \\ & (1/2))*b^4*c+3/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^{(1/2)}*\arctan((2*c \\ & *x+b)/(4*a*c-b^2)^{(1/2))*b^6 \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(c+a/x^2+b/x)^3/x^8,x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.55, size = 1255, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^8*(c + a/x^2 + b/x)^3),x)$

[Out]
$$\begin{aligned} & - (1/a + (x^2*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c)))/(a^3*(b^4 \\ & + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(2*a^ \\ & 2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2 \end{aligned}$$

$$\frac{b^3 c^3}{(2a^3(b^4 + 16a^2c^2 - 8ab^2c))} + \frac{(3c^2x^4(b^4 + 10a^2c^2 - 7ab^2c))}{(a^3(b^4 + 16a^2c^2 - 8ab^2c))} \frac{1}{(x^3(2ac + b^2) + a^2x + c^2x^5 + 2abx^2 + 2b^2cx^4) - (3b \log(x))/a^4 - (3 \log(2ab^{11} + 2b^{12}x + 2ab^6(-4ac - b^2)^5)^{1/2} - 39a^2b^9c - 1696a^6b^6c^5 + 320a^6c^6x + 2b^7x(-4ac - b^2)^5)^{1/2} + 303a^3b^7c^2 - 1170a^4b^5c^3 + 2240a^5b^3c^4 - 10a^4c^3(-4ac - b^2)^5)^{1/2} - 17a^2b^4c(-4ac - b^2)^5)^{1/2} + 321a^2b^8c^2x - 1296a^3b^6c^3x + 2660a^4b^4c^4x - 2336a^5b^2c^5x - 40ab^{10}cx + 39a^3b^2c^2(-4ac - b^2)^5)^{1/2} - 20ab^5cx(-4ac - b^2)^5)^{1/2} - 58a^3bc^3x(-4ac - b^2)^5)^{1/2} + 63a^2b^3c^2x(-4ac - b^2)^5)^{1/2}}{(b^{11} + b^6(-4ac - b^2)^5)^{1/2} - 1024a^5b^5c^5 + 160a^2b^7c^2 - 640a^3b^5c^3 + 1280a^4b^3c^4 - 20a^3c^3(-4ac - b^2)^5)^{1/2} - 20ab^9c + 30a^2b^2c^2(-4ac - b^2)^5)^{1/2} - 10ab^4c(-4ac - b^2)^5)^{1/2}} \frac{1}{(2a^4(4ac - b^2)^5) - (3 \log(2ab^{11} + 2b^{12}x - 2ab^6(-4ac - b^2)^5)^{1/2} - 39a^2b^9c - 1696a^6b^6c^5 + 320a^6c^6x - 2b^7x(-4ac - b^2)^5)^{1/2} + 303a^3b^7c^2 - 1170a^4b^5c^3 + 2240a^5b^3c^4 + 10a^4c^3(-4ac - b^2)^5)^{1/2} + 17a^2b^4c(-4ac - b^2)^5)^{1/2} + 321a^2b^8c^2x - 1296a^3b^6c^3x + 2660a^4b^4c^4x - 2336a^5b^2c^5x - 40ab^{10}cx - 39a^3b^2c^2(-4ac - b^2)^5)^{1/2} + 20ab^5cx(-4ac - b^2)^5)^{1/2} + 58a^3bc^3x(-4ac - b^2)^5)^{1/2} - 63a^2b^3c^2x(-4ac - b^2)^5)^{1/2}}{(b^{11} - b^6(-4ac - b^2)^5)^{1/2} - 1024a^5b^5c^5 + 160a^2b^7c^2 - 640a^3b^5c^3 + 1280a^4b^3c^4 + 20a^3c^3(-4ac - b^2)^5)^{1/2} - 20ab^9c - 30a^2b^2c^2(-4ac - b^2)^5)^{1/2} + 10ab^4c(-4ac - b^2)^5)^{1/2}} \frac{1}{(2a^4(4ac - b^2)^5)}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**2+b/x)**3/x**8,x)

[Out] Timed out

$$3.379 \quad \int \frac{x^2}{15 + \frac{2}{x^2} + \frac{13}{x}} dx$$

Optimal. Leaf size=40

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x + 2) + \frac{\log(5x + 1)}{4375}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1354, 701, 632, 31}

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x + 2) + \frac{\log(5x + 1)}{4375}$$

Antiderivative was successfully verified.

[In] Int[x^2/(15 + 2/x^2 + 13/x), x]

[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]/4375

Rule 31

Int[((a_) + (b_)*(x_))^(n_)/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 701

Int[((d_) + (e_)*(x_))^(m_)/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 1354

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}

`}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{15 + \frac{2}{x^2} + \frac{13}{x}} dx &= \int \frac{x^4}{2 + 13x + 15x^2} dx \\
 &= \int \left(\frac{139}{3375} - \frac{13x}{225} + \frac{x^2}{15} - \frac{278 + 1417x}{3375(2 + 13x + 15x^2)} \right) dx \\
 &= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{\int \frac{278+1417x}{2+13x+15x^2} dx}{3375} \\
 &= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} + \frac{3}{875} \int \frac{1}{3 + 15x} dx - \frac{80}{189} \int \frac{1}{10 + 15x} dx \\
 &= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567} \log(2 + 3x) + \frac{\log(1 + 5x)}{4375}
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x + 2) + \frac{\log(5x + 1)}{4375}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(15 + 2/x^2 + 13/x), x]

[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]/4375

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{15 + \frac{2}{x^2} + \frac{13}{x}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(15 + 2/x^2 + 13/x), x]

[Out] IntegrateAlgebraic[x^2/(15 + 2/x^2 + 13/x), x]

fricas [A] time = 1.72, size = 30, normalized size = 0.75

$$\frac{1}{45} x^3 - \frac{13}{450} x^2 + \frac{139}{3375} x + \frac{1}{4375} \log(5x + 1) - \frac{16}{567} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="fricas")

[Out] $\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(5x + 1) - \frac{16}{567}\log(3x + 2)$

giac [A] time = 0.26, size = 32, normalized size = 0.80

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(|5x + 1|) - \frac{16}{567}\log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="giac")

[Out] $\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(\text{abs}(5x + 1)) - \frac{16}{567}\log(\text{abs}(3x + 2))$

maple [A] time = 0.01, size = 31, normalized size = 0.78

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\ln(5x + 1)}{4375} - \frac{16\ln(3x + 2)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(15+2/x^2+13/x),x)

[Out] $\frac{139}{3375}x - \frac{13}{450}x^2 + \frac{1}{45}x^3 - \frac{16}{567}\ln(2+3*x) + \frac{1}{4375}\ln(1+5*x)$

maxima [A] time = 0.42, size = 30, normalized size = 0.75

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(5x + 1) - \frac{16}{567}\log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="maxima")

[Out] $\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(5x + 1) - \frac{16}{567}\log(3x + 2)$

mupad [B] time = 0.05, size = 26, normalized size = 0.65

$$\frac{139x}{3375} - \frac{16\ln\left(x + \frac{2}{3}\right)}{567} + \frac{\ln\left(x + \frac{1}{5}\right)}{4375} - \frac{13x^2}{450} + \frac{x^3}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(13/x + 2/x^2 + 15),x)`

[Out] $(139x)/3375 - (16\log(x + 2/3))/567 + \log(x + 1/5)/4375 - (13x^2)/450 + x^3/45$

sympy [A] time = 0.12, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x + \frac{1}{5}\right)}{4375} - \frac{16\log\left(x + \frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(15+2/x**2+13/x),x)`

[Out] $x^3/45 - 13x^2/450 + 139x/3375 + \log(x + 1/5)/4375 - 16\log(x + 2/3)/567$

$$3.380 \quad \int \frac{x}{15 + \frac{2}{x^2} + \frac{13}{x}} dx$$

Optimal. Leaf size=33

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189} \log(3x + 2) - \frac{1}{875} \log(5x + 1)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1354, 701, 632, 31}

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189} \log(3x + 2) - \frac{1}{875} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[x/(15 + 2/x^2 + 13/x), x]

[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 701

Int[((d_) + (e_)*(x_))^(m_)/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 1354

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}

`}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]`

Rubi steps

$$\begin{aligned}
 \int \frac{x}{15 + \frac{2}{x^2} + \frac{13}{x}} dx &= \int \frac{x^3}{2 + 13x + 15x^2} dx \\
 &= \int \left(-\frac{13}{225} + \frac{x}{15} + \frac{26 + 139x}{225(2 + 13x + 15x^2)} \right) dx \\
 &= -\frac{13x}{225} + \frac{x^2}{30} + \frac{1}{225} \int \frac{26 + 139x}{2 + 13x + 15x^2} dx \\
 &= -\frac{13x}{225} + \frac{x^2}{30} - \frac{3}{175} \int \frac{1}{3 + 15x} dx + \frac{40}{63} \int \frac{1}{10 + 15x} dx \\
 &= -\frac{13x}{225} + \frac{x^2}{30} + \frac{8}{189} \log(2 + 3x) - \frac{1}{875} \log(1 + 5x)
 \end{aligned}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189} \log(3x + 2) - \frac{1}{875} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x/(15 + 2/x^2 + 13/x), x]

[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{15 + \frac{2}{x^2} + \frac{13}{x}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(15 + 2/x^2 + 13/x), x]

[Out] IntegrateAlgebraic[x/(15 + 2/x^2 + 13/x), x]

fricas [A] time = 0.76, size = 25, normalized size = 0.76

$$\frac{1}{30} x^2 - \frac{13}{225} x - \frac{1}{875} \log(5x + 1) + \frac{8}{189} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(15+2/x^2+13/x),x, algorithm="fricas")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)

giac [A] time = 0.28, size = 27, normalized size = 0.82

$$\frac{1}{30}x^2 - \frac{13}{225}x - \frac{1}{875}\log(|5x + 1|) + \frac{8}{189}\log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(15+2/x^2+13/x),x, algorithm="giac")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))

maple [A] time = 0.00, size = 26, normalized size = 0.79

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\ln(5x + 1)}{875} + \frac{8\ln(3x + 2)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(15+2/x^2+13/x),x)

[Out] -13/225*x+1/30*x^2+8/189*ln(3*x+2)-1/875*ln(5*x+1)

maxima [A] time = 0.42, size = 25, normalized size = 0.76

$$\frac{1}{30}x^2 - \frac{13}{225}x - \frac{1}{875}\log(5x + 1) + \frac{8}{189}\log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(15+2/x^2+13/x),x, algorithm="maxima")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)

mupad [B] time = 1.31, size = 21, normalized size = 0.64

$$\frac{8\ln\left(x + \frac{2}{3}\right)}{189} - \frac{13x}{225} - \frac{\ln\left(x + \frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(13/x + 2/x^2 + 15),x)

[Out] $(8 \cdot \log(x + 2/3))/189 - (13 \cdot x)/225 - \log(x + 1/5)/875 + x^2/30$

sympy [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x + \frac{1}{5}\right)}{875} + \frac{8 \log\left(x + \frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(15+2/x**2+13/x),x)

[Out] $x^2/30 - 13 \cdot x/225 - \log(x + 1/5)/875 + 8 \cdot \log(x + 2/3)/189$

$$3.381 \quad \int \frac{1}{15 + \frac{2}{x^2} + \frac{13}{x}} dx$$

Optimal. Leaf size=26

$$\frac{x}{15} - \frac{4}{63} \log(3x + 2) + \frac{1}{175} \log(5x + 1)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1340, 703, 632, 31}

$$\frac{x}{15} - \frac{4}{63} \log(3x + 2) + \frac{1}{175} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[(15 + 2/x^2 + 13/x)^(-1), x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1340

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[x^(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]

] && LtQ[n, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{15 + \frac{2}{x^2} + \frac{13}{x}} dx &= \int \frac{x^2}{2 + 13x + 15x^2} dx \\ &= \frac{x}{15} + \frac{1}{15} \int \frac{-2 - 13x}{2 + 13x + 15x^2} dx \\ &= \frac{x}{15} + \frac{3}{35} \int \frac{1}{3 + 15x} dx - \frac{20}{21} \int \frac{1}{10 + 15x} dx \\ &= \frac{x}{15} - \frac{4}{63} \log(2 + 3x) + \frac{1}{175} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{15} - \frac{4}{63} \log(3x + 2) + \frac{1}{175} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(15 + 2/x^2 + 13/x)^(-1), x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{15 + \frac{2}{x^2} + \frac{13}{x}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(15 + 2/x^2 + 13/x)^(-1), x]

[Out] IntegrateAlgebraic[(15 + 2/x^2 + 13/x)^(-1), x]

fricas [A] time = 1.22, size = 20, normalized size = 0.77

$$\frac{1}{15} x + \frac{1}{175} \log(5x + 1) - \frac{4}{63} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x),x, algorithm="fricas")

[Out] $1/15*x + 1/175*\log(5*x + 1) - 4/63*\log(3*x + 2)$

giac [A] time = 0.28, size = 22, normalized size = 0.85

$$\frac{1}{15}x + \frac{1}{175} \log(|5x + 1|) - \frac{4}{63} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(15+2/x^2+13/x),x, algorithm="giac")`

[Out] $1/15*x + 1/175*\log(\text{abs}(5*x + 1)) - 4/63*\log(\text{abs}(3*x + 2))$

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$\frac{x}{15} + \frac{\ln(5x + 1)}{175} - \frac{4 \ln(3x + 2)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(15+2/x^2+13/x),x)`

[Out] $1/15*x - 4/63*\ln(3*x+2) + 1/175*\ln(5*x+1)$

maxima [A] time = 0.42, size = 20, normalized size = 0.77

$$\frac{1}{15}x + \frac{1}{175} \log(5x + 1) - \frac{4}{63} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(15+2/x^2+13/x),x, algorithm="maxima")`

[Out] $1/15*x + 1/175*\log(5*x + 1) - 4/63*\log(3*x + 2)$

mupad [B] time = 0.08, size = 16, normalized size = 0.62

$$\frac{x}{15} - \frac{4 \ln\left(x + \frac{2}{3}\right)}{63} + \frac{\ln\left(x + \frac{1}{5}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(13/x + 2/x^2 + 15),x)`

[Out] $x/15 - (4*\log(x + 2/3))/63 + \log(x + 1/5)/175$

sympy [A] time = 0.12, size = 20, normalized size = 0.77

$$\frac{x}{15} + \frac{\log\left(x + \frac{1}{5}\right)}{175} - \frac{4 \log\left(x + \frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

$$3.382 \quad \int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x} dx$$

Optimal. Leaf size=21

$$\frac{2}{21} \log(3x + 2) - \frac{1}{35} \log(5x + 1)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1354, 632, 31}

$$\frac{2}{21} \log(3x + 2) - \frac{1}{35} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/((15 + 2/x^2 + 13/x)*x),x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1354

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))]^(p), x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x} dx &= \int \frac{x}{2 + 13x + 15x^2} dx \\ &= -\left(\frac{3}{7} \int \frac{1}{3 + 15x} dx\right) + \frac{10}{7} \int \frac{1}{10 + 15x} dx \\ &= \frac{2}{21} \log(2 + 3x) - \frac{1}{35} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{21} \log(3x + 2) - \frac{1}{35} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/((15 + 2/x^2 + 13/x)*x), x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x), x]

[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x), x]

fricas [A] time = 1.40, size = 17, normalized size = 0.81

$$-\frac{1}{35} \log(5x + 1) + \frac{2}{21} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="fricas")

[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)

giac [A] time = 0.28, size = 19, normalized size = 0.90

$$-\frac{1}{35} \log(|5x + 1|) + \frac{2}{21} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="giac")

[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{\ln(5x + 1)}{35} + \frac{2 \ln(3x + 2)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(15+2/x^2+13/x)/x,x)

[Out] 2/21*ln(3*x+2)-1/35*ln(5*x+1)

maxima [A] time = 0.42, size = 17, normalized size = 0.81

$$-\frac{1}{35} \log(5x + 1) + \frac{2}{21} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="maxima")

[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)

mupad [B] time = 0.07, size = 13, normalized size = 0.62

$$\frac{2 \ln\left(x + \frac{2}{3}\right)}{21} - \frac{\ln\left(x + \frac{1}{5}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(13/x + 2/x^2 + 15)),x)

[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35

sympy [A] time = 0.11, size = 17, normalized size = 0.81

$$-\frac{\log\left(x + \frac{1}{5}\right)}{35} + \frac{2 \log\left(x + \frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x**2+13/x)/x,x)

[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21

$$3.383 \quad \int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^2} dx$$

Optimal. Leaf size=23

$$\frac{1}{7} \log\left(\frac{1}{x} + 5\right) - \frac{1}{7} \log\left(\frac{2}{x} + 3\right)$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1352, 616, 31}

$$\frac{1}{7} \log\left(\frac{1}{x} + 5\right) - \frac{1}{7} \log\left(\frac{2}{x} + 3\right)$$

Antiderivative was successfully verified.

[In] Int[1/((15 + 2/x^2 + 13/x)*x^2),x]

[Out] Log[5 + x^(-1)]/7 - Log[3 + 2/x]/7

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^2} dx &= -\text{Subst}\left(\int \frac{1}{15 + 13x + 2x^2} dx, x, \frac{1}{x}\right) \\
&= -\left(\frac{2}{7}\text{Subst}\left(\int \frac{1}{3 + 2x} dx, x, \frac{1}{x}\right)\right) + \frac{2}{7}\text{Subst}\left(\int \frac{1}{10 + 2x} dx, x, \frac{1}{x}\right) \\
&= \frac{1}{7}\log\left(5 + \frac{1}{x}\right) - \frac{1}{7}\log\left(3 + \frac{2}{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.91

$$\frac{1}{7}\log(5x + 1) - \frac{1}{7}\log(3x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^2), x]

[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^2), x]

[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^2), x]

fricas [A] time = 1.04, size = 17, normalized size = 0.74

$$\frac{1}{7}\log(5x + 1) - \frac{1}{7}\log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="fricas")

[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)

giac [A] time = 0.22, size = 19, normalized size = 0.83

$$\frac{1}{7}\log(5x + 1) - \frac{1}{7}\log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="giac")

[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{\ln(5x + 1)}{7} - \frac{\ln(3x + 2)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(15+2/x^2+13/x)/x^2,x)

[Out] -1/7*ln(3*x+2)+1/7*ln(5*x+1)

maxima [A] time = 0.42, size = 17, normalized size = 0.74

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="maxima")

[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)

mupad [B] time = 1.37, size = 8, normalized size = 0.35

$$-\frac{2 \operatorname{atanh}\left(\frac{30x}{7} + \frac{13}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(13/x + 2/x^2 + 15)),x)

[Out] -(2*atanh((30*x)/7 + 13/7))/7

sympy [A] time = 0.11, size = 15, normalized size = 0.65

$$\frac{\log\left(x + \frac{1}{5}\right)}{7} - \frac{\log\left(x + \frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x**2+13/x)/x**2,x)

[Out] log(x + 1/5)/7 - log(x + 2/3)/7

$$3.384 \quad \int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^3} dx$$

Optimal. Leaf size=27

$$\frac{\log(x)}{2} + \frac{3}{14} \log(3x + 2) - \frac{5}{7} \log(5x + 1)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1354, 705, 29, 632, 31}

$$\frac{\log(x)}{2} + \frac{3}{14} \log(3x + 2) - \frac{5}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/((15 + 2/x^2 + 13/x)*x^3), x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^3} dx &= \int \frac{1}{x(2 + 13x + 15x^2)} dx \\ &= \frac{1}{2} \int \frac{1}{x} dx + \frac{1}{2} \int \frac{-13 - 15x}{2 + 13x + 15x^2} dx \\ &= \frac{\log(x)}{2} + \frac{45}{14} \int \frac{1}{10 + 15x} dx - \frac{75}{7} \int \frac{1}{3 + 15x} dx \\ &= \frac{\log(x)}{2} + \frac{3}{14} \log(2 + 3x) - \frac{5}{7} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{2} + \frac{3}{14} \log(3x + 2) - \frac{5}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^3), x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^3), x]
```

```
[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^3), x]
```

fricas [A] time = 1.30, size = 21, normalized size = 0.78

$$-\frac{5}{7} \log(5x + 1) + \frac{3}{14} \log(3x + 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="fricas")

[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)

giac [A] time = 0.25, size = 24, normalized size = 0.89

$$-\frac{5}{7} \log(|5x + 1|) + \frac{3}{14} \log(|3x + 2|) + \frac{1}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="giac")

[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{\ln(x)}{2} - \frac{5 \ln(5x + 1)}{7} + \frac{3 \ln(3x + 2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(15+2/x^2+13/x)/x^3,x)

[Out] 1/2*ln(x)+3/14*ln(3*x+2)-5/7*ln(5*x+1)

maxima [A] time = 0.42, size = 21, normalized size = 0.78

$$-\frac{5}{7} \log(5x + 1) + \frac{3}{14} \log(3x + 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="maxima")

[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)

mupad [B] time = 1.39, size = 17, normalized size = 0.63

$$\frac{3 \ln\left(x + \frac{2}{3}\right)}{14} - \frac{5 \ln\left(x + \frac{1}{5}\right)}{7} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(13/x + 2/x^2 + 15)),x)

[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2

sympy [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2} - \frac{5 \log\left(x + \frac{1}{5}\right)}{7} + \frac{3 \log\left(x + \frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x**2+13/x)/x**3,x)

[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14

$$3.385 \quad \int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^4} dx$$

Optimal. Leaf size=34

$$-\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(3x + 2) + \frac{25}{7} \log(5x + 1)$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1354, 709, 800}

$$-\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(3x + 2) + \frac{25}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/((15 + 2/x^2 + 13/x)*x^4), x]

[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
  ] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
  t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
  x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
  4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
  , -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
  (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
  + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
  c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
  ] :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
  }, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^4} dx &= \int \frac{1}{x^2(2 + 13x + 15x^2)} dx \\
&= -\frac{1}{2x} + \frac{1}{2} \int \frac{-13 - 15x}{x(2 + 13x + 15x^2)} dx \\
&= -\frac{1}{2x} + \frac{1}{2} \int \left(-\frac{13}{2x} - \frac{27}{14(2 + 3x)} + \frac{250}{7(1 + 5x)} \right) dx \\
&= -\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(2 + 3x) + \frac{25}{7} \log(1 + 5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(3x + 2) + \frac{25}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^4), x]

[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^4), x]

[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^4), x]

fricas [A] time = 1.30, size = 30, normalized size = 0.88

$$\frac{100x \log(5x + 1) - 9x \log(3x + 2) - 91x \log(x) - 14}{28x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="fricas")

[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x

giac [A] time = 0.31, size = 29, normalized size = 0.85

$$-\frac{1}{2x} + \frac{25}{7} \log(|5x + 1|) - \frac{9}{28} \log(|3x + 2|) - \frac{13}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="giac")

[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))

maple [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{13 \ln(x)}{4} + \frac{25 \ln(5x + 1)}{7} - \frac{9 \ln(3x + 2)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(15+2/x^2+13/x)/x^4,x)

[Out] -1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(5*x+1)

maxima [A] time = 0.43, size = 26, normalized size = 0.76

$$-\frac{1}{2x} + \frac{25}{7} \log(5x + 1) - \frac{9}{28} \log(3x + 2) - \frac{13}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="maxima")

[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)

mupad [B] time = 0.04, size = 22, normalized size = 0.65

$$\frac{25 \ln\left(x + \frac{1}{5}\right)}{7} - \frac{9 \ln\left(x + \frac{2}{3}\right)}{28} - \frac{13 \ln(x)}{4} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(13/x + 2/x^2 + 15)),x)

[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)

sympy [A] time = 0.16, size = 31, normalized size = 0.91

$$-\frac{13 \log(x)}{4} + \frac{25 \log\left(x + \frac{1}{5}\right)}{7} - \frac{9 \log\left(x + \frac{2}{3}\right)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**4,x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```


$$3.386 \quad \int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^5} dx$$

Optimal. Leaf size=41

$$-\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(3x + 2) - \frac{125}{7} \log(5x + 1)$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1354, 709, 800}

$$-\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(3x + 2) - \frac{125}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/((15 + 2/x^2 + 13/x)*x^5),x]

[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1 + 5*x])/7

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1354

Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^5} dx &= \int \frac{1}{x^3(2 + 13x + 15x^2)} dx \\
&= -\frac{1}{4x^2} + \frac{1}{2} \int \frac{-13 - 15x}{x^2(2 + 13x + 15x^2)} dx \\
&= -\frac{1}{4x^2} + \frac{1}{2} \int \left(-\frac{13}{2x^2} + \frac{139}{4x} + \frac{81}{28(2 + 3x)} - \frac{1250}{7(1 + 5x)} \right) dx \\
&= -\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(2 + 3x) - \frac{125}{7} \log(1 + 5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(3x + 2) - \frac{125}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^5),x]

[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1 + 5*x])/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^5),x]

[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^5), x]

fricas [A] time = 1.30, size = 39, normalized size = 0.95

$$\frac{1000x^2 \log(5x + 1) - 27x^2 \log(3x + 2) - 973x^2 \log(x) - 182x + 14}{56x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="fricas")

[Out] $-1/56*(1000*x^2*\log(5*x + 1) - 27*x^2*\log(3*x + 2) - 973*x^2*\log(x) - 182*x + 14)/x^2$

giac [A] time = 0.30, size = 34, normalized size = 0.83

$$\frac{13x-1}{4x^2} - \frac{125}{7} \log(|5x+1|) + \frac{27}{56} \log(|3x+2|) + \frac{139}{8} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="giac")`

[Out] $1/4*(13*x - 1)/x^2 - 125/7*\log(\text{abs}(5*x + 1)) + 27/56*\log(\text{abs}(3*x + 2)) + 139/8*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{139 \ln(x)}{8} - \frac{125 \ln(5x+1)}{7} + \frac{27 \ln(3x+2)}{56} + \frac{13}{4x} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(15+2/x^2+13/x)/x^5,x)`

[Out] $-1/4/x^2+13/4/x+139/8*\ln(x)+27/56*\ln(3*x+2)-125/7*\ln(5*x+1)$

maxima [A] time = 0.42, size = 31, normalized size = 0.76

$$\frac{13x-1}{4x^2} - \frac{125}{7} \log(5x+1) + \frac{27}{56} \log(3x+2) + \frac{139}{8} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="maxima")`

[Out] $1/4*(13*x - 1)/x^2 - 125/7*\log(5*x + 1) + 27/56*\log(3*x + 2) + 139/8*\log(x)$

mupad [B] time = 1.31, size = 26, normalized size = 0.63

$$\frac{27 \ln\left(x + \frac{2}{3}\right)}{56} - \frac{125 \ln\left(x + \frac{1}{5}\right)}{7} + \frac{139 \ln(x)}{8} + \frac{\frac{13x}{4} - \frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(13/x + 2/x^2 + 15)),x)`

[Out] $(27*\log(x + 2/3))/56 - (125*\log(x + 1/5))/7 + (139*\log(x))/8 + ((13*x)/4 - 1/4)/x^2$

sympy [A] time = 0.17, size = 36, normalized size = 0.88

$$\frac{139 \log(x)}{8} - \frac{125 \log\left(x + \frac{1}{5}\right)}{7} + \frac{27 \log\left(x + \frac{2}{3}\right)}{56} + \frac{13x - 1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x**2+13/x)/x**5,x)

[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2)
)

$$3.387 \quad \int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^6} dx$$

Optimal. Leaf size=48

$$-\frac{1}{6x^3} + \frac{13}{8x^2} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(3x+2) + \frac{625}{7} \log(5x+1)$$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1354, 709, 800}

$$\frac{13}{8x^2} - \frac{1}{6x^3} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(3x+2) + \frac{625}{7} \log(5x+1)$$

Antiderivative was successfully verified.

[In] Int[1/((15 + 2/x^2 + 13/x)*x^6), x]

[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/112 + (625*Log[1 + 5*x])/7

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1354

Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^6} dx &= \int \frac{1}{x^4(2 + 13x + 15x^2)} dx \\
&= -\frac{1}{6x^3} + \frac{1}{2} \int \frac{-13 - 15x}{x^3(2 + 13x + 15x^2)} dx \\
&= -\frac{1}{6x^3} + \frac{1}{2} \int \left(-\frac{13}{2x^3} + \frac{139}{4x^2} - \frac{1417}{8x} - \frac{243}{56(2 + 3x)} + \frac{6250}{7(1 + 5x)} \right) dx \\
&= -\frac{1}{6x^3} + \frac{13}{8x^2} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(2 + 3x) + \frac{625}{7} \log(1 + 5x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{1}{6x^3} + \frac{13}{8x^2} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(3x + 2) + \frac{625}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^6), x]

[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/112 + (625*Log[1 + 5*x])/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(15 + \frac{2}{x^2} + \frac{13}{x}\right)x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^6), x]

[Out] IntegrateAlgebraic[1/((15 + 2/x^2 + 13/x)*x^6), x]

fricas [A] time = 1.14, size = 44, normalized size = 0.92

$$\frac{30000x^3 \log(5x + 1) - 243x^3 \log(3x + 2) - 29757x^3 \log(x) - 5838x^2 + 546x - 56}{336x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="fricas")

[Out] $1/336*(30000*x^3*\log(5*x + 1) - 243*x^3*\log(3*x + 2) - 29757*x^3*\log(x) - 5838*x^2 + 546*x - 56)/x^3$

giac [A] time = 0.29, size = 39, normalized size = 0.81

$$-\frac{417x^2 - 39x + 4}{24x^3} + \frac{625}{7} \log(|5x + 1|) - \frac{81}{112} \log(|3x + 2|) - \frac{1417}{16} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="giac")`

[Out] $-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*\log(\text{abs}(5*x + 1)) - 81/112*\log(\text{abs}(3*x + 2)) - 1417/16*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{1417 \ln(x)}{16} + \frac{625 \ln(5x + 1)}{7} - \frac{81 \ln(3x + 2)}{112} - \frac{139}{8x} + \frac{13}{8x^2} - \frac{1}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(15+2/x^2+13/x)/x^6,x)`

[Out] $-1/6/x^3+13/8/x^2-139/8/x-1417/16*\ln(x)-81/112*\ln(3*x+2)+625/7*\ln(5*x+1)$

maxima [A] time = 0.65, size = 36, normalized size = 0.75

$$-\frac{417x^2 - 39x + 4}{24x^3} + \frac{625}{7} \log(5x + 1) - \frac{81}{112} \log(3x + 2) - \frac{1417}{16} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="maxima")`

[Out] $-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*\log(5*x + 1) - 81/112*\log(3*x + 2) - 1417/16*\log(x)$

mupad [B] time = 0.05, size = 32, normalized size = 0.67

$$\frac{625 \ln\left(x + \frac{1}{5}\right)}{7} - \frac{81 \ln\left(x + \frac{2}{3}\right)}{112} - \frac{1417 \ln(x)}{16} - \frac{\frac{139x^2}{8} - \frac{13x}{8} + \frac{1}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^6*(13/x + 2/x^2 + 15)),x)`

[Out] $(625 \cdot \log(x + 1/5))/7 - (81 \cdot \log(x + 2/3))/112 - (1417 \cdot \log(x))/16 - ((139 \cdot x^2)/8 - (13 \cdot x)/8 + 1/6)/x^3$

sympy [A] time = 0.18, size = 41, normalized size = 0.85

$$-\frac{1417 \log(x)}{16} + \frac{625 \log\left(x + \frac{1}{5}\right)}{7} - \frac{81 \log\left(x + \frac{2}{3}\right)}{112} + \frac{-417x^2 + 39x - 4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(15+2/x**2+13/x)/x**6,x)

[Out] $-1417 \cdot \log(x)/16 + 625 \cdot \log(x + 1/5)/7 - 81 \cdot \log(x + 2/3)/112 + (-417 \cdot x^2 + 39 \cdot x - 4)/(24 \cdot x^3)$

$$3.388 \quad \int \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{5/2} dx$$

Optimal. Leaf size=204

$$\frac{5}{2} a^{3/2} b \tanh^{-1} \left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right) + \frac{5(-48a^2c^2 - 24ab^2c + b^4) \tanh^{-1} \left(\frac{b + \frac{2c}{x}}{2\sqrt{c} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right)}{128c^{3/2}} - \frac{5 \left(\frac{2c(12ac + b^2)}{x} + b(44ac + b^2) \right)}{64c}$$

Rubi [A] time = 0.23, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1342, 732, 814, 843, 621, 206, 724}

$$\frac{5(-48a^2c^2 - 24ab^2c + b^4) \tanh^{-1} \left(\frac{b + \frac{2c}{x}}{2\sqrt{c} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right)}{128c^{3/2}} + \frac{5}{2} a^{3/2} b \tanh^{-1} \left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right) - \frac{5 \left(\frac{2c(12ac + b^2)}{x} + b(44ac + b^2) \right) \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{64c} + x \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{5/2} - \frac{5}{24} \left(7b + \frac{6c}{x} \right) \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a + c/x^2 + b/x)^(5/2), x]

[Out] (-5*(a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/24 - (5*Sqrt[a + c/x^2 + b/x]*(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/(64*c) + (a + c/x^2 + b/x)^(5/2)*x + (5*a^(3/2)*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/2 + (5*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(128*c^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 732

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 814

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1342

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{5/2} dx &= -\text{Subst} \left(\int \frac{(a + bx + cx^2)^{5/2}}{x^2} dx, x, \frac{1}{x} \right) \\
&= \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{5/2} x - \frac{5}{2} \text{Subst} \left(\int \frac{(b + 2cx)(a + bx + cx^2)^{3/2}}{x} dx, x, \frac{1}{x} \right) \\
&= -\frac{5}{24} \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} \left(7b + \frac{6c}{x} \right) + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{5/2} x + \frac{5 \text{Subst} \left(\int \frac{(-8abc - c(b^2 + 12ac)x + \sqrt{a + \frac{c}{x^2} + \frac{b}{x}})}{x} dx, x, \frac{1}{x} \right)}{16c} \\
&= -\frac{5}{24} \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} \left(7b + \frac{6c}{x} \right) - \frac{5 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \left(b(b^2 + 44ac) + \frac{2c(b^2 + 12ac)}{x} \right)}{64c} + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{5/2} x \\
&= -\frac{5}{24} \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} \left(7b + \frac{6c}{x} \right) - \frac{5 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \left(b(b^2 + 44ac) + \frac{2c(b^2 + 12ac)}{x} \right)}{64c} + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{5/2} x \\
&= -\frac{5}{24} \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} \left(7b + \frac{6c}{x} \right) - \frac{5 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \left(b(b^2 + 44ac) + \frac{2c(b^2 + 12ac)}{x} \right)}{64c} + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{5/2} x \\
&= -\frac{5}{24} \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} \left(7b + \frac{6c}{x} \right) - \frac{5 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \left(b(b^2 + 44ac) + \frac{2c(b^2 + 12ac)}{x} \right)}{64c} + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{5/2} x \\
&= -\frac{5}{24} \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} \left(7b + \frac{6c}{x} \right) - \frac{5 \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \left(b(b^2 + 44ac) + \frac{2c(b^2 + 12ac)}{x} \right)}{64c} + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{5/2} x
\end{aligned}$$

Mathematica [A] time = 0.52, size = 213, normalized size = 1.04

$$\frac{\sqrt{a + \frac{bx+c}{x^2}} \left(960a^{3/2}bc^{3/2}x^4 \tanh^{-1} \left(\frac{2ax+b}{2\sqrt{a}\sqrt{ax+b+c}} \right) + 15x^4 (-48a^2c^2 - 24ab^2c + b^4) \tanh^{-1} \left(\frac{bx+2c}{2\sqrt{c}\sqrt{ax+b+c}} \right) - 2\sqrt{c}\sqrt{x(ax+b)+c} (2cx^2 (-96a^2x^2 + 278abx + 59b^2) + 8^2x(27ax + 17b) + 15b^3x^3 + 48c^3) \right)}{384c^{3/2}x^3\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c/x^2 + b/x)^(5/2), x]

[Out] (Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[c]*Sqrt[c + x*(b + a*x)]*(48*c^3 + 15*b^3*x^3 + 8*c^2*x*(17*b + 27*a*x) + 2*c*x^2*(59*b^2 + 278*a*b*x - 96*a^2*x^2)) + 960*a^(3/2)*b*c^(3/2)*x^4*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])] + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c + x*(b + a*x)])])/(384*c^(3/2)*x^3*Sqrt[c + x*(b + a*x)])

IntegrateAlgebraic [A] time = 8.56, size = 209, normalized size = 1.02

$$\frac{x^5 \left(a + \frac{bx+c}{x^2} \right)^{5/2} \left(-\frac{5}{2} a^{3/2} b \log \left(-2\sqrt{a} \sqrt{ax^2 + bx + c} + 2ax + b \right) + \frac{5(-48a^2c^2 - 24ab^2c + b^4) \tanh^{-1} \left(\frac{\sqrt{ax^2 + bx + c} - \sqrt{ax}}{\sqrt{c}} \right) + \frac{\sqrt{ax^2 + bx + c} (192a^2cx^4 - 556abcx^3 - 216ac^2x^2 - 15b^3x^3 - 118b^2cx^2 - 136bc^2x - 48c^3)}{192cx^4} \right)}{(x(ax+b)+c)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + c/x^2 + b/x)^(5/2), x]

[Out] (x^5*(a + (c + b*x)/x^2)^(5/2)*((Sqrt[c + b*x + a*x^2]*(-48*c^3 - 136*b*c^2*x - 118*b^2*c*x^2 - 216*a*c^2*x^2 - 15*b^3*x^3 - 556*a*b*c*x^3 + 192*a^2*c*x^4))/(192*c*x^4) + (5*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(-Sqrt[a]*x) + Sqrt[c + b*x + a*x^2])/Sqrt[c])/(64*c^(3/2)) - (5*a^(3/2)*b*Log[b + 2*a*x - 2*Sqrt[a]*Sqrt[c + b*x + a*x^2]]/2))/(c + x*(b + a*x))^(5/2)

fricas [A] time = 1.59, size = 959, normalized size = 4.70

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(5/2), x, algorithm="fricas")

[Out] [1/768*(960*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/768*(1920*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), 1/384*(480*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/384*(960*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.02, size = 701, normalized size = 3.44

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c/x^2+b/x)^(5/2),x)

[Out]
$$\begin{aligned} & 1/384*((a*x^2+b*x+c)/x^2)^(5/2)*x*(720*a^(7/2)*(a*x^2+b*x+c)^(1/2)*x^4*c^4+ \\ & 6*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x^3*b^3-6*a^(3/2)*(a*x^2+b*x+c)^(5/2)*x^4*b^4 \\ & -720*\ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(9/2)*a^(7/2)*x^4-6*a^(\\ & (5/2)*(a*x^2+b*x+c)^(5/2)*x^5*b^3+144*a^(7/2)*(a*x^2+b*x+c)^(5/2)*x^4*c^2-1 \\ & 44*a^(5/2)*(a*x^2+b*x+c)^(7/2)*x^2*c^2+240*a^(7/2)*(a*x^2+b*x+c)^(3/2)*x^4* \\ & c^3-360*\ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(7/2)*a^(5/2)*x^4*b \\ & ^2+660*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^4*b^2*c^3+960*\ln(1/2*(2*(a*x^2+b*x+c)^(\\ & (1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*x^4*b*c^4+148*a^(5/2)*(a*x^2+b*x+c)^(5/ \\ & 2)*x^4*b^2*c+280*a^(7/2)*(a*x^2+b*x+c)^(3/2)*x^5*b*c^2-10*a^(5/2)*(a*x^2+b* \\ & x+c)^(3/2)*x^5*b^3*c+15*\ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*c^(5/ \\ & 2)*a^(3/2)*x^4*b^4+260*a^(5/2)*(a*x^2+b*x+c)^(3/2)*x^4*b^2*c^2+600*a^(7/2)* \\ & (a*x^2+b*x+c)^(1/2)*x^5*b*c^3-30*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^5*b^3*c^2+15 \\ & 2*a^(7/2)*(a*x^2+b*x+c)^(5/2)*x^5*b*c-152*a^(5/2)*(a*x^2+b*x+c)^(7/2)*x^3*b \\ & *c+4*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x^2*b^2*c-10*a^(3/2)*(a*x^2+b*x+c)^(3/2)*x \\ & ^4*b^4*c+16*a^(3/2)*(a*x^2+b*x+c)^(7/2)*x*b*c^2-30*a^(3/2)*(a*x^2+b*x+c)^(1 \\ & /2)*x^4*b^4*c^2-96*(a*x^2+b*x+c)^(7/2)*c^3*a^(3/2)/(a*x^2+b*x+c)^(5/2)/c^4 \\ & /a^(3/2) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="maxima")

[Out] integrate((a + b/x + c/x^2)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x + c/x^2)^(5/2), x)`

[Out] `int((a + b/x + c/x^2)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c/x**2+b/x)**(5/2), x)`

[Out] `Integral((a + b/x + c/x**2)**(5/2), x)`

$$3.389 \quad \int \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} dx$$

Optimal. Leaf size=145

$$\frac{3(4ac + b^2) \tanh^{-1} \left(\frac{b + \frac{2c}{x}}{2\sqrt{c} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right)}{8\sqrt{c}} + x \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{3/2} - \frac{3}{4} \left(3b + \frac{2c}{x} \right) \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} + \frac{3}{2} \sqrt{a} b \tanh^{-1} \left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right)$$

Rubi [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1342, 732, 814, 843, 621, 206, 724}

$$\frac{3(4ac + b^2) \tanh^{-1} \left(\frac{b + \frac{2c}{x}}{2\sqrt{c} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right)}{8\sqrt{c}} + x \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{3/2} - \frac{3}{4} \left(3b + \frac{2c}{x} \right) \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} + \frac{3}{2} \sqrt{a} b \tanh^{-1} \left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + c/x^2 + b/x)^(3/2), x]

[Out] (-3*Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/4 + (a + c/x^2 + b/x)^(3/2)*x + (3*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/2 - (3*(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(8*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)),
Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
|| LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)),
Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1342

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> -Subst[Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} dx &= -\text{Subst} \left(\int \frac{(a + bx + cx^2)^{3/2}}{x^2} dx, x, \frac{1}{x} \right) \\
&= \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} x - \frac{3}{2} \text{Subst} \left(\int \frac{(b + 2cx)\sqrt{a + bx + cx^2}}{x} dx, x, \frac{1}{x} \right) \\
&= -\frac{3}{4} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \left(3b + \frac{2c}{x} \right) + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} x + \frac{3 \text{Subst} \left(\int \frac{-4abc - c(b^2 + 4ac)x}{x\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x} \right)}{8c} \\
&= -\frac{3}{4} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \left(3b + \frac{2c}{x} \right) + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} x - \frac{1}{2} (3ab) \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x} \right) \\
&= -\frac{3}{4} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \left(3b + \frac{2c}{x} \right) + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} x + (3ab) \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2}{\sqrt{a - \frac{b}{x}}} \right) \\
&= -\frac{3}{4} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \left(3b + \frac{2c}{x} \right) + \left(a + \frac{c}{x^2} + \frac{b}{x} \right)^{3/2} x + \frac{3}{2} \sqrt{a} b \tanh^{-1} \left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} \right)
\end{aligned}$$

Mathematica [A] time = 0.25, size = 163, normalized size = 1.12

$$\frac{\sqrt{a + \frac{bx+c}{x^2}} \left(-3x^2 (4ac + b^2) \tanh^{-1} \left(\frac{bx+2c}{2\sqrt{c} \sqrt{x(ax+b)+c}} \right) + 12\sqrt{a} b \sqrt{c} x^2 \tanh^{-1} \left(\frac{2ax+b}{2\sqrt{a} \sqrt{x(ax+b)+c}} \right) - 2\sqrt{c} (x(5b - 4ax) + 2c) \sqrt{x(ax+b)+c} \right)}{8\sqrt{c} x \sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c/x^2 + b/x)^(3/2), x]

[Out] (Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[c]*(2*c + x*(5*b - 4*a*x))*Sqrt[c + x*(b + a*x)] + 12*Sqrt[a]*b*Sqrt[c]*x^2*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])] - 3*(b^2 + 4*a*c)*x^2*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c + x*(b + a*x)])]))/(8*Sqrt[c]*x*Sqrt[c + x*(b + a*x)])

IntegrateAlgebraic [A] time = 7.17, size = 154, normalized size = 1.06

$$\frac{x^3 \left(a + \frac{bx+c}{x^2} \right)^{3/2} \left(\frac{3(4ac+b^2) \tanh^{-1} \left(\frac{\sqrt{a}x - \sqrt{ax^2+bx+c}}{\sqrt{c}} \right)}{4\sqrt{c}} + \frac{\sqrt{ax^2+bx+c} (4ax^2-5bx-2c)}{4x^2} - \frac{3}{2} \sqrt{a} b \log \left(-2\sqrt{a} \sqrt{ax^2+bx+c} + 2ax + b \right) \right)}{(x(ax+b)+c)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + c/x^2 + b/x)^(3/2),x]

[Out] $(x^3*(a + (c + b*x)/x^2)^{(3/2)}*((\text{Sqrt}[c + b*x + a*x^2]*(-2*c - 5*b*x + 4*a*x^2))/(4*x^2) + (3*(b^2 + 4*a*c)*\text{ArcTanh}[(\text{Sqrt}[a]*x - \text{Sqrt}[c + b*x + a*x^2])]/\text{Sqrt}[c]))/(4*\text{Sqrt}[c]) - (3*\text{Sqrt}[a]*b*\text{Log}[b + 2*a*x - 2*\text{Sqrt}[a]*\text{Sqrt}[c + b*x + a*x^2]])/2)/(c + x*(b + a*x))^{(3/2)}$

fricas [A] time = 1.36, size = 709, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="fricas")

[Out] $[1/16*(12*\text{sqrt}(a)*b*c*x*\text{log}(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*\text{sqrt}(a)*\text{sqrt}((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*\text{sqrt}(c)*x*\text{log}(-8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*\text{sqrt}(c)*\text{sqrt}((a*x^2 + b*x + c)/x^2))/x^2 + 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(c*x), -1/16*(24*\text{sqrt}(-a)*b*c*x*\text{arctan}(1/2*(2*a*x^2 + b*x)*\text{sqrt}(-a)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 3*(b^2 + 4*a*c)*\text{sqrt}(c)*x*\text{log}(-8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*\text{sqrt}(c)*\text{sqrt}((a*x^2 + b*x + c)/x^2))/x^2 - 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(c*x), 1/8*(6*\text{sqrt}(a)*b*c*x*\text{log}(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*\text{sqrt}(a)*\text{sqrt}((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*\text{sqrt}(-c)*x*\text{arctan}(1/2*(b*x^2 + 2*c*x)*\text{sqrt}(-c)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(c*x), -1/8*(12*\text{sqrt}(-a)*b*c*x*\text{arctan}(1/2*(2*a*x^2 + b*x)*\text{sqrt}(-a)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 3*(b^2 + 4*a*c)*\text{sqrt}(-c)*x*\text{arctan}(1/2*(b*x^2 + 2*c*x)*\text{sqrt}(-c)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - 2*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(c*x)]$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 334, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+c/x^2+b/x)^(3/2),x)`

[Out]
$$-1/8*((a*x^2+b*x+c)/x^2)^(3/2)*x*(12*a^(5/2)*c^(5/2)*\ln((b*x+2*c+2*(a*x^2+b*x+c)^(1/2)*c^(1/2))/x)*x^2-2*a^(5/2)*(a*x^2+b*x+c)^(3/2)*x^3*b-4*a^(5/2)*(a*x^2+b*x+c)^(3/2)*x^2*c-6*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^3*b*c+3*a^(3/2)*c^(3/2)*\ln((b*x+2*c+2*(a*x^2+b*x+c)^(1/2)*c^(1/2))/x)*x^2*b^2-12*a^(5/2)*(a*x^2+b*x+c)^(1/2)*x^2*c^2+2*a^(3/2)*(a*x^2+b*x+c)^(5/2)*x*b-2*a^(3/2)*(a*x^2+b*x+c)^(3/2)*x^2*b^2+4*(a*x^2+b*x+c)^(5/2)*c*a^(3/2)-6*a^(3/2)*(a*x^2+b*x+c)^(1/2)*x^2*b^2*c-12*a^2*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^(1/2)*a^(1/2)))/a^(1/2))*x^2*b*c^2)/(a*x^2+b*x+c)^(3/2)/c^2/a^(3/2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c/x^2+b/x)^(3/2),x, algorithm="maxima")`

[Out] `integrate((a + b/x + c/x^2)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x + c/x^2)^(3/2),x)`

[Out] `int((a + b/x + c/x^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c/x**2+b/x)**(3/2),x)`

[Out] `Integral((a + b/x + c/x**2)**(3/2), x)`

$$3.390 \quad \int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} dx$$

Optimal. Leaf size=105

$$x\sqrt{a + \frac{b}{x} + \frac{c}{x^2}} + \frac{b \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{2\sqrt{a}} - \sqrt{c} \tanh^{-1}\left(\frac{b + \frac{2c}{x}}{2\sqrt{c}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1342, 732, 843, 621, 206, 724}

$$x\sqrt{a + \frac{b}{x} + \frac{c}{x^2}} + \frac{b \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{2\sqrt{a}} - \sqrt{c} \tanh^{-1}\left(\frac{b + \frac{2c}{x}}{2\sqrt{c}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c/x^2 + b/x], x]

[Out] Sqrt[a + c/x^2 + b/x]*x + (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x]))/(2*Sqrt[a]) - Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 732

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Dist[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1342

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} dx &= -\text{Subst} \left(\int \frac{\sqrt{a + bx + cx^2}}{x^2} dx, x, \frac{1}{x} \right) \\
&= \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x - \frac{1}{2} \text{Subst} \left(\int \frac{b + 2cx}{x\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x} \right) \\
&= \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x - \frac{1}{2} b \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x} \right) - c \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x} \right) \\
&= \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x + b \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + \frac{b}{x}}{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} \right) - (2c) \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{1}{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} \right) \\
&= \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x + \frac{b \tanh^{-1} \left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} \right)}{2\sqrt{a}} - \sqrt{c} \tanh^{-1} \left(\frac{b + \frac{2c}{x}}{2\sqrt{c} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} \right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 128, normalized size = 1.22

$$\frac{x \sqrt{a + \frac{bx+c}{x^2}} \left(b \tanh^{-1} \left(\frac{2ax+b}{2\sqrt{a} \sqrt{x(ax+b)+c}} \right) + 2\sqrt{a} \left(\sqrt{x(ax+b)+c} - \sqrt{c} \tanh^{-1} \left(\frac{bx+2c}{2\sqrt{c} \sqrt{x(ax+b)+c}} \right) \right) \right)}{2\sqrt{a} \sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c/x^2 + b/x], x]

[Out] (x*Sqrt[a + (c + b*x)/x^2]*(b*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])] + 2*Sqrt[a]*(Sqrt[c + x*(b + a*x)] - Sqrt[c]*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c + x*(b + a*x)])]))/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])

IntegrateAlgebraic [A] time = 5.60, size = 125, normalized size = 1.19

$$\frac{x \sqrt{a + \frac{bx+c}{x^2}} \left(\sqrt{ax^2 + bx + c} - \frac{b \log(-2\sqrt{a} \sqrt{ax^2 + bx + c} + 2ax + b)}{2\sqrt{a}} + 2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{a}x}{\sqrt{c}} - \frac{\sqrt{ax^2 + bx + c}}{\sqrt{c}} \right) \right)}{\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + c/x^2 + b/x], x]

[Out] $(x\sqrt{a + (c + b*x)/x^2}) * (\sqrt{c + b*x + a*x^2} + 2\sqrt{c}) * \text{ArcTanh}[(\sqrt{a}*x)/\sqrt{c} - \sqrt{c + b*x + a*x^2}/\sqrt{c}] - (b*\text{Log}[b + 2*a*x - 2*\sqrt{a}*\sqrt{c + b*x + a*x^2}]) / (2*\sqrt{a}) / \sqrt{c + x*(b + a*x)}$

fricas [A] time = 1.25, size = 590, normalized size = 5.62

$$\frac{4a\sqrt{\frac{a^2+c^2}{4a^2}} + \sqrt{c}\log\left(\frac{-8a^2b^2-8ab^2c-4a^2c^2-4(2a^2+bx)\sqrt{\frac{a^2+c^2}{4a^2}}}{-8a^2b^2-8ab^2c-4a^2c^2}\right) + 2a^2\log\left(\frac{8a^2b^2+8a^2bx+8a^2c^2}{-8a^2b^2-8ab^2c-4a^2c^2}\right)}{2a\sqrt{\frac{a^2+c^2}{4a^2}} - \sqrt{c}\arctan\left(\frac{b^2+bx+c}{2a^2}\right) + a^2\log\left(\frac{8a^2b^2+8a^2bx+8a^2c^2}{-8a^2b^2-8ab^2c-4a^2c^2}\right)} + \frac{4a\sqrt{\frac{a^2+c^2}{4a^2}} + 4a^2\arctan\left(\frac{b^2+bx+c}{2a^2}\right) + \sqrt{c}\log\left(\frac{-8a^2b^2-8ab^2c-4a^2c^2-4(2a^2+bx)\sqrt{\frac{a^2+c^2}{4a^2}}}{-8a^2b^2-8ab^2c-4a^2c^2}\right)}{2a\sqrt{\frac{a^2+c^2}{4a^2}} - \sqrt{c}\arctan\left(\frac{b^2+bx+c}{2a^2}\right) + a^2\log\left(\frac{8a^2b^2+8a^2bx+8a^2c^2}{-8a^2b^2-8ab^2c-4a^2c^2}\right)} - 2a^2\arctan\left(\frac{b^2+bx+c}{2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="fricas")

[Out] $[1/4*(4*a*x*\sqrt{(a*x^2 + b*x + c)/x^2}) + \sqrt{a}*b*\log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*\sqrt{a}*\sqrt{(a*x^2 + b*x + c)/x^2}) + 2*a*\sqrt{c}*\log(-8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*\sqrt{c}*\sqrt{(a*x^2 + b*x + c)/x^2})/x^2)/a, 1/2*(2*a*x*\sqrt{(a*x^2 + b*x + c)/x^2}) - \sqrt{-a}*b*\arctan(1/2*(2*a*x^2 + b*x)*\sqrt{-a}*\sqrt{(a*x^2 + b*x + c)/x^2})/(a^2*x^2 + a*b*x + a*c) + a*\sqrt{c}*\log(-8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*\sqrt{c}*\sqrt{(a*x^2 + b*x + c)/x^2})/x^2)/a, 1/4*(4*a*x*\sqrt{(a*x^2 + b*x + c)/x^2}) + 4*a*\sqrt{-c}*\arctan(1/2*(b*x^2 + 2*c*x)*\sqrt{-c}*\sqrt{(a*x^2 + b*x + c)/x^2})/(a*c*x^2 + b*c*x + c^2) + \sqrt{a}*b*\log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*\sqrt{a}*\sqrt{(a*x^2 + b*x + c)/x^2})/a, 1/2*(2*a*x*\sqrt{(a*x^2 + b*x + c)/x^2}) - \sqrt{-a}*b*\arctan(1/2*(2*a*x^2 + b*x)*\sqrt{-a}*\sqrt{(a*x^2 + b*x + c)/x^2})/(a^2*x^2 + a*b*x + a*c) + 2*a*\sqrt{-c}*\arctan(1/2*(b*x^2 + 2*c*x)*\sqrt{-c}*\sqrt{(a*x^2 + b*x + c)/x^2})/(a*c*x^2 + b*c*x + c^2))/a]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] index.cc index_m operator + Error: Bad Argument Value

maple [A] time = 0.01, size = 121, normalized size = 1.15

$$\frac{\sqrt{\frac{ax^2+bx+c}{x^2}} \left(-2\sqrt{a}\sqrt{c} \ln\left(\frac{bx+2c+2\sqrt{ax^2+bx+c}\sqrt{c}}{x}\right) + b \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx+c}\sqrt{a}}{2\sqrt{a}}\right) + 2\sqrt{ax^2+bx+c}\sqrt{a} \right) x}{2\sqrt{ax^2+bx+c}\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+c/x^2+b/x)^(1/2),x)`

[Out] $\frac{1}{2} * ((a*x^2+b*x+c)/x^2)^{(1/2)} * x * (-2*c^{(1/2)} * \ln((b*x+2*c+2*(a*x^2+b*x+c)^{(1/2)} * c^{(1/2)})/x) * a^{(1/2)} + b * \ln(1/2 * (2*a*x+b+2*(a*x^2+b*x+c)^{(1/2)} * a^{(1/2)})/a^{(1/2)}) + 2*(a*x^2+b*x+c)^{(1/2)} * a^{(1/2)}) / (a*x^2+b*x+c)^{(1/2)} / a^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c/x^2+b/x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a + b/x + c/x^2), x)`

mupad [B] time = 0.13, size = 100, normalized size = 0.95

$$x \sqrt{\frac{1}{x^2}} \sqrt{ax^2 + bx + c} - \sqrt{c} x \ln\left(\frac{2c + 2\sqrt{c} \sqrt{ax^2 + bx + c} + bx}{x}\right) \sqrt{\frac{1}{x^2}} + \frac{bx \ln\left(\frac{\frac{b}{2} + \sqrt{a} \sqrt{ax^2 + bx + c} + ax}{\sqrt{a}}\right) \sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x + c/x^2)^(1/2),x)`

[Out] $x * (1/x^2)^{(1/2)} * (c + b*x + a*x^2)^{(1/2)} - c^{(1/2)} * x * \log((2*c + 2*c^{(1/2)} * (c + b*x + a*x^2)^{(1/2)} + b*x)/x) * (1/x^2)^{(1/2)} + (b*x * \log((b/2 + a^{(1/2)} * (c + b*x + a*x^2)^{(1/2)} + a*x)/a^{(1/2)}) * (1/x^2)^{(1/2)}) / (2*a^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c/x**2+b/x)**(1/2),x)`

[Out] `Integral(sqrt(a + b/x + c/x**2), x)`

$$3.391 \quad \int \frac{1}{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} dx$$

Optimal. Leaf size=67

$$\frac{x\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{a} - \frac{b \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{2a^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1342, 730, 724, 206}

$$\frac{x\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{a} - \frac{b \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + c/x^2 + b/x], x]

[Out] (Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x]))/(2*a^(3/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 730

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),

`Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]`

Rule 1342

`Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[n2, 2*n] && ILtQ[n, 0]`

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} dx &= -\text{Subst} \left(\int \frac{1}{x^2 \sqrt{a + bx + cx^2}} dx, x, \frac{1}{x} \right) \\ &= \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{a} + \frac{b \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx + cx^2}} dx, x, \frac{1}{x} \right)}{2a} \\ &= \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{a} - \frac{b \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + \frac{b}{x}}{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} \right)}{a} \\ &= \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{a} - \frac{b \tanh^{-1} \left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} \right)}{2a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.33

$$\frac{2\sqrt{a}(x(ax+b)+c) - b\sqrt{x(ax+b)+c} \tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)}{2a^{3/2}x\sqrt{a + \frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + c/x^2 + b/x], x]

[Out] (2*Sqrt[a]*(c + x*(b + a*x)) - b*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(2*a^(3/2)*x*Sqrt[a + (c + b*x)/x^2])

IntegrateAlgebraic [A] time = 4.70, size = 93, normalized size = 1.39

$$\frac{\sqrt{x(ax+b)+c} \left(\frac{b \log(-2a^{3/2} \sqrt{ax^2+bx+c} + 2a^2x+ab)}{2a^{3/2}} + \frac{\sqrt{ax^2+bx+c}}{a} \right)}{x \sqrt{a + \frac{bx+c}{x^2}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[a + c/x^2 + b/x], x]

[Out] (Sqrt[c + x*(b + a*x)]*(Sqrt[c + b*x + a*x^2]/a + (b*Log[a*b + 2*a^2*x - 2*a^(3/2)*Sqrt[c + b*x + a*x^2]])/(2*a^(3/2))))/(x*Sqrt[a + (c + b*x)/x^2])

fricas [A] time = 1.29, size = 171, normalized size = 2.55

$$\left[\frac{4ax\sqrt{\frac{ax^2+bx+c}{x^2}} + \sqrt{a}b \log\left(-8a^2x^2 - 8abx - b^2 - 4ac + 4(2ax^2 + bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4a^2}, \frac{2ax\sqrt{\frac{ax^2+bx+c}{x^2}} + \sqrt{-a}b \arctan\left(\frac{(2ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(a^2x^2+abx+ac)}\right)}{2a^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(1/2), x, algorithm="fricas")

[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))/a^2, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)))/a^2]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] index.cc index_m operator + Error: Bad Argument Value

maple [A] time = 0.00, size = 88, normalized size = 1.31

$$\frac{\sqrt{ax^2+bx+c} \left(-ab \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx+c}\sqrt{a}}{2\sqrt{a}}\right) + 2\sqrt{ax^2+bx+c}a^{\frac{3}{2}} \right)}{2\sqrt{\frac{ax^2+bx+c}{x^2}}a^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+c/x^2+b/x)^(1/2),x)`

[Out] $1/2*(a*x^2+b*x+c)^{(1/2)}*(2*(a*x^2+b*x+c)^{(1/2)}*a^{(3/2)}-b*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^{(1/2)}*a^{(1/2))}/a^{(1/2)})))/((a*x^2+b*x+c)/x^2)^{(1/2)}/x/a^{(5/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(a + b/x + c/x^2), x)`

mupad [B] time = 1.45, size = 53, normalized size = 0.79

$$\frac{x \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{a} - \frac{b \operatorname{atanh}\left(\frac{a + \frac{b}{2x}}{\sqrt{a} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{2 a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b/x + c/x^2)^(1/2),x)`

[Out] $(x*(a + b/x + c/x^2)^{(1/2)})/a - (b*\operatorname{atanh}((a + b/(2*x))/(a^{(1/2)}*(a + b/x + c/x^2)^{(1/2)})))/(2*a^{(3/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+c/x**2+b/x)**(1/2),x)`

[Out] `Integral(1/sqrt(a + b/x + c/x**2), x)`

$$3.392 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2}} dx$$

Optimal. Leaf size=133

$$-\frac{3b \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{2a^{5/2}} + \frac{x(3b^2 - 8ac)\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{a^2(b^2 - 4ac)} - \frac{2x(-2ac + b^2 + \frac{bc}{x})}{a(b^2 - 4ac)\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}$$

Rubi [A] time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1342, 740, 806, 724, 206}

$$\frac{x(3b^2 - 8ac)\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{a^2(b^2 - 4ac)} - \frac{3b \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{2a^{5/2}} - \frac{2x(-2ac + b^2 + \frac{bc}{x})}{a(b^2 - 4ac)\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + c/x^2 + b/x)^(-3/2), x]

[Out] ((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/(a^2*(b^2 - 4*a*c)) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) - (3*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(5/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

```

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 806

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

```

Rule 1342

```

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> -Subst[Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2}} dx &= -\text{Subst} \left(\int \frac{1}{x^2 (a + bx + cx^2)^{3/2}} dx, x, \frac{1}{x} \right) \\
&= -\frac{2 \left(b^2 - 2ac + \frac{bc}{x}\right) x}{a (b^2 - 4ac) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} + \frac{2 \text{Subst} \left(\int \frac{\frac{1}{2}(-3b^2 + 8ac) - bcx}{x^2 \sqrt{a + bx + cx^2}} dx, x, \frac{1}{x} \right)}{a (b^2 - 4ac)} \\
&= \frac{(3b^2 - 8ac) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{a^2 (b^2 - 4ac)} - \frac{2 \left(b^2 - 2ac + \frac{bc}{x}\right) x}{a (b^2 - 4ac) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} + \frac{(3b) \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx + cx^2}} dx, x, \frac{1}{x} \right)}{2a^2} \\
&= \frac{(3b^2 - 8ac) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{a^2 (b^2 - 4ac)} - \frac{2 \left(b^2 - 2ac + \frac{bc}{x}\right) x}{a (b^2 - 4ac) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} - \frac{(3b) \text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2}{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} \right)}{a^2} \\
&= \frac{(3b^2 - 8ac) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{a^2 (b^2 - 4ac)} - \frac{2 \left(b^2 - 2ac + \frac{bc}{x}\right) x}{a (b^2 - 4ac) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} - \frac{3b \tanh^{-1} \left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} \right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 138, normalized size = 1.04

$$\frac{3b(b^2 - 4ac) \sqrt{x(ax + b) + c} \tanh^{-1} \left(\frac{2ax + b}{2\sqrt{a} \sqrt{x(ax + b) + c}} \right) + 2\sqrt{a} (-b^2(ax^2 + 3c) + 10abcx + 4ac(ax^2 + 2c) - 3b^3x)}{2a^{5/2}x(b^2 - 4ac) \sqrt{a + \frac{bx + c}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c/x^2 + b/x)^(-3/2), x]

[Out] -1/2*(2*Sqrt[a]*(-3*b^3*x + 10*a*b*c*x + 4*a*c*(2*c + a*x^2) - b^2*(3*c + a*x^2)) + 3*b*(b^2 - 4*a*c)*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(a^(5/2)*(b^2 - 4*a*c)*x*Sqrt[a + (c + b*x)/x^2])

IntegrateAlgebraic [A] time = 5.93, size = 150, normalized size = 1.13

$$\frac{(x(ax + b) + c)^{3/2} \left(\frac{4a^2cx^2 - ab^2x^2 + 10abcx + 8ac^2 - 3b^3x - 3b^2c}{a^2(4ac - b^2)\sqrt{ax^2 + bx + c}} + \frac{3b \log(-2a^{5/2}\sqrt{ax^2 + bx + c} + 2a^3x + a^2b)}{2a^{5/2}} \right)}{x^3 \left(a + \frac{bx + c}{x^2} \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + c/x^2 + b/x)^(-3/2), x]

[Out] ((c + x*(b + a*x))^(3/2)*((-3*b^2*c + 8*a*c^2 - 3*b^3*x + 10*a*b*c*x - a*b^2*x^2 + 4*a^2*c*x^2)/(a^2*(-b^2 + 4*a*c)*Sqrt[c + b*x + a*x^2])) + (3*b*Log[a^2*b + 2*a^3*x - 2*a^(5/2)*Sqrt[c + b*x + a*x^2]]/(2*a^(5/2)))/(x^3*(a + (c + b*x)/x^2)^(3/2))

fricas [A] time = 1.35, size = 465, normalized size = 3.50

$$\frac{3 \left(b^2c - 4ab^2c + (ab^2 - 4a^2b^2)c^2 + (b^2 - 4ab^2)c \right) \sqrt{a} \log \left(\frac{-8a^2x^2 - 8a^2bx - b^2 - 4a^2c + 4(2ax^2 + bx)\sqrt{a}}{2(a^2x^2 + bx + c)} \right) + 4 \left((a^2b^2 - 4a^2c^2) + 4 \left((a^2b^2 - 4a^2c^2) + (3ab^2c - 8a^2c^2) \right) \sqrt{\frac{a^2 + 2ax + c}{a}} \right) \arctan \left(\frac{2(a^2 + 2ax + c)\sqrt{a}}{2(a^2x^2 + bx + c)} \right) + 2 \left((a^2b^2 - 4a^2c^2) + (3ab^2c - 8a^2c^2) \right) \sqrt{\frac{a^2 + 2ax + c}{a}}}{4(a^2b^2c - 4a^2c^2 + (a^2b^2 - 4a^2c^2)c^2 + (a^2b^2 - 4a^2c^2)c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(3/2), x, algorithm="fricas")

[Out] [1/4*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 4*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x), 1/2*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2))/(a^2*x^2 + a*b*x + a*c) + 2*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes

constant sign by intervals (correct if the argument is real): Check [abs(t_n ostep)]index.cc index_m operator + Error: Bad Argument Value

maple [A] time = 0.01, size = 197, normalized size = 1.48

$$\frac{(ax^2 + bx + c) \left(8a^{\frac{7}{2}}cx^2 - 2a^{\frac{5}{2}}b^2x^2 + 20a^{\frac{5}{2}}bcx - 6a^{\frac{3}{2}}b^3x - 12\sqrt{ax^2 + bx + c} a^2bc \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx+c}\sqrt{a}}{2\sqrt{a}}\right) + 3\sqrt{ax^2 + bx + c} ab^3 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx+c}\sqrt{a}}{2\sqrt{a}}\right) + 16a^{\frac{5}{2}}c^2 - 6a^{\frac{3}{2}}b^2c \right)}{2\left(\frac{ax^2+bx+c}{x^2}\right)^{\frac{3}{2}}(4ac - b^2)a^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)^(3/2), x)

[Out] $\frac{1}{2} \frac{(ax^2+bx+c)^{7/2} (8a^{7/2}x^2c - 2a^{5/2}x^2b^2 + 20a^{5/2}x^2bc - 6a^{3/2}x^2b^3 + 16a^{5/2}c^2 - 6a^{3/2}b^2c - 12 \ln(1/2(2ax+b+2\sqrt{ax^2+bx+c})/a^{1/2})) a^{1/2}}{(ax^2+bx+c)^{1/2} a^2 b^3 c + 3 \ln(1/2(2ax+b+2\sqrt{ax^2+bx+c})/a^{1/2})) a^{1/2}}}{(ax^2+bx+c)^{3/2} x^3} + \frac{1}{4} \frac{a^{7/2} x^3}{(ax^2+bx+c)^{3/2} x^3}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(3/2), x, algorithm="maxima")

[Out] integrate((a + b/x + c/x^2)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/x + c/x^2)^(3/2), x)

[Out] int(1/(a + b/x + c/x^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(3/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(-3/2), x)
```

$$3.393 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{5/2}} dx$$

Optimal. Leaf size=220

$$\frac{5b \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{2a^{7/2}} - \frac{2x\left(32a^2c^2 + \frac{bc(5b^2 - 28ac)}{x} - 32ab^2c + 5b^4\right)}{3a^2(b^2 - 4ac)^2\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} + \frac{x(128a^2c^2 - 100ab^2c + 15b^4)\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{3a^3(b^2 - 4ac)^2}$$

Rubi [A] time = 0.19, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1342, 740, 822, 806, 724, 206}

$$-\frac{2x\left(32a^2c^2 + \frac{bc(5b^2 - 28ac)}{x} - 32ab^2c + 5b^4\right)}{3a^2(b^2 - 4ac)^2\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}} + \frac{x(128a^2c^2 - 100ab^2c + 15b^4)\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{3a^3(b^2 - 4ac)^2} - \frac{5b \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{2a^{7/2}} - \frac{2x(-2ac + b^2 + \frac{bc}{x})}{3a(b^2 - 4ac)\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c/x^2 + b/x)^(-5/2), x]

[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + c/x^2 + b/x]*x)/(3*a^3*(b^2 - 4*a*c)^2) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 - 4*a*c)*(a + c/x^2 + b/x)^(3/2)) - (2*(5*b^4 - 32*a*b^2*c + 32*a^2*c^2 + (b*c*(5*b^2 - 28*a*c))/x)*x)/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + c/x^2 + b/x]) - (5*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(7/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

```

Int[((d._) + (e._)*(x_))^(m_)*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 806

```

Int[((d._) + (e._)*(x_))^(m_)*((f._) + (g._)*(x_))*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

```

Rule 822

```

Int[((d._) + (e._)*(x_))^(m_)*((f._) + (g._)*(x_))*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 1342

```

Int[((a_) + (c._)*(x_)^(n2_)) + (b._)*(x_)^(n_)]^(p_), x_Symbol]
:> -Subst[Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{5/2}} dx &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^{5/2}} dx, x, \frac{1}{x}\right) \\
&= -\frac{2\left(b^2 - 2ac + \frac{bc}{x}\right)x}{3a(b^2 - 4ac)\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}(-5b^2+16ac)-3bcx}{x^2(a+bx+cx^2)^{3/2}} dx, x, \frac{1}{x}\right)}{3a(b^2 - 4ac)} \\
&= -\frac{2\left(b^2 - 2ac + \frac{bc}{x}\right)x}{3a(b^2 - 4ac)\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4 - 32ab^2c + 32a^2c^2 + \frac{bc(5b^2-28ac)}{x}\right)x}{3a^2(b^2 - 4ac)^2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} - \frac{4\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^{3/2}} dx, x, \frac{1}{x}\right)}{3a^2} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{3a^3(b^2 - 4ac)^2} - \frac{2\left(b^2 - 2ac + \frac{bc}{x}\right)x}{3a(b^2 - 4ac)\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4 - 32ab^2c + 32a^2c^2 + \frac{bc(5b^2-28ac)}{x}\right)x}{3a^2} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{3a^3(b^2 - 4ac)^2} - \frac{2\left(b^2 - 2ac + \frac{bc}{x}\right)x}{3a(b^2 - 4ac)\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4 - 32ab^2c + 32a^2c^2 + \frac{bc(5b^2-28ac)}{x}\right)x}{3a^2} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{3a^3(b^2 - 4ac)^2} - \frac{2\left(b^2 - 2ac + \frac{bc}{x}\right)x}{3a(b^2 - 4ac)\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4 - 32ab^2c + 32a^2c^2 + \frac{bc(5b^2-28ac)}{x}\right)x}{3a^2}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 256, normalized size = 1.16

$$\frac{2\sqrt{a}\left(3b^4(a^2x^4 - 30acx^2 + 5c^2) - 4ab^2c(6a^2x^4 - 12acx^2 + 25c^2) + 8a^2bc^2x(32ax^2 + 39c) + 16a^2c^2(3a^2x^4 + 12acx^2 + 8c^2) + 10b^5(2ax^3 + 3cx) - 2ab^3cx(74ax^2 + 105c) + 15b^6x^2 - 15b(b^2 - 4ac)^2(x(ax+b) + c)^{3/2} \tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{(ax+b)c}}\right)\right)}{6a^{7/2}x(b^2 - 4ac)^2(x(ax+b) + c)\sqrt{a + \frac{bc}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c/x^2 + b/x)^(-5/2), x]

[Out] (2*sqrt[a]*(15*b^6*x^2 + 8*a^2*b*c^2*x*(39*c + 32*a*x^2) - 2*a*b^3*c*x*(105*c + 74*a*x^2) + 10*b^5*(3*c*x + 2*a*x^3) + 3*b^4*(5*c^2 - 30*a*c*x^2 + a^2*x^4) + 16*a^2*c^2*(8*c^2 + 12*a*c*x^2 + 3*a^2*x^4) - 4*a*b^2*c*(25*c^2 - 12*a*c*x^2 + 6*a^2*x^4)) - 15*b*(b^2 - 4*a*c)^2*(c + x*(b + a*x))^(3/2)*ArcT

$\text{anh}[(b + 2*a*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[c + x*(b + a*x)])]/(6*a^{(7/2)}*(b^2 - 4*a*c)^2*x*(c + x*(b + a*x))*\text{Sqrt}[a + (c + b*x)/x^2])$

IntegrateAlgebraic [A] time = 7.91, size = 273, normalized size = 1.24

$$\frac{(x(ax+b)+c)^{5/2} \left(\frac{5b \log(-2a^{7/2} \sqrt{ax^2+bx+c} + 2a^4 x + a^3 b)}{2a^{7/2}} + \frac{48a^4 c^2 x^4 - 24a^3 b^2 c x^4 + 256a^3 b c^2 x^3 + 192a^3 c^3 x^2 + 3a^2 b^4 x^4 - 148a^2 b^3 c x^3 + 48a^2 b^2 c^2 x^2 + 312a^2 b c^3 x + 128a^2 c^4 + 20ab^5 x^3 - 90ab^4 c x^2 - 210ab^3 c^2 x - 100ab^2 c^3 + 15b^6 x^2 + 30b^5 c x + 15b^4 c^2}{3a^3(4ac-b^2)^2(ax^2+bx+c)^{3/2}} \right)}{x^5 \left(a + \frac{bx+c}{x^2} \right)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + c/x^2 + b/x)^(-5/2), x]

[Out] ((c + x*(b + a*x))^(5/2)*((15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4 + 30*b^5*c*x - 210*a*b^3*c^2*x + 312*a^2*b*c^3*x + 15*b^6*x^2 - 90*a*b^4*c*x^2 + 48*a^2*b^2*c^2*x^2 + 192*a^3*c^3*x^2 + 20*a*b^5*x^3 - 148*a^2*b^3*c*x^3 + 256*a^3*b*c^2*x^3 + 3*a^2*b^4*x^4 - 24*a^3*b^2*c*x^4 + 48*a^4*c^2*x^4)/(3*a^3*(-b^2 + 4*a*c)^2*(c + b*x + a*x^2)^(3/2)) + (5*b*Log[a^3*b + 2*a^4*x - 2*a^(7/2)*Sqrt[c + b*x + a*x^2]]/(2*a^(7/2))))/(x^5*(a + (c + b*x)/x^2)^(5/2))

fricas [B] time = 1.86, size = 1081, normalized size = 4.91

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(5/2), x, algorithm="fricas")

[Out] [1/12*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 4*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x), 1/6*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35

$$*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x)*\sqrt{(a*x^2 + b*x + c)/x^2)}/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x)]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]index.cc index_m operator + Error: Bad Argument Value

maple [A] time = 0.01, size = 376, normalized size = 1.71

$$\frac{(x^2 + bx + c) \left(-96a^{\frac{13}{2}}c^2 + 48a^{\frac{11}{2}}b^2c^2 - 6a^{\frac{9}{2}}b^4c^2 + 512a^{\frac{7}{2}}b^2c^3 + 296a^{\frac{5}{2}}b^4c^3 - 40a^{\frac{3}{2}}b^6c^3 - 384a^{\frac{11}{2}}c^4 - 96a^{\frac{9}{2}}b^2c^4 + 180a^{\frac{7}{2}}b^4c^4 - 30a^{\frac{5}{2}}b^6c^4 - 624a^{\frac{13}{2}}c^5 + 420a^{\frac{11}{2}}b^2c^5 - 60a^{\frac{9}{2}}b^4c^5 - 256a^{\frac{7}{2}}c^6 + 200a^{\frac{5}{2}}b^2c^6 - 30a^{\frac{3}{2}}b^4c^6 + 240(a^2 + bx + c)^{\frac{1}{2}}a^{\frac{13}{2}}c^2 \ln\left(\frac{20a^2 + \sqrt{40a^2c} - c}{2a}\right) - 120(a^2 + bx + c)^{\frac{1}{2}}a^{\frac{11}{2}}b^2c^2 \ln\left(\frac{20a^2 + \sqrt{40a^2c} - c}{2a}\right) + 15(a^2 + bx + c)^{\frac{1}{2}}a^{\frac{9}{2}}b^4c^2 \ln\left(\frac{20a^2 + \sqrt{40a^2c} - c}{2a}\right) \right)}{6\left(\frac{a^2 + bx + c}{x}\right)^{\frac{5}{2}}(4a^2 - b^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)^(5/2),x)

[Out] $-1/6*(a*x^2+b*x+c)*(-96*a^{(13/2)}*x^4*c^2+48*a^{(11/2)}*x^4*b^2*c-512*a^{(11/2)}*x^3*b*c^2-6*a^{(9/2)}*x^4*b^4-384*a^{(11/2)}*x^2*c^3+296*a^{(9/2)}*x^3*b^3*c-96*a^{(9/2)}*x^2*b^2*c^2-40*a^{(7/2)}*x^3*b^5-624*a^{(9/2)}*x*b*c^3+180*a^{(7/2)}*x^2*b^4*c-256*a^{(9/2)}*c^4+420*a^{(7/2)}*x*b^3*c^2-30*a^{(5/2)}*x^2*b^6+200*a^{(7/2)}*b^2*c^3-60*a^{(5/2)}*x*b^5*c+240*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^{(1/2)}*a^{(1/2)}))/a^{(1/2)}*(a*x^2+b*x+c)^{(3/2)}*a^4*b*c^2-120*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^{(1/2)}*a^{(1/2)}))/a^{(1/2)}*(a*x^2+b*x+c)^{(3/2)}*a^3*b^3*c+15*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x+c)^{(1/2)}*a^{(1/2)}))/a^{(1/2)}*(a*x^2+b*x+c)^{(3/2)}*a^2*b^5-30*a^{(5/2)}*b^4*c^2)/a^{(11/2)}/((a*x^2+b*x+c)/x^2)^{(5/2)}/x^5/(4*a*c-b^2)^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="maxima")

[Out] integrate((a + b/x + c/x^2)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/x + c/x^2)^(5/2), x)

[Out] int(1/(a + b/x + c/x^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)**(5/2), x)

[Out] Integral((a + b/x + c/x**2)**(-5/2), x)

$$3.394 \quad \int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}} dx$$

Optimal. Leaf size=73

$$\frac{ax\sqrt{a^2 + \frac{2ab}{x} + \frac{b^2}{x^2}}}{a + \frac{b}{x}} - \frac{b \log\left(\frac{1}{x}\right)\sqrt{a^2 + \frac{2ab}{x} + \frac{b^2}{x^2}}}{a + \frac{b}{x}}$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1342, 646, 43}

$$\frac{ax\sqrt{a^2 + \frac{2ab}{x} + \frac{b^2}{x^2}}}{a + \frac{b}{x}} - \frac{b \log\left(\frac{1}{x}\right)\sqrt{a^2 + \frac{2ab}{x} + \frac{b^2}{x^2}}}{a + \frac{b}{x}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x], x]

[Out] (a*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*x)/(a + b/x) - (b*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*Log[x^(-1)])/(a + b/x)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1342

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := -Subst[Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}} dx &= -\text{Subst} \left(\int \frac{\sqrt{a^2 + 2abx + b^2x^2}}{x^2} dx, x, \frac{1}{x} \right) \\
&= -\frac{\sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}} \text{Subst} \left(\int \frac{ab+b^2x}{x^2} dx, x, \frac{1}{x} \right)}{ab + \frac{b^2}{x}} \\
&= -\frac{\sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}} \text{Subst} \left(\int \left(\frac{ab}{x^2} + \frac{b^2}{x} \right) dx, x, \frac{1}{x} \right)}{ab + \frac{b^2}{x}} \\
&= \frac{a\sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}} x}{a + \frac{b}{x}} + \frac{b\sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}} \log(x)}{a + \frac{b}{x}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.44

$$\frac{x\sqrt{\frac{(ax+b)^2}{x^2}}(ax + b \log(x))}{ax + b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x], x]

[Out] (x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)

IntegrateAlgebraic [A] time = 4.98, size = 32, normalized size = 0.44

$$\frac{x\sqrt{\frac{(ax+b)^2}{x^2}}(ax + b \log(x))}{ax + b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x], x]

[Out] (x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)

fricas [A] time = 1.60, size = 8, normalized size = 0.11

$$ax + b \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="fricas")

[Out] a*x + b*log(x)

giac [A] time = 0.34, size = 29, normalized size = 0.40

$$ax \operatorname{sgn}(ax^2 + bx) + b \log(|x|) \operatorname{sgn}(ax^2 + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="giac")

[Out] a*x*sgn(a*x^2 + b*x) + b*log(abs(x))*sgn(a*x^2 + b*x)

maple [A] time = 0.01, size = 40, normalized size = 0.55

$$\frac{\sqrt{\frac{a^2x^2+2abx+b^2}{x^2}} (ax + b \ln(x)) x}{ax + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+b^2/x^2+2*a*b/x)^(1/2),x)

[Out] ((a^2*x^2+2*a*b*x+b^2)/x^2)^(1/2)/(a*x+b)*x*(a*x+b*ln(x))

maxima [A] time = 0.78, size = 8, normalized size = 0.11

$$ax + b \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="maxima")

[Out] a*x + b*log(x)

mupad [B] time = 0.11, size = 134, normalized size = 1.84

$$x \sqrt{\frac{1}{x^2}} \sqrt{a^2x^2 + 2abx + b^2} - x \ln\left(\frac{2\sqrt{b^2} \sqrt{a^2x^2 + 2abx + b^2} + 2b^2 + 2abx}{x}\right) \sqrt{b^2} \sqrt{\frac{1}{x^2}} + \frac{abx \ln\left(\frac{ab + \sqrt{a^2} \sqrt{a^2x^2 + 2abx + b^2} + a^2x}{\sqrt{a^2}}\right) \sqrt{\frac{1}{x^2}}}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2/x^2 + (2*a*b)/x)^(1/2),x)

[Out] x*(1/x^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) - x*log((2*(b^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) + 2*b^2 + 2*a*b*x)/x)*(b^2)^(1/2)*(1/x^2)^(1/2) + (a*b*x*log((a*b + (a^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) + a^2*x)/(a^2)^(1/2))*(1/x^2)^(1/2))/(a^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + \frac{2ab}{x} + \frac{b^2}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b/x + b**2/x**2), x)
```

$$3.395 \quad \int \frac{1}{c + \frac{a}{x^4} + \frac{b}{x^2}} dx$$

Optimal. Leaf size=179

$$-\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Rubi [A] time = 0.29, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1340, 1122, 1166, 205}

$$-\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[(c + a/x^4 + b/x^2)^(-1),x]

[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1122

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{c + \frac{a}{x^4} + \frac{b}{x^2}} dx &= \int \frac{x^4}{a + bx^2 + cx^4} dx \\ &= \frac{x}{c} - \frac{\int \frac{a+bx^2}{a+bx^2+cx^4} dx}{c} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2c} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 202, normalized size = 1.13

$$\frac{\left(b\sqrt{b^2-4ac} + 2ac - b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b\sqrt{b^2-4ac} - 2ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^4 + b/x^2)^(-1), x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
```

$c] * x) / \text{Sqrt}[b + \text{Sqrt}[b^2 - 4 * a * c]]] / (\text{Sqrt}[2] * c^{(3/2)} * \text{Sqrt}[b^2 - 4 * a * c] * \text{Sqrt}[b + \text{Sqrt}[b^2 - 4 * a * c]])$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

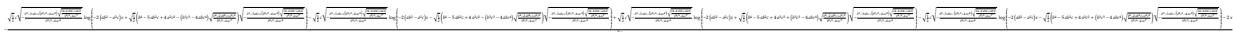
$$\int \frac{1}{c + \frac{a}{x^4} + \frac{b}{x^2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + a/x^4 + b/x^2)^(-1), x]

[Out] IntegrateAlgebraic[(c + a/x^4 + b/x^2)^(-1), x]

fricas [B] time = 0.96, size = 1059, normalized size = 5.92



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/2 * (\text{sqrt}(1/2) * c * \text{sqrt}(-(b^3 - 3 * a * b * c + (b^2 * c^3 - 4 * a * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) / (b^2 * c^3 - 4 * a * c^4)) * \log(-2 * (a * b^2 - a^2 * c) * x + \text{sqrt}(1/2) * (b^4 - 5 * a * b^2 * c + 4 * a^2 * c^2 - (b^3 * c^3 - 4 * a * b * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) * \text{sqrt}(-(b^3 - 3 * a * b * c + (b^2 * c^3 - 4 * a * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) / (b^2 * c^3 - 4 * a * c^4)) - \text{sqrt}(1/2) * c * \text{sqrt}(-(b^3 - 3 * a * b * c + (b^2 * c^3 - 4 * a * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) / (b^2 * c^3 - 4 * a * c^4)) * \log(-2 * (a * b^2 - a^2 * c) * x - \text{sqrt}(1/2) * (b^4 - 5 * a * b^2 * c + 4 * a^2 * c^2 - (b^3 * c^3 - 4 * a * b * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) * \text{sqrt}(-(b^3 - 3 * a * b * c + (b^2 * c^3 - 4 * a * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) / (b^2 * c^3 - 4 * a * c^4)) + \text{sqrt}(1/2) * c * \text{sqrt}(-(b^3 - 3 * a * b * c - (b^2 * c^3 - 4 * a * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) / (b^2 * c^3 - 4 * a * c^4)) * \log(-2 * (a * b^2 - a^2 * c) * x + \text{sqrt}(1/2) * (b^4 - 5 * a * b^2 * c + 4 * a^2 * c^2 + (b^3 * c^3 - 4 * a * b * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) * \text{sqrt}(-(b^3 - 3 * a * b * c - (b^2 * c^3 - 4 * a * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) / (b^2 * c^3 - 4 * a * c^4)) - \text{sqrt}(1/2) * c * \text{sqrt}(-(b^3 - 3 * a * b * c - (b^2 * c^3 - 4 * a * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) / (b^2 * c^3 - 4 * a * c^4)) * \log(-2 * (a * b^2 - a^2 * c) * x - \text{sqrt}(1/2) * (b^4 - 5 * a * b^2 * c + 4 * a^2 * c^2 + (b^3 * c^3 - 4 * a * b * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) * \text{sqrt}(-(b^3 - 3 * a * b * c - (b^2 * c^3 - 4 * a * c^4) * \text{sqrt}((b^4 - 2 * a * b^2 * c + a^2 * c^2) / (b^2 * c^6 - 4 * a * c^7)))) / (b^2 * c^3 - 4 * a * c^4)) - 2 * x) / c \end{aligned}$$

giac [B] time = 1.66, size = 2109, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^
```


$$2 + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^2 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^3 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^4 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*\text{abs}(c))*\arctan(2*\sqrt{1/2}*x/\sqrt{(b*c - \sqrt{b^2*c^2 - 4*a*c^3})/c^2}))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)$$

maple [B] time = 0.01, size = 343, normalized size = 1.92

$$\frac{\sqrt{2} a \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{\sqrt{-4ac+b^2} \sqrt{(-b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2} a \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{\sqrt{-4ac+b^2} \sqrt{(b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} b^2 \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2} \sqrt{(-b+\sqrt{-4ac+b^2})c} c} - \frac{\sqrt{2} b^2 \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2} \sqrt{(b+\sqrt{-4ac+b^2})c} c} + \frac{\sqrt{2} b \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{(-b+\sqrt{-4ac+b^2})c} c} - \frac{\sqrt{2} b \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{(b+\sqrt{-4ac+b^2})c} c} + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^4+b/x^2),x)

[Out] $\frac{1}{c}x + \frac{1}{2}c^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} c * x) * b + 1 / (-4ac + b^2)^{1/2} * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} c * x) * a - 1/2 / (-4ac + b^2)^{1/2} / c * 2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2})c)^{1/2} c * x) * b - 1/2 / c * 2^{1/2} / ((b + (-4ac + b^2)^{1/2})c)^{1/2} \operatorname{arctan}(2^{1/2} / ((b + (-4ac + b^2)^{1/2})c)^{1/2} c * x) * b + 1 / (-4ac + b^2)^{1/2} * 2^{1/2} / ((b + (-4ac + b^2)^{1/2})c)^{1/2} \operatorname{arctan}(2^{1/2} / ((b + (-4ac + b^2)^{1/2})c)^{1/2} c * x) * a - 1/2 / (-4ac + b^2)^{1/2} / c * 2^{1/2} / ((b + (-4ac + b^2)^{1/2})c)^{1/2} \operatorname{arctan}(2^{1/2} / ((b + (-4ac + b^2)^{1/2})c)^{1/2} c * x) * b^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="maxima")

[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c

mupad [B] time = 2.08, size = 3026, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c + a/x^4 + b/x^2),x)

$$3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)}/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)}/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)} + (2*a^2*b)/c))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^{(1/2)}*2i$$

sympy [A] time = 2.12, size = 129, normalized size = 0.72

$$\text{RootSum}\left(t^4(256a^2c^5 - 128ab^2c^4 + 16b^4c^3) + t^2(48a^2bc^2 - 28ab^3c + 4b^5) + a^3, \left(t \mapsto t \log\left(x + \frac{32t^3abc^4 - 8t^3b^3c^3 - 4ta^2c^2 + 8tab^2c - 2tb^4}{a^2c - ab^2}\right)\right)\right) + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**4+b/x**2),x)

[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)/(a**2*c - a*b**2)))) + x/c

$$3.396 \quad \int \frac{1}{c + \frac{a}{x^6} + \frac{b}{x^3}} dx$$

Optimal. Leaf size=631

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2-4ac}\right)^{2/3}}$$

Rubi [A] time = 1.17, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1340, 1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(\frac{b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2-4ac}} + \left(b - \sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[3]{b - \sqrt{b^2-4ac}}}{\sqrt[3]{2} \sqrt[3]{c} x}\right)}{\sqrt[3]{2} \sqrt[3]{c} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt[3]{b - \sqrt{b^2-4ac}}}{\sqrt[3]{2} \sqrt[3]{c} x}\right)}{\sqrt[3]{2} \sqrt[3]{c} \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{2}{c}$$

Antiderivative was successfully verified.

[In] Int[(c + a/x^6 + b/x^3)^(-1), x]

[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1367

```
Int[((d_.)*(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(
p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
```

&& IntegerQ[p]

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{c + \frac{a}{x^6} + \frac{b}{x^3}} dx &= \int \frac{x^6}{a + bx^3 + cx^6} dx \\
&= \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6} dx}{c} \\
&= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2c} \\
&= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\sqrt[3]{2}c\left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}} - \sqrt[3]{c}x}{\left(b-\sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x + c^{2/3}x^2} dx}{3\sqrt[3]{2}c\left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
&= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}}\right)}{3\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)} \\
&= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}}\right)}{3\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)} \\
&= \frac{x}{c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1^3b \log(x-\#1) + a \log(x-\#1)}{2\#1^5c + \#1^2b}\right]}{3c}$$

Antiderivative was successfully verified.

[In] Integrate[(c + a/x^6 + b/x^3)^(-1), x]

[Out] $x/c - \text{RootSum}[a + b\#1^3 + c\#1^6 \& , (a*\text{Log}[x - \#1] + b*\text{Log}[x - \#1]\#1^3)/ (b\#1^2 + 2*c\#1^5) \&]/(3*c)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c + \frac{a}{x^6} + \frac{b}{x^3}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + a/x^6 + b/x^3)^(-1),x]

[Out] IntegrateAlgebraic[(c + a/x^6 + b/x^3)^(-1), x]

fricas [B] time = 3.18, size = 5260, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="fricas")

[Out] $\frac{1}{6} * (4 * \sqrt{3}) * (1/2)^{(1/3)} * c * (- (b^3 - 2 * a * b * c + (b^2 * c^4 - 4 * a * c^5) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (b^6 * c^8 - 12 * a * b^4 * c^9 + 48 * a^2 * b^2 * c^{10} - 64 * a^3 * c^{11})) / (b^2 * c^4 - 4 * a * c^5))^{(1/3)} * \arctan(-1/6 * (2 * (1/2)^{(2/3)} * (\sqrt{3}) * (b^8 * c^4 - 13 * a * b^6 * c^5 + 60 * a^2 * b^4 * c^6 - 112 * a^3 * b^2 * c^7 + 64 * a^4 * c^8)) * x * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (b^6 * c^8 - 12 * a * b^4 * c^9 + 48 * a^2 * b^2 * c^{10} - 64 * a^3 * c^{11})) - \sqrt{3} * (b^9 - 11 * a * b^7 * c + 42 * a^2 * b^5 * c^2 - 62 * a^3 * b^3 * c^3 + 24 * a^4 * b * c^4) * x) * (- (b^3 - 2 * a * b * c + (b^2 * c^4 - 4 * a * c^5) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (b^6 * c^8 - 12 * a * b^4 * c^9 + 48 * a^2 * b^2 * c^{10} - 64 * a^3 * c^{11})) / (b^2 * c^4 - 4 * a * c^5))^{(2/3)} - (1/2)^{(1/6)} * (\sqrt{3}) * (b^8 * c^4 - 13 * a * b^6 * c^5 + 60 * a^2 * b^4 * c^6 - 112 * a^3 * b^2 * c^7 + 64 * a^4 * c^8) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (b^6 * c^8 - 12 * a * b^4 * c^9 + 48 * a^2 * b^2 * c^{10} - 64 * a^3 * c^{11})) - \sqrt{3} * (b^9 - 11 * a * b^7 * c + 42 * a^2 * b^5 * c^2 - 62 * a^3 * b^3 * c^3 + 24 * a^4 * b * c^4) * (- (b^3 - 2 * a * b * c + (b^2 * c^4 - 4 * a * c^5) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (b^6 * c^8 - 12 * a * b^4 * c^9 + 48 * a^2 * b^2 * c^{10} - 64 * a^3 * c^{11})) / (b^2 * c^4 - 4 * a * c^5))^{(2/3)} * \sqrt{(2 * (a^2 * b^4 - 4 * a^3 * b^2 * c + 2 * a^4 * c^2)) * x^2 + (1/2)^{(2/3)} * (b^8 - 10 * a * b^6 * c + 34 * a^2 * b^4 * c^2 - 44 * a^3 * b^2 * c^3 + 16 * a^4 * c^4 - (b^7 * c^4 - 12 * a * b^5 * c^5 + 48 * a^2 * b^3 * c^6 - 64 * a^3 * b * c^7) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (b^6 * c^8 - 12 * a * b^4 * c^9 + 48 * a^2 * b^2 * c^{10} - 64 * a^3 * c^{11})) * (- (b^3 - 2 * a * b * c + (b^2 * c^4 - 4 * a * c^5) * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (b^6 * c^8 - 12 * a * b^4 * c^9 + 48 * a^2 * b^2 * c^{10} - 64 * a^3 * c^{11})) / (b^2 * c^4 - 4 * a * c^5))^{(2/3)} + (1/2)^{(1/3)} * ((a * b^5 * c^4 - 8 * a^2 * b^3 * c^5 + 16 * a^3 * b * c^6) * x * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)}) / (b^6 * c^8 - 12 * a * b^4 * c^9 + 48 * a^2 * b^2 * c^{10} - 64 * a^3 * c^{11}))$

$$\begin{aligned}
& ^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^6 - 8*a^2*b^4*c \\
& + 18*a^3*b^2*c^2 - 8*a^4*c^3)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*\text{sqrt} \\
& \text{t}((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 \\
& - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^{(1/3 \\
&))/(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)) + 2*\text{sqrt}(3)*(a^3*b^4 - 4*a^4*b^2*c \\
& + 2*a^5*c^2))/(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)) - 4*\text{sqrt}(3)*(1/2)^{(1/3)* \\
& c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*\text{sqrt}((b^8 - 8*a*b^6*c + 20*a^2*b^4 \\
& *c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1 \\
& 0 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^{(1/3)*\arctan(-1/6*(2*(1/2)^{(2/3)*(s \\
& \text{qrt}(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8 \\
& c^8)*x*\text{sqrt}((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4) \\
& / (b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) + \text{sqrt}(3)*(b^9 - \\
& 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)*x)*(-(b^3 - 2 \\
& *a*b*c - (b^2*c^4 - 4*a*c^5)*\text{sqrt}((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^ \\
& 3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c \\
& ^11)))/(b^2*c^4 - 4*a*c^5))^{(2/3)} - (1/2)^{(1/6)*(\text{sqrt}(3)*(b^8*c^4 - 13*a*b^ \\
& 6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*\text{sqrt}((b^8 - 8*a*b^6* \\
& c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + \\
& 48*a^2*b^2*c^10 - 64*a^3*c^11)) + \text{sqrt}(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^ \\
& 2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)* \\
& \text{sqrt}((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c \\
& ^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^{(\\
& 2/3)*\text{sqrt}((2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^{(2/3)*(b^8 - 1 \\
& 0*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 + (b^7*c^4 - 12*a \\
& b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*\text{sqrt}((b^8 - 8*a*b^6*c + 20*a^2*b^4 \\
& *c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1 \\
& 0 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*\text{sqrt}((b^8 - 8*a*b \\
& ^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 \\
& + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^{(2/3)} - (1/2)^{(1/3 \\
&)*((a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*x*\text{sqrt}((b^8 - 8*a*b^6*c + 20* \\
& a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2* \\
& b^2*c^10 - 64*a^3*c^11)) + (a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^ \\
& 3)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*\text{sqrt}((b^8 - 8*a*b^6*c + 20*a^2 \\
& *b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2 \\
& *c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^{(1/3)))/(a^2*b^4 - 4*a^3*b^2*c + \\
& 2*a^4*c^2)) - 2*\text{sqrt}(3)*(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2))/(a^3*b^4 - 4* \\
& a^4*b^2*c + 2*a^5*c^2)) - (1/2)^{(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c \\
& ^5)*\text{sqrt}((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b \\
& ^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5 \\
&))^{(1/3)*\log(2*(a^2*b^4 - 4*a^3*b^2*c + 2*a^4*c^2)*x^2 + (1/2)^{(2/3)*(b^8 - \\
& 10*a*b^6*c + 34*a^2*b^4*c^2 - 44*a^3*b^2*c^3 + 16*a^4*c^4 - (b^7*c^4 - 12* \\
& a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*\text{sqrt}((b^8 - 8*a*b^6*c + 20*a^2*b \\
& ^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c \\
& ^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*\text{sqrt}((b^8 - 8*a \\
& *b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
\end{aligned}$$

$$\begin{aligned}
& \left(b^9 + 48a^2b^2c^{10} - 64a^3c^{11} \right) / \left(b^2c^4 - 4a^2c^5 \right)^{2/3} + \left(1/2 \right)^{1/3} \\
& \left((ab^5c^4 - 8a^2b^3c^5 + 16a^3b^2c^6) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11}) \right) \\
& - (ab^6 - 8a^2b^4c + 18a^3b^2c^2 - 8a^4c^3) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11}) \\
& \left. \right) / \left(b^2c^4 - 4a^2c^5 \right)^{1/3} - \left(1/2 \right)^{1/3} c \left(- (b^3 - 2ab^2c - (b^2c^4 - 4a^2c^5) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})) \right) \\
& \left. \right) / \left(b^2c^4 - 4a^2c^5 \right)^{1/3} \log \left(2(a^2b^4 - 4a^3b^2c + 2a^4c^2) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11}) \right) \\
& + (1/2)^{2/3} (b^8 - 10a^2b^6c + 34a^2b^4c^2 - 44a^3b^2c^3 + 16a^4c^4 + (b^7c^4 - 12a^2b^5c^5 + 48a^2b^3c^6 - 64a^3b^2c^7) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})) \\
& \left. \right) \left(- (b^3 - 2ab^2c - (b^2c^4 - 4a^2c^5) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})) \right) \\
& \left. \right) / \left(b^2c^4 - 4a^2c^5 \right)^{2/3} - \left(1/2 \right)^{1/3} \left((ab^5c^4 - 8a^2b^3c^5 + 16a^3b^2c^6) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11}) \right) \\
& + (ab^6 - 8a^2b^4c + 18a^3b^2c^2 - 8a^4c^3) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11}) \\
& \left. \right) / \left(b^2c^4 - 4a^2c^5 \right)^{1/3} + 2 \left(1/2 \right)^{1/3} c \left(- (b^3 - 2ab^2c + (b^2c^4 - 4a^2c^5) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})) \right) \\
& \left. \right) / \left(b^2c^4 - 4a^2c^5 \right)^{1/3} \log \left(2(a^2b^4 - 4a^3b^2c + 2a^4c^2) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11}) \right) \\
& + (1/2)^{1/3} (b^6 - 8a^2b^4c + 18a^2b^2c^2 - 8a^3c^3 - (b^5c^4 - 8a^2b^3c^5 + 16a^2b^2c^6) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})) \\
& \left. \right) \left(- (b^3 - 2ab^2c + (b^2c^4 - 4a^2c^5) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})) \right) \\
& \left. \right) / \left(b^2c^4 - 4a^2c^5 \right)^{1/3} + 2 \left(1/2 \right)^{1/3} c \left(- (b^3 - 2ab^2c - (b^2c^4 - 4a^2c^5) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})) \right) \\
& \left. \right) / \left(b^2c^4 - 4a^2c^5 \right)^{1/3} \log \left(2(a^2b^4 - 4a^3b^2c + 2a^4c^2) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11}) \right) \\
& + (1/2)^{1/3} (b^6 - 8a^2b^4c + 18a^2b^2c^2 - 8a^3c^3 + (b^5c^4 - 8a^2b^3c^5 + 16a^2b^2c^6) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})) \\
& \left. \right) \left(- (b^3 - 2ab^2c - (b^2c^4 - 4a^2c^5) \sqrt{(b^8 - 8a^2b^6c + 20a^3b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)} / (b^6c^8 - 12a^2b^4c^9 + 48a^2b^2c^{10} - 64a^3c^{11})) \right) \\
& \left. \right) / \left(b^2c^4 - 4a^2c^5 \right)^{1/3} + 6x/c
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c + \frac{b}{x^3} + \frac{a}{x^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="giac")

[Out] integrate(1/(c + b/x^3 + a/x^6), x)

maple [C] time = 0.01, size = 59, normalized size = 0.09

$$\frac{x}{c} + \frac{\left(-\text{RootOf}\left(-Z^6c + Z^3b + a\right)^3 b - a\right) \ln\left(-\text{RootOf}\left(-Z^6c + Z^3b + a\right) + x\right)}{3c \left(2 \text{RootOf}\left(-Z^6c + Z^3b + a\right)^5 c + \text{RootOf}\left(-Z^6c + Z^3b + a\right)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^6+b/x^3),x)

[Out] 1/c*x+1/3/c*sum((-R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(-R+x),_R=RootOf(-Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{c} - \frac{\int \frac{bx^3+a}{cx^6+bx^3+a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="maxima")

[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c

mupad [B] time = 4.54, size = 2280, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c + a/x^6 + b/x^3),x)

[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)))/(c^4*(4*a*c - b^2)^3))^(1/3)

$$\begin{aligned}
& \left(\frac{1}{2} + b^4 + 16a^2c^2 - 8ab^2c \right) / (4c(4ac - b^2)) * \left(-(b^4 * (-4ac - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3 \right)^{(1/2)} \\
& + 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (54 * (64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} + x/c + \log((3a^2 * x * (b^4 + 2a^2c^2 - 4ab^2c)) / c + (3 * 2^{(2/3)} * a * ((b^7 + b^4 * (-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (c^4 * (4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b * (-4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c) / (4c(4ac - b^2)) * ((b^7 + b^4 * (-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (54 * (64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} + \log((3a^2 * x * (b^4 + 2a^2c^2 - 4ab^2c)) / c + (3 * 2^{(2/3)} * a * (3^{(1/2)} * i - 1) * ((b^7 + b^4 * (-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (c^4 * (4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b * (-4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c) / (8c(4ac - b^2)) * ((3^{(1/2)} * i) / 2 - 1/2) * ((b^7 + b^4 * (-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (54 * (64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} - \log((3a^2 * x * (b^4 + 2a^2c^2 - 4ab^2c)) / c - (3 * 2^{(2/3)} * a * (3^{(1/2)} * i + 1) * ((b^7 + b^4 * (-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (c^4 * (4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b * (-4ac - b^2)^3)^{(1/2)} - b^4 - 16a^2c^2 + 8ab^2c) / (8c(4ac - b^2)) * ((3^{(1/2)} * i) / 2 + 1/2) * ((b^7 + b^4 * (-4ac - b^2)^3)^{(1/2)} - 32a^3b^3c^3 + 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} - 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (54 * (64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} + \log((3a^2 * x * (b^4 + 2a^2c^2 - 4ab^2c)) / c - (3 * 2^{(2/3)} * a * (3^{(1/2)} * i - 1) * (-b^4 * (-4ac - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} + 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (c^4 * (4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b * (-4ac - b^2)^3)^{(1/2)} + b^4 + 16a^2c^2 - 8ab^2c) / (8c(4ac - b^2)) * ((3^{(1/2)} * i) / 2 - 1/2) * (-b^4 * (-4ac - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} + 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (54 * (64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)} - \log((3a^2 * x * (b^4 + 2a^2c^2 - 4ab^2c)) / c + (3 * 2^{(2/3)} * a * (3^{(1/2)} * i + 1) * (-b^4 * (-4ac - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} + 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (c^4 * (4ac - b^2)^3))^{(1/3)} * (b^4 + 2a^2c^2 - 4ab^2c) * (b * (-4ac - b^2)^3)^{(1/2)} + b^4 + 16a^2c^2 - 8ab^2c) / (8c(4ac - b^2)) * ((3^{(1/2)} * i) / 2 + 1/2) * (-b^4 * (-4ac - b^2)^3)^{(1/2)} - b^7 + 32a^3b^3c^3 - 32a^2b^3c^2 + 2a^2c^2 * (-4ac - b^2)^3)^{(1/2)} + 10ab^5c - 4ab^2c * (-4ac - b^2)^3)^{(1/2)} / (54 * (64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6))^{(1/3)}
\end{aligned}$$

1/3)

sympy [A] time = 6.97, size = 196, normalized size = 0.31

$$\text{RootSum}\left(i^6(46656a^3c^7 - 34992a^2b^2c^6 + 8748ab^4c^5 - 729b^6c^4) + i^3(864a^2bc^3 - 864a^2b^3c^2 + 270ab^5c - 27b^7) + a^4\left(t \mapsto t \log\left(x + \frac{1296t^4a^2bc^6 - 648t^4ab^3c^5 + 81t^4b^5c^4 - 12ta^3c^3 + 39ta^2b^2c^2 - 21tab^4c + 3tb^6}{2a^3c^2 - 4a^2b^2c + ab^4}\right)\right)\right) + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**6+b/x**3),x)

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
- 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) +
x/c
```

$$3.397 \quad \int \frac{1}{c + \frac{a}{x^8} + \frac{b}{x^4}} dx$$

Optimal. Leaf size=376

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) + \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) + \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) + \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac}-b\right)^{3/4} + 2\sqrt[4]{2} c^{5/4} \left(\sqrt{b^2-4ac}-b\right)^{3/4} + 2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac}-b\right)^{3/4} + 2\sqrt[4]{2} c^{5/4} \left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Rubi [A] time = 0.67, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1340, 1367, 1422, 212, 208, 205}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) + \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) + \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) + \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) + \frac{x}{c}}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac}-b\right)^{3/4} + 2\sqrt[4]{2} c^{5/4} \left(\sqrt{b^2-4ac}-b\right)^{3/4} + 2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac}-b\right)^{3/4} + 2\sqrt[4]{2} c^{5/4} \left(\sqrt{b^2-4ac}-b\right)^{3/4} + \frac{x}{c}}$$

Antiderivative was successfully verified.

[In] Int[(c + a/x^8 + b/x^4)^(-1), x]

[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x]/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x]/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x]/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x]/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],

$x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{!GtQ}[a/b, 0]$

Rule 1340

$\text{Int}[(a_ + (c_)*(x_)^{(n2_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Int}[x^{(2*n*p)}*(c + b/x^n + a/x^{(2*n)})^p, x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{LtQ}[n, 0] \&\& \text{IntegerQ}[p]$

Rule 1367

$\text{Int}[(d_)*(x_)^{(m_)}*((a_ + (c_)*(x_)^{(n2_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(d^{(2*n - 1)}*(d*x)^{(m - 2*n + 1)}*(a + b*x^n + c*x^{(2*n)})^{(p + 1)})/(c*(m + 2*n*p + 1)), x] - \text{Dist}[d^{(2*n)}/(c*(m + 2*n*p + 1)), \text{Int}[(d*x)^{(m - 2*n)}*\text{Simp}[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^{(2*n)})^p, x], x] /; \text{FreeQ}\{a, b, c, d, p\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, 2*n - 1] \&\& \text{NeQ}[m + 2*n*p + 1, 0] \&\& \text{IntegerQ}[p]$

Rule 1422

$\text{Int}[(d_ + (e_)*(x_)^{(n_)})/((a_ + (b_)*(x_)^{(n_) + (c_)*(x_)^{(n2_)})}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[e/2 + (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 - q/2 + c*x^n), x], x] + \text{Dist}[e/2 - (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 + q/2 + c*x^n), x], x]] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& (\text{PosQ}[b^2 - 4*a*c] || \text{!IGtQ}[n/2, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{1}{c + \frac{a}{x^8} + \frac{b}{x^4}} dx &= \int \frac{x^8}{a + bx^4 + cx^8} dx \\
&= \frac{x}{c} - \frac{\int \frac{a+bx^4}{a+bx^4+cx^8} dx}{c} \\
&= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2c} \\
&= \frac{x}{c} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \dots \\
&= \frac{x}{c} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}c^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.19

$$\frac{x}{c} - \frac{\text{RootSum}\left[\#1^8 c + \#1^4 b + a \&, \frac{\#1^4 b \log(x-\#1) + a \log(x-\#1)}{2\#1^7 c + \#1^3 b} \&\right]}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[(c + a/x^8 + b/x^4)^(-1), x]

[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 &, (a*Log[x - #1] + b*Log[x - #1]*#1^4)/ (b*#1^3 + 2*c*#1^7) &]/(4*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c + \frac{a}{x^8} + \frac{b}{x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + a/x^8 + b/x^4)^(-1), x]

[Out] IntegrateAlgebraic[(c + a/x^8 + b/x^4)^(-1), x]

fricas [B] time = 2.63, size = 5082, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="fricas")

[Out]
$$-1/4*(4*c*\sqrt{\sqrt{1/2}}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}})/((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*\arctan(1/4*(2*\sqrt{1/2}*((b^{10}*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^{10})*x*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}) + (b^{11} - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*x)*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}})/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) - (b^{11} - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (b^{10}*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^{10})*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}))*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}})/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2))*x^2 + \sqrt{1/2}*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 + (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}))*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}})/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/((a^2*b^4 - 3*a^3*b^2*c + a^4*c^2))*\sqrt{\sqrt{1/2}}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}})/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/((a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)) - 4*c*\sqrt{\sqrt{1/2}}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}})/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*\arctan(1/4*(2*\sqrt{1/2}*((b^{10}*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^{10})*x*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}) - (b^{11} - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*x)*\sqrt{\sqrt{1/2}}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})}})/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/((a^4*b^4 - 3*a^5*b^2*c + a^6*c^2))$$

$$\begin{aligned}
& \frac{2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \frac{1}{(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \frac{1}{(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)} + \frac{(b^{11} - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (b^{10}*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b^2*c^9 - 128*a^5*c^{10}) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \sqrt{\sqrt{1/2} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})}} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \frac{1}{(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \frac{1}{(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)} \cdot \sqrt{((2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x^2 + \sqrt{1/2}*(b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^3*b^2*c^3 + 8*a^4*c^4 - (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)}))} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})} \\
& \frac{1}{(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)} \cdot \frac{1}{(a^4*b^4 - 3*a^5*b^2*c + a^6*c^2)} - c \cdot \sqrt{\sqrt{1/2} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})}} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \frac{1}{(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)} \cdot \log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \sqrt{\sqrt{1/2} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})}} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \frac{1}{(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \frac{1}{(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)} \cdot \log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \sqrt{\sqrt{1/2} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})}} \\
& \frac{1}{(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})} \cdot \frac{1}{(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})} \\
& - c \cdot \sqrt{\sqrt{1/2} \cdot \sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7) \cdot \sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)})}}
\end{aligned}$$

$$t(1/2)*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*\log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x + 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*\sqrt{\sqrt{1/2}}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))} + c*\sqrt{\sqrt{1/2}}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*\log((a*b^4 - 3*a^2*b^2*c + a^3*c^2)*x - 1/2*(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*\sqrt{\sqrt{1/2}}*\sqrt{-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))*\sqrt{(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))} - 4*x)/c$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Evaluation time: 6.96Unable to convert to real 1/4 Error: Bad Argument Value

maple [C] time = 0.00, size = 59, normalized size = 0.16

$$\frac{x}{c} + \frac{\left(-\text{RootOf}\left(-Z^8c + Z^4b + a\right)^4 b - a\right) \ln\left(-\text{RootOf}\left(-Z^8c + Z^4b + a\right) + x\right)}{4c\left(2\text{RootOf}\left(-Z^8c + Z^4b + a\right)^7 c + \text{RootOf}\left(-Z^8c + Z^4b + a\right)^3 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+a/x^8+b/x^4),x)

[Out] 1/c*x+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(-Z^8*c+_Z^4*b+a))

$$\begin{aligned}
& 12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8) \\
&))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)/c)*(-(b^9 \\
& + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 \\
& + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c \\
& - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 \\
& - 256*a^3*b^2*c^8)))^{(1/4)} - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c) \\
& *(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12 \\
& 0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(\\
& -(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^ \\
& 2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b \\
& ^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80 \\
& *a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^ \\
& (1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 \\
& + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3/4)}*(4096* \\
& a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - \\
& b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2* \\
& -(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}) \\
& / (512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2* \\
& c^8)))^{(1/4)} + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(- \\
& (4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + \\
& a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5 \\
&)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256* \\
& a^3*b^2*c^8)))^{(1/4)})))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 \\
& + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13 \\
& *a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 \\
& - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*2i + atan((((16 \\
& *(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^ \\
& 4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^ \\
& 3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^ \\
& 2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - \\
& 256*a^3*b^2*c^8)))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^ \\
& ^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^ \\
& ^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a* \\
& b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 \\
& + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4 \\
& *a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a \\
& ^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7* \\
& c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a* \\
& b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i - (((16*(a^3*b^6 - 4 \\
& *a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 - b^4*(-(4*a*c - \\
& b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(\\
& -(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/ \\
& (512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^ \\
& ^8)))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^ \\
& 9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*
\end{aligned}$$

$$\begin{aligned}
& b^2c)) / c) * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - b^2)^5)^{1/2} - 13ab^7c - 3 \\
& * ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{1/4} - (((16 * (a^3b^6 - 4a^6c^3 \\
& - 7a^4b^4c + 13a^5b^2c^2)) / c + (x * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - \\
& b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{3/4} \\
&) * (4096a^5b^3c^6 + 256a^3b^5c^4 - 2048a^4b^3c^5) * 4i) / c) * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 \\
& + a^2c^2 * (- (4ac - b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 2 \\
& 56a^3b^2c^8)))^{1/4} * 1i - (4 * x * (a^4b^4 + 2a^6c^2 - 4a^5b^2c)) / c) * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - \\
& b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{1/4} / (((16 * (a^3b^6 - 4a^6c^3 - 7a^4b^4 \\
& * c + 13a^5b^2c^2)) / c - (x * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - b^2)^5)^{1/2} \\
&) - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{3/4} * (4096a^5b^3c^6 + 256a^3b^5c^4 - 2048a^4b^3c^5) * 4i) / c) * (- (b^9 + b^4 * (- (4ac - \\
& b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / \\
& (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{1/4} * 1i + (4 * x * (a^4b^4 + 2a^6c^2 - 4a^5b^2c)) / c) * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 \\
& + a^2c^2 * (- (4ac - b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 25 \\
& 6a^3b^2c^8)))^{1/4} * 1i + (((16 * (a^3b^6 - 4a^6c^3 - 7a^4b^4c + 13a^5b^2c^2)) / c + (x * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + \\
& 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 1 \\
& 6ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{3/4} * (4096a^5b^3c^6 + 2 \\
& 56a^3b^5c^4 - 2048a^4b^3c^5) * 4i) / c) * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - \\
& b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{1/4} * 1i - (4 * x * (a^4b^4 + 2a^6c^2 - 4a^5b^2c)) / c) * (- (b^9 + b^4 * (- (4ac - \\
& b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 1 \\
& 6ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{3/4} * (4096a^5b^3c^6 + 2 \\
& 56a^3b^5c^4 - 2048a^4b^3c^5) * 4i) / c) * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + 61a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - \\
& b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{1/4} * 1i) * (- (b^9 + b^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4 + 6 \\
& 1a^2b^5c^2 - 120a^3b^3c^3 + a^2c^2 * (- (4ac - b^2)^5)^{1/2} - 13ab^7c - 3ab^2c * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^4c^9 + b^8c^5 - 16
\end{aligned}$$

$$\begin{aligned}
& *a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} - 2*\operatorname{atan}((((16*(a^3 \\
& *b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4* \\
& a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2 \\
& *c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(\\
& 1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3 \\
& *b^2*c^8)))^{(3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i) \\
& /c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - \\
& 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c \\
& *(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96 \\
& *a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4* \\
& a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^ \\
& 2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c \\
& + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b \\
& ^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} - (((16*(a^3*b^6 - 4*a^6 \\
& *c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a* \\
& c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(\\
& 256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(\\
& 3/4)}*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - \\
& b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3 \\
& *c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - \\
& b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 \\
& - 256*a^3*b^2*c^8)))^{(1/4)}*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/ \\
& c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - \\
& 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c \\
& *(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96* \\
& a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)})/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4 \\
& *b^4*c + 13*a^5*b^2*c^2))/c - (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80 \\
& *a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^ \\
& (1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 \\
& + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3/4)}*(4096* \\
& a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a* \\
& c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c \\
& ^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/ \\
& 2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b \\
& ^2*c^8)))^{(1/4)}*1i + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - \\
& b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3* \\
& c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - \\
& b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 \\
& - 256*a^3*b^2*c^8)))^{(1/4)}*1i + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + \\
& 13*a^5*b^2*c^2))/c + (x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^ \\
& 4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 1 \\
& 3*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)}))/(512*(256*a^4*c^9 + b^8*c^5 \\
& - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3/4)}*(4096*a^5*b*c^6 \\
& + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5)*4i)/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^
\end{aligned}$$

$$\begin{aligned}
& 5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512* \\
& (256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))) \\
& ^{(1/4)}*1i - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4* \\
& a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2 \\
& *c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(\\
& 1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3 \\
& *b^2*c^8)))^{(1/4)}*1i))*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4 \\
& + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^{(1/2)} - 13 \\
& *a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 \\
& - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} + x/c
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x**8+b/x**4),x)

[Out] Timed out

$$3.398 \quad \int \frac{\sqrt{a+b\sqrt{x}+cx}}{x} dx$$

Optimal. Leaf size=106

$$2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a} \tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b \tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1357, 734, 843, 621, 206, 724}

$$2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a} \tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b \tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sqrt[x] + c*x]/x,x]

[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqrt[a + b*Sqrt[x] + c*x])])/Sqrt[c]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,

$d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

Rule 734

$\text{Int}[(d + e*x)^m * (a + b*x + c*x^2)^p, x_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1} * (a + b*x + c*x^2)^p / (e*(m + 2*p + 1)), x] - \text{Dist}[p / (e*(m + 2*p + 1)), \text{Int}[(d + e*x)^m * \text{Simp}[b*d - 2*a*e + (2*c*d - b*e)*x, x] * (a + b*x + c*x^2)^{p-1}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, m\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[m + 2*p + 1, 0] \&\& (!\text{RationalQ}[m] \mid\mid \text{LtQ}[m, 1]) \&\& !\text{ILtQ}[m + 2*p, 0] \&\& \text{IntQuadraticQ}[a, b, c, d, e, m, p, x]$

Rule 843

$\text{Int}[(d + e*x)^m * (f + g*x) * (a + b*x + c*x^2)^p, x_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + e*x)^{m+1} * (a + b*x + c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m * (a + b*x + c*x^2)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 1357

$\text{Int}[x^m * (a + c*x^{n2} + b*x^n)^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1) * (a + b*x + c*x^2)^p}, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + b\sqrt{x} + cx}}{x} dx &= 2 \operatorname{Subst} \left(\int \frac{\sqrt{a + bx + cx^2}}{x} dx, x, \sqrt{x} \right) \\
&= 2\sqrt{a + b\sqrt{x} + cx} - \operatorname{Subst} \left(\int \frac{-2a - bx}{x\sqrt{a + bx + cx^2}} dx, x, \sqrt{x} \right) \\
&= 2\sqrt{a + b\sqrt{x} + cx} + (2a) \operatorname{Subst} \left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, \sqrt{x} \right) + b \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, \sqrt{x} \right) \\
&= 2\sqrt{a + b\sqrt{x} + cx} - (4a) \operatorname{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + b\sqrt{x}}{\sqrt{a + b\sqrt{x} + cx}} \right) + (2b) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, \sqrt{x} \right) \\
&= 2\sqrt{a + b\sqrt{x} + cx} - 2\sqrt{a} \tanh^{-1} \left(\frac{2a + b\sqrt{x}}{2\sqrt{a} \sqrt{a + b\sqrt{x} + cx}} \right) + \frac{b \tanh^{-1} \left(\frac{b + 2c\sqrt{x}}{2\sqrt{c} \sqrt{a + b\sqrt{x} + cx}} \right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 1.00

$$2\sqrt{a + b\sqrt{x} + cx} - 2\sqrt{a} \tanh^{-1} \left(\frac{2a + b\sqrt{x}}{2\sqrt{a} \sqrt{a + b\sqrt{x} + cx}} \right) + \frac{b \tanh^{-1} \left(\frac{b + 2c\sqrt{x}}{2\sqrt{c} \sqrt{a + b\sqrt{x} + cx}} \right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sqrt[x] + c*x]/x,x]

[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqrt[a + b*Sqrt[x] + c*x])])/Sqrt[c]

IntegrateAlgebraic [A] time = 0.24, size = 109, normalized size = 1.03

$$2\sqrt{a + b\sqrt{x} + cx} - \frac{b \log \left(-2\sqrt{c} \sqrt{a + b\sqrt{x} + cx} + b + 2c\sqrt{x} \right)}{\sqrt{c}} + 4\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{x}}{\sqrt{a}} - \frac{\sqrt{a + b\sqrt{x} + cx}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[x] + c*x]/x,x]

[Out] $2\sqrt{a + b\sqrt{x} + c*x} + 4\sqrt{a}*\text{ArcTanh}[(\sqrt{c}*\sqrt{x})/\sqrt{a} - \sqrt{a + b\sqrt{x} + c*x}/\sqrt{a}] - (b*\text{Log}[b + 2*c*\sqrt{x} - 2*\sqrt{c}*\sqrt{a + b\sqrt{x} + c*x}])/\sqrt{c}$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")`

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:index.cc index_m operator + Error: Bad Argument Value

maple [A] time = 0.01, size = 84, normalized size = 0.79

$$-2\sqrt{a} \ln\left(\frac{b\sqrt{x} + 2a + 2\sqrt{cx + b\sqrt{x} + a} \sqrt{a}}{\sqrt{x}}\right) + \frac{b \ln\left(\frac{c\sqrt{x} + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx + b\sqrt{x} + a}\right)}{\sqrt{c}} + 2\sqrt{cx + b\sqrt{x} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+c*x+b*x^(1/2))^(1/2)/x,x)`

[Out] $2*(a+c*x+b*x^(1/2))^(1/2)+b*\ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+c*x+b*x^(1/2))^(1/2))/c^(1/2)-2*a^(1/2)*\ln((2*a+b*x^(1/2)+2*a^(1/2)*(a+c*x+b*x^(1/2))^(1/2))/x^(1/2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx + b\sqrt{x} + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(c*x + b*sqrt(x) + a)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + cx + b\sqrt{x}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x + b*x^(1/2))^(1/2)/x,x)

[Out] int((a + c*x + b*x^(1/2))^(1/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{x} + cx}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*x+b*x**(1/2))**(1/2)/x,x)

[Out] Integral(sqrt(a + b*sqrt(x) + c*x)/x, x)

$$3.399 \quad \int \left(\frac{b^2}{4c} + b\sqrt{x} + cx \right)^2 dx$$

Optimal. Leaf size=40

$$\frac{(b + 2c\sqrt{x})^6}{192c^4} - \frac{b(b + 2c\sqrt{x})^5}{160c^4}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 190, 43}

$$\frac{(b + 2c\sqrt{x})^6}{192c^4} - \frac{b(b + 2c\sqrt{x})^5}{160c^4}$$

Antiderivative was successfully verified.

[In] Int[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]

[Out] -(b*(b + 2*c*Sqrt[x])^5)/(160*c^4) + (b + 2*c*Sqrt[x])^6/(192*c^4)

Rule 28

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 43

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned}
\int \left(\frac{b^2}{4c} + b\sqrt{x} + cx \right)^2 dx &= \frac{\int \left(\frac{b}{2} + c\sqrt{x} \right)^4 dx}{c^2} \\
&= \frac{2 \operatorname{Subst} \left(\int x \left(\frac{b}{2} + cx \right)^4 dx, x, \sqrt{x} \right)}{c^2} \\
&= \frac{2 \operatorname{Subst} \left(\int \left(-\frac{b \left(\frac{b}{2} + cx \right)^4}{2c} + \frac{\left(\frac{b}{2} + cx \right)^5}{c} \right) dx, x, \sqrt{x} \right)}{c^2} \\
&= -\frac{b \left(b + 2c\sqrt{x} \right)^5}{160c^4} + \frac{\left(b + 2c\sqrt{x} \right)^6}{192c^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.72

$$-\frac{(b - 10c\sqrt{x})(b + 2c\sqrt{x})^5}{960c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]

[Out] -1/960*((b - 10*c*Sqrt[x])*(b + 2*c*Sqrt[x])^5)/c^4

IntegrateAlgebraic [A] time = 0.03, size = 55, normalized size = 1.38

$$\frac{15b^4x + 80b^3cx^{3/2} + 180b^2c^2x^2 + 192bc^3x^{5/2} + 80c^4x^3}{240c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]

[Out] (15*b^4*x + 80*b^3*c*x^(3/2) + 180*b^2*c^2*x^2 + 192*b*c^3*x^(5/2) + 80*c^4*x^3)/(240*c^2)

fricas [A] time = 1.07, size = 53, normalized size = 1.32

$$\frac{80c^4x^3 + 180b^2c^2x^2 + 15b^4x + 16(12bc^3x^2 + 5b^3cx)\sqrt{x}}{240c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="fricas")

[Out] 1/240*(80*c^4*x^3 + 180*b^2*c^2*x^2 + 15*b^4*x + 16*(12*b*c^3*x^2 + 5*b^3*c*x)*sqrt(x))/c^2

giac [A] time = 0.33, size = 49, normalized size = 1.22

$$\frac{80c^4x^3 + 192bc^3x^{\frac{5}{2}} + 180b^2c^2x^2 + 80b^3cx^{\frac{3}{2}} + 15b^4x}{240c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="giac")

[Out] 1/240*(80*c^4*x^3 + 192*b*c^3*x^(5/2) + 180*b^2*c^2*x^2 + 80*b^3*c*x^(3/2) + 15*b^4*x)/c^2

maple [A] time = 0.00, size = 52, normalized size = 1.30

$$\frac{b^2x^2}{2} + \frac{\left(\frac{8c^2x^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{3}{2}}}{3}\right)b}{2c} + \frac{\left(cx + \frac{b^2}{4c}\right)^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/4*b^2/c+c*x+b*x^(1/2))^2,x)

[Out] 1/2*b^2*x^2+1/2*b/c*(8/5*c^2*x^(5/2)+2/3*x^(3/2)*b^2)+1/3*(1/4*b^2/c+c*x)^3/c

maxima [A] time = 0.87, size = 54, normalized size = 1.35

$$\frac{1}{3}c^2x^3 + \frac{4}{5}bcx^{\frac{5}{2}} + \frac{1}{2}b^2x^2 + \frac{b^4x}{16c^2} + \frac{\left(3cx^2 + 4bx^{\frac{3}{2}}\right)b^2}{12c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4/c*b^2+c*x+b*x^(1/2))^2,x, algorithm="maxima")

[Out] 1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + 1/16*b^4*x/c^2 + 1/12*(3*c*x^2 + 4*b*x^(3/2))*b^2/c

mupad [B] time = 0.04, size = 44, normalized size = 1.10

$$\frac{3b^2x^2}{4} + \frac{c^2x^3}{3} + \frac{b^4x}{16c^2} + \frac{b^3x^{3/2}}{3c} + \frac{4bcx^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x + b*x^(1/2) + b^2/(4*c))^2,x)`

[Out] $(3*b^2*x^2)/4 + (c^2*x^3)/3 + (b^4*x)/(16*c^2) + (b^3*x^{(3/2)})/(3*c) + (4*b*c*x^{(5/2)})/5$

sympy [A] time = 0.33, size = 51, normalized size = 1.28

$$\frac{b^4x}{16c^2} + \frac{b^3x^{\frac{3}{2}}}{3c} + \frac{3b^2x^2}{4} + \frac{4bcx^{\frac{5}{2}}}{5} + \frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/4/c*b**2+c*x+b*x**(1/2))**2,x)`

[Out] $b**4*x/(16*c**2) + b**3*x**(3/2)/(3*c) + 3*b**2*x**2/4 + 4*b*c*x**(5/2)/5 + c**2*x**3/3$

$$3.400 \quad \int \frac{1}{\sqrt{a^2 + 2ab\sqrt{x} + b^2x}} dx$$

Optimal. Leaf size=75

$$\frac{2\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}{b^2} - \frac{2a(a + b\sqrt{x}) \log(a + b\sqrt{x})}{b^2\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}$$

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1341, 640, 608, 31}

$$\frac{2\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}{b^2} - \frac{2a(a + b\sqrt{x}) \log(a + b\sqrt{x})}{b^2\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x], x]

[Out] (2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])/b^2 - (2*a*(a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 608

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[(b/2 + c*x)/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k
= Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{a^2 + 2ab\sqrt{x} + b^2x}} dx &= 2 \operatorname{Subst} \left(\int \frac{x}{\sqrt{a^2 + 2abx + b^2x^2}} dx, x, \sqrt{x} \right) \\
 &= \frac{2\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}{b^2} - \frac{(2a) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a^2 + 2abx + b^2x^2}} dx, x, \sqrt{x} \right)}{b} \\
 &= \frac{2\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}{b^2} - \frac{(2a(a + b\sqrt{x})) \operatorname{Subst} \left(\int \frac{1}{ab + b^2x} dx, x, \sqrt{x} \right)}{\sqrt{a^2 + 2ab\sqrt{x} + b^2x}} \\
 &= \frac{2\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}{b^2} - \frac{2a(a + b\sqrt{x}) \log(a + b\sqrt{x})}{b^2 \sqrt{a^2 + 2ab\sqrt{x} + b^2x}}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.67

$$\frac{2(a + b\sqrt{x})(b\sqrt{x} - a \log(a + b\sqrt{x}))}{b^2 \sqrt{(a + b\sqrt{x})^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x], x]
```

```
[Out] (2*(a + b*Sqrt[x])*(b*Sqrt[x] - a*Log[a + b*Sqrt[x]]))/(b^2*Sqrt[(a + b*Sqr
t[x])^2])
```

IntegrateAlgebraic [B] time = 0.24, size = 151, normalized size = 2.01

$$\frac{\sqrt{a^2 + 2ab\sqrt{x} + b^2x}}{b^2} + \frac{a(\sqrt{b^2} + b) \log\left(\sqrt{a^2 + 2ab\sqrt{x} + b^2x} - a - \sqrt{b^2} \sqrt{x}\right)}{b^3} + \frac{a(\sqrt{b^2} - b) \log\left(\sqrt{a^2 + 2ab\sqrt{x} + b^2x} + a - \sqrt{b^2} \sqrt{x}\right)}{b^3} - \frac{\sqrt{x}}{\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x], x]
```

[Out] $-(\text{Sqrt}[x]/\text{Sqrt}[b^2]) + \text{Sqrt}[a^2 + 2*a*b*\text{Sqrt}[x] + b^2*x]/b^2 + (a*(b + \text{Sqrt}[b^2])*\text{Log}[-a - \text{Sqrt}[b^2]*\text{Sqrt}[x] + \text{Sqrt}[a^2 + 2*a*b*\text{Sqrt}[x] + b^2*x]])/b^3 + (a*(-b + \text{Sqrt}[b^2])*\text{Log}[a - \text{Sqrt}[b^2]*\text{Sqrt}[x] + \text{Sqrt}[a^2 + 2*a*b*\text{Sqrt}[x] + b^2*x]])/b^3$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="fricas")`

[Out] Timed out

giac [A] time = 0.40, size = 45, normalized size = 0.60

$$-\frac{2|a|\log\left(\left|\sqrt{b^2x}\operatorname{sgn}(a)\operatorname{sgn}(b)+|a|\right|\right)}{b^2} + \frac{2\sqrt{b^2x}}{b^2\operatorname{sgn}(a)\operatorname{sgn}(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="giac")`

[Out] $-2*\operatorname{abs}(a)*\log(\operatorname{abs}(\operatorname{sqrt}(b^2*x)*\operatorname{sgn}(a)*\operatorname{sgn}(b)+\operatorname{abs}(a)))/b^2 + 2*\operatorname{sqrt}(b^2*x)/(b^2*\operatorname{sgn}(a)*\operatorname{sgn}(b))$

maple [A] time = 0.01, size = 50, normalized size = 0.67

$$\frac{2\sqrt{b^2x + 2ab\sqrt{x} + a^2} (-a \ln(b\sqrt{x} + a) + b\sqrt{x})}{(b\sqrt{x} + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x)`

[Out] $2*(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)*(b*x^(1/2)-a*\ln(a+b*x^(1/2)))/(a+b*x^(1/2))/b^2$

maxima [A] time = 0.96, size = 23, normalized size = 0.31

$$-\frac{2a \log(b\sqrt{x} + a)}{b^2} + \frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="maxima")

[Out] -2*a*log(b*sqrt(x) + a)/b^2 + 2*sqrt(x)/b

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b^2 x + a^2 + 2 a b \sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2),x)

[Out] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2 + 2ab\sqrt{x} + b^2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+b**2*x+2*a*b*x**(1/2))**(1/2),x)

[Out] Integral(1/sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x), x)

$$3.401 \quad \int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^{7/2} dx$$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^9}{10b^3} - \frac{2a\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^8}{3b^3} + \frac{3a^2\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^7}{8b^3}$$

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$\frac{3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^9}{10b^3} - \frac{2a\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^8}{3b^3} + \frac{3a^2\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^7}{8b^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2), x]

[Out] (3*a^2*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]/(8*b^3) - (2*a*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]/(3*b^3) + (3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]/(10*b^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{7/2} dx &= 3 \text{Subst} \left(\int x^2 (a^2 + 2abx + b^2x^2)^{7/2} dx, x, \sqrt[3]{x} \right) \\
&= \frac{\left(3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} \right) \text{Subst} \left(\int x^2 (ab + b^2x)^7 dx, x, \sqrt[3]{x} \right)}{b^7 (a + b\sqrt[3]{x})} \\
&= \frac{\left(3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} \right) \text{Subst} \left(\int \left(\frac{a^2(ab+b^2x)^7}{b^2} - \frac{2a(ab+b^2x)^8}{b^3} + \frac{(ab+b^2x)^9}{b^4} \right) dx, x, \sqrt[3]{x} \right)}{b^7 (a + b\sqrt[3]{x})} \\
&= \frac{3a^2 (a + b\sqrt[3]{x})^7 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{8b^3} - \frac{2a (a + b\sqrt[3]{x})^8 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{3b^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.41

$$\frac{(a + b\sqrt[3]{x})^7 \sqrt{(a + b\sqrt[3]{x})^2 (a^2 - 8ab\sqrt[3]{x} + 36b^2x^{2/3})}}{120b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2), x]

[Out] ((a + b*x^(1/3))^7*Sqrt[(a + b*x^(1/3))^2*(a^2 - 8*a*b*x^(1/3) + 36*b^2*x^(2/3))])/(120*b^3)

IntegrateAlgebraic [F] time = 2.56, size = 0, normalized size = 0.00

$$\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{7/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2), x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2), x]

fricas [A] time = 1.47, size = 84, normalized size = 0.61

$$\frac{7}{3} ab^6x^3 + \frac{35}{2} a^4b^3x^2 + a^7x + \frac{63}{40} (5a^2b^5x^2 + 8a^5b^2x)x^{\frac{2}{3}} + \frac{3}{20} (2b^7x^3 + 100a^3b^4x^2 + 35a^6bx)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")

[Out] $7/3*a*b^6*x^3 + 35/2*a^4*b^3*x^2 + a^7*x + 63/40*(5*a^2*b^5*x^2 + 8*a^5*b^2*x)*x^{2/3} + 3/20*(2*b^7*x^3 + 100*a^3*b^4*x^2 + 35*a^6*b*x)*x^{1/3}$

giac [A] time = 0.51, size = 140, normalized size = 1.02

$\frac{3}{10} b^7 x^{\frac{10}{3}} \operatorname{sgn}(bx^{\frac{1}{3}} + a) + \frac{7}{3} ab^6 x^3 \operatorname{sgn}(bx^{\frac{1}{3}} + a) + \frac{63}{8} a^2 b^5 x^{\frac{8}{3}} \operatorname{sgn}(bx^{\frac{1}{3}} + a) + 15 a^3 b^4 x^{\frac{7}{3}} \operatorname{sgn}(bx^{\frac{1}{3}} + a) + \frac{35}{2} a^4 b^3 x^2 \operatorname{sgn}(bx^{\frac{1}{3}} + a) + \frac{63}{5} a^5 b^2 x^{\frac{5}{3}} \operatorname{sgn}(bx^{\frac{1}{3}} + a) + \frac{21}{4} a^6 b x^{\frac{4}{3}} \operatorname{sgn}(bx^{\frac{1}{3}} + a) + a^7 x \operatorname{sgn}(bx^{\frac{1}{3}} + a)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")

[Out] $3/10*b^7*x^{10/3}*sgn(b*x^{1/3} + a) + 7/3*a*b^6*x^3*sgn(b*x^{1/3} + a) + 63/8*a^2*b^5*x^{8/3}*sgn(b*x^{1/3} + a) + 15*a^3*b^4*x^{7/3}*sgn(b*x^{1/3} + a) + 35/2*a^4*b^3*x^2*sgn(b*x^{1/3} + a) + 63/5*a^5*b^2*x^{5/3}*sgn(b*x^{1/3} + a) + 21/4*a^6*b*x^{4/3}*sgn(b*x^{1/3} + a) + a^7*x*sgn(b*x^{1/3} + a)$

maple [A] time = 0.02, size = 109, normalized size = 0.80

$$\frac{\sqrt{b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2} \left(36 b^7 x^{\frac{10}{3}} + 280 a b^6 x^3 + 945 a^2 b^5 x^{\frac{8}{3}} + 1800 a^3 b^4 x^{\frac{7}{3}} + 2100 a^4 b^3 x^2 + 1512 a^5 b^2 x^{\frac{5}{3}} + 630 a^6 b x^{\frac{4}{3}} + 120 a^7 x \right)}{120 b x^{\frac{1}{3}} + 120 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x)

[Out] $1/120*(a^2+2*a*b*x^{1/3}+b^2*x^{2/3})^{1/2}*(36*b^7*x^{10/3}+945*a^2*b^5*x^{8/3}+1800*a^3*b^4*x^{7/3}+1512*a^5*b^2*x^{5/3}+630*a^6*b*x^{4/3}+280*a*b^6*x^3+2100*a^4*b^3*x^2+120*a^7*x)/(a+b*x^{1/3})$

maxima [A] time = 0.91, size = 114, normalized size = 0.83

$$\frac{3 \left(b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2 \right)^{\frac{7}{2}} a^2 x^{\frac{1}{3}}}{8 b^2} + \frac{3 \left(b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2 \right)^{\frac{7}{2}} a^3}{8 b^3} + \frac{3 \left(b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2 \right)^{\frac{9}{2}} x^{\frac{1}{3}}}{10 b^2} - \frac{11 \left(b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2 \right)^{\frac{9}{2}} a}{30 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")

[Out] $3/8*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{7/2}*a^2*x^{1/3}/b^2 + 3/8*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{7/2}*a^3/b^3 + 3/10*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{9/2}*x^{1/3}/b^2 - 11/30*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{9/2}*a/b^3$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a^2 + b^2 x^{2/3} + 2 a b x^{1/3})^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2), x)`

[Out] `int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}} \right)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2), x)`

[Out] `Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(7/2), x)`

$$3.402 \quad \int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^{5/2} dx$$

Optimal. Leaf size=137

$$\frac{3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^7}{8b^3} - \frac{6a\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^6}{7b^3} + \frac{a^2\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^5}{2b^3}$$

Rubi [A] time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1341, 646, 43}

$$\frac{3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^7}{8b^3} - \frac{6a\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^6}{7b^3} + \frac{a^2\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^5}{2b^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2), x]

[Out] (a^2*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3) - (6*a*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(7*b^3) + (3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{5/2} dx &= 3 \operatorname{Subst} \left(\int x^2 (a^2 + 2abx + b^2x^2)^{5/2} dx, x, \sqrt[3]{x} \right) \\
&= \frac{\left(3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} \right) \operatorname{Subst} \left(\int x^2 (ab + b^2x)^5 dx, x, \sqrt[3]{x} \right)}{b^5 (a + b\sqrt[3]{x})} \\
&= \frac{\left(3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} \right) \operatorname{Subst} \left(\int \left(\frac{a^2(ab+b^2x)^5}{b^2} - \frac{2a(ab+b^2x)^6}{b^3} + \frac{(ab+b^2x)^7}{b^4} \right) dx, x, \sqrt[3]{x} \right)}{b^5 (a + b\sqrt[3]{x})} \\
&= \frac{a^2 (a + b\sqrt[3]{x})^5 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{2b^3} - \frac{6a (a + b\sqrt[3]{x})^6 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{7b^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.41

$$\frac{(a + b\sqrt[3]{x})^5 \sqrt{(a + b\sqrt[3]{x})^2 (a^2 - 6ab\sqrt[3]{x} + 21b^2x^{2/3})}}{56b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2), x]

[Out] ((a + b*x^(1/3))^5*Sqrt[(a + b*x^(1/3))^2]*(a^2 - 6*a*b*x^(1/3) + 21*b^2*x^(2/3)))/(56*b^3)

IntegrateAlgebraic [F] time = 2.40, size = 0, normalized size = 0.00

$$\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{5/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2), x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2), x]

fricas [A] time = 1.12, size = 61, normalized size = 0.45

$$5a^2b^3x^2 + a^5x + \frac{3}{8}(b^5x^2 + 16a^3b^2x)x^{\frac{2}{3}} + \frac{15}{28}(4ab^4x^2 + 7a^4bx)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")

[Out] 5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*b^4*x^2 + 7*a^4*b*x)*x^(1/3)

giac [A] time = 0.53, size = 102, normalized size = 0.74

$$\frac{3}{8}b^5x^{\frac{8}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{7}ab^4x^{\frac{7}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)+5a^2b^3x^2\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)+6a^3b^2x^{\frac{5}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{4}a^4bx^{\frac{4}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^5x\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")

[Out] 3/8*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15/7*a*b^4*x^(7/3)*sgn(b*x^(1/3) + a) + 5*a^2*b^3*x^2*sgn(b*x^(1/3) + a) + 6*a^3*b^2*x^(5/3)*sgn(b*x^(1/3) + a) + 15/4*a^4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^5*x*sgn(b*x^(1/3) + a)

maple [A] time = 0.00, size = 87, normalized size = 0.64

$$\frac{\sqrt{b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2} \left(21b^5x^{\frac{8}{3}} + 120ab^4x^{\frac{7}{3}} + 280a^2b^3x^2 + 336a^3b^2x^{\frac{5}{3}} + 210a^4bx^{\frac{4}{3}} + 56a^5x \right)}{56bx^{\frac{1}{3}} + 56a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(5/2),x)

[Out] 1/56*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(21*b^5*x^(8/3)+120*a*b^4*x^(7/3)+336*a^3*b^2*x^(5/3)+210*a^4*b*x^(4/3)+280*a^2*b^3*x^2+56*a^5*x)/(b*x^(1/3)+a)

maxima [A] time = 0.89, size = 114, normalized size = 0.83

$$\frac{\left(b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^{\frac{5}{2}}a^{\frac{1}{3}}}{2b^2} + \frac{\left(b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^{\frac{5}{2}}a^3}{2b^3} + \frac{3\left(b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^{\frac{7}{2}}x^{\frac{1}{3}}}{8b^2} - \frac{27\left(b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^{\frac{7}{2}}a}{56b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")

[Out] 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^2*x^(1/3)/b^2 + 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^3/b^3 + 3/8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*x^(1/3)/b^2 - 27/56*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a/b^3

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + b^2 x^{2/3} + 2 a b x^{1/3} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2), x)`

[Out] `int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2), x)`

[Out] `Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(5/2), x)`

$$3.403 \quad \int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^{3/2} dx$$

Optimal. Leaf size=137

$$\frac{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^5}{2b^3} - \frac{6a\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^4}{5b^3} + \frac{3a^2\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^3}{4b^3}$$

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1341, 645}

$$\frac{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^5}{2b^3} - \frac{6a\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^4}{5b^3} + \frac{3a^2\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (a + b\sqrt[3]{x})^3}{4b^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2), x]

[Out] (3*a^2*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]/(4*b^3) - (6*a*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]/(5*b^3) + ((a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]/(2*b^3))

Rule 645

Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{3/2} dx &= 3 \operatorname{Subst} \left(\int x^2 (a^2 + 2abx + b^2x^2)^{3/2} dx, x, \sqrt[3]{x} \right) \\
&= \frac{\left(3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} \right) \operatorname{Subst} \left(\int \left(\frac{a^2(ab+b^2x)^3}{b^2} - \frac{2a(ab+b^2x)^4}{b^3} + \frac{(ab+b^2x)^5}{b^4} \right) dx, x \right)}{b^3(a + b\sqrt[3]{x})} \\
&= \frac{3a^2(a + b\sqrt[3]{x})^3 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{4b^3} - \frac{6a(a + b\sqrt[3]{x})^4 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{5b^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.47

$$\frac{x\sqrt{(a + b\sqrt[3]{x})^2 (20a^3 + 45a^2b\sqrt[3]{x} + 36ab^2x^{2/3} + 10b^3x)}}{20(a + b\sqrt[3]{x})}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2), x]

[Out] (Sqrt[(a + b*x^(1/3))^2]*x*(20*a^3 + 45*a^2*b*x^(1/3) + 36*a*b^2*x^(2/3) + 10*b^3*x))/(20*(a + b*x^(1/3)))

IntegrateAlgebraic [F] time = 1.68, size = 0, normalized size = 0.00

$$\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{3/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2), x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2), x]

fricas [A] time = 1.40, size = 32, normalized size = 0.23

$$\frac{1}{2}b^3x^2 + \frac{9}{5}ab^2x^{5/3} + \frac{9}{4}a^2bx^{4/3} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2), x, algorithm="fricas")

[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x

giac [A] time = 0.37, size = 64, normalized size = 0.47

$$\frac{1}{2} b^3 x^2 \operatorname{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{9}{5} ab^2 x^{\frac{5}{3}} \operatorname{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{9}{4} a^2 b x^{\frac{4}{3}} \operatorname{sgn}\left(bx^{\frac{1}{3}} + a\right) + a^3 x \operatorname{sgn}\left(bx^{\frac{1}{3}} + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")

[Out] 1/2*b^3*x^2*sgn(b*x^(1/3) + a) + 9/5*a*b^2*x^(5/3)*sgn(b*x^(1/3) + a) + 9/4*a^2*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^3*x*sgn(b*x^(1/3) + a)

maple [A] time = 0.00, size = 65, normalized size = 0.47

$$\frac{\sqrt{b^2 x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2} \left(10b^3 x^2 + 36a b^2 x^{\frac{5}{3}} + 45a^2 b x^{\frac{4}{3}} + 20a^3 x\right)}{20b x^{\frac{1}{3}} + 20a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(3/2),x)

[Out] 1/20*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(36*a*b^2*x^(5/3)+45*a^2*b*x^(4/3)+10*b^3*x^2+20*a^3*x)/(b*x^(1/3)+a)

maxima [A] time = 0.91, size = 114, normalized size = 0.83

$$\frac{3\left(b^2 x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^{\frac{3}{2}} a^2 x^{\frac{1}{3}}}{4b^2} + \frac{3\left(b^2 x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^{\frac{3}{2}} a^3}{4b^3} + \frac{\left(b^2 x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^{\frac{5}{2}} x^{\frac{1}{3}}}{2b^2} - \frac{7\left(b^2 x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^{\frac{5}{2}} a}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")

[Out] 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^2*x^(1/3)/b^2 + 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^3/b^3 + 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*x^(1/3)/b^2 - 7/10*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a/b^3

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + b^2 x^{2/3} + 2abx^{1/3}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2), x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(3/2), x)
```

$$3.404 \quad \int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} dx$$

Optimal. Leaf size=88

$$\frac{3bx^{4/3}\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{4(a + b\sqrt[3]{x})} + \frac{ax\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{a + b\sqrt[3]{x}}$$

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$\frac{3bx^{4/3}\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{4(a + b\sqrt[3]{x})} + \frac{ax\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{a + b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)], x]

[Out] (a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x)/(a + b*x^(1/3)) + (3*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3))/(4*(a + b*x^(1/3)))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} dx &= 3 \operatorname{Subst} \left(\int x^2 \sqrt{a^2 + 2abx + b^2x^2} dx, x, \sqrt[3]{x} \right) \\
&= \frac{\left(3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} \right) \operatorname{Subst} \left(\int x^2 (ab + b^2x) dx, x, \sqrt[3]{x} \right)}{b(a + b\sqrt[3]{x})} \\
&= \frac{\left(3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} \right) \operatorname{Subst} \left(\int (abx^2 + b^2x^3) dx, x, \sqrt[3]{x} \right)}{b(a + b\sqrt[3]{x})} \\
&= \frac{a\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} x}{a + b\sqrt[3]{x}} + \frac{3b\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} x^{4/3}}{4(a + b\sqrt[3]{x})}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.49

$$\frac{\sqrt{(a + b\sqrt[3]{x})^2} (4ax + 3bx^{4/3})}{4(a + b\sqrt[3]{x})}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)], x]

[Out] (Sqrt[(a + b*x^(1/3))^2]*(4*a*x + 3*b*x^(4/3)))/(4*(a + b*x^(1/3)))

IntegrateAlgebraic [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)], x]

fricas [A] time = 0.95, size = 10, normalized size = 0.11

$$\frac{3}{4}bx^{\frac{4}{3}} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")

[Out] 3/4*b*x^(4/3) + a*x

giac [A] time = 0.39, size = 26, normalized size = 0.30

$$\frac{3}{4}bx^{\frac{4}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)+ax\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")

[Out] 3/4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a*x*sgn(b*x^(1/3) + a)

maple [A] time = 0.00, size = 43, normalized size = 0.49

$$\frac{\sqrt{b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2}\left(3bx^{\frac{4}{3}}+4ax\right)}{4bx^{\frac{1}{3}}+4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2),x)

[Out] 1/4*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(3*b*x^(4/3)+4*a*x)/(b*x^(1/3)+a)

maxima [A] time = 0.88, size = 114, normalized size = 1.30

$$\frac{3\sqrt{b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2}a^{\frac{1}{3}}}{2b^2} + \frac{3\sqrt{b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2}a^{\frac{1}{3}}}{2b^3} + \frac{3\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}x^{\frac{1}{3}}}{4b^2} - \frac{5\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")

[Out] 3/2*sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^2*x^(1/3)/b^2 + 3/2*sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^3/b^3 + 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*x^(1/3)/b^2 - 5/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a/b^3

mupad [B] time = 1.56, size = 71, normalized size = 0.81

$$\frac{\sqrt{a^2+b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}}\left(a^3-4a^2bx^{\frac{1}{3}}-5ab^2x^{\frac{2}{3}}+3bx^{\frac{1}{3}}\left(a^2+b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}\right)\right)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2), x)`

[Out] $((a^2 + b^2x^{2/3} + 2abx^{1/3})^{1/2}(a^3 - 4a^2bx^{1/3} - 5a^2bx^{2/3} + 3bx^{1/3}(a^2 + b^2x^{2/3} + 2abx^{1/3}))) / (4b^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2), x)`

[Out] `Integral(sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)`

$$3.405 \quad \int \frac{1}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} dx$$

Optimal. Leaf size=147

$$-\frac{3a\sqrt[3]{x}(a+b\sqrt[3]{x})}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3x^{2/3}(a+b\sqrt[3]{x})}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3a^2(a+b\sqrt[3]{x})\log(a+b\sqrt[3]{x})}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$-\frac{3a\sqrt[3]{x}(a+b\sqrt[3]{x})}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3x^{2/3}(a+b\sqrt[3]{x})}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3a^2(a+b\sqrt[3]{x})\log(a+b\sqrt[3]{x})}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)], x]

[Out] (-3*a*(a + b*x^(1/3))*x^(1/3))/(b^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3*(a + b*x^(1/3))*x^(2/3))/(2*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3*a^2*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n)

))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{a^2 + 2abx + b^2x^2}} dx, x, \sqrt[3]{x} \right) \\
 &= \frac{(3b(a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \frac{x^2}{ab + b^2x} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
 &= \frac{(3b(a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \left(-\frac{a}{b^3} + \frac{x}{b^2} + \frac{a^2}{b^3(a+bx)} \right) dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
 &= -\frac{3a(a + b\sqrt[3]{x})\sqrt[3]{x}}{b^2\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} + \frac{3(a + b\sqrt[3]{x})x^{2/3}}{2b\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} + \frac{3a^2(a + b\sqrt[3]{x})\log(a + b\sqrt[3]{x})}{b^3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.44

$$\frac{3(a + b\sqrt[3]{x})(2a^2 \log(a + b\sqrt[3]{x}) + b\sqrt[3]{x}(b\sqrt[3]{x} - 2a))}{2b^3\sqrt{(a + b\sqrt[3]{x})^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)], x]

[Out] (3*(a + b*x^(1/3))*(b*(-2*a + b*x^(1/3))*x^(1/3) + 2*a^2*Log[a + b*x^(1/3)]))/(2*b^3*Sqrt[(a + b*x^(1/3))^2])

IntegrateAlgebraic [A] time = 0.36, size = 204, normalized size = 1.39

$$\frac{3a^2(\sqrt{b^2 + b})\log\left(\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} - a - \sqrt{b^2}\sqrt[3]{x}\right)}{2b^4} - \frac{3a^2(\sqrt{b^2 - b})\log\left(\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} + a - \sqrt{b^2}\sqrt[3]{x}\right)}{2b^4} - \frac{3(3a - b\sqrt[3]{x})\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}{4b^3} - \frac{3(bx^{2/3} - 2a\sqrt[3]{x})}{4b\sqrt{b^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)], x]

[Out] (-3*(-2*a*x^(1/3) + b*x^(2/3)))/(4*b*Sqrt[b^2]) - (3*(3*a - b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(4*b^3) - (3*a^2*(b + Sqrt[b^2])*Log[-

$$a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2*x^{(1/3)}] / (2*b^4) - (3*a^2*(-b + \text{Sqrt}[b^2])*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2*x^{(1/3)}]) / (2*b^4)$$

fricas [A] time = 1.37, size = 33, normalized size = 0.22

$$\frac{3 \left(2 a^2 \log \left(b x^{\frac{1}{3}} + a \right) + b^2 x^{\frac{2}{3}} - 2 a b x^{\frac{1}{3}} \right)}{2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")

[Out] 3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3

giac [A] time = 0.49, size = 61, normalized size = 0.41

$$\frac{3 \left(b x^{\frac{2}{3}} \text{sgn} \left(b x^{\frac{1}{3}} + a \right) - 2 a x^{\frac{1}{3}} \text{sgn} \left(b x^{\frac{1}{3}} + a \right) \right)}{2 b^2} + \frac{3 a^2 \log \left(\left| b x^{\frac{1}{3}} + a \right| \right)}{b^3 \text{sgn} \left(b x^{\frac{1}{3}} + a \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")

[Out] 3/2*(b*x^(2/3)*sgn(b*x^(1/3) + a) - 2*a*x^(1/3)*sgn(b*x^(1/3) + a))/b^2 + 3*a^2*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a))

maple [A] time = 0.02, size = 103, normalized size = 0.70

$$\frac{\sqrt{b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2} \left(4 a^2 \ln \left(b x^{\frac{1}{3}} + a \right) - 2 a^2 \ln \left(b^2 x^{\frac{2}{3}} - a b x^{\frac{1}{3}} + a^2 \right) + 2 a^2 \ln \left(b^3 x + a^3 \right) + 3 b^2 x^{\frac{2}{3}} - 6 a b x^{\frac{1}{3}} \right)}{2 \left(b x^{\frac{1}{3}} + a \right) b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2),x)

[Out] 1/2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(3*b^2*x^(2/3)-6*a*b*x^(1/3)+2*a^2*ln(b^3*x+a^3)-2*a^2*ln(b^2*x^(2/3)-a*b*x^(1/3)+a^2)+4*a^2*ln(b*x^(1/3)+a))/(b*x^(1/3)+a)/b^3

maxima [A] time = 0.88, size = 36, normalized size = 0.24

$$\frac{3 a^2 \log \left(x^{\frac{1}{3}} + \frac{a}{b} \right)}{b^3} + \frac{3 x^{\frac{2}{3}}}{2 b} - \frac{3 a x^{\frac{1}{3}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")

[Out] 3*a^2*log(x^(1/3) + a/b)/b^3 + 3/2*x^(2/3)/b - 3*a*x^(1/3)/b^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2 + b^2 x^{2/3} + 2 a b x^{1/3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2), x)

[Out] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)

[Out] Integral(1/sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)

$$3.406 \quad \int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{3/2}} dx$$

Optimal. Leaf size=130

$$-\frac{3a^2}{2b^3(a+b\sqrt[3]{x})\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3(a+b\sqrt[3]{x})\log(a+b\sqrt[3]{x})}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{2b^3(a+b\sqrt[3]{x})\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3(a+b\sqrt[3]{x})\log(a+b\sqrt[3]{x})}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2), x]

[Out] (6*a)/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - (3*a^2)/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra

ctionQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{3/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^2}{(a^2 + 2abx + b^2x^2)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
 &= \frac{(3b^3 (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \frac{x^2}{(ab+b^2x)^3} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
 &= \frac{(3b^3 (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \left(\frac{a^2}{b^5(a+bx)^3} - \frac{2a}{b^5(a+bx)^2} + \frac{1}{b^5(a+bx)} \right) dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
 &= \frac{6a}{b^3 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} - \frac{3a^2}{2b^3 (a + b\sqrt[3]{x}) \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} + \frac{3(a + b\sqrt[3]{x})}{b^3 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.55

$$\frac{3a(3a + 4b\sqrt[3]{x}) + 6(a + b\sqrt[3]{x})^2 \log(a + b\sqrt[3]{x})}{2b^3 (a + b\sqrt[3]{x}) \sqrt{(a + b\sqrt[3]{x})^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2), x]

[Out] (3*a*(3*a + 4*b*x^(1/3)) + 6*(a + b*x^(1/3))^2*Log[a + b*x^(1/3)])/(2*b^3*(a + b*x^(1/3))*Sqrt[(a + b*x^(1/3))^2])

IntegrateAlgebraic [B] time = 1.31, size = 1439, normalized size = 11.07

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2), x]

[Out] ((12*a^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]/b^3 - (36*a^3*x^(1/3))/(b*Sqrt[b^2])) + (24*a^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(1/3))/b^2

$$\begin{aligned}
& - (72*a^2*x^{(2/3)})/\text{Sqrt}[b^2] + (48*a*\text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] \\
&]*x^{(2/3)})/b - (48*a*b*x)/\text{Sqrt}[b^2] + (24*a^2*x^{(2/3)}*\text{ArcTanh}[(\text{Sqrt}[a^2 + 2 \\
& *a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)})/a])/b - (24*a*\text{Sqrt}[a^2 + 2 \\
& *a*b*x^{(1/3)} + b^2*x^{(2/3)}]*x^{(2/3)}*\text{ArcTanh}[(\text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2 \\
& *x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)})/a])/ \text{Sqrt}[b^2] + 48*a*x*\text{ArcTanh}[(\text{Sqrt}[a^2 + 2 \\
& *a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)})/a] - (24*b*\text{Sqrt}[a^2 + 2*a* \\
& b*x^{(1/3)} + b^2*x^{(2/3)}]*x*\text{ArcTanh}[(\text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] \\
& - \text{Sqrt}[b^2]*x^{(1/3)})/a])/ \text{Sqrt}[b^2] + 24*b*x^{(4/3)}*\text{ArcTanh}[(\text{Sqrt}[a^2 + 2*a* \\
& b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)})/a])/((-a + \text{Sqrt}[a^2 + 2*a*b*x \\
& ^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)})^2*(a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} \\
& + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)})^2) + ((12*a^4*\text{Sqrt}[b^2])/b^4 + (36*a^3* \\
& \text{Sqrt}[b^2]*x^{(1/3)})/b^3 - (36*a^2*\text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}]*x^{(\\
& 1/3)})/b^2 + (36*a^2*(b^2)^{(3/2)}*x^{(2/3)})/b^4 - (12*a^2*(b^2)^{(3/2)}*x^{(2/3)} \\
& * \text{Log}[-a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)}])/b^4 \\
& + (12*a*\text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}]*x^{(2/3)}*\text{Log}[-a + \text{Sqrt}[a^2 \\
& + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)}])/b - (24*a*\text{Sqrt}[b^2]*x* \\
& \text{Log}[-a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)}])/b + \\
& 12*\text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}]*x*\text{Log}[-a + \text{Sqrt}[a^2 + 2*a*b*x^{(1 \\
& /3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)}] - 12*\text{Sqrt}[b^2]*x^{(4/3)}*\text{Log}[-a + \text{Sqr \\
& t}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)}] - (12*a^2*(b^2)^{(\\
& 3/2)}*x^{(2/3)}*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{ \\
& (1/3)}])/b^4 + (12*a*\text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}]*x^{(2/3)}*\text{Log}[a + \\
& \text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)}])/b - (24*a*\text{Sq \\
& rt}[b^2]*x*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/ \\
& 3)}])/b + 12*\text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}]*x*\text{Log}[a + \text{Sqrt}[a^2 + 2* \\
& a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)}] - 12*\text{Sqrt}[b^2]*x^{(4/3)}*\text{Log}[\\
& a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)}])/((-a + \text{Sq \\
& rt}[a^2 + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)})^2*(a + \text{Sqrt}[a^2 \\
& + 2*a*b*x^{(1/3)} + b^2*x^{(2/3)}] - \text{Sqrt}[b^2]*x^{(1/3)})^2)
\end{aligned}$$

fricas [A] time = 1.23, size = 113, normalized size = 0.87

$$\frac{3 \left(6 a^3 b^3 x + 3 a^6 + 2 (b^6 x^2 + 2 a^3 b^3 x + a^6) \log \left(b x^{\frac{1}{3}} + a \right) + (4 a b^5 x + a^4 b^2) x^{\frac{2}{3}} - (5 a^2 b^4 x + 2 a^5 b) x^{\frac{1}{3}} \right)}{2 (b^9 x^2 + 2 a^3 b^6 x + a^6 b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")

[Out] 3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) + a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^9*x^2 + 2*a^3*b^6*x + a^6*b^3)

giac [A] time = 0.53, size = 64, normalized size = 0.49

$$\frac{3 \log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{b^3 \operatorname{sgn}\left(bx^{\frac{1}{3}} + a\right)} + \frac{3\left(4ax^{\frac{1}{3}} + \frac{3a^2}{b}\right)}{2\left(bx^{\frac{1}{3}} + a\right)^2 b^2 \operatorname{sgn}\left(bx^{\frac{1}{3}} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")

[Out] 3*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a)) + 3/2*(4*a*x^(1/3) + 3*a^2/b)/((b*x^(1/3) + a)^2*b^2*sgn(b*x^(1/3) + a))

maple [A] time = 0.01, size = 92, normalized size = 0.71

$$\frac{3\sqrt{b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2} \left(2b^2x^{\frac{2}{3}} \ln\left(bx^{\frac{1}{3}} + a\right) + 4abx^{\frac{1}{3}} \ln\left(bx^{\frac{1}{3}} + a\right) + 2a^2 \ln\left(bx^{\frac{1}{3}} + a\right) + 4abx^{\frac{1}{3}} + 3a^2\right)}{2\left(bx^{\frac{1}{3}} + a\right)^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(3/2),x)

[Out] 3/2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(2*x^(2/3)*ln(b*x^(1/3)+a)*b^2+4*x^(1/3)*ln(b*x^(1/3)+a)*a*b+4*a*b*x^(1/3)+2*a^2*ln(b*x^(1/3)+a)+3*a^2)/(b*x^(1/3)+a)^3/b^3

maxima [A] time = 0.62, size = 55, normalized size = 0.42

$$\frac{3 \log\left(x^{\frac{1}{3}} + \frac{a}{b}\right)}{b^3} + \frac{6ax^{\frac{1}{3}}}{b^4\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^2} + \frac{9a^2}{2b^5\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")

[Out] 3*log(x^(1/3) + a/b)/b^3 + 6*a*x^(1/3)/(b^4*(x^(1/3) + a/b)^2) + 9/2*a^2/(b^5*(x^(1/3) + a/b)^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2 + b^2 x^{2/3} + 2 a b x^{1/3}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)`

[Out] `int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2), x)`

[Out] `Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-3/2), x)`

$$3.407 \quad \int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{5/2}} dx$$

Optimal. Leaf size=135

$$\frac{3a^2}{4b^3(a+b\sqrt[3]{x})^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{b^3(a+b\sqrt[3]{x})^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{2b^3(a+b\sqrt[3]{x})\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Rubi [A] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$\frac{3a^2}{4b^3(a+b\sqrt[3]{x})^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{b^3(a+b\sqrt[3]{x})^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{2b^3(a+b\sqrt[3]{x})\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2), x]

[Out] (-3*a^2)/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (2*a)/(b^3*(a + b*x^(1/3))^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{5/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^2}{(a^2 + 2abx + b^2x^2)^{5/2}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{(3b^5 (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \frac{x^2}{(ab+b^2x)^5} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
&= \frac{(3b^5 (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \left(\frac{a^2}{b^7(a+bx)^5} - \frac{2a}{b^7(a+bx)^4} + \frac{1}{b^7(a+bx)^3} \right) dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
&= -\frac{3a^2}{4b^3 (a + b\sqrt[3]{x})^3 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} + \frac{2a}{b^3 (a + b\sqrt[3]{x})^2 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.43

$$\frac{-a^2 - 4ab\sqrt[3]{x} - 6b^2x^{2/3}}{4b^3 (a + b\sqrt[3]{x})^3 \sqrt{(a + b\sqrt[3]{x})^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2), x]

[Out] (-a^2 - 4*a*b*x^(1/3) - 6*b^2*x^(2/3))/(4*b^3*(a + b*x^(1/3))^3*Sqrt[(a + b*x^(1/3))^2])

IntegrateAlgebraic [B] time = 0.77, size = 284, normalized size = 2.10

$$\frac{2\sqrt{b^2} (3a^6 + a^2b^4x^{4/3} + 4ab^5x^{5/3} + 6b^6x^2) + 2\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (3a^5b - 3a^4b^2\sqrt[3]{x} + 3a^3b^3x^{2/3} - 3a^2b^4x + 2ab^5x^{4/3} - 6b^6x^{5/3})}{\sqrt{b^2} b^4x^{4/3} \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (-8a^3b^3 - 24a^2b^4\sqrt[3]{x} - 24ab^5x^{2/3} - 8b^6x) + b^4x^{4/3} (8a^4b^4 + 32a^3b^5\sqrt[3]{x} + 48a^2b^6x^{2/3} + 32ab^7x + 8b^8x^{4/3})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2), x]

[Out] (2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(3*a^5*b - 3*a^4*b^2*x^(1/3) + 3*a^3*b^3*x^(2/3) - 3*a^2*b^4*x + 2*a*b^5*x^(4/3) - 6*b^6*x^(5/3)) + 2*Sqrt[b^2]*(3*a^6 + a^2*b^4*x^(4/3) + 4*a*b^5*x^(5/3) + 6*b^6*x^2))/(b^4*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3)*(-8*a^3*b^3 - 24*a^2*b^4*x

$x^{1/3} - 24ab^5x^{2/3} - 8b^6x + b^4x^{4/3}(8a^4b^4 + 32a^3b^5x^{1/3} + 48a^2b^6x^{2/3} + 32ab^7x + 8b^8x^{4/3})$

fricas [A] time = 1.14, size = 136, normalized size = 1.01

$$\frac{20ab^9x^3 - 60a^4b^6x^2 - a^{10} - 9(5a^2b^8x^2 - 4a^5b^5x)x^{2/3} - 3(2b^{10}x^3 - 20a^3b^7x^2 + 5a^6b^4x)x^{1/3}}{4(b^{15}x^4 + 4a^3b^{12}x^3 + 6a^6b^9x^2 + 4a^9b^6x + a^{12}b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")

[Out] 1/4*(20*a*b^9*x^3 - 60*a^4*b^6*x^2 - a^10 - 9*(5*a^2*b^8*x^2 - 4*a^5*b^5*x)*x^(2/3) - 3*(2*b^10*x^3 - 20*a^3*b^7*x^2 + 5*a^6*b^4*x)*x^(1/3))/(b^15*x^4 + 4*a^3*b^12*x^3 + 6*a^6*b^9*x^2 + 4*a^9*b^6*x + a^12*b^3)

giac [A] time = 0.55, size = 43, normalized size = 0.32

$$-\frac{6b^2x^{2/3} + 4abx^{1/3} + a^2}{4\left(bx^{1/3} + a\right)^4 b^3 \operatorname{sgn}\left(bx^{1/3} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")

[Out] -1/4*(6*b^2*x^(2/3) + 4*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^4*b^3*sgn(b*x^(1/3) + a))

maple [A] time = 0.01, size = 54, normalized size = 0.40

$$-\frac{\sqrt{b^2x^{2/3} + 2abx^{1/3} + a^2} \left(6b^2x^{2/3} + 4abx^{1/3} + a^2\right)}{4\left(bx^{1/3} + a\right)^5 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(5/2),x)

[Out] -1/4*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(6*b^2*x^(2/3)+4*a*b*x^(1/3)+a^2)/(b*x^(1/3)+a)^5/b^3

maxima [A] time = 0.72, size = 53, normalized size = 0.39

$$-\frac{3}{2b^5\left(x^{1/3} + \frac{a}{b}\right)^2} + \frac{2a}{b^6\left(x^{1/3} + \frac{a}{b}\right)^3} - \frac{3a^2}{4b^7\left(x^{1/3} + \frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")`

[Out] $-3/2/(b^5*(x^{1/3} + a/b)^2) + 2*a/(b^6*(x^{1/3} + a/b)^3) - 3/4*a^2/(b^7*(x^{1/3} + a/b)^4)$

mupad [B] time = 2.80, size = 53, normalized size = 0.39

$$\frac{\sqrt{a^2 + b^2 x^{2/3} + 2 a b x^{1/3}} (a^2 + 6 b^2 x^{2/3} + 4 a b x^{1/3})}{4 b^3 (a + b x^{1/3})^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)`

[Out] $-((a^2 + b^2*x^{2/3} + 2*a*b*x^{1/3})^{1/2}*(a^2 + 6*b^2*x^{2/3} + 4*a*b*x^{1/3}))/ (4*b^3*(a + b*x^{1/3})^5)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)`

[Out] `Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-5/2), x)`

$$3.408 \quad \int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{7/2}} dx$$

Optimal. Leaf size=137

$$\frac{a^2}{2b^3(a+b\sqrt[3]{x})^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{6a}{5b^3(a+b\sqrt[3]{x})^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{4b^3(a+b\sqrt[3]{x})^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$\frac{a^2}{2b^3(a+b\sqrt[3]{x})^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{6a}{5b^3(a+b\sqrt[3]{x})^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{4b^3(a+b\sqrt[3]{x})^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2), x]

[Out] -a^2/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (6*a)/(5*b^3*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{7/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^2}{(a^2 + 2abx + b^2x^2)^{7/2}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{(3b^7 (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \frac{x^2}{(ab+b^2x)^7} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
&= \frac{(3b^7 (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \left(\frac{a^2}{b^9(a+bx)^7} - \frac{2a}{b^9(a+bx)^6} + \frac{1}{b^9(a+bx)^5} \right) dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
&= -\frac{a^2}{2b^3 (a + b\sqrt[3]{x})^5 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} + \frac{6a}{5b^3 (a + b\sqrt[3]{x})^4 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.42

$$\frac{-a^2 - 6ab\sqrt[3]{x} - 15b^2x^{2/3}}{20b^3 (a + b\sqrt[3]{x})^5 \sqrt{(a + b\sqrt[3]{x})^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2), x]

[Out] (-a^2 - 6*a*b*x^(1/3) - 15*b^2*x^(2/3))/(20*b^3*(a + b*x^(1/3))^5*Sqrt[(a + b*x^(1/3))^2])

IntegrateAlgebraic [B] time = 0.89, size = 356, normalized size = 2.60

$$\frac{8\sqrt{b^2} (10a^8 + a^2b^6x^2 + 6ab^7x^{7/3} + 15b^8x^{8/3}) + 8\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (10a^7b - 10a^6b^2\sqrt[3]{x} + 10a^5b^3x^{2/3} - 10a^4b^4x + 10a^3b^5x^{4/3} - 10a^2b^6x^{5/3} + 9ab^7x^2 - 15b^8x^{7/3})}{5\sqrt{b^2} b^4 x^2 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (-32a^5b^5 - 160a^4b^6\sqrt[3]{x} - 320a^3b^7x^{2/3} - 320a^2b^8x - 160ab^9x^{4/3} - 32b^{10}x^{5/3}) + 5b^4x^2 (32a^6b^6 + 192a^5b^7\sqrt[3]{x} + 480a^4b^8x^{2/3} + 640a^3b^9x + 480a^2b^{10}x^{4/3} + 192ab^{11}x^{5/3} + 32b^{12}x^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2), x]

[Out] (8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(10*a^7*b - 10*a^6*b^2*x^(1/3) + 10*a^5*b^3*x^(2/3) - 10*a^4*b^4*x + 10*a^3*b^5*x^(4/3) - 10*a^2*b^6*x^(5/3) + 9*a*b^7*x^2 - 15*b^8*x^(7/3)) + 8*Sqrt[b^2]*(10*a^8 + a^2*b^6*x^2 + 6*a*b^7*x^(7/3) + 15*b^8*x^(8/3)))/(5*b^4*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])

$$b^2x^{(2/3)}]x^2*(-32a^5b^5 - 160a^4b^6x^{(1/3)} - 320a^3b^7x^{(2/3)} - 320a^2b^8x - 160ab^9x^{(4/3)} - 32b^{10}x^{(5/3)}) + 5b^4x^2(32a^6b^6 + 192a^5b^7x^{(1/3)} + 480a^4b^8x^{(2/3)} + 640a^3b^9x + 480a^2b^{10}x^{(4/3)} + 192ab^{11}x^{(5/3)} + 32b^{12}x^2)$$

fricas [A] time = 1.31, size = 209, normalized size = 1.53

$$\frac{280a^2b^{12}x^4 - 1400a^5b^9x^3 + 735a^8b^6x^2 - 14a^{11}b^3x + a^{14} + 3(5b^{14}x^4 - 210a^3b^{11}x^3 + 483a^6b^8x^2 - 112a^9b^5x)x^{2/3} - 3(28ab^{13}x^4 - 357a^4b^{10}x^3 + 390a^7b^7x^2 - 35a^{10}b^4x)x^{1/3}}{20(b^{21}x^6 + 6a^3b^{18}x^5 + 15a^6b^{15}x^4 + 20a^9b^{12}x^3 + 15a^{12}b^9x^2 + 6a^{15}b^6x + a^{18}b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")

[Out] -1/20*(280*a^2*b^12*x^4 - 1400*a^5*b^9*x^3 + 735*a^8*b^6*x^2 - 14*a^11*b^3*x + a^14 + 3*(5*b^14*x^4 - 210*a^3*b^11*x^3 + 483*a^6*b^8*x^2 - 112*a^9*b^5*x)*x^(2/3) - 3*(28*a*b^13*x^4 - 357*a^4*b^10*x^3 + 390*a^7*b^7*x^2 - 35*a^10*b^4*x)*x^(1/3))/(b^21*x^6 + 6*a^3*b^18*x^5 + 15*a^6*b^15*x^4 + 20*a^9*b^12*x^3 + 15*a^12*b^9*x^2 + 6*a^15*b^6*x + a^18*b^3)

giac [A] time = 0.59, size = 43, normalized size = 0.31

$$-\frac{15b^2x^{2/3} + 6abx^{1/3} + a^2}{20\left(bx^{1/3} + a\right)^6 b^3 \operatorname{sgn}\left(bx^{1/3} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")

[Out] -1/20*(15*b^2*x^(2/3) + 6*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^6*b^3*sgn(b*x^(1/3) + a))

maple [A] time = 0.01, size = 54, normalized size = 0.39

$$-\frac{\sqrt{b^2x^{2/3} + 2abx^{1/3} + a^2} \left(15b^2x^{2/3} + 6abx^{1/3} + a^2\right)}{20\left(bx^{1/3} + a\right)^7 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(7/2),x)

[Out] -1/20*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(1/2)*(15*b^2*x^(2/3)+6*a*b*x^(1/3)+a^2)/(b*x^(1/3)+a)^7/b^3

maxima [A] time = 0.54, size = 53, normalized size = 0.39

$$-\frac{3}{4b^7\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^4} + \frac{6a}{5b^8\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^5} - \frac{a^2}{2b^9\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")

[Out] -3/4/(b^7*(x^(1/3) + a/b)^4) + 6/5*a/(b^8*(x^(1/3) + a/b)^5) - 1/2*a^2/(b^9*(x^(1/3) + a/b)^6)

mupad [B] time = 3.23, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2 + b^2 x^{2/3} + 2 a b x^{1/3}} \left(a^2 + 15 b^2 x^{2/3} + 6 a b x^{1/3}\right)}{20 b^3 \left(a + b x^{1/3}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)

[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 15*b^2*x^(2/3) + 6*a*b*x^(1/3)))/(20*b^3*(a + b*x^(1/3))^7)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)

[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-7/2), x)

$$3.409 \quad \int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{9/2}} dx$$

Optimal. Leaf size=137

$$\frac{3a^2}{8b^3(a+b\sqrt[3]{x})^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{6a}{7b^3(a+b\sqrt[3]{x})^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{1}{2b^3(a+b\sqrt[3]{x})^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$\frac{3a^2}{8b^3(a+b\sqrt[3]{x})^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{6a}{7b^3(a+b\sqrt[3]{x})^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{1}{2b^3(a+b\sqrt[3]{x})^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(9/2), x]

[Out] (-3*a^2)/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (6*a)/(7*b^3*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 1/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{9/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^2}{(a^2 + 2abx + b^2x^2)^{9/2}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{(3b^9 (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \frac{x^2}{(ab+b^2x)^9} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
&= \frac{(3b^9 (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \left(\frac{a^2}{b^{11}(a+bx)^9} - \frac{2a}{b^{11}(a+bx)^8} + \frac{1}{b^{11}(a+bx)^7} \right) dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
&= -\frac{3a^2}{8b^3 (a + b\sqrt[3]{x})^7 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} + \frac{6a}{7b^3 (a + b\sqrt[3]{x})^6 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.42

$$\frac{-a^2 - 8ab\sqrt[3]{x} - 28b^2x^{2/3}}{56b^3 (a + b\sqrt[3]{x})^7 \sqrt{(a + b\sqrt[3]{x})^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(9/2), x]

[Out] (-a^2 - 8*a*b*x^(1/3) - 28*b^2*x^(2/3))/(56*b^3*(a + b*x^(1/3))^7*Sqrt[(a + b*x^(1/3))^2])

IntegrateAlgebraic [B] time = 1.00, size = 434, normalized size = 3.17

$$\frac{16\sqrt{2} (21a^{10} + a^2b^8x^8 + 8ab^9x^7 + 28b^{10}x^{10}) + 16\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (21a^9b - 21a^7b^2\sqrt[3]{x} + 21a^7b^3x^{2/3} - 21a^6b^4x + 21a^6b^5x^{4/3} - 21a^5b^6x^2 - 21a^5b^7x^{2/3} + 20ab^8x^{8/3} - 28b^{10}x^3)}{7\sqrt{2} b^4 x^8 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}} (-128a^7b^7 - 896a^6b^8\sqrt[3]{x} - 2688a^5b^9x^{2/3} - 4480a^4b^{10}x - 4480a^3b^{11}x^{4/3} - 2688a^2b^{12}x^{2/3} - 896ab^{13}x^2 - 128b^{14}x^{7/3}) + 7b^4x^8(128a^8b^8 + 1024a^7b^9\sqrt[3]{x} + 3584a^6b^{10}x^{2/3} + 7168a^5b^{11}x + 8960a^4b^{12}x^{4/3} + 7168a^3b^{13}x^2 + 3584a^2b^{14}x^{2/3} + 1024ab^{15}x^{5/3} + 128b^{16}x^3)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(9/2), x]

[Out] (16*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(21*a^9*b - 21*a^8*b^2*x^(1/3) + 21*a^7*b^3*x^(2/3) - 21*a^6*b^4*x + 21*a^5*b^5*x^(4/3) - 21*a^4*b^6*x^(5/3) + 21*a^3*b^7*x^2 - 21*a^2*b^8*x^(7/3) + 20*a*b^9*x^(8/3) - 28*b^10*x^3) + 16*Sqrt[b^2]*(21*a^10 + a^2*b^8*x^(8/3) + 8*a*b^9*x^3 + 28*b^10*x^(10/3)))/(7*b^4*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(8/3)*(-128*a^7

$7*b^7 - 896*a^6*b^8*x^{(1/3)} - 2688*a^5*b^9*x^{(2/3)} - 4480*a^4*b^{10}*x - 4480$
 $*a^3*b^{11}*x^{(4/3)} - 2688*a^2*b^{12}*x^{(5/3)} - 896*a*b^{13}*x^2 - 128*b^{14}*x^{(7/3)}$
 $+ 7*b^4*x^{(8/3)}*(128*a^8*b^8 + 1024*a^7*b^9*x^{(1/3)} + 3584*a^6*b^{10}*x^{(2/3)}$
 $+ 7168*a^5*b^{11}*x + 8960*a^4*b^{12}*x^{(4/3)} + 7168*a^3*b^{13}*x^{(5/3)} + 35$
 $84*a^2*b^{14}*x^2 + 1024*a*b^{15}*x^{(7/3)} + 128*b^{16}*x^{(8/3)})$

fricas [B] time = 1.57, size = 275, normalized size = 2.01

$$\frac{28 b^{18} x^6 - 2856 a^3 b^{15} x^5 + 18186 a^6 b^{12} x^4 - 20608 a^9 b^9 x^3 + 4200 a^{12} b^6 x^2 - 48 a^{15} b^3 x + a^{18} - 27 (8 a b^{17} x^5 - 244 a^4 b^{14} x^4 + 840 a^7 b^{11} x^3 - 553 a^{10} b^8 x^2 + 56 a^{13} b^5 x) x^{2/3} + 27 (35 a^2 b^{16} x^5 - 448 a^5 b^{13} x^4 + 876 a^8 b^{10} x^3 - 328 a^{11} b^7 x^2 + 14 a^{14} b^4 x) x^{1/3}}{56 (b^2 x^8 + 8 a^3 b^2 x^7 + 28 a^6 b^2 x^6 + 56 a^9 b^2 x^5 + 70 a^{12} b^2 x^4 + 56 a^{15} b^2 x^3 + 28 a^{18} b^2 x^2 + 8 a^{21} b^2 x + a^{24} b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="fricas")

[Out] -1/56*(28*b^18*x^6 - 2856*a^3*b^15*x^5 + 18186*a^6*b^12*x^4 - 20608*a^9*b^9*x^3 + 4200*a^12*b^6*x^2 - 48*a^15*b^3*x + a^18 - 27*(8*a*b^17*x^5 - 244*a^4*b^14*x^4 + 840*a^7*b^11*x^3 - 553*a^10*b^8*x^2 + 56*a^13*b^5*x)*x^(2/3) + 27*(35*a^2*b^16*x^5 - 448*a^5*b^13*x^4 + 876*a^8*b^10*x^3 - 328*a^11*b^7*x^2 + 14*a^14*b^4*x)*x^(1/3))/(b^27*x^8 + 8*a^3*b^24*x^7 + 28*a^6*b^21*x^6 + 56*a^9*b^18*x^5 + 70*a^12*b^15*x^4 + 56*a^15*b^12*x^3 + 28*a^18*b^9*x^2 + 8*a^21*b^6*x + a^24*b^3)

giac [A] time = 0.65, size = 43, normalized size = 0.31

$$-\frac{28 b^2 x^{\frac{2}{3}} + 8 a b x^{\frac{1}{3}} + a^2}{56 \left(b x^{\frac{1}{3}} + a \right)^8 b^3 \operatorname{sgn} \left(b x^{\frac{1}{3}} + a \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="giac")

[Out] -1/56*(28*b^2*x^(2/3) + 8*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^8*b^3*sgn(b*x^(1/3) + a))

maple [A] time = 0.01, size = 54, normalized size = 0.39

$$\frac{\sqrt{b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2} \left(28 b^2 x^{\frac{2}{3}} + 8 a b x^{\frac{1}{3}} + a^2 \right)}{56 \left(b x^{\frac{1}{3}} + a \right)^9 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(9/2),x)

[Out] $-1/56*(b^2*x^{(2/3)}+2*a*b*x^{(1/3)}+a^2)^{(1/2)}*(28*b^2*x^{(2/3)}+8*a*b*x^{(1/3)}+a^2)/(b*x^{(1/3)}+a)^9/b^3$

maxima [A] time = 0.72, size = 53, normalized size = 0.39

$$-\frac{1}{2b^9\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^6} + \frac{6a}{7b^{10}\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^7} - \frac{3a^2}{8b^{11}\left(x^{\frac{1}{3}} + \frac{a}{b}\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="maxima")`

[Out] $-1/2/(b^9*(x^{(1/3)} + a/b)^6) + 6/7*a/(b^{10}*(x^{(1/3)} + a/b)^7) - 3/8*a^2/(b^{11}*(x^{(1/3)} + a/b)^8)$

mupad [B] time = 3.65, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2 + b^2 x^{2/3} + 2 a b x^{1/3}} (a^2 + 28 b^2 x^{2/3} + 8 a b x^{1/3})}{56 b^3 (a + b x^{1/3})^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(9/2),x)`

[Out] $-((a^2 + b^2*x^{(2/3)} + 2*a*b*x^{(1/3)})^{(1/2)}*(a^2 + 28*b^2*x^{(2/3)} + 8*a*b*x^{(1/3)}))/(56*b^3*(a + b*x^{(1/3)})^9)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(9/2),x)`

[Out] `Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-9/2), x)`

$$3.410 \quad \int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{11/2}} dx$$

Optimal. Leaf size=137

$$-\frac{3a^2}{10b^3(a+b\sqrt[3]{x})^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{3b^3(a+b\sqrt[3]{x})^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{8b^3(a+b\sqrt[3]{x})^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Rubi [A] time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$-\frac{3a^2}{10b^3(a+b\sqrt[3]{x})^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{3b^3(a+b\sqrt[3]{x})^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{8b^3(a+b\sqrt[3]{x})^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(11/2), x]

[Out] (-3*a^2)/(10*b^3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (2*a)/(3*b^3*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^{11/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^2}{(a^2 + 2abx + b^2x^2)^{11/2}} dx, x, \sqrt[3]{x} \right) \\
&= \frac{(3b^{11} (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \frac{x^2}{(ab+b^2x)^{11}} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
&= \frac{(3b^{11} (a + b\sqrt[3]{x})) \operatorname{Subst} \left(\int \left(\frac{a^2}{b^{13}(a+bx)^{11}} - \frac{2a}{b^{13}(a+bx)^{10}} + \frac{1}{b^{13}(a+bx)^9} \right) dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} \\
&= -\frac{3a^2}{10b^3 (a + b\sqrt[3]{x})^9 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}} + \frac{2a}{3b^3 (a + b\sqrt[3]{x})^8 \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.42

$$\frac{-a^2 - 10ab\sqrt[3]{x} - 45b^2x^{2/3}}{120b^3 (a + b\sqrt[3]{x})^9 \sqrt{(a + b\sqrt[3]{x})^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2), x]

[Out] (-a^2 - 10*a*b*x^(1/3) - 45*b^2*x^(2/3))/(120*b^3*(a + b*x^(1/3))^9*Sqrt[(a + b*x^(1/3))^2])

IntegrateAlgebraic [B] time = 1.22, size = 507, normalized size = 3.70

$$\frac{-64(-36a^{10} - 231a^9b - 10ab^{12} - 45b^{11}x) - 64\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}(-36a^{11} + 36a^{10}b\sqrt[3]{x} - 36a^9b^2x^{2/3} + 36a^8b^3x - 36a^7b^4x^{4/3} + 36a^6b^5x^{5/3} - 36a^5b^6x^2 + 36a^4b^7x^{7/3} - 36a^3b^8x^{8/3} + 36a^2b^9x^3 - 35ab^{10}x^{10/3} + 45b^{11}x^{11/3})}{15b^3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}(-912a^{10}b - 4608a^9b^2 - 18432a^8b^3 - 43008a^7b^4 - 64512a^6b^5 - 43008a^5b^6 - 18432a^4b^7 - 4608a^3b^8 - 912a^2b^9) + 15b^3\sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}(912a^{10}b + 9120a^9b^2 + 23040a^8b^3 + 61440a^7b^4 + 107520a^6b^5 + 128024a^5b^6 + 107520a^4b^7 + 61440a^3b^8 + 23040a^2b^9 + 9120ab^{10} + 912b^{11})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2), x]

[Out] (-64*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(-36*a^11 + 36*a^10*b*x^(1/3) - 36*a^9*b^2*x^(2/3) + 36*a^8*b^3*x - 36*a^7*b^4*x^(4/3) + 36*a^6*b^5*x^(5/3) - 36*a^5*b^6*x^2 + 36*a^4*b^7*x^(7/3) - 36*a^3*b^8*x^(8/3) + 36*a^2*b^9*x^3 - 35*a*b^10*x^(10/3) + 45*b^11*x^(11/3)) - 64*(-36*a^12*b - a^2*b^11*x^(10/3) - 10*a*b^12*x^(11/3) - 45*b^13*x^4))/(15*b^3*Sqrt[a^2 + 2*

$a*b*x^{(1/3)} + b^2*x^{(2/3)}] * x^{(10/3)} * (-512*a^9*b^{11} - 4608*a^8*b^{12}*x^{(1/3)}$
 $- 18432*a^7*b^{13}*x^{(2/3)} - 43008*a^6*b^{14}*x - 64512*a^5*b^{15}*x^{(4/3)} - 6451$
 $2*a^4*b^{16}*x^{(5/3)} - 43008*a^3*b^{17}*x^2 - 18432*a^2*b^{18}*x^{(7/3)} - 4608*a*b$
 $^{19}*x^{(8/3)} - 512*b^{20}*x^3) + 15*b^3*sqrt[b^2]*x^{(10/3)} * (512*a^{10}*b^{10} + 51$
 $20*a^9*b^{11}*x^{(1/3)} + 23040*a^8*b^{12}*x^{(2/3)} + 61440*a^7*b^{13}*x + 107520*a^$
 $6*b^{14}*x^{(4/3)} + 129024*a^5*b^{15}*x^{(5/3)} + 107520*a^4*b^{16}*x^2 + 61440*a^3*$
 $b^{17}*x^{(7/3)} + 23040*a^2*b^{18}*x^{(8/3)} + 5120*a*b^{19}*x^3 + 512*b^{20}*x^{(10/3)}$
 $))$

fricas [B] time = 1.41, size = 343, normalized size = 2.50

$$\frac{440a^{21}x^7 - 25630a^{19}b^2x^6 + 186252a^{17}b^4x^5 - 326150a^{15}b^6x^4 + 154000a^{13}b^8x^3 - 16005a^{11}b^{10}x^2 + 110a^9b^{12}x - 27(88a^7b^{14}x^6 - 2200a^5b^{16}x^5 + 9625a^3b^{18}x^4 - 10910a^{11}b^{11}x^3 + 3245a^{14}b^8x^2 - 176a^{17}b^5x)x^{(2/3)} - 9(5b^{22}x^7 - 990a^3b^{19}x^6 + 12705a^6b^{16}x^5 - 34760a^9b^{13}x^4 + 25542a^{12}b^{10}x^3 - 4620a^{15}b^7x^2 + 110a^{18}b^4x)x^{(1/3)}}{120(b^{33}x^{10} + 10a^3b^{30}x^9 + 45a^6b^{27}x^8 + 120a^9b^{24}x^7 + 210a^{12}b^{21}x^6 + 252a^{15}b^{18}x^5 + 210a^{18}b^{15}x^4 + 120a^{21}b^{12}x^3 + 45a^{24}b^9x^2 + 10a^{27}b^6x + a^{30}b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="fricas")

[Out] 1/120*(440*a*b^21*x^7 - 25630*a^4*b^18*x^6 + 186252*a^7*b^15*x^5 - 326150*a^10*b^12*x^4 + 154000*a^13*b^9*x^3 - 16005*a^16*b^6*x^2 + 110*a^19*b^3*x - a^22 - 27*(88*a^2*b^20*x^6 - 2200*a^5*b^17*x^5 + 9625*a^8*b^14*x^4 - 10910*a^11*b^11*x^3 + 3245*a^14*b^8*x^2 - 176*a^17*b^5*x)*x^(2/3) - 9*(5*b^22*x^7 - 990*a^3*b^19*x^6 + 12705*a^6*b^16*x^5 - 34760*a^9*b^13*x^4 + 25542*a^12*b^10*x^3 - 4620*a^15*b^7*x^2 + 110*a^18*b^4*x)*x^(1/3))/(b^33*x^10 + 10*a^3*b^30*x^9 + 45*a^6*b^27*x^8 + 120*a^9*b^24*x^7 + 210*a^12*b^21*x^6 + 252*a^15*b^18*x^5 + 210*a^18*b^15*x^4 + 120*a^21*b^12*x^3 + 45*a^24*b^9*x^2 + 10*a^27*b^6*x + a^30*b^3)

giac [A] time = 0.74, size = 43, normalized size = 0.31

$$-\frac{45b^2x^{\frac{2}{3}} + 10abx^{\frac{1}{3}} + a^2}{120\left(bx^{\frac{1}{3}} + a\right)^{10}b^3\operatorname{sgn}\left(bx^{\frac{1}{3}} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="giac")

[Out] -1/120*(45*b^2*x^(2/3) + 10*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^10*b^3*sgn(b*x^(1/3) + a))

maple [A] time = 0.01, size = 54, normalized size = 0.39

$$-\frac{\sqrt{b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2} \left(45b^2x^{\frac{2}{3}} + 10abx^{\frac{1}{3}} + a^2\right)}{120\left(bx^{\frac{1}{3}} + a\right)^{11}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^(11/2),x)`

[Out] $-1/120*(b^2*x^{2/3}+2*a*b*x^{1/3}+a^2)^{1/2}*(45*b^2*x^{2/3}+10*a*b*x^{1/3}+a^2)/(b*x^{1/3}+a)^{11}/b^3$

maxima [A] time = 0.54, size = 53, normalized size = 0.39

$$-\frac{3}{8b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^8} + \frac{2a}{3b^{12}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^9} - \frac{3a^2}{10b^{13}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="maxima")`

[Out] $-3/8/(b^{11}*(x^{1/3} + a/b)^8) + 2/3*a/(b^{12}*(x^{1/3} + a/b)^9) - 3/10*a^2/(b^{13}*(x^{1/3} + a/b)^{10})$

mupad [B] time = 4.34, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2 + b^2 x^{2/3} + 2 a b x^{1/3}} (a^2 + 45 b^2 x^{2/3} + 10 a b x^{1/3})}{120 b^3 (a + b x^{1/3})^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(11/2),x)`

[Out] $-((a^2 + b^2*x^{2/3} + 2*a*b*x^{1/3})^{1/2}*(a^2 + 45*b^2*x^{2/3} + 10*a*b*x^{1/3}))/((120*b^3*(a + b*x^{1/3}))^{11})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}}\right)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(11/2),x)`

[Out] `Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-11/2), x)`

$$3.411 \quad \int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p x^2 dx$$

Optimal. Leaf size=468

$$\frac{3a^9 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^9 \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p}{b^9(2p+9)} - \frac{12a^9 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^8 \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p}{b^9(p+4)} + \frac{84a^9 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^7 \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p}{b^9(2p+7)}$$

Rubi [A] time = 0.22, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1356, 266, 43}

$$\frac{3a^9 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^9 \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p}{b^9(2p+9)} - \frac{12a^9 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^8 \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p}{b^9(p+4)} + \frac{84a^9 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^7 \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p}{b^9(2p+7)} - \frac{84a^9 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^6 \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p}{b^9(2p+5)} + \frac{210a^9 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^5 \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p}{b^9(2p+3)} - \frac{84a^9 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^4 \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p}{b^9(2p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]

[Out] (3*a^9*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(1 + 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(1 + p)) + (84*a^9*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(2 + p)) + (210*a^9*(1 + (b*x^(1/3))/a)^5*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(5 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + p)) + (84*a^9*(1 + (b*x^(1/3))/a)^7*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(7 + 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^8*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(4 + p)) + (3*a^9*(1 + (b*x^(1/3))/a)^9*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(9 + 2*p))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1356


```

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[2*p]

```

Rubi steps

$$\begin{aligned}
\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p x^2 dx &= \left(\left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p \right) \int \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{2p} x^2 dx \\
&= \left(3 \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p \right) \text{Subst} \left(\int x^8 \left(1 + \frac{bx}{a} \right)^{2p} dx, x, \sqrt[3]{x} \right) \\
&= \left(3 \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p \right) \text{Subst} \left(\int \left(\frac{a^8 \left(1 + \frac{bx}{a} \right)^{2p}}{b^8} - \frac{8a^8 \left(1 + \frac{bx}{a} \right)^{2p-1}}{b^8} \right) dx, x, \sqrt[3]{x} \right) \\
&= \frac{3a^9 \left(1 + \frac{b\sqrt[3]{x}}{a} \right) (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^9(1+2p)} - \frac{12a^9 \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^2 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^9(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 207, normalized size = 0.44

$$\frac{3(a + b\sqrt[3]{x}) \left(\frac{a^8}{2p+1} - \frac{4a^7(a+b\sqrt[3]{x})}{p+1} + \frac{28a^6(a+b\sqrt[3]{x})^2}{2p+3} - \frac{28a^5(a+b\sqrt[3]{x})^3}{p+2} + \frac{70a^4(a+b\sqrt[3]{x})^4}{2p+5} - \frac{28a^3(a+b\sqrt[3]{x})^5}{p+3} + \frac{28a^2(a+b\sqrt[3]{x})^6}{2p+7} - \frac{4a(a+b\sqrt[3]{x})^7}{p+4} + \frac{(a+b\sqrt[3]{x})^8}{2p+9} \right) (a + b\sqrt[3]{x})^{2p}}{b^9}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2, x]

[Out] (3*(a^8/(1 + 2*p) - (4*a^7*(a + b*x^(1/3)))/(1 + p) + (28*a^6*(a + b*x^(1/3))^2)/(3 + 2*p) - (28*a^5*(a + b*x^(1/3))^3)/(2 + p) + (70*a^4*(a + b*x^(1/3))^4)/(5 + 2*p) - (28*a^3*(a + b*x^(1/3))^5)/(3 + p) + (28*a^2*(a + b*x^(1/3))^6)/(7 + 2*p) - (4*a*(a + b*x^(1/3))^7)/(4 + p) + (a + b*x^(1/3))^8/(9 + 2*p))*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p/b^9

IntegrateAlgebraic [F] time = 0.54, size = 0, normalized size = 0.00

$$\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p x^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2, x]

fricas [A] time = 1.66, size = 579, normalized size = 1.24

[[[...]]]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="fricas")

[Out] $3*(2520*a^9 + (16*b^9*p^8 + 288*b^9*p^7 + 2184*b^9*p^6 + 9072*b^9*p^5 + 22449*b^9*p^4 + 33642*b^9*p^3 + 29531*b^9*p^2 + 13698*b^9*p + 2520*b^9)*x^3 + 28*(8*a^3*b^6*p^6 + 60*a^3*b^6*p^5 + 170*a^3*b^6*p^4 + 225*a^3*b^6*p^3 + 137*a^3*b^6*p^2 + 30*a^3*b^6*p)*x^2 - 1680*(2*a^6*b^3*p^3 + 3*a^6*b^3*p^2 + a^6*b^3*p)*x + (5040*a^7*b^2*p^2 + 2520*a^7*b^2*p + (16*a*b^8*p^8 + 224*a*b^8*p^7 + 1288*a*b^8*p^6 + 3920*a*b^8*p^5 + 6769*a*b^8*p^4 + 6566*a*b^8*p^3 + 3267*a*b^8*p^2 + 630*a*b^8*p)*x^2 - 168*(4*a^4*b^5*p^5 + 20*a^4*b^5*p^4 + 35*a^4*b^5*p^3 + 25*a^4*b^5*p^2 + 6*a^4*b^5*p)*x)*x^{2/3} - 4*(1260*a^8*b*p + 2*(8*a^2*b^7*p^7 + 84*a^2*b^7*p^6 + 350*a^2*b^7*p^5 + 735*a^2*b^7*p^4 + 812*a^2*b^7*p^3 + 441*a^2*b^7*p^2 + 90*a^2*b^7*p)*x^2 - 105*(4*a^5*b^4*p^4 + 12*a^5*b^4*p^3 + 11*a^5*b^4*p^2 + 3*a^5*b^4*p)*x)*x^{1/3})*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^7 + 37800*b^9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293175*b^9*p^2 + 128322*b^9*p + 22680*b^9)$

giac [B] time = 0.63, size = 1564, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="giac")

[Out] $3*(16*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*b^9*p^8*x^3 + 16*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a*b^8*p^8*x^{8/3} + 288*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*b^9*p^7*x^3 + 224*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a*b^8*p^7*x^{8/3} - 64*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a^2*b^7*p^7*x^{7/3} + 2184*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*b^9*p^6*x^3 + 1288*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a*b^8*p^6*x^{8/3} - 672*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a^2*b^7*p^6*x^{7/3} + 224*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a^3*b^6*p^6*x^2 + 9072*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*b^9*p^5*x^3 + 3920*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a*b^8*p^5*x^{8/3} - 2800*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a^2*b^7*p^5*x^{7/3} + 1680*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a^3*b^6*p^5*x^2 + 22449*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*b^9*p^4*x^3 - 672*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a^4*b^5*p^5*x^{5/3} + 6769*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a*b^8*p^4*x^{8/3} -$

$$\begin{aligned}
& 5880*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^2*b^7*p^4*x^{(7/3)} + 4760*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^3*b^6*p^4*x^2 + 33642*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*b^9*p^3*x^3 - 3360*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^4*b^5*p^4*x^{(5/3)} + 6566*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a*b^8*p^3*x^{(8/3)} + 1680*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^5*b^4*p^4*x^{(4/3)} \\
& - 6496*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^2*b^7*p^3*x^{(7/3)} + 6300*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^3*b^6*p^3*x^2 + 29531*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*b^9*p^2*x^3 - 5880*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^4*b^5*p^3*x^{(5/3)} + 3267*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a*b^8*p^2*x^{(8/3)} + 5040*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^5*b^4*p^3*x^{(4/3)} \\
&) - 3528*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^2*b^7*p^2*x^{(7/3)} - 3360*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^6*b^3*p^3*x + 3836*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^3*b^6*p^2*x^2 + 13698*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*b^9*p*x^3 - 4200*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^4*b^5*p^2*x^{(5/3)} + 630*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a*b^8*p*x^{(8/3)} + 4620*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^5*b^4*p^2*x^{(4/3)} - 720*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^2*b^7*p*x^{(7/3)} - 5040*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^6*b^3*p^2*x + 840*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^3*b^6*p*x^2 + 2520*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*b^9*x^3 + 5040*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^7*b^2*p^2*x^{(2/3)} - 1008*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^4*b^5*p*x^{(5/3)} + 1260*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^5*b^4*p*x^{(4/3)} - 1680*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^6*b^3*p*x + 2520*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^7*b^2*p*x^{(2/3)} - 5040*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^8*b*p*x^{(1/3)} + 2520*(b^2*x^{(2/3)} + 2*a*b*x^{(1/3)} + a^2)^p*a^9)/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^7 + 37800*b^9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293175*b^9*p^2 + 128322*b^9*p + 22680*b^9)
\end{aligned}$$

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^2 \left(b^2 x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2 \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x^2,x)

[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x^2,x)

maxima [A] time = 0.70, size = 362, normalized size = 0.77

3(16*p^4 + 288*p^3 + 2304*p^2 + 1072*p + 2240)*a^5 + 33642*p^4 + 26532*p^3 + 13680*p^2 + 2520*p - (16*p^4 + 224*p^3 + 1296*p^2 + 6768*p + 6566)*a^4 + 630*p^4 - 3(16*p^4 + 288*p^3 + 2304*p^2 + 1072*p + 2240)*a^3 + 33642*p^4 + 26532*p^3 + 13680*p^2 + 2520*p - (16*p^4 + 288*p^3 + 2304*p^2 + 1072*p + 2240)*a^2 + 630*p^4 - 3(16*p^4 + 288*p^3 + 2304*p^2 + 1072*p + 2240)*a + 630*p^4

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="maxima")

```
[Out] 3*((16*p^8 + 288*p^7 + 2184*p^6 + 9072*p^5 + 22449*p^4 + 33642*p^3 + 29531*
p^2 + 13698*p + 2520)*b^9*x^3 + (16*p^8 + 224*p^7 + 1288*p^6 + 3920*p^5 + 6
769*p^4 + 6566*p^3 + 3267*p^2 + 630*p)*a*b^8*x^(8/3) - 8*(8*p^7 + 84*p^6 +
350*p^5 + 735*p^4 + 812*p^3 + 441*p^2 + 90*p)*a^2*b^7*x^(7/3) + 28*(8*p^6 +
60*p^5 + 170*p^4 + 225*p^3 + 137*p^2 + 30*p)*a^3*b^6*x^2 - 168*(4*p^5 + 20
*p^4 + 35*p^3 + 25*p^2 + 6*p)*a^4*b^5*x^(5/3) + 420*(4*p^4 + 12*p^3 + 11*p^
2 + 3*p)*a^5*b^4*x^(4/3) - 1680*(2*p^3 + 3*p^2 + p)*a^6*b^3*x + 2520*(2*p^2
+ p)*a^7*b^2*x^(2/3) - 5040*a^8*b*p*x^(1/3) + 2520*a^9)*(b*x^(1/3) + a)^(2
*p)/((32*p^9 + 720*p^8 + 6960*p^7 + 37800*p^6 + 126546*p^5 + 269325*p^4 + 3
61840*p^3 + 293175*p^2 + 128322*p + 22680)*b^9)
```

mupad [B] time = 3.52, size = 777, normalized size = 1.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p, x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^3*(13698*p + 29531*p^2 + 33642*
p^3 + 22449*p^4 + 9072*p^5 + 2184*p^6 + 288*p^7 + 16*p^8 + 2520))/(128322*p
+ 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
+ 720*p^8 + 32*p^9 + 22680) + (7560*a^9)/(b^9*(128322*p + 293175*p^2 + 361
840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9
+ 22680)) - (15120*a^8*p*x^(1/3))/(b^8*(128322*p + 293175*p^2 + 361840*p^3
+ 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2268
0)) + (3*a*p*x^(8/3)*(3267*p + 6566*p^2 + 6769*p^3 + 3920*p^4 + 1288*p^5 +
224*p^6 + 16*p^7 + 630))/(b*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^
4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)) + (84*a^
3*p*x^2*(137*p + 225*p^2 + 170*p^3 + 60*p^4 + 8*p^5 + 30))/(b^3*(128322*p +
293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 +
720*p^8 + 32*p^9 + 22680)) - (5040*a^6*p*x*(3*p + 2*p^2 + 1))/(b^6*(128322
*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p
^7 + 720*p^8 + 32*p^9 + 22680)) - (24*a^2*p*x^(7/3)*(441*p + 812*p^2 + 735*
p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90))/(b^2*(128322*p + 293175*p^2 + 361840*
p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2
2680)) + (7560*a^7*p*x^(2/3)*(2*p + 1))/(b^7*(128322*p + 293175*p^2 + 36184
0*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 +
22680)) + (1260*a^5*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(b^5*(128322*p
+ 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
+ 720*p^8 + 32*p^9 + 22680)) - (504*a^4*p*x^(5/3)*(25*p + 35*p^2 + 20*p^3 +
4*p^4 + 6))/(b^4*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546
*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

$$3.412 \quad \int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p x dx$$

Optimal. Leaf size=315

$$\frac{3a^6 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^6 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{2b^6(p+3)} - \frac{15a^6 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^5 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^6(2p+5)} + \frac{15a^6 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^4 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^6(p+2)}$$

Rubi [A] time = 0.14, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1356, 266, 43}

$$\frac{3a^6 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^6 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{2b^6(p+3)} - \frac{15a^6 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^5 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^6(2p+5)} + \frac{15a^6 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^4 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^6(p+2)} - \frac{30a^6 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^3 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^6(2p+3)} + \frac{15a^6 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right)^2 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{2b^6(p+1)} - \frac{3a^6 \left(\frac{b\sqrt[3]{x}}{a} + 1 \right) (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^6(2p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]

[Out] (-3*a^6*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(1 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(1 + p)) - (30*a^6*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(3 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(2 + p)) - (15*a^6*(1 + (b*x^(1/3))/a)^5*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(5 + 2*p)) + (3*a^6*(1 + (b*x^(1/3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(3 + p))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1356

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^p_, x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p]]/(1 + (2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&

! IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p x dx &= \left(\left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p \right) \int \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{2p} x dx \\
&= \left(3 \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p \right) \text{Subst} \left(\int x^5 \left(1 + \frac{bx}{a} \right)^{2p} dx, x, \sqrt[3]{x} \right) \\
&= \left(3 \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p \right) \text{Subst} \left(\int \left(-\frac{a^5 \left(1 + \frac{bx}{a} \right)^{2p}}{b^5} + \frac{5a^5 \left(1 + \frac{bx}{a} \right)^{2p} bx}{b^5} \right) dx, x, \sqrt[3]{x} \right) \\
&= -\frac{3a^6 \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^6(1+2p)} + \frac{15a^6 \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{2b^6(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 143, normalized size = 0.45

$$\frac{3(a + b\sqrt[3]{x}) \left(-\frac{2a^5}{2p+1} + \frac{5a^4(a+b\sqrt[3]{x})}{p+1} - \frac{20a^3(a+b\sqrt[3]{x})^2}{2p+3} + \frac{10a^2(a+b\sqrt[3]{x})^3}{p+2} - \frac{10a(a+b\sqrt[3]{x})^4}{2p+5} + \frac{(a+b\sqrt[3]{x})^5}{p+3} \right) \left((a + b\sqrt[3]{x})^2 \right)^p}{2b^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]

[Out] (3*((-2*a^5)/(1 + 2*p) + (5*a^4*(a + b*x^(1/3)))/(1 + p) - (20*a^3*(a + b*x^(1/3))^2)/(3 + 2*p) + (10*a^2*(a + b*x^(1/3))^3)/(2 + p) - (10*a*(a + b*x^(1/3))^4)/(5 + 2*p) + (a + b*x^(1/3))^5/(3 + p))*(a + b*x^(1/3))^2)^p)/(2*b^6)

IntegrateAlgebraic [F] time = 0.19, size = 0, normalized size = 0.00

$$\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p x dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x, x]

fricas [A] time = 1.31, size = 297, normalized size = 0.94

$$\frac{3(30a^6 - (8b^6p^5 + 60b^6p^4 + 170b^6p^3 + 225b^6p^2 + 137b^6p + 30b^6)x^2 - 20(2a^3b^3p^3 + 3a^3b^3p^2 + a^3b^3p)x + 2(30a^4b^2p^2 + 15a^4b^2p - (4ab^5p^5 + 20ab^5p^4 + 35ab^5p^3 + 25ab^5p^2 + 6ab^5p)x)^{\frac{1}{3}} - 5(12a^5bp - (4a^2b^4p^4 + 12a^2b^4p^3 + 11a^2b^4p^2 + 3a^2b^4p)x)^{\frac{1}{3}})(b^2x^{\frac{2}{3}} + a^2)^{\frac{p}{3}}}{2(8b^6p^6 + 84b^6p^5 + 350b^6p^4 + 735b^6p^3 + 812b^6p^2 + 441b^6p + 90b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="fricas")

[Out] $-3/2*(30*a^6 - (8*b^6*p^5 + 60*b^6*p^4 + 170*b^6*p^3 + 225*b^6*p^2 + 137*b^6*p + 30*b^6)*x^2 - 20*(2*a^3*b^3*p^3 + 3*a^3*b^3*p^2 + a^3*b^3*p)*x + 2*(30*a^4*b^2*p^2 + 15*a^4*b^2*p - (4*a*b^5*p^5 + 20*a*b^5*p^4 + 35*a*b^5*p^3 + 25*a*b^5*p^2 + 6*a*b^5*p)*x)*x^{2/3} - 5*(12*a^5*b*p - (4*a^2*b^4*p^4 + 12*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 3*a^2*b^4*p)*x)*x^{1/3})*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^6*p^2 + 441*b^6*p + 90*b^6)}$

giac [B] time = 0.48, size = 745, normalized size = 2.37

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="giac")

[Out] $3/2*(8*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*b^6*p^5*x^2 + 8*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a*b^5*p^5*x^{5/3} + 60*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*b^6*p^4*x^2 + 40*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a*b^5*p^4*x^{5/3} - 20*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^2*b^4*p^4*x^{4/3} + 170*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*b^6*p^3*x^2 + 70*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a*b^5*p^3*x^{5/3} - 60*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^2*b^4*p^3*x^{4/3} + 40*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^3*b^3*p^3*x + 225*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*b^6*p^2*x^2 + 50*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a*b^5*p^2*x^{5/3} - 55*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^2*b^4*p^2*x^{4/3} + 60*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^3*b^3*p^2*x + 137*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*b^6*p*x^2 - 60*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^4*b^2*p^2*x^{2/3} + 12*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a*b^5*p*x^{5/3} - 15*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^2*b^4*p*x^{4/3} + 20*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^3*b^3*p*x + 30*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*b^6*x^2 - 30*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^4*b^2*p*x^{2/3} + 60*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^5*b*p*x^{1/3} - 30*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^{p*a^6}}/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^6*p^2 + 441*b^6*p + 90*b^6)}$

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x \left(b^2 x^{\frac{2}{3}} + 2ab x^{\frac{1}{3}} + a^2 \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x,x)`

[Out] `int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p*x,x)`

maxima [A] time = 0.64, size = 198, normalized size = 0.63

$$\frac{3 \left((8p^5 + 60p^4 + 170p^3 + 225p^2 + 137p + 30)b^6x^2 + 2(4p^5 + 20p^4 + 35p^3 + 25p^2 + 6p)ab^5x^{\frac{5}{3}} - 5(4p^4 + 12p^3 + 11p^2 + 3p)a^2b^4x^{\frac{4}{3}} + 20(2p^3 + 3p^2 + p)a^3b^3x - 30(2p^2 + p)a^4b^2x^{\frac{2}{3}} + 60a^5bpx^{\frac{1}{3}} - 30a^6 \right) (bx^{\frac{1}{3}} + a)^{2p}}{2(8p^6 + 84p^5 + 350p^4 + 735p^3 + 812p^2 + 441p + 90)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="maxima")`

[Out] `3/2*((8*p^5 + 60*p^4 + 170*p^3 + 225*p^2 + 137*p + 30)*b^6*x^2 + 2*(4*p^5 + 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a*b^5*x^(5/3) - 5*(4*p^4 + 12*p^3 + 11*p^2 + 3*p)*a^2*b^4*x^(4/3) + 20*(2*p^3 + 3*p^2 + p)*a^3*b^3*x - 30*(2*p^2 + p)*a^4*b^2*x^(2/3) + 60*a^5*b*p*x^(1/3) - 30*a^6)*(b*x^(1/3) + a)^(2*p)/((8*p^6 + 84*p^5 + 350*p^4 + 735*p^3 + 812*p^2 + 441*p + 90)*b^6)`

mupad [B] time = 2.17, size = 390, normalized size = 1.24

$$\frac{(p^6 + 84p^5 + 350p^4 + 735p^3 + 812p^2 + 441p + 90) \left(\frac{3a^2(b^2x^{2/3} + 2abx^{1/3} + a^2)^{2p}}{2(8p^6 + 84p^5 + 350p^4 + 735p^3 + 812p^2 + 441p + 90)} - \frac{45a^6}{b^6(441p + 812p^2 + 735p^3 + 350p^4 + 84p^5 + 8p^6 + 90)} + \frac{90a^5p}{b^5(441p + 812p^2 + 735p^3 + 350p^4 + 84p^5 + 8p^6 + 90)} - \frac{15a^2p}{2b^2(441p + 812p^2 + 735p^3 + 350p^4 + 84p^5 + 8p^6 + 90)} + \frac{30a^3p}{b^3(441p + 812p^2 + 735p^3 + 350p^4 + 84p^5 + 8p^6 + 90)} - \frac{45a^4p}{b^4(441p + 812p^2 + 735p^3 + 350p^4 + 84p^5 + 8p^6 + 90)} + \frac{3a^5p}{b(441p + 812p^2 + 735p^3 + 350p^4 + 84p^5 + 8p^6 + 90)} \right)}{(8p^6 + 84p^5 + 350p^4 + 735p^3 + 812p^2 + 441p + 90)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)`

[Out] `(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^2*(137*p + 225*p^2 + 170*p^3 + 60*p^4 + 8*p^5 + 30))/(2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) - (45*a^6)/(b^6*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (90*a^5*p*x^(1/3))/(b^5*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) - (15*a^2*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(2*b^2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (30*a^3*p*x*(3*p + 2*p^2 + 1))/(b^3*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) - (45*a^4*p*x^(2/3)*(2*p + 1))/(b^4*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (3*a*p*x^(5/3)*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6))/(b*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)))`

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))*p*x,x)`

[Out] Exception raised: HeuristicGCDFailed

$$3.413 \quad \int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3} \right)^p dx$$

Optimal. Leaf size=142

$$\frac{3(a + b\sqrt[3]{x})^3 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^3(2p + 3)} - \frac{3a(a + b\sqrt[3]{x})^2 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^3(p + 1)} + \frac{3a^2(a + b\sqrt[3]{x})(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^3(2p + 1)}$$

Rubi [A] time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1341, 646, 43}

$$\frac{3(a + b\sqrt[3]{x})^3 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^3(2p + 3)} - \frac{3a(a + b\sqrt[3]{x})^2 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^3(p + 1)} + \frac{3a^2(a + b\sqrt[3]{x})(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^3(2p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p, x]

[Out] (3*a^2*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1 + 2*p)) - (3*a*(a + b*x^(1/3))^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1 + p)) + (3*(a + b*x^(1/3))^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(3 + 2*p))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p dx &= 3 \operatorname{Subst} \left(\int x^2 (a^2 + 2abx + b^2x^2)^p dx, x, \sqrt[3]{x} \right) \\
&= \left(3 (b(a + b\sqrt[3]{x}))^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p \right) \operatorname{Subst} \left(\int x^2 (ab + b^2x)^{2p} dx, x, \right. \\
&= \left(3 (b(a + b\sqrt[3]{x}))^{-2p} (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p \right) \operatorname{Subst} \left(\int \left(\frac{a^2 (ab + b^2x)^{2p}}{b^2} - \frac{2a}{b} \right) dx, x, \right. \\
&= \frac{3a^2 (a + b\sqrt[3]{x}) (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^3(1 + 2p)} - \frac{3a (a + b\sqrt[3]{x})^2 (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{b^3(1 + p)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 0.58

$$\frac{3(a + b\sqrt[3]{x}) \left((a + b\sqrt[3]{x})^2 \right)^p (a^2 - ab(2p + 1)\sqrt[3]{x} + b^2(2p^2 + 3p + 1)x^{2/3})}{b^3(p + 1)(2p + 1)(2p + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p, x]

[Out] (3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(1/3) + b^2*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p, x]

[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p, x]

fricas [A] time = 1.50, size = 110, normalized size = 0.77

$$\frac{3 \left(2a^2bp^{\frac{1}{3}} - a^3 - (2b^3p^2 + 3b^3p + b^3)x - (2ab^2p^2 + ab^2p)x^{\frac{2}{3}} \right) \left(b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2 \right)^p}{4b^3p^3 + 12b^3p^2 + 11b^3p + 3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="fricas")

[Out] $-3*(2*a^2*b*p*x^{1/3} - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*a*b^2*p^2 + a*b^2*p)*x^{2/3})*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)$

giac [A] time = 0.48, size = 229, normalized size = 1.61

$$\frac{3\left(2\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p b^3 p^2 x + 2\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p ab^2 p^2 x^{\frac{2}{3}} + 3\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p b^3 p x + \left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p ab^2 p x^{\frac{2}{3}} - 2\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p a^2 b p x^{\frac{1}{3}} + \left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p b^3 x + \left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p a^3\right)}{4b^3p^3+12b^3p^2+11b^3p+3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="giac")

[Out] $3*(2*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*b^3*p^2*x + 2*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a*b^2*p^2*x^{2/3} + 3*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*b^3*p*x + (b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a*b^2*p*x^{2/3} - 2*(b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a^2*b*p*x^{1/3} + (b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*b^3*x + (b^2*x^{2/3} + 2*a*b*x^{1/3} + a^2)^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)$

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b^2 x^{\frac{2}{3}} + 2ab x^{\frac{1}{3}} + a^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p,x)

[Out] int((b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p,x)

maxima [A] time = 0.80, size = 77, normalized size = 0.54

$$\frac{3\left(\left(2p^2+3p+1\right)b^3x+\left(2p^2+p\right)ab^2x^{\frac{2}{3}}-2a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2p}}{\left(4p^3+12p^2+11p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="maxima")

[Out] $3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^{2/3} - 2*a^2*b*p*x^{1/3} + a^3)*(b*x^{1/3} + a)^{(2*p)}/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)$

mupad [B] time = 1.54, size = 138, normalized size = 0.97

$$\left(a^2 + b^2 x^{2/3} + 2abx^{1/3}\right)^p \left(\frac{3x(2p^2+3p+1)}{4p^3+12p^2+11p+3} + \frac{3a^3}{b^3(4p^3+12p^2+11p+3)} - \frac{6a^2px^{1/3}}{b^2(4p^3+12p^2+11p+3)} + \frac{3apx^{2/3}(2p+1)}{b(4p^3+12p^2+11p+3)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)`

[Out] $(a^2 + b^2x^{2/3} + 2abx^{1/3})^p \left(\frac{3x(3p + 2p^2 + 1)}{(11p + 12p^2 + 4p^3 + 3)} + \frac{3a^3}{b^3(11p + 12p^2 + 4p^3 + 3)} - \frac{6a^2p x^{1/3}}{b^2(11p + 12p^2 + 4p^3 + 3)} + \frac{3a p x^{2/3}(2p + 1)}{b(11p + 12p^2 + 4p^3 + 3)} \right)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{\frac{2}{3}} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)`

[Out] `Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p, x)`

$$3.414 \quad \int \left(\frac{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{x^2} - \frac{2b^3(1-2p)(1-p)p(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{3a^3x} \right) dx$$

Optimal. Leaf size=146

$$\frac{b(1-p)(a+b\sqrt[3]{x})(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{a^2x^{2/3}} - \frac{(a+b\sqrt[3]{x})(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{ax} - \frac{b^2(1-2p)(1-p)(a+b\sqrt[3]{x})(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p}{a^3\sqrt[3]{x}}$$

Rubi [C] time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 3, integrand size = 77, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {1356, 266, 65}

$$\frac{2b^3(1-2p)(1-p)p\left(\frac{b\sqrt[3]{x}}{a}+1\right)(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p {}_2F_1\left(1, 2p+1; 2(p+1); \frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)} + \frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)(a^2+2ab\sqrt[3]{x}+b^2x^{2/3})^p {}_2F_1\left(4, 2p+1; 2(p+1); \frac{\sqrt[3]{x}b}{a}+1\right)}{a^3(2p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x), x]

[Out] (2*b^3*(1 - 2*p)*(1 - p)*p*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p)) + (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p))

Rule 65

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1356

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&

! IntegerQ[2*p]

Rubi steps

$$\int \left(\frac{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{x^2} - \frac{2b^3(1-2p)(1-p)p(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{3a^3x} \right) dx = -\frac{(2b^3(1-2p)(1-p)p) \int \frac{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{x^2} dx}{3a^3}$$

$$= \left(\left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x}) \right)$$

$$= \left(3 \left(1 + \frac{b\sqrt[3]{x}}{a} \right)^{-2p} (a^2 + 2ab\sqrt[3]{x}) \right)$$

$$= \frac{2b^3(1-2p)(1-p)p \left(1 + \frac{b\sqrt[3]{x}}{a} \right)}{3a^3}$$

Mathematica [C] time = 0.08, size = 101, normalized size = 0.69

$$\frac{b^3(a + b\sqrt[3]{x}) \left((a + b\sqrt[3]{x})^2 \right)^p \left(2p(2p^2 - 3p + 1) {}_2F_1 \left(1, 2p + 1; 2(p + 1); \frac{\sqrt[3]{x}b}{a} + 1 \right) + 3 {}_2F_1 \left(4, 2p + 1; 2(p + 1); \frac{\sqrt[3]{x}b}{a} + 1 \right) \right)}{a^3(2ap + a)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x), x]

[Out] (b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(2*p*(1 - 3*p + 2*p^2)*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a] + 3*Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a]))/(a^3*(a + 2*a*p))

IntegrateAlgebraic [F] time = 5.79, size = 0, normalized size = 0.00

$$\int \left(\frac{(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{x^2} - \frac{2b^3(1-2p)(1-p)p(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3})^p}{3a^3x} \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x), x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x), x]

fricas [A] time = 1.67, size = 82, normalized size = 0.56

$$\frac{\left(a^2 b p x^{\frac{1}{3}} + a^3 + \left(2 b^3 p^2 - 3 b^3 p + b^3\right) x + 2 \left(a b^2 p^2 - a b^2 p\right) x^{\frac{2}{3}}\right) \left(b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2\right)^p}{a^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="fricas")

[Out] -(a^2*b*p*x^(1/3) + a^3 + (2*b^3*p^2 - 3*b^3*p + b^3)*x + 2*(a*b^2*p^2 - a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(a^3*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2 \left(b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2\right)^p b^3 (2 p - 1) (p - 1) p}{3 a^3 x} + \frac{\left(b^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + a^2\right)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="giac")

[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int -\frac{2(-2p+1)(-p+1)b^3p\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p}{3a^3x} + \frac{\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p-2/3*b^3*(1-2*p)*(1-p)*p*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p/a^3/x,x)

[Out] int(1/x^2*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p-2/3*b^3*(1-2*p)*(1-p)*p*(b^2*x^(2/3)+2*a*b*x^(1/3)+a^2)^p/a^3/x,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\left(b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^p b^3(2p-1)(p-1)p}{3a^3x} + \frac{\left(b^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + a^2\right)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="maxima")

[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)

mupad [B] time = 1.65, size = 69, normalized size = 0.47

$$\frac{\left(a^2 + b^2 x^{2/3} + 2 a b x^{1/3}\right)^p \left(\frac{b^3 x(2 p^2-3 p+1)}{a^3} + \frac{b p x^{1/3}}{a} + \frac{2 b^2 p x^{2/3}(p-1)}{a^2} + 1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2 - (2*b^3*p*(2*p - 1)*(p - 1)*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p)/(3*a^3*x), x)

[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((b^3*x*(2*p^2 - 3*p + 1))/a^3 + (b*p*x^(1/3))/a + (2*b^2*p*x^(2/3)*(p - 1))/a^2 + 1))/x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \left(-\frac{3a^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x^2}\right) dx + \int \frac{2b^3p\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x} dx + \int \left(-\frac{6b^3p^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x}\right) dx + \int \frac{4b^3p^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p}{x} dx}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-2*p)*(1-p)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)

[Out] -(Integral(-3*a**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x**2, x) + Integral(2*b**3*p*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral(-6*b**3*p**2*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral(4*b**3*p**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x))/(3*a**3)

$$3.415 \quad \int \frac{1}{(a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x})^{3/2}} dx$$

Optimal. Leaf size=176

$$-\frac{12a^2}{b^4\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} - \frac{12a(a + b\sqrt[4]{x})\log(a + b\sqrt[4]{x})}{b^4\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} + \frac{4\sqrt[4]{x}(a + b\sqrt[4]{x})}{b^3\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} + \frac{2a^3}{b^4(a + b\sqrt[4]{x})\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}}$$

Rubi [A] time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$\frac{2a^3}{b^4(a + b\sqrt[4]{x})\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} - \frac{12a^2}{b^4\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} + \frac{4\sqrt[4]{x}(a + b\sqrt[4]{x})}{b^3\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} - \frac{12a(a + b\sqrt[4]{x})\log(a + b\sqrt[4]{x})}{b^4\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2), x]

[Out] (-12*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (2*a^3)/(b^4*(a + b*x^(1/4))*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (4*(a + b*x^(1/4))*x^(1/4))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) - (12*a*(a + b*x^(1/4))*Log[a + b*x^(1/4)]/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra

ctionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x})^{3/2}} dx &= 4 \operatorname{Subst} \left(\int \frac{x^3}{(a^2 + 2abx + b^2x^2)^{3/2}} dx, x, \sqrt[4]{x} \right) \\
&= \frac{(4b^3 (a + b\sqrt[4]{x})) \operatorname{Subst} \left(\int \frac{x^3}{(ab+b^2x)^3} dx, x, \sqrt[4]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} \\
&= \frac{(4b^3 (a + b\sqrt[4]{x})) \operatorname{Subst} \left(\int \left(\frac{1}{b^6} - \frac{a^3}{b^6(a+bx)^3} + \frac{3a^2}{b^6(a+bx)^2} - \frac{3a}{b^6(a+bx)} \right) dx, x, \sqrt[4]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} \\
&= -\frac{12a^2}{b^4\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} + \frac{2a^3}{b^4(a + b\sqrt[4]{x})\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}} + \frac{4}{b^3\sqrt{a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x}}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.53

$$\frac{2 \left(-5a^3 - 4a^2b\sqrt[4]{x} + 4ab^2\sqrt{x} - 6a(a + b\sqrt[4]{x})^2 \log(a + b\sqrt[4]{x}) + 2b^3x^{3/4} \right)}{b^4(a + b\sqrt[4]{x})\sqrt{(a + b\sqrt[4]{x})^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2), x]

[Out] (2*(-5*a^3 - 4*a^2*b*x^(1/4) + 4*a*b^2*Sqrt[x] + 2*b^3*x^(3/4) - 6*a*(a + b*x^(1/4))^2*Log[a + b*x^(1/4)]))/(b^4*(a + b*x^(1/4))*Sqrt[(a + b*x^(1/4))^2])

IntegrateAlgebraic [B] time = 1.26, size = 1582, normalized size = 8.99

result too large to display

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2), x]

[Out] ((-16*a^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])/b^4 + (64*a^4*Sqrt[b^2]*x^(1/4))/b^4 - (48*a^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]*x^(1/4))/b^3

$$\begin{aligned}
& + (128*a^3*\text{Sqrt}[b^2]*\text{Sqrt}[x])/b^3 - (80*a^2*\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2 \\
& * \text{Sqrt}[x]]*\text{Sqrt}[x])/b^2 + (32*a^2*(b^2)^{(3/2)}*x^{(3/4)})/b^4 + (48*a*\text{Sqrt}[a^2 \\
& + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]]*x^{(3/4)})/b - (80*a*\text{Sqrt}[b^2]*x)/b + 32*\text{Sqrt}[\\
& a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]]*x - 32*\text{Sqrt}[b^2]*x^{(5/4)} - (96*a^3*\text{Sqrt}[\\
& x]*\text{ArcTanh}[(\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)})/a] \\
&)/b^2 + (96*a^2*\text{Sqrt}[b^2]*\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]]*\text{Sqrt}[x]*\text{A} \\
& \text{rcTanh}[(\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)})/a])/b^3 \\
& - (192*a^2*x^{(3/4)}*\text{ArcTanh}[(\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqr} \\
& \text{t}[b^2]*x^{(1/4)})/a])/b + (96*a*(b^2)^{(3/2)}*\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqr} \\
& \text{t}[x]]*x^{(3/4)}*\text{ArcTanh}[(\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2] \\
& *x^{(1/4)})/a])/b^4 - 96*a*x*\text{ArcTanh}[(\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] \\
& - \text{Sqrt}[b^2]*x^{(1/4)})/a])/((-a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \\
& \text{Sqrt}[b^2]*x^{(1/4)})^2*(a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^ \\
& 2]*x^{(1/4)})^2) + ((-16*a^5)/(b^3*\text{Sqrt}[b^2]) - (64*a^4*x^{(1/4)})/(b^2)^{(3/2)} \\
& + (64*a^3*\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]]*x^{(1/4)})/b^3 - (64*a^3*\text{Sqr} \\
& \text{t}[x])/(b*\text{Sqrt}[b^2]) + (48*a^3*\text{Sqrt}[x]*\text{Log}[-a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + \\
& b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/(b*\text{Sqrt}[b^2]) - (48*a^2*\text{Sqrt}[a^2 + 2*a*b \\
& *x^{(1/4)} + b^2*\text{Sqrt}[x]]*\text{Sqrt}[x]*\text{Log}[-a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqr} \\
& \text{t}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/b^2 + (96*a^2*x^{(3/4)}*\text{Log}[-a + \text{Sqrt}[a^2 + 2*a*b \\
& *x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/ \text{Sqrt}[b^2] - (48*a*\text{Sqrt}[a^2 + \\
& 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]]*x^{(3/4)}*\text{Log}[-a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^ \\
& 2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/b + (48*a*b*x*\text{Log}[-a + \text{Sqrt}[a^2 + 2*a*b*x^ \\
& (1/4) + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/ \text{Sqrt}[b^2] + (48*a^3*\text{Sqrt}[x]*\text{Log}[\\
& a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/(b*\text{Sqrt}[b \\
& ^2]) - (48*a^2*\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]]*\text{Sqrt}[x]*\text{Log}[a + \text{Sqr} \\
& \text{t}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/b^2 + (96*a^2*x^{(\\
& 3/4)}*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/ \\
& \text{Sqrt}[b^2] - (48*a*\text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]]*x^{(3/4)}*\text{Log}[a + \text{S} \\
& \text{qrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/b + (48*a*b*x* \\
& \text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)}])/ \text{Sqrt}[\\
& b^2])/((-a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)})^2 \\
& *(a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/4)} + b^2*\text{Sqrt}[x]] - \text{Sqrt}[b^2]*x^{(1/4)})^2)
\end{aligned}$$

fricas [A] time = 11.03, size = 147, normalized size = 0.84

$$\frac{2\left(9a^5b^4x - 5a^9 - 6(ab^8x^2 - 2a^5b^4x + a^9)\log\left(bx^{\frac{1}{4}} + a\right) - 2(3a^2b^7x - a^6b^3)x^{\frac{3}{4}} + (7a^3b^6x - 3a^7b^2)\sqrt{x} + 2(b^9x^2 - 6a^4b^5x + 3a^8b)x^{\frac{1}{4}}\right)}{b^{12}x^2 - 2a^4b^8x + a^8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="fricas")

[Out] 2*(9*a^5*b^4*x - 5*a^9 - 6*(a*b^8*x^2 - 2*a^5*b^4*x + a^9)*log(b*x^(1/4) + a) - 2*(3*a^2*b^7*x - a^6*b^3)*x^(3/4) + (7*a^3*b^6*x - 3*a^7*b^2)*sqrt(x) + 2*(b^9*x^2 - 6*a^4*b^5*x + 3*a^8*b)*x^(1/4))/(b^12*x^2 - 2*a^4*b^8*x + a^8*b^4)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.02, size = 114, normalized size = 0.65

$$\frac{2\sqrt{b^2\sqrt{x} + 2abx^{\frac{1}{4}} + a^2} \left(-6ab^2\sqrt{x} \ln\left(bx^{\frac{1}{4}} + a\right) - 12a^2bx^{\frac{1}{4}} \ln\left(bx^{\frac{1}{4}} + a\right) - 6a^3 \ln\left(bx^{\frac{1}{4}} + a\right) + 2b^3x^{\frac{3}{4}} + 4ab^2\sqrt{x} - 4a^2bx^{\frac{1}{4}} - 5a^3 \right)}{\left(bx^{\frac{1}{4}} + a\right)^3 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x)

[Out] 2*(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)*(2*x^(3/4)*b^3-6*x^(1/2)*ln(a+b*x^(1/4))*a*b^2+4*x^(1/2)*a*b^2-12*x^(1/4)*ln(a+b*x^(1/4))*a^2*b-4*x^(1/4)*a^2*b-6*ln(a+b*x^(1/4))*a^3-5*a^3)/(a+b*x^(1/4))^3/b^4

maxima [A] time = 0.58, size = 114, normalized size = 0.65

$$\frac{4\sqrt{x}}{\sqrt{b^2\sqrt{x} + 2abx^{\frac{1}{4}} + a^2} b^2} - \frac{12a \log\left(x^{\frac{1}{4}} + \frac{a}{b}\right)}{b^4} + \frac{8a^2}{\sqrt{b^2\sqrt{x} + 2abx^{\frac{1}{4}} + a^2} b^4} - \frac{24a^2x^{\frac{1}{4}}}{b^5\left(x^{\frac{1}{4}} + \frac{a}{b}\right)^2} - \frac{22a^3}{b^6\left(x^{\frac{1}{4}} + \frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="maxima")

[Out] 4*sqrt(x)/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^2) - 12*a*log(x^(1/4) + a/b)/b^4 + 8*a^2/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^4) - 24*a^2*x^(1/4)/(b^5*(x^(1/4) + a/b)^2) - 22*a^3/(b^6*(x^(1/4) + a/b)^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2 + b^2 \sqrt{x} + 2abx^{1/4}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2),x)

[Out] `int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 + 2ab\sqrt[4]{x} + b^2\sqrt{x})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2), x)`

[Out] `Integral((a**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))**(-3/2), x)`

$$3.416 \quad \int \frac{1}{(a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x})^{5/2}} dx$$

Optimal. Leaf size=268

$$\frac{60a^2}{b^6\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} - \frac{30a(a + b\sqrt[6]{x})\log(a + b\sqrt[6]{x})}{b^6\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} + \frac{6\sqrt[6]{x}(a + b\sqrt[6]{x})}{b^5\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} + \frac{3a^5}{2b^6(a + b\sqrt[6]{x})^3\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}}$$

Rubi [A] time = 0.15, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$\frac{3a^5}{2b^6(a + b\sqrt[6]{x})^3\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} - \frac{10a^4}{b^6(a + b\sqrt[6]{x})^2\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} + \frac{30a^3}{b^6(a + b\sqrt[6]{x})\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} - \frac{60a^2}{b^6\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} + \frac{6\sqrt[6]{x}(a + b\sqrt[6]{x})}{b^5\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} - \frac{30a(a + b\sqrt[6]{x})\log(a + b\sqrt[6]{x})}{b^6\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(5/2), x]

[Out] (-60*a^2)/(b^6*sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (3*a^5)/(2*b^6*(a + b*x^(1/6))^3*sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (10*a^4)/(b^6*(a + b*x^(1/6))^2*sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (30*a^3)/(b^6*(a + b*x^(1/6))*sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (6*(a + b*x^(1/6))*x^(1/6))/(b^5*sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (30*a*(a + b*x^(1/6))*Log[a + b*x^(1/6)])/(b^6*sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n)

))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x})^{5/2}} dx &= 6 \operatorname{Subst} \left(\int \frac{x^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx, x, \sqrt[6]{x} \right) \\
 &= \frac{(6b^5(a + b\sqrt[6]{x})) \operatorname{Subst} \left(\int \frac{x^5}{(ab+b^2x)^5} dx, x, \sqrt[6]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} \\
 &= \frac{(6b^5(a + b\sqrt[6]{x})) \operatorname{Subst} \left(\int \left(\frac{1}{b^{10}} - \frac{a^5}{b^{10}(a+bx)^5} + \frac{5a^4}{b^{10}(a+bx)^4} - \frac{10a^3}{b^{10}(a+bx)^3} + \frac{10a^2}{b^{10}(a+bx)^2} \right) dx, x, \sqrt[6]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} \\
 &= -\frac{60a^2}{b^6\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} + \frac{3a^5}{2b^6(a + b\sqrt[6]{x})^3\sqrt{a^2 + 2ab\sqrt[6]{x} + b^2\sqrt[3]{x}}} - \frac{1}{b^6(a + b\sqrt[6]{x})}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 121, normalized size = 0.45

$$\frac{-77a^5 - 248a^4b\sqrt[6]{x} - 252a^3b^2\sqrt[3]{x} - 48a^2b^3\sqrt{x} + 48ab^4x^{2/3} - 60a(a + b\sqrt[6]{x})^4 \log(a + b\sqrt[6]{x}) + 12b^5x^{5/6}}{2b^6(a + b\sqrt[6]{x})^3\sqrt{(a + b\sqrt[6]{x})^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(5/2), x]

[Out] (-77*a^5 - 248*a^4*b*x^(1/6) - 252*a^3*b^2*x^(1/3) - 48*a^2*b^3*Sqrt[x] + 48*a*b^4*x^(2/3) + 12*b^5*x^(5/6) - 60*a*(a + b*x^(1/6))^4*Log[a + b*x^(1/6)])/(2*b^6*(a + b*x^(1/6))^3*Sqrt[(a + b*x^(1/6))^2])

IntegrateAlgebraic [B] time = 2.59, size = 2887, normalized size = 10.77

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(5/2), x]


```

[Out] ((-192*a^8*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])/b^6 + (1024*a^8*x^(1/6)
)/(b^4*Sqrt[b^2]) - (832*a^7*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(1/6
))/b^5 + (5056*a^7*x^(1/3))/(b^3*Sqrt[b^2]) - (4224*a^6*Sqrt[a^2 + 2*a*b*x^(
1/6) + b^2*x^(1/3)]*x^(1/3))/b^4 + (15744*a^6*Sqrt[x])/(b^2)^(3/2) - (1152
0*a^5*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*Sqrt[x])/b^3 + (31616*a^5*x^(
2/3))/(b*Sqrt[b^2]) - (20096*a^4*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(
2/3))/b^2 + (37376*a^4*x^(5/6))/Sqrt[b^2] - (17280*a^3*Sqrt[a^2 + 2*a*b*x^(
1/6) + b^2*x^(1/3)]*x^(5/6))/b + (21504*a^3*b*x)/Sqrt[b^2] - 4224*a^2*Sqrt
[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x + 1536*a^2*Sqrt[b^2]*x^(7/6) + 2688*a
*b*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(7/6) - (3456*a*b^3*x^(4/3))/S
qrt[b^2] + 768*b^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(4/3) - (768*b
^4*x^(3/2))/Sqrt[b^2] - (3840*a^5*x^(2/3)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6)
+ b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a])/b^2 + (3840*a^4*Sqrt[a^2 + 2*a*b*x
^(1/6) + b^2*x^(1/3)]*x^(2/3)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/
3)] - Sqrt[b^2]*x^(1/6))/a])/(b*Sqrt[b^2]) - (15360*a^4*x^(5/6)*ArcTanh[(Sq
rt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a])/b + (11520*a
^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(5/6)*ArcTanh[(Sqrt[a^2 + 2*a*
b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a])/Sqrt[b^2] - 23040*a^3*x*A
rcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a] + (
11520*a^2*b*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x*ArcTanh[(Sqrt[a^2 + 2
*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a])/Sqrt[b^2] - 15360*a^2*
b*x^(7/6)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1
/6))/a] + 3840*a*Sqrt[b^2]*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(7/6)*
ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6))/a] -
3840*a*b^2*x^(4/3)*ArcTanh[(Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[
b^2]*x^(1/6))/a])/((-a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2
]*x^(1/6))^4*(a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/
6))^4) + ((-192*a^9)/(b^5*Sqrt[b^2]) - (1024*a^8*x^(1/6))/(b^4*Sqrt[b^2]) +
(1024*a^7*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(1/6))/b^5 - (5056*a^7
*x^(1/3))/(b^3*Sqrt[b^2]) + (4032*a^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3
)]*x^(1/3))/b^4 - (15744*a^6*Sqrt[x])/(b^2)^(3/2) + (11712*a^5*Sqrt[a^2 + 2
*a*b*x^(1/6) + b^2*x^(1/3)]*Sqrt[x])/b^3 - (27072*a^5*x^(2/3))/(b*Sqrt[b^2
]) + (15360*a^4*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(2/3))/b^2 - (2304
0*a^4*x^(5/6))/Sqrt[b^2] + (7680*a^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)
]*x^(5/6))/b - (7680*a^3*b*x)/Sqrt[b^2] + (1920*a^5*x^(2/3)*Log[-a + Sqrt[a
^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)])/(b*Sqrt[b^2]) - (19
20*a^4*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(2/3)*Log[-a + Sqrt[a^2 +
2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)]/b^2 + (7680*a^4*x^(5/6)*
Log[-a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)]/Sqrt
[b^2] - (5760*a^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x^(5/6)*Log[-a +
Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)]/b + (11520*a^
3*b*x*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)
])/Sqrt[b^2] - 5760*a^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]*x*Log[-a + S
qrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*x^(1/6)] + 7680*a^2*Sqrt
[b^2]*x^(7/6)*Log[-a + Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)] - Sqrt[b^2]*

```

$$\begin{aligned}
& x^{(1/6)}] - 1920*a*b*\text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}]*x^{(7/6)}*\text{Log}[-a \\
& + \text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)}] + (1920*a*b^3 \\
& *x^{(4/3)}*\text{Log}[-a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)}]) \\
& / \text{Sqrt}[b^2] + (1920*a^5*x^{(2/3)}*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}] \\
& - \text{Sqrt}[b^2]*x^{(1/6)}]) / (b*\text{Sqrt}[b^2]) - (1920*a^4*\text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} \\
& + b^2*x^{(1/3)}]*x^{(2/3)}*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}] \\
& - \text{Sqrt}[b^2]*x^{(1/6)}]) / b^2 + (7680*a^4*x^{(5/6)}*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} \\
& + b^2*x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)}]) / \text{Sqrt}[b^2] - (5760*a^3*\text{Sqrt}[a^2 \\
& + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}]*x^{(5/6)}*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} + b^2 \\
& *x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)}]) / b + (11520*a^3*b*x*\text{Log}[a + \text{Sqrt}[a^2 + 2*a \\
& *b*x^{(1/6)} + b^2*x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)}]) / \text{Sqrt}[b^2] - 5760*a^2*\text{Sqrt}[a^2 \\
& + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}]*x*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}] \\
& - \text{Sqrt}[b^2]*x^{(1/6)}] + 7680*a^2*\text{Sqrt}[b^2]*x^{(7/6)}*\text{Log}[a + \text{Sqrt}[a^2 \\
& + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)}] - 1920*a*b*\text{Sqrt}[a^2 + \\
& 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}]*x^{(7/6)}*\text{Log}[a + \text{Sqrt}[a^2 + 2*a*b*x^{(1/6)} + b^2 \\
& *x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)}] + (1920*a*b^3*x^{(4/3)}*\text{Log}[a + \text{Sqrt}[a^2 + 2*a \\
& *b*x^{(1/6)} + b^2*x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)}]) / \text{Sqrt}[b^2] / ((-a + \text{Sqrt}[a^2 \\
& + 2*a*b*x^{(1/6)} + b^2*x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)})^4*(a + \text{Sqrt}[a^2 + 2*a*b \\
& *x^{(1/6)} + b^2*x^{(1/3)}] - \text{Sqrt}[b^2]*x^{(1/6)})^4)
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.87, size = 237, normalized size = 0.88

$$\frac{3a^4|a|\log\left(\left|x^{\frac{1}{6}}|b|\text{sgn}(a)\text{sgn}(b)+|a|\right.\right)}{4(a^2b^2|a|b|\text{sgn}(a)\text{sgn}(b)-a^4b^6)} + \frac{3(24a^7b^2|b|\text{sgn}(a)\text{sgn}(b)-25a^4b^3|a|)\log\left(\left|bx^{\frac{1}{6}}+a\right.\right)}{4(a^2b^8|a|b|\text{sgn}(a)\text{sgn}(b)-a^4b^6)} + \frac{6x^{\frac{1}{6}}}{b^4|b|\text{sgn}(a)\text{sgn}(b)} + \frac{70a^2|b|\text{sgn}(a)\text{sgn}(b)-70a^4|b|+93(a^2b^2|b|\text{sgn}(a)\text{sgn}(b)-a^2b^3|a|)^{\frac{1}{3}}+159(a^4b|b|\text{sgn}(a)\text{sgn}(b)-a^2b^2|a|)^{\frac{1}{6}}}{4(|a|b|\text{sgn}(a)\text{sgn}(b)-ab)\left(bx^{\frac{1}{6}}+a\right)^{\frac{5}{6}}b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="giac")

[Out] $\frac{3}{4}a^4*abs(a)*\log(abs(x^{(1/6)}*abs(b)*sgn(a)*sgn(b) + abs(a)))/(a^3*b^5*abs(a)*abs(b)*sgn(a)*sgn(b) - a^4*b^6) + \frac{3}{4}*(24*a^5*b^2*abs(b)*sgn(a)*sgn(b) - 25*a^4*b^3*abs(a))*\log(abs(b*x^{(1/6)} + a))/(a^3*b^8*abs(a)*abs(b)*sgn(a)*sgn(b) - a^4*b^9) + 6*x^{(1/6)}/(b^4*abs(b)*sgn(a)*sgn(b)) + \frac{1}{4}*(70*a^5*abs(b)*sgn(a)*sgn(b) - 70*a^4*b*abs(a) + 93*(a^3*b^2*abs(b)*sgn(a)*sgn(b) - a^2*b^3*abs(a))*x^{(1/3)} + 159*(a^4*b*abs(b)*sgn(a)*sgn(b) - a^3*b^2*abs(a))*x^{(1/6)}/((abs(a)*abs(b)*sgn(a)*sgn(b) - a*b)*(b*x^{(1/6)} + a)^3*b^6)$

maple [A] time = 0.02, size = 174, normalized size = 0.65

$$\frac{\sqrt{b^2x^{\frac{1}{3}} + 2abx^{\frac{1}{6}} + a^2} \left(-60ab^4x^{\frac{2}{3}} \ln(bx^{\frac{1}{6}} + a) - 240a^2b^3\sqrt{x} \ln(bx^{\frac{1}{6}} + a) - 360a^3b^2x^{\frac{1}{3}} \ln(bx^{\frac{1}{6}} + a) - 240a^4bx^{\frac{1}{6}} \ln(bx^{\frac{1}{6}} + a) - 60a^5 \ln(bx^{\frac{1}{6}} + a) + 12b^5x^{\frac{5}{6}} + 48ab^4x^{\frac{2}{3}} - 48a^2b^3\sqrt{x} - 252a^3b^2x^{\frac{1}{3}} - 248a^4bx^{\frac{1}{6}} - 77a^5 \right)}{2(bx^{\frac{1}{6}} + a)^5 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2), x)

[Out] $\frac{1}{2} \cdot (a^2 + 2abx^{1/6} + b^2x^{1/3})^{1/2} \cdot (12x^{5/6}b^5 - 60x^{2/3} \ln(a + bx^{1/6})ab^4 + 48x^{2/3}ab^4 - 240x^{1/2} \ln(a + bx^{1/6})a^2b^3 - 48x^{1/2}a^2b^3 - 360x^{1/3} \ln(a + bx^{1/6})a^3b^2 - 252x^{1/3}a^3b^2 - 240x^{1/6} \ln(a + bx^{1/6})a^4b - 248x^{1/6}a^4b - 60 \ln(a + bx^{1/6})a^5 - 77a^5) / (a + bx^{1/6})^5 / b^6$

maxima [A] time = 0.88, size = 119, normalized size = 0.44

$$\frac{12b^5x^{\frac{5}{6}} + 48ab^4x^{\frac{2}{3}} - 48a^2b^3\sqrt{x} - 252a^3b^2x^{\frac{1}{3}} - 248a^4bx^{\frac{1}{6}} - 77a^5}{2(b^{10}x^{\frac{2}{3}} + 4ab^9\sqrt{x} + 6a^2b^8x^{\frac{1}{3}} + 4a^3b^7x^{\frac{1}{6}} + a^4b^6)} - \frac{30a \log(bx^{\frac{1}{6}} + a)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2), x, algorithm="maxima")

[Out] $\frac{1}{2} \cdot (12b^5x^{5/6} + 48ab^4x^{2/3} - 48a^2b^3\sqrt{x} - 252a^3b^2x^{1/3} - 248a^4bx^{1/6} - 77a^5) / (b^{10}x^{2/3} + 4a^2b^8x^{1/3} + 4a^3b^7x^{1/6} + a^4b^6) - 30a \log(bx^{1/6} + a) / b^6$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a^2 + b^2x^{1/3} + 2abx^{1/6})^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2), x)

[Out] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2), x)

[Out] Timed out

$$3.417 \quad \int \left(a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}} \right)^{3/2} dx$$

Optimal. Leaf size=179

$$\frac{6a^2b\sqrt{x}\sqrt{a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}}}{a + \frac{b}{\sqrt{x}}} + \frac{6ab^2 \log(\sqrt{x})\sqrt{a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}}}{a + \frac{b}{\sqrt{x}}} - \frac{2b^3\sqrt{a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}}}{\sqrt{x}\left(a + \frac{b}{\sqrt{x}}\right)} + \frac{a^3x\sqrt{a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}}}{a + \frac{b}{\sqrt{x}}}$$

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1341, 1355, 263, 43}

$$\frac{a^3x\sqrt{a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}}}{a + \frac{b}{\sqrt{x}}} + \frac{6a^2b\sqrt{x}\sqrt{a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}}}{a + \frac{b}{\sqrt{x}}} - \frac{2b^3\sqrt{a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}}}{\sqrt{x}\left(a + \frac{b}{\sqrt{x}}\right)} + \frac{6ab^2 \log(\sqrt{x})\sqrt{a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}}}{a + \frac{b}{\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2), x]

[Out] (-2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]/((a + b/Sqrt[x])*Sqrt[x]) + (6*a^2*b*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Sqrt[x])/(a + b/Sqrt[x]) + (a^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*x)/(a + b/Sqrt[x]) + (6*a*b^2*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Log[Sqrt[x]])/(a + b/Sqrt[x])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra

ctionQ[n]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]

Rubi steps

$$\begin{aligned}
 \int \left(a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}} \right)^{3/2} dx &= 2 \operatorname{Subst} \left(\int \left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x} \right)^{3/2} x dx, x, \sqrt{x} \right) \\
 &= \frac{\left(2\sqrt{a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}}} \right) \operatorname{Subst} \left(\int \left(ab + \frac{b^2}{x} \right)^3 x dx, x, \sqrt{x} \right)}{b^2 \left(ab + \frac{b^2}{\sqrt{x}} \right)} \\
 &= \frac{\left(2\sqrt{a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}}} \right) \operatorname{Subst} \left(\int \frac{(b^2+abx)^3}{x^2} dx, x, \sqrt{x} \right)}{b^2 \left(ab + \frac{b^2}{\sqrt{x}} \right)} \\
 &= \frac{\left(2\sqrt{a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}}} \right) \operatorname{Subst} \left(\int \left(3a^2b^4 + \frac{b^6}{x^2} + \frac{3ab^5}{x} + a^3b^3x \right) dx, x, \sqrt{x} \right)}{b^2 \left(ab + \frac{b^2}{\sqrt{x}} \right)} \\
 &= -\frac{2b^4 \sqrt{a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}}}}{\left(ab + \frac{b^2}{\sqrt{x}} \right) \sqrt{x}} + \frac{6a^2b^2 \sqrt{a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}}} \sqrt{x}}{ab + \frac{b^2}{\sqrt{x}}} + \frac{a^3 \sqrt{a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}}} x}{a + \frac{b}{\sqrt{x}}} + \frac{3ab^3 \sqrt{a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}}}}{a + \frac{b}{\sqrt{x}}}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.37

$$\frac{\sqrt{\frac{(a\sqrt{x}+b)^2}{x}} \left(a^3x^{3/2} + 6a^2bx + 3ab^2\sqrt{x} \log(x) - 2b^3 \right)}{a\sqrt{x} + b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2), x]

[Out] (Sqrt[(b + a*Sqrt[x])^2/x]*(-2*b^3 + 6*a^2*b*x + a^3*x^(3/2) + 3*a*b^2*Sqrt[x]*Log[x]))/(b + a*Sqrt[x])

IntegrateAlgebraic [A] time = 5.93, size = 77, normalized size = 0.43

$$\frac{\sqrt{x} \sqrt{\frac{(a\sqrt{x}+b)^2}{x}} \left(\frac{a^3 x^{3/2} + 6a^2 b x - 2b^3}{\sqrt{x}} + 6ab^2 \log(\sqrt{x}) \right)}{a\sqrt{x} + b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2), x]

[Out] (Sqrt[(b + a*Sqrt[x])^2/x]*Sqrt[x]*((-2*b^3 + 6*a^2*b*x + a^3*x^(3/2))/Sqrt[x] + 6*a*b^2*Log[Sqrt[x]]))/(b + a*Sqrt[x])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.39, size = 80, normalized size = 0.45

$$a^3 x \operatorname{sgn}(ax + b\sqrt{x}) \operatorname{sgn}(x) + 3ab^2 \log(|x|) \operatorname{sgn}(ax + b\sqrt{x}) \operatorname{sgn}(x) + 6a^2 b \sqrt{x} \operatorname{sgn}(ax + b\sqrt{x}) \operatorname{sgn}(x) - \frac{2b^3 \operatorname{sgn}(ax + b\sqrt{x}) \operatorname{sgn}(x)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2), x, algorithm="giac")

[Out] a^3*x*sgn(a*x + b*sqrt(x))*sgn(x) + 3*a*b^2*log(abs(x))*sgn(a*x + b*sqrt(x))*sgn(x) + 6*a^2*b*sqrt(x)*sgn(a*x + b*sqrt(x))*sgn(x) - 2*b^3*sgn(a*x + b*sqrt(x))*sgn(x)/sqrt(x)

maple [A] time = 0.03, size = 68, normalized size = 0.38

$$\frac{\sqrt{\frac{a^2 x^{\frac{3}{2}} + 2abx + b^2 \sqrt{x}}{x^2}} \left(a^3 x^{\frac{3}{2}} + 3a b^2 \sqrt{x} \ln(x) + 6a^2 b x - 2b^3 \right)}{a\sqrt{x} + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x)`

[Out] $((a^2x^{3/2}+b^2x^{1/2}+2axb)/x^{3/2})^{1/2}*(3ab^2\ln(x)x^{1/2}+6a^2bx+x^{3/2}a^3-2b^3)/(ax^{1/2}+b)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3x + 3ab^2 \int \frac{1}{x} dx + 6a^2b\sqrt{x} - \frac{2b^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="maxima")`

[Out] $a^3x + 3ab^2\text{integrate}(1/x, x) + 6a^2b\text{sqrt}(x) - 2b^3/\text{sqrt}(x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + \frac{b^2}{x} + \frac{2ab}{\sqrt{x}} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2),x)`

[Out] `int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)`

[Out] `Integral((a**2 + 2*a*b/sqrt(x) + b**2/x)**(3/2), x)`

$$3.418 \quad \int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}} \right)^{7/2} dx$$

Optimal. Leaf size=391

$$\frac{3b^7 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{4x^{4/3} \left(a + \frac{b}{\sqrt[3]{x}} \right)} - \frac{7ab^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{x \left(a + \frac{b}{\sqrt[3]{x}} \right)} - \frac{63a^2b^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2x^{2/3} \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{a^7 x \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} + \frac{21a^6bx^{2/3} \sqrt{a^2}}{2 \left(a + \frac{b}{\sqrt[3]{x}} \right)}$$

Rubi [A] time = 0.19, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1341, 1355, 263, 43}

$$\frac{a^7 x \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} + \frac{21a^6bx^{2/3} \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2 \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{63a^2b^5 \sqrt[3]{x} \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} - \frac{105a^3b^4 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{\sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}} \right)} - \frac{63a^2b^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2x^{2/3} \left(a + \frac{b}{\sqrt[3]{x}} \right)} - \frac{7ab^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{x \left(a + \frac{b}{\sqrt[3]{x}} \right)} - \frac{3b^7 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{4x^{4/3} \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{105a^4b^3 \log(\sqrt[3]{x}) \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x]

[Out] (-3*b^7*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]/(4*(a + b/x^(1/3))*x^(4/3)) - (7*a*b^6*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]/((a + b/x^(1/3))*x) - (63*a^2*b^5*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]/(2*(a + b/x^(1/3))*x^(2/3)) - (105*a^3*b^4*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]/((a + b/x^(1/3))*x^(1/3)) + (63*a^5*b^2*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3)) + (21*a^6*b*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^7*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3)) + (105*a^4*b^3*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)]/(a + b/x^(1/3)))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341


```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k
= Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n)
)^(p), x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
 \int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}} \right)^{7/2} dx &= 3 \operatorname{Subst} \left(\int \left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x} \right)^{7/2} x^2 dx, x, \sqrt[3]{x} \right) \\
 &= \frac{\left(3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \left(ab + \frac{b^2}{x} \right)^7 x^2 dx, x, \sqrt[3]{x} \right)}{b^6 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} \\
 &= \frac{\left(3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \frac{(b^2 + abx)^7}{x^5} dx, x, \sqrt[3]{x} \right)}{b^6 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} \\
 &= \frac{\left(3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \left(21a^5b^9 + \frac{b^{14}}{x^5} + \frac{7ab^{13}}{x^4} + \frac{21a^2b^{12}}{x^3} + \frac{35a^3b^{11}}{x^2} + \frac{35a^4b^{10}}{x} + \right) dx, x, \sqrt[3]{x} \right)}{b^6 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} \\
 &= \frac{3b^8 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{4 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right) x^{4/3}} - \frac{7ab^7 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{\left(ab + \frac{b^2}{\sqrt[3]{x}} \right) x} - \frac{63a^2b^6 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{2 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right) x^{2/3}} - \frac{105a^3b^5 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{\left(ab + \frac{b^2}{\sqrt[3]{x}} \right) x^{1/3}} - \frac{35a^4b^4 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{\left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} - \frac{35a^5b^3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{\left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} - \frac{35a^6b^2 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{\left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} - \frac{35a^7b \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{\left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} - \frac{35a^8 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{\left(ab + \frac{b^2}{\sqrt[3]{x}} \right)}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 125, normalized size = 0.32

$$\frac{\sqrt{\frac{(a\sqrt[3]{x}+b)^2}{x^{2/3}}} (4a^7x^{7/3} + 42a^6bx^2 + 252a^5b^2x^{5/3} + 140a^4b^3x^{4/3} \log(x) - 420a^3b^4x - 126a^2b^5x^{2/3} - 28ab^6\sqrt[3]{x} - 3b^7)}{4x(a\sqrt[3]{x}+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x]

[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(-3*b^7 - 28*a*b^6*x^(1/3) - 126*a^2*b^5*x^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^7*x^(7/3) + 140*a^4*b^3*x^(4/3)*Log[x]))/(4*(b + a*x^(1/3))*x)

IntegrateAlgebraic [A] time = 6.54, size = 133, normalized size = 0.34

$$\frac{\sqrt[3]{x} \sqrt{\frac{(a\sqrt[3]{x}+b)^2}{x^{2/3}}} \left(105a^4b^3 \log(\sqrt[3]{x}) + \frac{4a^7x^{7/3} + 42a^6bx^2 + 252a^5b^2x^{5/3} - 420a^3b^4x - 126a^2b^5x^{2/3} - 28ab^6\sqrt[3]{x} - 3b^7}{4x^{4/3}} \right)}{a\sqrt[3]{x} + b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x]

[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((-3*b^7 - 28*a*b^6*x^(1/3) - 126*a^2*b^5*x^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^7*x^(7/3))/(4*x^(4/3)) + 105*a^4*b^3*Log[x^(1/3)]))/(b + a*x^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.54, size = 173, normalized size = 0.44

$$a^7x\operatorname{sgn}(ax+bx^{\frac{2}{3}})+35a^4b^3\log(x)\operatorname{sgn}(ax+bx^{\frac{2}{3}})+\frac{21}{2}a^6b^2x^{\frac{2}{3}}\operatorname{sgn}(ax+bx^{\frac{2}{3}})+63a^5b^2x^{\frac{1}{3}}\operatorname{sgn}(ax+bx^{\frac{2}{3}})-\frac{420a^3b^4x+126a^2b^5x^{\frac{2}{3}}+28ab^6x^{\frac{1}{3}}+3b^7}{4x^{\frac{4}{3}}}\operatorname{sgn}(ax+bx^{\frac{2}{3}})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2), x, algorithm="giac")

[Out] a^7*x*sgn(a*x + b*x^(2/3))*sgn(x) + 35*a^4*b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*sgn(x) + 21/2*a^6*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 63*a^5*b^2*x^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 1/4*(420*a^3*b^4*x*sgn(a*x + b*x^(2/3))*sgn(x) + 126*a^2*b^5*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 28*a*b^6*x^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 3*b^7*sgn(a*x + b*x^(2/3))*sgn(x))/x^(4/3)

maple [A] time = 0.03, size = 115, normalized size = 0.29

$$\frac{\left(\frac{a^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} \left(4a^7x^{\frac{10}{3}} + 140a^4b^3x^{\frac{7}{3}} \ln(x) + 42a^6bx^3 + 252a^5b^2x^{\frac{8}{3}} - 420a^3b^4x^2 - 126a^2b^5x^{\frac{5}{3}} - 28ab^6x^{\frac{4}{3}} - 3b^7x\right)}{4\left(ax^{\frac{1}{3}} + b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2), x)

[Out] 1/4*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(7/2)*(42*a^6*b*x^3+140*a^4*b^3*ln(x)*x^(7/3)+252*a^5*b^2*x^(8/3)+4*a^7*x^(10/3)-28*a*b^6*x^(4/3)-420*a^3*b^4*x^2-126*a^2*b^5*x^(5/3)-3*b^7*x)/(b+a*x^(1/3))^7

maxima [A] time = 0.60, size = 79, normalized size = 0.20

$$35a^4b^3 \log(x) + \frac{4a^7x^{\frac{7}{3}} + 42a^6bx^2 + 252a^5b^2x^{\frac{5}{3}} - 420a^3b^4x - 126a^2b^5x^{\frac{2}{3}} - 28ab^6x^{\frac{1}{3}} - 3b^7}{4x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2), x, algorithm="maxima")

[Out] 35*a^4*b^3*log(x) + 1/4*(4*a^7*x^(7/3) + 42*a^6*b*x^2 + 252*a^5*b^2*x^(5/3) - 420*a^3*b^4*x - 126*a^2*b^5*x^(2/3) - 28*a*b^6*x^(1/3) - 3*b^7)/x^(4/3)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{x^{1/3}}\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x)

[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2), x)

[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(7/2), x)

$$3.419 \quad \int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}} \right)^{5/2} dx$$

Optimal. Leaf size=291

$$\frac{3b^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2x^{2/3} \left(a + \frac{b}{\sqrt[3]{x}} \right)} - \frac{15ab^4 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{\sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{30a^2b^3 \log(\sqrt[3]{x}) \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} + \frac{a^5x \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} + \frac{15a^4bx^{2/3} \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2 \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{30a^3b^2 \sqrt[3]{x} \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} - \frac{15ab^4 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{\sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}} \right)} - \frac{3b^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2x^{2/3} \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{30a^2b^3 \log(\sqrt[3]{x}) \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}}$$

Rubi [A] time = 0.14, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1341, 1355, 263, 43}

$$\frac{a^5x \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} + \frac{15a^4bx^{2/3} \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2 \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{30a^3b^2 \sqrt[3]{x} \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} - \frac{15ab^4 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{\sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}} \right)} - \frac{3b^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2x^{2/3} \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{30a^2b^3 \log(\sqrt[3]{x}) \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x]

[Out] (-3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]/(2*(a + b/x^(1/3))*x^(2/3)) - (15*a*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]/((a + b/x^(1/3))*x^(1/3)) + (30*a^3*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3)) + (15*a^4*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3)) + (30*a^2*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)]/(a + b/x^(1/3)))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n)

)^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p]))], Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
 \int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}} \right)^{5/2} dx &= 3 \operatorname{Subst} \left(\int \left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x} \right)^{5/2} x^2 dx, x, \sqrt[3]{x} \right) \\
 &= \frac{\left(3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \left(ab + \frac{b^2}{x} \right)^5 x^2 dx, x, \sqrt[3]{x} \right)}{b^4 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} \\
 &= \frac{\left(3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \frac{(b^2 + abx)^5}{x^3} dx, x, \sqrt[3]{x} \right)}{b^4 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} \\
 &= \frac{\left(3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \left(10a^3b^7 + \frac{b^{10}}{x^3} + \frac{5ab^9}{x^2} + \frac{10a^2b^8}{x} + 5a^4b^6x + a^5b^5x^2 \right) dx, x, \sqrt[3]{x} \right)}{b^4 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} \\
 &= -\frac{3b^6 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{2 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right) x^{2/3}} - \frac{15ab^5 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{\left(ab + \frac{b^2}{\sqrt[3]{x}} \right) \sqrt[3]{x}} + \frac{30a^3b^3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}{ab + \frac{b^2}{\sqrt[3]{x}}} \sqrt[3]{x}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.34

$$\frac{(a\sqrt[3]{x} + b) \left(2a^5x^{5/3} + 15a^4bx^{4/3} + 60a^3b^2x + 20a^2b^3x^{2/3} \log(x) - 30ab^4\sqrt[3]{x} - 3b^5 \right)}{2x \sqrt{\frac{(a\sqrt[3]{x} + b)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x]

[Out] ((b + a*x^(1/3))*(-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a^4*b*x^(4/3) + 2*a^5*x^(5/3) + 20*a^2*b^3*x^(2/3)*Log[x]))/(2*sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x)

IntegrateAlgebraic [A] time = 6.28, size = 109, normalized size = 0.37

$$\frac{\sqrt[3]{x} \sqrt{\frac{(a\sqrt[3]{x}+b)^2}{x^{2/3}}} \left(30a^2b^3 \log(\sqrt[3]{x}) + \frac{2a^5x^{5/3}+15a^4bx^{4/3}+60a^3b^2x-30ab^4\sqrt[3]{x}-3b^5}{2x^{2/3}} \right)}{a\sqrt[3]{x} + b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x]

[Out] (sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a^4*b*x^(4/3) + 2*a^5*x^(5/3))/(2*x^(2/3)) + 30*a^2*b^3*Log[x^(1/3)]))/(b + a*x^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.51, size = 128, normalized size = 0.44

$$a^5x\operatorname{sgn}(ax+bx^{2/3})\operatorname{sgn}(x) + 10a^2b^3\log(|x|)\operatorname{sgn}(ax+bx^{2/3})\operatorname{sgn}(x) + \frac{15}{2}a^4bx^{2/3}\operatorname{sgn}(ax+bx^{2/3})\operatorname{sgn}(x) + 30a^3b^2x^{1/3}\operatorname{sgn}(ax+bx^{2/3})\operatorname{sgn}(x) - \frac{3(10ab^4x^{1/3}\operatorname{sgn}(ax+bx^{2/3})\operatorname{sgn}(x) + b^5\operatorname{sgn}(ax+bx^{2/3})\operatorname{sgn}(x))}{2x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2), x, algorithm="giac")

[Out] a^5*x*sgn(a*x + b*x^(2/3))*sgn(x) + 10*a^2*b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*sgn(x) + 15/2*a^4*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 30*a^3*b^2*x^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 3/2*(10*a*b^4*x^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) + b^5*sgn(a*x + b*x^(2/3))*sgn(x))/x^(2/3)

maple [A] time = 0.01, size = 91, normalized size = 0.31

$$\frac{\left(\frac{a^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}} \left(2a^5x^{\frac{5}{3}} + 20a^2b^3x^{\frac{2}{3}} \ln(x) + 15a^4bx^{\frac{4}{3}} + 60a^3b^2x - 30ab^4x^{\frac{1}{3}} - 3b^5\right)x}{2\left(ax^{\frac{1}{3}} + b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2), x)

[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)*x*(15*a^4*b*x^(4/3)+60*a^3*b^2*x+20*a^2*b^3*ln(x)*x^(2/3)+2*a^5*x^(5/3)-30*a*b^4*x^(1/3)-3*b^5)/(a*x^(1/3)+b)^5

maxima [A] time = 0.64, size = 57, normalized size = 0.20

$$10a^2b^3 \log(x) + \frac{2a^5x^{\frac{5}{3}} + 15a^4bx^{\frac{4}{3}} + 60a^3b^2x - 30ab^4x^{\frac{1}{3}} - 3b^5}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2), x, algorithm="maxima")

[Out] 10*a^2*b^3*log(x) + 1/2*(2*a^5*x^(5/3) + 15*a^4*b*x^(4/3) + 60*a^3*b^2*x - 30*a*b^4*x^(1/3) - 3*b^5)/x^(2/3)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{x^{1/3}} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)

[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2), x)
```


$$3.420 \quad \int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}} \right)^{3/2} dx$$

Optimal. Leaf size=189

$$\frac{9a^2bx^{2/3}\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2\left(a + \frac{b}{\sqrt[3]{x}}\right)} + \frac{9ab^2\sqrt[3]{x}\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} + \frac{3b^3 \log(\sqrt[3]{x})\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} + \frac{a^3x\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}}$$

Rubi [A] time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1341, 1355, 263, 43}

$$\frac{a^3x\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} + \frac{9a^2bx^{2/3}\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2\left(a + \frac{b}{\sqrt[3]{x}}\right)} + \frac{9ab^2\sqrt[3]{x}\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}} + \frac{3b^3 \log(\sqrt[3]{x})\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x]

[Out] (9*a*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3)) + (9*a^2*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3)) + (3*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra

ctionQ[n]

Rule 1355

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_),
 x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
 c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
 a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
 [p - 1/2]

Rubi steps

$$\begin{aligned}
 \int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}} \right)^{3/2} dx &= 3 \operatorname{Subst} \left(\int \left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x} \right)^{3/2} x^2 dx, x, \sqrt[3]{x} \right) \\
 &= \frac{\left(3\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \left(ab + \frac{b^2}{x} \right)^3 x^2 dx, x, \sqrt[3]{x} \right)}{b^2 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} \\
 &= \frac{\left(3\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \frac{(b^2+abx)^3}{x} dx, x, \sqrt[3]{x} \right)}{b^2 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} \\
 &= \frac{\left(3\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \left(3ab^5 + \frac{b^6}{x} + 3a^2b^4x + a^3b^3x^2 \right) dx, x, \sqrt[3]{x} \right)}{b^2 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} \\
 &= \frac{9ab^3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \sqrt[3]{x}}{ab + \frac{b^2}{\sqrt[3]{x}}} + \frac{9a^2b^2 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} x^{2/3}}{2 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} + \frac{a^3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} x}{a + \frac{b}{\sqrt[3]{x}}} +
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.41

$$\frac{\sqrt{\frac{(a\sqrt[3]{x}+b)^2}{x^{2/3}}} (2a^3x^{4/3} + 9a^2bx + 18ab^2x^{2/3} + 2b^3\sqrt[3]{x} \log(x))}{2(a\sqrt[3]{x} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x]

[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(18*a*b^2*x^(2/3) + 9*a^2*b*x + 2*a^3*x^(4/3) + 2*b^3*x^(1/3)*Log[x]))/(2*(b + a*x^(1/3)))

IntegrateAlgebraic [A] time = 5.98, size = 85, normalized size = 0.45

$$\frac{\sqrt[3]{x} \sqrt{\frac{(a\sqrt[3]{x}+b)^2}{x^{2/3}}} \left(\frac{1}{2} a \sqrt[3]{x} (2a^2 x^{2/3} + 9ab\sqrt[3]{x} + 18b^2) + 3b^3 \log(\sqrt[3]{x}) \right)}{a\sqrt[3]{x} + b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x]

[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((a*(18*b^2 + 9*a*b*x^(1/3) + 2*a^2*x^(2/3))*x^(1/3))/2 + 3*b^3*Log[x^(1/3)]))/(b + a*x^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.41, size = 79, normalized size = 0.42

$$a^3 x \operatorname{sgn}(ax + bx^{\frac{2}{3}}) \operatorname{sgn}(x) + b^3 \log(|x|) \operatorname{sgn}(ax + bx^{\frac{2}{3}}) \operatorname{sgn}(x) + \frac{9}{2} a^2 b x^{\frac{2}{3}} \operatorname{sgn}(ax + bx^{\frac{2}{3}}) \operatorname{sgn}(x) + 9 a b^2 x^{\frac{1}{3}} \operatorname{sgn}(ax + bx^{\frac{2}{3}}) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2), x, algorithm="giac")

[Out] a^3*x*sgn(a*x + b*x^(2/3))*sgn(x) + b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*sgn(x) + 9/2*a^2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 9*a*b^2*x^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x)

maple [A] time = 0.00, size = 69, normalized size = 0.37

$$\frac{\left(\frac{a^2 x^{\frac{2}{3}} + 2 a b x^{\frac{1}{3}} + b^2}{x^{\frac{2}{3}}} \right)^{\frac{3}{2}} \left(2 a^3 x + 2 b^3 \ln(x) + 9 a^2 b x^{\frac{2}{3}} + 18 a b^2 x^{\frac{1}{3}} \right) x}{2 \left(a x^{\frac{1}{3}} + b \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)`

[Out] $\frac{1}{2} * ((a^2 * x^{2/3} + 2 * a * b * x^{1/3} + b^2) / x^{2/3})^{3/2} * x * (9 * a^2 * b * x^{2/3} + 18 * a * b^2 * x^{1/3} + 2 * b^3 * \ln(x) + 2 * a^3 * x) / (a * x^{1/3} + b)^3$

maxima [A] time = 0.66, size = 30, normalized size = 0.16

$$a^3x + b^3 \log(x) + \frac{9}{2} a^2bx^{\frac{2}{3}} + 9ab^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")`

[Out] $a^3x + b^3 \log(x) + 9/2 * a^2 * b * x^{2/3} + 9 * a * b^2 * x^{1/3}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{x^{1/3}} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)`

[Out] `int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)`

[Out] `Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2), x)`

$$3.421 \quad \int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} dx$$

Optimal. Leaf size=88

$$\frac{3bx^{2/3} \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2 \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{ax \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}}$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 1355, 14}

$$\frac{3bx^{2/3} \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{2 \left(a + \frac{b}{\sqrt[3]{x}} \right)} + \frac{ax \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}{a + \frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]

[Out] (3*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))

Rule 14

Int[(u_)*((c_)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1341

Int[((a_) + (c_)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rule 1355

Int[((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.) + (c_)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ

[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} dx &= 3 \operatorname{Subst} \left(\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}} x^2 dx, x, \sqrt[3]{x} \right) \\
&= \frac{\left(3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int \left(ab + \frac{b^2}{x} \right) x^2 dx, x, \sqrt[3]{x} \right)}{ab + \frac{b^2}{\sqrt[3]{x}}} \\
&= \frac{\left(3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} \right) \operatorname{Subst} \left(\int (b^2 x + ab x^2) dx, x, \sqrt[3]{x} \right)}{ab + \frac{b^2}{\sqrt[3]{x}}} \\
&= \frac{3b^2 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} x^{2/3}}{2 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right)} + \frac{a \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} x}{a + \frac{b}{\sqrt[3]{x}}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.56

$$\frac{\sqrt{\frac{(a\sqrt[3]{x}+b)^2}{x^{2/3}}} (2ax^{4/3} + 3bx)}{2(a\sqrt[3]{x} + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)], x]

[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(3*b*x + 2*a*x^(4/3)))/(2*(b + a*x^(1/3)))

IntegrateAlgebraic [A] time = 5.30, size = 54, normalized size = 0.61

$$\frac{\sqrt[3]{x} \sqrt{\frac{(a\sqrt[3]{x}+b)^2}{x^{2/3}}} (2ax + 3bx^{2/3})}{2(a\sqrt[3]{x} + b)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)], x]

[Out] $(\text{Sqrt}[(b + a*x^{(1/3)})^2/x^{(2/3)}]*x^{(1/3)}*(3*b*x^{(2/3)} + 2*a*x))/(2*(b + a*x^{(1/3)}))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")`

[Out] Timed out

giac [A] time = 0.40, size = 34, normalized size = 0.39

$$ax \operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sgn}(x) + \frac{3}{2} bx^{\frac{2}{3}} \operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")`

[Out] $a*x*\operatorname{sgn}(a*x + b*x^{(2/3)})*\operatorname{sgn}(x) + 3/2*b*x^{(2/3)}*\operatorname{sgn}(a*x + b*x^{(2/3)})*\operatorname{sgn}(x)$

maple [A] time = 0.00, size = 50, normalized size = 0.57

$$\frac{\sqrt{\frac{a^2 x^{\frac{2}{3}} + 2ab x^{\frac{1}{3}} + b^2}{x^{\frac{2}{3}}}} \left(2ax + 3b x^{\frac{2}{3}}\right) x^{\frac{1}{3}}}{2a x^{\frac{1}{3}} + 2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)`

[Out] $1/2*((a^2*x^{(2/3)}+2*a*b*x^{(1/3)}+b^2)/x^{(2/3)})^{(1/2)}*x^{(1/3)}*(3*x^{(2/3)}*b+2*a*x)/(a*x^{(1/3)}+b)$

maxima [A] time = 0.73, size = 10, normalized size = 0.11

$$ax + \frac{3}{2} bx^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")`

[Out] $a*x + 3/2*b*x^{(2/3)}$

mupad [B] time = 1.43, size = 39, normalized size = 0.44

$$\frac{x \left(a + \frac{3b}{2x^{1/3}} \right) \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{x^{1/3}}}}{a + \frac{b}{x^{1/3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2), x)

[Out] (x*(a + (3*b)/(2*x^(1/3)))*(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2))/(a + b/x^(1/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2), x)

[Out] Integral(sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)

$$3.422 \quad \int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} dx$$

Optimal. Leaf size=190

$$\frac{3bx^{2/3} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{2a^2 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} + \frac{x \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} - \frac{3b^3 \left(a + \frac{b}{\sqrt[3]{x}}\right) \log(a\sqrt[3]{x} + b)}{a^4 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} + \frac{3b^2 \sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^3 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}$$

Rubi [A] time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^2 \sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^3 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} - \frac{3bx^{2/3} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{2a^2 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} + \frac{x \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} - \frac{3b^3 \left(a + \frac{b}{\sqrt[3]{x}}\right) \log(a\sqrt[3]{x} + b)}{a^4 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]

[Out] (3*b^2*(a + b/x^(1/3))*x^(1/3))/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (3*b*(a + b/x^(1/3))*x^(2/3))/(2*a^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (3*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n)

)^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}}} dx, x, \sqrt[3]{x} \right) \\
 &= \frac{\left(3 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right) \right) \operatorname{Subst} \left(\int \frac{x^2}{ab + \frac{b^2}{x}} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \\
 &= \frac{\left(3 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right) \right) \operatorname{Subst} \left(\int \frac{x^3}{b^2 + abx} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \\
 &= \frac{\left(3 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right) \right) \operatorname{Subst} \left(\int \left(\frac{b}{a^3} - \frac{x}{a^2} + \frac{x^2}{ab} - \frac{b^2}{a^3(b+ax)} \right) dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \\
 &= \frac{3 \left(ab^2 + \frac{b^3}{\sqrt[3]{x}} \right) \sqrt[3]{x}}{a^3 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} - \frac{3 \left(ab + \frac{b^2}{\sqrt[3]{x}} \right) x^{2/3}}{2a^2 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a + \frac{b}{\sqrt[3]{x}} \right) x}{a \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} - \frac{3 \left(ab^3 + \frac{b^4}{\sqrt[3]{x}} \right) \log}{a^4 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 86, normalized size = 0.45

$$\frac{\left(a \sqrt[3]{x} + b \right) \left(2a^3 x - 3a^2 b x^{2/3} - 6b^3 \log \left(a \sqrt[3]{x} + b \right) + 6ab^2 \sqrt[3]{x} \right)}{2a^4 \sqrt[3]{x} \sqrt{\frac{\left(a \sqrt[3]{x} + b \right)^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]

[Out] ((b + a*x^(1/3))*(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6*b^3*Log[b + a*x^(1/3)]))/(2*a^4*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))

IntegrateAlgebraic [A] time = 4.80, size = 94, normalized size = 0.49

$$\frac{\sqrt[3]{x} \sqrt{\frac{(a\sqrt[3]{x}+b)^2}{x^{2/3}}} \left(\frac{\sqrt[3]{x}(2a^2x^{2/3}-3ab\sqrt[3]{x}+6b^2)}{2a^3} - \frac{3b^3 \log(a\sqrt[3]{x}+b)}{a^4} \right)}{a\sqrt[3]{x} + b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]

[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*(((6*b^2 - 3*a*b*x^(1/3) + 2*a^2*x^(2/3))*x^(1/3))/(2*a^3) - (3*b^3*Log[b + a*x^(1/3)]/a^4))/(b + a*x^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.59, size = 77, normalized size = 0.41

$$-\frac{3b^3 \log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^4 \operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sgn}(x)} + \frac{2a^2x - 3abx^{\frac{2}{3}} + 6b^2x^{\frac{1}{3}}}{2a^3 \operatorname{sgn}\left(ax + bx^{\frac{2}{3}}\right) \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")

[Out] -3*b^3*log(abs(a*x^(1/3) + b))/(a^4*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/(a^3*sgn(a*x + b*x^(2/3))*sgn(x))

maple [A] time = 0.01, size = 78, normalized size = 0.41

$$\frac{\left(ax^{\frac{1}{3}} + b\right)\left(-2a^3x + 6b^3 \ln\left(ax^{\frac{1}{3}} + b\right) + 3a^2bx^{\frac{2}{3}} - 6ab^2x^{\frac{1}{3}}\right)}{2\sqrt{\frac{a^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + b^2}{x^{\frac{2}{3}}}} a^4x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2), x)

[Out] -1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)/x^(1/3)*(a*x^(1/3)+b)*(3*a^2*b*x^(2/3)-6*a*b^2*x^(1/3)+6*b^3*ln(a*x^(1/3)+b)-2*a^3*x)/a^4

maxima [A] time = 0.74, size = 44, normalized size = 0.23

$$-\frac{3b^3 \log\left(ax^{\frac{1}{3}} + b\right)}{a^4} + \frac{2a^2x - 3abx^{\frac{2}{3}} + 6b^2x^{\frac{1}{3}}}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2), x, algorithm="maxima")

[Out] -3*b^3*log(a*x^(1/3) + b)/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/a^3

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{x^{1/3}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2), x)

[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2), x)

[Out] Integral(1/sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)

$$3.423 \quad \int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}\right)^{3/2}} dx$$

Optimal. Leaf size=300

$$\frac{3b^5 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{2a^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)^2} - \frac{15b^4 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)} - \frac{30b^3 \left(a + \frac{b}{\sqrt[3]{x}}\right) \log(a\sqrt[3]{x} + b)}{a^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} + \frac{18b^2 \sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}$$

Rubi [A] time = 0.19, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^5 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{2a^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)^2} - \frac{15b^4 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)} + \frac{18b^2 \sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} - \frac{9bx^{2/3} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{2a^4 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} + \frac{x \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^3 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} - \frac{30b^3 \left(a + \frac{b}{\sqrt[3]{x}}\right) \log(a\sqrt[3]{x} + b)}{a^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x]

[Out] (3*b^5*(a + b/x^(1/3)))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^2 - (15*b^4*(a + b/x^(1/3)))/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (18*b^2*(a + b/x^(1/3))*x^(1/3))/(a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (9*b*(a + b/x^(1/3))*x^(2/3))/(2*a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (30*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n)

)^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rule 1355

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}\right)^{3/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^2}{\left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}\right)^{3/2}} dx, x, \sqrt[3]{x} \right) \\
 &= \frac{\left(3b^2 \left(ab + \frac{b^2}{\sqrt[3]{x}}\right)\right) \operatorname{Subst} \left(\int \frac{x^2}{\left(ab + \frac{b^2}{x}\right)^3} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \\
 &= \frac{\left(3b^2 \left(ab + \frac{b^2}{\sqrt[3]{x}}\right)\right) \operatorname{Subst} \left(\int \frac{x^5}{(b^2 + abx)^3} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \\
 &= \frac{\left(3b^2 \left(ab + \frac{b^2}{\sqrt[3]{x}}\right)\right) \operatorname{Subst} \left(\int \left(\frac{6}{a^5b} - \frac{3x}{a^4b^2} + \frac{x^2}{a^3b^3} - \frac{b^2}{a^5(b+ax)^3} + \frac{5b}{a^5(b+ax)^2} - \frac{10}{a^5(b+ax)} \right) dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \\
 &= \frac{3 \left(ab^5 + \frac{b^6}{\sqrt[3]{x}}\right)}{2a^6 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} (b + a\sqrt[3]{x})^2} - \frac{15 \left(ab^4 + \frac{b^5}{\sqrt[3]{x}}\right)}{a^6 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} (b + a\sqrt[3]{x})} + \frac{18 \left(ab^2 + \frac{b^3}{\sqrt[3]{x}}\right)}{a^5 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 126, normalized size = 0.42

$$\frac{(a\sqrt[3]{x} + b) \left(2a^5x^{5/3} - 5a^4bx^{4/3} + 20a^3b^2x + 63a^2b^3x^{2/3} + 6ab^4\sqrt[3]{x} - 60b^3(a\sqrt[3]{x} + b)^2 \log(a\sqrt[3]{x} + b) - 27b^5 \right)}{2a^6x \left(\frac{(a\sqrt[3]{x} + b)^2}{x^{2/3}} \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x]

[Out] ((b + a*x^(1/3))*(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*b^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3) - 60*b^3*(b + a*x^(1/3))^2*Log[b + a*x^(1/3)]))/(2*a^6*(b + a*x^(1/3))^2/x^(2/3))^(3/2)*x)

IntegrateAlgebraic [A] time = 6.44, size = 135, normalized size = 0.45

$$\frac{\sqrt[3]{x} \sqrt{\frac{(a\sqrt[3]{x} + b)^2}{x^{2/3}}} \left(\frac{2a^5x^{5/3} - 5a^4bx^{4/3} + 20a^3b^2x + 63a^2b^3x^{2/3} + 6ab^4\sqrt[3]{x} - 27b^5}{2a^6(a\sqrt[3]{x} + b)^2} - \frac{30b^3 \log(a\sqrt[3]{x} + b)}{a^6} \right)}{a\sqrt[3]{x} + b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x]

[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*b^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3))/(2*a^6*(b + a*x^(1/3))^2) - (30*b^3*Log[b + a*x^(1/3)]/a^6))/(b + a*x^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.66, size = 121, normalized size = 0.40

$$\frac{30b^3 \log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^6 \operatorname{sgn}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)} - \frac{3\left(10ab^4x^{\frac{1}{3}} + 9b^5\right)}{2\left(ax^{\frac{1}{3}} + b\right)^2 a^6 \operatorname{sgn}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)} + \frac{2a^6x - 9a^5bx^{\frac{2}{3}} + 36a^4b^2x^{\frac{1}{3}}}{2a^9 \operatorname{sgn}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")

[Out] $-30*b^3*\log(\text{abs}(a*x^{(1/3)} + b))/(a^6*\text{sgn}(a*x^{(2/3)} + b*x^{(1/3)})) - 3/2*(10*a*b^4*x^{(1/3)} + 9*b^5)/((a*x^{(1/3)} + b)^2*a^6*\text{sgn}(a*x^{(2/3)} + b*x^{(1/3)})) + 1/2*(2*a^6*x - 9*a^5*b*x^{(2/3)} + 36*a^4*b^2*x^{(1/3)})/(a^9*\text{sgn}(a*x^{(2/3)} + b*x^{(1/3)}))$

maple [A] time = 0.01, size = 141, normalized size = 0.47

$$\frac{(2a^5x^{\frac{5}{3}} - 60a^2b^3x^{\frac{2}{3}} \ln(ax^{\frac{1}{3}} + b) - 5a^4bx^{\frac{4}{3}} - 120ab^4x^{\frac{1}{3}} \ln(ax^{\frac{1}{3}} + b) + 20a^3b^2x - 60b^5 \ln(ax^{\frac{1}{3}} + b) + 63a^2b^3x^{\frac{2}{3}} + 6ab^4x^{\frac{1}{3}} - 27b^5)(ax^{\frac{1}{3}} + b)}{2\left(\frac{a^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} a^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)

[Out] $1/2/((a^2*x^{(2/3)}+2*a*b*x^{(1/3)}+b^2)/x^{(2/3)})^{(3/2)}/x*(2*a^5*x^{(5/3)}-5*a^4*b*x^{(4/3)}-60*x^{(2/3)}*\ln(a*x^{(1/3)}+b)*a^2*b^3+63*x^{(2/3)}*a^2*b^3-120*x^{(1/3)}*\ln(a*x^{(1/3)}+b)*a*b^4+6*a*b^4*x^{(1/3)}-60*\ln(a*x^{(1/3)}+b)*b^5+20*a^3*b^2*x-27*b^5)*(a*x^{(1/3)}+b)/a^6$

maxima [A] time = 1.07, size = 97, normalized size = 0.32

$$\frac{2a^5x^{\frac{5}{3}} - 5a^4bx^{\frac{4}{3}} + 20a^3b^2x + 63a^2b^3x^{\frac{2}{3}} + 6ab^4x^{\frac{1}{3}} - 27b^5}{2\left(a^8x^{\frac{2}{3}} + 2a^7bx^{\frac{1}{3}} + a^6b^2\right)} - \frac{30b^3 \log\left(ax^{\frac{1}{3}} + b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")

[Out] $1/2*(2*a^5*x^{(5/3)} - 5*a^4*b*x^{(4/3)} + 20*a^3*b^2*x + 63*a^2*b^3*x^{(2/3)} + 6*a*b^4*x^{(1/3)} - 27*b^5)/(a^8*x^{(2/3)} + 2*a^7*b*x^{(1/3)} + a^6*b^2) - 30*b^3*\log(a*x^{(1/3)} + b)/a^6$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{x^{1/3}}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)`

[Out] `int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2), x)`

[Out] `Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-3/2), x)`

$$3.424 \quad \int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}\right)^{5/2}} dx$$

Optimal. Leaf size=410

$$\frac{3b^7 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{4a^8 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)^4} - \frac{7b^6 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^8 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)^3} + \frac{63b^5 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{2a^8 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)^2} - \frac{105b^4 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^7 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)} + \frac{45b^2 \sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{2a^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} - \frac{15bx^{2/3} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} + \frac{x \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} - \frac{105b^3 \left(a + \frac{b}{\sqrt[3]{x}}\right) \log(a\sqrt[3]{x} + b)}{a^8 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}$$

Rubi [A] time = 0.27, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1341, 1355, 263, 43}

$$\frac{3b^7 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{4a^8 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)^4} - \frac{7b^6 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^8 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)^3} + \frac{63b^5 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{2a^8 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)^2} - \frac{105b^4 \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^7 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}} (a\sqrt[3]{x} + b)} + \frac{45b^2 \sqrt[3]{x} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{2a^6 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} - \frac{15bx^{2/3} \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} + \frac{x \left(a + \frac{b}{\sqrt[3]{x}}\right)}{a^5 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}} - \frac{105b^3 \left(a + \frac{b}{\sqrt[3]{x}}\right) \log(a\sqrt[3]{x} + b)}{a^8 \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x]

[Out] (3*b^7*(a + b/x^(1/3)))/(4*a^8*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^4) - (7*b^6*(a + b/x^(1/3)))/(a^8*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^3) + (63*b^5*(a + b/x^(1/3)))/(2*a^8*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^2) - (105*b^4*(a + b/x^(1/3)))/(a^8*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (45*b^2*(a + b/x^(1/3))*x^(1/3))/(a^7*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (15*b*(a + b/x^(1/3))*x^(2/3))/(2*a^6*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^5*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (105*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^8*sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341

```
Int[((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k
= Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}\right)^{5/2}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^2}{\left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}\right)^{5/2}} dx, x, \sqrt[3]{x} \right) \\
 &= \frac{\left(3b^4 \left(ab + \frac{b^2}{\sqrt[3]{x}}\right)\right) \operatorname{Subst} \left(\int \frac{x^2}{\left(ab + \frac{b^2}{x}\right)^5} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \\
 &= \frac{\left(3b^4 \left(ab + \frac{b^2}{\sqrt[3]{x}}\right)\right) \operatorname{Subst} \left(\int \frac{x^7}{(b^2 + abx)^5} dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \\
 &= \frac{\left(3b^4 \left(ab + \frac{b^2}{\sqrt[3]{x}}\right)\right) \operatorname{Subst} \left(\int \left(\frac{15}{a^7 b^3} - \frac{5x}{a^6 b^4} + \frac{x^2}{a^5 b^5} - \frac{b^2}{a^7 (b+ax)^5} + \frac{7b}{a^7 (b+ax)^4} - \frac{21}{a^7 (b+ax)^3} + \frac{a^7 b^6}{a^7 (b+ax)^2} \right) dx, x, \sqrt[3]{x} \right)}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \\
 &= \frac{3 \left(ab^7 + \frac{b^8}{\sqrt[3]{x}}\right)}{4a^8 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} (b + a\sqrt[3]{x})^4} - \frac{7 \left(ab^6 + \frac{b^7}{\sqrt[3]{x}}\right)}{a^8 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}} (b + a\sqrt[3]{x})^3} + \frac{63}{2a^8 \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}}
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 152, normalized size = 0.37

$$\frac{(a\sqrt[3]{x} + b) \left(4a^7 x^{7/3} - 14a^6 b x^2 + 84a^5 b^2 x^{5/3} + 556a^4 b^3 x^{4/3} + 544a^3 b^4 x - 444a^2 b^5 x^{2/3} - 856ab^6 \sqrt[3]{x} - 420b^3 (a\sqrt[3]{x} + b)^4 \log(a\sqrt[3]{x} + b) - 319b^7 \right)}{4a^8 x^{5/3} \left(\frac{(a\sqrt[3]{x} + b)^2}{x^{2/3}} \right)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x]

[Out] ((b + a*x^(1/3))*(-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2/3) + 544*a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 4*a^7*x^(7/3) - 420*b^3*(b + a*x^(1/3))^4*Log[b + a*x^(1/3)]))/(4*a^8*((b + a*x^(1/3))^2/x^(2/3))^(5/2)*x^(5/3))

IntegrateAlgebraic [A] time = 9.55, size = 159, normalized size = 0.39

$$\frac{\sqrt[3]{x} \sqrt{\frac{(a\sqrt[3]{x} + b)^2}{x^{2/3}}} \left(\frac{4a^7 x^{7/3} - 14a^6 b x^2 + 84a^5 b^2 x^{5/3} + 556a^4 b^3 x^{4/3} + 544a^3 b^4 x - 444a^2 b^5 x^{2/3} - 856ab^6 \sqrt[3]{x} - 319b^7}{4a^8 (a\sqrt[3]{x} + b)^4} - \frac{105b^3 \log(a\sqrt[3]{x} + b)}{a^8} \right)}{a\sqrt[3]{x} + b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x]

[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3)*((-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2/3) + 544*a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 4*a^7*x^(7/3))/(4*a^8*(b + a*x^(1/3))^4) - (105*b^3*Log[b + a*x^(1/3)])/a^8))/(b + a*x^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.72, size = 141, normalized size = 0.34

$$-\frac{105b^3 \log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^8 \operatorname{sgn}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)} - \frac{420a^3b^4x + 1134a^2b^5x^{\frac{2}{3}} + 1036ab^6x^{\frac{1}{3}} + 319b^7}{4\left(ax^{\frac{1}{3}} + b\right)^4 a^8 \operatorname{sgn}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)} + \frac{2a^{10}x - 15a^9bx^{\frac{2}{3}} + 90a^8b^2x^{\frac{1}{3}}}{2a^{15} \operatorname{sgn}\left(ax^{\frac{2}{3}} + bx^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")

[Out] $-105*b^3*\log(\text{abs}(a*x^{(1/3)} + b))/(a^8*\text{sgn}(a*x^{(2/3)} + b*x^{(1/3)})) - 1/4*(420*a^3*b^4*x + 1134*a^2*b^5*x^{(2/3)} + 1036*a*b^6*x^{(1/3)} + 319*b^7)/((a*x^{(1/3)} + b)^4*a^8*\text{sgn}(a*x^{(2/3)} + b*x^{(1/3)})) + 1/2*(2*a^{10}*x - 15*a^9*b*x^{(2/3)} + 90*a^8*b^2*x^{(1/3)})/(a^{15}*\text{sgn}(a*x^{(2/3)} + b*x^{(1/3)}))$

maple [A] time = 0.01, size = 199, normalized size = 0.49

$$\frac{(4a^7x^{\frac{7}{3}} - 420a^6b^2x^{\frac{4}{3}} \ln(ax^{\frac{1}{3}} + b) - 14a^6bx^2 - 1680a^5b^4x \ln(ax^{\frac{1}{3}} + b) + 84a^5b^2x^{\frac{5}{3}} - 2520a^4b^5x^{\frac{2}{3}} \ln(ax^{\frac{1}{3}} + b) + 556a^4b^3x^{\frac{4}{3}} - 1680a^3b^6x^{\frac{1}{3}} \ln(ax^{\frac{1}{3}} + b) + 544a^3b^4x - 420b^7 \ln(ax^{\frac{1}{3}} + b) - 444a^2b^7x^{\frac{2}{3}} - 856ab^6x^{\frac{1}{3}} - 319b^7)(ax^{\frac{1}{3}} + b)}{4 \left(\frac{a^2x^{\frac{2}{3}} + 2abx^{\frac{1}{3}} + b^2}{x^{\frac{2}{3}}} \right)^{\frac{5}{2}} a^8x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)

[Out] $1/4/((a^2*x^{(2/3)}+2*a*b*x^{(1/3)}+b^2)/x^{(2/3)})^{(5/2)}/x^{(5/3)}*(4*x^{(7/3)}*a^7+84*a^5*b^2*x^{(5/3)}-420*x^{(4/3)}*\ln(a*x^{(1/3)}+b)*a^4*b^3+556*x^{(4/3)}*a^4*b^3-2520*x^{(2/3)}*\ln(a*x^{(1/3)}+b)*a^2*b^5-444*x^{(2/3)}*a^2*b^5-1680*x^{(1/3)}*\ln(a*x^{(1/3)}+b)*a*b^6-1680*x*\ln(a*x^{(1/3)}+b)*a^3*b^4-14*a^6*b*x^2-856*x^{(1/3)}*a*b^6-420*\ln(a*x^{(1/3)}+b)*b^7+544*x*a^3*b^4-319*b^7)*(a*x^{(1/3)}+b)/a^8$

maxima [A] time = 0.85, size = 139, normalized size = 0.34

$$\frac{4a^7x^{\frac{7}{3}} - 14a^6bx^2 + 84a^5b^2x^{\frac{5}{3}} + 556a^4b^3x^{\frac{4}{3}} + 544a^3b^4x - 444a^2b^5x^{\frac{2}{3}} - 856ab^6x^{\frac{1}{3}} - 319b^7}{4 \left(a^{12}x^{\frac{4}{3}} + 4a^{11}bx + 6a^{10}b^2x^{\frac{2}{3}} + 4a^9b^3x^{\frac{1}{3}} + a^8b^4 \right)} - \frac{105b^3 \log(ax^{\frac{1}{3}} + b)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")

[Out] $1/4*(4*a^7*x^{(7/3)} - 14*a^6*b*x^2 + 84*a^5*b^2*x^{(5/3)} + 556*a^4*b^3*x^{(4/3)} + 544*a^3*b^4*x - 444*a^2*b^5*x^{(2/3)} - 856*a*b^6*x^{(1/3)} - 319*b^7)/(a^{12}*x^{(4/3)} + 4*a^{11}*b*x + 6*a^{10}*b^2*x^{(2/3)} + 4*a^9*b^3*x^{(1/3)} + a^8*b^4) - 105*b^3*\log(a*x^{(1/3)} + b)/a^8$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{x^{1/3}} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)`

[Out] `int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2), x)`

[Out] `Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-5/2), x)`

$$3.425 \quad \int \left(a^2 + \frac{b^2}{\sqrt{x}} + \frac{2ab}{\sqrt[4]{x}} \right)^{5/2} dx$$

Optimal. Leaf size=289

$$\frac{4b^5 \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{\sqrt[4]{x} \left(a + \frac{b}{\sqrt[4]{x}} \right)} + \frac{20ab^4 \log(\sqrt[4]{x}) \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{a + \frac{b}{\sqrt[4]{x}}} + \frac{40a^2b^3 \sqrt[4]{x} \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{a + \frac{b}{\sqrt[4]{x}}} + \frac{a^5 x \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{a + \frac{b}{\sqrt[4]{x}}}$$

Rubi [A] time = 0.14, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1341, 1355, 263, 43}

$$\frac{20a^4bx^{3/4} \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{3 \left(a + \frac{b}{\sqrt[4]{x}} \right)} + \frac{a^5x \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{a + \frac{b}{\sqrt[4]{x}}} + \frac{20a^3b^2 \sqrt{x} \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{a + \frac{b}{\sqrt[4]{x}}} + \frac{40a^2b^3 \sqrt[4]{x} \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{a + \frac{b}{\sqrt[4]{x}}} - \frac{4b^5 \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{\sqrt[4]{x} \left(a + \frac{b}{\sqrt[4]{x}} \right)} + \frac{20ab^4 \log(\sqrt[4]{x}) \sqrt{a^2 + \frac{2ab}{\sqrt[4]{x}} + \frac{b^2}{\sqrt{x}}}}{a + \frac{b}{\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2), x]

[Out] (-4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]/((a + b/x^(1/4))*x^(1/4)) + (40*a^2*b^3*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(1/4))/(a + b/x^(1/4)) + (20*a^3*b^2*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Sqrt[x])/(a + b/x^(1/4)) + (20*a^4*b*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(3/4))/(3*(a + b/x^(1/4))) + (a^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x)/(a + b/x^(1/4)) + (20*a*b^4*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Log[x^(1/4)])/(a + b/x^(1/4))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n)

$)^p, x], x, x^{(1/k)}, x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{FractionQ}[n]$

Rule 1355

$\text{Int}[(d_*)(x_*)^{(m_*)} * ((a_*) + (b_*)(x_*)^{(n_*)} + (c_*)(x_*)^{(n2_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[(a + b*x^n + c*x^{(2*n)})^{\text{FracPart}[p]} / (c^{\text{IntPart}[p]} * (b/2 + c*x^n)^{(2*\text{FracPart}[p])}), \text{Int}[(d*x)^m * (b/2 + c*x^n)^{(2*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[p - 1/2]$

Rubi steps

$$\begin{aligned}
 \int \left(a^2 + \frac{b^2}{\sqrt{x}} + \frac{2ab}{\sqrt[4]{x}} \right)^{5/2} dx &= 4 \text{Subst} \left(\int \left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x} \right)^{5/2} x^3 dx, x, \sqrt[4]{x} \right) \\
 &= \frac{\left(4\sqrt{a^2 + \frac{b^2}{\sqrt{x}} + \frac{2ab}{\sqrt[4]{x}}} \right) \text{Subst} \left(\int \left(ab + \frac{b^2}{x} \right)^5 x^3 dx, x, \sqrt[4]{x} \right)}{b^4 \left(ab + \frac{b^2}{\sqrt[4]{x}} \right)} \\
 &= \frac{\left(4\sqrt{a^2 + \frac{b^2}{\sqrt{x}} + \frac{2ab}{\sqrt[4]{x}}} \right) \text{Subst} \left(\int \frac{(b^2+abx)^5}{x^2} dx, x, \sqrt[4]{x} \right)}{b^4 \left(ab + \frac{b^2}{\sqrt[4]{x}} \right)} \\
 &= \frac{\left(4\sqrt{a^2 + \frac{b^2}{\sqrt{x}} + \frac{2ab}{\sqrt[4]{x}}} \right) \text{Subst} \left(\int \left(10a^2b^8 + \frac{b^{10}}{x^2} + \frac{5ab^9}{x} + 10a^3b^7x + 5a^4b^6x^2 + a^5b^5x^3 \right) dx, x, \sqrt[4]{x} \right)}{b^4 \left(ab + \frac{b^2}{\sqrt[4]{x}} \right)} \\
 &= -\frac{4b^6 \sqrt{a^2 + \frac{b^2}{\sqrt{x}} + \frac{2ab}{\sqrt[4]{x}}}}{\left(ab + \frac{b^2}{\sqrt[4]{x}} \right) \sqrt[4]{x}} + \frac{40a^2b^4 \sqrt{a^2 + \frac{b^2}{\sqrt{x}} + \frac{2ab}{\sqrt[4]{x}}} \sqrt[4]{x}}{ab + \frac{b^2}{\sqrt[4]{x}}} + \frac{20a^3b^3 \sqrt{a^2 + \frac{b^2}{\sqrt{x}} + \frac{2ab}{\sqrt[4]{x}}} \sqrt[4]{x}}{ab + \frac{b^2}{\sqrt[4]{x}}}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 98, normalized size = 0.34

$$\frac{\sqrt{\frac{(a\sqrt[4]{x}+b)^2}{\sqrt{x}}}}{3(a\sqrt[4]{x}+b)} \left(3a^5x^{5/4} + 20a^4bx + 60a^3b^2x^{3/4} + 120a^2b^3\sqrt{x} + 15ab^4\sqrt[4]{x} \log(x) - 12b^5 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2), x]

[Out] (Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*(-12*b^5 + 120*a^2*b^3*Sqrt[x] + 60*a^3*b^2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4) + 15*a*b^4*x^(1/4)*Log[x]))/(3*(b + a*x^(1/4)))

IntegrateAlgebraic [A] time = 6.28, size = 109, normalized size = 0.38

$$\frac{\sqrt[4]{x} \sqrt{\frac{(a\sqrt[4]{x}+b)^2}{\sqrt{x}}}}{a\sqrt[4]{x}+b} \left(\frac{3a^5x^{5/4}+20a^4bx+60a^3b^2x^{3/4}+120a^2b^3\sqrt{x}-12b^5}{3\sqrt[4]{x}} + 20ab^4 \log\left(\sqrt[4]{x}\right) \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2), x]

[Out] (Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*x^(1/4)*((-12*b^5 + 120*a^2*b^3*Sqrt[x] + 60*a^3*b^2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4))/(3*x^(1/4)) + 20*a*b^4*Log[x^(1/4)]))/(b + a*x^(1/4))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.49, size = 126, normalized size = 0.44

$$a^5 \operatorname{sgn}(ax + bx^{\frac{3}{4}}) \operatorname{sgn}(x) + 5ab^4 \log(|x|) \operatorname{sgn}(ax + bx^{\frac{3}{4}}) \operatorname{sgn}(x) + \frac{20}{3} a^4 bx^{\frac{3}{4}} \operatorname{sgn}(ax + bx^{\frac{3}{4}}) \operatorname{sgn}(x) + 20a^3 b^2 \sqrt{x} \operatorname{sgn}(ax + bx^{\frac{3}{4}}) \operatorname{sgn}(x) + 40a^2 b^3 x^{\frac{1}{4}} \operatorname{sgn}(ax + bx^{\frac{3}{4}}) \operatorname{sgn}(x) - \frac{4b^5 \operatorname{sgn}(ax + bx^{\frac{3}{4}}) \operatorname{sgn}(x)}{x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2), x, algorithm="giac")

[Out] a^5*x*sgn(a*x + b*x^(3/4))*sgn(x) + 5*a*b^4*log(abs(x))*sgn(a*x + b*x^(3/4))*sgn(x) + 20/3*a^4*b*x^(3/4)*sgn(a*x + b*x^(3/4))*sgn(x) + 20*a^3*b^2*sqrt(x)*sgn(a*x + b*x^(3/4))*sgn(x) + 40*a^2*b^3*x^(1/4)*sgn(a*x + b*x^(3/4))*sgn(x) - 4*b^5*sgn(a*x + b*x^(3/4))*sgn(x)/x^(1/4)

maple [A] time = 0.04, size = 94, normalized size = 0.33

$$\frac{\sqrt{\frac{a^2x^{\frac{3}{4}}+2ab\sqrt{x}+b^2x^{\frac{1}{4}}}{x^{\frac{3}{4}}}}}{3ax^{\frac{1}{4}}+3b} \left(3a^5x^{\frac{5}{4}} + 15ab^4x^{\frac{1}{4}} \ln(x) + 20a^4bx + 60a^3b^2x^{\frac{3}{4}} + 120a^2b^3\sqrt{x} - 12b^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x)`

[Out] $\frac{1}{3} \left((a^2 x^{3/4} + b^2 x^{1/4} + 2 a b x^{1/2}) / x^{3/4} \right)^{1/2} (20 x a^4 b + 15 \ln(x) x^{1/4} a b^4 + 120 a^2 b^3 x^{1/2} + 60 x^{3/4} a^3 b^2 + 3 x^{5/4} a^5 - 12 b^5) / (a x^{1/4} + b)$

maxima [A] time = 0.99, size = 57, normalized size = 0.20

$$5 a b^4 \log(x) + \frac{3 a^5 x^{\frac{5}{4}} + 20 a^4 b x + 60 a^3 b^2 x^{\frac{3}{4}} + 120 a^2 b^3 \sqrt{x} - 12 b^5}{3 x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="maxima")`

[Out] $5 a^4 b^4 \log(x) + \frac{1}{3} (3 a^5 x^{5/4} + 20 a^4 b x + 60 a^3 b^2 x^{3/4} + 120 a^2 b^3 \sqrt{x} - 12 b^5) / x^{1/4}$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2 + \frac{b^2}{\sqrt{x}} + \frac{2 a b}{x^{1/4}} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2),x)`

[Out] `int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+2*a*b/x**(1/4)+b**2/x**(1/2))**(5/2),x)`

[Out] Timed out

$$3.426 \quad \int \left(a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{\sqrt[5]{x}} \right)^{5/2} dx$$

Optimal. Leaf size=291

$$\frac{5b^5 \log(\sqrt[5]{x}) \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{a + \frac{b}{\sqrt[5]{x}}} + \frac{25ab^4 \sqrt[5]{x} \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{a + \frac{b}{\sqrt[5]{x}}} + \frac{25a^2 b^3 x^{2/5} \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{a + \frac{b}{\sqrt[5]{x}}} + \frac{a^5 x \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{a + \frac{b}{\sqrt[5]{x}}}$$

Rubi [A] time = 0.14, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1341, 1355, 263, 43}

$$\frac{a^5 x \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{a + \frac{b}{\sqrt[5]{x}}} + \frac{25a^4 b x^{4/5} \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{4 \left(a + \frac{b}{\sqrt[5]{x}} \right)} + \frac{50a^3 b^2 x^{3/5} \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{3 \left(a + \frac{b}{\sqrt[5]{x}} \right)} + \frac{25a^2 b^3 x^{2/5} \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{a + \frac{b}{\sqrt[5]{x}}} + \frac{25ab^4 \sqrt[5]{x} \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{a + \frac{b}{\sqrt[5]{x}}} + \frac{5b^5 \log(\sqrt[5]{x}) \sqrt{a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^{2/5}}}}{a + \frac{b}{\sqrt[5]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x]

[Out] (25*a*b^4*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(1/5))/(a + b/x^(1/5)) + (25*a^2*b^3*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(2/5))/(a + b/x^(1/5)) + (50*a^3*b^2*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(3/5))/(3*(a + b/x^(1/5))) + (25*a^4*b*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(4/5))/(4*(a + b/x^(1/5))) + (a^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x)/(a + b/x^(1/5)) + (5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*Log[x^(1/5)])/(a + b/x^(1/5))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n)

)^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_)+(b_)*(x_)^(n_)+(c_)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a+b*x^n+c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2+c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2+c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2-4*a*c, 0] && IntegerQ[p-1/2]

Rubi steps

$$\begin{aligned}
 \int \left(a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{\sqrt[5]{x}} \right)^{5/2} dx &= 5 \operatorname{Subst} \left(\int \left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x} \right)^{5/2} x^4 dx, x, \sqrt[5]{x} \right) \\
 &= \frac{\left(5 \sqrt{a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{\sqrt[5]{x}}} \right) \operatorname{Subst} \left(\int \left(ab + \frac{b^2}{x} \right)^5 x^4 dx, x, \sqrt[5]{x} \right)}{b^4 \left(ab + \frac{b^2}{\sqrt[5]{x}} \right)} \\
 &= \frac{\left(5 \sqrt{a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{\sqrt[5]{x}}} \right) \operatorname{Subst} \left(\int \frac{(b^2+abx)^5}{x} dx, x, \sqrt[5]{x} \right)}{b^4 \left(ab + \frac{b^2}{\sqrt[5]{x}} \right)} \\
 &= \frac{\left(5 \sqrt{a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{\sqrt[5]{x}}} \right) \operatorname{Subst} \left(\int \left(5ab^9 + \frac{b^{10}}{x} + 10a^2b^8x + 10a^3b^7x^2 + 5a^4b^6x^3 + a^5b^5x^4 \right) dx, x, \sqrt[5]{x} \right)}{b^4 \left(ab + \frac{b^2}{\sqrt[5]{x}} \right)} \\
 &= \frac{25ab^5 \sqrt{a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{\sqrt[5]{x}}} \sqrt[5]{x}}{ab + \frac{b^2}{\sqrt[5]{x}}} + \frac{25a^2b^4 \sqrt{a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{\sqrt[5]{x}}} x^{2/5}}{ab + \frac{b^2}{\sqrt[5]{x}}} + \frac{50a^3b^3 \sqrt{a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{\sqrt[5]{x}}}}{3 \left(ab + \frac{b^2}{\sqrt[5]{x}} \right)}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 103, normalized size = 0.35

$$\frac{\sqrt{\frac{(a\sqrt[5]{x}+b)^2}{x^{2/5}}} (12a^5x^{6/5} + 75a^4bx + 200a^3b^2x^{4/5} + 300a^2b^3x^{3/5} + 300ab^4x^{2/5} + 12b^5\sqrt[5]{x} \log(x))}{12(a\sqrt[5]{x}+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x]

[Out] (Sqrt[(b + a*x^(1/5))^2/x^(2/5)]*(300*a*b^4*x^(2/5) + 300*a^2*b^3*x^(3/5) + 200*a^3*b^2*x^(4/5) + 75*a^4*b*x + 12*a^5*x^(6/5) + 12*b^5*x^(1/5)*Log[x])/(12*(b + a*x^(1/5)))

IntegrateAlgebraic [A] time = 6.30, size = 109, normalized size = 0.37

$$\frac{\sqrt[5]{x} \sqrt{\frac{(a\sqrt[5]{x}+b)^2}{x^{2/5}}} \left(\frac{1}{12} (12a^5x + 75a^4bx^{4/5} + 200a^3b^2x^{3/5} + 300a^2b^3x^{2/5} + 300ab^4\sqrt[5]{x}) + 5b^5 \log(\sqrt[5]{x}) \right)}{a\sqrt[5]{x} + b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x]

[Out] (Sqrt[(b + a*x^(1/5))^2/x^(2/5)]*x^(1/5)*((300*a*b^4*x^(1/5) + 300*a^2*b^3*x^(2/5) + 200*a^3*b^2*x^(3/5) + 75*a^4*b*x^(4/5) + 12*a^5*x)/12 + 5*b^5*Log[x^(1/5)]))/(b + a*x^(1/5))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.48, size = 125, normalized size = 0.43

$$a^5x\operatorname{sgn}(ax + bx^{4/5})\operatorname{sgn}(x) + b^5\log(|x|)\operatorname{sgn}(ax + bx^{4/5})\operatorname{sgn}(x) + \frac{25}{4}a^4bx^{4/5}\operatorname{sgn}(ax + bx^{4/5})\operatorname{sgn}(x) + \frac{50}{3}a^3b^2x^{3/5}\operatorname{sgn}(ax + bx^{4/5})\operatorname{sgn}(x) + 25a^2b^3x^{2/5}\operatorname{sgn}(ax + bx^{4/5})\operatorname{sgn}(x) + 25ab^4x^{1/5}\operatorname{sgn}(ax + bx^{4/5})\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2), x, algorithm="giac")

[Out] a^5*x*sgn(a*x + b*x^(4/5))*sgn(x) + b^5*log(abs(x))*sgn(a*x + b*x^(4/5))*sgn(x) + 25/4*a^4*b*x^(4/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 50/3*a^3*b^2*x^(3/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 25*a^2*b^3*x^(2/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 25*a*b^4*x^(1/5)*sgn(a*x + b*x^(4/5))*sgn(x)

maple [A] time = 0.03, size = 91, normalized size = 0.31

$$\frac{\left(\frac{a^2x^{2/5}+2abx^{1/5}+b^2}{x^{2/5}}\right)^{5/2} \left(12a^5x + 12b^5\ln(x) + 75a^4bx^{4/5} + 200a^3b^2x^{3/5} + 300a^2b^3x^{2/5} + 300ab^4x^{1/5}\right)x}{12\left(ax^{1/5} + b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x)`

[Out] $\frac{1}{12} * ((a^2 * x^{(2/5)} + 2 * a * b * x^{(1/5)} + b^2) / x^{(2/5)})^{(5/2)} * x * (75 * a^4 * b * x^{(4/5)} + 20 * a^3 * b^2 * x^{(3/5)} + 300 * a^2 * b^3 * x^{(2/5)} + 12 * b^5 * \ln(x) + 300 * a * b^4 * x^{(1/5)} + 12 * a^5 * x) / (x^{(1/5)} * a + b)^5$

maxima [A] time = 0.94, size = 52, normalized size = 0.18

$$a^5 x + b^5 \log(x) + \frac{25}{4} a^4 b x^{\frac{4}{5}} + \frac{50}{3} a^3 b^2 x^{\frac{3}{5}} + 25 a^2 b^3 x^{\frac{2}{5}} + 25 a b^4 x^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="maxima")`

[Out] $a^5 * x + b^5 * \log(x) + 25/4 * a^4 * b * x^{(4/5)} + 50/3 * a^3 * b^2 * x^{(3/5)} + 25 * a^2 * b^3 * x^{(2/5)} + 25 * a * b^4 * x^{(1/5)}$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2 + \frac{b^2}{x^{2/5}} + \frac{2ab}{x^{1/5}} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x)`

[Out] `int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2ab}{\sqrt[5]{x}} + \frac{b^2}{x^5} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)`

[Out] `Integral((a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(5/2), x)`

$$3.427 \quad \int \frac{1}{(a^2 + 2ab\sqrt[5]{x} + b^2x^{2/5})^{5/2}} dx$$

Optimal. Leaf size=222

$$\frac{15a^2}{b^5(a+b\sqrt[5]{x})\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{5(a+b\sqrt[5]{x})\log(a+b\sqrt[5]{x})}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} - \frac{1}{4b^5(a+b\sqrt[5]{x})}$$

Rubi [A] time = 0.13, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1341, 646, 43}

$$\frac{5a^4}{4b^5(a+b\sqrt[5]{x})^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{20a^3}{3b^5(a+b\sqrt[5]{x})^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} - \frac{15a^2}{b^5(a+b\sqrt[5]{x})\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{5(a+b\sqrt[5]{x})\log(a+b\sqrt[5]{x})}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(5/2), x]

[Out] (20*a)/(b^5*sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (5*a^4)/(4*b^5*(a + b*x^(1/5))^3*sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (20*a^3)/(3*b^5*(a + b*x^(1/5))^2*sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (15*a^2)/(b^5*(a + b*x^(1/5))*sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (5*(a + b*x^(1/5))*Log[a + b*x^(1/5)])/(b^5*sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 646

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0]

Rule 1341

Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra

ctionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a^2 + 2ab\sqrt[5]{x} + b^2x^{2/5})^{5/2}} dx &= 5 \operatorname{Subst} \left(\int \frac{x^4}{(a^2 + 2abx + b^2x^2)^{5/2}} dx, x, \sqrt[5]{x} \right) \\
&= \frac{(5b^5 (a + b\sqrt[5]{x})) \operatorname{Subst} \left(\int \frac{x^4}{(ab+b^2x)^5} dx, x, \sqrt[5]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[5]{x} + b^2x^{2/5}}} \\
&= \frac{(5b^5 (a + b\sqrt[5]{x})) \operatorname{Subst} \left(\int \left(\frac{a^4}{b^9(a+bx)^5} - \frac{4a^3}{b^9(a+bx)^4} + \frac{6a^2}{b^9(a+bx)^3} - \frac{4a}{b^9(a+bx)^2} + \frac{1}{b^9(a+bx)} \right) dx, x, \sqrt[5]{x} \right)}{\sqrt{a^2 + 2ab\sqrt[5]{x} + b^2x^{2/5}}} \\
&= \frac{20a}{b^5 \sqrt{a^2 + 2ab\sqrt[5]{x} + b^2x^{2/5}}} - \frac{5a^4}{4b^5 (a + b\sqrt[5]{x})^3 \sqrt{a^2 + 2ab\sqrt[5]{x} + b^2x^{2/5}}} + \frac{1}{3b^5 (a + b\sqrt[5]{x})}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 98, normalized size = 0.44

$$\frac{5a(25a^3 + 88a^2b\sqrt[5]{x} + 108ab^2x^{2/5} + 48b^3x^{3/5}) + 60(a + b\sqrt[5]{x})^4 \log(a + b\sqrt[5]{x})}{12b^5(a + b\sqrt[5]{x})^3 \sqrt{(a + b\sqrt[5]{x})^2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(5/2), x]`

```
[Out] (5*a*(25*a^3 + 88*a^2*b*x^(1/5) + 108*a*b^2*x^(2/5) + 48*b^3*x^(3/5)) + 60*(a + b*x^(1/5))^4*Log[a + b*x^(1/5)])/(12*b^5*(a + b*x^(1/5))^3*Sqrt[(a + b*x^(1/5))^2])
```

IntegrateAlgebraic [B] time = 2.81, size = 2765, normalized size = 12.45

Result too large to show

Antiderivative was successfully verified.

`[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(5/2), x]`

```
[Out] ((160*a^7*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])/b^5 - (2240*a^7*x^(1/5))/(3*b^3*Sqrt[b^2]) + (1760*a^6*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(1/5)))/b^5
```


$$\begin{aligned}
& /5)) / (3b^4) - (9440a^6x^{(2/5)}) / (3(b^2)^{(3/2)}) + (2560a^5\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(2/5)}) / b^3 - (8000a^5x^{(3/5)}) / (b*\sqrt{b^2}) \\
& + (5440a^4*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(3/5)}) / b^2 - (4000a^4*x^{(4/5)}) / (3*\sqrt{b^2}) + (23680a^3*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(4/5)}) / (3*b) \\
& - (42880a^3*b*x) / (3*\sqrt{b^2}) + 6400a^2*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x - 8960a^2*\sqrt{b^2} * x^{(6/5)} + 2560a*b*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(6/5)} \\
& - (2560a*b^3*x^{(7/5)}) / \sqrt{b^2} + (640a^4*x^{(4/5)} * \text{ArcTanh}[(\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] / a) / b \\
& - (640a^3*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(4/5)} * \text{ArcTanh}[(\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] / a) / \sqrt{b^2} \\
& + 2560a^3*x * \text{ArcTanh}[(\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] / a - (1920a^2*b*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) * x * \text{ArcTanh}[(\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] / a) / \sqrt{b^2} \\
& + 3840a^2*b*x^{(6/5)} * \text{ArcTanh}[(\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] / a - 1920a*\sqrt{b^2} * \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(6/5)} * \text{ArcTanh}[(\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] / a \\
& + 2560a*b^2*x^{(7/5)} * \text{ArcTanh}[(\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] / a - (640*b^3*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(7/5)} * \text{ArcTanh}[(\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] / a) / \sqrt{b^2} \\
& + 640*b^3*x^{(8/5)} * \text{ArcTanh}[(\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] / a) / ((-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)})^4 * (a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)})^4 + ((160*a^8) / (b^4*\sqrt{b^2})) + (2240*a^7*x^{(1/5)}) / (3*b^3*\sqrt{b^2}) \\
& - (2240a^6*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(1/5)}) / (3*b^4) + (9440a^6*x^{(2/5)}) / (3*(b^2)^{(3/2)}) - (2400a^5*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(2/5)}) / b^3 + (8000a^5*x^{(3/5)}) / (b*\sqrt{b^2}) \\
& - (5600a^4*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(3/5)}) / b^2 + (12000a^4*x^{(4/5)}) / \sqrt{b^2} - (6400a^3*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(4/5)}) / b + (9600a^3*b*x) / \sqrt{b^2} - 3200a^2*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x + 3200a^2*\sqrt{b^2} * x^{(6/5)} \\
& - (320a^4*x^{(4/5)} * \text{Log}[-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}]) / \sqrt{b^2} + (320a^3*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(4/5)} * \text{Log}[-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}]) / b \\
& - (1280a^3*b*x * \text{Log}[-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}]) / \sqrt{b^2} + 960a^2*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x * \text{Log}[-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] - 1920a^2*\sqrt{b^2} * x^{(6/5)} * \text{Log}[-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] - \sqrt{b^2} * x^{(1/5)} \\
& + 960a*b*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(6/5)} * \text{Log}[-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] - (1280a*b^3*x^{(7/5)} * \text{Log}[-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}]) / \sqrt{b^2} \\
& + 320*b^2*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}} * x^{(7/5)} * \text{Log}[-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}] - (320*b^4*x^{(8/5)} * \text{Log}[-a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}]) / \sqrt{b^2} \\
& - (320a^4*x^{(4/5)} * \text{Log}[a + \sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}]) / \sqrt{b^2} + (320a^3*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}]) / \sqrt{b^2} + (320a^3*\sqrt{a^2 + 2a*b*x^{(1/5)} + b^2*x^{(2/5)}}) - \sqrt{b^2} * x^{(1/5)}]) / \sqrt{b^2}
\end{aligned}$$

5) + b^2*x^(2/5)]*x^(4/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)]/b - (1280*a^3*b*x*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)]/Sqrt[b^2] + 960*a^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)] - 1920*a^2*Sqrt[b^2]*x^(6/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)] + 960*a*b*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(6/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)] - (1280*a*b^3*x^(7/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)]/Sqrt[b^2] + 320*b^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]*x^(7/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)] - (320*b^4*x^(8/5)*Log[a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5)]/Sqrt[b^2])/((-a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5))^4*(a + Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)] - Sqrt[b^2]*x^(1/5))^4)

fricas [A] time = 0.91, size = 302, normalized size = 1.36

$$\frac{5(300a^5b^5x^3 + 100a^{15}b^5x + 25a^{20} + 12(b^{20}x^4 + 4a^5b^{15}x^3 + 6a^{10}b^{10}x^2 + 4a^{15}b^5x + a^{20})\log\left(\frac{bx^{1/5} + a}{b}\right) + (48ab^2x^3 - 226a^6b^{14}x^2 + 104a^{11}b^9x + 3a^{16}b^4)x^{4/5} - (84a^2b^{18}x^3 - 228a^7b^{13}x^2 + 67a^{12}b^8x + 4a^{17}b^3)x^{3/5} + (136a^3b^{17}x^3 - 197a^8b^{12}x^2 + 48a^{13}b^7x + 6a^{18}b^2)x^{2/5} - (207a^4b^{16}x^3 - 124a^9b^{11}x^2 + 56a^{14}b^6x + 12a^{19}b)x^{1/5})}{12(b^{25}x^4 + 4a^5b^{20}x^3 + 6a^{10}b^{15}x^2 + 4a^{15}b^{10}x + a^{20}b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="fricas")

[Out] 5/12*(300*a^5*b^15*x^3 + 100*a^15*b^5*x + 25*a^20 + 12*(b^20*x^4 + 4*a^5*b^15*x^3 + 6*a^10*b^10*x^2 + 4*a^15*b^5*x + a^20)*log(b*x^(1/5) + a) + (48*a*b^19*x^3 - 226*a^6*b^14*x^2 + 104*a^11*b^9*x + 3*a^16*b^4)*x^(4/5) - (84*a^2*b^18*x^3 - 228*a^7*b^13*x^2 + 67*a^12*b^8*x + 4*a^17*b^3)*x^(3/5) + (136*a^3*b^17*x^3 - 197*a^8*b^12*x^2 + 48*a^13*b^7*x + 6*a^18*b^2)*x^(2/5) - (207*a^4*b^16*x^3 - 124*a^9*b^11*x^2 + 56*a^14*b^6*x + 12*a^19*b)*x^(1/5))/(b^25*x^4 + 4*a^5*b^20*x^3 + 6*a^10*b^15*x^2 + 4*a^15*b^10*x + a^20*b^5)

giac [A] time = 0.59, size = 84, normalized size = 0.38

$$\frac{5 \log\left(\left|bx^{\frac{1}{5}} + a\right|\right)}{b^5 \operatorname{sgn}\left(bx^{\frac{1}{5}} + a\right)} + \frac{5\left(48ab^2x^{\frac{3}{5}} + 108a^2bx^{\frac{2}{5}} + 88a^3x^{\frac{1}{5}} + \frac{25a^4}{b}\right)}{12\left(bx^{\frac{1}{5}} + a\right)^4 b^4 \operatorname{sgn}\left(bx^{\frac{1}{5}} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="giac")

[Out] 5*log(abs(b*x^(1/5) + a))/(b^5*sgn(b*x^(1/5) + a)) + 5/12*(48*a*b^2*x^(3/5) + 108*a^2*b*x^(2/5) + 88*a^3*x^(1/5) + 25*a^4/b)/((b*x^(1/5) + a)^4*b^4*sgn(b*x^(1/5) + a))

maple [A] time = 0.01, size = 152, normalized size = 0.68

$$\frac{5\sqrt{b^2x^{\frac{2}{5}} + 2abx^{\frac{1}{5}} + a^2} \left(12b^4x^{\frac{4}{5}} \ln(bx^{\frac{1}{5}} + a) + 48ab^3x^{\frac{3}{5}} \ln(bx^{\frac{1}{5}} + a) + 72a^2b^2x^{\frac{2}{5}} \ln(bx^{\frac{1}{5}} + a) + 48a^3bx^{\frac{1}{5}} \ln(bx^{\frac{1}{5}} + a) + 12a^4 \ln(bx^{\frac{1}{5}} + a) + 48ab^3x^{\frac{3}{5}} + 108a^2b^2x^{\frac{2}{5}} + 88a^3bx^{\frac{1}{5}} + 25a^4 \right)}{12(bx^{\frac{1}{5}} + a)^5 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2), x)`

[Out] $5/12*(a^2+2*a*b*x^{(1/5)}+b^2*x^{(2/5)})^{(1/2)}*(12*x^{(4/5)}*\ln(a+b*x^{(1/5)})*b^4+48*x^{(3/5)}*\ln(a+b*x^{(1/5)})*a*b^3+48*x^{(3/5)}*a*b^3+72*x^{(2/5)}*\ln(a+b*x^{(1/5)})*a^2*b^2+108*x^{(2/5)}*a^2*b^2+48*x^{(1/5)}*\ln(a+b*x^{(1/5)})*a^3*b+88*x^{(1/5)}*a^3*b+12*\ln(a+b*x^{(1/5)})*a^4+25*a^4)/(a+b*x^{(1/5)})^5/b^5$

maxima [A] time = 1.02, size = 99, normalized size = 0.45

$$\frac{5 \left(48 ab^3 x^{\frac{3}{5}} + 108 a^2 b^2 x^{\frac{2}{5}} + 88 a^3 b x^{\frac{1}{5}} + 25 a^4 \right)}{12 \left(b^9 x^{\frac{4}{5}} + 4 ab^8 x^{\frac{3}{5}} + 6 a^2 b^7 x^{\frac{2}{5}} + 4 a^3 b^6 x^{\frac{1}{5}} + a^4 b^5 \right)} + \frac{5 \log(bx^{\frac{1}{5}} + a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2), x, algorithm="maxima")`

[Out] $5/12*(48*a*b^3*x^{(3/5)} + 108*a^2*b^2*x^{(2/5)} + 88*a^3*b*x^{(1/5)} + 25*a^4)/(b^9*x^{(4/5)} + 4*a*b^8*x^{(3/5)} + 6*a^2*b^7*x^{(2/5)} + 4*a^3*b^6*x^{(1/5)} + a^4*b^5) + 5*\log(b*x^{(1/5)} + a)/b^5$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a^2 + b^2 x^{2/5} + 2 a b x^{1/5})^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2), x)`

[Out] `int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 + 2ab\sqrt[5]{x} + b^2x^{\frac{2}{5}})^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(-5/2), x)
```

$$3.428 \quad \int \left(a^2 + \frac{b^2}{\sqrt[3]{x}} + \frac{2ab}{\sqrt[6]{x}} \right)^{7/2} dx$$

Optimal. Leaf size=391

$$\frac{6b^7 \sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{\sqrt[6]{x} \left(a + \frac{b}{\sqrt[6]{x}} \right)} + \frac{42ab^6 \log(\sqrt[6]{x}) \sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{a + \frac{b}{\sqrt[6]{x}}} + \frac{126a^2b^5 \sqrt[6]{x} \sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{a + \frac{b}{\sqrt[6]{x}}} + \frac{a^7 x \sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{a + \frac{b}{\sqrt[6]{x}}}$$

Rubi [A] time = 0.18, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1341, 1355, 263, 43}

$$\frac{42a^6b^5\sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{5\left(a + \frac{b}{\sqrt[6]{x}}\right)} + \frac{63a^5b^2x^{2/3}\sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{2\left(a + \frac{b}{\sqrt[6]{x}}\right)} + \frac{a^7x\sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{a + \frac{b}{\sqrt[6]{x}}} + \frac{70a^4b^3\sqrt{x}\sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{a + \frac{b}{\sqrt[6]{x}}} + \frac{105a^3b^4\sqrt[3]{x}\sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{a + \frac{b}{\sqrt[6]{x}}} + \frac{126a^2b^5\sqrt[6]{x}\sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{a + \frac{b}{\sqrt[6]{x}}} - \frac{6b^7\sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{\sqrt[6]{x}\left(a + \frac{b}{\sqrt[6]{x}}\right)} + \frac{42ab^6\log(\sqrt[6]{x})\sqrt{a^2 + \frac{2ab}{\sqrt[6]{x}} + \frac{b^2}{\sqrt[3]{x}}}}{a + \frac{b}{\sqrt[6]{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x]

[Out] (-6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]/((a + b/x^(1/6))*x^(1/6)) + (126*a^2*b^5*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/6))/(a + b/x^(1/6)) + (105*a^3*b^4*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/3))/(a + b/x^(1/6)) + (70*a^4*b^3*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Sqrt[x])/(a + b/x^(1/6)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(2/3))/(2*(a + b/x^(1/6))) + (42*a^6*b*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(5/6))/(5*(a + b/x^(1/6))) + (a^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x)/(a + b/x^(1/6)) + (42*a*b^6*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Log[x^(1/6)])/(a + b/x^(1/6))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 263

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k
= Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^(p), x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1355

```
Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^(FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p]))), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \left(a^2 + \frac{b^2}{\sqrt[3]{x}} + \frac{2ab}{\sqrt[6]{x}} \right)^{7/2} dx &= 6 \operatorname{Subst} \left(\int \left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x} \right)^{7/2} x^5 dx, x, \sqrt[6]{x} \right) \\
&= \frac{\left(6 \sqrt{a^2 + \frac{b^2}{\sqrt[3]{x}} + \frac{2ab}{\sqrt[6]{x}}} \right) \operatorname{Subst} \left(\int \left(ab + \frac{b^2}{x} \right)^7 x^5 dx, x, \sqrt[6]{x} \right)}{b^6 \left(ab + \frac{b^2}{\sqrt[6]{x}} \right)} \\
&= \frac{\left(6 \sqrt{a^2 + \frac{b^2}{\sqrt[3]{x}} + \frac{2ab}{\sqrt[6]{x}}} \right) \operatorname{Subst} \left(\int \frac{(b^2+abx)^7}{x^2} dx, x, \sqrt[6]{x} \right)}{b^6 \left(ab + \frac{b^2}{\sqrt[6]{x}} \right)} \\
&= \frac{\left(6 \sqrt{a^2 + \frac{b^2}{\sqrt[3]{x}} + \frac{2ab}{\sqrt[6]{x}}} \right) \operatorname{Subst} \left(\int \left(21a^2b^{12} + \frac{b^{14}}{x^2} + \frac{7ab^{13}}{x} + 35a^3b^{11}x + 35a^4b^{10}x^2 + 21a^5b^9x^3 + 7a^6b^8x^4 \right) dx, x, \sqrt[6]{x} \right)}{b^6 \left(ab + \frac{b^2}{\sqrt[6]{x}} \right)} \\
&= -\frac{6b^8 \sqrt{a^2 + \frac{b^2}{\sqrt[3]{x}} + \frac{2ab}{\sqrt[6]{x}}}}{\left(ab + \frac{b^2}{\sqrt[6]{x}} \right) \sqrt[6]{x}} + \frac{126a^2b^6 \sqrt{a^2 + \frac{b^2}{\sqrt[3]{x}} + \frac{2ab}{\sqrt[6]{x}}} \sqrt[6]{x}}{ab + \frac{b^2}{\sqrt[6]{x}}} + \frac{105a^3b^5 \sqrt{a^2 + \frac{b^2}{\sqrt[3]{x}} + \frac{2ab}{\sqrt[6]{x}}}}{ab + \frac{b^2}{\sqrt[6]{x}}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.32

$$\frac{\sqrt{\frac{(a\sqrt[6]{x}+b)^2}{\sqrt[3]{x}}} \left(10a^7x^{7/6} + 84a^6bx + 315a^5b^2x^{5/6} + 700a^4b^3x^{2/3} + 1050a^3b^4\sqrt{x} + 1260a^2b^5\sqrt[3]{x} + 70ab^6\sqrt[6]{x} \log(x) - 60b^7 \right)}{10(a\sqrt[6]{x}+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x]

[Out] (Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*(-60*b^7 + 1260*a^2*b^5*x^(1/3) + 1050*a^3*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*b*x + 10*a^7*x^(7/6) + 70*a*b^6*x^(1/6)*Log[x]))/(10*(b + a*x^(1/6)))

IntegrateAlgebraic [A] time = 6.78, size = 135, normalized size = 0.35

$$\frac{\sqrt[6]{x} \sqrt{\frac{(a\sqrt[6]{x}+b)^2}{\sqrt[3]{x}}}}{a\sqrt[6]{x}+b} \left(\frac{10a^7x^{7/6}+84a^6bx+315a^5b^2x^{5/6}+700a^4b^3x^{2/3}+1050a^3b^4\sqrt{x}+1260a^2b^5\sqrt[3]{x}-60b^7}{10\sqrt[6]{x}} + 42ab^6 \log(\sqrt[6]{x}) \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x]

[Out] (Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*x^(1/6)*((-60*b^7 + 1260*a^2*b^5*x^(1/3) + 1050*a^3*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*b*x + 10*a^7*x^(7/6))/(10*x^(1/6)) + 42*a*b^6*Log[x^(1/6)]))/(b + a*x^(1/6))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.46, size = 172, normalized size = 0.44

$$a^7 \operatorname{sgn}(ax + bx^{\frac{1}{6}}) \operatorname{sgn}(x) + 7ab^6 \log(|x|) \operatorname{sgn}(ax + bx^{\frac{1}{6}}) \operatorname{sgn}(x) + \frac{42}{5} a^6 b^5 \operatorname{sgn}(ax + bx^{\frac{1}{6}}) \operatorname{sgn}(x) + \frac{63}{2} a^5 b^4 \operatorname{sgn}(ax + bx^{\frac{1}{6}}) \operatorname{sgn}(x) + 70 a^4 b^3 \sqrt{x} \operatorname{sgn}(ax + bx^{\frac{1}{6}}) \operatorname{sgn}(x) + 105 a^3 b^2 x^{\frac{1}{3}} \operatorname{sgn}(ax + bx^{\frac{1}{6}}) \operatorname{sgn}(x) + 126 a^2 b^5 x^{\frac{1}{6}} \operatorname{sgn}(ax + bx^{\frac{1}{6}}) \operatorname{sgn}(x) - \frac{6b^7 \operatorname{sgn}(ax + bx^{\frac{1}{6}}) \operatorname{sgn}(x)}{x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2), x, algorithm="giac")

[Out] a^7*x*sgn(a*x + b*x^(5/6))*sgn(x) + 7*a*b^6*log(abs(x))*sgn(a*x + b*x^(5/6))*sgn(x) + 42/5*a^6*b*x^(5/6)*sgn(a*x + b*x^(5/6))*sgn(x) + 63/2*a^5*b^2*x^(2/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 70*a^4*b^3*sqrt(x)*sgn(a*x + b*x^(5/6))*sgn(x) + 105*a^3*b^4*x^(1/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 126*a^2*b^5*x^(1/6)*sgn(a*x + b*x^(5/6))*sgn(x) - 6*b^7*sgn(a*x + b*x^(5/6))*sgn(x)/x^(1/6)

maple [A] time = 0.03, size = 116, normalized size = 0.30

$$\frac{\sqrt{\frac{a^2\sqrt{x}+2abx^{\frac{1}{3}}+b^2x^{\frac{1}{6}}}{\sqrt{x}}}\left(10a^7x^{\frac{7}{6}}+70ab^6x^{\frac{1}{6}}\ln(x)+84a^6bx+315a^5b^2x^{\frac{5}{6}}+700a^4b^3x^{\frac{2}{3}}+1050a^3b^4\sqrt{x}+1260a^2b^5x^{\frac{1}{3}}-60b^7\right)}{10ax^{\frac{1}{6}}+10b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x)

[Out] 1/10*((a^2*x^(1/2)+2*a*b*x^(1/3)+b^2*x^(1/6))/x^(1/2))^(1/2)*(84*a^6*b*x+315*a^5*b^2*x^(5/6)+70*a*b^6*ln(x)*x^(1/6)+1050*a^3*b^4*x^(1/2)+1260*a^2*b^5*x^(1/3)+700*a^4*b^3*x^(2/3)+10*a^7*x^(7/6)-60*b^7)/(a*x^(1/6)+b)

maxima [A] time = 0.95, size = 79, normalized size = 0.20

$$7ab^6\log(x) + \frac{10a^7x^{\frac{7}{6}} + 84a^6bx + 315a^5b^2x^{\frac{5}{6}} + 700a^4b^3x^{\frac{2}{3}} + 1050a^3b^4\sqrt{x} + 1260a^2b^5x^{\frac{1}{3}} - 60b^7}{10x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="maxima")

[Out] 7*a*b^6*log(x) + 1/10*(10*a^7*x^(7/6) + 84*a^6*b*x + 315*a^5*b^2*x^(5/6) + 700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt(x) + 1260*a^2*b^5*x^(1/3) - 60*b^7)/x^(1/6)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2 + \frac{b^2}{x^{1/3}} + \frac{2ab}{x^{1/6}} \right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x)

[Out] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)

[Out] Timed out

$$3.429 \quad \int \frac{x^{-1+4n}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=46

$$\frac{b^2 \log(b + cx^n)}{c^3 n} - \frac{bx^n}{c^2 n} + \frac{x^{2n}}{2cn}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 266, 43}

$$\frac{b^2 \log(b + cx^n)}{c^3 n} - \frac{bx^n}{c^2 n} + \frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)), x]

[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b^2*Log[b + c*x^n])/(c^3*n)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+4n}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1+3n}}{b + cx^n} dx \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{b+cx} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2} + \frac{x}{c} + \frac{b^2}{c^2(b+cx)}\right) dx, x, x^n\right)}{n} \\
&= -\frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} + \frac{b^2 \log(b + cx^n)}{c^3n}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.83

$$\frac{2b^2 \log(b + cx^n) + cx^n (cx^n - 2b)}{2c^3n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)), x]

[Out] (c*x^n*(-2*b + c*x^n) + 2*b^2*Log[b + c*x^n])/(2*c^3*n)

IntegrateAlgebraic [A] time = 0.06, size = 41, normalized size = 0.89

$$\frac{b^2 \log(b + cx^n)}{c^3n} + \frac{x^n (cx^n - 2b)}{2c^2n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)), x]

[Out] (x^n*(-2*b + c*x^n))/(2*c^2*n) + (b^2*Log[b + c*x^n])/(c^3*n)

fricas [A] time = 1.64, size = 38, normalized size = 0.83

$$\frac{c^2x^{2n} - 2bcx^n + 2b^2 \log(cx^n + b)}{2c^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] 1/2*(c^2*x^(2*n) - 2*b*c*x^n + 2*b^2*log(c*x^n + b))/(c^3*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4n-1}}{cx^{2n} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+4*n)/(b*xⁿ+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*xⁿ), x)

maple [A] time = 0.03, size = 62, normalized size = 1.35

$$\left(-\frac{b e^{2n \ln(x)}}{c^2 n} + \frac{e^{3n \ln(x)}}{2cn} \right) e^{-n \ln(x)} + \frac{b^2 \ln(c e^{n \ln(x)} + b)}{c^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+4*n)/(b*xⁿ+c*x^(2*n)),x)

[Out] (1/2/c/n*exp(n*ln(x))³-b/c²/n*exp(n*ln(x))²)/exp(n*ln(x))+b²/c³/n*ln(c*exp(n*ln(x))+b)

maxima [A] time = 0.93, size = 45, normalized size = 0.98

$$\frac{b^2 \log\left(\frac{cx^n+b}{c}\right)}{c^3 n} + \frac{cx^{2n} - 2bx^n}{2c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+4*n)/(b*xⁿ+c*x^(2*n)),x, algorithm="maxima")

[Out] b²*log((c*xⁿ + b)/c)/(c³*n) + 1/2*(c*x^(2*n) - 2*b*xⁿ)/(c²*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{4n-1}}{bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(4*n - 1)/(b*xⁿ + c*x^(2*n)),x)

[Out] int(x^(4*n - 1)/(b*xⁿ + c*x^(2*n)), x)

sympy [A] time = 19.83, size = 42, normalized size = 0.91

$$\frac{b^2 \left(\begin{cases} \frac{x^n}{b} & \text{for } c = 0 \\ \frac{\log(b+cx^n)}{c} & \text{otherwise} \end{cases} \right)}{c^{2n}} - \frac{bx^n}{c^{2n}} + \frac{x^{2n}}{2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)

[Out] b**2*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c**2*n) - b*x**n/(c**2*n) + x**(2*n)/(2*c*n)

$$3.430 \quad \int \frac{x^{-1+3n}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=28

$$\frac{x^n}{cn} - \frac{b \log(b + cx^n)}{c^2 n}$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 266, 43}

$$\frac{x^n}{cn} - \frac{b \log(b + cx^n)}{c^2 n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)), x]

[Out] x^n/(c*n) - (b*Log[b + c*x^n])/(c^2*n)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1+2n}}{b + cx^n} dx \\
&= \frac{\text{Subst}\left(\int \frac{x}{b+cx} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{c} - \frac{b}{c(b+cx)}\right) dx, x, x^n\right)}{n} \\
&= \frac{x^n}{cn} - \frac{b \log(b + cx^n)}{c^2 n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$\frac{\frac{x^n}{c} - \frac{b \log(b+cx^n)}{c^2}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)), x]

[Out] (x^n/c - (b*Log[b + c*x^n])/c^2)/n

IntegrateAlgebraic [A] time = 0.07, size = 34, normalized size = 1.21

$$\frac{x^n}{cn} - \frac{b \log(bcn + c^2 nx^n)}{c^2 n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)), x]

[Out] x^n/(c*n) - (b*Log[b*c*n + c^2*n*x^n])/(c^2*n)

fricas [A] time = 2.06, size = 24, normalized size = 0.86

$$\frac{cx^n - b \log(cx^n + b)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] (c*x^n - b*log(c*x^n + b))/(c^2*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{cx^{2n} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+3*n)/(b*x[^]n+c*x[^](2*n)),x, algorithm="giac")

[Out] integrate(x[^](3*n - 1)/(c*x[^](2*n) + b*x[^]n), x)

maple [A] time = 0.02, size = 33, normalized size = 1.18

$$-\frac{b \ln(c e^{n \ln(x)} + b)}{c^2 n} + \frac{e^{n \ln(x)}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x[^](-1+3*n)/(b*x[^]n+c*x[^](2*n)),x)

[Out] 1/c/n*exp(n*ln(x))-b/c[^]2/n*ln(c*exp(n*ln(x))+b)

maxima [A] time = 0.87, size = 32, normalized size = 1.14

$$\frac{x^n}{cn} - \frac{b \log\left(\frac{cx^n+b}{c}\right)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+3*n)/(b*x[^]n+c*x[^](2*n)),x, algorithm="maxima")

[Out] x[^]n/(c*n) - b*log((c*x[^]n + b)/c)/(c[^]2*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{3n-1}}{bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x[^](3*n - 1)/(b*x[^]n + c*x[^](2*n)),x)

[Out] int(x[^](3*n - 1)/(b*x[^]n + c*x[^](2*n)), x)

sympy [A] time = 13.89, size = 26, normalized size = 0.93

$$-\frac{b \left(\begin{cases} \frac{x^n}{b} & \text{for } c = 0 \\ \frac{\log(b+cx^n)}{c} & \text{otherwise} \end{cases} \right)}{cn} + \frac{x^n}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -b*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c*n) + x**n/(c*n)
```


$$3.431 \quad \int \frac{x^{-1+2n}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=15

$$\frac{\log(b + cx^n)}{cn}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1584, 260}

$$\frac{\log(b + cx^n)}{cn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)), x]

[Out] Log[b + c*x^n]/(c*n)

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+2n}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1+n}}{b + cx^n} dx \\ &= \frac{\log(b + cx^n)}{cn} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(b + cx^n)}{cn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/(b*xⁿ + c*x^(2*n)), x]

[Out] Log[b + c*xⁿ]/(c*n)

IntegrateAlgebraic [A] time = 0.05, size = 18, normalized size = 1.20

$$\frac{\log(bn + cnx^n)}{cn}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/(b*xⁿ + c*x^(2*n)), x]

[Out] Log[b*n + c*n*xⁿ]/(c*n)

fricas [A] time = 1.13, size = 15, normalized size = 1.00

$$\frac{\log(cx^n + b)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(b*xⁿ+c*x^(2*n)), x, algorithm="fricas")

[Out] log(c*xⁿ + b)/(c*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{cx^{2n} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(b*xⁿ+c*x^(2*n)), x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*xⁿ), x)

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\frac{\ln\left(c e^{n \ln(x)} + b\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+2*n)/(b*xⁿ+c*x^(2*n)), x)

[Out] 1/c/n*ln(c*exp(n*ln(x))+b)

maxima [A] time = 0.89, size = 19, normalized size = 1.27

$$\frac{\log\left(\frac{cx^n+b}{c}\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(b*xⁿ+c*x^(2*n)), x, algorithm="maxima")

[Out] log((c*xⁿ + b)/c)/(c*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^{2n-1}}{bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n - 1)/(b*xⁿ + c*x^(2*n)), x)

[Out] int(x^(2*n - 1)/(b*xⁿ + c*x^(2*n)), x)

sympy [A] time = 11.28, size = 37, normalized size = 2.47

$$\begin{cases} \frac{\log(x)}{b} & \text{for } c = 0 \wedge n = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{x^n}{bn} & \text{for } c = 0 \\ -\frac{\log(x)}{c} + \frac{\log\left(\frac{bx^n}{c} + x^{2n}\right)}{cn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(b*xⁿ+c*x^(2*n)), x)

[Out] Piecewise((log(x)/b, Eq(c, 0) & Eq(n, 0)), (log(x)/(b + c), Eq(n, 0)), (xⁿ/b*n, Eq(c, 0)), (-log(x)/c + log(b*xⁿ/c + x^(2*n))/(c*n), True))

$$3.432 \quad \int \frac{x^{-1+n}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=23

$$\frac{\log(x)}{b} - \frac{\log(b + cx^n)}{bn}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1584, 266, 36, 29, 31}

$$\frac{\log(x)}{b} - \frac{\log(b + cx^n)}{bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]

[Out] Log[x]/b - Log[b + c*x^n]/(b*n)

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}}{bx^n + cx^{2n}} dx &= \int \frac{1}{x(b + cx^n)} dx \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(b+cx)} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, x^n\right)}{bn} - \frac{c \text{Subst}\left(\int \frac{1}{b+cx} dx, x, x^n\right)}{bn} \\
&= \frac{\log(x)}{b} - \frac{\log(b + cx^n)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{n \log(x) - \log(b + cx^n)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)/(b*x^n + c*x^(2*n)), x]

[Out] (n*Log[x] - Log[b + c*x^n])/(b*n)

IntegrateAlgebraic [A] time = 0.05, size = 34, normalized size = 1.48

$$\frac{\log(x^n)}{bn} - \frac{\log(b^2n + bcnx^n)}{bn}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + n)/(b*x^n + c*x^(2*n)), x]

[Out] Log[x^n]/(b*n) - Log[b^2*n + b*c*n*x^n]/(b*n)

fricas [A] time = 1.46, size = 22, normalized size = 0.96

$$\frac{n \log(x) - \log(cx^n + b)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] (n*log(x) - log(c*x^n + b))/(b*n)

giac [A] time = 0.25, size = 25, normalized size = 1.09

$$\frac{\log(|x|)}{b} - \frac{\log(|cx^n + b|)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+n)/(b*x[^]n+c*x[^](2*n)),x, algorithm="giac")

[Out] log(abs(x))/b - log(abs(c*x[^]n + b))/(b*n)

maple [A] time = 0.02, size = 26, normalized size = 1.13

$$\frac{\ln(x)}{b} - \frac{\ln(c e^{n \ln(x)} + b)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x[^](-1+n)/(b*x[^]n+c*x[^](2*n)),x)

[Out] ln(x)/b-1/b/n*ln(c*exp(n*ln(x))+b)

maxima [A] time = 0.90, size = 27, normalized size = 1.17

$$\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+n)/(b*x[^]n+c*x[^](2*n)),x, algorithm="maxima")

[Out] log(x)/b - log((c*x[^]n + b)/c)/(b*n)

mupad [B] time = 1.37, size = 20, normalized size = 0.87

$$-\frac{2 \operatorname{atanh}\left(\frac{2cx^n}{b} + 1\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x[^](n - 1)/(b*x[^]n + c*x[^](2*n)),x)

[Out] -(2*atanh((2*c*x[^]n)/b + 1))/(b*n)

sympy [A] time = 16.49, size = 66, normalized size = 2.87

$$\left\{ \begin{array}{ll} \infty \log(x) & \text{for } b = 0 \wedge c = 0 \wedge n = 0 \\ -\frac{x^{-n}}{cn} & \text{for } b = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{\frac{n^2 \log(x)}{n^2-n} - \frac{n \log(x)}{n^2-n}}{b} & \text{for } c = 0 \\ \frac{2 \log(x)}{b} - \frac{\log\left(\frac{bx^n}{c} + x^{2n}\right)}{bn} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)

[Out] Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-x**(-n)/(c*n), Eq(b, 0)), (log(x)/(b + c), Eq(n, 0)), ((n**2*log(x)/(n**2 - n) - n*log(x)/(n**2 - n))/b, Eq(c, 0)), (2*log(x)/b - log(b*x**n/c + x**(2*n))/(b*n), True)
)

$$3.433 \quad \int \frac{x^{-1-n}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=57

$$-\frac{c^2 \log(b + cx^n)}{b^3 n} + \frac{c^2 \log(x)}{b^3} + \frac{cx^{-n}}{b^2 n} - \frac{x^{-2n}}{2bn}$$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 266, 44}

$$-\frac{c^2 \log(b + cx^n)}{b^3 n} + \frac{c^2 \log(x)}{b^3} + \frac{cx^{-n}}{b^2 n} - \frac{x^{-2n}}{2bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]

[Out] -1/(2*b*n*x^(2*n)) + c/(b^2*n*x^n) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^n])/b^3*n

Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-n}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1-2n}}{b + cx^n} dx \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^3(b+cx)} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^3} - \frac{c}{b^2x^2} + \frac{c^2}{b^3x} - \frac{c^3}{b^3(b+cx)}\right) dx, x, x^n\right)}{n} \\
&= -\frac{x^{-2n}}{2bn} + \frac{cx^{-n}}{b^2n} + \frac{c^2 \log(x)}{b^3} - \frac{c^2 \log(b + cx^n)}{b^3n}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 49, normalized size = 0.86

$$\frac{-2c^2 \log(b + cx^n) + bx^{-2n}(2cx^n - b) + 2c^2n \log(x)}{2b^3n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n)/(b*x^n + c*x^(2*n)), x]

[Out] ((b*(-b + 2*c*x^n))/x^(2*n) + 2*c^2*n*Log[x] - 2*c^2*Log[b + c*x^n])/(2*b^3*n)

IntegrateAlgebraic [A] time = 0.07, size = 59, normalized size = 1.04

$$\frac{c^2 \log(x^n)}{b^3n} - \frac{c^2 \log(b + cx^n)}{b^3n} + \frac{x^{-2n}(2cx^n - b)}{2b^2n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 - n)/(b*x^n + c*x^(2*n)), x]

[Out] (-b + 2*c*x^n)/(2*b^2*n*x^(2*n)) + (c^2*Log[x^n])/(b^3*n) - (c^2*Log[b + c*x^n])/(b^3*n)

fricas [A] time = 1.39, size = 59, normalized size = 1.04

$$\frac{2c^2nx^{2n} \log(x) - 2c^2x^{2n} \log(cx^n + b) + 2bcx^n - b^2}{2b^3nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] $1/2*(2*c^2*n*x^(2*n)*\log(x) - 2*c^2*x^(2*n)*\log(c*x^n + b) + 2*b*c*x^n - b^2)/(b^3*n*x^(2*n))$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{cx^{2n} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")`

[Out] `integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n), x)`

maple [A] time = 0.02, size = 69, normalized size = 1.21

$$\left(\frac{c^2 e^{2n \ln(x)} \ln(x)}{b^3} + \frac{c e^{n \ln(x)}}{b^2 n} - \frac{1}{2bn} \right) e^{-2n \ln(x)} - \frac{c^2 \ln(c e^{n \ln(x)} + b)}{b^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-n)/(b*x^n+c*x^(2*n)),x)`

[Out] $(c/b^2/n*\exp(n*\ln(x))-1/2/b/n+c^2/b^3*\ln(x)*\exp(n*\ln(x))^2)/\exp(n*\ln(x))^2-c^2/b^3/n*\ln(c*\exp(n*\ln(x))+b)$

maxima [A] time = 0.91, size = 58, normalized size = 1.02

$$\frac{c^2 \log(x)}{b^3} - \frac{c^2 \log\left(\frac{cx^n+b}{c}\right)}{b^3 n} + \frac{2cx^n - b}{2b^2 n x^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")`

[Out] $c^2*\log(x)/b^3 - c^2*\log((c*x^n + b)/c)/(b^3*n) + 1/2*(2*c*x^n - b)/(b^2*n*x^(2*n))$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{n+1} (bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))),x)`

```
[Out] int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))), x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

```
Exception raised: HeuristicGCDFailed
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

$$3.434 \quad \int \frac{x^{-1-2n}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=76

$$\frac{c^3 \log(b + cx^n)}{b^4 n} - \frac{c^3 \log(x)}{b^4} - \frac{c^2 x^{-n}}{b^3 n} + \frac{cx^{-2n}}{2b^2 n} - \frac{x^{-3n}}{3bn}$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 266, 44}

$$-\frac{c^2 x^{-n}}{b^3 n} + \frac{c^3 \log(b + cx^n)}{b^4 n} - \frac{c^3 \log(x)}{b^4} + \frac{cx^{-2n}}{2b^2 n} - \frac{x^{-3n}}{3bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]

[Out] -1/(3*b*n*x^(3*n)) + c/(2*b^2*n*x^(2*n)) - c^2/(b^3*n*x^n) - (c^3*Log[x])/b^4 + (c^3*Log[b + c*x^n])/b^4

Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-2n}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1-3n}}{b + cx^n} dx \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^4(b+cx)} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^4} - \frac{c}{b^2x^3} + \frac{c^2}{b^3x^2} - \frac{c^3}{b^4x} + \frac{c^4}{b^4(b+cx)}\right) dx, x, x^n\right)}{n} \\
&= -\frac{x^{-3n}}{3bn} + \frac{cx^{-2n}}{2b^2n} - \frac{c^2x^{-n}}{b^3n} - \frac{c^3 \log(x)}{b^4} + \frac{c^3 \log(b + cx^n)}{b^4n}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 62, normalized size = 0.82

$$\frac{bx^{-3n} (2b^2 - 3bcx^n + 6c^2x^{2n}) - 6c^3 \log(b + cx^n) + 6c^3n \log(x)}{6b^4n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)), x]

[Out] -1/6*((b*(2*b^2 - 3*b*c*x^n + 6*c^2*x^(2*n)))/x^(3*n) + 6*c^3*n*Log[x] - 6*c^3*Log[b + c*x^n])/(b^4*n)

IntegrateAlgebraic [A] time = 0.10, size = 72, normalized size = 0.95

$$-\frac{c^3 \log(x^n)}{b^4n} + \frac{c^3 \log(b + cx^n)}{b^4n} + \frac{x^{-3n} (-2b^2 + 3bcx^n - 6c^2x^{2n})}{6b^3n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)), x]

[Out] (-2*b^2 + 3*b*c*x^n - 6*c^2*x^(2*n))/(6*b^3*n*x^(3*n)) - (c^3*Log[x^n])/(b^4*n) + (c^3*Log[b + c*x^n])/(b^4*n)

fricas [A] time = 1.47, size = 72, normalized size = 0.95

$$\frac{6c^3nx^{3n} \log(x) - 6c^3x^{3n} \log(cx^n + b) + 6bc^2x^{2n} - 3b^2cx^n + 2b^3}{6b^4nx^{3n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] $-1/6*(6*c^3*n*x^{(3*n)}*\log(x) - 6*c^3*x^{(3*n)}*\log(c*x^n + b) + 6*b*c^2*x^{(2*n)} - 3*b^2*c*x^n + 2*b^3)/(b^4*n*x^{(3*n)})$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n-1}}{cx^{2n} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")`

[Out] `integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n), x)`

maple [A] time = 0.03, size = 88, normalized size = 1.16

$$\left(-\frac{c^3 e^{3n \ln(x)} \ln(x)}{b^4} + \frac{c e^{n \ln(x)}}{2b^2 n} - \frac{c^2 e^{2n \ln(x)}}{b^3 n} - \frac{1}{3bn} \right) e^{-3n \ln(x)} + \frac{c^3 \ln(c e^{n \ln(x)} + b)}{b^4 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x)`

[Out] $(-1/3/b/n+1/2/b^2*c/n*\exp(n*\ln(x))-c^2/b^3/n*\exp(n*\ln(x))^2-c^3/b^4*\ln(x)*\exp(n*\ln(x))^3)/\exp(n*\ln(x))^3+c^3/b^4/n*\ln(c*\exp(n*\ln(x))+b)$

maxima [A] time = 0.91, size = 71, normalized size = 0.93

$$-\frac{c^3 \log(x)}{b^4} + \frac{c^3 \log\left(\frac{cx^n+b}{c}\right)}{b^4 n} - \frac{6c^2 x^{2n} - 3bcx^n + 2b^2}{6b^3 n x^{3n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")`

[Out] $-c^3*\log(x)/b^4 + c^3*\log((c*x^n + b)/c)/(b^4*n) - 1/6*(6*c^2*x^{(2*n)} - 3*b*c*x^n + 2*b^2)/(b^3*n*x^{(3*n)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2n+1} (bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))),x)`

[Out] `int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))), x)`

sympy [A] time = 52.91, size = 73, normalized size = 0.96

$$-\frac{x^{-3n}}{3bn} + \frac{cx^{-2n}}{2b^2n} - \frac{c^2x^{-n}}{b^3n} + \frac{c^4 \begin{cases} \frac{x^n}{b} & \text{for } c = 0 \\ \frac{\log(b+cx^n)}{c} & \text{otherwise} \end{cases}}{b^4n} - \frac{c^3 \log(x^n)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)), x)`

[Out] `-x**(-3*n)/(3*b*n) + c*x**(-2*n)/(2*b**2*n) - c**2*x**(-n)/(b**3*n) + c**4*
Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(b**4*n) - c**3*log(x**n)/(b**4*n)`

$$3.435 \quad \int \frac{x^{-1-3n}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=93

$$-\frac{c^4 \log(b + cx^n)}{b^5 n} + \frac{c^4 \log(x)}{b^5} + \frac{c^3 x^{-n}}{b^4 n} - \frac{c^2 x^{-2n}}{2b^3 n} + \frac{cx^{-3n}}{3b^2 n} - \frac{x^{-4n}}{4bn}$$

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 266, 44}

$$-\frac{c^2 x^{-2n}}{2b^3 n} + \frac{c^3 x^{-n}}{b^4 n} - \frac{c^4 \log(b + cx^n)}{b^5 n} + \frac{c^4 \log(x)}{b^5} + \frac{cx^{-3n}}{3b^2 n} - \frac{x^{-4n}}{4bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]

[Out] -1/(4*b*n*x^(4*n)) + c/(3*b^2*n*x^(3*n)) - c^2/(2*b^3*n*x^(2*n)) + c^3/(b^4*n*x^n) + (c^4*Log[x])/b^5 - (c^4*Log[b + c*x^n])/(b^5*n)

Rule 44

Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-3n}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1-4n}}{b + cx^n} dx \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^5(b+cx)} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^5} - \frac{c}{b^2x^4} + \frac{c^2}{b^3x^3} - \frac{c^3}{b^4x^2} + \frac{c^4}{b^5x} - \frac{c^5}{b^5(b+cx)}\right) dx, x, x^n\right)}{n} \\
&= -\frac{x^{-4n}}{4bn} + \frac{cx^{-3n}}{3b^2n} - \frac{c^2x^{-2n}}{2b^3n} + \frac{c^3x^{-n}}{b^4n} + \frac{c^4 \log(x)}{b^5} - \frac{c^4 \log(b + cx^n)}{b^5n}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 0.81

$$\frac{bx^{-4n} (3b^3 - 4b^2cx^n + 6bc^2x^{2n} - 12c^3x^{3n}) + 12c^4 \log(b + cx^n) - 12c^4n \log(x)}{12b^5n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)), x]

[Out] -1/12*((b*(3*b^3 - 4*b^2*c*x^n + 6*b*c^2*x^(2*n) - 12*c^3*x^(3*n)))/x^(4*n) - 12*c^4*n*Log[x] + 12*c^4*Log[b + c*x^n])/(b^5*n)

IntegrateAlgebraic [A] time = 0.12, size = 85, normalized size = 0.91

$$\frac{c^4 \log(x^n)}{b^5n} - \frac{c^4 \log(b + cx^n)}{b^5n} + \frac{x^{-4n} (-3b^3 + 4b^2cx^n - 6bc^2x^{2n} + 12c^3x^{3n})}{12b^4n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)), x]

[Out] (-3*b^3 + 4*b^2*c*x^n - 6*b*c^2*x^(2*n) + 12*c^3*x^(3*n))/(12*b^4*n*x^(4*n)) + (c^4*Log[x^n])/(b^5*n) - (c^4*Log[b + c*x^n])/(b^5*n)

fricas [A] time = 1.22, size = 85, normalized size = 0.91

$$\frac{12c^4nx^{4n} \log(x) - 12c^4x^{4n} \log(cx^n + b) + 12bc^3x^{3n} - 6b^2c^2x^{2n} + 4b^3cx^n - 3b^4}{12b^5nx^{4n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] $\frac{1}{12} \cdot (12 \cdot c^4 \cdot n \cdot x^{(4 \cdot n)} \cdot \log(x) - 12 \cdot c^4 \cdot x^{(4 \cdot n)} \cdot \log(c \cdot x^n + b) + 12 \cdot b \cdot c^3 \cdot x^{(3 \cdot n)} - 6 \cdot b^2 \cdot c^2 \cdot x^{(2 \cdot n)} + 4 \cdot b^3 \cdot c \cdot x^n - 3 \cdot b^4) / (b^5 \cdot n \cdot x^{(4 \cdot n)})$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3n-1}}{cx^{2n} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")`

[Out] `integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n), x)`

maple [A] time = 0.03, size = 105, normalized size = 1.13

$$\left(\frac{c^4 e^{4n \ln(x)} \ln(x)}{b^5} + \frac{c e^{n \ln(x)}}{3b^2 n} - \frac{c^2 e^{2n \ln(x)}}{2b^3 n} + \frac{c^3 e^{3n \ln(x)}}{b^4 n} - \frac{1}{4bn} \right) e^{-4n \ln(x)} - \frac{c^4 \ln(c e^{n \ln(x)} + b)}{b^5 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x)`

[Out] $(c^3/b^4/n \cdot \exp(n \cdot \ln(x))^{3-1/4} / b/n + 1/3/b^2 \cdot c/n \cdot \exp(n \cdot \ln(x)) - 1/2 \cdot c^2/b^3/n \cdot \exp(n \cdot \ln(x))^{2+c^4/b^5 \cdot \ln(x) \cdot \exp(n \cdot \ln(x))^{4}} / \exp(n \cdot \ln(x))^{4-c^4/b^5/n \cdot \ln(c \cdot \exp(n \cdot \ln(x)) + b)})$

maxima [A] time = 0.92, size = 84, normalized size = 0.90

$$\frac{c^4 \log(x)}{b^5} - \frac{c^4 \log\left(\frac{cx^n+b}{c}\right)}{b^5 n} + \frac{12c^3x^{3n} - 6bc^2x^{2n} + 4b^2cx^n - 3b^3}{12b^4nx^{4n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")`

[Out] $c^4 \cdot \log(x) / b^5 - c^4 \cdot \log((c \cdot x^n + b) / c) / (b^5 \cdot n) + 1/12 \cdot (12 \cdot c^3 \cdot x^{(3 \cdot n)} - 6 \cdot b \cdot c^2 \cdot x^{(2 \cdot n)} + 4 \cdot b^2 \cdot c \cdot x^n - 3 \cdot b^3) / (b^4 \cdot n \cdot x^{(4 \cdot n)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3n+1} (bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))),x)`

[Out] `int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))), x)`

sympy [A] time = 105.73, size = 88, normalized size = 0.95

$$-\frac{x^{-4n}}{4bn} + \frac{cx^{-3n}}{3b^2n} - \frac{c^2x^{-2n}}{2b^3n} + \frac{c^3x^{-n}}{b^4n} - \frac{c^5 \left(\begin{array}{ll} \frac{x^n}{b} & \text{for } c = 0 \\ \frac{\log(b+cx^n)}{c} & \text{otherwise} \end{array} \right)}{b^5n} + \frac{c^4 \log(x^n)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)), x)`

[Out] `-x**(-4*n)/(4*b*n) + c*x**(-3*n)/(3*b**2*n) - c**2*x**(-2*n)/(2*b**3*n) + c**3*x**(-n)/(b**4*n) - c**5*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(b**5*n) + c**4*log(x**n)/(b**5*n)`

$$3.436 \quad \int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}} dx$$

Optimal. Leaf size=236

$$\frac{c^{3/4} \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{n/4} + \sqrt{b} + \sqrt{c} x^{n/2}\right)}{\sqrt{2} b^{7/4} n} - \frac{c^{3/4} \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{n/4} + \sqrt{b} + \sqrt{c} x^{n/2}\right)}{\sqrt{2} b^{7/4} n} + \frac{\sqrt{2} c^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4} n}$$

Rubi [A] time = 0.21, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1584, 362, 345, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4} \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{n/4} + \sqrt{b} + \sqrt{c} x^{n/2}\right)}{\sqrt{2} b^{7/4} n} - \frac{c^{3/4} \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{n/4} + \sqrt{b} + \sqrt{c} x^{n/2}\right)}{\sqrt{2} b^{7/4} n} + \frac{\sqrt{2} c^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4} n} - \frac{\sqrt{2} c^{3/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{b}} + 1\right)}{b^{7/4} n} - \frac{4x^{-3n/4}}{3bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n/4)/(b*x^n + c*x^(2*n)), x]

[Out] -4/(3*b*n*x^((3*n)/4)) + (Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x^(n/4))/b^(1/4)]/(b^(7/4)*n) - (Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x^(n/4))/b^(1/4)]/(b^(7/4)*n) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)]/(Sqrt[2]*b^(7/4)*n) - (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)]/(Sqrt[2]*b^(7/4)*n)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 345

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[x^(m + 1)/(a*(m + 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n_, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+\frac{n}{4}}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1-\frac{3n}{4}}}{b + cx^n} dx \\
&= -\frac{4x^{-3n/4}}{3bn} - \frac{c \int \frac{x^{\frac{1}{4}(-4+n)}}{b+cx^n} dx}{b} \\
&= -\frac{4x^{-3n/4}}{3bn} - \frac{(4c) \text{Subst} \left(\int \frac{1}{b+cx^4} dx, x, x^{1+\frac{1}{4}(-4+n)} \right)}{bn} \\
&= -\frac{4x^{-3n/4}}{3bn} - \frac{(2c) \text{Subst} \left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4} dx, x, x^{1+\frac{1}{4}(-4+n)} \right)}{b^{3/2}n} - \frac{(2c) \text{Subst} \left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4} dx, x, x^{1+\frac{1}{4}(-4+n)} \right)}{b^{3/2}n} \\
&= -\frac{4x^{-3n/4}}{3bn} - \frac{\sqrt{c} \text{Subst} \left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, x^{1+\frac{1}{4}(-4+n)} \right)}{b^{3/2}n} - \frac{\sqrt{c} \text{Subst} \left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{c}} + x^2} dx, x, x^{1+\frac{1}{4}(-4+n)} \right)}{b^{3/2}n} \\
&= -\frac{4x^{-3n/4}}{3bn} + \frac{c^{3/4} \log(\sqrt{b} - \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{n/4} + \sqrt{c} x^{n/2})}{\sqrt{2} b^{7/4} n} - \frac{c^{3/4} \log(\sqrt{b} + \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{n/4} + \sqrt{c} x^{n/2})}{\sqrt{2} b^{7/4} n} \\
&= -\frac{4x^{-3n/4}}{3bn} + \frac{\sqrt{2} c^{3/4} \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{b}} \right)}{b^{7/4} n} - \frac{\sqrt{2} c^{3/4} \tan^{-1} \left(1 + \frac{\sqrt{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{b}} \right)}{b^{7/4} n} + \frac{c^{3/4} \log(\sqrt{b} - \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{n/4} + \sqrt{c} x^{n/2})}{\sqrt{2} b^{7/4} n} - \frac{c^{3/4} \log(\sqrt{b} + \sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{n/4} + \sqrt{c} x^{n/2})}{\sqrt{2} b^{7/4} n}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.14

$$-\frac{4x^{-3n/4} {}_2F_1\left(-\frac{3}{4}, 1; \frac{1}{4}; -\frac{cx^n}{b}\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n/4)/(b*x^n + c*x^(2*n)), x]

[Out] (-4*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^n)/b)])/(3*b*n*x^((3*n)/4))

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + n/4)/(b*x^n + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + n/4)/(b*x^n + c*x^(2*n)), x]

fricas [A] time = 0.92, size = 272, normalized size = 1.15

$$\frac{12 b n x^3 x^{4 n-3} \left(-\frac{c^3}{b^3 n^4}\right)^{\frac{1}{4}} \arctan\left(\frac{b^5 c n^3 x x^{\frac{1}{4} n-1} \left(-\frac{c^3}{b^3 n^4}\right)^{\frac{3}{4}} - b^5 n^3 x \sqrt{\frac{b^4 n^2 \sqrt{\frac{c^3}{b^3 n^4} + 2 x^{\frac{1}{2} n-2}}}{x^2}} \left(-\frac{c^3}{b^3 n^4}\right)^{\frac{3}{4}}}{c^3}\right) + 3 b n x^3 x^{4 n-3} \left(-\frac{c^3}{b^3 n^4}\right)^{\frac{1}{4}} \log\left(\frac{b^2 n \left(-\frac{c^3}{b^3 n^4}\right)^{\frac{1}{4}} + c x x^{\frac{1}{4} n-1}}{x}\right) - 3 b n x^3 x^{4 n-3} \left(-\frac{c^3}{b^3 n^4}\right)^{\frac{1}{4}} \log\left(\frac{b^2 n \left(-\frac{c^3}{b^3 n^4}\right)^{\frac{1}{4}} - c x x^{\frac{1}{4} n-1}}{x}\right) + 4}{3 b n x^3 x^{4 n-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] $-1/3*(12*b*n*x^3*x^{(3/4*n - 3)}*(-c^3/(b^7*n^4))^{(1/4)}*\arctan(-b^5*c*n^3*x*x^{(1/4*n - 1)}*(-c^3/(b^7*n^4))^{(3/4)} - b^5*n^3*x*\sqrt{(b^4*n^2*\sqrt{-c^3/(b^7*n^4)} + c^2*x^2*x^{(1/2*n - 2)})/x^2}*(-c^3/(b^7*n^4))^{(3/4)})/c^3 + 3*b*n*x^3*x^{(3/4*n - 3)}*(-c^3/(b^7*n^4))^{(1/4)}*\log((b^2*n*(-c^3/(b^7*n^4))^{(1/4)} + c*x*x^{(1/4*n - 1)})/x) - 3*b*n*x^3*x^{(3/4*n - 3)}*(-c^3/(b^7*n^4))^{(1/4)}*\log(-b^2*n*(-c^3/(b^7*n^4))^{(1/4)} - c*x*x^{(1/4*n - 1)})/x) + 4)/(b*n*x^3*x^{(3/4*n - 3)})$

giac [A] time = 0.35, size = 203, normalized size = 0.86

$$\frac{6 \sqrt{2} (bc^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\frac{b}{c}\right)^{\frac{1}{4}} + 2 (x^n)^{\frac{1}{4}}}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2} + \frac{6 \sqrt{2} (bc^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\frac{b}{c}\right)^{\frac{1}{4}} - 2 (x^n)^{\frac{1}{4}}}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2} + \frac{3 \sqrt{2} (bc^3)^{\frac{1}{4}} \log\left(x^{\frac{1}{2} n} + \sqrt{2} (x^n)^{\frac{1}{4}} \left(\frac{b}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{b}{c}}\right)}{b^2} - \frac{3 \sqrt{2} (bc^3)^{\frac{1}{4}} \log\left(x^{\frac{1}{2} n} - \sqrt{2} (x^n)^{\frac{1}{4}} \left(\frac{b}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{b}{c}}\right)}{b^2} + \frac{8}{b x^{\frac{3}{4} n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)), x, algorithm="giac")

[Out] $-1/6*(6*\sqrt{2}*(b*c^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} + 2*(x^n)^{(1/4)})/b^2 + 6*\sqrt{2}*(b*c^3)^{(1/4)}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(b/c)^{(1/4)} - 2*(x^n)^{(1/4)})/b^2 + 3*\sqrt{2}*(b*c^3)^{(1/4)}*\log(x^{(1/2*n)} + \sqrt{2}*(x^n)^{(1/4)}*(b/c)^{(1/4)} + \sqrt{b/c})/b^2 - 3*\sqrt{2}*(b*c^3)^{(1/4)}*\log(x^{(1/2*n)} - \sqrt{2}*(x^n)^{(1/4)}*(b/c)^{(1/4)} + \sqrt{b/c})/b^2 + 8/(b*x^{(3/4*n)}))/n$

maple [C] time = 0.12, size = 54, normalized size = 0.23

$$\text{RootOf}(b^7 n^4 _Z^4 + c^3) \ln\left(-\frac{\text{RootOf}(b^7 n^4 _Z^4 + c^3) b^2 n}{c} + x^{\frac{n}{4}}\right) - \frac{4 x^{-\frac{3n}{4}}}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x)`

[Out] `-4/3/b/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-b^2*n/c*_R),_R=RootOf(_Z^4*b^7*n^4+c^3))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \int \frac{x^{\frac{1}{4}n}}{bcx^n + b^2x} dx - \frac{4}{3bnx^{\frac{3}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")`

[Out] `-c*integrate(x^(1/4*n)/(b*c*x*x^n + b^2*x), x) - 4/3/(b*n*x^(3/4*n))`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{4}-1}}{bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)),x)`

[Out] `int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n}x^{\frac{n}{4}-1}}{b + cx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)`

[Out] `Integral(x**(-n)*x**(n/4 - 1)/(b + c*x**n), x)`

$$3.437 \quad \int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}} dx$$

Optimal. Leaf size=160

$$-\frac{c^{2/3} \log(\sqrt[3]{b} + \sqrt[3]{c} x^{n/3})}{b^{5/3}n} + \frac{c^{2/3} \log(b^{2/3} - \sqrt[3]{b} \sqrt[3]{c} x^{n/3} + c^{2/3} x^{2n/3})}{2b^{5/3}n} + \frac{\sqrt{3} c^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n} - \frac{3x^{-2n/3}}{2bn}$$

Rubi [A] time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1584, 362, 345, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{2/3} \log(\sqrt[3]{b} + \sqrt[3]{c} x^{n/3})}{b^{5/3}n} + \frac{c^{2/3} \log(b^{2/3} - \sqrt[3]{b} \sqrt[3]{c} x^{n/3} + c^{2/3} x^{2n/3})}{2b^{5/3}n} + \frac{\sqrt{3} c^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n} - \frac{3x^{-2n/3}}{2bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n/3)/(b*x^n + c*x^(2*n)), x]

[Out] $-\frac{3}{(2*b*n*x^{(2*n)/3})} + \frac{(\text{Sqrt}[3]*c^{(2/3)}*\text{ArcTan}[(b^{(1/3)} - 2*c^{(1/3)}*x^{(n/3)})/(\text{Sqrt}[3]*b^{(1/3)})])}{(b^{(5/3)*n})} - \frac{(c^{(2/3)}*\text{Log}[b^{(1/3)} + c^{(1/3)}*x^{(n/3)}])}{(b^{(5/3)*n})} + \frac{(c^{(2/3)}*\text{Log}[b^{(2/3)} - b^{(1/3)}*c^{(1/3)}*x^{(n/3)} + c^{(2/3)}*x^{((2*n)/3)}])}{(2*b^{(5/3)*n})}$

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[x^(m + 1)/(a*(m +
1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1-\frac{2n}{3}}}{b + cx^n} dx \\
&= -\frac{3x^{-2n/3}}{2bn} - \frac{c \int \frac{x^{\frac{1}{3}(-3+n)}}{b+cx^n} dx}{b} \\
&= -\frac{3x^{-2n/3}}{2bn} - \frac{(3c) \text{Subst} \left(\int \frac{1}{b+cx^3} dx, x, x^{1+\frac{1}{3}(-3+n)} \right)}{bn} \\
&= -\frac{3x^{-2n/3}}{2bn} - \frac{c \text{Subst} \left(\int \frac{1}{\sqrt[3]{b} + \sqrt[3]{c}x} dx, x, x^{1+\frac{1}{3}(-3+n)} \right)}{b^{5/3}n} - \frac{c \text{Subst} \left(\int \frac{2\sqrt[3]{b} - \sqrt[3]{c}x}{b^{2/3} - \sqrt[3]{b}\sqrt[3]{c}x + c^{2/3}x^2} dx, x, x^{1+\frac{1}{3}(-3+n)} \right)}{b^{5/3}n} \\
&= -\frac{3x^{-2n/3}}{2bn} - \frac{c^{2/3} \log(\sqrt[3]{b} + \sqrt[3]{c}x^{n/3})}{b^{5/3}n} + \frac{c^{2/3} \text{Subst} \left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c} + 2c^{2/3}x}{b^{2/3} - \sqrt[3]{b}\sqrt[3]{c}x + c^{2/3}x^2} dx, x, x^{1+\frac{1}{3}(-3+n)} \right)}{2b^{5/3}n} \quad (3) \\
&= -\frac{3x^{-2n/3}}{2bn} - \frac{c^{2/3} \log(\sqrt[3]{b} + \sqrt[3]{c}x^{n/3})}{b^{5/3}n} + \frac{c^{2/3} \log(b^{2/3} - \sqrt[3]{b}\sqrt[3]{c}x^{n/3} + c^{2/3}x^{2n/3})}{2b^{5/3}n} - \frac{(3c^{2/3}) \text{Subst} \left(\int \frac{1}{b^{2/3} - \sqrt[3]{b}\sqrt[3]{c}x + c^{2/3}x^2} dx, x, x^{1+\frac{1}{3}(-3+n)} \right)}{2b^{5/3}n} \\
&= -\frac{3x^{-2n/3}}{2bn} + \frac{\sqrt{3}c^{2/3} \tan^{-1} \left(\frac{\sqrt[3]{b} - 2\sqrt[3]{c}x^{n/3}}{\sqrt{3}\sqrt[3]{b}} \right)}{b^{5/3}n} - \frac{c^{2/3} \log(\sqrt[3]{b} + \sqrt[3]{c}x^{n/3})}{b^{5/3}n} + \frac{c^{2/3} \log(b^{2/3} - \sqrt[3]{b}\sqrt[3]{c}x^{n/3} + c^{2/3}x^{2n/3})}{2b^{5/3}n}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.21

$$-\frac{3x^{-2n/3} {}_2F_1\left(-\frac{2}{3}, 1; \frac{1}{3}; -\frac{cx^n}{b}\right)}{2bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n/3)/(b*x^n + c*x^(2*n)), x]

[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((c*x^n)/b)])/(2*b*n*x^((2*n)/3))

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + n/3)/(b*x^n + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + n/3)/(b*x^n + c*x^(2*n)), x]

fricas [A] time = 1.33, size = 212, normalized size = 1.32

$$\frac{2\sqrt{3}x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bxx^{\frac{1}{3}n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)+2x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{c^{\frac{1}{3}}x^{\frac{1}{3}n-1}-b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}}{x}\right)-x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{c^2x^{\frac{2}{3}n-2}+bcx^{\frac{1}{3}n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}+b^2\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}}{x^2}\right)-3}{2bnx^2x^{\frac{2}{3}n-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] $\frac{1}{2}*(2*\sqrt{3}*x^2*x^{(2/3*n - 2)}*(-c^2/b^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*b*x^{(1/3*n - 1)}*(-c^2/b^2)^{(2/3)} - \sqrt{3}*c)/c + 2*x^2*x^{(2/3*n - 2)}*(-c^2/b^2)^{(1/3)}*\log((c*x*x^{(1/3*n - 1)} - b*(-c^2/b^2)^{(1/3)})/x) - x^2*x^{(2/3*n - 2)}*(-c^2/b^2)^{(1/3)}*\log((c^2*x^2*x^{(2/3*n - 2)} + b*c*x*x^{(1/3*n - 1)}*(-c^2/b^2)^{(1/3)} + b^2*(-c^2/b^2)^{(2/3)})/x^2) - 3)/(b*n*x^2*x^{(2/3*n - 2)})$

giac [A] time = 0.42, size = 136, normalized size = 0.85

$$\frac{2c\left(-\frac{b}{c}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}n}-\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right)}{b^2} - \frac{2\sqrt{3}(-bc^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}n}+\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{c}\right)^{\frac{1}{3}}}\right)}{b^2} - \frac{(-bc^2)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}n}\left(-\frac{b}{c}\right)^{\frac{1}{3}}+(x^n)^{\frac{2}{3}}+\left(-\frac{b}{c}\right)^{\frac{2}{3}}\right)}{b^2} - \frac{3}{b(x^n)^{\frac{2}{3}}}$$

$2n$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] $\frac{1}{2}*(2*c*(-b/c)^{(1/3)}*\log(\text{abs}(x^{(1/3*n)} - (-b/c)^{(1/3)}))/b^2 - 2*\sqrt{3}*(-b*c^2)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*x^{(1/3*n)} + (-b/c)^{(1/3)})/(-b/c)^{(1/3)})/b^2 - (-b*c^2)^{(1/3)}*\log(x^{(1/3*n)}*(-b/c)^{(1/3)} + (x^n)^{(2/3)} + (-b/c)^{(2/3}))/b^2 - 3/(b*(x^n)^{(2/3)}))/n$

maple [C] time = 0.07, size = 54, normalized size = 0.34

$$\text{RootOf}(b^5n^3_Z^3 + c^2)\ln\left(-\frac{\text{RootOf}(b^5n^3_Z^3 + c^2)b^2n}{c} + x^{\frac{n}{3}}\right) - \frac{3x^{-\frac{2n}{3}}}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x)

[Out] $-3/2/b/n/(x^{(1/3*n)})^2 + \text{sum}(_R * \ln(x^{(1/3*n)}) - _R * b^2/c*n), _R = \text{RootOf}(_Z^3 * b^5 * n^3 + c^2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \int \frac{x^{\frac{1}{3}n}}{bcxx^n + b^2x} dx - \frac{3}{2bnx^{\frac{2}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")`

[Out] $-c * \text{integrate}(x^{(1/3*n)} / (b * c * x * x^n + b^2 * x), x) - 3/2 / (b * n * x^{(2/3*n)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{n}{3}-1}}{bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)),x)`

[Out] `int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)`

[Out] Timed out

$$3.438 \quad \int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}} dx$$

Optimal. Leaf size=50

$$\frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n} - \frac{2x^{-n/2}}{bn}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1584, 345, 193, 321, 205}

$$\frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n} - \frac{2x^{-n/2}}{bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]

[Out] -2/(b*n*x^(n/2)) + (2*sqrt[c]*ArcTan[sqrt[b]/(sqrt[c]*x^(n/2))])/(b^(3/2)*n)

Rule 193

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 321

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 345

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m+1), Subst[Int[(a+b*x^Simplify[n/(m+1)])^p, x], x, x^(m+1)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m+1)]] && !IntegerQ[n]

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1-\frac{n}{2}}}{b + cx^n} dx \\ &= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}} dx, x, x^{-n/2}\right)}{n} \\ &= -\frac{2 \operatorname{Subst}\left(\int \frac{x^2}{c+bx^2} dx, x, x^{-n/2}\right)}{n} \\ &= -\frac{2x^{-n/2}}{bn} + \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{c+bx^2} dx, x, x^{-n/2}\right)}{bn} \\ &= -\frac{2x^{-n/2}}{bn} + \frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n} \end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.64

$$-\frac{2x^{-n/2} {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\frac{cx^n}{b}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/(b*x^n + c*x^(2*n)), x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((c*x^n)/b)])/(b*n*x^(n/2))
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n/2)/(b*x^n + c*x^(2*n)), x]
```

[Out] Defer[IntegrateAlgebraic][x^(-1 + n/2)/(b*xⁿ + c*x^(2*n)), x]

fricas [A] time = 1.28, size = 151, normalized size = 3.02

$$\left[\frac{xx^{\frac{1}{2}n-1} \sqrt{-\frac{c}{b}} \log\left(\frac{cx^2x^{n-2}-2bxx^{\frac{1}{2}n-1}\sqrt{-\frac{c}{b}}-b}{cx^2x^{n-2}+b}\right) - 2 \left(xx^{\frac{1}{2}n-1} \sqrt{\frac{c}{b}} \arctan\left(\frac{b\sqrt{\frac{c}{b}}}{cxx^{\frac{1}{2}n-1}}\right) - 1 \right)}{bnxx^{\frac{1}{2}n-1}}, \frac{2 \left(xx^{\frac{1}{2}n-1} \sqrt{\frac{c}{b}} \arctan\left(\frac{b\sqrt{\frac{c}{b}}}{cxx^{\frac{1}{2}n-1}}\right) - 1 \right)}{bnxx^{\frac{1}{2}n-1}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/2*n)/(b*xⁿ+c*x^(2*n)),x, algorithm="fricas")

[Out] [(x*x^(1/2*n - 1)*sqrt(-c/b)*log((c*x²*x^(n - 2) - 2*b*x*x^(1/2*n - 1)*sqrt(-c/b) - b)/(c*x²*x^(n - 2) + b) - 2)/(b*n*x*x^(1/2*n - 1)), 2*(x*x^(1/2*n - 1)*sqrt(c/b)*arctan(b*sqrt(c/b)/(c*x*x^(1/2*n - 1))) - 1)/(b*n*x*x^(1/2*n - 1))]

giac [A] time = 0.38, size = 38, normalized size = 0.76

$$-\frac{2 \left(\frac{c \arctan\left(\frac{c\sqrt{x^n}}{\sqrt{bc}}\right)}{\sqrt{bc} b} + \frac{1}{b\sqrt{x^n}} \right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/2*n)/(b*xⁿ+c*x^(2*n)),x, algorithm="giac")

[Out] -2*(c*arctan(c*sqrt(xⁿ)/sqrt(b*c))/(sqrt(b*c)*b) + 1/(b*sqrt(xⁿ)))/n

maple [A] time = 0.07, size = 79, normalized size = 1.58

$$-\frac{2x^{-\frac{n}{2}}}{bn} + \frac{\sqrt{-bc} \ln\left(x^{\frac{n}{2}} - \frac{\sqrt{-bc}}{c}\right)}{b^2n} - \frac{\sqrt{-bc} \ln\left(x^{\frac{n}{2}} + \frac{\sqrt{-bc}}{c}\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+1/2*n)/(b*xⁿ+c*x^(2*n)),x)

[Out] -2/b/n/(x^(1/2*n))+1/b²*(-b*c)^(1/2)/n*ln(x^(1/2*n)-(-b*c)^(1/2)/c)-1/b²*(-b*c)^(1/2)/n*ln(x^(1/2*n)+(-b*c)^(1/2)/c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \int \frac{x^{\frac{1}{2}n}}{bcx^n + b^2x} dx - \frac{2}{bnx^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+1/2*n)/(b*x[^]n+c*x[^](2*n)),x, algorithm="maxima")

[Out] -c*integrate(x[^](1/2*n)/(b*c*x*x[^]n + b[^]2*x), x) - 2/(b*n*x[^](1/2*n))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{\frac{n}{2}-1}}{bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x[^](n/2 - 1)/(b*x[^]n + c*x[^](2*n)),x)

[Out] int(x[^](n/2 - 1)/(b*x[^]n + c*x[^](2*n)), x)

sympy [A] time = 13.00, size = 36, normalized size = 0.72

$$-\frac{2 \operatorname{atan}\left(\frac{x^{\frac{n}{2}}}{\sqrt{\frac{b}{c}}}\right)}{bn\sqrt{\frac{b}{c}}} - \frac{2x^{-\frac{n}{2}}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x[^](-1+1/2*n)/(b*x[^]n+c*x[^](2*n)),x)

[Out] -2*atan(x[^](n/2)/sqrt(b/c))/(b*n*sqrt(b/c)) - 2*x[^](-n/2)/(b*n)

$$3.439 \quad \int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=68

$$-\frac{2c^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n} + \frac{2cx^{-n/2}}{b^2n} - \frac{2x^{-3n/2}}{3bn}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1584, 362, 345, 193, 321, 205}

$$-\frac{2c^{3/2} \tan^{-1}\left(\frac{\sqrt{b}x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n} + \frac{2cx^{-n/2}}{b^2n} - \frac{2x^{-3n/2}}{3bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]

[Out] -2/(3*b*n*x^((3*n)/2)) + (2*c)/(b^2*n*x^(n/2)) - (2*c^(3/2)*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2)))]/(b^(5/2)*n)

Rule 193

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 321

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 345

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{

a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]

Rule 362

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[x^(m + 1)/(a*(m + 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1-\frac{3n}{2}}}{b + cx^n} dx \\
 &= -\frac{2x^{-3n/2}}{3bn} - \frac{c \int \frac{x^{-1-\frac{n}{2}}}{b+cx^n} dx}{b} \\
 &= -\frac{2x^{-3n/2}}{3bn} + \frac{(2c) \text{Subst} \left(\int \frac{1}{b+\frac{c}{x^2}} dx, x, x^{-n/2} \right)}{bn} \\
 &= -\frac{2x^{-3n/2}}{3bn} + \frac{(2c) \text{Subst} \left(\int \frac{x^2}{c+bx^2} dx, x, x^{-n/2} \right)}{bn} \\
 &= -\frac{2x^{-3n/2}}{3bn} + \frac{2cx^{-n/2}}{b^2n} - \frac{(2c^2) \text{Subst} \left(\int \frac{1}{c+bx^2} dx, x, x^{-n/2} \right)}{b^2n} \\
 &= -\frac{2x^{-3n/2}}{3bn} + \frac{2cx^{-n/2}}{b^2n} - \frac{2c^{3/2} \tan^{-1} \left(\frac{\sqrt{b} x^{-n/2}}{\sqrt{c}} \right)}{b^{5/2}n}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.50

$$-\frac{2x^{-3n/2} {}_2F_1 \left(-\frac{3}{2}, 1; -\frac{1}{2}; -\frac{cx^n}{b} \right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^{-1 - n/2}/(b*xⁿ + c*x^(2*n)), x]

[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((c*xⁿ)/b)]/(3*b*n*x^{((3*n)/2)})

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^{-1 - n/2}/(b*xⁿ + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic][x^{-1 - n/2}/(b*xⁿ + c*x^(2*n)), x]

fricas [A] time = 1.39, size = 161, normalized size = 2.37

$$\left[\frac{2bx^3x^{-\frac{3}{2}n-3} - 6cxx^{-\frac{1}{2}n-1} - 3c\sqrt{\frac{c}{b}} \log\left(\frac{bx^2x^{-n-2} - 2bxx^{-\frac{1}{2}n-1}\sqrt{\frac{c}{b}-c}}{bx^2x^{-n-2}+c}\right)}{3b^2n}, \frac{2\left(bx^3x^{-\frac{3}{2}n-3} - 3cxx^{-\frac{1}{2}n-1} - 3c\sqrt{\frac{c}{b}} \arctan\left(\frac{\sqrt{\frac{c}{b}}}{xx^{-\frac{1}{2}n-1}}\right)\right)}{3b^2n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{-1-1/2*n}/(b*xⁿ+c*x^(2*n)), x, algorithm="fricas")

[Out] [-1/3*(2*b*x³*x^(-3/2*n - 3) - 6*c*x*x^(-1/2*n - 1) - 3*c*sqrt(-c/b)*log((b*x²*x^(-n - 2) - 2*b*x*x^(-1/2*n - 1)*sqrt(-c/b) - c)/(b*x²*x^(-n - 2) + c))/(b²*n), -2/3*(b*x³*x^(-3/2*n - 3) - 3*c*x*x^(-1/2*n - 1) - 3*c*sqrt(c/b)*arctan(sqrt(c/b)/(x*x^(-1/2*n - 1))))/(b²*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}n-1}}{cx^{2n} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{-1-1/2*n}/(b*xⁿ+c*x^(2*n)), x, algorithm="giac")

[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*xⁿ), x)

maple [A] time = 0.10, size = 97, normalized size = 1.43

$$-\frac{2x^{-\frac{3n}{2}}}{3bn} + \frac{2cx^{-\frac{n}{2}}}{b^2n} - \frac{\sqrt{-bc} c \ln\left(x^{\frac{n}{2}} - \frac{\sqrt{-bc}}{c}\right)}{b^3n} + \frac{\sqrt{-bc} c \ln\left(x^{\frac{n}{2}} + \frac{\sqrt{-bc}}{c}\right)}{b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x)`

[Out] $2*c/b^2/n/(x^{(1/2*n)})-2/3/b/n/(x^{(1/2*n)})^3+1/b^3*(-b*c)^{(1/2)*c/n*\ln(x^{(1/2*n)}+(-b*c)^{(1/2)/c})-1/b^3*(-b*c)^{(1/2)*c/n*\ln(x^{(1/2*n)}-(-b*c)^{(1/2)/c})}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \frac{x^{\frac{1}{2}n}}{b^2 c x^n + b^3 x} dx + \frac{2(3 c x^n - b)}{3 b^2 n x^{\frac{3}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")`

[Out] $c^2*\text{integrate}(x^{(1/2*n)}/(b^2*c*x*x^n + b^3*x), x) + 2/3*(3*c*x^n - b)/(b^2*n*x^{(3/2*n)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{n}{2}+1} (b x^n + c x^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))),x)`

[Out] `int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))), x)`

sympy [A] time = 24.92, size = 58, normalized size = 0.85

$$-\frac{2x^{-\frac{3n}{2}}}{3bn} + \frac{2cx^{-\frac{n}{2}}}{b^2n} - \frac{2c^2 \operatorname{atan}\left(\frac{x^{-\frac{n}{2}}}{\sqrt{\frac{c}{b}}}\right)}{b^3n\sqrt{\frac{c}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)`

[Out] $-2*x^{(-3*n/2)}/(3*b*n) + 2*c*x^{(-n/2)}/(b**2*n) - 2*c**2*\operatorname{atan}(x^{(-n/2)}/\operatorname{sqrt}(c/b))/(b**3*n*\operatorname{sqrt}(c/b))$

$$3.440 \quad \int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=176

$$-\frac{c^{4/3} \log(\sqrt[3]{b} x^{-n/3} + \sqrt[3]{c})}{b^{7/3n}} + \frac{c^{4/3} \log(b^{2/3} x^{-2n/3} - \sqrt[3]{b} \sqrt[3]{c} x^{-n/3} + c^{2/3})}{2b^{7/3n}} + \frac{\sqrt{3} c^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{b} x^{-n/3}}{\sqrt{3} \sqrt[3]{c}}\right)}{b^{7/3n}} + \frac{3cx^{-n/3}}{b^{2n}} - \frac{3x^{-4n/3}}{4bn}$$

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1584, 362, 345, 193, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{c^{4/3} \log(\sqrt[3]{b} x^{-n/3} + \sqrt[3]{c})}{b^{7/3n}} + \frac{c^{4/3} \log(b^{2/3} x^{-2n/3} - \sqrt[3]{b} \sqrt[3]{c} x^{-n/3} + c^{2/3})}{2b^{7/3n}} + \frac{\sqrt{3} c^{4/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{b} x^{-n/3}}{\sqrt{3} \sqrt[3]{c}}\right)}{b^{7/3n}} + \frac{3cx^{-n/3}}{b^{2n}} - \frac{3x^{-4n/3}}{4bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]

[Out] $-\frac{3}{(4*b*n*x^{(4*n)/3})} + \frac{(3*c)}{(b^2*n*x^{(n/3)})} + \frac{(\text{Sqrt}[3]*c^{(4/3)}*\text{ArcTan}[(c^{(1/3)} - (2*b^{(1/3)})/x^{(n/3)})/(\text{Sqrt}[3]*c^{(1/3)})])}{(b^{(7/3)*n})} - \frac{(c^{(4/3)}*\text{Log}[c^{(1/3)} + b^{(1/3)}/x^{(n/3)})]}{(b^{(7/3)*n})} + \frac{(c^{(4/3)}*\text{Log}[c^{(2/3)} + b^{(2/3)}/x^{((2*n)/3)} - (b^{(1/3)}*c^{(1/3)})/x^{(n/3)})]}{(2*b^{(7/3)*n})}$

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 193

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 345

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]

Rule 362

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[x^(m + 1)/(a*(m + 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1-\frac{4n}{3}}}{b + cx^n} dx \\
&= -\frac{3x^{-4n/3}}{4bn} - \frac{c \int \frac{x^{-1-\frac{n}{3}}}{b+cx^n} dx}{b} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c) \text{Subst}\left(\int \frac{1}{b+\frac{c}{x^3}} dx, x, x^{-n/3}\right)}{bn} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c) \text{Subst}\left(\int \frac{x^3}{c+bx^3} dx, x, x^{-n/3}\right)}{bn} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{(3c^2) \text{Subst}\left(\int \frac{1}{c+bx^3} dx, x, x^{-n/3}\right)}{b^2n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3} \text{Subst}\left(\int \frac{1}{\sqrt[3]{c} + \sqrt[3]{b}x} dx, x, x^{-n/3}\right)}{b^2n} - \frac{c^{4/3} \text{Subst}\left(\int \frac{2\sqrt[3]{c} - \sqrt[3]{b}x}{c^{2/3} - \sqrt[3]{b}\sqrt[3]{c}x + b^{2/3}x^2} dx, x, x^{-n/3}\right)}{b^2n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{b}x^{-n/3})}{b^{7/3}n} + \frac{c^{4/3} \text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c} + 2b^{2/3}x}{c^{2/3} - \sqrt[3]{b}\sqrt[3]{c}x + b^{2/3}x^2} dx, x, x^{-n/3}\right)}{2b^{7/3}n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{b}x^{-n/3})}{b^{7/3}n} + \frac{c^{4/3} \log(c^{2/3} + b^{2/3}x^{-2n/3} - \sqrt[3]{b}\sqrt[3]{c}x^{-n/3})}{2b^{7/3}n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} + \frac{\sqrt{3}c^{4/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}x^{-n/3}}{\sqrt[3]{c}}\right)}{b^{7/3}n} - \frac{c^{4/3} \log(\sqrt[3]{c} + \sqrt[3]{b}x^{-n/3})}{b^{7/3}n} + \frac{c^{4/3} \log(c^{2/3} + b^{2/3}x^{-2n/3} - \sqrt[3]{b}\sqrt[3]{c}x^{-n/3})}{2b^{7/3}n}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.19

$$-\frac{3x^{-4n/3} {}_2F_1\left(-\frac{4}{3}, 1; -\frac{1}{3}; -\frac{cx^n}{b}\right)}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n/3)/(b*xⁿ + c*x^(2*n)), x]

[Out] (-3*Hypergeometric2F1[-4/3, 1, -1/3, -(c*xⁿ)/b])/(4*b*n*x^{((4*n)/3)})

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{3}}}{bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 - n/3)/(b*xⁿ + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 - n/3)/(b*xⁿ + c*x^(2*n)), x]

fricas [A] time = 1.42, size = 171, normalized size = 0.97

$$\frac{3bx^4x^{-\frac{4}{3}n-4} - 12cxx^{-\frac{1}{3}n-1} - 4\sqrt{3}c\left(-\frac{c}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}bxx^{-\frac{1}{3}n-1}\left(-\frac{c}{b}\right)^{\frac{2}{3}} - \sqrt{3}c}{3c}\right) - 4c\left(-\frac{c}{b}\right)^{\frac{1}{3}} \log\left(\frac{xx^{-\frac{1}{3}n-1}\left(-\frac{c}{b}\right)^{\frac{1}{3}}}{x}\right) + 2c\left(-\frac{c}{b}\right)^{\frac{1}{3}} \log\left(\frac{x^2x^{\frac{2}{3}n-2} + xx^{-\frac{1}{3}n-1}\left(-\frac{c}{b}\right)^{\frac{1}{3}} + \left(-\frac{c}{b}\right)^{\frac{2}{3}}}{x^2}\right)}{4b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/3*n)/(b*xⁿ+c*x^(2*n)), x, algorithm="fricas")

[Out] -1/4*(3*b*x⁴*x^(-4/3*n - 4) - 12*c*x*x^(-1/3*n - 1) - 4*sqrt(3)*c*(-c/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*x^(-1/3*n - 1)*(-c/b)^(2/3) - sqrt(3)*c)/c - 4*c*(-c/b)^(1/3)*log((x*x^(-1/3*n - 1) - (-c/b)^(1/3))/x) + 2*c*(-c/b)^(1/3)*log((x²*x^(-2/3*n - 2) + x*x^(-1/3*n - 1)*(-c/b)^(1/3) + (-c/b)^(2/3))/x²)/(b²*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/3*n)/(b*xⁿ+c*x^(2*n)), x, algorithm="giac")

[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*xⁿ), x)

maple [C] time = 0.08, size = 73, normalized size = 0.41

$$\text{RootOf}(b^7n^3_Z^3 + c^4) \ln\left(\frac{\text{RootOf}(b^7n^3_Z^3 + c^4)^2 b^5n^2}{c^3} + x^{\frac{n}{3}}\right) - \frac{3x^{-\frac{4n}{3}}}{4bn} + \frac{3cx^{-\frac{n}{3}}}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x)`

[Out] `3*c/b^2/n/(x^(1/3*n))-3/4/b/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+b^5*n^2/c^3*_R^2),_R=RootOf(_Z^3*b^7*n^3+c^4))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \frac{x^{\frac{2}{3}n}}{b^2 c x^n + b^3 x} dx + \frac{3(4cx^n - b)}{4b^2 n x^{\frac{4}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")`

[Out] `c^2*integrate(x^(2/3*n)/(b^2*c*x*x^n + b^3*x), x) + 3/4*(4*c*x^n - b)/(b^2*n*x^(4/3*n))`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{n}{3}+1} (b x^n + c x^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))),x)`

[Out] `int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)`

[Out] Timed out

$$3.441 \quad \int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}} dx$$

Optimal. Leaf size=252

$$\frac{c^{5/4} \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{-n/4} + \sqrt{b} x^{-n/2} + \sqrt{c}\right)}{\sqrt{2} b^{9/4} n} - \frac{c^{5/4} \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{-n/4} + \sqrt{b} x^{-n/2} + \sqrt{c}\right)}{\sqrt{2} b^{9/4} n} + \frac{\sqrt{2} c^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}}{\sqrt[4]{c}}\right)}{b^{9/4} n}$$

Rubi [A] time = 0.22, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1584, 362, 345, 193, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{5/4} \log\left(-\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{-n/4} + \sqrt{b} x^{-n/2} + \sqrt{c}\right)}{\sqrt{2} b^{9/4} n} - \frac{c^{5/4} \log\left(\sqrt{2} \sqrt[4]{b} \sqrt[4]{c} x^{-n/4} + \sqrt{b} x^{-n/2} + \sqrt{c}\right)}{\sqrt{2} b^{9/4} n} + \frac{\sqrt{2} c^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4} n} - \frac{\sqrt{2} c^{5/4} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x^{-n/4}}{\sqrt[4]{c}} + 1\right)}{b^{9/4} n} + \frac{4cx^{-n/4}}{b^2 n} - \frac{4x^{-5n/4}}{5bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n/4)/(b*x^n + c*x^(2*n)), x]

[Out] $-4/(5*b*n*x^{((5*n)/4)}) + (4*c)/(b^2*n*x^{(n/4)}) + (\text{Sqrt}[2]*c^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)})/(c^{(1/4)}*x^{(n/4)})])/(b^{(9/4)}*n) - (\text{Sqrt}[2]*c^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)})/(c^{(1/4)}*x^{(n/4)})])/(b^{(9/4)}*n) + (c^{(5/4)}*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[b]/x^{(n/2)} - (\text{Sqrt}[2]*b^{(1/4)}*c^{(1/4)})/x^{(n/4)}])/(b^{(9/4)}*n) - (c^{(5/4)}*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[b]/x^{(n/2)} + (\text{Sqrt}[2]*b^{(1/4)}*c^{(1/4)})/x^{(n/4)}])/(b^{(9/4)}*n)$

Rule 193

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[x^(m + 1)/(a*(m +
1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
 && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}} dx &= \int \frac{x^{-1-\frac{5n}{4}}}{b + cx^n} dx \\
 &= -\frac{4x^{-5n/4}}{5bn} - \frac{c \int \frac{x^{-1-\frac{n}{4}}}{b+cx^n} dx}{b} \\
 &= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c) \operatorname{Subst}\left(\int \frac{1}{b+\frac{c}{x^4}} dx, x, x^{-n/4}\right)}{bn} \\
 &= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c) \operatorname{Subst}\left(\int \frac{x^4}{c+bx^4} dx, x, x^{-n/4}\right)}{bn} \\
 &= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{(4c^2) \operatorname{Subst}\left(\int \frac{1}{c+bx^4} dx, x, x^{-n/4}\right)}{b^2n} \\
 &= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{(2c^{3/2}) \operatorname{Subst}\left(\int \frac{\sqrt{c}-\sqrt{b}x^2}{c+bx^4} dx, x, x^{-n/4}\right)}{b^2n} - \frac{(2c^{3/2}) \operatorname{Subst}\left(\int \frac{\sqrt{c}+\sqrt{b}x^2}{c+bx^4} dx, x, x^{-n/4}\right)}{b^2n} \\
 &= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4} \operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}+2x}{\sqrt[4]{b}}}{-\frac{\sqrt{c}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{b}}-x^2} dx, x, x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} + \frac{c^{5/4} \operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}-2x}{\sqrt[4]{b}}}{-\frac{\sqrt{c}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{b}}-x^2} dx, x, x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} \\
 &= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4} \log\left(\sqrt{c} + \sqrt{b}x^{-n/2} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} - \frac{c^{5/4} \log\left(\sqrt{c} + \sqrt{b}x^{-n/2} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}x^{-n/4}\right)}{\sqrt{2}b^{9/4}n} \\
 &= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{\sqrt{2}c^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} - \frac{\sqrt{2}c^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} + \frac{c^{5/4}}{b^{9/4}n}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.13

$$\frac{4x^{-5n/4} {}_2F_1\left(-\frac{5}{4}, 1; -\frac{1}{4}; -\frac{cx^n}{b}\right)}{5bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n/4)/(b*x^n + c*x^(2*n)), x]

[Out] (-4*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^n)/b)])/(5*b*n*x^((5*n)/4))

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 - n/4)/(b*x^n + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 - n/4)/(b*x^n + c*x^(2*n)), x]

fricas [A] time = 1.38, size = 259, normalized size = 1.03

$$\frac{4bx^5x^{-\frac{5}{4}n-5} + 20b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}} \arctan\left(\frac{b^7cn^3xx^{-\frac{1}{4}n-1}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}} - b^7n^3x\sqrt{\frac{b^4n^2\sqrt{-\frac{5}{b^9n^4} + c^2x^2x^{-\frac{1}{2}n-2}}}{x^2}}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}}}{c^5}\right)}{5b^2n} + 5b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}} \log\left(\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}} + cxx^{-\frac{1}{4}n-1}}{x}\right) - 5b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}} \log\left(\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}} - cxx^{-\frac{1}{4}n-1}}{x}\right) - 20c^5x^{-\frac{1}{4}n-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] -1/5*(4*b*x^5*x^(-5/4*n - 5) + 20*b^2*n*(-c^5/(b^9*n^4))^(1/4)*arctan(-(b^7*c*n^3*x*x^(-1/4*n - 1)*(-c^5/(b^9*n^4))^(3/4) - b^7*n^3*x*sqrt((b^4*n^2*sqrt(-c^5/(b^9*n^4) + c^2*x^2*x^(-1/2*n - 2))/x^2)*(-c^5/(b^9*n^4))^(3/4))/c^5) + 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((b^2*n*(-c^5/(b^9*n^4))^(1/4) + c*x*x^(-1/4*n - 1))/x) - 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log(-(b^2*n*(-c^5/(b^9*n^4))^(1/4) - c*x*x^(-1/4*n - 1))/x) - 20*c*x*x^(-1/4*n - 1))/(b^2*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/4*n)/(b*xⁿ+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*xⁿ), x)

maple [C] time = 0.09, size = 73, normalized size = 0.29

$$\text{RootOf}(b^9 n^4 Z^4 + c^5) \ln \left(\frac{\text{RootOf}(b^9 n^4 Z^4 + c^5)^3 b^7 n^3}{c^4} + x^{\frac{n}{4}} \right) - \frac{4x^{-\frac{5n}{4}}}{5bn} + \frac{4cx^{-\frac{n}{4}}}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-1/4*n)/(b*xⁿ+c*x^(2*n)),x)

[Out] 4*c/b²n/(x^(1/4*n))-4/5/b/n/(x^(1/4*n))⁵+sum(_R*ln(x^(1/4*n))+b⁷*n³/c⁴*_R³),_R=RootOf(_Z⁴*b⁹*n⁴+c⁵)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \frac{x^{\frac{3}{4}n}}{b^2 c x^n + b^3 x} dx + \frac{4(5cx^n - b)}{5b^2 n x^{\frac{5}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/4*n)/(b*xⁿ+c*x^(2*n)),x, algorithm="maxima")

[Out] c²*integrate(x^(3/4*n)/(b²*c*x*xⁿ + b³*x), x) + 4/5*(5*c*xⁿ - b)/(b²*n*x^(5/4*n))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{4}+1} (bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(n/4 + 1)*(b*xⁿ + c*x^(2*n))),x)

[Out] int(1/(x^(n/4 + 1)*(b*xⁿ + c*x^(2*n))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/4*n)/(b*xⁿ+c*x^(2*n)),x)

[Out] Timed out

$$3.442 \quad \int x^{-1-n(-1+p)} (bx^n + cx^{2n})^p dx$$

Optimal. Leaf size=37

$$\frac{x^{-n(p+1)} (bx^n + cx^{2n})^{p+1}}{cn(p+1)}$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2014}

$$\frac{x^{-n(p+1)} (bx^n + cx^{2n})^{p+1}}{cn(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p, x]

[Out] (b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(-1+p)} (bx^n + cx^{2n})^p dx = \frac{x^{-n(1+p)} (bx^n + cx^{2n})^{1+p}}{cn(1+p)}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.03

$$\frac{x^{-np} (b + cx^n) (x^n (b + cx^n))^p}{cn(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p, x]

[Out] ((b + c*x^n)*(x^n*(b + c*x^n))^p)/(c*n*(1 + p)*x^(n*p))

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int x^{-1-n(-1+p)} (bx^n + cx^{2n})^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]

[Out] Defer[IntegrateAlgebraic][x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p, x]

fricas [A] time = 1.34, size = 59, normalized size = 1.59

$$\frac{(cxx^{-np+n-1}x^n + bxx^{-np+n-1})(cx^{2n} + bx^n)^p}{(cnp + cn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")

[Out] (c*x*x^(-n*p + n - 1)*x^n + b*x*x^(-n*p + n - 1))*(c*x^(2*n) + b*x^n)^p/((c*n*p + c*n)*x^n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^{2n} + bx^n)^p x^{-n(p-1)-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1), x)

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^{-(p-1)n-1} (bx^n + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)

[Out] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)

maxima [A] time = 1.11, size = 43, normalized size = 1.16

$$\frac{(cx^n + b)e^{(-np \log(x) + p \log(cx^n + b) + p \log(x^n))}}{cn(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")

[Out] (c*x^n + b)*e^(-n*p*log(x) + p*log(c*x^n + b) + p*log(x^n))/(c*n*(p + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(bx^n + cx^{2n})^p}{x^{n(p-1)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1),x)

[Out] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)

[Out] Timed out

$$3.443 \quad \int x^{-1-n(1+2p)} (bx^n + cx^{2n})^p dx$$

Optimal. Leaf size=38

$$-\frac{x^{-2n(p+1)} (bx^n + cx^{2n})^{p+1}}{bn(p+1)}$$

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2014}

$$-\frac{x^{-2n(p+1)} (bx^n + cx^{2n})^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]

[Out] -((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
  :-> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(1+2p)} (bx^n + cx^{2n})^p dx = -\frac{x^{-2n(1+p)} (bx^n + cx^{2n})^{1+p}}{bn(1+p)}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.13

$$-\frac{x^{-n(2p+1)} (b + cx^n) (x^n (b + cx^n))^p}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]

[Out] -(((b + c*x^n)*(x^n*(b + c*x^n))^p)/(b*n*(1 + p)*x^(n*(1 + 2*p))))

IntegrateAlgebraic [F] time = 0.15, size = 0, normalized size = 0.00

$$\int x^{-1-n(1+2p)} (bx^n + cx^{2n})^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]

[Out] Defer[IntegrateAlgebraic][x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p, x]

fricas [A] time = 1.15, size = 59, normalized size = 1.55

$$\frac{(c x x^{-2np-n-1} x^n + b x x^{-2np-n-1}) (c x^{2n} + b x^n)^p}{bnp + bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")

[Out] -(c*x*x^(-2*n*p - n - 1)*x^n + b*x*x^(-2*n*p - n - 1))*(c*x^(2*n) + b*x^n)^p/(b*n*p + b*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^{2n} + bx^n)^p x^{-n(2p+1)-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^{-(2p+1)n-1} (bx^n + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-n*(2*p+1))*(b*x^n+c*x^(2*n))^p,x)

[Out] int(x^(-1-n*(2*p+1))*(b*x^n+c*x^(2*n))^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^{2n} + bx^n)^p x^{-n(2p+1)-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{(-1-n*(1+2*p))}*(b*xⁿ+c*x^(2*n))^p,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + b*xⁿ)^p*x^{(-n*(2*p + 1) - 1)}, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(bx^n + cx^{2n})^p}{x^{n(2p+1)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*xⁿ + c*x^(2*n))^p/x^{(n*(2*p + 1) + 1)},x)

[Out] int((b*xⁿ + c*x^(2*n))^p/x^{(n*(2*p + 1) + 1)}, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{**(-1-n*(1+2*p))}*(b*x^{**n}+c*x^{** (2*n)})^{**p},x)

[Out] Timed out

$$3.444 \quad \int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^{5/2} dx$$

Optimal. Leaf size=112

$$\frac{(a + bx^n)^7 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{7n(ab^2 + b^3x^n)} - \frac{a(a + bx^n)^6 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{6n(ab^2 + b^3x^n)}$$

Rubi [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1355, 266, 43}

$$\frac{(a + bx^n)^7 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{7n(ab^2 + b^3x^n)} - \frac{a(a + bx^n)^6 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{6n(ab^2 + b^3x^n)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2), x]

[Out] -(a*(a + b*x^n)^6*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(6*n*(a*b^2 + b^3*x^n)) + ((a + b*x^n)^7*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(7*n*(a*b^2 + b^3*x^n))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^{5/2} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int x^{-1+2n} (ab + b^2x^n)^5 dx}{b^4 (ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \text{Subst} \left(\int x (ab + b^2x)^5 dx, x, x^n \right)}{b^4 n (ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \text{Subst} \left(\int \left(-\frac{a(ab+b^2x)^5}{b} + \frac{(ab+b^2x)^6}{b^2} \right) dx, x, x^n \right)}{b^4 n (ab + b^2x^n)} \\
&= -\frac{a(a+bx^n)^6 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{6n(ab^2 + b^3x^n)} + \frac{(a+bx^n)^7 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{7n(ab^2 + b^3x^n)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 0.36

$$\frac{(a - 6bx^n)(a + bx^n)^5 \sqrt{(a + bx^n)^2}}{42b^2n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2), x]

[Out] -1/42*((a - 6*b*x^n)*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])/(b^2*n)

IntegrateAlgebraic [A] time = 0.07, size = 96, normalized size = 0.86

$$\frac{x^{2n} \sqrt{(a + bx^n)^2} (21a^5 + 70a^4bx^n + 105a^3b^2x^{2n} + 84a^2b^3x^{3n} + 35ab^4x^{4n} + 6b^5x^{5n})}{42n(a + bx^n)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2), x]

[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(21*a^5 + 70*a^4*b*x^n + 105*a^3*b^2*x^(2*n) + 84*a^2*b^3*x^(3*n) + 35*a*b^4*x^(4*n) + 6*b^5*x^(5*n)))/(42*n*(a + b*x^n))

fricas [A] time = 1.30, size = 74, normalized size = 0.66

$$\frac{6b^5x^{7n} + 35ab^4x^{6n} + 84a^2b^3x^{5n} + 105a^3b^2x^{4n} + 70a^4bx^{3n} + 21a^5x^{2n}}{42n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a^{2+2*a*b*xⁿ+b²*x^(2*n))^(5/2),x, algorithm="fricas")}

[Out] 1/42*(6*b⁵*x^(7*n) + 35*a*b⁴*x^(6*n) + 84*a²*b³*x^(5*n) + 105*a³*b²*x^(4*n) + 70*a⁴*b*x^(3*n) + 21*a⁵*x^(2*n))/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b^2 x^{2n} + 2abx^n + a^2)^{\frac{5}{2}} x^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a^{2+2*a*b*xⁿ+b²*x^(2*n))^(5/2),x, algorithm="giac")}

[Out] integrate((b²*x^(2*n) + 2*a*b*xⁿ + a²)^(5/2)*x^(2*n - 1), x)

maple [A] time = 0.05, size = 208, normalized size = 1.86

$$\frac{\sqrt{(bx^n+a)^2} a^5 x^{2n}}{2(bx^n+a)n} + \frac{5\sqrt{(bx^n+a)^2} a^4 b x^{3n}}{3(bx^n+a)n} + \frac{5\sqrt{(bx^n+a)^2} a^3 b^2 x^{4n}}{2(bx^n+a)n} + \frac{2\sqrt{(bx^n+a)^2} a^2 b^3 x^{5n}}{(bx^n+a)n} + \frac{5\sqrt{(bx^n+a)^2} a b^4 x^{6n}}{6(bx^n+a)n} + \frac{\sqrt{(bx^n+a)^2} b^5 x^{7n}}{7(bx^n+a)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+2*n)*(a^{2+2*a*b*xⁿ+b²*x^(2*n))^(5/2),x)}

[Out] 1/7*((a+b*xⁿ)²)^(1/2)/(a+b*xⁿ)*b⁵/n*(xⁿ)^{7+5/6}+5/6*((a+b*xⁿ)²)^(1/2)/(a+b*xⁿ)*a*b⁴/n*(xⁿ)⁶⁺²+2*((a+b*xⁿ)²)^(1/2)/(a+b*xⁿ)*a²*b³/n*(xⁿ)^{5+5/2}+2*((a+b*xⁿ)²)^(1/2)/(a+b*xⁿ)*a³*b²/n*(xⁿ)^{4+5/3}+5/3*((a+b*xⁿ)²)^(1/2)/(a+b*xⁿ)*a⁴*b/n*(xⁿ)^{3+1/2}+1/2*((a+b*xⁿ)²)^(1/2)/(a+b*xⁿ)*a⁵/n*(xⁿ)²

maxima [A] time = 0.92, size = 74, normalized size = 0.66

$$\frac{6b^5x^{7n} + 35ab^4x^{6n} + 84a^2b^3x^{5n} + 105a^3b^2x^{4n} + 70a^4bx^{3n} + 21a^5x^{2n}}{42n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a^{2+2*a*b*xⁿ+b²*x^(2*n))^(5/2),x, algorithm="maxima")}

[Out] 1/42*(6*b⁵*x^(7*n) + 35*a*b⁴*x^(6*n) + 84*a²*b³*x^(5*n) + 105*a³*b²*x^(4*n) + 70*a⁴*b*x^(3*n) + 21*a⁵*x^(2*n))/n

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2n-1} (a^2 + b^2 x^{2n} + 2abx^n)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2), x)
```

```
[Out] Timed out
```

$$3.445 \quad \int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$$

Optimal. Leaf size=112

$$\frac{(a + bx^n)^5 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{5n(ab^2 + b^3x^n)} - \frac{a(a + bx^n)^4 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{4n(ab^2 + b^3x^n)}$$

Rubi [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1355, 266, 43}

$$\frac{(a + bx^n)^5 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{5n(ab^2 + b^3x^n)} - \frac{a(a + bx^n)^4 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{4n(ab^2 + b^3x^n)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)*(a² + 2*a*b*xⁿ + b²*x^(2*n))^(3/2), x]

[Out] -(a*(a + b*xⁿ)⁴*Sqrt[a² + 2*a*b*xⁿ + b²*x^(2*n)])/(4*n*(a*b² + b³*xⁿ)) + ((a + b*xⁿ)⁵*Sqrt[a² + 2*a*b*xⁿ + b²*x^(2*n)])/(5*n*(a*b² + b³*xⁿ))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)ⁿ, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^{(Simplify[(m + 1)/n] - 1)}*(a + b*x)^p, x], x, xⁿ], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*xⁿ + c*x^(2*n))^{FracPart[p]}/(c^{IntPart[p]}*(b/2 + c*xⁿ)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*xⁿ)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b² - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int x^{-1+2n} (ab + b^2x^n)^3 dx}{b^2 (ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \text{Subst} \left(\int x (ab + b^2x)^3 dx, x, x^n \right)}{b^2 n (ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \text{Subst} \left(\int \left(-\frac{a(ab+b^2x)^3}{b} + \frac{(ab+b^2x)^4}{b^2} \right) dx, x, x^n \right)}{b^2 n (ab + b^2x^n)} \\
&= -\frac{a(a+bx^n)^4 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{4n(ab^2 + b^3x^n)} + \frac{(a+bx^n)^5 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{5n(ab^2 + b^3x^n)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.36

$$\frac{(a - 4bx^n)(a + bx^n)^3 \sqrt{(a + bx^n)^2}}{20b^2n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] -1/20*((a - 4*b*x^n)*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])/(b^2*n)

IntegrateAlgebraic [A] time = 0.06, size = 70, normalized size = 0.62

$$\frac{x^{2n} \sqrt{(a + bx^n)^2} (10a^3 + 20a^2bx^n + 15ab^2x^{2n} + 4b^3x^{3n})}{20n(a + bx^n)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(10*a^3 + 20*a^2*b*x^n + 15*a*b^2*x^(2*n) + 4*b^3*x^(3*n)))/(20*n*(a + b*x^n))

fricas [A] time = 1.33, size = 48, normalized size = 0.43

$$\frac{4b^3x^{5n} + 15ab^2x^{4n} + 20a^2bx^{3n} + 10a^3x^{2n}}{20n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a²+2*a*b*xⁿ+b²*x^(2*n))^(3/2),x, algorithm="fricas")

[Out] 1/20*(4*b³*x^(5*n) + 15*a*b²*x^(4*n) + 20*a²*b*x^(3*n) + 10*a³*x^(2*n))/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b^2 x^{2n} + 2 a b x^n + a^2)^{\frac{3}{2}} x^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a²+2*a*b*xⁿ+b²*x^(2*n))^(3/2),x, algorithm="giac")

[Out] integrate((b²*x^(2*n) + 2*a*b*xⁿ + a²)^(3/2)*x^(2*n - 1), x)

maple [A] time = 0.03, size = 135, normalized size = 1.21

$$\frac{\sqrt{(b x^n + a)^2} a^3 x^{2n}}{2 (b x^n + a) n} + \frac{\sqrt{(b x^n + a)^2} a^2 b x^{3n}}{(b x^n + a) n} + \frac{3 \sqrt{(b x^n + a)^2} a b^2 x^{4n}}{4 (b x^n + a) n} + \frac{\sqrt{(b x^n + a)^2} b^3 x^{5n}}{5 (b x^n + a) n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+2*n)*(a²+2*a*b*xⁿ+b²*x^(2*n))^(3/2),x)

[Out] 1/5*((b*xⁿ+a)²)^(1/2)/(b*xⁿ+a)*b³/n*(xⁿ)⁵+3/4*((b*xⁿ+a)²)^(1/2)/(b*xⁿ+a)*a*b²/n*(xⁿ)⁴+((b*xⁿ+a)²)^(1/2)/(b*xⁿ+a)*a²*b/n*(xⁿ)³+1/2*((b*xⁿ+a)²)^(1/2)/(b*xⁿ+a)*a³/n*(xⁿ)²

maxima [A] time = 0.92, size = 48, normalized size = 0.43

$$\frac{4 b^3 x^{5n} + 15 a b^2 x^{4n} + 20 a^2 b x^{3n} + 10 a^3 x^{2n}}{20 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a²+2*a*b*xⁿ+b²*x^(2*n))^(3/2),x, algorithm="maxima")

[Out] 1/20*(4*b³*x^(5*n) + 15*a*b²*x^(4*n) + 20*a²*b*x^(3*n) + 10*a³*x^(2*n))/n

mapad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2n-1} (a^2 + b^2 x^{2n} + 2 a b x^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2), x)
```

```
[Out] Timed out
```

$$3.446 \quad \int x^{-1+2n} \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$$

Optimal. Leaf size=99

$$\frac{ax^{2n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2n(a + bx^n)} + \frac{b^2x^{3n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{3n(ab + b^2x^n)}$$

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1355, 14}

$$\frac{ax^{2n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2n(a + bx^n)} + \frac{b^2x^{3n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{3n(ab + b^2x^n)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (a*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a + b*x^n)) + (b^2*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^{-1+2n} \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int x^{-1+2n} (ab + b^2x^n) dx}{ab + b^2x^n} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (abx^{-1+2n} + b^2x^{-1+3n}) dx}{ab + b^2x^n} \\
&= \frac{ax^{2n} \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2n(a + bx^n)} + \frac{b^2x^{3n} \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{3n(ab + b^2x^n)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.44

$$\frac{x^{2n} \sqrt{(a + bx^n)^2 (3a + 2bx^n)}}{6n(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))

IntegrateAlgebraic [A] time = 0.05, size = 44, normalized size = 0.44

$$\frac{x^{2n} \sqrt{(a + bx^n)^2 (3a + 2bx^n)}}{6n(a + bx^n)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))

fricas [A] time = 1.25, size = 22, normalized size = 0.22

$$\frac{2bx^{3n} + 3ax^{2n}}{6n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2), x, algorithm="fricas")

[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b^2 x^{2n} + 2 a b x^n + a^2} x^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a²+2*a*b*xⁿ+b²*x^(2*n))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b²*x^(2*n) + 2*a*b*xⁿ + a²)*x^(2*n - 1), x)

maple [A] time = 0.02, size = 64, normalized size = 0.65

$$\frac{\sqrt{(b x^n + a)^2} a x^{2n}}{2 (b x^n + a) n} + \frac{\sqrt{(b x^n + a)^2} b x^{3n}}{3 (b x^n + a) n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+2*n)*(a²+2*a*b*xⁿ+b²*x^(2*n))^(1/2),x)

[Out] 1/3*((b*xⁿ+a)²)^(1/2)/(b*xⁿ+a)*b/n*(xⁿ)³+1/2*((b*xⁿ+a)²)^(1/2)/(b*xⁿ+a)*a/n*(xⁿ)²

maxima [A] time = 0.89, size = 22, normalized size = 0.22

$$\frac{2 b x^{3n} + 3 a x^{2n}}{6 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a²+2*a*b*xⁿ+b²*x^(2*n))^(1/2),x, algorithm="maxima")

[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2n-1} \sqrt{a^2 + b^2 x^{2n} + 2 a b x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n - 1)*(a² + b²*x^(2*n) + 2*a*b*xⁿ)^(1/2),x)

[Out] int(x^(2*n - 1)*(a² + b²*x^(2*n) + 2*a*b*xⁿ)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2n-1} \sqrt{(a + b x^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)*sqrt((a + b*x**n)**2), x)
```

$$3.447 \quad \int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}} dx$$

Optimal. Leaf size=90

$$\frac{x^n (a + bx^n)}{bn\sqrt{a^2 + 2abx^n + b^2x^{2n}}} - \frac{a (a + bx^n) \log (a + bx^n)}{b^2n\sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1355, 266, 43}

$$\frac{x^n (a + bx^n)}{bn\sqrt{a^2 + 2abx^n + b^2x^{2n}}} - \frac{a (a + bx^n) \log (a + bx^n)}{b^2n\sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (x^n*(a + b*x^n))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - (a*(a + b*x^n)*Log[a + b*x^n])/(b^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} dx &= \frac{(ab + b^2x^n) \int \frac{x^{-1+2n}}{ab+b^2x^n} dx}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{(ab + b^2x^n) \text{Subst} \left(\int \frac{x}{ab+b^2x} dx, x, x^n \right)}{n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{(ab + b^2x^n) \text{Subst} \left(\int \left(\frac{1}{b^2} - \frac{a}{b^2(a+bx)} \right) dx, x, x^n \right)}{n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{x^n (a + bx^n)}{bn\sqrt{a^2 + 2abx^n + b^2x^{2n}}} - \frac{a (a + bx^n) \log(a + bx^n)}{b^2n\sqrt{a^2 + 2abx^n + b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.51

$$\frac{(a + bx^n) \left(\frac{x^n}{b} - \frac{a \log(a+bx^n)}{b^2} \right)}{n\sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] ((a + b*x^n)*(x^n/b - (a*Log[a + b*x^n])/b^2))/(n*Sqrt[(a + b*x^n)^2])

IntegrateAlgebraic [A] time = 0.07, size = 55, normalized size = 0.61

$$\frac{(a + bx^n) \left(\frac{x^n}{bn} - \frac{a \log(abn+b^2nx^n)}{b^2n} \right)}{\sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] ((a + b*x^n)*(x^n/(b*n) - (a*Log[a*b*n + b^2*n*x^n]/(b^2*n)))/Sqrt[(a + b*x^n)^2])

fricas [A] time = 1.31, size = 24, normalized size = 0.27

$$\frac{bx^n - a \log(bx^n + a)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a²+2*a*b*xⁿ+b²*x^(2*n))^(1/2),x, algorithm="fricas")

[Out] (b*xⁿ - a*log(b*xⁿ + a))/(b²*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{b^2 x^{2n} + 2 a b x^n + a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a²+2*a*b*xⁿ+b²*x^(2*n))^(1/2),x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/sqrt(b²*x^(2*n) + 2*a*b*xⁿ + a²), x)

maple [A] time = 0.03, size = 71, normalized size = 0.79

$$-\frac{\sqrt{(b x^n + a)^2} a \ln\left(x^n + \frac{a}{b}\right)}{(b x^n + a) b^2 n} + \frac{\sqrt{(b x^n + a)^2} x^n}{(b x^n + a) b n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+2*n)/(a²+2*a*b*xⁿ+b²*x^(2*n))^(1/2),x)

[Out] ((b*xⁿ+a)²)^(1/2)/(b*xⁿ+a)/b/n*xⁿ-((b*xⁿ+a)²)^(1/2)/(b*xⁿ+a)*a/b²/n*ln(xⁿ+a/b)

maxima [A] time = 0.89, size = 32, normalized size = 0.36

$$\frac{x^n}{b n} - \frac{a \log\left(\frac{b x^n + a}{b}\right)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a²+2*a*b*xⁿ+b²*x^(2*n))^(1/2),x, algorithm="maxima")

[Out] xⁿ/(b*n) - a*log((b*xⁿ + a)/b)/(b²*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{\sqrt{a^2 + b^2 x^{2n} + 2 a b x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)`

[Out] `int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{(a + bx^n)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2), x)`

[Out] `Integral(x**(2*n - 1)/sqrt((a + b*x**n)**2), x)`

$$3.448 \quad \int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}} dx$$

Optimal. Leaf size=48

$$\frac{x^{2n}}{2an(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1355, 264}

$$\frac{x^{2n}}{2an(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] x^(2*n)/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{3/2}} dx &= \frac{(b^2(ab+b^2x^n)) \int \frac{x^{-1+2n}}{(ab+b^2x^n)^3} dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}} \\ &= \frac{x^{2n}}{2an(a+bx^n)\sqrt{a^2+2abx^n+b^2x^{2n}}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.73

$$\frac{x^{2n} (a + bx^n)}{2an \left((a + bx^n)^2 \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (x^(2*n)*(a + b*x^n))/(2*a*n*((a + b*x^n)^2)^(3/2))

IntegrateAlgebraic [A] time = 0.06, size = 42, normalized size = 0.88

$$\frac{-a - 2bx^n}{2b^2n (a + bx^n) \sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (-a - 2*b*x^n)/(2*b^2*n*(a + b*x^n)*Sqrt[(a + b*x^n)^2])

fricas [A] time = 1.30, size = 41, normalized size = 0.85

$$-\frac{2bx^n + a}{2(b^4nx^{2n} + 2ab^3nx^n + a^2b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2), x, algorithm="fricas")

[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2), x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)

maple [A] time = 0.03, size = 37, normalized size = 0.77

$$-\frac{\sqrt{(bx^n + a)^2} (2bx^n + a)}{2(bx^n + a)^3 b^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2), x)`

[Out] `-1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^3*(2*b*x^n+a)/b^2/n`

maxima [A] time = 0.92, size = 41, normalized size = 0.85

$$\frac{2bx^n + a}{2(b^4nx^{2n} + 2ab^3nx^n + a^2b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2), x, algorithm="maxima")`

[Out] `-1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2n-1}}{(a^2 + b^2 x^{2n} + 2abx^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)`

[Out] `int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{((a + bx^n)^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2), x)`

[Out] `Integral(x**(2*n - 1)/((a + b*x**n)**2)**(3/2), x)`

$$3.449 \quad \int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}} dx$$

Optimal. Leaf size=88

$$\frac{a}{4b^2n(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{3b^2n(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1355, 266, 43}

$$\frac{a}{4b^2n(a+bx^n)^3\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{3b^2n(a+bx^n)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2), x]

[Out] a/(4*b^2*n*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(3*b^2*n*(a + b*x^n)^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{(a^2 + 2abx^n + b^2x^{2n})^{5/2}} dx &= \frac{(b^4(ab + b^2x^n)) \int \frac{x^{-1+2n}}{(ab+b^2x^n)^5} dx}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{(b^4(ab + b^2x^n)) \text{Subst}\left(\int \frac{x}{(ab+b^2x)^5} dx, x, x^n\right)}{n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{(b^4(ab + b^2x^n)) \text{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5} + \frac{1}{b^6(a+bx)^4}\right) dx, x, x^n\right)}{n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{a}{4b^2n(a + bx^n)^3 \sqrt{a^2 + 2abx^n + b^2x^{2n}}} - \frac{1}{3b^2n(a + bx^n)^2 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.45

$$-\frac{a + 4bx^n}{12b^2n(a + bx^n)^3 \sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2), x]

[Out] -1/12*(a + 4*b*x^n)/(b^2*n*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])

IntegrateAlgebraic [A] time = 0.14, size = 42, normalized size = 0.48

$$\frac{-a - 4bx^n}{12b^2n(a + bx^n)^3 \sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2), x]

[Out] (-a - 4*b*x^n)/(12*b^2*n*(a + b*x^n)^3*Sqrt[(a + b*x^n)^2])

fricas [A] time = 1.18, size = 69, normalized size = 0.78

$$-\frac{4bx^n + a}{12(b^6nx^{4n} + 4ab^5nx^{3n} + 6a^2b^4nx^{2n} + 4a^3b^3nx^n + a^4b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a^{2+2*a*b*xⁿ+b²*x^(2*n))^(5/2),x, algorithm="fricas")}

[Out] -1/12*(4*b*xⁿ + a)/(b^{6*n}*x^(4*n) + 4*a*b^{5*n}*x^(3*n) + 6*a²*b^{4*n}*x^(2*n) + 4*a³*b^{3*n}*xⁿ + a⁴*b^{2*n})

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a^{2+2*a*b*xⁿ+b²*x^(2*n))^(5/2),x, algorithm="giac")}

[Out] integrate(x^(2*n - 1)/(b²*x^(2*n) + 2*a*b*xⁿ + a²)^(5/2), x)

maple [A] time = 0.03, size = 37, normalized size = 0.42

$$-\frac{\sqrt{(bx^n + a)^2} (4bx^n + a)}{12 (bx^n + a)^5 b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+2*n)/(a^{2+2*a*b*xⁿ+b²*x^(2*n))^(5/2),x)}

[Out] -1/12*((b*xⁿ+a)²)^(1/2)/(b*xⁿ+a)⁵*(4*b*xⁿ+a)/b²/n

maxima [A] time = 0.95, size = 69, normalized size = 0.78

$$-\frac{4bx^n + a}{12(b^6nx^{4n} + 4ab^5nx^{3n} + 6a^2b^4nx^{2n} + 4a^3b^3nx^n + a^4b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a^{2+2*a*b*xⁿ+b²*x^(2*n))^(5/2),x, algorithm="maxima")}

[Out] -1/12*(4*b*xⁿ + a)/(b^{6*n}*x^(4*n) + 4*a*b^{5*n}*x^(3*n) + 6*a²*b^{4*n}*x^(2*n) + 4*a³*b^{3*n}*xⁿ + a⁴*b^{2*n})

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a^2 + b^2x^{2n} + 2abx^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2), x)
```

```
[Out] Timed out
```

$$3.450 \quad \int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}} dx$$

Optimal. Leaf size=88

$$\frac{a}{6b^2n(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{5b^2n(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1355, 266, 43}

$$\frac{a}{6b^2n(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{5b^2n(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2), x]

[Out] a/(6*b^2*n*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(5*b^2*n*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{(a^2 + 2abx^n + b^2x^{2n})^{7/2}} dx &= \frac{(b^6(ab + b^2x^n)) \int \frac{x^{-1+2n}}{(ab+b^2x^n)^7} dx}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{(b^6(ab + b^2x^n)) \text{Subst}\left(\int \frac{x}{(ab+b^2x)^7} dx, x, x^n\right)}{n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{(b^6(ab + b^2x^n)) \text{Subst}\left(\int \left(-\frac{a}{b^8(a+bx)^7} + \frac{1}{b^8(a+bx)^6}\right) dx, x, x^n\right)}{n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{a}{6b^2n(a + bx^n)^5 \sqrt{a^2 + 2abx^n + b^2x^{2n}}} - \frac{1}{5b^2n(a + bx^n)^4 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.45

$$-\frac{a + 6bx^n}{30b^2n(a + bx^n)^5 \sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2), x]

[Out] -1/30*(a + 6*b*x^n)/(b^2*n*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])

IntegrateAlgebraic [A] time = 0.15, size = 42, normalized size = 0.48

$$\frac{-a - 6bx^n}{30b^2n(a + bx^n)^5 \sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2), x]

[Out] (-a - 6*b*x^n)/(30*b^2*n*(a + b*x^n)^5*Sqrt[(a + b*x^n)^2])

fricas [A] time = 1.21, size = 97, normalized size = 1.10

$$-\frac{6bx^n + a}{30(b^8nx^{6n} + 6ab^7nx^{5n} + 15a^2b^6nx^{4n} + 20a^3b^5nx^{3n} + 15a^4b^4nx^{2n} + 6a^5b^3nx^n + a^6b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="fricas")

[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^2*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(b^2x^{2n} + 2abx^n + a^2)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2), x)

maple [A] time = 0.03, size = 37, normalized size = 0.42

$$-\frac{\sqrt{(bx^n + a)^2} (6bx^n + a)}{30 (bx^n + a)^7 b^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x)

[Out] -1/30*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^7*(6*b*x^n+a)/b^2/n

maxima [A] time = 0.97, size = 97, normalized size = 1.10

$$-\frac{6bx^n + a}{30(b^8nx^{6n} + 6ab^7nx^{5n} + 15a^2b^6nx^{4n} + 20a^3b^5nx^{3n} + 15a^4b^4nx^{2n} + 6a^5b^3nx^n + a^6b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="maxima")

[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^2*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a^2 + b^2x^{2n} + 2abx^n)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2), x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2), x)
```

```
[Out] Timed out
```


$$3.451 \quad \int (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$$

Optimal. Leaf size=108

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(m+n+1)(ab + b^2x^n)} + \frac{a(dx)^{m+1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{d(m+1)(a + bx^n)}$$

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1355, 14, 20, 30}

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(m+n+1)(ab + b^2x^n)} + \frac{a(dx)^{m+1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{d(m+1)(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x^n)) + (b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m + n)*(a*b + b^2*x^n))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2n))^FracPart[p]/(c^IntPart[p]*(b/2 +

$c*x^n)^{(2*\text{FracPart}[p])}$, Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (dx)^m (ab + b^2x^n) dx}{ab + b^2x^n} \\ &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (ab(dx)^m + b^2x^n(dx)^m) dx}{ab + b^2x^n} \\ &= \frac{a(dx)^{1+m} \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{d(1+m)(a + bx^n)} + \frac{\left(b^2 \sqrt{a^2 + 2abx^n + b^2x^{2n}}\right) \int x^n (dx)^m dx}{ab + b^2x^n} \\ &= \frac{a(dx)^{1+m} \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{d(1+m)(a + bx^n)} + \frac{\left(b^2 x^{-m} (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}}\right) \int x^{m+n}}{ab + b^2x^n} \\ &= \frac{a(dx)^{1+m} \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{d(1+m)(a + bx^n)} + \frac{b^2 x^{1+n} (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1+m+n)(ab + b^2x^n)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.51

$$\frac{x(dx)^m \sqrt{(a + bx^n)^2} (a(m+n+1) + b(m+1)x^n)}{(m+1)(m+n+1)(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (x*(d*x)^m*Sqrt[(a + b*x^n)^2]*(a*(1 + m + n) + b*(1 + m)*x^n))/((1 + m)*(1 + m + n)*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.19, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

fricas [A] time = 1.45, size = 57, normalized size = 0.53

$$\frac{(bm + b)xx^n e^{(m \log(d) + m \log(x))} + (am + an + a)xe^{(m \log(d) + m \log(x))}}{m^2 + (m + 1)n + 2m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")

[Out] ((b*m + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m + a*n + a)*x*e^(m*log(d) + m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)

giac [A] time = 0.35, size = 173, normalized size = 1.60

$$\frac{bmx^n e^{(m \log(d) + m \log(x))} \operatorname{sgn}(bx^n + a) + amx e^{(m \log(d) + m \log(x))} \operatorname{sgn}(bx^n + a) + bmx e^{(m \log(d) + m \log(x))} \operatorname{sgn}(bx^n + a) + amx e^{(m \log(d) + m \log(x))} \operatorname{sgn}(bx^n + a) + bxx^n e^{(m \log(d) + m \log(x))} \operatorname{sgn}(bx^n + a) + ax e^{(m \log(d) + m \log(x))} \operatorname{sgn}(bx^n + a) + bxe^{(m \log(d) + m \log(x))} \operatorname{sgn}(bx^n + a)}{m^2 + mn + 2m + n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")

[Out] (b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a))/(m^2 + m*n + 2*m + n + 1)

maple [C] time = 0.04, size = 132, normalized size = 1.22

$$\frac{\sqrt{(bx^n + a)^2} (bm x^n + am + an + bx^n + a) x e^{\frac{(-i\pi \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx) + i\pi \operatorname{csgn}(id) \operatorname{csgn}(idx)^2 + i\pi \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2 - i\pi \operatorname{csgn}(idx)^3 + 2 \ln(d) + 2 \ln(x))m}{2}}}}{(bx^n + a)(m + 1)(m + n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)

[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x*(m*b*x^n+a*m+a*n+b*x^n+a)/(m+1)/(1+m+n)*exp(1/2*m*(-I*Pi*csgn(I*d*x)^3+I*Pi*csgn(I*d*x)^2*csgn(I*d)+I*Pi*csgn(I*d*x)^2*csgn(I*x)-I*Pi*csgn(I*d*x)*csgn(I*d)*csgn(I*x)+2*ln(d)+2*ln(x)))

maxima [A] time = 0.94, size = 47, normalized size = 0.44

$$\frac{ad^m(m + n + 1)xx^m + bd^m(m + 1)xe^{(m \log(x) + n \log(x))}}{m^2 + m(n + 2) + n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")

[Out] (a*d^m*(m + n + 1)*x*x^m + b*d^m*(m + 1)*x*e^(m*log(x) + n*log(x)))/(m^2 + m*(n + 2) + n + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (dx)^m \sqrt{a^2 + b^2 x^{2n} + 2abx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)

[Out] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)

[Out] Integral((d*x)**m*sqrt((a + b*x**n)**2), x)

$$3.452 \quad \int x^2 \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$$

Optimal. Leaf size=93

$$\frac{b^2x^{n+3}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(n+3)(ab + b^2x^n)} + \frac{ax^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{3(a + bx^n)}$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 14}

$$\frac{b^2x^{n+3}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(n+3)(ab + b^2x^n)} + \frac{ax^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{3(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b^2*x^n))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_ + (b_)*(x_)^(n_)) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int x^2 (ab + b^2x^n) dx}{ab + b^2x^n} \\ &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (abx^2 + b^2x^{2+n}) dx}{ab + b^2x^n} \\ &= \frac{ax^3 \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{3(a + bx^n)} + \frac{b^2x^{3+n} \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(3+n)(ab + b^2x^n)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.49

$$\frac{x^3 \sqrt{(a + bx^n)^2 (a(n+3) + 3bx^n)}}{3(n+3)(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a*(3 + n) + 3*b*x^n))/(3*(3 + n)*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] Defer[IntegrateAlgebraic][x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

fricas [A] time = 1.11, size = 28, normalized size = 0.30

$$\frac{3bx^3x^n + (an + 3a)x^3}{3(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2), x, algorithm="fricas")

[Out] 1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)

giac [A] time = 0.26, size = 53, normalized size = 0.57

$$\frac{3bx^3x^n \operatorname{sgn}(bx^n + a) + anx^3 \operatorname{sgn}(bx^n + a) + 3ax^3 \operatorname{sgn}(bx^n + a)}{3(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")

[Out] 1/3*(3*b*x^3*x^n*sgn(b*x^n + a) + a*n*x^3*sgn(b*x^n + a) + 3*a*x^3*sgn(b*x^n + a))/(n + 3)

maple [A] time = 0.02, size = 61, normalized size = 0.66

$$\frac{\sqrt{(bx^n + a)^2} bx^3x^n}{(bx^n + a)(n + 3)} + \frac{\sqrt{(bx^n + a)^2} ax^3}{3bx^n + 3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)

[Out] 1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*x^3+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/(3+n)*x^3*x^n

maxima [A] time = 0.95, size = 25, normalized size = 0.27

$$\frac{3bx^3x^n + a(n+3)x^3}{3(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")

[Out] 1/3*(3*b*x^3*x^n + a*(n + 3)*x^3)/(n + 3)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sqrt{a^2 + b^2 x^{2n} + 2abx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)

[Out] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)

[Out] Integral(x**2*sqrt((a + b*x**n)**2), x)

3.453 $\int x\sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$

Optimal. Leaf size=93

$$\frac{b^2x^{n+2}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(n+2)(ab + b^2x^n)} + \frac{ax^2\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(a + bx^n)}$$

Rubi [A] time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 14}

$$\frac{b^2x^{n+2}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(n+2)(ab + b^2x^n)} + \frac{ax^2\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x\sqrt{a^2 + 2abx^n + b^2x^{2n}} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int x(ab + b^2x^n) dx}{ab + b^2x^n} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (abx + b^2x^{1+n}) dx}{ab + b^2x^n} \\
&= \frac{ax^2\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(a + bx^n)} + \frac{b^2x^{2+n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2+n)(ab + b^2x^n)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.49

$$\frac{x^2\sqrt{(a + bx^n)^2 (a(n + 2) + 2bx^n)}}{2(n + 2)(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a*(2 + n) + 2*b*x^n))/(2*(2 + n)*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x\sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] Defer[IntegrateAlgebraic][x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

fricas [A] time = 1.15, size = 28, normalized size = 0.30

$$\frac{2bx^2x^n + (an + 2a)x^2}{2(n + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2), x, algorithm="fricas")

[Out] 1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)

giac [A] time = 0.29, size = 53, normalized size = 0.57

$$\frac{2bx^2x^n\operatorname{sgn}(bx^n + a) + anx^2\operatorname{sgn}(bx^n + a) + 2ax^2\operatorname{sgn}(bx^n + a)}{2(n + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")

[Out] 1/2*(2*b*x^2*x^n*sgn(b*x^n + a) + a*n*x^2*sgn(b*x^n + a) + 2*a*x^2*sgn(b*x^n + a))/(n + 2)

maple [A] time = 0.02, size = 61, normalized size = 0.66

$$\frac{\sqrt{(bx^n + a)^2} bx^2x^n}{(bx^n + a)(n + 2)} + \frac{\sqrt{(bx^n + a)^2} ax^2}{2bx^n + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)

[Out] 1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*x^2+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/(2+n)*x^2*x^n

maxima [A] time = 0.88, size = 25, normalized size = 0.27

$$\frac{2bx^2x^n + a(n+2)x^2}{2(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")

[Out] 1/2*(2*b*x^2*x^n + a*(n + 2)*x^2)/(n + 2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{a^2 + b^2 x^{2n} + 2abx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)

[Out] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)

[Out] Integral(x*sqrt((a + b*x**n)**2), x)

$$3.454 \quad \int \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$$

Optimal. Leaf size=88

$$\frac{b^2x^{n+1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(n+1)(ab + b^2x^n)} + \frac{ax\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{a + bx^n}$$

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1343}

$$\frac{b^2x^{n+1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(n+1)(ab + b^2x^n)} + \frac{ax\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{a + bx^n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a + b*x^n) + (b^2*x^(1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + n)*(a*b + b^2*x^n))

Rule 1343

Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (2ab + 2b^2x^n) dx}{2ab + 2b^2x^n} \\ &= \frac{ax\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{a + bx^n} + \frac{b^2x^{1+n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1+n)(ab + b^2x^n)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.44

$$\frac{x\sqrt{(a + bx^n)^2} (an + a + bx^n)}{(n+1)(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] (x*Sqrt[(a + b*x^n)^2]*(a + a*n + b*x^n))/((1 + n)*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)], x]

fricas [A] time = 1.32, size = 20, normalized size = 0.23

$$\frac{bx^n + (an + a)x}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2), x, algorithm="fricas")

[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)

giac [A] time = 0.22, size = 25, normalized size = 0.28

$$\left(ax + \frac{bx^{n+1}}{n+1}\right) \operatorname{sgn}(bx^n + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2), x, algorithm="giac")

[Out] (a*x + b*x^(n + 1)/(n + 1))*sgn(b*x^n + a)

maple [A] time = 0.02, size = 56, normalized size = 0.64

$$\frac{\sqrt{(bx^n + a)^2} bx^n}{(bx^n + a)(n + 1)} + \frac{\sqrt{(bx^n + a)^2} ax}{bx^n + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2), x)

[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/(1+n)*x*x^n

maxima [A] time = 1.04, size = 19, normalized size = 0.22

$$\frac{a(n+1)x + bxx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")

[Out] (a*(n + 1)*x + b*x*x^n)/(n + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a^2 + b^2 x^{2n} + 2 a b x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)

[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + 2abx^n + b^2x^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)

[Out] Integral(sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)

$$3.455 \quad \int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x} dx$$

Optimal. Leaf size=85

$$\frac{b^2x^n\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{n(ab + b^2x^n)} + \frac{a \log(x)\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{a + bx^n}$$

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 14}

$$\frac{b^2x^n\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{n(ab + b^2x^n)} + \frac{a \log(x)\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{a + bx^n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]

[Out] (b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \frac{ab+b^2x^n}{x} dx}{ab + b^2x^n} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \left(\frac{ab}{x} + b^2x^{-1+n}\right) dx}{ab + b^2x^n} \\
&= \frac{b^2x^n \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{n(ab + b^2x^n)} + \frac{a\sqrt{a^2 + 2abx^n + b^2x^{2n}} \log(x)}{a + bx^n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.44

$$\frac{\sqrt{(a + bx^n)^2 (an \log(x) + bx^n)}}{n(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]

[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n + a*n*Log[x]))/(n*(a + b*x^n))

IntegrateAlgebraic [A] time = 0.04, size = 41, normalized size = 0.48

$$\frac{\sqrt{(a + bx^n)^2 \left(\frac{a \log(x^n)}{n} + \frac{bx^n}{n}\right)}}{a + bx^n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]

[Out] (Sqrt[(a + b*x^n)^2]*((b*x^n)/n + (a*Log[x^n])/n))/(a + b*x^n)

fricas [A] time = 1.36, size = 15, normalized size = 0.18

$$\frac{an \log(x) + bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="fricas")

[Out] (a*n*log(x) + b*x^n)/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2 x^{2n} + 2 abx^n + a^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x, x)

maple [A] time = 0.02, size = 54, normalized size = 0.64

$$\frac{\sqrt{(bx^n + a)^2} a \ln(x)}{bx^n + a} + \frac{\sqrt{(bx^n + a)^2} bx^n}{(bx^n + a)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x)

[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*ln(x)+((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b/n*x^n

maxima [A] time = 0.80, size = 13, normalized size = 0.15

$$a \log(x) + \frac{bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="maxima")

[Out] a*log(x) + b*x^n/n

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + b^2 x^{2n} + 2 a b x^n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x,x)

[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a + bx^n)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x, x)
```

$$3.456 \quad \int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2} dx$$

Optimal. Leaf size=94

$$\frac{b^2x^{n-1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-n)(ab + b^2x^n)} - \frac{a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x(a + bx^n)}$$

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 14}

$$\frac{b^2x^{n-1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-n)(ab + b^2x^n)} - \frac{a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2, x]

[Out] -((a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (b^2*x^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \frac{ab+b^2x^n}{x^2} dx}{ab + b^2x^n} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \left(\frac{ab}{x^2} + b^2x^{-2+n}\right) dx}{ab + b^2x^n} \\
&= -\frac{a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x(a + bx^n)} - \frac{b^2x^{-1+n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-n)(ab + b^2x^n)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.45

$$\frac{\sqrt{(a + bx^n)^2} (-an + a + bx^n)}{(n - 1)x(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]

[Out] (Sqrt[(a + b*x^n)^2]*(a - a*n + b*x^n))/((-1 + n)*x*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2, x]

fricas [A] time = 1.29, size = 23, normalized size = 0.24

$$-\frac{an - bx^n - a}{(n - 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")

[Out] -(a*n - b*x^n - a)/((n - 1)*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2 x^{2n} + 2 a b x^n + a^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2, x)

maple [A] time = 0.02, size = 61, normalized size = 0.65

$$\frac{\sqrt{(b x^n + a)^2} b x^n}{(b x^n + a) (n - 1) x} - \frac{\sqrt{(b x^n + a)^2} a}{(b x^n + a) x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x)

[Out] -((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a/x+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-1+n)*b/x*x^n

maxima [A] time = 0.67, size = 22, normalized size = 0.23

$$-\frac{a(n-1) - b x^n}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")

[Out] -(a*(n - 1) - b*x^n)/((n - 1)*x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + b^2 x^{2n} + 2 a b x^n}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2,x)

[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a + bx^n)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)

[Out] Integral(sqrt((a + b*x**n)**2)/x**2, x)

$$3.457 \quad \int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3} dx$$

Optimal. Leaf size=96

$$\frac{b^2x^{n-2}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2-n)(ab + b^2x^n)} - \frac{a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2x^2(a + bx^n)}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 14}

$$\frac{b^2x^{n-2}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2-n)(ab + b^2x^n)} - \frac{a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2x^2(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3, x]

[Out] -(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (b^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a*b + b^2*x^n))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \frac{ab+b^2x^n}{x^3} dx}{ab + b^2x^n} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \left(\frac{ab}{x^3} + b^2x^{-3+n}\right) dx}{ab + b^2x^n} \\
&= -\frac{a\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2x^2(a + bx^n)} - \frac{b^2x^{-2+n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2-n)(ab + b^2x^n)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.49

$$\frac{\sqrt{(a + bx^n)^2 (2bx^n - a(n - 2))}}{2(n - 2)x^2(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]

[Out] (Sqrt[(a + b*x^n)^2]*(-(a*(-2 + n)) + 2*b*x^n))/(2*(-2 + n)*x^2*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3, x]

fricas [A] time = 1.33, size = 23, normalized size = 0.24

$$-\frac{an - 2bx^n - 2a}{2(n - 2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")

[Out] -1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b^2 x^{2n} + 2 a b x^n + a^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3, x)

maple [A] time = 0.02, size = 61, normalized size = 0.64

$$\frac{\sqrt{(b x^n + a)^2} b x^n}{(b x^n + a)(n - 2)x^2} - \frac{\sqrt{(b x^n + a)^2} a}{2(b x^n + a)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x)

[Out] -1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a/x^2+((b*x^n+a)^2)^(1/2)/(b*x^n+a)/(-2+n)*b/x^2*x^n

maxima [A] time = 0.91, size = 22, normalized size = 0.23

$$\frac{a(n - 2) - 2 b x^n}{2(n - 2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")

[Out] -1/2*(a*(n - 2) - 2*b*x^n)/((n - 2)*x^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + b^2 x^{2n} + 2 a b x^n}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3,x)

[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a + bx^n)^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)

[Out] Integral(sqrt((a + b*x**n)**2)/x**3, x)

$$3.458 \quad \int (dx)^m (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$$

Optimal. Leaf size=238

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)(ab+b^2x^n)} + \frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)(ab+b^2x^n)} + \frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)(ab+b^2x^n)}$$

Rubi [A] time = 0.10, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1355, 270, 20, 30}

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)(ab+b^2x^n)} + \frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)(ab+b^2x^n)} + \frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)(ab+b^2x^n)} + \frac{a^3(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^(m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (a^3*(d*x)^(1+m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/(d*(1+m)*(a + b*x^n)) + (3*a^2*b^2*x^(1+n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/((1+m+n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(1+2*n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/((1+m+2*n)*(a*b + b^2*x^n)) + (b^4*x^(1+3*n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/((1+m+3*n)*(a*b + b^2*x^n))

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +

$c*x^n)^{(2*\text{FracPart}[p])}$, Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int (dx)^m (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (dx)^m (ab + b^2x^n)^3 dx}{b^2 (ab + b^2x^n)} \\ &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (a^3b^3(dx)^m + 3a^2b^4x^n(dx)^m + 3ab^5x^{2n}(dx)^m + b^6x^{3n}(dx)^m) dx}{b^2 (ab + b^2x^n)} \\ &= \frac{a^3(dx)^{1+m} \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{d(1+m)(a + bx^n)} + \frac{(3a^2b^2 \sqrt{a^2 + 2abx^n + b^2x^{2n}}) \int x^n(dx)^m}{ab + b^2x^n} \\ &= \frac{a^3(dx)^{1+m} \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{d(1+m)(a + bx^n)} + \frac{(3a^2b^2x^{-m}(dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}})}{ab + b^2x^n} \\ &= \frac{a^3(dx)^{1+m} \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{d(1+m)(a + bx^n)} + \frac{3a^2b^2x^{1+n}(dx)^m \sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1+m+n)(ab + b^2x^n)} \end{aligned}$$

Mathematica [A] time = 0.11, size = 90, normalized size = 0.38

$$\frac{x(dx)^m ((a + bx^n)^2)^{3/2} \left(\frac{a^3}{m+1} + \frac{3a^2bx^n}{m+n+1} + \frac{3ab^2x^{2n}}{m+2n+1} + \frac{b^3x^{3n}}{m+3n+1} \right)}{(a + bx^n)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (x*(d*x)^m*((a + b*x^n)^2)^(3/2)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*b^2*x^(2*n))/(1 + m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n)))/(a + b*x^n)^3

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (dx)^m (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

fricas [A] time = 1.01, size = 390, normalized size = 1.64

(b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 + 2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*(a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 + 3*a^3*m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m + a^3)*n)*x*e^(m*log(d) + m*log(x))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m + 1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")

[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 + 2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*(a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 + 3*a^3*m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m + a^3)*n)*x*e^(m*log(d) + m*log(x))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m + 1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)

giac [B] time = 0.95, size = 2719, normalized size = 11.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")

[Out] (b^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*m^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*m^2*n*x*e^(m*log(d) + m*log(x))

) + 3*a*b^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a))/(m^4 + 6*m^3*n + 11*m^2*n^2 + 6*m*n^3 + 4*m^3 + 18*m^2*n + 22*m*n^2 + 6*n^3 + 6*m^2 + 18*m*n + 11*n^2 + 4*m + 6*n + 1)

maple [C] time = 0.06, size = 532, normalized size = 2.24

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)

[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*x*(b^3*(x^n)^3+a^3*m^3+3*a^3*m^2+11*a^3*n^2+6*a^3*n+6*a^3*n^3+a^3+15*a^2*b*n*x^n+3*b^3*m^2*n*(x^n)^3+2*b^3*m*n^2*(x^n)^3+3*a*b^2*m^3*(x^n)^2+6*b^3*m*n*(x^n)^3+3*a^2*b*m^3*x^n+9*a*b^2*m^2*(x^n)^2+9*a*b^2*n^2*(x^n)^2+9*a^2*b*m^2*x^n+18*a^2*b*n^2*x^n+9*m*a*b^2*(x^n)^2+12*a*b^2*(x^n)^2*n+9*m*a^2*b*x^n+12*a*b^2*m^2*n*(x^n)^2+b^3*m^3*(x^n)^3+3*b^3*m^2*(x^n)^3+2*b^3*n^2*(x^n)^3+3*m*b^3*(x^n)^3+3*b^3*(x^n)^3*n+3*a^2*b*x^n+3*(x^n)^2*a*b^2+6*a^3*m^2*n+11*a^3*m*n^2+12*a^3*m*n+9*a*b^2*m*n^2*(x^n)^2+15*a^2*b*m^2*n*x^n+18*a^2*b*m*n^2*x^n+3*m*a^3+24*a*b^2*m*n*(x^n)^2+30*a^2*b*m*n*x^n)/(m+1)/(m+n+1)/(1+m+2*n)/(1+m+3*n)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*d*x)^3+2*ln(d)+2*ln(x))*m)

maxima [A] time = 0.99, size = 276, normalized size = 1.16

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")

[Out] ((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1) * a^3*d^m*x*x^m + (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1) * b^3*d^m*x*e^(m*log(x) + 3*n*log(x)) + 3*(m^3 + m^2*(4*n + 3) + (3*n^2 + 8*n + 3)*m + 3*n^2 + 4*n + 1) * a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x)) + 3*(m^3 + m^2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1) * a^2*b*d^m*x*e^(m*log(x) + n*log(x)))/(m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m^2 + 6*n^3 + 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (dx)^m (a^2 + b^2 x^{2n} + 2 a b x^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)`

[Out] `int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + bx^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2), x)`

[Out] `Integral((d*x)**m*((a + b*x**n)**2)**(3/2), x)`

$$3.459 \quad \int x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$$

Optimal. Leaf size=212

$$\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)(ab+b^2x^n)} + \frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)(ab+b^2x^n)} + \frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)(ab+b^2x^n)} + \frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+b^2x^n)}$$

Rubi [A] time = 0.06, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)(ab+b^2x^n)} + \frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)(ab+b^2x^n)} + \frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)(ab+b^2x^n)} + \frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+b^2x^n)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/(3*(a + b*x^n)) + (b^4*x^(3*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/(3*(1 + n)*(a*b + b^2*x^n)) + (3*a^2*b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/((3 + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(3 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/((3 + 2*n)*(a*b + b^2*x^n)))

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int x^2 (ab + b^2x^n)^3 dx}{b^2 (ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (a^3b^3x^2 + 3ab^5x^{2(1+n)} + 3a^2b^4x^{2+n} + b^6x^{2+3n}) dx}{b^2 (ab + b^2x^n)} \\
&= \frac{a^3x^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{3(a + bx^n)} + \frac{b^4x^{3(1+n)}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{3(1+n)(ab + b^2x^n)} + \frac{3a^2b^2x^{3+n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(3+n)(ab + b^2x^n)}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 123, normalized size = 0.58

$$\frac{x^3 \sqrt{(a + bx^n)^2} (a^3 (2n^3 + 11n^2 + 18n + 9) + 9a^2b (2n^2 + 5n + 3)x^n + 9ab^2 (n^2 + 4n + 3)x^{2n} + b^3 (2n^2 + 9n + 9)x^{3n})}{3(n+1)(n+3)(2n+3)(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a^3*(9 + 18*n + 11*n^2 + 2*n^3) + 9*a^2*b*(3 + 5*n + 2*n^2)*x^n + 9*a*b^2*(3 + 4*n + n^2)*x^(2*n) + b^3*(9 + 9*n + 2*n^2)*x^(3*n)))/(3*(1 + n)*(3 + n)*(3 + 2*n)*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.15, size = 0, normalized size = 0.00

$$\int x^2 (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] Defer[IntegrateAlgebraic][x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

fricas [A] time = 1.10, size = 144, normalized size = 0.68

$$\frac{(2b^3n^2 + 9b^3n + 9b^3)x^3x^{3n} + 9(ab^2n^2 + 4ab^2n + 3ab^2)x^3x^{2n} + 9(2a^2bn^2 + 5a^2bn + 3a^2b)x^3x^n + (2a^3n^3 + 11a^3n^2 + 18a^3n + 9a^3)x^3}{3(2n^3 + 11n^2 + 18n + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2), x, algorithm="fricas")

[Out] $\frac{1}{3} * ((2 * b^3 * n^2 + 9 * b^3 * n + 9 * b^3) * x^3 * x^{(3 * n)} + 9 * (a * b^2 * n^2 + 4 * a * b^2 * n + 3 * a * b^2) * x^3 * x^{(2 * n)} + 9 * (2 * a^2 * b * n^2 + 5 * a^2 * b * n + 3 * a^2 * b) * x^3 * x^n + (2 * a^3 * n^3 + 11 * a^3 * n^2 + 18 * a^3 * n + 9 * a^3) * x^3) / (2 * n^3 + 11 * n^2 + 18 * n + 9)$

giac [A] time = 0.43, size = 292, normalized size = 1.38

$\frac{2 * b^3 * n^3 * \text{sgn}(b * x^n + a) + 9 * a * b^2 * n^2 * \text{sgn}(b * x^n + a) + 18 * a^2 * b * n * \text{sgn}(b * x^n + a) + 2 * a^3 * n^3 * \text{sgn}(b * x^n + a) + 9 * b^3 * n^3 * \text{sgn}(b * x^n + a) + 36 * a * b^2 * n^2 * \text{sgn}(b * x^n + a) + 45 * a^2 * b * n * \text{sgn}(b * x^n + a) + 11 * a^3 * n^2 * \text{sgn}(b * x^n + a) + 9 * b^3 * n^3 * \text{sgn}(b * x^n + a) + 27 * a * b^2 * n^2 * \text{sgn}(b * x^n + a) + 27 * a^2 * b * n * \text{sgn}(b * x^n + a) + 18 * a^3 * n^3 * \text{sgn}(b * x^n + a) + 9 * a^3 * n^3 * \text{sgn}(b * x^n + a)}{3 * (2 * n^3 + 11 * n^2 + 18 * n + 9)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")`

[Out] $\frac{1}{3} * (2 * b^3 * n^2 * x^3 * x^{(3 * n)} * \text{sgn}(b * x^n + a) + 9 * a * b^2 * n^2 * x^3 * x^{(2 * n)} * \text{sgn}(b * x^n + a) + 18 * a^2 * b * n^2 * x^3 * x^n * \text{sgn}(b * x^n + a) + 2 * a^3 * n^3 * x^3 * \text{sgn}(b * x^n + a) + 9 * b^3 * n^3 * x^3 * x^{(3 * n)} * \text{sgn}(b * x^n + a) + 36 * a * b^2 * n^2 * x^3 * x^{(2 * n)} * \text{sgn}(b * x^n + a) + 45 * a^2 * b * n^2 * x^3 * x^n * \text{sgn}(b * x^n + a) + 11 * a^3 * n^2 * x^3 * \text{sgn}(b * x^n + a) + 9 * b^3 * n^3 * x^3 * x^{(3 * n)} * \text{sgn}(b * x^n + a) + 27 * a * b^2 * n^2 * x^3 * x^{(2 * n)} * \text{sgn}(b * x^n + a) + 27 * a^2 * b * n^2 * x^3 * x^n * \text{sgn}(b * x^n + a) + 18 * a^3 * n^3 * x^3 * \text{sgn}(b * x^n + a) + 9 * a^3 * x^3 * \text{sgn}(b * x^n + a)) / (2 * n^3 + 11 * n^2 + 18 * n + 9)$

maple [A] time = 0.02, size = 146, normalized size = 0.69

$$\frac{3\sqrt{(bx^n+a)^2} a^2 b x^3 x^n}{(bx^n+a)(n+3)} + \frac{3\sqrt{(bx^n+a)^2} a b^2 x^3 x^{2n}}{(bx^n+a)(2n+3)} + \frac{\sqrt{(bx^n+a)^2} b^3 x^3 x^{3n}}{3(bx^n+a)(n+1)} + \frac{\sqrt{(bx^n+a)^2} a^3 x^3}{3bx^n+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)`

[Out] $\frac{1}{3} * ((b * x^n + a)^2)^{(1/2)} / (b * x^n + a) * x^3 * a^3 + \frac{1}{3} * ((b * x^n + a)^2)^{(1/2)} / (b * x^n + a) * b^3 * x^3 / (n + 1) * (x^n)^3 + 3 * ((b * x^n + a)^2)^{(1/2)} / (b * x^n + a) * a * b^2 / (3 + 2 * n) * x^3 * (x^n)^2 + 3 * ((b * x^n + a)^2)^{(1/2)} / (b * x^n + a) * a^2 * b / (n + 3) * x^3 * x^n$

maxima [A] time = 0.93, size = 108, normalized size = 0.51

$$\frac{(2n^2 + 9n + 9)b^3 x^3 x^{3n} + 9(n^2 + 4n + 3)ab^2 x^3 x^{2n} + 9(2n^2 + 5n + 3)a^2 b x^3 x^n + (2n^3 + 11n^2 + 18n + 9)a^3 x^3}{3(2n^3 + 11n^2 + 18n + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")`

[Out] $\frac{1}{3} * ((2 * n^2 + 9 * n + 9) * b^3 * x^3 * x^{(3 * n)} + 9 * (n^2 + 4 * n + 3) * a * b^2 * x^3 * x^{(2 * n)} + 9 * (2 * n^2 + 5 * n + 3) * a^2 * b * x^3 * x^n + (2 * n^3 + 11 * n^2 + 18 * n + 9) * a^3 * x^3) / (2 * n^3 + 11 * n^2 + 18 * n + 9)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a^2 + b^2 x^{2n} + 2 a b x^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)`

[Out] `int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 ((a + bx^n)^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2), x)`

[Out] `Integral(x**2*((a + b*x**n)**2)**(3/2), x)`

$$3.460 \quad \int x \left(a^2 + 2abx^n + b^2x^{2n} \right)^{3/2} dx$$

Optimal. Leaf size=211

$$\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)(ab+b^2x^n)} + \frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)(ab+b^2x^n)} + \frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)(ab+b^2x^n)} + \frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}$$

Rubi [A] time = 0.06, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 270}

$$\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)(ab+b^2x^n)} + \frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)(ab+b^2x^n)} + \frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)(ab+b^2x^n)} + \frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (3*a*b^3*x^(2*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 + n)*(a*b + b^2*x^n)) + (3*a^2*b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n)) + (b^4*x^(2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + 3*n)*(a*b + b^2*x^n))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int x (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int x (ab + b^2x^n)^3 dx}{b^2 (ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int (a^3b^3x + 3a^2b^4x^{1+n} + 3ab^5x^{1+2n} + b^6x^{1+3n}) dx}{b^2 (ab + b^2x^n)} \\
&= \frac{a^3x^2\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(a + bx^n)} + \frac{3ab^3x^{2(1+n)}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(1+n)(ab + b^2x^n)} + \frac{3a^2b^2x^{2+n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(1+2n)(ab + b^2x^n)} + \frac{b^6x^{2+3n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(1+3n)(ab + b^2x^n)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.59

$$\frac{x^2\sqrt{(a + bx^n)^2} (a^3(3n^3 + 11n^2 + 12n + 4) + 6a^2b(3n^2 + 5n + 2)x^n + 3ab^2(3n^2 + 8n + 4)x^{2n} + 2b^3(n^2 + 3n + 2)x^{3n})}{2(n+1)(n+2)(3n+2)(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (x^2*sqrt[(a + b*x^n)^2]*(a^3*(4 + 12*n + 11*n^2 + 3*n^3) + 6*a^2*b*(2 + 5*n + 3*n^2)*x^n + 3*a*b^2*(4 + 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 + 3*n + n^2)*x^(3*n)))/(2*(1 + n)*(2 + n)*(2 + 3*n)*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int x (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] Defer[IntegrateAlgebraic][x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

fricas [A] time = 1.32, size = 145, normalized size = 0.69

$$\frac{2(b^3n^2 + 3b^3n + 2b^3)x^2x^{3n} + 3(3ab^2n^2 + 8ab^2n + 4ab^2)x^2x^{2n} + 6(3a^2bn^2 + 5a^2bn + 2a^2b)x^2x^n + (3a^3n^3 + 11a^3n^2 + 12a^3n + 4a^3)x^2}{2(3n^3 + 11n^2 + 12n + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2), x, algorithm="fricas")

[Out] 1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*n + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n + (3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)

giac [A] time = 0.37, size = 292, normalized size = 1.38

$$\frac{2b^3x^{2n+1}\operatorname{sgn}(bx^n+a) + 9ab^2x^{2n}\operatorname{sgn}(bx^n+a) + 18a^2bx^{2n}\operatorname{sgn}(bx^n+a) + 3a^3x^{2n}\operatorname{sgn}(bx^n+a) + 6b^3x^{2n}\operatorname{sgn}(bx^n+a) + 24ab^2x^{2n}\operatorname{sgn}(bx^n+a) + 30a^2bx^{2n}\operatorname{sgn}(bx^n+a) + 11a^3x^{2n}\operatorname{sgn}(bx^n+a) + 4b^3x^{2n}\operatorname{sgn}(bx^n+a) + 12ab^2x^{2n}\operatorname{sgn}(bx^n+a) + 12a^2bx^{2n}\operatorname{sgn}(bx^n+a) + 4a^3x^{2n}\operatorname{sgn}(bx^n+a)}{2(3n^3 + 11n^2 + 12n + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")

[Out] $\frac{1}{2}*(2*b^3*n^2*x^2*x^{(3*n)}*\operatorname{sgn}(b*x^n + a) + 9*a*b^2*n^2*x^2*x^{(2*n)}*\operatorname{sgn}(b*x^n + a) + 18*a^2*b*n^2*x^2*x^n*\operatorname{sgn}(b*x^n + a) + 3*a^3*n^3*x^2*\operatorname{sgn}(b*x^n + a) + 6*b^3*n*x^2*x^{(3*n)}*\operatorname{sgn}(b*x^n + a) + 24*a*b^2*n*x^2*x^{(2*n)}*\operatorname{sgn}(b*x^n + a) + 30*a^2*b*n*x^2*x^n*\operatorname{sgn}(b*x^n + a) + 11*a^3*n^2*x^2*\operatorname{sgn}(b*x^n + a) + 4*b^3*x^2*x^{(3*n)}*\operatorname{sgn}(b*x^n + a) + 12*a*b^2*x^2*x^{(2*n)}*\operatorname{sgn}(b*x^n + a) + 12*a^2*b*x^2*x^n*\operatorname{sgn}(b*x^n + a) + 12*a^3*n*x^2*\operatorname{sgn}(b*x^n + a) + 4*a^3*x^2*\operatorname{sgn}(b*x^n + a))/(3*n^3 + 11*n^2 + 12*n + 4)$

maple [A] time = 0.02, size = 145, normalized size = 0.69

$$\frac{3\sqrt{(bx^n+a)^2} a^2 b x^2 x^n}{(bx^n+a)(n+2)} + \frac{3\sqrt{(bx^n+a)^2} a b^2 x^2 x^{2n}}{2(bx^n+a)(n+1)} + \frac{\sqrt{(bx^n+a)^2} b^3 x^2 x^{3n}}{(bx^n+a)(3n+2)} + \frac{\sqrt{(bx^n+a)^2} a^3 x^2}{2bx^n+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)

[Out] $\frac{1}{2}*((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)*x^2*a^3+((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)*b^3/(2+3*n)*x^2*(x^n)^3+3/2*((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)*a*b^2*x^2/(n+1)*(x^n)^2+3*((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)*a^2*b/(n+2)*x^2*x^n$

maxima [A] time = 0.94, size = 109, normalized size = 0.52

$$\frac{2(n^2+3n+2)b^3x^2x^{3n}+3(3n^2+8n+4)ab^2x^2x^{2n}+6(3n^2+5n+2)a^2bx^2x^n+(3n^3+11n^2+12n+4)a^3x^2}{2(3n^3+11n^2+12n+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{2}*(2*(n^2 + 3*n + 2)*b^3*x^2*x^{(3*n)} + 3*(3*n^2 + 8*n + 4)*a*b^2*x^2*x^{(2*n)} + 6*(3*n^2 + 5*n + 2)*a^2*b*x^2*x^n + (3*n^3 + 11*n^2 + 12*n + 4)*a^3*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x(a^2 + b^2 x^{2n} + 2 a b x^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)`

[Out] `int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left((a + bx^n)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2), x)`

[Out] `Integral(x*((a + b*x**n)**2)**(3/2), x)`

$$3.461 \quad \int (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$$

Optimal. Leaf size=206

$$\frac{b^6x^{3n+1}(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(3n+1)(ab + b^2x^n)^3} + \frac{3ab^5x^{2n+1}(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(2n+1)(ab + b^2x^n)^3} + \frac{3a^2b^4x^{n+1}(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(n+1)(ab + b^2x^n)^3} + \frac{a^3x(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(a + bx^n)^3}$$

Rubi [A] time = 0.05, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1343, 244}

$$\frac{3a^2b^4x^{n+1}(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(n+1)(ab + b^2x^n)^3} + \frac{3ab^5x^{2n+1}(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(2n+1)(ab + b^2x^n)^3} + \frac{b^6x^{3n+1}(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(3n+1)(ab + b^2x^n)^3} + \frac{a^3x(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(a + bx^n)^3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (a^3*x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/(a + b*x^n)^3 + (3*a^2*b^4*x^(1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + n)*(a*b + b^2*x^n)^3) + (3*a*b^5*x^(1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 2*n)*(a*b + b^2*x^n)^3) + (b^6*x^(1 + 3*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 3*n)*(a*b + b^2*x^n)^3)

Rule 244

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]

Rule 1343

Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx &= \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2} \int (2ab + 2b^2x^n)^3 dx}{(2ab + 2b^2x^n)^3} \\
&= \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2} \int (8a^3b^3 + 24a^2b^4x^n + 24ab^5x^{2n} + 8b^6x^{3n}) dx}{(2ab + 2b^2x^n)^3} \\
&= \frac{a^3x(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(a + bx^n)^3} + \frac{3a^2b^4x^{1+n}(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{(1+n)(ab + b^2x^n)^3} + \frac{3ab^5x^{1+2n}}{(1+n)(ab + b^2x^n)^3} + \frac{b^6x^{1+3n}}{(1+n)(ab + b^2x^n)^3}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 122, normalized size = 0.59

$$\frac{x\sqrt{(a+bx^n)^2} (a^3(6n^3+11n^2+6n+1) + 3a^2b(6n^2+5n+1)x^n + 3ab^2(3n^2+4n+1)x^{2n} + b^3(2n^2+3n+1)x^{3n})}{(n+1)(2n+1)(3n+1)(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] (x*Sqrt[(a + b*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*b*(1 + 5*n + 6*n^2)*x^n + 3*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n) + b^3*(1 + 3*n + 2*n^2)*x^(3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (a^2 + 2abx^n + b^2x^{2n})^{3/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

[Out] Defer[IntegrateAlgebraic][(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2), x]

fricas [A] time = 1.36, size = 130, normalized size = 0.63

$$\frac{(2b^3n^2 + 3b^3n + b^3)xx^{3n} + 3(3ab^2n^2 + 4ab^2n + ab^2)xx^{2n} + 3(6a^3bn^2 + 5a^2bn + a^2b)xx^n + (6a^3n^3 + 11a^3n^2 + 6a^3n + a^3)x}{6n^3 + 11n^2 + 6n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2), x, algorithm="fricas")

[Out] $((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^{(3*n)} + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^2)*x*x^{(2*n)} + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11*a^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)$

giac [A] time = 0.49, size = 263, normalized size = 1.28

$\frac{6a^2n^3\text{sgn}(bx^a+a) + 2b^3n^2x^3\text{sgn}(bx^a+a) + 9ab^2n^2x^2\text{sgn}(bx^a+a) + 18a^2b^2x^2\text{sgn}(bx^a+a) + 11a^2n^2\text{sgn}(bx^a+a) + 3b^3x^3\text{sgn}(bx^a+a) + 12ab^2n^2x^2\text{sgn}(bx^a+a) + 15a^2bn^2x\text{sgn}(bx^a+a) + 6a^2n\text{sgn}(bx^a+a) + b^3x^3\text{sgn}(bx^a+a) + 3ab^2x^2\text{sgn}(bx^a+a) + 3a^2b^2x\text{sgn}(bx^a+a) + a^2\text{sgn}(bx^a+a)}{6n^3+11n^2+6n+1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")`

[Out] $(6*a^3*n^3*x*\text{sgn}(b*x^n + a) + 2*b^3*n^2*x*x^{(3*n)}*\text{sgn}(b*x^n + a) + 9*a*b^2*n^2*x*x^{(2*n)}*\text{sgn}(b*x^n + a) + 18*a^2*b*n^2*x*x^n*\text{sgn}(b*x^n + a) + 11*a^3*n^2*x*\text{sgn}(b*x^n + a) + 3*b^3*n*x*x^{(3*n)}*\text{sgn}(b*x^n + a) + 12*a*b^2*n*x*x^{(2*n)}*\text{sgn}(b*x^n + a) + 15*a^2*b*n*x*x^n*\text{sgn}(b*x^n + a) + 6*a^3*n*x*\text{sgn}(b*x^n + a) + b^3*x*x^{(3*n)}*\text{sgn}(b*x^n + a) + 3*a*b^2*x*x^{(2*n)}*\text{sgn}(b*x^n + a) + 3*a^2*b*x*x^n*\text{sgn}(b*x^n + a) + a^3*x*\text{sgn}(b*x^n + a))/(6*n^3 + 11*n^2 + 6*n + 1)$

maple [A] time = 0.02, size = 138, normalized size = 0.67

$$\frac{3\sqrt{(bx^n+a)^2} a^2 b x x^n}{(bx^n+a)(n+1)} + \frac{3\sqrt{(bx^n+a)^2} a b^2 x x^{2n}}{(bx^n+a)(2n+1)} + \frac{\sqrt{(bx^n+a)^2} b^3 x x^{3n}}{(bx^n+a)(3n+1)} + \frac{\sqrt{(bx^n+a)^2} a^3 x}{bx^n+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)`

[Out] $((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)*a^3*x + ((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)*b^3/(1+3*n)*x*(x^n)^3 + 3*((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)*a*b^2/(1+2*n)*x*(x^n)^2 + 3*((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)*a^2*b/(n+1)*x*x^n$

maxima [A] time = 0.95, size = 101, normalized size = 0.49

$$\frac{(2n^2 + 3n + 1)b^3 x x^{3n} + 3(3n^2 + 4n + 1)ab^2 x x^{2n} + 3(6n^2 + 5n + 1)a^2 b x x^n + (6n^3 + 11n^2 + 6n + 1)a^3 x}{6n^3 + 11n^2 + 6n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")`

[Out] $((2*n^2 + 3*n + 1)*b^3*x*x^{(3*n)} + 3*(3*n^2 + 4*n + 1)*a*b^2*x*x^{(2*n)} + 3*(6*n^2 + 5*n + 1)*a^2*b*x*x^n + (6*n^3 + 11*n^2 + 6*n + 1)*a^3*x)/(6*n^3 + 11*n^2 + 6*n + 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a^2 + b^2 x^{2n} + 2 a b x^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)

[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^2 + 2abx^n + b^2x^{2n})^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2), x)

[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2), x)

$$3.462 \quad \int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx$$

Optimal. Leaf size=196

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n(ab+b^2x^n)} + \frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n(ab+b^2x^n)} + \frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(ab+b^2x^n)} + \frac{a^3\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Rubi [A] time = 0.05, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1355, 266, 43}

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n(ab+b^2x^n)} + \frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(ab+b^2x^n)} + \frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n(ab+b^2x^n)} + \frac{a^3\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]

[Out] (3*a^2*b^2*x^n*sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (3*a*b^3*x^(2*n)*sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a*b + b^2*x^n)) + (b^4*x^(3*n)*sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n)) + (a^3*sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \frac{(ab+b^2x^n)^3}{x} dx}{b^2(ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \text{Subst}\left(\int \frac{(ab+b^2x)^3}{x} dx, x, x^n\right)}{b^2n(ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \text{Subst}\left(\int \left(3a^2b^4 + \frac{a^3b^3}{x} + 3ab^5x + b^6x^2\right) dx, x, x^n\right)}{b^2n(ab + b^2x^n)} \\
&= \frac{3a^2b^2x^n\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{n(ab + b^2x^n)} + \frac{3ab^3x^{2n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2n(ab + b^2x^n)} + \frac{b^4x^{3n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{3n(ab + b^2x^n)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.35

$$\frac{((a + bx^n)^2)^{3/2} \left(a^3 n \log(x) + 3a^2 bx^n + \frac{3}{2} ab^2 x^{2n} + \frac{1}{3} b^3 x^{3n} \right)}{n(a + bx^n)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]

[Out] (((a + b*x^n)^2)^(3/2)*(3*a^2*b*x^n + (3*a*b^2*x^(2*n))/2 + (b^3*x^(3*n))/3 + a^3*n*Log[x]))/(n*(a + b*x^n)^3)

IntegrateAlgebraic [A] time = 0.04, size = 69, normalized size = 0.35

$$\frac{\sqrt{(a + bx^n)^2} \left(\frac{a^3 \log(x^n)}{n} + \frac{bx^n(18a^2 + 9abx^n + 2b^2x^{2n})}{6n} \right)}{a + bx^n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]

[Out] (Sqrt[(a + b*x^n)^2]*((b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)))/(6*n) + (a^3*Log[x^n])/n))/(a + b*x^n)

fricas [A] time = 0.80, size = 44, normalized size = 0.22

$$\frac{6 a^3 n \log(x) + 2 b^3 x^{3n} + 9 a b^2 x^{2n} + 18 a^2 b x^n}{6 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="fricas")

[Out] 1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b^2 x^{2n} + 2 a b x^n + a^2)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="giac")

[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x, x)

maple [A] time = 0.02, size = 127, normalized size = 0.65

$$\frac{\sqrt{(b x^n + a)^2} a^3 \ln(x)}{b x^n + a} + \frac{3 \sqrt{(b x^n + a)^2} a^2 b x^n}{(b x^n + a) n} + \frac{3 \sqrt{(b x^n + a)^2} a b^2 x^{2n}}{2 (b x^n + a) n} + \frac{\sqrt{(b x^n + a)^2} b^3 x^{3n}}{3 (b x^n + a) n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x)

[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^3*ln(x)+1/3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*b^3/n*(x^n)^3+3/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a*b^2/n*(x^n)^2+3*((b*x^n+a)^2)^(1/2)/(b*x^n+a)*a^2*b/n*x^n

maxima [A] time = 0.90, size = 43, normalized size = 0.22

$$a^3 \log(x) + \frac{2 b^3 x^{3n} + 9 a b^2 x^{2n} + 18 a^2 b x^n}{6 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="maxima")

[Out] a^3*log(x) + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + b^2 x^{2n} + 2 a b x^n)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x, x)

[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b x^n)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x, x)

[Out] Integral(((a + b*x**n)**2)**(3/2)/x, x)

$$3.463 \quad \int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} dx$$

Optimal. Leaf size=212

$$\frac{3a^2b^2x^{n-1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-n)(ab + b^2x^n)} - \frac{b^4x^{3n-1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-3n)(ab + b^2x^n)} - \frac{3ab^3x^{2n-1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-2n)(ab + b^2x^n)} - \frac{a^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x(ab + b^2x^n)}$$

Rubi [A] time = 0.07, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$\frac{3a^2b^2x^{n-1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-n)(ab + b^2x^n)} - \frac{3ab^3x^{2n-1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-2n)(ab + b^2x^n)} - \frac{b^4x^{3n-1}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-3n)(ab + b^2x^n)} - \frac{a^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x(ab + b^2x^n)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]

[Out] -((a^3*sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (3*a^2*b^2*x^(-1 + n)*sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n)) - (3*a*b^3*x^(-1 + 2*n)*sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 2*n)*(a*b + b^2*x^n)) - (b^4*x^(-1 + 3*n)*sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 3*n)*(a*b + b^2*x^n))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \frac{(ab+b^2x^n)^3}{x^2} dx}{b^2(ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \left(\frac{a^3b^3}{x^2} + 3a^2b^4x^{-2+n} + 3ab^5x^{2(-1+n)} + b^6x^{-2+3n} \right) dx}{b^2(ab + b^2x^n)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{x(a + bx^n)} - \frac{3a^2b^2x^{-1+n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-n)(ab + b^2x^n)} - \frac{3ab^3x^{-1+2n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(1-2n)(ab + b^2x^n)}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 124, normalized size = 0.58

$$\frac{\sqrt{(a + bx^n)^2} (a^3(-6n^3 + 11n^2 - 6n + 1) + 3a^2b(6n^2 - 5n + 1)x^n + 3ab^2(3n^2 - 4n + 1)x^{2n} + b^3(2n^2 - 3n + 1)x^{3n})}{(n-1)(2n-1)(3n-1)x(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2, x]

[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(1 - 6*n + 11*n^2 - 6*n^3) + 3*a^2*b*(1 - 5*n + 6*n^2)*x^n + 3*a*b^2*(1 - 4*n + 3*n^2)*x^(2*n) + b^3*(1 - 3*n + 2*n^2)*x^(3*n)))/((-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2, x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2, x]

fricas [A] time = 1.34, size = 131, normalized size = 0.62

$$\frac{6a^3n^3 - 11a^3n^2 + 6a^3n - a^3 - (2b^3n^2 - 3b^3n + b^3)x^{3n} - 3(3ab^2n^2 - 4ab^2n + ab^2)x^{2n} - 3(6a^2bn^2 - 5a^2bn + a^2b)x^n}{(6n^3 - 11n^2 + 6n - 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")

[Out] $-(6a^{3n^3} - 11a^{3n^2} + 6a^{3n} - a^3 - (2b^{3n^2} - 3b^{3n} + b^3)x^{(3n)} - 3(3ab^{2n^2} - 4ab^{2n} + ab^2)x^{(2n)} - 3(6a^2b^{n^2} - 5a^2b^n + a^2b)x^n) / ((6n^3 - 11n^2 + 6n - 1)x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^{2n} + 2abx^n + a^2)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="giac")`

[Out] `integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2, x)`

maple [A] time = 0.02, size = 147, normalized size = 0.69

$$\frac{3\sqrt{(bx^n+a)^2} a^2bx^n}{(bx^n+a)(n-1)x} + \frac{3\sqrt{(bx^n+a)^2} ab^2x^{2n}}{(bx^n+a)(2n-1)x} + \frac{\sqrt{(bx^n+a)^2} b^3x^{3n}}{(bx^n+a)(3n-1)x} - \frac{\sqrt{(bx^n+a)^2} a^3}{(bx^n+a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x)`

[Out] $-\frac{(bx^n+a)^2)^{(1/2)}}{(bx^n+a)a^3/x} + \frac{(bx^n+a)^2)^{(1/2)}}{(bx^n+a)} / (-1+3n) * b^3/x * (x^n)^3 + 3 * \frac{(bx^n+a)^2)^{(1/2)}}{(bx^n+a)} / (-1+2n) * a * b^2/x * (x^n)^2 + 3 * \frac{(bx^n+a)^2)^{(1/2)}}{(bx^n+a)} / (n-1) * a^2 * b/x * x^n$

maxima [A] time = 0.94, size = 101, normalized size = 0.48

$$\frac{(2n^2 - 3n + 1)b^3x^{3n} + 3(3n^2 - 4n + 1)ab^2x^{2n} + 3(6n^2 - 5n + 1)a^2bx^n - (6n^3 - 11n^2 + 6n - 1)a^3}{(6n^3 - 11n^2 + 6n - 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")`

[Out] $((2n^2 - 3n + 1)b^3x^{(3n)} + 3(3n^2 - 4n + 1)a^2b^2x^{(2n)} + 3(6n^2 - 5n + 1)a^2b^2x^n - (6n^3 - 11n^2 + 6n - 1)a^3) / ((6n^3 - 11n^2 + 6n - 1)x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + b^2x^{2n} + 2abx^n)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2,x)`

[Out] `int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^n)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)`

[Out] `Integral(((a + b*x**n)**2)**(3/2)/x**2, x)`

$$3.464 \quad \int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3} dx$$

Optimal. Leaf size=218

$$\frac{3a^2b^2x^{n-2}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2-n)(ab + b^2x^n)} - \frac{b^4x^{3n-2}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2-3n)(ab + b^2x^n)} - \frac{3ab^3x^{-2(1-n)}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(1-n)(ab + b^2x^n)} - \frac{a^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2x^2(a + bx^n)}$$

Rubi [A] time = 0.07, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1355, 270}

$$\frac{3ab^3x^{-2(1-n)}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(1-n)(ab + b^2x^n)} - \frac{3a^2b^2x^{n-2}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2-n)(ab + b^2x^n)} - \frac{b^4x^{3n-2}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2-3n)(ab + b^2x^n)} - \frac{a^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2x^2(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3, x]

[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*x^2*(a + b*x^n)) - (3*a*b^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1-n)*x^(2*(1-n))*(a*b + b^2*x^n)) - (3*a^2*b^2*x^(-2+n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2-n)*(a*b + b^2*x^n)) - (b^4*x^(-2+3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2-3*n)*(a*b + b^2*x^n))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3} dx &= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \frac{(ab+b^2x^n)^3}{x^3} dx}{b^2(ab + b^2x^n)} \\
&= \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n}} \int \left(\frac{a^3b^3}{x^3} + 3a^2b^4x^{-3+n} + b^6x^{3(-1+n)} + 3ab^5x^{-3+2n} \right) dx}{b^2(ab + b^2x^n)} \\
&= -\frac{a^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2x^2(a + bx^n)} - \frac{3ab^3x^{-2(1-n)}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{2(1-n)(ab + b^2x^n)} - \frac{3a^2b^2x^{-2+n}\sqrt{a^2 + 2abx^n + b^2x^{2n}}}{(2-n)(ab + b^2x^n)}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 124, normalized size = 0.57

$$\frac{\sqrt{(a + bx^n)^2} (a^3(-3n^3 + 11n^2 - 12n + 4) + 6a^2b(3n^2 - 5n + 2)x^n + 3ab^2(3n^2 - 8n + 4)x^{2n} + 2b^3(n^2 - 3n + 2)x^{3n})}{2(n-2)(n-1)(3n-2)x^2(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3, x]

[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(4 - 12*n + 11*n^2 - 3*n^3) + 6*a^2*b*(2 - 5*n + 3*n^2)*x^n + 3*a*b^2*(4 - 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 - 3*n + n^2)*x^(3*n)))/(2*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2*(a + b*x^n))

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(a^2 + 2abx^n + b^2x^{2n})^{3/2}}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3, x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3, x]

fricas [A] time = 1.33, size = 134, normalized size = 0.61

$$\frac{3a^3n^3 - 11a^3n^2 + 12a^3n - 4a^3 - 2(b^3n^2 - 3b^3n + 2b^3)x^{3n} - 3(3ab^2n^2 - 8ab^2n + 4ab^2)x^{2n} - 6(3a^2bn^2 - 5a^2bn + 2a^2b)x^n}{2(3n^3 - 11n^2 + 12n - 4)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")

[Out] $-1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2*b^3)*x^{(3*n)} - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^{(2*n)} - 6*(3*a^2*b*n^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b^2 x^{2n} + 2 a b x^n + a^2)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="giac")

[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3, x)

maple [A] time = 0.03, size = 145, normalized size = 0.67

$$\frac{3\sqrt{(b x^n + a)^2} a^2 b x^n}{(b x^n + a)(n - 2) x^2} + \frac{3\sqrt{(b x^n + a)^2} a b^2 x^{2n}}{2(b x^n + a)(n - 1) x^2} + \frac{\sqrt{(b x^n + a)^2} b^3 x^{3n}}{(b x^n + a)(3n - 2) x^2} - \frac{\sqrt{(b x^n + a)^2} a^3}{2(b x^n + a) x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x)

[Out] $-1/2*((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)*a^3/x^2+((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)/(-2+3*n)*b^3/x^2*(x^n)^3+3/2*((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)/(n-1)*a*b^2/x^2*(x^n)^2+3*((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)/(n-2)*a^2*b/x^2*x^n$

maxima [A] time = 1.17, size = 101, normalized size = 0.46

$$\frac{2(n^2 - 3n + 2)b^3x^{3n} + 3(3n^2 - 8n + 4)ab^2x^{2n} + 6(3n^2 - 5n + 2)a^2bx^n - (3n^3 - 11n^2 + 12n - 4)a^3}{2(3n^3 - 11n^2 + 12n - 4)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")

[Out] $1/2*(2*(n^2 - 3*n + 2)*b^3*x^{(3*n)} + 3*(3*n^2 - 8*n + 4)*a*b^2*x^{(2*n)} + 6*(3*n^2 - 5*n + 2)*a^2*b*x^n - (3*n^3 - 11*n^2 + 12*n - 4)*a^3)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + b^2 x^{2n} + 2 a b x^n)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3,x)`

[Out] `int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^n)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)`

[Out] `Integral(((a + b*x**n)**2)**(3/2)/x**3, x)`

$$3.465 \quad \int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}} dx$$

Optimal. Leaf size=85

$$\frac{\log(x)(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1355, 266, 36, 29, 31}

$$\frac{\log(x)(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]

[Out] ((a + b*x^n)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b*x^n])/(a*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +

$c*x^n)^{(2*\text{FracPart}[p])}$), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a^2 + 2abx^n + b^2x^{2n}}} dx &= \frac{(ab + b^2x^n) \int \frac{1}{x(ab+b^2x^n)} dx}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\ &= \frac{(ab + b^2x^n) \text{Subst}\left(\int \frac{1}{x(ab+b^2x)} dx, x, x^n\right)}{n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\ &= \frac{(ab + b^2x^n) \text{Subst}\left(\int \frac{1}{x} dx, x, x^n\right) - (b(ab + b^2x^n)) \text{Subst}\left(\int \frac{1}{ab+b^2x} dx, x, x^n\right)}{an\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\ &= \frac{(a + bx^n) \log(x)}{a\sqrt{a^2 + 2abx^n + b^2x^{2n}}} - \frac{(a + bx^n) \log(a + bx^n)}{an\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.49

$$\frac{(a + bx^n) (n \log(x) - \log(a + bx^n))}{an\sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]), x]

[Out] ((a + b*x^n)*(n*Log[x] - Log[a + b*x^n]))/(a*n*Sqrt[(a + b*x^n)^2])

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 0.65

$$\frac{(a + bx^n) \left(\frac{\log(x^n)}{an} - \frac{\log(a^2n + abnx^n)}{an} \right)}{\sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]), x]

[Out] $((a + b*x^n)*(Log[x^n]/(a*n) - Log[a^2*n + a*b*n*x^n]/(a*n)))/Sqrt[(a + b*x^n)^2]$

fricas [A] time = 1.22, size = 22, normalized size = 0.26

$$\frac{n \log(x) - \log(bx^n + a)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")`

[Out] $(n*\log(x) - \log(b*x^n + a))/(a*n)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^{2n} + 2abx^n + a^2}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x), x)`

maple [A] time = 0.02, size = 66, normalized size = 0.78

$$\frac{\sqrt{(bx^n + a)^2} \ln(x)}{(bx^n + a)a} - \frac{\sqrt{(bx^n + a)^2} \ln\left(x^n + \frac{a}{b}\right)}{(bx^n + a)an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2),x)`

[Out] $((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)/a*\ln(x) - ((b*x^n+a)^2)^{(1/2)}/(b*x^n+a)/a/n*\ln(x^n+a/b)$

maxima [A] time = 1.07, size = 27, normalized size = 0.32

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")`

[Out] $\log(x)/a - \log((b*x^n + a)/b)/(a*n)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \sqrt{a^2 + b^2 x^{2n} + 2 a b x^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)

[Out] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{(a + b x^n)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2), x)

[Out] Integral(1/(x*sqrt((a + b*x**n)**2)), x)

$$3.466 \quad \int \frac{1}{x(a^2 + 2abx^n + b^2x^{2n})^{3/2}} dx$$

Optimal. Leaf size=159

$$\frac{1}{a^2 n \sqrt{a^2 + 2abx^n + b^2x^{2n}}} + \frac{1}{2an(a + bx^n) \sqrt{a^2 + 2abx^n + b^2x^{2n}}} + \frac{\log(x)(a + bx^n)}{a^3 \sqrt{a^2 + 2abx^n + b^2x^{2n}}} - \frac{(a + bx^n) \log(a + bx^n)}{a^3 n \sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

Rubi [A] time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1355, 266, 44}

$$\frac{1}{a^2 n \sqrt{a^2 + 2abx^n + b^2x^{2n}}} + \frac{1}{2an(a + bx^n) \sqrt{a^2 + 2abx^n + b^2x^{2n}}} + \frac{\log(x)(a + bx^n)}{a^3 \sqrt{a^2 + 2abx^n + b^2x^{2n}}} - \frac{(a + bx^n) \log(a + bx^n)}{a^3 n \sqrt{a^2 + 2abx^n + b^2x^{2n}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)), x]

[Out] 1/(a^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + 1/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + ((a + b*x^n)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b*x^n])/(a^3*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1355

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a^2 + 2abx^n + b^2x^{2n})^{3/2}} dx &= \frac{(b^2(ab + b^2x^n)) \int \frac{1}{x(ab+b^2x^n)^3} dx}{\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{(b^2(ab + b^2x^n)) \text{Subst}\left(\int \frac{1}{x(ab+b^2x)^3} dx, x, x^n\right)}{n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{(b^2(ab + b^2x^n)) \text{Subst}\left(\int \left(\frac{1}{a^3b^3x} - \frac{1}{ab^2(a+bx)^3} - \frac{1}{a^2b^2(a+bx)^2} - \frac{1}{a^3b^2(a+bx)}\right) dx, x, x^n\right)}{n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} \\
&= \frac{1}{a^2n\sqrt{a^2 + 2abx^n + b^2x^{2n}}} + \frac{1}{2an(a + bx^n)\sqrt{a^2 + 2abx^n + b^2x^{2n}}} + \frac{(a + b^2x^n)}{a^3\sqrt{a^2 + 2abx^n + b^2x^{2n}}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.48

$$\frac{(a + bx^n)^3 \left(-\frac{\log(a+bx^n)}{a^3} + \frac{n \log(x)}{a^3} + \frac{1}{a^2(a+bx^n)} + \frac{1}{2a(a+bx^n)^2} \right)}{n \left((a + bx^n)^2 \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)), x]

[Out] ((a + b*x^n)^3*(1/(2*a*(a + b*x^n)^2) + 1/(a^2*(a + b*x^n)) + (n*Log[x])/a^3 - Log[a + b*x^n]/a^3))/(n*((a + b*x^n)^2)^(3/2))

IntegrateAlgebraic [A] time = 0.05, size = 78, normalized size = 0.49

$$\frac{(a + bx^n) \left(-\frac{\log(a+bx^n)}{a^3n} + \frac{\log(x^n)}{a^3n} + \frac{3a+2bx^n}{2a^2n(a+bx^n)^2} \right)}{\sqrt{(a + bx^n)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)), x]

[Out] ((a + b*x^n)*((3*a + 2*b*x^n)/(2*a^2*n*(a + b*x^n)^2) + Log[x^n]/(a^3*n) - Log[a + b*x^n]/(a^3*n)))/Sqrt[(a + b*x^n)^2]

fricas [A] time = 1.49, size = 106, normalized size = 0.67

$$\frac{2b^2nx^{2n}\log(x) + 2a^2n\log(x) + 3a^2 + 2(2abn\log(x) + ab)x^n - 2(b^2x^{2n} + 2abx^n + a^2)\log(bx^n + a)}{2(a^3b^2nx^{2n} + 2a^4bnx^n + a^5n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")

[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) + a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x^(2*n) + 2*a^4*b*n*x^n + a^5*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")

[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x), x)

maple [A] time = 0.02, size = 104, normalized size = 0.65

$$\frac{\sqrt{(bx^n + a)^2} \ln(x)}{(bx^n + a)a^3} + \frac{\sqrt{(bx^n + a)^2} (2bx^n + 3a)}{2(bx^n + a)^3 a^2 n} - \frac{\sqrt{(bx^n + a)^2} \ln\left(x^n + \frac{a}{b}\right)}{(bx^n + a)a^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(2*a*b*x^n+b^2*x^(2*n)+a^2)^(3/2),x)

[Out] ((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a^3*ln(x)+1/2*((b*x^n+a)^2)^(1/2)/(b*x^n+a)^3*(2*b*x^n+3*a)/a^2/n-((b*x^n+a)^2)^(1/2)/(b*x^n+a)/a^3/n*ln(x^n+a/b)

maxima [A] time = 1.16, size = 70, normalized size = 0.44

$$\frac{2bx^n + 3a}{2(a^2b^2nx^{2n} + 2a^3bnx^n + a^4n)} + \frac{\log(x)}{a^3} - \frac{\log\left(\frac{bx^n+a}{b}\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{2} \frac{(2bx^n + 3a)}{(a^2 + b^2x^{2n} + 2abx^n)^{3/2}} + \frac{2a^3bx^n + a^4n}{a^3} + \frac{\log(x)}{a^3} - \frac{\log((bx^n + a)/b)}{a^3n}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x(a^2 + b^2x^{2n} + 2abx^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)`

[Out] `int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x((a + bx^n)^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2), x)`

[Out] `Integral(1/(x*((a + b*x**n)**2)**(3/2)), x)`

$$3.467 \quad \int \left(a^2 + b^2 x^{-\frac{2}{1+2p}} + 2abx^{-\frac{1}{1+2p}} \right)^p dx$$

Optimal. Leaf size=52

$$\frac{x \left(a + bx^{-\frac{1}{2p-1}} \right) \left(a^2 + 2abx^{-\frac{1}{2p-1}} + b^2 x^{-\frac{2}{2p+1}} \right)^p}{a}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1343, 191}

$$\frac{x \left(a + bx^{-\frac{1}{2p-1}} \right) \left(a^2 + 2abx^{-\frac{1}{2p-1}} + b^2 x^{-\frac{2}{2p+1}} \right)^p}{a}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x^(2/(1 + 2*p))) + (2*a*b)/x^(1 + 2*p)^(-1))^p, x]

[Out] (x*(a + b*x^(-1 - 2*p))^(-1))*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2/x^(2/(1 + 2*p)))^p/a

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 1343

Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \left(a^2 + b^2 x^{-\frac{2}{1+2p}} + 2abx^{-\frac{1}{1+2p}} \right)^p dx &= \left(\left(a^2 + b^2 x^{-\frac{2}{1+2p}} + 2abx^{-\frac{1}{1+2p}} \right)^p \left(2ab + 2b^2 x^{-\frac{1}{1+2p}} \right)^{-2p} \right) \int \left(2ab + 2b^2 x^{-\frac{1}{1+2p}} \right) \\ &= \frac{x \left(a + bx^{-\frac{1}{1-2p}} \right) \left(a^2 + 2abx^{-\frac{1}{1-2p}} + b^2 x^{-\frac{2}{1+2p}} \right)^p}{a} \end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.12

$$\frac{x^{\frac{2p}{2p+1}} \left(ax^{\frac{1}{2p+1}} + b \right) \left(x^{-\frac{2}{2p+1}} \left(ax^{\frac{1}{2p+1}} + b \right)^2 \right)^p}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x^(2/(1 + 2*p))) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]

[Out] (x^((2*p)/(1 + 2*p))*(b + a*x^(1 + 2*p)^(-1))*((b + a*x^(1 + 2*p)^(-1))^2/x^(2/(1 + 2*p))))^p/a

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(a^2 + b^2 x^{-\frac{2}{1+2p}} + 2abx^{-\frac{1}{1+2p}} \right)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + b^2/x^(2/(1 + 2*p))) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + b^2/x^(2/(1 + 2*p))) + (2*a*b)/x^(1 + 2*p)^(-1))^p, x]

fricas [A] time = 1.09, size = 79, normalized size = 1.52

$$\frac{\left(axx^{\left(\frac{1}{2p+1}\right)} + bx \right) \left(\frac{a^2 x^{\frac{2}{2p+1}} + 2abx^{\left(\frac{1}{2p+1}\right)} + b^2}{x^{\frac{2}{2p+1}}} \right)^p}{ax^{\left(\frac{1}{2p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/(x^(2/(1+2*p))))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="fricas")

[Out] (a*x*x^(1/(2*p + 1)) + b*x)*((a^2*x^(2/(2*p + 1)) + 2*a*b*x^(1/(2*p + 1)) + b^2)/x^(2/(2*p + 1)))^p/(a*x^(1/(2*p + 1)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{b^2}{x^{\frac{2}{2p+1}}} + \frac{2ab}{x^{\left(\frac{1}{2p+1}\right)}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/(x^(2/(1+2*p))))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="giac")

[Out] integrate((a^2 + b^2/x^(2/(2*p + 1))) + 2*a*b/x^(1/(2*p + 1)))^p, x)

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(2ab x^{-\frac{1}{2p+1}} + b^2 x^{-\frac{2}{2p+1}} + a^2 \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+b^2/(x^(2/(2*p+1))))+2*a*b/(x^(1/(2*p+1))))^p,x)

[Out] int((a^2+b^2/(x^(2/(2*p+1))))+2*a*b/(x^(1/(2*p+1))))^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{b^2}{x^{\frac{2}{2p+1}}} + \frac{2ab}{x^{\left(\frac{1}{2p+1}\right)}} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/(x^(2/(1+2*p))))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="maxima")

[Out] integrate((a^2 + b^2/x^(2/(2*p + 1))) + 2*a*b/x^(1/(2*p + 1)))^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a^2 + \frac{b^2}{x^{\frac{2}{2p+1}}} + \frac{2ab}{x^{\frac{1}{2p+1}}} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2/x^(2/(2*p + 1))) + (2*a*b)/x^(1/(2*p + 1)))^p,x)

[Out] int((a^2 + b^2/x^(2/(2*p + 1))) + (2*a*b)/x^(1/(2*p + 1)))^p, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+b**2/(x**(2/(1+2*p))))+2*a*b/(x**(1/(1+2*p))))**p,x)

[Out] Timed out

$$3.468 \quad \int \left(a^2 + 2abx^n + b^2x^{2n} \right)^{\frac{-1-n}{2n}} dx$$

Optimal. Leaf size=43

$$\frac{x(a + bx^n)(a^2 + 2abx^n + b^2x^{2n})^{-\frac{n+1}{2n}}}{a}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1343, 191}

$$\frac{x(a + bx^n)(a^2 + 2abx^n + b^2x^{2n})^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + n)/(2*n)), x]

[Out] (x*(a + b*x^n))/(a*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n)))

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 1343

Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \left(a^2 + 2abx^n + b^2x^{2n} \right)^{\frac{-1-n}{2n}} dx &= \left((2ab + 2b^2x^n)^{-\frac{-1-n}{n}} \left(a^2 + 2abx^n + b^2x^{2n} \right)^{\frac{-1-n}{2n}} \right) \int \left(2ab + 2b^2x^n \right)^{\frac{-1-n}{n}} dx \\ &= \frac{x(a + bx^n)(a^2 + 2abx^n + b^2x^{2n})^{-\frac{1+n}{2n}}}{a} \end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.74

$$\frac{x(a + bx^n) \left((a + bx^n)^2 \right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)), x]

[Out] (x*(a + b*x^n))/(a*((a + b*x^n)^2)^((1 + n)/(2*n)))

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (a^2 + 2abx^n + b^2x^{2n})^{\frac{-1-n}{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)), x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)), x]

fricas [A] time = 1.28, size = 45, normalized size = 1.05

$$\frac{bxx^n + ax}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{n+1}{2n}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="fricas")

[Out] (b*x*x^n + a*x)/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)*a)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{n+1}{2n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="giac")

[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)

maple [A] time = 0.04, size = 51, normalized size = 1.19

$$\left(\frac{bx e^{n \ln(x)}}{a} + x\right) e^{-\frac{(n+1) \ln(2ab e^{n \ln(x)} + b^2 e^{2n \ln(x)} + a^2)}{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2*(n+1)/n)),x)`

[Out] `(x+1/a*b*x*exp(n*ln(x)))/exp(1/2*(n+1)/n*ln(a^2+2*a*b*exp(n*ln(x))+b^2*exp(n*ln(x))^2))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^2 x^{2n} + 2 abx^n + a^2)^{\frac{n+1}{2n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="maxima")`

[Out] `integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a^2 + b^2 x^{2n} + 2 abx^n)^{\frac{\frac{n}{2} + \frac{1}{2}}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n),x)`

[Out] `int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^2 + 2abx^n + b^2x^{2n})^{-\frac{\frac{n}{2} + \frac{1}{2}}{n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+n)/n)),x)`

[Out] `Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-(n/2 + 1/2)/n), x)`

$$3.469 \quad \int \left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}} \right)^p dx$$

Optimal. Leaf size=130

$$\frac{2(p+1)x \left(a + bx^{-\frac{1}{2(p+1)}} \right) \left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}} \right)^p}{a(2p+1)} - \frac{x \left(a + bx^{-\frac{1}{2(p+1)}} \right)^2 \left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}} \right)^p}{a^2(2p+1)}$$

Rubi [A] time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {1343, 192, 191}

$$\frac{2(p+1)x \left(a + bx^{-\frac{1}{2(p+1)}} \right) \left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}} \right)^p}{a(2p+1)} - \frac{x \left(a + bx^{-\frac{1}{2(p+1)}} \right)^2 \left(a^2 + 2abx^{-\frac{1}{2(p+1)}} + b^2 x^{-\frac{1}{p+1}} \right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + b^2/x^(1 + p))^(-1) + (2*a*b)/x^(1/(2*(1 + p)))]^p,x]

[Out] (2*(1 + p)*x*(a + b/x^(1/(2*(1 + p))))*(a^2 + b^2/x^(1 + p))^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p/(a*(1 + 2*p)) - (x*(a + b/x^(1/(2*(1 + p))))^2*(a^2 + b^2/x^(1 + p))^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p/(a^2*(1 + 2*p))

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 192

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 1343

Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}} \right)^p dx &= \left(\left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}} \right)^p \left(2ab + 2b^2 x^{-\frac{1}{2(1+p)}} \right)^{-2p} \right) \int \left(2ab + 2b^2 x^{-\frac{1}{2(1+p)}} \right)^{2p} dx \\
&= \frac{2(1+p)x \left(a + bx^{-\frac{1}{2(1+p)}} \right) \left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}} \right)^p}{a(1+2p)} - \frac{\left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}} \right)^p}{a(1+2p)} \\
&= \frac{2(1+p)x \left(a + bx^{-\frac{1}{2(1+p)}} \right) \left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}} \right)^p}{a(1+2p)} - \frac{\left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}} \right)^p}{a(1+2p)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.62

$$\frac{x^{\frac{p}{p+1}} \left(ax^{\frac{1}{2p+2}} + b \right) \left(x^{-\frac{1}{p+1}} \left(ax^{\frac{1}{2p+2}} + b \right)^2 \right)^p \left(a(2p+1)x^{\frac{1}{2p+2}} - b \right)}{a^2(2p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + b^2/x^(1+p))^(-1) + (2*a*b)/x^(1/(2*(1+p)))]^p,x]

[Out] (x^(p/(1+p))*(b + a*x^(2+2*p))^(-1))*((b + a*x^(2+2*p))^(-1))^2/x^(1+p)^(-1))^p*(-b + a*(1+2*p)*x^(2+2*p)^(-1))/(a^2*(1+2*p))

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(a^2 + b^2 x^{-\frac{1}{1+p}} + 2abx^{-\frac{1}{2(1+p)}} \right)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + b^2/x^(1+p))^(-1) + (2*a*b)/x^(1/(2*(1+p)))]^p,x]

[Out] Defer[IntegrateAlgebraic][(a^2 + b^2/x^(1+p))^(-1) + (2*a*b)/x^(1/(2*(1+p)))]^p,x]

fricas [A] time = 0.93, size = 103, normalized size = 0.79

$$\frac{\left(2abpxx^{\frac{1}{2(p+1)}} - b^2x + (2a^2p + a^2)xx^{\left(\frac{1}{p+1}\right)} \right) \left(\frac{2abx^{\frac{1}{2(p+1)}} + a^2x^{\left(\frac{1}{p+1}\right)} + b^2}{x^{\left(\frac{1}{p+1}\right)}} \right)^p}{(2a^2p + a^2)x^{\left(\frac{1}{p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/(x^(1/(1+p))))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="fricas")

[Out] (2*a*b*p*x*x^(1/2/(p + 1)) - b^2*x + (2*a^2*p + a^2)*x*x^(1/(p + 1)))*((2*a*b*x^(1/2/(p + 1)) + a^2*x^(1/(p + 1)) + b^2)/x^(1/(p + 1)))^p/((2*a^2*p + a^2)*x^(1/(p + 1)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2ab}{x^{2(p+1)}} + \frac{b^2}{x^{\left(\frac{1}{p+1}\right)}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/(x^(1/(1+p))))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="giac")

[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1))))^p, x)

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \left(2abx^{-\frac{1}{2(p+1)}} + b^2x^{-\frac{1}{p+1}} + a^2 \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+b^2/(x^(1/(p+1))))+2*a*b/(x^(1/2/(p+1))))^p,x)

[Out] int((a^2+b^2/(x^(1/(p+1))))+2*a*b/(x^(1/2/(p+1))))^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2ab}{x^{2(p+1)}} + \frac{b^2}{x^{\left(\frac{1}{p+1}\right)}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+b^2/(x^(1/(1+p))))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="maxima")

[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1))))^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b^2}{x^{\frac{1}{p+1}}} + a^2 + \frac{2ab}{x^{\frac{1}{2(p+1)}}} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p,x)

[Out] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+b**2/(x**(1/(1+p)))+2*a*b/(x**(1/2/(1+p))))**p,x)

[Out] Timed out

$$3.470 \quad \int \left(a^2 + 2abx^n + b^2x^{2n} \right)^{\frac{-1-2n}{2n}} dx$$

Optimal. Leaf size=102

$$\frac{nx(a+bx^n)^2(a^2+2abx^n+b^2x^{2n})^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)} + \frac{x(a+bx^n)(a^2+2abx^n+b^2x^{2n})^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1343, 192, 191}

$$\frac{nx(a+bx^n)^2(a^2+2abx^n+b^2x^{2n})^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)} + \frac{x(a+bx^n)(a^2+2abx^n+b^2x^{2n})^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-(1 + 2*n)/(2*n)), x]

[Out] (x*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/(a*(1 + n)) + (n*x*(a + b*x^n)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/(a^2*(1 + n))

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 192

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 1343

Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] := Dist[(a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int (a^2 + 2abx^n + b^2x^{2n})^{\frac{-1-2n}{2n}} dx &= \left((2ab + 2b^2x^n)^{\frac{-1-2n}{n}} (a^2 + 2abx^n + b^2x^{2n})^{\frac{-1-2n}{2n}} \right) \int (2ab + 2b^2x^n)^{\frac{-1-2n}{n}} dx \\
&= \frac{x(a + bx^n)(a^2 + 2abx^n + b^2x^{2n})^{\frac{1}{2}\left(-2 - \frac{1}{n}\right)}}{a(1+n)} + \frac{\left(n(2ab + 2b^2x^n)^{\frac{-1-2n}{n}} (a^2 + 2abx^n + b^2x^{2n})^{\frac{-1-2n}{2n}}\right)}{a^2(1+n)} \\
&= \frac{x(a + bx^n)(a^2 + 2abx^n + b^2x^{2n})^{\frac{1}{2}\left(-2 - \frac{1}{n}\right)}}{a(1+n)} + \frac{nx(a + bx^n)^2(a^2 + 2abx^n + b^2x^{2n})^{\frac{-1-2n}{2n}}}{a^2(1+n)}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 0.58

$$\frac{x((a + bx^n)^2)^{-\frac{1}{2n}} \left(\frac{bx^n}{a} + 1\right)^{\frac{1}{n}} {}_2F_1\left(2 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)), x]

[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*((a + b*x^n)^2)^(1/(2*n)))

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (a^2 + 2abx^n + b^2x^{2n})^{\frac{-1-2n}{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)), x]

[Out] Defer[IntegrateAlgebraic] [(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)), x]

fricas [A] time = 1.13, size = 82, normalized size = 0.80

$$\frac{b^2nxx^{2n} + (2abn + ab)xx^n + (a^2n + a^2)x}{(a^2n + a^2)(b^2x^{2n} + 2abx^n + a^2)^{\frac{2n+1}{2n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="fricas")

[Out] (b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2*n + a^2)*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{2n+1}{2n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="giac")

[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (2abx^n + b^2x^{2n} + a^2)^{-\frac{2n+1}{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2*(2*n+1)/n)),x)

[Out] int(1/((2*a*b*x^n+b^2*x^(2*n)+a^2)^(1/2*(2*n+1)/n)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^2x^{2n} + 2abx^n + a^2)^{\frac{2n+1}{2n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="maxima")

[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a^2 + b^2x^{2n} + 2abx^n)^{\frac{n+\frac{1}{2}}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n), x)`

[Out] `int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+2*n)/n)), x)`

[Out] Timed out

$$3.471 \quad \int (dx)^{-1-2n(1+p)} (a^2 + 2abx^n + b^2x^{2n})^p dx$$

Optimal. Leaf size=117

$$\frac{(dx)^{-2n(p+1)} (a^2 + 2abx^n + b^2x^{2n})^{p+1}}{2a^2dn(p+1)(2p+1)} - \frac{(a + bx^n)(dx)^{-2n(p+1)} (a^2 + 2abx^n + b^2x^{2n})^p}{adn(2p+1)}$$

Rubi [A] time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1356, 273, 264}

$$\frac{\left(\frac{bx^n}{a} + 1\right)^2 (dx)^{-2n(p+1)} (a^2 + 2abx^n + b^2x^{2n})^p}{2dn(2p^2 + 3p + 1)} - \frac{\left(\frac{bx^n}{a} + 1\right) (dx)^{-2n(p+1)} (a^2 + 2abx^n + b^2x^{2n})^p}{dn(2p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p, x]

[Out] -(((1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(d*n*(1 + 2*p)*(d*x)^(2*n*(1 + p)))) + (((1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*d*n*(1 + 3*p + 2*p^2)*(d*x)^(2*n*(1 + p))))

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rule 273

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m + n*(p+1) + 1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m, n, p}, x] && ILtQ[Simplify[(m+1)/n + p + 1], 0] && NeQ[p, -1]

Rule 1356

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p]]/(1 + (2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
\int (dx)^{-1-2n(1+p)} (a^2 + 2abx^n + b^2x^{2n})^p dx &= \left(\left(1 + \frac{bx^n}{a}\right)^{-2p} (a^2 + 2abx^n + b^2x^{2n})^p \right) \int (dx)^{-1-2n(1+p)} \left(1 + \frac{bx^n}{a}\right)^{2p} \\
&= -\frac{(dx)^{-2n(1+p)} \left(1 + \frac{bx^n}{a}\right) (a^2 + 2abx^n + b^2x^{2n})^p}{dn(1+2p)} + \frac{\left((-2n(1+p) + n\right)}{dn(1+2p)} \\
&= -\frac{(dx)^{-2n(1+p)} \left(1 + \frac{bx^n}{a}\right) (a^2 + 2abx^n + b^2x^{2n})^p}{dn(1+2p)} + \frac{(dx)^{-2n(1+p)} \left(1 + \frac{bx^n}{a}\right)^{2p}}{2dn}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.64

$$\frac{x(dx)^{-2n(p+1)-1} \left((a + bx^n)^2\right)^p \left(\frac{bx^n}{a} + 1\right)^{-2p} {}_2F_1\left(-2p, -2(p+1); 1 - 2(p+1); -\frac{bx^n}{a}\right)}{2n(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]

[Out] -1/2*(x*(d*x)^(-1 - 2*n*(1 + p))*((a + b*x^n)^2)^p*Hypergeometric2F1[-2*p, -2*(1 + p), 1 - 2*(1 + p), -(b*x^n)/a])/(n*(1 + p)*(1 + (b*x^n)/a)^(2*p))

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (dx)^{-1-2n(1+p)} (a^2 + 2abx^n + b^2x^{2n})^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p, x]

fricas [A] time = 1.08, size = 165, normalized size = 1.41

$$\frac{\left(2abpx^n e^{-(2np+2n+1)\log(d)-(2np+2n+1)\log(x)} - b^2xx^{2n} e^{-(2np+2n+1)\log(d)-(2np+2n+1)\log(x)} + (2a^2p + a^2)xe^{-(2np+2n+1)\log(d)-(2np+2n+1)\log(x)}\right)(b^2x^{2n} + 2abx^n + a^2)^p}{2(2a^2np^2 + 3a^2np + a^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="fricas")

[Out] $-1/2*(2*a*b*p*x*x^n*e^{-(2*n*p + 2*n + 1)*\log(d) - (2*n*p + 2*n + 1)*\log(x)} - b^2*x*x^{(2*n)}*e^{-(2*n*p + 2*n + 1)*\log(d) - (2*n*p + 2*n + 1)*\log(x)} + (2*a^2*p + a^2)*x*e^{-(2*n*p + 2*n + 1)*\log(d) - (2*n*p + 2*n + 1)*\log(x)})*(b^2*x^{(2*n)} + 2*a*b*x^n + a^2)^p/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b^2x^{2n} + 2abx^n + a^2)^p (dx)^{-2n(p+1)-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="giac")

[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (dx)^{-2(p+1)n-1} (2abx^n + b^2x^{2n} + a^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(-1-2*n*(p+1))*(2*a*b*x^n+b^2*x^(2*n)+a^2)^p,x)

[Out] int((d*x)^(-1-2*n*(p+1))*(2*a*b*x^n+b^2*x^(2*n)+a^2)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b^2x^{2n} + 2abx^n + a^2)^p (dx)^{-2n(p+1)-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="maxima")

[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + b^2x^{2n} + 2abx^n)^p}{(dx)^{2n(p+1)+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1),x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1-2*n*(1+p))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

$$3.472 \quad \int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^p dx$$

Optimal. Leaf size=103

$$\frac{a^2 \left(\frac{bx^n}{a} + 1\right)^2 (a^2 + 2abx^n + b^2x^{2n})^p}{2b^2n(p+1)} - \frac{a^2 \left(\frac{bx^n}{a} + 1\right) (a^2 + 2abx^n + b^2x^{2n})^p}{b^2n(2p+1)}$$

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1356, 266, 43}

$$\frac{a^2 \left(\frac{bx^n}{a} + 1\right)^2 (a^2 + 2abx^n + b^2x^{2n})^p}{2b^2n(p+1)} - \frac{a^2 \left(\frac{bx^n}{a} + 1\right) (a^2 + 2abx^n + b^2x^{2n})^p}{b^2n(2p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]

[Out] -((a^2*(1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*(1 + 2*p)) + (a^2*(1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*b^2*n*(1 + p)))

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1356

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/(1 + (
2*c*x^n)/b)^(2*FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/b)^(2*p), x], x] /;
FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^p dx &= \left(\left(1 + \frac{bx^n}{a} \right)^{-2p} (a^2 + 2abx^n + b^2x^{2n})^p \right) \int x^{-1+2n} \left(1 + \frac{bx^n}{a} \right)^{2p} dx \\
&= \frac{\left(\left(1 + \frac{bx^n}{a} \right)^{-2p} (a^2 + 2abx^n + b^2x^{2n})^p \right) \text{Subst} \left(\int x \left(1 + \frac{bx}{a} \right)^{2p} dx, x, x^n \right)}{n} \\
&= \frac{\left(\left(1 + \frac{bx^n}{a} \right)^{-2p} (a^2 + 2abx^n + b^2x^{2n})^p \right) \text{Subst} \left(\int \left(-\frac{a \left(1 + \frac{bx}{a} \right)^{2p}}{b} + \frac{a \left(1 + \frac{bx}{a} \right)^{1+2p}}{b} \right) dx, x, x^n \right)}{n} \\
&= -\frac{a^2 \left(1 + \frac{bx^n}{a} \right) (a^2 + 2abx^n + b^2x^{2n})^p}{b^2n(1+2p)} + \frac{a^2 \left(1 + \frac{bx^n}{a} \right)^2 (a^2 + 2abx^n + b^2x^{2n})^p}{2b^2n(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.52

$$\frac{(a + bx^n) \left((a + bx^n)^2 \right)^p (b(2p + 1)x^n - a)}{2b^2n(p + 1)(2p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]

[Out] ((a + b*x^n)*((a + b*x^n)^2)^p*(-a + b*(1 + 2*p)*x^n))/(2*b^2*n*(1 + p)*(1 + 2*p))

IntegrateAlgebraic [F] time = 0.23, size = 0, normalized size = 0.00

$$\int x^{-1+2n} (a^2 + 2abx^n + b^2x^{2n})^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p, x]

fricas [A] time = 1.08, size = 78, normalized size = 0.76

$$\frac{(2abpx^n - a^2 + (2b^2p + b^2)x^{2n})(b^2x^{2n} + 2abx^n + a^2)^p}{2(2b^2np^2 + 3b^2np + b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a^{2+2*a*b*xⁿ+b²*x^(2*n))^p,x, algorithm="fricas")}

[Out] 1/2*(2*a*b*p*xⁿ - a² + (2*b²*p + b²)*x^(2*n))*(b²*x^(2*n) + 2*a*b*xⁿ + a²)^p/(2*b²*n*p² + 3*b²*n*p + b²*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b^2 x^{2n} + 2 abx^n + a^2)^p x^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a^{2+2*a*b*xⁿ+b²*x^(2*n))^p,x, algorithm="giac")}

[Out] integrate((b²*x^(2*n) + 2*a*b*xⁿ + a²)^p*x^(2*n - 1), x)

maple [C] time = 0.07, size = 148, normalized size = 1.44

$$\frac{(-2abp x^n - 2b^2 p x^{2n} - b^2 x^{2n} + a^2) e^{\frac{\left(-i\pi \operatorname{csgn}(i(bx^n+a))^2 \operatorname{csgn}(i(bx^n+a)^2) + 2i\pi \operatorname{csgn}(i(bx^n+a)) \operatorname{csgn}(i(bx^n+a))^2\right)^2 - i\pi \operatorname{csgn}(i(bx^n+a))^3 + 4 \ln(bx^n+a)}{2}}}{2(2p+1)(p+1)b^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)*(2*a*b*xⁿ+b²*x^(2*n)+a²)^p,x)

[Out] -1/2*(-2*b²*p*(xⁿ)²-2*a*p*xⁿ*b-b²*(xⁿ)²+a²)/(2*p+1)/(p+1)/n/b²*exp(1/2*p*(-I*Pi*csgn(I*(b*xⁿ+a)²)³+2*I*Pi*csgn(I*(b*xⁿ+a)²)*csgn(I*(b*xⁿ+a))-I*Pi*csgn(I*(b*xⁿ+a)²)*csgn(I*(b*xⁿ+a))²+4*ln(b*xⁿ+a))

maxima [A] time = 1.23, size = 59, normalized size = 0.57

$$\frac{(b^2(2p+1)x^{2n} + 2abpx^n - a^2)(bx^n + a)^{2p}}{2(2p^2 + 3p + 1)b^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a^{2+2*a*b*xⁿ+b²*x^(2*n))^p,x, algorithm="maxima")}

[Out] 1/2*(b²*(2*p + 1)*x^(2*n) + 2*a*b*p*xⁿ - a²)*(b*xⁿ + a)^(2*p)/((2*p² + 3*p + 1)*b²*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2n-1} (a^2 + b^2 x^{2n} + 2 a b x^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p, x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p, x)
```

```
[Out] Timed out
```

$$3.473 \quad \int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=111

$$\frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3 n \sqrt{b^2 - 4ac}} + \frac{(b^2 - ac) \log(a + bx^n + cx^{2n})}{2c^3 n} - \frac{bx^n}{c^2 n} + \frac{x^{2n}}{2cn}$$

Rubi [A] time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 701, 634, 618, 206, 628}

$$\frac{(b^2 - ac) \log(a + bx^n + cx^{2n})}{2c^3 n} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3 n \sqrt{b^2 - 4ac}} - \frac{bx^n}{c^2 n} + \frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*c^3*n)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\text{t}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 701

$\text{Int}[(d + e*x)^m / (a + b*x + c*x^2), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[(d + e*x)^m, a + b*x + c*x^2, x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{IGtQ}[m, 1] \&\& (\text{NeQ}[d, 0] \parallel \text{GtQ}[m, 2])$

Rule 1357

$\text{Int}[x^m * ((a + c*x^{n2}) + (b*x^n)^{p}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{\text{Simplify}[(m+1)/n] - 1} * (a + b*x + c*x^2)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x\} \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+4n}}{a + bx^n + cx^{2n}} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2} + \frac{x}{c} + \frac{ab+(b^2-ac)x}{c^2(a+bx+cx^2)}\right) dx, x, x^n\right)}{n} \\ &= -\frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} + \frac{\text{Subst}\left(\int \frac{ab+(b^2-ac)x}{a+bx+cx^2} dx, x, x^n\right)}{c^2n} \\ &= -\frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} - \frac{(b(b^2-3ac)) \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^n\right)}{2c^3n} + \frac{(b^2-ac) \text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^n\right)}{2c^3n} \\ &= -\frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} + \frac{(b^2-ac) \log(a + bx^n + cx^{2n})}{2c^3n} + \frac{(b(b^2-3ac)) \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, x^n\right)}{c^3n} \\ &= -\frac{bx^n}{c^2n} + \frac{x^{2n}}{2cn} + \frac{b(b^2-3ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}n} + \frac{(b^2-ac) \log(a + bx^n + cx^{2n})}{2c^3n} \end{aligned}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 0.84

$$\frac{(b^2 - ac) \log(a + x^n (b + cx^n)) + \frac{2b(b^2 - 3ac) \tanh^{-1}\left(\frac{b + 2cx^n}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{b^2 - 4ac}} + cx^n (cx^n - 2b)}{2c^3n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] (c*x^n*(-2*b + c*x^n) + (2*b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] + (b^2 - a*c)*Log[a + x^n*(b + c*x^n)]/(2*c^3*n)

IntegrateAlgebraic [A] time = 0.25, size = 126, normalized size = 1.14

$$\frac{(b^2 - ac) \log(a + bx^n + cx^{2n})}{2c^3n} + \frac{(3abc - b^3) \tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac - b^2}} + \frac{b}{\sqrt{4ac - b^2}}\right)}{c^3n\sqrt{4ac - b^2}} + \frac{x^n (cx^n - 2b)}{2c^2n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] (x^n*(-2*b + c*x^n))/(2*c^2*n) + ((-b^3 + 3*a*b*c)*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*n) + ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*c^3*n)

fricas [A] time = 1.18, size = 353, normalized size = 3.18

$$\left[\frac{(b^3 - 3abc)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^{2n} + b^2 - 2ac + 2(b^2 - 4ac)x^n}{c^2x^{2n} + b^2}\right) - (b^2 - 4ac)^2x^{2n} + 2(b^2 - 4ac)x^n - (b^4 - 5ab^2c + 4a^2c^2) \log(cx^n + bx^n + a)}{2(b^2 - 4ac)^2n}, \frac{2(b^3 - 3abc)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-2\sqrt{-b^2 + 4ac}cx^n + \sqrt{-b^2 + 4ac}b}{b^2 - 4ac}\right) + (b^2 - 4ac)^2x^{2n} - 2(b^2 - 4ac)x^n + (b^4 - 5ab^2c + 4a^2c^2) \log(cx^{2n} + bx^n + a)}{2(b^2 - 4ac)^2n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) - (b^2*c^2 - 4*a*c^3)*x^(2*n) + 2*(b^3*c - 4*a*b*c^2)*x^n - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b*x^n + a)]/((b^2*c^3 - 4*a*c^4)*n), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan((-2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^2*c^2 - 4*a*c^3)*x^(2*n) - 2*(b^3*c - 4*a*b*c^2)*x^n + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b*x^n + a)]/((b^2*c^3 - 4*a*c^4)*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

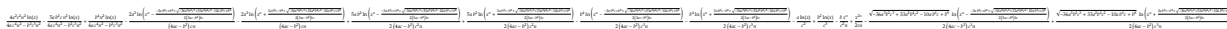
$$\int \frac{x^{4n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [B] time = 0.16, size = 973, normalized size = 8.77



Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x)

[Out]
$$\begin{aligned} & -1/c^2 \ln(x) * a + 1/c^3 \ln(x) * b^2 + 1/2/c/n * (x^n)^{-2} * b * x^n / c^2 / n + 4 / (4 * a * c^4 * n^2 - b^2 * c^3 * n^2) * n^2 * \ln(x) * a^2 * c^2 - 5 / (4 * a * c^4 * n^2 - b^2 * c^3 * n^2) * n^2 * \ln(x) * a * b^2 * c \\ & + 1 / (4 * a * c^4 * n^2 - b^2 * c^3 * n^2) * n^2 * \ln(x) * b^4 - 2/c / (4 * a * c - b^2) / n * \ln(x^n + 1/2 * (3 * a * b^2 * c - b^4 + (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2})) / c / b / (3 * a * c - b^2) * a^2 + 5/2/c^2 / (4 * a * c - b^2) / n * \ln(x^n + 1/2 * (3 * a * b^2 * c - b^4 + (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2})) / c / b / (3 * a * c - b^2) * a * b^2 - 1/2/c^3 / (4 * a * c - b^2) / n * \ln(x^n + 1/2 * (3 * a * b^2 * c - b^4 + (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2})) / c / b / (3 * a * c - b^2) * b^4 + 1/2/c^3 / (4 * a * c - b^2) / n * \ln(x^n + 1/2 * (3 * a * b^2 * c - b^4 + (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2})) / c / b / (3 * a * c - b^2) * (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2} - 2/c / (4 * a * c - b^2) / n * \ln(x^n - 1/2 * (-3 * a * b^2 * c + b^4 + (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2})) / c / b / (3 * a * c - b^2) * a^2 + 5/2/c^2 / (4 * a * c - b^2) / n * \ln(x^n - 1/2 * (-3 * a * b^2 * c + b^4 + (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2})) / c / b / (3 * a * c - b^2) * a * b^2 - 1/2/c^3 / (4 * a * c - b^2) / n * \ln(x^n - 1/2 * (-3 * a * b^2 * c + b^4 + (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2})) / c / b / (3 * a * c - b^2) * b^4 - 1/2/c^3 / (4 * a * c - b^2) / n * \ln(x^n - 1/2 * (-3 * a * b^2 * c + b^4 + (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2})) / c / b / (3 * a * c - b^2) * (-36 * a^3 * b^2 * c^3 + 33 * a^2 * b^4 * c^2 - 10 * a * b^6 * c + b^8)^{1/2} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(b^2 - ac) \log(x)}{c^3} + \frac{cx^{2n} - 2bx^n}{2c^2n} + \int -\frac{ab^2 - a^2c + (b^3 - 2abc)x^n}{c^4xx^{2n} + bc^3xx^n + ac^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] $(b^2 - a*c)*\log(x)/c^3 + 1/2*(c*x^{(2*n)} - 2*b*x^n)/(c^{2*n}) + \text{integrate}(-(a*b^2 - a^2*c + (b^3 - 2*a*b*c)*x^n)/(c^4*x*x^{(2*n)} + b*c^3*x*x^n + a*c^3*x), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4n-1}}{a + b x^n + c x^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{(4*n - 1)}/(a + b*x^n + c*x^{(2*n)}), x)$

[Out] $\text{int}(x^{(4*n - 1)}/(a + b*x^n + c*x^{(2*n)}), x)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{*(-1+4*n)}/(a+b*x^{*n}+c*x^{*(2*n)}), x)$

[Out] Timed out

$$3.474 \quad \int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=87

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}} - \frac{b \log(a + bx^n + cx^{2n})}{2c^2n} + \frac{x^n}{cn}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1357, 703, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}} - \frac{b \log(a + bx^n + cx^{2n})}{2c^2n} + \frac{x^n}{cn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] x^n/(c*n) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2*n)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+3n}}{a + bx^n + cx^{2n}} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2} dx, x, x^n\right)}{n} \\ &= \frac{x^n}{cn} + \frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2} dx, x, x^n\right)}{cn} \\ &= \frac{x^n}{cn} - \frac{b \text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^n\right)}{2c^2n} + \frac{(b^2 - 2ac) \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^n\right)}{2c^2n} \\ &= \frac{x^n}{cn} - \frac{b \log(a + bx^n + cx^{2n})}{2c^2n} - \frac{(b^2 - 2ac) \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^n\right)}{c^2n} \\ &= \frac{x^n}{cn} - \frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}n} - \frac{b \log(a + bx^n + cx^{2n})}{2c^2n} \end{aligned}$$

Mathematica [A] time = 0.13, size = 80, normalized size = 0.92

$$\frac{\frac{(b^2-2ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} - \frac{b \log(a+x^n(b+cx^n))}{2c} + x^n}{cn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] (x^n - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) - (b*Log[a + x^n*(b + c*x^n)])/(2*c))/(c*n)

IntegrateAlgebraic [A] time = 0.16, size = 104, normalized size = 1.20

$$\frac{(b^2 - 2ac) \tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}}\right)}{c^2n\sqrt{4ac-b^2}} - \frac{b \log(a + bx^n + cx^{2n})}{2c^2n} + \frac{x^n}{cn}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] x^n/(c*n) + ((b^2 - 2*a*c)*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2*n)

fricas [A] time = 1.08, size = 285, normalized size = 3.28

$$\left[\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^{2n} + b^2 - 2ac + 2\left(\frac{bc + \sqrt{b^2 - 4ac}}{c^2n + bx^n + a}\right)^n + \sqrt{b^2 - 4ac}b}{c^2n + bx^n + a}\right) - 2(b^2c - 4ac^2)x^n + (b^3 - 4abc) \log(cx^{2n} + bx^n + a)}{2(b^2c^2 - 4ac^3)n}, \frac{2(b^2 - 2ac)\sqrt{-b^2 + 4ac} \arctan\left(\frac{2\sqrt{-b^2 + 4ac}cx^n + \sqrt{-b^2 + 4ac}b}{b^2 - 4ac}\right) - 2(b^2c - 4ac^2)x^n + (b^3 - 4abc) \log(cx^{2n} + bx^n + a)}{2(b^2c^2 - 4ac^3)n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) - 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^2 - 4*a*c^3)*n), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan((-2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^2 - 4*a*c^3)*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)), x, algorithm="giac")

[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [B] time = 0.13, size = 664, normalized size = 7.63

$$\frac{4bc^2 \ln(x)}{4a^2c^2 - b^2c^2} + \frac{b^2 \ln(x)}{4a^2c^2 - b^2c^2} - \frac{2ab \ln\left(\frac{c - \sqrt{4ac - b^2} + \sqrt{4ac - b^2} + \sqrt{4ac - b^2}}{2(4ac - b^2)}\right)}{(4ac - b^2)c} + \frac{2ab \ln\left(\frac{c + \sqrt{4ac - b^2} + \sqrt{4ac - b^2} + \sqrt{4ac - b^2}}{2(4ac - b^2)}\right)}{(4ac - b^2)c} + \frac{b^2 \ln\left(\frac{c - \sqrt{4ac - b^2} + \sqrt{4ac - b^2} + \sqrt{4ac - b^2}}{2(4ac - b^2)}\right)}{2(4ac - b^2)c} + \frac{b^2 \ln\left(\frac{c + \sqrt{4ac - b^2} + \sqrt{4ac - b^2} + \sqrt{4ac - b^2}}{2(4ac - b^2)}\right)}{2(4ac - b^2)c} + \frac{3 \ln(x)}{c^2} + \frac{\sqrt{4ac - b^2} + 20b^2c^2 - 8b^2ac + b^2 \ln\left(\frac{c - \sqrt{4ac - b^2} + \sqrt{4ac - b^2} + \sqrt{4ac - b^2}}{2(4ac - b^2)}\right)}{2(4ac - b^2)c} + \frac{\sqrt{4ac - b^2} + 20b^2c^2 - 8b^2ac + b^2 \ln\left(\frac{c + \sqrt{4ac - b^2} + \sqrt{4ac - b^2} + \sqrt{4ac - b^2}}{2(4ac - b^2)}\right)}{2(4ac - b^2)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n-1)/(a+b*x^n+c*x^(2*n)),x)

[Out]
$$-b/c^2 \ln(x) + x^n/c/n + 4/(4*a*c^3*n^2 - b^2*c^2*n^2)*n^2*\ln(x)*a*b*c - 1/(4*a*c^3*n^2 - b^2*c^2*n^2)*n^2*\ln(x)*b^3 - 2/c/(4*a*c - b^2)/n*\ln(x^{n-1/2}*(-2*a*b*c + b^3 + (-16*a^3*c^3 + 20*a^2*b^2*c^2 - 8*a*b^4*c + b^6)^{(1/2)}))/c/(2*a*c - b^2))*a*b + 1/2/c^2/(4*a*c - b^2)/n*\ln(x^{n-1/2}*(-2*a*b*c + b^3 + (-16*a^3*c^3 + 20*a^2*b^2*c^2 - 8*a*b^4*c + b^6)^{(1/2)}))/c/(2*a*c - b^2))*b^3 + 1/2/c^2/(4*a*c - b^2)/n*\ln(x^{n-1/2}*(-2*a*b*c + b^3 + (-16*a^3*c^3 + 20*a^2*b^2*c^2 - 8*a*b^4*c + b^6)^{(1/2)}))/c/(2*a*c - b^2))*(-16*a^3*c^3 + 20*a^2*b^2*c^2 - 8*a*b^4*c + b^6)^{(1/2)} - 2/c/(4*a*c - b^2)/n*\ln(x^{n+1/2}*(2*a*b*c - b^3 + (-16*a^3*c^3 + 20*a^2*b^2*c^2 - 8*a*b^4*c + b^6)^{(1/2)}))/c/(2*a*c - b^2))*a*b + 1/2/c^2/(4*a*c - b^2)/n*\ln(x^{n+1/2}*(2*a*b*c - b^3 + (-16*a^3*c^3 + 20*a^2*b^2*c^2 - 8*a*b^4*c + b^6)^{(1/2)}))/c/(2*a*c - b^2))*b^3 - 1/2/c^2/(4*a*c - b^2)/n*\ln(x^{n+1/2}*(2*a*b*c - b^3 + (-16*a^3*c^3 + 20*a^2*b^2*c^2 - 8*a*b^4*c + b^6)^{(1/2)}))/c/(2*a*c - b^2))*(-16*a^3*c^3 + 20*a^2*b^2*c^2 - 8*a*b^4*c + b^6)^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b \log(x)}{c^2} + \frac{x^n}{cn} - \int -\frac{ab + (b^2 - ac)x^n}{c^3xx^{2n} + bc^2xx^n + ac^2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out]
$$-b*\log(x)/c^2 + x^n/(c*n) - \text{integrate}(-(a*b + (b^2 - a*c)*x^n)/(c^3*x*x^(2*n) + b*c^2*x*x^n + a*c^2*x), x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)),x)

[Out] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

$$3.475 \quad \int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=68

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}} + \frac{\log(a+bx^n+cx^{2n})}{2cn}$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1357, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}} + \frac{\log(a+bx^n+cx^{2n})}{2cn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]

[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]]/(c*Sqrt[b^2 - 4*a*c]*n) + Log[a + b*x^n + c*x^(2*n)]/(2*c*n)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1357

$\text{Int}[(x_)^{(m_.)*((a_) + (c_.)*(x_)^{(n2_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}], x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+2n}}{a + bx^n + cx^{2n}} dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^n\right)}{2cn} - \frac{b \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^n\right)}{2cn} \\ &= \frac{\log(a + bx^n + cx^{2n})}{2cn} + \frac{b \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^n\right)}{cn} \\ &= \frac{b \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}n} + \frac{\log(a + bx^n + cx^{2n})}{2cn} \end{aligned}$$

Mathematica [A] time = 0.07, size = 62, normalized size = 0.91

$$\frac{\frac{2b \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + \log(a + x^n (b + cx^n))}{2cn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] ((2*b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] + Log[a + x^n*(b + c*x^n)])/(2*c*n)

IntegrateAlgebraic [A] time = 0.12, size = 87, normalized size = 1.28

$$\frac{\log(a + bx^n + cx^{2n})}{2cn} - \frac{b \tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}}\right)}{cn\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] -((b*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(c*Sqrt[-b^2 + 4*a*c]*n)) + Log[a + b*x^n + c*x^(2*n)]/(2*c*n)

fricas [A] time = 1.58, size = 231, normalized size = 3.40

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2 x^{2n} + b^2 - 2ac + 2(bc + \sqrt{b^2 - 4ac}c)x^n + \sqrt{b^2 - 4ac}b}{cx^{2n} + bx^n + a}\right) + (b^2 - 4ac) \log(cx^{2n} + bx^n + a)}{2(b^2c - 4ac^2)n}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(\frac{2\sqrt{-b^2 + 4ac}cx^n + \sqrt{-b^2 + 4ac}b}{b^2 - 4ac}\right) + (b^2 - 4ac) \log(cx^{2n} + bx^n + a)}{2(b^2c - 4ac^2)n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a)/((b^2*c - 4*a*c^2)*n), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a)/((b^2*c - 4*a*c^2)*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)), x, algorithm="giac")

[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [B] time = 0.10, size = 402, normalized size = 5.91

$$\frac{4ac^2n^2 \ln(x)}{4ac^2n^2 - b^2cn^2} + \frac{b^2n^2 \ln(x)}{4ac^2n^2 - b^2cn^2} + \frac{2a \ln\left(x^n - \frac{-b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{(4ac - b^2)n} + \frac{2a \ln\left(x^n + \frac{b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{(4ac - b^2)n} - \frac{b^2 \ln\left(x^n - \frac{-b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{2(4ac - b^2)cn} - \frac{b^2 \ln\left(x^n + \frac{b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{2(4ac - b^2)cn} + \frac{\ln(x)}{c} + \frac{\sqrt{-4ab^2c + b^4} \ln\left(x^n - \frac{-b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{2(4ac - b^2)cn} - \frac{\sqrt{-4ab^2c + b^4} \ln\left(x^n + \frac{b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{2(4ac - b^2)cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)/(a+b*x^n+c*x^(2*n)), x)

[Out] 1/c*ln(x)-4/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*a*c+1/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*b^2+2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2)))/b/c)*a-1/2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2)))/b/c)*b^2+1/2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2)))/b/c)*(-4*a*b^2*c+b^4)^(1/2)+2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2)))/b/c)*a

$$-1/2/(4*a*c-b^2)/c/n*\ln(x^{n+1/2}*(b^2+(-4*a*b^2*c+b^4)^{(1/2)})/b/c)*b^2-1/2/(4*a*c-b^2)/c/n*\ln(x^{n+1/2}*(b^2+(-4*a*b^2*c+b^4)^{(1/2)})/b/c)*(-4*a*b^2*c+b^4)^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log(x)}{c} - \int \frac{bx^n + a}{c^2xx^{2n} + bcxx^n + acx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*xⁿ+c*x^(2*n)), x, algorithm="maxima")

[Out] log(x)/c - integrate((b*xⁿ + a)/(c²*x*x^(2*n) + b*c*x*xⁿ + a*c*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n - 1)/(a + b*xⁿ + c*x^(2*n)), x)

[Out] int(x^(2*n - 1)/(a + b*xⁿ + c*x^(2*n)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)/(a+b*xⁿ+c*x^(2*n)), x)

[Out] Timed out

$$3.476 \quad \int \frac{x^{-1+n}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=39

$$\frac{2 \tanh^{-1} \left(\frac{b+2cx^n}{\sqrt{b^2-4ac}} \right)}{n\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1352, 618, 206}

$$\frac{2 \tanh^{-1} \left(\frac{b+2cx^n}{\sqrt{b^2-4ac}} \right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]

[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\int \frac{x^{-1+n}}{a + bx^n + cx^{2n}} dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^n\right)}{n}$$

$$= -\frac{2 \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^n\right)}{n}$$

$$= -\frac{2 \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} n}$$

Mathematica [A] time = 0.06, size = 39, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)/(a + b*x^n + c*x^(2*n)), x]

[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)

IntegrateAlgebraic [A] time = 0.08, size = 57, normalized size = 1.46

$$\frac{2 \tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}}\right)}{n\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + n)/(a + b*x^n + c*x^(2*n)), x]

[Out] (2*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*n)

fricas [B] time = 1.22, size = 159, normalized size = 4.08

$$\left[\frac{\log\left(\frac{2c^2x^{2n}+b^2-2ac+2(bc-\sqrt{b^2-4ac})x^n-\sqrt{b^2-4ac}b}{cx^{2n}+bx^n+a}\right)}{\sqrt{b^2-4ac}n}, -\frac{2\sqrt{-b^2+4ac} \arctan\left(-\frac{2\sqrt{-b^2+4ac}cx^n+\sqrt{-b^2+4ac}b}{b^2-4ac}\right)}{(b^2-4ac)n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{(-1+n)/(a+b*xⁿ+c*x^(2*n))),x, algorithm="fricas")}

[Out] [log((2*c²*x^(2*n) + b² - 2*a*c + 2*(b*c - sqrt(b² - 4*a*c))*c)*xⁿ - sqrt(b² - 4*a*c)*b)/(c*x^(2*n) + b*xⁿ + a))/(sqrt(b² - 4*a*c)*n), -2*sqrt(-b² + 4*a*c)*arctan(-(2*sqrt(-b² + 4*a*c))*c*xⁿ + sqrt(-b² + 4*a*c)*b)/(b² - 4*a*c))/((b² - 4*a*c)*n)]

giac [A] time = 0.41, size = 39, normalized size = 1.00

$$\frac{2 \arctan\left(\frac{2cx^n+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{(-1+n)/(a+b*xⁿ+c*x^(2*n))),x, algorithm="giac")}

[Out] 2*arctan((2*c*xⁿ + b)/sqrt(-b² + 4*a*c))/(sqrt(-b² + 4*a*c)*n)

maple [B] time = 0.06, size = 113, normalized size = 2.90

$$-\frac{\ln\left(x^n + \frac{-4ac+b^2+\sqrt{-4ac+b^2}b}{2\sqrt{-4ac+b^2}c}\right)}{\sqrt{-4ac+b^2}n} + \frac{\ln\left(x^n + \frac{4ac-b^2+\sqrt{-4ac+b^2}b}{2\sqrt{-4ac+b^2}c}\right)}{\sqrt{-4ac+b^2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^{(n-1)/(a+b*xⁿ+c*x^(2*n))),x)}

[Out] -1/(-4*a*c+b²)^(1/2)/n*ln(x^{n+1/2}*((-4*a*c+b²)^(1/2)*b-4*a*c+b²)/c/(-4*a*c+b²)^(1/2))+1/(-4*a*c+b²)^(1/2)/n*ln(x^{n+1/2}*((-4*a*c+b²)^(1/2)*b+4*a*c-b²)/c/(-4*a*c+b²)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{(-1+n)/(a+b*xⁿ+c*x^(2*n))),x, algorithm="maxima")}

[Out] integrate(x^{(n - 1)/(c*x^(2*n) + b*xⁿ + a)}, x)

mupad [B] time = 1.47, size = 39, normalized size = 1.00

$$\frac{2 \operatorname{atan}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{n\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

```
[Out] (2*atan((b + 2*c*x^n)/(4*a*c - b^2)^(1/2)))/(n*(4*a*c - b^2)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)), x)
```

```
[Out] Timed out
```

$$3.477 \quad \int \frac{x^{-1-n}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=98

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2 n \sqrt{b^2 - 4ac}} + \frac{b \log(a + bx^n + cx^{2n})}{2a^2 n} - \frac{b \log(x)}{a^2} - \frac{x^{-n}}{an}$$

Rubi [A] time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2 n \sqrt{b^2 - 4ac}} + \frac{b \log(a + bx^n + cx^{2n})}{2a^2 n} - \frac{b \log(x)}{a^2} - \frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]

[Out] -(1/(a*n*x^n)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[x])/a^2 + (b*Log[a + b*x^n + c*x^(2*n)])/(2*a^2*n)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)(a + bx + cx^2)}{(a + bx + cx^2)^2}, x \int ; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2cd - b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 709

$\text{Int}[\frac{(d + ex)^m}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Simp}[e(d + ex)^{m+1}/((m+1)(cd^2 - bde + ae^2)), x] + \text{Dist}[1/(cd^2 - bde + ae^2), \text{Int}[\frac{(d + ex)^{m+1} \text{Simp}[cd - b^2e - c^2ex, x]}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{NeQ}[2cd - b^2e, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 800

$\text{Int}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 1357

$\text{Int}[(x^m)^n (a + cx^{n2} + b)^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)(a + bx + cx^2)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, m, n, p, x\} \ \&\& \ \text{EqQ}[n2, 2n] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-n}}{a + bx^n + cx^{2n}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)} dx, x, x^n\right)}{n} \\
&= -\frac{x^{-n}}{an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)} dx, x, x^n\right)}{an} \\
&= -\frac{x^{-n}}{an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right) dx, x, x^n\right)}{an} \\
&= -\frac{x^{-n}}{an} - \frac{b \log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx, x, x^n\right)}{a^2n} \\
&= -\frac{x^{-n}}{an} - \frac{b \log(x)}{a^2} + \frac{b \text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^n\right)}{2a^2n} + \frac{(b^2-2ac) \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^n\right)}{2a^2n} \\
&= -\frac{x^{-n}}{an} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx^n + cx^{2n})}{2a^2n} - \frac{(b^2-2ac) \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^n\right)}{a^2n} \\
&= -\frac{x^{-n}}{an} - \frac{(b^2-2ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}n} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx^n + cx^{2n})}{2a^2n}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 135, normalized size = 1.38

$$\frac{-\frac{4c^2 \log\left(x^{-n}\left(b-\sqrt{b^2-4ac}\right)+2c\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^2} + \frac{4c^2 \log\left(x^{-n}\left(\sqrt{b^2-4ac}+b\right)+2c\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^2} + \frac{x^{-n}}{a}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n)/(a + b*x^n + c*x^(2*n)), x]

[Out] -((1/(a*x^n) - (4*c^2*Log[2*c + (b - Sqrt[b^2 - 4*a*c])/x^n])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^2) + (4*c^2*Log[2*c + (b + Sqrt[b^2 - 4*a*c])/x^n])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^2))/n)

IntegrateAlgebraic [A] time = 0.19, size = 120, normalized size = 1.22

$$\frac{(b^2-2ac) \tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}}\right)}{a^2n\sqrt{4ac-b^2}} + \frac{b \log(a + bx^n + cx^{2n})}{2a^2n} - \frac{b \log(x^n)}{a^2n} - \frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]

[Out] $-(1/(a*n*x^n)) + ((b^2 - 2*a*c)*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(a^2*sqrt[-b^2 + 4*a*c]*n) - (b*Log[x^n])/(a^2*n) + (b*Log[a + b*x^n + c*x^(2*n)])/(2*a^2*n)$

fricas [A] time = 1.41, size = 333, normalized size = 3.40

$$\frac{2(b^3 - 4abc)nx^{\log(x)} + (b^2 - 2ac)\sqrt{b^2 - 4ac}x^{\log\left(\frac{x^2 + \sqrt{b^2 - 4ac}}{x^2 - \sqrt{b^2 - 4ac}}\right)} + 2ab^2 - 8a^2c - (b^3 - 4abc)x^{\log(cx^n + bx^n + a)}}{2(a^2b^2 - 4a^2c)nx^n} + \frac{2(b^3 - 4abc)nx^{\log(x)} + 2(b^2 - 2ac)\sqrt{-b^2 + 4ac}x^n \arctan\left(\frac{2\sqrt{-b^2 + 4ac}(cx^n + \sqrt{-b^2 + 4ac})}{b^2 - 4ac}\right) + 2ab^2 - 8a^2c - (b^3 - 4abc)x^{\log(cx^{2n} + bx^n + a)}}{2(a^2b^2 - 4a^2c)nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] $[-1/2*(2*(b^3 - 4*a*b*c)*n*x^n*\log(x) + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^n*\log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x^n*\log(c*x^(2*n) + b*x^n + a))/((a^2*b^2 - 4*a^3*c)*n*x^n), -1/2*(2*(b^3 - 4*a*b*c)*n*x^n*\log(x) + 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^n*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x^n*\log(c*x^(2*n) + b*x^n + a))/((a^2*b^2 - 4*a^3*c)*n*x^n)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [B] time = 0.15, size = 658, normalized size = 6.71

$$\frac{4abc^2 \ln(x)}{4a^2c^2 - a^2b^2} - \frac{b^2 \ln(x)}{4a^2c^2 - a^2b^2} + \frac{2bc \ln\left(x - \frac{2abx^n + \sqrt{4a^2c^2 - b^2x^{2n}}}{4ac - b^2}\right)}{(4ac - b^2)an} + \frac{2bc \ln\left(x + \frac{2abx^n + \sqrt{4a^2c^2 - b^2x^{2n}}}{4ac - b^2}\right)}{(4ac - b^2)an} - \frac{b^2 \ln\left(x - \frac{2abx^n + \sqrt{4a^2c^2 - b^2x^{2n}}}{4ac - b^2}\right)}{2(4ac - b^2)a} - \frac{b^2 \ln\left(x + \frac{2abx^n + \sqrt{4a^2c^2 - b^2x^{2n}}}{4ac - b^2}\right)}{2(4ac - b^2)a} - \frac{x^{-n}}{an} + \frac{\sqrt{-36ac^2 + 20b^2c^2 - 8b^2ac + b^4} \ln\left(x - \frac{2abx^n + \sqrt{4a^2c^2 - b^2x^{2n}}}{4ac - b^2}\right)}{2(4ac - b^2)a} - \frac{\sqrt{-36ac^2 + 20b^2c^2 - 8b^2ac + b^4} \ln\left(x + \frac{2abx^n + \sqrt{4a^2c^2 - b^2x^{2n}}}{4ac - b^2}\right)}{2(4ac - b^2)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x)

[Out] $-1/a/n/(x^n) - 4/(4*a^3*c*n^2 - a^2*b^2*n^2)*n^2*\ln(x)*a*b*c + 1/(4*a^3*c*n^2 - a^2*b^2*n^2)*n^2*\ln(x)*b^3 + 2/a/(4*a*c - b^2)/n*\ln(x^n - 1/2*(-2*a*b*c + b^3 + (-16*a^3*c^3 + 20*a^2*b^2*c^2 - 8*a*b^4*c + b^6)^(1/2)))/(2*a*c - b^2)/c)*b*c - 1/2/a^2/(4*a*c$

$$-b^2)/n*\ln(x^{n-1/2}*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^{1/2}))/((2*a*c-b^2)/c)*b^3+1/2/a^2/(4*a*c-b^2)/n*\ln(x^{n-1/2}*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^{1/2}))/((2*a*c-b^2)/c)*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^{1/2}+2/a/(4*a*c-b^2)/n*\ln(x^{n+1/2}*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^{1/2}))/((2*a*c-b^2)/c)*b*c-1/2/a^2/(4*a*c-b^2)/n*\ln(x^{n+1/2}*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^{1/2}))/((2*a*c-b^2)/c)*b^3-1/2/a^2/(4*a*c-b^2)/n*\ln(x^{n+1/2}*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^{1/2}))/((2*a*c-b^2)/c)*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^{1/2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{ax^n} - \int \frac{cx^n + b}{acxx^{2n} + abxx^n + a^2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] -1/(a*n*x^n) - integrate((c*x^n + b)/(a*c*x*x^(2*n) + a*b*x*x^n + a^2*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{n+1} (a + b x^n + c x^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))),x)

[Out] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out

$$3.478 \quad \int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=126

$$\frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3 n \sqrt{b^2 - 4ac}} - \frac{(b^2 - ac) \log(a + bx^n + cx^{2n})}{2a^3 n} + \frac{\log(x)(b^2 - ac)}{a^3} + \frac{bx^{-n}}{a^2 n} - \frac{x^{-2n}}{2an}$$

Rubi [A] time = 0.17, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - ac) \log(a + bx^n + cx^{2n})}{2a^3 n} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3 n \sqrt{b^2 - 4ac}} + \frac{\log(x)(b^2 - ac)}{a^3} + \frac{bx^{-n}}{a^2 n} - \frac{x^{-2n}}{2an}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] -1/(2*a*n*x^(2*n)) + b/(a^2*n*x^n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^3*n)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)/(a + bx + cx^2)}{x} dx$; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 709

$\int \frac{(d + ex)^m}{(a + bx + cx^2)} dx$:> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

$\int \frac{(d + ex)^m (f + gx)}{a + bx + cx^2} dx$:> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1357

$\int (x^m) * ((a + c*x^{n2}) + (b*x^n)^p) dx$:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-2n}}{a + bx^n + cx^{2n}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx+cx^2)} dx, x, x^n\right)}{n} \\
&= -\frac{x^{-2n}}{2an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^2(a+bx+cx^2)} dx, x, x^n\right)}{an} \\
&= -\frac{x^{-2n}}{2an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right) dx, x, x^n\right)}{an} \\
&= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\text{Subst}\left(\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2} dx, x, x^n\right)}{a^3n} \\
&= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac))\text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^n\right)}{2a^3n} - \frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^3n} \\
&= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^3n} + \frac{(b(b^2-3ac))\text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^n\right)}{2a^3n} \\
&= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx^n+cx^{2n})}{2a^3n}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 112, normalized size = 0.89

$$\frac{-a^2x^{-2n} - (b^2 - ac)\log(a + x^n(b + cx^n)) + \frac{2b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + 2n\log(x)(b^2 - ac) + 2abx^{-n}}{2a^3n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] $(-a^2/x^{(2*n)}) + (2*a*b)/x^n + (2*b*(b^2 - 3*a*c))*\text{ArcTanh}[(b + 2*c*x^n)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[b^2 - 4*a*c] + 2*(b^2 - a*c)*n*\text{Log}[x] - (b^2 - a*c)*\text{Log}[a + x^n*(b + c*x^n)]/(2*a^3*n)$

IntegrateAlgebraic [A] time = 0.33, size = 149, normalized size = 1.18

$$\frac{(b^2 - ac)\log(x^n)}{a^3n} + \frac{(ac - b^2)\log(a + bx^n + cx^{2n})}{2a^3n} + \frac{(3abc - b^3)\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}}\right)}{a^3n\sqrt{4ac-b^2}} + \frac{x^{-2n}(2bx^n - a)}{2a^2n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]

[Out] $(-a + 2*b*x^n)/(2*a^2*n*x^(2*n)) + ((-b^3 + 3*a*b*c)*\text{ArcTan}[b/\text{Sqrt}[-b^2 + 4*a*c] + (2*c*x^n)/\text{Sqrt}[-b^2 + 4*a*c]])/(a^3*\text{Sqrt}[-b^2 + 4*a*c]*n) + ((b^2 - a*c)*\text{Log}[x^n])/(a^3*n) + ((-b^2 + a*c)*\text{Log}[a + b*x^n + c*x^(2*n)])/(2*a^3*n)$

fricas [A] time = 1.41, size = 429, normalized size = 3.40

$$\frac{\frac{a^2 b^2 - 4 a^2 c^2 - 2 (b^4 - 5 a b^2 c + 4 a^2 c^2) \log(x) + (b^4 - 3 a b^2) \sqrt{b^2 - 4 a c} \log\left(\frac{3 a^2 b^2 - 2 a c (b^2 - \sqrt{b^2 - 4 a c}) + c^2 \sqrt{b^2 - 4 a c}}{2 (a^2 b^2 - 4 a^2 c^2) n x^{2 n}}\right) + (b^4 - 5 a b^2 c + 4 a^2 c^2) \log(x^2 + b x^n + a) - 2 (a b^3 - 4 a^2 b c) x^n}{2 (a^2 b^2 - 4 a^2 c^2) n x^{2 n}} + \frac{a^2 b^2 - 4 a^2 c^2 - 2 (b^4 - 5 a b^2 c + 4 a^2 c^2) \log(x) - 2 (b^4 - 3 a b^2) \sqrt{b^2 - 4 a c} \arctan\left(\frac{b \sqrt{b^2 - 4 a c} + c x^n}{a}\right) + (b^4 - 5 a b^2 c + 4 a^2 c^2) \log(x^2 + b x^n + a) - 2 (a b^3 - 4 a^2 b c) x^n}{2 (a^2 b^2 - 4 a^2 c^2) n x^{2 n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] $[-1/2*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*\log(x) + (b^3 - 3*a*b*c)*\text{sqrt}(b^2 - 4*a*c)*x^(2*n)*\log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*x^n - \text{sqrt}(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*n)*\log(c*x^(2*n) + b*x^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n]/((a^3*b^2 - 4*a^4*c)*n*x^(2*n)), -1/2*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*\log(x) - 2*(b^3 - 3*a*b*c)*\text{sqrt}(-b^2 + 4*a*c)*x^(2*n)*\arctan(-(2*\text{sqrt}(-b^2 + 4*a*c))*c*x^n + \text{sqrt}(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*n)*\log(c*x^(2*n) + b*x^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n]/((a^3*b^2 - 4*a^4*c)*n*x^(2*n))]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [B] time = 0.17, size = 958, normalized size = 7.60

$$\frac{a^2 b^2 - 4 a^2 c^2 - 2 (b^4 - 5 a b^2 c + 4 a^2 c^2) \log(x) + (b^4 - 3 a b^2) \sqrt{b^2 - 4 a c} \log\left(\frac{3 a^2 b^2 - 2 a c (b^2 - \sqrt{b^2 - 4 a c}) + c^2 \sqrt{b^2 - 4 a c}}{2 (a^2 b^2 - 4 a^2 c^2) n x^{2 n}}\right) + (b^4 - 5 a b^2 c + 4 a^2 c^2) \log(x^2 + b x^n + a) - 2 (a b^3 - 4 a^2 b c) x^n}{2 (a^2 b^2 - 4 a^2 c^2) n x^{2 n}} + \frac{a^2 b^2 - 4 a^2 c^2 - 2 (b^4 - 5 a b^2 c + 4 a^2 c^2) \log(x) - 2 (b^4 - 3 a b^2) \sqrt{b^2 - 4 a c} \arctan\left(\frac{b \sqrt{b^2 - 4 a c} + c x^n}{a}\right) + (b^4 - 5 a b^2 c + 4 a^2 c^2) \log(x^2 + b x^n + a) - 2 (a b^3 - 4 a^2 b c) x^n}{2 (a^2 b^2 - 4 a^2 c^2) n x^{2 n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x)


```
[Out] b/a^2/n/(x^n)-1/2/a/n/(x^n)^2-4/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a^2*c^2
+5/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a*b^2*c-1/(4*a^4*c*n^2-a^3*b^2*n^2)*
n^2*ln(x)*b^4+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+
33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)))/(3*a*c-b^2)/b/c)*c^2-5/2/a^2/(4*a*c-b
^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2)))/(3*a*c-b^2)/b/c)*b^2*c+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^
2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)))/(3*a*c-b^2)/
b/c)*b^4+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+3
3*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)))/(3*a*c-b^2)/b/c)*(-36*a^3*b^2*c^3+33*a
^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b
^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)))/(3*a*c-b^2)/b/c)*
c^2-5/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^
2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)))/(3*a*c-b^2)/b/c)*b^2*c+1/2/a^3/(4*a*c-b^2)
/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^
8)^(1/2)))/(3*a*c-b^2)/b/c)*b^4-1/2/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c
+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)))/(3*a*c-b^2)/b/c
)*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2bx^n - a}{2a^2nx^{2n}} + \int \frac{bcx^n + b^2 - ac}{a^2cx^{2n} + a^2bxx^n + a^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n - a)/(a^2*n*x^(2*n)) + integrate((b*c*x^n + b^2 - a*c)/(a^2*c*
x*x^(2*n) + a^2*b*x*x^n + a^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2n+1} (a + bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

$$3.479 \quad \int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=164

$$\frac{b(b^2 - 2ac) \log(a + bx^n + cx^{2n})}{2a^4n} - \frac{b \log(x)(b^2 - 2ac)}{a^4} - \frac{x^{-n}(b^2 - ac)}{a^3n} + \frac{bx^{-2n}}{2a^2n} - \frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1357, 709, 800, 634, 618, 206, 628}

$$\frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}} - \frac{x^{-n}(b^2 - ac)}{a^3n} + \frac{b(b^2 - 2ac) \log(a + bx^n + cx^{2n})}{2a^4n} - \frac{b \log(x)(b^2 - 2ac)}{a^4} + \frac{bx^{-2n}}{2a^2n} - \frac{x^{-3n}}{3an}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] -1/(3*a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - (b^2 - a*c)/(a^3*n*x^n) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]*n) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^4*n)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)(a + bx + cx^2)}{(a + bx + cx^2)^2}, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 709

$\text{Int}[\frac{(d + e)x^m}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Simp}[\frac{e(d + ex)^{m+1}}{(m+1)(cd^2 - bde + ae^2)}, x] + \text{Dist}[\frac{1}{cd^2 - bde + ae^2}, \text{Int}[\frac{(d + ex)^{m+1} \text{Simp}[cd - be - cex, x]}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 800

$\text{Int}[\frac{(d + e)x^m (f + gx)}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 1357

$\text{Int}[x^m (a + cx^{n2}) + (bx)^n]^p, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{\text{Simplify}[(m+1)/n] - 1} (a + bx + cx^2)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, m, n, p, x\} \ \&\& \ \text{EqQ}[n2, 2n] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx+cx^2)} dx, x, x^n\right)}{n} \\
&= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^3(a+bx+cx^2)} dx, x, x^n\right)}{an} \\
&= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right) dx, x, x^n\right)}{an} \\
&= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{\text{Subst}\left(\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2} dx, x, x^n\right)}{a^4n} \\
&= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{(b(b^2-2ac))\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^n\right)}{2a^4n} \\
&= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log(a+bx^n+cx^{2n})}{2a^4n} \\
&= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)x^{-n}}{a^3n} - \frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}n} - \frac{b(b^2-2ac)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 143, normalized size = 0.87

$$\frac{-2a^3x^{-3n} - \frac{6(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} + 3a^2bx^{-2n} + 6ax^{-n}(ac-b^2) + 3b(b^2-2ac)\log(a+x^n(b+cx^n)) - 6bn\log(x)(b^2-2ac)}{6a^4n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)), x]

[Out] ((-2*a^3)/x^(3*n) + (3*a^2*b)/x^(2*n) + (6*a*(-b^2 + a*c))/x^n - (6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] - 6*b*(b^2 - 2*a*c)*n*Log[x] + 3*b*(b^2 - 2*a*c)*Log[a + x^n*(b + c*x^n)]/(6*a^4*n)

IntegrateAlgebraic [A] time = 0.32, size = 182, normalized size = 1.11

$$\frac{(2abc-b^3)\log(x^n)}{a^4n} + \frac{(b^3-2abc)\log(a+bx^n+cx^{2n})}{2a^4n} + \frac{(2a^2c^2-4ab^2c+b^4)\tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}}\right)}{a^4n\sqrt{4ac-b^2}} + \frac{x^{-3n}(-2a^2+3abx^n+6acx^{2n}-6b^2x^{2n})}{6a^3n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]

[Out] $(-2*a^2 + 3*a*b*x^n - 6*b^2*x^{2*n} + 6*a*c*x^{2*n})/(6*a^3*n*x^{3*n}) + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*\text{ArcTan}[b/\text{Sqrt}[-b^2 + 4*a*c] + (2*c*x^n)/\text{Sqrt}[-b^2 + 4*a*c]])/(a^4*\text{Sqrt}[-b^2 + 4*a*c]*n) + ((-b^3 + 2*a*b*c)*\text{Log}[x^n])/(a^4*n) + ((b^3 - 2*a*b*c)*\text{Log}[a + b*x^n + c*x^{2*n}])/(2*a^4*n)$

fricas [A] time = 1.36, size = 522, normalized size = 3.18

$$\frac{2*a^2*b^4 - 4*a^2*c^2 + 6*a*b^2*c^2 - 6*a*b^2*c^2*\log(-b^2 + 4*a*c) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*\sqrt{-b^2 + 4*a*c}*\log\left(\frac{b + \sqrt{-b^2 + 4*a*c}}{2}\right) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*\log\left(\frac{b - \sqrt{-b^2 + 4*a*c}}{2}\right) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*\log\left(\frac{b + \sqrt{-b^2 + 4*a*c}}{2}\right) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*\log\left(\frac{b - \sqrt{-b^2 + 4*a*c}}{2}\right) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*\log\left(\frac{b + \sqrt{-b^2 + 4*a*c}}{2}\right) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*\log\left(\frac{b - \sqrt{-b^2 + 4*a*c}}{2}\right)}{6*a^4*n*\sqrt{-b^2 + 4*a*c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] $[-1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^{3*n})*\log(x) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*\sqrt{b^2 - 4*a*c}*x^{3*n}*\log((2*c^2*x^{2*n} + b^2 - 2*a*c + 2*(b*c - \sqrt{b^2 - 4*a*c})*c)*x^n - \sqrt{b^2 - 4*a*c}*b)/(c*x^{2*n} + b*x^n + a) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^{3*n}*\log(c*x^{2*n} + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^{2*n} - 3*(a^2*b^3 - 4*a^3*b*c)*x^n]/((a^4*b^2 - 4*a^5*c)*n*x^{3*n}), -1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^{3*n})*\log(x) + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*\sqrt{-b^2 + 4*a*c}*x^{3*n}*\arctan(-(2*\sqrt{-b^2 + 4*a*c})*c*x^n + \sqrt{-b^2 + 4*a*c}*b)/(b^2 - 4*a*c) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^{3*n}*\log(c*x^{2*n} + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^{2*n} - 3*(a^2*b^3 - 4*a^3*b*c)*x^n]/((a^4*b^2 - 4*a^5*c)*n*x^{3*n})]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [B] time = 0.20, size = 1300, normalized size = 7.93

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x)

```
[Out] 1/a^2/n/(x^n)*c-1/a^3/n/(x^n)*b^2+1/2/a^2*b/n/(x^n)^2-1/3/a/n/(x^n)^3+8/(4*
a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a^2*b*c^2-6/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*
ln(x)*a*b^3*c+1/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*b^5-4/a^2/(4*a*c-b^2)/n
*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b
^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*
b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^
5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)))/c/(2
*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^
2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-1
2*a*b^8*c+b^10)^(1/2)))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^5+1/2/a^4/(4*a*c-b^2)
/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3
*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)))/c/(2*a^2*c^2-4*a*b^2*c+b^4)
)*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10
)^(1/2)-4/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5
*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)))/c
/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*
c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2
-12*a*b^8*c+b^10)^(1/2)))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-
b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-9
6*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)))/c/(2*a^2*c^2-4*a*b^2*c
+b^4))*b^5-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-1
6*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/
2)))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3
+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3 abx^n - 2a^2 - 6(b^2 - ac)x^{2n}}{6a^3nx^{3n}} + \int -\frac{b^3 - 2abc + (b^2c - ac^2)x^n}{a^3cx^{2n} + a^3bxx^n + a^4x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*x^n - 2*a^2 - 6*(b^2 - a*c)*x^(2*n))/(a^3*n*x^(3*n)) + integrate
(-(b^3 - 2*a*b*c + (b^2*c - a*c^2)*x^n)/(a^3*c*x*x^(2*n) + a^3*b*x*x^n + a^
4*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3n+1} (a + bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))), x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```


$$3.480 \quad \int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=353

$$\frac{2^{3/4} c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{2^{3/4} c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{2^{3/4} c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{2^{3/4} c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Rubi [A] time = 0.63, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1381, 1347, 212, 208, 205}

$$\frac{2^{3/4} c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{2^{3/4} c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{2^{3/4} c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac} \left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{2^{3/4} c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)), x]

[Out] (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) + (2*2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTan[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],

$x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /; \text{FreeQ}[\{a, b\}, x] \&\& !\text{GtQ}[a/b, 0]$

Rule 1347

$\text{Int}[(a_) + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(n2_)}]^{(-1)}, x_Symbol] := \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[c/q, \text{Int}[1/(b/2 - q/2 + c*x^n), x], x] - \text{Dist}[c/q, \text{Int}[1/(b/2 + q/2 + c*x^n), x], x]] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 1381

$\text{Int}[(x_)^{(m_)}*((a_) + (c_)*(x_)^{(n2_)} + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[(a + b*x^{\text{Simplify}[n/(m + 1)]} + c*x^{\text{Simplify}[(2*n)/(m + 1)])^p, x], x, x^{(m + 1)}], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IntegerQ}[\text{Simplify}[n/(m + 1)]] \&\& !\text{IntegerQ}[n]$

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+\frac{n}{4}}}{a + bx^n + cx^{2n}} dx &= \frac{4 \text{Subst}\left(\int \frac{1}{a+bx^4+cx^8} dx, x, x^{n/4}\right)}{n} \\ &= \frac{(4c) \text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4} dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac} n} - \frac{(4c) \text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4} dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac} n} \\ &= \frac{(4c) \text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2} dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac} \sqrt{-b-\sqrt{b^2-4ac}} n} + \frac{(4c) \text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2} dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac} \sqrt{-b-\sqrt{b^2-4ac}} n} \\ &= \frac{2 \cdot 2^{3/4} c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac} \left(-b-\sqrt{b^2-4ac}\right)^{3/4} n} - \frac{2 \cdot 2^{3/4} c^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac} \left(-b+\sqrt{b^2-4ac}\right)^{3/4} n} + \frac{2 \cdot 2^{3/4} c^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac} \left(-b-\sqrt{b^2-4ac}\right)^{3/4} n} \end{aligned}$$

Mathematica [A] time = 0.90, size = 340, normalized size = 0.96

$$2^{2^{3/4}} c^{3/4} \left(\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{b\sqrt{b^2-4ac-4ac+b^2}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{\sqrt[4]{-\sqrt{b^2-4ac}-b} \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{b\sqrt{b^2-4ac-4ac+b^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} \right) \frac{1}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^{-1 + n/4}/(a + b*xⁿ + c*x^(2*n)), x]

[Out] (2*2^(3/4)*c^(3/4)*(-(((-b - Sqrt[b² - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4)]/(-b - Sqrt[b² - 4*a*c])^(1/4))]/(b² - 4*a*c + b*Sqrt[b² - 4*a*c])) - ArcTan[(2^(1/4)*c^(1/4)*x^(n/4)]/(-b + Sqrt[b² - 4*a*c])^(1/4)]/(Sqrt[b² - 4*a*c]*(-b + Sqrt[b² - 4*a*c])^(3/4)) - ((-b - Sqrt[b² - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4)]/(-b - Sqrt[b² - 4*a*c])^(1/4))]/(b² - 4*a*c + b*Sqrt[b² - 4*a*c]) - ArcTanh[(2^(1/4)*c^(1/4)*x^(n/4)]/(-b + Sqrt[b² - 4*a*c])^(1/4)]/(Sqrt[b² - 4*a*c]*(-b + Sqrt[b² - 4*a*c])^(3/4))))/n

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{4}}}{a + bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^{-1 + n/4}/(a + b*xⁿ + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic][x^{-1 + n/4}/(a + b*xⁿ + c*x^(2*n)), x]

fricas [B] time = 2.56, size = 4426, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{-1+1/4*n}/(a+b*xⁿ+c*x^(2*n)),x, algorithm="fricas")

[Out] -2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a³*b⁴ - 8*a⁴*b²*c + 16*a⁵*c²)*n⁴*sqrt((b⁴ - 2*a*b²*c + a²*c²)/((a⁶*b⁶ - 12*a⁷*b⁴*c + 48*a⁸*b²*c² - 64*a⁹*c³)*n⁸)) + b³ - 3*a*b*c)/((a³*b⁴ - 8*a⁴*b²*c + 16*a⁵*c²)*n⁴))*arctan(1/16*sqrt(2)*(2*sqrt(2)*((a³*b¹⁰*c - 15*a⁴*b⁸*c² + 86*a⁵*b⁶*c³ - 232*a⁶*b⁴*c⁴ + 288*a⁷*b²*c⁵ - 128*a⁸*c⁶)*n⁷*x*sqrt((b⁴ - 2*a*b²*c + a²*c²)/((a⁶*b⁶ - 12*a⁷*b⁴*c + 48*a⁸*b²*c² - 64*a⁹*c³)*n⁸)) - (b⁹*c - 10*a*b⁷*c² + 33*a²*b⁵*c³ - 40*a³*b³*c⁴ + 16*a⁴*b*c⁵)*n³*x)*x^(1/4*n - 1)*sqrt(-((a³*b⁴ - 8*a⁴*b²*c + 16*a⁵*c²)*n⁴*sqrt((b⁴ - 2*a*b²*c + a²*c²)/((a⁶*b⁶ - 12*a⁷*b⁴*c + 48*a⁸*b²*c² - 64*a⁹*c³)*n⁸))

$$\begin{aligned}
& 2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2) \\
&)*n^4)) + \text{sqrt}(2)*((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c \\
& ^3 + 128*a^7*c^4)*n^7*x*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7 \\
& *b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3 \\
& *c^2 - 16*a^3*b*c^3)*n^3*x)*\text{sqrt}((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x \\
& ^{(1/2*n - 2)} - \text{sqrt}(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6* \\
& b^3*c^3 + 64*a^7*b*c^4)*n^6*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12 \\
& *a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^8 - 8*a*b^6*c + 21*a^2*b^4* \\
& *c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*\text{sqrt}(-((a^3*b^4 - 8*a^4*b^2*c + \\
& 16*a^5*c^2)*n^4*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c \\
& + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2* \\
& *c + 16*a^5*c^2)*n^4))/x^2)*\text{sqrt}(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^ \\
& 4*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 \\
& - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2) \\
&)*n^4)))*\text{sqrt}(\text{sqrt}(2)*\text{sqrt}(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*\text{sqrt} \\
& (b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64* \\
& a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)) \\
&)/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 2*\text{sqrt}(2)*\text{sqrt}(\text{sqrt}(2)*\text{sqrt}(((a^3*b^ \\
& 4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^ \\
& 6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^ \\
& 3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*\arctan(1/8*(2*((a^3*b^10*c - 15*a^ \\
& 4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^ \\
& 6)*n^7*x*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8 \\
& *b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40* \\
& a^3*b^3*c^4 + 16*a^4*b*c^5)*n^3*x)*x^{(1/4*n - 1)}*\text{sqrt}(\text{sqrt}(2)*\text{sqrt}(((a^3*b^ \\
& 4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^ \\
& 6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^ \\
& 3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*\text{sqrt}(((a^3*b^4 - 8*a^4*b^2*c + 16* \\
& a^5*c^2)*n^4*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48 \\
& *a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + \\
& 16*a^5*c^2)*n^4)) + ((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^ \\
& 2*c^3 + 128*a^7*c^4)*n^7*x*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12* \\
& a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^7 - 9*a*b^5*c + 24*a^2* \\
& b^3*c^2 - 16*a^3*b*c^3)*n^3*x)*\text{sqrt}(\text{sqrt}(2)*\text{sqrt}(((a^3*b^4 - 8*a^4*b^2*c + \\
& 16*a^5*c^2)*n^4*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + \\
& 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2* \\
& c + 16*a^5*c^2)*n^4)))*\text{sqrt}((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x^{(1/2 \\
& *n - 2)} + \text{sqrt}(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c \\
& ^3 + 64*a^7*b*c^4)*n^6*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7* \\
& b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^8 - 8*a*b^6*c + 21*a^2*b^4* \\
& c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*\text{sqrt}(((a^3*b^4 - 8*a^4*b^2*c + 16*a^ \\
& 5*c^2)*n^4*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a \\
& ^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 1 \\
& 6*a^5*c^2)*n^4)))/x^2)*\text{sqrt}(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*\text{sqrt} \\
& (b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
\end{aligned}$$

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a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))
)/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-(a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/
4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-(a^3*b^4 - 8
*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c
)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^
(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-(a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*
b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b
*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*
x^(1/4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt
(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*
b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x
*x^(1/4*n - 1) - sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a
^9*c^3)*n^8)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(((a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^
3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/4*n)/(a+b*xⁿ+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*xⁿ + a), x)

maple [C] time = 0.75, size = 280, normalized size = 0.79

RootOf(256*x^{4*n} - 256*b*x^{3*n} + 320*b²*x^{2*n} - 128*b³*xⁿ + 256*c*x^{2*n} - (48*b²*c*xⁿ + 64*b*c²)*x^{n/2} + 256*c²), 2ⁿ + 1) | (256*c²*x^{n/2} - 256*b*c*x^{n/2} + 256*c²)*x^{n/2} + 256*c² | RootOf(256*x^{4*n} - 256*b*x^{3*n} + 320*b²*x^{2*n} - 128*b³*xⁿ + 256*c*x^{2*n} - (48*b²*c*xⁿ + 64*b*c²)*x^{n/2} + 256*c²), 2ⁿ + 1) | (256*c²*x^{n/2} - 256*b*c*x^{n/2} + 256*c²)*x^{n/2} + 256*c² | RootOf(256*x^{4*n} - 256*b*x^{3*n} + 320*b²*x^{2*n} - 128*b³*xⁿ + 256*c*x^{2*n} - (48*b²*c*xⁿ + 64*b*c²)*x^{n/2} + 256*c²), 2ⁿ + 1)

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+1/4*n)/(a+b*xⁿ+c*x^(2*n)),x)

[Out] sum(_R*ln(x^(1/4*n))+(16/(a*c²-b²*c)*n⁵*b*a⁵*c²-8/(a*c²-b²*c)*n⁵*b³*a⁴*c+1/(a*c²-b²*c)*n⁵*b⁵*a³)*_R⁵+(2/(a*c²-b²*c)*n*a²*c²-4/(a*c²-b²*c)*n*b²*a*c+1/(a*c²-b²*c)*n*b⁴)*_R, _R=RootOf((256*a⁷*c⁴*n⁸-256*a⁶*b²*c³*n⁸+96*a⁵*b⁴*c²*n⁸-16*a⁴*b⁶*c*n⁸+a³*b⁸*n⁸)*_Z⁸+(-48*a³*b*c³*n⁴+40*a²*b³*c²*n⁴-11*a*b⁵*c*n⁴+b⁷*n⁴)*_Z⁴+c³)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{4}n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/4*n)/(a+b*xⁿ+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*xⁿ + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{4}-1}}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n/4 - 1)/(a + b*xⁿ + c*x^(2*n)),x)

[Out] int(x^(n/4 - 1)/(a + b*xⁿ + c*x^(2*n)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{n}{4}-1}}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/4*n)/(a+b*xⁿ+c*x^(2*n)),x)

[Out] Integral(x^(n/4 - 1)/(a + b*xⁿ + c*x^(2*n)), x)

$$3.481 \quad \int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=610

$$\frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x^{n/3}\right)}{n\sqrt{b^2-4ac} \left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{\sqrt{b^2-4ac}+b} + \sqrt[3]{2} \sqrt[3]{c} x^{n/3}\right)}{n\sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}+b\right)^{2/3}} - c^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x\right)$$

Rubi [A] time = 1.15, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1381, 1347, 200, 31, 634, 617, 204, 628}

$$\frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c} x^{n/3}\right)}{n\sqrt{b^2-4ac} \left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{\sqrt{b^2-4ac}+b} + \sqrt[3]{2} \sqrt[3]{c} x^{n/3}\right)}{n\sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{c^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x\right)}{\sqrt[3]{2} n \sqrt{b^2-4ac} \left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3} \log\left(-\sqrt[3]{2} \sqrt[3]{c} x\right) \sqrt[3]{\sqrt{b^2-4ac}+b} + \left(\sqrt{b^2-4ac}+b\right)^{2/3} + 2^{2/3}c^{2/3}x^{n/3}}{\sqrt[3]{2} n \sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{2^{2/3} \sqrt[3]{3} c^{2/3} \tan^{-1}\left(\frac{1-\sqrt[3]{3} \sqrt[3]{c} x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac} \left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{2^{2/3} \sqrt[3]{3} c^{2/3} \tan^{-1}\left(\frac{1-\sqrt[3]{3} \sqrt[3]{c} x^{n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac} \left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)), x]

[Out] -((2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3)]/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) + (2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3)))/(b + Sqrt[b^2 - 4*a*c])^(1/3)]/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)]/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) - (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)]/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)]/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)

Rule 31

Int[((a_) + (b_.)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R

$t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[\{a, b\}, x]$

Rule 204

$Int[((a_) + (b_)*(x_)^2)^{-1}, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[\{a, b\}, x] \&\& PosQ[a/b] \&\& (LtQ[a, 0] || LtQ[b, 0])$

Rule 617

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] := With[\{q = 1 - 4*Simplify[(a*c)/b^2]\}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] \&\& (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[\{a, b, c\}, x] \&\& NeQ[b^2 - 4*a*c, 0]$

Rule 628

$Int(((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& EqQ[2*c*d - b*e, 0]$

Rule 634

$Int(((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& NeQ[2*c*d - b*e, 0] \&\& NeQ[b^2 - 4*a*c, 0] \&\& !NiceSqrtQ[b^2 - 4*a*c]$

Rule 1347

$Int(((a_) + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(n2_)})^{-1}, x_Symbol] := With[\{q = Rt[b^2 - 4*a*c, 2]\}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[\{a, b, c\}, x] \&\& EqQ[n2, 2*n] \&\& NeQ[b^2 - 4*a*c, 0]$

Rule 1381

$Int((x_)^{(m_)}*((a_) + (c_)*(x_)^{(n2_)}) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[\{a, b, c, m, n, p\}, x] \&\& EqQ[n2, 2*n] \&\& NeQ[b^2 - 4*a*c, 0] \&\& IntegerQ[Simplify[n/(m + 1)]] \&\& !IntegerQ[n]$

Rubi steps

$$\begin{aligned}
 \int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}} dx &= \frac{3 \operatorname{Subst}\left(\int \frac{1}{a+bx^3+cx^6} dx, x, x^{n/3}\right)}{n} \\
 &= \frac{(3c) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3} dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac} n} - \frac{(3c) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3} dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac} n} \\
 &= \frac{(2^{2/3}c) \operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x} dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})^{2/3} n} + \frac{(2^{2/3}c) \operatorname{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}}{(b-\sqrt{b^2-4ac})^{2/3}-\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}}{2^{2/3}} dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})^{2/3} n} \\
 &= \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})^{2/3} n} - \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac} (b+\sqrt{b^2-4ac})^{2/3} n} \\
 &= \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})^{2/3} n} - \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac} (b+\sqrt{b^2-4ac})^{2/3} n} \\
 &= -\frac{2^{2/3}\sqrt{3}c^{2/3} \tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})^{2/3} n} + \frac{2^{2/3}\sqrt{3}c^{2/3} \tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac} (b+\sqrt{b^2-4ac})^{2/3} n} + \frac{2^{2/3}c^{2/3} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right)}{\sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})^{2/3} n}
 \end{aligned}$$

Mathematica [A] time = 0.76, size = 526, normalized size = 0.86

$$\frac{c^{2/3} \left(-(\sqrt{b^2-4ac}+b)^{2/3} \log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+(b-\sqrt{b^2-4ac})^{2/3}\right) + (b-\sqrt{b^2-4ac})^{2/3} \log\left(-\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+b+(\sqrt{b^2-4ac}+b)^{2/3}\right) + 2(\sqrt{b^2-4ac}+b)^{2/3} \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right) - 2(b-\sqrt{b^2-4ac})^{2/3} \log\left(\sqrt[3]{b^2-4ac}+b+\sqrt[3]{2}\sqrt[3]{c}x^{n/3}\right) - 2\sqrt{3}(\sqrt{b^2-4ac}+b)^{2/3} \tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right) + 2\sqrt{3}(b-\sqrt{b^2-4ac})^{2/3} \tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right) \right)}{\sqrt[3]{2}\sqrt{b^2-4ac} (b-\sqrt{b^2-4ac})^{2/3} (\sqrt{b^2-4ac}+b)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)), x]

```
[Out] (c^(2/3)*(-2*Sqrt[3]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqrt[b^2 - 4*a*c])^(1/3)]/Sqrt[3]] + 2*Sqrt[3]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b + Sqrt[b^2 - 4*a*c])^(1/3)]/Sqrt[3]] + 2*(b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - 2*(b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x^(n/3)] - (b + Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)] + (b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{3}}}{a + bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)), x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)), x]
```

fricas [B] time = 1.98, size = 4699, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)), x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3)^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 + 128*a^6*c^5)*n^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 20*a^2*b^3*c^3 - 16*a^3*b*c^4)*n^2*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + sqrt(2)*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*n^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n^2*x)*sqrt((2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) - (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)))
```

$$\begin{aligned}
& - (b^6c - 8ab^4c^2 + 20a^2b^2c^3 - 16a^3c^4)nx) * x^{(1/3n - 1)} * ((\\
& (a^2b^2 - 4a^3c) * n^3 * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a \\
& ^5b^4c + 48a^6b^2c^2 - 64a^7c^3) * n^6)) + b) / ((a^2b^2 - 4a^3c) * n^3 \\
&))^{(1/3)} - (1/2)^{(2/3)} * ((a^2b^9 - 14a^3b^7c + 72a^4b^5c^2 - 160a^5 \\
& b^3c^3 + 128a^6b^2c^4) * n^5 * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - \\
& 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3) * n^6)) - (b^8 - 10ab^6c + 36 \\
& a^2b^4c^2 - 56a^3b^2c^3 + 32a^4c^4) * n^2) * (((a^2b^2 - 4a^3c) * n^3 * \\
& \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 \\
& - 64a^7c^3) * n^6)) + b) / ((a^2b^2 - 4a^3c) * n^3))^{(2/3)} / x^2) * (((a^2b^ \\
& 2 - 4a^3c) * n^3 * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^4c \\
& c + 48a^6b^2c^2 - 64a^7c^3) * n^6)) + b) / ((a^2b^2 - 4a^3c) * n^3))^{(2/3)} \\
&) + 2 * \sqrt{3} * (b^4c^2 - 4ab^2c^3 + 4a^2c^4) / (b^4c^2 - 4ab^2c^3 + \\
& 4a^2c^4) - 2 * \sqrt{3} * (1/2)^{(1/3)} * (-((a^2b^2 - 4a^3c) * n^3 * \sqrt{(b^4 - \\
& 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7 \\
& c^3) * n^6)) - b) / ((a^2b^2 - 4a^3c) * n^3))^{(1/3)} * \arctan(-1/6 * (2 * (1/2)^{(2/3)} \\
& * (\sqrt{3} * (a^2b^8c - 14a^3b^6c^2 + 72a^4b^4c^3 - 160a^5b^2c^4 + \\
& 128a^6c^5) * n^5 * x * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^ \\
& 4c + 48a^6b^2c^2 - 64a^7c^3) * n^6)) + \sqrt{3} * (b^7c - 8ab^5c^2 + 2 \\
& 0a^2b^3c^3 - 16a^3b^2c^4) * n^2 * x) * x^{(1/3n - 1)} * (-((a^2b^2 - 4a^3c) * n \\
& ^3 * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^4c + 48a^6b^2 \\
& * c^2 - 64a^7c^3) * n^6)) - b) / ((a^2b^2 - 4a^3c) * n^3))^{(2/3)} + \sqrt{2} * (1 \\
& / 2)^{(2/3)} * (\sqrt{3} * (a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3) * n \\
& ^5 * x * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^4c + 48a^6b \\
& ^2c^2 - 64a^7c^3) * n^6)) + \sqrt{3} * (b^5 - 6ab^3c + 8a^2b^2c) * n^2 * x) \\
& * \sqrt{(2 * (b^4c^2 - 4ab^2c^3 + 4a^2c^4) * x^2 * x^{(2/3n - 2)} + (1/2)^{(1/3)} \\
&) * ((a^2b^7c - 10a^3b^5c^2 + 32a^4b^3c^3 - 32a^5b^2c^4) * n^4 * x * \sqrt{(\\
& b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 6 \\
& 4a^7c^3) * n^6)) + (b^6c - 8ab^4c^2 + 20a^2b^2c^3 - 16a^3c^4) * nx) \\
& * x^{(1/3n - 1)} * (-((a^2b^2 - 4a^3c) * n^3 * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 \\
&) / ((a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3) * n^6)) - b) / ((a^2 * \\
& b^2 - 4a^3c) * n^3))^{(1/3)} + (1/2)^{(2/3)} * ((a^2b^9 - 14a^3b^7c + 72a^4 * \\
& b^5c^2 - 160a^5b^3c^3 + 128a^6b^2c^4) * n^5 * \sqrt{(b^4 - 4ab^2c + 4a^ \\
& 2c^2)} / ((a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3) * n^6)) + (b^8 \\
& - 10ab^6c + 36a^2b^4c^2 - 56a^3b^2c^3 + 32a^4c^4) * n^2) * (-((a^2 * \\
& b^2 - 4a^3c) * n^3 * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^ \\
& 4c + 48a^6b^2c^2 - 64a^7c^3) * n^6)) - b) / ((a^2b^2 - 4a^3c) * n^3))^{(2 \\
& / 3)} / x^2) * (-((a^2b^2 - 4a^3c) * n^3 * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a \\
& ^4b^6 - 12a^5b^4c + 48a^6b^2c^2 - 64a^7c^3) * n^6)) - b) / ((a^2b^2 - \\
& 4a^3c) * n^3))^{(2/3)} - 2 * \sqrt{3} * (b^4c^2 - 4ab^2c^3 + 4a^2c^4) / (b^4 \\
& * c^2 - 4ab^2c^3 + 4a^2c^4) + (1/2)^{(1/3)} * (((a^2b^2 - 4a^3c) * n^3 * \sqrt{ \\
& (b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^4c + 48a^6b^2c^2 \\
& - 64a^7c^3) * n^6)) + b) / ((a^2b^2 - 4a^3c) * n^3))^{(1/3)} * \log(-2 * (b^2c - \\
& 2a^2c^2) * x * x^{(1/3n - 1)} + (1/2)^{(1/3)} * ((a^2b^5 - 8a^3b^3c + 16a^4b^2c \\
& ^2) * n^4 * \sqrt{(b^4 - 4ab^2c + 4a^2c^2)} / ((a^4b^6 - 12a^5b^4c + 48a^ \\
& 6b^2c^2 - 64a^7c^3) * n^6)) - (b^4 - 6ab^2c + 8a^2c^2) * n) * (((a^2b^2
\end{aligned}$$

$$\begin{aligned}
& - 4*a^3*c)*n^3*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} + b)/((a^2*b^2 - 4*a^3*c)*n^3)^{(1/3)} \\
&)/x) + (1/2)^{(1/3)}*(-((a^2*b^2 - 4*a^3*c)*n^3*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} - b)/((a^2*b^2 - 4*a^3*c)*n^3)^{(1/3)}*\log(-2*(b^2*c - 2*a*c^2)*x*x^{(1/3)*n - 1} - \\
& (1/2)^{(1/3)}*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)})) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(-((a^2*b^2 - 4*a^3*c)*n^3*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} - b)/((a^2*b^2 - 4*a^3*c)*n^3)^{(1/3)})/x) - 1/2*(1/2)^{(1/3)}*((a^2*b^2 - 4*a^3*c)*n^3*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} + b)/((a^2*b^2 - 4*a^3*c)*n^3)^{(1/3)}*\log(8*(2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^{(2/3)*n - 2} - (1/2)^{(1/3)}*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} - (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^{(1/3)*n - 1}*((a^2*b^2 - 4*a^3*c)*n^3*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} + b)/((a^2*b^2 - 4*a^3*c)*n^3)^{(1/3)} - (1/2)^{(2/3)}*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} - (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*((a^2*b^2 - 4*a^3*c)*n^3*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} + b)/((a^2*b^2 - 4*a^3*c)*n^3)^{(2/3)})/x^2) - 1/2*(1/2)^{(1/3)}*(-((a^2*b^2 - 4*a^3*c)*n^3*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} - b)/((a^2*b^2 - 4*a^3*c)*n^3)^{(1/3)}*\log(8*(2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^{(2/3)*n - 2} + (1/2)^{(1/3)}*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} + (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^{(1/3)*n - 1}*(-((a^2*b^2 - 4*a^3*c)*n^3*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} - b)/((a^2*b^2 - 4*a^3*c)*n^3)^{(1/3)} + (1/2)^{(2/3)}*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} + (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*\sqrt{(b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)} - b)/((a^2*b^2 - 4*a^3*c)*n^3)^{(2/3)})/x^2)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/3*n)/(a+b*xⁿ+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*xⁿ + a), x)

maple [C] time = 0.46, size = 260, normalized size = 0.43

RootOf((64*a^2*b^4 - 48*b^2*c^4 + 12*b^4*c^4 - a^2*b^4)*z^2 + (16*b^2*c^4 - 8*b^2*c^4 + b^4*c^4)*z + a^2) in ((16*b^2*c^4 - 8*b^2*c^4 + b^4*c^4)/2a^2 - b^2, (16*b^2*c^4 - 8*b^2*c^4 + b^4*c^4)/2a^2 - b^2) RootOf((64*a^2*b^4 - 48*b^2*c^4 + 12*b^4*c^4 - a^2*b^4)*z^2 + (16*b^2*c^4 - 8*b^2*c^4 + b^4*c^4)*z + a^2) in ((16*b^2*c^4 - 8*b^2*c^4 + b^4*c^4)/2a^2 - b^2, (16*b^2*c^4 - 8*b^2*c^4 + b^4*c^4)/2a^2 - b^2) RootOf((64*a^2*b^4 - 48*b^2*c^4 + 12*b^4*c^4 - a^2*b^4)*z^2 + (16*b^2*c^4 - 8*b^2*c^4 + b^4*c^4)*z + a^2) in ((16*b^2*c^4 - 8*b^2*c^4 + b^4*c^4)/2a^2 - b^2, (16*b^2*c^4 - 8*b^2*c^4 + b^4*c^4)/2a^2 - b^2)

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+1/3*n)/(a+b*xⁿ+c*x^(2*n)),x)

[Out] sum(_R*ln(x^(1/3*n)+(-16/(2*a*c²-b²*c)*n⁴*b*a⁴*c²+8/(2*a*c²-b²*c)*n⁴*b³*a³*c-1/(2*a*c²-b²*c)*n⁴*b⁵*a²)*_R⁴+ (4/(2*a*c²-b²*c)*n*a²*c²-5/(2*a*c²-b²*c)*n*b²*a*c+1/(2*a*c²-b²*c)*n*b⁴)*_R),_R=RootOf((64*a⁵*c³*n⁶-48*a⁴*b²*c²*n⁶+12*a³*b⁴*c*n⁶-a²*b⁶*n⁶)*_Z⁶+ (16*a²*b*c²*n³-8*a*b³*c*n³+b⁵*n³)*_Z³+c²)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{3}n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/3*n)/(a+b*xⁿ+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*xⁿ + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{3}-1}}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n/3 - 1)/(a + b*xⁿ + c*x^(2*n)),x)

[Out] int(x^(n/3 - 1)/(a + b*xⁿ + c*x^(2*n)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/3*n)/(a+b*xⁿ+c*x^(2*n)),x)

[Out] Timed out

$$3.482 \quad \int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=169

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1381, 1093, 205}

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]

[Out] (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) - (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1381

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&

EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}} dx &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{a+bx^2+cx^4} dx, x, x^{n/2}\right)}{n} \\ &= \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx, x, x^{n/2}\right)}{\sqrt{b^2-4ac}n} - \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx, x, x^{n/2}\right)}{\sqrt{b^2-4ac}n} \\ &= \frac{2\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}n} - \frac{2\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}n} \end{aligned}$$

Mathematica [A] time = 0.26, size = 145, normalized size = 0.86

$$\frac{2\sqrt{2}\sqrt{c} \left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)), x]

[Out] (2*Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[b^2 - 4*a*c]*n)

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + n/2)/(a + b*xⁿ + c*x^(2*n)), x]

fricas [B] time = 1.36, size = 801, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/2*n)/(a+b*xⁿ+c*x^(2*n)),x, algorithm="fricas")

[Out] $\frac{1}{2}\sqrt{2}\sqrt{-((a*b^2 - 4*a^2*c)*n^2\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) + b}/((a*b^2 - 4*a^2*c)*n^2))\log((4*c*x*x^{(1/2*n - 1)} + \sqrt{2})*((a*b^3 - 4*a^2*b*c)*n^3\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 - 4*a*c)*n)\sqrt{-((a*b^2 - 4*a^2*c)*n^2\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) + b}/((a*b^2 - 4*a^2*c)*n^2)))/x - \frac{1}{2}\sqrt{2}\sqrt{-((a*b^2 - 4*a^2*c)*n^2\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) + b}/((a*b^2 - 4*a^2*c)*n^2))\log((4*c*x*x^{(1/2*n - 1)} - \sqrt{2})*((a*b^3 - 4*a^2*b*c)*n^3\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 - 4*a*c)*n)\sqrt{-((a*b^2 - 4*a^2*c)*n^2\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) + b}/((a*b^2 - 4*a^2*c)*n^2)))/x - \frac{1}{2}\sqrt{2}\sqrt{((a*b^2 - 4*a^2*c)*n^2\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) - b}/((a*b^2 - 4*a^2*c)*n^2))\log((4*c*x*x^{(1/2*n - 1)} + \sqrt{2})*((a*b^3 - 4*a^2*b*c)*n^3\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)\sqrt{((a*b^2 - 4*a^2*c)*n^2\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) - b}/((a*b^2 - 4*a^2*c)*n^2)))/x + \frac{1}{2}\sqrt{2}\sqrt{((a*b^2 - 4*a^2*c)*n^2\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) - b}/((a*b^2 - 4*a^2*c)*n^2))\log((4*c*x*x^{(1/2*n - 1)} - \sqrt{2})*((a*b^3 - 4*a^2*b*c)*n^3\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)\sqrt{((a*b^2 - 4*a^2*c)*n^2\sqrt{1/((a^2*b^2 - 4*a^3*c)*n^4)) - b}/((a*b^2 - 4*a^2*c)*n^2)))/x$

giac [B] time = 1.85, size = 1035, normalized size = 6.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/2*n)/(a+b*xⁿ+c*x^(2*n)),x, algorithm="giac")

[Out] $\frac{1}{2}*((\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4 - 8*\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c - 2*\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c - 2*b^4*c + 16*\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 + 8*\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 + \sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*\sqrt{2}\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + \sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3 - 4*\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c - 2*\sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^2*c + \sqrt{2}\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)$

$$\begin{aligned}
 & *b*c^2) * \arctan(2*\sqrt{1/2}*\sqrt{x^n}) / \sqrt{(b + \sqrt{b^2 - 4*a*c})/c} / ((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3) * \text{abs}(c)) + (\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * b^4 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * a*b^2*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * b^3*c + 2*b^4*c + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * a^2*c^2 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * a*b*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * b^3 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * a*b*c - 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * b^2*c + \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c) * b*c^2 - 2*(b^2 - 4*a*c) * b^2*c + 8*(b^2 - 4*a*c) * a*c^2 + 2*(b^2 - 4*a*c) * b*c^2) * \arctan(2*\sqrt{1/2}*\sqrt{x^n}) / \sqrt{(b - \sqrt{b^2 - 4*a*c})/c} / ((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3) * \text{abs}(c))) / n
 \end{aligned}$$

maple [C] time = 0.19, size = 114, normalized size = 0.67

$$\text{RootOf}((16a^3c^2n^4 - 8a^2b^2cn^4 + ab^4n^4)Z^4 + (-4abcn^2 + b^3n^2)Z^2 + c) \ln\left(\frac{4a^2bn^3 - ab^3n^3}{c}\right) \text{RootOf}((16a^3c^2n^4 - 8a^2b^2cn^4 + ab^4n^4)Z^4 + (-4abcn^2 + b^3n^2)Z^2 + c)^3 + \left(2an - \frac{b^2n}{c}\right) \text{RootOf}((16a^3c^2n^4 - 8a^2b^2cn^4 + ab^4n^4)Z^4 + (-4abcn^2 + b^3n^2)Z^2 + c) + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)), x)

[Out] sum(_R*ln(x^(1/2*n)+(4*n^3*b*a^2-1/c*n^3*b^3*a)*_R^3+(2*a*n-1/c*n*b^2)*_R), _R=RootOf((16*a^3*c^2*n^4-8*a^2*b^2*c*n^4+a*b^4*n^4)*_Z^4+(-4*a*b*c*n^2+b^3*n^2)*_Z^2+c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{2}n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)), x, algorithm="maxima")

[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{n}{2}-1}}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)), x)

```
[Out] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^{\frac{n}{2}-1}}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)), x)
```

```
[Out] Integral(x**(n/2 - 1)/(a + b*x**n + c*x**(2*n)), x)
```

$$3.483 \quad \int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=205

$$\frac{\sqrt{2} \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{a} x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{a^{3/2} n \sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{a} x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{a^{3/2} n \sqrt{\sqrt{b^2-4ac}+b}} - \frac{2x^{-n/2}}{an}$$

Rubi [A] time = 0.40, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1381, 1340, 1122, 1166, 205}

$$\frac{\sqrt{2} \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{a} x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{a^{3/2} n \sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{a} x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{a^{3/2} n \sqrt{\sqrt{b^2-4ac}+b}} - \frac{2x^{-n/2}}{an}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)), x]

[Out] -2/(a*n*x^(n/2)) + (Sqrt[2]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) + (Sqrt[2]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1122

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1340

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:= Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] && !I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{2}}}{a + bx^n + cx^{2n}} dx &= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{a + \frac{c}{x^4} + \frac{b}{x^2}} dx, x, x^{-n/2}\right)}{n} \\
&= -\frac{2 \operatorname{Subst}\left(\int \frac{x^4}{c + bx^2 + ax^4} dx, x, x^{-n/2}\right)}{n} \\
&= -\frac{2x^{-n/2}}{an} + \frac{2 \operatorname{Subst}\left(\int \frac{c + bx^2}{c + bx^2 + ax^4} dx, x, x^{-n/2}\right)}{an} \\
&= -\frac{2x^{-n/2}}{an} + \frac{\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + ax^2} dx, x, x^{-n/2}\right)}{an} + \frac{\left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + ax^2} dx, x, x^{-n/2}\right)}{an} \\
&= -\frac{2x^{-n/2}}{an} + \frac{\sqrt{2} \left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{a} x^{-n/2}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{a^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}} n} + \frac{\sqrt{2} \left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{a} x^{-n/2}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{a^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}} n}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 127, normalized size = 0.62

$$\frac{4cx^{-n/2} \left(\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} + \frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2} \right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)), x]

[Out] (4*c*(Hypergeometric2F1[-1/2, 1, 1/2, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/2, 1, 1/2, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]))/(n*x^(n/2))

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{2}}}{a + bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)), x]

fricas [B] time = 1.25, size = 1229, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] 1/2*(sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x - sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) - sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x - sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sqrt((

$$b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*\log(-4*(b^2*c - a*c^2)*x*x^{(-1/2*n - 1)} + \sqrt{2}*(a^3*b^3 - 4*a^4*b*c)*n^3*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))} + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*\sqrt{((a^3*b^2 - 4*a^4*c)*n^2*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))} - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) + \sqrt{2}*a*n*\sqrt{((a^3*b^2 - 4*a^4*c)*n^2*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))} - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*\log(-4*(b^2*c - a*c^2)*x*x^{(-1/2*n - 1)} - \sqrt{2}*((a^3*b^3 - 4*a^4*b*c)*n^3*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))} + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*\sqrt{((a^3*b^2 - 4*a^4*c)*n^2*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))} - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) - 4*x*x^{(-1/2*n - 1))/(a*n)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [C] time = 0.35, size = 268, normalized size = 1.31

RootOf((16a^5c^2n^4 - 8a^4b^2cn^4 + a^2b^4n^4)_Z^4 + (12a^2b^2n^2 - 7a^3b^2cn^2 + b^4n^2)_Z^2) ln((frac(8a^2c^2n^2, a^2c^2 - b^2c^2) + frac(6a^4b^2cn^2, a^2c^2 - b^2c^2) - frac(a^2b^4n^2, a^2c^2 - b^2c^2))RootOf((16a^5c^2n^4 - 8a^4b^2cn^4 + a^2b^4n^4)_Z^4 + (12a^2b^2n^2 - 7a^3b^2cn^2 + b^4n^2)_Z^2) + (frac(5a^2b^2n, a^2c^2 - b^2c^2) - frac(5a^4b^2cn, a^2c^2 - b^2c^2) - frac(b^4n, a^2c^2 - b^2c^2))RootOf((16a^5c^2n^4 - 8a^4b^2cn^4 + a^2b^4n^4)_Z^4 + (12a^2b^2n^2 - 7a^3b^2cn^2 + b^4n^2)_Z^2) + i^2) - frac(2x^(-1/2), a)

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x)

[Out] -2/a/n/(x^(1/2*n))+sum(_R*ln(x^(1/2*n))+(-8/(a*c^3-b^2*c^2)*n^3*a^5*c^2+6/(a*c^3-b^2*c^2)*n^3*b^2*a^4*c-1/(a*c^3-b^2*c^2)*n^3*b^4*a^3)*_R^3+(-5/(a*c^3-b^2*c^2)*n*b*a^2*c^2+5/(a*c^3-b^2*c^2)*n*b^3*a*c-1/(a*c^3-b^2*c^2)*n*b^5)*_R), _R=RootOf((16*a^5*c^2*n^4-8*a^4*b^2*c*n^4+a^3*b^4*n^4)*_Z^4+(12*a^2*b^2*c^2*n^2-7*a*b^3*c*n^2+b^5*n^2)*_Z^2+c^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{anx^{\frac{1}{2}n}} - \int \frac{cx^{\frac{3}{2}n} + bx^{\frac{1}{2}n}}{acxx^{2n} + abxx^n + a^2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] $-2/(a*n*x^{(1/2*n)}) - \text{integrate}((c*x^{(3/2*n)} + b*x^{(1/2*n)})/(a*c*x*x^{(2*n)} + a*b*x*x^n + a^2*x), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{2}+1} (a + b x^n + c x^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^{(n/2 + 1)}*(a + b*x^n + c*x^{(2*n)})), x)$

[Out] $\text{int}(1/(x^{(n/2 + 1)}*(a + b*x^n + c*x^{(2*n)})), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{n}{2}-1}}{a + b x^n + c x^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{*(-1-1/2*n)}/(a+b*x**n+c*x**(2*n)), x)$

[Out] $\text{Integral}(x^{*(-n/2 - 1)}/(a + b*x**n + c*x**(2*n)), x)$

$$3.484 \quad \int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=699

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right)}{\sqrt[3]{2} a^{4/3} n \left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \log\left(\sqrt[3]{\sqrt{b^2-4ac} + b} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right)}{\sqrt[3]{2} a^{4/3} n \left(\sqrt{b^2-4ac} + b\right)^{2/3}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right)}{\sqrt[3]{2} a^{4/3} n \left(b - \sqrt{b^2-4ac}\right)^{2/3}}$$

Rubi [A] time = 1.49, antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 26, number of rules / integrand size = 0.385, Rules used = {1381, 1340, 1367, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(\frac{b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}}{\sqrt[3]{2} a^{4/3} n \left(b - \sqrt{b^2-4ac}\right)^{2/3}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right) + \left(\frac{\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b}{\sqrt[3]{2} a^{4/3} n \left(\sqrt{b^2-4ac} + b\right)^{2/3}}\right) \log\left(\sqrt[3]{\sqrt{b^2-4ac} + b} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right) - \left(\frac{b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}}{\sqrt[3]{2} a^{4/3} n \left(b - \sqrt{b^2-4ac}\right)^{2/3}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right)}{\sqrt[3]{2} a^{4/3} n \left(b - \sqrt{b^2-4ac}\right)^{2/3} + \sqrt[3]{2} a^{4/3} n \left(\sqrt{b^2-4ac} + b\right)^{2/3} - \sqrt[3]{2} a^{4/3} n \left(b - \sqrt{b^2-4ac}\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)), x]

[Out]
$$\begin{aligned} & -3/(a*n*x^{(n/3)}) - (\text{Sqrt}[3]*(b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*ArcTan[(1 - (2*2^{(1/3)}*a^{(1/3)})/((b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}*x^{(n/3)})]/\text{Sqrt}[3]])/(2^{(1/3)}*a^{(4/3)}*(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)*n}) - (\text{Sqrt}[3]*(b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*ArcTan[(1 - (2*2^{(1/3)}*a^{(1/3)})/((b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}*x^{(n/3)})]/\text{Sqrt}[3]])/(2^{(1/3)}*a^{(4/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)*n}) + ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*Log[(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + (2^{(1/3)}*a^{(1/3)})/x^{(n/3)}])/(2^{(1/3)}*a^{(4/3)}*(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)*n}) + ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*Log[(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + (2^{(1/3)}*a^{(1/3)})/x^{(n/3)}])/(2^{(1/3)}*a^{(4/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)*n}) - ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*Log[(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} + (2^{(2/3)}*a^{(2/3)})/x^{((2*n)/3)} - (2^{(1/3)}*a^{(1/3)}*(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/x^{(n/3)}])/(2*2^{(1/3)}*a^{(4/3)}*(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)*n}) - ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*Log[(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} + (2^{(2/3)}*a^{(2/3)})/x^{((2*n)/3)} - (2^{(1/3)}*a^{(1/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/x^{(n/3)}])/(2*2^{(1/3)}*a^{(4/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)*n}) \end{aligned}$$

Rule 31

Int[((a_) + (b_.)*(x_))^(−1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1340

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && LtQ[n, 0] && IntegerQ[p]

Rule 1367

Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(d^(2*n - 1)*(d*x)^(m - 2*n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + 2*n*p + 1)), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]

&& IntegerQ[p]

Rule 1381

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p, x_Symbol]
  :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[(
2*n)/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] && !I
ntegerQ[n]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] || !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}} dx &= -\frac{3 \operatorname{Subst}\left(\int \frac{1}{a+\frac{c}{x^6}+\frac{b}{x^3}} dx, x, x^{-n/3}\right)}{n} \\
&= -\frac{3 \operatorname{Subst}\left(\int \frac{x^6}{c+bx^3+ax^6} dx, x, x^{-n/3}\right)}{n} \\
&= -\frac{3x^{-n/3}}{an} + \frac{3 \operatorname{Subst}\left(\int \frac{c+bx^3}{c+bx^3+ax^6} dx, x, x^{-n/3}\right)}{an} \\
&= -\frac{3x^{-n/3}}{an} + \frac{\left(3\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^3} dx, x, x^{-n/3}\right)}{2an} + \frac{\left(3\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+ax^3} dx, x, x^{-n/3}\right)}{2an} \\
&= -\frac{3x^{-n/3}}{an} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{a}x} dx, x, x^{-n/3}\right)}{\sqrt[3]{2}a\left(b - \sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{a}x} dx, x, x^{-n/3}\right)}{\sqrt[3]{2}a\left(b + \sqrt{b^2-4ac}\right)^{2/3}n} \\
&= -\frac{3x^{-n/3}}{an} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}n} \\
&= -\frac{3x^{-n/3}}{an} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}n} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{a} x^{-n/3}\right)}{\sqrt[3]{2}a^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}n} \\
&= -\frac{3x^{-n/3}}{an} - \frac{\sqrt{3}\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}n} - \frac{\sqrt{3}\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}a^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}n}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 127, normalized size = 0.18

$$\frac{6cx^{-n/3} \left(\frac{{}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} + \frac{{}_2F_1\left(-\frac{1}{3}, 1; \frac{2}{3}; \frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2} \right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)), x]

[Out] (6*c*(Hypergeometric2F1[-1/3, 1, 2/3, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/3, 1, 2/3, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])))/(n*x^(n/3))

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{3}}}{a + bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic][x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)), x]

fricas [B] time = 7.49, size = 6279, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] 1/2*(4*sqrt(3)*(1/2)^(1/3)*a*n*((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3)^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^4*b^12*c - 17*a^5*b^10*c^2 + 114*a^6*b^8*c^3 - 378*a^7*b^6*c^4 + 632*a^8*b^4*c^5 - 480*a^9*b^2*c^6 + 128*a^10*c^7)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - sqrt(3)*(b^13*c - 15*a*b^11*c^2 + 88*a^2*b^9*c^3 - 252*a^3*b^7*c^4 + 356*a^4*b^5*c^5 - 220*a^5*b^3*c^6 + 48*a^6*b*c^7)*n^2*x)*x^(-1/3*n - 1)*((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3)^(2/3) - sqrt(2)*(1/2)^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3

$$\begin{aligned}
& + 64a^8c^4)n^5x\sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 \\
& + 4a^4c^4)/((a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3)n^6)} \\
&) - \sqrt{3}(b^9 - 11a^2b^7c + 42a^2b^5c^2 - 62a^3b^3c^3 + 24a^4b \\
& *c^4)n^2x)\sqrt{(2(b^8c^2 - 8a^2b^6c^3 + 20a^2b^4c^4 - 16a^3b^2c^5 \\
& + 4a^4c^6)x^2x^{(-2/3n - 2)} - (1/2)^{(1/3)}((a^4b^9c - 12a^5b^7c \\
& ^2 + 50a^6b^5c^3 - 80a^7b^3c^4 + 32a^8b^2c^5)n^4x\sqrt{(b^8 - 8a^2 \\
& b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/((a^8b^6 - 12a^9b^4 \\
& *c + 48a^{10}b^2c^2 - 64a^{11}c^3)n^6)) - (b^{10}c - 12a^2b^8c^2 + 52a^2 \\
& *b^6c^3 - 96a^3b^4c^4 + 68a^4b^2c^5 - 16a^5c^6)n^2x)x^{(-1/3n - 1)} \\
&)*((a^4b^2 - 4a^5c)n^3\sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3 \\
& *b^2c^3 + 4a^4c^4)/((a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11} \\
& c^3)n^6)) + b^3 - 2a^2b^2c)/((a^4b^2 - 4a^5c)n^3))^{(1/3)} - (1/2)^{(2/3)} \\
& ((a^4b^{11} - 16a^5b^9c + 98a^6b^7c^2 - 280a^7b^5c^3 + 352a^8b^3 \\
& c^4 - 128a^9b^2c^5)n^5\sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2 \\
& c^3 + 4a^4c^4)/((a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3) \\
&)n^6)) - (b^{12} - 14a^2b^{10}c + 76a^2b^8c^2 - 200a^3b^6c^3 + 260a^4 \\
& b^4c^4 - 152a^5b^2c^5 + 32a^6c^6)n^2)*((a^4b^2 - 4a^5c)n^3\sqrt{ \\
& ((b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/((a^8b^6 \\
& - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3)n^6)) + b^3 - 2a^2b^2c)/((a^ \\
& 4b^2 - 4a^5c)n^3))^{(2/3)}/x^2)*((a^4b^2 - 4a^5c)n^3\sqrt{(b^8 - 8 \\
& a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/((a^8b^6 - 12a^9b^ \\
& ^4c + 48a^{10}b^2c^2 - 64a^{11}c^3)n^6)) + b^3 - 2a^2b^2c)/((a^4b^2 - 4 \\
& a^5c)n^3))^{(2/3)} + 2\sqrt{3}(b^8c^4 - 8a^2b^6c^5 + 20a^2b^4c^6 - 16 \\
& *a^3b^2c^7 + 4a^4c^8))/(b^8c^4 - 8a^2b^6c^5 + 20a^2b^4c^6 - 16a^3 \\
& *b^2c^7 + 4a^4c^8)) - 4\sqrt{3}(1/2)^{(1/3)}a^n*(-((a^4b^2 - 4a^5c)n \\
& ^3\sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/((a \\
& ^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3)n^6)) - b^3 + 2a^2b^2 \\
& c)/((a^4b^2 - 4a^5c)n^3))^{(1/3)}\arctan(-1/6*(2(1/2)^{(2/3)}(\sqrt{3}(a^ \\
& 4b^{12}c - 17a^5b^{10}c^2 + 114a^6b^8c^3 - 378a^7b^6c^4 + 632a^8b^ \\
& 4c^5 - 480a^9b^2c^6 + 128a^{10}c^7)n^5x\sqrt{(b^8 - 8a^2b^6c + 20a^ \\
& 2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/((a^8b^6 - 12a^9b^4c + 48a^{10} \\
& b^2c^2 - 64a^{11}c^3)n^6)) + \sqrt{3}(b^{13}c - 15a^2b^{11}c^2 + 88a^2b^9 \\
& *c^3 - 252a^3b^7c^4 + 356a^4b^5c^5 - 220a^5b^3c^6 + 48a^6b^2c^7) \\
&)n^2x)x^{(-1/3n - 1)}*(-((a^4b^2 - 4a^5c)n^3\sqrt{(b^8 - 8a^2b^6c + 20 \\
& *a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4)/((a^8b^6 - 12a^9b^4c + 48a^{10} \\
& b^2c^2 - 64a^{11}c^3)n^6)) - b^3 + 2a^2b^2c)/((a^4b^2 - 4a^5c)n^3)) \\
& ^{(2/3)} - \sqrt{2}(1/2)^{(2/3)}(\sqrt{3}(a^4b^8 - 13a^5b^6c + 60a^6b^4 \\
& c^2 - 112a^7b^2c^3 + 64a^8c^4)n^5x\sqrt{(b^8 - 8a^2b^6c + 20a^2b^ \\
& 4c^2 - 16a^3b^2c^3 + 4a^4c^4)/((a^8b^6 - 12a^9b^4c + 48a^{10}b^2 \\
& c^2 - 64a^{11}c^3)n^6)) + \sqrt{3}(b^9 - 11a^2b^7c + 42a^2b^5c^2 - 62 \\
& a^3b^3c^3 + 24a^4b^2c^4)n^2x)\sqrt{(2(b^8c^2 - 8a^2b^6c^3 + 20a^2 \\
& b^4c^4 - 16a^3b^2c^5 + 4a^4c^6)x^2x^{(-2/3n - 2)} + (1/2)^{(1/3)}((a^ \\
& 4b^9c - 12a^5b^7c^2 + 50a^6b^5c^3 - 80a^7b^3c^4 + 32a^8b^2c^5) \\
&)n^4x\sqrt{(b^8 - 8a^2b^6c + 20a^2b^4c^2 - 16a^3b^2c^3 + 4a^4c^4) / \\
& ((a^8b^6 - 12a^9b^4c + 48a^{10}b^2c^2 - 64a^{11}c^3)n^6)) + (b^{10}c -
\end{aligned}$$

$$\begin{aligned}
& 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^{(-1/3*n - 1)}*(-((a^4*b^2 - 4*a^5*c)*n^3*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^{\frac{1}{3}} + (1/2)^{\frac{2}{3}}*((a^4*b^{11} - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) + (b^{12} - 14*a*b^{10}*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(-((a^4*b^2 - 4*a^5*c)*n^3*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^{\frac{2}{3}}/x^2)*(-((a^4*b^2 - 4*a^5*c)*n^3*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^{\frac{2}{3}} - 2*\sqrt{3}*(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^8))/(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^8)) + 2*(1/2)^{\frac{1}{3}}*a*n*((a^4*b^2 - 4*a^5*c)*n^3*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^{\frac{1}{3}}*\log((2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^{(-1/3*n - 1)} + (1/2)^{\frac{1}{3}}*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) - (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*((a^4*b^2 - 4*a^5*c)*n^3*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^{\frac{1}{3}}/x) + 2*(1/2)^{\frac{1}{3}}*a*n*(-((a^4*b^2 - 4*a^5*c)*n^3*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^{\frac{1}{3}}*\log((2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*x*x^{(-1/3*n - 1)} - (1/2)^{\frac{1}{3}}*((a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*n^4*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) + (b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*n)*(-((a^4*b^2 - 4*a^5*c)*n^3*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^{\frac{1}{3}}/x) - (1/2)^{\frac{1}{3}}*a*n*((a^4*b^2 - 4*a^5*c)*n^3*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^{\frac{1}{3}}*\log(8*(2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^{(-2/3*n - 2)} - (1/2)^{\frac{1}{3}}*((a^4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4*x*\sqrt{(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)})) - (b^{10}*c - 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a
\end{aligned}$$

$$\begin{aligned} &^5c^6) * n * x) * x^{(-1/3 * n - 1)} * (((a^4 * b^2 - 4 * a^5 * c) * n^3 * \sqrt{(b^8 - 8 * a * b^6 * c \\ &+ 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)} / ((a^8 * b^6 - 12 * a^9 * b^4 * c + \\ &48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3) * n^6)) + b^3 - 2 * a * b * c) / ((a^4 * b^2 - 4 * a^5 * c) * \\ &n^3))^{(1/3)} - (1/2)^{(2/3)} * ((a^4 * b^{11} - 16 * a^5 * b^9 * c + 98 * a^6 * b^7 * c^2 - 280 * \\ &a^7 * b^5 * c^3 + 352 * a^8 * b^3 * c^4 - 128 * a^9 * b * c^5) * n^5 * \sqrt{(b^8 - 8 * a * b^6 * c + \\ &20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)} / ((a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * \\ &a^{10} * b^2 * c^2 - 64 * a^{11} * c^3) * n^6)) - (b^{12} - 14 * a * b^{10} * c + 76 * a^2 * b^8 * c^2 - \\ &200 * a^3 * b^6 * c^3 + 260 * a^4 * b^4 * c^4 - 152 * a^5 * b^2 * c^5 + 32 * a^6 * c^6) * n^2) * (((a \\ &^4 * b^2 - 4 * a^5 * c) * n^3 * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c \\ &^3 + 4 * a^4 * c^4)} / ((a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3) * n \\ &^6)) + b^3 - 2 * a * b * c) / ((a^4 * b^2 - 4 * a^5 * c) * n^3))^{(2/3)} / x^2) - (1/2)^{(1/3)} * \\ &a * n * (-((a^4 * b^2 - 4 * a^5 * c) * n^3 * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * \\ &a^3 * b^2 * c^3 + 4 * a^4 * c^4)} / ((a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3) * n^6)) \\ &- b^3 + 2 * a * b * c) / ((a^4 * b^2 - 4 * a^5 * c) * n^3))^{(1/3)} * \log(8 * (2 * (b \\ &^8 * c^2 - 8 * a * b^6 * c^3 + 20 * a^2 * b^4 * c^4 - 16 * a^3 * b^2 * c^5 + 4 * a^4 * c^6) * x^2 * x^{(\\ &-2/3 * n - 2)} + (1/2)^{(1/3)} * ((a^4 * b^9 * c - 12 * a^5 * b^7 * c^2 + 50 * a^6 * b^5 * c^3 - 8 \\ &0 * a^7 * b^3 * c^4 + 32 * a^8 * b * c^5) * n^4 * x * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 \\ &- 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)} / ((a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - \\ &64 * a^{11} * c^3) * n^6)) + (b^{10} * c - 12 * a * b^8 * c^2 + 52 * a^2 * b^6 * c^3 - 96 * a^3 * b^4 * c \\ &^4 + 68 * a^4 * b^2 * c^5 - 16 * a^5 * c^6) * n * x) * x^{(-1/3 * n - 1)} * (-((a^4 * b^2 - 4 * a^5 * c \\ &) * n^3 * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)} / \\ &((a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3) * n^6)) - b^3 + 2 * a \\ &* b * c) / ((a^4 * b^2 - 4 * a^5 * c) * n^3))^{(1/3)} + (1/2)^{(2/3)} * ((a^4 * b^{11} - 16 * a^5 * b^9 * c + \\ &98 * a^6 * b^7 * c^2 - 280 * a^7 * b^5 * c^3 + 352 * a^8 * b^3 * c^4 - 128 * a^9 * b * c^5) * n \\ &^5 * \sqrt{(b^8 - 8 * a * b^6 * c + 20 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)} / ((a \\ &^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a^{10} * b^2 * c^2 - 64 * a^{11} * c^3) * n^6)) + (b^{12} - 14 * a \\ &* b^{10} * c + 76 * a^2 * b^8 * c^2 - 200 * a^3 * b^6 * c^3 + 260 * a^4 * b^4 * c^4 - 152 * a^5 * b^2 * \\ &c^5 + 32 * a^6 * c^6) * n^2) * (-((a^4 * b^2 - 4 * a^5 * c) * n^3 * \sqrt{(b^8 - 8 * a * b^6 * c + 2 \\ &0 * a^2 * b^4 * c^2 - 16 * a^3 * b^2 * c^3 + 4 * a^4 * c^4)} / ((a^8 * b^6 - 12 * a^9 * b^4 * c + 48 * a \\ &^10 * b^2 * c^2 - 64 * a^{11} * c^3) * n^6)) - b^3 + 2 * a * b * c) / ((a^4 * b^2 - 4 * a^5 * c) * n^3) \\ &)^{(2/3)} / x^2) - 6 * x * x^{(-1/3 * n - 1)} / (a * n) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{3}n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{(-1-1/3*n)}/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^{(-1/3*n - 1)}/(c*x^(2*n) + b*x^n + a), x)

maple [C] time = 0.75, size = 534, normalized size = 0.76

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x)`

[Out]
$$-3/a/n/(x^{(1/3*n)})+\text{sum}(_R*\ln(x^{(1/3*n)})+(-64/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*a^8*c^4+112/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^2*a^7*c^3-60/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^4*a^6*c^2+13/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^6*a^5*c-1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^8*a^4)*_R^5+(28/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b*a^4*c^4-63/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^3*a^3*c^3+42/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^5*a^2*c^2-11/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^7*a*c+1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^9)*_R^2), _R=\text{RootOf}((64*a^7*c^3*n^6-48*a^6*b^2*c^2*n^6+12*a^5*b^4*c*n^6-a^4*b^6*n^6)*_Z^6+(-32*a^3*b*c^3*n^3+32*a^2*b^3*c^2*n^3-10*a*b^5*c*n^3+b^7*n^3)*_Z^3+c^4))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{anx^{\frac{1}{3}n}} - \int \frac{cx^{\frac{5}{3}n} + bx^{\frac{2}{3}n}}{acxx^{2n} + abxx^n + a^2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")`

[Out]
$$-3/(a*n*x^{(1/3*n)}) - \text{integrate}((c*x^{(5/3*n)} + b*x^{(2/3*n)})/(a*c*x*x^{(2*n)} + a*b*x*x^n + a^2*x), x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{3}+1} (a + b x^n + c x^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))),x)`

[Out] `int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)`

[Out] Timed out

$$3.485 \quad \int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=414

$$\frac{2^{3/4} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{a} x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{a^{5/4} n \left(-\sqrt{b^2-4ac} - b \right)^{3/4}} - \frac{2^{3/4} \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{a} x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}} \right)}{a^{5/4} n \left(\sqrt{b^2-4ac} - b \right)^{3/4}} - \frac{2^{3/4} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{a} x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{a^{5/4} n \left(-\sqrt{b^2-4ac} - b \right)^{3/4}}$$

Rubi [A] time = 0.79, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, number of rules / integrand size = 0.269, Rules used = {1381, 1340, 1367, 1422, 212, 208, 205}

$$\frac{2^{3/4} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{a} x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{a^{5/4} n \left(-\sqrt{b^2-4ac} - b \right)^{3/4}} - \frac{2^{3/4} \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{a} x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}} \right)}{a^{5/4} n \left(\sqrt{b^2-4ac} - b \right)^{3/4}} - \frac{2^{3/4} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{a} x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{a^{5/4} n \left(-\sqrt{b^2-4ac} - b \right)^{3/4}} - \frac{2^{3/4} \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{a} x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}} \right)}{a^{5/4} n \left(\sqrt{b^2-4ac} - b \right)^{3/4}} - \frac{4x^{-n/4}}{an}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)), x]

[Out]
$$-4/(a*n*x^{(n/4)}) - (2^{(3/4)}*(b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*a^{(1/4)})/((-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}*x^{(n/4)})])/(a^{(5/4)}*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}*n) - (2^{(3/4)}*(b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*a^{(1/4)})/((-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}*x^{(n/4)})])/(a^{(5/4)}*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}*n) - (2^{(3/4)}*(b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{(1/4)}*a^{(1/4)})/((-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}*x^{(n/4)})])/(a^{(5/4)}*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}*n) - (2^{(3/4)}*(b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{(1/4)}*a^{(1/4)})/((-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}*x^{(n/4)})])/(a^{(5/4)}*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}*n)$$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],

$x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{!GtQ}[a/b, 0]$

Rule 1340

$\text{Int}[\{(a_)+(c_)*(x_)^{(n2_)}+(b_)*(x_)^{(n_)}\}^{(p_)}, x_Symbol] \rightarrow \text{Int}[x^{(2*n*p)}*(c + b/x^n + a/x^{(2*n)})^p, x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{LtQ}[n, 0] \&\& \text{IntegerQ}[p]$

Rule 1367

$\text{Int}[\{(d_)*(x_)^{(m_)}\}*(\{(a_)+(c_)*(x_)^{(n2_)}+(b_)*(x_)^{(n_)}\}^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(d^{(2*n-1)}*(d*x)^{(m-2*n+1)}*(a + b*x^n + c*x^{(2*n)})^{(p+1)})/(c*(m + 2*n*p + 1)), x] - \text{Dist}[d^{(2*n)}/(c*(m + 2*n*p + 1)), \text{Int}[(d*x)^{(m-2*n)}*\text{Simp}[a*(m-2*n+1) + b*(m+n*(p-1)+1)*x^n, x]*(a + b*x^n + c*x^{(2*n)})^p, x], x] /; \text{FreeQ}[\{a, b, c, d, p\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, 2*n-1] \&\& \text{NeQ}[m + 2*n*p + 1, 0] \&\& \text{IntegerQ}[p]$

Rule 1381

$\text{Int}[(x_)^{(m_)}*(\{(a_)+(c_)*(x_)^{(n2_)}+(b_)*(x_)^{(n_)}\}^{(p_)}), x_Symbol] \rightarrow \text{Dist}[1/(m+1), \text{Subst}[\text{Int}[(a + b*x^{\text{Simplify}[n/(m+1)]} + c*x^{\text{Simplify}[(2*n)/(m+1)])^p, x], x, x^{(m+1)}], x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IntegerQ}[\text{Simplify}[n/(m+1)]] \&\& \text{!IntegerQ}[n]$

Rule 1422

$\text{Int}[\{(d_)+(e_)*(x_)^{(n_)}\}/\{(a_)+(b_)*(x_)^{(n_)}+(c_)*(x_)^{(n2_)}\}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[e/2 + (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 - q/2 + c*x^n), x], x] + \text{Dist}[e/2 - (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 + q/2 + c*x^n), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& (\text{PosQ}[b^2 - 4*a*c] || \text{!IGtQ}[n/2, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{4}}}{a + bx^n + cx^{2n}} dx &= -\frac{4 \operatorname{Subst}\left(\int \frac{1}{a + \frac{c}{x^8} + \frac{b}{x^4}} dx, x, x^{-n/4}\right)}{n} \\
&= -\frac{4 \operatorname{Subst}\left(\int \frac{x^8}{c + bx^4 + ax^8} dx, x, x^{-n/4}\right)}{n} \\
&= -\frac{4x^{-n/4}}{an} + \frac{4 \operatorname{Subst}\left(\int \frac{c + bx^4}{c + bx^4 + ax^8} dx, x, x^{-n/4}\right)}{an} \\
&= -\frac{4x^{-n/4}}{an} + \frac{\left(2\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + ax^4} dx, x, x^{-n/4}\right)}{an} + \frac{\left(2\left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + ax^4} dx, x, x^{-n/4}\right)}{an} \\
&= -\frac{4x^{-n/4}}{an} - \frac{\left(2\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{-b + \sqrt{b^2 - 4ac}} - \sqrt{2}\sqrt{a}x^2} dx, x, x^{-n/4}\right)}{an} - \frac{\left(2\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right)\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{-b + \sqrt{b^2 - 4ac}} + \sqrt{2}\sqrt{a}x^2} dx, x, x^{-n/4}\right)}{an} \\
&= -\frac{4x^{-n/4}}{an} - \frac{2^{3/4}\left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt{-b - \sqrt{b^2 - 4ac}}}\right)}{a^{5/4}\left(-b - \sqrt{b^2 - 4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{-n/4}}{\sqrt{-b + \sqrt{b^2 - 4ac}}}\right)}{a^{5/4}\left(-b + \sqrt{b^2 - 4ac}\right)^{3/4}n}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 127, normalized size = 0.31

$$\frac{8cx^{-n/4} \left(\frac{{}_2F_1\left(-\frac{1}{4}, 1, \frac{3}{4}; \frac{2cx^n}{\sqrt{b^2 - 4ac} - b}\right)}{-b\sqrt{b^2 - 4ac} - 4ac + b^2} + \frac{{}_2F_1\left(-\frac{1}{4}, 1, \frac{3}{4}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)}{b\sqrt{b^2 - 4ac} - 4ac + b^2} \right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)), x]

[Out] (8*c*(Hypergeometric2F1[-1/4, 1, 3/4, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + Hypergeometric2F1[-1/4, 1, 3/4, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]/(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c]))/(n*x^(n/4))

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^{-1-\frac{n}{4}}}{a + bx^n + cx^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]

[Out] Defer[IntegrateAlgebraic][x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)), x]

fricas [B] time = 4.18, size = 5712, normalized size = 13.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out]
$$\frac{1}{2} \cdot (4 \cdot \sqrt{2}) \cdot a \cdot n \cdot \sqrt{\sqrt{2} \cdot \sqrt{((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4 \cdot \sqrt{(b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) / ((a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3) \cdot n^8)) - b^5 + 5 a b^3 c - 5 a^2 b c^2} / ((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4))} \cdot \arctan\left(\frac{1}{16} \cdot \sqrt{2} \cdot (2 \cdot \sqrt{2}) \cdot ((a^5 b^{14} c - 19 a^6 b^{12} c^2 + 147 a^7 b^{10} c^3 - 590 a^8 b^8 c^4 + 1290 a^9 b^6 c^5 - 1464 a^{10} b^4 c^6 + 736 a^{11} b^2 c^7 - 128 a^{12} c^8) \cdot n^7 \cdot x \cdot \sqrt{(b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) / ((a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3) \cdot n^8)}) + (b^{15} c - 16 a b^{13} c^2 + 103 a^2 b^{11} c^3 - 340 a^3 b^9 c^4 + 605 a^4 b^7 c^5 - 554 a^5 b^5 c^6 + 224 a^6 b^3 c^7 - 32 a^7 b c^8) \cdot n^3 \cdot x) \cdot x^{-1/4 \cdot n - 1} \cdot \sqrt{((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4 \cdot \sqrt{(b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) / ((a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3) \cdot n^8)}) - b^5 + 5 a b^3 c - 5 a^2 b c^2} / ((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4)} - \sqrt{2} \cdot ((a^5 b^{10} - 16 a^6 b^8 c + 98 a^7 b^6 c^2 - 280 a^8 b^4 c^3 + 352 a^9 b^2 c^4 - 128 a^{10} c^5) \cdot n^7 \cdot x \cdot \sqrt{(b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) / ((a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3) \cdot n^8)}) + (b^{11} - 13 a b^9 c + 63 a^2 b^7 c^2 - 138 a^3 b^5 c^3 + 128 a^4 b^3 c^4 - 32 a^5 b c^5) \cdot n^3 \cdot x) \cdot \sqrt{(4 \cdot (b^8 c^2 - 6 a b^6 c^3 + 11 a^2 b^4 c^4 - 6 a^3 b^2 c^5 + a^4 c^6) \cdot x^2 \cdot x^{-1/2 \cdot n - 2} + \sqrt{2} \cdot ((a^5 b^{11} - 15 a^6 b^9 c + 85 a^7 b^7 c^2 - 220 a^8 b^5 c^3 + 240 a^9 b^3 c^4 - 64 a^{10} b c^5) \cdot n^6 \cdot \sqrt{(b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) / ((a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3) \cdot n^8)}) + (b^{12} - 12 a b^{10} c + 55 a^2 b^8 c^2 - 120 a^3 b^6 c^3 + 125 a^4 b^4 c^4 - 54 a^5 b^2 c^5 + 8 a^6 c^6) \cdot n^2) \cdot \sqrt{((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4 \cdot \sqrt{(b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) / ((a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3) \cdot n^8)}) - b^5 + 5 a b^3 c - 5 a^2 b c^2} / ((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4)} \cdot \sqrt{((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4 \cdot \sqrt{(b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) / ((a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3) \cdot n^8)}) - b^5 + 5 a b^3 c - 5 a^2 b c^2} / ((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4)} \cdot \sqrt{\sqrt{2} \cdot \sqrt{((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4 \cdot \sqrt{(b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) / ((a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3) \cdot n^8)}) - b^5 + 5 a b^3 c - 5 a^2 b c^2} / ((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4)} \cdot \sqrt{\sqrt{2} \cdot \sqrt{((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4 \cdot \sqrt{(b^8 - 6 a b^6 c + 11 a^2 b^4 c^2 - 6 a^3 b^2 c^3 + a^4 c^4) / ((a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3) \cdot n^8)}) - b^5 + 5 a b^3 c - 5 a^2 b c^2} / ((a^5 b^4 - 8 a^6 b^2 c + 16 a^7 c^2) \cdot n^4)}}$$

$$\begin{aligned}
& ^{12}b^2c^2 - 64a^{13}c^3)n^8)) - b^5 + 5a^6b^3c - 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)))/(b^8c^5 - 6a^6b^6c^6 + 11a^2b^4c^7 - 6a^3b^2c^8 + a^4c^9)) - 4\sqrt{2}a^n\sqrt{\sqrt{2}\sqrt{-(a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4}\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8))} + b^5 - 5a^6b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4))\arctan(1/8*(2*((a^5b^{14}c - 19a^6b^{12}c^2 + 147a^7b^{10}c^3 - 590a^8b^8c^4 + 1290a^9b^6c^5 - 1464a^{10}b^4c^6 + 736a^{11}b^2c^7 - 128a^{12}c^8)n^7x\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)) - (b^{15}c - 16a^6b^{13}c^2 + 103a^2b^{11}c^3 - 340a^3b^9c^4 + 605a^4b^7c^5 - 554a^5b^5c^6 + 224a^6b^3c^7 - 32a^7b^2c^8)n^3x)x^{(-1/4)n - 1})\sqrt{\sqrt{2}\sqrt{-(a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4}\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8))} + b^5 - 5a^6b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4))\sqrt{-(a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4}\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8))} + b^5 - 5a^6b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)) - ((a^5b^{10} - 16a^6b^8c + 98a^7b^6c^2 - 280a^8b^4c^3 + 352a^9b^2c^4 - 128a^{10}c^5)n^7x\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)) - (b^{11} - 13a^6b^9c + 63a^2b^7c^2 - 138a^3b^5c^3 + 128a^4b^3c^4 - 32a^5b^2c^5)n^3x)\sqrt{\sqrt{2}\sqrt{-(a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4}\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8))} + b^5 - 5a^6b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4))\sqrt{(4*(b^8c^2 - 6a^6b^6c^3 + 11a^2b^4c^4 - 6a^3b^2c^5 + a^4c^6)x^2x^{(-1/2)n - 2} - \sqrt{2}*(a^5b^{11} - 15a^6b^9c + 85a^7b^7c^2 - 220a^8b^5c^3 + 240a^9b^3c^4 - 64a^{10}b^2c^5)n^6\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)) - (b^{12} - 12a^6b^10c + 55a^2b^8c^2 - 120a^3b^6c^3 + 125a^4b^4c^4 - 54a^5b^2c^5 + 8a^6c^6)n^2)\sqrt{-(a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4}\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8))} + b^5 - 5a^6b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)))/x^2)\sqrt{-(a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4}\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8))} + b^5 - 5a^6b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)))/(b^8c^5 - 6a^6b^6c^6 + 11a^2b^4c^7 - 6a^3b^2c^8 + a^4c^9)) + \sqrt{2}a^n\sqrt{\sqrt{2}\sqrt{-(a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4}\sqrt{(b^8 - 6a^6b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/(a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8))} + b^5 - 5a^6b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4))\log((4*(b
\end{aligned}$$

$$\begin{aligned}
& ^4c - 3ab^2c^2 + a^2c^3)xx^{(-1/4n - 1)} + \sqrt{2}*((a^5b^5 - 8a^6b^3c + 16a^7b^2c^2)n^5\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} - (b^6 - 7a^2b^4c + 13a^2b^2c^2 - 4a^3c^3)n)\sqrt{\sqrt{2}\sqrt{-((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} + b^5 - 5a^2b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)}})/x) - \sqrt{2}a^n\sqrt{\sqrt{2}\sqrt{-((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} + b^5 - 5a^2b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)}})*\log((4*(b^4c - 3a^2b^2c^2 + a^2c^3)xx^{(-1/4n - 1)} - \sqrt{2}*((a^5b^5 - 8a^6b^3c + 16a^7b^2c^2)n^5\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} - (b^6 - 7a^2b^4c + 13a^2b^2c^2 - 4a^3c^3)n)\sqrt{\sqrt{2}\sqrt{-((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} + b^5 - 5a^2b^3c + 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)}})/x) - \sqrt{2}a^n\sqrt{\sqrt{2}\sqrt{((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} - b^5 + 5a^2b^3c - 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)}})*\log((4*(b^4c - 3a^2b^2c^2 + a^2c^3)xx^{(-1/4n - 1)} + \sqrt{2}*((a^5b^5 - 8a^6b^3c + 16a^7b^2c^2)n^5\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} + (b^6 - 7a^2b^4c + 13a^2b^2c^2 - 4a^3c^3)n)\sqrt{\sqrt{2}\sqrt{((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} - b^5 + 5a^2b^3c - 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)}})/x) + \sqrt{2}a^n\sqrt{\sqrt{2}\sqrt{((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} - b^5 + 5a^2b^3c - 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)}})*\log((4*(b^4c - 3a^2b^2c^2 + a^2c^3)xx^{(-1/4n - 1)} - \sqrt{2}*((a^5b^5 - 8a^6b^3c + 16a^7b^2c^2)n^5\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} + (b^6 - 7a^2b^4c + 13a^2b^2c^2 - 4a^3c^3)n)\sqrt{\sqrt{2}\sqrt{((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4\sqrt{(b^8 - 6a^2b^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4)/((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)n^8)} - b^5 + 5a^2b^3c - 5a^2b^2c^2)/((a^5b^4 - 8a^6b^2c + 16a^7c^2)n^4)}})/x) - 8xx^{(-1/4n - 1))/(a^n)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{4}n-1}}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)

maple [C] time = 1.22, size = 630, normalized size = 1.52

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x)

[Out]
$$-4/a/n/(x^{(1/4*n)})+\text{sum}(_R*\ln(x^{(1/4*n)})+(-128/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*a^{10}*c^5+352/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^2*a^9*c^4-280/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^4*a^8*c^3+98/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^6*a^7*c^2-16/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^8*a^6*c+1/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^{10}*a^5)*_R^7+(-36/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b*a^5*c^5+129/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^3*a^4*c^4-138/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^5*a^3*c^3+63/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^7*a^2*c^2-13/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^9*a*c+1/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^{11})*_R^3), _R=\text{RootOf}((256*a^9*c^4*n^8-256*a^8*b^2*c^3*n^8+96*a^7*b^4*c^2*n^8-16*a^6*b^6*c*n^8+a^5*b^8*n^8)*_Z^8+(80*a^4*b*c^4*n^4-120*a^3*b^3*c^3*n^4+61*a^2*b^5*c^2*n^4-13*a*b^7*c*n^4+b^9*n^4)*_Z^4+c^5))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4}{anx^{\frac{1}{4}n}} - \int \frac{cx^{\frac{7}{4}n} + bx^{\frac{3}{4}n}}{acxx^{2n} + abxx^n + a^2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out]
$$-4/(a*n*x^{(1/4*n)}) - \text{integrate}((c*x^{(7/4*n)} + b*x^{(3/4*n)})/(a*c*x*x^{(2*n)} + a*b*x*x^n + a^2*x), x)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{4}+1} (a + b x^n + c x^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

```
[Out] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)), x)
```

```
[Out] Timed out
```


$$3.486 \quad \int \frac{1}{x(a+bx^n+cx^{2n})} dx$$

Optimal. Leaf size=74

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}} - \frac{\log(a+bx^n+cx^{2n})}{2an} + \frac{\log(x)}{a}$$

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1357, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}} - \frac{\log(a+bx^n+cx^{2n})}{2an} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^n + c*x^(2*n))),x]

[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]]/(a*Sqrt[b^2 - 4*a*c]*n) + Log[x]/a - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a + bx^n + cx^{2n})} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, x^n\right)}{an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2} dx, x, x^n\right)}{an} \\
&= \frac{\log(x)}{a} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^n\right)}{2an} - \frac{b \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, x^n\right)}{2an} \\
&= \frac{\log(x)}{a} - \frac{\log(a + bx^n + cx^{2n})}{2an} + \frac{b \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^n\right)}{an} \\
&= \frac{b \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}n} + \frac{\log(x)}{a} - \frac{\log(a + bx^n + cx^{2n})}{2an}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 74, normalized size = 1.00

$$\frac{2b \tan^{-1}\left(\frac{b+2cx^n}{\sqrt{4ac-b^2}}\right)}{n\sqrt{4ac-b^2}} + \frac{\log(a+x^n(b+cx^n))}{n} - 2 \log(x)$$

$$2a$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))), x]

[Out] -1/2*((2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*n) - 2*Log[x] + Log[a + x^n*(b + c*x^n)]/n)/a

IntegrateAlgebraic [A] time = 0.15, size = 98, normalized size = 1.32

$$-\frac{b \tan^{-1}\left(\frac{2cx^n}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}}\right)}{an\sqrt{4ac-b^2}} - \frac{\log(a + bx^n + cx^{2n})}{2an} + \frac{\log(x^n)}{an}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^n + c*x^(2*n))), x]

[Out] -((b*ArcTan[b/Sqrt[-b^2 + 4*a*c] + (2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(a*Sqrt[-b^2 + 4*a*c]*n)) + Log[x^n]/(a*n) - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)

fricas [A] time = 1.01, size = 259, normalized size = 3.50

$$\left[\frac{2(b^2 - 4ac)n \log(x) + \sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^{2n} + b^2 - 2ac + 2\left(\sqrt{b^2 - 4ac}c\right)^n + \sqrt{b^2 - 4ac}b}{cx^{2n} + bx^n + a}\right) - (b^2 - 4ac) \log(cx^{2n} + bx^n + a)}{2(ab^2 - 4a^2c)n}, \frac{2(b^2 - 4ac)n \log(x) + 2\sqrt{-b^2 + 4ac} b \arctan\left(\frac{2\sqrt{-b^2 + 4ac}cx^n + \sqrt{-b^2 + 4ac}b}{b^2 - 4ac}\right) - (b^2 - 4ac) \log(cx^{2n} + bx^n + a)}{2(ab^2 - 4a^2c)n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] [1/2*(2*(b^2 - 4*a*c)*n*log(x) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) - (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4*a^2*c)*n), 1/2*(2*(b^2 - 4*a*c)*n*log(x) + 2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4*a^2*c)*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^{2n} + bx^n + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)

maple [B] time = 0.10, size = 397, normalized size = 5.36

$$\frac{4acn^2 \ln(x)}{4a^2cn^2 - ab^2n^2} - \frac{b^2n^2 \ln(x)}{4a^2cn^2 - ab^2n^2} + \frac{b^2 \ln\left(x^n - \frac{-b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{2(4ac - b^2)an} + \frac{b^2 \ln\left(x^n + \frac{b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{2(4ac - b^2)an} - \frac{2c \ln\left(x^n - \frac{-b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{(4ac - b^2)n} - \frac{2c \ln\left(x^n + \frac{b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{(4ac - b^2)n} + \frac{\sqrt{-4ab^2c + b^4} \ln\left(x^n - \frac{-b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{2(4ac - b^2)an} - \frac{\sqrt{-4ab^2c + b^4} \ln\left(x^n + \frac{b^2 + \sqrt{-4ab^2c + b^4}}{2bc}\right)}{2(4ac - b^2)an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^n+c*x^(2*n)+a),x)

[Out]
$$\frac{4}{(4a^2c*n^2 - a*b^2*n^2)*n^2*\ln(x)*a*c - 1/(4a^2c*n^2 - a*b^2*n^2)*n^2*\ln(x)*b^2 - 2/(4a*c - b^2)/n*\ln(x^n - 1/2*(-b^2 + (-4a*b^2*c + b^4)^(1/2))/b/c)*c + 1/2/a/(4a*c - b^2)/n*\ln(x^n - 1/2*(-b^2 + (-4a*b^2*c + b^4)^(1/2))/b/c)*b^2 + 1/2/a/(4a*c - b^2)/n*\ln(x^n - 1/2*(-b^2 + (-4a*b^2*c + b^4)^(1/2))/b/c)*(-4a*b^2*c + b^4)^(1/2) - 2/(4a*c - b^2)/n*\ln(x^n + 1/2*(b^2 + (-4a*b^2*c + b^4)^(1/2))/b/c)*c + 1/2/a/(4a*c - b^2)/n*\ln(x^n + 1/2*(b^2 + (-4a*b^2*c + b^4)^(1/2))/b/c)*b^2 - 1/2/a/(4a*c - b^2)/n*\ln(x^n + 1/2*(b^2 + (-4a*b^2*c + b^4)^(1/2))/b/c)*(-4a*b^2*c + b^4)^(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^{2n} + bx^n + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)

mupad [B] time = 1.61, size = 224, normalized size = 3.03

$$\frac{\ln\left(\frac{-1}{cx} - \frac{(2an + bn^2)(4ac + b\sqrt{b^2 - 4ac - b^2})}{2cx(a^2n - 4a^2cn)}\right)(4ac + b\sqrt{b^2 - 4ac - b^2})}{2(a^2n - 4a^2cn)} - \frac{\ln\left(\frac{(2an + bn^2)(b\sqrt{b^2 - 4ac - 4ac + b^2})}{2cx(a^2n - 4a^2cn)} - \frac{1}{cx}\right)(b\sqrt{b^2 - 4ac - 4ac + b^2})}{2(a^2n - 4a^2cn)} + \frac{\ln(x)(n-1)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^n + c*x^(2*n))),x)

[Out]
$$\frac{\log(-1/(c*x) - ((2*a*n + b*n*x^n)*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/(2*c*x*(a*b^2*n - 4*a^2*c*n)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2)}{(2*(a*b^2*n - 4*a^2*c*n))} - \frac{\log(((2*a*n + b*n*x^n)*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2))/(2*c*x*(a*b^2*n - 4*a^2*c*n)) - 1/(c*x))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2)}{(2*(a*b^2*n - 4*a^2*c*n))} + \frac{\log(x)*(n-1)}{(a*n)}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out

$$3.487 \quad \int \frac{\sqrt{a+bx^n+cx^{2n}}}{x} dx$$

Optimal. Leaf size=119

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b \tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1357, 734, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b \tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]

[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n - (Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])])/n + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/(2*Sqrt[c]*n)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

```

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x]
- Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]
*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
&& GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x]
+ Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1357

```

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x]
/; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
&& IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x} dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x} dx, x, x^n\right)}{n} \\
&= \frac{\sqrt{a + bx^n + cx^{2n}}}{n} - \frac{\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}} dx, x, x^n\right)}{2n} \\
&= \frac{\sqrt{a + bx^n + cx^{2n}}}{n} + \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^n\right)}{n} + \frac{b \text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^n\right)}{2n} \\
&= \frac{\sqrt{a + bx^n + cx^{2n}}}{n} - \frac{(2a) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n} \\
&= \frac{\sqrt{a + bx^n + cx^{2n}}}{n} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b \tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}n}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 110, normalized size = 0.92

$$\frac{\sqrt{a + x^n(b + cx^n)} - \sqrt{a} \tanh^{-1}\left(\frac{2a + bx^n}{2\sqrt{a}\sqrt{a + x^n(b + cx^n)}}\right) + \frac{b \tanh^{-1}\left(\frac{b + 2cx^n}{2\sqrt{c}\sqrt{a + x^n(b + cx^n)}}\right)}{2\sqrt{c}}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]

[Out] (Sqrt[a + x^n*(b + c*x^n)] - Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + x^n*(b + c*x^n)])] + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)])]))/(2*Sqrt[c])/n

IntegrateAlgebraic [A] time = 0.35, size = 125, normalized size = 1.05

$$\frac{\sqrt{a + bx^n + cx^{2n}}}{n} - \frac{b \log\left(-2\sqrt{cn}\sqrt{a + bx^n + cx^{2n}} + bn + 2cnx^n\right)}{2\sqrt{cn}} + \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{c}x^n}{\sqrt{a}} - \frac{\sqrt{a + bx^n + cx^{2n}}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]

[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n + (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x^n)/Sqrt[a] - Sqrt[a + b*x^n + c*x^(2*n)]/Sqrt[a]])/n - (b*Log[b*n + 2*c*n*x^n - 2*Sqrt[c]*n*Sqrt[a + b*x^n + c*x^(2*n)]])/(2*Sqrt[c]*n)

fricas [A] time = 1.61, size = 658, normalized size = 5.53

$$\frac{1}{4} \sqrt{c} \log(-8c^2x^{2n} - 8bcx^n - b^2 - 4ac - 4(2c^{3/2})x^n + b\sqrt{c})\sqrt{cx^{2n} + bx^n + a} + 2\sqrt{a}c \log(-8abx^n + 8a^2 + (b^2 + 4ac)x^{2n} - 4(\sqrt{a}bx^n + 2a^{3/2})\sqrt{cx^{2n} + bx^n + a})/x^{2n} + 4\sqrt{cx^{2n} + bx^n + a}c/(cn), -1/2 * (b\sqrt{-c})\arctan(1/2(2\sqrt{-c})cx^n + b\sqrt{-c})\sqrt{cx^{2n} + bx^n + a}/(c^2x^{2n} + bcx^n + ac) - \sqrt{a}c \log(-8abx^n + 8a^2 + (b^2 + 4ac)x^{2n} - 4(\sqrt{a}bx^n + 2a^{3/2})\sqrt{cx^{2n} + bx^n + a})/x^{2n} - 2\sqrt{cx^{2n} + bx^n + a}c/(cn), 1/4(4\sqrt{-a}c\arctan(1/2(\sqrt{-a}bx^n + 2\sqrt{-a}a)\sqrt{cx^{2n} + bx^n + a})/(acx^{2n} + abx^n + a^2)) + b\sqrt{c}\log(-8c^2x^{2n} - 8bcx^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="fricas")

[Out] [1/4*(b*sqrt(c))*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2))*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a) + 2*sqrt(a)*c*log(-8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c/(c*n), -1/2*(b*sqrt(-c))*arctan(1/2*(2*sqrt(-c))*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c) - sqrt(a)*c*log(-8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n) - 2*sqrt(c*x^(2*n) + b*x^n + a)*c/(c*n), 1/4*(4*sqrt(-a)*c*arctan(1/2*(sqrt(-a))*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n

- b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/2*(2*sqrt(-a)*c*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)

maple [A] time = 0.12, size = 125, normalized size = 1.05

$$-\frac{\sqrt{a} \ln\left(\left(b e^{n \ln(x)} + 2a + 2\sqrt{b e^{n \ln(x)} + c e^{2n \ln(x)} + a} \sqrt{a}\right) e^{-n \ln(x)}\right)}{n} + \frac{b \ln\left(\frac{c e^{n \ln(x)} + \frac{b}{2}}{\sqrt{c}} + \sqrt{b e^{n \ln(x)} + c e^{2n \ln(x)} + a}\right)}{2\sqrt{c} n} + \frac{\sqrt{b e^{n \ln(x)} + c e^{2n \ln(x)} + a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^n+c*x^(2*n)+a)^(1/2)/x,x)

[Out] 1/n*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)+1/2/n*b*ln((1/2*b+c*exp(n*ln(x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/c^(1/2)-1/n*a^(1/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/exp(n*ln(x))))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2n} + bx^n + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b x^n + c x^{2n}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^n + c*x^(2*n))^(1/2)/x, x)`

[Out] `int((a + b*x^n + c*x^(2*n))^(1/2)/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n+c*x**(2*n))**(1/2)/x, x)`

[Out] `Integral(sqrt(a + b*x**n + c*x**(2*n))/x, x)`

$$3.488 \quad \int \frac{(a+bx^n+cx^{2n})^{3/2}}{x} dx$$

Optimal. Leaf size=173

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} - \frac{b(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n} + \frac{(8ac+b^2+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn}$$

Rubi [A] time = 0.16, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1357, 734, 814, 843, 621, 206, 724}

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n} - \frac{b(b^2-12ac) \tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n} + \frac{(8ac+b^2+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x, x]

[Out] ((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*x^n + c*x^(2*n)])/(8*c*n) + (a + b*x^n + c*x^(2*n))^(3/2)/(3*n) - (a^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])])/n - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[a + b*x^n + c*x^(2*n)])])/(16*c^(3/2)*n)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

```

Int[((d._) + (e._)*(x_))^(m_)*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
&& GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0]
&& IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 814

```

Int[((d._) + (e._)*(x_))^(m_)*((f._) + (g._)*(x_))*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 843

```

Int[((d._) + (e._)*(x_))^(m_)*((f._) + (g._)*(x_))*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1357

```

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x]
&& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^n + cx^{2n})^{3/2}}{x} dx &= \frac{\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x} dx, x, x^n\right)}{n} \\
&= \frac{(a + bx^n + cx^{2n})^{3/2}}{3n} - \frac{\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x} dx, x, x^n\right)}{2n} \\
&= \frac{(b^2 + 8ac + 2bcx^n)\sqrt{a + bx^n + cx^{2n}}}{8cn} + \frac{(a + bx^n + cx^{2n})^{3/2}}{3n} + \frac{\text{Subst}\left(\int \frac{8a^2c - \frac{1}{2}b(b^2-1)}{x\sqrt{a+bx+cx^2}} dx, x, x^n\right)}{8cn} \\
&= \frac{(b^2 + 8ac + 2bcx^n)\sqrt{a + bx^n + cx^{2n}}}{8cn} + \frac{(a + bx^n + cx^{2n})^{3/2}}{3n} + \frac{a^2 \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^n\right)}{n} \\
&= \frac{(b^2 + 8ac + 2bcx^n)\sqrt{a + bx^n + cx^{2n}}}{8cn} + \frac{(a + bx^n + cx^{2n})^{3/2}}{3n} - \frac{(2a^2) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, x^n\right)}{n} \\
&= \frac{(b^2 + 8ac + 2bcx^n)\sqrt{a + bx^n + cx^{2n}}}{8cn} + \frac{(a + bx^n + cx^{2n})^{3/2}}{3n} - \frac{a^{3/2} \tanh^{-1}\left(\frac{2a+b}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 158, normalized size = 0.91

$$\frac{-48a^{3/2}c^{3/2} \tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+x^n(b+cx^n)}}\right) - 3b(b^2 - 12ac) \tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+x^n(b+cx^n)}}\right) + 2\sqrt{c}\sqrt{a+x^n(b+cx^n)}(8c(4a+cx^{2n}) + 3b^2 + 14bcx^n)}{48c^{3/2}n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x, x]

[Out] (2*sqrt[c]*sqrt[a + x^n*(b + c*x^n)]*(3*b^2 + 14*b*c*x^n + 8*c*(4*a + c*x^(2*n))) - 48*a^(3/2)*c^(3/2)*ArcTanh[(2*a + b*x^n)/(2*sqrt[a]*sqrt[a + x^n*(b + c*x^n)])] - 3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*sqrt[c]*sqrt[a + x^n*(b + c*x^n)])])/(48*c^(3/2)*n)

IntegrateAlgebraic [A] time = 0.89, size = 169, normalized size = 0.98

$$\frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}x^n}{\sqrt{a}} - \frac{\sqrt{a+bx^n+cx^{2n}}}{\sqrt{a}}\right)}{n} + \frac{(b^3 - 12abc) \log\left(-2c^{3/2}n\sqrt{a + bx^n + cx^{2n}} + bcn + 2c^2nx^n\right)}{16c^{3/2}n} + \frac{\sqrt{a + bx^n + cx^{2n}}(32ac + 3b^2 + 14bcx^n + 8c^2x^{2n})}{24cn}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^n + c*x^(2*n))^(3/2)/x, x]

```
[Out] (Sqrt[a + b*x^n + c*x^(2*n)]*(3*b^2 + 32*a*c + 14*b*c*x^n + 8*c^2*x^(2*n)))
/(24*c*n) + (2*a^(3/2)*ArcTanh[(Sqrt[c]*x^n)/Sqrt[a] - Sqrt[a + b*x^n + c*x
^(2*n)]/Sqrt[a]])/n + ((b^3 - 12*a*b*c)*Log[b*c*n + 2*c^2*n*x^n - 2*c^(3/2)
*n*Sqrt[a + b*x^n + c*x^(2*n)]])/(16*c^(3/2)*n)
```

fricas [A] time = 1.80, size = 827, normalized size = 4.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(
sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) - 3*(b^3 -
12*a*b*c)*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3
/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b
*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(
24*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)
*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 3*(b^3 - 12*a*b
*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*c^2*x^n +
3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/96*(96*sqrt(-a)
)*a*c^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n +
a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*
x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^
(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)
*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(48*sqrt(-a)*a*c^2*arctan(1/2*(
sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) +
a*b*x^n + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n +
b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2
*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n
+ a))/(c^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

maple [A] time = 0.05, size = 209, normalized size = 1.21

$$\frac{a^{\frac{3}{2}} \ln\left(\frac{b e^{n \ln(x)} + 2a + 2\sqrt{b e^{n \ln(x)} + c e^{2n \ln(x)} + a} \sqrt{a}}{n}\right) e^{-n \ln(x)} + \frac{3ab \ln\left(\frac{c e^{n \ln(x)} + \frac{b}{\sqrt{c}} + \sqrt{b e^{n \ln(x)} + c e^{2n \ln(x)} + a}}{\sqrt{c}}\right)}{4\sqrt{c} n} - \frac{b^3 \ln\left(\frac{c e^{n \ln(x)} + \frac{b}{\sqrt{c}} + \sqrt{b e^{n \ln(x)} + c e^{2n \ln(x)} + a}}{\sqrt{c}}\right)}{16c^{\frac{3}{2}} n} + \frac{(14bc e^{n \ln(x)} + 8c^2 e^{2n \ln(x)} + 32ac + 3b^2) \sqrt{b e^{n \ln(x)} + c e^{2n \ln(x)} + a}}{24cn}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^n+c*x^(2*n)+a)^(3/2)/x,x)

[Out] 1/24*(8*c^2*exp(n*ln(x))^2+14*b*c*exp(n*ln(x))+32*a*c+3*b^2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)/c/n+3/4/c^(1/2)/n*a*b*ln((c*exp(n*ln(x))+1/2*b)/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/16/c^(3/2)/n*b^3*ln((c*exp(n*ln(x))+1/2*b)/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/n*a^(3/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/exp(n*ln(x))))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + bx^n + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b x^n + c x^{2n})^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x,x)

[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b x^n + c x^{2n})^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)

[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x, x)

$$3.489 \quad \int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}} dx$$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1357, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}} dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^n\right)}{n}$$

$$= -\frac{2 \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]), x]

[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n))

IntegrateAlgebraic [A] time = 0.17, size = 51, normalized size = 1.09

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c}x^n}{\sqrt{a}} - \frac{\sqrt{a+bx^n+cx^{2n}}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]), x]

[Out] (2*ArcTanh[(Sqrt[c]*x^n)/Sqrt[a] - Sqrt[a + b*x^n + c*x^(2*n)]/Sqrt[a]]/(Sqrt[a]*n))

fricas [A] time = 1.34, size = 148, normalized size = 3.15

$$\left[\frac{\log\left(\frac{8abx^n+8a^2+(b^2+4ac)x^{2n}-4\left(\sqrt{a}bx^n+2a^{\frac{3}{2}}\right)\sqrt{cx^{2n}+bx^n+a}}{x^{2n}}\right)}{2\sqrt{a}n}, \frac{\sqrt{-a} \arctan\left(\frac{(\sqrt{-a}bx^n+2\sqrt{-a}a)\sqrt{cx^{2n}+bx^n+a}}{2(acx^{2n}+abx^n+a^2)}\right)}{an} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n))/(sqrt(a)*n), sqrt(-a)*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2))/(a*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2n} + bx^n + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + cx^{2n} + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^n+c*x^(2*n)+a)^(1/2),x)

[Out] int(1/x/(b*x^n+c*x^(2*n)+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^{2n} + bx^n + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\sqrt{a + bx^n + cx^{2n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)), x)`

[Out] `int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + bx^n + cx^{2n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2), x)`

[Out] `Integral(1/(x*sqrt(a + b*x**n + c*x**(2*n))), x)`

$$3.490 \quad \int \frac{1}{x(a+bx^n+cx^{2n})^{3/2}} dx$$

Optimal. Leaf size=98

$$\frac{2(-2ac + b^2 + bcx^n)}{an(b^2 - 4ac)\sqrt{a + bx^n + cx^{2n}}} - \frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1357, 740, 12, 724, 206}

$$\frac{2(-2ac + b^2 + bcx^n)}{an(b^2 - 4ac)\sqrt{a + bx^n + cx^{2n}}} - \frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*sqrt[a + b*x^n + c*x^(2*n)]) - ArcTanh[(2*a + b*x^n)/(2*sqrt[a]*sqrt[a + b*x^n + c*x^(2*n)])]/(a^(3/2)*n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 740

```

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 1357

```

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a + bx^n + cx^{2n})^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}} dx, x, x^n\right)}{n} \\
&= \frac{2(b^2 - 2ac + bcx^n)}{a(b^2 - 4ac)n\sqrt{a + bx^n + cx^{2n}}} - \frac{2 \text{Subst}\left(\int \frac{-\frac{b^2}{2} + 2ac}{x\sqrt{a+bx+cx^2}} dx, x, x^n\right)}{a(b^2 - 4ac)n} \\
&= \frac{2(b^2 - 2ac + bcx^n)}{a(b^2 - 4ac)n\sqrt{a + bx^n + cx^{2n}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^n\right)}{an} \\
&= \frac{2(b^2 - 2ac + bcx^n)}{a(b^2 - 4ac)n\sqrt{a + bx^n + cx^{2n}}} - \frac{2 \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{an} \\
&= \frac{2(b^2 - 2ac + bcx^n)}{a(b^2 - 4ac)n\sqrt{a + bx^n + cx^{2n}}} - \frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 94, normalized size = 0.96

$$\frac{2(-2ac+b^2+bcx^n)}{a(b^2-4ac)\sqrt{a+cx^n}} - \frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+cx^n}}\right)}{a^{3/2}}$$

n

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)), x]

[Out] ((2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*Sqrt[a + x^n*(b + c*x^n)]) - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + x^n*(b + c*x^n)])])/a^(3/2))/n

IntegrateAlgebraic [A] time = 0.59, size = 107, normalized size = 1.09

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c}x^n}{\sqrt{a}} - \frac{\sqrt{a+bx^n+cx^{2n}}}{\sqrt{a}}\right)}{a^{3/2}n} + \frac{2(2ac - b^2 - bcx^n)}{an(4ac - b^2)\sqrt{a + bx^n + cx^{2n}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)), x]

[Out] (2*(-b^2 + 2*a*c - b*c*x^n))/(a*(-b^2 + 4*a*c)*n*Sqrt[a + b*x^n + c*x^(2*n)]) + (2*ArcTanh[(Sqrt[c]*x^n)/Sqrt[a] - Sqrt[a + b*x^n + c*x^(2*n)]/Sqrt[a]])/(a^(3/2)*n)

fricas [B] time = 1.70, size = 449, normalized size = 4.58

$$\frac{\left(\frac{((b^2c - 4ac^2)\sqrt{ax^2 + (b^2 - 4abc)\sqrt{a}} + (ab^2 - 4a^2c)\sqrt{a}) \log\left(\frac{8abx^n + b^2 + 4ac}{x^2}\right) + 4(abcx^n + ab^2 - 2a^2c)\sqrt{cx^{2n} + bx^n + a}}{2((a^2bc - 4a^2c^2)nx^{2n} + (a^2b^3 - 4a^2bc)nx^n + (a^2b^2 - 4a^2c)n)}\right) + \frac{((b^2c - 4ac^2)\sqrt{-a}x^{2n} + (b^3 - 4abc)\sqrt{-a}x^n + (ab^2 - 4a^2c)\sqrt{-a}) \arctan\left(\frac{(\sqrt{cx^{2n} + bx^n + a})\sqrt{-a}}{2(acx^n + ab^2 - 2a^2c)}\right) + 2(abcx^n + ab^2 - 2a^2c)\sqrt{cx^{2n} + bx^n + a}}{(a^2bc - 4a^2c^2)nx^{2n} + (a^2b^3 - 4a^2bc)nx^n + (a^2b^2 - 4a^2c)n}}{2((a^2bc - 4a^2c^2)nx^{2n} + (a^2b^3 - 4a^2bc)nx^n + (a^2b^2 - 4a^2c)n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2), x, algorithm="fricas")

[Out] [1/2*((b^2*c - 4*a*c^2)*sqrt(a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(a)*x^n + (a*b^2 - 4*a^2*c)*sqrt(a))*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*(a*b*c*x^n + a*b^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a)/((a^2*b^2*c - 4*a^3*c^2)*n*x^(2*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n), (((b^2*c - 4*a*c^2)*sqrt(-a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(-a)*x^n + (a*b^2 - 4*a^2*c)*sqrt(-a))*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + 2*(a*b*c*x^n + a*b^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a)/((a^2*b^2*c - 4*a^3*c^2)*n*x^(2*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + cx^{2n} + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x^n+c*x^(2*n)+a)^(3/2),x)

[Out] int(1/x/(b*x^n+c*x^(2*n)+a)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^{2n} + bx^n + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x(a + bx^n + cx^{2n})^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x)

[Out] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + bx^n + cx^{2n})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2), x)

[Out] Integral(1/(x*(a + b*x**n + c*x**(2*n))**(3/2)), x)

$$3.491 \quad \int (dx)^m (a + bx^n + cx^{2n})^3 dx$$

Optimal. Leaf size=182

$$\frac{a^3(dx)^{m+1}}{d(m+1)} + \frac{3a^2bx^{n+1}(dx)^m}{m+n+1} + \frac{3ax^{2n+1}(ac+b^2)(dx)^m}{m+2n+1} + \frac{bx^{3n+1}(6ac+b^2)(dx)^m}{m+3n+1} + \frac{3cx^{4n+1}(ac+b^2)(dx)^m}{m+4n+1} + \frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1} + \frac{c^3x^{6n+1}(dx)^m}{m+6n+1}$$

Rubi [A] time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1353, 20, 30}

$$\frac{3a^2bx^{n+1}(dx)^m}{m+n+1} + \frac{a^3(dx)^{m+1}}{d(m+1)} + \frac{3ax^{2n+1}(ac+b^2)(dx)^m}{m+2n+1} + \frac{bx^{3n+1}(6ac+b^2)(dx)^m}{m+3n+1} + \frac{3cx^{4n+1}(ac+b^2)(dx)^m}{m+4n+1} + \frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1} + \frac{c^3x^{6n+1}(dx)^m}{m+6n+1}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]

[Out] (3*a^2*b*x^(1+n)*(d*x)^m)/(1+m+n) + (3*a*(b^2 + a*c)*x^(1+2*n)*(d*x)^m)/(1+m+2*n) + (b*(b^2 + 6*a*c)*x^(1+3*n)*(d*x)^m)/(1+m+3*n) + (3*c*(b^2 + a*c)*x^(1+4*n)*(d*x)^m)/(1+m+4*n) + (3*b*c^2*x^(1+5*n)*(d*x)^m)/(1+m+5*n) + (c^3*x^(1+6*n)*(d*x)^m)/(1+m+6*n) + (a^3*(d*x)^(1+m))/(d*(1+m))

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 1353

Int[((d_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ[Simplify[(m+1)/n]]

Rubi steps

$$\begin{aligned}
\int (dx)^m (a + bx^n + cx^{2n})^3 dx &= \int \left(a^3(dx)^m + 3a^2bx^n(dx)^m + 3ab^2 \left(1 + \frac{ac}{b^2}\right) x^{2n}(dx)^m + b^3 \left(1 + \frac{6ac}{b^2}\right) x^{3n}(dx)^m + \right. \\
&= \frac{a^3(dx)^{1+m}}{d(1+m)} + (3a^2b) \int x^n(dx)^m dx + (3bc^2) \int x^{5n}(dx)^m dx + c^3 \int x^{6n}(dx)^m dx + \\
&= \frac{a^3(dx)^{1+m}}{d(1+m)} + (3a^2bx^{-m}(dx)^m) \int x^{m+n} dx + (3bc^2x^{-m}(dx)^m) \int x^{m+5n} dx + (c^3x^{-m}(dx)^m) \int x^{m+6n} dx + \\
&= \frac{3a^2bx^{1+n}(dx)^m}{1+m+n} + \frac{3a(b^2+ac)x^{1+2n}(dx)^m}{1+m+2n} + \frac{b(b^2+6ac)x^{1+3n}(dx)^m}{1+m+3n} + \frac{3c(b^2+6ac)x^{1+4n}(dx)^m}{1+m+4n} + \frac{c^3x^{1+5n}(dx)^m}{1+m+5n} + \frac{c^3x^{1+6n}(dx)^m}{1+m+6n}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 137, normalized size = 0.75

$$x(dx)^m \left(\frac{a^3}{m+1} + \frac{3a^2bx^n}{m+n+1} + \frac{3ax^{2n}(ac+b^2)}{m+2n+1} + \frac{bx^{3n}(6ac+b^2)}{m+3n+1} + \frac{3cx^{4n}(ac+b^2)}{m+4n+1} + \frac{3bc^2x^{5n}}{m+5n+1} + \frac{c^3x^{6n}}{m+6n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]

[Out] x*(d*x)^m*(a^3/(1+m) + (3*a^2*b*x^n)/(1+m+n) + (3*a*(b^2+a*c)*x^(2*n))/(1+m+2*n) + (b*(b^2+6*a*c)*x^(3*n))/(1+m+3*n) + (3*c*(b^2+a*c)*x^(4*n))/(1+m+4*n) + (3*b*c^2*x^(5*n))/(1+m+5*n) + (c^3*x^(6*n))/(1+m+6*n))

IntegrateAlgebraic [F] time = 0.18, size = 0, normalized size = 0.00

$$\int (dx)^m (a + bx^n + cx^{2n})^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(a + b*x^n + c*x^(2*n))^3, x]

fricas [B] time = 1.39, size = 2303, normalized size = 12.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")

[Out] ((c^3*m^6 + 6*c^3*m^5 + 15*c^3*m^4 + 20*c^3*m^3 + 120*(c^3*m + c^3)*n^5 + 15*c^3*m^2 + 274*(c^3*m^2 + 2*c^3*m + c^3)*n^4 + 6*c^3*m + 225*(c^3*m^3 + 3*

$$\begin{aligned}
& c^3 m^2 + 3c^3 m + c^3) n^3 + c^3 + 85(c^3 m^4 + 4c^3 m^3 + 6c^3 m^2 + \\
& 4c^3 m + c^3) n^2 + 15(c^3 m^5 + 5c^3 m^4 + 10c^3 m^3 + 10c^3 m^2 + 5c^3 m + c^3) n) * x^x^{(6n)} * e^{(m \log(d) + m \log(x))} + 3(b^2 c^2 m^6 + 6b^2 c^2 m^5 + 15b^2 c^2 m^4 + 20b^2 c^2 m^3 + 144(b^2 c^2 m + b^2 c^2) n^5 + 15b^2 c^2 m^2 + 324(b^2 c^2 m^2 + 2b^2 c^2 m + b^2 c^2) n^4 + 6b^2 c^2 m + 260(b^2 c^2 m^3 + 3b^2 c^2 m^2 + 3b^2 c^2 m + b^2 c^2) n^3 + b^2 c^2 + 95(b^2 c^2 m^4 + 4b^2 c^2 m^3 + 6b^2 c^2 m^2 + 4b^2 c^2 m + b^2 c^2) n^2 + 16(b^2 c^2 m^5 + 5b^2 c^2 m^4 + 10b^2 c^2 m^3 + 10b^2 c^2 m^2 + 5b^2 c^2 m + b^2 c^2) n) * x^x^{(5n)} * e^{(m \log(d) + m \log(x))} + 3((b^2 c + a^2 c) m^6 + 6(b^2 c + a^2 c) m^5 + 180(b^2 c + a^2 c) m^4 + (b^2 c + a^2 c) m) n^5 + 15(b^2 c + a^2 c) m^4 + 396(b^2 c + a^2 c + (b^2 c + a^2 c) m^2 + 2(b^2 c + a^2 c) m) n^4 + 20(b^2 c + a^2 c) m^3 + 307((b^2 c + a^2 c) m^3 + b^2 c + a^2 c + 3(b^2 c + a^2 c) m^2 + 3(b^2 c + a^2 c) m) n^3 + b^2 c + a^2 c + 15(b^2 c + a^2 c) m^2 + 107((b^2 c + a^2 c) m^4 + 4(b^2 c + a^2 c) m^3 + b^2 c + a^2 c + 6(b^2 c + a^2 c) m^2 + 4(b^2 c + a^2 c) m) n^2 + 6(b^2 c + a^2 c) m + 17((b^2 c + a^2 c) m^5 + 5(b^2 c + a^2 c) m^4 + 10(b^2 c + a^2 c) m^3 + b^2 c + a^2 c + 10(b^2 c + a^2 c) m^2 + 5(b^2 c + a^2 c) m) n) * x^x^{(4n)} * e^{(m \log(d) + m \log(x))} + ((b^3 + 6a^2 b^2 c) m^6 + 6(b^3 + 6a^2 b^2 c) m^5 + 240(b^3 + 6a^2 b^2 c + (b^3 + 6a^2 b^2 c) m) n^5 + 15(b^3 + 6a^2 b^2 c) m^4 + 508(b^3 + 6a^2 b^2 c + (b^3 + 6a^2 b^2 c) m) m^2 + 2(b^3 + 6a^2 b^2 c) m) n^4 + 20(b^3 + 6a^2 b^2 c) m^3 + 372((b^3 + 6a^2 b^2 c) m^3 + b^3 + 6a^2 b^2 c + 3(b^3 + 6a^2 b^2 c) m^2 + 3(b^3 + 6a^2 b^2 c) m) n^3 + b^3 + 6a^2 b^2 c + 15(b^3 + 6a^2 b^2 c) m^2 + 121((b^3 + 6a^2 b^2 c) m^4 + 4(b^3 + 6a^2 b^2 c) m^3 + b^3 + 6a^2 b^2 c + 6(b^3 + 6a^2 b^2 c) m^2 + 4(b^3 + 6a^2 b^2 c) m) n^2 + 6(b^3 + 6a^2 b^2 c) m + 18((b^3 + 6a^2 b^2 c) m^5 + 5(b^3 + 6a^2 b^2 c) m^4 + 10(b^3 + 6a^2 b^2 c) m^3 + b^3 + 6a^2 b^2 c + 10(b^3 + 6a^2 b^2 c) m^2 + 5(b^3 + 6a^2 b^2 c) m) n) * x^x^{(3n)} * e^{(m \log(d) + m \log(x))} + 3((a^2 b^2 + a^2 c) m^6 + 6(a^2 b^2 + a^2 c) m^5 + 360(a^2 b^2 + a^2 c + (a^2 b^2 + a^2 c) m) n^5 + 15(a^2 b^2 + a^2 c) m^4 + 702(a^2 b^2 + a^2 c + (a^2 b^2 + a^2 c) m^2 + 2(a^2 b^2 + a^2 c) m) n^4 + 20(a^2 b^2 + a^2 c) m^3 + 461((a^2 b^2 + a^2 c) m^3 + a^2 b^2 + a^2 c + 3(a^2 b^2 + a^2 c) m^2 + 3(a^2 b^2 + a^2 c) m) n^3 + a^2 b^2 + a^2 c + 15(a^2 b^2 + a^2 c) m^2 + 137((a^2 b^2 + a^2 c) m^4 + 4(a^2 b^2 + a^2 c) m^3 + a^2 b^2 + a^2 c + 6(a^2 b^2 + a^2 c) m^2 + 4(a^2 b^2 + a^2 c) m) n^2 + 6(a^2 b^2 + a^2 c) m + 19((a^2 b^2 + a^2 c) m^5 + 5(a^2 b^2 + a^2 c) m^4 + 10(a^2 b^2 + a^2 c) m^3 + a^2 b^2 + a^2 c + 10(a^2 b^2 + a^2 c) m^2 + 5(a^2 b^2 + a^2 c) m) n) * x^x^{(2n)} * e^{(m \log(d) + m \log(x))} + 3(a^2 b^2 m^6 + 6a^2 b^2 m^5 + 15a^2 b^2 m^4 + 20a^2 b^2 m^3 + 720(a^2 b^2 m + a^2 b^2) n^5 + 15a^2 b^2 m^2 + 1044(a^2 b^2 m^2 + 2a^2 b^2 m + a^2 b^2) n^4 + 6a^2 b^2 m + 580(a^2 b^2 m^3 + 3a^2 b^2 m^2 + 3a^2 b^2 m + a^2 b^2) n^3 + a^2 b^2 + 155(a^2 b^2 m^4 + 4a^2 b^2 m^3 + 6a^2 b^2 m^2 + 4a^2 b^2 m + a^2 b^2) n^2 + 20(a^2 b^2 m^5 + 5a^2 b^2 m^4 + 10a^2 b^2 m^3 + 10a^2 b^2 m^2 + 5a^2 b^2 m + a^2 b^2) n) * x^x^n * e^{(m \log(d) + m \log(x))} + (a^3 m^6 + 720a^3 n^6 + 6a^3 m^5 + 15a^3 m^4 + 20a^3 m^3 + 1764(a^3 m + a^3) n^5 + 15a^3 m^2 + 1624(a^3 m^2 + 2a^3 m + a^3) n^4 + 6a^3 m + 735(a^3 m^3 + 3a^3 m^2 + 3a^3 m + a^3) n^3 + a^3 + 175(a^3 m^4 + 4a^3 m^3 + 6a^3 m^2 + 4a^3 m + a^3) n^2 + 21(a^3 m^5 + 5a^3 m^4 + 10a^3 m^3 + 10a^3 m^2 + 5a^3 m + a^3) n) * x * e^{(m \log(d) + m \log(x))} / (m^7 +
\end{aligned}$$

$$720*(m + 1)*n^6 + 7*m^6 + 1764*(m^2 + 2*m + 1)*n^5 + 21*m^5 + 1624*(m^3 + 3*m^2 + 3*m + 1)*n^4 + 35*m^4 + 735*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*n^3 + 35*m^3 + 175*(m^5 + 5*m^4 + 10*m^3 + 10*m^2 + 5*m + 1)*n^2 + 21*m^2 + 21*(m^6 + 6*m^5 + 15*m^4 + 20*m^3 + 15*m^2 + 6*m + 1)*n + 7*m + 1$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.11, size = 3798, normalized size = 20.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a)^3,x)

[Out] $x*(c^3*(x^n)^6+a^3*m^6+6*a^3*m^5+1764*a^3*n^5+15*a^3*m^4+1624*a^3*n^4+720*a^3*n^3+3*b^3*(x^n)^3+20*a^3*m^3+15*a^3*m^2+175*a^3*n^2+21*a^3*n+735*a^3*n^3+a^3+972*b*c^2*n^4*(x^n)^5+45*b*c^2*m^4*(x^n)^5+240*b^3*m*n^5*(x^n)^3+18*b^2*c*m^5*(x^n)^4+540*b^2*c*n^5*(x^n)^4+150*c^3*m^3*n*(x^n)^6+510*c^3*m^2*n^2*(x^n)^6+675*c^3*m*n^3*(x^n)^6+18*a*c^2*m^5*(x^n)^4+540*a*c^2*n^5*(x^n)^4+18*b^3*m^5*n*(x^n)^3+508*b^3*m^2*n^4*(x^n)^3+15*c^3*m^5*n*(x^n)^6+85*c^3*m^4*n^2*(x^n)^6+225*c^3*m^3*n^3*(x^n)^6+274*c^3*m^2*n^4*(x^n)^6+120*c^3*m*n^5*(x^n)^6+3*b*c^2*m^6*(x^n)^5+75*c^3*m^4*n*(x^n)^6+340*c^3*m^3*n^2*(x^n)^6+675*c^3*m^2*n^3*(x^n)^6+548*c^3*m*n^4*(x^n)^6+3*a*c^2*m^6*(x^n)^4+3*b^2*c*m^6*(x^n)^4+18*b*c^2*m^5*(x^n)^5+432*b*c^2*n^5*(x^n)^5+1116*b^3*m^2*n^3*(x^n)^3+1016*b^3*m*n^4*(x^n)^3+45*b^2*c*m^4*(x^n)^4+1188*b^2*c*n^4*(x^n)^4+60*b*c^2*m^3*(x^n)^5+780*b*c^2*n^3*(x^n)^5+75*c^3*m*n*(x^n)^6+3*a^2*b*m^6*x^n+18*a^2*c*m^5*(x^n)^2+1080*a^2*c*n^5*(x^n)^2+18*a*b^2*m^5*(x^n)^2+1080*a*b^2*n^5*(x^n)^2+60*a*c^2*m^3*(x^n)^4+921*a*c^2*n^3*(x^n)^4+18*m*b*c^2*(x^n)^5+48*b*c^2*(x^n)^5*n+45*a^2*b*m^4*x^n+3132*a^2*b*n^4*x^n+121*b^3*m^4*n^2*(x^n)^3+372*b^3*m^3*n^3*(x^n)^3+340*c^3*m*n^2*(x^n)^6+3*a^2*c*m^6*(x^n)^2+3*a*b^2*m^6*(x^n)^2+45*a*c^2*m^4*(x^n)^4+1188*a*c^2*n^4*(x^n)^4+90*b^3*m^4*n*(x^n)^3+484*b^3*m^3*n^2*(x^n)^3+45*b*c^2*m^2*(x^n)^5+285*b*c^2*n^2*(x^n)^5+18*a^2*b*m^5*x^n+2160*a^2*b*n^5*x^n+45*a^2*c*m^4*(x^n)^2+2106*a^2*c*n^4*(x^n)^2+45*a*b^2*m^4*(x^n)^2+2106*a*b^2*n^4*(x^n)^2+45*a*c^2*m^2*(x^n)^4+321*a*c^2*n^2*(x^n)^4+45*b^2*c*m^2*(x^n)^4+321*b^2*c*n^2*(x^n)^4+18*b^2*c*(x^n)^4*m+51*b^2*c*(x^n)^4*n+1740*a^2*b*n^3*x^n+45*a^2*c*m^2*(x^n)^2+411*a^2*c*n^2*(x^n)^2+180*b^3*m^3*n*(x^n)^3+726*b^3*m^2*n^2*(x^n)^3+1116*b^3*m*n^3*(x^n)^3+60*b$

$$\begin{aligned}
&^2*c*m^3*(x^n)^4+921*b^2*c*n^3*(x^n)^4+18*a^2*c*(x^n)^2*m+57*a^2*c*(x^n)^2* \\
&n+6*a*b*c*(x^n)^3+60*a^2*c*m^3*(x^n)^2+1383*a^2*c*n^3*(x^n)^2+1383*a*b^2*n^ \\
&3*(x^n)^2+18*a*c^2*(x^n)^4*m+51*a*c^2*(x^n)^4*n+60*a^2*b*n*x^n+180*b^3*m^2* \\
&n*(x^n)^3+484*b^3*m*n^2*(x^n)^3+60*a*b^2*m^3*(x^n)^2+90*b^3*m*n*(x^n)^3+60* \\
&a^2*b*m^3*x^n+45*a*b^2*m^2*(x^n)^2+411*a*b^2*n^2*(x^n)^2+45*a^2*b*m^2*x^n+4 \\
&65*a^2*b*n^2*x^n+18*m*a*b^2*(x^n)^2+57*a*b^2*(x^n)^2*n+18*a^2*b*m*x^n+1080* \\
&a*b*c*m^3*n*(x^n)^3+570*a*b^2*m^2*n*(x^n)^2+2904*a*b*c*m*n^2*(x^n)^3+540*a* \\
&b*c*m*n*(x^n)^3+240*b^3*n^5*(x^n)^3+15*c^3*m^2*(x^n)^6+85*c^3*n^2*(x^n)^6+1 \\
&5*b^3*m^4*(x^n)^3+1284*b^2*c*m*n^2*(x^n)^4+240*b*c^2*m*n*(x^n)^5+300*a^2*b* \\
&m^4*n*x^n+1860*a^2*b*m^3*n^2*x^n+5220*a^2*b*m^2*n^3*x^n+6264*a^2*b*m*n^4*x^ \\
&n+570*a^2*c*m^3*n*(x^n)^2+2466*a^2*c*m^2*n^2*(x^n)^2+4149*a^2*c*m*n^3*(x^n) \\
&^2+570*a*b^2*m^3*n*(x^n)^2+2466*a*b^2*m^2*n^2*(x^n)^2+4149*a*b^2*m*n^3*(x^n) \\
&)^2+120*a*b*c*m^3*(x^n)^3+2232*a*b*c*n^3*(x^n)^3+6696*a*b*c*m^2*n^3*(x^n)^3 \\
&+150*c^3*m^2*n*(x^n)^6+105*a^3*m^4*n+700*a^3*m^3*n^2+2205*a^3*m^2*n^3+3248* \\
&a^3*m*n^4+210*a^3*m^3*n+1050*a^3*m^2*n^2+2205*a^3*m*n^3+20*b^3*m^3*(x^n)^3+ \\
&15*b^3*m^2*(x^n)^3+121*b^3*n^2*(x^n)^3+6*m*b^3*(x^n)^3+18*b^3*(x^n)^3*n+3*a \\
&^2*b*x^n+3*(x^n)^2*a*b^2+57*a*b^2*m^5*n*(x^n)^2+411*a*b^2*m^4*n^2*(x^n)^2+1 \\
&383*a*b^2*m^3*n^3*(x^n)^2+2106*a*b^2*m^2*n^4*(x^n)^2+1080*a*b^2*m*n^5*(x^n) \\
&^2+36*a*b*c*m^5*(x^n)^3+1440*a*b*c*n^5*(x^n)^3+510*a*c^2*m^3*n*(x^n)^4+1926 \\
&*a*c^2*m^2*n^2*(x^n)^4+2763*a*c^2*m*n^3*(x^n)^4+510*b^2*c*m^3*n*(x^n)^4+192 \\
&6*b^2*c*m^2*n^2*(x^n)^4+2763*b^2*c*m*n^3*(x^n)^4+480*b*c^2*m^2*n*(x^n)^5+43 \\
&56*a*b*c*m^2*n^2*(x^n)^3+6696*a*b*c*m*n^3*(x^n)^3+1080*a*b*c*m^2*n*(x^n)^3+ \\
&1140*b*c^2*m*n^2*(x^n)^5+60*a^2*b*m^5*n*x^n+465*a^2*b*m^4*n^2*x^n+1740*a^2* \\
&b*m^3*n^3*x^n+3132*a^2*b*m^2*n^4*x^n+2160*a^2*b*m*n^5*x^n+285*a^2*c*m^4*n*(\\
&x^n)^2+1644*a^2*c*m^3*n^2*(x^n)^2+4149*a^2*c*m^2*n^3*(x^n)^2+4212*a^2*c*m*n \\
&^4*(x^n)^2+285*a*b^2*m^4*n*(x^n)^2+1644*a*b^2*m^3*n^2*(x^n)^2+4149*a*b^2*m^ \\
&2*n^3*(x^n)^2+4212*a*b^2*m*n^4*(x^n)^2+3*(x^n)^5*b*c^2+108*a*b*c*m^5*n*(x^n) \\
&)^3+90*a*b*c*m^4*(x^n)^3+3048*a*b*c*n^4*(x^n)^3+510*a*c^2*m^2*n*(x^n)^4+128 \\
&4*a*c^2*m*n^2*(x^n)^4+510*b^2*c*m^2*n*(x^n)^4+210*a^3*m^2*n+700*a^3*m*n^2+1 \\
&05*a^3*m*n+1644*a*b^2*m*n^2*(x^n)^2+600*a^2*b*m^2*n*x^n+1860*a^2*b*m*n^2*x^ \\
&n+6*a^3*m+1440*a*b*c*m*n^5*(x^n)^3+2904*a*b*c*m^3*n^2*(x^n)^3+48*b*c^2*m^5* \\
&n*(x^n)^5+285*b*c^2*m^4*n^2*(x^n)^5+780*b*c^2*m^3*n^3*(x^n)^5+972*b*c^2*m^2 \\
&*n^4*(x^n)^5+432*b*c^2*m*n^5*(x^n)^5+51*a*c^2*m^5*n*(x^n)^4+321*a*c^2*m^4*n \\
&^2*(x^n)^4+21*a^3*m^5*n+175*a^3*m^4*n^2+735*a^3*m^3*n^3+1624*a^3*m^2*n^4+17 \\
&64*a^3*m*n^5+540*a*b*c*m^4*n*(x^n)^3+c^3*m^6*(x^n)^6+6*c^3*m^5*(x^n)^6+120* \\
&c^3*n^5*(x^n)^6+15*c^3*m^4*(x^n)^6+274*c^3*n^4*(x^n)^6+b^3*m^6*(x^n)^3+20*c \\
&^3*m^3*(x^n)^6+225*c^3*n^3*(x^n)^6+6*b^3*m^5*(x^n)^3+255*a*c^2*m*n*(x^n)^4+ \\
&255*b^2*c*m*n*(x^n)^4+600*a^2*b*m^3*n*x^n+2790*a^2*b*m^2*n^2*x^n+5220*a^2*b \\
&*m*n^3*x^n+570*a^2*c*m^2*n*(x^n)^2+1644*a^2*c*m*n^2*(x^n)^2+90*a*b*c*m^2*(x \\
&n)^3+726*a*b*c*n^2*(x^n)^3+285*a^2*c*m*n*(x^n)^2+36*a*b*c*(x^n)^3*m+108*a* \\
&b*c*(x^n)^3*n+1284*a*c^2*m^3*n^2*(x^n)^4+2763*a*c^2*m^2*n^3*(x^n)^4+2376*a* \\
&c^2*m*n^4*(x^n)^4+255*b^2*c*m^4*n*(x^n)^4+1284*b^2*c*m^3*n^2*(x^n)^4+2763*b \\
&^2*c*m^2*n^3*(x^n)^4+2376*b^2*c*m*n^4*(x^n)^4+480*b*c^2*m^3*n*(x^n)^5+1710* \\
&b*c^2*m^2*n^2*(x^n)^5+2340*b*c^2*m*n^3*(x^n)^5+57*a^2*c*m^5*n*(x^n)^2+411*a \\
&^2*c*m^4*n^2*(x^n)^2+1383*a^2*c*m^3*n^3*(x^n)^2+2106*a^2*c*m^2*n^4*(x^n)^2+
\end{aligned}$$

$1080*a^2*c*m*n^5*(x^n)^2+372*b^3*n^3*(x^n)^3+3*(x^n)^2*a^2*c+3*(x^n)^4*a*c^2+3*(x^n)^4*b^2*c+6096*a*b*c*m*n^4*(x^n)^3+508*b^3*n^4*(x^n)^3+6*m*c^3*(x^n)^6+15*c^3*(x^n)^6*n+726*a*b*c*m^4*n^2*(x^n)^3+2232*a*b*c*m^3*n^3*(x^n)^3+3048*a*b*c*m^2*n^4*(x^n)^3+921*a*c^2*m^3*n^3*(x^n)^4+1188*a*c^2*m^2*n^4*(x^n)^4+540*a*c^2*m*n^5*(x^n)^4+51*b^2*c*m^5*n*(x^n)^4+321*b^2*c*m^4*n^2*(x^n)^4+921*b^2*c*m^3*n^3*(x^n)^4+1188*b^2*c*m^2*n^4*(x^n)^4+540*b^2*c*m*n^5*(x^n)^4+240*b*c^2*m^4*n*(x^n)^5+1140*b*c^2*m^3*n^2*(x^n)^5+2340*b*c^2*m^2*n^3*(x^n)^5+1944*b*c^2*m*n^4*(x^n)^5+6*a*b*c*m^6*(x^n)^3+255*a*c^2*m^4*n*(x^n)^4+285*a*b^2*m*n*(x^n)^2+300*a^2*b*m*n*x^n)/(m+1)/(m+n+1)/(m+2*n+1)/(m+3*n+1)/(1+m+4*n)/(1+m+5*n)/(1+m+6*n)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*d*x)^3+2*ln(d)+2*ln(x))*m)$

maxima [A] time = 1.30, size = 273, normalized size = 1.50

$$\frac{c^3 d^m x^{m+6n \log(x)}}{m+6n+1} + \frac{3 b c^2 d^m x^{m \log(x)+5n \log(x)}}{m+5n+1} + \frac{3 b^2 c d^m x^{m \log(x)+4n \log(x)}}{m+4n+1} + \frac{3 a c^2 d^m x^{m \log(x)+4n \log(x)}}{m+4n+1} + \frac{b^3 d^m x^{m \log(x)+3n \log(x)}}{m+3n+1} + \frac{6 a b c d^m x^{m \log(x)+3n \log(x)}}{m+3n+1} + \frac{3 a b^2 d^m x^{m \log(x)+2n \log(x)}}{m+2n+1} + \frac{3 a^2 c d^m x^{m \log(x)+2n \log(x)}}{m+2n+1} + \frac{3 a^2 b d^m x^{m \log(x)+n \log(x)}}{m+n+1} + \frac{(d x)^{m+1} a^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")

[Out] $c^3*d^m*x*e^{(m*\log(x) + 6*n*\log(x))/(m + 6*n + 1)} + 3*b*c^2*d^m*x*e^{(m*\log(x) + 5*n*\log(x))/(m + 5*n + 1)} + 3*b^2*c*d^m*x*e^{(m*\log(x) + 4*n*\log(x))/(m + 4*n + 1)} + 3*a*c^2*d^m*x*e^{(m*\log(x) + 4*n*\log(x))/(m + 4*n + 1)} + b^3*d^m*x*e^{(m*\log(x) + 3*n*\log(x))/(m + 3*n + 1)} + 6*a*b*c*d^m*x*e^{(m*\log(x) + 3*n*\log(x))/(m + 3*n + 1)} + 3*a*b^2*d^m*x*e^{(m*\log(x) + 2*n*\log(x))/(m + 2*n + 1)} + 3*a^2*c*d^m*x*e^{(m*\log(x) + 2*n*\log(x))/(m + 2*n + 1)} + 3*a^2*b*d^m*x*e^{(m*\log(x) + n*\log(x))/(m + n + 1)} + (d*x)^{(m + 1)}*a^3/(d*(m + 1))$

mupad [B] time = 2.16, size = 1734, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^3,x)

[Out] $(a^3*x*(d*x)^m)/(m+1) + (c^3*x*x^{(6*n)}*(d*x)^m*(5*m+15*n+60*m*n+255*m*n^2+90*m^2*n+450*m*n^3+60*m^3*n+274*m*n^4+15*m^4*n+10*m^2+10*m^3+5*m^4+m^5+85*n^2+225*n^3+274*n^4+120*n^5+255*m^2*n^2+225*m^2*n^3+85*m^3*n^2+1))/(6*m+21*n+105*m*n+700*m*n^2+210*m^2*n+2205*m*n^3+210*m^3*n+3248*m*n^4+105*m^4*n+1764*m*n^5+21*m^5*n+15*m^2+20*m^3+15*m^4+6*m^5+m^6+175*n^2+735*n^3+1624*n^4+1764*n^5+720*n^6+1050*m^2*n^2+2205*m^2*n^3+700*m^3*n^2+1624*m^2*n^4+735*m^3*n^3+175*m^4*n^2+1) + (3*a*x*x^{(2*n)}*(d*x)^m*(a*c + b^2)*(5*m+19*n+76*m*n+411*m*n^2+114*m^2*n+922*m*n^3+76*m^3*n+702*m*n^4+19*m^4*n+10*m^2+10*m^3+5*m^4+m^5+137*n^2+461*n^3+702*n^4+360*n^5+411*m^2*n^2+461*m^2*n^3+137*m^3*n^2+1))/(6*m+21*n+1$

$$\begin{aligned}
& 05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105* \\
& m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17 \\
& 5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n \\
& ^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (b*x*x^(\\
& 3*n)*(d*x)^m*(6*a*c + b^2)*(5*m + 18*n + 72*m*n + 363*m*n^2 + 108*m^2*n + 7 \\
& 44*m*n^3 + 72*m^3*n + 508*m*n^4 + 18*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 \\
& + 121*n^2 + 372*n^3 + 508*n^4 + 240*n^5 + 363*m^2*n^2 + 372*m^2*n^3 + 121*m \\
& ^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 2 \\
& 10*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 \\
& + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 \\
& + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + \\
& 175*m^4*n^2 + 1) + (3*c*x*x^(4*n)*(d*x)^m*(a*c + b^2)*(5*m + 17*n + 68*m*n \\
& + 321*m*n^2 + 102*m^2*n + 614*m*n^3 + 68*m^3*n + 396*m*n^4 + 17*m^4*n + 10 \\
& *m^2 + 10*m^3 + 5*m^4 + m^5 + 107*n^2 + 307*n^3 + 396*n^4 + 180*n^5 + 321*m \\
& ^2*n^2 + 307*m^2*n^3 + 107*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 \\
& + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 \\
& + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1 \\
& 624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + \\
& 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a^2*b*x*x^n*(d*x)^m*(5*m \\
& + 20*n + 80*m*n + 465*m*n^2 + 120*m^2*n + 1160*m*n^3 + 80*m^3*n + 1044*m*n \\
& ^4 + 20*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 155*n^2 + 580*n^3 + 1044*n^ \\
& 4 + 720*n^5 + 465*m^2*n^2 + 580*m^2*n^3 + 155*m^3*n^2 + 1))/(6*m + 21*n + 1 \\
& 05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105* \\
& m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17 \\
& 5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n \\
& ^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*b*c^2 \\
& *x*x^(5*n)*(d*x)^m*(5*m + 16*n + 64*m*n + 285*m*n^2 + 96*m^2*n + 520*m*n^3 \\
& + 64*m^3*n + 324*m*n^4 + 16*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 95*n^2 \\
& + 260*n^3 + 324*n^4 + 144*n^5 + 285*m^2*n^2 + 260*m^2*n^3 + 95*m^3*n^2 + 1) \\
&)/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + \\
& 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + \\
& 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2 \\
& *n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^ \\
& 2 + 1)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)

[Out] Timed out

$$3.492 \quad \int (dx)^m (a + bx^n + cx^{2n})^2 dx$$

Optimal. Leaf size=117

$$\frac{a^2(dx)^{m+1}}{d(m+1)} + \frac{x^{2n+1}(2ac+b^2)(dx)^m}{m+2n+1} + \frac{2abx^{n+1}(dx)^m}{m+n+1} + \frac{2bcx^{3n+1}(dx)^m}{m+3n+1} + \frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1353, 20, 30}

$$\frac{a^2(dx)^{m+1}}{d(m+1)} + \frac{x^{2n+1}(2ac+b^2)(dx)^m}{m+2n+1} + \frac{2abx^{n+1}(dx)^m}{m+n+1} + \frac{2bcx^{3n+1}(dx)^m}{m+3n+1} + \frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]

[Out] (2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) + (c^2*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m))/(d*(1 + m))

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 1353

Int[((d_)*(x_))^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ[Simplify[(m+1)/n]]

Rubi steps

$$\begin{aligned}
\int (dx)^m (a + bx^n + cx^{2n})^2 dx &= \int \left(a^2(dx)^m + 2abx^n(dx)^m + b^2 \left(1 + \frac{2ac}{b^2} \right) x^{2n}(dx)^m + 2bcx^{3n}(dx)^m + c^2x^{4n}(dx)^m \right) \\
&= \frac{a^2(dx)^{1+m}}{d(1+m)} + (2ab) \int x^n(dx)^m dx + (2bc) \int x^{3n}(dx)^m dx + c^2 \int x^{4n}(dx)^m dx + \dots \\
&= \frac{a^2(dx)^{1+m}}{d(1+m)} + (2abx^{-m}(dx)^m) \int x^{m+n} dx + (2bcx^{-m}(dx)^m) \int x^{m+3n} dx + (c^2x^{-m}(dx)^m) \int x^{m+4n} dx + \dots \\
&= \frac{2abx^{1+n}(dx)^m}{1+m+n} + \frac{(b^2+2ac)x^{1+2n}(dx)^m}{1+m+2n} + \frac{2bcx^{1+3n}(dx)^m}{1+m+3n} + \frac{c^2x^{1+4n}(dx)^m}{1+m+4n} + \dots
\end{aligned}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.74

$$x(dx)^m \left(\frac{a^2}{m+1} + \frac{x^{2n}(2ac+b^2)}{m+2n+1} + \frac{2abx^n}{m+n+1} + \frac{2bcx^{3n}}{m+3n+1} + \frac{c^2x^{4n}}{m+4n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]

[Out] x*(d*x)^m*(a^2/(1+m) + (2*a*b*x^n)/(1+m+n) + ((b^2+2*a*c)*x^(2*n))/(1+m+2*n) + (2*b*c*x^(3*n))/(1+m+3*n) + (c^2*x^(4*n))/(1+m+4*n))

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx)^m (a + bx^n + cx^{2n})^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(a + b*x^n + c*x^(2*n))^2, x]

fricas [B] time = 1.25, size = 706, normalized size = 6.03

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")

[Out] ((c^2*m^4 + 4*c^2*m^3 + 6*c^2*m^2 + 6*(c^2*m + c^2)*n^3 + 4*c^2*m + 11*(c^2*m^2 + 2*c^2*m + c^2)*n^2 + c^2 + 6*(c^2*m^3 + 3*c^2*m^2 + 3*c^2*m + c^2)*n

$$\begin{aligned} &) * x * x^{(4*n)} * e^{(m*\log(d) + m*\log(x))} + 2*(b*c*m^4 + 4*b*c*m^3 + 6*b*c*m^2 + \\ & 8*(b*c*m + b*c)*n^3 + 4*b*c*m + 14*(b*c*m^2 + 2*b*c*m + b*c)*n^2 + b*c + 7* \\ & (b*c*m^3 + 3*b*c*m^2 + 3*b*c*m + b*c)*n) * x * x^{(3*n)} * e^{(m*\log(d) + m*\log(x))} \\ & + ((b^2 + 2*a*c)*m^4 + 4*(b^2 + 2*a*c)*m^3 + 12*(b^2 + 2*a*c + (b^2 + 2*a*c) \\ &) * m) * n^3 + 6*(b^2 + 2*a*c)*m^2 + 19*((b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 2*(b \\ & ^2 + 2*a*c)*m) * n^2 + b^2 + 2*a*c + 4*(b^2 + 2*a*c)*m + 8*((b^2 + 2*a*c)*m^3 \\ & + 3*(b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 3*(b^2 + 2*a*c)*m) * n) * x * x^{(2*n)} * e^{(m \\ & * \log(d) + m*\log(x))} + 2*(a*b*m^4 + 4*a*b*m^3 + 6*a*b*m^2 + 24*(a*b*m + a*b) \\ & * n^3 + 4*a*b*m + 26*(a*b*m^2 + 2*a*b*m + a*b)*n^2 + a*b + 9*(a*b*m^3 + 3*a* \\ & b*m^2 + 3*a*b*m + a*b)*n) * x * x^n * e^{(m*\log(d) + m*\log(x))} + (a^2*m^4 + 24*a^2 \\ & * n^4 + 4*a^2*m^3 + 6*a^2*m^2 + 50*(a^2*m + a^2)*n^3 + 4*a^2*m + 35*(a^2*m^2 \\ & + 2*a^2*m + a^2)*n^2 + a^2 + 10*(a^2*m^3 + 3*a^2*m^2 + 3*a^2*m + a^2)*n) * x \\ & * e^{(m*\log(d) + m*\log(x))} / (m^5 + 24*(m + 1)*n^4 + 5*m^4 + 50*(m^2 + 2*m + 1) \\ &) * n^3 + 10*m^3 + 35*(m^3 + 3*m^2 + 3*m + 1)*n^2 + 10*m^2 + 10*(m^4 + 4*m^3 \\ & + 6*m^2 + 4*m + 1)*n + 5*m + 1) \end{aligned}$$

giac [B] time = 0.80, size = 5454, normalized size = 46.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")

[Out] $(c^2*m^4*x*x^{(4*n)}*e^{(m*\log(d) + m*\log(x))} + 6*c^2*m^3*n*x*x^{(4*n)}*e^{(m*\log(d) + m*\log(x))} + 11*c^2*m^2*n^2*x*x^{(4*n)}*e^{(m*\log(d) + m*\log(x))} + 6*c^2*m*n^3*x*x^{(4*n)}*e^{(m*\log(d) + m*\log(x))} + 2*b*c*m^4*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + c^2*m^4*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 14*b*c*m^3*n*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 6*c^2*m^3*n*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 28*b*c*m^2*n^2*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 11*c^2*m^2*n^2*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 16*b*c*m*n^3*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 6*c^2*m*n^3*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + b^2*m^4*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 2*a*c*m^4*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 2*b*c*m^4*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + c^2*m^4*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 8*b^2*m^3*n*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 16*a*c*m^3*n*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 14*b*c*m^3*n*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 6*c^2*m^3*n*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 19*b^2*m^2*n^2*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 38*a*c*m^2*n^2*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 28*b*c*m^2*n^2*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 11*c^2*m^2*n^2*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 12*b^2*m*n^3*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 24*a*c*m*n^3*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 16*b*c*m*n^3*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 6*c^2*m*n^3*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 2*a*b*m^4*x*x^n*e^{(m*\log(d) + m*\log(x))} + b^2*m^4*x*x^n*e^{(m*\log(d) + m*\log(x))} + 2*a*c*m^4*x*x^n*e^{(m*\log(d) + m*\log(x))} + 2*b*c*m^4*x*x^n*e^{(m*\log(d) + m*\log(x))} + c^2*m^4*x*x^n*e^{(m*\log(d) + m*\log(x))} + 18*a*b*m^3*n*x*x^n*e^{(m*\log(d) + m*\log(x))} + 8*b^2*m^3*n*x*x^n*e^{(m*\log(d) + m*\log(x))} + 16*a$

$$\begin{aligned}
& *c^m^3*n*x*x^n*e^{(m*\log(d) + m*\log(x))} + 14*b*c^m^3*n*x*x^n*e^{(m*\log(d) + m} \\
& * \log(x)) + 6*c^2*m^3*n*x*x^n*e^{(m*\log(d) + m*\log(x))} + 52*a*b*m^2*n^2*x*x^n \\
& *e^{(m*\log(d) + m*\log(x))} + 19*b^2*m^2*n^2*x*x^n*e^{(m*\log(d) + m*\log(x))} + 3 \\
& 8*a*c^m^2*n^2*x*x^n*e^{(m*\log(d) + m*\log(x))} + 28*b*c^m^2*n^2*x*x^n*e^{(m*\log} \\
& (d) + m*\log(x)) + 11*c^2*m^2*n^2*x*x^n*e^{(m*\log(d) + m*\log(x))} + 48*a*b*m*n \\
& ^3*x*x^n*e^{(m*\log(d) + m*\log(x))} + 12*b^2*m^n^3*x*x^n*e^{(m*\log(d) + m*\log(x} \\
&)) + 24*a*c*m^n^3*x*x^n*e^{(m*\log(d) + m*\log(x))} + 16*b*c*m^n^3*x*x^n*e^{(m*\log} \\
& (d) + m*\log(x)) + 6*c^2*m^n^3*x*x^n*e^{(m*\log(d) + m*\log(x))} + a^2*m^4*x*e \\
& ^{(m*\log(d) + m*\log(x))} + 2*a*b*m^4*x*e^{(m*\log(d) + m*\log(x))} + b^2*m^4*x*e \\
& ^{(m*\log(d) + m*\log(x))} + 2*a*c^m^4*x*e^{(m*\log(d) + m*\log(x))} + 2*b*c^m^4*x*e \\
& ^{(m*\log(d) + m*\log(x))} + c^2*m^4*x*e^{(m*\log(d) + m*\log(x))} + 10*a^2*m^3*n*x \\
& *e^{(m*\log(d) + m*\log(x))} + 18*a*b*m^3*n*x*e^{(m*\log(d) + m*\log(x))} + 8*b^2*m \\
& ^3*n*x*e^{(m*\log(d) + m*\log(x))} + 16*a*c^m^3*n*x*e^{(m*\log(d) + m*\log(x))} + 1 \\
& 4*b*c^m^3*n*x*e^{(m*\log(d) + m*\log(x))} + 6*c^2*m^3*n*x*e^{(m*\log(d) + m*\log(x} \\
&)) + 35*a^2*m^2*n^2*x*e^{(m*\log(d) + m*\log(x))} + 52*a*b*m^2*n^2*x*e^{(m*\log(d} \\
&) + m*\log(x)) + 19*b^2*m^2*n^2*x*e^{(m*\log(d) + m*\log(x))} + 38*a*c^m^2*n^2*x \\
& *e^{(m*\log(d) + m*\log(x))} + 28*b*c^m^2*n^2*x*e^{(m*\log(d) + m*\log(x))} + 11*c^ \\
& 2*m^2*n^2*x*e^{(m*\log(d) + m*\log(x))} + 50*a^2*m^n^3*x*e^{(m*\log(d) + m*\log(x} \\
&) + 48*a*b*m^n^3*x*e^{(m*\log(d) + m*\log(x))} + 12*b^2*m^n^3*x*e^{(m*\log(d) + m} \\
& * \log(x)) + 24*a*c*m^n^3*x*e^{(m*\log(d) + m*\log(x))} + 16*b*c*m^n^3*x*e^{(m*\log} \\
& (d) + m*\log(x)) + 6*c^2*m^n^3*x*e^{(m*\log(d) + m*\log(x))} + 24*a^2*n^4*x*e^{(m} \\
& * \log(d) + m*\log(x)) + 4*c^2*m^3*x*x^{(4*n)}*e^{(m*\log(d) + m*\log(x))} + 18*c^2*m \\
& ^2*n*x*x^{(4*n)}*e^{(m*\log(d) + m*\log(x))} + 22*c^2*m^n^2*x*x^{(4*n)}*e^{(m*\log(d} \\
&) + m*\log(x)) + 6*c^2*n^3*x*x^{(4*n)}*e^{(m*\log(d) + m*\log(x))} + 8*b*c^m^3*x*x \\
& ^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 4*c^2*m^3*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x} \\
&) + 42*b*c^m^2*n*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 18*c^2*m^2*n*x*x^{(3*n)} \\
& *e^{(m*\log(d) + m*\log(x))} + 56*b*c^m^n^2*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + \\
& 22*c^2*m^n^2*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 16*b*c^n^3*x*x^{(3*n)}*e^{(m} \\
& * \log(d) + m*\log(x)) + 6*c^2*n^3*x*x^{(3*n)}*e^{(m*\log(d) + m*\log(x))} + 4*b^2*m \\
& ^3*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 8*a*c^m^3*x*x^{(2*n)}*e^{(m*\log(d) + m*} \\
& \log(x)) + 8*b*c^m^3*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 4*c^2*m^3*x*x^{(2*n)} \\
& *e^{(m*\log(d) + m*\log(x))} + 24*b^2*m^2*n*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + \\
& 48*a*c^m^2*n*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 42*b*c^m^2*n*x*x^{(2*n)}*e^{ \\
& (m*\log(d) + m*\log(x))} + 18*c^2*m^2*n*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 38 \\
& *b^2*m^n^2*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 76*a*c^m^n^2*x*x^{(2*n)}*e^{(m*} \\
& \log(d) + m*\log(x)) + 56*b*c^m^n^2*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 22*c^ \\
& 2*m^n^2*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 12*b^2*n^3*x*x^{(2*n)}*e^{(m*\log(d} \\
&) + m*\log(x)) + 24*a*c^n^3*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 16*b*c^n^3*x \\
& *x^{(2*n)}*e^{(m*\log(d) + m*\log(x))} + 6*c^2*n^3*x*x^{(2*n)}*e^{(m*\log(d) + m*\log(} \\
& x)) + 8*a*b*m^3*x*x^n*e^{(m*\log(d) + m*\log(x))} + 4*b^2*m^3*x*x^n*e^{(m*\log(d} \\
& + m*\log(x))} + 8*a*c^m^3*x*x^n*e^{(m*\log(d) + m*\log(x))} + 8*b*c^m^3*x*x^n*e^{ \\
& (m*\log(d) + m*\log(x))} + 4*c^2*m^3*x*x^n*e^{(m*\log(d) + m*\log(x))} + 54*a*b*m^ \\
& 2*n*x*x^n*e^{(m*\log(d) + m*\log(x))} + 24*b^2*m^2*n*x*x^n*e^{(m*\log(d) + m*\log(} \\
& x)) + 48*a*c^m^2*n*x*x^n*e^{(m*\log(d) + m*\log(x))} + 42*b*c^m^2*n*x*x^n*e^{(m*} \\
& \log(d) + m*\log(x)) + 18*c^2*m^2*n*x*x^n*e^{(m*\log(d) + m*\log(x))} + 104*a*b*m
\end{aligned}$$

$$\begin{aligned} & n^2 * x * x^n * e^{(m \log(d) + m \log(x))} + 38 * b^2 * m * n^2 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 76 * a * c * m * n^2 * x * x^n * e^{(m \log(d) + m \log(x))} + 56 * b * c * m * n^2 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 22 * c^2 * m * n^2 * x * x^n * e^{(m \log(d) + m \log(x))} + 48 * a * b * n^3 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 12 * b^2 * n^3 * x * x^n * e^{(m \log(d) + m \log(x))} + 24 * a * c * n^3 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 16 * b * c * n^3 * x * x^n * e^{(m \log(d) + m \log(x))} + 6 * c^2 * n^3 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 4 * a^2 * m^3 * x * e^{(m \log(d) + m \log(x))} + 8 * a * b * m^3 * x * e^{(m \log(d) + m \log(x))} \\ & + 4 * b^2 * m^3 * x * e^{(m \log(d) + m \log(x))} + 8 * a * c * m^3 * x * e^{(m \log(d) + m \log(x))} \\ & + 8 * b * c * m^3 * x * e^{(m \log(d) + m \log(x))} + 4 * c^2 * m^3 * x * e^{(m \log(d) + m \log(x))} \\ & + 30 * a^2 * m^2 * n * x * e^{(m \log(d) + m \log(x))} + 54 * a * b * m^2 * n * x * e^{(m \log(d) + m \log(x))} \\ & + 24 * b^2 * m^2 * n * x * e^{(m \log(d) + m \log(x))} + 48 * a * c * m^2 * n * x * e^{(m \log(d) + m \log(x))} \\ & + 42 * b * c * m^2 * n * x * e^{(m \log(d) + m \log(x))} + 18 * c^2 * m^2 * n * x * e^{(m \log(d) + m \log(x))} \\ & + 70 * a^2 * m * n^2 * x * e^{(m \log(d) + m \log(x))} + 104 * a * b * m * n^2 * x * e^{(m \log(d) + m \log(x))} \\ & + 38 * b^2 * m * n^2 * x * e^{(m \log(d) + m \log(x))} + 76 * a * c * m * n^2 * x * e^{(m \log(d) + m \log(x))} \\ & + 56 * b * c * m * n^2 * x * e^{(m \log(d) + m \log(x))} + 22 * c^2 * m * n^2 * x * e^{(m \log(d) + m \log(x))} \\ & + 50 * a^2 * n^3 * x * e^{(m \log(d) + m \log(x))} + 48 * a * b * n^3 * x * e^{(m \log(d) + m \log(x))} \\ & + 12 * b^2 * n^3 * x * e^{(m \log(d) + m \log(x))} + 24 * a * c * n^3 * x * e^{(m \log(d) + m \log(x))} \\ & + 16 * b * c * n^3 * x * e^{(m \log(d) + m \log(x))} + 6 * c^2 * n^3 * x * e^{(m \log(d) + m \log(x))} \\ & + 6 * c^2 * m^2 * x * x^{(4 * n)} * e^{(m \log(d) + m \log(x))} + 18 * c^2 * m * n * x * x^{(4 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 11 * c^2 * n^2 * x * x^{(4 * n)} * e^{(m \log(d) + m \log(x))} + 12 * b * c * m^2 * x * x^{(3 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 6 * c^2 * m^2 * x * x^{(3 * n)} * e^{(m \log(d) + m \log(x))} + 42 * b * c * m * n * x * x^{(3 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 18 * c^2 * m * n * x * x^{(3 * n)} * e^{(m \log(d) + m \log(x))} + 28 * b * c * n^2 * x * x^{(3 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 11 * c^2 * n^2 * x * x^{(3 * n)} * e^{(m \log(d) + m \log(x))} + 6 * b^2 * m^2 * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 12 * a * c * m^2 * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} + 12 * b * c * m^2 * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 6 * c^2 * m^2 * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} + 24 * b^2 * m * n * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 48 * a * c * m * n * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} + 42 * b * c * m * n * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 18 * c^2 * m * n * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} + 19 * b^2 * n^2 * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 38 * a * c * n^2 * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} + 28 * b * c * n^2 * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} \\ & + 11 * c^2 * n^2 * x * x^{(2 * n)} * e^{(m \log(d) + m \log(x))} + 12 * a * b * m^2 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 6 * b^2 * m^2 * x * x^n * e^{(m \log(d) + m \log(x))} + 12 * a * c * m^2 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 12 * b * c * m^2 * x * x^n * e^{(m \log(d) + m \log(x))} + 6 * c^2 * m^2 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 54 * a * b * m * n * x * x^n * e^{(m \log(d) + m \log(x))} + 24 * b^2 * m * n * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 48 * a * c * m * n * x * x^n * e^{(m \log(d) + m \log(x))} + 42 * b * c * m * n * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 18 * c^2 * m * n * x * x^n * e^{(m \log(d) + m \log(x))} + 52 * a * b * n^2 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 19 * b^2 * n^2 * x * x^n * e^{(m \log(d) + m \log(x))} + 38 * a * c * n^2 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 28 * b * c * n^2 * x * x^n * e^{(m \log(d) + m \log(x))} + 11 * c^2 * n^2 * x * x^n * e^{(m \log(d) + m \log(x))} \\ & + 6 * a^2 * m^2 * x * e^{(m \log(d) + m \log(x))} + 12 * a * b * m^2 * x * e^{(m \log(d) + m \log(x))} \\ & + 6 * b^2 * m^2 * x * e^{(m \log(d) + m \log(x))} + 12 * a * c * m^2 * x * e^{(m \log(d) + m \log(x))} \\ & + 6 * c^2 * m^2 * x * e^{(m \log(d) + m \log(x))} + 30 * a^2 * m * n * x * e^{(m \log(d) + m \log(x))} \\ & + 54 * a * b * m * n * x * e^{(m \log(d) + m \log(x))} + 24 * b^2 * m * n * x * e^{(m \log(d) + m \log(x))} \\ & + 48 * a * c * m * n * x * e^{(m \log(d) + m \log(x))} + \end{aligned}$$

$$\begin{aligned}
& m \log(d) + m \log(x)) + 42*b*c*m*n*x*e^{(m \log(d) + m \log(x))} + 18*c^2*m*n*x* \\
& e^{(m \log(d) + m \log(x))} + 35*a^2*n^2*x*e^{(m \log(d) + m \log(x))} + 52*a*b*n^2 \\
& *x*e^{(m \log(d) + m \log(x))} + 19*b^2*n^2*x*e^{(m \log(d) + m \log(x))} + 38*a*c* \\
& n^2*x*e^{(m \log(d) + m \log(x))} + 28*b*c*n^2*x*e^{(m \log(d) + m \log(x))} + 11*c \\
& ^2*n^2*x*e^{(m \log(d) + m \log(x))} + 4*c^2*m*x*x^{(4*n)}*e^{(m \log(d) + m \log(x))} \\
&) + 6*c^2*n*x*x^{(4*n)}*e^{(m \log(d) + m \log(x))} + 8*b*c*m*x*x^{(3*n)}*e^{(m \log(d) \\
& + m \log(x))} + 4*c^2*m*x*x^{(3*n)}*e^{(m \log(d) + m \log(x))} + 14*b*c*n*x*x^{(3 \\
& *n)}*e^{(m \log(d) + m \log(x))} + 6*c^2*n*x*x^{(3*n)}*e^{(m \log(d) + m \log(x))} + \\
& 4*b^2*m*x*x^{(2*n)}*e^{(m \log(d) + m \log(x))} + 8*a*c*m*x*x^{(2*n)}*e^{(m \log(d) + \\
& m \log(x))} + 8*b*c*m*x*x^{(2*n)}*e^{(m \log(d) + m \log(x))} + 4*c^2*m*x*x^{(2*n)}* \\
& e^{(m \log(d) + m \log(x))} + 8*b^2*n*x*x^{(2*n)}*e^{(m \log(d) + m \log(x))} + 16*a* \\
& c*n*x*x^{(2*n)}*e^{(m \log(d) + m \log(x))} + 14*b*c*n*x*x^{(2*n)}*e^{(m \log(d) + m \\
& \log(x))} + 6*c^2*n*x*x^{(2*n)}*e^{(m \log(d) + m \log(x))} + 8*a*b*m*x*x^n*e^{(m \log \\
& (d) + m \log(x))} + 4*b^2*m*x*x^n*e^{(m \log(d) + m \log(x))} + 8*a*c*m*x*x^n*e^{(\\
& m \log(d) + m \log(x))} + 8*b*c*m*x*x^n*e^{(m \log(d) + m \log(x))} + 4*c^2*m*x*x \\
& ^n*e^{(m \log(d) + m \log(x))} + 18*a*b*n*x*x^n*e^{(m \log(d) + m \log(x))} + 8*b^2 \\
& *n*x*x^n*e^{(m \log(d) + m \log(x))} + 16*a*c*n*x*x^n*e^{(m \log(d) + m \log(x))} + \\
& 14*b*c*n*x*x^n*e^{(m \log(d) + m \log(x))} + 6*c^2*n*x*x^n*e^{(m \log(d) + m \log \\
& (x))} + 4*a^2*m*x*e^{(m \log(d) + m \log(x))} + 8*a*b*m*x*e^{(m \log(d) + m \log(x))} \\
&) + 4*b^2*m*x*e^{(m \log(d) + m \log(x))} + 8*a*c*m*x*e^{(m \log(d) + m \log(x))} + \\
& 8*b*c*m*x*e^{(m \log(d) + m \log(x))} + 4*c^2*m*x*e^{(m \log(d) + m \log(x))} + 10 \\
& *a^2*n*x*e^{(m \log(d) + m \log(x))} + 18*a*b*n*x*e^{(m \log(d) + m \log(x))} + 8*b \\
& ^2*n*x*e^{(m \log(d) + m \log(x))} + 16*a*c*n*x*e^{(m \log(d) + m \log(x))} + 14*b* \\
& c*n*x*e^{(m \log(d) + m \log(x))} + 6*c^2*n*x*e^{(m \log(d) + m \log(x))} + c^2*x*x \\
& ^{(4*n)}*e^{(m \log(d) + m \log(x))} + 2*b*c*x*x^{(3*n)}*e^{(m \log(d) + m \log(x))} + \\
& c^2*x*x^{(3*n)}*e^{(m \log(d) + m \log(x))} + b^2*x*x^{(2*n)}*e^{(m \log(d) + m \log(x) \\
&)} + 2*a*c*x*x^{(2*n)}*e^{(m \log(d) + m \log(x))} + 2*b*c*x*x^{(2*n)}*e^{(m \log(d) \\
& + m \log(x))} + c^2*x*x^{(2*n)}*e^{(m \log(d) + m \log(x))} + 2*a*b*x*x^n*e^{(m \log(d) \\
& + m \log(x))} + b^2*x*x^n*e^{(m \log(d) + m \log(x))} + 2*a*c*x*x^n*e^{(m \log(d) \\
& + m \log(x))} + 2*b*c*x*x^n*e^{(m \log(d) + m \log(x))} + c^2*x*x^n*e^{(m \log(d) \\
& + m \log(x))} + a^2*x*e^{(m \log(d) + m \log(x))} + 2*a*b*x*e^{(m \log(d) + m \log(x) \\
& + m \log(x))} + b^2*x*e^{(m \log(d) + m \log(x))} + 2*a*c*x*e^{(m \log(d) + m \log(x))} + 2*b \\
& *c*x*e^{(m \log(d) + m \log(x))} + c^2*x*e^{(m \log(d) + m \log(x))})/(m^5 + 10*m^4 \\
& *n + 35*m^3*n^2 + 50*m^2*n^3 + 24*m*n^4 + 5*m^4 + 40*m^3*n + 105*m^2*n^2 + \\
& 100*m*n^3 + 24*n^4 + 10*m^3 + 60*m^2*n + 105*m*n^2 + 50*n^3 + 10*m^2 + 40*m \\
& *n + 35*n^2 + 5*m + 10*n + 1)
\end{aligned}$$

maple [C] time = 0.07, size = 1065, normalized size = 9.10

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x)^m*(b*x^n+c*x^{(2*n)}+a)^2,x)$

[Out] $x*(2*(x^n)^3*b*c+a^2+c^2*(x^n)^4+a^2*m^4+4*a^2*m^3+50*a^2*n^3+6*a^2*m^2+35*$

$$\begin{aligned}
& a^2 * n^2 + 24 * a^2 * n^4 + 4 * a^2 * m + 10 * a^2 * n + b^2 * (x^n)^2 + 6 * c^2 * m^3 * n * (x^n)^4 + 11 * c^2 * m^2 * n^2 * (x^n)^4 + 6 * c^2 * m * n^3 * (x^n)^4 + 2 * b * c * m^4 * (x^n)^3 + 18 * c^2 * m^2 * n * (x^n)^4 + \\
& 22 * c^2 * m * n^2 * (x^n)^4 + 52 * a * b * n^2 * x^n + 8 * a * c * (x^n)^2 * m + 12 * a * b * m^2 * x^n + 12 * a * c * m^2 * (x^n)^2 + 38 * a * c * n^2 * (x^n)^2 + 24 * b^2 * m * n * (x^n)^2 + 8 * m * b * c * (x^n)^3 + 14 * b * c * (x^n)^3 * n + \\
& 18 * c^2 * m * n * (x^n)^4 + 2 * a * b * m^4 * x^n + 8 * a * c * m^3 * (x^n)^2 + 24 * a * c * n^3 * (x^n)^2 + 24 * b^2 * m^2 * n * (x^n)^2 + 38 * b^2 * m * n^2 * (x^n)^2 + 12 * b * c * m^2 * (x^n)^3 + 28 * b * c * n^2 * (x^n)^3 + \\
& 8 * a * b * m^3 * x^n + 48 * a * b * n^3 * x^n + 16 * a * c * (x^n)^2 * n + 8 * a * b * x^n * m + 18 * a * b * x^n * n + 2 * a * c * m^4 * (x^n)^2 + 8 * b^2 * m^3 * n * (x^n)^2 + 19 * b^2 * m^2 * n^2 * (x^n)^2 + 12 * b^2 * m * n^3 * (x^n)^2 + \\
& 8 * b * c * m^3 * (x^n)^3 + 16 * b * c * n^3 * (x^n)^3 + 2 * (x^n)^2 * a * c + c^2 * m^4 * (x^n)^4 + 4 * c^2 * m^3 * (x^n)^4 + 56 * b * c * m * n^2 * (x^n)^3 + 18 * a * b * m^3 * n * x^n + 52 * a * b * m^2 * n^2 * x^n + \\
& 48 * a * b * m * n^3 * x^n + 48 * a * c * m^2 * n * (x^n)^2 + 76 * a * c * m * n^2 * (x^n)^2 + 42 * b * c * m * n * (x^n)^3 + 54 * a * b * m^2 * n * x^n + 104 * a * b * m * n^2 * x^n + 48 * a * c * m * n * (x^n)^2 + 54 * a * b * m * n * x^n + \\
& 6 * c^2 * n^3 * (x^n)^4 + b^2 * m^4 * (x^n)^2 + 6 * c^2 * m^2 * (x^n)^4 + 11 * c^2 * n^2 * (x^n)^4 + 4 * b^2 * m^3 * (x^n)^2 + 12 * b^2 * n^3 * (x^n)^2 + 4 * m * c^2 * (x^n)^4 + 6 * c^2 * (x^n)^4 * n + 6 * b^2 * m^2 * (x^n)^2 + \\
& 14 * b * c * m^3 * n * (x^n)^3 + 28 * b * c * m^2 * n^2 * (x^n)^3 + 16 * b * c * m * n^3 * (x^n)^3 + 16 * a * c * m^3 * n * (x^n)^2 + 38 * a * c * m^2 * n^2 * (x^n)^2 + 24 * a * c * m * n^3 * (x^n)^2 + 42 * b * c * m^2 * n * (x^n)^3 + \\
& 19 * b^2 * n^2 * (x^n)^2 + 4 * b^2 * (x^n)^2 * m + 8 * b^2 * (x^n)^2 * n + 30 * a^2 * m * n + 2 * a * b * x^n + 10 * a^2 * m^3 * n + 35 * a^2 * m^2 * n^2 + 50 * a^2 * m * n^3 + 30 * a^2 * m^2 * n + 70 * a^2 * m * n^2) / (m + 1) / (m + n + 1) / (m + 2 * n + 1) / (m + 3 * n + 1) / (1 + m + 4 * n) * \exp(1/2 * (-I * \pi * \operatorname{csgn}(I * d) * \operatorname{csgn}(I * x)) * \operatorname{csgn}(I * d * x) + I * \pi * \operatorname{csgn}(I * d) * \operatorname{csgn}(I * d * x))^2 + I * \pi * \operatorname{csgn}(I * x) * \operatorname{csgn}(I * d * x))^2 - I * \pi * \operatorname{csgn}(I * d * x))^3 + 2 * \ln(d) + 2 * \ln(x)) * m)
\end{aligned}$$

maxima [A] time = 1.27, size = 152, normalized size = 1.30

$$\frac{c^2 d^m x e^{(m \log(x) + 4n \log(x))}}{m + 4n + 1} + \frac{2 b c d^m x e^{(m \log(x) + 3n \log(x))}}{m + 3n + 1} + \frac{b^2 d^m x e^{(m \log(x) + 2n \log(x))}}{m + 2n + 1} + \frac{2 a c d^m x e^{(m \log(x) + 2n \log(x))}}{m + 2n + 1} + \frac{2 a b d^m x e^{(m \log(x) + n \log(x))}}{m + n + 1} + \frac{(dx)^{m+1} a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")

[Out] $c^2 d^m x^m e^{(m \log(x) + 4n \log(x))} / (m + 4n + 1) + 2 * b * c * d^m x^m e^{(m \log(x) + 3n \log(x))} / (m + 3n + 1) + b^2 d^m x^m e^{(m \log(x) + 2n \log(x))} / (m + 2n + 1) + 2 * a * c * d^m x^m e^{(m \log(x) + 2n \log(x))} / (m + 2n + 1) + 2 * a * b * d^m x^m e^{(m \log(x) + n \log(x))} / (m + n + 1) + (d * x)^{(m + 1)} * a^2 / (d * (m + 1))$

mupad [B] time = 1.62, size = 543, normalized size = 4.64

$$\frac{c^2 d^m x^m e^{(m \log(x) + 4n \log(x))}}{m + 4n + 1} + \frac{2 b c d^m x^m e^{(m \log(x) + 3n \log(x))}}{m + 3n + 1} + \frac{b^2 d^m x^m e^{(m \log(x) + 2n \log(x))}}{m + 2n + 1} + \frac{2 a c d^m x^m e^{(m \log(x) + 2n \log(x))}}{m + 2n + 1} + \frac{2 a b d^m x^m e^{(m \log(x) + n \log(x))}}{m + n + 1} + \frac{(d x)^{m+1} a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^2,x)

[Out] $(a^2 * x * (d * x)^m) / (m + 1) + (x * x^n * (2 * n) * (d * x)^m * (2 * a * c + b^2) * (3 * m + 8 * n + 16 * m * n + 19 * m * n^2 + 8 * m^2 * n + 3 * m^2 + m^3 + 19 * n^2 + 12 * n^3 + 1)) / (4 * m + 10 * n + 30 * m * n + 70 * m * n^2 + 30 * m^2 * n + 50 * m * n^3 + 10 * m^3 * n + 6 * m^2 + 4 * m^3 + m^4$

$$\begin{aligned}
& + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (c^2*x*x^{(4*n)}*(d*x)^m*(3*m \\
& + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1))/(4 \\
& *m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4* \\
& m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*a*b*x*x^n*(d*x) \\
& ^m*(3*m + 9*n + 18*m*n + 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*n^2 + 24*n^3 \\
& + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6 \\
& *m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*b*c*x* \\
& x^{(3*n)}*(d*x)^m*(3*m + 7*n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2 + m^3 + 14 \\
& *n^2 + 8*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + \\
& 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)

[Out] Timed out

$$3.493 \quad \int (dx)^m (a + bx^n + cx^{2n}) dx$$

Optimal. Leaf size=58

$$\frac{a(dx)^{m+1}}{d(m+1)} + \frac{bx^{n+1}(dx)^m}{m+n+1} + \frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {14, 20, 30}

$$\frac{a(dx)^{m+1}}{d(m+1)} + \frac{bx^{n+1}(dx)^m}{m+n+1} + \frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

Antiderivative was successfully verified.

[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]

[Out] (b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) + (a*(d*x)^(1 + m))/(d*(1 + m))

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
\int (dx)^m (a + bx^n + cx^{2n}) dx &= \int (a(dx)^m + bx^n(dx)^m + cx^{2n}(dx)^m) dx \\
&= \frac{a(dx)^{1+m}}{d(1+m)} + b \int x^n(dx)^m dx + c \int x^{2n}(dx)^m dx \\
&= \frac{a(dx)^{1+m}}{d(1+m)} + (bx^{-m}(dx)^m) \int x^{m+n} dx + (cx^{-m}(dx)^m) \int x^{m+2n} dx \\
&= \frac{bx^{1+n}(dx)^m}{1+m+n} + \frac{cx^{1+2n}(dx)^m}{1+m+2n} + \frac{a(dx)^{1+m}}{d(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.71

$$x(dx)^m \left(\frac{a}{m+1} + x^n \left(\frac{b}{m+n+1} + \frac{cx^n}{m+2n+1} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n)), x]

[Out] x*(d*x)^m*(a/(1 + m) + x^n*(b/(1 + m + n) + (c*x^n)/(1 + m + 2*n)))

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx)^m (a + bx^n + cx^{2n}) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x)^m*(a + b*x^n + c*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic] [(d*x)^m*(a + b*x^n + c*x^(2*n)), x]

fricas [B] time = 1.28, size = 142, normalized size = 2.45

$$\frac{(cm^2 + 2cm + (cm + c)n + c)xx^{2n}e^{(m \log(d) + m \log(x))} + (bm^2 + 2bm + 2(bm + b)n + b)xx^n e^{(m \log(d) + m \log(x))} + (am^2 + 2an^2 + 2am + 3(am + a)n + a)xe^{(m \log(d) + m \log(x))}}{m^3 + 2(m+1)n^2 + 3m^2 + 3(m^2 + 2m + 1)n + 3m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)), x, algorithm="fricas")

[Out] ((c*m^2 + 2*c*m + (c*m + c)*n + c)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + (b*m^2 + 2*b*m + 2*(b*m + b)*n + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m^2 + 2*a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*x*x*e^(m*log(d) + m*log(x)))/(m^3 + 2*(m + 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)

giac [B] time = 0.40, size = 557, normalized size = 9.60

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] (c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b*m^2*x*x^n*e^(m*log(d) + m*log(x)) + c*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + c*m*n*x*x^n*e^(m*log(d) + m*log(x)) + a*m^2*x*e^(m*log(d) + m*log(x)) + b*m^2*x*e^(m*log(d) + m*log(x)) + c*m^2*x*e^(m*log(d) + m*log(x)) + 3*a*m*n*x*e^(m*log(d) + m*log(x)) + 2*b*m*n*x*e^(m*log(d) + m*log(x)) + c*m*n*x*e^(m*log(d) + m*log(x)) + 2*a*n^2*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*m*x*x^n*e^(m*log(d) + m*log(x)) + 2*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*n*x*x^n*e^(m*log(d) + m*log(x)) + c*n*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*m*x*e^(m*log(d) + m*log(x)) + 2*b*m*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*e^(m*log(d) + m*log(x)) + 3*a*n*x*e^(m*log(d) + m*log(x)) + 2*b*n*x*e^(m*log(d) + m*log(x)) + c*n*x*e^(m*log(d) + m*log(x)) + c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b*x*x^n*e^(m*log(d) + m*log(x)) + c*x*x^n*e^(m*log(d) + m*log(x)) + a*x*e^(m*log(d) + m*log(x)) + b*x*e^(m*log(d) + m*log(x)) + c*x*e^(m*log(d) + m*log(x)))/(m^3 + 3*m^2*n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)

maple [C] time = 0.05, size = 205, normalized size = 3.53

$$\frac{(b m^2 x^n + 2 b m n x^n + c m^2 x^{2n} + c m n x^{2n} + a m^2 + 3 a m n + 2 a n^2 + 2 b m x^n + 2 b n x^n + 2 c m x^{2n} + c n x^{2n} + 2 a m + 3 a n + b x^n + c x^{2n} + a) x e^{\frac{(-i \pi \operatorname{sgn}(d) \operatorname{sgn}(x) \operatorname{sgn}(d x) + i \pi \operatorname{sgn}(d) \operatorname{sgn}(d x)^2 + i \pi \operatorname{sgn}(x) \operatorname{sgn}(d x)^2 - i \pi \operatorname{sgn}(d x)^3 + 2 \ln(d) + 2 \ln(x)) m}{2}}}{(m+1)(m+n+1)(m+2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(b*x^n+c*x^(2*n)+a),x)

[Out] x*(c*m^2*(x^n)^2+c*m*n*(x^n)^2+b*m^2*x^n+2*b*m*n*x^n+2*m*c*(x^n)^2+c*(x^n)^2*2*n+a*m^2+3*a*m*n+2*a*n^2+2*b*m*x^n+2*b*x^n*n+c*(x^n)^2+2*a*m+3*a*n+b*x^n+a)/(m+1)/(m+n+1)/(m+2*n+1)*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*d*x)^3+2*ln(d)+2*ln(x))*m)

maxima [A] time = 1.12, size = 65, normalized size = 1.12

$$\frac{cd^m x e^{(m \log(x) + 2n \log(x))}}{m + 2n + 1} + \frac{bd^m x e^{(m \log(x) + n \log(x))}}{m + n + 1} + \frac{(dx)^{m+1} a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] $c*d^m*x*e^{(m*\log(x) + 2*n*\log(x))/(m + 2*n + 1)} + b*d^m*x*e^{(m*\log(x) + n*\log(x))/(m + n + 1)} + (d*x)^{(m + 1)}*a/(d*(m + 1))$

mupad [B] time = 1.41, size = 83, normalized size = 1.43

$$(dx)^m \left(\frac{ax}{m+1} + \frac{bx x^n (m+2n+1)}{m^2 + 3mn + 2m + 2n^2 + 3n + 1} + \frac{cx x^{2n} (m+n+1)}{m^2 + 3mn + 2m + 2n^2 + 3n + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a + b*x^n + c*x^(2*n)),x)

[Out] $(d*x)^m*((a*x)/(m + 1) + (b*x*x^n*(m + 2*n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2 + 1) + (c*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2 + 1))$

sympy [A] time = 43.00, size = 1239, normalized size = 21.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)

[Out] Piecewise(((a + b + c)*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a*log(x) + b*x**n/n + c*x**(2*n)/(2*n))/d, Eq(m, -1)), (a*Piecewise((log(x), Eq(n, 0)), (-x**(-2*n)*(0**(1/n))**(-2*n)/(2*n), Eq(d, 0**(1/n))), (-d**(-2*n)*x**(-2*n)/(2*n), True))/d + b*Piecewise((log(x), Eq(n, 0)), (-x**n/(2*0**(1/n)*zoo**(1/n)*n*x**(2*n)*(0**(1/n))**(2*n) - n*x**(2*n)*(0**(1/n))**(2*n)), Eq(d, 0**(1/n))), (-d**(-2*n)*x**(-n)/n, True))/d + c*Piecewise((d**(-2*n)*log(x), Abs(x) < 1), (-d**(-2*n)*log(1/x), 1/Abs(x) < 1), (-d**(-2*n)*meijerg(((), (1, 1)), ((0, 0), ()), x) + d**(-2*n)*meijerg(((1, 1), ()), (((), (0, 0)), x), True))/d, Eq(m, -2*n - 1)), (a*Piecewise((log(x), Eq(n, 0)), (-x**(-n)*(0**(1/n))**(-n)/n, Eq(d, 0**(1/n))), (-d**(-n)*x**(-n)/n, True))/d + b*Piecewise((d**(-n)*log(x), Abs(x) < 1), (-d**(-n)*log(1/x), 1/Abs(x) < 1), (-d**(-n)*meijerg(((), (1, 1)), ((0, 0), ()), x) + d**(-n)*meijerg(((1, 1), ()), (((), (0, 0)), x), True))/d + c*Piecewise((log(x), Eq(n, 0)), (-x**(2*n)/(0**(1/n)*zoo**(1/n)*n*x**n*(0**(1/n))**n - 2*n*x**n*(0**(1/n))**n), Eq(d, 0**(1/n))), (d**(-n)*x**n/n, True))/d, Eq(m, -n - 1)), (a*d**m**m**2*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*d**m*m*n*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a*d**m*m*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a*d**m*n**2*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a*d**m*n*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + a*d**m*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1))

```

+ b*d**m*m**2*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*
m + 2*n**2 + 3*n + 1) + 2*b*d**m*m*n*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2
+ 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*d**m*m*x*x**m*x**n/(m**3
+ 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*d**
m*n*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2
+ 3*n + 1) + b*d**m*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m
*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*m**2*x*x**m*x**(2*n)/(m**3 + 3*m**2*n
+ 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*m*n*x*x**m*
x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n
+ 1) + 2*c*d**m*m*x*x**m*x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*
m*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*n*x*x**m*x**(2*n)/(m**3 + 3*m**2*n +
3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + c*d**m*x*x**m*x**(2*
n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1),
True))

```

$$3.494 \quad \int (d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4) dx$$

Optimal. Leaf size=46

$$\frac{a(d + ex)^4}{4e} + \frac{b(d + ex)^6}{6e} + \frac{c(d + ex)^8}{8e}$$

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1142, 14}

$$\frac{a(d + ex)^4}{4e} + \frac{b(d + ex)^6}{6e} + \frac{c(d + ex)^8}{8e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] (a*(d + e*x)^4)/(4*e) + (b*(d + e*x)^6)/(6*e) + (c*(d + e*x)^8)/(8*e)

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1142

Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned} \int (d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4) dx &= \frac{\text{Subst} \left(\int x^3 (a + bx^2 + cx^4) dx, x, d + ex \right)}{e} \\ &= \frac{\text{Subst} \left(\int (ax^3 + bx^5 + cx^7) dx, x, d + ex \right)}{e} \\ &= \frac{a(d + ex)^4}{4e} + \frac{b(d + ex)^6}{6e} + \frac{c(d + ex)^8}{8e} \end{aligned}$$

Mathematica [B] time = 0.04, size = 150, normalized size = 3.26

$$\frac{1}{4}e^3x^4(a+10bd^2+35cd^4)+\frac{1}{3}de^2x^3(3a+10bd^2+21cd^4)+\frac{1}{2}d^2ex^2(3a+5bd^2+7cd^4)+d^3x(a+bd^2+cd^4)+\frac{1}{6}e^5x^6(b+21cd^2)+de^4x^5(b+7cd^2)+cde^6x^7+\frac{1}{8}ce^7x^8$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] $d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 + (d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7 + (c*e^7*x^8)/8$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

fricas [B] time = 1.16, size = 175, normalized size = 3.80

$$\frac{1}{8}x^8e^7c+x^7e^6dc+\frac{7}{2}x^6e^5d^2c+7x^5e^4d^3c+\frac{35}{4}x^4e^3d^4c+\frac{1}{6}x^6e^5b+7x^3e^2d^5c+x^5e^4db+\frac{7}{2}x^2ed^6c+\frac{5}{2}x^4e^3d^2b+xd^7c+\frac{10}{3}x^3e^2d^3b+\frac{5}{2}x^2ed^4b+\frac{1}{4}x^4e^3a+xd^5b+x^3e^2da+\frac{3}{2}x^2ed^2a+xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="fricas")

[Out] $\frac{1}{8}x^8e^7c + x^7e^6d^2c + \frac{7}{2}x^6e^5d^2c + 7x^5e^4d^3c + \frac{35}{4}x^4e^3d^4c + \frac{1}{6}x^6e^5b + 7x^3e^2d^5c + x^5e^4db + \frac{7}{2}x^2ed^6c + \frac{5}{2}x^4e^3d^2b + xd^7c + \frac{10}{3}x^3e^2d^3b + \frac{5}{2}x^2ed^4b + \frac{1}{4}x^4e^3a + xd^5b + x^3e^2da + \frac{3}{2}x^2ed^2a + xd^3a$

giac [B] time = 0.41, size = 169, normalized size = 3.67

$$\frac{1}{2}(x^2e+2dx)cd^6+\frac{3}{4}(x^2e+2dx)^2cd^4e+\frac{1}{2}(x^2e+2dx)^3cd^2e^2+\frac{1}{2}(x^2e+2dx)bd^4+\frac{1}{8}(x^2e+2dx)^4ce^3+\frac{1}{2}(x^2e+2dx)^2bd^2e+\frac{1}{6}(x^2e+2dx)^3be^2+\frac{1}{2}(x^2e+2dx)ad^2+\frac{1}{4}(x^2e+2dx)^2ae$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="giac")

[Out] $\frac{1}{2}*(x^2*e + 2*d*x)*c*d^6 + \frac{3}{4}*(x^2*e + 2*d*x)^2*c*d^4*e + \frac{1}{2}*(x^2*e + 2*d*x)^3*c*d^2*e^2 + \frac{1}{2}*(x^2*e + 2*d*x)*b*d^4 + \frac{1}{8}*(x^2*e + 2*d*x)^4*c*e^3$

$$+ 1/2*(x^2*e + 2*d*x)^2*b*d^2*e + 1/6*(x^2*e + 2*d*x)^3*b*e^2 + 1/2*(x^2*e + 2*d*x)*a*d^2 + 1/4*(x^2*e + 2*d*x)^2*a*e$$

maple [B] time = 0.00, size = 298, normalized size = 6.48

$$\frac{c^2x^8}{8} + cd^6x^7 + \frac{(15cd^2 + (6cd^2 + b^2)e^2)x^6}{6} + \frac{(13cd^4 + 3(6cd^2 + b^2)d^2 + (4cd^2 + 2bd)e^2)x^5}{5} + (cd^4 + bd^2 + a)d^3x^4 + \frac{(4cd^2 + 3(6cd^2 + b^2)d^2 + 3(4cd^2 + 2bd)d^2 + (cd^4 + b^2 + a)e^2)x^3}{4} + \frac{(6cd^2 + b^2)d^2 + 3(4cd^2 + 2bd)d^2 + 3(cd^4 + b^2 + a)d^2x^2}{3} + \frac{(4cd^2 + 2bd)d^2 + 3(cd^4 + b^2 + a)d^2x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4), x)

$$[Out] 1/8*e^7*c*x^8+d*e^6*c*x^7+1/6*(15*d^2*e^5*c+e^3*(6*c*d^2*e^2+b*e^2))*x^6+1/5*(13*d^3*c*e^4+3*d*e^2*(6*c*d^2*e^2+b*e^2)+e^3*(4*c*d^3*e+2*b*d*e))*x^5+1/4*(4*d^4*c*e^3+3*d^2*e*(6*c*d^2*e^2+b*e^2)+3*d*e^2*(4*c*d^3*e+2*b*d*e)+e^3*(c*d^4+b*d^2+a))*x^4+1/3*(d^3*(6*c*d^2*e^2+b*e^2)+3*d^2*e*(4*c*d^3*e+2*b*d*e)+3*d*e^2*(c*d^4+b*d^2+a))*x^3+1/2*(d^3*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4+b*d^2+a))*x^2+d^3*(c*d^4+b*d^2+a)*x$$

maxima [B] time = 1.02, size = 142, normalized size = 3.09

$$\frac{1}{8}ce^7x^8 + cd^6x^7 + \frac{1}{6}(21cd^2 + b)e^5x^6 + (7cd^3 + bd)e^4x^5 + \frac{1}{4}(35cd^4 + 10bd^2 + a)e^3x^4 + \frac{1}{3}(21cd^5 + 10bd^3 + 3ad)e^2x^3 + \frac{1}{2}(7cd^6 + 5bd^4 + 3ad^2)ex^2 + (cd^7 + bd^5 + ad^3)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="maxima")

$$[Out] 1/8*c*e^7*x^8 + c*d*e^6*x^7 + 1/6*(21*c*d^2 + b)*e^5*x^6 + (7*c*d^3 + b*d)*e^4*x^5 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*e^3*x^4 + 1/3*(21*c*d^5 + 10*b*d^3 + 3*a*d)*e^2*x^3 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*e*x^2 + (c*d^7 + b*d^5 + a*d^3)*x$$

mupad [B] time = 0.08, size = 141, normalized size = 3.07

$$x(c d^7 + b d^5 + a d^3) + \frac{e^5 x^6 (21 c d^2 + b)}{6} + \frac{c e^7 x^8}{8} + \frac{e^3 x^4 (35 c d^4 + 10 b d^2 + a)}{4} + \frac{d^2 e x^2 (7 c d^4 + 5 b d^2 + 3 a)}{2} + \frac{d e^2 x^3 (21 c d^4 + 10 b d^2 + 3 a)}{3} + d e^4 x^5 (7 c d^2 + b) + c d e^6 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x)

$$[Out] x*(a*d^3 + b*d^5 + c*d^7) + (e^5*x^6*(b + 21*c*d^2))/6 + (c*e^7*x^8)/8 + (e^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*x^2*(3*a + 5*b*d^2 + 7*c*d^4))/2 + (d*e^2*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*e^4*x^5*(b + 7*c*d^2) + c*d*e^6*x^7$$

sympy [B] time = 0.11, size = 178, normalized size = 3.87

$$cd^6e^7x^7 + \frac{ce^7x^8}{8} + x^6\left(\frac{be^5}{6} + \frac{7cd^2e^5}{2}\right) + x^5(bde^4 + 7cd^3e^4) + x^4\left(\frac{ae^3}{4} + \frac{5bd^2e^3}{2} + \frac{35cd^4e^3}{4}\right) + x^3\left(ade^2 + \frac{10bd^3e^2}{3} + 7cd^5e^2\right) + x^2\left(\frac{3ad^2e}{2} + \frac{5bd^4e}{2} + \frac{7cd^6e}{2}\right) + x(ad^3 + bd^5 + cd^7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*x**7 + c*e**7*x**8/8 + x**6*(b*e**5/6 + 7*c*d**2*e**5/2) + x**5*(b*d*e**4 + 7*c*d**3*e**4) + x**4*(a*e**3/4 + 5*b*d**2*e**3/2 + 35*c*d**4*e**3/4) + x**3*(a*d*e**2 + 10*b*d**3*e**2/3 + 7*c*d**5*e**2) + x**2*(3*a*d**2*e/2 + 5*b*d**4*e/2 + 7*c*d**6*e/2) + x*(a*d**3 + b*d**5 + c*d**7)
```


$$3.495 \quad \int (d + ex)^3 \left(a + b(d + ex)^2 + c(d + ex)^4 \right)^2 dx$$

Optimal. Leaf size=89

$$\frac{a^2(d + ex)^4}{4e} + \frac{(2ac + b^2)(d + ex)^8}{8e} + \frac{ab(d + ex)^6}{3e} + \frac{bc(d + ex)^{10}}{5e} + \frac{c^2(d + ex)^{12}}{12e}$$

Rubi [A] time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1142, 1114, 631}

$$\frac{a^2(d + ex)^4}{4e} + \frac{(2ac + b^2)(d + ex)^8}{8e} + \frac{ab(d + ex)^6}{3e} + \frac{bc(d + ex)^{10}}{5e} + \frac{c^2(d + ex)^{12}}{12e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] (a^2*(d + e*x)^4)/(4*e) + (a*b*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*(d + e*x)^8)/(8*e) + (b*c*(d + e*x)^10)/(5*e) + (c^2*(d + e*x)^12)/(12*e)

Rule 631

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int (d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^2 dx &= \frac{\text{Subst}\left(\int x^3 (a+bx^2+cx^4)^2 dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int x (a+bx+cx^2)^2 dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{\text{Subst}\left(\int (a^2x+2abx^2+(b^2+2ac)x^3+2bcx^4+c^2x^5) dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{a^2(d+ex)^4}{4e} + \frac{ab(d+ex)^6}{3e} + \frac{(b^2+2ac)(d+ex)^8}{8e} + \frac{bc(d+ex)^{10}}{5e}
\end{aligned}$$

Mathematica [B] time = 0.11, size = 401, normalized size = 4.51

$\frac{1}{4}a^2(e^4 d^4 + 20abcd^3 + 35b^2d^2 + 188bc^2d + 165c^3) + \frac{1}{3}ab^2(e^6 d^6 + 42abcd^5 + 21b^2d^4 + 72b^2c^2d^3 + 55c^2d^2) + \frac{1}{8}(b^2+2ac)^2(e^8 d^8 + 14abcd^7 + 70a^2cd^6 + 168abc^2d^5 + 165c^2d^4) + \frac{1}{5}bc^2(e^{10} d^{10} + 330abcd^9 + 252a^2cd^8 + 330c^2d^7) + \frac{1}{10}e^{10}d^{10} + \frac{1}{5}e^8 d^8 + \frac{1}{5}e^6 d^6 + \frac{1}{5}e^4 d^4 + \frac{1}{5}e^2 d^2 + \frac{1}{5}d^0$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] $d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 14*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8))*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8))*e^2*x^3)/3 + ((a^2 + 20*a*b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8))*e^3*x^4)/4 + (d*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6))*e^4*x^5)/5 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6))*e^5*x^6)/6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4))*e^6*x^7 + ((b^2 + 2*a*c + 72*b*c*d^2 + 330*c^2*d^4))*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2))*e^8*x^9)/3 + (c*(2*b + 55*c*d^2))*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]

fricas [B] time = 1.14, size = 571, normalized size = 6.42

$\frac{1}{4}a^2(e^4 d^4 + 20abcd^3 + 35b^2d^2 + 188bc^2d + 165c^3) + \frac{1}{3}ab^2(e^6 d^6 + 42abcd^5 + 21b^2d^4 + 72b^2c^2d^3 + 55c^2d^2) + \frac{1}{8}(b^2+2ac)^2(e^8 d^8 + 14abcd^7 + 70a^2cd^6 + 168abc^2d^5 + 165c^2d^4) + \frac{1}{5}bc^2(e^{10} d^{10} + 330abcd^9 + 252a^2cd^8 + 330c^2d^7) + \frac{1}{10}e^{10}d^{10} + \frac{1}{5}e^8 d^8 + \frac{1}{5}e^6 d^6 + \frac{1}{5}e^4 d^4 + \frac{1}{5}e^2 d^2 + \frac{1}{5}d^0$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out] $\frac{1}{12}x^{12}e^{11}c^2 + x^{11}e^{10}d^2c^2 + \frac{11}{2}x^{10}e^9d^2c^2 + \frac{55}{3}x^9e^8d^3c^2 + \frac{165}{4}x^8e^7d^4c^2 + \frac{1}{5}x^{10}e^9c^2b + 66x^7e^6d^5c^2 + 2x^9e^8d^2cb + 77x^6e^5d^6c^2 + 9x^8e^7d^2cb + 66x^5e^4d^7c^2 + 24x^7e^6d^3cb + \frac{165}{4}x^4e^3d^8c^2 + 42x^6e^5d^4cb + \frac{1}{8}x^8e^7b^2 + \frac{1}{4}x^8e^7ca + \frac{55}{3}x^3e^2d^9c^2 + \frac{252}{5}x^5e^4d^5cb + x^7e^6db^2 + 2x^7e^6dca + \frac{11}{2}x^2e^10c^2 + 42x^4e^3d^6cb + \frac{7}{2}x^6e^5d^2b^2 + 7x^6e^5d^2ca + xd^{11}c^2 + 24x^3e^2d^7cb + 7x^5e^4d^3b^2 + 14x^5e^4d^3ca + 9x^2e^8d^8cb + \frac{35}{4}x^4e^3d^4b^2 + \frac{35}{2}x^4e^3d^4ca + \frac{1}{3}x^6e^5b^2a + 2xd^9c^2b + 7x^3e^2d^5b^2 + 14x^3e^2d^5ca + 2x^5e^4d^2b^2a + \frac{7}{2}x^2e^6d^6b^2 + 7x^2e^6d^6ca + 5x^4e^3d^2b^2a + xd^7b^2 + 2xd^7ca + \frac{20}{3}x^3e^2d^3b^2a + 5x^2e^4d^4b^2a + \frac{1}{4}x^4e^3a^2 + 2xd^5b^2a + x^3e^2da^2 + \frac{3}{2}x^2e^2d^2a^2 + xd^3a^2$

giac [B] time = 0.41, size = 493, normalized size = 5.54

$\frac{1}{12}x^{12}e^{11}c^2 + \frac{1}{5}x^{10}e^9c^2b + \frac{11}{2}x^{10}e^9d^2c^2 + \frac{55}{3}x^9e^8d^3c^2 + \frac{165}{4}x^8e^7d^4c^2 + \frac{1}{8}x^8e^7b^2 + \frac{1}{4}x^8e^7ca + \frac{55}{3}x^3e^2d^9c^2 + \frac{252}{5}x^5e^4d^5cb + x^7e^6db^2 + 2x^7e^6dca + \frac{11}{2}x^2e^10c^2 + 42x^4e^3d^6cb + \frac{7}{2}x^6e^5d^2b^2 + 7x^6e^5d^2ca + xd^{11}c^2 + 24x^3e^2d^7cb + 7x^5e^4d^3b^2 + 14x^5e^4d^3ca + 9x^2e^8d^8cb + \frac{35}{4}x^4e^3d^4b^2 + \frac{35}{2}x^4e^3d^4ca + \frac{1}{3}x^6e^5b^2a + 2xd^9c^2b + 7x^3e^2d^5b^2 + 14x^3e^2d^5ca + 2x^5e^4d^2b^2a + \frac{7}{2}x^2e^6d^6b^2 + 7x^2e^6d^6ca + 5x^4e^3d^2b^2a + xd^7b^2 + 2xd^7ca + \frac{20}{3}x^3e^2d^3b^2a + 5x^2e^4d^4b^2a + \frac{1}{4}x^4e^3a^2 + 2xd^5b^2a + x^3e^2da^2 + \frac{3}{2}x^2e^2d^2a^2 + xd^3a^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] $\frac{1}{2}(x^2e + 2d*x)^2c^2d^{10} + \frac{5}{4}(x^2e + 2d*x)^2c^2d^8e + \frac{5}{3}(x^2e + 2d*x)^3c^2d^6e^2 + (x^2e + 2d*x)^2b^2c^2d^8 + \frac{5}{4}(x^2e + 2d*x)^4c^2d^4e^3 + 2(x^2e + 2d*x)^2b^2c^2d^6e + \frac{1}{2}(x^2e + 2d*x)^5c^2d^2e^4 + 2(x^2e + 2d*x)^3b^2c^2d^4e^2 + \frac{1}{2}(x^2e + 2d*x)^2b^2d^6 + (x^2e + 2d*x)^2a^2c^2d^6 + \frac{1}{12}(x^2e + 2d*x)^6c^2e^5 + (x^2e + 2d*x)^4b^2c^2d^2e^3 + \frac{3}{4}(x^2e + 2d*x)^2b^2d^4e + \frac{3}{2}(x^2e + 2d*x)^2a^2c^2d^4e + \frac{1}{5}(x^2e + 2d*x)^5b^2c^2e^4 + \frac{1}{2}(x^2e + 2d*x)^3b^2d^2e^2 + (x^2e + 2d*x)^3a^2c^2d^2e^2 + (x^2e + 2d*x)^2a^2b^2d^4 + \frac{1}{8}(x^2e + 2d*x)^4b^2e^3 + \frac{1}{4}(x^2e + 2d*x)^4a^2c^2e^3 + (x^2e + 2d*x)^2a^2b^2d^2e + \frac{1}{3}(x^2e + 2d*x)^3a^2b^2e^2 + \frac{1}{2}(x^2e + 2d*x)^2a^2d^2 + \frac{1}{4}(x^2e + 2d*x)^2a^2e$

maple [B] time = 0.00, size = 1314, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out] $\frac{1}{12}e^{11}c^2x^{12} + d^2e^{10}c^2x^{11} + \frac{1}{10}(27d^2e^9c^2 + e^3(2(6cd^2e^2 + b^2e^2))c^2e^4 + 16c^2d^2e^6)x^{10} + \frac{1}{9}(25d^3c^2e^8 + 3d^2e^2(2(6cd^2$

$$\begin{aligned}
& *e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+e^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)) *x^9+1/8*(8*d^4*c^2*e^7+3*d^2*e*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+3*d*e^2*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)+e^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)) *x^8+1/7*(d^3*(2*(6*c*d^2*e^2+b*e^2)*c*e^4+16*c^2*d^2*e^6)+3*d^2*e*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)+3*d*e^2*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+e^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))) *x^7+1/6*(d^3*(2*(4*c*d^3*e+2*b*d*e)*c*e^4+8*(6*c*d^2*e^2+b*e^2)*c*d*e^3)+3*d^2*e*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+3*d*e^2*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+e^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)) *x^6+1/5*(d^3*(2*(c*d^4+b*d^2+a)*c*e^4+8*(4*c*d^3*e+2*b*d*e)*c*d*e^3+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*e*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d*e^2*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)) *x^5+1/4*(d^3*(8*(c*d^4+b*d^2+a)*c*d*e^3+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d^2*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d*e^2*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+e^3*(c*d^4+b*d^2+a)^2) *x^4+1/3*(d^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d^2*e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d*e^2*(c*d^4+b*d^2+a)^2) *x^3+1/2*(2*d^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4+b*d^2+a)^2) *x^2+d^3*(c*d^4+b*d^2+a)^2*x
\end{aligned}$$

maxima [B] time = 1.06, size = 403, normalized size = 4.53

$\frac{1}{12}c^2e^{11}x^{12} + \frac{1}{10}(55c^2d^2 + 2b^2c)e^9x^{10} + \frac{1}{8}(330c^2d^4 + 72b^2c^2d^2 + b^2 + 2a^2c)e^7x^8 + \frac{1}{6}(462c^2d^6 + 252b^2c^2d^4 + 21(b^2 + 2a^2c)d^2 + 2a^2b)d^5x^6 + \frac{1}{4}(330c^2d^7 + 252b^2c^2d^5 + 35(b^2 + 2a^2c)d^3 + 10a^2bd^2 + a^2)e^3x^4 + \frac{1}{3}(55c^2d^9 + 72b^2c^2d^7 + 21(b^2 + 2a^2c)d^5 + 20a^2bd^3 + 3a^2d^2)e^2x^3 + \frac{1}{2}(11c^2d^{10} + 18b^2c^2d^8 + 7(b^2 + 2a^2c)d^6 + 10a^2bd^4 + 3a^2d^2)e^2x^2 + (c^2d^{11} + 2b^2c^2d^9 + (b^2 + 2a^2c)d^7 + 2a^2bd^5 + a^2d^3)x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] 1/12*c^2*e^11*x^12 + c^2*d*e^10*x^11 + 1/10*(55*c^2*d^2 + 2*b*c)*e^9*x^10 + 1/3*(55*c^2*d^3 + 6*b*c*d)*e^8*x^9 + 1/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2 + 2*a*c)*e^7*x^8 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*e^6*x^7 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*e^5*x^6 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*e^4*x^5 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*e^3*x^4 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*x

mupad [B] time = 1.48, size = 383, normalized size = 4.30

$\frac{1}{12}c^2e^{11}x^{12} + \frac{1}{10}(55c^2d^2 + 2b^2c)e^9x^{10} + \frac{1}{8}(330c^2d^4 + 72b^2c^2d^2 + b^2 + 2a^2c)e^7x^8 + \frac{1}{6}(462c^2d^6 + 252b^2c^2d^4 + 21(b^2 + 2a^2c)d^2 + 2a^2b)d^5x^6 + \frac{1}{4}(330c^2d^7 + 252b^2c^2d^5 + 35(b^2 + 2a^2c)d^3 + 10a^2bd^2 + a^2)e^3x^4 + \frac{1}{3}(55c^2d^9 + 72b^2c^2d^7 + 21(b^2 + 2a^2c)d^5 + 20a^2bd^3 + 3a^2d^2)e^2x^3 + \frac{1}{2}(11c^2d^{10} + 18b^2c^2d^8 + 7(b^2 + 2a^2c)d^6 + 10a^2bd^4 + 3a^2d^2)e^2x^2 + (c^2d^{11} + 2b^2c^2d^9 + (b^2 + 2a^2c)d^7 + 2a^2bd^5 + a^2d^3)x$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x)$

[Out] $(e^7*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*x^6*(2*a*b + 21*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (e^3*x^4*(a^2 + 35*b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 168*b*c*d^6))/4 + (c^2*e^{11}*x^{12})/12 + d^3*x*(a + b*d^2 + c*d^4)^2 + (c*e^9*x^{10}*(2*b + 55*c*d^2))/10 + c^2*d*e^{10}*x^{11} + (d^2*e*x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d^2 + 14*a*c*d^4 + 18*b*c*d^6))/2 + (d*e^2*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*d^8 + 20*a*b*d^2 + 42*a*c*d^4 + 72*b*c*d^6))/3 + d*e^6*x^7*(2*a*c + b^2 + 66*c^2*d^4 + 24*b*c*d^2) + (d*e^4*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 70*a*c*d^2 + 252*b*c*d^4))/5 + (c*d*e^8*x^9*(6*b + 55*c*d^2))/3$

sympy [B] time = 0.19, size = 559, normalized size = 6.28

$\int (d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2 dx = \frac{e^7 x^8 (2 a c + b^2 + 330 c^2 d^4 + 72 b c d^2)}{8} + \frac{e^5 x^6 (2 a b + 21 b^2 d^2 + 462 c^2 d^6 + 42 a c d^2 + 252 b c d^4)}{6} + \frac{e^3 x^4 (a^2 + 35 b^2 d^4 + 165 c^2 d^8 + 20 a b d^2 + 70 a c d^4 + 168 b c d^6)}{4} + \frac{c^2 e^{11} x^{12}}{12} + d^3 x (a + b d^2 + c d^4)^2 + \frac{c e^9 x^{10} (2 b + 55 c d^2)}{10} + c^2 d e^{10} x^{11} + \frac{d^2 e x^2 (3 a^2 + 7 b^2 d^4 + 11 c^2 d^8 + 10 a b d^2 + 14 a c d^4 + 18 b c d^6)}{2} + \frac{d e^2 x^3 (3 a^2 + 21 b^2 d^4 + 55 c^2 d^8 + 20 a b d^2 + 42 a c d^4 + 72 b c d^6)}{3} + d e^6 x^7 (2 a c + b^2 + 66 c^2 d^4 + 24 b c d^2) + \frac{d e^4 x^5 (10 a b + 35 b^2 d^2 + 330 c^2 d^6 + 70 a c d^2 + 252 b c d^4)}{5} + \frac{c d e^8 x^9 (6 b + 55 c d^2)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2, x)$

[Out] $c**2*d*e**10*x**11 + c**2*e**11*x**12/12 + x**10*(b*c*e**9/5 + 11*c**2*d**2*e**9/2) + x**9*(2*b*c*d*e**8 + 55*c**2*d**3*e**8/3) + x**8*(a*c*e**7/4 + b**2*e**7/8 + 9*b*c*d**2*e**7 + 165*c**2*d**4*e**7/4) + x**7*(2*a*c*d*e**6 + b**2*d*e**6 + 24*b*c*d**3*e**6 + 66*c**2*d**5*e**6) + x**6*(a*b*e**5/3 + 7*a*c*d**2*e**5 + 7*b**2*d**2*e**5/2 + 42*b*c*d**4*e**5 + 77*c**2*d**6*e**5) + x**5*(2*a*b*d*e**4 + 14*a*c*d**3*e**4 + 7*b**2*d**3*e**4 + 252*b*c*d**5*e**4/5 + 66*c**2*d**7*e**4) + x**4*(a**2*e**3/4 + 5*a*b*d**2*e**3 + 35*a*c*d**4*e**3/2 + 35*b**2*d**4*e**3/4 + 42*b*c*d**6*e**3 + 165*c**2*d**8*e**3/4) + x**3*(a**2*d*e**2 + 20*a*b*d**3*e**2/3 + 14*a*c*d**5*e**2 + 7*b**2*d**5*e**2 + 24*b*c*d**7*e**2 + 55*c**2*d**9*e**2/3) + x**2*(3*a**2*d**2*e/2 + 5*a*b*d**4*e + 7*a*c*d**6*e + 7*b**2*d**6*e/2 + 9*b*c*d**8*e + 11*c**2*d**10*e/2) + x*(a**2*d**3 + 2*a*b*d**5 + 2*a*c*d**7 + b**2*d**7 + 2*b*c*d**9 + c**2*d**11)$

$$3.496 \quad \int (d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)^3 dx$$

Optimal. Leaf size=138

$$\frac{a^3(d + ex)^4}{4e} + \frac{a^2b(d + ex)^6}{2e} + \frac{c(ac + b^2)(d + ex)^{12}}{4e} + \frac{b(6ac + b^2)(d + ex)^{10}}{10e} + \frac{3a(ac + b^2)(d + ex)^8}{8e} + \frac{3bc^2(d + ex)^{14}}{14e}$$

Rubi [A] time = 0.37, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1142, 1114, 631}

$$\frac{a^2b(d + ex)^6}{2e} + \frac{a^3(d + ex)^4}{4e} + \frac{c(ac + b^2)(d + ex)^{12}}{4e} + \frac{b(6ac + b^2)(d + ex)^{10}}{10e} + \frac{3a(ac + b^2)(d + ex)^8}{8e} + \frac{3bc^2(d + ex)^{14}}{14e} + \frac{c^3(d + ex)^{16}}{16e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (a^3*(d + e*x)^4)/(4*e) + (a^2*b*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a*c)*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*(d + e*x)^10)/(10*e) + (c*(b^2 + a*c)*(d + e*x)^12)/(4*e) + (3*b*c^2*(d + e*x)^14)/(14*e) + (c^3*(d + e*x)^16)/(16*e)

Rule 631

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1142

Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int (d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^3 dx &= \frac{\text{Subst}\left(\int x^3 (a+bx^2+cx^4)^3 dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int x (a+bx+cx^2)^3 dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{\text{Subst}\left(\int (a^3x+3a^2bx^2+3a(b^2+ac)x^3+b(b^2+6ac)x^4+3c^2x^5) dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{a^3(d+ex)^4}{4e} + \frac{a^2b(d+ex)^6}{2e} + \frac{3a(b^2+ac)(d+ex)^8}{8e} + \frac{b(b^2+6ac)(d+ex)^{10}}{10e} + \frac{c^2(d+ex)^{12}}{12e}
\end{aligned}$$

Mathematica [B] time = 0.28, size = 797, normalized size = 5.78

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] $d^3(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^{10} + 35*c^3*d^{12})*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2*d^{10} + 455*c^3*d^{12})*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^2*d^8 + 455*c^3*d^{10})*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8 + 1001*c^3*d^{10})*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 + ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^3*d^6)*e^9*x^{10})/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^{10}*x^{11} + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^{11}*x^{12})/4 + c^2*d*(3*b + 35*c*d^2)*e^{12}*x^{13} + (3*c^2*(b + 35*c*d^2)*e^{13}*x^{14})/14 + c^3*d*e^{14}*x^{15} + (c^3*e^{15}*x^{16})/16$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] IntegrateAlgebraic[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]

fricas [B] time = 1.24, size = 1335, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] $\frac{1}{16}x^{16}e^{15}c^3 + x^{15}e^{14}d^2c^3 + \frac{15}{2}x^{14}e^{13}d^2c^3 + 35x^{13}e^{12}d^3c^3 + \frac{455}{4}x^{12}e^{11}d^4c^3 + \frac{3}{14}x^{14}e^{13}c^2b + 273x^{11}e^{10}d^5c^3 + 3x^{13}e^{12}d^2c^2b + \frac{1001}{2}x^{10}e^9d^6c^3 + \frac{39}{2}x^{12}e^{11}d^2c^2b + 715x^9e^8d^7c^3 + 78x^{11}e^{10}d^3c^2b + \frac{6435}{8}x^8e^7d^8c^3 + \frac{429}{2}x^{10}e^9d^4c^2b + \frac{1}{4}x^{12}e^{11}c^2b^2 + \frac{1}{4}x^{12}e^{11}c^2a + 715x^7e^6d^9c^3 + 429x^9e^8d^5c^2b + 3x^{11}e^{10}d^2c^2b + 3x^{11}e^{10}d^2c^2a + \frac{1001}{2}x^6e^5d^{10}c^3 + \frac{1287}{2}x^8e^7d^6c^2b + \frac{33}{2}x^{10}e^9d^2c^2b^2 + \frac{33}{2}x^{10}e^9d^2c^2a + 273x^5e^4d^{11}c^3 + \frac{5148}{7}x^7e^6d^7c^2b + 55x^9e^8d^3c^2b^2 + 55x^9e^8d^3c^2a + \frac{455}{4}x^4e^3d^{12}c^3 + \frac{1287}{2}x^6e^5d^8c^2b + \frac{495}{4}x^8e^7d^4c^2b^2 + \frac{1}{10}x^{10}e^9b^3 + \frac{495}{4}x^8e^7d^4c^2a + \frac{3}{5}x^{10}e^9c^2b^2a + 35x^3e^2d^{13}c^3 + 429x^5e^4d^9c^2b + 198x^7e^6d^5c^2b^2 + x^9e^8d^2b^3 + 198x^7e^6d^5c^2a + 6x^9e^8d^2c^2b^2a + \frac{15}{2}x^2e^4d^{14}c^3 + \frac{429}{2}x^4e^3d^{10}c^2b + 231x^6e^5d^6c^2b^2 + \frac{9}{2}x^8e^7d^2b^3 + 231x^6e^5d^6c^2a + 27x^8e^7d^2c^2b^2a + xd^{15}c^3 + 78x^3e^2d^{11}c^2b + 198x^5e^4d^7c^2b^2 + 12x^7e^6d^3b^3 + 198x^5e^4d^7c^2a + 72x^7e^6d^3c^2b^2a + 39x^2e^4d^{12}c^2b + \frac{495}{4}x^4e^3d^8c^2b^2 + 21x^6e^5d^4b^3 + \frac{495}{4}x^4e^3d^8c^2a + 126x^6e^5d^4c^2b^2a + \frac{3}{8}x^8e^7b^2a + \frac{3}{8}x^8e^7c^2a^2 + 3xd^{13}c^2b + 55x^3e^2d^9c^2b^2 + \frac{126}{5}x^5e^4d^5b^3 + 55x^3e^2d^9c^2a + \frac{756}{5}x^5e^4d^5c^2b^2a + 3x^7e^6d^2b^2a + 3x^7e^6d^2c^2a^2 + \frac{33}{2}x^2e^4d^{10}c^2b^2 + 21x^4e^3d^6b^3 + \frac{3}{2}x^2e^4d^{10}c^2a + 126x^4e^3d^6c^2b^2a + \frac{21}{2}x^6e^5d^2b^2a + \frac{21}{2}x^6e^5d^2c^2a^2 + 3xd^{11}c^2b^2 + 12x^3e^2d^7b^3 + 3xd^{11}c^2a + 72x^3e^2d^7c^2b^2a + 21x^5e^4d^3b^2a + 21x^5e^4d^3c^2a^2 + \frac{9}{2}x^2e^4d^8b^3 + 27x^2e^4d^8c^2b^2a + \frac{105}{4}x^4e^3d^4b^2a + \frac{105}{4}x^4e^3d^4c^2a^2 + \frac{1}{2}x^6e^5b^2a^2 + xd^9b^3 + 6xd^9c^2b^2a + 21x^3e^2d^5b^2a + 21x^3e^2d^5c^2a^2 + 3x^5e^4d^2b^2a^2 + \frac{21}{2}x^2e^4d^6b^2a + \frac{21}{2}x^2e^4d^6c^2a^2 + \frac{15}{2}x^4e^3d^2b^2a^2 + 3xd^7b^2a + 3xd^7c^2a^2 + 10x^3e^2d^3b^2a^2 + \frac{15}{2}x^2e^4d^4b^2a^2 + \frac{1}{4}x^4e^3a^3 + 3xd^5b^2a^2 + x^3e^2d^3a^3 + \frac{3}{2}x^2e^4d^2a^3 + xd^3a^3$

giac [B] time = 0.62, size = 1109, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] $\frac{1}{2}(x^2e + 2dx)^3c^3d^{14} + \frac{7}{4}(x^2e + 2dx)^2c^3d^{12}e + \frac{7}{2}(x^2e + 2dx)^3c^3d^{10}e^2 + \frac{3}{2}(x^2e + 2dx)bc^2d^{12} + \frac{35}{8}(x^2e + 2dx)^4c^3d^8e^3 + \frac{9}{2}(x^2e + 2dx)^2b^2c^2d^{10}e + \frac{7}{2}(x^2e + 2dx)^5c^3d^6e^4 + \frac{15}{2}(x^2e + 2dx)^3b^2c^2d^8e^2 + \frac{3}{2}(x^2e + 2dx)bc^2d^{10} + \frac{3}{2}(x^2e + 2dx)ac^2d^{10} + \frac{7}{4}(x^2e + 2dx)^6c^3d^4e^5 + \frac{15}{2}(x^2e + 2dx)^4b^2c^2d^6e^3 + \frac{15}{4}(x^2e + 2dx)^2b^2c^2d^8e + \frac{15}{4}(x^2e + 2dx)^2ac^2d^8e + \frac{1}{2}(x^2e + 2dx)^7c^3d^2e^6 + \frac{9}{2}(x^2e + 2dx)^5b^2c^2d^4e^4 + 5(x^2e + 2dx)^3b^2c^2d^6e^2 + 5(x^2e + 2dx)^3ac^2d^6e^2 + \frac{1}{2}(x^2e + 2dx)b^3d^8 + 3(x^2e + 2dx)ab^2c^2d^8 + \frac{1}{16}(x^2e + 2dx)^8c^3e^7 + \frac{3}{2}(x^2e + 2dx)^6b^2c^2d^2e^5 + \frac{15}{4}(x^2e + 2dx)^4b^2c^2d^4e^3 + \frac{15}{4}(x^2e + 2dx)^4ac^2d^4e^3 + (x^2e + 2dx)^2b^3d^6e + 6(x^2e + 2dx)^2ab^2c^2d^6e + \frac{3}{14}(x^2e + 2dx)^7b^2c^2e^6 + \frac{3}{2}(x^2e + 2dx)^5b^2c^2d^2e^4 + \frac{3}{2}(x^2e + 2dx)^5ac^2d^2e^4 + (x^2e + 2dx)^3b^3d^4e^2 + 6(x^2e + 2dx)^3ab^2c^2d^4e^2 + \frac{3}{2}(x^2e + 2dx)ab^2d^6 + \frac{3}{2}(x^2e + 2dx)a^2c^2d^6 + \frac{1}{4}(x^2e + 2dx)^6b^2c^2e^5 + \frac{1}{4}(x^2e + 2dx)^6ac^2e^5 + \frac{1}{2}(x^2e + 2dx)^4b^3d^2e^3 + 3(x^2e + 2dx)^4ab^2c^2d^2e^3 + \frac{9}{4}(x^2e + 2dx)^2ab^2d^4e + \frac{9}{4}(x^2e + 2dx)^2a^2c^2d^4e + \frac{1}{10}(x^2e + 2dx)^5b^3e^4 + \frac{3}{5}(x^2e + 2dx)^5ab^2c^2e^4 + \frac{3}{2}(x^2e + 2dx)^3ab^2d^2e^2 + \frac{3}{2}(x^2e + 2dx)^3a^2c^2d^2e^2 + \frac{3}{2}(x^2e + 2dx)a^2b^2d^4 + \frac{3}{8}(x^2e + 2dx)^4ab^2e^3 + \frac{3}{8}(x^2e + 2dx)^4a^2c^2e^3 + \frac{3}{2}(x^2e + 2dx)^2a^2b^2d^2e + \frac{1}{2}(x^2e + 2dx)^3a^2b^2e^2 + \frac{1}{2}(x^2e + 2dx)a^3d^2 + \frac{1}{4}(x^2e + 2dx)^2a^3e$

maple [B] time = 0.00, size = 7550, normalized size = 54.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)

[Out] result too large to display

maxima [B] time = 1.13, size = 872, normalized size = 6.32

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

```
[Out] 1/16*c^3*e^15*x^16 + c^3*d*e^14*x^15 + 3/14*(35*c^3*d^2 + b*c^2)*e^13*x^14
+ (35*c^3*d^3 + 3*b*c^2*d)*e^12*x^13 + 1/4*(455*c^3*d^4 + 78*b*c^2*d^2 + b^
2*c + a*c^2)*e^11*x^12 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*d)*
e^10*x^11 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(b^2*
c + a*c^2)*d^2)*e^9*x^10 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c + a*c^2
)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*x^9 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 3
30*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*x^8 +
1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6
*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*x^7 + 1/2*(1001*c^3*d^10 + 1287*b*c
^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b
^2 + a^2*c)*d^2)*e^5*x^6 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c +
a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*
e^4*x^5 + 1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84
*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*x^
4 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b
*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*x^3 + 3/2*(5*c
^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*e*x^2 + (c^3*d^15 + 3*b*c^2
*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b
^2 + a^2*c)*d^7 + a^3*d^3)*x
```

mupad [B] time = 1.66, size = 777, normalized size = 5.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] (3*e^7*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 + 330
*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*x^6*(a^2*b + 42*b^3*d
^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 + 462*b^2*
c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*x^10*(b^3 + 5005*c^3*d^6
+ 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10 + (c^3*e^15
*x^16)/16 + d^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*x^4*(a^3 + 84*b^3*d^6 + 455*
c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 495*a*c^2*d^8 + 4
95*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 + (3*c^2*e^13*x^14*(b + 3
5*c*d^2))/14 + c^3*d*e^14*x^15 + d*e^2*x^3*(a^3 + 12*b^3*d^6 + 35*c^3*d^12
+ 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 55*a*c^2*d^8 + 55*b^2*c*d^8
+ 78*b*c^2*d^10 + 72*a*b*c*d^6) + (c*e^11*x^12*(a*c + b^2 + 455*c^2*d^4 + 7
8*b*c*d^2))/4 + (d*e^6*x^7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8
+ 1386*a*c^2*d^4 + 1386*b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (
3*d*e^4*x^5*(5*a^2*b + 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*
d^2 + 330*a*c^2*d^6 + 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d
*e^8*x^9*(b^3 + 715*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 +
6*a*b*c) + (3*d^2*e*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 +
```

$$c^2 d e^{12} x^{13} (3b + 35 c d^2) + 3 c d e^{10} x^{11} (a c + b^2 + 91 c^2 d^4 + 26 b c d^2)$$

sympy [B] time = 0.36, size = 1314, normalized size = 9.52

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] $c^3 d e^{14} x^{15} + c^3 e^{15} x^{16} / 16 + x^{14} (3 b^2 c^2 e^{13} / 14 + 15 c^3 d^2 e^{13} / 2) + x^{13} (3 b^2 c^2 d e^{12} + 35 c^3 d^3 e^{12}) + x^{12} (a^2 c^2 e^{11} / 4 + b^2 c^2 e^{11} / 4 + 39 b^2 c^2 d^2 e^{11} / 2 + 455 c^3 d^4 e^{11} / 4) + x^{11} (3 a^2 c^2 d e^{10} + 3 b^2 c^2 d e^{10} + 78 b^2 c^2 d^3 e^{10} + 273 c^3 d^5 e^{10}) + x^{10} (3 a^2 b^2 c^2 e^9 / 5 + 33 a^2 c^2 d^2 e^9 / 2 + b^2 c^2 e^9 / 10 + 33 b^2 c^2 d^2 e^9 / 2 + 429 b^2 c^2 d^4 e^9 / 2 + 1001 c^3 d^6 e^9 / 2) + x^9 (6 a^2 b^2 c^2 d e^8 + 55 a^2 c^2 d^3 e^8 + b^2 c^2 d e^8 + 55 b^2 c^2 d^3 e^8 + 429 b^2 c^2 d^5 e^8 + 715 c^3 d^7 e^8) + x^8 (3 a^2 c^2 e^7 / 8 + 3 a^2 b^2 e^7 / 8 + 27 a^2 b^2 c^2 d e^7 + 495 a^2 c^2 d^4 e^7 / 4 + 9 b^2 c^2 d^2 e^7 / 2 + 495 b^2 c^2 d^4 e^7 / 4 + 1287 b^2 c^2 d^6 e^7 / 2 + 6435 c^3 d^8 e^7 / 8) + x^7 (3 a^2 c^2 d e^6 + 3 a^2 b^2 d e^6 + 72 a^2 b^2 c^2 d^3 e^6 + 198 a^2 c^2 d^5 e^6 + 12 b^2 c^2 d^3 e^6 + 198 b^2 c^2 d^5 e^6 + 5148 b^2 c^2 d^7 e^6 / 7 + 715 c^3 d^9 e^6) + x^6 (a^2 b^2 e^5 / 2 + 21 a^2 c^2 d^2 e^5 / 2 + 21 a^2 b^2 d^2 e^5 / 2 + 126 a^2 b^2 c^2 d^4 e^5 + 231 a^2 c^2 d^6 e^5 + 21 b^2 c^2 d^4 e^5 + 231 b^2 c^2 d^6 e^5 + 1287 b^2 c^2 d^8 e^5 / 2 + 1001 c^3 d^10 e^5 / 2) + x^5 (3 a^2 b^2 d e^4 + 21 a^2 c^2 d^3 e^4 + 21 a^2 b^2 d^3 e^4 + 756 a^2 b^2 c^2 d^5 e^4 / 5 + 198 a^2 c^2 d^7 e^4 + 126 b^2 c^2 d^5 e^4 / 5 + 198 b^2 c^2 d^7 e^4 + 429 b^2 c^2 d^9 e^4 + 273 c^3 d^11 e^4) + x^4 (a^2 c^2 e^3 / 4 + 15 a^2 b^2 d^2 e^3 / 2 + 105 a^2 c^2 d^4 e^3 / 4 + 105 a^2 b^2 d^4 e^3 / 4 + 126 a^2 b^2 c^2 d^6 e^3 + 495 a^2 c^2 d^8 e^3 / 4 + 21 b^2 c^2 d^6 e^3 + 495 b^2 c^2 d^8 e^3 / 4 + 429 b^2 c^2 d^10 e^3 / 2 + 455 c^3 d^12 e^3 / 4) + x^3 (a^2 c^2 d e^2 + 10 a^2 b^2 d^3 e^2 + 21 a^2 c^2 d^5 e^2 + 21 a^2 b^2 d^5 e^2 + 72 a^2 b^2 c^2 d^7 e^2 + 55 a^2 c^2 d^9 e^2 + 12 b^2 c^2 d^7 e^2 + 55 b^2 c^2 d^9 e^2 + 78 b^2 c^2 d^11 e^2 + 35 c^3 d^13 e^2) + x^2 (3 a^2 c^2 d^2 e / 2 + 15 a^2 b^2 d^4 e / 2 + 21 a^2 c^2 d^6 e / 2 + 21 a^2 b^2 d^6 e / 2 + 27 a^2 b^2 c^2 d^8 e + 33 a^2 c^2 d^10 e / 2 + 9 b^2 c^2 d^8 e / 2 + 33 b^2 c^2 d^10 e / 2 + 39 b^2 c^2 d^12 e / 2 + 15 c^3 d^14 e / 2) + x (a^2 c^2 d^3 + 3 a^2 b^2 d^5 + 3 a^2 c^2 d^7 + 3 a^2 b^2 d^7 + 6 a^2 b^2 c^2 d^9 + 3 a^2 c^2 d^11 + b^2 c^2 d^9 + 3 b^2 c^2 d^11 + 3 b^2 c^2 d^13 + c^3 d^15)$

$$3.497 \quad \int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4) dx$$

Optimal. Leaf size=55

$$\frac{af^3(d + ex)^4}{4e} + \frac{bf^3(d + ex)^6}{6e} + \frac{cf^3(d + ex)^8}{8e}$$

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1142, 14}

$$\frac{af^3(d + ex)^4}{4e} + \frac{bf^3(d + ex)^6}{6e} + \frac{cf^3(d + ex)^8}{8e}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] (a*f^3*(d + e*x)^4)/(4*e) + (b*f^3*(d + e*x)^6)/(6*e) + (c*f^3*(d + e*x)^8)/(8*e)

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 1142

Int[(u_)^(m_)*((a_) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned} \int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4) dx &= \frac{f^3 \text{Subst}\left(\int x^3 (a + bx^2 + cx^4) dx, x, d + ex\right)}{e} \\ &= \frac{f^3 \text{Subst}\left(\int (ax^3 + bx^5 + cx^7) dx, x, d + ex\right)}{e} \\ &= \frac{af^3(d + ex)^4}{4e} + \frac{bf^3(d + ex)^6}{6e} + \frac{cf^3(d + ex)^8}{8e} \end{aligned}$$

Mathematica [B] time = 0.01, size = 154, normalized size = 2.80

$$f^3 \left(\frac{1}{4} e^3 x^4 (a + 10bd^2 + 35cd^4) + \frac{1}{3} d e^2 x^3 (3a + 10bd^2 + 21cd^4) + \frac{1}{2} d^2 e x^2 (3a + 5bd^2 + 7cd^4) + d^3 x (a + bd^2 + cd^4) + \frac{1}{6} e^5 x^6 (b + 21cd^2) + d e^4 x^5 (b + 7cd^2) + c d e^6 x^7 + \frac{1}{8} c e^7 x^8 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]

[Out] $f^3(d^3(a + b*d^2 + c*d^4)*x + (d^2(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 + (d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7 + (c*e^7*x^8)/8$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (df + ef x)^3 (a + b(d + ex)^2 + c(d + ex)^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

fricas [B] time = 1.09, size = 229, normalized size = 4.16

$$\frac{1}{8} x^8 f^3 e^7 c + x^7 f^3 e^6 d c + \frac{7}{2} x^6 f^3 e^5 d^2 c + 7 x^5 f^3 e^4 d^3 c + \frac{35}{4} x^4 f^3 e^3 d^4 c + \frac{1}{6} x^6 f^3 e^5 b + 7 x^3 f^3 e^2 d^6 c + x^5 f^3 e^4 d b + \frac{7}{2} x^2 f^3 e d^6 c + \frac{5}{2} x^4 f^3 e^3 d^2 b + x f^3 d^7 c + \frac{10}{3} x^3 f^3 e^2 d^3 b + \frac{5}{2} x^2 f^3 e d^4 b + \frac{1}{4} x^4 f^3 e^3 a + x f^3 d^6 b + x^3 f^3 e^2 d a + \frac{3}{2} x^2 f^3 e d a + x f^3 d^5 a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] $1/8*x^8*f^3*e^7*c + x^7*f^3*e^6*d*c + 7/2*x^6*f^3*e^5*d^2*c + 7*x^5*f^3*e^4*d^3*c + 35/4*x^4*f^3*e^3*d^4*c + 1/6*x^6*f^3*e^5*b + 7*x^3*f^3*e^2*d^5*c + x^5*f^3*e^4*d*b + 7/2*x^2*f^3*e*d^6*c + 5/2*x^4*f^3*e^3*d^2*b + x*f^3*d^7*c + 10/3*x^3*f^3*e^2*d^3*b + 5/2*x^2*f^3*e*d^4*b + 1/4*x^4*f^3*e^3*a + x*f^3*d^5*b + x^3*f^3*e^2*d*a + 3/2*x^2*f^3*e*d^2*a + x*f^3*d^3*a$

giac [B] time = 0.30, size = 213, normalized size = 3.87

$$\frac{1}{2} (f^2 e + 2 d f x) c d^6 f^2 + \frac{1}{2} (f^2 e + 2 d f x) b d^4 f^2 + \frac{1}{2} (f^2 e + 2 d f x) a d^2 f^2 + \frac{18 (f^2 e + 2 d f x)^2 c d^4 f^2 e + 12 (f^2 e + 2 d f x)^3 c d^2 f^2 e + 12 (f^2 e + 2 d f x)^2 b d^2 f^2 e + 3 (f^2 e + 2 d f x)^4 c e^3 + 4 (f^2 e + 2 d f x)^3 b f e^2 + 6 (f^2 e + 2 d f x)^2 a f e^2}{24 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] $1/2*(f*x^2*e + 2*d*f*x)*c*d^6*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*b*d^4*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*a*d^2*f^2 + 1/24*(18*(f*x^2*e + 2*d*f*x)^2*c*d^4*f^2*e$

$$+ 12*(f*x^2*e + 2*d*f*x)^3*c*d^2*f*e^2 + 12*(f*x^2*e + 2*d*f*x)^2*b*d^2*f^2*e + 3*(f*x^2*e + 2*d*f*x)^4*c*e^3 + 4*(f*x^2*e + 2*d*f*x)^3*b*f*e^2 + 6*(f*x^2*e + 2*d*f*x)^2*a*f^2*e)/f$$

maple [B] time = 0.00, size = 349, normalized size = 6.35

$$\frac{c^2 d^2 e^2 + c d e^2 f^2 + (c^2 d + b^2 e) d^2 f^2 + \frac{(13 c d^2 e^2 + (6 c d^2 + b^2 e) d^2 f^2)^2}{6} + \frac{(13 c d^2 e^2 + 3(6 c d^2 + b^2 e) d^2 f^2 + (4 d^2 + 2 a b) d^2 f^2)^2}{5} + \frac{(4 d^2 e^2 + 3(6 c d^2 + b^2 e) d^2 f^2 + 3(4 d^2 e + 2 a b) d^2 f^2 + (c^2 + b^2 e + a) d^2 f^2)^2}{4} + \frac{((6 c d^2 + b^2 e) d^2 f^2 + 3(4 d^2 e + 2 a b) d^2 f^2 + 3(c^2 + b^2 e + a) d^2 f^2)^2}{3} + \frac{((4 d^2 + 2 a b) d^2 f^2 + 3(c^2 + b^2 e + a) d^2 f^2)^2}{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4), x)`

[Out] $\frac{1}{8}e^7f^3c^2x^8 + d^2f^3e^6cx^7 + \frac{1}{6}(15d^2f^3e^5c + e^3f^3(6cd^2e^2 + b^2e^2))x^6 + \frac{1}{5}(13d^3f^3c^2e^4 + 3d^2f^3e^2(6cd^2e^2 + b^2e^2) + e^3f^3(4c^2d^3e + 2b^2d^2e))x^5 + \frac{1}{4}(4d^4f^3c^2e^3 + 3d^2f^3e(6cd^2e^2 + b^2e^2) + 3d^2f^3e^2(4c^2d^3e + 2b^2d^2e) + e^3f^3(c^2d^4 + b^2d^2 + a))x^4 + \frac{1}{3}(d^3f^3(6cd^2e^2 + b^2e^2) + 3d^2f^3e(4c^2d^3e + 2b^2d^2e) + 3d^2f^3e^2(c^2d^4 + b^2d^2 + a))x^3 + \frac{1}{2}(d^3f^3(4c^2d^3e + 2b^2d^2e) + 3d^2f^3e(c^2d^4 + b^2d^2 + a))x^2 + d^3f^3(c^2d^4 + b^2d^2 + a)x$

maxima [B] time = 1.08, size = 166, normalized size = 3.02

$$\frac{1}{8}ce^7f^3x^8 + cde^6f^3x^7 + \frac{1}{6}(21cd^2 + b)e^5f^3x^6 + (7cd^3 + bd)e^4f^3x^5 + \frac{1}{4}(35cd^4 + 10bd^2 + a)e^3f^3x^4 + \frac{1}{3}(21cd^5 + 10bd^3 + 3ad)e^2f^3x^3 + \frac{1}{2}(7cd^6 + 5bd^4 + 3ad^2)ef^3x^2 + (cd^7 + bd^5 + ad^3)f^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="maxima")`

[Out] $\frac{1}{8}c^2e^7f^3x^8 + c^2d^2e^6f^3x^7 + \frac{1}{6}(21c^2d^2 + b^2)e^5f^3x^6 + (7c^2d^3 + b^2d)e^4f^3x^5 + \frac{1}{4}(35c^2d^4 + 10b^2d^2 + a^2)e^3f^3x^4 + \frac{1}{3}(21c^2d^5 + 10b^2d^3 + 3a^2d)e^2f^3x^3 + \frac{1}{2}(7c^2d^6 + 5b^2d^4 + 3a^2d^2)e^2f^3x^2 + (cd^7 + b^2d^5 + ad^3)f^3x$

mupad [B] time = 0.08, size = 164, normalized size = 2.98

$$\frac{e^5 f^3 x^6 (21 c d^2 + b)}{6} + \frac{c e^7 f^3 x^8}{8} + d^3 f^3 x (c d^4 + b d^2 + a) + \frac{e^3 f^3 x^4 (35 c d^4 + 10 b d^2 + a)}{4} + \frac{d^2 e f^3 x^2 (7 c d^4 + 5 b d^2 + 3 a)}{2} + \frac{d e^2 f^3 x^3 (21 c d^4 + 10 b d^2 + 3 a)}{3} + d e^4 f^3 x^5 (7 c d^2 + b) + c d e^6 f^3 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4), x)`

[Out] $\frac{(e^5 f^3 x^6 (b + 21 c d^2))}{6} + \frac{(c e^7 f^3 x^8)}{8} + d^3 f^3 x (a + b d^2 + c d^4) + \frac{(e^3 f^3 x^4 (a + 10 b d^2 + 35 c d^4))}{4} + \frac{(d^2 e f^3 x^2 (3 a + 5 b d^2 + 7 c d^4))}{2} + \frac{(d e^2 f^3 x^3 (3 a + 10 b d^2 + 21 c d^4))}{3} + d e^4 f^3 x^5 (b + 7 c d^2) + c d e^6 f^3 x^7$

sympy [B] time = 0.11, size = 240, normalized size = 4.36

$$c d e^6 f^3 x^7 + \frac{c e^7 f^3 x^8}{8} + x^6 \left(\frac{b e^5 f^3}{6} + \frac{7 c d^2 e^5 f^3}{2} \right) + x^5 (b d e^4 f^3 + 7 c d^3 e^4 f^3) + x^4 \left(\frac{a e^3 f^3}{4} + \frac{5 b d^2 e^3 f^3}{2} + \frac{35 c d^4 e^3 f^3}{4} \right) + x^3 \left(a d e^2 f^3 + \frac{10 b d^2 e^2 f^3}{3} + 7 c d^5 e^2 f^3 \right) + x^2 \left(\frac{3 a d^2 e f^3}{2} + \frac{5 b d^4 e f^3}{2} + \frac{7 c d^6 e f^3}{2} \right) + x (a d^3 f^3 + b d^5 f^3 + c d^7 f^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)`

[Out] `c*d*e**6*f**3*x**7 + c*e**7*f**3*x**8/8 + x**6*(b*e**5*f**3/6 + 7*c*d**2*e*
*5*f**3/2) + x**5*(b*d*e**4*f**3 + 7*c*d**3*e**4*f**3) + x**4*(a*e**3*f**3/
4 + 5*b*d**2*e**3*f**3/2 + 35*c*d**4*e**3*f**3/4) + x**3*(a*d*e**2*f**3 + 1
0*b*d**3*e**2*f**3/3 + 7*c*d**5*e**2*f**3) + x**2*(3*a*d**2*e*f**3/2 + 5*b*
d**4*e*f**3/2 + 7*c*d**6*e*f**3/2) + x*(a*d**3*f**3 + b*d**5*f**3 + c*d**7*
f**3)`

$$3.498 \quad \int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^2 dx$$

Optimal. Leaf size=104

$$\frac{a^2 f^3 (d + ex)^4}{4e} + \frac{f^3 (2ac + b^2) (d + ex)^8}{8e} + \frac{abf^3 (d + ex)^6}{3e} + \frac{bcf^3 (d + ex)^{10}}{5e} + \frac{c^2 f^3 (d + ex)^{12}}{12e}$$

Rubi [A] time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1142, 1114, 631}

$$\frac{a^2 f^3 (d + ex)^4}{4e} + \frac{f^3 (2ac + b^2) (d + ex)^8}{8e} + \frac{abf^3 (d + ex)^6}{3e} + \frac{bcf^3 (d + ex)^{10}}{5e} + \frac{c^2 f^3 (d + ex)^{12}}{12e}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] (a^2*f^3*(d + e*x)^4)/(4*e) + (a*b*f^3*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*f^3*(d + e*x)^8)/(8*e) + (b*c*f^3*(d + e*x)^10)/(5*e) + (c^2*f^3*(d + e*x)^12)/(12*e)

Rule 631

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
 \int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^2 dx &= \frac{f^3 \text{Subst}\left(\int x^3 (a + bx^2 + cx^4)^2 dx, x, d + ex\right)}{e} \\
 &= \frac{f^3 \text{Subst}\left(\int x (a + bx + cx^2)^2 dx, x, (d + ex)^2\right)}{2e} \\
 &= \frac{f^3 \text{Subst}\left(\int (a^2x + 2abx^2 + (b^2 + 2ac)x^3 + 2bcx^4 + c^2x^5) dx, x, (d + ex)^2\right)}{2e} \\
 &= \frac{a^2 f^3 (d + ex)^4}{4e} + \frac{ab f^3 (d + ex)^6}{3e} + \frac{(b^2 + 2ac) f^3 (d + ex)^8}{8e} + \frac{bc f^3 (d + ex)^{10}}{5e} + \frac{c^2 f^3 (d + ex)^{12}}{12e}
 \end{aligned}$$

Mathematica [B] time = 0.07, size = 405, normalized size = 3.89

$f^3 \left(\frac{1}{12} a^2 x^{12} + 20 a b x^{10} + 35 b^2 x^8 + 165 c^2 x^8 \right) - \frac{1}{3} b^2 c x^{10} + 20 a b c x^8 + 235 c^2 x^8 + 85 c^2 x^8 + \frac{1}{2} b^2 c x^{10} (b^2 + 2 a c) + \frac{1}{2} b^2 c x^{10} (b^2 + 2 a c) + 14 a b^2 c x^8 + 18 a b^2 c x^8 + 11 c^2 x^8 (2 a c + b^2) + 4 b^2 c x^8 (2 a c + b^2) + 24 a b^2 c x^8 + 66 c^2 x^8 (2 a c + b^2) + \frac{1}{2} b^2 c x^{10} (2 a c + 2 b^2) + 46 c^2 x^8 (2 a c + b^2) + \frac{1}{2} b^2 c x^{10} (2 a c + 2 b^2) + 35 b^2 c x^8 + 25 c^2 x^8 + 330 c^2 x^8 (2 a c + b^2) - d^3 (x + d e)^3 + \frac{1}{2} b^2 c x^{10} (2 b + 5 c) + \frac{1}{2} b^2 c x^{10} (2 b + 5 c) + \frac{1}{2} b^2 c x^{10} (2 b + 5 c) + \frac{1}{2} b^2 c x^{10} (2 b + 5 c) \right)$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] $f^3(d^3(a + b*d^2 + c*d^4)^2*x + (d^2(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 14*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8))*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8))*e^2*x^3)/3 + ((a^2 + 20*a*b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8))*e^3*x^4)/4 + (d*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6))*e^4*x^5)/5 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6))*e^5*x^6)/6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*b*c*d^2 + 330*c^2*d^4))*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2))*e^8*x^9)/3 + (c*(2*b + 55*c*d^2))*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]

fricas [B] time = 1.02, size = 715, normalized size = 6.88

.....

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*f^3*e^11*c^2 + x^11*f^3*e^10*d*c^2 + 11/2*x^10*f^3*e^9*d^2*c^2 +
55/3*x^9*f^3*e^8*d^3*c^2 + 165/4*x^8*f^3*e^7*d^4*c^2 + 1/5*x^10*f^3*e^9*c*b
+ 66*x^7*f^3*e^6*d^5*c^2 + 2*x^9*f^3*e^8*d*c*b + 77*x^6*f^3*e^5*d^6*c^2 +
9*x^8*f^3*e^7*d^2*c*b + 66*x^5*f^3*e^4*d^7*c^2 + 24*x^7*f^3*e^6*d^3*c*b + 1
65/4*x^4*f^3*e^3*d^8*c^2 + 42*x^6*f^3*e^5*d^4*c*b + 1/8*x^8*f^3*e^7*b^2 + 1
/4*x^8*f^3*e^7*c*a + 55/3*x^3*f^3*e^2*d^9*c^2 + 252/5*x^5*f^3*e^4*d^5*c*b +
x^7*f^3*e^6*d*b^2 + 2*x^7*f^3*e^6*d*c*a + 11/2*x^2*f^3*e*d^10*c^2 + 42*x^4
*f^3*e^3*d^6*c*b + 7/2*x^6*f^3*e^5*d^2*b^2 + 7*x^6*f^3*e^5*d^2*c*a + x*f^3*
d^11*c^2 + 24*x^3*f^3*e^2*d^7*c*b + 7*x^5*f^3*e^4*d^3*b^2 + 14*x^5*f^3*e^4*
d^3*c*a + 9*x^2*f^3*e*d^8*c*b + 35/4*x^4*f^3*e^3*d^4*b^2 + 35/2*x^4*f^3*e^3
*d^4*c*a + 1/3*x^6*f^3*e^5*b*a + 2*x*f^3*d^9*c*b + 7*x^3*f^3*e^2*d^5*b^2 +
14*x^3*f^3*e^2*d^5*c*a + 2*x^5*f^3*e^4*d*b*a + 7/2*x^2*f^3*e*d^6*b^2 + 7*x^
2*f^3*e*d^6*c*a + 5*x^4*f^3*e^3*d^2*b*a + x*f^3*d^7*b^2 + 2*x*f^3*d^7*c*a +
20/3*x^3*f^3*e^2*d^3*b*a + 5*x^2*f^3*e*d^4*b*a + 1/4*x^4*f^3*e^3*a^2 + 2*x
*f^3*d^5*b*a + x^3*f^3*e^2*d*a^2 + 3/2*x^2*f^3*e*d^2*a^2 + x*f^3*d^3*a^2
```

giac [B] time = 0.43, size = 615, normalized size = 5.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c^2*d^10*f^2 + (f*x^2*e + 2*d*f*x)*b*c*d^8*f^2 + 1/
2*(f*x^2*e + 2*d*f*x)*b^2*d^6*f^2 + (f*x^2*e + 2*d*f*x)*a*c*d^6*f^2 + (f*x^
2*e + 2*d*f*x)*a*b*d^4*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*a^2*d^2*f^2 + 1/120*(1
50*(f*x^2*e + 2*d*f*x)^2*c^2*d^8*f^4*e + 200*(f*x^2*e + 2*d*f*x)^3*c^2*d^6*
f^3*e^2 + 240*(f*x^2*e + 2*d*f*x)^2*b*c*d^6*f^4*e + 150*(f*x^2*e + 2*d*f*x)
^4*c^2*d^4*f^2*e^3 + 240*(f*x^2*e + 2*d*f*x)^3*b*c*d^4*f^3*e^2 + 90*(f*x^2*
e + 2*d*f*x)^2*b^2*d^4*f^4*e + 180*(f*x^2*e + 2*d*f*x)^2*a*c*d^4*f^4*e + 60
*(f*x^2*e + 2*d*f*x)^5*c^2*d^2*f*e^4 + 120*(f*x^2*e + 2*d*f*x)^4*b*c*d^2*f^
2*e^3 + 60*(f*x^2*e + 2*d*f*x)^3*b^2*d^2*f^3*e^2 + 120*(f*x^2*e + 2*d*f*x)^
3*a*c*d^2*f^3*e^2 + 120*(f*x^2*e + 2*d*f*x)^2*a*b*d^2*f^4*e + 10*(f*x^2*e +
2*d*f*x)^6*c^2*e^5 + 24*(f*x^2*e + 2*d*f*x)^5*b*c*f*e^4 + 15*(f*x^2*e + 2*
d*f*x)^4*b^2*f^2*e^3 + 30*(f*x^2*e + 2*d*f*x)^4*a*c*f^2*e^3 + 40*(f*x^2*e +
2*d*f*x)^3*a*b*f^3*e^2 + 30*(f*x^2*e + 2*d*f*x)^2*a^2*f^4*e)/f^3
```

maple [B] time = 0.00, size = 1413, normalized size = 13.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)$

[Out] $\frac{1}{12}e^{11}f^3c^2x^{12}+d^3f^3e^{10}c^2x^{11}+\frac{1}{10}(27d^2f^3e^9c^2+e^3f^3(16c^2d^2e^6+2(6cd^2e^2+be^2)ce^4))x^{10}+\frac{1}{9}(25d^3f^3c^2e^8+3d^2f^3e^2(16c^2d^2e^6+2(6cd^2e^2+be^2)ce^4)+e^3f^3(8(6cd^2e^2+be^2)cd^3e+2bde)ce^4)x^9+\frac{1}{8}(8d^4f^3c^2e^7+3d^2f^3e*(16c^2d^2e^6+2(6cd^2e^2+be^2)ce^4)+3d^2f^3e^2(8(6cd^2e^2+be^2)cd^3e+2bde)ce^4)+e^3f^3(8(4cd^3e+2bde)cd^3e+2(c^4d+b^2d+a)ce^4+(6cd^2e^2+be^2)^2))x^8+\frac{1}{7}(d^3f^3(16c^2d^2e^6+2(6cd^2e^2+be^2)ce^4)+3d^2f^3e*(8(6cd^2e^2+be^2)cd^3e+2bde)ce^4)+3d^2f^3e^2(8(4cd^3e+2bde)cd^3e+2(c^4d+b^2d+a)ce^4+(6cd^2e^2+be^2)^2)+e^3f^3(8(c^4d+b^2d+a)cd^3e+2(4cd^3e+2bde)(6cd^2e^2+be^2)))x^7+\frac{1}{6}(d^3f^3(8(6cd^2e^2+be^2)cd^3e+2(4cd^3e+2bde)ce^4)+3d^2f^3e*(8(4cd^3e+2bde)cd^3e+2(c^4d+b^2d+a)ce^4+(6cd^2e^2+be^2)^2)+3d^2f^3e^2(8(c^4d+b^2d+a)cd^3e+2(4cd^3e+2bde)(6cd^2e^2+be^2))+e^3f^3(2(c^4d+b^2d+a)(6cd^2e^2+be^2)+(4cd^3e+2bde)^2))x^6+\frac{1}{5}(d^3f^3(8(4cd^3e+2bde)cd^3e+2(c^4d+b^2d+a)ce^4+(6cd^2e^2+be^2)^2)+3d^2f^3e*(8(c^4d+b^2d+a)cd^3e+2(4cd^3e+2bde)(6cd^2e^2+be^2))+3d^2f^3e^2(2(c^4d+b^2d+a)(6cd^2e^2+be^2)+(4cd^3e+2bde)^2)+2e^3f^3(c^4d+b^2d+a)(4cd^3e+2bde))x^5+\frac{1}{4}(d^3f^3(8(c^4d+b^2d+a)cd^3e+2(4cd^3e+2bde)(6cd^2e^2+be^2))+3d^2f^3e*(2(c^4d+b^2d+a)(6cd^2e^2+be^2)+(4cd^3e+2bde)^2)+6d^2f^3e^2(c^4d+b^2d+a)(4cd^3e+2bde)+e^3f^3(c^4d+b^2d+a)^2)x^4+\frac{1}{3}(d^3f^3(2(c^4d+b^2d+a)(6cd^2e^2+be^2)+(4cd^3e+2bde)^2)+6d^2f^3e*(c^4d+b^2d+a)(4cd^3e+2bde)+3d^2f^3e^2(c^4d+b^2d+a)^2)x^3+\frac{1}{2}(2d^3f^3(c^4d+b^2d+a)(4cd^3e+2bde)+3d^2f^3e*(c^4d+b^2d+a)^2)x^2+d^3f^3(c^4d+b^2d+a)^2x$

maxima [B] time = 1.03, size = 439, normalized size = 4.22

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, \text{algorithm}=\text{"maxima"})$

[Out] $\frac{1}{12}c^2e^{11}f^3x^{12} + c^2d^3e^{10}f^3x^{11} + \frac{1}{10}(55c^2d^2 + 2b^2c)e^9f^3x^{10} + \frac{1}{3}(55c^2d^3 + 6b^2cd)e^8f^3x^9 + \frac{1}{8}(330c^2d^4 + 72b^2cd^2 + b^2 + 2a^2c)e^7f^3x^8 + (66c^2d^5 + 24b^2cd^3 + (b^2 + 2a^2c)d)e^6f^3x^7 + \frac{1}{6}(462c^2d^6 + 252b^2cd^4 + 21(b^2 + 2a^2c)d^2 + 2a^2b)e^5f^3x^6 + \frac{1}{5}(330c^2d^7 + 252b^2cd^5 + 35(b^2 + 2a^2c)d^3 + 10a^2bd)e^4f^3x^5 + \frac{1}{4}(165c^2d^8 + 168b^2cd^6 + 35(b^2 + 2a^2c)d^4 + 20a^2bd^2 + a^2)e^3f^3x^4 + \frac{1}{3}(55c^2d^9 + 72b^2cd^7 + 21c^2d^8 + 2b^2cd^6 + a^2d^4)e^2f^3x^3 + \frac{1}{2}(55c^2d^{10} + 72b^2cd^8 + 21c^2d^9 + 2b^2cd^7 + a^2d^5)e^1f^3x^2 + d^3f^3(c^2d^8 + 2b^2cd^6 + a^2d^4)x$

$$(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*e^2*f^3*x^3 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*e*f^3*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*x$$

mupad [B] time = 1.47, size = 419, normalized size = 4.03

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)`

[Out] $(e^3*f^3*x^4*(a^2 + 35*b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 16*8*b*c*d^6))/4 + (c^2*e^{11}*f^3*x^{12})/12 + d^3*f^3*x*(a + b*d^2 + c*d^4)^2 + (e^7*f^3*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*f^3*x^6*(2*a*b + 21*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (d^2*e*f^3*x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d^2 + 14*a*c*d^4 + 18*b*c*d^6))/2 + (d*e^2*f^3*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*d^8 + 20*a*b*d^2 + 42*a*c*d^4 + 72*b*c*d^6))/3 + d*e^6*f^3*x^7*(2*a*c + b^2 + 66*c^2*d^4 + 24*b*c*d^2) + (d*e^4*f^3*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 70*a*c*d^2 + 252*b*c*d^4))/5 + (c*e^9*f^3*x^{10}*(2*b + 55*c*d^2))/10 + c^2*d*e^{10}*f^3*x^{11} + (c*d*e^8*f^3*x^9*(6*b + 55*c*d^2))/3$

sympy [B] time = 0.21, size = 722, normalized size = 6.94

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)`

[Out] $c**2*d*e**10*f**3*x**11 + c**2*e**11*f**3*x**12/12 + x**10*(b*c*e**9*f**3/5 + 11*c**2*d**2*e**9*f**3/2) + x**9*(2*b*c*d*e**8*f**3 + 55*c**2*d**3*e**8*f**3/3) + x**8*(a*c*e**7*f**3/4 + b**2*e**7*f**3/8 + 9*b*c*d**2*e**7*f**3 + 165*c**2*d**4*e**7*f**3/4) + x**7*(2*a*c*d*e**6*f**3 + b**2*d*e**6*f**3 + 24*b*c*d**3*e**6*f**3 + 66*c**2*d**5*e**6*f**3) + x**6*(a*b*e**5*f**3/3 + 7*a*c*d**2*e**5*f**3 + 7*b**2*d**2*e**5*f**3/2 + 42*b*c*d**4*e**5*f**3 + 77*c**2*d**6*e**5*f**3) + x**5*(2*a*b*d*e**4*f**3 + 14*a*c*d**3*e**4*f**3 + 7*b**2*d**3*e**4*f**3 + 252*b*c*d**5*e**4*f**3/5 + 66*c**2*d**7*e**4*f**3) + x**4*(a**2*e**3*f**3/4 + 5*a*b*d**2*e**3*f**3 + 35*a*c*d**4*e**3*f**3/2 + 3*5*b**2*d**4*e**3*f**3/4 + 42*b*c*d**6*e**3*f**3 + 165*c**2*d**8*e**3*f**3/4) + x**3*(a**2*d*e**2*f**3 + 20*a*b*d**3*e**2*f**3/3 + 14*a*c*d**5*e**2*f**3 + 7*b**2*d**5*e**2*f**3 + 24*b*c*d**7*e**2*f**3 + 55*c**2*d**9*e**2*f**3/3) + x**2*(3*a**2*d**2*e*f**3/2 + 5*a*b*d**4*e*f**3 + 7*a*c*d**6*e*f**3 + 7*b**2*d**6*e*f**3/2 + 9*b*c*d**8*e*f**3 + 11*c**2*d**10*e*f**3/2) + x*(a**2*d**3*f**3 + 2*a*b*d**5*f**3 + 2*a*c*d**7*f**3 + b**2*d**7*f**3 + 2*b*c*d**9*f**3 + c**2*d**11*f**3)$

$$3.499 \quad \int (df + efx)^3 \left(a + b(d + ex)^2 + c(d + ex)^4 \right)^3 dx$$

Optimal. Leaf size=159

$$\frac{a^3 f^3 (d + ex)^4}{4e} + \frac{a^2 b f^3 (d + ex)^6}{2e} + \frac{c f^3 (ac + b^2) (d + ex)^{12}}{4e} + \frac{b f^3 (6ac + b^2) (d + ex)^{10}}{10e} + \frac{3a f^3 (ac + b^2) (d + ex)^8}{8e} + \dots$$

Rubi [A] time = 0.32, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1142, 1114, 631}

$$\frac{a^2 b f^3 (d + ex)^6}{2e} + \frac{a^3 f^3 (d + ex)^4}{4e} + \frac{c f^3 (ac + b^2) (d + ex)^{12}}{4e} + \frac{b f^3 (6ac + b^2) (d + ex)^{10}}{10e} + \frac{3a f^3 (ac + b^2) (d + ex)^8}{8e} + \frac{3bc^2 f^3 (d + ex)^{14}}{14e} + \frac{c^3 f^3 (d + ex)^{16}}{16e}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (a^3*f^3*(d + e*x)^4)/(4*e) + (a^2*b*f^3*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a*c)*f^3*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*f^3*(d + e*x)^10)/(10*e) + (c*(b^2 + a*c)*f^3*(d + e*x)^12)/(4*e) + (3*b*c^2*f^3*(d + e*x)^14)/(14*e) + (c^3*f^3*(d + e*x)^16)/(16*e)

Rule 631

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1142

Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^3 dx &= \frac{f^3 \text{Subst}\left(\int x^3 (a + bx^2 + cx^4)^3 dx, x, d + ex\right)}{e} \\
&= \frac{f^3 \text{Subst}\left(\int x (a + bx + cx^2)^3 dx, x, (d + ex)^2\right)}{2e} \\
&= \frac{f^3 \text{Subst}\left(\int (a^3x + 3a^2bx^2 + 3a(b^2 + ac)x^3 + b(b^2 + 6ac)x^4\right)}{2e} \\
&= \frac{a^3 f^3 (d + ex)^4}{4e} + \frac{a^2 b f^3 (d + ex)^6}{2e} + \frac{3a(b^2 + ac) f^3 (d + ex)^8}{8e} + \dots
\end{aligned}$$

Mathematica [B] time = 0.04, size = 801, normalized size = 5.04

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] $f^3(d^3(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10 + 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2*d^10 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 + ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35*c*d^2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 + (c^3*e^15*x^16)/16$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x
]

fricas [B] time = 0.73, size = 1635, normalized size = 10.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] $1/16*x^{16}*f^3*e^{15}*c^3 + x^{15}*f^3*e^{14}*d*c^3 + 15/2*x^{14}*f^3*e^{13}*d^2*c^3 + 35*x^{13}*f^3*e^{12}*d^3*c^3 + 455/4*x^{12}*f^3*e^{11}*d^4*c^3 + 3/14*x^{14}*f^3*e^{13}*c^2*b + 273*x^{11}*f^3*e^{10}*d^5*c^3 + 3*x^{13}*f^3*e^{12}*d*c^2*b + 1001/2*x^{10}*f^3*e^9*d^6*c^3 + 39/2*x^{12}*f^3*e^{11}*d^2*c^2*b + 715*x^9*f^3*e^8*d^7*c^3 + 78*x^{11}*f^3*e^{10}*d^3*c^2*b + 6435/8*x^8*f^3*e^7*d^8*c^3 + 429/2*x^{10}*f^3*e^9*d^4*c^2*b + 1/4*x^{12}*f^3*e^{11}*c*b^2 + 1/4*x^{12}*f^3*e^{11}*c^2*a + 715*x^7*f^3*e^6*d^9*c^3 + 429*x^9*f^3*e^8*d^5*c^2*b + 3*x^{11}*f^3*e^{10}*d*c*b^2 + 3*x^{11}*f^3*e^{10}*d*c^2*a + 1001/2*x^6*f^3*e^5*d^{10}*c^3 + 1287/2*x^8*f^3*e^7*d^6*c^2*b + 33/2*x^{10}*f^3*e^9*d^2*c*b^2 + 33/2*x^{10}*f^3*e^9*d^2*c^2*a + 273*x^5*f^3*e^4*d^{11}*c^3 + 5148/7*x^7*f^3*e^6*d^7*c^2*b + 55*x^9*f^3*e^8*d^3*c*b^2 + 55*x^9*f^3*e^8*d^3*c^2*a + 455/4*x^4*f^3*e^3*d^{12}*c^3 + 1287/2*x^6*f^3*e^5*d^8*c^2*b + 495/4*x^8*f^3*e^7*d^4*c*b^2 + 1/10*x^{10}*f^3*e^9*b^3 + 495/4*x^8*f^3*e^7*d^4*c^2*a + 3/5*x^{10}*f^3*e^9*c*b*a + 35*x^3*f^3*e^2*d^{13}*c^3 + 429*x^5*f^3*e^4*d^9*c^2*b + 198*x^7*f^3*e^6*d^5*c*b^2 + x^9*f^3*e^8*d*b^3 + 198*x^7*f^3*e^6*d^5*c^2*a + 6*x^9*f^3*e^8*d*c*b*a + 15/2*x^2*f^3*e*d^{14}*c^3 + 429/2*x^4*f^3*e^3*d^{10}*c^2*b + 231*x^6*f^3*e^5*d^6*c*b^2 + 9/2*x^8*f^3*e^7*d^2*b^3 + 231*x^6*f^3*e^5*d^6*c^2*a + 27*x^8*f^3*e^7*d^2*c*b*a + x*f^3*d^{15}*c^3 + 78*x^3*f^3*e^2*d^{11}*c^2*b + 198*x^5*f^3*e^4*d^7*c*b^2 + 12*x^7*f^3*e^6*d^3*b^3 + 198*x^5*f^3*e^4*d^7*c^2*a + 72*x^7*f^3*e^6*d^3*c*b*a + 39/2*x^2*f^3*e*d^{12}*c^2*b + 495/4*x^4*f^3*e^3*d^8*c*b^2 + 21*x^6*f^3*e^5*d^4*b^3 + 495/4*x^4*f^3*e^3*d^8*c^2*a + 126*x^6*f^3*e^5*d^4*c*b*a + 3/8*x^8*f^3*e^7*b^2*a + 3/8*x^8*f^3*e^7*c*a^2 + 3*x*f^3*d^{13}*c^2*b + 55*x^3*f^3*e^2*d^9*c*b^2 + 126/5*x^5*f^3*e^4*d^5*b^3 + 55*x^3*f^3*e^2*d^9*c^2*a + 756/5*x^5*f^3*e^4*d^5*c*b*a + 3*x^7*f^3*e^6*d*b^2*a + 3*x^7*f^3*e^6*d*c*a^2 + 33/2*x^2*f^3*e*d^{10}*c*b^2 + 21*x^4*f^3*e^3*d^6*b^3 + 33/2*x^2*f^3*e*d^{10}*c^2*a + 126*x^4*f^3*e^3*d^6*c*b*a + 21/2*x^6*f^3*e^5*d^2*b^2*a + 21/2*x^6*f^3*e^5*d^2*c*a^2 + 3*x*f^3*d^{11}*c*b^2 + 12*x^3*f^3*e^2*d^7*b^3 + 3*x*f^3*d^{11}*c^2*a + 72*x^3*f^3*e^2*d^7*c*b*a + 21*x^5*f^3*e^4*d^3*b^2*a + 21*x^5*f^3*e^4*d^3*c*a^2 + 9/2*x^2*f^3*e^8*d*b^3 + 27*x^2*f^3*e^8*d^8*c*b*a + 105/4*x^4*f^3*e^3*d^4*b^2*a + 105/4*x^4*f^3*e^3*d^4*c*a^2 + 1/2*x^6*f^3*e^5*b*a^2 + x*f^3*d^9*b^3 + 6*x*f^3*d^9*c*b*a + 21*x^3*f^3*e^2*d^5*b^2*a + 21*x^3*f^3*e^2*d^5*c*$

$$\begin{aligned}
& a^2 + 3*x^5*f^3*e^4*d*b*a^2 + 21/2*x^2*f^3*e*d^6*b^2*a + 21/2*x^2*f^3*e*d^6 \\
& *c*a^2 + 15/2*x^4*f^3*e^3*d^2*b*a^2 + 3*x*f^3*d^7*b^2*a + 3*x*f^3*d^7*c*a^2 \\
& + 10*x^3*f^3*e^2*d^3*b*a^2 + 15/2*x^2*f^3*e*d^4*b*a^2 + 1/4*x^4*f^3*e^3*a^3 \\
& + 3*x*f^3*d^5*b*a^2 + x^3*f^3*e^2*d*a^3 + 3/2*x^2*f^3*e*d^2*a^3 + x*f^3*d^3*a^3
\end{aligned}$$

giac [B] time = 0.51, size = 1360, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] 1/2*(f*x^2*e + 2*d*f*x)*c^3*d^14*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*b*c^2*d^12*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*b^2*c*d^10*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a*c^2*d^10*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*b^3*d^8*f^2 + 3*(f*x^2*e + 2*d*f*x)*a*b*c*d^8*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a*b^2*d^6*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a^2*c*d^6*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a^2*b*d^4*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*a^3*d^2*f^2 + 1/560*(980*(f*x^2*e + 2*d*f*x)^2*c^3*d^12*f^6*e + 1960*(f*x^2*e + 2*d*f*x)^3*c^3*d^10*f^5*e^2 + 2520*(f*x^2*e + 2*d*f*x)^2*b*c^2*d^10*f^6*e + 2450*(f*x^2*e + 2*d*f*x)^4*c^3*d^8*f^4*e^3 + 4200*(f*x^2*e + 2*d*f*x)^3*b*c^2*d^8*f^5*e^2 + 2100*(f*x^2*e + 2*d*f*x)^2*b^2*c*d^8*f^6*e + 2100*(f*x^2*e + 2*d*f*x)^2*a*c^2*d^8*f^6*e + 1960*(f*x^2*e + 2*d*f*x)^5*c^3*d^6*f^3*e^4 + 4200*(f*x^2*e + 2*d*f*x)^4*b*c^2*d^6*f^4*e^3 + 2800*(f*x^2*e + 2*d*f*x)^3*b^2*c*d^6*f^5*e^2 + 2800*(f*x^2*e + 2*d*f*x)^3*a*c^2*d^6*f^5*e^2 + 560*(f*x^2*e + 2*d*f*x)^2*b^3*d^6*f^6*e + 3360*(f*x^2*e + 2*d*f*x)^2*a*b*c*d^6*f^6*e + 980*(f*x^2*e + 2*d*f*x)^6*c^3*d^4*f^2*e^5 + 2520*(f*x^2*e + 2*d*f*x)^5*b*c^2*d^4*f^3*e^4 + 2100*(f*x^2*e + 2*d*f*x)^4*b^2*c*d^4*f^4*e^3 + 2100*(f*x^2*e + 2*d*f*x)^4*a*c^2*d^4*f^4*e^3 + 560*(f*x^2*e + 2*d*f*x)^3*b^3*d^4*f^5*e^2 + 3360*(f*x^2*e + 2*d*f*x)^3*a*b*c*d^4*f^5*e^2 + 1260*(f*x^2*e + 2*d*f*x)^2*a*b^2*d^4*f^6*e + 1260*(f*x^2*e + 2*d*f*x)^2*a^2*c*d^4*f^6*e + 280*(f*x^2*e + 2*d*f*x)^7*c^3*d^2*f^6*e + 840*(f*x^2*e + 2*d*f*x)^6*b*c^2*d^2*f^3*e^4 + 840*(f*x^2*e + 2*d*f*x)^5*a*c^2*d^2*f^3*e^4 + 280*(f*x^2*e + 2*d*f*x)^4*b^3*d^2*f^4*e^3 + 1680*(f*x^2*e + 2*d*f*x)^4*a*b*c*d^2*f^4*e^3 + 840*(f*x^2*e + 2*d*f*x)^3*a*b^2*d^2*f^5*e^2 + 840*(f*x^2*e + 2*d*f*x)^3*a^2*c*d^2*f^5*e^2 + 840*(f*x^2*e + 2*d*f*x)^2*a^2*b*d^2*f^6*e + 35*(f*x^2*e + 2*d*f*x)^8*c^3*e^7 + 120*(f*x^2*e + 2*d*f*x)^7*b*c^2*f^6*e + 140*(f*x^2*e + 2*d*f*x)^6*b^2*c*f^2*e^5 + 140*(f*x^2*e + 2*d*f*x)^6*a*c^2*f^2*e^5 + 56*(f*x^2*e + 2*d*f*x)^5*b^3*f^3*e^4 + 336*(f*x^2*e + 2*d*f*x)^5*a*b*c*f^3*e^4 + 210*(f*x^2*e + 2*d*f*x)^4*a*b^2*f^4*e^3 + 210*(f*x^2*e + 2*d*f*x)^4*a^2*c*f^4*e^3 + 280*(f*x^2*e + 2*d*f*x)^3*a^2*b*f^5*e^2 + 140*(f*x^2*e + 2*d*f*x)^2*a^3*f^6*e)/f^5

maple [B] time = 0.00, size = 7697, normalized size = 48.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)$

[Out] result too large to display

maxima [B] time = 1.16, size = 920, normalized size = 5.79

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & 1/16*c^3*e^{15}*f^3*x^{16} + c^3*d*e^{14}*f^3*x^{15} + 3/14*(35*c^3*d^2 + b*c^2)*e^{13}*f^3*x^{14} + (35*c^3*d^3 + 3*b*c^2*d)*e^{12}*f^3*x^{13} + 1/4*(455*c^3*d^4 + 7 \\ & 8*b*c^2*d^2 + b^2*c + a*c^2)*e^{11}*f^3*x^{12} + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*d)*e^{10}*f^3*x^{11} + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b \\ & ^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*e^9*f^3*x^{10} + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*e^8*f^3*x^9 + 3/8*(2 \\ & 145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*e^7*f^3*x^8 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 13 \\ & 86*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*e^6*f^3*x^7 + 1/2*(1001*c^3*d^{10} + 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + \\ & 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*e^5*f^3*x^6 + 3/5*(455*c^3*d^{11} + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c) \\ &)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*e^4*f^3*x^5 + 1/4*(455*c^3*d^{12} + 858*b*c^2*d^{10} + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b*c)*d^6 + 30*a \\ & ^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*e^3*f^3*x^4 + (35*c^3*d^{13} + 78*b*c^2*d^{11} + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3 \\ & + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*e^2*f^3*x^3 + 3/2*(5*c^3*d^{14} + 13*b*c^2*d^{12} + 11*(b^2*c + a*c^2)*d^{10} + 3*(b^3 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a \\ & *b^2 + a^2*c)*d^6 + a^3*d^2)*e*f^3*x^2 + (c^3*d^{15} + 3*b*c^2*d^{13} + 3*(b^2*c + a*c^2)*d^{11} + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7 \\ & + a^3*d^3)*f^3*x \end{aligned}$$

mupad [B] time = 1.65, size = 825, normalized size = 5.19

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)$

[Out] $(3*e^7*f^3*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 + 330*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*f^3*x^6*(a^2*b +$

$$\begin{aligned}
& 42*b^3*d^4 + 1001*c^3*d^{10} + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 + \\
& 462*b^2*c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4)/2 + (e^9*f^3*x^{10}*(b^3 + 5 \\
& 005*c^3*d^6 + 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10 \\
& + (c^3*e^{15}*f^3*x^{16})/16 + d^3*f^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*f^3*x^4* \\
& (a^3 + 84*b^3*d^6 + 455*c^3*d^{12} + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c \\
& *d^4 + 495*a*c^2*d^8 + 495*b^2*c*d^8 + 858*b*c^2*d^{10} + 504*a*b*c*d^6))/4 + \\
& (c*e^{11}*f^3*x^{12}*(a*c + b^2 + 455*c^2*d^4 + 78*b*c*d^2))/4 + (d*e^6*f^3*x^ \\
& 7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8 + 1386*a*c^2*d^4 + 1386* \\
& b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (3*d*e^4*f^3*x^5*(5*a^2*b \\
& + 42*b^3*d^4 + 455*c^3*d^{10} + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 330*a*c^2*d^6 + \\
& 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d*e^8*f^3*x^9*(b^3 + 7 \\
& 15*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 + 6*a*b*c) + (3*c^ \\
& 2*e^{13}*f^3*x^{14}*(b + 35*c*d^2))/14 + c^3*d*e^{14}*f^3*x^{15} + d*e^2*f^3*x^3*(a \\
& ^3 + 12*b^3*d^6 + 35*c^3*d^{12} + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 \\
& + 55*a*c^2*d^8 + 55*b^2*c*d^8 + 78*b*c^2*d^{10} + 72*a*b*c*d^6) + (3*d^2*e*f^ \\
& 3*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 + c^2*d*e^{12}*f^3*x^1 \\
& 3*(3*b + 35*c*d^2) + 3*c*d*e^{10}*f^3*x^{11}*(a*c + b^2 + 91*c^2*d^4 + 26*b*c*d \\
& ^2)
\end{aligned}$$

sympy [B] time = 0.39, size = 1654, normalized size = 10.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] c**3*d*e**14*f**3*x**15 + c**3*e**15*f**3*x**16/16 + x**14*(3*b*c**2*e**13*f**3/14 + 15*c**3*d**2*e**13*f**3/2) + x**13*(3*b*c**2*d*e**12*f**3 + 35*c**3*d**3*e**12*f**3) + x**12*(a*c**2*e**11*f**3/4 + b**2*c*e**11*f**3/4 + 39*b*c**2*d**2*e**11*f**3/2 + 455*c**3*d**4*e**11*f**3/4) + x**11*(3*a*c**2*d*e**10*f**3 + 3*b**2*c*d*e**10*f**3 + 78*b*c**2*d**3*e**10*f**3 + 273*c**3*d**5*e**10*f**3) + x**10*(3*a*b*c*e**9*f**3/5 + 33*a*c**2*d**2*e**9*f**3/2 + b**3*e**9*f**3/10 + 33*b**2*c*d**2*e**9*f**3/2 + 429*b*c**2*d**4*e**9*f**3/2 + 1001*c**3*d**6*e**9*f**3/2) + x**9*(6*a*b*c*d*e**8*f**3 + 55*a*c**2*d**3*e**8*f**3 + b**3*d*e**8*f**3 + 55*b**2*c*d**3*e**8*f**3 + 429*b*c**2*d**5*e**8*f**3 + 715*c**3*d**7*e**8*f**3) + x**8*(3*a**2*c*e**7*f**3/8 + 3*a*b**2*e**7*f**3/8 + 27*a*b*c*d**2*e**7*f**3 + 495*a*c**2*d**4*e**7*f**3/4 + 9*b**3*d**2*e**7*f**3/2 + 495*b**2*c*d**4*e**7*f**3/4 + 1287*b*c**2*d**6*e**7*f**3/2 + 6435*c**3*d**8*e**7*f**3/8) + x**7*(3*a**2*c*d*e**6*f**3 + 3*a*b**2*d*e**6*f**3 + 72*a*b*c*d**3*e**6*f**3 + 198*a*c**2*d**5*e**6*f**3 + 12*b**3*d**3*e**6*f**3 + 198*b**2*c*d**5*e**6*f**3 + 5148*b*c**2*d**7*e**6*f**3/7 + 715*c**3*d**9*e**6*f**3) + x**6*(a**2*b*e**5*f**3/2 + 21*a**2*c*d**2*e**5*f**3/2 + 21*a*b**2*d**2*e**5*f**3/2 + 126*a*b*c*d**4*e**5*f**3 + 231*a*c**2*d**6*e**5*f**3 + 21*b**3*d**4*e**5*f**3 + 231*b**2*c*d**6*e**5*f**3 + 1287*b*c**2*d**8*e**5*f**3/2 + 1001*c**3*d**10*e**5*f**3/2) + x**5*(3*a**2

$$\begin{aligned}
& 2*b*d*e^{4*f^3} + 21*a^2*c*d^3*e^{4*f^3} + 21*a*b^2*d^3*e^{4*f^3} + 756 \\
& *a*b*c*d^5*e^{4*f^3/5} + 198*a*c^2*d^7*e^{4*f^3} + 126*b^3*d^5*e^{4*f^3} \\
& *3/5 + 198*b^2*c*d^7*e^{4*f^3} + 429*b*c^2*d^9*e^{4*f^3} + 273*c^3*d^{11} \\
& *e^{4*f^3}) + x^4*(a^3*e^{3*f^3/4} + 15*a^2*b*d^2*e^{3*f^3/2} + 105*a^2 \\
& *c*d^4*e^{3*f^3/4} + 105*a*b^2*d^4*e^{3*f^3/4} + 126*a*b*c*d^6*e^{3*f^3} \\
& *f^3 + 495*a*c^2*d^8*e^{3*f^3/4} + 21*b^3*d^6*e^{3*f^3} + 495*b^2*c*d^8 \\
& *e^{3*f^3/4} + 429*b*c^2*d^{10}*e^{3*f^3/2} + 455*c^3*d^{12}*e^{3*f^3/4}) \\
& + x^3*(a^3*d*e^{2*f^3} + 10*a^2*b*d^3*e^{2*f^3} + 21*a^2*c*d^5*e^{2*f^3} \\
& *f^3 + 21*a*b^2*d^5*e^{2*f^3} + 72*a*b*c*d^7*e^{2*f^3} + 55*a*c^2*d^9* \\
& e^{2*f^3} + 12*b^3*d^7*e^{2*f^3} + 55*b^2*c*d^9*e^{2*f^3} + 78*b*c^2*d^{11} \\
& *e^{2*f^3} + 35*c^3*d^{13}*e^{2*f^3}) + x^2*(3*a^3*d^2*e^{f^3/2} + 15 \\
& *a^2*b*d^4*e^{f^3/2} + 21*a^2*c*d^6*e^{f^3/2} + 21*a*b^2*d^6*e^{f^3/2} + \\
& 27*a*b*c*d^8*e^{f^3} + 33*a*c^2*d^{10}*e^{f^3/2} + 9*b^3*d^8*e^{f^3/2} + 3 \\
& 3*b^2*c*d^{10}*e^{f^3/2} + 39*b*c^2*d^{12}*e^{f^3/2} + 15*c^3*d^{14}*e^{f^3/2} \\
&) + x*(a^3*d^3*f^3 + 3*a^2*b*d^5*f^3 + 3*a^2*c*d^7*f^3 + 3*a*b^2*d^7 \\
& *f^3 + 6*a*b*c*d^9*f^3 + 3*a*c^2*d^{11}*f^3 + b^3*d^9*f^3 + 3*b^2 \\
& *c*d^{11}*f^3 + 3*b*c^2*d^{13}*f^3 + c^3*d^{15}*f^3)
\end{aligned}$$

$$3.500 \quad \int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4} dx$$

Optimal. Leaf size=193

$$-\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Rubi [A] time = 0.44, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1142, 1122, 1166, 205}

$$-\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1122

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4} dx, x, d+ex\right)}{e} \\ &= \frac{x}{c} - \frac{\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4} dx, x, d+ex\right)}{ce} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx, x, d+ex\right)}{2ce} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) S}{2ce} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}e} \end{aligned}$$

Mathematica [A] time = 0.14, size = 219, normalized size = 1.13

$$\frac{\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + 2\sqrt{c}(d+ex)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

```
[Out] (2*sqrt[c]*(d + e*x) - (sqrt[2]*(-b^2 + 2*a*c + b*sqrt[b^2 - 4*a*c]))*ArcTan
((sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]])/(sqrt[b^2 - 4*a*c]
```

$c)\sqrt{b - \sqrt{b^2 - 4ac}} - (\sqrt{2}(b^2 - 2ac + b\sqrt{b^2 - 4ac}))\operatorname{ArcTan}[\sqrt{2}\sqrt{c}(d + ex)/\sqrt{b + \sqrt{b^2 - 4ac}}]/(\sqrt{b^2 - 4ac}\sqrt{b + \sqrt{b^2 - 4ac}})]/(2c^{3/2}e)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^4}{a + b(d + ex)^2 + c(d + ex)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

fricas [B] time = 2.18, size = 1231, normalized size = 6.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="fricas")

[Out] $\frac{1}{2}(\sqrt{1/2}c\sqrt{-(b^2c^3 - 4a^2c^4)e^2}\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) + b^3 - 3ab^2c)/((b^2c^3 - 4a^2c^4)e^2) \log(-2(ab^2 - a^2c)ex - 2(ab^2 - a^2c)d + \sqrt{1/2}((b^3c^3 - 4a^2b^2c^4)e^3\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) - (b^4 - 5ab^2c + 4a^2c^2)e)\sqrt{-(b^2c^3 - 4a^2c^4)e^2}\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) + b^3 - 3ab^2c)/((b^2c^3 - 4a^2c^4)e^2)) - \sqrt{1/2}c\sqrt{-(b^2c^3 - 4a^2c^4)e^2}\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) + b^3 - 3ab^2c)/((b^2c^3 - 4a^2c^4)e^2) \log(-2(ab^2 - a^2c)ex - 2(ab^2 - a^2c)d - \sqrt{1/2}((b^3c^3 - 4a^2b^2c^4)e^3\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) - (b^4 - 5ab^2c + 4a^2c^2)e)\sqrt{-(b^2c^3 - 4a^2c^4)e^2}\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) + b^3 - 3ab^2c)/((b^2c^3 - 4a^2c^4)e^2)) - \sqrt{1/2}c\sqrt{((b^2c^3 - 4a^2c^4)e^2}\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) - b^3 + 3ab^2c)/((b^2c^3 - 4a^2c^4)e^2)) \log(-2(ab^2 - a^2c)ex - 2(ab^2 - a^2c)d + \sqrt{1/2}((b^3c^3 - 4a^2b^2c^4)e^3\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) + (b^4 - 5ab^2c + 4a^2c^2)e)\sqrt{((b^2c^3 - 4a^2c^4)e^2}\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) - b^3 + 3ab^2c)/((b^2c^3 - 4a^2c^4)e^2)) + \sqrt{1/2}c\sqrt{((b^2c^3 - 4a^2c^4)e^2}\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) - b^3 + 3ab^2c)/((b^2c^3 - 4a^2c^4)e^2)) \log(-2(ab^2 - a^2c)ex - 2(ab^2 - a^2c)d - \sqrt{1/2}((b^3c^3 - 4a^2b^2c^4)e^3\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) - (b^4 - 5ab^2c + 4a^2c^2)e)\sqrt{((b^2c^3 - 4a^2c^4)e^2}\sqrt{(b^4 - 2ab^2c + a^2c^2)}/((b^2c^6 - 4a^2c^7)e^4) - b^3 + 3ab^2c)/((b^2c^3 - 4a^2c^4)e^2))$

$(b^2*c^6 - 4*a*c^7)*e^4)) - b^3 + 3*a*b*c)/((b^2*c^3 - 4*a*c^4)*e^2))) + 2*x)/c$

giac [B] time = 0.43, size = 1194, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] $1/2*(((d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^2*b*e^6 - 2*(d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c)*b*d*e^5 + b*d^2*e^4 + a*e^4)*\log(d*e^{-1} + x + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c)/((2*(d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^3*c*e^4 - 6*(d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))) + ((d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^2*b*e^6 - 2*(d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c)*b*d*e^5 + b*d^2*e^4 + a*e^4)*\log(d*e^{-1} + x - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c)/((2*(d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^3*c*e^4 - 6*(d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))) + ((d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^2*b*e^6 - 2*(d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c)*b*d*e^5 + b*d^2*e^4 + a*e^4)*\log(d*e^{-1} + x + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c)/((2*(d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^3*c*e^4 - 6*(d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))) + ((d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^2*b*e^6 - 2*(d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c)*b*d*e^5 + b*d^2*e^4 + a*e^4)*\log(d*e^{-1} + x - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c)/((2*(d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^3*c*e^4 - 6*(d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{-4}/c))))*e^{-4}/c + x/c$

maple [C] time = 0.07, size = 158, normalized size = 0.82

$\frac{(-\operatorname{RootOf}(2z^4 + 4z^2d^2 + c^2 + b^2 + (cd^2 + b^2)z^2 + (bd^2 + 2bd)z + a^2))b^2 - 2\operatorname{RootOf}(2z^4 + 4z^2d^2 + c^2 + b^2 + (cd^2 + b^2)z^2 + (bd^2 + 2bd)z + a^2))b^2 - b^2 - a)}{2c(\operatorname{RootOf}(2z^4 + 4z^2d^2 + c^2 + b^2 + (cd^2 + b^2)z^2 + (bd^2 + 2bd)z + a^2)) - \operatorname{RootOf}(2z^4 + 4z^2d^2 + c^2 + b^2 + (cd^2 + b^2)z^2 + (bd^2 + 2bd)z + a^2))} \ln(-\operatorname{RootOf}(2z^4 + 4z^2d^2 + c^2 + b^2 + (cd^2 + b^2)z^2 + (bd^2 + 2bd)z + a^2)) - \operatorname{RootOf}(2z^4 + 4z^2d^2 + c^2 + b^2 + (cd^2 + b^2)z^2 + (bd^2 + 2bd)z + a^2))$

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& - a*c*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c - ((-b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}*((2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)})*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) + (2*a^2*b*e^10)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)})*2i + atan(((b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)})*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)})*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + ((-b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)})*((2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)})*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i)/((-b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)})*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)})*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i)/((-b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c +
\end{aligned}$$

$$\begin{aligned} & (2*x*(b^4*e^{12} + 2*a^2*c^2*e^{12} - 4*a*b^2*c*e^{12}))/c - (- (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})) / (8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} * ((2*b^4*d*e^{11} + 4*a^2*c^2*d*e^{11} - 8*a*b^2*c*d*e^{11})/c - ((16*a^2*c^3*e^{12} - 4*a*b^2*c^2*e^{12})/c - ((8*b^3*c^3*d*e^{13} - 32*a*b*c^4*d*e^{13})/c + (2*x*(4*b^3*c^3*e^{14} - 16*a*b*c^4*e^{14}))/c) * (- (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})) / (8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}) * (- (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})) / (8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*x*(b^4*e^{12} + 2*a^2*c^2*e^{12} - 4*a*b^2*c*e^{12}))/c + (2*a^2*b*e^{10})/c) * (- (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})) / (8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} * 2i + x/c \end{aligned}$$

sympy [A] time = 3.07, size = 178, normalized size = 0.92

$$\text{RootSum}\left(t^4(256a^2c^5e^4 - 128ab^2c^4e^4 + 16b^4c^3e^4) + t^2(48a^2bc^2e^2 - 28ab^3ce^2 + 4b^5e^2) + a^3\left(t \mapsto t \log\left(x + \frac{32t^3abc^4e^3 - 8t^3b^3c^3e^3 - 4ta^2c^2e + 8tab^2ce - 2tb^4e + a^2cd - ab^2d}{a^2ce - ab^2e}\right)\right)\right) + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)

[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + a**3, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3*b**3*c**3*e**3 - 4*_t*a**2*c**2*e + 8*_t*a*b**2*c*e - 2*_t*b**4*e + a**2*c*d - a*b**2*d)/(a**2*c*e - a*b**2*e)))) + x/c

$$3.501 \quad \int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4} dx$$

Optimal. Leaf size=81

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}} + \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4ce}$$

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1142, 1114, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}} + \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4ce}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(2*c*Sqrt[b^2 - 4*a*c]*e) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*c*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1114

$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m - 1)/2)*(a + b*x + c*x^2)^p}, x], x, x^2], x] /;$ FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1142

$\text{Int}[(u_)^{(m_.)}*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[u^m/(\text{Coefficient}[v, x, 1]*v^m), \text{Subst}[\text{Int}[x^m*(a + b*x^2 + c*x^{(2*2)})^p, x], x, v], x] /;$ FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex)^3}{a + b(d + ex)^2 + c(d + ex)^4} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a + bx^2 + cx^4} dx, x, d + ex\right)}{e} \\ &= \frac{\text{Subst}\left(\int \frac{x}{a + bx + cx^2} dx, x, (d + ex)^2\right)}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, (d + ex)^2\right)}{4ce} - \frac{b \text{Subst}\left(\int \frac{1}{a + bx + cx^2} dx, x, (d + ex)^2\right)}{4ce} \\ &= \frac{\log(a + b(d + ex)^2 + c(d + ex)^4)}{4ce} + \frac{b \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2c(d + ex)^2\right)}{2ce} \\ &= \frac{b \tanh^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{b^2 - 4ac}}\right)}{2c\sqrt{b^2 - 4ac}e} + \frac{\log(a + b(d + ex)^2 + c(d + ex)^4)}{4ce} \end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.95

$$\frac{\log(a + b(d + ex)^2 + c(d + ex)^4) - \frac{2b \tan^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}}}{4ce}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] $((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3}{a + b(d + ex)^2 + c(d + ex)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

fricas [B] time = 1.01, size = 434, normalized size = 5.36

$$\frac{\sqrt{b^2 - 4ac} \log\left(\frac{(2c^2d^2 + 2b^2d + b^2c) \sqrt{b^2 - 4ac} + (b^2 - 4ac) \log\left(\frac{(2c^2d^2 + 2b^2d + b^2c) \sqrt{b^2 - 4ac}}{2c^2d^2 + 2b^2d + b^2c}\right) + (b^2 - 4ac) \log\left(\frac{(2c^2d^2 + 2b^2d + b^2c) \sqrt{b^2 - 4ac}}{2c^2d^2 + 2b^2d + b^2c}\right)}{4(b^2 - 4ac)^2}\right) + (b^2 - 4ac) \log\left(\frac{(2c^2d^2 + 2b^2d + b^2c) \sqrt{b^2 - 4ac}}{2c^2d^2 + 2b^2d + b^2c}\right) + (b^2 - 4ac) \log\left(\frac{(2c^2d^2 + 2b^2d + b^2c) \sqrt{b^2 - 4ac}}{2c^2d^2 + 2b^2d + b^2c}\right)}{4(b^2 - 4ac)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="fricas")

[Out] $[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*d^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^2*e^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c)))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2)*e), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^2*c - 4*a*c^2)*e)]$

giac [A] time = 0.40, size = 130, normalized size = 1.60

$$\frac{b \arctan\left(\frac{2cd^2 + 2(x^2e + 2dx)ce + b}{\sqrt{-b^2 + 4ac}}\right) e^{(-1)} + \frac{e^{(-1)} \log\left(cd^4 + 2(x^2e + 2dx)cd^2e + (x^2e + 2dx)^2ce^2 + bd^2 + (x^2e + 2dx)be + a\right)}{4c}}{2\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="giac")

[Out] $-1/2*b*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^{(-1)}/(sqrt(-b^2 + 4*a*c)*c) + 1/4*e^{(-1)}*log(c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a)/c$

maple [C] time = 0.00, size = 151, normalized size = 1.86

$$\frac{\left(\text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right)^2 + 3\text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right)^2 d^2 + 3\text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right)^2 \ln\left(-\text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right)\right) + 2\left(2c\text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right) + 4c d^2 \text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right) + 4c d^2 \text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right)\right)}{2\left(2c\text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right) + 4c d^2 \text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right) + 4c d^2 \text{RootOf}\left(\mathcal{Z}^4 + 4\mathcal{Z}^3 d^2 + c d^2 + b d^2 + (4c d^2 + b^2)\mathcal{Z}^2 + (4c d^2 + 2ab)\mathcal{Z} + a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)`

[Out] `1/2/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex + d)^3}{(ex + d)^4 c + (ex + d)^2 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")`

[Out] `integrate((e*x + d)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)`

mupad [B] time = 1.76, size = 278, normalized size = 3.43

$$\frac{4ace \ln(cd^4 + 4cd^3ex + 6cd^2e^2x^2 + bd^2 + 4cd^3x^3 + 2bdex + ce^4x^4 + b^2x^2 + a)}{16a^2c^2 - 4b^2c^2} - \frac{b^2e \ln(cd^4 + 4cd^3ex + 6cd^2e^2x^2 + bd^2 + 4cd^3x^3 + 2bdex + ce^4x^4 + b^2x^2 + a)}{16a^2c^2 - 4b^2c^2} - \frac{b \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cd^2}{\sqrt{4ac-b^2}} + \frac{2c^2x^2}{\sqrt{4ac-b^2}} + \frac{4cdex}{\sqrt{4ac-b^2}}\right)}{2ce\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)`

[Out] `(4*a*c*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b^2*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*e^2*x^2)/(4*a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*a*c - b^2)^(1/2))`

sympy [B] time = 1.64, size = 280, normalized size = 3.46

$$\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)} + \frac{1}{4ce}\right) \log\left(\frac{2dx}{e} + x^2 + \frac{-8ace\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)} + \frac{1}{4ce}\right) + 2a + 2b^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)} + \frac{1}{4ce}\right) + bd^2}{be^2}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)} + \frac{1}{4ce}\right) \log\left(\frac{2dx}{e} + x^2 + \frac{-8ace\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)} + \frac{1}{4ce}\right) + 2a + 2b^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)} + \frac{1}{4ce}\right) + bd^2}{be^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)`

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))*log(2*d*x/e + x
**2 + (-8*a*c*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))
+ 2*a + 2*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e
)) + b*d**2)/(b*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/
(4*c*e))*log(2*d*x/e + x**2 + (-8*a*c*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*
c - b**2)) + 1/(4*c*e)) + 2*a + 2*b**2*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a
*c - b**2)) + 1/(4*c*e)) + b*d**2)/(b*e**2))
```

$$3.502 \quad \int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} dx$$

Optimal. Leaf size=164

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.15, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1142, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1130

Int[((d_)*(x_)^(m_))/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 1142

Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)
) + b)/((b^2*c - 4*a*c^2)*e^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-
(b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*
c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sq
rt(-((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c -
4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(-((b^2*c - 4*a*c^2
)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(
1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x + d) - 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*
c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*lo
g(sqrt(1/2)*(b^2*c - 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*
c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c
^3)*e^4)) + e*x + d) + 1/2*sqrt(1/2)*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b
^2*c^2 - 4*a*c^3)*e^4)) - b)/((b^2*c - 4*a*c^2)*e^2))*log(-sqrt(1/2)*(b^2*c
- 4*a*c^2)*e^3*sqrt(((b^2*c - 4*a*c^2)*e^2*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4
)) - b)/((b^2*c - 4*a*c^2)*e^2))*sqrt(1/((b^2*c^2 - 4*a*c^3)*e^4)) + e*x +
d)
```

giac [B] time = 0.45, size = 1285, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-
4)/c))*d*e + d^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-
4)/c))*d*e + d^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-
```

$$-4/c)) * d * e + d^2 * \log(d * e^{-1} + x + \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c}) / (2 * (d * e^{-1} + \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c}))^3 * c * e^4 - 6 * (d * e^{-1} + \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c})^2 * c * d * e^3 + 6 * (d * e^{-1} + \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c}) * c * d^2 * e^2 - 2 * c * d^3 * e + (d * e^{-1} + \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c}) * b * e^2 - b * d * e - 1/2 * ((d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c})^2 * e^2 - 2 * (d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c}) * d * e + d^2 * \log(d * e^{-1} + x - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c}) / (2 * (d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c}))^3 * c * e^4 - 6 * (d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c})^2 * c * d * e^3 + 6 * (d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c}) * c * d^2 * e^2 - 2 * c * d^3 * e + (d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4/c}) * b * e^2 - b * d * e$$

maple [C] time = 0.01, size = 140, normalized size = 0.85

$$\frac{(\text{RootOf}(_Z^4 * e^4 + 4 * _Z^3 * c * d * e^3 + c * d^4 + (4 * a * c * e^2 + 4 * d^2 * e^2) * _Z^2 + (4 * d^2 * e^2 + 2 * \text{RootOf}(_Z^2 * e^2 + 4 * _Z * d * e + c * d + b * d^2 + (4 * d^2 * e^2 + b^2) * _Z + a)) * _Z + a)) * \ln(-\text{RootOf}(_Z^4 * e^4 + 4 * _Z^3 * c * d * e^3 + c * d^4 + (4 * a * c * e^2 + 4 * d^2 * e^2) * _Z^2 + (4 * d^2 * e^2 + 2 * \text{RootOf}(_Z^2 * e^2 + 4 * _Z * d * e + c * d + b * d^2 + (4 * d^2 * e^2 + b^2) * _Z + a))) - \text{RootOf}(_Z^4 * e^4 + 4 * _Z^3 * c * d * e^3 + c * d^4 + (4 * a * c * e^2 + 4 * d^2 * e^2) * _Z^2 + (4 * d^2 * e^2 + 2 * \text{RootOf}(_Z^2 * e^2 + 4 * _Z * d * e + c * d + b * d^2 + (4 * d^2 * e^2 + b^2) * _Z + a)))}{2 * (\text{RootOf}(_Z^4 * e^4 + 4 * _Z^3 * c * d * e^3 + c * d^4 + (4 * a * c * e^2 + 4 * d^2 * e^2) * _Z^2 + (4 * d^2 * e^2 + 2 * \text{RootOf}(_Z^2 * e^2 + 4 * _Z * d * e + c * d + b * d^2 + (4 * d^2 * e^2 + b^2) * _Z + a))) + 6 * d^2 * \text{RootOf}(_Z^2 * e^2 + 4 * _Z * d * e + c * d + b * d^2 + (4 * d^2 * e^2 + b^2) * _Z + a)) + 6 * c * d * \text{RootOf}(_Z^2 * e^2 + 4 * _Z * d * e + c * d + b * d^2 + (4 * d^2 * e^2 + b^2) * _Z + a)) + 2 * d * b * \text{RootOf}(_Z^2 * e^2 + 4 * _Z * d * e + c * d + b * d^2 + (4 * d^2 * e^2 + b^2) * _Z + a)) + b * d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)

[Out] 1/2/e*sum((R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex + d)^2}{(ex + d)^4 c + (ex + d)^2 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] integrate((e*x + d)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)

mupad [B] time = 1.74, size = 590, normalized size = 3.60

$$-2 \operatorname{atanh} \left(\frac{\sqrt{\frac{d^2 + (4ac - b^2) - 4ad}{4(ad^2 + b^2)c^2}}}{\sqrt{\frac{d^2 + (4ac - b^2) - 4ad}{4(ad^2 + b^2)c^2}}} \right) \left(\frac{(b^2 d^2 - 2ad^2) \sqrt{\frac{d^2 + (4ac - b^2) - 4ad}{4(ad^2 + b^2)c^2}}}{4ad^2} + 4ad^2 d^2 - 2b^2 d^2 \right) \left(\frac{b^2 + \sqrt{(4ac - b^2) - 4ad}}{\sqrt{8(b^2 d^2 - 4ad^2 c^2 + b^2 c^2)}} - 2 \operatorname{atanh} \left(\frac{\sqrt{\frac{d^2 + (4ac - b^2) - 4ad}{4(ad^2 + b^2)c^2}}}{\sqrt{\frac{d^2 + (4ac - b^2) - 4ad}{4(ad^2 + b^2)c^2}}} \right) \left(\frac{(b^2 d^2 - 2ad^2) \sqrt{\frac{d^2 + (4ac - b^2) - 4ad}{4(ad^2 + b^2)c^2}}}{4ad^2} + 4ad^2 d^2 - 2b^2 d^2 \right) \right) \sqrt{8(b^2 d^2 - 4ad^2 c^2 + b^2 c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)

```
[Out] - 2*atanh((((-(b^3 + (-4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12 - 2*b^2*c*e^12) + (x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^13 - 32*a*b*c^3*d*e^13)*(b^3 + (-4*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*e^11))/(a*c*e^10))*(-(b^3 + (-4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2) - 2*atanh((((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12 - 2*b^2*c*e^12) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^13 - 32*a*b*c^3*d*e^13))*((-4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*e^11))/(a*c*e^10))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)
```

sympy [A] time = 1.35, size = 104, normalized size = 0.63

$$\text{RootSum}\left(t^4(256a^2c^3e^4 - 128ab^2c^2e^4 + 16b^4ce^4) + t^2(-16abce^2 + 4b^3e^2) + a, \left(t \mapsto t \log\left(x + \frac{64t^3ac^2e^3 - 16t^3b^2ce^3 - 2tbe + d}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**2 + 4*b**3*e**2) + a, Lambda(_t, _t*log(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e + d)/e)))
```

$$3.503 \quad \int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4} dx$$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1142, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]

[Out] -(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*a*c]*e))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4} dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx^2+cx^4} dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, (d+ex)^2\right)}{2e} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2c(d+ex)^2\right)}{e} \\
&= -\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}e}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{e\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]]/(Sqrt[-b^2 + 4*a*c]*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

fricas [A] time = 1.30, size = 272, normalized size = 6.33

$$\left[\frac{\log\left(\frac{2c^2e^4x^4+8c^2de^3x^3+2c^2d^4+2(6c^2d^2+bc)e^2x^2+2bcd^2+4(2c^2d^3+bcd)ex+b^2-2ac-(2ce^2x^2+4cdex+2cd^2+b)\sqrt{b^2-4ac}}{ce^4x^4+4cde^3x^3+cd^4+(6cd^2+b)e^2x^2+bd^2+2(2cd^3+bd)ex+a}\right)}{2\sqrt{b^2-4ac}e}, -\frac{\sqrt{-b^2+4ac} \arctan\left(-\frac{(2ce^2x^2+4cdex+2cd^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{(b^2-4ac)e} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="fricas")

[Out] $\left[\frac{1}{2} \log\left((2c^2e^4x^4 + 8c^2de^3x^3 + 2c^2d^4 + 2(6c^2d^2 + b^2c)e^2x^2 + 2b^2cd^2 + 4(2c^2d^3 + b^2cd)e^2x + b^2 - 2a^2c - (2c^2e^2x^2 + 4c^2de^2x + 2c^2d^2 + b)^2 \sqrt{b^2 - 4ac} \right) / (c^2e^4x^4 + 4c^2de^3x^3 + c^2d^4 + (6c^2d^2 + b)e^2x^2 + b^2d^2 + 2(2c^2d^3 + b^2d)e^2x + a) \right] / \left(\sqrt{b^2 - 4ac} e \right), -\sqrt{b^2 - 4ac} \arctan\left(\frac{(2c^2e^2x^2 + 4c^2de^2x + 2c^2d^2 + b)\sqrt{b^2 - 4ac}}{b^2 - 4ac} \right) / \left((b^2 - 4ac)e \right)]$

giac [A] time = 0.45, size = 53, normalized size = 1.23

$$\frac{\arctan\left(\frac{2cd^2 + 2(x^2e + 2dx)ce + b}{\sqrt{-b^2 + 4ac}}\right)e^{(-1)}}{\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] $\arctan\left(\frac{2c^2d^2 + 2(x^2e + 2d^2x)ce + b}{\sqrt{-b^2 + 4ac}}\right)e^{(-1)} / \sqrt{-b^2 + 4ac}$

maple [C] time = 0.01, size = 129, normalized size = 3.00

$\frac{\text{RootOf}\left(_Z^4 + 4_Z^3cd + c^2d^2 + (bc^2d^2 + b^2d^2 + (4cd^2 + 2ab))_Z + d\right)\ln\left(-\text{RootOf}\left(_Z^4 + 4_Z^3cd + c^2d^2 + (bc^2d^2 + b^2d^2 + (4cd^2 + 2ab))_Z + d\right)\right)}{2\left(2c^2\text{RootOf}\left(_Z^4 + 4_Z^3cd + c^2d^2 + (bc^2d^2 + b^2d^2 + (4cd^2 + 2ab))_Z + d\right) + 6cd^2\text{RootOf}\left(_Z^4 + 4_Z^3cd + c^2d^2 + (bc^2d^2 + b^2d^2 + (4cd^2 + 2ab))_Z + d\right) + 6cd^2\text{RootOf}\left(_Z^4 + 4_Z^3cd + c^2d^2 + (bc^2d^2 + b^2d^2 + (4cd^2 + 2ab))_Z + d\right) + 2d^2 + bc\text{RootOf}\left(_Z^4 + 4_Z^3cd + c^2d^2 + (bc^2d^2 + b^2d^2 + (4cd^2 + 2ab))_Z + d\right) + 2d^2 + bc\text{RootOf}\left(_Z^4 + 4_Z^3cd + c^2d^2 + (bc^2d^2 + b^2d^2 + (4cd^2 + 2ab))_Z + d\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)

[Out] $\frac{1}{2}e^{\sum\left(\frac{_R^2e+d}{2_R^3c^2e^3+6_R^2c^2d^2e^2+6_R^2c^2d^2e+2c^2d^3+_R^2b^2e+b^2d}\right)\ln(-_R+x), _R=\text{RootOf}\left(_Z^4+c^2e^4+4_Z^3c^2d^2e^3+c^2d^4+b^2d^2+(6c^2d^2e^2+2+b^2e^2)_Z^2+(4c^2d^3e+2b^2d^2e)_Z+a\right)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex + d}{(ex + d)^4c + (ex + d)^2b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] integrate((e*x + d)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)

mupad [B] time = 0.09, size = 61, normalized size = 1.42

$$\frac{\text{atan}\left(\frac{2acd^2 + 4acdex + 2ace^2x^2 + ab}{a\sqrt{4ac - b^2}}\right)}{e\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x)`

[Out] `atan((a*b + 2*a*c*d^2 + 2*a*c*e^2*x^2 + 4*a*c*d*e*x)/(a*(4*a*c - b^2)^(1/2)))/(e*(4*a*c - b^2)^(1/2))`

sympy [B] time = 1.04, size = 168, normalized size = 3.91

$$\frac{\sqrt{-\frac{1}{4ac-b^2}} \log\left(\frac{2dx}{e} + x^2 + \frac{-4ac\sqrt{-\frac{1}{4ac-b^2}} + b^2\sqrt{-\frac{1}{4ac-b^2}} + b+2cd^2}{2ce^2}\right)}{2e} + \frac{\sqrt{-\frac{1}{4ac-b^2}} \log\left(\frac{2dx}{e} + x^2 + \frac{4ac\sqrt{-\frac{1}{4ac-b^2}} - b^2\sqrt{-\frac{1}{4ac-b^2}} + b+2cd^2}{2ce^2}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)`

[Out] `-sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e) + sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e)`

$$3.504 \quad \int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal. Leaf size=94

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4ae} + \frac{\log(d+ex)}{ae}$$

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1142, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4ae} + \frac{\log(d+ex)}{ae}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(2*a*Sqrt[b^2 - 4*a*c] * e) + Log[d + e*x]/(a*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*e)

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)} dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)} dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, (d+ex)^2\right)}{2ae} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2} dx, x, (d+ex)^2\right)}{2ae} \\
&= \frac{\log(d+ex)}{ae} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, (d+ex)^2\right)}{4ae} - \frac{b \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, (d+ex)^2\right)}{2ae} \\
&= \frac{\log(d+ex)}{ae} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4ae} + \frac{b \text{Subst}\left(\int \frac{1}{b^2-4ac} dx, x, (d+ex)^2\right)}{2ae} \\
&= \frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}e} + \frac{\log(d+ex)}{ae} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4ae}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 128, normalized size = 1.36

$$\frac{4\sqrt{b^2-4ac} \log(d+ex) - (\sqrt{b^2-4ac} + b) \log(-\sqrt{b^2-4ac} + b + 2c(d+ex)^2) + (b - \sqrt{b^2-4ac}) \log(\sqrt{b^2-4ac} + b + 2c(d+ex)^2)}{4ae\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

[Out] IntegrateAlgebraic[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

fricas [A] time = 1.30, size = 468, normalized size = 4.98

$$\frac{\sqrt{b^2-4ac} \log\left(\frac{(2c^2d^2e^2+2bdce+\sqrt{b^2-4ac}x^2+2\sqrt{b^2-4ac}dx+bd^2+2d)\sqrt{b^2-4ac}}{2c^2d^2e^2+2bdce+\sqrt{b^2-4ac}x^2+2\sqrt{b^2-4ac}dx+bd^2+2d}\right) - (b^2-4ac)\log(c^2e^4+4cd^2e^2+cd^4+(6cd^2+b)^2e^2+bd^2+2(2cd^2+bd)ex+a)+4(b^2-4ac)\log(ex+d)}{4(b^2-4ac)^2} + \frac{2\sqrt{b^2-4ac} \arctan\left(\frac{(2c^2d^2e^2+2bdce+\sqrt{b^2-4ac}x^2+2\sqrt{b^2-4ac}dx+bd^2+2d)\sqrt{b^2-4ac}}{2c^2d^2e^2+2bdce+\sqrt{b^2-4ac}x^2+2\sqrt{b^2-4ac}dx+bd^2+2d}\right)}{4(b^2-4ac)^2} - \frac{(b^2-4ac)\log(c^2e^4+4cd^2e^2+cd^4+(6cd^2+b)^2e^2+bd^2+2(2cd^2+bd)ex+a)+4(b^2-4ac)\log(ex+d)}{4(b^2-4ac)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(ex + d)]/(a*b^2 - 4*a^2*c)*e, 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(ex + d)]/(a*b^2 - 4*a^2*c)*e)]

giac [B] time = 1.15, size = 274, normalized size = 2.91

$$\frac{e^{(-1)} \log\left(\frac{cx^4e^4 + 4cd^2e^2 + 6cd^2e^2 + 4cd^2xe + cd^4 + bx^2e^2 + 2bdxe + bd^2 + d}{a}\right) + \frac{e^{(-1)} \log\left(\frac{cx^4e^4 + 4cd^2e^2 + 6cd^2e^2 + 4cd^2xe + cd^4 + bx^2e^2 + 2bdxe + bd^2 + d}{a}\right)}{a} - \frac{\left(\frac{abc^3 \log\left(\frac{bx^2e^2 + 2bdxe + \sqrt{b^2-4ac}x^2 + 2\sqrt{b^2-4ac}dx + bd^2 + 2d}{\sqrt{b^2-4ac}}\right) - abc^3 \log\left(\frac{-bx^2e^2 - 2bdxe + \sqrt{b^2-4ac}x^2 + 2\sqrt{b^2-4ac}dx - bd^2 + 2d}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\right)}{4a^2c}}{4a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] -1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/a + e^(-1)*log(abs(x*e + d))/a - 1/4*(a*b*c*e^3*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sqrt(b^2 - 4*a*c) - a*b*c*e^3*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^2*c)

maple [C] time = 0.01, size = 184, normalized size = 1.96

$$\frac{\ln\left(\frac{cx^4e^4 + 4cd^2e^2 + 6cd^2e^2 + 4cd^2xe + cd^4 + bx^2e^2 + 2bdxe + bd^2 + d}{a}\right)}{a} - \frac{\ln\left(\frac{cx^4e^4 + 4cd^2e^2 + 6cd^2e^2 + 4cd^2xe + cd^4 + bx^2e^2 + 2bdxe + bd^2 + d}{a}\right)}{a} - \frac{\left(\frac{abc^3 \operatorname{Re}\left(\frac{bx^2e^2 + 2bdxe + \sqrt{b^2-4ac}x^2 + 2\sqrt{b^2-4ac}dx + bd^2 + 2d}{\sqrt{b^2-4ac}}\right) - abc^3 \operatorname{Im}\left(\frac{bx^2e^2 + 2bdxe + \sqrt{b^2-4ac}x^2 + 2\sqrt{b^2-4ac}dx + bd^2 + 2d}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\right)}{4a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)

[Out] 1/2/a/e*sum((-c*e^3*_R^3-3*c*d*e^2*_R^2+e*(-3*c*d^2-b)*_R-c*d^3-b*d)/(2*_R^3+c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf

$(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))+\ln(e*x+d)/a/e$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 2.50, size = 2173, normalized size = 23.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)

[Out] $\log(d + e*x)/(a*e) - (\log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e - 8*a*c*e))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b*\operatorname{atan}((16*a^3*x^2*((3*b^3 - 8*a*b*c)*((b^2*(10*b*c^3*e^{18} + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^{19} - 40*a*b*c^3*e^{19}))/2*(4*a*b^2*e^2 - 16*a^2*c*e^2)))))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2*(10*b*c^3*e^{18} + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^{19} - 40*a*b*c^3*e^{19}))/2*(4*a*b^2*e^2 - 16*a^2*c*e^2)))))/(4*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^{19} - 40*a*b*c^3*e^{19}))/16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))/8*a^3*c^2*(25*a*c - 6*b^2) - ((b*(2*b^2*e - 8*a*c*e)^2*(12*b^3*c^2*e^{19} - 40*a*b*c^3*e^{19}))/16*a*e*(4*a*c - b^2)^{1/2}*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2 - (b^3*(12*b^3*c^2*e^{19} - 40*a*b*c^3*e^{19}))/64*a^3*e^3*(4*a*c - b^2)^{3/2}) + (b*(2*b^2*e - 8*a*c*e)*(10*b*c^3*e^{18} + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^{19} - 40*a*b*c^3*e^{19}))/2*(4*a*b^2*e^2 - 16*a^2*c*e^2)))/(4*a*e*(4*a*c - b^2)^{1/2}*(4*a*b^2*e^2 - 16*a^2*c*e^2))*((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/8*a^3*c^2*(4*a*c - b^2)^{1/2}*(25*a*c - 6*b^2))*((4*a*c - b^2)^{3/2})/b^2*c^2*e^{14} + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^{3/2}*((b^2*((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^{17} + 12*b^3*c^2*d^2*e^{17} - 40*a*b*c^3*d^2*e^{17}))/2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^{16} + 10*b*c^3*d^2*e^{16}))/16*a^2*e^2*(4*a*c - b^2) - ((2*b^2*e - 8*a*c*e)^2*((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^{17} + 12*b^3*c^2*d^2*e^{17} - 40*a*b*c^3*d^2*e^{17}))/2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^{16} + 10*b*c^3*d^2*e^{16}))/4*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^{17} + 12*b^3*c^2*d^2*e^{17} - 40*a*b*c^3*d^2*e^{17}))/16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)))/b^2*c^4*e^{14}*(25*a*c - 6*b^2) - (2*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*(2*b^2*e - 8*a*c*e)*((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^{17} + 12*b^3*c^2*d^2*e^{17} - 40*a*b*c^3*d^2*e^{17}))/2*(4*a*b^2*e^2 - 16*a^2*c$

$$\begin{aligned}
& 2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(4*a*e*(4*a*c - b^2)^{(1/2)} \\
& *(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b^3*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17)))/(64*a^3*e^3*(4*a*c - b^2)^{(3/2)}) + (b*(2*b^2*e - 8*a*c*e)^2*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17)) \\
& /(16*a*e*(4*a*c - b^2)^{(1/2)}*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2))/(b^2*c^4*e^14*(25*a*c - 6*b^2)) + (16*a^3*x*((3*b^3 - 8*a*b*c)*(b^2*((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18)))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2*((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18)))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(b*(2*b^2*e - 8*a*c*e)*((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18)))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*a*e*(4*a*c - b^2)^{(1/2)}*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b^3*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(64*a^3*e^3*(4*a*c - b^2)^{(3/2)}) + (b*(2*b^2*e - 8*a*c*e)^2*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(16*a*e*(4*a*c - b^2)^{(1/2)}*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2))/(8*a^3*c^2*(4*a*c - b^2)^{(1/2)}*(25*a*c - 6*b^2)))*(4*a*c - b^2)^{(3/2)}/(b^2*c^2*e^14))/(2*a*e*(4*a*c - b^2)^{(1/2)})
\end{aligned}$$

sympy [B] time = 6.04, size = 320, normalized size = 3.40

$$\left(\frac{b\sqrt{-4ac+b^2}}{4ac(4ac-b^2)} - \frac{1}{4ac} \right) \log \left(\frac{2dx}{c} + x^2 + \frac{-8a^2cc \left(\frac{b\sqrt{-4ac+b^2}}{4ac(4ac-b^2)} - \frac{1}{4ac} \right) + 2ab^2c \left(\frac{b\sqrt{-4ac+b^2}}{4ac(4ac-b^2)} - \frac{1}{4ac} \right) - 2ac + b^2 + bcd^2}{bcc^2} \right) + \left(\frac{b\sqrt{-4ac+b^2}}{4ac(4ac-b^2)} - \frac{1}{4ac} \right) \log \left(\frac{2dx}{c} + x^2 + \frac{-8a^2cc \left(\frac{b\sqrt{-4ac+b^2}}{4ac(4ac-b^2)} - \frac{1}{4ac} \right) + 2ab^2c \left(\frac{b\sqrt{-4ac+b^2}}{4ac(4ac-b^2)} - \frac{1}{4ac} \right) - 2ac + b^2 + bcd^2}{bcc^2} \right) + \log \left(\frac{d}{e} + x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)

[Out] $(-b*\sqrt{-4*a*c + b**2})/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e))*\log(2*d*x/e + x**2 + (-8*a**2*c*e*(-b*\sqrt{-4*a*c + b**2})/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(-b*\sqrt{-4*a*c + b**2})/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*\sqrt{-4*a*c + b**2})/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e))*\log(2*d*x/e + x**2 + (-8*a**2*c*e*(b*\sqrt{-4*a*c + b**2})/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(b*\sqrt{-4*a*c + b**2})/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + \log(d/e + x)/(a*e)$

$$3.505 \quad \int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal. Leaf size=195

$$\frac{\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} ae \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} ae \sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{ae(d+ex)}$$

Rubi [A] time = 0.29, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1142, 1123, 1166, 205}

$$\frac{\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} ae \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} ae \sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{ae(d+ex)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] -(1/(a*e*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1123

Int[((d_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,

$x], x, v], x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{LinearPairQ}[u, v, x]$

Rule 1166

$\text{Int}[(d + e \cdot x) \cdot (x^2) / ((a + b \cdot x) \cdot (x^2) + c \cdot (x^4)), x_Symbol] :$
 $> \text{With}\{q = \text{Rt}[b^2 - 4 \cdot a \cdot c, 2]\}, \text{Dist}[e/2 + (2 \cdot c \cdot d - b \cdot e) / (2 \cdot q), \text{Int}[1 / (b/2$
 $- q/2 + c \cdot x^2), x], x] + \text{Dist}[e/2 - (2 \cdot c \cdot d - b \cdot e) / (2 \cdot q), \text{Int}[1 / (b/2 + q/2$
 $+ c \cdot x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\& \text{NeQ}[c \cdot d^2 - a \cdot e^2, 0] \&\& \text{PosQ}[b^2 - 4 \cdot a \cdot c]$

Rubi steps

$$\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)} dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)} dx, x, d+ex\right)}{e}$$

$$= -\frac{1}{ae(d+ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4} dx, x, d+ex\right)}{ae}$$

$$= -\frac{1}{ae(d+ex)} - \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx, x, d+ex\right)}{2ae}$$

$$= -\frac{1}{ae(d+ex)} - \frac{\sqrt{c}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}e} - \frac{\sqrt{c}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}e}$$

Mathematica [A] time = 0.36, size = 206, normalized size = 1.06

$$\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2}{d+ex}$$

$$2ae$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[

$(\sqrt{2} \sqrt{c} (d + ex)) / \sqrt{b + \sqrt{b^2 - 4ac}} / (\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}) / (ae)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

fricas [B] time = 1.36, size = 1339, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] $\frac{1}{2} * (\sqrt{1/2} * (a * e^{2x} + a * d * e)) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) + b^3 - 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2) * \log(-2 * (b^2 * c^2 - a * c^3) * e * x - 2 * (b^2 * c^2 - a * c^3) * d + \sqrt{1/2} * ((a^3 * b^4 - 6 * a^4 * b^2 * c + 8 * a^5 * c^2) * e^3 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) - (b^5 - 5 * a * b^3 * c + 4 * a^2 * b * c^2) * e) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) + b^3 - 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2)) - \sqrt{1/2} * (a * e^{2x} + a * d * e) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) + b^3 - 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2)) * \log(-2 * (b^2 * c^2 - a * c^3) * e * x - 2 * (b^2 * c^2 - a * c^3) * d - \sqrt{1/2} * ((a^3 * b^4 - 6 * a^4 * b^2 * c + 8 * a^5 * c^2) * e^3 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) - (b^5 - 5 * a * b^3 * c + 4 * a^2 * b * c^2) * e) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) + b^3 - 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2)) - \sqrt{1/2} * (a * e^{2x} + a * d * e) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) - b^3 + 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2)) * \log(-2 * (b^2 * c^2 - a * c^3) * e * x - 2 * (b^2 * c^2 - a * c^3) * d + \sqrt{1/2} * ((a^3 * b^4 - 6 * a^4 * b^2 * c + 8 * a^5 * c^2) * e^3 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) + (b^5 - 5 * a * b^3 * c + 4 * a^2 * b * c^2) * e) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) - b^3 + 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2)) * \log(-2 * (b^2 * c^2 - a * c^3) * e * x - 2 * (b^2 * c^2 - a * c^3) * d - \sqrt{1/2} * ((a^3 * b^4 - 6 * a^4 * b^2 * c + 8 * a^5 * c^2) * e^3 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) + (b^5 - 5 * a * b^3 * c + 4 * a^2 * b * c^2) * e) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2)} / ((a^6 * b^2 - 4 * a^7 * c) * e^4)) - b^3 + 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2))$

```
)*e)*sqrt(((a^3*b^2 - 4*a^4*c)*e^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2))) - 2)/(a*e^2*x + a*d*e)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Error index.cc index_gcd Error: Bad Argument
ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

maple [C] time = 0.01, size = 168, normalized size = 0.86

$$\frac{(-\operatorname{RootOf}(Z^2+4Z^2d^2+cd^2+b^2d^2+(4d^2+2d))Z+a)^2c^2-2\operatorname{RootOf}(Z^2+4Z^2d^2+cd^2+b^2d^2+(4d^2+2d))Z+(4d^2+2d))\ln(-\operatorname{RootOf}(Z^2+4Z^2d^2+cd^2+b^2d^2+(4d^2+2d))Z+a)+1}{2a(\operatorname{RootOf}(Z^2+4Z^2d^2+cd^2+b^2d^2+(4d^2+2d))Z+a)+\operatorname{RootOf}(Z^2+4Z^2d^2+cd^2+b^2d^2+(4d^2+2d))Z+a)+\operatorname{RootOf}(Z^2+4Z^2d^2+cd^2+b^2d^2+(4d^2+2d))Z+a)+2d^2+4\operatorname{RootOf}(Z^2+4Z^2d^2+cd^2+b^2d^2+(4d^2+2d))Z+a)+1} \frac{1}{(d+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)
```

```
[Out] 1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/a/e/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.39, size = 3844, normalized size = 19.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] - atan(((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*
```


$$\begin{aligned} & *c*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2))^{(1/2)} + 4*a^4*c^4*d*e^{11} - 2*a^3*b^2*c^3*d*e^{11})*i + (- (b^5 - b^2* \\ & (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}))^{(1/2)}/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2))^{(1/2)}*(x*(4* \\ & a^4*c^4*e^{12} - 2*a^3*b^2*c^3*e^{12}) + ((x*(32*a^6*b*c^3*e^{14} - 8*a^5*b^3*c^2*e^{14}) + 32*a^6*b*c^3*d*e^{13} - 8*a^5*b^3*c^2*d*e^{13})*(- (b^5 - b^2* \\ & (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}))^{(1/2)} + 16*a^5*b*c^3* \\ & e^{12} - 4*a^4*b^3*c^2*e^{12})*(- (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}))^{(1/2)} + 4*a^4*c^4*d*e^{11} - 2*a^3*b^2*c^3*d*e^{11} \\ &)*i)/((- (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}))^{(1/2)}/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2))^{(1/2)}*(x*(4*a^4*c^4*e^{12} - 2*a^3*b^2*c^3*e^{12}) + ((x*(32*a^6*b*c^3* \\ & e^{14} - 8*a^5*b^3*c^2*e^{14}) + 32*a^6*b*c^3*d*e^{13} - 8*a^5*b^3*c^2*d*e^{13})*(- (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a* \\ & c - b^2)^3)^{(1/2)}))^{(1/2)}/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2))^{(1/2)} + 16*a^5*b*c^3*e^{12} - 4*a^4*b^3*c^2*e^{12})*(- (b^5 - b^2*(-(4*a*c - b^2)^ \\ & ^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}))^{(1/2)}/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2))^{(1/2)} + 4*a^4*c^4*d*e^{11} - \\ & 2*a^3*b^2*c^3*d*e^{11}) - (- (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}))^{(1/2)}/(8*(a^3*b^4*e^2 + 16*a^5*c^2* \\ & e^2 - 8*a^4*b^2*c*e^2))^{(1/2)}*(x*(4*a^4*c^4*e^{12} - 2*a^3*b^2*c^3*e^{12}) + (\\ & (x*(32*a^6*b*c^3*e^{14} - 8*a^5*b^3*c^2*e^{14}) + 32*a^6*b*c^3*d*e^{13} - 8*a^5*b^3*c^2*d*e^{13})*(- (b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^ \\ & ^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}))^{(1/2)}/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2))^{(1/2)} - 16*a^5*b*c^3*e^{12} + 4*a^4*b^3*c^2*e^{12})*(- (b^5 - b^ \\ & ^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)}))^{(1/2)}/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2))^{(1/2)} + 4* \\ & a^4*c^4*d*e^{11} - 2*a^3*b^2*c^3*d*e^{11}) + 2*a^3*c^4*e^{10}))*(- (b^5 - b^2*(-(4 \\ & *a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1 \\ & /2)))/((8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2))^{(1/2)}*2i - 1/(a* \\ & e*(d + e*x))) \end{aligned}$$

sympy [A] time = 4.27, size = 211, normalized size = 1.08

$$\text{RootSum}\left(t^4(256a^5c^2e^4 - 128a^4b^2ce^4 + 16a^3b^4e^4) + t^2(48a^2bc^2e^2 - 28ab^3ce^2 + 4b^5e^2) + c^3, \left(t \mapsto t \log\left(x + \frac{-64t^3a^5c^2e^3 + 48t^3a^4b^2ce^3 - 8t^3a^3b^4e^3 - 10ta^2bc^2e + 10tab^3ce - 2tb^5e + ac^3d - b^2c^2d}{ac^3e - b^2c^2e}\right)\right)\right) - \frac{1}{ade + ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)

[Out] RootSum(_t**4*(256*a**5*c**2*e**4 - 128*a**4*b**2*c*e**4 + 16*a**3*b**4*e**4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**3 + 48*_t**3*a**4*b**2*c*e**3 - 8*_t**3*a**3*b**4*e**3 - 10*_t*a**2*b*c**2*e + 10*_t*a*b**3*c*e - 2*_t*b

$$\frac{5e + a^3c - b^2c^2}{(a^3e - b^2c^2e)}} - \frac{1}{(ade + a^2e^2x)}$$

$$3.506 \quad \int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal. Leaf size=121

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}} + \frac{b \log(a + b(d+ex)^2 + c(d+ex)^4)}{4a^2e} - \frac{b \log(d+ex)}{a^2e} - \frac{1}{2ae(d+ex)^2}$$

Rubi [A] time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1142, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}} + \frac{b \log(a + b(d+ex)^2 + c(d+ex)^4)}{4a^2e} - \frac{b \log(d+ex)}{a^2e} - \frac{1}{2ae(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] -1/(2*a*e*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e) - (b*Log[d + e*x])/(a^2*e) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)/(a + bx + cx^2), x}{x} /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2cd - b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 709

$\text{Int}[\frac{(d + ex)^m}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Simp}[\frac{e(d + ex)^{m+1}}{(m+1)(cd^2 - bde + ae^2)}, x] + \text{Dist}[\frac{1}{cd^2 - bde + ae^2}, \text{Int}[\frac{(d + ex)^{m+1} \text{Simp}[cd - b^2e - c^2ex, x]}{a + bx + cx^2}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{NeQ}[2cd - b^2e, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 800

$\text{Int}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(d + ex)^m (f + gx)}{a + bx + cx^2}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 1114

$\text{Int}[(x^m)((a + bx + cx^2)^p), x_Symbol] \rightarrow \text{Dist}[\frac{1}{2}, \text{Subst}[\text{Int}[x^{(m-1)/2}(a + bx + cx^2)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$

Rule 1142

$\text{Int}[(u^m)((a + bv + cv^2)^p), x_Symbol] \rightarrow \text{Dist}[\frac{u^m}{\text{Coefficient}[v, x, 1]v^m}, \text{Subst}[\text{Int}[x^m(a + bx^2 + cx^{2*2})^p, x], x, v], x] /; \text{FreeQ}[\{a, b, c, m, p\}, x] \ \&\& \ \text{LinearPairQ}[u, v, x]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)} dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)} dx, x, (d+ex)^2\right)}{2e} \\
&= -\frac{1}{2ae(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)} dx, x, (d+ex)^2\right)}{2ae} \\
&= -\frac{1}{2ae(d+ex)^2} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right) dx, x, (d+ex)^2\right)}{2ae} \\
&= -\frac{1}{2ae(d+ex)^2} - \frac{b \log(d+ex)}{a^2e} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx, x, (d+ex)^2\right)}{2a^2e} \\
&= -\frac{1}{2ae(d+ex)^2} - \frac{b \log(d+ex)}{a^2e} + \frac{b \text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, (d+ex)^2\right)}{4a^2e} \\
&= -\frac{1}{2ae(d+ex)^2} - \frac{b \log(d+ex)}{a^2e} + \frac{b \log(a+b(d+ex)^2+c(d+ex)^4)}{4a^2e} \\
&= -\frac{1}{2ae(d+ex)^2} - \frac{(b^2-2ac) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}e} - \frac{b \log(d+ex)}{a^2e} + \dots
\end{aligned}$$

Mathematica [A] time = 0.13, size = 154, normalized size = 1.27

$$\frac{\frac{(b\sqrt{b^2-4ac}-2ac+b^2)\log(-\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{\sqrt{b^2-4ac}} + \frac{(b\sqrt{b^2-4ac}+2ac-b^2)\log(\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{\sqrt{b^2-4ac}}}{4a^2e} - \frac{2a}{(d+ex)^2} - 4b \log(d+ex)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c])/(4*a^2*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

fricas [B] time = 1.10, size = 810, normalized size = 6.69

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d)]/(a^2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*x + (a^2*b^2 - 4*a^3*c)*d^2*e), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*log(e*x + d)]/(a^2*b^2 - 4*a^3*c)*e^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*x + (a^2*b^2 - 4*a^3*c)*d^2*e)]

giac [A] time = 0.40, size = 102, normalized size = 0.84

$$\frac{be^{(-1)} \log\left(c + \frac{b}{(xe+d)^2} + \frac{a}{(xe+d)^4}\right)}{4a^2} + \frac{(b^2 - 2ac) \arctan\left(-\frac{b + \frac{2a}{(xe+d)^2}}{\sqrt{-b^2 + 4ac}}\right) e^{(-1)}}{2\sqrt{-b^2 + 4ac} a^2} - \frac{e^{(-1)}}{2(xe + d)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& 4*a*c - b^2)))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + (((((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (6*b*c^4*e^17)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)^3*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(64*a^9*e^3*(4*a*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14) + (16*a^6*x*((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((2*b^3*e - 8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4*d*e^16)/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (2*c^5*d*e^15)/a^3 - (((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e))/(16*a^7*e^2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + (((((((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (((2*b^3*e - 8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4*d*e^16)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^3)/(32*a^9*e^3*(4*a*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^2 - 9*a*b^2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/a^3 + (((4*a*b^2*c^3*e^15 - a^2*c^4*e^15 + 6*a*b*c^4*d^2*e^15)/a^3 + (((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*
\end{aligned}$$

$$\begin{aligned}
& e^2 - 4a^2b^2e^2)) * (2b^3e - 8a*b*c*e)) / (2 * (16a^3c^2e^2 - 4a^2b^2e^2) \\
& e^2) - ((2a*c - b^2) * (((4a^2b^3c^2e^16 - 4a^3b^3c^3e^16 + 20a^3c^4d^2e^16 + 2a^2b^2c^3d^2e^16) / a^3 - ((2b^3e - 8a*b*c*e) * (4a^4b^2c^2e^17 + 12a^3b^3c^2d^2e^17 - 40a^4b^3c^3d^2e^17)) / (2a^3 * (16a^3c^2e^2 - 4a^2b^2e^2))) * (2a*c - b^2)) / (4a^2e * (4a*c - b^2)^{(1/2)}) - \\
& ((2a*c - b^2) * (2b^3e - 8a*b*c*e) * (4a^4b^2c^2e^17 + 12a^3b^3c^2d^2e^17 - 40a^4b^3c^3d^2e^17)) / (8a^5e * (16a^3c^2e^2 - 4a^2b^2e^2) * (4a*c - b^2)^{(1/2)})) / (4a^2e * (4a*c - b^2)^{(1/2)}) + ((2a*c - b^2)^2 * (2b^3e - 8a*b*c*e) * (4a^4b^2c^2e^17 + 12a^3b^3c^2d^2e^17 - 40a^4b^3c^3d^2e^17)) / (32a^7e^2 * (16a^3c^2e^2 - 4a^2b^2e^2) * (4a*c - b^2))) / (c^2 * (a^2c^2 - 6b^4 + 24a*b^2c) * (4a^2c^4e^14 + b^4c^2e^14 - 4a*b^2c^3e^14)) + (2a^3 * (4a*c - b^2) * (((2b^3e - 8a*b*c*e) * (((4a^2b^3c^2e^16 - 4a^3b^3c^3e^16 + 20a^3c^4d^2e^16 + 2a^2b^2c^3d^2e^16) / a^3 - ((2b^3e - 8a*b*c*e) * (4a^4b^2c^2e^17 + 12a^3b^3c^2d^2e^17 - 40a^4b^3c^3d^2e^17)) / (2a^3 * (16a^3c^2e^2 - 4a^2b^2e^2))) * (2a*c - b^2)) / (4a^2e * (4a*c - b^2)^{(1/2)}) - ((2a*c - b^2) * (2b^3e - 8a*b*c*e) * (4a^4b^2c^2e^17 + 12a^3b^3c^2d^2e^17 - 40a^4b^3c^3d^2e^17)) / (8a^5e * (16a^3c^2e^2 - 4a^2b^2e^2) * (4a*c - b^2)^{(1/2)})) / (2 * (16a^3c^2e^2 - 4a^2b^2e^2) + (((4a*b^2c^3e^15 - a^2c^4e^15 + 6a*b*c^4d^2e^15) / a^3 + (((4a^2b^3c^2e^16 - 4a^3b^3c^3e^16 + 20a^3c^4d^2e^16 + 2a^2b^2c^3d^2e^16) / a^3 - ((2b^3e - 8a*b*c*e) * (4a^4b^2c^2e^17 + 12a^3b^3c^2d^2e^17 - 40a^4b^3c^3d^2e^17)) / (2a^3 * (16a^3c^2e^2 - 4a^2b^2e^2))) * (2b^3e - 8a*b*c*e)) / (2 * (16a^3c^2e^2 - 4a^2b^2e^2))) * (2a*c - b^2)) / (4a^2e * (4a*c - b^2)^{(1/2)}) + ((2a*c - b^2)^3 * (4a^4b^2c^2e^17 + 12a^3b^3c^2d^2e^17 - 40a^4b^3c^3d^2e^17)) / (64a^9e^3 * (4a*c - b^2)^{(3/2)})) * (3b^5 + 13a^2b^3c^2 - 15a*b^3c)) / (c^2 * (a^2c^2 - 6b^4 + 24a*b^2c) * (4a^2c^4e^14 + b^4c^2e^14 - 4a*b^2c^3e^14)) * (2a*c - b^2)) / (2a^2e * (4a*c - b^2)^{(1/2)}) - (\log(((c^5e^16*x^2) / a^3 - ((b + a^2e * (-2a*c - b^2)^2 / (a^4e^2 * (4a*c - b^2)))^2)^{(1/2)}) * ((c^3e^15 * (4b^2 - a*c + 6b*c*d^2)) / a^2 - ((b + a^2e * (-2a*c - b^2)^2 / (a^4e^2 * (4a*c - b^2)))^2)^{(1/2)}) * ((2c^2e^16 * (2b^3 + 10a*c^2d^2 + b^2c*d^2 - 2a*b*c)) / a + (2c^3e^18*x^2 * (10a*c + b^2)) / a + (b*c^2e^16 * (b + a^2e * (-2a*c - b^2)^2 / (a^4e^2 * (4a*c - b^2)))^2)^{(1/2)}) * (a*b + 3b^2d^2 + 3b^2e^2*x^2 - 10a*c*d^2 + 6b^2d*e*x - 10a*c*e^2*x^2 - 20a*c*d*e*x)) / a^2 + (4c^3d*e^17*x * (10a*c + b^2)) / a)) / (4a^2e) + (6b*c^4e^17*x^2) / a^2 + (12b*c^4d*e^16*x) / a^2)) / (4a^2e) + (c^4e^14 * (b + c*d^2)) / a^3 + (2c^5d*e^15*x) / a^3 * ((c^5e^16*x^2) / a^3 - ((b - a^2e * (-2a*c - b^2)^2 / (a^4e^2 * (4a*c - b^2)))^2)^{(1/2)}) * ((c^3e^15 * (4b^2 - a*c + 6b*c*d^2)) / a^2 - ((b - a^2e * (-2a*c - b^2)^2 / (a^4e^2 * (4a*c - b^2)))^2)^{(1/2)}) * ((2c^2e^16 * (2b^3 + 10a*c^2d^2 + b^2c*d^2 - 2a*b*c)) / a + (2c^3e^18*x^2 * (10a*c + b^2)) / a + (b*c^2e^16 * (b - a^2e * (-2a*c - b^2)^2 / (a^4e^2 * (4a*c - b^2)))^2)^{(1/2)}) * (a*b + 3b^2d^2 + 3b^2e^2*x^2 - 10a*c*d^2 + 6b^2d*e*x - 10a*c*e^2*x^2 - 20a*c*d*e*x)) / a^2 + (4c^3d*e^17*x * (10a*c + b^2)) / a)) / (4a^2e) + (6b*c^4e^17*x^2) / a^2 + (12b*c^4d*e^16*x) / a^2)) / (4a^2e) + (c^4e^14 * (b + c*d^2)) / a^3 + (2c^5d*e^15*x) / a^3)) * (2b^3e - 8a*b*c*e)) / (2 * (16a^3c^2e^2 - 4a^2b^2e^2)
\end{aligned}$$

$\wedge 2)) - (b \cdot \log(d + e \cdot x)) / (a^2 \cdot e) - 1 / (2 \cdot a \cdot e \cdot (d^2 + e^2 \cdot x^2 + 2 \cdot d \cdot e \cdot x))$

sympy [B] time = 146.46, size = 464, normalized size = 3.83

$$\left(\frac{b}{4a^2c} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2c(4ac - b^2)} \right) \log \left(\frac{2dx}{c} + x^2 + \frac{-8a^3cx \left(\frac{b}{4a^2c} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2c(4ac - b^2)} \right) + 2a^2b^2c \left(\frac{b}{4a^2c} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2c(4ac - b^2)} \right) + 3ahc + 2ac^2b^2 - b^3 - b^2cx^2}{2a^2c^2 - b^2cx^2} \right) + \left(\frac{b}{4a^2c} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2c(4ac - b^2)} \right) \log \left(\frac{2dx}{c} + x^2 + \frac{-8a^3cx \left(\frac{b}{4a^2c} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2c(4ac - b^2)} \right) + 2a^2b^2c \left(\frac{b}{4a^2c} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2c(4ac - b^2)} \right) + 3ahc + 2ac^2b^2 - b^3 - b^2cx^2}{2a^2c^2 - b^2cx^2} \right) - \frac{1}{2a^2c + 4a^2b^2c + 2a^2b^2} - \frac{b \log \left(\frac{d}{e} + x \right)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)

[Out] $(b/(4*a**2*e) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) * \log(2*d*x/e + x**2 + (-8*a**3*c*e*(b/(4*a**2*e) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)))) + 2*a**2*b**2*e*(b/(4*a**2*e) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)))) + 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) + (b/(4*a**2*e) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2))) * \log(2*d*x/e + x**2 + (-8*a**3*c*e*(b/(4*a**2*e) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)))) + 2*a**2*b**2*e*(b/(4*a**2*e) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*(4*a*c - b**2)))) + 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) - 1/(2*a*d**2*e + 4*a*d*e**2*x + 2*a*e**3*x**2) - b*log(d/e + x)/(a**2*e)$

$$3.507 \quad \int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal. Leaf size=224

$$\frac{\sqrt{c} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \sqrt{c} \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} a^2 e \sqrt{b-\sqrt{b^2-4ac}}} + \frac{b}{a^2 e (d+ex)} - \frac{1}{3 a e (d+ex)^3}$$

Rubi [A] time = 0.50, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1142, 1123, 1281, 1166, 205}

$$\frac{\sqrt{c} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right) + \sqrt{c} \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} a^2 e \sqrt{b-\sqrt{b^2-4ac}}} + \frac{b}{a^2 e (d+ex)} - \frac{1}{3 a e (d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] -1/(3*a*e*(d + e*x)^3) + b/(a^2*e*(d + e*x)) + (Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1123

Int[((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,

`x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]`

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^(2*(m + 1))), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4 (a+b(d+ex)^2+c(d+ex)^4)} dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)} dx, x, d+ex\right)}{e}$$

$$= -\frac{1}{3ae(d+ex)^3} + \frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2(a+bx^2+cx^4)} dx, x, d+ex\right)}{3ae}$$

$$= -\frac{1}{3ae(d+ex)^3} + \frac{b}{a^2e(d+ex)} - \frac{\text{Subst}\left(\int \frac{-3(b^2-ac)-3bcx^2}{a+bx^2+cx^4} dx, x, d+ex\right)}{3a^2e}$$

$$= -\frac{1}{3ae(d+ex)^3} + \frac{b}{a^2e(d+ex)} + \frac{\left(c\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac}}{x^2} dx, x, d+ex\right)}{2a^2e}$$

$$= -\frac{1}{3ae(d+ex)^3} + \frac{b}{a^2e(d+ex)} + \frac{\sqrt{c}\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}e}$$

Mathematica [A] time = 0.21, size = 235, normalized size = 1.05

$$\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{2a}{(d+ex)^3} + \frac{6b}{d+ex}$$

$$6a^2e$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(6*a^2*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^4 (a + b(d + ex)^2 + c(d + ex)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] IntegrateAlgebraic[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

fricas [B] time = 1.22, size = 2044, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*x^3 + 3*a^2*d*e^3*x^2 + 3*a^2*d^2*e^2*x + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)))/((a^5*b^2 - 4*a^6*c)*e^2))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*e^4)) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*e^2*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*

$$\begin{aligned}
& a^{11c}e^4) / ((a^5b^2 - 4a^6c)e^2)) - 3\sqrt{1/2} * (a^2e^4x^3 + 3a^2d^3e^3x^2 + 3a^2d^2e^2x + a^2d^3e) * \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^2 - 4a^6c)e^2 * \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / ((a^{10}b^2 - 4a^{11c})e^4)}) / ((a^5b^2 - 4a^6c)e^2))} * \log(2(b^4c^3 - 3ab^2c^4 + a^2c^5)e^x + 2(b^4c^3 - 3ab^2c^4 + a^2c^5)d - \sqrt{1/2} * ((a^5b^5 - 7a^6b^3c + 12a^7b^2c^2)e^3 * \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / ((a^{10}b^2 - 4a^{11c})e^4)} - (b^8 - 8ab^6c + 20a^2b^4c^2 - 17a^3b^2c^3 + 4a^4c^4)e) * \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 + (a^5b^2 - 4a^6c)e^2 * \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / ((a^{10}b^2 - 4a^{11c})e^4)}) / ((a^5b^2 - 4a^6c)e^2))} - 3\sqrt{1/2} * (a^2e^4x^3 + 3a^2d^3e^3x^2 + 3a^2d^2e^2x + a^2d^3e) * \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 - (a^5b^2 - 4a^6c)e^2 * \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / ((a^{10}b^2 - 4a^{11c})e^4)}) / ((a^5b^2 - 4a^6c)e^2))} * \log(2(b^4c^3 - 3ab^2c^4 + a^2c^5)e^x + 2(b^4c^3 - 3ab^2c^4 + a^2c^5)d + \sqrt{1/2} * ((a^5b^5 - 7a^6b^3c + 12a^7b^2c^2)e^3 * \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / ((a^{10}b^2 - 4a^{11c})e^4)} + (b^8 - 8ab^6c + 20a^2b^4c^2 - 17a^3b^2c^3 + 4a^4c^4)e) * \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 - (a^5b^2 - 4a^6c)e^2 * \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / ((a^{10}b^2 - 4a^{11c})e^4)}) / ((a^5b^2 - 4a^6c)e^2))} + 3\sqrt{1/2} * (a^2e^4x^3 + 3a^2d^3e^3x^2 + 3a^2d^2e^2x + a^2d^3e) * \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 - (a^5b^2 - 4a^6c)e^2 * \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / ((a^{10}b^2 - 4a^{11c})e^4)}) / ((a^5b^2 - 4a^6c)e^2))} * \log(2(b^4c^3 - 3ab^2c^4 + a^2c^5)e^x + 2(b^4c^3 - 3ab^2c^4 + a^2c^5)d - \sqrt{1/2} * ((a^5b^5 - 7a^6b^3c + 12a^7b^2c^2)e^3 * \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / ((a^{10}b^2 - 4a^{11c})e^4)} + (b^8 - 8ab^6c + 20a^2b^4c^2 - 17a^3b^2c^3 + 4a^4c^4)e) * \sqrt{-(b^5 - 5ab^3c + 5a^2b^2c^2 - (a^5b^2 - 4a^6c)e^2 * \sqrt{(b^8 - 6ab^6c + 11a^2b^4c^2 - 6a^3b^2c^3 + a^4c^4) / ((a^{10}b^2 - 4a^{11c})e^4)}) / ((a^5b^2 - 4a^6c)e^2))} - 2a) / (a^2e^4x^3 + 3a^2d^3e^3x^2 + 3a^2d^2e^2x + a^2d^3e)
\end{aligned}$$

giac [B] time = 0.49, size = 1243, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] $-1/2 * (((d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^{-2} + \sqrt{b^2 - 4 * a * c}) * e^{-2}}) * e^{-4} / c)^2 * b * c * e^{-2} - 2 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^{-2} + \sqrt{b^2 - 4 * a * c}) * e^{-2}}) * e^{-4} / c) * b * c * d * e + b * c * d^2 + b^2 - a * c) * \log(d * e^{-1} + x + \sqrt{1/2} * \sqrt{-(b * e^{-2} + \sqrt{b^2 - 4 * a * c}) * e^{-2}}) * e^{-4} / c) / (2 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^{-2} + \sqrt{b^2 - 4 * a * c}) * e^{-2}}) * e^{-4} / c)^3 * c * e^4 - 6 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^{-2} + \sqrt{b^2 - 4 * a * c}) * e^{-2}}) * e^{-4} / c$

[Out] Timed out

mupad [B] time = 2.83, size = 5214, normalized size = 23.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x)$

[Out]
$$\begin{aligned} & ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3 + e^3*x^3 + \\ & 3*d*e^2*x^2 + 3*d^2*e*x) - \text{atan}\left(\frac{(b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2})}{(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2}}\right) * \\ & (x*(4*a^8*c^5*e^{12} + 2*a^6*b^4*c^3*e^{12} - 8*a^7*b^2*c^4*e^{12}) - ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} * \\ & ((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13}) * ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} - \\ & 16*a^{10}*c^4*e^{12} - 4*a^8*b^4*c^2*e^{12} + 20*a^9*b^2*c^3*e^{12}) + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11}) * i + ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} * \\ & (x*(4*a^8*c^5*e^{12} + 2*a^6*b^4*c^3*e^{12} - 8*a^7*b^2*c^4*e^{12}) - ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} * \\ & ((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13}) * ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} + \\ & 16*a^{10}*c^4*e^{12} + 4*a^8*b^4*c^2*e^{12} - 20*a^9*b^2*c^3*e^{12}) + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11}) * i) / (((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} * \\ & (x*(4*a^8*c^5*e^{12} + 2*a^6*b^4*c^3*e^{12} - 8*a^7*b^2*c^4*e^{12}) - ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} * \\ & ((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13}) * ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} * \\ & ((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13}) * ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} + \\ & 16*a^{10}*c^4*e^{12} + 4*a^8*b^4*c^2*e^{12} - 20*a^9*b^2*c^3*e^{12}) + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11}) * i) / (((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} * \\ & (x*(4*a^8*c^5*e^{12} + 2*a^6*b^4*c^3*e^{12} - 8*a^7*b^2*c^4*e^{12}) - ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} * \\ & ((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13}) * ((b^4*(-(4*a*c - b^2)^3)^{1/2} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{1/2} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{1/2}) / (8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{1/2} + \\ & 16*a^{10}*c^4*e^{12} + 4*a^8*b^4*c^2*e^{12} - 20*a^9*b^2*c^3*e^{12}) + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11}) * i) \end{aligned}$$

$$\begin{aligned}
& c - b^2)^3)^{(1/2)}/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)} + 16*a^{10}*c^4*e^{12} + 4*a^8*b^4*c^2*e^{12} - 20*a^9*b^2*c^3*e^{12} + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11} - ((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*(x*(4*a^8*c^5*e^{12} + 2*a^6*b^4*c^3*e^{12} - 8*a^7*b^2*c^4*e^{12}) - ((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13})*((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)} - 16*a^{10}*c^4*e^{12} - 4*a^8*b^4*c^2*e^{12} + 20*a^9*b^2*c^3*e^{12} + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11} + 2*a^6*b*c^5*e^{10}))*((b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*2i - \operatorname{atan}(((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*(x*(4*a^8*c^5*e^{12} + 2*a^6*b^4*c^3*e^{12} - 8*a^7*b^2*c^4*e^{12}) - ((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13})*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)} - 16*a^{10}*c^4*e^{12} - 4*a^8*b^4*c^2*e^{12} + 20*a^9*b^2*c^3*e^{12} + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11})*1i + ((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*(x*(4*a^8*c^5*e^{12} + 2*a^6*b^4*c^3*e^{12} - 8*a^7*b^2*c^4*e^{12}) - ((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13})*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)} + 16*a^{10}*c^4*e^{12} + 4*a^8*b^4*c^2*e^{12} - 20*a^9*b^2*c^3*e^{12} + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11})*1i)/(((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)} - 16*a^{10}*c^4*e^{12} - 4*a^8*b^4*c^2*e^{12} + 20*a^9*b^2*c^3*e^{12} + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11})*1i)
\end{aligned}$$

$$\begin{aligned}
& 5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*(x*(4*a^8*c^5*e^{12} + 2*a^6*b^4*c^3*e^{12} - 8*a^7*b^2*c^4*e^{12}) - ((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13})*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)} + 16*a^{10}*c^4*e^{12} + 4*a^8*b^4*c^2*e^{12} - 20*a^9*b^2*c^3*e^{12}) + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11}) - ((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*(x*(4*a^8*c^5*e^{12} + 2*a^6*b^4*c^3*e^{12} - 8*a^7*b^2*c^4*e^{12}) - ((b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)}*((x*(32*a^{11}*b*c^3*e^{14} - 8*a^{10}*b^3*c^2*e^{14}) + 32*a^{11}*b*c^3*d*e^{13} - 8*a^{10}*b^3*c^2*d*e^{13})*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)} - 16*a^{10}*c^4*e^{12} - 4*a^8*b^4*c^2*e^{12} + 20*a^9*b^2*c^3*e^{12}) + 4*a^8*c^5*d*e^{11} + 2*a^6*b^4*c^3*d*e^{11} - 8*a^7*b^2*c^4*d*e^{11}) + 2*a^6*b*c^5*e^{10}))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2))^{(1/2)})*2i
\end{aligned}$$

sympy [A] time = 12.77, size = 347, normalized size = 1.55

$$\frac{-a + 3bf^2 + 6bdex + 3b^2d^2}{3a^2b^2e + 9a^2b^2c^2x + 9a^2d^2c^2 + 3a^2d^2e^3} + \text{RootSum}\left(f^4(256a^7c^2e^4 - 128a^6b^2c^4 + 16a^5b^4e^4) + f^2(-80a^3b^2c^2 + 100a^2b^3c^2 - 36ab^5c^2 + 4b^7e^2) + e^5, \left(1 + 11\log\left(x + \frac{-96a^3a^7b^2c^3 + 56a^2a^6b^3c^3 - 8f^2a^5b^4c^3 - 41a^4e^4e + 321a^3b^2c^2e - 401a^2b^4c^2e + 161ab^6ce - 21b^8e + a^2c^2d - 3ab^2c^2d + b^4c^2d}{a^2c^2e - 3ab^2c^2e + b^4c^2e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)

[Out] $(-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e + 9*a**2*d**2*e**2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3) + \text{RootSum}(_t**4*(256*a**7*c**2*e**4 - 128*a**6*b**2*c*e**4 + 16*a**5*b**4*e**4) + _t**2*(-80*a**3*b*c**3*e**2 + 100*a**2*b**3*c**2*e**2 - 36*a*b**5*c*e**2 + 4*b**7*e**2) + c**5, \text{Lambda}(_t, _t*\log(x + (-96*_t**3*a**7*b*c**2*e**3 + 56*_t**3*a**6*b**3*c*e**3 - 8*_t**3*a**5*b**5*e**3 - 4*_t*a**4*c**4*e + 32*_t*a**3*b**2*c**3*e - 40*_t*a**2*b**4*c**2*e + 16*_t*a*b**6*c*e - 2*_t*b**8*e + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2*c**4*e + b**4*c**3*e)))$

$$3.508 \quad \int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=270

$$\frac{(d+ex)(2a+b(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\left(b - \frac{4ac+b^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b\sqrt{b^2-4ac} + 4ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Rubi [A] time = 0.58, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1142, 1120, 1166, 205}

$$\frac{(d+ex)(2a+b(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\left(b - \frac{4ac+b^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b\sqrt{b^2-4ac} + 4ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1120

Int[((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(d^3*(d*x)^(m-3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^(m-4)*(2*a*(m-3) + b*(m+4*p+3)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx = \frac{\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4} dx, x, d+ex\right)}{2(b^2-4ac)e}$$

$$= \frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{(b^2+4ac-b\sqrt{b^2-4ac})S}{4(b^2-4ac)}$$

$$= \frac{(d+ex)(2a+b(d+ex)^2)}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{(b^2+4ac-b\sqrt{b^2-4ac})t}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.47, size = 263, normalized size = 0.97

$$\frac{\frac{2(-2a(d+ex)-b(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}(b\sqrt{b^2-4ac}-4ac-b^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}(b\sqrt{b^2-4ac}+4ac+b^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [B] time = 1.11, size = 2454, normalized size = 9.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e^3*x^3 + 6*b*d*e^2*x^2 + 2*b*d^3 + 2*(3*b*d^2 + 2*a)*e*x + sqrt(1
/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
- 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4
a*b*c)*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*
e^2*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) +
b^3 + 12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))
*log((3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d + sqrt(1/2)*(2*(b^7*c - 12*a*b
^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*e)*s
qrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sqrt(1/((b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 + 12*a*b*c)/
((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((
b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c +
6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a
*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)
*d^2)*e)*sqrt(-((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2*sq
rt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4)) + b^3 +
12*a*b*c)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log((
3*b^2 + 4*a*c)*e*x + (3*b^2 + 4*a*c)*d - sqrt(1/2)*(2*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3*sqrt(1/((b^6*c^2 - 12*a*b^4*c^3 + 48*
```


$$\begin{aligned}
& a^2 b^2 c^4 - 64 a^3 c^5) e^4)) + (b^4 - 8 a b^2 c + 16 a^2 c^2) e) \sqrt{-(b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2 \sqrt{1 / ((b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) e^4)} + b^3 + 12 a b c) / ((b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2)) - \sqrt{1 / 2} * ((b^2 c - 4 a c^2) e^5 x^4 + 4 (b^2 c - 4 a c^2) d e^4 x^3 + (b^3 - 4 a b c + 6 (b^2 c - 4 a c^2) d^2) e^3 x^2 + 2 (2 (b^2 c - 4 a c^2) d^3 + (b^3 - 4 a b c) d) e^2 x + ((b^2 c - 4 a c^2) d^4 + a b^2 - 4 a^2 c + (b^3 - 4 a b c) d^2) e) \sqrt{((b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2 \sqrt{1 / ((b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) e^4)} - b^3 - 12 a b c) / ((b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2)} * \log((3 b^2 + 4 a c) e x + (3 b^2 + 4 a c) d + \sqrt{1 / 2} * (2 (b^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - 64 a^3 b c^4) e^3 \sqrt{1 / ((b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) e^4)} - (b^4 - 8 a b^2 c + 16 a^2 c^2) e) \sqrt{((b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2 \sqrt{1 / ((b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) e^4)} - b^3 - 12 a b c) / ((b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2))} + \sqrt{1 / 2} * ((b^2 c - 4 a c^2) e^5 x^4 + 4 (b^2 c - 4 a c^2) d e^4 x^3 + (b^3 - 4 a b c + 6 (b^2 c - 4 a c^2) d^2) e^3 x^2 + 2 (2 (b^2 c - 4 a c^2) d^3 + (b^3 - 4 a b c) d) e^2 x + ((b^2 c - 4 a c^2) d^4 + a b^2 - 4 a^2 c + (b^3 - 4 a b c) d^2) e) \sqrt{((b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2 \sqrt{1 / ((b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) e^4)} - b^3 - 12 a b c) / ((b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2)} * \log((3 b^2 + 4 a c) e x + (3 b^2 + 4 a c) d - \sqrt{1 / 2} * (2 (b^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - 64 a^3 b c^4) e^3 \sqrt{1 / ((b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) e^4)} - (b^4 - 8 a b^2 c + 16 a^2 c^2) e) \sqrt{((b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2 \sqrt{1 / ((b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) e^4)} - b^3 - 12 a b c) / ((b^6 c - 12 a b^4 c^2 + 48 a^2 b^2 c^3 - 64 a^3 c^4) e^2))} + 4 a d) / ((b^2 c - 4 a c^2) e^5 x^4 + 4 (b^2 c - 4 a c^2) d e^4 x^3 + (b^3 - 4 a b c + 6 (b^2 c - 4 a c^2) d^2) e^3 x^2 + 2 (2 (b^2 c - 4 a c^2) d^3 + (b^3 - 4 a b c) d) e^2 x + ((b^2 c - 4 a c^2) d^4 + a b^2 - 4 a^2 c + (b^3 - 4 a b c) d^2) e)
\end{aligned}$$

giac [B] time = 0.52, size = 1304, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] $-1/4 * (((d e^{-1}) + \sqrt{1/2} * \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2}) e^{-4} / c)^2 * b e^2 - 2 * (d e^{-1}) + \sqrt{1/2} * \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2}) * e^{-4} / c) * b d e + b d^2 - 2 a) * \log(d e^{-1} + x + \sqrt{1/2} * \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2}) e^{-4} / c) / (2 * (d e^{-1}) + \sqrt{1/2} * \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2}) e^{-4} / c)^3 * c e^4 - 6 * (d e^{-1}) + \sqrt{1/2} * \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2}) e^{-4} / c)^2 * c d e^3 - 2 * c d^3 e - b d e + ($

$$\begin{aligned}
& 6*c*d^2*e^2 + b*e^2)*(d*e^{-1} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))) + ((d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))^2*b*e^2 - 2*(d*e^{-1} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))*b*d*e + b*d^2 - 2*a)*\log(d*e^{-1} + x - \sqrt{1/2})* \\
& \sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})})*e^{-4}/c))/(2*(d*e^{-1} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))^3*c*e^4 - 6*(d*e^{-1} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{-1} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))) + ((d*e^{-1} + \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))^2*b*e^2 - 2*(d*e^{-1} + \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))*b*d*e + b*d^2 - 2*a)*\log(d*e^{-1} + \\
& x + \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})})*e^{-4}/c))/(2*(d*e^{-1} \\
& + \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})})*e^{-4}/c))^3*c*e^4 - 6*(\\
& d*e^{-1} + \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})})*e^{-4}/c))^2*c*d \\
& *e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{-1} + \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))) + ((d*e^{-1} - \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))^2*b*e^2 - 2*(d*e^{-1} - \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))*b*d*e + b*d^2 - 2*a)*\log(d*e^{-1} + x - \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))/(2*(d*e^{-1} - \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))^3*c*e^4 - 6*(d*e^{-1} - \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{-1} - \sqrt{1/2})*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{-4}/c))))/(b^2 - 4*a*c) \\
& + 1/2*(b*x^3*e^3 + 3*b*d*x^2*e^2 + 3*b*d^2*x*e + b*d^3 + 2*a*x*e + 2*a*d)/(\\
& (c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2* \\
& e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
\end{aligned}$$

maple [C] time = 0.02, size = 323, normalized size = 1.20

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2, x$

[Out]
$$\begin{aligned}
& (-1/2*b*e^2/(4*a*c-b^2)*x^3-3/2*d*b*e/(4*a*c-b^2)*x^2-1/2*(3*b*d^2+2*a)/(4* \\
& a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^ \\
& 2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e* \\
& \text{sum}((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c* \\
& d^2*e+2*c*d^3+_R*b*e+b*d)*\ln(-_R+x), _R=\text{RootOf}(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d \\
& ^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& (b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2} + (4 a^2 b^3 c^2 e^{10} + 3 a^3 b^3 c^3 e^{10}) / (4 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) * (- (b^9 + (-4 a^3 c - b^2)^9)^{1/2} - 768 a^4 b^3 c^4 - 96 a^2 b^5 c^2 + 512 a^3 b^3 c^3) / (32 (b^{12} c^2 e^2 + 4096 a^6 c^7 e^2 - 24 a^2 b^{10} c^2 e^2 + 240 a^2 b^8 c^3 e^2 - 1280 a^3 b^6 c^4 e^2 + 3840 a^4 b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2} \\
& * 2i - ((2 a d + b d^3) / (2 e (4 a^3 c - b^2)) + (x (2 a + 3 b d^2)) / (2 (4 a^3 c - b^2)) + (b e^2 x^3) / (2 (4 a^3 c - b^2)) + (3 b d e x^2) / (2 (4 a^3 c - b^2))) / (a + x^2 (b e^2 + 6 c d^2 e^2) + b d^2 + c d^4 + x (2 b d e + 4 c d^3 e) + c e^4 x^4 + 4 c d e^3 x^3) + \operatorname{atan}(\frac{(((-4 a^3 c - b^2)^9)^{1/2} - b^9 + 768 a^4 b^3 c^4 + 96 a^2 b^5 c^2 - 512 a^3 b^3 c^3) / (32 (b^{12} c^2 e^2 + 4096 a^6 c^7 e^2 - 24 a^2 b^{10} c^2 e^2 + 240 a^2 b^8 c^3 e^2 - 1280 a^3 b^6 c^4 e^2 + 3840 a^4 b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2}}{(2048 a^4 c^5 e^{12} - 32 a^3 b^6 c^2 e^{12} + 384 a^2 b^4 c^3 e^{12} - 1536 a^3 b^2 c^4 e^{12}) / (8 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) + ((64 b^9 c^2 d e^{13} - 1024 a^2 b^7 c^3 d e^{13} + 16384 a^4 b^3 c^6 d e^{13} + 6144 a^2 b^5 c^4 d e^{13} - 16384 a^3 b^3 c^5 d e^{13}) / (8 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) + (x (16 b^7 c^2 e^{14} - 192 a^2 b^5 c^3 e^{14} - 1024 a^3 b^3 c^5 e^{14} + 768 a^2 b^3 c^4 e^{14})) / (2 (b^4 + 16 a^2 c^2 - 8 a b^2 c))} * ((-4 a^3 c - b^2)^9)^{1/2} - b^9 + 768 a^4 b^3 c^4 + 96 a^2 b^5 c^2 - 512 a^3 b^3 c^3) / (32 (b^{12} c^2 e^2 + 4096 a^6 c^7 e^2 - 24 a^2 b^{10} c^2 e^2 + 240 a^2 b^8 c^3 e^2 - 1280 a^3 b^6 c^4 e^2 + 3840 a^4 b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2} * (((-4 a^3 c - b^2)^9)^{1/2} - b^9 + 768 a^4 b^3 c^4 + 96 a^2 b^5 c^2 - 512 a^3 b^3 c^3) / (32 (b^{12} c^2 e^2 + 4096 a^6 c^7 e^2 - 24 a^2 b^{10} c^2 e^2 + 240 a^2 b^8 c^3 e^2 - 1280 a^3 b^6 c^4 e^2 + 3840 a^4 b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2} * (((-4 a^3 c - b^2)^9)^{1/2} - b^9 + 768 a^4 b^3 c^4 + 96 a^2 b^5 c^2 - 512 a^3 b^3 c^3) / (32 (b^{12} c^2 e^2 + 4096 a^6 c^7 e^2 - 24 a^2 b^{10} c^2 e^2 + 240 a^2 b^8 c^3 e^2 - 1280 a^3 b^6 c^4 e^2 + 3840 a^4 b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2} - (128 a^3 c^4 d e^{11} - 4 b^6 c d e^{11} + 8 a b^4 c^2 d e^{11}) / (8 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) + (x (b^4 c^2 e^{12} + 8 a^2 c^3 e^{12} + 2 a b^2 c^2 e^{12})) / (2 (b^4 + 16 a^2 c^2 - 8 a b^2 c)) * i - (((-4 a^3 c - b^2)^9)^{1/2} - b^9 + 768 a^4 b^3 c^4 + 96 a^2 b^5 c^2 - 512 a^3 b^3 c^3) / (32 (b^{12} c^2 e^2 + 4096 a^6 c^7 e^2 - 24 a^2 b^{10} c^2 e^2 + 240 a^2 b^8 c^3 e^2 - 1280 a^3 b^6 c^4 e^2 + 3840 a^4 b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2} * (((2048 a^4 c^5 e^{12} - 32 a^3 b^6 c^2 e^{12} + 384 a^2 b^4 c^3 e^{12} - 1536 a^3 b^2 c^4 e^{12}) / (8 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) - ((64 b^9 c^2 d e^{13} - 1024 a^2 b^7 c^3 d e^{13} + 16384 a^4 b^3 c^6 d e^{13} + 6144 a^2 b^5 c^4 d e^{13} - 16384 a^3 b^3 c^5 d e^{13}) / (8 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) + (x (16 b^7 c^2 e^{14} - 192 a^2 b^5 c^3 e^{14} - 1024 a^3 b^3 c^5 e^{14} + 768 a^2 b^3 c^4 e^{14})) / (2 (b^4 + 16 a^2 c^2 - 8 a b^2 c))} * (((-4 a^3 c - b^2)^9)^{1/2} - b^9 + 768 a^4 b^3 c^4 + 96 a^2 b^5 c^2 - 512 a^3 b^3 c^3) / (32 (b^{12} c^2 e^2 + 4096 a^6 c^7 e^2 - 24 a^2 b^{10} c^2 e^2 + 240 a^2 b^8 c^3 e^2 - 1280 a^3 b^6 c^4 e^2 + 3840 a^4 b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2} * (((-4 a^3 c - b^2)^9)^{1/2} - b^9 + 768 a^4 b^3 c^4 + 96 a^2 b^5 c^2 - 512 a^3 b^3 c^3) / (32 (b^{12} c^2 e^2 + 4096 a^6 c^7 e^2 - 24 a^2 b^{10} c^2 e^2 + 240 a^2 b^8 c^3 e^2 - 1280 a^3 b^6 c^4 e^2 + 3840 a^4 b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2} * (((-4 a^3 c - b^2)^9)^{1/2} - b^9 + 768 a^4 b^3 c^4 + 96 a^2 b^5 c^2 - 512 a^3 b^3 c^3) / (32 (b^{12} c^2 e^2 + 4096 a^6 c^7 e^2 - 24 a^2 b^{10} c^2 e^2 + 240 a^2 b^8 c^3 e^2 - 1280 a^3 b^6 c^4 e^2 + 3840 a^4 b^4 c^5 e^2 - 6144 a^5 b^2 c^6 e^2))^{1/2} + (128 a^3 c^4 d e^{11} - 4 b^6 c d e^{11} + 8 a b^4 c^2 d e^{11}) / (8 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) - (x (b^4 c^2 e^{12} + 8 a^2 c^3 e^{12} + 2 a b^2 c^2 e^{12})) / (2 (b^4 + 16 a^2 c^2 - 8 a b^2 c))
\end{aligned}$$

$$\begin{aligned}
& a^2 b^2 c^2)) * i) / ((((-4ac - b^2)^9)^{1/2} - b^9 + 768a^4 b^2 c^4 + 96a^2 b^5 c^2 - 512a^3 b^3 c^3) / (32(b^{12} c^2 e^2 + 4096a^6 c^7 e^2 - 24a^2 b^{10} c^2 e^2 + 240a^2 b^8 c^3 e^2 - 1280a^3 b^6 c^4 e^2 + 3840a^4 b^4 c^5 e^2 - 6144a^5 b^2 c^6 e^2)))^{1/2} * (((2048a^4 c^5 e^{12} - 32a^2 b^6 c^2 e^{12} + 384a^2 b^4 c^3 e^{12} - 1536a^3 b^2 c^4 e^{12}) / (8(b^6 - 64a^3 c^3 + 48a^2 b^2 c^2 - 12a^2 b^4 c)) + ((64b^9 c^2 d e^{13} - 1024a^2 b^7 c^3 d e^{13} + 16384a^4 b^2 c^6 d e^{13} + 6144a^2 b^5 c^4 d e^{13} - 16384a^3 b^3 c^5 d e^{13}) / (8(b^6 - 64a^3 c^3 + 48a^2 b^2 c^2 - 12a^2 b^4 c)) + (x(16b^7 c^2 e^{14} - 192a^2 b^5 c^3 e^{14} - 1024a^3 b^2 c^4 e^{14} + 768a^2 b^3 c^4 e^{14})) / (2(b^4 + 16a^2 c^2 - 8a^2 b^2 c))) * ((((-4ac - b^2)^9)^{1/2} - b^9 + 768a^4 b^2 c^4 + 96a^2 b^5 c^2 - 512a^3 b^3 c^3) / (32(b^{12} c^2 e^2 + 4096a^6 c^7 e^2 - 24a^2 b^{10} c^2 e^2 + 240a^2 b^8 c^3 e^2 - 1280a^3 b^6 c^4 e^2 + 3840a^4 b^4 c^5 e^2 - 6144a^5 b^2 c^6 e^2)))^{1/2} * ((((-4ac - b^2)^9)^{1/2} - b^9 + 768a^4 b^2 c^4 + 96a^2 b^5 c^2 - 512a^3 b^3 c^3) / (32(b^{12} c^2 e^2 + 4096a^6 c^7 e^2 - 24a^2 b^{10} c^2 e^2 + 240a^2 b^8 c^3 e^2 - 1280a^3 b^6 c^4 e^2 + 3840a^4 b^4 c^5 e^2 - 6144a^5 b^2 c^6 e^2)))^{1/2} - (128a^3 c^4 d e^{11} - 4b^6 c d e^{11} + 8a^2 b^4 c^2 d e^{11}) / (8(b^6 - 64a^3 c^3 + 48a^2 b^2 c^2 - 12a^2 b^4 c)) + (x(b^4 c^2 e^{12} + 8a^2 c^3 e^{12} + 2a^2 b^2 c^2 e^{12})) / (2(b^4 + 16a^2 c^2 - 8a^2 b^2 c))) + ((((-4ac - b^2)^9)^{1/2} - b^9 + 768a^4 b^2 c^4 + 96a^2 b^5 c^2 - 512a^3 b^3 c^3) / (32(b^{12} c^2 e^2 + 4096a^6 c^7 e^2 - 24a^2 b^{10} c^2 e^2 + 240a^2 b^8 c^3 e^2 - 1280a^3 b^6 c^4 e^2 + 3840a^4 b^4 c^5 e^2 - 6144a^5 b^2 c^6 e^2)))^{1/2} * (((2048a^4 c^5 e^{12} - 32a^2 b^6 c^2 e^{12} + 384a^2 b^4 c^3 e^{12} - 1536a^3 b^2 c^4 e^{12}) / (8(b^6 - 64a^3 c^3 + 48a^2 b^2 c^2 - 12a^2 b^4 c)) - ((64b^9 c^2 d e^{13} - 1024a^2 b^7 c^3 d e^{13} + 16384a^4 b^2 c^6 d e^{13} + 6144a^2 b^5 c^4 d e^{13} - 16384a^3 b^3 c^5 d e^{13}) / (8(b^6 - 64a^3 c^3 + 48a^2 b^2 c^2 - 12a^2 b^4 c)) + (x(16b^7 c^2 e^{14} - 192a^2 b^5 c^3 e^{14} - 1024a^3 b^2 c^4 e^{14} + 768a^2 b^3 c^4 e^{14})) / (2(b^4 + 16a^2 c^2 - 8a^2 b^2 c))) * ((((-4ac - b^2)^9)^{1/2} - b^9 + 768a^4 b^2 c^4 + 96a^2 b^5 c^2 - 512a^3 b^3 c^3) / (32(b^{12} c^2 e^2 + 4096a^6 c^7 e^2 - 24a^2 b^{10} c^2 e^2 + 240a^2 b^8 c^3 e^2 - 1280a^3 b^6 c^4 e^2 + 3840a^4 b^4 c^5 e^2 - 6144a^5 b^2 c^6 e^2)))^{1/2} * ((((-4ac - b^2)^9)^{1/2} - b^9 + 768a^4 b^2 c^4 + 96a^2 b^5 c^2 - 512a^3 b^3 c^3) / (32(b^{12} c^2 e^2 + 4096a^6 c^7 e^2 - 24a^2 b^{10} c^2 e^2 + 240a^2 b^8 c^3 e^2 - 1280a^3 b^6 c^4 e^2 + 3840a^4 b^4 c^5 e^2 - 6144a^5 b^2 c^6 e^2)))^{1/2} + (128a^3 c^4 d e^{11} - 4b^6 c d e^{11} + 8a^2 b^4 c^2 d e^{11}) / (8(b^6 - 64a^3 c^3 + 48a^2 b^2 c^2 - 12a^2 b^4 c)) - (x(b^4 c^2 e^{12} + 8a^2 c^3 e^{12} + 2a^2 b^2 c^2 e^{12})) / (2(b^4 + 16a^2 c^2 - 8a^2 b^2 c))) - (4a^2 b^2 c^2 e^{10} + 3a^2 b^3 c^2 e^{10}) / (4(b^6 - 64a^3 c^3 + 48a^2 b^2 c^2 - 12a^2 b^4 c)))) * ((((-4ac - b^2)^9)^{1/2} - b^9 + 768a^4 b^2 c^4 + 96a^2 b^5 c^2 - 512a^3 b^3 c^3) / (32(b^{12} c^2 e^2 + 4096a^6 c^7 e^2 - 24a^2 b^{10} c^2 e^2 + 240a^2 b^8 c^3 e^2 - 1280a^3 b^6 c^4 e^2 + 3840a^4 b^4 c^5 e^2 - 6144a^5 b^2 c^6 e^2)))^{1/2} * 2i
\end{aligned}$$

sympy [B] time = 8.97, size = 573, normalized size = 2.12

$$\frac{-2d^2x^2 - 3bd^2x + (3d^2 - 3b^2d)x^2 + (3d^2 - 3b^2d)x^2}{(4a^2c^2e^2 + 3b^2d^2e^2)^2} + \text{RootSum}\left(\frac{(-1048576a^6c^7e^4 - 1572864a^5b^2c^6e^4 + 983040a^4b^4c^5e^4 - 327680a^3b^6c^4e^4 + 61440a^2b^8c^3e^4 - 6144ab^{10}c^2e^4 + 256b^{12}c^2e^4) + t^2(-12288a^4b^4c^4e^2 + 8192a^3b^3c^3e^2 - 1536a^2b^5c^2e^2 + 16b^9e^2) + 16a^3c^2 + 24a^2b^2c + 9ab^4, \text{Lambda}(t, t \log(x + (16384t^3a^3b^4c^4e^3 - 12288t^3a^2b^3c^3e^3 + 3072t^3ab^5c^2e^3 - 256t^3b^7c^2e^3 + 64t^2a^2c^2e - 128t^2ab^2c^2e - 4t^2b^4e + 4ac^2d + 3b^2d)/t))\right)}{(4a^2c^2e^2 + 3b^2d^2e^2)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out]
$$\frac{(-2ad - b^2d^3 - 3bd^2e^2x^2 - b^3e^3x^3 + x(-2ae - 3bd^2e^2))}{(8a^2c^2e - 2ab^2e + 8abc^2d^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2c^2d^4e + x^4(8a^2c^2e^5 - 2b^2c^2e^5) + x^3(32a^2c^2d^4e - 8b^2c^2d^4e) + x^2(8ab^3c^2e^3 + 48ac^2d^2e^3 - 2b^3e^3 - 12b^2c^2d^2e^3) + x(16abc^2d^2e^2 + 32ac^2d^3e^2 - 4b^3d^2e^2 - 8b^2c^2d^3e^2)) + \text{RootSum}(_t^4(1048576a^6c^7e^4 - 1572864a^5b^2c^6e^4 + 983040a^4b^4c^5e^4 - 327680a^3b^6c^4e^4 + 61440a^2b^8c^3e^4 - 6144ab^{10}c^2e^4 + 256b^{12}c^2e^4) + _t^2(-12288a^4b^4c^4e^2 + 8192a^3b^3c^3e^2 - 1536a^2b^5c^2e^2 + 16b^9e^2) + 16a^3c^2 + 24a^2b^2c + 9ab^4, \text{Lambda}(_t, _t \log(x + (16384_t^3a^3b^4c^4e^3 - 12288_t^3a^2b^3c^3e^3 + 3072_t^3ab^5c^2e^3 - 256_t^3b^7c^2e^3 + 64_t^2a^2c^2e - 128_t^2ab^2c^2e - 4_t^2b^4e + 4ac^2d + 3b^2d)/t)))/((4a^2c^2e + 3b^2d^2e)^2)}$$

$$3.509 \quad \int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=97

$$\frac{2a + b(d + ex)^2}{2e(b^2 - 4ac)(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1142, 1114, 638, 618, 206}

$$\frac{2a + b(d + ex)^2}{2e(b^2 - 4ac)(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] (2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{e} \\ &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2} dx, x, (d+ex)^2\right)}{2e} \\ &= \frac{2a+b(d+ex)^2}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{b \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, (d+ex)^2\right)}{2(b^2-4ac)e} \\ &= \frac{2a+b(d+ex)^2}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{b \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, (d+ex)^2\right)}{(b^2-4ac)e} \\ &= \frac{2a+b(d+ex)^2}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}e} \end{aligned}$$

Mathematica [A] time = 0.13, size = 100, normalized size = 1.03

$$\frac{\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} - \frac{2b \tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]

[Out] $((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2]/Sqrt[-b^2 + 4*a*c]))/(-b^2 + 4*a*c)^{(3/2)})/(2*e)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]

fricas [B] time = 1.03, size = 1021, normalized size = 10.53

(b^2 - 4*a*c)^2 + 2*(b - 4*a*c)*e + 2*(b^2 - 4*a*c)*e^2 + 2*(b - 4*a*c)*e^3 + 2*(b^2 - 4*a*c)*e^4 + 2*(b - 4*a*c)*e^5 + 2*(b^2 - 4*a*c)*e^6 + 2*(b - 4*a*c)*e^7 + 2*(b^2 - 4*a*c)*e^8 + 2*(b - 4*a*c)*e^9 + 2*(b^2 - 4*a*c)*e^10 + 2*(b - 4*a*c)*e^11 + 2*(b^2 - 4*a*c)*e^12 + 2*(b - 4*a*c)*e^13 + 2*(b^2 - 4*a*c)*e^14 + 2*(b - 4*a*c)*e^15 + 2*(b^2 - 4*a*c)*e^16 + 2*(b - 4*a*c)*e^17 + 2*(b^2 - 4*a*c)*e^18 + 2*(b - 4*a*c)*e^19 + 2*(b^2 - 4*a*c)*e^20 + 2*(b - 4*a*c)*e^21 + 2*(b^2 - 4*a*c)*e^22 + 2*(b - 4*a*c)*e^23 + 2*(b^2 - 4*a*c)*e^24 + 2*(b - 4*a*c)*e^25 + 2*(b^2 - 4*a*c)*e^26 + 2*(b - 4*a*c)*e^27 + 2*(b^2 - 4*a*c)*e^28 + 2*(b - 4*a*c)*e^29 + 2*(b^2 - 4*a*c)*e^30 + 2*(b - 4*a*c)*e^31 + 2*(b^2 - 4*a*c)*e^32 + 2*(b - 4*a*c)*e^33 + 2*(b^2 - 4*a*c)*e^34 + 2*(b - 4*a*c)*e^35 + 2*(b^2 - 4*a*c)*e^36 + 2*(b - 4*a*c)*e^37 + 2*(b^2 - 4*a*c)*e^38 + 2*(b - 4*a*c)*e^39 + 2*(b^2 - 4*a*c)*e^40 + 2*(b - 4*a*c)*e^41 + 2*(b^2 - 4*a*c)*e^42 + 2*(b - 4*a*c)*e^43 + 2*(b^2 - 4*a*c)*e^44 + 2*(b - 4*a*c)*e^45 + 2*(b^2 - 4*a*c)*e^46 + 2*(b - 4*a*c)*e^47 + 2*(b^2 - 4*a*c)*e^48 + 2*(b - 4*a*c)*e^49 + 2*(b^2 - 4*a*c)*e^50 + 2*(b - 4*a*c)*e^51 + 2*(b^2 - 4*a*c)*e^52 + 2*(b - 4*a*c)*e^53 + 2*(b^2 - 4*a*c)*e^54 + 2*(b - 4*a*c)*e^55 + 2*(b^2 - 4*a*c)*e^56 + 2*(b - 4*a*c)*e^57 + 2*(b^2 - 4*a*c)*e^58 + 2*(b - 4*a*c)*e^59 + 2*(b^2 - 4*a*c)*e^60 + 2*(b - 4*a*c)*e^61 + 2*(b^2 - 4*a*c)*e^62 + 2*(b - 4*a*c)*e^63 + 2*(b^2 - 4*a*c)*e^64 + 2*(b - 4*a*c)*e^65 + 2*(b^2 - 4*a*c)*e^66 + 2*(b - 4*a*c)*e^67 + 2*(b^2 - 4*a*c)*e^68 + 2*(b - 4*a*c)*e^69 + 2*(b^2 - 4*a*c)*e^70 + 2*(b - 4*a*c)*e^71 + 2*(b^2 - 4*a*c)*e^72 + 2*(b - 4*a*c)*e^73 + 2*(b^2 - 4*a*c)*e^74 + 2*(b - 4*a*c)*e^75 + 2*(b^2 - 4*a*c)*e^76 + 2*(b - 4*a*c)*e^77 + 2*(b^2 - 4*a*c)*e^78 + 2*(b - 4*a*c)*e^79 + 2*(b^2 - 4*a*c)*e^80 + 2*(b - 4*a*c)*e^81 + 2*(b^2 - 4*a*c)*e^82 + 2*(b - 4*a*c)*e^83 + 2*(b^2 - 4*a*c)*e^84 + 2*(b - 4*a*c)*e^85 + 2*(b^2 - 4*a*c)*e^86 + 2*(b - 4*a*c)*e^87 + 2*(b^2 - 4*a*c)*e^88 + 2*(b - 4*a*c)*e^89 + 2*(b^2 - 4*a*c)*e^90 + 2*(b - 4*a*c)*e^91 + 2*(b^2 - 4*a*c)*e^92 + 2*(b - 4*a*c)*e^93 + 2*(b^2 - 4*a*c)*e^94 + 2*(b - 4*a*c)*e^95 + 2*(b^2 - 4*a*c)*e^96 + 2*(b - 4*a*c)*e^97 + 2*(b^2 - 4*a*c)*e^98 + 2*(b - 4*a*c)*e^99 + 2*(b^2 - 4*a*c)*e^100

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out] $[1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2 - (b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*c*d^4 + (6*b*c*d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + 2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2 - 2*(b*c*e^4*x^4 + 4*b*c*d*e^3*x^3 + b*c*d^4 + (6*b*c*d^2 + b^2)*e^2*x^2 + b^2*d^2 + 2*(2*b*c*d^3 + b^2*d)*e*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e]]$

giac [A] time = 0.57, size = 171, normalized size = 1.76

$$\frac{b \arctan\left(\frac{2cd^2 + 2(x^2e + 2dx)ce + b}{\sqrt{-b^2 + 4ac}}\right) e^{(-1)}}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} + \frac{bd^2 + (x^2e + 2dx)be + 2a}{2\left(cd^4 + 2(x^2e + 2dx)cd^2e + (x^2e + 2dx)^2ce^2 + bd^2 + (x^2e + 2dx)be + a\right)(b^2e - 4ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] b*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*d^2 + (x^2*e + 2*d*x)*b*e + 2*a)/((c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))

maple [C] time = 0.02, size = 276, normalized size = 2.85

$$\frac{b \arctan\left(\frac{2cd^2 + 2(x^2e + 2dx)ce + b}{\sqrt{-b^2 + 4ac}}\right) e^{(-1)}}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} + \frac{bd^2 + (x^2e + 2dx)be + 2a}{2\left(cd^4 + 2(x^2e + 2dx)cd^2e + (x^2e + 2dx)^2ce^2 + bd^2 + (x^2e + 2dx)be + a\right)(b^2e - 4ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out] (-1/2*b*e/(4*a*c-b^2)*x^2-b*d/(4*a*c-b^2)*x-1/2/e*(b*d^2+2*a)/(4*a*c-b^2))/((c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/2/(4*a*c-b^2)*b/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e^x + d}{(bc - 4ac^2)x^4 + 4(bc - 4ac^2)dx^3 + (b^3 - 4abc + 6(bc - 4ac^2)d)x^2 + ab^2 - 4ac^2 + (b^3 - 4abc)d + 2(2(bc - 4ac^2)d^2 + (b^3 - 4abc)d)} dx + \frac{b^2x^2 + 2bdex + b^2 + 2a}{2((bc - 4ac^2)d^2 + 4(bc - 4ac^2)dx^3 + (b^3 - 4abc + 6(bc - 4ac^2)d)^2 + 2(2(bc - 4ac^2)d^2 + (b^3 - 4abc)d)^2 + ((bc - 4ac^2)d^2 + ab^2 - 4ac^2 + (b^3 - 4abc)d))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] -b*integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*x^2 + 2*b*d*e*x + b*d^2 + 2*a)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)

mupad [B] time = 1.77, size = 427, normalized size = 4.40

$$b \operatorname{atan} \left(\frac{\left((4ac-b^2)^4 \left(x \left(\frac{b^3(2b^2d^3-8ab^2d^2)}{a^2(4ac-b^2)^{11/2}} - \frac{2b^2d^2}{a(4ac-b^2)^{7/2}} \right) + x^2 \left(\frac{b^3(2b^2d^3-8ab^2d^2)}{2a^2(4ac-b^2)^{11/2}} - \frac{b^2d^2}{a(4ac-b^2)^{7/2}} \right) \right) \frac{b^3(16a^2c^3b^3-4ab^2c^2b^2+8ab^2b^2b-2b^3d^2d^2)}{2a^2(4ac-b^2)^{11/2}} - \frac{b^2d^2}{a(4ac-b^2)^{7/2}} \right)}{e(4ac-b^2)^{3/2}} \right) - \frac{\frac{bd^2+2a}{2c(4ac-b^2)} + \frac{bcx^2}{2(4ac-b^2)} + \frac{bdx}{4ac-b^2}}{a+x^2(6cd^2e^2+be^2)+bd^2+cd^4+x(4ced^3+2bed)+ce^4x^4+4cd^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x)`

[Out] $(b \operatorname{atan}(((4ac - b^2)^4 (x((b^3(2b^2c^2d^2e^9 - 8ab^2c^3d^2e^9)) / (a^2(4ac - b^2)^{11/2}) - (2b^2c^2d^2e^7) / (a(4ac - b^2)^{7/2}))) + x^2((b^3(2b^2c^2d^2e^{10} - 8ab^2c^3d^2e^{10})) / (2a^2(4ac - b^2)^{11/2}) - (b^2c^2d^2e^8) / (a(4ac - b^2)^{7/2})) - (b^3(16a^2c^3e^8 - 4ab^2c^2e^8 - 2b^3c^2d^2e^8 + 8ab^2c^3d^2e^8)) / (2a^2(4ac - b^2)^{11/2}) - (b^2c^2d^2e^6) / (a(4ac - b^2)^{7/2}))) / (2b^2c^2e^6))) / (e(4ac - b^2)^{3/2}) - ((2a + bd^2) / (2e(4ac - b^2)) + (bex^2) / (2(4ac - b^2)) + (bd*x) / (4ac - b^2)) / (a + x^2(b^2e^2 + 6cd^2e^2) + bd^2 + cd^4 + x(2bd^2e + 4cd^3e) + ce^4x^4 + 4cd^3e^3x^3)$

sympy [B] time = 4.88, size = 495, normalized size = 5.10

$$b \sqrt{\frac{-1}{(4ac-b^2)}} \log \left(\frac{\frac{bd^2+2a}{2c(4ac-b^2)} + \frac{bcx^2}{2(4ac-b^2)} + \frac{bdx}{4ac-b^2}}{2a + bd^2} \right) + \frac{bd^2+2a}{2c(4ac-b^2)} + \frac{bcx^2}{2(4ac-b^2)} + \frac{bdx}{4ac-b^2} \sqrt{\frac{-1}{(4ac-b^2)}} \log \left(\frac{\frac{bd^2+2a}{2c(4ac-b^2)} + \frac{bcx^2}{2(4ac-b^2)} + \frac{bdx}{4ac-b^2}}{2a + bd^2} \right) + \frac{-2a - b^2 - 2bdx - b^2x^2}{8b^2c^2 - 2ab^2e + 8ab^2c^2e + 8a^2b^2e^2 - 2b^2d^2e - 2b^2d^2e^2 + x^2(8a^2c^2 - 2b^2c^2) + x^2(32a^2cd^2 - 8b^2cd^2) + x^2(8ab^2c^2 + 48a^2b^2c^2 - 2b^2d^2 - 12b^2d^2e^2) + x(16ab^2cd^2 + 32a^2cd^2 - 4b^2d^2 - 8b^2d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2, x)`

[Out] $b \sqrt{-1/(4ac - b^2)} \log(2dx/e + x^2 + (-16a^2b^2c^2 \sqrt{-1/(4ac - b^2)} + 8ab^3c \sqrt{-1/(4ac - b^2)} - b^5 \sqrt{-1/(4ac - b^2)} + b^2 + 2b^2cd^2) / (2b^2c^2e^2)) / (2e) - b \sqrt{-1/(4ac - b^2)} \log(2dx/e + x^2 + (16a^2b^2c^2 \sqrt{-1/(4ac - b^2)} - 8ab^3c \sqrt{-1/(4ac - b^2)} + b^5 \sqrt{-1/(4ac - b^2)} + b^2 + 2b^2cd^2) / (2b^2c^2e^2)) / (2e) + (-2a - bd^2 - 2bd^2e^2 - b^2e^2x^2) / (8a^2c^2e - 2ab^2e^2 + 8ab^2cd^2e + 8a^2c^2d^4e - 2b^3d^2e^2 - 2b^2c^2d^4e + x^4(8a^2c^2e^5 - 2b^2c^2e^5) + x^3(32a^2cd^2e^4 - 8b^2c^2d^4e^4) + x^2(8ab^2c^2e^3 + 48a^2c^2d^2e^3 - 2b^3c^2e^3 - 12b^2c^2d^2e^3) + x(16ab^2cd^2e^2 + 32a^2cd^2e^2 - 4b^3d^2e^2 - 8b^2c^2d^3e^2))$

$$3.510 \quad \int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=254

$$\frac{(d+ex)(b+2c(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c}(2b-\sqrt{b^2-4ac})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.39, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1142, 1119, 1166, 205}

$$\frac{(d+ex)(b+2c(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c}(2b-\sqrt{b^2-4ac})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1119

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(d*x)^(m-1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^(m-2)*(b*(m-1) + 2*c*(m+4*p+5)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{e} \\ &= -\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4} dx, x, d+ex\right)}{2(b^2-4ac)e} \\ &= -\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{(c(2b-\sqrt{b^2-4ac})) \text{Subst}\left(\int \frac{1}{\sqrt{b^2-4ac}+cx^2} dx, x, d+ex\right)}{2(b^2-4ac)e} \\ &= -\frac{(d+ex)(b+2c(d+ex)^2)}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c}(2b-\sqrt{b^2-4ac}) \tan^{-1}\left(\frac{\sqrt{b^2-4ac}+cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b^2-4ac}} \end{aligned}$$

Mathematica [A] time = 0.98, size = 247, normalized size = 0.97

$$\frac{\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}(\sqrt{b^2-4ac}-2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

[Out]
$$-1/2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(-2*b + \text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + e*x))/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(2*b + \text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + e*x))/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]))/e$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]

fricas [B] time = 1.32, size = 2474, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out]
$$-1/4*(4*c*e^3*x^3 + 12*c*d*e^2*x^2 + 4*c*d^3 + 2*(6*c*d^2 + b)*e*x - \text{sqrt}(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*\text{sqrt}(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*\text{sqrt}(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2)))*\log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*\text{sqrt}(1/2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*\text{sqrt}(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e)*\text{sqrt}(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*\text{sqrt}(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) + \text{sqrt}(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*\text{sqrt}(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2*\text{sqrt}(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*\log((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*\text{sqrt}(1/2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*\text{sqrt}(1/((a^2*b^6$$

$$\begin{aligned}
& - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) - (b^5 - 8*a*b^3*c + 16 \\
& *a^2*b*c^2)*e)*sqrt(-((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)* \\
& e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + \\
& b^3 + 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2)) \\
&) + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + \\
& (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)* \\
& d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + \\
& (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64* \\
& a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) \\
& *e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^ \\
& ^3)*e^2))*log(((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/ \\
& 2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a^2* \\
& b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3*c \\
& + 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^ \\
& 3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) \\
& - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^ \\
& 2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 \\
& + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^ \\
& 2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2* \\
& c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - \\
& 64*a^4*c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^ \\
& ^3)*e^4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^ \\
& 4*c^3)*e^2))*log(((3*b^2*c + 4*a*c^2)*e*x + (3*b^2*c + 4*a*c^2)*d - 1/2*sqrt \\
& (1/2)*((a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e^3*sqrt(1/((a \\
& ^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)) + (b^5 - 8*a*b^3 \\
& *c + 16*a^2*b*c^2)*e)*sqrt(((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4 \\
& *c^3)*e^2*sqrt(1/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^ \\
& 4)) - b^3 - 12*a*b*c)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3) \\
& *e^2))) + 2*b*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 \\
& + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^ \\
& 2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2* \\
& c + (b^3 - 4*a*b*c)*d^2)*e)
\end{aligned}$$

giac [B] time = 0.53, size = 1312, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] 1/4*((2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-

$$\begin{aligned}
& (b^2 e^2 + \sqrt{b^2 - 4ac}) e^{-4} / c)^2 c d e^3 - 2 c d^3 e - b d e + \\
& (6 c d^2 e^2 + b e^2) (d e^{-1} + \sqrt{1/2} \sqrt{-(b^2 + \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) + (2 (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 + \sqrt{b^2 - 4ac}) e^2} e^{-4} / c))^2 c e^2 - 4 (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 + \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) c d e + 2 c d^2 - b) \log(d e^{-1} + x - \sqrt{1/2} \sqrt{-(b^2 + \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) / (2 (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 + \sqrt{b^2 - 4ac}) e^2} e^{-4} / c))^3 c e^4 - 6 (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 + \sqrt{b^2 - 4ac}) e^2} e^{-4} / c))^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 + \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) + (2 (d e^{-1} + \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c))^2 c e^2 - 4 (d e^{-1} + \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) c d e + 2 c d^2 - b) \log(d e^{-1} + x + \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) / (2 (d e^{-1} + \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c))^3 c e^4 - 6 (d e^{-1} + \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c))^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} + \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) + (2 (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c))^2 c e^2 - 4 (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) c d e + 2 c d^2 - b) \log(d e^{-1} + x - \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) / (2 (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c))^3 c e^4 - 6 (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c))^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 - \sqrt{b^2 - 4ac}) e^2} e^{-4} / c)) / (b^2 - 4ac) - 1/2 (2 c x^3 e^3 + 6 c d x^2 e^2 + 6 c d^2 x e + 2 c d^3 + b x e + b d) / ((c x^4 e^4 + 4 c d x^3 e^3 + 6 c d^2 x^2 e^2 + 4 c d^3 x e + c d^4 + b x^2 e^2 + 2 b d x e + b d^2 + a) (b^2 e - 4 a c e))
\end{aligned}$$

maple [C] time = 0.02, size = 319, normalized size = 1.26

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out] $(c e^2 / (4 a c - b^2) x^3 + 3 d c e / (4 a c - b^2) x^2 + 1/2 (6 c d^2 + b) / (4 a c - b^2) x + 1/2 d e / (2 c d^2 + b) / (4 a c - b^2)) / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) + 1/4 / (4 a c - b^2) / e \sum((2 _R^2 c e^2 + 4 _R c d e + 2 c d^2 - b) / (2 _R^3 c e^3 + 6 _R^2 c d e^2 + 6 _R c d^2 e + 2 c d^3 + _R b e + b d) \ln(-_R + x), _R = \text{RootOf}(_Z^4 c e^4 + 4 _Z^3 c d e^3 + c d^4 + b d^2 + (6 c d^2 e^2 + b e^2) _Z^2 + (4 c d^3 e + 2 b d e) _Z + a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out]
$$-1/2*(2*c*e^3*x^3 + 6*c*d*e^2*x^2 + 2*c*d^3 + (6*c*d^2 + b)*e*x + b*d)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e) + 1/2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)$$

mupad [B] time = 3.95, size = 7200, normalized size = 28.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)

[Out]
$$\operatorname{atan}\left(\left(\left(-4*a*c - b^2\right)^9\right)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3\right)/\left(32*(a*b^{12}*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^{10}*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)\right)^{1/2} * \left(\left(64*a^2*c^5*d*e^{11} + 20*b^4*c^3*d*e^{11} - 96*a*b^2*c^4*d*e^{11}\right)/\left(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)\right) + \left(\left(32*b^9*c^2*d*e^{13} - 512*a*b^7*c^3*d*e^{13} + 8192*a^4*b*c^6*d*e^{13} + 3072*a^2*b^5*c^4*d*e^{13} - 8192*a^3*b^3*c^5*d*e^{13}\right)/\left(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)\right) + \left(x*(8*b^7*c^2*e^{14} - 96*a*b^5*c^3*e^{14} - 512*a^3*b*c^5*e^{14} + 384*a^2*b^3*c^4*e^{14})\right)/\left(b^4 + 16*a^2*c^2 - 8*a*b^2*c\right)\right) * \left(\left(-4*a*c - b^2\right)^9\right)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3\right)/\left(32*(a*b^{12}*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^{10}*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)\right)^{1/2} - \left(8*b^7*c^2*e^{12} - 96*a*b^5*c^3*e^{12} - 512*a^3*b*c^5*e^{12} + 384*a^2*b^3*c^4*e^{12}\right)/\left(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)\right) * \left(\left(-4*a*c - b^2\right)^9\right)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3\right)/\left(32*(a*b^{12}*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^{10}*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)\right)^{1/2} - \left(x*(4*a*c^4*e^{12} - 5*b^2*c^3*e^{12})\right)/\left(b^4 + 16*a^2*c^2 - 8*a*b^2*c\right) * i + \left(\left(-4*a*c - b^2\right)^9\right)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3\right)/\left(32*(a*b^{12}*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^{10}*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)\right)^{1/2} * \left(\left(64*a^2*c^5*d*e^{11} + 20*b^4*c^3*d*e^{11} - 96*a*b^2*c^4*d*e^{11}\right)/\left(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)\right) + \left(\left(32*b^9*c^2*d*e^{13} - 512*a*b^7*c^3*d*e^{13} + 8192*a^4*b*c^6*d*e^{13} + 3072*a^2*b^5*c^4*d*e^{13} - 8192*a^3*b^3*c^5*d*e^{13}\right)/\left(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)\right) + \left(x*(8*b^7*c^2*e^{14} - 96*a*b^5*c^3*e^{14} - 512*a^3*b*c^5*e^{14} + 384*a^2*b^3*c^4*e^{14})\right)/\left(b^4 + 16*a^2*c^2 - 8*a*b^2*c\right)\right)$$

$$\begin{aligned}
& \left(\frac{2b^3c^4e^{14}}{b^4 + 16a^2c^2 - 8ab^2c} \right) \left(\left((-4ac - b^2)^9 \right)^{1/2} - b^9 + 768a^4b^3c^4 + 96a^2b^5c^2 - 512a^3b^3c^3 \right) / \left(32(a^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2) \right)^{1/2} + (8b^7c^2e^{12} - 96ab^5c^3e^{12} - 512a^3b^3c^5e^{12} + 384a^2b^3c^4e^{12}) / (4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) \left(\left((-4ac - b^2)^9 \right)^{1/2} - b^9 + 768a^4b^3c^4 + 96a^2b^5c^2 - 512a^3b^3c^3 \right) / \left(32(a^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2) \right)^{1/2} - (x(4a^4c^4e^{12} - 5b^2c^3e^{12})) / (b^4 + 16a^2c^2 - 8ab^2c) * 1i) / \left(\left((-4ac - b^2)^9 \right)^{1/2} - b^9 + 768a^4b^3c^4 + 96a^2b^5c^2 - 512a^3b^3c^3 \right) / \left(32(a^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2) \right)^{1/2} * \left(\frac{64a^2c^5de^{11} + 20b^4c^3de^{11} - 96ab^2c^4de^{11}}{4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)} + \left(\frac{32b^9c^2de^{13} - 512a^3b^7c^3de^{13} + 8192a^4b^3c^6de^{13} + 3072a^2b^5c^4de^{13} - 8192a^3b^3c^5de^{13}}{4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)} + (x(8b^7c^2e^{14} - 96ab^5c^3e^{14} - 512a^3b^3c^5e^{14} + 384a^2b^3c^4e^{14})) / (b^4 + 16a^2c^2 - 8ab^2c) \right) \left(\left((-4ac - b^2)^9 \right)^{1/2} - b^9 + 768a^4b^3c^4 + 96a^2b^5c^2 - 512a^3b^3c^3 \right) / \left(32(a^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2) \right)^{1/2} + (8b^7c^2e^{12} - 96ab^5c^3e^{12} - 512a^3b^3c^5e^{12} + 384a^2b^3c^4e^{12}) / (4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) \left(\left((-4ac - b^2)^9 \right)^{1/2} - b^9 + 768a^4b^3c^4 + 96a^2b^5c^2 - 512a^3b^3c^3 \right) / \left(32(a^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2) \right)^{1/2} - (x(4a^4c^4e^{12} - 5b^2c^3e^{12})) / (b^4 + 16a^2c^2 - 8ab^2c) - \left(\left((-4ac - b^2)^9 \right)^{1/2} - b^9 + 768a^4b^3c^4 + 96a^2b^5c^2 - 512a^3b^3c^3 \right) / \left(32(a^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2) \right)^{1/2} * \left(\frac{64a^2c^5de^{11} + 20b^4c^3de^{11} - 96ab^2c^4de^{11}}{4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)} + \left(\frac{32b^9c^2de^{13} - 512a^3b^7c^3de^{13} + 8192a^4b^3c^6de^{13} + 3072a^2b^5c^4de^{13} - 8192a^3b^3c^5de^{13}}{4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)} + (x(8b^7c^2e^{14} - 96ab^5c^3e^{14} - 512a^3b^3c^5e^{14} + 384a^2b^3c^4e^{14})) / (b^4 + 16a^2c^2 - 8ab^2c) \right) \left(\left((-4ac - b^2)^9 \right)^{1/2} - b^9 + 768a^4b^3c^4 + 96a^2b^5c^2 - 512a^3b^3c^3 \right) / \left(32(a^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2) \right)^{1/2} - (8b^7c^2e^{12} - 96ab^5c^3e^{12} - 512a^3b^3c^5e^{12} + 384a^2b^3c^4e^{12}) / (4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) \left(\left((-4ac - b^2)^9 \right)^{1/2} - b^9 + 768a^4b^3c^4 + 96a^2b^5c^2 - 512a^3b^3c^3 \right) / \left(32(a^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2) \right)^{1/2} - (x(4a^4c^4e^{12} - 5b^2c^3e^{12})) / (b^4 + 16a^2c^2 - 8ab^2c) \right)
\end{aligned}$$

$$\begin{aligned}
& 2*c^3*e^{12}))/ (b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (4*a*c^4*e^{10} + 3*b^2*c^3*e^{10}) / (2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) * (((- (4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3) / (32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^{(1/2)} * 2 \\
& i - \operatorname{atan}(-((((32*b^9*c^2*d*e^{13} - 512*a*b^7*c^3*d*e^{13} + 8192*a^4*b*c^6*d*e^{13} + 3072*a^2*b^5*c^4*d*e^{13} - 8192*a^3*b^3*c^5*d*e^{13}) / (4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^{14} - 96*a*b^5*c^3*e^{14} - 512*a^3*b*c^5*e^{14} + 384*a^2*b^3*c^4*e^{14})) / (b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^{(1/2)} - (8*b^7*c^2*e^{12} - 96*a*b^5*c^3*e^{12} - 512*a^3*b*c^5*e^{12} + 384*a^2*b^3*c^4*e^{12}) / (4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^{(1/2)} + (64*a^2*c^5*d*e^{11} + 20*b^4*c^3*d*e^{11} - 96*a*b^2*c^4*d*e^{11}) / (4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a*c^4*e^{12} - 5*b^2*c^3*e^{12})) / (b^4 + 16*a^2*c^2 - 8*a*b^2*c)) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^{(1/2)} * 1i + (((32*b^9*c^2*d*e^{13} - 512*a*b^7*c^3*d*e^{13} + 8192*a^4*b*c^6*d*e^{13} + 3072*a^2*b^5*c^4*d*e^{13} - 8192*a^3*b^3*c^5*d*e^{13}) / (4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^{14} - 96*a*b^5*c^3*e^{14} - 512*a^3*b*c^5*e^{14} + 384*a^2*b^3*c^4*e^{14})) / (b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^{(1/2)} + (8*b^7*c^2*e^{12} - 96*a*b^5*c^3*e^{12} - 512*a^3*b*c^5*e^{12} + 384*a^2*b^3*c^4*e^{12}) / (4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^{(1/2)} + (64*a^2*c^5*d*e^{11} + 20*b^4*c^3*d*e^{11} - 96*a*b^2*c^4*d*e^{11}) / (4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(4*a*c^4*e^{12} - 5*b^2*c^3*e^{12})) / (b^4 + 16*a^2*c^2 - 8*a*b^2*c)) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^{(1/2)} * 1i) / ((4*a*c^4*e^{10} + 3*b^2*c^3*e^{10}) / (2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (((32*b^9*c^2*d*e^{13} - 512*a*b^7*c^3*d*e^{13} + 8192*a^4*b*c^6*d*e^{13} + 3072*a^2*b^5*c^4*d*e^{13} - 8192*a^3*b^3*c^5*d*e^{13}) / (4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7*c^2*e^{14} - 96*a*b^5*c^3
\end{aligned}$$

$$\begin{aligned}
& e^{14} - 512a^3b^3c^5e^{14} + 384a^2b^3c^4e^{14}) / (b^4 + 16a^2c^2 - 8ab^2c) * (- (b^9 + (-4ac - b^2)^9)^{1/2} - 768a^4b^3c^4 - 96a^2b^5c^2 \\
& + 512a^3b^3c^3) / (32(a^2b^12e^2 + 4096a^7c^6e^2 - 24a^2b^10c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{1/2} - (8b^7c^2e^{12} - 96ab^5c^3e^{12} - 512a^3b^3c^5e^{12} \\
& + 384a^2b^3c^4e^{12}) / (4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) * (- (b^9 + (-4ac - b^2)^9)^{1/2} - 768a^4b^3c^4 - 96a^2b^5c^2 + 512a^3b^3c^3) / (32(a^2b^12e^2 + 4096a^7c^6e^2 - 24a^2b^10c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{1/2} \\
& + (64a^2c^5de^{11} + 20b^4c^3de^{11} - 96ab^2c^4de^{11}) / (4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) - (x(4a^4c^4e^{12} - 5b^2c^3e^{12})) / (b^4 + 16a^2c^2 - 8ab^2c) * (- (b^9 + (-4ac - b^2)^9)^{1/2} - 768a^4b^3c^4 - 96a^2b^5c^2 + 512a^3b^3c^3) / (32(a^2b^12e^2 + 4096a^7c^6e^2 - 24a^2b^10c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{1/2} \\
& + (((32b^9c^2de^{13} - 512ab^7c^3de^{13} + 8192a^4b^3c^5de^{13}) / (4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) + (x(8b^7c^2e^{14} - 96ab^5c^3e^{14} - 512a^3b^3c^5e^{14} + 384a^2b^3c^4e^{14})) / (b^4 + 16a^2c^2 - 8ab^2c)) * (- (b^9 + (-4ac - b^2)^9)^{1/2} - 768a^4b^3c^4 - 96a^2b^5c^2 + 512a^3b^3c^3) / (32(a^2b^12e^2 + 4096a^7c^6e^2 - 24a^2b^10c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{1/2} \\
& + (8b^7c^2e^{12} - 96ab^5c^3e^{12} - 512a^3b^3c^5e^{12} + 384a^2b^3c^4e^{12}) / (4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) * (- (b^9 + (-4ac - b^2)^9)^{1/2} - 768a^4b^3c^4 - 96a^2b^5c^2 + 512a^3b^3c^3) / (32(a^2b^12e^2 + 4096a^7c^6e^2 - 24a^2b^10c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{1/2} \\
& + (64a^2c^5de^{11} + 20b^4c^3de^{11} - 96ab^2c^4de^{11}) / (4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) - (x(4a^4c^4e^{12} - 5b^2c^3e^{12})) / (b^4 + 16a^2c^2 - 8ab^2c) * (- (b^9 + (-4ac - b^2)^9)^{1/2} - 768a^4b^3c^4 - 96a^2b^5c^2 + 512a^3b^3c^3) / (32(a^2b^12e^2 + 4096a^7c^6e^2 - 24a^2b^10c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{1/2} \\
& * (- (b^9 + (-4ac - b^2)^9)^{1/2} - 768a^4b^3c^4 - 96a^2b^5c^2 + 512a^3b^3c^3) / (32(a^2b^12e^2 + 4096a^7c^6e^2 - 24a^2b^10c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{1/2} * 2i + ((x(b + 6cd^2)) / (2(4ac - b^2)) + (bd + 2cd^3) / (2e(4ac - b^2)) + (c^2e^2x^3) / (4ac - b^2) + (3cd^2e^2x^2) / (4ac - b^2)) / (a + x^2(b^2e^2 + 6cd^2e^2) + bd^2 + cd^4 + x(2bd^2e + 4cd^3e) + c^2e^4x^4 + 4cd^2e^3x^3)
\end{aligned}$$

sympy [B] time = 18.53, size = 578, normalized size = 2.28

$$\frac{1}{32} \sqrt{-4ac - b^2} \left(\frac{32b^9c^2de^{13} - 512ab^7c^3de^{13} + 8192a^4b^3c^5de^{13}}{4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)} + \frac{x(8b^7c^2e^{14} - 96ab^5c^3e^{14} - 512a^3b^3c^5e^{14} + 384a^2b^3c^4e^{14})}{b^4 + 16a^2c^2 - 8ab^2c} \right) - \frac{768a^4b^3c^4 - 96a^2b^5c^2 + 512a^3b^3c^3}{32(a^2b^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2)} \sqrt{-4ac - b^2} - \frac{8b^7c^2e^{12} - 96ab^5c^3e^{12} - 512a^3b^3c^5e^{12} + 384a^2b^3c^4e^{12}}{4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)} \sqrt{-4ac - b^2} + \frac{64a^2c^5de^{11} + 20b^4c^3de^{11} - 96ab^2c^4de^{11}}{4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)} - \frac{x(4a^4c^4e^{12} - 5b^2c^3e^{12})}{b^4 + 16a^2c^2 - 8ab^2c} \sqrt{-4ac - b^2} - \frac{768a^4b^3c^4 - 96a^2b^5c^2 + 512a^3b^3c^3}{32(a^2b^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2)} \sqrt{-4ac - b^2} + \frac{8b^7c^2e^{12} - 96ab^5c^3e^{12} - 512a^3b^3c^5e^{12} + 384a^2b^3c^4e^{12}}{4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)} \sqrt{-4ac - b^2} + \frac{64a^2c^5de^{11} + 20b^4c^3de^{11} - 96ab^2c^4de^{11}}{4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)} - \frac{x(4a^4c^4e^{12} - 5b^2c^3e^{12})}{b^4 + 16a^2c^2 - 8ab^2c} \sqrt{-4ac - b^2} - \frac{768a^4b^3c^4 - 96a^2b^5c^2 + 512a^3b^3c^3}{32(a^2b^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^{10}c^2e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2)} \sqrt{-4ac - b^2} * 2i + \frac{(x(b + 6cd^2))}{2(4ac - b^2)} + \frac{(bd + 2cd^3)}{2e(4ac - b^2)} + \frac{(c^2e^2x^3)}{4ac - b^2} + \frac{(3cd^2e^2x^2)}{4ac - b^2} \Big) / (a + x^2(b^2e^2 + 6cd^2e^2) + bd^2 + cd^4 + x(2bd^2e + 4cd^3e) + c^2e^4x^4 + 4cd^2e^3x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] (b*d + 2*c*d**3 + 6*c*d*e**2*x**2 + 2*c*e**3*x**3 + x*(b*e + 6*c*d**2*e))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**7*c**6*e**4 - 1572864*a**6*b**2*c**5*e**4 + 983040*a**5*b**4*c**4*e**4 - 327680*a**4*b**6*c**3*e**4 + 61440*a**3*b**8*c**2*e**4 - 6144*a**2*b**10*c*e**4 + 256*a*b**12*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*c**3*e**2 - 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**2*c**3 + 24*a*b**2*c**2 + 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a**4*b**2*c**3*e**3 + 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 128*_t*a**2*b*c**2*e - 16*_t*a*b**3*c*e - 4*_t*b**5*e + 4*a*c**2*d + 3*b**2*c*d)/(4*a*c**2*e + 3*b**2*c*e))))

$$3.511 \quad \int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=98

$$\frac{-b - 2c(d+ex)^2}{2e(b^2 - 4ac)(a + b(d+ex)^2 + c(d+ex)^4)} + \frac{2c \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{2c \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2 - 4ac)^{3/2}} - \frac{b + 2c(d+ex)^2}{2e(b^2 - 4ac)(a + b(d+ex)^2 + c(d+ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] -(b + 2*c*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (2*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

$\text{Int}[(x_*)*((a_*) + (b_*)*(x_*)^2 + (c_*)*(x_*)^4)^(p_*), x_Symbol] \rightarrow \text{Dist}[1/2,$
 $\text{Subst}[\text{Int}[(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, p\}, x]$

Rule 1142

$\text{Int}[(u_*)^(m_*)*((a_*) + (b_*)*(v_*)^2 + (c_*)*(v_*)^4)^(p_*), x_Symbol] \rightarrow \text{Di}$
 $\text{st}[u^m/(\text{Coefficient}[v, x, 1]*v^m), \text{Subst}[\text{Int}[x^m*(a + b*x^2 + c*x^(2*2))^p,$
 $x], x, v], x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{LinearPairQ}[u, v, x]$

Rubi steps

$$\int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2} dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{b+2c(d+ex)^2}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{c \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx, x, (d+ex)^2\right)}{(b^2-4ac)e}$$

$$= -\frac{b+2c(d+ex)^2}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{(2c) \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, (d+ex)^2\right)}{(b^2-4ac)}$$

$$= -\frac{b+2c(d+ex)^2}{2(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{2c \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}e}$$

Mathematica [A] time = 0.12, size = 98, normalized size = 1.00

$$-\frac{4c \tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}$$

$$2e(b^2-4ac)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]

[Out] $-1/2*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/((b^2 - 4*a*c)*e)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]

fricas [B] time = 0.87, size = 1042, normalized size = 10.63

21[0] - 4*a^2*d^2 + 4*b^2*c - 4*a*c^2) * e^2 * x^2 + 4*(b^2*c - 4*a*c^2) * d * e * x + b^3 - 4*a * b * c + 2*(b^2*c - 4*a*c^2) * d^2 + 2*(c^2 * e^4 * x^4 + 4*c^2 * d * e^3 * x^3 + c^2 * d^4 + (6*c^2 * d^2 + b*c) * e^2 * x^2 + b*c * d^2 + 2*(2*c^2 * d^3 + b*c*d) * e * x + a*c) * sqrt(b^2 - 4*a*c) * log((2*c^2 * e^4 * x^4 + 8*c^2 * d * e^3 * x^3 + 2*c^2 * d^4 + 2*(6*c^2 * d^2 + b*c) * e^2 * x^2 + 2*b*c * d^2 + 4*(2*c^2 * d^3 + b*c*d) * e * x + b^2 - 2*a*c - (2*c * e^2 * x^2 + 4*c * d * e * x + 2*c * d^2 + b) * sqrt(b^2 - 4*a*c)) / (c * e^4 * x^4 + 4*c * d * e^3 * x^3 + c * d^4 + (6*c * d^2 + b) * e^2 * x^2 + b * d^2 + 2*(2*c * d^3 + b * d) * e * x + a)) / ((b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * e^5 * x^4 + 4*(b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * d * e^4 * x^3 + (b^5 - 8*a * b^3 * c + 16*a^2 * b * c^2 + 6*(b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * d^2) * e^3 * x^2 + 2*(2*(b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * d^3 + (b^5 - 8*a * b^3 * c + 16*a^2 * b * c^2) * d) * e^2 * x + (a * b^4 - 8*a^2 * b^2 * c + 16*a^3 * c^2 + (b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * d^4 + (b^5 - 8*a * b^3 * c + 16*a^2 * b * c^2) * d^2) * e), -1/2*(2*(b^2*c - 4*a*c^2) * e^2 * x^2 + 4*(b^2*c - 4*a*c^2) * d * e * x + b^3 - 4*a * b * c + 2*(b^2*c - 4*a*c^2) * d^2 - 4*(c^2 * e^4 * x^4 + 4*c^2 * d * e^3 * x^3 + c^2 * d^4 + (6*c^2 * d^2 + b*c) * e^2 * x^2 + b*c * d^2 + 2*(2*c^2 * d^3 + b*c*d) * e * x + a*c) * sqrt(-b^2 + 4*a*c) * arctan(-(2*c * e^2 * x^2 + 4*c * d * e * x + 2 * c * d^2 + b) * sqrt(-b^2 + 4*a*c) / (b^2 - 4*a*c)) / ((b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * e^5 * x^4 + 4*(b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * d * e^4 * x^3 + (b^5 - 8*a * b^3 * c + 16*a^2 * b * c^2 + 6*(b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * d^2) * e^3 * x^2 + 2*(2*(b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * d^3 + (b^5 - 8*a * b^3 * c + 16*a^2 * b * c^2) * d) * e^2 * x + (a * b^4 - 8*a^2 * b^2 * c + 16*a^3 * c^2 + (b^4 * c - 8*a * b^2 * c^2 + 16*a^2 * c^3) * d^4 + (b^5 - 8*a * b^3 * c + 16*a^2 * b * c^2) * d^2) * e)]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out] $[-1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 + 2*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^4 + (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3 + b*c*d)*e*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c)) / (c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) / ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*x + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - 4*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + c^2*d^4 + (6*c^2*d^2 + b*c)*e^2*x^2 + b*c*d^2 + 2*(2*c^2*d^3 + b*c*d)*e*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) / ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e]$

giac [A] time = 0.46, size = 172, normalized size = 1.76

$$\frac{2c \arctan\left(\frac{2cd^2 + 2(x^2e + 2dx)ce + b}{\sqrt{-b^2 + 4ac}}\right)e^{(-1)}}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{2cd^2 + 2(x^2e + 2dx)ce + b}{2(cd^4 + 2(x^2e + 2dx)cd^2e + (x^2e + 2dx)^2ce^2 + bd^2 + (x^2e + 2dx)be + a)(b^2e - 4ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] $-2*c*\arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/\sqrt{-b^2 + 4*a*c})*e^{(-1)}/((b^2 - 4*a*c)*\sqrt{-b^2 + 4*a*c}) - 1/2*(2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/((c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))$

maple [C] time = 0.02, size = 270, normalized size = 2.76

$$\frac{(b^2 - 4ac)\sqrt{-b^2 + 4ac} \arctan\left(\frac{2cd^2 + 2(x^2e + 2dx)ce + b}{\sqrt{-b^2 + 4ac}}\right)e^{(-1)}}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{2cd^2 + 2(x^2e + 2dx)ce + b}{2(cd^4 + 2(x^2e + 2dx)cd^2e + (x^2e + 2dx)^2ce^2 + bd^2 + (x^2e + 2dx)be + a)(b^2e - 4ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out] $(c*e/(4*a*c - b^2)*x^2 + 2*c*d/(4*a*c - b^2)*x + 1/2/e*(2*c*d^2 + b)/(4*a*c - b^2))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + b*e^2*x^2 + c*d^4 + 2*b*d*e*x + b*d^2 + a) + c/(4*a*c - b^2)/e*\sum((_R*e + d)/(2*_R^3*c*e^3 + 6*_R^2*c*d*e^2 + 6*_R*c*d^2*e + 2*c*d^3 + _R*b*e + b*d)*\ln(-_R + x), _R = \text{RootOf}(_Z^4*c*e^4 + 4*_Z^3*c*d*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*_Z^2 + (4*c*d^3*e + 2*b*d*e)*_Z + a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2c \int \frac{cx + d}{(b^2 - 4ac)^2 x^4 + 4(b^2 - 4ac)d x^3 + (b^2 - 4ac)^2 d^2 + ab^2 - 4c^2 d + (b^2 - 4ac)d^2 + 2(2(b^2 - 4ac)d^2 + (b^2 - 4ac)d)x} dx - \frac{2c^2 d^2 + 4cdex + 2cd^2 + b}{2((b^2 - 4ac)^2 x^4 + 4(b^2 - 4ac)d x^3 + (b^2 - 4ac)^2 d^2 + ab^2 - 4c^2 d + (b^2 - 4ac)d^2 + 2(2(b^2 - 4ac)d^2 + (b^2 - 4ac)d)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] $2*c*\integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)$

mupad [B] time = 1.72, size = 417, normalized size = 4.26

$$\frac{\frac{2cx^2+b}{2e(4ac-b^2)} + \frac{ce^2}{4ac-b^2} + \frac{2cdx}{4ac-b^2}}{a+x^2(6cd^2e^2+be^2)+bd^2+cd^4+x(4ced^3+2bed)+ce^4x^4+4cde^3x^3} + \frac{2c \operatorname{atan}\left(\frac{\left(\frac{4ac-b^2}{a}\right)^{\frac{1}{2}} \left(x \left(\frac{8c^4d^2}{a(4ac-b^2)^{\frac{7}{2}}} - \frac{8b^2(\beta^2 d^2 - 4ab\beta^2 d^2)}{a^2(4ac-b^2)^{\frac{11}{2}}} \right) + x^2 \left(\frac{4c^4e^8}{a(4ac-b^2)^{\frac{7}{2}}} - \frac{4b^2(\beta^2 d^2 - 4ab\beta^2 d^2)}{a^2(4ac-b^2)^{\frac{11}{2}}} \right) + \frac{4c^4d^6}{a(4ac-b^2)^{\frac{7}{2}}} + \frac{4b^2(8a^2\beta^2 - 2ab\beta^2 d^2 - 4ab\beta^2 d^2 - \beta^2 d^2)}{a^2(4ac-b^2)^{\frac{11}{2}}} \right)}{8c^4e^6}}{e(4ac-b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x)`

[Out] $((b + 2*c*d^2)/(2*e*(4*a*c - b^2)) + (c*e*x^2)/(4*a*c - b^2) + (2*c*d*x)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*\operatorname{atan}(((4*a*c - b^2)^4*(x*((8*c^4*d*e^7)/(a*(4*a*c - b^2)^{(7/2)}) - (8*b*c^2*(b^3*c^2*d*e^9 - 4*a*b*c^3*d*e^9))/(a*e^2*(4*a*c - b^2)^{(11/2)})) + x^2*((4*c^4*e^8)/(a*(4*a*c - b^2)^{(7/2)}) - (4*b*c^2*(b^3*c^2*e^{10} - 4*a*b*c^3*e^{10}))/a^2*(4*a*c - b^2)^{(11/2)})) + (4*c^4*d^2*e^6)/(a*(4*a*c - b^2)^{(7/2)}) + (4*b*c^2*(8*a^2*c^3*e^8 - 2*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)^{(11/2)})))/(8*c^4*e^6)))/(e*(4*a*c - b^2)^{(3/2)})$

sympy [B] time = 4.68, size = 495, normalized size = 5.05

$$\frac{\sqrt{\frac{b}{(4ac-b^2)}} \log\left(\frac{2d+ex}{e} + x^2 + \frac{-2bd^2\sqrt{\frac{b}{(4ac-b^2)}} + 2cd^2\sqrt{\frac{b}{(4ac-b^2)}}}{2d^2}\right)}{e} + \frac{\sqrt{\frac{b}{(4ac-b^2)}} \log\left(\frac{2d+ex}{e} + x^2 + \frac{2bd^2\sqrt{\frac{b}{(4ac-b^2)}} + 2cd^2\sqrt{\frac{b}{(4ac-b^2)}}}{2d^2}\right)}{e} + \frac{b+2d^2+4cdex+2e^2}{8c^2e-2d^2e+8bd^2e+8c^2d^2e-2d^2e^2-x^2(8ac^2-2d^2e^2)+e^2(32a^2d^2-8d^2b^2)+x^2(8bd^2e+8a^2d^2e-2d^2e^2-12d^2e^2)+x(8bd^2d^2+32a^2d^2e-4d^2e^2-8d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2, x)`

[Out] $-c*\sqrt{-1/(4*a*c - b**2)**3}*\log(2*d*x/e + x**2 + (-16*a**2*c**3*\sqrt{-1/(4*a*c - b**2)**3} + 8*a*b**2*c**2*\sqrt{-1/(4*a*c - b**2)**3} - b**4*c*\sqrt{-1/(4*a*c - b**2)**3} + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + c*\sqrt{-1/(4*a*c - b**2)**3}*\log(2*d*x/e + x**2 + (16*a**2*c**3*\sqrt{-1/(4*a*c - b**2)**3} - 8*a*b**2*c**2*\sqrt{-1/(4*a*c - b**2)**3} + b**4*c*\sqrt{-1/(4*a*c - b**2)**3} + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + (b + 2*c*d**2 + 4*c*d*e*x + 2*c*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))$

$$3.512 \quad \int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=299

$$\frac{\left(\frac{d}{e} + x\right) \left(-2ac + b^2 + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{2a(b^2 - 4ac) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)} + \frac{\sqrt{c} \left(b\sqrt{b^2 - 4ac} - 12ac + b^2\right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2} ae (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(-b\sqrt{b^2 - 4ac} - 12ac + b^2\right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2} ae (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 0.70, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1106, 1092, 1166, 205}

$$\frac{\left(\frac{d}{e} + x\right) \left(-2ac + b^2 + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{2a(b^2 - 4ac) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)} + \frac{\sqrt{c} \left(b\sqrt{b^2 - 4ac} - 12ac + b^2\right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2} ae (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(-b\sqrt{b^2 - 4ac} - 12ac + b^2\right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2} ae (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2), x]

[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1],
c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx &= \text{Subst} \left(\int \frac{1}{(a + be^2x^2 + ce^4x^4)^2} dx, x, \frac{d}{e} + x \right) \\ &= \frac{\left(\frac{d}{e} + x\right) \left(b^2 - 2ac + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{2a \left(b^2 - 4ac\right) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)} - \text{Subst} \left(\int \frac{b^2e^4 - 2ace^4 - 2(b^2 - 2ac + bce^2 \left(\frac{d}{e} + x\right)^2)}{a + be^2x^2} dx, x, \frac{d}{e} + x \right) \\ &= \frac{\left(\frac{d}{e} + x\right) \left(b^2 - 2ac + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{2a \left(b^2 - 4ac\right) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)} - \frac{c \left(b^2 - 12ac - b\sqrt{b^2 - 4ac}\right)}{2a \left(b^2 - 4ac\right) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)} \\ &= \frac{\left(\frac{d}{e} + x\right) \left(b^2 - 2ac + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{2a \left(b^2 - 4ac\right) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)} + \frac{\sqrt{c} \left(b^2 - 12ac + b\sqrt{b^2 - 4ac}\right)}{2\sqrt{2} a \left(b^2 - 4ac\right) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)} \end{aligned}$$

Mathematica [A] time = 0.89, size = 271, normalized size = 0.91

$$\frac{2(d+ex)(-2ac+b^2+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

4ae

Antiderivative was successfully verified.

[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2), x]

[Out] ((2*(d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(4*a*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2), x]

[Out] IntegrateAlgebraic[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2), x]

fricas [B] time = 1.73, size = 3228, normalized size = 10.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out] 1/4*(2*b*c*e^3*x^3 + 6*b*c*d*e^2*x^2 + 2*b*c*d^3 + 2*(3*b*c*d^2 + b^2 - 2*a*c)*e*x - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*e*x + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*e^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)) - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4)))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2)) + sqrt(1/2)*((a*b^2*c

$$\begin{aligned} & *b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*e^4))/((a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*e^2))) + 2*(b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e) \end{aligned}$$

giac [B] time = 0.46, size = 1357, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/4*((d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*b*c*e^2 - 2*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c}) *b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*\log(d*e^{(-1)} + x + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})/(2*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^2*c*d*e^3 - 2*c*d^3 *e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})) + ((d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^2*b*c*e^2 - 2*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c}) *b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*\log(d*e^{(-1)} + x - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})/(2*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^2*c*d*e^3 - 2*c*d^3 *e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})) + ((d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^2*b*c*e^2 - 2*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c}) *b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*\log(d*e^{(-1)} + x + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})/(2*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^2*c*d*e^3 - 2*c*d^3 *e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})) + ((d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^2*b*c*e^2 - 2*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c}) *b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*\log(d*e^{(-1)} + x - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})/(2*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c})^2*c*d*e^3 - 2*c*d^3 *e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}) *e^{(-4)/c}))/((a*b^2 - 4*a^2*c) + 1/2*(b*c*x^3 *e^3 + 3*b*c*d*x^2*e^2 + 3*b*c*d^2*x*e + b*c*d^3 + b^2*x*e - 2*a*c*x*e + b^2 \end{aligned}$$

$$*d - 2*a*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(a*b^2*e - 4*a^2*c*e))$$

maple [C] time = 0.02, size = 364, normalized size = 1.22

$$\frac{\left[\text{RootOf}\left(\frac{Z^4 + 4Z^3c + 6Z^2cd + 4Zd^2 + d^4 + bZ^2e^2 + 2bZde + b^2d^2 + a}{(b^2e - 4a^2c)e}\right), Z + a \right] \text{RootOf}\left(\frac{Z^4 + 4Z^3c + 6Z^2cd + 4Zd^2 + d^4 + bZ^2e^2 + 2bZde + b^2d^2 + a}{(b^2e - 4a^2c)e}\right), Z + a \right]}{4(b^2e - 4a^2c)e \text{RootOf}\left(\frac{Z^4 + 4Z^3c + 6Z^2cd + 4Zd^2 + d^4 + bZ^2e^2 + 2bZde + b^2d^2 + a}{(b^2e - 4a^2c)e}\right), Z + a} \text{RootOf}\left(\frac{Z^4 + 4Z^3c + 6Z^2cd + 4Zd^2 + d^4 + bZ^2e^2 + 2bZde + b^2d^2 + a}{(b^2e - 4a^2c)e}\right), Z + a \right]}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out] $(-1/2*b*c*e^2/a/(4*a*c-b^2)*x^3-3/2*d*b*c*e/a/(4*a*c-b^2)*x^2+1/2*(-3*b*c*d^2+2*a*c-b^2)/a/(4*a*c-b^2)*x+1/2*d/e*(-b*c*d^2+2*a*c-b^2)/a/(4*a*c-b^2))/((c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/a/(4*a*c-b^2)/e*\text{sum}((-R^2*b*c*e^2-2*_R*b*c*d*e-b*c*d^2+6*a*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*\ln(-R+x),_R=\text{RootOf}(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bc^2d^3 + 3bcde^2 + bcd^3 + (3bcd^2 + b^2 - 2ac)cd + (b^2 - 2ac)d^2}{2((ab^2c - 4a^2c^2)d^2 + 4(ab^2c - 4a^2c^2)de^2 + (ab^3 - 4a^2bc + 6(ab^2c - 4a^2c^2)d^2 + 2(2(ab^2c - 4a^2c^2)d^2 + (ab^3 - 4a^2bc)d^2) + ((ab^2c - 4a^2c^2)d^2 + ab^2 - 4a^2c + (ab^3 - 4a^2bc)d^2))} - \frac{\int \frac{bc^2d^3 + 3bcde^2 + bcd^3 + (3bcd^2 + b^2 - 2ac)cd + (b^2 - 2ac)d^2}{(b^2c - 4a^2c^2)d^2 + 4(ab^2c - 4a^2c^2)de^2 + (ab^3 - 4a^2bc + 6(ab^2c - 4a^2c^2)d^2 + 2(2(ab^2c - 4a^2c^2)d^2 + (ab^3 - 4a^2bc)d^2) + ((ab^2c - 4a^2c^2)d^2 + ab^2 - 4a^2c + (ab^3 - 4a^2bc)d^2))} dx}{2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] $1/2*(b*c*e^3*x^3 + 3*b*c*d*e^2*x^2 + b*c*d^3 + (3*b*c*d^2 + b^2 - 2*a*c)*e*x + (b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*c^2)*e^5*x^4 + 4*(a*b^2*c - 4*a^2*c^2)*d*e^4*x^3 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*e^3*x^2 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*e^2*x + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e - 1/2*\text{integrate}(- (b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2 + b^2 - 6*a*c)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x)/a$

mupad [B] time = 4.85, size = 9056, normalized size = 30.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)

[Out] $\text{atan}\left(\frac{(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})}{(32*(a^3*b^{12}*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^{10}*c*e^2 + 2$

$$\begin{aligned}
& 40*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8 \\
& *b^2*c^5*e^2))^{(1/2)}*((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^ \\
& 6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 6 \\
& 4*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64 \\
& *a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 1 \\
& 6384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b \\
& ^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3* \\
& e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(- \\
& (b^11 + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15 \\
& 04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(\\
& 1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^ \\
& 8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5* \\
& e^2))^{(1/2)})*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288 \\
& *a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(\\
& 4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c \\
& *e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - \\
& 6144*a^8*b^2*c^5*e^2))^{(1/2)} - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 7 \\
& 2*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12* \\
& a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^ \\
& 2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))*1i - (-(b^11 + b^2*(\\
& -(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c \\
& ^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a \\
& ^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - \\
& 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)} \\
& *(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3 \\
& *b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^ \\
& 4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - \\
& 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^ \\
& 13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a \\
& ^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^ \\
& 4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840 \\
& *a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e \\
& ^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6* \\
& b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)})*(-(b^11 \\
& + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a \\
& ^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)} \\
&)/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^ \\
& 2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2) \\
&))^{(1/2)} + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - \\
& 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^ \\
& 2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2* \\
& b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))*1i)/((- (b^11 + b^2*(-(4*a*c - b^2)^9)^{(1/ \\
& 2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 \\
& - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^
\end{aligned}$$

$$\begin{aligned}
& 9c^6e^2 - 24a^4b^{10}c^2e^2 + 240a^5b^8c^2e^2 - 1280a^6b^6c^3e^2 \\
& + 3840a^7b^4c^4e^2 - 6144a^8b^2c^5e^2))^{(1/2)} * (((6144a^5c^6e^{12} \\
& + 16a^8b^2c^5e^{12} - 288a^2b^6c^3e^{12} + 1920a^3b^4c^4e^{12} - 5632a^4b^2c^5e^{12}) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2) \\
&) + ((16384a^6b^6c^6d^6e^{13} + 64a^2b^9c^2d^6e^{13} - 1024a^3b^7c^3d^6e^{13} + 6144a^4b^5c^4d^6e^{13} - 16384a^5b^3c^5d^6e^{13}) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) - (x(1024a^5b^6c^5e^{14} - 16a^2b^7c^2e^{14} + 192a^3b^5c^3e^{14} - 768a^4b^3c^4e^{14})) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) * (-b^{11} + b^2 * (-4ac - b^2)^9)^{(1/2)} - 3840a^5b^6c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^5b^9c - 9a^6c * (-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12}e^2 + 4096a^9c^6e^2 - 24a^4b^{10}c^2e^2 + 240a^5b^8c^2e^2 - 1280a^6b^6c^3e^2 + 3840a^7b^4c^4e^2 - 6144a^8b^2c^5e^2))^{(1/2)} * (-b^{11} + b^2 * (-4ac - b^2)^9)^{(1/2)} - 3840a^5b^6c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^5b^9c - 9a^6c * (-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12}e^2 + 4096a^9c^6e^2 - 24a^4b^{10}c^2e^2 + 240a^5b^8c^2e^2 - 1280a^6b^6c^3e^2 + 3840a^7b^4c^4e^2 - 6144a^8b^2c^5e^2))^{(1/2)} - (1152a^3c^6d^6e^{11} - 4b^6c^3d^6e^{11} + 72a^4b^4c^4d^6e^{11} - 512a^2b^2c^5d^6e^{11}) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) + (x(72a^2c^5e^{12} + b^4c^3e^{12} - 14a^2b^2c^4e^{12})) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) + (-b^{11} + b^2 * (-4ac - b^2)^9)^{(1/2)} - 3840a^5b^6c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^5b^9c - 9a^6c * (-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12}e^2 + 4096a^9c^6e^2 - 24a^4b^{10}c^2e^2 + 240a^5b^8c^2e^2 - 1280a^6b^6c^3e^2 + 3840a^7b^4c^4e^2 - 6144a^8b^2c^5e^2))^{(1/2)} * (((6144a^5c^6e^{12} + 16a^8b^2c^5e^{12} - 288a^2b^6c^3e^{12} + 1920a^3b^4c^4e^{12} - 5632a^4b^2c^5e^{12}) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) - ((16384a^6b^6c^6d^6e^{13} + 64a^2b^9c^2d^6e^{13} - 1024a^3b^7c^3d^6e^{13} + 6144a^4b^5c^4d^6e^{13} - 16384a^5b^3c^5d^6e^{13}) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) - (x(1024a^5b^6c^5e^{14} - 16a^2b^7c^2e^{14} + 192a^3b^5c^3e^{14} - 768a^4b^3c^4e^{14})) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) * (-b^{11} + b^2 * (-4ac - b^2)^9)^{(1/2)} - 3840a^5b^6c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^5b^9c - 9a^6c * (-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12}e^2 + 4096a^9c^6e^2 - 24a^4b^{10}c^2e^2 + 240a^5b^8c^2e^2 - 1280a^6b^6c^3e^2 + 3840a^7b^4c^4e^2 - 6144a^8b^2c^5e^2))^{(1/2)} * (-b^{11} + b^2 * (-4ac - b^2)^9)^{(1/2)} - 3840a^5b^6c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27a^5b^9c - 9a^6c * (-4ac - b^2)^9)^{(1/2)} / (32(a^3b^{12}e^2 + 4096a^9c^6e^2 - 24a^4b^{10}c^2e^2 + 240a^5b^8c^2e^2 - 1280a^6b^6c^3e^2 + 3840a^7b^4c^4e^2 - 6144a^8b^2c^5e^2))^{(1/2)} + (1152a^3c^6d^6e^{11} - 4b^6c^3d^6e^{11} + 72a^4b^4c^4d^6e^{11} - 512a^2b^2c^5d^6e^{11}) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) - (x(72a^2c^5e^{12} + b^4c^3e^{12} - 14a^2b^2c^4e^{12})) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) + (5b^3c^4e^{10} - 36a^2b^3c^5e^{10}) / (4(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2))) * (-b^{11} + b^2 * (-4ac - b^2)^9)^{(1/2)} - 3840a^5b^6c^5 + 288a
\end{aligned}$$

$$\begin{aligned}
& ^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)}*2i + \operatorname{atan}(((b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^{(1/2)}*((6144*a^5*c^6*e^{12} + 16*a*b^8*c^2*e^{12} - 288*a^2*b^6*c^3*e^{12} + 1920*a^3*b^4*c^4*e^{12} - 5632*a^4*b^2*c^5*e^{12})/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) + ((16384*a^6*b*c^6*d*e^{13} + 64*a^2*b^9*c^2*d*e^{13} - 1024*a^3*b^7*c^3*d*e^{13} + 6144*a^4*b^5*c^4*d*e^{13} - 16384*a^5*b^3*c^5*d*e^{13})/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) - (x*(1024*a^5*b*c^5*e^{14} - 16*a^2*b^7*c^2*e^{14} + 192*a^3*b^5*c^3*e^{14} - 768*a^4*b^3*c^4*e^{14}))/((2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) * (-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)} * (-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)} - (1152*a^3*c^6*d*e^{11} - 4*b^6*c^3*d*e^{11} + 72*a*b^4*c^4*d*e^{11} - 512*a^2*b^2*c^5*d*e^{11})/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) + (x*(72*a^2*c^5*e^{12} + b^4*c^3*e^{12} - 14*a*b^2*c^4*e^{12}))/((2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) * i - (-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)} * ((6144*a^5*c^6*e^{12} + 16*a*b^8*c^2*e^{12} - 288*a^2*b^6*c^3*e^{12} + 1920*a^3*b^4*c^4*e^{12} - 5632*a^4*b^2*c^5*e^{12})/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) - ((16384*a^6*b*c^6*d*e^{13} + 64*a^2*b^9*c^2*d*e^{13} - 1024*a^3*b^7*c^3*d*e^{13} + 6144*a^4*b^5*c^4*d*e^{13} - 16384*a^5*b^3*c^5*d*e^{13})/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) - (x*(1024*a^5*b*c^5*e^{14} - 16*a^2*b^7*c^2*e^{14} + 192*a^3*b^5*c^3*e^{14} - 768*a^4*b^3*c^4*e^{14}))/((2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))) * (-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)} * (-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)} * (-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)} * i - (-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2))^{(1/2)} * i
\end{aligned}$$

$$\begin{aligned}
& (a^6 b^6 c^3 e^2 + 3840 a^7 b^4 c^4 e^2 - 6144 a^8 b^2 c^5 e^2)^{1/2} + (1152 a^3 c^6 d e^{11} - 4 b^6 c^3 d e^{11} + 72 a b^4 c^4 d e^{11} - 512 a^2 b^2 c^5 d e^{11}) / (8 (a^2 b^6 - 64 a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) - (x \\
& (72 a^2 c^5 e^{12} + b^4 c^3 e^{12} - 14 a b^2 c^4 e^{12})) / (2 (a^2 b^4 + 16 a^4 c^2 - 8 a^3 b^2 c)) + (5 b^3 c^4 e^{10} - 36 a b c^5 e^{10}) / (4 (a^2 b^6 - 64 a^5 c^3 - 12 a^3 b^4 c + 48 a^4 b^2 c^2)) * (- (b^{11} - b^2 (- (4 a c - b^2)^9)^{1/2} - 3840 a^5 b c^5 + 288 a^2 b^7 c^2 - 1504 a^3 b^5 c^3 + 3840 a^4 b^3 c^4 - 27 a b^9 c + 9 a c (- (4 a c - b^2)^9)^{1/2})) / (32 (a^3 b^{12} e^2 + 4096 a^9 c^6 e^2 - 24 a^4 b^{10} c e^2 + 240 a^5 b^8 c^2 e^2 - 1280 a^6 b^6 c^3 e^2 + 3840 a^7 b^4 c^4 e^2 - 6144 a^8 b^2 c^5 e^2))^{1/2} * 2i - ((b^2 d - 2 a c d + b c d^3) / (2 a e (4 a c - b^2)) + (x (b^2 - 2 a c + 3 b c d^2)) / (2 a (4 a c - b^2)) + (b c e^2 x^3) / (2 a (4 a c - b^2)) + (3 b c d e x^2) / (2 a (4 a c - b^2))) / (a + x^2 (b e^2 + 6 c d^2 e^2) + b d^2 + c d^4 + x (2 b d e + 4 c d^3 e) + c e^4 x^4 + 4 c d e^3 x^3)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out

$$3.513 \quad \int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=162

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e(b^2 - 4ac)^{3/2}} - \frac{\log(a + b(d+ex)^2 + c(d+ex)^4)}{4a^2e} + \frac{\log(d+ex)}{a^2e} + \frac{-2ac + b^2 + bc(d+ex)^2}{2ae(b^2 - 4ac)(a + b(d+ex)^2 + c(d+ex)^4)}$$

Rubi [A] time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e(b^2 - 4ac)^{3/2}} - \frac{\log(a + b(d+ex)^2 + c(d+ex)^4)}{4a^2e} + \frac{\log(d+ex)}{a^2e} + \frac{-2ac + b^2 + bc(d+ex)^2}{2ae(b^2 - 4ac)(a + b(d+ex)^2 + c(d+ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2]/Sqrt[b^2 - 4*a*c])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e) + Log[d + e*x]/(a^2*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^(m)*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^2} dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac}{x(a+bx+cx^2)} dx, x, (d+ex)^2\right)}{2a(b^2-4ac)} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax}\right) dx, x, (d+ex)^2\right)}{2a(b^2-4ac)} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^2e} - \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, (d+ex)^2\right)}{2a(b^2-4ac)} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^2e} - \frac{\log(d+ex)}{2a(b^2-4ac)} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^2e} - \frac{\log(d+ex)}{2a(b^2-4ac)} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\log(d+ex)}{a^2e} - \frac{\log(d+ex)}{2a(b^2-4ac)} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)} + \frac{b(b^2-6ac) \tanh^{-1}\left(\frac{b+2c(d+ex)}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)}
\end{aligned}$$

Mathematica [A] time = 0.47, size = 235, normalized size = 1.45

$$\frac{2a(-2ac+b^2+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} - \frac{(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3)\log(-\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{(b^2-4ac)^{3/2}} + \frac{(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3)\log(\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{(b^2-4ac)^{3/2}} + 4\log(d+ex)}{4a^2e}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]

[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*sqrt[b^2 - 4*a*c]

$$+ 2*a*b*c*d*x*e + a*b*c*d^2 + a*b^2 - 2*a^2*c)*e^{(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*a^2)}$$

maple [C] time = 0.03, size = 693, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)`

[Out]
$$-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c*e/(4*a*c-b^2)*x^2-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c*d/(4*a*c-b^2)*x-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c*d^2+1/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c-1/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2-1/2/a^2/(4*a*c-b^2)/e*\text{sum}((c*e^3*(4*a*c-b^2)*_R^3+3*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*c*d^2+5*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*\ln(-_R+x),_R=\text{RootOf}(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))+\ln(e*x+d)/a^2/e$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 11.35, size = 11072, normalized size = 68.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)`

[Out]
$$((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 - 4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + \log(d + e*x)/(a^2*e) - (\log((((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^{(1/2)} - 1)*(((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^2)$$

$$\begin{aligned}
& 3))^{(1/2)} - 1) * ((b*c^2*e^{16*(a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))}^{(1/2)} - 1) * (a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x)) / a^2 + (2*b*c^2*e^{16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c)} / (a*(4*a*c - b^2)) - (2*b*c^3*e^{18*x^2*(10*a*c - b^2)} / (a*(4*a*c - b^2))) / (4*a^2*e) - (b*c^3*e^{15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c)} / (a^2*(4*a*c - b^2)^2) + (2*b*c^4*e^{17*x^2*(10*a*c - 3*b^2)} / (a^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^{16*x*(10*a*c - 3*b^2)} / (a^2*(4*a*c - b^2)^2)) / (4*a^2*e) + (b^3*c^5*e^{16*x^2}) / (a^3*(4*a*c - b^2)^3) + (b^2*c^4*e^{14*(b^2 - 4*a*c + b*c*d^2)}) / (a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^{15*x}) / (a^3*(4*a*c - b^2)^3)) * ((b^3*c^5*e^{16*x^2}) / (a^3*(4*a*c - b^2)^3) - ((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^{(1/2)} + 1) * (((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^{(1/2)} + 1) * ((b*c^2*e^{16*(a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))}^{(1/2)} + 1) * (a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x)) / a^2 - (2*b*c^2*e^{16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c)} / (a*(4*a*c - b^2)) + (2*b*c^3*e^{18*x^2*(10*a*c - b^2)} / (a*(4*a*c - b^2)) + (4*b*c^3*d*e^{17*x*(10*a*c - b^2)} / (a*(4*a*c - b^2)))) / (4*a^2*e) - (b*c^3*e^{15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c)} / (a^2*(4*a*c - b^2)^2) + (2*b*c^4*e^{17*x^2*(10*a*c - 3*b^2)} / (a^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^{16*x*(10*a*c - 3*b^2)} / (a^2*(4*a*c - b^2)^2))) / (4*a^2*e) + (b^2*c^4*e^{14*(b^2 - 4*a*c + b*c*d^2)}) / (a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^{15*x}) / (a^3*(4*a*c - b^2)^3))) * (2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e) / (2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*atan((((16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2)) * (x^2 * (((((b*((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18) / (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e) * (2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19)) / (2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) * (4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))) * (6*a*c - b^2)) / (4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2) * (2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e) * (2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19)) / (8*a^2*e*(4*a*c - b^2)^(3/2) * (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) * (4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))) * (2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e) / (2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*(6*a*c - b^2) * ((6*a*b^5*c^4*e^17 + 80*a^3*b*c^6*e^17 - 44*a^2*b^3*c^5*e^17) / (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18) / (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e) * (2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 +
\end{aligned}$$

$$\begin{aligned}
& 688*a^6*b^3*c^5*d*e^{18})/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)* \\
& (4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) * (6*a*c - b^2) / (4*a^2*e*(4*a*c - b^2)^{(3/2)}) - (b*(6*a*c - b^2)*(2* \\
& b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d* \\
& e^{18} + 12*a^3*b^9*c^2*d*e^{18} - 184*a^4*b^7*c^3*d*e^{18} + 1056*a^5*b^5*c^4*d* \\
& e^{18} - 2688*a^6*b^3*c^5*d*e^{18}) / (4*a^2*e*(4*a*c - b^2)^{(3/2)} * (a^3*b^6 - 64* \\
& a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) * (4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 \\
& - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) * (2*b^6*e - 128*a^3*c^3*e + 96*a \\
& ^2*b^2*c^2*e - 24*a*b^4*c*e) / (2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3* \\
& b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*(6*a*c - b^2)*((2*(6*a*b^5*c^4*d*e^1 \\
& 6 + 80*a^3*b*c^6*d*e^{16} - 44*a^2*b^3*c^5*d*e^{16})) / (a^3*b^6 - 64*a^6*c^3 - 1 \\
& 2*a^4*b^4*c + 48*a^5*b^2*c^2) + (((2*(320*a^5*b*c^6*d*e^{17} - 2*a^2*b^7*c^3*d* \\
& e^{17} + 36*a^3*b^5*c^4*d*e^{17} - 192*a^4*b^3*c^5*d*e^{17})) / (a^3*b^6 - 64*a^6* \\
& c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2* \\
& e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^{18} + 12*a^3*b^9*c^2*d*e^{18} - \\
& 184*a^4*b^7*c^3*d*e^{18} + 1056*a^5*b^5*c^4*d*e^{18} - 2688*a^6*b^3*c^5*d*e^{18} \\
&)) / ((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) * (4*a^2*b^6*e^2 - \\
& 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))) * (2*b^6*e - 128* \\
& a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e) / (2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 \\
& - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))) / (4*a^2*e*(4*a*c - b^2)^{(3/2)}) + (b^3*(6*a*c - b^2)^3*(2560*a^7*b*c^6*d*e^{18} + 12*a^3*b^9*c^2*d*e^{18} \\
& - 184*a^4*b^7*c^3*d*e^{18} + 1056*a^5*b^5*c^4*d*e^{18} - 2688*a^6*b^3*c^5*d*e^{18} \\
&)) / (32*a^6*e^3*(4*a*c - b^2)^{(9/2)} * (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c \\
& + 48*a^5*b^2*c^2)) * (3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c) / (8* \\
& a^3*c^2*(4*a*c - b^2)^{(7/2)} * (6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c) \\
&) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c) * (((2*(6*a*b^5*c^4*d*e^{16} + 80* \\
& a^3*b*c^6*d*e^{16} - 44*a^2*b^3*c^5*d*e^{16})) / (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c \\
& + 48*a^5*b^2*c^2) + (((2*(320*a^5*b*c^6*d*e^{17} - 2*a^2*b^7*c^3*d*e^{17} \\
& + 36*a^3*b^5*c^4*d*e^{17} - 192*a^4*b^3*c^5*d*e^{17})) / (a^3*b^6 - 64*a^6*c^3 - \\
& 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2* \\
& e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*d*e^{18} + 12*a^3*b^9*c^2*d*e^{18} - 184*a^4* \\
& b^7*c^3*d*e^{18} + 1056*a^5*b^5*c^4*d*e^{18} - 2688*a^6*b^3*c^5*d*e^{18})) / ((a^3* \\
& b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) * (4*a^2*b^6*e^2 - 256*a^5* \\
& c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))) * (2*b^6*e - 128*a^3*c^3* \\
& e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e) / (2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 \\
& - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2))) * (2*b^6*e - 128*a^3*c^3*e + 96* \\
& a^2*b^2*c^2*e - 24*a*b^4*c*e) / (2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3* \\
& b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) - (2*b^3*c^5*d*e^{15}) / (a^3*b^6 - 64*a^6*c^3 \\
& - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((2*(320*a^5*b*c^6*d*e^{17} \\
& - 2*a^2*b^7*c^3*d*e^{17} + 36*a^3*b^5*c^4*d*e^{17} - 192*a^4*b^3*c^5*d*e^{17})) / (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c \\
& + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e) * (2560*a^7*b*c^6*d*e^{18} \\
& + 12*a^3*b^9*c^2*d*e^{18} - 184*a^4*b^7*c^3*d*e^{18} + 1056*a^5*b^5*c^4*d*e^{18} \\
& - 2688*a^6*b^3*c^5*d*e^{18})) / ((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5* \\
& b^2*c^2) * (4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*
\end{aligned}$$

$$\begin{aligned}
& c^2e^2)))(6ac - b^2)/(4a^2e(4ac - b^2)^{3/2}) - (b(6ac - b^2) * \\
& (2b^6e - 128a^3c^3e + 96a^2b^2c^2e - 24ab^4ce) * (2560a^7b^6c^6 \\
& * d^18e + 12a^3b^9c^2d^18e - 184a^4b^7c^3d^18e + 1056a^5b^5c^4 \\
& * d^18e - 2688a^6b^3c^5d^18e)) / (4a^2e(4ac - b^2)^{3/2} * (a^3b^6 - \\
& 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) * (4a^2b^6e^2 - 256a^5c^3e \\
& ^2 - 48a^3b^4ce^2 + 192a^4b^2c^2e^2))) / (4a^2e(4ac - b^2)^{3/2} \\
&) + (b^2(6ac - b^2)^2(2b^6e - 128a^3c^3e + 96a^2b^2c^2e - 24a \\
& ab^4ce) * (2560a^7b^6c^6d^18e + 12a^3b^9c^2d^18e - 184a^4b^7c^3 \\
& * d^18e + 1056a^5b^5c^4d^18e - 2688a^6b^3c^5d^18e)) / (16a^4e^2 * (\\
& 4ac - b^2)^3 * (a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) * (4a^ \\
& 2b^6e^2 - 256a^5c^3e^2 - 48a^3b^4ce^2 + 192a^4b^2c^2e^2))) / (8 \\
& a^3c^2(4ac - b^2)^3(6b^6 - 400a^3c^3 + 291a^2b^2c^2 - 72ab^4c \\
& c)) + (((b((4ab^6c^3e^15 - 33a^2b^4c^4e^15 + 68a^3b^2c^5e^15 \\
& - 44a^2b^3c^5d^2e^15 + 6ab^5c^4d^2e^15 + 80a^3b^6c^6d^2e^15) / (\\
& a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) - (((4a^2b^8c^2e^ \\
& 16 - 52a^3b^6c^3e^16 + 224a^4b^4c^4e^16 - 320a^5b^2c^5e^16 + 2 \\
& a^2b^7c^3d^2e^16 - 36a^3b^5c^4d^2e^16 + 192a^4b^3c^5d^2e^16 - \\
& 320a^5b^6c^6d^2e^16) / (a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^ \\
& c^2) + ((2b^6e - 128a^3c^3e + 96a^2b^2c^2e - 24ab^4ce) * (4a^4b \\
& b^8c^2e^17 - 48a^5b^6c^3e^17 + 192a^6b^4c^4e^17 - 256a^7b^2c^5 \\
& * e^17 + 12a^3b^9c^2d^2e^17 - 184a^4b^7c^3d^2e^17 + 1056a^5b^5c^ \\
& ^4d^2e^17 - 2688a^6b^3c^5d^2e^17 + 2560a^7b^6c^6d^2e^17)) / (2 * (a^3 \\
& * b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) * (4a^2b^6e^2 - 256a^5 \\
& * c^3e^2 - 48a^3b^4ce^2 + 192a^4b^2c^2e^2))) * (2b^6e - 128a^3c^3 \\
& * e + 96a^2b^2c^2e - 24ab^4ce) / (2 * (4a^2b^6e^2 - 256a^5c^3e^2 \\
& - 48a^3b^4ce^2 + 192a^4b^2c^2e^2))) * (6ac - b^2) / (4a^2e(4ac \\
& - b^2)^{3/2}) - (((b(6ac - b^2) * ((4a^2b^8c^2e^16 - 52a^3b^6c^3e^ \\
& 16 + 224a^4b^4c^4e^16 - 320a^5b^2c^5e^16 + 2a^2b^7c^3d^2e^16 - \\
& 36a^3b^5c^4d^2e^16 + 192a^4b^3c^5d^2e^16 - 320a^5b^6c^6d^2e^1 \\
& 6) / (a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) + ((2b^6e - 128 \\
& a^3c^3e + 96a^2b^2c^2e - 24ab^4ce) * (4a^4b^8c^2e^17 - 48a^5b^ \\
& b^6c^3e^17 + 192a^6b^4c^4e^17 - 256a^7b^2c^5e^17 + 12a^3b^9c^2 \\
& * d^2e^17 - 184a^4b^7c^3d^2e^17 + 1056a^5b^5c^4d^2e^17 - 2688a^6 \\
& * b^3c^5d^2e^17 + 2560a^7b^6c^6d^2e^17)) / (2 * (a^3b^6 - 64a^6c^3 - 12 \\
& a^4b^4c + 48a^5b^2c^2) * (4a^2b^6e^2 - 256a^5c^3e^2 - 48a^3b^4c \\
& ce^2 + 192a^4b^2c^2e^2))) / (4a^2e(4ac - b^2)^{3/2}) + (b(6ac - \\
& b^2) * (2b^6e - 128a^3c^3e + 96a^2b^2c^2e - 24ab^4ce) * (4a^4b^ \\
& 8c^2e^17 - 48a^5b^6c^3e^17 + 192a^6b^4c^4e^17 - 256a^7b^2c^5e \\
& ^17 + 12a^3b^9c^2d^2e^17 - 184a^4b^7c^3d^2e^17 + 1056a^5b^5c^4 \\
& * d^2e^17 - 2688a^6b^3c^5d^2e^17 + 2560a^7b^6c^6d^2e^17)) / (8a^2e * \\
& (4ac - b^2)^{3/2} * (a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) * \\
& (4a^2b^6e^2 - 256a^5c^3e^2 - 48a^3b^4ce^2 + 192a^4b^2c^2e^2)) \\
&) * (2b^6e - 128a^3c^3e + 96a^2b^2c^2e - 24ab^4ce) / (2 * (4a^2b^ \\
& 6e^2 - 256a^5c^3e^2 - 48a^3b^4ce^2 + 192a^4b^2c^2e^2)) + (b^3 * (\\
& 6ac - b^2)^3 * (4a^4b^8c^2e^17 - 48a^5b^6c^3e^17 + 192a^6b^4c^4e
\end{aligned}$$

$$\begin{aligned}
& e^{17} - 256a^7b^2c^5e^{17} + 12a^3b^9c^2d^2e^{17} - 184a^4b^7c^3d^2 \\
& *e^{17} + 1056a^5b^5c^4d^2e^{17} - 2688a^6b^3c^5d^2e^{17} + 2560a^7b^* \\
& c^6d^2e^{17})/(64a^6e^3(4a*c - b^2)^{(9/2)}*(a^3b^6 - 64a^6c^3 - 12a^* \\
& ^4b^4c + 48a^5b^2c^2))*(3b^6 - 40a^3c^3 + 69a^2b^2c^2 - 27a*b^* \\
& 4c))/(8a^3c^2(4a*c - b^2)^{(7/2)}*(6b^6 - 400a^3c^3 + 291a^2b^2c^2 \\
& - 72a*b^4c)) + (3b*(b^4 + 11a^2c^2 - 7a*b^2c)*(((4a*b^6c^3e^{15} \\
& - 33a^2b^4c^4e^{15} + 68a^3b^2c^5e^{15} - 44a^2b^3c^5d^2e^{15} + 6a^* \\
& *b^5c^4d^2e^{15} + 80a^3b*c^6d^2e^{15})/(a^3b^6 - 64a^6c^3 - 12a^4b^* \\
& ^4c + 48a^5b^2c^2) - (((4a^2b^8c^2e^{16} - 52a^3b^6c^3e^{16} + 224a^* \\
& a^4b^4c^4e^{16} - 320a^5b^2c^5e^{16} + 2a^2b^7c^3d^2e^{16} - 36a^3b^* \\
& ^5c^4d^2e^{16} + 192a^4b^3c^5d^2e^{16} - 320a^5b*c^6d^2e^{16})/(a^3b^* \\
& ^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) + ((2b^6e - 128a^3c^3e \\
& e + 96a^2b^2c^2e - 24a*b^4c*e)*(4a^4b^8c^2e^{17} - 48a^5b^6c^3e \\
& ^{17} + 192a^6b^4c^4e^{17} - 256a^7b^2c^5e^{17} + 12a^3b^9c^2d^2e^{17} \\
& - 184a^4b^7c^3d^2e^{17} + 1056a^5b^5c^4d^2e^{17} - 2688a^6b^3c^5d^* \\
& ^2e^{17} + 2560a^7b^*c^6d^2e^{17}))/((2*(a^3b^6 - 64a^6c^3 - 12a^4b^4c \\
& + 48a^5b^2c^2)*(4a^2b^6e^2 - 256a^5c^3e^2 - 48a^3b^4c*e^2 + 1 \\
& 92a^4b^2c^2e^2)))*(2b^6e - 128a^3c^3e + 96a^2b^2c^2e - 24a*b^* \\
& 4c*e))/(2*(4a^2b^6e^2 - 256a^5c^3e^2 - 48a^3b^4c*e^2 + 192a^4b^* \\
& 2c^2e^2)))*(2b^6e - 128a^3c^3e + 96a^2b^2c^2e - 24a*b^4c*e))/(\\
& 2*(4a^2b^6e^2 - 256a^5c^3e^2 - 48a^3b^4c*e^2 + 192a^4b^2c^2e^2 \\
&)) - (b^4c^4e^{14} - 4a*b^2c^5e^{14} + b^3c^5d^2e^{14})/(a^3b^6 - 64a^6 \\
& *c^3 - 12a^4b^4c + 48a^5b^2c^2) + (b*(6a*c - b^2)*((b*(6a*c - b^2)* \\
& ((4a^2b^8c^2e^{16} - 52a^3b^6c^3e^{16} + 224a^4b^4c^4e^{16} - 320a^5 \\
& *b^2c^5e^{16} + 2a^2b^7c^3d^2e^{16} - 36a^3b^5c^4d^2e^{16} + 192a^4* \\
& b^3c^5d^2e^{16} - 320a^5b*c^6d^2e^{16})/(a^3b^6 - 64a^6c^3 - 12a^4b^* \\
& ^4c + 48a^5b^2c^2) + ((2b^6e - 128a^3c^3e + 96a^2b^2c^2e - 24a^* \\
& a*b^4c*e)*(4a^4b^8c^2e^{17} - 48a^5b^6c^3e^{17} + 192a^6b^4c^4e^{17} \\
& - 256a^7b^2c^5e^{17} + 12a^3b^9c^2d^2e^{17} - 184a^4b^7c^3d^2e^{17} \\
& 7 + 1056a^5b^5c^4d^2e^{17} - 2688a^6b^3c^5d^2e^{17} + 2560a^7b^*c^6 \\
& d^2e^{17}))/((2*(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2)*(4a^2 \\
& *b^6e^2 - 256a^5c^3e^2 - 48a^3b^4c*e^2 + 192a^4b^2c^2e^2))))/(4a^* \\
& ^2e*(4a*c - b^2)^{(3/2)}) + (b*(6a*c - b^2)*(2b^6e - 128a^3c^3e + 96 \\
& *a^2b^2c^2e - 24a*b^4c*e)*(4a^4b^8c^2e^{17} - 48a^5b^6c^3e^{17} + 192a^* \\
& 6b^4c^4e^{17} - 256a^7b^2c^5e^{17} + 12a^3b^9c^2d^2e^{17} - 184 \\
& *a^4b^7c^3d^2e^{17} + 1056a^5b^5c^4d^2e^{17} - 2688a^6b^3c^5d^2e^{17} \\
& 17 + 2560a^7b^*c^6d^2e^{17}))/((8a^2e*(4a*c - b^2)^{(3/2)}*(a^3b^6 - 64a^* \\
& ^6c^3 - 12a^4b^4c + 48a^5b^2c^2)*(4a^2b^6e^2 - 256a^5c^3e^2 - \\
& 48a^3b^4c*e^2 + 192a^4b^2c^2e^2))))/(4a^2e*(4a*c - b^2)^{(3/2)}) + \\
& (b^2*(6a*c - b^2)^2*(2b^6e - 128a^3c^3e + 96a^2b^2c^2e - 24a*b^4 \\
& *c*e)*(4a^4b^8c^2e^{17} - 48a^5b^6c^3e^{17} + 192a^6b^4c^4e^{17} - 25 \\
& 6a^7b^2c^5e^{17} + 12a^3b^9c^2d^2e^{17} - 184a^4b^7c^3d^2e^{17} + 1 \\
& 056a^5b^5c^4d^2e^{17} - 2688a^6b^3c^5d^2e^{17} + 2560a^7b^*c^6d^2e \\
& ^{17}))/((32a^4e^2*(4a*c - b^2)^3*(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48 \\
& *a^5b^2c^2)*(4a^2b^6e^2 - 256a^5c^3e^2 - 48a^3b^4c*e^2 + 192a^4
\end{aligned}$$

$$\frac{(b^2 c^2 e^2)))/(8 a^3 c^2 (4 a c - b^2)^3 (6 b^6 - 400 a^3 c^3 + 291 a^2 b^2 c^2 - 72 a b^4 c)))/(b^6 c^2 e^{14} - 12 a b^4 c^3 e^{14} + 36 a^2 b^2 c^4 e^{14}) (6 a c - b^2) / (2 a^2 e (4 a c - b^2)^{3/2})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out

$$3.514 \quad \int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=348

$$\frac{3b^2 - 10ac}{2a^2e(b^2 - 4ac)(d + ex)} - \frac{\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2 e (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(-(3b^2 - 10ac) \sqrt{b^2 - 4ac} + 16abc - 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2 e (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 1.65, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{3b^2 - 10ac}{2a^2e(b^2 - 4ac)(d + ex)} - \frac{\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2 e (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(-(3b^2 - 10ac) \sqrt{b^2 - 4ac} + 16abc - 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2 e (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{-2ac + b^2 + bc(d + ex)^2}{2ae(b^2 - 4ac)(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] $-(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (\text{Sqrt}[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + e*x))/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]])/(2*\text{Sqrt}[2]*a^2*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]*e) + (\text{Sqrt}[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + e*x))/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]])/(2*\text{Sqrt}[2]*a^2*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]*e)$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1121

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\right)}{\dots} \\
&= -\frac{3b^2-10ac}{2a^2(b^2-4ac)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)}{2a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} \\
&= -\frac{3b^2-10ac}{2a^2(b^2-4ac)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)}{2a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} \\
&= -\frac{3b^2-10ac}{2a^2(b^2-4ac)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)}{2a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}
\end{aligned}$$

Mathematica [A] time = 1.63, size = 339, normalized size = 0.97

$$\frac{2(d+ex)(-3abc-2a^2(d+ex)^2+b^3+b^2c(d+ex)^2)}{(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{4}{d+ex}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]

[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x
]

fricas [B] time = 1.86, size = 4330, normalized size = 12.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out]
$$-1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^2)*e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c - 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*e*x + \sqrt{1/2}*((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2)}*\log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*\sqrt{1/2}*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2)} - \sqrt{1/2}*((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3$$

$$\begin{aligned}
& + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - \\
& 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c \\
& ^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*\sqrt{-(9*b^7 \\
& 7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c \\
& + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*\sqrt{((81*b^8 - 918*a*b^6*c + 3051*a^2*b \\
& ^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^ \\
& 12*b^2*c^2 - 64*a^{13}*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 \\
& - 64*a^8*c^3)*e^2))*\log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - \\
& 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 250 \\
& 0*a^3*c^6)*d - 1/2*\sqrt{1/2}*((3*a^5*b^{10} - 55*a^6*b^8*c + 392*a^7*b^6*c^2 \\
& - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^{10}*c^5)*e^3*\sqrt{((81*b^8 - 9 \\
& 18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^{10}*b^6 \\
& - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)*e^4)) - (27*b^{11} - 486*a*b \\
& ^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b \\
& *b*c^5)*e)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a \\
& ^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*\sqrt{((81*b^8 - 918 \\
& *a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^{10}*b^6 - \\
& 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^ \\
& 4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2)) - \sqrt{1/2}*((a^2*b^2*c - 4*a^3*c \\
& ^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + \\
& 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + \\
& 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4 \\
& *a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^ \\
& 2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*\sqrt{-(9*b^7 \\
& - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c \\
& + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*\sqrt{((81*b^8 - 918*a*b^6*c + 3051*a^2*b^ \\
& 4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^1 \\
& 2*b^2*c^2 - 64*a^{13}*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - \\
& 64*a^8*c^3)*e^2))*\log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - \\
& 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500 \\
& *a^3*c^6)*d + 1/2*\sqrt{1/2}*((3*a^5*b^{10} - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - \\
& 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^{10}*c^5)*e^3*\sqrt{((81*b^8 - 91 \\
& 8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^{10}*b^6 - \\
& 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)*e^4)) + (27*b^{11} - 486*a*b^ \\
& 9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b \\
& *c^5)*e)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^ \\
& 5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*\sqrt{((81*b^8 - 918* \\
& a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^{10}*b^6 - 1 \\
& 2*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)*e^4)))/((a^5*b^6 - 12*a^6*b^4 \\
& *c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2)) + \sqrt{1/2}*((a^2*b^2*c - 4*a^3*c^ \\
& 2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 1 \\
& 0*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + \\
& 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4* \\
& a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2 \\
&)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*\sqrt{-(9*b^7
\end{aligned}$$

$$\begin{aligned}
& - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)*e^4)} \\
&)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2))*\log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*\sqrt{1/2}*((3*a^5*b^{10} - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^{10}*c^5)*e^3*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)*e^4)} + (27*b^{11} - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)*e^4)})))/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2)))/((a^2*b^2*c - 4*a^3*c^2)*e^6*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*x^4 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)
\end{aligned}$$

giac [B] time = 0.79, size = 847, normalized size = 2.43

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] 1/16*(2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*e^2 - (a^2*b^2*e^2 - 4*a^3*c*e^2)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a) + (3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2 + sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4*c*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a^4*c*e^4))))*e^(-3)/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*abs(a) + 1/16*(2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*e^2 + (a^2*b^2*e^2 - 4*a^3*c*e^2)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a) - (3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2 - sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4*c*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a^4*c*e^4))))*e^(-3)/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*abs(a)

$$a)) - 1/2*(b^2*c*e^{(-1)}/(x*e + d) - 2*a*c^2*e^{(-1)}/(x*e + d) + b^3*e^{(-1)}/(x*e + d)^3 - 3*a*b*c*e^{(-1)}/(x*e + d)^3)/((a^2*b^2 - 4*a^3*c)*(c + b/(x*e + d))^2 + a/(x*e + d)^4)) - e^{(-1)}/((x*e + d)*a^2)$$

maple [C] time = 0.03, size = 1304, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out]
$$-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*e^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e^2/(4*a*c-b^2)*x^3*b^2-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c^2*e/(4*a*c-b^2)*x^2+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c*e/(4*a*c-b^2)*x^2*b^2-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^2*d^2+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c*d^2-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*c+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*c^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^2*c-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b*c+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^3-1/4/a^2/(4*a*c-b^2)/e*sum((c*e^2*(10*a*c-3*b^2)*_R^2+2*c*d*e*(10*a*c-3*b^2)*_R+10*a*c^2*d^2-3*b^2*c*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/a^2/e/(e*x+d)$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 6.38, size = 10556, normalized size = 30.33

result too large to display

$$\begin{aligned}
& ^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2 \\
& *c^2*(-(4ac - b^2)^9)^{(1/2)} - 213a^2b^{11}c + 51a^2b^2c*(-(4ac - b^2)^9 \\
&)^{(1/2)})/(32*(a^5b^{12}e^2 + 4096a^{11}c^6e^2 - 24a^6b^{10}c^2e^2 + 240a^7 \\
& b^8c^2e^2 - 1280a^8b^6c^3e^2 + 3840a^9b^4c^4e^2 - 6144a^{10}b^2c^5e^2))^{(1/2)} \\
& *(x*(204800a^{12}c^9e^{12} + 144a^6b^{12}c^3e^{12} - 3264a^7b^{10}c^4e^{12} + 30112a^8b^8c^5e^{12} \\
& - 143360a^9b^6c^6e^{12} + 365568a^{10}b^4c^7e^{12} - 458752a^{11}b^2c^8e^{12}) + (-(9b^{13} - 9b^4 \\
& *(-(4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 \\
& + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2*(-(4ac - b^2)^9)^{(1/2)} - 213a^2b^{11}c \\
& + 51a^2b^2c*(-(4ac - b^2)^9)^{(1/2)})/(32*(a^5b^{12}e^2 + 4096a^{11}c^6e^2 - 24a^6b^{10}c^2e^2 \\
& + 240a^7b^8c^2e^2 - 1280a^8b^6c^3e^2 + 3840a^9b^4c^4e^2 - 6144a^{10}b^2c^5e^2))^{(1/2)} \\
& *((-9b^{13} - 9b^4*(-(4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 \\
& + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2*(-(4ac - b^2)^9)^{(1/2)} - 213a^2b^{11}c \\
& + 51a^2b^2c*(-(4ac - b^2)^9)^{(1/2)})/(32*(a^5b^{12}e^2 + 4096a^{11}c^6e^2 - 24a^6b^{10}c^2e^2 \\
& + 240a^7b^8c^2e^2 - 1280a^8b^6c^3e^2 + 3840a^9b^4c^4e^2 - 6144a^{10}b^2c^5e^2))^{(1/2)} \\
& *(x*(1048576a^{16}b^8c^8e^{14} + 256a^{10}b^{13}c^2e^{14} - 6144a^{11}b^{11}c^3e^{14} + 61440a^{12}b^9c^4e^{14} \\
& - 327680a^{13}b^7c^5e^{14} + 983040a^{14}b^5c^6e^{14} - 1572864a^{15}b^3c^7e^{14}) + 1048576a^{16}b^8c^8d^8e^{13} \\
& + 256a^{10}b^{13}c^2d^8e^{13} - 6144a^{11}b^{11}c^3d^8e^{13} + 61440a^{12}b^9c^4d^8e^{13} - 327680a^{13}b^7c^5d^8e^{13} \\
& + 983040a^{14}b^5c^6d^8e^{13} - 1572864a^{15}b^3c^7d^8e^{13}) - 851968a^{14}b^8c^8e^{12} - 192a^8b^{13}c^2e^{12} \\
& + 4672a^9b^{11}c^3e^{12} - 47360a^{10}b^9c^4e^{12} + 256000a^{11}b^7c^5e^{12} - 778240a^{12}b^5c^6e^{12} \\
& + 1261568a^{13}b^3c^7e^{12}) + 204800a^{12}c^9d^8e^{11} + 144a^6b^{12}c^3d^8e^{11} - 3264a^7b^{10}c^4d^8e^{11} \\
& + 30112a^8b^8c^5d^8e^{11} - 143360a^9b^6c^6d^8e^{11} + 365568a^{10}b^4c^7d^8e^{11} - 458752a^{11}b^2c^8d^8e^{11}) \\
& + 128000a^{10}c^9e^{10} + 504a^6b^8c^5e^{10} - 8112a^7b^6c^6e^{10} + 48704a^8b^4c^7e^{10} - 129280a^9b^2c^8e^{10})) \\
& *(-(9b^{13} - 9b^4*(-(4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 \\
& + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2*(-(4ac - b^2)^9)^{(1/2)} - 213a^2b^{11}c \\
& + 51a^2b^2c*(-(4ac - b^2)^9)^{(1/2)})/(32*(a^5b^{12}e^2 + 4096a^{11}c^6e^2 - 24a^6b^{10}c^2e^2 \\
& + 240a^7b^8c^2e^2 - 1280a^8b^6c^3e^2 + 3840a^9b^4c^4e^2 - 6144a^{10}b^2c^5e^2))^{(1/2)} \\
& *2i - \operatorname{atan}(((9b^{13} + 9b^4*(-(4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 \\
& + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2*(-(4ac - b^2)^9)^{(1/2)} - 213a^2b^{11}c - 51a^2b^2c \\
& *(-(4ac - b^2)^9)^{(1/2)})/(32*(a^5b^{12}e^2 + 4096a^{11}c^6e^2 - 24a^6b^{10}c^2e^2 + 240a^7b^8c^2e^2 \\
& - 1280a^8b^6c^3e^2 + 3840a^9b^4c^4e^2 - 6144a^{10}b^2c^5e^2))^{(1/2)}*(x*(204800a^{12}c^9e^{12} \\
& + 144a^6b^{12}c^3e^{12} - 3264a^7b^{10}c^4e^{12} + 30112a^8b^8c^5e^{12} - 143360a^9b^6c^6e^{12} \\
& + 365568a^{10}b^4c^7e^{12} - 458752a^{11}b^2c^8e^{12}) + (-(9b^{13} + 9b^4*(-(4ac - b^2)^9)^{(1/2)} \\
& + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 \\
& *(-(4ac - b^2)^9)^{(1/2)} - 213a^2b^{11}c - 51a^2b^2c*(-(4ac - b^2)^9)^{(1/2)})
\end{aligned}$$

$$\begin{aligned}
& /((32*(a^5*b^{12}*e^2 + 4096*a^{11}*c^6*e^2 - 24*a^6*b^{10}*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^{10}*b^2*c^5*e^2 \\
&))^{(1/2)}*((-(9*b^{13} + 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 \\
& + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^2 + 4096*a^{11}*c^6*e^2 - 24*a^6*b^{10}*c*e^2 \\
& + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^{10}*b^2*c^5*e^2)))^{(1/2)}*(x*(1048576*a^{16}*b*c^8*e^{14} + 256*a^{10}*b^{13}*c^2* \\
& e^{14} - 6144*a^{11}*b^{11}*c^3*e^{14} + 61440*a^{12}*b^9*c^4*e^{14} - 327680*a^{13}*b^7*c^5*e^{14} + 983040*a^{14}*b^5*c^6*e^{14} - 1572864*a^{15}*b^3*c^7*e^{14}) + 1048576* \\
& a^{16}*b*c^8*d*e^{13} + 256*a^{10}*b^{13}*c^2*d*e^{13} - 6144*a^{11}*b^{11}*c^3*d*e^{13} + 61440*a^{12}*b^9*c^4*d*e^{13} - 327680*a^{13}*b^7*c^5*d*e^{13} + 983040*a^{14}*b^5*c^6* \\
& d*e^{13} - 1572864*a^{15}*b^3*c^7*d*e^{13}) - 851968*a^{14}*b*c^8*e^{12} - 192*a^8*b^{13}*c^2*e^{12} + 4672*a^9*b^{11}*c^3*e^{12} - 47360*a^{10}*b^9*c^4*e^{12} + 256000*a^{11}*b^7*c^5* \\
& e^{12} - 778240*a^{12}*b^5*c^6*e^{12} + 1261568*a^{13}*b^3*c^7*e^{12}) + 204800*a^{12}*c^9*d*e^{11} + 144*a^6*b^{12}*c^3*d*e^{11} - 3264*a^7*b^{10}*c^4*d*e^{11} \\
& + 30112*a^8*b^8*c^5*d*e^{11} - 143360*a^9*b^6*c^6*d*e^{11} + 365568*a^{10}*b^4*c^7*d*e^{11} - 458752*a^{11}*b^2*c^8*d*e^{11})*i + (-(9*b^{13} + 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^2 + 4096*a^{11}*c^6*e^2 - 24*a^6*b^{10}*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^{10}*b^2*c^5*e^2)))^{(1/2)}*(x*(204800*a^{12}*c^9*e^{12} + 144*a^6*b^{12}*c^3*e^{12} - 3264*a^7*b^{10}*c^4*e^{12} + 30112*a^8*b^8*c^5*e^{12} - 143360*a^9*b^6*c^6*e^{12} + 365568*a^{10}*b^4*c^7*e^{12} - 458752*a^{11}*b^2*c^8*e^{12}) + (-(9*b^{13} + 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^2 + 4096*a^{11}*c^6*e^2 - 24*a^6*b^{10}*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^{10}*b^2*c^5*e^2)))^{(1/2)}*((-(9*b^{13} + 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^2 + 4096*a^{11}*c^6*e^2 - 24*a^6*b^{10}*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^{10}*b^2*c^5*e^2)))^{(1/2)}*(x*(1048576*a^{16}*b*c^8*e^{14} + 256*a^{10}*b^{13}*c^2*e^{14} - 6144*a^{11}*b^{11}*c^3*e^{14} + 61440*a^{12}*b^9*c^4*e^{14} - 327680*a^{13}*b^7*c^5*e^{14} + 983040*a^{14}*b^5*c^6*e^{14} - 1572864*a^{15}*b^3*c^7*e^{14}) + 1048576*a^{16}*b*c^8*d*e^{13} + 256*a^{10}*b^{13}*c^2*d*e^{13} - 6144*a^{11}*b^{11}*c^3*d*e^{13} + 61440*a^{12}*b^9*c^4*d*e^{13} - 327680*a^{13}*b^7*c^5*d*e^{13} + 983040*a^{14}*b^5*c^6*d*e^{13} - 1572864*a^{15}*b^3*c^7*d*e^{13}) + 851968*a^{14}*b*c^8*e^{12} + 192*a^8*b^{13}*c^2*e^{12} - 4672*a^9*b^{11}*c^3*e^{12} + 47360*a^{10}*b^9*c^4*e^{12} - 256000*a^{11}*b^7*c^5*e^{12} + 778240*a^{12}*b^5*c^6*e^{12} - 1261568*a^{13}*b^3*c^7*e^{12}) + 204800*a^{12}*c^9*d*e^{11} + 144*a^6*b^{12}*c^3*d*e^{11} - 3264*a^7*b^{10}*c^4*d*e^{11} + 30112*a^8*b^8*c^5*d*e^{11} - 143360*a^9*b^6*c^6*d*
\end{aligned}$$

$$\begin{aligned} &)^9)^{(1/2)}) / (32*(a^5*b^{12}*e^2 + 4096*a^{11}*c^6*e^2 - 24*a^6*b^{10}*c*e^2 + 240 \\ & *a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^{10}* \\ & b^2*c^5*e^2))^{(1/2)} * (x*(1048576*a^{16}*b*c^8*e^{14} + 256*a^{10}*b^{13}*c^2*e^{14} - \\ & 6144*a^{11}*b^{11}*c^3*e^{14} + 61440*a^{12}*b^9*c^4*e^{14} - 327680*a^{13}*b^7*c^5*e^{14} \\ & + 983040*a^{14}*b^5*c^6*e^{14} - 1572864*a^{15}*b^3*c^7*e^{14}) + 1048576*a^{16}*b \\ & *c^8*d*e^{13} + 256*a^{10}*b^{13}*c^2*d*e^{13} - 6144*a^{11}*b^{11}*c^3*d*e^{13} + 61440* \\ & a^{12}*b^9*c^4*d*e^{13} - 327680*a^{13}*b^7*c^5*d*e^{13} + 983040*a^{14}*b^5*c^6*d*e^{13} \\ & - 1572864*a^{15}*b^3*c^7*d*e^{13}) - 851968*a^{14}*b*c^8*e^{12} - 192*a^8*b^{13}*c \\ & ^2*e^{12} + 4672*a^9*b^{11}*c^3*e^{12} - 47360*a^{10}*b^9*c^4*e^{12} + 256000*a^{11}*b^7 \\ & *c^5*e^{12} - 778240*a^{12}*b^5*c^6*e^{12} + 1261568*a^{13}*b^3*c^7*e^{12}) + 204800 \\ & *a^{12}*c^9*d*e^{11} + 144*a^6*b^{12}*c^3*d*e^{11} - 3264*a^7*b^{10}*c^4*d*e^{11} + 301 \\ & 12*a^8*b^8*c^5*d*e^{11} - 143360*a^9*b^6*c^6*d*e^{11} + 365568*a^{10}*b^4*c^7*d*e \\ & ^{11} - 458752*a^{11}*b^2*c^8*d*e^{11}) + 128000*a^{10}*c^9*e^{10} + 504*a^6*b^8*c^5* \\ & e^{10} - 8112*a^7*b^6*c^6*e^{10} + 48704*a^8*b^4*c^7*e^{10} - 129280*a^9*b^2*c^8* \\ & e^{10})) * (- (9*b^{13} + 9*b^4 * (- (4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077* \\ & a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 2 \\ & 5*a^2*c^2 * (- (4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c * (- (4*a*c - b \\ & ^2)^9)^{(1/2)}) / (32*(a^5*b^{12}*e^2 + 4096*a^{11}*c^6*e^2 - 24*a^6*b^{10}*c*e^2 + 2 \\ & 40*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^{10}* \\ & b^2*c^5*e^2))^{(1/2)} * 2i - ((x*(3*b^3*d - 20*a*c^2*d^3 + 6*b^2*c*d^3 - 11* \\ & a*b*c*d)) / (a*(a*b^2 - 4*a^2*c)) - (x^4*(10*a*c^2*e^3 - 3*b^2*c*e^3)) / (2*a*(\\ & a*b^2 - 4*a^2*c)) - (2*x^3*(10*a*c^2*d*e^2 - 3*b^2*c*d*e^2)) / (a*(a*b^2 - 4* \\ & a^2*c)) + (2*a*b^2 - 8*a^2*c + 3*b^3*d^2 - 10*a*c^2*d^4 + 3*b^2*c*d^4 - 11* \\ & a*b*c*d^2) / (2*a*e*(a*b^2 - 4*a^2*c)) + (x^2*(3*b^3*e - 60*a*c^2*d^2*e + 18* \\ & b^2*c*d^2*e - 11*a*b*c*e)) / (2*a*(a*b^2 - 4*a^2*c))) / (a*d + x*(a*e + 3*b*d^2 \\ & *e + 5*c*d^4*e) + x^3*(b*e^3 + 10*c*d^2*e^3) + b*d^3 + c*d^5 + x^2*(10*c*d^3 \\ & *e^2 + 3*b*d*e^2) + c*e^5*x^5 + 5*c*d*e^4*x^4) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out

$$3.515 \quad \int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=213

$$\frac{b \log(a + b(d + ex)^2 + c(d + ex)^4)}{2a^3e} - \frac{2b \log(d + ex)}{a^3e} - \frac{b^2 - 3ac}{a^2e(b^2 - 4ac)(d + ex)^2} - \frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2c(d+ex)}{\sqrt{b^2-4ac}}\right)}{a^3e(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.39, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2c(d+ex)}{\sqrt{b^2-4ac}}\right)}{a^3e(b^2 - 4ac)^{3/2}} - \frac{b^2 - 3ac}{a^2e(b^2 - 4ac)(d + ex)^2} + \frac{b \log(a + b(d + ex)^2 + c(d + ex)^4)}{2a^3e} - \frac{2b \log(d + ex)}{a^3e} + \frac{-2ac + b^2 + bc(d + ex)^2}{2ae(b^2 - 4ac)(d + ex)^2(a + b(d + ex)^2 + c(d + ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^2)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e) - (2*b*Log[d + e*x])/(a^3*e) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634


```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^2} dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^2} dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^2} dx, x, (d+ex)^2\right)}{2e} \\
&= -\frac{b^2-3ac}{a^2(b^2-4ac)e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} \\
&= -\frac{b^2-3ac}{a^2(b^2-4ac)e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} \\
&= -\frac{b^2-3ac}{a^2(b^2-4ac)e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)} \\
&= -\frac{b^2-3ac}{a^2(b^2-4ac)e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 284, normalized size = 1.33

$$\frac{(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4)\log(-\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{(b^2-4ac)^{3/2}} + \frac{(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4)\log(\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{(b^2-4ac)^{3/2}} + \frac{a(-3abc-2ac^2(d+ex)^2+b^2+c(d+ex)^2)}{(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)} - \frac{a}{(d+ex)^2} - 4b\log(d+ex)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] $(-a/(d + e*x)^2 + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4) - 4*b*Log[d + e*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*sqrt[b^2 - 4*a*c] - 4*a*b*c*sqrt[$

$b^2 - 4ac$)*Log[b - Sqrt[b² - 4ac] + 2c*(d + ex)²]/(b² - 4ac)^(3/2) + ((-b⁴ + 6a*b²*c - 6a²*c² + b³*Sqrt[b² - 4ac] - 4a*b*c*Sqrt[b² - 4ac])*Log[b + Sqrt[b² - 4ac] + 2c*(d + ex)²]/(b² - 4ac)^(3/2))/(2a³*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + ex)³*(a + b*(d + ex)² + c*(d + ex)⁴)²), x]

[Out] IntegrateAlgebraic[1/((d + ex)³*(a + b*(d + ex)² + c*(d + ex)⁴)²), x]

fricas [B] time = 2.51, size = 4562, normalized size = 21.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)³/(a+b*(e*x+d)²+c*(e*x+d)⁴)²,x, algorithm="fricas")

[Out] [-1/2*(2*(a*b⁴*c - 7*a²*b²*c² + 12*a³*c³)*e⁴*x⁴ + 8*(a*b⁴*c - 7*a²*b²*c² + 12*a³*c³)*d*e³*x³ + a²*b⁴ - 8*a³*b²*c + 16*a⁴*c² + 2*(a*b⁴*c - 7*a²*b²*c² + 12*a³*c³)*d⁴ + (2*a*b⁵ - 15*a²*b³*c + 28*a³*b*c² + 12*(a*b⁴*c - 7*a²*b²*c² + 12*a³*c³)*d²)*e²*x² + (2*a*b⁵ - 15*a²*b³*c + 28*a³*b*c²)*d² + 2*(4*(a*b⁴*c - 7*a²*b²*c² + 12*a³*c³)*d³ + (2*a*b⁵ - 15*a²*b³*c + 28*a³*b*c²)*d)*e*x + ((b⁴*c - 6*a*b²*c² + 6*a²*c³)*e⁶*x⁶ + 6*(b⁴*c - 6*a*b²*c² + 6*a²*c³)*d*e⁵*x⁵ + (b⁵ - 6*a*b³*c + 6*a²*b*c² + 15*(b⁴*c - 6*a*b²*c² + 6*a²*c³)*d²)*e⁴*x⁴ + (b⁴*c - 6*a*b²*c² + 6*a²*c³)*d⁶ + 4*(5*(b⁴*c - 6*a*b²*c² + 6*a²*c³)*d³ + (b⁵ - 6*a*b³*c + 6*a²*b*c²)*d)*e³*x³ + (b⁵ - 6*a*b³*c + 6*a²*b*c²)*d⁴ + (a*b⁴ - 6*a²*b²*c + 6*a³*c² + 15*(b⁴*c - 6*a*b²*c² + 6*a²*c³)*d⁴ + 6*(b⁵ - 6*a*b³*c + 6*a²*b*c²)*d²)*e²*x² + (a*b⁴ - 6*a²*b²*c + 6*a³*c²)*d² + 2*(3*(b⁴*c - 6*a*b²*c² + 6*a²*c³)*d⁵ + 2*(b⁵ - 6*a*b³*c + 6*a²*b*c²)*d³ + (a*b⁴ - 6*a²*b²*c + 6*a³*c²)*d)*e*x)*sqrt(b² - 4a*c)*log((2*c²*e⁴*x⁴ + 8*c²*d*e³*x³ + 2*c²*d⁴ + 2*(6*c²*d² + b*c)*e²*x² + 2*b*c*d² + 4*(2*c²*d³ + b*c*d)*e*x + b² - 2*a*c + (2*c*e²*x² + 4*c*d*e*x + 2*c*d² + b)*sqrt(b² - 4a*c))/(c*e⁴*x⁴ + 4*c*d*e³*x³ + c*d⁴ + (6*c*d² + b)*e²*x² + b*d² + 2*(2*c*d³ + b*d)*e*x + a) - ((b⁵*c - 8*a*b³*c² + 16*a²*b*c³)*e⁶*x⁶ + 6*(b⁵*c - 8*a*b³*c² + 16*a²*b*c³)*d*e⁵*x⁵ + (b⁶ - 8*a*b⁴*c + 16*a²*b²*c² + 15*(b⁵*c - 8*a*b³*c² + 16*a²*b*c³)*d²)*e⁴*x⁴ + (b⁵*c - 8*a*b³*c² + 16*a²*b*c³)*d⁶ + 4*(5*(b⁵*c - 8*a*b³*c² +

$$\begin{aligned}
& 16a^2bc^3)d^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)d)e^3x^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)d^4 + (ab^5 - 8a^2b^3c + 16a^3bc^2 + 15(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^4 + 6(b^6 - 8a^2b^4c + 16a^2b^2c^2)d^2)e^2x^2 + (ab^5 - 8a^2b^3c + 16a^3bc^2)d^2 + 2(3(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^5 + 2(b^6 - 8a^2b^4c + 16a^2b^2c^2)d^3 + (ab^5 - 8a^2b^3c + 16a^3bc^2)d)e^2x) \log(c^2e^4x^4 + 4cd^2e^3x^3 + cd^4 + (6cd^2 + b)e^2x^2 + bd^2 + 2(2cd^3 + bd)e^2x + a) + 4((b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)e^6x^6 + 6(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^2e^5x^5 + (b^6 - 8a^2b^4c + 16a^2b^2c^2 + 15(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^2)e^4x^4 + (b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^6 + 4(5(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)d)e^3x^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)d^4 + (ab^5 - 8a^2b^3c + 16a^3bc^2 + 15(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^4 + 6(b^6 - 8a^2b^4c + 16a^2b^2c^2)d^2)e^2x^2 + (ab^5 - 8a^2b^3c + 16a^3bc^2)d^2 + 2(3(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^5 + 2(b^6 - 8a^2b^4c + 16a^2b^2c^2)d^3 + (ab^5 - 8a^2b^3c + 16a^3bc^2)d)e^2x) \log(ex + d) / ((a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)e^7x^6 + 6(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^2e^6x^5 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2 + 15(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^2)e^5x^4 + 4(5(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^3 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)d)e^4x^3 + (a^4b^4 - 8a^5b^2c + 16a^6c^2 + 15(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^4 + 6(a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)d^2)e^3x^2 + 2(3(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^5 + 2(a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)d^3 + (a^4b^4 - 8a^5b^2c + 16a^6c^2)d)e^2x + ((a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^6 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)d^4 + (a^4b^4 - 8a^5b^2c + 16a^6c^2)d^2)e), -1/2(2(ab^4c - 7a^2b^2c^2 + 12a^3c^3)e^4x^4 + 8(ab^4c - 7a^2b^2c^2 + 12a^3c^3)d^2e^3x^3 + a^2b^4 - 8a^3b^2c + 16a^4c^2 + 2(ab^4c - 7a^2b^2c^2 + 12a^3c^3)d^4 + (2ab^5 - 15a^2b^3c + 28a^3b^2c^2 + 12(a^2b^4c - 7a^2b^2c^2 + 12a^3c^3)d^2)e^2x^2 + (2ab^5 - 15a^2b^3c + 28a^3b^2c^2)d^2 + 2(4(ab^4c - 7a^2b^2c^2 + 12a^3c^3)d^3 + (2ab^5 - 15a^2b^3c + 28a^3b^2c^2)d)e^2x) \log(ex + d) + 6(b^4c - 6a^2b^2c^2 + 6a^2c^3)d^2e^5x^5 + (b^5 - 6a^2b^3c + 6a^2b^2c^2 + 15(b^4c - 6a^2b^2c^2 + 6a^2c^3)d^2)e^4x^4 + (b^4c - 6a^2b^2c^2 + 6a^2c^3)d^6 + 4(5(b^4c - 6a^2b^2c^2 + 6a^2c^3)d^3 + (b^5 - 6a^2b^3c + 6a^2b^2c^2)d)e^3x^3 + (b^5 - 6a^2b^3c + 6a^2b^2c^2)d^4 + (ab^4 - 6a^2b^2c + 6a^3c^2 + 15(b^4c - 6a^2b^2c^2 + 6a^2c^3)d^4 + 6(b^5 - 6a^2b^3c + 6a^2b^2c^2)d^2)e^2x^2 + (ab^4 - 6a^2b^2c + 6a^3c^2)d^2 + 2(3(b^4c - 6a^2b^2c^2 + 6a^2c^3)d^5 + 2(b^5 - 6a^2b^3c + 6a^2b^2c^2)d^3 + (ab^4 - 6a^2b^2c + 6a^3c^2)d)e^2x) \sqrt{-b^2 + 4ac} \arctan(-2c^2e^2x^2 + 4cd^2e^2x + 2cd^2 + b) \sqrt{-b^2 + 4ac} / (b^2 - 4ac) - ((b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)e^6x^6 + 6(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^2e^5x^5 + (b^6 - 8a^2b^4c + 16a^2b^2c^2 + 15(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^2)e^4x^4 + (b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^6 + 4(5(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)d)e^3x^3 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)d^4 + (ab^5 - 8a^2b^3c + 16a^3bc^2 + 15(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^4 + 6(b^6 - 8a^2b^4c + 16a^2b^2c^2)d^2)e^2x^2 + (ab^5 - 8a^2b^3c + 16a^3bc^2)d^2 + 2(3(b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)d^5 + 2(b^6 - 8a^2b^4c + 16a^2b^2c^2)d^3 + (ab^5 - 8a^2b^3c + 16a^3bc^2)d)e^2x) \log(ex + d) / ((a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)e^7x^6 + 6(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^2e^6x^5 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2 + 15(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^2)e^5x^4 + 4(5(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^3 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)d)e^4x^3 + (a^4b^4 - 8a^5b^2c + 16a^6c^2 + 15(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^4 + 6(a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)d^2)e^3x^2 + 2(3(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^5 + 2(a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)d^3 + (a^4b^4 - 8a^5b^2c + 16a^6c^2)d)e^2x + ((a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)d^6 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)d^4 + (a^4b^4 - 8a^5b^2c + 16a^6c^2)d^2)e)
\end{aligned}$$

$$\begin{aligned}
& d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*\log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*\log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*x^5 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e]
\end{aligned}$$

giac [A] time = 0.43, size = 224, normalized size = 1.05

$$\frac{(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(-\frac{b + \frac{2a}{(xe+d)^2}}{\sqrt{-b^2 + 4ac}}\right) e^{(-1)}}{(a^3b^2 - 4a^4c)\sqrt{-b^2 + 4ac}} + \frac{be^{(-1)} \log\left(c + \frac{b}{(xe+d)^2} + \frac{a}{(xe+d)^4}\right)}{2a^3} + \frac{\left(\frac{b^3c - 3abc^2}{a} + \frac{(b^4e - 4ab^2ce + 2a^2c^2e)e^{(-1)}}{(xe+d)^2a}\right) e^{(-1)}}{2(b^2 - 4ac)a^2\left(c + \frac{b}{(xe+d)^2} + \frac{a}{(xe+d)^4}\right)} - \frac{e^{(-1)}}{2(xe+d)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) + 1/2*b*e^(-1)*log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^3 + 1/2*((b^3*c - 3*a*b*c^2)/a + (b^4*e - 4*a*b^2*c*e + 2*a^2*c^2*e)*e^(-1)/((x*e + d)^2*a))*e^(-1)/((b^2 - 4*a*c)*a^2*(c + b/(x*e + d)^2 + a/(x*e + d)^4)) - 1/2*e^(-1)/((x*e + d)^2*a^2)

maple [C] time = 0.04, size = 1014, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)$

[Out]
$$\begin{aligned} & -1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2 \\ & *b*d*e*x+b*d^2+a)*c^2*e/(4*a*c-b^2)*x^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6* \\ & c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e/(4*a*c-b^2 \\ &)*x^2*b^2-2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^ \\ & 2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*d/(4*a*c-b^2)*x+1/a^2/(c*e^4*x^4+4*c*d*e^3*x \\ & ^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a* \\ & c-b^2)*x*b^2-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2 \\ & *x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c^2*d^2+1/2/a^2/(c*e^4*x^4+4*c* \\ & d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/ \\ & (4*a*c-b^2)*b^2*c*d^2-3/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^ \\ & 3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c+1/2/a^2/(c*e^4*x \\ & ^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^ \\ & 2+a)/e/(4*a*c-b^2)*b^3+1/a^3/(4*a*c-b^2)/e*\text{sum}((b*c*e^3*(4*a*c-b^2)*_R^3+3* \\ & b*c*d*e^2*(4*a*c-b^2)*_R^2+e*(12*a*b*c^2*d^2-3*b^3*c*d^2-3*a^2*c^2+5*a*b^2* \\ & c-b^4)*_R+4*a*b*c^2*d^3-b^3*c*d^3-3*a^2*c^2*d+5*a*b^2*c*d-b^4*d)/(2*_R^3*c* \\ & e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*\ln(-_R+x),_R=\text{RootOf}(_Z^ \\ & 4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b* \\ & d*e)*_Z+a))-1/2/a^2/e/(e*x+d)^2-2*b*\ln(e*x+d)/a^3/e \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, \text{algorithm}=\text{"maxima"})$

[Out] Timed out

mupad [B] time = 12.32, size = 12436, normalized size = 58.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)$

[Out]
$$\begin{aligned} & ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2) \\ & - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d* \\ & e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6* \\ & a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2* \\ & b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2 \\ &))/(x^4*(b*e^4 + 15*c*d^2*e^4) + a*d^2 + b*d^4 + c*d^6 + x*(2*a*d*e + 4*b* \end{aligned}$$

$$\begin{aligned}
& d^3e + 6cd^5e) + x^2*(a^2e^2 + 6bd^2e^2 + 15c^2d^4e^2) + x^3*(20c^2d^3e^3 + 4b^2d^2e^3) + c^2e^6x^6 + 6c^2d^5e^5x^5) + (\log((((b + a^3e*(-(b^4 + 6a^2c^2 - 6ab^2c)^2/(a^6e^2*(4ac - b^2)^3))^{1/2}))^{1/2})*((b + a^3e*(-(b^4 + 6a^2c^2 - 6ab^2c)^2/(a^6e^2*(4ac - b^2)^3))^{1/2}))*((4c^2e^{16}(2b^5 + 6a^2b^2c^2 + b^4cd^2 - 30a^2c^3d^2 - 10ab^3c + 2ab^2c^2d^2))/(a^2*(4ac - b^2)) + (4c^3e^{18}x^2*(b^4 - 30a^2c^2 + 2ab^2c))/(a^2*(4ac - b^2)) - (2b^2e^{16}(b + a^3e*(-(b^4 + 6a^2c^2 - 6ab^2c)^2/(a^6e^2*(4ac - b^2)^3))^{1/2})*(ab + 3b^2d^2 + 3b^2e^2x^2 - 10acd^2 + 6b^2d^2e^2x - 10ac^2e^2x^2 - 20acd^2e^2x))/a^3 + (8c^3d^2e^{17}x*(b^4 - 30a^2c^2 + 2ab^2c))/(a^2*(4ac - b^2))))/(2a^3e) - (4c^3e^{15}(3ac - b^2)*(4b^4 + 3a^2c^2 + 6b^3cd^2 - 17ab^2c - 23ab^2cd^2))/(a^4*(4ac - b^2)^2) + (4b^2c^4e^{17}x^2*(6b^4 + 69a^2c^2 - 41ab^2c))/(a^4*(4ac - b^2)^2) + (8b^2c^4d^2e^{16}x*(6b^4 + 69a^2c^2 - 41ab^2c))/(a^4*(4ac - b^2)^2)))/(2a^3e) - (8c^5e^{16}x^2*(3ac - b^2)^3)/(a^6*(4ac - b^2)^3) + (8c^4e^{14}(3ac - b^2)^2*(b^3 - 3ac^2d^2 + b^2cd^2 - 4ab^2c))/(a^6*(4ac - b^2)^3) - (16c^5d^2e^{15}x*(3ac - b^2)^3)/(a^6*(4ac - b^2)^3))*(((b - a^3e*(-(b^4 + 6a^2c^2 - 6ab^2c)^2/(a^6e^2*(4ac - b^2)^3))^{1/2}))^{1/2})*(((b - a^3e*(-(b^4 + 6a^2c^2 - 6ab^2c)^2/(a^6e^2*(4ac - b^2)^3))^{1/2}))*((4c^2e^{16}(2b^5 + 6a^2b^2c^2 + b^4cd^2 - 30a^2c^3d^2 - 10ab^3c + 2ab^2c^2d^2))/(a^2*(4ac - b^2)) + (4c^3e^{18}x^2*(b^4 - 30a^2c^2 + 2ab^2c))/(a^2*(4ac - b^2)) - (2b^2e^{16}(b - a^3e*(-(b^4 + 6a^2c^2 - 6ab^2c)^2/(a^6e^2*(4ac - b^2)^3))^{1/2})*(ab + 3b^2d^2 + 3b^2e^2x^2 - 10acd^2 + 6b^2d^2e^2x - 10ac^2e^2x^2 - 20acd^2e^2x))/a^3 + (8c^3d^2e^{17}x*(b^4 - 30a^2c^2 + 2ab^2c))/(a^2*(4ac - b^2))))/(2a^3e) - (4c^3e^{15}(3ac - b^2)*(4b^4 + 3a^2c^2 + 6b^3cd^2 - 17ab^2c - 23ab^2cd^2))/(a^4*(4ac - b^2)^2) + (4b^2c^4e^{17}x^2*(6b^4 + 69a^2c^2 - 41ab^2c))/(a^4*(4ac - b^2)^2) + (8b^2c^4d^2e^{16}x*(6b^4 + 69a^2c^2 - 41ab^2c))/(a^4*(4ac - b^2)^2)))/(2a^3e) - (8c^5e^{16}x^2*(3ac - b^2)^3)/(a^6*(4ac - b^2)^3) + (8c^4e^{14}(3ac - b^2)^2*(b^3 - 3ac^2d^2 + b^2cd^2 - 4ab^2c))/(a^6*(4ac - b^2)^3) - (16c^5d^2e^{15}x*(3ac - b^2)^3)/(a^6*(4ac - b^2)^3))*((b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^3c^3e))/(2*(a^3b^6e^2 - 64a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2)) - (2b*log(d + ex))/(a^3e) - (atan((((2a^9b^6(4ac - b^2)^{9/2} - 128a^12c^3(4ac - b^2)^{9/2} - 24a^10b^4c*(4ac - b^2)^{9/2} + 96a^11b^2c^2*(4ac - b^2)^{9/2})*((8*(54a^3c^8d^2e^{15} - 2b^6c^5d^2e^{15} + 18ab^4c^6d^2e^{15} - 54a^2b^2c^7d^2e^{15}))/a^6*b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) - (((8*(276a^5b^3c^7d^2e^{16} - 6a^2b^7c^4d^2e^{16} + 65a^3b^5c^5d^2e^{16} - 233a^4b^3c^6d^2e^{16} - 233a^4b^3c^6d^2e^{16}))/a^6*b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) - (((8*(480a^8c^7d^2e^{17} - a^4b^8c^3d^2e^{17} + 6a^5b^6c^4d^2e^{17} + 30a^6b^4c^5d^2e^{17} - 272a^7b^2c^6d^2e^{17}))/a^6*b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) - (4*(b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^3c^3e)*(640a^10b^2c^6d^2e^{18} + 3a^6b^9c^2d^2e^{18} - 46a^7b^7c^3d^2e^{18} + 264a^8b^5c^4d^2e^{18} - 672a^9b^3c^5d^2e^{18}))/((a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)))))
\end{aligned}$$

$$\begin{aligned}
& \left(a^4 e^{18} + 30 a^6 b^4 c^5 e^{18} - 272 a^7 b^2 c^6 e^{18} \right) / \left(a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2 \right) - \left(2 (b^7 e + 48 a^2 b^3 c^2 e - 12 a^5 c^3 e - 64 a^3 b^3 c^3 e) * (640 a^{10} b^6 c^6 e^{19} + 3 a^6 b^9 c^2 e^{19} - 46 a^7 b^7 c^3 e^{19} + 264 a^8 b^5 c^4 e^{19} - 672 a^9 b^3 c^5 e^{19}) \right) / \left((a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2) * (a^3 b^6 e^2 - 64 a^6 c^3 e^2 - 12 a^4 b^4 c e^2 + 48 a^5 b^2 c^2 e^2) \right) * \left(b^4 + 6 a^2 c^2 - 6 a b^2 c \right) / \left(2 a^3 e * (4 a c - b^2)^{(3/2)} \right) - \left((b^4 + 6 a^2 c^2 - 6 a b^2 c) * (b^7 e + 48 a^2 b^3 c^2 e - 12 a^5 c^3 e - 64 a^3 b^3 c^3 e) * (640 a^{10} b^6 c^6 e^{19} + 3 a^6 b^9 c^2 e^{19} - 46 a^7 b^7 c^3 e^{19} + 264 a^8 b^5 c^4 e^{19} - 672 a^9 b^3 c^5 e^{19}) \right) / \left(a^3 e * (4 a c - b^2)^{(3/2)} * (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2) * (a^3 b^6 e^2 - 64 a^6 c^3 e^2 - 12 a^4 b^4 c e^2 + 48 a^5 b^2 c^2 e^2) \right) * \left(b^7 e + 48 a^2 b^3 c^2 e - 12 a^5 c^3 e - 64 a^3 b^3 c^3 e \right) / \left(2 * (a^3 b^6 e^2 - 64 a^6 c^3 e^2 - 12 a^4 b^4 c e^2 + 48 a^5 b^2 c^2 e^2) \right) - \left(\left(4 * (276 a^5 b^6 c^7 e^{17} - 6 a^2 b^7 c^4 e^{17} + 65 a^3 b^5 c^5 e^{17} - 233 a^4 b^3 c^6 e^{17}) \right) / \left(a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2 \right) - \left(\left(4 * (480 a^8 c^7 e^{18} - a^4 b^8 c^3 e^{18} + 6 a^5 b^6 c^4 e^{18} + 30 a^6 b^4 c^5 e^{18} - 272 a^7 b^2 c^6 e^{18}) \right) / \left(a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2 \right) - \left(2 * (b^7 e + 48 a^2 b^3 c^2 e - 12 a^5 c^3 e - 64 a^3 b^3 c^3 e) * (640 a^{10} b^6 c^6 e^{19} + 3 a^6 b^9 c^2 e^{19} - 46 a^7 b^7 c^3 e^{19} + 264 a^8 b^5 c^4 e^{19} - 672 a^9 b^3 c^5 e^{19}) \right) / \left((a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2) * (a^3 b^6 e^2 - 64 a^6 c^3 e^2 - 12 a^4 b^4 c e^2 + 48 a^5 b^2 c^2 e^2) \right) * (b^7 e + 48 a^2 b^3 c^2 e - 12 a^5 c^3 e - 64 a^3 b^3 c^3 e) \right) / \left(2 * (a^3 b^6 e^2 - 64 a^6 c^3 e^2 - 12 a^4 b^4 c e^2 + 48 a^5 b^2 c^2 e^2) \right) * \left(b^4 + 6 a^2 c^2 - 6 a b^2 c \right) / \left(2 a^3 e * (4 a c - b^2)^{(3/2)} \right) + \left((b^4 + 6 a^2 c^2 - 6 a b^2 c)^3 * (640 a^{10} b^6 c^6 e^{19} + 3 a^6 b^9 c^2 e^{19} - 46 a^7 b^7 c^3 e^{19} + 264 a^8 b^5 c^4 e^{19} - 672 a^9 b^3 c^5 e^{19}) \right) / \left(2 a^9 e^3 * (4 a c - b^2)^{(9/2)} * (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2) \right) * \left(3 b^6 - 49 a^3 c^3 + 72 a^2 b^2 c^2 - 27 a b^4 c \right) / \left(8 a^3 c^2 * (4 a c - b^2)^{(7/2)} * (9 a^4 c^4 - 6 b^8 - 288 a^2 b^4 c^2 + 382 a^3 b^2 c^3 + 72 a b^6 c) \right) + \left(\left(\left(\left(4 * (36 a^6 c^7 e^{15} + 4 a^2 b^8 c^3 e^{15} - 45 a^3 b^6 c^4 e^{15} + 170 a^4 b^4 c^5 e^{15} - 225 a^5 b^2 c^6 e^{15} + 6 a^2 b^7 c^4 d^2 e^{15} - 65 a^3 b^5 c^5 d^2 e^{15} + 233 a^4 b^3 c^6 d^2 e^{15} - 276 a^5 b^6 c^4 d^2 e^{15}) \right) / \left(a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2 \right) - \left(\left(4 * (96 a^8 b^6 c^6 e^{16} + 2 a^4 b^9 c^2 e^{16} - 26 a^5 b^7 c^3 e^{16} + 118 a^6 b^5 c^4 e^{16} - 208 a^7 b^3 c^5 e^{16} - 480 a^8 c^7 d^2 e^{16} + a^4 b^8 c^3 d^2 e^{16} - 6 a^5 b^6 c^4 d^2 e^{16} - 30 a^6 b^4 c^5 d^2 e^{16} + 272 a^7 b^2 c^6 d^2 e^{16}) \right) / \left(a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2 \right) + \left(2 * (b^7 e + 48 a^2 b^3 c^2 e - 12 a^5 c^3 e - 64 a^3 b^3 c^3 e) * (a^7 b^8 c^2 e^{17} - 12 a^8 b^6 c^3 e^{17} + 48 a^9 b^4 c^4 e^{17} - 64 a^{10} b^2 c^5 e^{17} + 3 a^6 b^9 c^2 d^2 e^{17} - 46 a^7 b^7 c^3 d^2 e^{17} + 264 a^8 b^5 c^4 d^2 e^{17} - 672 a^9 b^3 c^5 d^2 e^{17} + 640 a^{10} b^6 c^6 d^2 e^{17}) \right) / \left((a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2) * (a^3 b^6 e^2 - 64 a^6 c^3 e^2 - 12 a^4 b^4 c e^2 + 48 a^5 b^2 c^2 e^2) \right) * (b^7 e + 48 a^2 b^3 c^2 e - 12 a^5 c^3 e - 64 a^3 b^3 c^3 e) \right) / \left(2 * (a^3 b^6 e^2 - 64 a^6 c^3 e^2 - 12 a^4 b^4 c e^2 + 48 a^5 b^2 c^2 e^2) \right) * \left(b^7 e + 48 a^2 b^3 c^2 e - 12 a^5 c^3 e - 64 a^3 b^3 c^3 e \right) / \left(2 * (a^3 b^6 e^2 - 64 a^6 c^3 e^2 - 12 a^4 b^4 c e^2 + 48 a^5 b^2 c^2 e^2) \right)
\end{aligned}$$

$$\begin{aligned}
& (4a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2) - (4(2b^7c^4e^{14} - 20ab^5c^5e^{14} - 72a^3b^3c^7e^{14} + 66a^2b^3c^6e^{14} - 54a^3c^8d^2e^{14} + 2b^6c^5d^2e^{14} + 54a^2b^2c^7d^2e^{14} - 18ab^4c^6d^2e^{14}))/((a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) + (((4(96a^8b^3c^6e^{16} + 2a^4b^9c^2e^{16} - 26a^5b^7c^3e^{16} + 118a^6b^5c^4e^{16} - 208a^7b^3c^5e^{16} - 480a^8c^7d^2e^{16} + a^4b^8c^3d^2e^{16} - 6a^5b^6c^4d^2e^{16} - 30a^6b^4c^5d^2e^{16} + 272a^7b^2c^6d^2e^{16}))/((a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) + (2(b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^3c^3e))(a^7b^8c^2e^{17} - 12a^8b^6c^3e^{17} + 48a^9b^4c^4e^{17} - 64a^{10}b^2c^5e^{17} + 3a^6b^9c^2d^2e^{17} - 46a^7b^7c^3d^2e^{17} + 264a^8b^5c^4d^2e^{17} - 672a^9b^3c^5d^2e^{17} + 640a^{10}b^3c^5d^2e^{17} + 640a^{10}b^3c^6d^2e^{17}))/((a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)(a^3b^6e^2 - 64a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2)))(b^4 + 6a^2c^2 - 6ab^2c))/(2a^3e(4ac - b^2))^{(3/2)} + ((b^4 + 6a^2c^2 - 6ab^2c)(b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^3c^3e)(a^7b^8c^2e^{17} - 12a^8b^6c^3e^{17} + 48a^9b^4c^4e^{17} - 64a^{10}b^2c^5e^{17} + 3a^6b^9c^2d^2e^{17} - 46a^7b^7c^3d^2e^{17} + 264a^8b^5c^4d^2e^{17} - 672a^9b^3c^5d^2e^{17} + 640a^{10}b^3c^5d^2e^{17} + 640a^{10}b^3c^6d^2e^{17}))/((a^3e(4ac - b^2))^{(3/2)}(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)(a^3b^6e^2 - 64a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2)))(b^4 + 6a^2c^2 - 6ab^2c))/(2a^3e(4ac - b^2))^{(3/2)} + ((b^4 + 6a^2c^2 - 6ab^2c)^2(b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^3c^3e)(a^7b^8c^2e^{17} - 12a^8b^6c^3e^{17} + 48a^9b^4c^4e^{17} - 64a^{10}b^2c^5e^{17} + 3a^6b^9c^2d^2e^{17} - 46a^7b^7c^3d^2e^{17} + 264a^8b^5c^4d^2e^{17} - 672a^9b^3c^5d^2e^{17} + 640a^{10}b^3c^5d^2e^{17} + 640a^{10}b^3c^6d^2e^{17}))/((2a^6e^2(4ac - b^2))^3(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)(a^3b^6e^2 - 64a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2)))(3b^6 - 3a^3c^3 + 36a^2b^2c^2 - 21ab^4c))/(8a^3c^2(4ac - b^2))^3(9a^4c^4 - 6b^8 - 288a^2b^4c^2 + 382a^3b^2c^3 + 72ab^6c)) - (b((((4(36a^6c^7e^{15} + 4a^2b^8c^3e^{15} - 45a^3b^6c^4e^{15} + 170a^4b^4c^5e^{15} - 225a^5b^2c^6e^{15} + 6a^2b^7c^4d^2e^{15} - 65a^3b^5c^5d^2e^{15} + 233a^4b^3c^6d^2e^{15} - 276a^5b^3c^7d^2e^{15}))/((a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) - (((4(96a^8b^3c^6e^{16} + 2a^4b^9c^2e^{16} - 26a^5b^7c^3e^{16} + 118a^6b^5c^4e^{16} - 208a^7b^3c^5e^{16} - 480a^8c^7d^2e^{16} + a^4b^8c^3d^2e^{16} - 6a^5b^6c^4d^2e^{16} - 30a^6b^4c^5d^2e^{16} + 272a^7b^2c^6d^2e^{16}))/((a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) + (2(b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^3c^3e))(a^7b^8c^2e^{17} - 12a^8b^6c^3e^{17} + 48a^9b^4c^4e^{17} - 64a^{10}b^2c^5e^{17} + 3a^6b^9c^2d^2e^{17} - 46a^7b^7c^3d^2e^{17} + 264a^8b^5c^4d^2e^{17} - 672a^9b^3c^5d^2e^{17} + 640a^{10}b^3c^5d^2e^{17} + 640a^{10}b^3c^6d^2e^{17}))/((a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)(a^3b^6e^2 - 64a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2)))(b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^3c^3e))/(2(a^3b^6e^2 - 64a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2)))(b^4 + 6a^2c^2 - 6ab^2c))/(2a^3e(4ac - b^2))^{(3/2)}
\end{aligned}$$

$$\frac{3}{2}) - \left(\frac{4(96a^8bc^6e^{16} + 2a^4b^9c^2e^{16} - 26a^5b^7c^3e^{16} + 118a^6b^5c^4e^{16} - 208a^7b^3c^5e^{16} - 480a^8c^7d^2e^{16} + a^4b^8c^3d^2e^{16} - 6a^5b^6c^4d^2e^{16} - 30a^6b^4c^5d^2e^{16} + 272a^7b^2c^6d^2e^{16})}{(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) + (2(b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^2c^3e))(a^7b^8c^2e^{17} - 12a^8b^6c^3e^{17} + 48a^9b^4c^4e^{17} - 64a^{10}b^2c^5e^{17} + 3a^6b^9c^2d^2e^{17} - 46a^7b^7c^3d^2e^{17} + 264a^8b^5c^4d^2e^{17} - 672a^9b^3c^5d^2e^{17} + 640a^{10}b^2c^6d^2e^{17})}{(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)(a^3b^6e^2 - 64a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2)} \right) (b^4 + 6a^2c^2 - 6ab^2c) / (2a^3e(4ac - b^2)^{3/2}) + ((b^4 + 6a^2c^2 - 6ab^2c)(b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^2c^3e))(a^7b^8c^2e^{17} - 12a^8b^6c^3e^{17} + 48a^9b^4c^4e^{17} - 64a^{10}b^2c^5e^{17} + 3a^6b^9c^2d^2e^{17} - 46a^7b^7c^3d^2e^{17} + 264a^8b^5c^4d^2e^{17} - 672a^9b^3c^5d^2e^{17} + 640a^{10}b^2c^6d^2e^{17}) / (a^3e(4ac - b^2)^{3/2}(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)(a^3b^6e^2 - 64a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2)) (b^7e + 48a^2b^3c^2e - 12ab^5c^2e - 64a^3b^2c^3e) / (2(a^3b^6e^2 - 64a^6c^3e^2 - 12a^4b^4c^2e^2 + 48a^5b^2c^2e^2)) + ((b^4 + 6a^2c^2 - 6ab^2c)^3(a^7b^8c^2e^{17} - 12a^8b^6c^3e^{17} + 48a^9b^4c^4e^{17} - 64a^{10}b^2c^5e^{17} + 3a^6b^9c^2d^2e^{17} - 46a^7b^7c^3d^2e^{17} + 264a^8b^5c^4d^2e^{17} - 672a^9b^3c^5d^2e^{17} + 640a^{10}b^2c^6d^2e^{17}) / (2a^9e^3(4ac - b^2)^{9/2}(a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2)) (3b^6 - 49a^3c^3 + 72a^2b^2c^2 - 27ab^4c) / (8a^3c^2(4ac - b^2)^{7/2}(9a^4c^4 - 6b^8 - 288a^2b^4c^2 + 382a^3b^2c^3 + 72ab^6c)) / (36a^4c^6e^{14} + b^8c^2e^{14} - 12ab^6c^3e^{14} + 48a^2b^4c^4e^{14} - 72a^3b^2c^5e^{14})(b^4 + 6a^2c^2 - 6ab^2c) / (a^3e(4ac - b^2)^{3/2})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out

$$3.516 \quad \int \frac{1}{(d+ex)^4(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=408

$$\frac{b(5b^2 - 19ac)}{2a^3e(b^2 - 4ac)(d + ex)} - \frac{5b^2 - 14ac}{6a^2e(b^2 - 4ac)(d + ex)^3} + \frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c + b(5b^2 - 19ac)\sqrt{b^2 - 4ac} + 5b^4 \right) \operatorname{arctan} \left(\frac{\sqrt{2}\sqrt{c(d+ex)}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2}a^3e(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 3.68, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c + b(5b^2 - 19ac)\sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c(d+ex)}}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2}a^3e(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c - b(5b^2 - 19ac)\sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c(d+ex)}}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{2\sqrt{2}a^3e(b^2 - 4ac)^{3/2}\sqrt{b^2 - 4ac} + b} + \frac{b(5b^2 - 19ac)}{2a^3e(b^2 - 4ac)(d + ex)} - \frac{5b^2 - 14ac}{6a^2e(b^2 - 4ac)(d + ex)^3} + \frac{-2ac + b^2 + bc(d + ex)^2}{2ac(b^2 - 4ac)(d + ex)^3(a + b(d + ex)^2 + c(d + ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] $-(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*(d + e*x)^3) + (b*(5*b^2 - 19*a*c))/(2*a^3*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (\operatorname{Sqrt}[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*\operatorname{Sqrt}[b^2 - 4*a*c])*\operatorname{ArcTan}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]*(d + e*x))/\operatorname{Sqrt}[b - \operatorname{Sqrt}[b^2 - 4*a*c]])]/(2*\operatorname{Sqrt}[2]*a^3*(b^2 - 4*a*c)^{(3/2)}*\operatorname{Sqrt}[b - \operatorname{Sqrt}[b^2 - 4*a*c]]*e) - (\operatorname{Sqrt}[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*\operatorname{Sqrt}[b^2 - 4*a*c])*\operatorname{ArcTan}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]*(d + e*x))/\operatorname{Sqrt}[b + \operatorname{Sqrt}[b^2 - 4*a*c]])]/(2*\operatorname{Sqrt}[2]*a^3*(b^2 - 4*a*c)^{(3/2)}*\operatorname{Sqrt}[b + \operatorname{Sqrt}[b^2 - 4*a*c]]*e)$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1121

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^4 (a+b(d+ex)^2+c(d+ex)^4)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)} - \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{2a(b^2-4ac)e(d+ex)^3} \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)e(d+ex)^3} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)} \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)e(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^3} \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)e(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^3} \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)e(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{2a(b^2-4ac)e(d+ex)^3}
\end{aligned}$$

Mathematica [A] time = 2.98, size = 384, normalized size = 0.94

$$\frac{6(d+ex)(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^4+b^3c(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} + \frac{3\sqrt{2}\sqrt{c}(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}\sqrt{b^2-4ac}} + \frac{3\sqrt{2}\sqrt{c}(-28a^2c^2+29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}-5b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b^2-4ac+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{b^2-4ac+b}} - \frac{4a}{(d+ex)^3} + \frac{24b}{d+ex}$$

$12a^3e$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] $((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) + (3*sqrt[2]*sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*sqrt[b^2 - 4*a*c] - 19*a*b*c*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) + (3*sqrt[2]*sqrt[c]*(-5*b^4 + 29*a*b^2*c - 28*a^2*c^2 + 5*b^3*sqrt[b^2 - 4*a*c] - 19*a*b*c*sqrt[b^2 - 4*a*c])*ArcTan$

$$\frac{(\sqrt{2} \sqrt{c} (d + ex)) / \sqrt{b + \sqrt{b^2 - 4ac}}}{((b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}})} / (12a^3e)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^4 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]

[Out] IntegrateAlgebraic[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x
]

fricas [B] time = 2.38, size = 5734, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out]
$$\frac{1}{12} (6(5b^3c - 19ab^2c^2)e^6x^6 + 36(5b^3c - 19ab^2c^2)d^6e^5x^5 + 2(15b^4 - 62ab^2c + 14a^2c^2 + 45(5b^3c - 19ab^2c^2)d^2)e^4x^4 + 6(5b^3c - 19ab^2c^2)d^6 + 8(15(5b^3c - 19ab^2c^2)d^3 + (15b^4 - 62ab^2c + 14a^2c^2)d)e^3x^3 + 2(15b^4 - 62ab^2c + 14a^2c^2)d^4 + 2(45(5b^3c - 19ab^2c^2)d^4 + 10ab^3 - 40a^2bc + 6(15b^4 - 62ab^2c + 14a^2c^2)d^2)e^2x^2 - 4a^2b^2 + 16a^3c + 20(ab^3 - 4a^2bc)d^2 + 4(9(5b^3c - 19ab^2c^2)d^5 + 2(15b^4 - 62ab^2c + 14a^2c^2)d^3 + 10(ab^3 - 4a^2bc)d)e^1x - 3\sqrt{1/2}((a^3b^2c - 4a^4c^2)e^8x^7 + 7(a^3b^2c - 4a^4c^2)d^6e^7x^6 + (a^3b^3 - 4a^4bc + 21(a^3b^2c - 4a^4c^2)d^2)e^6x^5 + 5(7(a^3b^2c - 4a^4c^2)d^3 + (a^3b^3 - 4a^4bc)d)e^5x^4 + (a^4b^2 - 4a^5c + 35(a^3b^2c - 4a^4c^2)d^4 + 10(a^3b^3 - 4a^4bc)d^2)e^4x^3 + (21(a^3b^2c - 4a^4c^2)d^5 + 10(a^3b^3 - 4a^4bc)d^3 + 3(a^4b^2 - 4a^5c)d)e^3x^2 + (7(a^3b^2c - 4a^4c^2)d^6 + 5(a^3b^3 - 4a^4bc)d^4 + 3(a^4b^2 - 4a^5c)d^2)e^2x + ((a^3b^2c - 4a^4c^2)d^7 + (a^3b^3 - 4a^4bc)d^5 + (a^4b^2 - 4a^5c)d^3)e^1) \sqrt{-25b^9 - 315ab^7c + 1386a^2b^5c^2 - 2415a^3b^3c^3 + 1260a^4b^2c^4 + (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3)e^2} \sqrt{(625b^{12} - 8250ab^{10}c + 39525a^2b^8c^2 - 83630a^3b^6c^3 + 76686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6)} / ((a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3)e^4)) / ((a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3)e^2)) \log((1125b^8c^4 - 12325ab^6c^5 + 43410a^2b^4c^6 - 50421a^3b^2c^7 + 9604a^4c^8)e^1x + (1125b^8c^4 - 12325ab^6c^5 + 43410a^2b^4c^6 - 50421a^3b^2c^7 + 9604a^4c^8)d + 1/2\sqrt{1/2}((5a^7c$$

$$\begin{aligned} & b^{11} - 94a^8b^9c + 700a^9b^7c^2 - 2576a^{10}b^5c^3 + 4672a^{11}b^3c^4 - 3328a^{12}b^1c^5) \cdot e^3 \cdot \sqrt{(625b^{12} - 8250a^2b^8c^2 - 83630a^3b^6c^3 + 76686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6) / ((a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3) \cdot e^4)} - (125 \\ & * b^{14} - 2425a^2b^{12}c + 18940a^2b^{10}c^2 - 75579a^3b^8c^3 + 160932a^4 \\ & * b^6c^4 - 172990a^5b^4c^5 + 79408a^6b^2c^6 - 10976a^7c^7) \cdot e) \cdot \sqrt{ \\ & - (25b^9 - 315a^2b^7c + 1386a^2b^5c^2 - 2415a^3b^3c^3 + 1260a^4b^1c^4 + (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) \cdot e^2) \cdot \sqrt{(625b^{12} - 8250a^2b^8c^2 - 83630a^3b^6c^3 + 76686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6) / ((a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3) \cdot e^4)})) / ((a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) \cdot e^2)) + 3 \cdot \sqrt{1/2} \cdot ((a^3b^2c - 4a^4c^2) \cdot e^8 \cdot x^7 + 7 \cdot (\\ & a^3b^2c - 4a^4c^2) \cdot d \cdot e^7 \cdot x^6 + (a^3b^3 - 4a^4b^2c + 21 \cdot (a^3b^2c - 4 \\ & * a^4c^2) \cdot d^2) \cdot e^6 \cdot x^5 + 5 \cdot (7 \cdot (a^3b^2c - 4a^4c^2) \cdot d^3 + (a^3b^3 - 4a^4 \\ & * b^2c) \cdot d) \cdot e^5 \cdot x^4 + (a^4b^2 - 4a^5c + 35 \cdot (a^3b^2c - 4a^4c^2) \cdot d^4 + 1 \\ & 0 \cdot (a^3b^3 - 4a^4b^2c) \cdot d^2) \cdot e^4 \cdot x^3 + (21 \cdot (a^3b^2c - 4a^4c^2) \cdot d^5 + 10 \\ & * (a^3b^3 - 4a^4b^2c) \cdot d^3 + 3 \cdot (a^4b^2 - 4a^5c) \cdot d) \cdot e^3 \cdot x^2 + (7 \cdot (a^3b^2 \\ & * c - 4a^4c^2) \cdot d^6 + 5 \cdot (a^3b^3 - 4a^4b^2c) \cdot d^4 + 3 \cdot (a^4b^2 - 4a^5c) \cdot d \\ & ^2) \cdot e^2 \cdot x + ((a^3b^2c - 4a^4c^2) \cdot d^7 + (a^3b^3 - 4a^4b^2c) \cdot d^5 + (a^4 \\ & * b^2 - 4a^5c) \cdot d^3) \cdot e) \cdot \sqrt{-(25b^9 - 315a^2b^7c + 1386a^2b^5c^2 - 24 \\ & 15a^3b^3c^3 + 1260a^4b^1c^4 + (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) \cdot e^2) \cdot \sqrt{(625b^{12} - 8250a^2b^8c^2 - 836 \\ & 30a^3b^6c^3 + 76686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6) / ((a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3) \cdot e^4)})) / ((a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) \cdot e^2)} \cdot \log((1125b^8c^4 - 1232 \\ & 5a^2b^6c^5 + 43410a^2b^4c^6 - 50421a^3b^2c^7 + 9604a^4c^8) \cdot e \cdot x + (\\ & 1125b^8c^4 - 12325a^2b^6c^5 + 43410a^2b^4c^6 - 50421a^3b^2c^7 + 96 \\ & 04a^4c^8) \cdot d - 1/2 \cdot \sqrt{1/2} \cdot ((5a^7b^{11} - 94a^8b^9c + 700a^9b^7c^2 \\ & - 2576a^{10}b^5c^3 + 4672a^{11}b^3c^4 - 3328a^{12}b^1c^5) \cdot e^3 \cdot \sqrt{(625b^{12} - 8250a^2b^8c^2 - 83630a^3b^6c^3 + 76686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6) / ((a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3) \cdot e^4)} - (125b^{14} - 2425a^2b^{12}c + 18940a^2b^{10}c^2 - 75579a^3b^8c^3 + 160932a^4b^6c^4 - 172990a^5b^4c^5 + 79408 \\ & * a^6b^2c^6 - 10976a^7c^7) \cdot e) \cdot \sqrt{-(25b^9 - 315a^2b^7c + 1386a^2b^5c^2 - 2415a^3b^3c^3 + 1260a^4b^1c^4 + (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) \cdot e^2) \cdot \sqrt{(625b^{12} - 8250a^2b^8c^2 - 83630a^3b^6c^3 + 76686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6) / ((a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3) \cdot e^4)})) / ((a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) \cdot e^2)} + 3 \cdot \sqrt{1/2} \cdot ((a^3b^2c - 4a^4c^2) \cdot e^8 \cdot x^7 + 7 \cdot (a^3b^2c - 4a^4c^2) \cdot d \cdot e^7 \cdot x^6 + (a^3b^3 - 4a^4b^2c + 21 \cdot (a^3b^2c - 4a^4c^2) \cdot d^2) \cdot e^6 \cdot x^5 + 5 \cdot (7 \cdot (a^3b^2c - 4a^4c^2) \cdot d^3 + (a^3b^3 - 4a^4b^2c) \cdot d) \cdot e^5 \cdot x^4 + (a^4b^2 - 4a^5c + 35 \cdot (a^3b^2c - 4a^4c^2) \cdot d^4 + 10 \cdot (a^3b^3 - 4a^4b^2c) \cdot d^2) \cdot e^4 \cdot x^3 + (21 \cdot (a^3b^2c - 4a^4c^2) \cdot d^5 + 10 \cdot (a^3b^3 - 4a^4b^2c) \cdot d^3 + 3 \cdot (a^4b^2 - 4a^5c) \cdot d) \cdot e^3 \cdot x^2 + (7 \cdot (a^3b^2c - 4a^4c^2) \cdot d^6 + 5 \cdot (a^3b^3 - 4a^4b^2c) \cdot d^4 + 3 \cdot (a^4b^2 - 4a^5c) \cdot d^2) \cdot e^2 \cdot x + ((a^3b^2c - 4a^4c^2) \cdot d^7 + (a^3b^3 - 4a^4b^2c) \cdot d^5 + (a^4b^2 - 4a^5c) \cdot d^3) \cdot e) \cdot \sqrt{-(25b^9 - 315a^2b^7c + 1386a^2b^5c^2 - 2415a^3b^3c^3 + 1260a^4b^1c^4 + (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) \cdot e^2) \cdot \sqrt{(625b^{12} - 8250a^2b^8c^2 - 83630a^3b^6c^3 + 76686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6) / ((a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3) \cdot e^4)})) / ((a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) \cdot e^2)} \end{aligned}$$

$$\begin{aligned}
& a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)* \\
& d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*\text{sqrt}(-(25*b^9 \\
& - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^ \\
& 7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*\text{sqrt}((625*b^12 - 8 \\
& 250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - \\
& 24108*a^5*b^2*c^5 + 2401*a^6*c^6))/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2* \\
& c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^ \\
& 10*c^3)*e^2))*\log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504 \\
& 21*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 4341 \\
& 0*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*\text{sqrt}(1/2)*((5*a^7 \\
& *b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3* \\
& c^4 - 3328*a^12*b*c^5)*e^3*\text{sqrt}((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c \\
& ^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c \\
& ^6))/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)) + (12 \\
& 5*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^ \\
& 4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*\text{sqrt} \\
& (-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b* \\
& c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*\text{sqrt}((625 \\
& *b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b \\
& ^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6))/(a^14*b^6 - 12*a^15*b^4*c + 48* \\
& a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^ \\
& 2 - 64*a^10*c^3)*e^2))) - 3*\text{sqrt}(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7* \\
& (a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - \\
& 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a \\
& ^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + \\
& 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 1 \\
& 0*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^ \\
& 2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)* \\
& d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^ \\
& 4*b^2 - 4*a^5*c)*d^3)*e)*\text{sqrt}(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2 \\
& 415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 \\
& - 64*a^10*c^3)*e^2*\text{sqrt}((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83 \\
& 630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6))/(a \\
& ^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4)))/((a^7*b^6 - \\
& 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2))*\log((1125*b^8*c^4 - 123 \\
& 25*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + \\
& (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9 \\
& 604*a^4*c^8)*d - 1/2*\text{sqrt}(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^ \\
& 2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*\text{sqrt}((625* \\
& b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^ \\
& 4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6))/(a^14*b^6 - 12*a^15*b^4*c + 48*a \\
& ^16*b^2*c^2 - 64*a^17*c^3)*e^4)) + (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^ \\
& 10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 7940 \\
& 8*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*\text{sqrt}(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^ \\
& 5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^
\end{aligned}$$

$$9*b^2*c^2 - 64*a^{10}*c^3)*e^2*\sqrt{((625*b^{12} - 8250*a*b^{10}*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^{14}*b^6 - 12*a^{15}*b^4*c + 48*a^{16}*b^2*c^2 - 64*a^{17}*c^3)*e^4)))/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^{10}*c^3)*e^2)))/((a^3*b^2*c - 4*a^4*c^2)*e^8*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*x^3 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)$$

giac [B] time = 0.55, size = 1987, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out]
$$-1/4*((5*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*b^3*c*e^2 - 19*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*a*b*c^2*e^2 - 10*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})*b^3*c*d*e + 38*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*\log(d*e^{(-1)} + x + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})/(2*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})) + (5*(d*e^{(-1)} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*b^3*c*e^2 - 19*(d*e^{(-1)} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*a*b*c^2*e^2 - 10*(d*e^{(-1)} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})*b^3*c*d*e + 38*(d*e^{(-1)} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*\log(d*e^{(-1)} + x - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})/(2*(d*e^{(-1)} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} - \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})) + (5*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*b^3*c*e^2 - 19*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*a*b*c^2*e^2 - 10*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})*b^3*c*d*e + 38*(d*e^{(-1)} + \sqrt{1/2})*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})$$

$$\begin{aligned}
& 2 - 4*a*c)*e^2)*e^{(-4)/c})*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 \\
& - 24*a*b^2*c + 14*a^2*c^2)*\log(d*e^{(-1)} + x + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})/(2*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})^3*c*e^4 - 6*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})) + (5*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})^2*b^3*c*e^2 - 19*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})^2*a*b*c^2*e^2 - 10*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})*b^3*c*d*e + 38*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*\log(d*e^{(-1)} + x - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})/(2*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})^3*c*e^4 - 6*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2)*e^{(-4)/c}})))/(a^3*b^2 - 4*a^4*c) + 1/2*(b^3*c*x^3*e^3 - 3*a*b*c^2*x^3*e^3 + 3*b^3*c*d*x^2*e^2 - 9*a*b*c^2*d*x^2*e^2 + 3*b^3*c*d^2*x*e - 9*a*b*c^2*d^2*x*e + b^3*c*d^3 - 3*a*b*c^2*d^3 + b^4*x*e - 4*a*b^2*c*x*e + 2*a^2*c^2*x*e + b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(a^3*b^2*e - 4*a^4*c*e)) + 1/3*(6*b*x^2*e^2 + 12*b*d*x*e + 6*b*d^2 - a)*e^{(-1)}/((x*e + d)^3*a^3)
\end{aligned}$$

maple [C] time = 0.04, size = 1518, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2, x)$

[Out]
$$\begin{aligned}
& 3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*c^2*e^2/(4*a*c-b^2)*x^3-1/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b^3*c*e^2/(4*a*c-b^2)*x^3+9/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*c^2*e/(4*a*c-b^2)*x^2-3/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b^3*c*e/(4*a*c-b^2)*x^2+9/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*c^2*d^2-3/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^2+2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c-1/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4
\end{aligned}$$

```

*a*c-b^2)*x*b^4+3/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*
x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b*c^2-1/2/a^3/(c*e^4
*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*
d^2+a)*d^3/e/(4*a*c-b^2)*b^3*c-1/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*c^2+2/a^2/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^2*c-1/2/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*
e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^4+
1/4/a^3/(4*a*c-b^2)/e*sum((b*c*e^2*(19*a*c-5*b^2)*_R^2+2*d*e*b*c*(19*a*c-5*
b^2)*_R+19*a*b*c^2*d^2-5*b^3*c*d^2-14*a^2*c^2+24*a*b^2*c-5*b^4)/(2*_R^3*c*e
^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4
*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d
*e)*_Z+a))-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d)

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 8.72, size = 12239, normalized size = 30.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)

```

[Out] atan((( -(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
+ 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
- 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2)))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9
*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^
2*c^5*e^2)))^(1/2)*((- (25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^
7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 2197
44*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) -
615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4
*a*c - b^2)^9)^(1/2)))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c
*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2
- 6144*a^12*b^2*c^5*e^2)))^(1/2)*((- (25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*
b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b
^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 16

```

$$\begin{aligned}
& 5*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - \\
& 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2))^{(1/2)}*(x*(1048576*a^21*b*c^8*e^14 \\
& + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) \\
& + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 \\
& + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^10*c^4*e^12 \\
& - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 \\
& + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) \\
& - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 \\
& - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)*1i + (-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 \\
& + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 \\
& - 6144*a^12*b^2*c^5*e^2))^{(1/2)}*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 \\
& - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)}) \\
&)/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2))^{(1/2)}* \\
& ((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 \\
& + 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 \\
& - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2))^{(1/2)}*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 \\
& + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 \\
& - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 \\
& - 320*a^12*b^14*c^2*e^12 + 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e^12) \\
& - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 14
\end{aligned}$$

$$\begin{aligned}
& 58688a^{13}b^6c^7e^{12} + 2401280a^{14}b^4c^8e^{12} - 1871872a^{15}b^2c^9e^{12} - 401408a^{16}c^{10}d^2e^{11} + 400a^9b^{14}c^3d^2e^{11} - 9440a^{10}b^{12}c^4d^2e^{11} + 92816a^{11}b^{10}c^5d^2e^{11} - 488096a^{12}b^8c^6d^2e^{11} + 1458688a^{13}b^6c^7d^2e^{11} - 2401280a^{14}b^4c^8d^2e^{11} + 1871872a^{15}b^2c^9d^2e^{11}) * i) / ((- (25b^{15} - 25b^6 * (- (4ac - b^2)^9)^{1/2}) - 80640a^7b^6c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 + 49a^3c^3 * (- (4ac - b^2)^9)^{1/2}) - 615a^2b^{13}c - 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2} + 165a^4b^4c * (- (4ac - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^2 + 4096a^{13}c^6e^2 - 24a^8b^{10}c^2e^2 + 240a^9b^8c^2e^2 - 1280a^{10}b^6c^3e^2 + 3840a^{11}b^4c^4e^2 - 6144a^{12}b^2c^5e^2))^{1/2} * ((- (25b^{15} - 25b^6 * (- (4ac - b^2)^9)^{1/2}) - 80640a^7b^6c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 + 49a^3c^3 * (- (4ac - b^2)^9)^{1/2}) - 615a^2b^{13}c - 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2} + 165a^4b^4c * (- (4ac - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^2 + 4096a^{13}c^6e^2 - 24a^8b^{10}c^2e^2 + 240a^9b^8c^2e^2 - 1280a^{10}b^6c^3e^2 + 3840a^{11}b^4c^4e^2 - 6144a^{12}b^2c^5e^2))^{1/2} * ((- (25b^{15} - 25b^6 * (- (4ac - b^2)^9)^{1/2}) - 80640a^7b^6c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 + 49a^3c^3 * (- (4ac - b^2)^9)^{1/2}) - 615a^2b^{13}c - 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2} + 165a^4b^4c * (- (4ac - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^2 + 4096a^{13}c^6e^2 - 24a^8b^{10}c^2e^2 + 240a^9b^8c^2e^2 - 1280a^{10}b^6c^3e^2 + 3840a^{11}b^4c^4e^2 - 6144a^{12}b^2c^5e^2))^{1/2} * (x * (1048576a^{21}b^8c^8e^{14} + 256a^{15}b^{13}c^2e^{14} - 6144a^{16}b^{11}c^3e^{14} + 61440a^{17}b^9c^4e^{14} - 327680a^{18}b^7c^5e^{14} + 983040a^{19}b^5c^6e^{14} - 1572864a^{20}b^3c^7e^{14}) + 1048576a^{21}b^8c^8d^2e^{13} + 256a^{15}b^{13}c^2d^2e^{13} - 6144a^{16}b^{11}c^3d^2e^{13} + 61440a^{17}b^9c^4d^2e^{13} - 327680a^{18}b^7c^5d^2e^{13} + 983040a^{19}b^5c^6d^2e^{13} - 1572864a^{20}b^3c^7d^2e^{13}) - 917504a^{19}c^9e^{12} + 320a^{12}b^{14}c^2e^{12} - 7936a^{13}b^{12}c^3e^{12} + 82816a^{14}b^{10}c^4e^{12} - 468480a^{15}b^8c^5e^{12} + 1536000a^{16}b^6c^6e^{12} - 2867200a^{17}b^4c^7e^{12} + 2719744a^{18}b^2c^8e^{12}) - x * (401408a^{16}c^{10}e^{12} - 400a^9b^{14}c^3e^{12} + 9440a^{10}b^{12}c^4e^{12} - 92816a^{11}b^{10}c^5e^{12} + 488096a^{12}b^8c^6e^{12} - 1458688a^{13}b^6c^7e^{12} + 2401280a^{14}b^4c^8e^{12} - 1871872a^{15}b^2c^9e^{12}) - 401408a^{16}c^{10}d^2e^{11} + 400a^9b^{14}c^3d^2e^{11} - 9440a^{10}b^{12}c^4d^2e^{11} + 92816a^{11}b^{10}c^5d^2e^{11} - 488096a^{12}b^8c^6d^2e^{11} + 1458688a^{13}b^6c^7d^2e^{11} - 2401280a^{14}b^4c^8d^2e^{11} + 1871872a^{15}b^2c^9d^2e^{11}) - ((- (25b^{15} - 25b^6 * (- (4ac - b^2)^9)^{1/2}) - 80640a^7b^6c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 + 49a^3c^3 * (- (4ac - b^2)^9)^{1/2}) - 615a^2b^{13}c - 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2} + 165a^4b^4c * (- (4ac - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^2 + 4096a^{13}c^6e^2 - 24a^8b^{10}c^2e^2 + 240a^9b^8c^2e^2 - 1280a^{10}b^6c^3e^2 + 3840a^{11}b^4c^4e^2 - 6144a^{12}b^2c^5e^2))^{1/2} * ((- (25b^{15} - 25b^6 * (- (4ac - b^2)^9)^{1/2}) - 80640a^7b^6c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6
\end{aligned}$$

$$\begin{aligned}
& *b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2 \\
& *c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(\\
& a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 \\
& - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))^{ \\
& (1/2)*((-25*b^{15} - 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 636 \\
& 6*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^ \\
& 5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c \\
& - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*(-(4*a*c - b^2)^9 \\
&)^{(1/2)})/(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^ \\
& 9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b \\
& ^2*c^5*e^2))^{(1/2)}*(x*(1048576*a^{21}*b*c^8*e^{14} + 256*a^{15}*b^{13}*c^2*e^{14} - \\
& 6144*a^{16}*b^{11}*c^3*e^{14} + 61440*a^{17}*b^9*c^4*e^{14} - 327680*a^{18}*b^7*c^5*e^{1 \\
& 4} + 983040*a^{19}*b^5*c^6*e^{14} - 1572864*a^{20}*b^3*c^7*e^{14}) + 1048576*a^{21}*b* \\
& c^8*d*e^{13} + 256*a^{15}*b^{13}*c^2*d*e^{13} - 6144*a^{16}*b^{11}*c^3*d*e^{13} + 61440*a \\
& ^{17}*b^9*c^4*d*e^{13} - 327680*a^{18}*b^7*c^5*d*e^{13} + 983040*a^{19}*b^5*c^6*d*e^{1 \\
& 3} - 1572864*a^{20}*b^3*c^7*d*e^{13}) + 917504*a^{19}*c^9*e^{12} - 320*a^{12}*b^{14}*c^2 \\
& *e^{12} + 7936*a^{13}*b^{12}*c^3*e^{12} - 82816*a^{14}*b^{10}*c^4*e^{12} + 468480*a^{15}*b^ \\
& 8*c^5*e^{12} - 1536000*a^{16}*b^6*c^6*e^{12} + 2867200*a^{17}*b^4*c^7*e^{12} - 271974 \\
& 4*a^{18}*b^2*c^8*e^{12}) - x*(401408*a^{16}*c^{10}*e^{12} - 400*a^9*b^{14}*c^3*e^{12} + 9 \\
& 440*a^{10}*b^{12}*c^4*e^{12} - 92816*a^{11}*b^{10}*c^5*e^{12} + 488096*a^{12}*b^8*c^6*e^{1 \\
& 2} - 1458688*a^{13}*b^6*c^7*e^{12} + 2401280*a^{14}*b^4*c^8*e^{12} - 1871872*a^{15}*b^ \\
& 2*c^9*e^{12}) - 401408*a^{16}*c^{10}*d*e^{11} + 400*a^9*b^{14}*c^3*d*e^{11} - 9440*a^{10} \\
& *b^{12}*c^4*d*e^{11} + 92816*a^{11}*b^{10}*c^5*d*e^{11} - 488096*a^{12}*b^8*c^6*d*e^{11} \\
& + 1458688*a^{13}*b^6*c^7*d*e^{11} - 2401280*a^{14}*b^4*c^8*d*e^{11} + 1871872*a^{15}* \\
& b^2*c^9*d*e^{11}) + 476672*a^{13}*b*c^{10}*e^{10} + 1800*a^9*b^9*c^6*e^{10} - 29080*a \\
& ^{10}*b^7*c^7*e^{10} + 176032*a^{11}*b^5*c^8*e^{10} - 473216*a^{12}*b^3*c^9*e^{10}))*(- \\
& (25*b^{15} - 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^1 \\
& 1*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 21504 \\
& 0*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^ \\
& 2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/ \\
& (32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2 \\
& *e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^ \\
& 2))^{(1/2)}*2i - ((x^4*(15*b^4*e^3 + 14*a^2*c^2*e^3 + 225*b^3*c*d^2*e^3 - 62 \\
& *a*b^2*c*e^3 - 855*a*b*c^2*d^2*e^3))/(6*a*(4*a^3*c - a^2*b^2)) + (3*x^5*(5* \\
& b^3*c*d*e^4 - 19*a*b*c^2*d*e^4))/(a*(4*a^3*c - a^2*b^2)) + (2*x^3*(15*b^4*d \\
& *e^2 + 14*a^2*c^2*d*e^2 + 75*b^3*c*d^3*e^2 - 62*a*b^2*c*d*e^2 - 285*a*b*c^2 \\
& *d^3*e^2))/(3*a*(4*a^3*c - a^2*b^2)) + (x*(30*b^4*d^3 + 45*b^3*c*d^5 + 28*a \\
& ^2*c^2*d^3 + 10*a*b^3*d - 40*a^2*b*c*d - 124*a*b^2*c*d^3 - 171*a*b*c^2*d^5) \\
&)/(3*a*(4*a^3*c - a^2*b^2)) + (x^6*(5*b^3*c*e^5 - 19*a*b*c^2*e^5))/(2*a*(4* \\
& a^3*c - a^2*b^2)) + (x^2*(90*b^4*d^2*e + 10*a*b^3*e + 84*a^2*c^2*d^2*e - 40 \\
& *a^2*b*c*e + 225*b^3*c*d^4*e - 372*a*b^2*c*d^2*e - 855*a*b*c^2*d^4*e))/(6*a \\
& *(4*a^3*c - a^2*b^2)) + (8*a^3*c - 2*a^2*b^2 + 15*b^4*d^4 + 10*a*b^3*d^2 + \\
& 15*b^3*c*d^6 + 14*a^2*c^2*d^4 - 40*a^2*b*c*d^2 - 62*a*b^2*c*d^4 - 57*a*b*c^ \\
& 2*d^6)/(6*a*e*(4*a^3*c - a^2*b^2)))/(x^2*(10*b*d^3*e^2 + 21*c*d^5*e^2 + 3*a \\
& *d*e^2) + x^5*(b*e^5 + 21*c*d^2*e^5) + a*d^3 + b*d^5 + c*d^7 + x^3*(a*e^3 +
\end{aligned}$$

$$\begin{aligned}
& 10*b*d^2*e^3 + 35*c*d^4*e^3) + x^4*(35*c*d^3*e^4 + 5*b*d*e^4) + x*(3*a*d^2 \\
& *e + 5*b*d^4*e + 7*c*d^6*e) + c*e^7*x^7 + 7*c*d*e^6*x^6) + \operatorname{atan}\left(\frac{(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{1/2} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2})}{(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))}\right)^{1/2} \\
& * \left(\frac{(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{1/2} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2})}{(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))}\right)^{1/2} \\
& * \left(\frac{(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{1/2} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2})}{(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))}\right)^{1/2} \\
& * \left(\frac{(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{1/2} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2})}{(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))}\right)^{1/2} \\
& * (x*(1048576*a^{21}*b*c^8*e^{14} + 256*a^{15}*b^{13}*c^2*e^{14} - 6144*a^{16}*b^{11}*c^3*e^{14} + 61440*a^{17}*b^9*c^4*e^{14} - 327680*a^{18}*b^7*c^5*e^{14} + 983040*a^{19}*b^5*c^6*e^{14} - 1572864*a^{20}*b^3*c^7*e^{14}) + 1048 \\
& 576*a^{21}*b*c^8*d*e^{13} + 256*a^{15}*b^{13}*c^2*d*e^{13} - 6144*a^{16}*b^{11}*c^3*d*e^{13} + 61440*a^{17}*b^9*c^4*d*e^{13} - 327680*a^{18}*b^7*c^5*d*e^{13} + 983040*a^{19}*b^5*c^6*d*e^{13} - 1572864*a^{20}*b^3*c^7*d*e^{13}) - 917504*a^{19}*c^9*e^{12} + 320*a^{12}*b^{14}*c^2*e^{12} - 7936*a^{13}*b^{12}*c^3*e^{12} + 82816*a^{14}*b^{10}*c^4*e^{12} - 468 \\
& 480*a^{15}*b^8*c^5*e^{12} + 1536000*a^{16}*b^6*c^6*e^{12} - 2867200*a^{17}*b^4*c^7*e^{12} + 2719744*a^{18}*b^2*c^8*e^{12}) - x*(401408*a^{16}*c^{10}*e^{12} - 400*a^9*b^{14}*c^3*e^{12} + 9440*a^{10}*b^{12}*c^4*e^{12} - 92816*a^{11}*b^{10}*c^5*e^{12} + 488096*a^{12}*b^8*c^6*e^{12} - 1458688*a^{13}*b^6*c^7*e^{12} + 2401280*a^{14}*b^4*c^8*e^{12} - 1871 \\
& 872*a^{15}*b^2*c^9*e^{12}) - 401408*a^{16}*c^{10}*d*e^{11} + 400*a^9*b^{14}*c^3*d*e^{11} - 9440*a^{10}*b^{12}*c^4*d*e^{11} + 92816*a^{11}*b^{10}*c^5*d*e^{11} - 488096*a^{12}*b^8*c^6*d*e^{11} + 1458688*a^{13}*b^6*c^7*d*e^{11} - 2401280*a^{14}*b^4*c^8*d*e^{11} + 18 \\
& 71872*a^{15}*b^2*c^9*d*e^{11})*i + \left(\frac{(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{1/2} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2})}{(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))}\right)^{1/2} \\
& * \left(\frac{(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{1/2} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2})}{(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))}\right)^{1/2} \\
& * \left(\frac{(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{1/2} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2})}{(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))}\right)^{1/2} \\
& * \left(\frac{(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{1/2} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2})}{(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))}\right)^{1/2} \\
& * \left(\frac{(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{1/2} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{1/2} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{1/2} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{1/2})}{(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))}\right)^{1/2}
\end{aligned}$$

$$\begin{aligned}
& 13c^6e^2 - 24a^8b^{10}c^4e^2 + 240a^9b^8c^2e^2 - 1280a^{10}b^6c^3e^2 + 3840a^{11}b^4c^4e^2 - 6144a^{12}b^2c^5e^2) \wedge (1/2) * ((- (25b^{15} + 25 \\
& * b^6 * (- (4ac - b^2)^9) \wedge (1/2) - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767 \\
& * a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 \\
& - 49a^3c^3 * (- (4ac - b^2)^9) \wedge (1/2) - 615a * b^{13}c + 246a^2b^2c^2 * (- (\\
& 4ac - b^2)^9) \wedge (1/2) - 165a * b^4c * (- (4ac - b^2)^9) \wedge (1/2)) / (32 * (a^7b^{12} \\
& * e^2 + 4096a^{13}c^6e^2 - 24a^8b^{10}c^4e^2 + 240a^9b^8c^2e^2 - 1280a^{10}b^6c^3e^2 + 3840a^{11}b^4c^4e^2 - 6144a^{12}b^2c^5e^2)) \wedge (1/2) * (x \\
& * (1048576a^{21}b^8c^8e^{14} + 256a^{15}b^{13}c^2e^{14} - 6144a^{16}b^{11}c^3e^{14} + 61440a^{17}b^9c^4e^{14} - 327680a^{18}b^7c^5e^{14} + 983040a^{19}b^5c^6e^{14} \\
& - 1572864a^{20}b^3c^7e^{14}) + 1048576a^{21}b^8c^8d^{13} + 256a^{15}b^{13}c^2d^{13} - 6144a^{16}b^{11}c^3d^{13} + 61440a^{17}b^9c^4d^{13} - 3 \\
& 27680a^{18}b^7c^5d^{13} + 983040a^{19}b^5c^6d^{13} - 1572864a^{20}b^3c^7d^{13}) + 917504a^{19}c^9e^{12} - 320a^{12}b^{14}c^2e^{12} + 7936a^{13}b^{12} \\
& * c^3e^{12} - 82816a^{14}b^{10}c^4e^{12} + 468480a^{15}b^8c^5e^{12} - 1536000a^{16}b^6c^6e^{12} + 2867200a^{17}b^4c^7e^{12} - 2719744a^{18}b^2c^8e^{12}) - \\
& x * (401408a^{16}c^{10}e^{12} - 400a^9b^{14}c^3e^{12} + 9440a^{10}b^{12}c^4e^{12} - 92816a^{11}b^{10}c^5e^{12} + 488096a^{12}b^8c^6e^{12} - 1458688a^{13}b^6c^7 \\
& * e^{12} + 2401280a^{14}b^4c^8e^{12} - 1871872a^{15}b^2c^9e^{12}) - 401408a^{16}c^{10}d^{11} + 400a^9b^{14}c^3d^{11} - 9440a^{10}b^{12}c^4d^{11} + 928 \\
& 16a^{11}b^{10}c^5d^{11} - 488096a^{12}b^8c^6d^{11} + 1458688a^{13}b^6c^7d^{11} - 2401280a^{14}b^4c^8d^{11} + 1871872a^{15}b^2c^9d^{11}) * i) / (\\
& - (25b^{15} + 25b^6 * (- (4ac - b^2)^9) \wedge (1/2) - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 2150 \\
& 40a^6b^3c^6 - 49a^3c^3 * (- (4ac - b^2)^9) \wedge (1/2) - 615a * b^{13}c + 246a^2b^2c^2 * (- (4ac - b^2)^9) \wedge (1/2) - 165a * b^4c * (- (4ac - \\
& b^2)^9) \wedge (1/2)) / (32 * (a^7b^{12}e^2 + 4096a^{13}c^6e^2 - 24a^8b^{10}c^4e^2 + 240a^9b^8c^2e^2 - 1280a^{10}b^6c^3e^2 + 3840a^{11}b^4c^4e^2 - 6144a^{12}b^2c^5e^2 \\
& ^2)) \wedge (1/2) * ((- (25b^{15} + 25b^6 * (- (4ac - b^2)^9) \wedge (1/2) - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 2150 \\
& 40a^6b^3c^6 - 49a^3c^3 * (- (4ac - b^2)^9) \wedge (1/2) - 615a * b^{13}c + 246a^2b^2c^2 * (- (4ac - b^2)^9) \wedge (1/2) - 165a * b^4c * (- (4ac - \\
& b^2)^9) \wedge (1/2)) / (32 * (a^7b^{12}e^2 + 4096a^{13}c^6e^2 - 24a^8b^{10}c^4e^2 + 240a^9b^8c^2e^2 - 1280a^{10}b^6c^3e^2 + 3840a^{11}b^4c^4e^2 - 6144a^{12}b^2c^5e^2 \\
& ^2)) \wedge (1/2) * ((- (25b^{15} + 25b^6 * (- (4ac - b^2)^9) \wedge (1/2) - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 2150 \\
& 40a^6b^3c^6 - 49a^3c^3 * (- (4ac - b^2)^9) \wedge (1/2) - 615a * b^{13}c + 246a^2b^2c^2 * (- (4ac - b^2)^9) \wedge (1/2) - 165a * b^4c * (- (4ac - \\
& b^2)^9) \wedge (1/2)) / (32 * (a^7b^{12}e^2 + 4096a^{13}c^6e^2 - 24a^8b^{10}c^4e^2 + 240a^9b^8c^2e^2 - 1280a^{10}b^6c^3e^2 + 3840a^{11}b^4c^4e^2 - 6144a^{12}b^2c^5e^2 \\
& ^2)) \wedge (1/2) * (x * (1048576a^{21}b^8c^8e^{14} + 256a^{15}b^{13}c^2e^{14} - 6144a^{16}b^{11}c^3e^{14} + 61440a^{17}b^9c^4e^{14} - 327 \\
& 680a^{18}b^7c^5e^{14} + 983040a^{19}b^5c^6e^{14} - 1572864a^{20}b^3c^7e^{14} + 1048576a^{21}b^8c^8d^{13} + 256a^{15}b^{13}c^2d^{13} - 6144a^{16}b^{11}c^3d^{13} + 61440a^{17}b^9c^4d^{13} - 327680a^{18}b^7c^5d^{13} + 983040a^{19}b^5c^6d^{13} - 1572864a^{20}b^3c^7d^{13}) + 1048576a^{21}b^8c^8d^{13} + 256a^{15}b^{13}c^2d^{13} - 6144a^{16}b^{11}c^3d^{13} + 61440a^{17}b^9c^4d^{13} - 327680a^{18}b^7c^5d^{13} + 983040a^{19}b^5c^6d^{13} - 1572864a^{20}b^3c^7d^{13}) + 917504a^{19}c^9e^{12} - 320a^{12}b^{14}c^2e^{12} + 7936a^{13}b^{12}c^3e^{12} - 82816a^{14}b^{10}c^4e^{12} + 468480a^{15}b^8c^5e^{12} - 1536000a^{16}b^6c^6e^{12} + 2867200a^{17}b^4c^7e^{12} - 2719744a^{18}b^2c^8e^{12}) - x * (401408a^{16}c^{10}e^{12} - 400a^9b^{14}c^3e^{12} + 9440a^{10}b^{12}c^4e^{12} - 92816a^{11}b^{10}c^5e^{12} + 488096a^{12}b^8c^6e^{12} - 1458688a^{13}b^6c^7e^{12} + 2401280a^{14}b^4c^8e^{12} - 1871872a^{15}b^2c^9e^{12}) - 401408a^{16}c^{10}d^{11} + 400a^9b^{14}c^3d^{11} - 9440a^{10}b^{12}c^4d^{11} + 92816a^{11}b^{10}c^5d^{11} - 488096a^{12}b^8c^6d^{11} + 1458688a^{13}b^6c^7d^{11} - 2401280a^{14}b^4c^8d^{11} + 1871872a^{15}b^2c^9d^{11}) * i) / (
\end{aligned}$$

$$\begin{aligned}
& 0*a^{19}*b^5*c^6*d*e^{13} - 1572864*a^{20}*b^3*c^7*d*e^{13}) - 917504*a^{19}*c^9*e^{12} \\
& + 320*a^{12}*b^{14}*c^2*e^{12} - 7936*a^{13}*b^{12}*c^3*e^{12} + 82816*a^{14}*b^{10}*c^4*e \\
& ^{12} - 468480*a^{15}*b^8*c^5*e^{12} + 1536000*a^{16}*b^6*c^6*e^{12} - 2867200*a^{17}*b \\
& ^4*c^7*e^{12} + 2719744*a^{18}*b^2*c^8*e^{12}) - x*(401408*a^{16}*c^{10}*e^{12} - 400*a \\
& ^9*b^{14}*c^3*e^{12} + 9440*a^{10}*b^{12}*c^4*e^{12} - 92816*a^{11}*b^{10}*c^5*e^{12} + 488 \\
& 096*a^{12}*b^8*c^6*e^{12} - 1458688*a^{13}*b^6*c^7*e^{12} + 2401280*a^{14}*b^4*c^8*e^ \\
& ^{12} - 1871872*a^{15}*b^2*c^9*e^{12}) - 401408*a^{16}*c^{10}*d*e^{11} + 400*a^9*b^{14}*c^ \\
& ^3*d*e^{11} - 9440*a^{10}*b^{12}*c^4*d*e^{11} + 92816*a^{11}*b^{10}*c^5*d*e^{11} - 488096* \\
& a^{12}*b^8*c^6*d*e^{11} + 1458688*a^{13}*b^6*c^7*d*e^{11} - 2401280*a^{14}*b^4*c^8*d* \\
& e^{11} + 1871872*a^{15}*b^2*c^9*d*e^{11}) - ((-25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9 \\
&)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928* \\
& a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e \\
& ^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840 \\
& *a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2)))^{(1/2)}*((-(25*b^{15} + 25*b^6*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9 \\
& *c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^ \\
& 3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12}*e^2 + 4 \\
& 096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6* \\
& c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2)))^{(1/2)}*((-(25*b^{1 \\
& 5} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - \\
& 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b \\
& ^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c \\
& ^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^ \\
& 7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - \\
& 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2)))^{(1 \\
& /2)}*(x*(1048576*a^{21}*b*c^8*e^{14} + 256*a^{15}*b^{13}*c^2*e^{14} - 6144*a^{16}*b^{11}*c \\
& ^3*e^{14} + 61440*a^{17}*b^9*c^4*e^{14} - 327680*a^{18}*b^7*c^5*e^{14} + 983040*a^{19}* \\
& b^5*c^6*e^{14} - 1572864*a^{20}*b^3*c^7*e^{14}) + 1048576*a^{21}*b*c^8*d*e^{13} + 256 \\
& *a^{15}*b^{13}*c^2*d*e^{13} - 6144*a^{16}*b^{11}*c^3*d*e^{13} + 61440*a^{17}*b^9*c^4*d*e^ \\
& ^{13} - 327680*a^{18}*b^7*c^5*d*e^{13} + 983040*a^{19}*b^5*c^6*d*e^{13} - 1572864*a^{20} \\
& *b^3*c^7*d*e^{13}) + 917504*a^{19}*c^9*e^{12} - 320*a^{12}*b^{14}*c^2*e^{12} + 7936*a^{1 \\
& 3}*b^{12}*c^3*e^{12} - 82816*a^{14}*b^{10}*c^4*e^{12} + 468480*a^{15}*b^8*c^5*e^{12} - 153 \\
& 6000*a^{16}*b^6*c^6*e^{12} + 2867200*a^{17}*b^4*c^7*e^{12} - 2719744*a^{18}*b^2*c^8*e \\
& ^{12}) - x*(401408*a^{16}*c^{10}*e^{12} - 400*a^9*b^{14}*c^3*e^{12} + 9440*a^{10}*b^{12}*c^ \\
& ^4*e^{12} - 92816*a^{11}*b^{10}*c^5*e^{12} + 488096*a^{12}*b^8*c^6*e^{12} - 1458688*a^{13} \\
& *b^6*c^7*e^{12} + 2401280*a^{14}*b^4*c^8*e^{12} - 1871872*a^{15}*b^2*c^9*e^{12}) - 40 \\
& 1408*a^{16}*c^{10}*d*e^{11} + 400*a^9*b^{14}*c^3*d*e^{11} - 9440*a^{10}*b^{12}*c^4*d*e^{11} \\
& + 92816*a^{11}*b^{10}*c^5*d*e^{11} - 488096*a^{12}*b^8*c^6*d*e^{11} + 1458688*a^{13}*b \\
& ^6*c^7*d*e^{11} - 2401280*a^{14}*b^4*c^8*d*e^{11} + 1871872*a^{15}*b^2*c^9*d*e^{11}) \\
& + 476672*a^{13}*b*c^{10}*e^{10} + 1800*a^9*b^9*c^6*e^{10} - 29080*a^{10}*b^7*c^7*e^{10} \\
& + 176032*a^{11}*b^5*c^8*e^{10} - 473216*a^{12}*b^3*c^9*e^{10}))*(-(25*b^{15} + 25*b^ \\
& 6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^
\end{aligned}$$

$$3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)}/(32*(a^7*b^{12}*e^2 + 4096*a^{13}*c^6*e^2 - 24*a^8*b^{10}*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^{10}*b^6*c^3*e^2 + 3840*a^{11}*b^4*c^4*e^2 - 6144*a^{12}*b^2*c^5*e^2))^{(1/2)}*2i$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out

$$3.517 \quad \int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=341

$$\frac{(d+ex)(2a+b(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{(d+ex)(-4ac+7b^2+12bc(d+ex)^2)}{8e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3\sqrt{c}(-2b\sqrt{b^2-4ac})}{4\sqrt{2}e(b^2-4ac)}$$

Rubi [A] time = 0.95, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1142, 1120, 1178, 1166, 205}

$$\frac{(d+ex)(2a+b(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{(d+ex)(-4ac+7b^2+12bc(d+ex)^2)}{8e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3\sqrt{c}(-2b\sqrt{b^2-4ac}+4ac+3b^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{c}(2b\sqrt{b^2-4ac}+4ac+3b^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2 - ((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (3*sqrt[c]*(3*b^2 + 4*a*c - 2*b*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]]])/(4*sqrt[2]*(b^2 - 4*a*c)^(5/2)*sqrt[b - sqrt[b^2 - 4*a*c]])*e - (3*sqrt[c]*(3*b^2 + 4*a*c + 2*b*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b + sqrt[b^2 - 4*a*c]]])/(4*sqrt[2]*(b^2 - 4*a*c)^(5/2)*sqrt[b + sqrt[b^2 - 4*a*c]])*e

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1120

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(d^3*(d*x)^(m-3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^(m-4)*(2*a*(m-3) + b*(m+4*p+3)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^3} dx, x, d+ex\right)}{e} \\
&= \frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{\text{Subst}\left(\int \frac{2a-5bx^2}{(a+bx^2+cx^4)^2} dx, x, d+ex\right)}{4(b^2-4ac)e} \\
&= \frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{(d+ex)(7b^2-4ac)}{8(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} \\
&= \frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{(d+ex)(7b^2-4ac)}{8(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} \\
&= \frac{(d+ex)(2a+b(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{(d+ex)(7b^2-4ac)}{8(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)}
\end{aligned}$$

Mathematica [A] time = 4.43, size = 328, normalized size = 0.96

$$\frac{\frac{(d+ex)(4ac-7b^2-12bc(d+ex)^2)}{(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{2(-2a(d+ex)-b(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}}{8e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (3*sqrt[2]*sqrt[c]*(3*b^2 + 4*a*c - 2*b*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) - (3*sqrt[2]*sqrt[c]*(3*b^2 + 4*a*c + 2*b*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b + sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*sqrt[b + sqrt[b^2 - 4*a*c]]))/(8*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] IntegrateAlgebraic[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]

fricas [B] time = 2.04, size = 6633, normalized size = 19.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/16*(24*b*c^2*e^7*x^7 + 168*b*c^2*d*e^6*x^6 + 2*(252*b*c^2*d^2 + 19*b^2*c \\ & - 4*a*c^2)*e^5*x^5 + 24*b*c^2*d^7 + 10*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2) \\ &)*d)*e^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2) \\ &)*d^2)*e^3*x^3 + 2*(19*b^2*c - 4*a*c^2)*d^5 + 2*(252*b*c^2*d^5 + 10*(19*b^2 \\ & *c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*e^2*x^2 + 2*(5*b^3 + 16*a*b*c)* \\ & d^3 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3 \\ & *(5*b^3 + 16*a*b*c)*d^2)*e*x - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2 \\ & *c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c \\ & - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2 \\ &)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a \\ & *b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b \\ & ^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b \\ & *c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b \\ & ^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)* \\ & e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^ \\ & 3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - \\ & 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^ \\ & 2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c \\ & + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c \\ & ^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3) \\ &)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)* \\ & d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c \\ & + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^ \\ & ^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*sqrt(1/((a^2*b^10 - 20*a^3*b^8*c \\ & + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4) \\ &))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^ \\ & ^2*c^4 - 1024*a^6*c^5)*e^2))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e* \end{aligned}$$

$$\begin{aligned}
& x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d + 3/2*\sqrt{1/2}*((a*b^{13} - 8*a^2*b^{11}*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*\sqrt{1/((a^2*b^{10} - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) - (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*\sqrt{-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^{10} - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*\sqrt{1/((a^2*b^{10} - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^{10} - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2)) + 3*\sqrt{1/2}*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*\sqrt{-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^{10} - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*\sqrt{1/((a^2*b^{10} - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^{10} - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))*\log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*e*x + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*\sqrt{1/2}*((a*b^{13} - 8*a^2*b^{11}*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*\sqrt{1/((a^2*b^{10} - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) - (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*\sqrt{-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^{10} - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*\sqrt{1/((a^2*b^{10} - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^{10} - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2)) + 3*\sqrt{1/2}*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
\end{aligned}$$

$$\begin{aligned}
& d^2) * e^5 * x^4 + 4 * (14 * (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * d^5 + 10 * (b^5 * c - \\
& 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * d^3 + (b^6 - 6 * a * b^4 * c + 32 * a^3 * c^3) * d) * e^4 * x^3 \\
& + 2 * (14 * (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * d^6 + a * b^5 - 8 * a^2 * b^3 * c + \\
& 16 * a^3 * b * c^2 + 15 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * d^4 + 3 * (b^6 - 6 * a * b^4 * \\
& c + 32 * a^3 * c^3) * d^2) * e^3 * x^2 + 4 * (2 * (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) \\
& * d^7 + 3 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * d^5 + (b^6 - 6 * a * b^4 * c + 32 * a^3 * \\
& c^3) * d^3 + (a * b^5 - 8 * a^2 * b^3 * c + 16 * a^3 * b * c^2) * d) * e^2 * x + ((b^4 * c^2 - 8 * \\
& a * b^2 * c^3 + 16 * a^2 * c^4) * d^8 + 2 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * d^6 + a^2 * b^4 - \\
& 8 * a^3 * b^2 * c + 16 * a^4 * c^2 + (b^6 - 6 * a * b^4 * c + 32 * a^3 * c^3) * d^4 + 2 * (a * b^5 - 8 * a^2 * b^3 * c + \\
& 16 * a^3 * b * c^2) * d^2) * e) * \text{sqrt}(-(b^5 + 40 * a * b^3 * c + 80 * a^2 * b * c^2 - (a * b^{10} - 20 * a^2 * b^8 * c + 160 * \\
& a^3 * b^6 * c^2 - 640 * a^4 * b^4 * c^3 + 1280 * a^5 * b^2 * c^4 - 1024 * a^6 * c^5) * e^2 * \text{sqrt}(1 / ((a^2 * b^{10} - 20 * a^3 * b^8 * c + 160 * \\
& a^4 * b^6 * c^2 - 640 * a^5 * b^4 * c^3 + 1280 * a^6 * b^2 * c^4 - 1024 * a^7 * c^5) * e^4))) / ((a * b^{10} - 20 * a^2 * b^8 * c + 160 * a^3 * b^6 * c^2 - 640 * a^4 * b^4 * c^3 + 1280 * a^5 * b^2 * c^4 \\
& - 1024 * a^6 * c^5) * e^2)) * \log(3 * (5 * b^4 * c + 40 * a * b^2 * c^2 + 16 * a^2 * c^3) * e * x + 3 * \\
& (5 * b^4 * c + 40 * a * b^2 * c^2 + 16 * a^2 * c^3) * d + 3 / 2 * \text{sqrt}(1 / 2) * ((a * b^{13} - 8 * a^2 * b^{11} * c - 80 * a^3 * b^9 * c^2 + 1280 * a^4 * b^7 * c^3 - 6400 * a^5 * b^5 * c^4 + 14336 * a^6 * b^3 * \\
& c^5 - 12288 * a^7 * b * c^6) * e^3 * \text{sqrt}(1 / ((a^2 * b^{10} - 20 * a^3 * b^8 * c + 160 * a^4 * b^6 * c^2 - 640 * a^5 * b^4 * c^3 + 1280 * a^6 * b^2 * c^4 - 1024 * a^7 * c^5) * e^4))) + (b^8 - 8 * a * \\
& b^6 * c + 128 * a^3 * b^2 * c^3 - 256 * a^4 * c^4) * e) * \text{sqrt}(-(b^5 + 40 * a * b^3 * c + 80 * a^2 * b * c^2 - (a * b^{10} - 20 * a^2 * b^8 * c + 160 * a^3 * b^6 * c^2 - 640 * a^4 * b^4 * c^3 + 1280 * \\
& a^5 * b^2 * c^4 - 1024 * a^6 * c^5) * e^2 * \text{sqrt}(1 / ((a^2 * b^{10} - 20 * a^3 * b^8 * c + 160 * a^4 * b^6 * c^2 - 640 * a^5 * b^4 * c^3 + 1280 * a^6 * b^2 * c^4 - 1024 * a^7 * c^5) * e^4))) / ((a * b^{10} \\
& - 20 * a^2 * b^8 * c + 160 * a^3 * b^6 * c^2 - 640 * a^4 * b^4 * c^3 + 1280 * a^5 * b^2 * c^4 - 1024 * a^6 * c^5) * e^2))) - 3 * \text{sqrt}(1 / 2) * ((b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * e^9 \\
& * x^8 + 8 * (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * d * e^8 * x^7 + 2 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3 + 14 * (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * d^2) * e^7 * x^6 \\
& + 4 * (14 * (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * d^3 + 3 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * d) * e^6 * x^5 + (b^6 - 6 * a * b^4 * c + 32 * a^3 * c^3 + 70 * (b^4 * c^2 - \\
& 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * d^4 + 30 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * d^2) * e^5 * x^4 + 4 * (14 * (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * d^5 + 10 * (b^5 * c - 8 * \\
& a * b^3 * c^2 + 16 * a^2 * b * c^3) * d^3 + (b^6 - 6 * a * b^4 * c + 32 * a^3 * c^3) * d) * e^4 * x^3 + 2 * (14 * (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * d^6 + a * b^5 - 8 * a^2 * b^3 * c + 16 * \\
& a^3 * b * c^2 + 15 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * d^4 + 3 * (b^6 - 6 * a * b^4 * c + 32 * a^3 * c^3) * d^2) * e^3 * x^2 + 4 * (2 * (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * d^7 \\
& + 3 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * d^5 + (b^6 - 6 * a * b^4 * c + 32 * a^3 * c^3) * d^3 + (a * b^5 - 8 * a^2 * b^3 * c + 16 * a^3 * b * c^2) * d) * e^2 * x + ((b^4 * c^2 - 8 * a * \\
& b^2 * c^3 + 16 * a^2 * c^4) * d^8 + 2 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * d^6 + a^2 * b^4 - 8 * a^3 * b^2 * c + 16 * a^4 * c^2 + (b^6 - 6 * a * b^4 * c + 32 * a^3 * c^3) * d^4 + 2 * (a * b^5 - 8 * a^2 * b^3 * c + 16 * a^3 * b * c^2) * d^2) * e) * \text{sqrt}(-(b^5 + 40 * a * b^3 * c + 80 * a^2 * b * c^2 - (a * b^{10} - 20 * a^2 * b^8 * c + 160 * a^3 * b^6 * c^2 - 640 * a^4 * b^4 * c^3 + 1280 * \\
& a^5 * b^2 * c^4 - 1024 * a^6 * c^5) * e^2 * \text{sqrt}(1 / ((a^2 * b^{10} - 20 * a^3 * b^8 * c + 160 * a^4 * b^6 * c^2 - 640 * a^5 * b^4 * c^3 + 1280 * a^6 * b^2 * c^4 - 1024 * a^7 * c^5) * e^4))) / ((a * b^{10} - 20 * a^2 * b^8 * c + 160 * a^3 * b^6 * c^2 - 640 * a^4 * b^4 * c^3 + 1280 * a^5 * b^2 * c^4 - \\
& 1024 * a^6 * c^5) * e^2)) * \log(3 * (5 * b^4 * c + 40 * a * b^2 * c^2 + 16 * a^2 * c^3) * e * x + 3 * (5 *
\end{aligned}$$

$$\begin{aligned}
& b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*\text{sqrt}(1/2)*((a*b^{13} - 8*a^2*b^{11}* \\
& c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e^3*\text{sqrt}(1/((a^2*b^{10} - 20*a^3*b^8*c + 160*a^4*b^6*c^2 \\
& - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)) + (b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e)*\text{sqrt}(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^{10} - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2*\text{sqrt}(1/((a^2*b^{10} - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5)*e^4)))/((a*b^{10} - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)*e^2))) + 6*(a*b^2 + 4*a^2*c)*d)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)
\end{aligned}$$

giac [B] time = 0.75, size = 1688, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] $3/16*((4*(d*e^{-1}) + \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c))^2*b*c*e^2 - 8*(d*e^{-1}) + \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c)*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*\log(d*e^{-1}) + x + \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c)/(2*(d*e^{-1}) + \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c))^3*c*e^4 - 6*(d*e^{-1}) + \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{-1}) + \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c))) + (4*(d*e^{-1}) - \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c))^2*b*c*e^2 - 8*(d*e^{-1}) - \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*\log(d*e^{-1}) + x - \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c)/(2*(d*e^{-1}) - \text{sqrt}(1/2)*\text{sqrt}(-(b*e^2 + \text{sqrt}(b^2 - 4*a*c))*e^2))*e^{-4}/c))$

$$\begin{aligned}
& (-4/c))^3*c*e^4 - 6*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})} \\
& *e^2)*e^{(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e \\
& ^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/c})) + (4*(d \\
& *e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/c}))^2*b*c* \\
& e^2 - 8*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/ \\
& c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*\log(d*e^{(-1)} + x + \sqrt{1/2}*\sqrt{-(\\
& b*e^2 - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/c}))/ (2*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b \\
& *e^2 - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/c}))^3*c*e^4 - 6*(d*e^{(-1)} + \sqrt{1/2} \\
& *\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/c}))^2*c*d*e^3 - 2*c*d^3*e - b* \\
& d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - \\
& 4*a*c})} *e^2)*e^{(-4)/c})) + (4*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - \\
& 4*a*c})} *e^2)*e^{(-4)/c}))^2*b*c*e^2 - 8*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 \\
& - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/c}))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*\log(\\
& d*e^{(-1)} + x - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/c}))/ (\\
& 2*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/c}))^3* \\
& c*e^4 - 6*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4 \\
&)/c}))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} - \sqrt{ \\
& t(1/2)*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})} *e^2)*e^{(-4)/c}))) / (b^4 - 8*a*b^2*c \\
& + 16*a^2*c^2) - 1/8*(12*b*c^2*x^7*e^7 + 84*b*c^2*d*x^6*e^6 + 252*b*c^2*d^2* \\
& x^5*e^5 + 420*b*c^2*d^3*x^4*e^4 + 420*b*c^2*d^4*x^3*e^3 + 252*b*c^2*d^5*x^2 \\
& *e^2 + 84*b*c^2*d^6*x*e + 12*b*c^2*d^7 + 19*b^2*c*x^5*e^5 - 4*a*c^2*x^5*e^5 \\
& + 95*b^2*c*d*x^4*e^4 - 20*a*c^2*d*x^4*e^4 + 190*b^2*c*d^2*x^3*e^3 - 40*a*c \\
& ^2*d^2*x^3*e^3 + 190*b^2*c*d^3*x^2*e^2 - 40*a*c^2*d^3*x^2*e^2 + 95*b^2*c*d^ \\
& 4*x*e - 20*a*c^2*d^4*x*e + 19*b^2*c*d^5 - 4*a*c^2*d^5 + 5*b^3*x^3*e^3 + 16* \\
& a*b*c*x^3*e^3 + 15*b^3*d*x^2*e^2 + 48*a*b*c*d*x^2*e^2 + 15*b^3*d^2*x*e + 48 \\
& *a*b*c*d^2*x*e + 5*b^3*d^3 + 16*a*b*c*d^3 + 3*a*b^2*x*e + 12*a^2*c*x*e + 3* \\
& a*b^2*d + 12*a^2*c*d) / ((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d \\
& ^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e \\
& + 16*a^2*c^2*e))
\end{aligned}$$

maple [C] time = 0.05, size = 704, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)$

[Out] $(-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d^6-20*a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*d^4+16*a*b*c$

$$\frac{d^2+5b^3d^2+12a^2c+3ab^2}{(16a^2c^2-8ab^2c+b^4)} \cdot \frac{1}{(ce^4x^4+4cd^2e^3x^3+6c^2d^2e^2x^2+4c^3d^3e^2x+b^2x^2+cd^4+2b^2d^2e^2x+b^2d^2+a)^2} + \frac{3/16}{(16a^2c^2-8ab^2c+b^4)} \cdot \frac{1}{e^{\sum((-4R^2b^2c^2e^2-8R^2b^2c^2d^2+4a^2c^2+2R^2b^2c^2e^3+6R^2b^2c^2d^2+6R^2b^2c^2e^2+2c^2d^3+R^2b^2e+b^2d) \ln(-R+x), R=\text{RootOf}(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))}}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 7.02, size = 12677, normalized size = 37.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)

[Out] atan((((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b^9*c^5*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b^7*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) * ((9*((-4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b^7*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c^2*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) * ((9*((-4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b^7*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c^2*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*

$$\begin{aligned}
& a^4c^7de^{11} + 936b^8c^3de^{11} - 6912a^2b^6c^4de^{11} + 11520a^2b^4c^5de^{11}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 \\
& + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24a^2b^{10}c)) + (x(144a^2c^5e^{12} + 117b^4c^3e^{12} + 72a^2b^2c^4e^{12})) / (16(b^8 + 256a^4c^4 + 96a^2b^4c^2 - 256a^3b^2c^3 - 16a^2b^6c)) \\
& * ((9((-(4ac - b^2)^{15})^{1/2}) - b^{15} + 81920a^7b^7c^7 + 560a^2b^{11}c^2 - 4160a^3b^9c^3 + 11520a^4b^7c^4 + 1024a^5b^5c^5 - 61440a^6b^3c^6 - 20a^2b^{13}c)) / (512(a^20e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^2e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2)))^{1/2} * i + ((18432a^4c^7de^{11} + 936b^8c^3de^{11} - 6912a^2b^6c^4de^{11} + 11520a^2b^4c^5de^{11}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24a^2b^{10}c)) - ((786432a^6c^8e^{12} - 192b^{12}c^2e^{12} + 3072a^2b^{10}c^3e^{12} - 15360a^2b^8c^4e^{12} + 245760a^4b^4c^6e^{12} - 786432a^5b^2c^7e^{12}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24a^2b^{10}c)) - ((1024b^{15}c^2de^{13} - 28672a^2b^{13}c^3de^{13} - 16777216a^7b^7c^9de^{13} + 344064a^2b^{11}c^4de^{13} - 2293760a^3b^9c^5de^{13} + 9175040a^4b^7c^6de^{13} - 22020096a^5b^5c^7de^{13} + 29360128a^6b^3c^8de^{13}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24a^2b^{10}c)) + (x(128b^{11}c^2e^{14} - 2560a^2b^9c^3e^{14} - 131072a^5b^7c^7e^{14} + 20480a^2b^7c^4e^{14} - 81920a^3b^5c^5e^{14} + 163840a^4b^3c^6e^{14})) / (16(b^8 + 256a^4c^4 + 96a^2b^4c^2 - 256a^3b^2c^3 - 16a^2b^6c)) * ((9((-(4ac - b^2)^{15})^{1/2}) - b^{15} + 81920a^7b^7c^7 + 560a^2b^{11}c^2 - 4160a^3b^9c^3 + 11520a^4b^7c^4 + 1024a^5b^5c^5 - 61440a^6b^3c^6 - 20a^2b^{13}c)) / (512(a^20e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^2e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2)))^{1/2} * ((9((-(4ac - b^2)^{15})^{1/2}) - b^{15} + 81920a^7b^7c^7 + 560a^2b^{11}c^2 - 4160a^3b^9c^3 + 11520a^4b^7c^4 + 1024a^5b^5c^5 - 61440a^6b^3c^6 - 20a^2b^{13}c)) / (512(a^20e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^2e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2)))^{1/2} * i) / ((135b^5c^3e^{10} + 1080a^2b^3c^4e^{10} + 432a^2b^5c^5e^{10}) / (64(b^{12} + 4096
\end{aligned}$$

$$\begin{aligned}
& a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c) - ((786432a^6c^8e^{12} - 192b^{12}c^2e^{12} + 3072ab^{10}c^3e^{12} - 15360a^2b^8c^4e^{12} + 245760a^4b^4c^6e^{12} - 786432a^5b^2c^7e^{12}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) + ((1024b^{15}c^2de^{13} - 28672ab^{13}c^3de^{13} - 16777216a^7b^9c^5de^{13} + 344064a^2b^{11}c^4de^{13} - 2293760a^3b^9c^5de^{13} + 9175040a^4b^7c^6de^{13} - 22020096a^5b^5c^7de^{13} + 29360128a^6b^3c^8de^{13}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) + (x(128b^{11}c^2e^{14} - 2560ab^9c^3e^{14} - 131072a^5b^7c^4e^{14} + 20480a^2b^7c^4e^{14} - 81920a^3b^5c^5e^{14} + 163840a^4b^3c^6e^{14})) / (16(b^8 + 256a^4c^4 + 96a^2b^4c^2 - 256a^3b^2c^3 - 16ab^6c)) * ((9((-4ac - b^2)^{15})^{1/2} - b^{15} + 81920a^7b^7c^7 + 560a^2b^{11}c^2 - 4160a^3b^9c^3 + 11520a^4b^7c^4 + 1024a^5b^5c^5 - 61440a^6b^3c^6 - 20ab^{13}c)) / (512(ab^{20}e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^4e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2))^{1/2} * ((9((-4ac - b^2)^{15})^{1/2} - b^{15} + 81920a^7b^7c^7 + 560a^2b^{11}c^2 - 4160a^3b^9c^3 + 11520a^4b^7c^4 + 1024a^5b^5c^5 - 61440a^6b^3c^6 - 20ab^{13}c)) / (512(ab^{20}e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^4e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2))^{1/2} + (18432a^4c^7de^{11} + 936b^8c^3de^{11} - 6912ab^6c^4de^{11} + 11520a^2b^4c^5de^{11}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) + (x(144a^2c^5e^{12} + 117b^4c^3e^{12} + 72ab^2c^4e^{12})) / (16(b^8 + 256a^4c^4 + 96a^2b^4c^2 - 256a^3b^2c^3 - 16ab^6c)) * ((9((-4ac - b^2)^{15})^{1/2} - b^{15} + 81920a^7b^7c^7 + 560a^2b^{11}c^2 - 4160a^3b^9c^3 + 11520a^4b^7c^4 + 1024a^5b^5c^5 - 61440a^6b^3c^6 - 20ab^{13}c)) / (512(ab^{20}e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^4e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2))^{1/2} + ((18432a^4c^7de^{11} + 936b^8c^3de^{11} - 6912ab^6c^4de^{11} + 11520a^2b^4c^5de^{11}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) - ((786432a^6c^8e^{12} - 192b^{12}c^2e^{12} + 3072ab^{10}c^3e^{12} - 15360a^2b^8c^4e^{12} + 245760a^4b^4c^6e^{12} - 786432a^5b^2c^7e^{12}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) - ((1024b^{15}c^2de^{13} - 28672ab^{13}c^3de^{13} - 16777216a^7b^9c^5de^{13} + 344064a^2b^{11}c^4de^{13} - 2293760a^3b^9c^5de^{13} + 9175040a^4b^7c^6de^{13} - 22020096a^5b^5c^7de^{13} + 29360128a^6b^3c^8de^{13}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3
\end{aligned}$$

$$\begin{aligned}
& 840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(128*b^{11}*c^2*e^{14} \\
& - 2560*a*b^9*c^3*e^{14} - 131072*a^5*b*c^7*e^{14} + 20480*a^2*b^7*c^4*e^{14} - 81 \\
& 920*a^3*b^5*c^5*e^{14} + 163840*a^4*b^3*c^6*e^{14}))/((16*(b^8 + 256*a^4*c^4 + 9 \\
& 6*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^{15})^{(1 \\
& /2) - b^{15} + 81920*a^7*b*c^7 + 560*a^2*b^{11}*c^2 - 4160*a^3*b^9*c^3 + 11520* \\
& a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^{13}*c)))/(512*(a* \\
& b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 \\
& - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 \\
& + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e \\
& ^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)})*((9*((-(4*a*c - b^2)^{15})^{(1/2) - b^ \\
& 15 + 81920*a^7*b*c^7 + 560*a^2*b^{11}*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7* \\
& c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^{13}*c)))/(512*(a*b^{20}*e^2 \\
& + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680* \\
& a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 86016 \\
& 0*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262 \\
& 1440*a^{10}*b^2*c^9*e^2)))^{(1/2)} + (x*(144*a^2*c^5*e^{12} + 117*b^4*c^3*e^{12} + \\
& 72*a*b^2*c^4*e^{12}))/((16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c \\
& ^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^{15})^{(1/2) - b^{15} + 81920*a^7*b*c^7 \\
& + 560*a^2*b^{11}*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^ \\
& 5 - 61440*a^6*b^3*c^6 - 20*a*b^{13}*c)))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10} \\
& e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53 \\
& 760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1 \\
& 966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2 \\
&)))^{(1/2)})*((9*((-(4*a*c - b^2)^{15})^{(1/2) - b^{15} + 81920*a^7*b*c^7 + 560*a \\
& ^2*b^{11}*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 614 \\
& 40*a^6*b^3*c^6 - 20*a*b^{13}*c)))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 4 \\
& 0*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5 \\
& *b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080* \\
& a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/ \\
& 2)}*2i + \operatorname{atan}((((786432*a^6*c^8*e^{12} - 192*b^{12}*c^2*e^{12} + 3072*a*b^{10}*c^3* \\
& e^{12} - 15360*a^2*b^8*c^4*e^{12} + 245760*a^4*b^4*c^6*e^{12} - 786432*a^5*b^2*c^ \\
& 7*e^{12}))/((128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38 \\
& 40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + ((1024*b^{15}*c^2*d*e^{13} \\
& - 28672*a*b^{13}*c^3*d*e^{13} - 16777216*a^7*b*c^9*d*e^{13} + 344064*a^2*b^{11}*c^4 \\
& *d*e^{13} - 2293760*a^3*b^9*c^5*d*e^{13} + 9175040*a^4*b^7*c^6*d*e^{13} - 2202009 \\
& 6*a^5*b^5*c^7*d*e^{13} + 29360128*a^6*b^3*c^8*d*e^{13}))/((128*(b^{12} + 4096*a^6*c \\
& ^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c \\
& ^5 - 24*a*b^{10}*c)) + (x*(128*b^{11}*c^2*e^{14} - 2560*a*b^9*c^3*e^{14} - 131072*a \\
& ^5*b*c^7*e^{14} + 20480*a^2*b^7*c^4*e^{14} - 81920*a^3*b^5*c^5*e^{14} + 163840*a^ \\
& 4*b^3*c^6*e^{14}))/((16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 \\
& - 16*a*b^6*c)))*(-(9*(b^{15} + (-(4*a*c - b^2)^{15})^{(1/2) - 81920*a^7*b*c^7 - \\
& 560*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 \\
& + 61440*a^6*b^3*c^6 + 20*a*b^{13}*c)))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^ \\
& 2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 5376 \\
& 0*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 196
\end{aligned}$$

$$\begin{aligned}
& 6080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)) \\
&)^{(1/2)}*(-(9*(b^{15} + (-4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7 - 560*a^2 \\
& *b^{11}*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440 \\
& *a^6*b^3*c^6 + 20*a*b^{13}*c)) / (512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40* \\
& a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b \\
& ^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^ \\
& 8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} \\
& + (18432*a^4*c^7*d*e^{11} + 936*b^8*c^3*d*e^{11} - 6912*a*b^6*c^4*d*e^{11} + 115 \\
& 20*a^2*b^4*c^5*d*e^{11}) / (128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a \\
& ^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(144* \\
& a^2*c^5*e^{12} + 117*b^4*c^3*e^{12} + 72*a*b^2*c^4*e^{12})) / (16*(b^8 + 256*a^4*c^ \\
& 4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^{15} + (-4*a*c \\
& - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7 - 560*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 - \\
& 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^{13}*c)) / (5 \\
& 12*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}* \\
& ^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}* \\
& ^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4 \\
& *c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)}*i + ((18432*a^4*c^7*d*e^{11} + \\
& 936*b^8*c^3*d*e^{11} - 6912*a*b^6*c^4*d*e^{11} + 11520*a^2*b^4*c^5*d*e^{11}) / (128 \\
& *(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^ \\
& ^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) - ((786432*a^6*c^8*e^{12} - 192*b^{12}*c^ \\
& 2*e^{12} + 3072*a*b^{10}*c^3*e^{12} - 15360*a^2*b^8*c^4*e^{12} + 245760*a^4*b^4*c^6 \\
& *e^{12} - 786432*a^5*b^2*c^7*e^{12}) / (128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^ \\
& 2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) \\
& - ((1024*b^{15}*c^2*d*e^{13} - 28672*a*b^{13}*c^3*d*e^{13} - 16777216*a^7*b*c^9*d*e \\
& ^{13} + 344064*a^2*b^{11}*c^4*d*e^{13} - 2293760*a^3*b^9*c^5*d*e^{13} + 9175040*a^4 \\
& *b^7*c^6*d*e^{13} - 22020096*a^5*b^5*c^7*d*e^{13} + 29360128*a^6*b^3*c^8*d*e^{13} \\
&) / (128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4 \\
& *b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(128*b^{11}*c^2*e^{14} - 2560* \\
& a*b^9*c^3*e^{14} - 131072*a^5*b*c^7*e^{14} + 20480*a^2*b^7*c^4*e^{14} - 81920*a^3 \\
& *b^5*c^5*e^{14} + 163840*a^4*b^3*c^6*e^{14})) / (16*(b^8 + 256*a^4*c^4 + 96*a^2*b \\
& ^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^{15} + (-4*a*c - b^2)^{15})^{(\\
& 1/2)} - 81920*a^7*b*c^7 - 560*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^ \\
& 7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^{13}*c)) / (512*(a*b^{20}* \\
& ^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 768 \\
& 0*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860 \\
& 160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2 \\
& 621440*a^{10}*b^2*c^9*e^2)))^{(1/2)}*(-(9*(b^{15} + (-4*a*c - b^2)^{15})^{(1/2)} - \\
& 81920*a^7*b*c^7 - 560*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - \\
& 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^{13}*c)) / (512*(a*b^{20}*e^2 + 10 \\
& 48576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b \\
& ^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7 \\
& *b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440* \\
& a^{10}*b^2*c^9*e^2)))^{(1/2)} + (x*(144*a^2*c^5*e^{12} + 117*b^4*c^3*e^{12} + 72*a* \\
& b^2*c^4*e^{12})) / (16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
\end{aligned}$$

$$\begin{aligned}
& 16*a*b^6*c)))*(-(9*(b^{15} + (-4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7 - 56 \\
& 0*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + \\
& 61440*a^6*b^3*c^6 + 20*a*b^{13}*c)))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 \\
& - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760* \\
& a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 19660 \\
& 80*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(\\
& (1/2)*1i)/((135*b^5*c^3*e^{10} + 1080*a*b^3*c^4*e^{10} + 432*a^2*b*c^5*e^{10})/(6 \\
& 4*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4* \\
& c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) - (((786432*a^6*c^8*e^{12} - 192*b^{12}* \\
& c^2*e^{12} + 3072*a*b^{10}*c^3*e^{12} - 15360*a^2*b^8*c^4*e^{12} + 245760*a^4*b^4*c \\
& ^6*e^{12} - 786432*a^5*b^2*c^7*e^{12})/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8* \\
& c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c) \\
&) + ((1024*b^{15}*c^2*d*e^{13} - 28672*a*b^{13}*c^3*d*e^{13} - 16777216*a^7*b*c^9*d \\
& *e^{13} + 344064*a^2*b^{11}*c^4*d*e^{13} - 2293760*a^3*b^9*c^5*d*e^{13} + 9175040*a \\
& ^4*b^7*c^6*d*e^{13} - 22020096*a^5*b^5*c^7*d*e^{13} + 29360128*a^6*b^3*c^8*d*e^{ \\
& 13})/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a \\
& ^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(128*b^{11}*c^2*e^{14} - 256 \\
& 0*a*b^9*c^3*e^{14} - 131072*a^5*b*c^7*e^{14} + 20480*a^2*b^7*c^4*e^{14} - 81920*a \\
& ^3*b^5*c^5*e^{14} + 163840*a^4*b^3*c^6*e^{14}))/((16*(b^8 + 256*a^4*c^4 + 96*a^2 \\
& *b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^{15} + (-4*a*c - b^2)^{15}) \\
& ^{(1/2)} - 81920*a^7*b*c^7 - 560*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4* \\
& b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^{13}*c)))/(512*(a*b^{20} \\
& *e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7 \\
& 680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 8 \\
& 60160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - \\
& 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2))*(-(9*(b^{15} + (-4*a*c - b^2)^{15})^{(1/2)} \\
& - 81920*a^7*b*c^7 - 560*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 \\
& - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^{13}*c)))/(512*(a*b^{20}*e^2 + \\
& 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4 \\
& *b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a \\
& ^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262144 \\
& 0*a^{10}*b^2*c^9*e^2)))^{(1/2)} + (18432*a^4*c^7*d*e^{11} + 936*b^8*c^3*d*e^{11} - \\
& 6912*a*b^6*c^4*d*e^{11} + 11520*a^2*b^4*c^5*d*e^{11})/(128*(b^{12} + 4096*a^6*c^6 \\
& + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 \\
& - 24*a*b^{10}*c)) + (x*(144*a^2*c^5*e^{12} + 117*b^4*c^3*e^{12} + 72*a*b^2*c^4*e \\
& ^{12}))/((16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6* \\
& c)))*(-(9*(b^{15} + (-4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7 - 560*a^2*b^1 \\
& 1*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6 \\
& *b^3*c^6 + 20*a*b^{13}*c)))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2* \\
& b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12} \\
& *c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^ \\
& 6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} + (\\
& (18432*a^4*c^7*d*e^{11} + 936*b^8*c^3*d*e^{11} - 6912*a*b^6*c^4*d*e^{11} + 11520* \\
& a^2*b^4*c^5*d*e^{11})/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3* \\
& b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) - ((786432*a^
\end{aligned}$$

$$\begin{aligned}
& 6*c^8*e^{12} - 192*b^{12}*c^2*e^{12} + 3072*a*b^{10}*c^3*e^{12} - 15360*a^2*b^8*c^4*e^{12} \\
& + 245760*a^4*b^4*c^6*e^{12} - 786432*a^5*b^2*c^7*e^{12}) / (128*(b^{12} + 4096*a^6*c^6 \\
& + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) \\
& - ((1024*b^{15}*c^2*d*e^{13} - 28672*a*b^{13}*c^3*d*e^{13} - 16777216*a^7*b*c^9*d*e^{13} \\
& + 344064*a^2*b^{11}*c^4*d*e^{13} - 2293760*a^3*b^9*c^5*d*e^{13} + 9175040*a^4*b^7*c^6*d*e^{13} \\
& - 22020096*a^5*b^5*c^7*d*e^{13} + 29360128*a^6*b^3*c^8*d*e^{13}) / (128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 128 \\
& 0*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(128*b^{11}*c^2*e^{14} \\
& - 2560*a*b^9*c^3*e^{14} - 131072*a^5*b*c^7*e^{14} + 20480*a^2*b^7*c^4*e^{14} - 81920*a^3*b^5*c^5*e^{14} \\
& + 163840*a^4*b^3*c^6*e^{14})) / (16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) \\
& * (- (9*(b^{15} + (- (4*a*c - b^2)^{15})^{1/2}) - 81920*a^7*b*c^7 - 560*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 \\
& - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^{13}*c)) / (512*(a*b^{20}*e^2 \\
& + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 \\
& + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 \\
& + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2))^{1/2}) * (- (9*(b^{15} + (- (4*a*c - b^2)^{15})^{1/2}) \\
& - 81920*a^7*b*c^7 - 560*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 \\
& + 61440*a^6*b^3*c^6 + 20*a*b^{13}*c)) / (512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 \\
& + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 \\
& + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2))^{1/2} \\
& + (x*(144*a^2*c^5*e^{12} + 117*b^4*c^3*e^{12} + 72*a*b^2*c^4*e^{12})) / (16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 \\
& - 256*a^3*b^2*c^3 - 16*a*b^6*c)) * (- (9*(b^{15} + (- (4*a*c - b^2)^{15})^{1/2}) - 81920*a^7*b*c^7 \\
& - 560*a^2*b^{11}*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 \\
& + 20*a*b^{13}*c)) / (512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 \\
& - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 \\
& - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2))^{1/2}) * 2i - ((x^2*(15*b^3*d*e \\
& - 40*a*c^2*d^3*e + 190*b^2*c*d^3*e + 252*b*c^2*d^5*e + 48*a*b*c*d*e)) / (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) \\
&) + (x^3*(5*b^3*e^2 - 40*a*c^2*d^2*e^2 + 190*b^2*c*d^2*e^2 + 420*b*c^2*d^4*e^2 + 16*a*b*c*e^2)) / (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) \\
& + (5*x^4*(84*b*c^2*d^3*e^3 - 4*a*c^2*d*e^3 + 19*b^2*c*d*e^3)) / (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) \\
& + (x^5*(19*b^2*c*e^4 - 4*a*c^2*e^4 + 252*b*c^2*d^2*e^4)) / (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) \\
& + (x*(3*a*b^2 + 12*a^2*c + 15*b^3*d^2 - 20*a*c^2*d^4 + 95*b^2*c*d^4 + 84*b*c^2*d^6 + 48*a*b*c*d^2)) / (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) \\
& + (5*b^3*d^3 - 4*a*c^2*d^5 + 19*b^2*c*d^5 + 12*b*c^2*d^7 + 3*a*b^2*d +
\end{aligned}$$

$$\begin{aligned}
& (12*a^2*c*d + 16*a*b*c*d^3)/(8*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2 \\
& *e^6*x^7)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (21*b*c^2*d*e^5*x^6)/(2*(b^4 \\
& + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e \\
& ^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + \\
& x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^ \\
& 3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c \\
& ^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30 \\
& *b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d \\
& ^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out

$$3.518 \quad \int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=150

$$\frac{2a + b(d + ex)^2}{4e(b^2 - 4ac)(a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{3b(b + 2c(d + ex)^2)}{4e(b^2 - 4ac)^2(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{3bc \tanh^{-1}\left(\frac{b+2c(d+ex)}{\sqrt{b^2-4ac}}\right)}{e(b^2 - 4ac)^{5/2}}$$

Rubi [A] time = 0.20, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1142, 1114, 638, 614, 618, 206}

$$\frac{2a + b(d + ex)^2}{4e(b^2 - 4ac)(a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{3b(b + 2c(d + ex)^2)}{4e(b^2 - 4ac)^2(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{3bc \tanh^{-1}\left(\frac{b+2c(d+ex)}{\sqrt{b^2-4ac}}\right)}{e(b^2 - 4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (2*a + b*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2 - (3*b*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (3*b*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)*e

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^3} dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^3} dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{2a+b(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2} dx, x, (d+ex)^2\right)}{4(b^2-4ac)} \\
&= \frac{2a+b(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{3b(b+2c(d+ex)^2)}{4(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} \\
&= \frac{2a+b(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{3b(b+2c(d+ex)^2)}{4(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} \\
&= \frac{2a+b(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{3b(b+2c(d+ex)^2)}{4(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 146, normalized size = 0.97

$$\frac{\frac{(b^2-4ac)(2a+b(d+ex)^2)}{(a+(d+ex)^2(b+c(d+ex)^2))^2} - \frac{12bc \tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{3b(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4}}{4e(b^2-4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] ((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] IntegrateAlgebraic[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]

fricas [B] time = 1.53, size = 3739, normalized size = 24.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*x^4 + 6*(b^3*c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*x - 6*(b*c^3*e^8*x^8 + 8*b*c^3*d*e^7*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*x^2 + a^2*b*c + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 +

$$\begin{aligned}
& 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10* \\
& a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a* \\
& b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4 \\
& *c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2 \\
& *b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^ \\
& 3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a \\
& ^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6 \\
& 4*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12 \\
& 8*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)* \\
& e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b \\
& ^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + \\
& 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + \\
& 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^ \\
& 2 - 64*a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*x^6 + 36*(b^3*c \\
& ^2 - 4*a*b*c^3)*d*e^5*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c \\
& ^3)*d^2)*e^4*x^4 + 6*(b^3*c^2 - 4*a*b*c^3)*d^6 + 12*(10*(b^3*c^2 - 4*a*b*c \\
& ^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*x^3 + a*b^4 + 4*a^2*b^2*c - 32*a^3* \\
& c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b \\
& ^3*c^2 - 4*a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*x^2 + 2*(b^5 + \\
& a*b^3*c - 20*a^2*b*c^2)*d^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4 \\
& *a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*x - 12*(b*c^3*e^8*x^8 \\
& + 8*b*c^3*d*e^7*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*x^6 + b*c^3*d^8 + 4*(\\
& 14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*x^5 + 2*b^2*c^2*d^6 + (70*b*c^3*d^4 + 30*b^ \\
& 2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + \\
& (b^3*c + 2*a*b*c^2)*d)*e^3*x^3 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + \\
& 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^ \\
& 2*x^2 + a^2*b*c + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a \\
& *b*c^2)*d^3)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c \\
& *d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48* \\
& a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2* \\
& c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64 \\
& *a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2) \\
& *e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 \\
& + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b \\
& ^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c \\
& ^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5 \\
& *c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a* \\
& b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48* \\
& a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a \\
& ^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3 \\
& *c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3* \\
& c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + \\
& 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e \\
& ^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + \\
& 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*
\end{aligned}$$

$$b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e]$$

giac [B] time = 0.69, size = 365, normalized size = 2.43

$$\frac{3bc \arctan\left(\frac{2cd^2 + 2(x^2 + 2dx)bc + b^2}{\sqrt{-b^2 + 4ac}}\right) e^{-1}}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2 + 4ac}} \cdot \frac{6bc^2d^6 + 18(x^2e + 2dx)bc^2d^4e + 18(x^2e + 2dx)^2bc^2d^2e^2 + 9b^2cd^4 + 6(x^2e + 2dx)^3bc^2e^3 + 18(x^2e + 2dx)b^2cd^2e + 9(x^2e + 2dx)^2b^2d^2 + 10abc d^2 + 2(x^2e + 2dx)b^3e + 10(x^2e + 2dx)abce + ab^2 + 8a^2c}{4(cd^4 + 2(x^2e + 2dx)cd^2e + (x^2e + 2dx)^2ce^2 + bcd^2 + (x^2e + 2dx)be + a)^2(b^4e - 8ab^2ce + 16a^2c^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] $-3*b*c*\arctan\left(\frac{2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b}{\sqrt{-b^2 + 4*a*c}}\right)*e^{-1}/\left(\frac{b^4 - 8*a*b^2*c + 16*a^2*c^2}{\sqrt{-b^2 + 4*a*c}}\right) - \frac{1}{4}*(6*b*c^2*d^6 + 18*(x^2*e + 2*d*x)*b*c^2*d^4*e + 18*(x^2*e + 2*d*x)^2*b*c^2*d^2*e^2 + 9*b^2*c*d^4 + 6*(x^2*e + 2*d*x)^3*b*c^2*e^3 + 18*(x^2*e + 2*d*x)*b^2*c*d^2*e + 9*(x^2*e + 2*d*x)^2*b^2*c*e^2 + 2*b^3*d^2 + 10*a*b*c*d^2 + 2*(x^2*e + 2*d*x)*b^3*e + 10*(x^2*e + 2*d*x)*a*b*c*e + a*b^2 + 8*a^2*c)/\left(\frac{c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a}{\sqrt{-b^2 + 4*a*c}}\right)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e)$

maple [C] time = 0.05, size = 544, normalized size = 3.63

$$\frac{3bc \arctan\left(\frac{2cd^2 + 2(x^2 + 2dx)bc + b^2}{\sqrt{-b^2 + 4ac}}\right) e^{-1}}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2 + 4ac}} \cdot \frac{6bc^2d^6 + 18(x^2e + 2dx)bc^2d^4e + 18(x^2e + 2dx)^2bc^2d^2e^2 + 9b^2cd^4 + 6(x^2e + 2dx)^3bc^2e^3 + 18(x^2e + 2dx)b^2cd^2e + 9(x^2e + 2dx)^2b^2d^2 + 10abc d^2 + 2(x^2e + 2dx)b^3e + 10(x^2e + 2dx)abce + ab^2 + 8a^2c}{4(cd^4 + 2(x^2e + 2dx)cd^2e + (x^2e + 2dx)^2ce^2 + bcd^2 + (x^2e + 2dx)be + a)^2(b^4e - 8ab^2ce + 16a^2c^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)

[Out] $\frac{-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-3*c*d*e^2*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45*c^2*d^4+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c*d^4+10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/\left(\frac{c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a}{\sqrt{-b^2 + 4*a*c}}\right)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*\sum\left(\frac{-_R*e-d}{2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d}\right)*\ln(-_R+x), _R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a)$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.85, size = 1182, normalized size = 7.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] - ((9*x^4*(b^2*c*e^3 + 10*b*c^2*d^2*e^3))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (a*b^2 + 8*a^2*c + 2*b^3*d^2 + 9*b^2*c*d^4 + 6*b*c^2*d^6 + 10*a*b*c*d^2
)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e + 27*b^2*c*d^2*e + 45*
b*c^2*d^4*e + 5*a*b*c*e))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*
b^2*c*e^2 + 10*b*c^2*d^2*e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3 +
9*b^2*c*d^2 + 9*b*c^2*d^4 + 5*a*b*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*
b*c^2*e^5*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*x^5)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*atan(((b^4*(4*a*c - b^2)^5 + 16*
a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*c^4*e^8)/
(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*
c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))) + x*((18*b^2*c^4*d*e^7)/(a*(4*a*c -
b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*c^2*d*e^9 -
16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8
- 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*
c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (9*b^2*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
))))/(18*b^2*c^4*e^6)))/(e*(4*a*c - b^2)^(5/2))
```

sympy [B] time = 14.45, size = 1671, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**4*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c + 6*b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*b*c**4*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c + 6*b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) + (-8*a**2*c - a*b**2 - 10*a*b*c*d**2 - 2*b**3*d**2 - 9*b**2*c*d**4 - 6*b*c**2*d**6 - 36*b*c**2*d*e**5*x**5 - 6*b*c**2*e**6*x**6 + x**4*(-9*b**2*c*e**4 - 90*b*c**2*d**2*e**4) + x**3*(-36*b**2*c*d*e**3 - 120*b*c**2*d**3*e**3) + x**2*(-10*a*b*c*e**2 - 2*b**3*e**2 - 54*b**2*c*d**2*e**2 - 90*b*c**2*d**4*e**2) + x*(-20*a*b*c*d*e - 4*b**3*d*e - 36*b**2*c*d**3*e - 36*b*c**2*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

$$3.519 \quad \int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=363

$$\frac{(d+ex)(b+2c(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(d+ex)(c(20ac+b^2)(d+ex)^2+b(8ac+b^2))}{8ae(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c} \left(\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}} \right)}{8\sqrt{2}ae(b^2-4ac)}$$

Rubi [A] time = 1.04, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1142, 1119, 1178, 1166, 205}

$$\frac{(d+ex)(b+2c(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(d+ex)(c(20ac+b^2)(d+ex)^2+b(8ac+b^2))}{8ae(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c} \left(\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}} + 20ac + b^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{8\sqrt{2}ae(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c} \left(-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}} + 20ac + b^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{8\sqrt{2}ae(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] -((d + e*x)*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1119

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(d*x)^(m-1)*(b+2*c*x^2)*(a+b*x^2+c*x^4)^(p+1))/(2*(p+1)*(b^2-4*a*c)), x] - Dist[d^2/(2*(p+1)*(b^2-4*a*c)), Int[(d*x)^(m-2)*(b*(m-1)+2*c*(m+4*p+5)*x^2)*(a+b*x^2+c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2-4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^3} dx, x, d+ex\right)}{e} \\
&= -\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{\text{Subst}\left(\int \frac{b-10cx^2}{(a+bx^2+cx^4)^2} dx\right)}{4(b^2-4ac)e} \\
&= -\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(d+ex)(b(b^2+8ac)+8ac^2)}{8a(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} \\
&= -\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(d+ex)(b(b^2+8ac)+8ac^2)}{8a(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} \\
&= -\frac{(d+ex)(b+2c(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{(d+ex)(b(b^2+8ac)+8ac^2)}{8a(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)}
\end{aligned}$$

Mathematica [A] time = 4.86, size = 382, normalized size = 1.05

$$\frac{\frac{4(b(d+ex)+2c(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2(d+ex)(8abc+20ac^2(d+ex)^2+b^3+b^2c(d+ex)^2)}{a(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}}{16e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] ((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 20*a*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] IntegrateAlgebraic[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]

fricas [B] time = 2.26, size = 7701, normalized size = 21.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*e^7*x^7 + 14*(b^2*c^2 + 20*a*c^3)*d*e^6*x^6 + 2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*e^5*x^5 + 10*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*e^4*x^4 + 2*(b^2*c^2 + 20*a*c^3)*d^7 + 2*(35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)*d^2)*e^3*x^3 + 4*(b^3*c + 14*a*b*c^2)*d^5 + 2*(21*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d)*e^2*x^2 + 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 + 2*(7*(b^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*e*x - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 10

$$\begin{aligned}
& 24*a^8*c^5)*e^2*\sqrt{(b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*\log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d + 1/2*\sqrt{1/2})*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*\sqrt{(b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*\sqrt{-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2)*\sqrt{(b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2)) + \sqrt{1/2})*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*\sqrt{-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2)*\sqrt{(b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*\log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d - 1/2*\sqrt{1/2})*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*\sqrt{(b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)) - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21
\end{aligned}$$

$$\begin{aligned}
& 824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*\text{sqrt}(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*\text{sqrt}((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) + \text{sqrt}(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*\text{sqrt}(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*\text{sqrt}((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*\log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d + 1/2*\text{sqrt}(1/2)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*e^3*\text{sqrt}((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4))) + (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*\text{sqrt}(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2*\text{sqrt}((b^4 - 50*a*b^2*c + 625*a^2*c^2)/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - \text{sqrt}(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x
\end{aligned}$$

$$\begin{aligned}
&^5 + (a^2b^6 - 6a^2b^4c + 32a^4c^3 + 70(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^4 + 30(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d^2)e^5x^4 + \\
&4(14(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^5 + 10(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d^3 + (a^2b^6 - 6a^2b^4c + 32a^4c^3)d)e^4x^3 + \\
&2(a^2b^5 - 8a^3b^3c + 16a^4b^2c^2 + 14(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^6 + 15(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d^4 + 3(a^2b^6 - 6a^2b^4c + 32a^4c^3)d^2)e^3x^2 + \\
&4(2(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^7 + 3(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d^5 + (a^2b^6 - 6a^2b^4c + 32a^4c^3)d^3 + (a^2b^5 - 8a^3b^3c + 16a^4b^2c^2)d)e^2x + \\
&((a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^8 + a^3b^4 - 8a^4b^2c + 16a^5c^2 + 2(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d^6 + (a^2b^6 - 6a^2b^4c + 32a^4c^3)d^4 + 2(a^2b^5 - 8a^3b^3c + 16a^4b^2c^2)d^2)e) \sqrt{-(b^7 - 35a^2b^5c + 280a^2b^3c^2 + 1680a^3b^2c^3 - (a^3b^10 - 20a^4b^8c + 160a^5b^6c^2 - 640a^6b^4c^3 + 1280a^7b^2c^4 - 1024a^8c^5)e^2} \sqrt{(b^4 - 50a^2b^2c + 625a^2c^2)} / ((a^6b^10 - 20a^7b^8c + 160a^8b^6c^2 - 640a^9b^4c^3 + 1280a^10b^2c^4 - 1024a^11c^5)e^4)) / ((a^3b^10 - 20a^4b^8c + 160a^5b^6c^2 - 640a^6b^4c^3 + 1280a^7b^2c^4 - 1024a^8c^5)e^2)) * \log((35b^6c^2 - 1491a^2b^4c^3 + 15000a^2b^2c^4 + 10000a^3c^5)e^2x + (35b^6c^2 - 1491a^2b^4c^3 + 15000a^2b^2c^4 + 10000a^3c^5)d - 1/2 \sqrt{1/2} * ((a^3b^14 - 38a^4b^12c + 480a^5b^10c^2 - 2720a^6b^8c^3 + 6400a^7b^6c^4 + 1536a^8b^4c^5 - 32768a^9b^2c^6 + 40960a^10c^7)e^3} \sqrt{(b^4 - 50a^2b^2c + 625a^2c^2)} / ((a^6b^10 - 20a^7b^8c + 160a^8b^6c^2 - 640a^9b^4c^3 + 1280a^10b^2c^4 - 1024a^11c^5)e^4)) + (b^11 - 53a^2b^9c + 940a^2b^7c^2 - 6832a^3b^5c^3 + 21824a^4b^3c^4 - 25600a^5b^2c^5)e) \sqrt{-(b^7 - 35a^2b^5c + 280a^2b^3c^2 + 1680a^3b^2c^3 - (a^3b^10 - 20a^4b^8c + 160a^5b^6c^2 - 640a^6b^4c^3 + 1280a^7b^2c^4 - 1024a^8c^5)e^2} \sqrt{(b^4 - 50a^2b^2c + 625a^2c^2)} / ((a^6b^10 - 20a^7b^8c + 160a^8b^6c^2 - 640a^9b^4c^3 + 1280a^10b^2c^4 - 1024a^11c^5)e^4)) / ((a^3b^10 - 20a^4b^8c + 160a^5b^6c^2 - 640a^6b^4c^3 + 1280a^7b^2c^4 - 1024a^8c^5)e^2)) - 2(a^2b^3 - 16a^2b^2c)d / ((a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)e^9x^8 + 8(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d e^8x^7 + 2(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3 + 14(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^2)e^7x^6 + 4(14(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^3 + 3(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d)e^6x^5 + (a^2b^6 - 6a^2b^4c + 32a^4c^3 + 70(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^4 + 30(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d^2)e^5x^4 + 4(14(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^5 + 10(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d^3 + (a^2b^6 - 6a^2b^4c + 32a^4c^3)d)e^4x^3 + 2(a^2b^5 - 8a^3b^3c + 16a^4b^2c^2 + 14(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^6 + 15(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d^4 + 3(a^2b^6 - 6a^2b^4c + 32a^4c^3)d^2)e^3x^2 + 4(2(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^7 + 3(a^2b^5c - 8a^2b^3c^2 + 16a^3b^2c^3)d^5 + (a^2b^6 - 6a^2b^4c + 32a^4c^3)d^3 + (a^2b^5 - 8a^3b^3c + 16a^4b^2c^2)d)e^2x + ((a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4)d^8 + a^3b^4 - 8a^4b^2c
\end{aligned}$$

+ 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)

giac [B] time = 0.87, size = 2295, normalized size = 6.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] -1/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*d^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*d^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*d^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-

$$\frac{(b^2 e^2 - \sqrt{b^2 - 4ac})e^{-4}/c) a^2 d e + b^2 c d^2 + 20 a^2 c^2 d^2 + b^3 - 16 a b c) \log(d e^{-1} + x - \sqrt{1/2} \sqrt{-(b^2 e^2 - \sqrt{b^2 - 4ac})e^{-4}/c}) / (2(d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 e^2 - \sqrt{b^2 - 4ac})e^{-4}/c})^3 c e^4 - 6(d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 e^2 - \sqrt{b^2 - 4ac})e^{-4}/c})^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} - \sqrt{1/2} \sqrt{-(b^2 e^2 - \sqrt{b^2 - 4ac})e^{-4}/c})) / (a b^4 - 8 a^2 b^2 c + 16 a^3 c^2) + 1/8 (b^2 c^2 x^7 e^7 + 20 a^2 c^3 x^7 e^7 + 7 b^2 c^2 d x^6 e^6 + 140 a^2 c^3 d x^6 e^6 + 21 b^2 c^2 d^2 x^5 e^5 + 420 a^2 c^3 d^2 x^5 e^5 + 35 b^2 c^2 d^3 x^4 e^4 + 700 a^2 c^3 d^3 x^4 e^4 + 35 b^2 c^2 d^4 x^3 e^3 + 700 a^2 c^3 d^4 x^3 e^3 + 21 b^2 c^2 d^5 x^2 e^2 + 420 a^2 c^3 d^5 x^2 e^2 + 7 b^2 c^2 d^6 x e + 140 a^2 c^3 d^6 x e + b^2 c^2 d^7 + 20 a^2 c^3 d^7 + 2 b^3 c x^5 e^5 + 28 a b c^2 x^5 e^5 + 10 b^3 c d x^4 e^4 + 140 a b c^2 d x^4 e^4 + 20 b^3 c d^2 x^3 e^3 + 280 a b c^2 d^2 x^3 e^3 + 20 b^3 c d^3 x^2 e^2 + 280 a b c^2 d^3 x^2 e^2 + 10 b^3 c d^4 x e + 140 a b c^2 d^4 x e + 2 b^3 c d^5 + 28 a b c^2 d^5 + b^4 x^3 e^3 + 5 a b^2 c x^3 e^3 + 36 a^2 c^2 x^3 e^3 + 3 b^4 d x^2 e^2 + 15 a b^2 c d x^2 e^2 + 108 a^2 c^2 d x^2 e^2 + 3 b^4 d^2 x e + 15 a b^2 c d^2 x e + 108 a^2 c^2 d^2 x e + b^4 d^3 + 5 a b^2 c d^3 + 36 a^2 c^2 d^3 - a b^3 x e + 16 a^2 b c x e - a b^3 d + 16 a^2 b c d) / ((c x^4 e^4 + 4 c d x^3 e^3 + 6 c d^2 x^2 e^2 + 4 c d^3 x e + c d^4 + b x^2 e^2 + 2 b d x e + b d^2 + a)^2 (a b^4 e - 8 a^2 b^2 c e + 16 a^3 c^2 e))$$

maple [C] time = 0.05, size = 885, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int ((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3, x)$

[Out] $(1/8 c^2 e^6 (20 a^2 c + b^2) / (16 a^2 c^2 - 8 a b^2 c + b^4) / a x^7 + 7/8 c^2 d e^5 (20 a^2 c + b^2) / (16 a^2 c^2 - 8 a b^2 c + b^4) / a x^6 + 1/8 (420 a^2 c^2 d^2 + 21 b^2 c d^2 + 28 a b c + 2 b^3) c e^4 / (16 a^2 c^2 - 8 a b^2 c + b^4) / a x^5 + 5/8 c d e^3 (140 a^2 c^2 d^2 + 7 b^2 c d^2 + 28 a b c + 2 b^3) / (16 a^2 c^2 - 8 a b^2 c + b^4) / a x^4 + 1/8 e^2 (700 a^2 c^3 d^4 + 35 b^2 c^2 d^4 + 280 a b c^2 d^2 + 20 b^3 c d^2 + 36 a^2 c^2 + 5 a b^2 c + b^4) / (16 a^2 c^2 - 8 a b^2 c + b^4) / a x^3 + 1/8 d e (420 a^2 c^3 d^4 + 21 b^2 c^2 d^4 + 280 a b c^2 d^2 + 20 b^3 c d^2 + 108 a^2 c^2 + 15 a b^2 c + 3 b^4) / (16 a^2 c^2 - 8 a b^2 c + b^4) / a x^2 + 1/8 (140 a^2 c^3 d^6 + 7 b^2 c^2 d^6 + 140 a b c^2 d^4 + 10 b^3 c d^4 + 108 a^2 c^2 d^2 + 15 a b^2 c d^2 + 3 b^4 d^2 + 16 a^2 b c - a b^3) / (16 a^2 c^2 - 8 a b^2 c + b^4) / a x + 1/8 d / e (20 a^2 c^3 d^6 + b^2 c^2 d^6 + 28 a b c^2 d^4 + 2 b^3 c d^4 + 36 a^2 c^2 d^2 + 5 a b^2 c d^2 + b^4 d^2 + 16 a^2 b c - a b^3) / (16 a^2 c^2 - 8 a b^2 c + b^4) / a) / (c e^4 x^4 + 4 c d x^3 e^3 + 6 c d^2 x^2 e^2 + 4 c d^3 x e + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 + 1/16 / (16 a^2 c^2 - 8 a b^2 c + b^4) / a / e * \operatorname{sum}((c e^2 (20 a^2 c + b^2) * _R^2 + 2 c d e (20 a^2 c + b^2) * _R + 20 a^2 c^2 d^2 + b^2 c d^2 - 16 a b c + b^3) / (2 * _R^3 c e^3 + 6 * _R^2 c d e^2 + 6 * _R c d^2 e + 2 c d^3 + _R b e + b d) * \ln(-_R + x), _R = \operatorname{RootOf}(_Z^4 c e^4 + 4 * _Z^3 c d e^3 + c d^4 + b d^2 + (6 c d^2 e^2 + b e$

$\wedge 2) * _Z^2 + (4 * c * d^3 * e + 2 * b * d * e) * _Z + a))$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 7.43, size = 14584, normalized size = 40.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)

[Out]
$$\begin{aligned} & ((x^5 * (2 * b^3 * c * e^4 + 420 * a * c^3 * d^2 * e^4 + 21 * b^2 * c^2 * d^2 * e^4 + 28 * a * b * c^2 * e^4) / (8 * a * (b^4 + 16 * a^2 * c^2 - 8 * a * b^2 * c)) + (x^2 * (3 * b^4 * d * e + 21 * b^2 * c^2 * d^5 * e + 108 * a^2 * c^2 * d * e + 420 * a * c^3 * d^5 * e + 20 * b^3 * c * d^3 * e + 280 * a * b * c^2 * d^3 * e + 15 * a * b^2 * c * d * e)) / (8 * a * (b^4 + 16 * a^2 * c^2 - 8 * a * b^2 * c)) + (7 * x^6 * (b^2 * c^2 * d * e^5 + 20 * a * c^3 * d * e^5)) / (8 * a * (b^4 + 16 * a^2 * c^2 - 8 * a * b^2 * c)) + (x^7 * (20 * a * c^3 * e^6 + b^2 * c^2 * e^6)) / (8 * a * (b^4 + 16 * a^2 * c^2 - 8 * a * b^2 * c)) + (x * (3 * b^4 * d^2 - a * b^3 + 140 * a * c^3 * d^6 + 10 * b^3 * c * d^4 + 108 * a^2 * c^2 * d^2 + 7 * b^2 * c^2 * d^6 + 16 * a^2 * b * c + 15 * a * b^2 * c * d^2 + 140 * a * b * c^2 * d^4)) / (8 * a * (b^4 + 16 * a^2 * c^2 - 8 * a * b^2 * c)) + (x^3 * (b^4 * e^2 + 36 * a^2 * c^2 * e^2 + 700 * a * c^3 * d^4 * e^2 + 20 * b^3 * c * d^2 * e^2 + 35 * b^2 * c^2 * d^4 * e^2 + 5 * a * b^2 * c * e^2 + 280 * a * b * c^2 * d^2 * e^2)) / (8 * a * (b^4 + 16 * a^2 * c^2 - 8 * a * b^2 * c)) + (b^4 * d^3 + 20 * a * c^3 * d^7 + 2 * b^3 * c * d^5 + 36 * a^2 * c^2 * d^3 + b^2 * c^2 * d^7 - a * b^3 * d + 16 * a^2 * b * c * d + 5 * a * b^2 * c * d^3 + 28 * a * b * c^2 * d^5) / (8 * a * e * (b^4 + 16 * a^2 * c^2 - 8 * a * b^2 * c)) + (5 * x^4 * (140 * a * c^3 * d^3 * e^3 + 7 * b^2 * c^2 * d^3 * e^3 + 2 * b^3 * c * d * e^3 + 28 * a * b * c^2 * d * e^3)) / (8 * a * (b^4 + 16 * a^2 * c^2 - 8 * a * b^2 * c)) / (x^2 * (6 * b^2 * d^2 * e^2 + 28 * c^2 * d^6 * e^2 + 2 * a * b * e^2 + 12 * a * c * d^2 * e^2 + 30 * b * c * d^4 * e^2) + x^6 * (28 * c^2 * d^2 * e^6 + 2 * b * c * e^6) + x * (4 * b^2 * d^3 * e + 8 * c^2 * d^7 * e + 8 * a * c * d^3 * e + 12 * b * c * d^5 * e + 4 * a * b * d * e) + x^3 * (4 * b^2 * d * e^3 + 56 * c^2 * d^5 * e^3 + 8 * a * c * d * e^3 + 40 * b * c * d^3 * e^3) + x^5 * (56 * c^2 * d^3 * e^5 + 12 * b * c * d * e^5) + x^4 * (b^2 * e^4 + 70 * c^2 * d^4 * e^4 + 2 * a * c * e^4 + 30 * b * c * d^2 * e^4) + a^2 + b^2 * d^4 + c^2 * d^8 + c^2 * e^8 * x^8 + 2 * a * b * d^2 + 2 * a * c * d^4 + 2 * b * c * d^6 + 8 * c^2 * d * e^7 * x^7) + \operatorname{atan}\left(\frac{(256 * a * b^{13} * c^2 * e^{12} + 4194304 * a^7 * b * c^8 * e^{12} - 9216 * a^2 * b^{11} * c^3 * e^{12} + 122880 * a^3 * b^9 * c^4 * e^{12} - 819200 * a^4 * b^7 * c^5 * e^{12} + 2949120 * a^5 * b^5 * c^6 * e^{12} - 5505024 * a^6 * b^3 * c^7 * e^{12})}{(512 * (a^2 * b^{12} + 4096 * a^8 * c^6 - 24 * a^3 * b^{10} * c + 240 * a^4 * b^8 * c^2 - 1280 * a^5 * b^6 * c^3 + 3840 * a^6 * b^4 * c^4 - 6144 * a^7 * b^2 * c^5)) + ((67108864 * a^9 * b * c^9 * d * e^{13} - 4096 * a^2 * b^{15} * c^2 * d * e^{13} + 114688 * a^3 * b^{13} * c^3 * d * e^{13} - 1376256 * a^4 * b^{11} * c^4 * d * e^{13} + 9175040 * a^5 * b^9 * c^5 * d * e^{13} - 36700160 * a^6 * b^7 * c^6 * d * e^{13} + 88080384 * a^7 * b^5 * c^7 * d * e^{13} - 117440512 * a^8 * b^3 * c^8 * d * e^{13})}{(512 * (a^2 * b^{12} + 4096 * a^8 * c^6 - 24 * a^3 * b^{10} * c + 240 * a^4 * b^8 * c^2 - 1280 * a^5 * b^6 * c^3 + 3840 * a^6 * b^4 * c^4 - 6144 * a^7 * b^2 * c^5)) + ((67108864 * a^9 * b * c^9 * d * e^{13} - 4096 * a^2 * b^{15} * c^2 * d * e^{13} + 114688 * a^3 * b^{13} * c^3 * d * e^{13} - 1376256 * a^4 * b^{11} * c^4 * d * e^{13} + 9175040 * a^5 * b^9 * c^5 * d * e^{13} - 36700160 * a^6 * b^7 * c^6 * d * e^{13} + 88080384 * a^7 * b^5 * c^7 * d * e^{13} - 117440512 * a^8 * b^3 * c^8 * d * e^{13})}\right) \end{aligned}$$

$$\begin{aligned}
& 4 - 327680*a^6*b^3*c^6*e^{14})/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 9 \\
& 6*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^{17} + b^2*(-(4*a*c - b^2)^{15})^{(1/2)} \\
& - 1720320*a^8*b*c^8 + 1140*a^2*b^{13}*c^2 - 10160*a^3*b^{11}*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a \\
& *b^{15}*c - 25*a*c*(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(a^3*b^{20}*e^2 + 1048576*a^{13}*c^{10}*e^2 - 40*a^4*b^{18}*c*e^2 + 720*a^5*b^{16}*c^2*e^2 - 7680*a^6*b^{14}*c^3* \\
& e^2 + 53760*a^7*b^{12}*c^4*e^2 - 258048*a^8*b^{10}*c^5*e^2 + 860160*a^9*b^8*c^6 \\
& *e^2 - 1966080*a^{10}*b^6*c^7*e^2 + 2949120*a^{11}*b^4*c^8*e^2 - 2621440*a^{12}*b^2*c^9*e^2)))^{(1/2)})*(-(b^{17} + b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*a^8* \\
& b*c^8 + 1140*a^2*b^{13}*c^2 - 10160*a^3*b^{11}*c^3 + 34880*a^4*b^9*c^4 + 43776* \\
& a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^{15}*c - 25*a \\
& *c*(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(a^3*b^{20}*e^2 + 1048576*a^{13}*c^{10}*e^2 - \\
& 40*a^4*b^{18}*c*e^2 + 720*a^5*b^{16}*c^2*e^2 - 7680*a^6*b^{14}*c^3*e^2 + 53760*a^7* \\
& b^{12}*c^4*e^2 - 258048*a^8*b^{10}*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080 \\
& *a^{10}*b^6*c^7*e^2 + 2949120*a^{11}*b^4*c^8*e^2 - 2621440*a^{12}*b^2*c^9*e^2)))^{(\\
& 1/2)} + (x*(800*a^3*c^6*e^{12} - b^6*c^3*e^{12} + 34*a*b^4*c^4*e^{12} - 1472*a^2* \\
& b^2*c^5*e^{12}))/((32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - \\
& 256*a^5*b^2*c^3)))*(-(b^{17} + b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*a^8*b \\
& *c^8 + 1140*a^2*b^{13}*c^2 - 10160*a^3*b^{11}*c^3 + 34880*a^4*b^9*c^4 + 43776*a \\
& ^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^{15}*c - 25*a* \\
& c*(-(4*a*c - b^2)^{15})^{(1/2)})/(512*(a^3*b^{20}*e^2 + 1048576*a^{13}*c^{10}*e^2 - 4 \\
& 0*a^4*b^{18}*c*e^2 + 720*a^5*b^{16}*c^2*e^2 - 7680*a^6*b^{14}*c^3*e^2 + 53760*a^7* \\
& b^{12}*c^4*e^2 - 258048*a^8*b^{10}*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080* \\
& a^{10}*b^6*c^7*e^2 + 2949120*a^{11}*b^4*c^8*e^2 - 2621440*a^{12}*b^2*c^9*e^2)))^{(\\
& 1/2)}*1i)/((8000*a^3*c^7*e^{10} - 35*b^6*c^4*e^{10} - 84*a*b^4*c^5*e^{10} + 12720* \\
& a^2*b^2*c^6*e^{10}))/((256*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240*a^4*b \\
& ^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (((256* \\
& a*b^{13}*c^2*e^{12} + 4194304*a^7*b*c^8*e^{12} - 9216*a^2*b^{11}*c^3*e^{12} + 122880* \\
& a^3*b^9*c^4*e^{12} - 819200*a^4*b^7*c^5*e^{12} + 2949120*a^5*b^5*c^6*e^{12} - 550 \\
& 5024*a^6*b^3*c^7*e^{12}))/((512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240* \\
& a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((\\
& 67108864*a^9*b*c^9*d*e^{13} - 4096*a^2*b^{15}*c^2*d*e^{13} + 114688*a^3*b^{13}*c^3* \\
& d*e^{13} - 1376256*a^4*b^{11}*c^4*d*e^{13} + 9175040*a^5*b^9*c^5*d*e^{13} - 3670016 \\
& 0*a^6*b^7*c^6*d*e^{13} + 88080384*a^7*b^5*c^7*d*e^{13} - 117440512*a^8*b^3*c^8* \\
& d*e^{13}))/((512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240*a^4*b^8*c^2 - 1 \\
& 280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b* \\
& c^7*e^{14} - 256*a^2*b^{11}*c^2*e^{14} + 5120*a^3*b^9*c^3*e^{14} - 40960*a^4*b^7*c^4 \\
& *e^{14} + 163840*a^5*b^5*c^5*e^{14} - 327680*a^6*b^3*c^6*e^{14}))/((32*(a^2*b^8 + \\
& 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^{17} + \\
& b^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*a^8*b*c^8 + 1140*a^2*b^{13}*c^2 - 10 \\
& 160*a^3*b^{11}*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c \\
& ^6 + 1863680*a^7*b^3*c^7 - 55*a*b^{15}*c - 25*a*c*(-(4*a*c - b^2)^{15})^{(1/2)})/ \\
& (512*(a^3*b^{20}*e^2 + 1048576*a^{13}*c^{10}*e^2 - 40*a^4*b^{18}*c*e^2 + 720*a^5*b^ \\
& 16*c^2*e^2 - 7680*a^6*b^{14}*c^3*e^2 + 53760*a^7*b^{12}*c^4*e^2 - 258048*a^8*b^ \\
& 10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^{10}*b^6*c^7*e^2 + 2949120*a^
\end{aligned}$$

$$\begin{aligned}
& (11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2))^{(1/2)} * (- (b^{17} + b^2 * (- (4*a*c - b^2)^{15})^{(1/2)} - 1720320*a^8*b*c^8 + 1140*a^2*b^{13}*c^2 - 10160*a^3*b^{11}*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^{15}*c - 25*a*c * (- (4*a*c - b^2)^{15})^{(1/2)}) / (512*(a^3*b^{20}*e^2 + 1048576*a^{13}*c^{10}*e^2 - 40*a^4*b^{18}*c*e^2 + 720*a^5*b^{16}*c^2*e^2 - 7680*a^6*b^{14}*c^3*e^2 + 53760*a^7*b^{12}*c^4*e^2 - 258048*a^8*b^{10}*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^{10}*b^6*c^7*e^2 + 2949120*a^{11}*b^4*c^8*e^2 - 2621440*a^{12}*b^2*c^9*e^2))^{(1/2)} + (204800*a^5*c^8*d*e^{11} - 16*b^{10}*c^3*d*e^{11} + 672*a*b^8*c^4*d*e^{11} - 28160*a^2*b^6*c^5*d*e^{11} + 209920*a^3*b^4*c^6*d*e^{11} - 479232*a^4*b^2*c^7*d*e^{11}) / (512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^{12} - b^6*c^3*e^{12} + 34*a*b^4*c^4*e^{12} - 1472*a^2*b^2*c^5*e^{12})) / (32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)) * (- (b^{17} + b^2 * (- (4*a*c - b^2)^{15})^{(1/2)} - 1720320*a^8*b*c^8 + 1140*a^2*b^{13}*c^2 - 10160*a^3*b^{11}*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^{15}*c - 25*a*c * (- (4*a*c - b^2)^{15})^{(1/2)}) / (512*(a^3*b^{20}*e^2 + 1048576*a^{13}*c^{10}*e^2 - 40*a^4*b^{18}*c*e^2 + 720*a^5*b^{16}*c^2*e^2 - 7680*a^6*b^{14}*c^3*e^2 + 53760*a^7*b^{12}*c^4*e^2 - 258048*a^8*b^{10}*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^{10}*b^6*c^7*e^2 + 2949120*a^{11}*b^4*c^8*e^2 - 2621440*a^{12}*b^2*c^9*e^2))^{(1/2)} + ((204800*a^5*c^8*d*e^{11} - 16*b^{10}*c^3*d*e^{11} + 672*a*b^8*c^4*d*e^{11} - 28160*a^2*b^6*c^5*d*e^{11} + 209920*a^3*b^4*c^6*d*e^{11} - 479232*a^4*b^2*c^7*d*e^{11}) / (512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^{13}*c^2*e^{12} + 4194304*a^7*b*c^8*e^{12} - 9216*a^2*b^{11}*c^3*e^{12} + 122880*a^3*b^9*c^4*e^{12} - 819200*a^4*b^7*c^5*e^{12} + 2949120*a^5*b^5*c^6*e^{12} - 5505024*a^6*b^3*c^7*e^{12}) / (512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((67108864*a^9*b*c^9*d*e^{13} - 4096*a^2*b^{15}*c^2*d*e^{13} + 114688*a^3*b^{13}*c^3*d*e^{13} - 1376256*a^4*b^{11}*c^4*d*e^{13} + 9175040*a^5*b^9*c^5*d*e^{13} - 36700160*a^6*b^7*c^6*d*e^{13} + 88080384*a^7*b^5*c^7*d*e^{13} - 117440512*a^8*b^3*c^8*d*e^{13}) / (512*(a^2*b^{12} + 4096*a^8*c^6 - 24*a^3*b^{10}*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^{14} - 256*a^2*b^{11}*c^2*e^{14} + 5120*a^3*b^9*c^3*e^{14} - 40960*a^4*b^7*c^4*e^{14} + 163840*a^5*b^5*c^5*e^{14} - 327680*a^6*b^3*c^6*e^{14})) / (32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)) * (- (b^{17} + b^2 * (- (4*a*c - b^2)^{15})^{(1/2)} - 1720320*a^8*b*c^8 + 1140*a^2*b^{13}*c^2 - 10160*a^3*b^{11}*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^{15}*c - 25*a*c * (- (4*a*c - b^2)^{15})^{(1/2)}) / (512*(a^3*b^{20}*e^2 + 1048576*a^{13}*c^{10}*e^2 - 40*a^4*b^{18}*c*e^2 + 720*a^5*b^{16}*c^2*e^2 - 7680*a^6*b^{14}*c^3*e^2 + 53760*a^7*b^{12}*c^4*e^2 - 258048*a^8*b^{10}*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^{10}*b^6*c^7*e^2 + 2949120*a^{11}*b^4*c^8*e^2 - 2621440*a^{12}*b^2*c^9*e^2))^{(1/2)} * (- (b^{17} + b^2 * (- (4*a*c - b^2)^{15})^{(1/2)} - 1720320*a^8*b*c^8 + 1140*a^2*b^{13}*c^2 - 10160*a^3*b^{11}*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*
\end{aligned}$$

$$\begin{aligned}
& b^3c^7 - 55ab^{15}c - 25a^2c^2(-4ac - b^2)^{15})^{1/2}) / (512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^2e^2 + 720a^5b^{16}c^2e^2 - 7680 \\
& a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - \\
& 2621440a^{12}b^2c^9e^2))^{1/2} + (x(800a^3c^6e^{12} - b^6c^3e^{12} + 3 \\
& 4a^2b^4c^4e^{12} - 1472a^2b^2c^5e^{12})) / (32(a^2b^8 + 256a^6c^4 - 16a^3b^6c \\
& + 96a^4b^4c^2 - 256a^5b^2c^3)) * (-b^{17} + b^2(-4ac - b^2)^{15})^{1/2} - 1720320a^8b^8c^8 + 1140a^2b^{13}c^2 - 10160a^3b^{11}c^3 + \\
& 34880a^4b^9c^4 + 43776a^5b^7c^5 - 680960a^6b^5c^6 + 1863680a^7b^3c^7 - 55a^2b^15c \\
& - 25a^2c^2(-4ac - b^2)^{15})^{1/2}) / (512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^2e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + \\
& 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{1/2} \\
& - 1720320a^8b^8c^8 + 1140a^2b^{13}c^2 - 10160a^3b^{11}c^3 + 34880a^4b^9c^4 + 43776a^5b^7c^5 - 680960a^6b^5c^6 + 1863680a^7b^3c^7 - 55a^2b^15c \\
& - 25a^2c^2(-4ac - b^2)^{15})^{1/2}) / (512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^2e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + \\
& 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{1/2} * 2i + \operatorname{atan}(\frac{((256a^2b^{13}c^2e^{12} + 4194304a^7b^8c^8e^{12} - 9216a^2b^{11}c^3e^{12} + 122880a^3b^9c^4e^{12} - 819200a^4b^7c^5e^{12} + 2949120a^5b^5c^6e^{12} - 5505024a^6b^3c^7e^{12}) / (512(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + ((67108864a^9b^8c^9d^2e^{13} - 4096a^2b^{15}c^2d^2e^{13} + 114688a^3b^{13}c^3d^2e^{13} - 1376256a^4b^{11}c^4d^2e^{13} + 9175040a^5b^9c^5d^2e^{13} - 36700160a^6b^7c^6d^2e^{13} + 88080384a^7b^5c^7d^2e^{13} - 117440512a^8b^3c^8d^2e^{13}) / (512(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + (x(262144a^7b^8c^7e^{14} - 256a^2b^{11}c^2e^{14} + 5120a^3b^9c^3e^{14} - 40960a^4b^7c^4e^{14} + 163840a^5b^5c^5e^{14} - 327680a^6b^3c^6e^{14})) / (32(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)) * (-b^{17} - b^2(-4ac - b^2)^{15})^{1/2} - 1720320a^8b^8c^8 + 1140a^2b^{13}c^2 - 10160a^3b^{11}c^3 + 34880a^4b^9c^4 + 43776a^5b^7c^5 - 680960a^6b^5c^6 + 1863680a^7b^3c^7 - 55a^2b^15c + 25a^2c^2(-4ac - b^2)^{15})^{1/2}) / (512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^2e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{1/2} * (-b^{17} - b^2(-4ac - b^2)^{15})^{1/2} - 1720320a^8b^8c^8 + 1140a^2b^{13}c^2 - 10160a^3b^{11}c^3 + 34880a^4b^9c^4 + 43776a^5b^7c^5 - 680960a^6b^5c^6 + 1863680a^7b^3c^7 - 55a^2b^15c + 25a^2c^2(-4ac - b^2)^{15})^{1/2}) / (512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^2e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{(1/2)} \\
& + (204800a^5c^8d^e^{11} - 16b^{10}c^3d^e^{11} + 672a^8b^4c^4d^e^{11} - 2816 \\
& 0a^2b^6c^5d^e^{11} + 209920a^3b^4c^6d^e^{11} - 479232a^4b^2c^7d^e^{11} \\
& 1)/(512(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5 \\
& b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + (x(800a^3c^6e^{12} - \\
& b^6c^3e^{12} + 34a^8b^4c^4e^{12} - 1472a^2b^2c^5e^{12}))/((32(a^2b^8 + \\
& 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)))*(-(b^{17} - \\
& b^2*(-(4ac - b^2)^{15})^{(1/2)} - 1720320a^8b^8c^8 + 1140a^2b^{13}c^2 - 101 \\
& 60a^3b^{11}c^3 + 34880a^4b^9c^4 + 43776a^5b^7c^5 - 680960a^6b^5c^6 \\
& + 1863680a^7b^3c^7 - 55a^8b^{15}c + 25a^9c*(-(4ac - b^2)^{15})^{(1/2)}))/ \\
& (512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^8e^2 + 720a^5b^{16} \\
& c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10} \\
& c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11} \\
& b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{(1/2)} * i + ((204800a^5c^8d^e^{11} \\
& - 16b^{10}c^3d^e^{11} + 672a^8b^4c^4d^e^{11} - 28160a^2b^6c^5d^e^{11} \\
& + 209920a^3b^4c^6d^e^{11} - 479232a^4b^2c^7d^e^{11}))/((512(a^2b^{12} + 4 \\
& 096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6 \\
& b^4c^4 - 6144a^7b^2c^5)) - ((256a^8b^{13}c^2e^{12} + 4194304a^7b^8c^8e \\
& ^{12} - 9216a^2b^{11}c^3e^{12} + 122880a^3b^9c^4e^{12} - 819200a^4b^7c^5 \\
& e^{12} + 2949120a^5b^5c^6e^{12} - 5505024a^6b^3c^7e^{12}))/((512(a^2b^{12} \\
& + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840 \\
& a^6b^4c^4 - 6144a^7b^2c^5)) - ((67108864a^9b^9c^9d^e^{13} - 4096a^2b^{15} \\
& c^2d^e^{13} + 114688a^3b^{13}c^3d^e^{13} - 1376256a^4b^{11}c^4d^e^{13} \\
& + 9175040a^5b^9c^5d^e^{13} - 36700160a^6b^7c^6d^e^{13} + 88080384a^7b^5 \\
& c^7d^e^{13} - 117440512a^8b^3c^8d^e^{13}))/((512(a^2b^{12} + 4096a^8c^6 \\
& - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - \\
& 6144a^7b^2c^5)) + (x(262144a^7b^7c^7e^{14} - 256a^2b^{11}c^2e^{14} + 51 \\
& 20a^3b^9c^3e^{14} - 40960a^4b^7c^4e^{14} + 163840a^5b^5c^5e^{14} - 32 \\
& 7680a^6b^3c^6e^{14}))/((32(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4 \\
& c^2 - 256a^5b^2c^3)))*(-(b^{17} - b^2*(-(4ac - b^2)^{15})^{(1/2)} - 1720 \\
& 320a^8b^8c^8 + 1140a^2b^{13}c^2 - 10160a^3b^{11}c^3 + 34880a^4b^9c^4 \\
& + 43776a^5b^7c^5 - 680960a^6b^5c^6 + 1863680a^7b^3c^7 - 55a^8b^{15} \\
& c + 25a^9c*(-(4ac - b^2)^{15})^{(1/2)}))/((512(a^3b^{20}e^2 + 1048576a^{13}c^{10} \\
& e^2 - 40a^4b^{18}c^8e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + \\
& 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - \\
& 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9 \\
& e^2))^{(1/2)})*(-(b^{17} - b^2*(-(4ac - b^2)^{15})^{(1/2)} - 1720320a^8b^8 \\
& c^8 + 1140a^2b^{13}c^2 - 10160a^3b^{11}c^3 + 34880a^4b^9c^4 + 43776a^5b^7 \\
& c^5 - 680960a^6b^5c^6 + 1863680a^7b^3c^7 - 55a^8b^{15}c + 25a^9c*(-(\\
& 4ac - b^2)^{15})^{(1/2)}))/((512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4 \\
& b^{18}c^8e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12} \\
& c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10} \\
& b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{(1/2)} \\
& + (x(800a^3c^6e^{12} - b^6c^3e^{12} + 34a^8b^4c^4e^{12} - 1472a^2b^2c^5 \\
& e^{12}))/((32(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a
\end{aligned}$$

$$0 \cdot e^2 - 40 \cdot a^4 \cdot b^{18} \cdot c \cdot e^2 + 720 \cdot a^5 \cdot b^{16} \cdot c^2 \cdot e^2 - 7680 \cdot a^6 \cdot b^{14} \cdot c^3 \cdot e^2 + 53760 \cdot a^7 \cdot b^{12} \cdot c^4 \cdot e^2 - 258048 \cdot a^8 \cdot b^{10} \cdot c^5 \cdot e^2 + 860160 \cdot a^9 \cdot b^8 \cdot c^6 \cdot e^2 - 1966080 \cdot a^{10} \cdot b^6 \cdot c^7 \cdot e^2 + 2949120 \cdot a^{11} \cdot b^4 \cdot c^8 \cdot e^2 - 2621440 \cdot a^{12} \cdot b^2 \cdot c^9 \cdot e^2))^{(1/2)} \cdot 2i$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out

$$3.520 \quad \int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=152

$$\frac{6c^2 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}} + \frac{3c(b+2c(d+ex)^2)}{2e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{-b-2c(d+ex)^2}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}$$

Rubi [A] time = 0.18, antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{6c^2 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}} + \frac{3c(b+2c(d+ex)^2)}{2e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{b+2c(d+ex)^2}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]

[Out] -(b + 2*c*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*c*(b + 2*c*(d + e*x)^2)/(2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (6*c^2*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1142

Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
 \int \frac{d+ex}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^3} dx, x, d+ex\right)}{e} \\
 &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^3} dx, x, (d+ex)^2\right)}{2e} \\
 &= -\frac{b+2c(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{(3c)\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2} dx, x, (d+ex)^2\right)}{2(b^2-4ac)} \\
 &= -\frac{b+2c(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3c(b+2c(d+ex)^2)}{2(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} \\
 &= -\frac{b+2c(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3c(b+2c(d+ex)^2)}{2(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)} \\
 &= -\frac{b+2c(d+ex)^2}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3c(b+2c(d+ex)^2)}{2(b^2-4ac)^2 e(a+b(d+ex)^2+c(d+ex)^4)}
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 147, normalized size = 0.97

$$\frac{24c^2 \tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{(b^2-4ac)(-b-2c(d+ex)^2)}{(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{6c(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4}$$

$$4e(b^2-4ac)^2$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] IntegrateAlgebraic[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]

fricas [B] time = 1.47, size = 3708, normalized size = 24.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*x^5 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*x^4 + 12*(b^2*c^3 - 4*a*c^4)*d^6 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*x^3 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^2*x^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*x + 12*(c^4*e^8*x^8 + 8*c^4*d*e^7*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*x^6 + c^4*d^8 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*x^5 + 2*b*c^3*d^6 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*x^3 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*x^2 + a^2*c^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -

$$\begin{aligned}
& a^2 b^3 c^3 - 64 a^3 b c^4) d^3 + (b^8 - 10 a b^6 c + 24 a^2 b^4 c^2 + 32 a^3 b^2 c^3 - 128 a^4 c^4) d) e^4 x^3 + 2(a b^7 - 12 a^2 b^5 c + 48 a^3 b^3 c^2 - 64 a^4 b c^3 + 14(b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) d^6 + 15(b^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - 64 a^3 b c^4) d^4 + 3(b^8 - 10 a b^6 c + 24 a^2 b^4 c^2 + 32 a^3 b^2 c^3 - 128 a^4 c^4) d^2) e^3 x^2 + 4(2(b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) d^7 + 3(b^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - 64 a^3 b c^4) d^5 + (b^8 - 10 a b^6 c + 24 a^2 b^4 c^2 + 32 a^3 b^2 c^3 - 128 a^4 c^4) d^3 + (a b^7 - 12 a^2 b^5 c + 48 a^3 b^3 c^2 - 64 a^4 b c^3) d) e^2 x + ((b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) d^8 + a^2 b^6 - 12 a^3 b^4 c + 48 a^4 b^2 c^2 - 64 a^5 c^3 + 2(b^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - 64 a^3 b c^4) d^6 + (b^8 - 10 a b^6 c + 24 a^2 b^4 c^2 + 32 a^3 b^2 c^3 - 128 a^4 c^4) d^4 + 2(a b^7 - 12 a^2 b^5 c + 48 a^3 b^3 c^2 - 64 a^4 b c^3) d^2) e)
\end{aligned}$$

giac [B] time = 0.63, size = 365, normalized size = 2.40

$$\frac{6c^2 \arctan\left(\frac{2cd^2 + (d^2 + 2dx)ce + b}{\sqrt{-b^2 + 4ac}}\right) e^{-1}}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2 + 4ac}} + \frac{12c^3 d^6 + 36(x^2e + 2dx)c^3 d^4 e + 36(x^2e + 2dx)^2 c^3 d^2 e^2 + 18bc^2 d^4 + 12(x^2e + 2dx)^3 c^3 e^3 + 36(x^2e + 2dx)bc^2 d^2 e + 18(x^2e + 2dx)^2 bc^2 e^2 + 4b^2 cd^2 + 20ac^2 d^2 + 4(x^2e + 2dx)^2 ce + 20(x^2e + 2dx)ac^2 e - b^3 + 10abc}{4(c^4 + 2(x^2e + 2dx)cd^2 e + (x^2e + 2dx)^2 ce + b^2 + (x^2e + 2dx)be + a)} \sqrt{(b^4 - 8ab^2c + 16a^2c^2)e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] $6c^2 \arctan\left(\frac{2cd^2 + 2(x^2e + 2d*x)ce + b}{\sqrt{-b^2 + 4ac}}\right) e^{-1} / ((b^4 - 8ab^2c + 16a^2c^2) \sqrt{-b^2 + 4ac}) + 1/4 * (12c^3 d^6 + 36(x^2e + 2d*x) c^3 d^4 e + 36(x^2e + 2d*x)^2 c^3 d^2 e^2 + 18b c^2 d^4 + 12(x^2e + 2d*x)^3 c^3 e^3 + 36(x^2e + 2d*x) b c^2 d^2 e + 18(x^2e + 2d*x)^2 b c^2 e^2 + 4b^2 c d^2 + 20a c^2 d^2 + 4(x^2e + 2d*x) b^2 c e + 20(x^2e + 2d*x) a c^2 e - b^3 + 10a b c) / ((c d^4 + 2(x^2e + 2d*x) c d^2 e + (x^2e + 2d*x)^2 c e^2 + b d^2 + (x^2e + 2d*x) b e + a)^2 (b^4 e - 8a b^2 c e + 16a^2 c^2 e))$

maple [C] time = 0.05, size = 541, normalized size = 3.56

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)

[Out] $(3c^3 e^5 / (16a^2 c^2 - 8a b^2 c + b^4) x^6 + 18e^4 c^3 d / (16a^2 c^2 - 8a b^2 c + b^4) x^5 + 9/2 c^2 e^3 (10c d^2 + b) / (16a^2 c^2 - 8a b^2 c + b^4) x^4 + 6c^2 d e^2 (10c d^2 + 3b) / (16a^2 c^2 - 8a b^2 c + b^4) x^3 + c e (45c^2 d^4 + 27b c d^2 + 5a c + b^2) / (16a^2 c^2 - 8a b^2 c + b^4) x^2 + 2c d (9c^2 d^4 + 9b c d^2 + 5a c + b^2) / (16a^2 c^2 - 8a b^2 c + b^4) x + 1/4 e (12c^3 d^6 + 18b c^2 d^4 + 20a c^2 d^2 + 4b^2 c d^2 + 10a b c - b^3) / (16a^2 c^2 - 8a b^2 c + b^4) / (c e^4 x^4 + 4c d e^3 x^3 + 6c d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 + 3$

```
*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d
*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

[Out] Timed out

mupad [B] time = 3.80, size = 1157, normalized size = 7.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((12*c^3*d^6 - b^3 + 20*a*c^2*d^2 + 4*b^2*c*d^2 + 18*b*c^2*d^4 + 10*a*b*c)/
(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(45*c^3*d^4*e + 5*a*c^2*e + b^2
*c*e + 27*b*c^2*d^2*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*x^4*(b*c^2*e^3
+ 10*c^3*d^2*e^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e^5*x^6)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*d*x*(5*a*c^2 + b^2*c + 9*c^3*d^4 + 9*b*c^2
*d^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(3*b*c^2*e^2 + 10*c^3*d^2*
e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (18*c^3*d*e^4*x^5)/(b^4 + 16*a^2*c^2
- 8*a*b^2*c)/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^
2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e
+ 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3
+ 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 1
2*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4)
+ a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6
+ 8*c^2*d*e^7*x^7) + (6*c^2*atan(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c
- b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x*((72*c^6*d*e^7)/(a*(4*a*c - b^2)^
(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72*b*c^4*(b^5*c^2*d*e^9 - 8*a*b^3*
c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c))) + x^2*((36*c^6*e^8)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*
c^4*e^10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (
36*c^6*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3
6*b*c^4*(32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^
2*e^8 - 8*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(72*c^6*e^6))/(e*(4*a*c - b^2)^(5/
2))
```

sympy [B] time = 13.93, size = 1646, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out]
$$-3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b*c**2 + 6*c**3*d**2)/(6*c**3*e**2))/e + 3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b*c**2 + 6*c**3*d**2)/(6*c**3*e**2))/e + (10*a*b*c + 20*a*c**2*d**2 - b**3 + 4*b**2*c*d**2 + 18*b*c**2*d**4 + 12*c**3*d**6 + 72*c**3*d*e**5*x**5 + 12*c**3*e**6*x**6 + x**4*(18*b*c**2*e**4 + 180*c**3*d**2*e**4) + x**3*(72*b*c**2*d*e**3 + 240*c**3*d**3*e**3) + x**2*(20*a*c**2*e**2 + 4*b**2*c*e**2 + 108*b*c**2*d**2*e**2 + 180*c**3*d**4*e**2) + x*(40*a*c**2*d*e + 8*b**2*c*d*e + 72*b*c**2*d**3*e + 72*c**3*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))$$

$$3.521 \quad \int \frac{1}{(a+b(dx)^2+c(dx)^4)^3} dx$$

Optimal. Leaf size=437

$$\frac{\left(\frac{d}{e} + x\right) \left(3bce^2 (b^2 - 8ac) \left(\frac{d}{e} + x\right)^2 + (b^2 - 7ac) (3b^2 - 4ac)\right) 3\sqrt{c} \left(56a^2c^2 - 10ab^2c + b(b^2 - 8ac) \sqrt{b^2 - 4ac} + 8a^2 (b^2 - 4ac)^2 \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)}{8\sqrt{2} a^2 e (b^2 - 4ac)^{5/2} \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 5.36, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1106, 1092, 1178, 1166, 205}

$$\frac{3\sqrt{c} \left(56a^2c^2 - 10ab^2c + b(b^2 - 8ac) \sqrt{b^2 - 4ac} + b^4\right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{d+ex}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - 3\sqrt{c} \left(56a^2c^2 - 10ab^2c - b(b^2 - 8ac) \sqrt{b^2 - 4ac} + b^4\right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{d+ex}}{\sqrt{b^2-4ac+b}}\right) + \left(\frac{d}{e} + x\right) \left(3bce^2 (b^2 - 8ac) \left(\frac{d}{e} + x\right)^2 + (b^2 - 7ac) (3b^2 - 4ac)\right) + \frac{\left(\frac{d}{e} + x\right) \left(-2ac + b^2 + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{4a(b^2 - 4ac) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)^2}}{8\sqrt{2} a^2 e (b^2 - 4ac)^{5/2} \sqrt{b - \sqrt{b^2 - 4ac}} - 8\sqrt{2} a^2 e (b^2 - 4ac)^{5/2} \sqrt{b^2 - 4ac + b} + 8a^2 (b^2 - 4ac)^2 \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right) + 4a(b^2 - 4ac) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3), x]

[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(4*a*(b^2 - 4*a*c)*(a + b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)^2) + ((d/e + x)*((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*e^2*(d/e + x)^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (3*sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]]])/(8*sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*sqrt[b - sqrt[b^2 - 4*a*c]]*e) - (3*sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b*(b^2 - 8*a*c)*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b + sqrt[b^2 - 4*a*c]]])/(8*sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*sqrt[b + sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1092

Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1],
c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symb
ol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
- 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx &= \text{Subst} \left(\int \frac{1}{(a + be^2x^2 + ce^4x^4)^3} dx, x, \frac{d}{e} + x \right) \\
&= \frac{\left(\frac{d}{e} + x\right) \left(b^2 - 2ac + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{4a(b^2 - 4ac) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)^2} - \text{Subst} \left(\int \frac{b^2e^4 - 2ace^4 - 4(b^2 - 2ac + bce^2 \left(\frac{d}{e} + x\right)^2)}{(a + be^2x^2 + ce^4x^4)^3} dx, x, \frac{d}{e} + x \right) \\
&= \frac{\left(\frac{d}{e} + x\right) \left(b^2 - 2ac + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{4a(b^2 - 4ac) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)^2} + \frac{(d + ex) \left((b^2 - 7ac)(3b^2 - 4ac) + 2ace^2(b^2 - 4ac)\right)}{8a^2(b^2 - 4ac)^2 e} \\
&= \frac{\left(\frac{d}{e} + x\right) \left(b^2 - 2ac + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{4a(b^2 - 4ac) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)^2} + \frac{(d + ex) \left((b^2 - 7ac)(3b^2 - 4ac) + 2ace^2(b^2 - 4ac)\right)}{8a^2(b^2 - 4ac)^2 e} \\
&= \frac{\left(\frac{d}{e} + x\right) \left(b^2 - 2ac + bce^2 \left(\frac{d}{e} + x\right)^2\right)}{4a(b^2 - 4ac) \left(a + be^2 \left(\frac{d}{e} + x\right)^2 + ce^4 \left(\frac{d}{e} + x\right)^4\right)^2} + \frac{(d + ex) \left((b^2 - 7ac)(3b^2 - 4ac) + 2ace^2(b^2 - 4ac)\right)}{8a^2(b^2 - 4ac)^2 e}
\end{aligned}$$

Mathematica [A] time = 6.17, size = 463, normalized size = 1.06

$$\frac{28a^2c^2(d+ex) - 25ab^2c(d+ex) - 24abc^2(d+ex)^2 + 3b^3(d+ex) + 3b^2c(d+ex)^3}{8a^2c(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{3\sqrt{c}(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b}\sqrt{b^2-4ac}}\right)}{8\sqrt{2}a^2c(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{c}(-56a^2c^2+10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b}\sqrt{b^2-4ac}}\right)}{8\sqrt{2}a^2c(b^2-4ac)^{3/2}\sqrt{b^2-4ac}+b} + \frac{2ac(d+ex)-b^2(d+ex)-bc(d+ex)^3}{4ac(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3), x]

[Out] $(-b^2(d + ex) + 2ac(d + ex) - b^3(d + ex)^3)/(4a(-b^2 + 4ac)e*(a + b(d + ex)^2 + c(d + ex)^4)^2) + (3b^4(d + ex) - 25ab^2c(d + ex) + 28a^2c^2(d + ex) + 3b^3c(d + ex)^3 - 24ab^2c^2(d + ex)^3)/(8a^2(-b^2 + 4ac)^2e*(a + b(d + ex)^2 + c(d + ex)^4)) + (3\sqrt{c}(b^4 - 10ab^2c + 56a^2c^2 + b^3\sqrt{b^2 - 4ac} - 8ab^2c\sqrt{b^2 - 4ac})\text{ArcTan}[\sqrt{2}\sqrt{c}(d + ex)/\sqrt{b - \sqrt{b^2 - 4ac}}])/(8\sqrt{2}a^2(b^2 - 4ac)^{5/2}\sqrt{b - \sqrt{b^2 - 4ac}})e + (3\sqrt{c}(-b^4 + 10ab^2c - 56a^2c^2 + b^3\sqrt{b^2 - 4ac} - 8ab^2c\sqrt{b^2 - 4ac})\text{ArcTan}[\sqrt{2}\sqrt{c}(d + ex)/\sqrt{b^2 - 4ac} + b])/(8\sqrt{2}a^2(b^2 - 4ac)^{5/2}\sqrt{b^2 - 4ac} + b) + (2ac(d + ex) - b^2(d + ex) - bc(d + ex)^3)/(4ac(4ac - b^2)(a + b(d + ex)^2 + c(d + ex)^4))$


```

sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*
a*c]]]/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3), x]
```

```
[Out] IntegrateAlgebraic[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3), x]
```

fricas [B] time = 2.55, size = 8554, normalized size = 19.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 42*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6
+ 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^
5*x^5 + 10*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2
*c^3)*d)*e^4*x^4 + 6*(b^3*c^2 - 8*a*b*c^3)*d^7 + 2*(3*b^5 - 20*a*b^3*c - 4*
a^2*b*c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28
*a^2*c^3)*d^2)*e^3*x^3 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + 2*(6
3*(b^3*c^2 - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3
+ 3*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + 2*(3*b^5 - 20*a*b^3*c -
4*a^2*b*c^2)*d^3 + 2*(21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*c
+ 44*a^3*c^2 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 2
0*a*b^3*c - 4*a^2*b*c^2)*d^2)*e*x + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c
^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*e
^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3
+ 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*e^6*x^
5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*e^5*x
^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c -
8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d
)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 + 4*(2*(a^2*b^4
*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*
a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^
4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c
```

$$\begin{aligned}
& ^4)d^8 + a^4b^4 - 8a^5b^2c + 16a^6c^2 + 2*(a^2b^5c - 8a^3b^3c^2 \\
& + 16a^4b^3c^3)*d^6 + (a^2b^6 - 6a^3b^4c + 32a^5c^3)*d^4 + 2*(a^3b^5 \\
& - 8a^4b^3c + 16a^5b^3c^2)*d^2)*e)*\sqrt{-(b^9 - 21a*b^7c + 189a^2b^5c^2 - 840a^3b^3c^3 + 1680a^4b^2c^4 + (a^5b^{10} - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^{10}c^5)*e^2*\sqrt{(b^8 - 22a*b^6c + 219a^2b^4c^2 - 1078a^3b^2c^3 + 2401a^4c^4)/((a^{10}b^{10} - 20a^{11}b^8c + 160a^{12}b^6c^2 - 640a^{13}b^4c^3 + 1280a^{14}b^2c^4 - 1024a^{15}c^5)*e^4)))/((a^5b^{10} - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^{10}c^5)*e^2))*\log(27*(21b^8c^3 - 447a*b^6c^4 + 4189a^2b^4c^5 - 19208a^3b^2c^6 + 38416a^4c^7)*e*x + 27*(21b^8c^3 - 447a*b^6c^4 + 4189a^2b^4c^5 - 19208a^3b^2c^6 + 38416a^4c^7)*d + 27/2*\sqrt{1/2})*((a^5b^{15} - 31a^6b^{13}c + 424a^7b^{11}c^2 - 3280a^8b^9c^3 + 15360a^9b^7c^4 - 43264a^{10}b^5c^5 + 67584a^{11}b^3c^6 - 45056a^{12}b^2c^7)*e^3*\sqrt{(b^8 - 22a*b^6c + 219a^2b^4c^2 - 1078a^3b^2c^3 + 2401a^4c^4)/((a^{10}b^{10} - 20a^{11}b^8c + 160a^{12}b^6c^2 - 640a^{13}b^4c^3 + 1280a^{14}b^2c^4 - 1024a^{15}c^5)*e^4)) - (b^{14} - 32a*b^{12}c + 464a^2b^{10}c^2 - 3885a^3b^8c^3 + 20088a^4b^6c^4 - 63680a^5b^4c^5 + 113792a^6b^2c^6 - 87808a^7c^7)*e)*\sqrt{-(b^9 - 21a*b^7c + 189a^2b^5c^2 - 840a^3b^3c^3 + 1680a^4b^2c^4 + (a^5b^{10} - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^{10}c^5)*e^2*\sqrt{(b^8 - 22a*b^6c + 219a^2b^4c^2 - 1078a^3b^2c^3 + 2401a^4c^4)/((a^{10}b^{10} - 20a^{11}b^8c + 160a^{12}b^6c^2 - 640a^{13}b^4c^3 + 1280a^{14}b^2c^4 - 1024a^{15}c^5)*e^4)))/((a^5b^{10} - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^{10}c^5)*e^2)) - 3*\sqrt{1/2})*((a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*e^9*x^8 + 8*(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d*e^8*x^7 + 2*(a^2b^5c - 8a^3b^3c^2 + 16a^4b^3c^3 + 14*(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^2)*e^7*x^6 + 4*(14*(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^3 + 3*(a^2b^5c - 8a^3b^3c^2 + 16a^4b^3c^3)*d)*e^6*x^5 + (a^2b^6 - 6a^3b^4c + 32a^5c^3 + 70*(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^4 + 30*(a^2b^5c - 8a^3b^3c^2 + 16a^4b^3c^3)*d^2)*e^5*x^4 + 4*(14*(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^5 + 10*(a^2b^5c - 8a^3b^3c^2 + 16a^4b^3c^3)*d^3 + (a^2b^6 - 6a^3b^4c + 32a^5c^3)*d)*e^4*x^3 + 2*(a^3b^5 - 8a^4b^3c + 16a^5b^3c^2 + 14*(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^6 + 15*(a^2b^5c - 8a^3b^3c^2 + 16a^4b^3c^3)*d^4 + 3*(a^2b^6 - 6a^3b^4c + 32a^5c^3)*d^2)*e^3*x^2 + 4*(2*(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^7 + 3*(a^2b^5c - 8a^3b^3c^2 + 16a^4b^3c^3)*d^5 + (a^2b^6 - 6a^3b^4c + 32a^5c^3)*d^3 + (a^3b^5 - 8a^4b^3c + 16a^5b^3c^2)*d)*e^2*x + ((a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^8 + a^4b^4 - 8a^5b^2c + 16a^6c^2 + 2*(a^2b^5c - 8a^3b^3c^2 + 16a^4b^3c^3)*d^6 + (a^2b^6 - 6a^3b^4c + 32a^5c^3)*d^4 + 2*(a^3b^5 - 8a^4b^3c + 16a^5b^3c^2)*d^2)*e)*\sqrt{-(b^9 - 21a*b^7c + 189a^2b^5c^2 - 840a^3b^3c^3 + 1680a^4b^2c^4 + (a^5b^{10} - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^{10}c^5)*e^2*\sqrt{(b^8 - 22a*b^6c + 219a^2b^4c^2 - 1078a^3b^2c^3 + 2401a^4c^4)/((a^{10}b^{10} - 20a^{11}b^8c
\end{aligned}$$

$$\begin{aligned}
& + 160*a^{12}*b^6*c^2 - 640*a^{13}*b^4*c^3 + 1280*a^{14}*b^2*c^4 - 1024*a^{15}*c^5) \\
& *e^4)))/((a^5*b^{10} - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 128 \\
& 0*a^9*b^2*c^4 - 1024*a^{10}*c^5)*e^2))*\log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4 \\
& 189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 - \\
& 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d - \\
& 27/2*\sqrt{1/2}*((a^5*b^{15} - 31*a^6*b^{13}*c + 424*a^7*b^{11}*c^2 - 3280*a^8*b^9 \\
& *c^3 + 15360*a^9*b^7*c^4 - 43264*a^{10}*b^5*c^5 + 67584*a^{11}*b^3*c^6 - 45056* \\
& a^{12}*b*c^7)*e^3*\sqrt{(b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 \\
& + 2401*a^4*c^4)/((a^{10}*b^{10} - 20*a^{11}*b^8*c + 160*a^{12}*b^6*c^2 - 640*a^{13} \\
& b^4*c^3 + 1280*a^{14}*b^2*c^4 - 1024*a^{15}*c^5)*e^4)) - (b^{14} - 32*a*b^{12}*c + \\
& 464*a^2*b^{10}*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 \\
& + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*\sqrt{-(b^9 - 21*a*b^7*c + 189*a^2 \\
& *b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^{10} - 20*a^6*b^8*c + 16 \\
& 0*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^{10}*c^5)*e^2)*\sqrt{ \\
& t((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/((\\
& a^{10}*b^{10} - 20*a^{11}*b^8*c + 160*a^{12}*b^6*c^2 - 640*a^{13}*b^4*c^3 + 1280*a^{14} \\
& *b^2*c^4 - 1024*a^{15}*c^5)*e^4)))/((a^5*b^{10} - 20*a^6*b^8*c + 160*a^7*b^6*c^ \\
& 2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^{10}*c^5)*e^2))) - 3*\sqrt{1/2} \\
&)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8* \\
& a^3*b^2*c^3 + 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4 \\
& *b*c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14 \\
& *(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3* \\
& c^2 + 16*a^4*b*c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(\\
& a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c \\
& ^2 + 16*a^4*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a \\
& ^4*c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 \\
& - 6*a^3*b^4*c + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5* \\
& b*c^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - \\
& 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3) \\
& *d^2)*e^3*x^2 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^ \\
& 2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a \\
& ^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c \\
& ^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + \\
& 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c \\
& + 32*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*\sqrt{-(\\
& b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a \\
& ^5*b^{10} - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c \\
& ^4 - 1024*a^{10}*c^5)*e^2)*\sqrt{(b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3 \\
& *b^2*c^3 + 2401*a^4*c^4)/((a^{10}*b^{10} - 20*a^{11}*b^8*c + 160*a^{12}*b^6*c^2 - 6 \\
& 40*a^{13}*b^4*c^3 + 1280*a^{14}*b^2*c^4 - 1024*a^{15}*c^5)*e^4)))/((a^5*b^{10} - 20 \\
& *a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^ \\
& 10*c^5)*e^2))*\log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208 \\
& *a^3*b^2*c^6 + 38416*a^4*c^7)*e*x + 27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a \\
& ^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*d + 27/2*\sqrt{1/2}*((a^5*b^ \\
& 15 - 31*a^6*b^{13}*c + 424*a^7*b^{11}*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^
\end{aligned}$$

$$\begin{aligned}
& 4 - 43264a^{10}b^5c^5 + 67584a^{11}b^3c^6 - 45056a^{12}b^2c^7)e^3\sqrt{(b^8 - 22ab^6c + 219a^2b^4c^2 - 1078a^3b^2c^3 + 2401a^4c^4)/((a^{10}b^{10} - 20a^{11}b^8c + 160a^{12}b^6c^2 - 640a^{13}b^4c^3 + 1280a^{14}b^2c^4 - 1024a^{15}c^5)e^4)} + (b^{14} - 32a^3b^{12}c + 464a^2b^{10}c^2 - 3885a^3b^8c^3 + 20088a^4b^6c^4 - 63680a^5b^4c^5 + 113792a^6b^2c^6 - 87808a^7c^7)e)\sqrt{-(b^9 - 21a^2b^7c + 189a^2b^5c^2 - 840a^3b^3c^3 + 1680a^4b^2c^4 - (a^5b^{10} - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^{10}c^5)e^2)\sqrt{(b^8 - 22ab^6c + 219a^2b^4c^2 - 1078a^3b^2c^3 + 2401a^4c^4)/((a^{10}b^{10} - 20a^{11}b^8c + 160a^{12}b^6c^2 - 640a^{13}b^4c^3 + 1280a^{14}b^2c^4 - 1024a^{15}c^5)e^4)))/((a^5b^{10} - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^{10}c^5)e^2))} + 3\sqrt{1/2}*((a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)e^9x^8 + 8(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d*e^8x^7 + 2(a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3 + 14(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^2)*e^7x^6 + 4(14(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^3 + 3(a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3)*d)*e^6x^5 + (a^2b^6 - 6a^3b^4c + 32a^5c^3 + 70(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^4 + 30(a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3)*d^2)*e^5x^4 + 4(14(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^5 + 10(a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3)*d^3 + (a^2b^6 - 6a^3b^4c + 32a^5c^3)*d)*e^4x^3 + 2(a^3b^5 - 8a^4b^3c + 16a^5b^2c^2 + 14(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^6 + 15(a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3)*d^4 + 3(a^2b^6 - 6a^3b^4c + 32a^5c^3)*d^2)*e^3x^2 + 4(2(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^7 + 3(a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3)*d^5 + (a^2b^6 - 6a^3b^4c + 32a^5c^3)*d^3 + (a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)*d)*e^2x + ((a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4)*d^8 + a^4b^4 - 8a^5b^2c + 16a^6c^2 + 2(a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3)*d^6 + (a^2b^6 - 6a^3b^4c + 32a^5c^3)*d^4 + 2(a^3b^5 - 8a^4b^3c + 16a^5b^2c^2)*d^2)*e)\sqrt{-(b^9 - 21a^2b^7c + 189a^2b^5c^2 - 840a^3b^3c^3 + 1680a^4b^2c^4 - (a^5b^{10} - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^{10}c^5)e^2)\sqrt{(b^8 - 22ab^6c + 219a^2b^4c^2 - 1078a^3b^2c^3 + 2401a^4c^4)/((a^{10}b^{10} - 20a^{11}b^8c + 160a^{12}b^6c^2 - 640a^{13}b^4c^3 + 1280a^{14}b^2c^4 - 1024a^{15}c^5)e^4)))/((a^5b^{10} - 20a^6b^8c + 160a^7b^6c^2 - 640a^8b^4c^3 + 1280a^9b^2c^4 - 1024a^{10}c^5)e^2)}*\log(27(21b^8c^3 - 447ab^6c^4 + 4189a^2b^4c^5 - 19208a^3b^2c^6 + 38416a^4c^7)*e*x + 27(21b^8c^3 - 447ab^6c^4 + 4189a^2b^4c^5 - 19208a^3b^2c^6 + 38416a^4c^7)*d - 27/2\sqrt{1/2}*((a^5b^{15} - 31a^6b^{13}c + 424a^7b^{11}c^2 - 3280a^8b^9c^3 + 15360a^9b^7c^4 - 43264a^{10}b^5c^5 + 67584a^{11}b^3c^6 - 45056a^{12}b^2c^7)e^3\sqrt{(b^8 - 22ab^6c + 219a^2b^4c^2 - 1078a^3b^2c^3 + 2401a^4c^4)/((a^{10}b^{10} - 20a^{11}b^8c + 160a^{12}b^6c^2 - 640a^{13}b^4c^3 + 1280a^{14}b^2c^4 - 1024a^{15}c^5)e^4)} + (b^{14} - 32a^3b^{12}c + 464a^2b^{10}c^2 - 3885a^3b^8c^3 + 20088a^4b^6c^4 - 63680a^5b^4c^5 + 113792a^6b^2c^6 - 87808a^7c^7)e)\sqrt{-(b^9 - 21a^2b^7c + 189a^2b^5c^2 - 840a^3b^3c^3 + 1680a^4b^2c^4 -
\end{aligned}$$

$$\begin{aligned} & (a^5 b^{10} - 20 a^6 b^8 c + 160 a^7 b^6 c^2 - 640 a^8 b^4 c^3 + 1280 a^9 b^2 c^4 - 1024 a^{10} c^5) e^{2 \sqrt{(b^8 - 22 a b^6 c + 219 a^2 b^4 c^2 - 1078 a^3 b^2 c^3 + 2401 a^4 c^4)}} / ((a^{10} b^{10} - 20 a^{11} b^8 c + 160 a^{12} b^6 c^2 - 640 a^{13} b^4 c^3 + 1280 a^{14} b^2 c^4 - 1024 a^{15} c^5) e^4) / ((a^5 b^{10} - 20 a^6 b^8 c + 160 a^7 b^6 c^2 - 640 a^8 b^4 c^3 + 1280 a^9 b^2 c^4 - 1024 a^{10} c^5) e^2)) + 2(5 a^5 b^4 - 37 a^2 b^2 c + 44 a^3 c^2) d / ((a^2 b^4 c^2 - 8 a^3 b^2 c^3 + 16 a^4 c^4) e^9 x^8 + 8(a^2 b^4 c^2 - 8 a^3 b^2 c^3 + 16 a^4 c^4) d e^8 x^7 + 2(a^2 b^5 c - 8 a^3 b^3 c^2 + 16 a^4 b c^3 + 14(a^2 b^4 c^2 - 8 a^3 b^2 c^3 + 16 a^4 c^4) d^2) e^7 x^6 + 4(14(a^2 b^4 c^2 - 8 a^3 b^2 c^3 + 16 a^4 c^4) d^3 + 3(a^2 b^5 c - 8 a^3 b^3 c^2 + 16 a^4 b c^3) d) e^6 x^5 + (a^2 b^6 - 6 a^3 b^4 c + 32 a^5 c^3 + 70(a^2 b^4 c^2 - 8 a^3 b^2 c^3 + 16 a^4 c^4) d^4 + 30(a^2 b^5 c - 8 a^3 b^3 c^2 + 16 a^4 b c^3) d^2) e^5 x^4 + 4(14(a^2 b^4 c^2 - 8 a^3 b^2 c^3 + 16 a^4 c^4) d^5 + 10(a^2 b^5 c - 8 a^3 b^3 c^2 + 16 a^4 b c^3) d^3 + (a^2 b^6 - 6 a^3 b^4 c + 32 a^5 c^3) d) e^4 x^3 + 2(a^3 b^5 - 8 a^4 b^3 c + 16 a^5 b c^2 + 14(a^2 b^4 c^2 - 8 a^3 b^2 c^3 + 16 a^4 c^4) d^6 + 15(a^2 b^5 c - 8 a^3 b^3 c^2 + 16 a^4 b c^3) d^4 + 3(a^2 b^6 - 6 a^3 b^4 c + 32 a^5 c^3) d^2) e^3 x^2 + 4(2(a^2 b^4 c^2 - 8 a^3 b^2 c^3 + 16 a^4 c^4) d^7 + 3(a^2 b^5 c - 8 a^3 b^3 c^2 + 16 a^4 b c^3) d^5 + (a^2 b^6 - 6 a^3 b^4 c + 32 a^5 c^3) d^3 + (a^3 b^5 - 8 a^4 b^3 c + 16 a^5 b c^2) d) e^2 x + ((a^2 b^4 c^2 - 8 a^3 b^2 c^3 + 16 a^4 c^4) d^8 + a^4 b^4 - 8 a^5 b^2 c + 16 a^6 c^2 + 2(a^2 b^5 c - 8 a^3 b^3 c^2 + 16 a^4 b c^3) d^6 + (a^2 b^6 - 6 a^3 b^4 c + 32 a^5 c^3) d^4 + 2(a^3 b^5 - 8 a^4 b^3 c + 16 a^5 b c^2) d^2) e) \end{aligned}$$

giac [B] time = 0.56, size = 2487, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -3/16 * (((d e^{-1}) + \sqrt{1/2}) \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2}) e^{-4} / c))^2 b^3 c e^2 - 8(d e^{-1}) \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c))^2 a b c^2 e^2 - 2(d e^{-1}) \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c) b^3 c d e + 16(d e^{-1}) \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c) a b c^2 d e + b^3 c d^2 - 8 a b c^2 d^2 + b^4 - 9 a b^2 c + 28 a^2 c^2) \log(d e^{-1}) + x + \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c) / (2(d e^{-1}) \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c))^3 c e^4 - 6(d e^{-1}) \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c))^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1}) \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c) + ((d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c))^2 b^3 c e^2 - 8(d e^{-1}) \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c))^2 a b c^2 e^2 - 2(d e^{-1}) \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c) b^3 c d e + 16(d e^{-1}) \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4} / c) \end{aligned}$$

$$\begin{aligned}
& \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c} \cdot a b c^2 d e + b^3 c d^2 - 8 a b c^2 d^2 + b^4 - 9 a b^2 c + 28 a^2 c^2 \cdot \log(d e^{-1} + x - \\
& \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) / (2 (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^3 c e^4 - 6 (d e^{-1} - \\
& \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})) + ((d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 b^3 c e^2 - 8 (d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 a b c^2 e^2 - 2 (d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) b^3 c d e + 16 (d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) a b c^2 d e + b^3 c d^2 - 8 a b c^2 d^2 + b^4 - 9 a b^2 c + 28 a^2 c^2) \cdot \log(d e^{-1} + x + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) / (2 (d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^3 c e^4 - 6 (d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})) + ((d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 b^3 c e^2 - 8 (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 a b c^2 e^2 - 2 (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) b^3 c d e + 16 (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) a b c^2 d e + b^3 c d^2 - 8 a b c^2 d^2 + b^4 - 9 a b^2 c + 28 a^2 c^2) \cdot \log(d e^{-1} + x - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) / (2 (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^3 c e^4 - 6 (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})))) / (a^2 b^4 - 8 a^3 b^2 c + 16 a^4 c^2) + 1/8 (3 b^3 c^2 x^7 e^7 - 24 a b c^3 x^7 e^7 + 21 b^3 c^2 d x^6 e^6 - 168 a b c^3 d x^6 e^6 + 63 b^3 c^2 d^2 x^5 e^5 - 504 a b c^3 d^2 x^5 e^5 + 105 b^3 c^2 d^3 x^4 e^4 - 840 a b c^3 d^3 x^4 e^4 + 105 b^3 c^2 d^4 x^3 e^3 - 840 a b c^3 d^4 x^3 e^3 + 63 b^3 c^2 d^5 x^2 e^2 - 504 a b c^3 d^5 x^2 e^2 + 21 b^3 c^2 d^6 x e - 168 a b c^3 d^6 x e + 3 b^3 c^2 d^7 - 24 a b c^3 d^7 + 6 b^4 c x^5 e^5 - 49 a b^2 c^2 x^5 e^5 + 28 a^2 c^3 x^5 e^5 + 30 b^4 c d x^4 e^4 - 245 a b^2 c^2 d x^4 e^4 + 140 a^2 c^3 d x^4 e^4 + 60 b^4 c d^2 x^3 e^3 - 490 a b^2 c^2 d^2 x^3 e^3 + 280 a^2 c^3 d^2 x^3 e^3 + 60 b^4 c d^3 x^2 e^2 - 490 a b^2 c^2 d^3 x^2 e^2 + 280 a^2 c^3 d^3 x^2 e^2 + 30 b^4 c d^4 x e - 245 a b^2 c^2 d^4 x e + 140 a^2 c^3 d^4 x e + 6 b^4 c d^5 - 49 a b^2 c^2 d^5 + 28 a^2 c^3 d^5 + 3 b^5 x^3 e^3 - 20 a b^3 c x^3 e^3 - 4 a^2 b c^2 x^3 e^3 + 9 b^5 d x^2 e^2 - 60 a b^3 c d x^2 e^2 - 12 a^2 b c^2 d x^2 e^2 + 9 b^5 d^2 x e - 60 a b^3 c d^2 x e - 12 a^2 b c^2 d^2 x e + 3 b^5 d^3 - 20 a b^3 c d^3 - 4 a^2 b c^2 d^3 + 5 a b^4 x e - 37 a^2 b^2 c x e + 44 a^3 c^2 x e + 5 a b^4 d - 37 a^2 b^2 c d + 44 a^3 c^2 d) / ((a^2 b^4 e - 8 a^3 b^2 c e + 16 a^4 c^2 e) (c x^4 e^4 + 4 c d x^3 e^3 + 6 c d^2 x^2 e^2 + 4 c d^3 x e + c d^4 + b x^2 e^2 + 2 b d x e + b d^2 + a)^2)
\end{aligned}$$

maple [C] time = 0.05, size = 1010, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)$

[Out]
$$\begin{aligned} & (-3/8*c^2*e^6*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^7-21/8*c^2*d*e \\ & ^5*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^6+1/8*(-504*a*b*c^2*d^2+6 \\ & 3*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b^4)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a \\ & ^2*x^5+5/8*c*d*e^3*(-168*a*b*c^2*d^2+21*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b \\ & ^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^4-1/8*e^2*(840*a*b*c^3*d^4-105*b^3*c^2 \\ & *d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+4*a^2*b*c^2+20*a*b^3*c- \\ & 3*b^5)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(504*a*b*c^3*d^4-63*b^3*c \\ & ^2*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+12*a^2*b*c^2+60*a*b^3 \\ & *c-9*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^2+1/8*(-168*a*b*c^3*d^6+21*b^3*c \\ & ^2*d^6+140*a^2*c^3*d^4-245*a*b^2*c^2*d^4+30*b^4*c*d^4-12*a^2*b*c^2*d^2-60*a \\ & *b^3*c*d^2+9*b^5*d^2+44*a^3*c^2-37*a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c \\ & +b^4)/a^2*x+1/8*d/e*(-24*a*b*c^3*d^6+3*b^3*c^2*d^6+28*a^2*c^3*d^4-49*a*b^2*c \\ & ^2*d^4+6*b^4*c*d^4-4*a^2*b*c^2*d^2-20*a*b^3*c*d^2+3*b^5*d^2+44*a^3*c^2-37* \\ & a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2/(c*e^4*x^4+4*c*d*e^3*x^3 \\ & +6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/(16* \\ & a^2*c^2-8*a*b^2*c+b^4)/a^2/e*\text{sum}((b*c*e^2*(-8*a*c+b^2)*_R^2+2*b*c*d*e*(-8*a \\ & *c+b^2)*_R-8*a*b*c^2*d^2+b^3*c*d^2+28*a^2*c^2-9*a*b^2*c+b^4)/(2*_R^3*c*e^3+ \\ & 6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*\ln(-_R+x),_R=\text{RootOf}(_Z^4*c \\ & e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e) \\ & *_Z+a)) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, \text{algorithm}=\text{"maxima"})$

[Out]
$$\begin{aligned} & 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*e^7*x^7 + 21*(b^3*c^2 - 8*a*b*c^3)*d*e^6*x^6 + \\ & (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63*(b^3*c^2 - 8*a*b*c^3)*d^2)*e^5*x \\ & ^5 + 5*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 \\ &)*d)*e^4*x^4 + 3*(b^3*c^2 - 8*a*b*c^3)*d^7 + (3*b^5 - 20*a*b^3*c - 4*a^2*b* \\ & c^2 + 105*(b^3*c^2 - 8*a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c \\ & ^3)*d^2)*e^3*x^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^5 + (63*(b^3*c^2 \\ & - 8*a*b*c^3)*d^5 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3 + 3*(3*b^5 \\ & - 20*a*b^3*c - 4*a^2*b*c^2)*d)*e^2*x^2 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2 \\ &)*d^3 + (21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2 \\ & + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 20*a*b^3*c - 4* \\ & a^2*b*c^2)*d^2)*e*x + (5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d)/((a^2*b^4*c^ \end{aligned}$$

$$\begin{aligned}
& 2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^9*x^8 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + \\
& 16*a^4*c^4)*d*e^8*x^7 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a \\
& ^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*e^7*x^6 + 4*(14*(a^2*b^4*c^2 \\
& - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b \\
& *c^3)*d)*e^6*x^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - \\
& 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b \\
& *c^3)*d^2)*e^5*x^4 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + \\
& 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c \\
& + 32*a^5*c^3)*d)*e^4*x^3 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^ \\
& 2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 \\
& + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*e^3*x^2 \\
& + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^ \\
& 3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + \\
& (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^2*x + ((a^2*b^4*c^2 - 8*a^3*b^2 \\
& *c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c \\
& - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)* \\
& d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e) - 3/8*integrate(-((b \\
& ^3*c - 8*a*b*c^2)*e^2*x^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2 + 2*(b^3*c - 8*a*b \\
& *c^2)*d*e*x + (b^3*c - 8*a*b*c^2)*d^2)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + \\
& (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a), x)/(a^2*b^4 - \\
& 8*a^3*b^2*c + 16*a^4*c^2)
\end{aligned}$$

mupad [B] time = 7.80, size = 16086, normalized size = 36.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x)$

[Out]
$$\begin{aligned}
& ((3*b^5*d^3 + 44*a^3*c^2*d + 6*b^4*c*d^5 + 28*a^2*c^3*d^5 + 3*b^3*c^2*d^7 + \\
& 5*a*b^4*d - 4*a^2*b*c^2*d^3 - 49*a*b^2*c^2*d^5 - 37*a^2*b^2*c*d - 20*a*b^3 \\
& *c*d^3 - 24*a*b*c^3*d^7)/(8*a^2*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(3 \\
& *b^5*e^2 - 4*a^2*b*c^2*e^2 + 60*b^4*c*d^2*e^2 + 280*a^2*c^3*d^2*e^2 + 105*b \\
& ^3*c^2*d^4*e^2 - 20*a*b^3*c*e^2 - 840*a*b*c^3*d^4*e^2 - 490*a*b^2*c^2*d^2*e \\
& ^2))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(6*b^4*c*e^4 + 28*a^2*c^ \\
& 3*e^4 - 49*a*b^2*c^2*e^4 + 63*b^3*c^2*d^2*e^4 - 504*a*b*c^3*d^2*e^4))/(8*a^ \\
& 2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(9*b^5*d*e + 280*a^2*c^3*d^3*e + 6 \\
& 3*b^3*c^2*d^5*e + 60*b^4*c*d^3*e - 12*a^2*b*c^2*d*e - 504*a*b*c^3*d^5*e - 4 \\
& 90*a*b^2*c^2*d^3*e - 60*a*b^3*c*d*e))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c) \\
&) + (5*x^4*(28*a^2*c^3*d*e^3 + 21*b^3*c^2*d^3*e^3 + 6*b^4*c*d*e^3 - 49*a*b^ \\
& 2*c^2*d*e^3 - 168*a*b*c^3*d^3*e^3))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) \\
& + (21*x^6*(b^3*c^2*d*e^5 - 8*a*b*c^3*d*e^5))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a \\
& *b^2*c)) + (x*(5*a*b^4 + 44*a^3*c^2 + 9*b^5*d^2 - 37*a^2*b^2*c + 30*b^4*c*d \\
& ^4 + 140*a^2*c^3*d^4 + 21*b^3*c^2*d^6 - 12*a^2*b*c^2*d^2 - 245*a*b^2*c^2*d^ \\
& 4 - 60*a*b^3*c*d^2 - 168*a*b*c^3*d^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
\end{aligned}$$

$$\begin{aligned}
&)) + (3*x^7*(b^3*c^2*e^6 - 8*a*b*c^3*e^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b \\
&^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + \\
&30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^ \\
&2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c \\
&^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d \\
&*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + \\
&b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^ \\
&2*d*e^7*x^7) - \operatorname{atan}\left(\frac{(3612672*a^6*c^9*d*e^{11} + 144*b^{12}*c^3*d*e^{11} - 4032* \\
&a*b^{10}*c^4*d*e^{11} + 49824*a^2*b^8*c^5*d*e^{11} - 340992*a^3*b^6*c^6*d*e^{11} + \\
&1410048*a^4*b^4*c^7*d*e^{11} - 3391488*a^5*b^2*c^8*d*e^{11})}{(512*(a^4*b^{12} + 4 \\
&096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^ \\
&8*b^4*c^4 - 6144*a^9*b^2*c^5)) - \left(\frac{(67108864*a^{11}*b*c^9*d*e^{13} - 4096*a^4*b \\
&^{15}*c^2*d*e^{13} + 114688*a^5*b^{13}*c^3*d*e^{13} - 1376256*a^6*b^{11}*c^4*d*e^{13} + \\
&9175040*a^7*b^9*c^5*d*e^{13} - 36700160*a^8*b^7*c^6*d*e^{13} + 88080384*a^9*b^ \\
&5*c^7*d*e^{13} - 117440512*a^{10}*b^3*c^8*d*e^{13})}{(512*(a^4*b^{12} + 4096*a^{10}*c^ \\
&6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - \\
&6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^{14} - 256*a^4*b^{11}*c^2*e^{14} + 5 \\
&120*a^5*b^9*c^3*e^{14} - 40960*a^6*b^7*c^4*e^{14} + 163840*a^7*b^5*c^5*e^{14} - 3 \\
&27680*a^8*b^3*c^6*e^{14}))}{(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6 \\
&*b^4*c^2 - 256*a^7*b^2*c^3))}*(-(9*(b^{19} + b^4*(-(4*a*c - b^2)^{15})^{1/2} - \\
&1720320*a^9*b*c^9 + 769*a^2*b^{15}*c^2 - 8620*a^3*b^{13}*c^3 + 63440*a^4*b^{11}*c \\
&^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 30105 \\
&60*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^{15})^{1/2} - 41*a*b^{17}*c - 11*a* \\
&b^2*c*(-(4*a*c - b^2)^{15})^{1/2}))/((512*(a^5*b^{20}*e^2 + 1048576*a^{15}*c^{10}*e^ \\
&2 - 40*a^6*b^{18}*c*e^2 + 720*a^7*b^{16}*c^2*e^2 - 7680*a^8*b^{14}*c^3*e^2 + 5376 \\
&0*a^9*b^{12}*c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 + 860160*a^{11}*b^8*c^6*e^2 - 1 \\
&966080*a^{12}*b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^8*e^2 - 2621440*a^{14}*b^2*c^9*e \\
&^2)))^{1/2} - (22020096*a^9*c^9*e^{12} - 768*a^2*b^{14}*c^2*e^{12} + 22272*a^3*b^ \\
&12*c^3*e^{12} - 282624*a^4*b^{10}*c^4*e^{12} + 2027520*a^5*b^8*c^5*e^{12} - 8847360 \\
&*a^6*b^6*c^6*e^{12} + 23396352*a^7*b^4*c^7*e^{12} - 34603008*a^8*b^2*c^8*e^{12})/ \\
&(512*(a^4*b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7 \\
&*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(9*(b^{19} + b^4*(-(4*a*c \\
&- b^2)^{15})^{1/2} - 1720320*a^9*b*c^9 + 769*a^2*b^{15}*c^2 - 8620*a^3*b^{13}*c^ \\
&3 + 63440*a^4*b^{11}*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936 \\
&*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^{15})^{1/2} - \\
&41*a*b^{17}*c - 11*a*b^2*c*(-(4*a*c - b^2)^{15})^{1/2}))/((512*(a^5*b^{20}*e^2 + \\
&1048576*a^{15}*c^{10}*e^2 - 40*a^6*b^{18}*c*e^2 + 720*a^7*b^{16}*c^2*e^2 - 7680*a^8 \\
&*b^{14}*c^3*e^2 + 53760*a^9*b^{12}*c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 + 860160* \\
&a^{11}*b^8*c^6*e^2 - 1966080*a^{12}*b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^8*e^2 - 26 \\
&21440*a^{14}*b^2*c^9*e^2)))^{1/2} + (x*(14112*a^4*c^7*e^{12} + 9*b^8*c^3*e^{12} - \\
&180*a*b^6*c^4*e^{12} + 1530*a^2*b^4*c^5*e^{12} - 6192*a^3*b^2*c^6*e^{12}))/((32*(\\
&a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))* \\
&(-(9*(b^{19} + b^4*(-(4*a*c - b^2)^{15})^{1/2} - 1720320*a^9*b*c^9 + 769*a^2*b^ \\
&15*c^2 - 8620*a^3*b^{13}*c^3 + 63440*a^4*b^{11}*c^4 - 316864*a^5*b^9*c^5 + 1069 \\
&824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-
\end{aligned}$$

$$\begin{aligned}
& ((4ac - b^2)^{15})^{1/2} - 41ab^{17}c - 11a^2b^2c(-(4ac - b^2)^{15})^{1/2} \\
&)) / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^2e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{1/2} * i + ((3612672a^6c^9d^{11} + 144b^{12}c^3d^{11} - 4032a^2b^{10}c^4d^{11} + 49824a^2b^8c^5d^{11} - 340992a^3b^6c^6d^{11} + 1410048a^4b^4c^7d^{11} - 3391488a^5b^2c^8d^{11}) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) - ((67108864a^{11}b^2c^9d^{13} - 4096a^4b^{15}c^2d^{13} + 114688a^5b^{13}c^3d^{13} - 1376256a^6b^{11}c^4d^{13} + 9175040a^7b^9c^5d^{13} - 36700160a^8b^7c^6d^{13} + 88080384a^9b^5c^7d^{13} - 117440512a^{10}b^3c^8d^{13}) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) + (x*(262144a^9b^2c^7e^{14} - 256a^4b^{11}c^2e^{14} + 5120a^5b^9c^3e^{14} - 40960a^6b^7c^4e^{14} + 163840a^7b^5c^5e^{14} - 327680a^8b^3c^6e^{14})) / (32(a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3))) * (-9(b^{19} + b^4(-(4ac - b^2)^{15})^{1/2} - 1720320a^9b^2c^9 + 769a^2b^{15}c^2 - 8620a^3b^{13}c^3 + 63440a^4b^{11}c^4 - 316864a^5b^9c^5 + 1069824a^6b^7c^6 - 2343936a^7b^5c^7 + 3010560a^8b^3c^8 + 49a^2c^2(-(4ac - b^2)^{15})^{1/2} - 41ab^{17}c - 11a^2b^2c(-(4ac - b^2)^{15})^{1/2}))) / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^2e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{1/2} + (22020096a^9c^9e^{12} - 768a^2b^{14}c^2e^{12} + 22272a^3b^{12}c^3e^{12} - 282624a^4b^{10}c^4e^{12} + 2027520a^5b^8c^5e^{12} - 8847360a^6b^6c^6e^{12} + 23396352a^7b^4c^7e^{12} - 34603008a^8b^2c^8e^{12}) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) * (-9(b^{19} + b^4(-(4ac - b^2)^{15})^{1/2} - 1720320a^9b^2c^9 + 769a^2b^{15}c^2 - 8620a^3b^{13}c^3 + 63440a^4b^{11}c^4 - 316864a^5b^9c^5 + 1069824a^6b^7c^6 - 2343936a^7b^5c^7 + 3010560a^8b^3c^8 + 49a^2c^2(-(4ac - b^2)^{15})^{1/2} - 41ab^{17}c - 11a^2b^2c(-(4ac - b^2)^{15})^{1/2}))) / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^2e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{1/2} + (x*(14112a^4c^7e^{12} + 9b^8c^3e^{12} - 180a^2b^6c^4e^{12} + 1530a^2b^4c^5e^{12} - 6192a^3b^2c^6e^{12})) / (32(a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3))) * (-9(b^{19} + b^4(-(4ac - b^2)^{15})^{1/2} - 1720320a^9b^2c^9 + 769a^2b^{15}c^2 - 8620a^3b^{13}c^3 + 63440a^4b^{11}c^4 - 316864a^5b^9c^5 + 1069824a^6b^7c^6 - 2343936a^7b^5c^7 + 3010560a^8b^3c^8 + 49a^2c^2(-(4ac - b^2)^{15})^{1/2} - 41ab^{17}c - 11a^2b^2c(-(4ac - b^2)^{15})^{1/2}))) / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^2e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 + 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{1/2} +
\end{aligned}$$

$$\begin{aligned}
& e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2) \\
& \left((567b^7c^5e^{10} - 10368ab^5c^6e^{10} - 169344a^3b^3c^8e^{10} + 67824a^2b^3c^7e^{10}) / (256(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) \right. \\
& \left. + ((3612672a^6c^9d^{11} + 144b^{12}c^3d^{11} - 4032ab^{10}c^4d^{11} + 49824a^2b^8c^5d^{11} - 340992a^3b^6c^6d^{11} + 1410048a^4b^4c^7d^{11} - 3391488a^5b^2c^8d^{11}) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) \right. \\
& \left. - (((67108864a^{11}b^3c^9d^{13} - 4096a^4b^{15}c^2d^{13} + 114688a^5b^{13}c^3d^{13} - 1376256a^6b^{11}c^4d^{13} + 9175040a^7b^9c^5d^{13} - 36700160a^8b^7c^6d^{13} + 88080384a^9b^5c^7d^{13} - 117440512a^{10}b^3c^8d^{13}) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) \right. \\
& \left. + (x(262144a^9b^3c^7e^{14} - 256a^4b^{11}c^2e^{14} + 5120a^5b^9c^3e^{14} - 40960a^6b^7c^4e^{14} + 163840a^7b^5c^5e^{14} - 327680a^8b^3c^6e^{14}) / (32(a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3))) \right) \\
& \left(-(9(b^{19} + b^4(-(4ac - b^2)^{15})^{1/2}) - 1720320a^9b^3c^9 + 769a^2b^{15}c^2 - 8620a^3b^{13}c^3 + 63440a^4b^{11}c^4 - 316864a^5b^9c^5 + 1069824a^6b^7c^6 - 2343936a^7b^5c^7 + 3010560a^8b^3c^8 + 49a^2c^2(-(4ac - b^2)^{15})^{1/2} - 41ab^{17}c - 11a^2b^2c(-(4ac - b^2)^{15})^{1/2}) \right) \\
& / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^3e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2) \\
& \left((22020096a^9c^9e^{12} - 768a^2b^{14}c^2e^{12} + 22272a^3b^{12}c^3e^{12} - 282624a^4b^{10}c^4e^{12} + 2027520a^5b^8c^5e^{12} - 8847360a^6b^6c^6e^{12} + 23396352a^7b^4c^7e^{12} - 34603008a^8b^2c^8e^{12}) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) \right) \\
& \left(-(9(b^{19} + b^4(-(4ac - b^2)^{15})^{1/2}) - 1720320a^9b^3c^9 + 769a^2b^{15}c^2 - 8620a^3b^{13}c^3 + 63440a^4b^{11}c^4 - 316864a^5b^9c^5 + 1069824a^6b^7c^6 - 2343936a^7b^5c^7 + 3010560a^8b^3c^8 + 49a^2c^2(-(4ac - b^2)^{15})^{1/2} - 41ab^{17}c - 11a^2b^2c(-(4ac - b^2)^{15})^{1/2}) \right) \\
& / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^3e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2) \\
& \left((14112a^4c^7e^{12} + 9b^8c^3e^{12} - 180ab^6c^4e^{12} + 1530a^2b^4c^5e^{12} - 6192a^3b^2c^6e^{12}) / (32(a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3))) \right) \\
& \left(-(9(b^{19} + b^4(-(4ac - b^2)^{15})^{1/2}) - 1720320a^9b^3c^9 + 769a^2b^{15}c^2 - 8620a^3b^{13}c^3 + 63440a^4b^{11}c^4 - 316864a^5b^9c^5 + 1069824a^6b^7c^6 - 2343936a^7b^5c^7 + 3010560a^8b^3c^8 + 49a^2c^2(-(4ac - b^2)^{15})^{1/2} - 41ab^{17}c - 11a^2b^2c(-(4ac - b^2)^{15})^{1/2}) \right) \\
& / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^3e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2)
\end{aligned}$$

$$\begin{aligned} & *b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{(1/2)} - ((3612672a^6c^9 * d^11 + 144b^{12}c^3d^11 - 4032a^8b^{10}c^4d^11 + 49824a^2b^8c^5 * d^11 - 340992a^3b^6c^6d^11 + 1410048a^4b^4c^7d^11 - 3391488a^5 * b^2c^8d^11)/(512*(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6 * b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) - (((67 * 108864a^{11}b^9c^3d^13 - 4096a^4b^{15}c^2d^13 + 114688a^5b^{13}c^3 * d^13 - 1376256a^6b^{11}c^4d^13 + 9175040a^7b^9c^5d^13 - 36700160 * a^8b^7c^6d^13 + 88080384a^9b^5c^7d^13 - 117440512a^{10}b^3c^8 * d^13)/(512*(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6 * b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) + (x*(262144a^9 * b^7c^14 - 256a^4b^{11}c^2e^{14} + 5120a^5b^9c^3e^{14} - 40960a^6b^7 * c^4e^{14} + 163840a^7b^5c^5e^{14} - 327680a^8b^3c^6e^{14}))/((32 * (a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3))) * (-9 * (b^{19} + b^4 * (-4ac - b^2)^{15})^{(1/2)} - 1720320a^9b^9c^9 + 769a^2b^{15}c^2 * 8620a^3b^{13}c^3 + 63440a^4b^{11}c^4 - 316864a^5b^9c^5 + 1069824a^6 * b^7c^6 - 2343936a^7b^5c^7 + 3010560a^8b^3c^8 + 49a^2c^2 * (-4ac - b^2)^{15})^{(1/2)} - 41 * a * b^{17}c - 11 * a * b^2 * c * (-4ac - b^2)^{15})^{(1/2)})) / (512 * (a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c * e^2 + 720a^7b^{16} * c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10} * c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13} * b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{(1/2)} + (22020096a^9c^9e^{12} - 768a^2b^{14}c^2e^{12} + 22272a^3b^{12}c^3e^{12} - 282624a^4b^{10}c^4e^{12} + 2027520a^5b^8c^5e^{12} - 8847360a^6b^6c^6e^{12} + 23396352a^7b^4c^7 * e^{12} - 34603008a^8b^2c^8e^{12}) / (512 * (a^4b^{12} + 4096a^{10}c^6 - 24a^5 * b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9 * b^2c^5))) * (-9 * (b^{19} + b^4 * (-4ac - b^2)^{15})^{(1/2)} - 1720320a^9b^9 * c^9 + 769a^2b^{15}c^2 - 8620a^3b^{13}c^3 + 63440a^4b^{11}c^4 - 316864a^5 * b^9c^5 + 1069824a^6b^7c^6 - 2343936a^7b^5c^7 + 3010560a^8b^3c^8 + 4 * 9a^2c^2 * (-4ac - b^2)^{15})^{(1/2)} - 41 * a * b^{17}c - 11 * a * b^2 * c * (-4ac - b^2)^{15})^{(1/2)})) / (512 * (a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18} * c * e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10} * c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13} * b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{(1/2)} + (x * (14112a^4c^7e^{12} + 9 * b^8c^3e^{12} - 180 * a * b^6c^4e^{12} + 1530a^2b^4c^5 * e^{12} - 6192a^3b^2c^6e^{12})) / (32 * (a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3))) * (-9 * (b^{19} + b^4 * (-4ac - b^2)^{15})^{(1/2)} - 1720320a^9b^9 * c^9 + 769a^2b^{15}c^2 - 8620a^3b^{13}c^3 + 63440a^4 * b^{11}c^4 - 316864a^5b^9c^5 + 1069824a^6b^7c^6 - 2343936a^7b^5c^7 + 3010560a^8b^3c^8 + 49a^2c^2 * (-4ac - b^2)^{15})^{(1/2)} - 41 * a * b^{17}c - 11 * a * b^2 * c * (-4ac - b^2)^{15})^{(1/2)})) / (512 * (a^5b^{20}e^2 + 1048576a^{15} * c^{10}e^2 - 40a^6b^{18}c * e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10} * c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13} * b^4c^8e^2 - 2621440a^{14}b^2c^9e^2) \end{aligned}$$

$$\begin{aligned}
& 2*c^9*e^2))^{(1/2)})) * (- (9*(b^{19} + b^4*(-(4*a*c - b^2)^{15})^{(1/2)} - 1720320*a \\
& ^9*b*c^9 + 769*a^2*b^{15}*c^2 - 8620*a^3*b^{13}*c^3 + 63440*a^4*b^{11}*c^4 - 3168 \\
& 64*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 \\
& + 49*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 41*a*b^{17}*c - 11*a*b^2*c*(-(4 \\
& *a*c - b^2)^{15})^{(1/2)})) / (512*(a^5*b^{20}*e^2 + 1048576*a^{15}*c^{10}*e^2 - 40*a^6 \\
& *b^{18}*c*e^2 + 720*a^7*b^{16}*c^2*e^2 - 7680*a^8*b^{14}*c^3*e^2 + 53760*a^9*b^{12} \\
& *c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 + 860160*a^{11}*b^8*c^6*e^2 - 1966080*a^{12} \\
& *b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^8*e^2 - 2621440*a^{14}*b^2*c^9*e^2))^{(1/2)} * 2i - \operatorname{atan}(((9*(b^4*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{19} + 1720320*a^9*b*c^9 \\
& - 769*a^2*b^{15}*c^2 + 8620*a^3*b^{13}*c^3 - 63440*a^4*b^{11}*c^4 + 316864*a^5*b^9*c^5 \\
& - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 41*a*b^{17}*c - 11*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)})) / (512*(a^5*b^{20}*e^2 + 1048576*a^{15}*c^{10}*e^2 - 40*a^6*b^{18}*c \\
& *e^2 + 720*a^7*b^{16}*c^2*e^2 - 7680*a^8*b^{14}*c^3*e^2 + 53760*a^9*b^{12}*c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 + 860160*a^{11}*b^8*c^6*e^2 - 1966080*a^{12}*b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^8*e^2 - 2621440*a^{14}*b^2*c^9*e^2))^{(1/2)} * ((22 \\
& 020096*a^9*c^9*e^{12} - 768*a^2*b^{14}*c^2*e^{12} + 22272*a^3*b^{12}*c^3*e^{12} - 282 \\
& 624*a^4*b^{10}*c^4*e^{12} + 2027520*a^5*b^8*c^5*e^{12} - 8847360*a^6*b^6*c^6*e^{12} \\
& + 23396352*a^7*b^4*c^7*e^{12} - 34603008*a^8*b^2*c^8*e^{12}) / (512*(a^4*b^{12} + \\
& 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((67108864*a^{11}*b*c^9*d*e^{13} - 4096*a^4*b^15*c^2*d*e^{13} + 114688*a^5*b^{13}*c^3*d*e^{13} - 1376256*a^6*b^{11}*c^4*d*e^{13} + 9175040*a^7*b^9*c^5*d*e^{13} - 36700160*a^8*b^7*c^6*d*e^{13} + 88080384*a^9*b^5*c^7*d*e^{13} - 117440512*a^{10}*b^3*c^8*d*e^{13}) / (512*(a^4*b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^{14} - 256*a^4*b^{11}*c^2*e^{14} + 5120*a^5*b^9*c^3*e^{14} - 40960*a^6*b^7*c^4*e^{14} + 163840*a^7*b^5*c^5*e^{14} - 327680*a^8*b^3*c^6*e^{14})) / (32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) * ((9*(b^4*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{19} + 1720320*a^9*b*c^9 - 769*a^2*b^{15}*c^2 + 8620*a^3*b^{13}*c^3 - 63440*a^4*b^{11}*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 41*a*b^{17}*c - 11*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)})) / (512*(a^5*b^{20}*e^2 + 1048576*a^{15}*c^{10}*e^2 - 40*a^6*b^{18}*c*e^2 + 720*a^7*b^{16}*c^2*e^2 - 7680*a^8*b^{14}*c^3*e^2 + 53760*a^9*b^{12}*c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 + 860160*a^{11}*b^8*c^6*e^2 - 1966080*a^{12}*b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^8*e^2 - 2621440*a^{14}*b^2*c^9*e^2))^{(1/2)} * ((9*(b^4*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{19} + 1720320*a^9*b*c^9 - 769*a^2*b^{15}*c^2 + 8620*a^3*b^{13}*c^3 - 63440*a^4*b^{11}*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 41*a*b^{17}*c - 11*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)})) / (512*(a^5*b^{20}*e^2 + 1048576*a^{15}*c^{10}*e^2 - 40*a^6*b^{18}*c*e^2 + 720*a^7*b^{16}*c^2*e^2 - 7680*a^8*b^{14}*c^3*e^2 + 53760*a^9*b^{12}*c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 + 860160*a^{11}*b^8*c^6*e^2 - 1966080*a^{12}*b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^8*e^2 - 2621440*a^{14}*b^2*c^9*e^2))^{(1/2)} + (3612 \\
& 672*a^6*c^9*d*e^{11} + 144*b^{12}*c^3*d*e^{11} - 4032*a*b^{10}*c^4*d*e^{11} + 49824*a
\end{aligned}$$

$$\begin{aligned}
& ^2*b^8*c^5*d*e^{11} - 340992*a^3*b^6*c^6*d*e^{11} + 1410048*a^4*b^4*c^7*d*e^{11} \\
& - 3391488*a^5*b^2*c^8*d*e^{11})/(512*(a^4*b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^{10}* \\
& c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^ \\
& 5)) + (x*(14112*a^4*c^7*e^{12} + 9*b^8*c^3*e^{12} - 180*a*b^6*c^4*e^{12} + 1530*a \\
& ^2*b^4*c^5*e^{12} - 6192*a^3*b^2*c^6*e^{12}))/((32*(a^4*b^8 + 256*a^8*c^4 - 16*a \\
& ^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*i + ((9*(b^4*(-(4*a*c - b^2 \\
&)^15)^{(1/2)} - b^{19} + 1720320*a^9*b*c^9 - 769*a^2*b^{15}*c^2 + 8620*a^3*b^{13}*c \\
& ^3 - 63440*a^4*b^{11}*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 234393 \\
& 6*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^{(1/2)} \\
& + 41*a*b^{17}*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^{(1/2}))/((512*(a^5*b^{20}*e^2 + \\
& 1048576*a^{15}*c^{10}*e^2 - 40*a^6*b^{18}*c*e^2 + 720*a^7*b^{16}*c^2*e^2 - 7680*a^ \\
& 8*b^{14}*c^3*e^2 + 53760*a^9*b^{12}*c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 + 860160 \\
& *a^{11}*b^8*c^6*e^2 - 1966080*a^{12}*b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^8*e^2 - 2 \\
& 621440*a^{14}*b^2*c^9*e^2))^{(1/2)}*((3612672*a^6*c^9*d*e^{11} + 144*b^{12}*c^3*d* \\
& e^{11} - 4032*a*b^{10}*c^4*d*e^{11} + 49824*a^2*b^8*c^5*d*e^{11} - 340992*a^3*b^6*c \\
& ^6*d*e^{11} + 1410048*a^4*b^4*c^7*d*e^{11} - 3391488*a^5*b^2*c^8*d*e^{11})/(512*(\\
& a^4*b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c \\
& ^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((22020096*a^9*c^9*e^{12} - 768* \\
& a^2*b^{14}*c^2*e^{12} + 22272*a^3*b^{12}*c^3*e^{12} - 282624*a^4*b^{10}*c^4*e^{12} + 20 \\
& 27520*a^5*b^8*c^5*e^{12} - 8847360*a^6*b^6*c^6*e^{12} + 23396352*a^7*b^4*c^7*e^ \\
& 12 - 34603008*a^8*b^2*c^8*e^{12}))/((512*(a^4*b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^1 \\
& 0*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2* \\
& c^5)) + ((67108864*a^{11}*b*c^9*d*e^{13} - 4096*a^4*b^{15}*c^2*d*e^{13} + 114688*a^ \\
& 5*b^{13}*c^3*d*e^{13} - 1376256*a^6*b^{11}*c^4*d*e^{13} + 9175040*a^7*b^9*c^5*d*e^{1 \\
& 3 - 36700160*a^8*b^7*c^6*d*e^{13} + 88080384*a^9*b^5*c^7*d*e^{13} - 117440512*a \\
& ^10*b^3*c^8*d*e^{13}))/((512*(a^4*b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^ \\
& 6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(\\
& 262144*a^9*b*c^7*e^{14} - 256*a^4*b^{11}*c^2*e^{14} + 5120*a^5*b^9*c^3*e^{14} - 409 \\
& 60*a^6*b^7*c^4*e^{14} + 163840*a^7*b^5*c^5*e^{14} - 327680*a^8*b^3*c^6*e^{14}))/ \\
& (32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3 \\
&)))*((9*(b^4*(-(4*a*c - b^2)^15)^{(1/2)} - b^{19} + 1720320*a^9*b*c^9 - 769*a^2 \\
& *b^{15}*c^2 + 8620*a^3*b^{13}*c^3 - 63440*a^4*b^{11}*c^4 + 316864*a^5*b^9*c^5 - 1 \\
& 069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2 \\
& *(-(4*a*c - b^2)^15)^{(1/2)} + 41*a*b^{17}*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^{(\\
& 1/2}))/((512*(a^5*b^{20}*e^2 + 1048576*a^{15}*c^{10}*e^2 - 40*a^6*b^{18}*c*e^2 + 720 \\
& *a^7*b^{16}*c^2*e^2 - 7680*a^8*b^{14}*c^3*e^2 + 53760*a^9*b^{12}*c^4*e^2 - 258048 \\
& *a^{10}*b^{10}*c^5*e^2 + 860160*a^{11}*b^8*c^6*e^2 - 1966080*a^{12}*b^6*c^7*e^2 + 2 \\
& 949120*a^{13}*b^4*c^8*e^2 - 2621440*a^{14}*b^2*c^9*e^2))^{(1/2)}*((9*(b^4*(-(4* \\
& a*c - b^2)^15)^{(1/2)} - b^{19} + 1720320*a^9*b*c^9 - 769*a^2*b^{15}*c^2 + 8620*a \\
& ^3*b^{13}*c^3 - 63440*a^4*b^{11}*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 \\
& + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^1 \\
& 5)^{(1/2)} + 41*a*b^{17}*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^{(1/2}))/((512*(a^5*b \\
& ^{20}*e^2 + 1048576*a^{15}*c^{10}*e^2 - 40*a^6*b^{18}*c*e^2 + 720*a^7*b^{16}*c^2*e^2 \\
& - 7680*a^8*b^{14}*c^3*e^2 + 53760*a^9*b^{12}*c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 \\
& + 860160*a^{11}*b^8*c^6*e^2 - 1966080*a^{12}*b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^
\end{aligned}$$

$$\begin{aligned}
& 8e^2 - 2621440a^{14}b^2c^9e^2))^{(1/2)} + (x*(14112a^4c^7e^{12} + 9b^8c^3e^{12} - 180a^6b^2c^4e^{12} + 1530a^2b^4c^5e^{12} - 6192a^3b^2c^6e^{12} - 12)) / (32(a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3)) * i) / ((567b^7c^5e^{10} - 10368a^5b^5c^6e^{10} - 169344a^3b^3c^8e^{10} + 67824a^2b^3c^7e^{10}) / (256(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) + ((9(b^4(-4ac - b^2)^{15})^{(1/2)} - b^{19} + 1720320a^9b^9c^9 - 769a^2b^{15}c^2 + 8620a^3b^{13}c^3 - 63440a^4b^{11}c^4 + 316864a^5b^9c^5 - 1069824a^6b^7c^6 + 2343936a^7b^5c^7 - 3010560a^8b^3c^8 + 49a^2c^2(-4ac - b^2)^{15})^{(1/2)} + 41a^2b^{17}c - 11a^2b^2c(-4ac - b^2)^{15})^{(1/2)}) / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^8e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{(1/2)} * (((22020096a^9c^9e^{12} - 768a^2b^{14}c^2e^{12} + 22272a^3b^{12}c^3e^{12} - 282624a^4b^{10}c^4e^{12} + 2027520a^5b^8c^5e^{12} - 8847360a^6b^6c^6e^{12} + 23396352a^7b^4c^7e^{12} - 34603008a^8b^2c^8e^{12}) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) - ((67108864a^{11}b^9c^9d^13 - 4096a^4b^{15}c^2d^13 + 114688a^5b^{13}c^3d^13 - 1376256a^6b^{11}c^4d^13 + 9175040a^7b^9c^5d^13 - 36700160a^8b^7c^6d^13 + 88080384a^9b^5c^7d^13 - 117440512a^{10}b^3c^8d^13) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240a^6b^8c^2 - 1280a^7b^6c^3 + 3840a^8b^4c^4 - 6144a^9b^2c^5)) + (x*(262144a^9b^9c^7e^{14} - 256a^4b^{11}c^2e^{14} + 5120a^5b^9c^3e^{14} - 40960a^6b^7c^4e^{14} + 163840a^7b^5c^5e^{14} - 327680a^8b^3c^6e^{14}) / (32(a^4b^8 + 256a^8c^4 - 16a^5b^6c + 96a^6b^4c^2 - 256a^7b^2c^3)) * ((9(b^4(-4ac - b^2)^{15})^{(1/2)} - b^{19} + 1720320a^9b^9c^9 - 769a^2b^{15}c^2 + 8620a^3b^{13}c^3 - 63440a^4b^{11}c^4 + 316864a^5b^9c^5 - 1069824a^6b^7c^6 + 2343936a^7b^5c^7 - 3010560a^8b^3c^8 + 49a^2c^2(-4ac - b^2)^{15})^{(1/2)} + 41a^2b^{17}c - 11a^2b^2c(-4ac - b^2)^{15})^{(1/2)}) / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^8e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{(1/2)} * ((9(b^4(-4ac - b^2)^{15})^{(1/2)} - b^{19} + 1720320a^9b^9c^9 - 769a^2b^{15}c^2 + 8620a^3b^{13}c^3 - 63440a^4b^{11}c^4 + 316864a^5b^9c^5 - 1069824a^6b^7c^6 + 2343936a^7b^5c^7 - 3010560a^8b^3c^8 + 49a^2c^2(-4ac - b^2)^{15})^{(1/2)} + 41a^2b^{17}c - 11a^2b^2c(-4ac - b^2)^{15})^{(1/2)}) / (512(a^5b^{20}e^2 + 1048576a^{15}c^{10}e^2 - 40a^6b^{18}c^8e^2 + 720a^7b^{16}c^2e^2 - 7680a^8b^{14}c^3e^2 + 53760a^9b^{12}c^4e^2 - 258048a^{10}b^{10}c^5e^2 + 860160a^{11}b^8c^6e^2 - 1966080a^{12}b^6c^7e^2 + 2949120a^{13}b^4c^8e^2 - 2621440a^{14}b^2c^9e^2))^{(1/2)} + (3612672a^6c^9d^11 + 144b^{12}c^3d^11 - 4032a^2b^{10}c^4d^11 + 49824a^2b^8c^5d^11 - 340992a^3b^6c^6d^11 + 1410048a^4b^4c^7d^11 - 3391488a^5b^2c^8d^11) / (512(a^4b^{12} + 4096a^{10}c^6 - 24a^5b^{10}c + 240
\end{aligned}$$

$$\begin{aligned}
& *a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (\\
& x*(14112*a^4*c^7*e^{12} + 9*b^8*c^3*e^{12} - 180*a*b^6*c^4*e^{12} + 1530*a^2*b^4* \\
& c^5*e^{12} - 6192*a^3*b^2*c^6*e^{12}))/((32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6* \\
& c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))) - ((9*(b^4*(-(4*a*c - b^2)^{15})^{(1/2)} \\
&) - b^{19} + 1720320*a^9*b*c^9 - 769*a^2*b^{15}*c^2 + 8620*a^3*b^{13}*c^3 - 63440 \\
& *a^4*b^{11}*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5* \\
& c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 41*a*b^{17} \\
& *c - 11*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)}))/((512*(a^5*b^{20}*e^2 + 1048576*a \\
& ^{15}*c^{10}*e^2 - 40*a^6*b^{18}*c*e^2 + 720*a^7*b^{16}*c^2*e^2 - 7680*a^8*b^{14}*c^3 \\
& *e^2 + 53760*a^9*b^{12}*c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 + 860160*a^{11}*b^8* \\
& c^6*e^2 - 1966080*a^{12}*b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^8*e^2 - 2621440*a^{14} \\
& *b^2*c^9*e^2)))^{(1/2)}*((3612672*a^6*c^9*d*e^{11} + 144*b^{12}*c^3*d*e^{11} - 403 \\
& 2*a*b^{10}*c^4*d*e^{11} + 49824*a^2*b^8*c^5*d*e^{11} - 340992*a^3*b^6*c^6*d*e^{11} \\
& + 1410048*a^4*b^4*c^7*d*e^{11} - 3391488*a^5*b^2*c^8*d*e^{11}))/((512*(a^4*b^{12} + \\
& 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840* \\
& a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - ((22020096*a^9*c^9*e^{12} - 768*a^2*b^{14}*c \\
& ^2*e^{12} + 22272*a^3*b^{12}*c^3*e^{12} - 282624*a^4*b^{10}*c^4*e^{12} + 2027520*a^5* \\
& b^8*c^5*e^{12} - 8847360*a^6*b^6*c^6*e^{12} + 23396352*a^7*b^4*c^7*e^{12} - 34603 \\
& 008*a^8*b^2*c^8*e^{12}))/((512*(a^4*b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240* \\
& a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + ((\\
& 67108864*a^{11}*b*c^9*d*e^{13} - 4096*a^4*b^{15}*c^2*d*e^{13} + 114688*a^5*b^{13}*c^3 \\
& *d*e^{13} - 1376256*a^6*b^{11}*c^4*d*e^{13} + 9175040*a^7*b^9*c^5*d*e^{13} - 367001 \\
& 60*a^8*b^7*c^6*d*e^{13} + 88080384*a^9*b^5*c^7*d*e^{13} - 117440512*a^{10}*b^3*c^8 \\
& *d*e^{13}))/((512*(a^4*b^{12} + 4096*a^{10}*c^6 - 24*a^5*b^{10}*c + 240*a^6*b^8*c^2 \\
& - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(262144*a^9 \\
& *b*c^7*e^{14} - 256*a^4*b^{11}*c^2*e^{14} + 5120*a^5*b^9*c^3*e^{14} - 40960*a^6*b^7 \\
& *c^4*e^{14} + 163840*a^7*b^5*c^5*e^{14} - 327680*a^8*b^3*c^6*e^{14}))/((32*(a^4*b^8 \\
& + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))) * ((9*(b^4*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{19} + 1720320*a^9*b*c^9 - 769*a^2*b^{15}*c^2 + 8620*a^3*b^{13}*c^3 - 63440*a^4*b^{11}*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 41*a*b^{17} *c - 11*a*b^2*c*(-(4*a*c - b^2)^{15})^{(1/2)}))/((512*(a^5*b^{20}*e^2 + 1048576*a^{15}*c^{10}*e^2 - 40*a^6*b^{18}*c*e^2 + 720*a^7*b^{16}*c^2*e^2 - 7680*a^8*b^{14}*c^3*e^2 + 53760*a^9*b^{12}*c^4*e^2 - 258048*a^{10}*b^{10}*c^5*e^2 + 860160*a^{11}*b^8*c^6*e^2 - 1966080*a^{12}*b^6*c^7*e^2 + 2949120*a^{13}*b^4*c^8*e^2 - 2621440*a^{14}*b^2*c^9*e^2)))^{(1/2)} + (x*(14112*a^4*c^7*e^{12} + 9*b^8*c^3*e^{12} - 180*a*b^6*c^4*e^{12} + 1530*a^2*b^4*c^5*e^{12} - 6192*a^3*b^2*c^6*e^{12}))/((32*(
\end{aligned}$$

$$\begin{aligned} & a^4 b^8 + 256 a^8 c^4 - 16 a^5 b^6 c + 96 a^6 b^4 c^2 - 256 a^7 b^2 c^3) \\ &) * ((9 (b^4 (-4 a c - b^2)^{15})^{1/2} - b^{19} + 1720320 a^9 b c^9 - 769 a^2 b^{15} c^2 \\ & + 8620 a^3 b^{13} c^3 - 63440 a^4 b^{11} c^4 + 316864 a^5 b^9 c^5 - 1069824 a^6 b^7 c^6 \\ & + 2343936 a^7 b^5 c^7 - 3010560 a^8 b^3 c^8 + 49 a^2 c^2 (-4 a c - b^2)^{15})^{1/2} \\ & + 41 a b^{17} c - 11 a b^2 c (-4 a c - b^2)^{15})^{1/2})) / (512 (a^5 b^{20} e^2 + 1048576 a^{15} c^{10} e^2 \\ & - 40 a^6 b^{18} c e^2 + 720 a^7 b^{16} c^2 e^2 - 7680 a^8 b^{14} c^3 e^2 + 53760 a^9 b^{12} c^4 e^2 - 258048 a^{10} b^{10} c^5 e^2 \\ & + 860160 a^{11} b^8 c^6 e^2 - 1966080 a^{12} b^6 c^7 e^2 + 2949120 a^{13} b^4 c^8 e^2 - 2621440 a^{14} b^2 c^9 e^2))^{1/2} * 2i \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out

$$3.522 \quad \int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=255

$$-\frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^3e} + \frac{\log(d+ex)}{a^3e} + \frac{16a^2c^2+2bc(b^2-7ac)(d+ex)^2-15ab^2c+2b^4}{4a^2e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{b(30a^2c^2-10ab^2c+b^4)\operatorname{tanh}^{-1}\left(\frac{b+2c(d+ex)}{\sqrt{b^2-4ac}}\right)}{2a^3e(b^2-4ac)^{5/2}} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^3e} + \frac{\log(d+ex)}{a^3e} + \frac{-2ac+b^2+bc(d+ex)^2}{4ae(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Rubi [A] time = 0.49, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, number of rules / integrand size = 0.300, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bc(b^2-7ac)(d+ex)^2-15ab^2c+2b^4}{4a^2e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{b(30a^2c^2-10ab^2c+b^4)\operatorname{tanh}^{-1}\left(\frac{b+2c(d+ex)}{\sqrt{b^2-4ac}}\right)}{2a^3e(b^2-4ac)^{5/2}} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^3e} + \frac{\log(d+ex)}{a^3e} + \frac{-2ac+b^2+bc(d+ex)^2}{4ae(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2 + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2]/Sqrt[b^2 - 4*a*c])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e) + Log[d + e*x]/(a^3*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^3} dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^3} dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{\text{Subst}\left(\int \frac{-2(b^2-4ac)}{x(a+b(d+ex)^2+c(d+ex)^4)} dx, x, (d+ex)^2\right)}{4a} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+10a^2b^2-10a^2c^2}{4a^2(b^2-4ac)^2} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+10a^2b^2-10a^2c^2}{4a^2(b^2-4ac)^2} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+10a^2b^2-10a^2c^2}{4a^2(b^2-4ac)^2} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+10a^2b^2-10a^2c^2}{4a^2(b^2-4ac)^2} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+10a^2b^2-10a^2c^2}{4a^2(b^2-4ac)^2} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2b^4-15ab^2c+10a^2b^2-10a^2c^2}{4a^2(b^2-4ac)^2}
\end{aligned}$$

Mathematica [A] time = 3.95, size = 391, normalized size = 1.53

$$\frac{a^2(2ac-b^2-4c(d+ex)^2)}{(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{a(16a^2c^2-15ab^2c-14ab^2c(d+ex)^2+2b^4+2b^2c(d+ex)^2)}{(b^2-4ac)^2(a+(d+ex)^2(b+4c(d+ex)^2))} - \frac{(16a^2c^2\sqrt{b^2-4ac}+30a^2b^2c^2-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac})\log(-\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{(b^2-4ac)^{5/2}} + \frac{(-16a^2c^2\sqrt{b^2-4ac}+30a^2b^2c^2-10ab^3c+8ab^2c\sqrt{b^2-4ac}-b^4\sqrt{b^2-4ac})\log(\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{(b^2-4ac)^{5/2}} + 4\log(d+ex)$$

Antiderivative was successfully verified.

$$\begin{aligned}
& 2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8* \\
& a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a \\
& ^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a \\
& ^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2 \\
& *c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - \\
& 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 6 \\
& 0*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3) \\
& *d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^ \\
& 3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3* \\
& (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - \\
& 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2 \\
& *b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^ \\
& 5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b \\
& ^2*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2* \\
& c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e \\
& *x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c \\
&))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(\\
& 2*c*d^3 + b*d)*e*x + a) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a \\
& ^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)* \\
& d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b \\
& ^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(\\
& b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a \\
& *b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4* \\
& c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 \\
& + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c \\
& ^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b \\
& *c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + \\
& 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6* \\
& c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^ \\
& 5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4 \\
& *c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^ \\
& 2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 4 \\
& 8*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 \\
& - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^ \\
& 4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^ \\
& 4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)* \\
& d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(\\
& b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6 \\
& *c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b \\
& ^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 \\
& + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(\\
& (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 \\
& - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a* \\
& b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a \\
& ^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*
\end{aligned}$$

$$\begin{aligned}
& a^2 b^2 c^4 - 64 a^3 c^5) d^3 + 3(b^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - \\
& 64 a^3 b c^4) d) e^5 x^5 + (b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 \\
& 3 c^5) d^8 + (b^8 - 10 a b^6 c + 24 a^2 b^4 c^2 + 32 a^3 b^2 c^3 - 128 a^4 c^4 \\
& c^4 + 70(b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) d^4 + 30(b \\
& ^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - 64 a^3 b c^4) d^2) e^4 x^4 + a^2 b^6 \\
& - 12 a^3 b^4 c + 48 a^4 b^2 c^2 - 64 a^5 c^3 + 2(b^7 c - 12 a b^5 c^2 + 4 \\
& 8 a^2 b^3 c^3 - 64 a^3 b c^4) d^6 + 4(14(b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - \\
& 64 a^3 c^5) d^5 + 10(b^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - 64 a^3 b c^4) d^3 + (b^8 - 10 a b^6 c + 24 a^2 b^4 c^2 + 32 a^3 b^2 c^3 - 128 a^4 c^4) d) e^3 x^3 + (b^8 - 10 a b^6 c + 24 a^2 b^4 c^2 + 32 a^3 b^2 c^3 - 128 a^4 c^4) d^4 + 2(a b^7 - 12 a^2 b^5 c + 48 a^3 b^3 c^2 - 64 a^4 b c^3 + 14(b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) d^6 + 15(b^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - 64 a^3 b c^4) d^4 + 3(b^8 - 10 a b^6 c + 24 a^2 b^4 c^2 + 32 a^3 b^2 c^3 - 128 a^4 c^4) d^2) e^2 x^2 + 2(a b^7 - 12 a^2 b^5 c + 48 a^3 b^3 c^2 - 64 a^4 b c^3) d^2 + 4(2(b^6 c^2 - 12 a b^4 c^3 + 48 a^2 b^2 c^4 - 64 a^3 c^5) d^7 + 3(b^7 c - 12 a b^5 c^2 + 48 a^2 b^3 c^3 - 64 a^3 b c^4) d^5 + (b^8 - 10 a b^6 c + 24 a^2 b^4 c^2 + 32 a^3 b^2 c^3 - 128 a^4 c^4) d^3 + (a b^7 - 12 a^2 b^5 c + 48 a^3 b^3 c^2 - 64 a^4 b c^3) d) e x) \log(e x + d) / ((a^3 b^6 c^2 - 12 a^4 b^4 c^3 + 48 a^5 b^2 c^4 - 64 a^6 c^5) e^9 x^8 + 8(a^3 b^6 c^2 - 12 a^4 b^4 c^3 + 48 a^5 b^2 c^4 - 64 a^6 c^5) d e^8 x^7 + 2(a^3 b^7 c - 12 a^4 b^5 c^2 + 48 a^5 b^3 c^3 - 64 a^6 b c^4 + 14(a^3 b^6 c^2 - 12 a^4 b^4 c^3 + 48 a^5 b^2 c^4 - 64 a^6 c^5) d^2) e^7 x^6 + 4(14(a^3 b^6 c^2 - 12 a^4 b^4 c^3 + 48 a^5 b^2 c^4 - 64 a^6 c^5) d^3 + 3(a^3 b^7 c - 12 a^4 b^5 c^2 + 48 a^5 b^3 c^3 - 64 a^6 b c^4) d) e^6 x^5 + (a^3 b^8 - 10 a^4 b^6 c + 24 a^5 b^4 c^2 + 32 a^6 b^2 c^3 - 128 a^7 c^4 + 70(a^3 b^6 c^2 - 12 a^4 b^4 c^3 + 48 a^5 b^2 c^4 - 64 a^6 c^5) d^4 + 30(a^3 b^7 c - 12 a^4 b^5 c^2 + 48 a^5 b^3 c^3 - 64 a^6 b c^4) d^2) e^5 x^4 + 4(14(a^3 b^6 c^2 - 12 a^4 b^4 c^3 + 48 a^5 b^2 c^4 - 64 a^6 c^5) d^5 + 10(a^3 b^7 c - 12 a^4 b^5 c^2 + 48 a^5 b^3 c^3 - 64 a^6 b c^4) d^3 + (a^3 b^8 - 10 a^4 b^6 c + 24 a^5 b^4 c^2 + 32 a^6 b^2 c^3 - 128 a^7 c^4) d) e^4 x^3 + 2(a^4 b^7 - 12 a^5 b^5 c + 48 a^6 b^3 c^2 - 64 a^7 b c^3 + 14(a^3 b^6 c^2 - 12 a^4 b^4 c^3 + 48 a^5 b^2 c^4 - 64 a^6 c^5) d^6 + 15(a^3 b^7 c - 12 a^4 b^5 c^2 + 48 a^5 b^3 c^3 - 64 a^6 b c^4) d^4 + 3(a^3 b^8 - 10 a^4 b^6 c + 24 a^5 b^4 c^2 + 32 a^6 b^2 c^3 - 128 a^7 c^4) d^2) e^3 x^2 + 4(2(a^3 b^6 c^2 - 12 a^4 b^4 c^3 + 48 a^5 b^2 c^4 - 64 a^6 c^5) d^7 + 3(a^3 b^7 c - 12 a^4 b^5 c^2 + 48 a^5 b^3 c^3 - 64 a^6 b c^4) d^5 + (a^3 b^8 - 10 a^4 b^6 c + 24 a^5 b^4 c^2 + 32 a^6 b^2 c^3 - 128 a^7 c^4) d^3 + (a^4 b^7 - 12 a^5 b^5 c + 48 a^6 b^3 c^2 - 64 a^7 b c^3) d) e^2 x + (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3 + (a^3 b^6 c^2 - 12 a^4 b^4 c^3 + 48 a^5 b^2 c^4 - 64 a^6 c^5) d^8 + 2(a^3 b^7 c - 12 a^4 b^5 c^2 + 48 a^5 b^3 c^3 - 64 a^6 b c^4) d^6 + (a^3 b^8 - 10 a^4 b^6 c + 24 a^5 b^4 c^2 + 32 a^6 b^2 c^3 - 128 a^7 c^4) d^4 + 2(a^4 b^7 - 12 a^5 b^5 c + 48 a^6 b^3 c^2 - 64 a^7 b c^3) d^2) e), 1/4(2(a b^5 c^2 - 11 a^2 b^3 c^3 + 28 a^3 b c^4) e^6 x^6 + 12(a b^5 c^2 - 11 a^2 b^3 c^3 + 28 a^3 b c^4) d e^5 x^5 + (4 a b^6 c - 45 a^2 b^4 c^2 + 132 a^3 b^2 c^3 - 64 a^4 c^4 + 30(
\end{aligned}$$

$$\begin{aligned}
& a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + 2*((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*sqrt(-b^2 + 4*a*c) *arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6
\end{aligned}$$

$$\begin{aligned}
& + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + \\
& 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*\log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*\log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*e^9*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d*e^8*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^2)*e^7*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d)*e^6*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2)*e^5*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d)*e^4*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^4 + 3*(a^3*b^8 - 10*a^4
\end{aligned}$$

```

*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^2)*e^3*x^2 + 4*(2
*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*
b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^5 + (a^3*b^8 - 10
*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^3 + (a^4*b^7
- 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d)*e^2*x + (a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*
a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*
c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6
*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 -
64*a^7*b*c^3)*d^2)*e)]

```

giac [B] time = 1.72, size = 1012, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*((a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*
c^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c
)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a
)) - (a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*c
^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c
)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a
)))/(a^6*b^8*c*e^4 - 16*a^7*b^6*c^2*e^4 + 96*a^8*b^4*c^3*e^4 - 256*a^9*b^2*
c^4*e^4 + 256*a^10*c^5*e^4) - 1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3
+ 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a
))/a^3 + e^(-1)*log(abs(x*e + d))/a^3 + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3
*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 -
12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^
6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c
^3*d*e^5)*x^5 + (30*a*b^3*c^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e
^4 - 29*a^2*b^2*c^2*e^4 + 16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 7
0*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d
*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^
2*e^2 - 87*a^2*b^2*c^2*d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3
*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4
*a*b^4*c*d^3*e - 29*a^2*b^2*c^2*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^
2*b^3*c*d*e - a^3*b*c^2*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^
2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2
- 4*a*c)^2*a^3)
```

maple [C] time = 0.08, size = 4477, normalized size = 17.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)$

[Out]
$$-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/e*\text{sum}((c*e^3*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c^2*d^3+b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*\ln(-_R+x),_R=\text{RootOf}(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3*c-21/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c+3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c^2-29/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2+4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^4*c-6/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3*c-7/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c^2*d^6+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c^2*d^4+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4*c*d^4-3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^3*c*d^2-29/4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2-7/2/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^c^2-1/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^c^2-1/2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^c^2*d^2+1/2/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^5+1/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b$$

$$\begin{aligned}
& e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 / e / (16 a^2 c^2 - 8 a b^2 c + b^4) b^5 d^2 + 1 / \\
& a^2 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b \\
& d e x + b d^2 + a)^2 e^3 c / (16 a^2 c^2 - 8 a b^2 c + b^4) x^4 b^4 + 16 / (c e^4 x^4 + 4 \\
& c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) \\
& ^2 c^3 d e^2 / (16 a^2 c^2 - 8 a b^2 c + b^4) x^3 + 24 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c \\
& d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 e / (16 a^2 c^2 \\
& - 8 a b^2 c + b^4) x^2 c^3 d^2 + 6 a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c \\
& d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 / e / (16 a^2 c^2 - 8 a b^2 c + b^4) \\
& c^2 + 3 / 4 / a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c \\
& d^4 + 2 b d e x + b d^2 + a)^2 / e / (16 a^2 c^2 - 8 a b^2 c + b^4) b^4 + 4 / (c e^4 x^4 + 4 c \\
& d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 \\
& e^3 c^3 / (16 a^2 c^2 - 8 a b^2 c + b^4) x^4 + 16 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 \\
& e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 d^3 / (16 a^2 c^2 - 8 \\
& a b^2 c + b^4) x x c^3 + 4 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + \\
& b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 / e / (16 a^2 c^2 - 8 a b^2 c + b^4) c^3 d^4 - 2 \\
& 1 / 4 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b \\
& d e x + b d^2 + a)^2 / e / (16 a^2 c^2 - 8 a b^2 c + b^4) b^2 c + \ln(e x + d) / a^3 / e - 21 / a / \\
& (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e \\
& x + b d^2 + a)^2 b c^3 d e^4 / (16 a^2 c^2 - 8 a b^2 c + b^4) x^5 + 3 / a^2 / (c e^4 x^4 + \\
& 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a \\
&)^2 b^3 c^2 d e^4 / (16 a^2 c^2 - 8 a b^2 c + b^4) x^5 - 105 / 2 / a / (c e^4 x^4 + 4 c d e \\
& ^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 e^3 \\
& c^3 / (16 a^2 c^2 - 8 a b^2 c + b^4) x^4 b d^2 + 15 / 2 / a^2 / (c e^4 x^4 + 4 c d e^3 x^3 \\
& + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 e^3 c^2 / (\\
& 16 a^2 c^2 - 8 a b^2 c + b^4) x^4 b^3 d^2 - 70 / a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 \\
& e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 c^3 d^3 e^2 / (16 a \\
& ^2 c^2 - 8 a b^2 c + b^4) x^3 b + 10 / a^2 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 \\
& + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 c^2 d^3 e^2 / (16 a^2 c^2 - 8 \\
& a b^2 c + b^4) x^3 b^3 - 29 / a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 \\
& e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 c^2 d e^2 / (16 a^2 c^2 - 8 a b^2 c + b \\
& ^4) x^3 b^2 + 4 / a^2 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^ \\
& 2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 c d e^2 / (16 a^2 c^2 - 8 a b^2 c + b^4) x^3 b^4 \\
& - 105 / 2 / a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d \\
& ^4 + 2 b d e x + b d^2 + a)^2 e / (16 a^2 c^2 - 8 a b^2 c + b^4) x^2 b c^3 d^4 + 15 / 2 / a^2 \\
& / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d \\
& e x + b d^2 + a)^2 e / (16 a^2 c^2 - 8 a b^2 c + b^4) x^2 b^3 c^2 d^4 - 87 / 2 / a / (c e^4 x \\
& ^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d \\
& ^2 + a)^2 e / (16 a^2 c^2 - 8 a b^2 c + b^4) x^2 b^2 c^2 d^2 + 6 / a^2 / (c e^4 x^4 + 4 c d \\
& e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a)^2 e \\
& / (16 a^2 c^2 - 8 a b^2 c + b^4) x^2 b^4 c d^2
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 17.98, size = 19440, normalized size = 76.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)

[Out]
$$\begin{aligned} & ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e)) / (2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3)) / (4*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2)) / (a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4)) / (a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5)) / (2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5)) / (a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 - 21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6) / (4*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) / (x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d^3*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d^3*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d^5*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d^7*x^7) + \log(d + e*x) / (a^3*e) - (\log((((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))^2) / (a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1) * (((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))^2) / (a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1) * ((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2)) / (a^2*(4*a*c - b^2)^2) + (b*c^2*e^16*(a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))^2) / (a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1) * (a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x)) / a^3 + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c)) / (a^2*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c)) / (a^2*(4*a*c - b^2)^2))) / (4*a^3*e) + (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2)) / (a^4*(4*a*c - b^2)^4) - (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c)) / (a^4*(4*a*c - b^2)^4) - (2*b*c^4*d*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c)) / (a^4*(4*a*c - b^2)^4)) / (4*a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3) / (a^6*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c$$

$$\begin{aligned}
& \cdot 2*d^2)) / (a^6*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c - b^2)^3) / (a^6* \\
& (4*a*c - b^2)^6) * (((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2) / (a^6*e \\
& ^2*(4*a*c - b^2)^5))^{(1/2)} - 1) * (((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^ \\
& 2*c)^2) / (a^6*e^2*(4*a*c - b^2)^5))^{(1/2)} - 1) * ((2*b*c^2*e^16*(2*b^5 + 46*a^ \\
& 2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2)) / (a^2* \\
& (4*a*c - b^2)^2) - (b*c^2*e^16*(a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c \\
&)^2) / (a^6*e^2*(4*a*c - b^2)^5))^{(1/2)} - 1) * (a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 \\
& - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x)) / a^3 + (2*b*c^ \\
& 3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c)) / (a^2*(4*a*c - b^2)^2) + (4*b*c^3 \\
& *d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c)) / (a^2*(4*a*c - b^2)^2))) / (4*a^3*e) \\
& - (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c \\
& ^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2)) / (a^4*(4*a*c - b^2)^4) + (b*c^4*e^1 \\
& 7*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c)) / (a^4*(4*a*c - b \\
& ^2)^4) + (2*b*c^4*d*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^ \\
& 4*c)) / (a^4*(4*a*c - b^2)^4)) / (4*a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3 \\
&) / (a^6*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + \\
& b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c^2*d^2)) / (a^6*(4*a*c - b^2)^6) - (2*b^3*c^5 \\
& *d*e^15*x*(7*a*c - b^2)^3) / (a^6*(4*a*c - b^2)^6))) * (2*b^10*e - 2048*a^5*c^5 \\
& *e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8 \\
& *c*e)) / (2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b \\
& ^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*atan((x*((\\
& ((b*((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4* \\
& d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^ \\
& 7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17)) / (a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^1 \\
& 0*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^ \\
& 2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3 \\
& *e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e) * (163840*a^13*b*c^9*d*e^18 - 12*a^6* \\
& b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 2496 \\
& 0*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^ \\
& 18 - 294912*a^12*b^3*c^8*d*e^18)) / ((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80* \\
& a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c \\
& ^4*e^2) * (a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280* \\
& a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5))) * (b^4 + 30*a^2*c^2 - \\
& 10*a*b^2*c)) / (4*a^3*e*(4*a*c - b^2)^{(5/2)}) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^ \\
& 2*c) * (2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + \\
& 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e) * (163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15* \\
& c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9 \\
& *b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - \\
& 294912*a^12*b^3*c^8*d*e^18)) / (4*a^3*e*(4*a*c - b^2)^{(5/2)}) * (4*a^3*b^10*e^2 - \\
& 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c \\
& ^3*e^2 + 5120*a^7*b^2*c^4*e^2) * (a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + \\
& 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) \\
&)) * (2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 25 \\
& 60*a^4*b^2*c^4*e - 40*a*b^8*c*e)) / (2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 8 \\
& 0*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2
\end{aligned}$$

$$\begin{aligned}
& *c^4*e^2)) + (b*((2*(8960*a^7*b*c^9*d*e^16 - 6*a^2*b^11*c^4*d*e^16 + 137*a^3*b^9*c^5*d*e^16 - 1217*a^4*b^7*c^6*d*e^16 + 5256*a^5*b^5*c^7*d*e^16 - 1102 \\
& 4*a^6*b^3*c^8*d*e^16)))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2* \\
& (5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 \\
& - 6144*a^9*b^3*c^8*d*e^17)))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240 \\
& *a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - \\
& ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560 \\
& *a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d \\
& *e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9 \\
& *c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 2949 \\
& 12*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c \\
& *e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(\\
& a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c \\
& ^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(2*b^10*e - 2048*a^5*c^5*e + \\
& 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e) \\
&)/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^ \\
& 2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(b^4 + 30*a^2*c^2 - \\
& 10*a*b^2*c))/(4*a^3*e*(4*a*c - b^2)^(5/2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a* \\
& b^2*c)^3*(163840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13* \\
& c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^ \\
& 10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18 \\
&))/(32*a^9*e^3*(4*a*c - b^2)^(15/2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10 \\
& *c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2 \\
& *c^5)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6 \\
& *c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c \\
& ^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((2*(b^9*c^ \\
& 5*d*e^15 - 21*a*b^7*c^6*d*e^15 + 147*a^2*b^5*c^7*d*e^15 - 343*a^3*b^3*c^8*d \\
& *e^15)))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280* \\
& a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*(8960*a^7*b*c^9 \\
& *d*e^16 - 6*a^2*b^11*c^4*d*e^16 + 137*a^3*b^9*c^5*d*e^16 - 1217*a^4*b^7*c^6 \\
& *d*e^16 + 5256*a^5*b^5*c^7*d*e^16 - 11024*a^6*b^3*c^8*d*e^16)))/(a^6*b^12 + \\
& 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a \\
& ^10*b^4*c^4 - 6144*a^11*b^2*c^5) + (((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^1 \\
& 3*c^3*d*e^17 - 36*a^5*b^11*c^4*d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b \\
& ^7*c^6*d*e^17 + 3456*a^8*b^5*c^7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17)))/(a^6*b^ \\
& 12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3 \\
& 840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a \\
& ^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163 \\
& 840*a^13*b*c^9*d*e^18 - 12*a^6*b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - \\
& 3840*a^8*b^11*c^4*d*e^18 + 24960*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d* \\
& e^18 + 227328*a^11*b^5*c^7*d*e^18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^ \\
& 10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a \\
& ^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b
\end{aligned}$$

$$\begin{aligned}
& ^{10}c + 240a^8b^8c^2 - 1280a^9b^6c^3 + 3840a^{10}b^4c^4 - 6144a^{11}b^2c^5)) \cdot (2b^{10}e - 2048a^5c^5e + 320a^2b^6c^2e - 1280a^3b^4c^3e + 2560a^4b^2c^4e - 40ab^8c^3e)) / (2(4a^3b^{10}e^2 - 4096a^8c^5e^2 - 80a^4b^8c^3e^2 + 640a^5b^6c^2e^2 - 2560a^6b^4c^3e^2 + 5120a^7b^2c^4e^2)) \cdot (2b^{10}e - 2048a^5c^5e + 320a^2b^6c^2e - 1280a^3b^4c^3e + 2560a^4b^2c^4e - 40ab^8c^3e)) / (2(4a^3b^{10}e^2 - 4096a^8c^5e^2 - 80a^4b^8c^3e^2 + 640a^5b^6c^2e^2 - 2560a^6b^4c^3e^2 + 5120a^7b^2c^4e^2)) - (b((b((2(5120a^{10}b^3c^9d^5e^{17} + 2a^4b^{13}c^3d^5e^{17} - 36a^5b^{11}c^4d^5e^{17} + 276a^6b^9c^5d^5e^{17} - 1216a^7b^7c^6d^5e^{17} + 3456a^8b^5c^7d^5e^{17} - 6144a^9b^3c^8d^5e^{17}))) / (a^6b^{12} + 4096a^{12}c^6 - 24a^7b^{10}c + 240a^8b^8c^2 - 1280a^9b^6c^3 + 3840a^{10}b^4c^4 - 6144a^{11}b^2c^5) - ((2b^{10}e - 2048a^5c^5e + 320a^2b^6c^2e - 1280a^3b^4c^3e + 2560a^4b^2c^4e - 40ab^8c^3e)) \cdot (163840a^{13}b^3c^9d^5e^{18} - 12a^6b^{15}c^2d^5e^{18} + 328a^7b^{13}c^3d^5e^{18} - 3840a^8b^{11}c^4d^5e^{18} + 24960a^9b^9c^5d^5e^{18} - 97280a^{10}b^7c^6d^5e^{18} + 227328a^{11}b^5c^7d^5e^{18} - 294912a^{12}b^3c^8d^5e^{18}))) / ((4a^3b^{10}e^2 - 4096a^8c^5e^2 - 80a^4b^8c^3e^2 + 640a^5b^6c^2e^2 - 2560a^6b^4c^3e^2 + 5120a^7b^2c^4e^2)) \cdot (a^6b^{12} + 4096a^{12}c^6 - 24a^7b^{10}c + 240a^8b^8c^2 - 1280a^9b^6c^3 + 3840a^{10}b^4c^4 - 6144a^{11}b^2c^5)) \cdot (b^4 + 30a^2c^2 - 10ab^2c)) / (4a^3e(4ac - b^2)^{(5/2)}) - (b(b^4 + 30a^2c^2 - 10ab^2c)) \cdot (2b^{10}e - 2048a^5c^5e + 320a^2b^6c^2e - 1280a^3b^4c^3e + 2560a^4b^2c^4e - 40ab^8c^3e)) \cdot (163840a^{13}b^3c^9d^5e^{18} - 12a^6b^{15}c^2d^5e^{18} + 328a^7b^{13}c^3d^5e^{18} - 3840a^8b^{11}c^4d^5e^{18} + 24960a^9b^9c^5d^5e^{18} - 97280a^{10}b^7c^6d^5e^{18} + 227328a^{11}b^5c^7d^5e^{18} - 294912a^{12}b^3c^8d^5e^{18}))) / (4a^3e(4ac - b^2)^{(5/2)) \cdot (4a^3b^{10}e^2 - 4096a^8c^5e^2 - 80a^4b^8c^3e^2 + 640a^5b^6c^2e^2 - 2560a^6b^4c^3e^2 + 5120a^7b^2c^4e^2)) \cdot (a^6b^{12} + 4096a^{12}c^6 - 24a^7b^{10}c + 240a^8b^8c^2 - 1280a^9b^6c^3 + 3840a^{10}b^4c^4 - 6144a^{11}b^2c^5)) \cdot (b^4 + 30a^2c^2 - 10ab^2c)) / (4a^3e(4ac - b^2)^{(5/2)}) + (b^2(b^4 + 30a^2c^2 - 10ab^2c))^2 \cdot (2b^{10}e - 2048a^5c^5e + 320a^2b^6c^2e - 1280a^3b^4c^3e + 2560a^4b^2c^4e - 40ab^8c^3e)) \cdot (163840a^{13}b^3c^9d^5e^{18} - 12a^6b^{15}c^2d^5e^{18} + 328a^7b^{13}c^3d^5e^{18} - 3840a^8b^{11}c^4d^5e^{18} + 24960a^9b^9c^5d^5e^{18} - 97280a^{10}b^7c^6d^5e^{18} + 227328a^{11}b^5c^7d^5e^{18} - 294912a^{12}b^3c^8d^5e^{18}))) / (16a^6e^2(4ac - b^2)^{5/2} \cdot (4a^3b^{10}e^2 - 4096a^8c^5e^2 - 80a^4b^8c^3e^2 + 640a^5b^6c^2e^2 - 2560a^6b^4c^3e^2 + 5120a^7b^2c^4e^2)) \cdot (a^6b^{12} + 4096a^{12}c^6 - 24a^7b^{10}c + 240a^8b^8c^2 - 1280a^9b^6c^3 + 3840a^{10}b^4c^4 - 6144a^{11}b^2c^5)) \cdot (b^6 - 45a^3c^3 + 40a^2b^2c^2 - 11ab^4c)) / (8a^3c^2(4ac - b^2)^6(6b^{10} - 6400a^5c^5 + 960a^2b^6c^2 - 3850a^3b^4c^3 + 7775a^4b^2c^4 - 120ab^8c)) \cdot (16a^9b^{12}(4ac - b^2)^{(15/2)} + 65536a^{15}c^6(4ac - b^2)^{(15/2)} - 384a^{10}b^{10}c(4ac - b^2)^{(15/2)} + 3840a^{11}b^8c^2(4ac - b^2)^{(15/2)} - 20480a^{12}b^6c^3(4ac - b^2)^{(15/2)} + 61440a^{13}b^4c^4(4ac - b^2)^{(15/2)} - 98304a^{14}b^2c^5(4ac - b^2)^{(15/2}))) / (b^{10}c^2e^{14} - 20ab^8c^3e^{14} + 160a^2b^6c^4e^{14} - 600a^3b^4c^5e^{14} + 900
\end{aligned}$$

$$\begin{aligned}
& *a^4*b^2*c^6*e^{14}) + (x^2*(((b*((5120*a^{10}*b*c^9*e^{18} + 2*a^4*b^{13}*c^3*e^{18} \\
& - 36*a^5*b^{11}*c^4*e^{18} + 276*a^6*b^9*c^5*e^{18} - 1216*a^7*b^7*c^6*e^{18} + \\
& 3456*a^8*b^5*c^7*e^{18} - 6144*a^9*b^3*c^8*e^{18))/(a^6*b^{12} + 4096*a^{12}*c^6 - \\
& 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6 \\
& 144*a^{11}*b^2*c^5) - ((2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^{13}*b*c^9*e^{19} \\
& - 12*a^6*b^{15}*c^2*e^{19} + 328*a^7*b^{13}*c^3*e^{19} - 3840*a^8*b^{11}*c^4*e^{19} + 2 \\
& 4960*a^9*b^9*c^5*e^{19} - 97280*a^{10}*b^7*c^6*e^{19} + 227328*a^{11}*b^5*c^7*e^{19} \\
& - 294912*a^{12}*b^3*c^8*e^{19}))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4 \\
& *b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4* \\
& e^2)*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9 \\
& *b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5)))*(b^4 + 30*a^2*c^2 - 10* \\
& a*b^2*c))/((4*a^3*e*(4*a*c - b^2)^{(5/2)}) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c \\
&)*(2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 256 \\
& 0*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^{13}*b*c^9*e^{19} - 12*a^6*b^{15}*c^2*e \\
& ^{19} + 328*a^7*b^{13}*c^3*e^{19} - 3840*a^8*b^{11}*c^4*e^{19} + 24960*a^9*b^9*c^5*e^{19} \\
& - 97280*a^{10}*b^7*c^6*e^{19} + 227328*a^{11}*b^5*c^7*e^{19} - 294912*a^{12}*b^3*c^8 \\
& *e^{19}))/((8*a^3*e*(4*a*c - b^2)^{(5/2})*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - \\
& 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4* \\
& e^2)*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1 \\
& 280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5)))*(2*b^{10}*e - 2048 \\
& *a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - \\
& 40*a*b^8*c*e))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 6 \\
& 40*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) + (b*((8 \\
& 960*a^7*b*c^9*e^{17} - 6*a^2*b^{11}*c^4*e^{17} + 137*a^3*b^9*c^5*e^{17} - 1217*a^4* \\
& b^7*c^6*e^{17} + 5256*a^5*b^5*c^7*e^{17} - 11024*a^6*b^3*c^8*e^{17}))/((a^6*b^{12} + \\
& 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a \\
& ^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5) + (((5120*a^{10}*b*c^9*e^{18} + 2*a^4*b^{13}*c^3 \\
& *e^{18} - 36*a^5*b^{11}*c^4*e^{18} + 276*a^6*b^9*c^5*e^{18} - 1216*a^7*b^7*c^6*e^{18} \\
& + 3456*a^8*b^5*c^7*e^{18} - 6144*a^9*b^3*c^8*e^{18}))/((a^6*b^{12} + 4096*a^{12}*c^6 \\
& - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - \\
& 6144*a^{11}*b^2*c^5) - ((2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 128 \\
& 0*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^{13}*b*c^9*e^{19} \\
& - 12*a^6*b^{15}*c^2*e^{19} + 328*a^7*b^{13}*c^3*e^{19} - 3840*a^8*b^{11}*c^4*e^{19} + \\
& 24960*a^9*b^9*c^5*e^{19} - 97280*a^{10}*b^7*c^6*e^{19} + 227328*a^{11}*b^5*c^7*e^{19} \\
& - 294912*a^{12}*b^3*c^8*e^{19}))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4 \\
& *b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4* \\
& e^2)*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9 \\
& *b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5)))*(2*b^{10}*e - 2048*a^5* \\
& c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a* \\
& b^8*c*e))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5 \\
& *b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(b^4 + 30*a^2 \\
& *c^2 - 10*a*b^2*c))/((4*a^3*e*(4*a*c - b^2)^{(5/2)}) + (b^3*(b^4 + 30*a^2*c^2 \\
& - 10*a*b^2*c))^3*(163840*a^{13}*b*c^9*e^{19} - 12*a^6*b^{15}*c^2*e^{19} + 328*a^7*b \\
& ^{13}*c^3*e^{19} - 3840*a^8*b^{11}*c^4*e^{19} + 24960*a^9*b^9*c^5*e^{19} - 97280*a^{10}
\end{aligned}$$

$$\begin{aligned}
& *b^7*c^6*e^{19} + 227328*a^{11}*b^5*c^7*e^{19} - 294912*a^{12}*b^3*c^8*e^{19}))/((64*a^9*e^3*(4*a*c - b^2)^{(15/2)}*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5))) * \\
& (3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c))/((8*a^3*c^2*(4*a*c - b^2)^{(13/2)}*(6*b^{10} - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) + (3*b*((b^9*c^5*e^{16} - 21*a*b^7*c^6*e^{16} + 147*a^2*b^5*c^7*e^{16} - 343*a^3*b^3*c^8*e^{16}))/((a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5) + ((8960*a^7*b*c^9*e^{17} - 6*a^2*b^{11}*c^4*e^{17} + 137*a^3*b^9*c^5*e^{17} - 1217*a^4*b^7*c^6*e^{17} + 5256*a^5*b^5*c^7*e^{17} - 11024*a^6*b^3*c^8*e^{17}))/((a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5) + ((5120*a^{10}*b*c^9*e^{18} + 2*a^4*b^{13}*c^3*e^{18} - 36*a^5*b^{11}*c^4*e^{18} + 276*a^6*b^9*c^5*e^{18} - 1216*a^7*b^7*c^6*e^{18} + 3456*a^8*b^5*c^7*e^{18} - 6144*a^9*b^3*c^8*e^{18}))/((a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5) - ((2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^{13}*b*c^9*e^{19} - 12*a^6*b^{15}*c^2*e^{19} + 328*a^7*b^{13}*c^3*e^{19} - 3840*a^8*b^{11}*c^4*e^{19} + 24960*a^9*b^9*c^5*e^{19} - 97280*a^{10}*b^7*c^6*e^{19} + 227328*a^{11}*b^5*c^7*e^{19} - 294912*a^{12}*b^3*c^8*e^{19}))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5))) * (2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) * (2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*((b*((5120*a^{10}*b*c^9*e^{18} + 2*a^4*b^{13}*c^3*e^{18} - 36*a^5*b^{11}*c^4*e^{18} + 276*a^6*b^9*c^5*e^{18} - 1216*a^7*b^7*c^6*e^{18} + 3456*a^8*b^5*c^7*e^{18} - 6144*a^9*b^3*c^8*e^{18}))/((a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5) - ((2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^{13}*b*c^9*e^{19} - 12*a^6*b^{15}*c^2*e^{19} + 328*a^7*b^{13}*c^3*e^{19} - 3840*a^8*b^{11}*c^4*e^{19} + 24960*a^9*b^9*c^5*e^{19} - 97280*a^{10}*b^7*c^6*e^{19} + 227328*a^{11}*b^5*c^7*e^{19} - 294912*a^{12}*b^3*c^8*e^{19}))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5))) * (b^4 + 30*a^2*c^2 - 10*a*b^2*c))/((4*a^3*e*(4*a*c - b^2)^{(5/2)}) - (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^{13}*b*c^9*e^{19} - 12*a^6*b^{15}*c^2*e^{19} + 328*a^7*b^{13}*c^3*e^{19} - 3840*a^8*b^{11}*c^4*e^{19} + 24960*a^9*
\end{aligned}$$

$$\begin{aligned}
& b^9 c^5 e^{19} - 97280 a^{10} b^7 c^6 e^{19} + 227328 a^{11} b^5 c^7 e^{19} - 294912 a^{12} b^3 c^8 e^{19}) / (8 a^3 e (4 a c - b^2)^{(5/2)} (4 a^3 b^{10} e^2 - 4096 a^8 c^5 e^2 - 80 a^4 b^8 c e^2 + 640 a^5 b^6 c^2 e^2 - 2560 a^6 b^4 c^3 e^2 + 5120 a^7 b^2 c^4 e^2) (a^6 b^{12} + 4096 a^{12} c^6 - 24 a^7 b^{10} c + 240 a^8 b^8 c^2 - 1280 a^9 b^6 c^3 + 3840 a^{10} b^4 c^4 - 6144 a^{11} b^2 c^5))) (b^4 + 30 a^2 c^2 - 10 a b^2 c) / (4 a^3 e (4 a c - b^2)^{(5/2)}) + (b^2 (b^4 + 30 a^2 c^2 - 10 a b^2 c)^2 (2 b^{10} e - 2048 a^5 c^5 e + 320 a^2 b^6 c^2 e - 1280 a^3 b^4 c^3 e + 2560 a^4 b^2 c^4 e - 40 a b^8 c e) * (163840 a^{13} b^9 c^9 e^{19} - 12 a^6 b^{15} c^2 e^{19} + 328 a^7 b^{13} c^3 e^{19} - 3840 a^8 b^{11} c^4 e^{19} + 24960 a^9 b^9 c^5 e^{19} - 97280 a^{10} b^7 c^6 e^{19} + 227328 a^{11} b^5 c^7 e^{19} - 294912 a^{12} b^3 c^8 e^{19})) / (32 a^6 e^2 (4 a c - b^2)^5 (4 a^3 b^{10} e^2 - 4096 a^8 c^5 e^2 - 80 a^4 b^8 c e^2 + 640 a^5 b^6 c^2 e^2 - 2560 a^6 b^4 c^3 e^2 + 5120 a^7 b^2 c^4 e^2) (a^6 b^{12} + 4096 a^{12} c^6 - 24 a^7 b^{10} c + 240 a^8 b^8 c^2 - 1280 a^9 b^6 c^3 + 3840 a^{10} b^4 c^4 - 6144 a^{11} b^2 c^5))) (b^6 - 45 a^3 c^3 + 40 a^2 b^2 c^2 - 11 a b^4 c) / (8 a^3 c^2 (4 a c - b^2)^6 (6 b^{10} - 6400 a^5 c^5 + 960 a^2 b^6 c^2 - 3850 a^3 b^4 c^3 + 7775 a^4 b^2 c^4 - 120 a b^8 c)) * (16 a^9 b^{12} (4 a c - b^2)^{(15/2)} + 65536 a^{15} c^6 (4 a c - b^2)^{(15/2)} - 384 a^{10} b^{10} c (4 a c - b^2)^{(15/2)} + 3840 a^{11} b^8 c^2 (4 a c - b^2)^{(15/2)} - 20480 a^{12} b^6 c^3 (4 a c - b^2)^{(15/2)} + 61440 a^{13} b^4 c^4 (4 a c - b^2)^{(15/2)} - 98304 a^{14} b^2 c^5 (4 a c - b^2)^{(15/2)})) / (b^{10} c^2 e^{14} - 20 a b^8 c^3 e^{14} + 160 a^2 b^6 c^4 e^{14} - 600 a^3 b^4 c^5 e^{14} + 900 a^4 b^2 c^6 e^{14}) - (((b^4 ((4 a^2 b^{12} c^3 e^{15} - 93 a^3 b^{10} c^4 e^{15} + 854 a^4 b^8 c^5 e^{15} - 3889 a^5 b^6 c^6 e^{15} + 8808 a^6 b^4 c^7 e^{15} - 7952 a^7 b^2 c^8 e^{15} + 6 a^2 b^{11} c^4 d^2 e^{15} - 137 a^3 b^9 c^5 d^2 e^{15} + 1217 a^4 b^7 c^6 d^2 e^{15} - 5256 a^5 b^5 c^7 d^2 e^{15} + 11024 a^6 b^3 c^8 d^2 e^{15} - 8960 a^7 b c^9 d^2 e^{15})) / (a^6 b^{12} + 4096 a^{12} c^6 - 24 a^7 b^{10} c + 240 a^8 b^8 c^2 - 1280 a^9 b^6 c^3 + 3840 a^{10} b^4 c^4 - 6144 a^{11} b^2 c^5) - (((4 a^4 b^{14} c^2 e^{16} - 100 a^5 b^{12} c^3 e^{16} + 1052 a^6 b^{10} c^4 e^{16} - 5952 a^7 b^8 c^5 e^{16} + 19072 a^8 b^6 c^6 e^{16} - 32768 a^9 b^4 c^7 e^{16} + 23552 a^{10} b^2 c^8 e^{16} + 2 a^4 b^{13} c^3 d^2 e^{16} - 36 a^5 b^{11} c^4 d^2 e^{16} + 276 a^6 b^9 c^5 d^2 e^{16} - 1216 a^7 b^7 c^6 d^2 e^{16} + 3456 a^8 b^5 c^7 d^2 e^{16} - 6144 a^9 b^3 c^8 d^2 e^{16} + 5120 a^{10} b c^9 d^2 e^{16})) / (a^6 b^{12} + 4096 a^{12} c^6 - 24 a^7 b^{10} c + 240 a^8 b^8 c^2 - 1280 a^9 b^6 c^3 + 3840 a^{10} b^4 c^4 - 6144 a^{11} b^2 c^5) + ((2 b^{10} e - 2048 a^5 c^5 e + 320 a^2 b^6 c^2 e - 1280 a^3 b^4 c^3 e + 2560 a^4 b^2 c^4 e - 40 a b^8 c e) * (4 a^7 b^{14} c^2 e^{17} - 96 a^8 b^{12} c^3 e^{17} + 960 a^9 b^{10} c^4 e^{17} - 5120 a^{10} b^8 c^5 e^{17} + 15360 a^{11} b^6 c^6 e^{17} - 24576 a^{12} b^4 c^7 e^{17} + 16384 a^{13} b^2 c^8 e^{17} + 12 a^6 b^{15} c^2 d^2 e^{17} - 328 a^7 b^{13} c^3 d^2 e^{17} + 3840 a^8 b^{11} c^4 d^2 e^{17} - 24960 a^9 b^9 c^5 d^2 e^{17} + 97280 a^{10} b^7 c^6 d^2 e^{17} - 227328 a^{11} b^5 c^7 d^2 e^{17} + 294912 a^{12} b^3 c^8 d^2 e^{17} - 163840 a^{13} b c^9 d^2 e^{17})) / (2 (4 a^3 b^{10} e^2 - 4096 a^8 c^5 e^2 - 80 a^4 b^8 c e^2 + 640 a^5 b^6 c^2 e^2 - 2560 a^6 b^4 c^3 e^2 + 5120 a^7 b^2 c^4 e^2) (a^6 b^{12} + 4096 a^{12} c^6 - 24 a^7 b^{10} c + 240 a^8 b^8 c^2 - 1280 a^9 b^6 c^3 + 3840 a^{10} b^4 c^4 - 6144 a^{11} b^2 c^5))) (2 b^{10} e - 2048 a^5 c^5 e + 320 a^2 b^6 c^2 e - 1280 a^3 b^4 c^3 e + 2560 a^4 b^2 c^4 e -
\end{aligned}$$

$$\begin{aligned}
& *c^4e - 40*a*b^8*c*e)) / (2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) \\
&)*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)) / (4*a^3*e*(4*a*c - b^2)^{(5/2)}) - (((b*((4 \\
& *a^4*b^14*c^2*e^16 - 100*a^5*b^12*c^3*e^16 + 1052*a^6*b^10*c^4*e^16 - 5952* \\
& a^7*b^8*c^5*e^16 + 19072*a^8*b^6*c^6*e^16 - 32768*a^9*b^4*c^7*e^16 + 23552* \\
& a^10*b^2*c^8*e^16 + 2*a^4*b^13*c^3*d^2*e^16 - 36*a^5*b^11*c^4*d^2*e^16 + 27 \\
& 6*a^6*b^9*c^5*d^2*e^16 - 1216*a^7*b^7*c^6*d^2*e^16 + 3456*a^8*b^5*c^7*d^2*e \\
& ^16 - 6144*a^9*b^3*c^8*d^2*e^16 + 5120*a^10*b*c^9*d^2*e^16)) / (a^6*b^12 + 409 \\
& 6*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10 \\
& *b^4*c^4 - 6144*a^11*b^2*c^5) + ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c \\
& ^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14* \\
& c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5 \\
& *e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + 16384*a^13*b^2* \\
& c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e^17 + 3840*a^8* \\
& b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10*b^7*c^6*d^2*e^1 \\
& 7 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e^17 - 163840*a^ \\
& 13*b*c^9*d^2*e^17)) / (2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^ \\
& 2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6 \\
& *b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 \\
& + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)) \\
& / (4*a^3*e*(4*a*c - b^2)^{(5/2)}) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10 \\
& *e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2 \\
& *c^4*e - 40*a*b^8*c*e)*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^ \\
& 9*b^10*c^4*e^17 - 5120*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576* \\
& a^12*b^4*c^7*e^17 + 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 32 \\
& 8*a^7*b^13*c^3*d^2*e^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^ \\
& 2*e^17 + 97280*a^10*b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 29491 \\
& 2*a^12*b^3*c^8*d^2*e^17 - 163840*a^13*b*c^9*d^2*e^17)) / (8*a^3*e*(4*a*c - b^ \\
& 2)^{(5/2)}*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^ \\
& 6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^12 + 4096*a^ \\
& ^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^ \\
& 4*c^4 - 6144*a^11*b^2*c^5))*(2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e \\
& - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)) / (2*(4*a^3*b^10* \\
& e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6* \\
& b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c) \\
& ^3*(4*a^7*b^14*c^2*e^17 - 96*a^8*b^12*c^3*e^17 + 960*a^9*b^10*c^4*e^17 - 51 \\
& 20*a^10*b^8*c^5*e^17 + 15360*a^11*b^6*c^6*e^17 - 24576*a^12*b^4*c^7*e^17 + \\
& 16384*a^13*b^2*c^8*e^17 + 12*a^6*b^15*c^2*d^2*e^17 - 328*a^7*b^13*c^3*d^2*e \\
& ^17 + 3840*a^8*b^11*c^4*d^2*e^17 - 24960*a^9*b^9*c^5*d^2*e^17 + 97280*a^10* \\
& b^7*c^6*d^2*e^17 - 227328*a^11*b^5*c^7*d^2*e^17 + 294912*a^12*b^3*c^8*d^2*e \\
& ^17 - 163840*a^13*b*c^9*d^2*e^17)) / (64*a^9*e^3*(4*a*c - b^2)^{(15/2)}*(a^6*b^ \\
& 12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3 \\
& 840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4* \\
& c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^9*b^12*(4*a*c - b^2)^{(15/2)} + 655 \\
& 36*a^15*c^6*(4*a*c - b^2)^{(15/2)} - 384*a^10*b^10*c*(4*a*c - b^2)^{(15/2)} + 3
\end{aligned}$$

$$\begin{aligned}
& 840*a^{11}*b^8*c^2*(4*a*c - b^2)^{(15/2)} - 20480*a^{12}*b^6*c^3*(4*a*c - b^2)^{(15/2)} + 61440*a^{13}*b^4*c^4*(4*a*c - b^2)^{(15/2)} - 98304*a^{14}*b^2*c^5*(4*a*c - b^2)^{(15/2)})/(8*a^3*c^2*(4*a*c - b^2)^{(13/2)}*(b^{10}*c^2*e^{14} - 20*a*b^8*c^3*e^{14} + 160*a^2*b^6*c^4*e^{14} - 600*a^3*b^4*c^5*e^{14} + 900*a^4*b^2*c^6*e^{14})*((6*b^{10} - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) - (3*b*(((4*a^2*b^{12}*c^3*e^{15} - 93*a^3*b^{10}*c^4*e^{15} + 854*a^4*b^8*c^5*e^{15} - 3889*a^5*b^6*c^6*e^{15} + 8808*a^6*b^4*c^7*e^{15} - 7952*a^7*b^2*c^8*e^{15} + 6*a^2*b^{11}*c^4*d^2*e^{15} - 137*a^3*b^9*c^5*d^2*e^{15} + 1217*a^4*b^7*c^6*d^2*e^{15} - 5256*a^5*b^5*c^7*d^2*e^{15} + 11024*a^6*b^3*c^8*d^2*e^{15} - 8960*a^7*b*c^9*d^2*e^{15}))/((a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5) - (((4*a^4*b^{14}*c^2*e^{16} - 100*a^5*b^{12}*c^3*e^{16} + 1052*a^6*b^{10}*c^4*e^{16} - 5952*a^7*b^8*c^5*e^{16} + 19072*a^8*b^6*c^6*e^{16} - 32768*a^9*b^4*c^7*e^{16} + 23552*a^{10}*b^2*c^8*e^{16} + 2*a^4*b^{13}*c^3*d^2*e^{16} - 36*a^5*b^{11}*c^4*d^2*e^{16} + 276*a^6*b^9*c^5*d^2*e^{16} - 1216*a^7*b^7*c^6*d^2*e^{16} + 3456*a^8*b^5*c^7*d^2*e^{16} - 6144*a^9*b^3*c^8*d^2*e^{16} + 5120*a^{10}*b*c^9*d^2*e^{16}))/((a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5) + ((2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e) * (4*a^7*b^{14}*c^2*e^{17} - 96*a^8*b^{12}*c^3*e^{17} + 960*a^9*b^{10}*c^4*e^{17} - 5120*a^{10}*b^8*c^5*e^{17} + 15360*a^{11}*b^6*c^6*e^{17} - 24576*a^{12}*b^4*c^7*e^{17} + 16384*a^{13}*b^2*c^8*e^{17} + 12*a^6*b^{15}*c^2*d^2*e^{17} - 328*a^7*b^{13}*c^3*d^2*e^{17} + 3840*a^8*b^{11}*c^4*d^2*e^{17} - 24960*a^9*b^9*c^5*d^2*e^{17} + 97280*a^{10}*b^7*c^6*d^2*e^{17} - 227328*a^{11}*b^5*c^7*d^2*e^{17} + 294912*a^{12}*b^3*c^8*d^2*e^{17} - 163840*a^{13}*b*c^9*d^2*e^{17}))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)*(a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5)))*(2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)))*(2*b^{10}*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e))/((2*(4*a^3*b^{10}*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b^{10}*c^4*e^{14} - 22*a*b^8*c^5*e^{14} + 177*a^2*b^6*c^6*e^{14} - 616*a^3*b^4*c^7*e^{14} + 784*a^4*b^2*c^8*e^{14} + b^9*c^5*d^2*e^{14} + 147*a^2*b^5*c^7*d^2*e^{14} - 343*a^3*b^3*c^8*d^2*e^{14} - 21*a*b^7*c^6*d^2*e^{14}))/((a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5) + (b*(((4*a^4*b^{14}*c^2*e^{16} - 100*a^5*b^{12}*c^3*e^{16} + 1052*a^6*b^{10}*c^4*e^{16} - 5952*a^7*b^8*c^5*e^{16} + 19072*a^8*b^6*c^6*e^{16} - 32768*a^9*b^4*c^7*e^{16} + 23552*a^{10}*b^2*c^8*e^{16} + 2*a^4*b^{13}*c^3*d^2*e^{16} - 36*a^5*b^{11}*c^4*d^2*e^{16} + 276*a^6*b^9*c^5*d^2*e^{16} - 1216*a^7*b^7*c^6*d^2*e^{16} + 3456*a^8*b^5*c^7*d^2*e^{16} - 6144*a^9*b^3*c^8*d^2*e^{16} + 5120*a^{10}*b*c^9*d^2*e^{16}))/((a^6*b^{12} + 4096*a^{12}*c^6 - 24*a^7*b^{10}*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^{10}*b^4*c^4 - 6144*a^{11}*b^2*c^5) + ((2*b^{10}*
\end{aligned}$$

$$\begin{aligned}
& e - 2048a^5c^5e + 320a^2b^6c^2e - 1280a^3b^4c^3e + 2560a^4b^2c^4e - 40ab^8c^5e) \cdot (4a^7b^{14}c^2e^{17} - 96a^8b^{12}c^3e^{17} + 960a^9b^{10}c^4e^{17} - 5120a^{10}b^8c^5e^{17} + 15360a^{11}b^6c^6e^{17} - 24576a^{12}b^4c^7e^{17} + 16384a^{13}b^2c^8e^{17} + 12a^6b^{15}c^2d^2e^{17} - 328a^7b^{13}c^3d^2e^{17} + 3840a^8b^{11}c^4d^2e^{17} - 24960a^9b^9c^5d^2e^{17} + 97280a^{10}b^7c^6d^2e^{17} - 227328a^{11}b^5c^7d^2e^{17} + 294912a^{12}b^3c^8d^2e^{17} - 163840a^{13}b^1c^9d^2e^{17})) / (2(4a^3b^{10}e^2 - 4096a^8c^5e^2 - 80a^4b^8c^5e^2 + 640a^5b^6c^2e^2 - 2560a^6b^4c^3e^2 + 5120a^7b^2c^4e^2) \cdot (a^6b^{12} + 4096a^{12}c^6 - 24a^7b^{10}c + 240a^8b^8c^2 - 1280a^9b^6c^3 + 3840a^{10}b^4c^4 - 6144a^{11}b^2c^5)) \cdot (b^4 + 30a^2c^2 - 10ab^2c)) / (4a^3e \cdot (4ac - b^2)^{(5/2)}) + (b \cdot (b^4 + 30a^2c^2 - 10ab^2c) \cdot (2b^{10}e - 2048a^5c^5e + 320a^2b^6c^2e - 1280a^3b^4c^3e + 2560a^4b^2c^4e - 40ab^8c^5e) \cdot (4a^7b^{14}c^2e^{17} - 96a^8b^{12}c^3e^{17} + 960a^9b^{10}c^4e^{17} - 5120a^{10}b^8c^5e^{17} + 15360a^{11}b^6c^6e^{17} - 24576a^{12}b^4c^7e^{17} + 16384a^{13}b^2c^8e^{17} + 12a^6b^{15}c^2d^2e^{17} - 328a^7b^{13}c^3d^2e^{17} + 3840a^8b^{11}c^4d^2e^{17} - 24960a^9b^9c^5d^2e^{17} + 97280a^{10}b^7c^6d^2e^{17} - 227328a^{11}b^5c^7d^2e^{17} + 294912a^{12}b^3c^8d^2e^{17} - 163840a^{13}b^1c^9d^2e^{17})) / (8a^3e \cdot (4ac - b^2)^{(5/2)} \cdot (4a^3b^{10}e^2 - 4096a^8c^5e^2 - 80a^4b^8c^5e^2 + 640a^5b^6c^2e^2 - 2560a^6b^4c^3e^2 + 5120a^7b^2c^4e^2) \cdot (a^6b^{12} + 4096a^{12}c^6 - 24a^7b^{10}c + 240a^8b^8c^2 - 1280a^9b^6c^3 + 3840a^{10}b^4c^4 - 6144a^{11}b^2c^5)) \cdot (b^4 + 30a^2c^2 - 10ab^2c)) / (4a^3e \cdot (4ac - b^2)^{(5/2)}) + (b^2 \cdot (b^4 + 30a^2c^2 - 10ab^2c)^2 \cdot (2b^{10}e - 2048a^5c^5e + 320a^2b^6c^2e - 1280a^3b^4c^3e + 2560a^4b^2c^4e - 40ab^8c^5e) \cdot (4a^7b^{14}c^2e^{17} - 96a^8b^{12}c^3e^{17} + 960a^9b^{10}c^4e^{17} - 5120a^{10}b^8c^5e^{17} + 15360a^{11}b^6c^6e^{17} - 24576a^{12}b^4c^7e^{17} + 16384a^{13}b^2c^8e^{17} + 12a^6b^{15}c^2d^2e^{17} - 328a^7b^{13}c^3d^2e^{17} + 3840a^8b^{11}c^4d^2e^{17} - 24960a^9b^9c^5d^2e^{17} + 97280a^{10}b^7c^6d^2e^{17} - 227328a^{11}b^5c^7d^2e^{17} + 294912a^{12}b^3c^8d^2e^{17} - 163840a^{13}b^1c^9d^2e^{17})) / (32a^6e^2 \cdot (4ac - b^2)^5 \cdot (4a^3b^{10}e^2 - 4096a^8c^5e^2 - 80a^4b^8c^5e^2 + 640a^5b^6c^2e^2 - 2560a^6b^4c^3e^2 + 5120a^7b^2c^4e^2) \cdot (a^6b^{12} + 4096a^{12}c^6 - 24a^7b^{10}c + 240a^8b^8c^2 - 1280a^9b^6c^3 + 3840a^{10}b^4c^4 - 6144a^{11}b^2c^5)) \cdot (b^6 - 45a^3c^3 + 40a^2b^2c^2 - 11ab^4c) \cdot (16a^9b^{12} \cdot (4ac - b^2)^{(15/2)} + 65536a^{15}c^6 \cdot (4ac - b^2)^{(15/2)} - 384a^{10}b^{10}c \cdot (4ac - b^2)^{(15/2)} + 3840a^{11}b^8c^2 \cdot (4ac - b^2)^{(15/2)} - 20480a^{12}b^6c^3 \cdot (4ac - b^2)^{(15/2)} + 61440a^{13}b^4c^4 \cdot (4ac - b^2)^{(15/2)} - 98304a^{14}b^2c^5 \cdot (4ac - b^2)^{(15/2)})) / (8a^3c^2 \cdot (4ac - b^2)^6 \cdot (b^{10}c^2e^{14} - 20ab^8c^3e^{14} + 160a^2b^6c^4e^{14} - 600a^3b^4c^5e^{14} + 900a^4b^2c^6e^{14}) \cdot (6b^{10} - 6400a^5c^5 + 960a^2b^6c^2 - 3850a^3b^4c^3 + 7775a^4b^2c^4 - 120ab^8c)) \cdot (b^4 + 30a^2c^2 - 10ab^2c)) / (2a^3e \cdot (4ac - b^2)^{(5/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out

$$3.523 \quad \int \frac{1}{(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=484

$$\frac{3(5b^2 - 12ac)(b^2 - 5ac)}{8a^3e(b^2 - 4ac)^2(d+ex)} + \frac{36a^2c^2 + bc(5b^2 - 32ac)(d+ex)^2 - 35ab^2c + 5b^4}{8a^2e(b^2 - 4ac)^2(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3\sqrt{c} \left(\frac{b(124a^2c^2 - 47ab^2c + 5b^4)}{\sqrt{b^2 - 4ac}} + \dots \right)}{8\sqrt{2}a^3e}$$

Rubi [A] time = 1.23, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1142, 1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2 + bc(5b^2 - 32ac)(d+ex)^2 - 35ab^2c + 5b^4}{8a^2e(b^2 - 4ac)^2(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3\sqrt{c} \left(\frac{b(124a^2c^2 - 47ab^2c + 5b^4)}{\sqrt{b^2 - 4ac}} + (5b^2 - 12ac)(b^2 - 5ac) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b^2 - 4ac}} \right) \right)}{8\sqrt{2}a^3e(b^2 - 4ac)^2\sqrt{b^2 - 4ac}} - \frac{3\sqrt{c} \left((5b^2 - 12ac)(b^2 - 5ac) - \frac{124a^2c^2 - 47ab^2c + 5b^4}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b^2 - 4ac}} \right)}{8\sqrt{2}a^3e(b^2 - 4ac)^2\sqrt{b^2 - 4ac} + b} - \frac{3(5b^2 - 12ac)(b^2 - 5ac)}{8a^2e(b^2 - 4ac)^2(d+ex)} + \frac{-2ac + b^2 + bc(d+ex)^2}{4ae(b^2 - 4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2)))/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]]]/(8*sqrt[2]*a^3*(b^2 - 4*a*c)^2*sqrt[b - sqrt[b^2 - 4*a*c]]*e) - (3*sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2))/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b + sqrt[b^2 - 4*a*c]]]/(8*sqrt[2]*a^3*(b^2 - 4*a*c)^2*sqrt[b + sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1121

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[((d*x)^(m+1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*a*d*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p+1)*Simp[b^2*(m+2*p+3) - 2*a*c*(m+4*p+5) + b*c*(m+4*p+7)*x^2, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In

tegerQ[m])

Rule 1142

Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1277

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1281

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^2 (a+b(d+ex)^2+c(d+ex)^4)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^3} dx, x, d+ex\right)}{e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^3} dx, x, d+ex\right)}{4a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{5b^4-12ac^2}{8a^2(b^2-4ac)^2e(d+ex)} \\
&= -\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3(b^2-4ac)^2e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2} \\
&= -\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3(b^2-4ac)^2e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2} \\
&= -\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3(b^2-4ac)^2e(d+ex)} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)(a+b(d+ex)^2+c(d+ex)^4)^2}
\end{aligned}$$

Mathematica [A] time = 6.24, size = 560, normalized size = 1.16

$$\frac{1}{2\sqrt{d+ex}} - \frac{3ab(d+ex) - 2a^2(d+ex)^2 + b^2(d+ex) + c^2(d+ex)^2}{4ab(4ac-b^2)(e+bd+cx^2+cd+ex^3)} - \frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac} + 12ab^2c^2 - 47ab^2c - 37ab^2c\sqrt{b^2-4ac} + 5b^4\sqrt{b^2-4ac} + 5b^3\right)\text{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{b^2-4ac}}\right)}{8\sqrt{c}\sqrt{b^2-4ac}^3\sqrt{b^2-4ac}} - \frac{3\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac} - 12ab^2c^2 + 47ab^2c - 37ab^2c\sqrt{b^2-4ac} + 5b^4\sqrt{b^2-4ac} + 5b^3\right)\text{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{b^2-4ac}}\right)}{8\sqrt{c}\sqrt{b^2-4ac}^3\sqrt{b^2-4ac}} + \frac{-84a^2c^2(d+ex) + 52ab^2c(d+ex) + 47ab^2c^2(d+ex)^2 - 7b^4(d+ex) - 7b^4cd + cx^3}{8ab(4ac-b^2)(e+bd+cx^2+cd+ex^3)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

[Out] $-(1/(a^3e(d+ex))) + (b^3(d+ex) - 3ab^2c(d+ex) + b^2c(d+ex)^3 - 2ac^2(d+ex)^3)/(4a^2(-b^2+4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2) + (-7b^5(d+ex) + 52ab^3c(d+ex) - 84a^2b^2c^2(d+ex) - 7b^4c(d+ex)^3 + 47a^2b^2c^2(d+ex)^3 - 52a^2c^3(d+ex)^3)/(8a^3(-b^2+4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)) - (3\sqrt{c}(5b^5 - 47ab^3c + 124a^2b^2c^2 + 5b^4\sqrt{b^2-4ac} - 37ab^2c\sqrt{b^2-4ac} + 60a^2c^2\sqrt{b^2-4ac})*\text{ArcTan}[(\sqrt{2}\sqrt{c}(d+ex))/\sqrt{b-\sqrt{b^2-4ac}}])/(8\sqrt{2}a^3(b^2-4ac))$

$$\begin{aligned} & *c)^{(5/2)} * \text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]] * e) - (3 * \text{Sqrt}[c] * (-5*b^5 + 47*a*b^3*c \\ & - 124*a^2*b*c^2 + 5*b^4 * \text{Sqrt}[b^2 - 4*a*c] - 37*a*b^2*c * \text{Sqrt}[b^2 - 4*a*c] + \\ & 60*a^2*c^2 * \text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTan}[(\text{Sqrt}[2] * \text{Sqrt}[c] * (d + e*x)) / \text{Sqrt}[b + \text{S} \\ & \text{qrt}[b^2 - 4*a*c]])] / (8 * \text{Sqrt}[2] * a^3 * (b^2 - 4*a*c)^{(5/2)} * \text{Sqrt}[b + \text{Sqrt}[b^2 - \\ & 4*a*c]] * e) \end{aligned}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x
]

fricas [B] time = 3.92, size = 10260, normalized size = 21.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*b^4*c^2 - \\ & 37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2 \\ & *b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 12*(28* \\ & (5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 3 \\ & 92*a^2*b*c^3)*d)*e^5*x^5 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + \\ & 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37 \\ & *a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3 \\ &)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 8*(42*(\\ & 5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 + \\ & 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d \\ &)*e^3*x^3 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4 \\ & *c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 + \\ & 60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c - \\ & 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c \\ & ^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 \\ &)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c - \\ & 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 \\ & + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x - \\ & 3*\text{sqrt}(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3*b^4*c^2 \\ & - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 \\ & + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8* \end{aligned}$$

$$\begin{aligned}
& x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8 \\
& *a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c \\
& ^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8 \\
& *a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c \\
& ^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + \\
& 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c \\
& + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(\\
& a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + \\
& 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4) \\
& *d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a \\
& ^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^ \\
& 3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^ \\
& 3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5 \\
& *(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a \\
& ^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2* \\
& (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 3 \\
& 2*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - \\
& 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7 \\
& *c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 \\
& - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1 \\
& 024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351 \\
& 310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/ \\
& ((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^ \\
& 18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6* \\
& c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))*log(-27*(\\
& 4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 \\
& + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^10*c^4 - 77825*a*b \\
& ^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 8 \\
& 10000*a^5*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9* \\
& b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + \\
& 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*e^3*sqrt((625 \\
& *b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^ \\
& 4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c \\
& + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5) \\
& *e^4)) - (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c \\
& ^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684 \\
& 672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894 \\
& *a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a \\
& ^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2 \\
& *c^4 - 1024*a^12*c^5)*e^2*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c \\
& ^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a \\
& ^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + \\
& 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4)))/((a^7*b^10 - 20*a^8*b^8*c + 160* \\
& a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e^2))) \\
& + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*x^9 + 9*(a^3
\end{aligned}$$

$$\begin{aligned}
& 0*a^9*b^6*c^2 - 640*a^{10}*b^4*c^3 + 1280*a^{11}*b^2*c^4 - 1024*a^{12}*c^5)*e^2)) \\
&) + 3*\sqrt{1/2}*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^{10*x^9} + 9*(a \\
& ^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b \\
& ^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)* \\
& e^8*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c \\
& - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a \\
& ^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c \\
& - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b \\
& ^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^ \\
& 3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5* \\
& b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + \\
& 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c \\
& + 32*a^6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5* \\
& c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - \\
& 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d \\
&)*e^3*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^ \\
& 2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 \\
& + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + \\
& 16*a^6*b*c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 \\
& + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c \\
& + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^ \\
& 4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*\sqrt{-(25*b^{11} - 495*a*b^9*c + 3894*a^2 \\
& *b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b \\
& ^{10} - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^{10}*b^4*c^3 + 1280*a^{11}*b^2*c^4 \\
& - 1024*a^{12}*c^5)*e^2*\sqrt{((625*b^{12} - 12250*a*b^{10}*c + 94725*a^2*b^8*c^2 - \\
& 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c \\
& ^6)/((a^{14}*b^{10} - 20*a^{15}*b^8*c + 160*a^{16}*b^6*c^2 - 640*a^{17}*b^4*c^3 + 128 \\
& 0*a^{18}*b^2*c^4 - 1024*a^{19}*c^5)*e^4)))/((a^7*b^{10} - 20*a^8*b^8*c + 160*a^9* \\
& b^6*c^2 - 640*a^{10}*b^4*c^3 + 1280*a^{11}*b^2*c^4 - 1024*a^{12}*c^5)*e^2))*\log(- \\
& 27*(4125*b^{10}*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4* \\
& c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^{10}*c^4 - 77825 \\
& *a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 \\
& - 810000*a^5*c^9)*d + 27/2*\sqrt{1/2}*((5*a^7*b^{16} - 152*a^8*b^{14}*c + 2006* \\
& a^9*b^{12}*c^2 - 14960*a^{10}*b^{10}*c^3 + 68640*a^{11}*b^8*c^4 - 197120*a^{12}*b^6*c \\
& ^5 + 342528*a^{13}*b^4*c^6 - 323584*a^{14}*b^2*c^7 + 122880*a^{15}*c^8)*e^3*\sqrt{ \\
& (625*b^{12} - 12250*a*b^{10}*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 59188 \\
& 6*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^{14}*b^{10} - 20*a^{15}*b \\
& ^8*c + 160*a^{16}*b^6*c^2 - 640*a^{17}*b^4*c^3 + 1280*a^{18}*b^2*c^4 - 1024*a^{19} \\
& c^5)*e^4)) + (125*b^{17} - 3775*a*b^{15}*c + 49360*a^2*b^{13}*c^2 - 362733*a^3*b^ \\
& 11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - \\
& 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8)*e)*\sqrt{-(25*b^{11} - 495*a*b^9*c + \\
& 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 \\
& - (a^7*b^{10} - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^{10}*b^4*c^3 + 1280*a^{11} \\
& *b^2*c^4 - 1024*a^{12}*c^5)*e^2*\sqrt{((625*b^{12} - 12250*a*b^{10}*c + 94725*a^2*b \\
& ^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 506
\end{aligned}$$

$$\begin{aligned}
& 25a^6c^6)/((a^{14}b^{10} - 20a^{15}b^8c + 160a^{16}b^6c^2 - 640a^{17}b^4c^3 + 1280a^{18}b^2c^4 - 1024a^{19}c^5)e^4))/((a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5)e^2)) - 3\sqrt{1/2}*((a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)e^{10}x^9 + 9(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)d^9e^{9}x^8 + 2(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3 + 18(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)d^2)e^{8}x^7 + 14(6(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)d^3 + (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d)e^{7}x^6 + (a^3b^6 - 6a^4b^4c + 32a^6c^3 + 126(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)d^4 + 42(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^2)e^{6}x^5 + (126(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)d^5 + 70(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^3 + 5(a^3b^6 - 6a^4b^4c + 32a^6c^3)d)e^{5}x^4 + 2(a^4b^5 - 8a^5b^3c + 16a^6b^2c^2 + 42(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)d^6 + 35(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^4 + 5(a^3b^6 - 6a^4b^4c + 32a^6c^3)d^2)e^{4}x^3 + 2(18(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)d^7 + 21(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^5 + 5(a^3b^6 - 6a^4b^4c + 32a^6c^3)d^3 + 3(a^4b^5 - 8a^5b^3c + 16a^6b^2c^2)d)e^{3}x^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2 + 9(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)d^8 + 14(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^6 + 5(a^3b^6 - 6a^4b^4c + 32a^6c^3)d^4 + 6(a^4b^5 - 8a^5b^3c + 16a^6b^2c^2)d^2)e^{2}x + ((a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)d^9 + 2(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^7 + (a^3b^6 - 6a^4b^4c + 32a^6c^3)d^5 + 2(a^4b^5 - 8a^5b^3c + 16a^6b^2c^2)d^3 + (a^5b^4 - 8a^6b^2c + 16a^7c^2)d)e)*\sqrt{-(25b^{11} - 495ab^9c + 3894a^2b^7c^2 - 15015a^3b^5c^3 + 27720a^4b^3c^4 - 18480a^5b^2c^5 - (a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5)e^2}\sqrt{(625b^{12} - 12250ab^{10}c + 94725a^2b^8c^2 - 351310a^3b^6c^3 + 591886a^4b^4c^4 - 312300a^5b^2c^5 + 50625a^6c^6)/((a^{14}b^{10} - 20a^{15}b^8c + 160a^{16}b^6c^2 - 640a^{17}b^4c^3 + 1280a^{18}b^2c^4 - 1024a^{19}c^5)e^4))/((a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5)e^2))*\log(-27(4125b^{10}c^4 - 77825ab^8c^5 + 571030a^2b^6c^6 - 1957349a^3b^4c^7 + 2835000a^4b^2c^8 - 810000a^5c^9))e^x - 27(4125b^{10}c^4 - 77825ab^8c^5 + 571030a^2b^6c^6 - 1957349a^3b^4c^7 + 2835000a^4b^2c^8 - 810000a^5c^9)d - 27/2\sqrt{1/2}*((5a^7b^{16} - 152a^8b^{14}c + 2006a^9b^{12}c^2 - 14960a^{10}b^{10}c^3 + 68640a^{11}b^8c^4 - 197120a^{12}b^6c^5 + 342528a^{13}b^4c^6 - 323584a^{14}b^2c^7 + 122880a^{15}c^8)e^3\sqrt{(625b^{12} - 12250ab^{10}c + 94725a^2b^8c^2 - 351310a^3b^6c^3 + 591886a^4b^4c^4 - 312300a^5b^2c^5 + 50625a^6c^6)/((a^{14}b^{10} - 20a^{15}b^8c + 160a^{16}b^6c^2 - 640a^{17}b^4c^3 + 1280a^{18}b^2c^4 - 1024a^{19}c^5)e^4)) + (125b^{17} - 3775ab^{15}c + 49360a^2b^{13}c^2 - 362733a^3b^{11}c^3 + 1623534a^4b^9c^4 - 4463140a^5b^7c^5 + 7146736a^6b^5c^6 - 5684672a^7b^3c^7 + 1324800a^8b^2c^8)e)*\sqrt{-(25b^{11} - 495ab^9c + 3894a^2b^7c^2 - 15015a^3b^5c^3 + 27720a^4b^3c^4 - 18480a^5b^2c^5 - (a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5)e^2)}
\end{aligned}$$

$$\begin{aligned}
& 11*b^2*c^4 - 1024*a^{12}*c^5)*e^2*\sqrt{(625*b^{12} - 12250*a*b^{10}*c + 94725*a^2 \\
& *b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 5 \\
& 0625*a^6*c^6)/((a^{14}*b^{10} - 20*a^{15}*b^8*c + 160*a^{16}*b^6*c^2 - 640*a^{17}*b^4 \\
& *c^3 + 1280*a^{18}*b^2*c^4 - 1024*a^{19}*c^5)*e^4)))/((a^7*b^{10} - 20*a^8*b^8*c \\
& + 160*a^9*b^6*c^2 - 640*a^{10}*b^4*c^3 + 1280*a^{11}*b^2*c^4 - 1024*a^{12}*c^5)*e \\
& ^2)))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^{10}*x^9 + 9*(a^3*b^4*c^ \\
& ^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*x^8 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + \\
& 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*e^8*x^7 + \\
& 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8*a^4* \\
& b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3 + \\
& 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8*a^4* \\
& b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*x^5 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + \\
& 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^ \\
& 3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 1 \\
& 6*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b \\
& ^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^ \\
& 6*c^3)*d^2)*e^4*x^3 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 \\
& + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^ \\
& 4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*x^2 \\
& + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 1 \\
& 6*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3 \\
& *b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b* \\
& c^2)*d^2)*e^2*x + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3* \\
& b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6 \\
& *c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6 \\
& *b^2*c + 16*a^7*c^2)*d)*e)
\end{aligned}$$

giac [B] time = 1.22, size = 1412, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] $\begin{aligned}
& 3/64*(2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*\sqrt{ \\
& (2*a*b + 2*\sqrt{b^2 - 4*a*c})*a}*\sqrt{b^2 - 4*a*c}*abs(a^3*b^4*e^2 - 8*a^4*b \\
& ^2*c*e^2 + 16*a^5*c^2*e^2)*e^2 - (a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^ \\
& 2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*\sqrt{2*a*b + 2*\sqrt{b^2 - 4*a* \\
& c})*a} + (5*a^6*b^{13} - 112*a^7*b^{11}*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3 \\
& + 12736*a^{10}*b^5*c^4 - 16384*a^{11}*b^3*c^5 + 7680*a^{12}*b*c^6)*\sqrt{2*a*b + 2 \\
& *\sqrt{b^2 - 4*a*c})*a}*e^4)*\arctan(2*\sqrt{1/2}*e^{(-1)/((x*e + d)*\sqrt{(a^3*b \\
& ^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 + \sqrt{(a^3*b^5*e^2 - 8*a^4*b^3 \\
& *c*e^2 + 16*a^5*b*c^2*e^2)}^2 - 4*(a^4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^ \\
& 2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*b^4*e^4 - 8*a^5*b^2* \\
& c*e^4 + 16*a^6*c^2*e^4)))*e^{(-3)/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2
\end{aligned}$

$$\begin{aligned}
& *c^3 - 64*a^{10}*c^4)*\sqrt{b^2 - 4*a*c}*\text{abs}(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 1 \\
& 6*a^5*c^2*e^2)*\text{abs}(a) + 3/64*(2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^ \\
& 2*c^3 - 240*a^7*c^4)*\sqrt{2*a*b - 2*\sqrt{b^2 - 4*a*c}}*a)*\sqrt{b^2 - 4*a*c}* \\
& \text{abs}(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*e^2 + (a^3*b^4*e^2 - 8* \\
& a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*\sqrt{ \\
& 2*a*b - 2*\sqrt{b^2 - 4*a*c}}*a) - (5*a^6*b^{13} - 112*a^7*b^{11}*c + 1030*a^8*b^ \\
& 9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^{10}*b^5*c^4 - 16384*a^{11}*b^3*c^5 + 7680*a \\
& ^{12}*b*c^6)*\sqrt{2*a*b - 2*\sqrt{b^2 - 4*a*c}}*a)*e^4)*\arctan(2*\sqrt{1/2})*e^{(- \\
& 1)/((x*e + d)*\sqrt{(a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 - \sqrt{ \\
& ((a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8* \\
& a^5*b^2*c*e^4 + 16*a^6*c^2*e^4)}*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/ \\
& (a^4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^2*e^4)))*e^{(-3)/((a^7*b^6*c - 12 \\
& *a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^{10}*c^4)*\sqrt{b^2 - 4*a*c}*\text{abs}(a^3*b^4* \\
& e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*\text{abs}(a)) - 1/8*(7*b^4*c^2*e^{(-1)/(x* \\
& e + d) - 47*a*b^2*c^3*e^{(-1)/(x*e + d) + 52*a^2*c^4*e^{(-1)/(x*e + d) + 14*b \\
& ^5*c*e^{(-1)/(x*e + d)^3 - 99*a*b^3*c^2*e^{(-1)/(x*e + d)^3 + 136*a^2*b*c^3*e \\
& ^{(-1)/(x*e + d)^3 + 7*b^6*e^{(-1)/(x*e + d)^5 - 43*a*b^4*c*e^{(-1)/(x*e + d)^ \\
& 5 + 25*a^2*b^2*c^2*e^{(-1)/(x*e + d)^5 + 68*a^3*c^3*e^{(-1)/(x*e + d)^5 + 9*a \\
& *b^5*e^{(-1)/(x*e + d)^7 - 66*a^2*b^3*c*e^{(-1)/(x*e + d)^7 + 108*a^3*b*c^2*e \\
& ^{(-1)/(x*e + d)^7}/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b/(x*e + d))^2 \\
& + a/(x*e + d)^4)^2) - e^{(-1)/((x*e + d)*a^3)}
\end{aligned}$$

maple [C] time = 0.07, size = 6821, normalized size = 14.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)`

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 14.38, size = 18112, normalized size = 37.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)

[Out] - ((x^4*(15*b^6*e^3 + 324*a^3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2*e^3 + 12600*a^2*c^4*d^4*e^3 + 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 3405*a*b^3*c^2*d^2*e^3 + 5880*a^2*b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*c^2*e^5 + 392*a^2*b*c^3*e^5 + 5040*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 - 3108*a*b^2*c^3*d^2*e^5))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(30*b^6*d^3 + 90*b^5*c*d^5 + 648*a^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^2*d^7 + 25*a*b^5*d - 681*a*b^3*c^2*d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^3*d^7 + 50*a^2*b^2*c^2*d^3 - 194*a^2*b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4*c*d^3))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4*d^3*e^4 + 140*b^4*c^2*d^3*e^4 + 30*b^5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 392*a^2*b*c^3*d*e^4 - 1036*a*b^2*c^3*d^3*e^4))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^8*(60*a^2*c^4*e^7 + 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^2*(90*b^6*d^2*e + 25*a*b^5*e + 1944*a^3*c^3*d^2*e + 5040*a^2*c^4*d^6*e + 420*b^4*c^2*d^6*e - 194*a^2*b^3*c*e + 364*a^3*b*c^2*e + 450*b^5*c*d^4*e - 546*a*b^4*c*d^2*e - 3405*a*b^3*c^2*d^4*e + 5880*a^2*b*c^3*d^4*e - 3108*a*b^2*c^3*d^6*e + 150*a^2*b^2*c^2*d^2*e))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(15*b^6*d*e^2 + 324*a^3*c^3*d*e^2 + 150*b^5*c*d^3*e^2 + 2520*a^2*c^4*d^5*e^2 + 210*b^4*c^2*d^5*e^2 - 91*a*b^4*c*d*e^2 + 25*a^2*b^2*c^2*d*e^2 - 1135*a*b^3*c^2*d^3*e^2 + 1960*a^2*b*c^3*d^3*e^2 - 1554*a*b^2*c^3*d^5*e^2))/(2*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^7*(60*a^2*c^4*d*e^6 + 5*b^4*c^2*d*e^6 - 37*a*b^2*c^3*d*e^6))/(a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (8*a^2*b^4 + 128*a^4*c^2 + 15*b^6*d^4 - 64*a^3*b^2*c + 25*a*b^5*d^2 + 30*b^5*c*d^6 + 324*a^3*c^3*d^4 + 180*a^2*c^4*d^8 + 15*b^4*c^2*d^8 - 194*a^2*b^3*c*d^2 + 364*a^3*b*c^2*d^2 - 227*a*b^3*c^2*d^6 + 392*a^2*b*c^3*d^6 - 111*a*b^2*c^3*d^8 + 25*a^2*b^2*c^2*d^4 - 91*a*b^4*c*d^4)/(8*a*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(10*b^2*d^2*e^3 + 84*c^2*d^6*e^3 + 2*a*b*e^3 + 20*a*c*d^2*e^3 + 70*b*c*d^4*e^3) + x^7*(36*c^2*d^2*e^7 + 2*b*c*e^7) + x*(a^2*e + 5*b^2*d^4*e + 9*c^2*d^8*e + 6*a*b*d^2*e + 10*a*c*d^4*e + 14*b*c*d^6*e) + x^4*(5*b^2*d*e^4 + 126*c^2*d^5*e^4 + 10*a*c*d*e^4 + 70*b*c*d^3*e^4) + a^2*d + x^2*(10*b^2*d^3*e^2 + 36*c^2*d^7*e^2 + 6*a*b*d*e^2 + 20*a*c*d^3*e^2 + 42*b*c*d^5*e^2) + x^6*(84*c^2*d^3*e^6 + 14*b*c*d*e^6) + x^5*(b^2*e^5 + 126*c^2*d^4*e^5 + 2*a*c*e^5 + 42*b*c*d^2*e^5) + b^2*d^5 + c^2*d^9 + c^2*e^9*x^9 + 2*a*b*d^3 + 2*a*c*d^5 + 2*b*c*d^7 + 9*c^2*d*e^8*x^8) - atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 1048576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 26

$$\begin{aligned}
& 21440*a^{16}*b^2*c^9*e^2))^{(1/2)}*((-9*(25*b^{21} - 25*b^6*(-(4*a*c - b^2)^{15}) \\
& ^{(1/2)} + 18923520*a^{10}*b*c^{10} + 17794*a^2*b^{17}*c^2 - 188095*a^3*b^{15}*c^3 + \\
& 1299860*a^4*b^{13}*c^4 - 6126640*a^5*b^{11}*c^5 + 19905600*a^6*b^9*c^6 - 439042 \\
& 56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3* \\
& (-4*a*c - b^2)^{15})^{(1/2)} - 995*a*b^{19}*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 245*a*b^4*c*(-(4*a*c - b^2)^{15})^{(1/2)})) / (512*(a^7*b^{20}*e^2 + 10 \\
& 48576*a^{17}*c^{10}*e^2 - 40*a^8*b^{18}*c*e^2 + 720*a^9*b^{16}*c^2*e^2 - 7680*a^{10}* \\
& b^{14}*c^3*e^2 + 53760*a^{11}*b^{12}*c^4*e^2 - 258048*a^{12}*b^{10}*c^5*e^2 + 860160* \\
& a^{13}*b^8*c^6*e^2 - 1966080*a^{14}*b^6*c^7*e^2 + 2949120*a^{15}*b^4*c^8*e^2 - 26 \\
& 21440*a^{16}*b^2*c^9*e^2))^{(1/2)}*((-9*(25*b^{21} - 25*b^6*(-(4*a*c - b^2)^{15}) \\
& ^{(1/2)} + 18923520*a^{10}*b*c^{10} + 17794*a^2*b^{17}*c^2 - 188095*a^3*b^{15}*c^3 + \\
& 1299860*a^4*b^{13}*c^4 - 6126640*a^5*b^{11}*c^5 + 19905600*a^6*b^9*c^6 - 439042 \\
& 56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3* \\
& (-4*a*c - b^2)^{15})^{(1/2)} - 995*a*b^{19}*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} + 245*a*b^4*c*(-(4*a*c - b^2)^{15})^{(1/2)})) / (512*(a^7*b^{20}*e^2 + 10 \\
& 48576*a^{17}*c^{10}*e^2 - 40*a^8*b^{18}*c*e^2 + 720*a^9*b^{16}*c^2*e^2 - 7680*a^{10}* \\
& b^{14}*c^3*e^2 + 53760*a^{11}*b^{12}*c^4*e^2 - 258048*a^{12}*b^{10}*c^5*e^2 + 860160* \\
& a^{13}*b^8*c^6*e^2 - 1966080*a^{14}*b^6*c^7*e^2 + 2949120*a^{15}*b^4*c^8*e^2 - 26 \\
& 21440*a^{16}*b^2*c^9*e^2))^{(1/2)}*(x*(1099511627776*a^{26}*b*c^{13}*e^{14} - 262144 \\
& *a^{15}*b^{23}*c^2*e^{14} + 11534336*a^{16}*b^{21}*c^3*e^{14} - 230686720*a^{17}*b^{19}*c^4 \\
& *e^{14} + 2768240640*a^{18}*b^{17}*c^5*e^{14} - 22145925120*a^{19}*b^{15}*c^6*e^{14} + 12 \\
& 4017180672*a^{20}*b^{13}*c^7*e^{14} - 496068722688*a^{21}*b^{11}*c^8*e^{14} + 141733920 \\
& 7680*a^{22}*b^9*c^9*e^{14} - 2834678415360*a^{23}*b^7*c^{10}*e^{14} + 3779571220480*a \\
& ^{24}*b^5*c^{11}*e^{14} - 3023656976384*a^{25}*b^3*c^{12}*e^{14}) + 1099511627776*a^{26}* \\
& b*c^{13}*d*e^{13} - 262144*a^{15}*b^{23}*c^2*d*e^{13} + 11534336*a^{16}*b^{21}*c^3*d*e^{13} \\
& - 230686720*a^{17}*b^{19}*c^4*d*e^{13} + 2768240640*a^{18}*b^{17}*c^5*d*e^{13} - 22145 \\
& 925120*a^{19}*b^{15}*c^6*d*e^{13} + 124017180672*a^{20}*b^{13}*c^7*d*e^{13} - 496068722 \\
& 688*a^{21}*b^{11}*c^8*d*e^{13} + 1417339207680*a^{22}*b^9*c^9*d*e^{13} - 283467841536 \\
& 0*a^{23}*b^7*c^{10}*d*e^{13} + 3779571220480*a^{24}*b^5*c^{11}*d*e^{13} - 3023656976384 \\
& *a^{25}*b^3*c^{12}*d*e^{13}) - 1185410973696*a^{23}*b*c^{13}*e^{12} + 245760*a^{12}*b^{23}* \\
& c^2*e^{12} - 10911744*a^{13}*b^{21}*c^3*e^{12} + 220397568*a^{14}*b^{19}*c^4*e^{12} - 267 \\
& 3082368*a^{15}*b^{17}*c^5*e^{12} + 21630025728*a^{16}*b^{15}*c^6*e^{12} - 122607894528* \\
& a^{17}*b^{13}*c^7*e^{12} + 496773365760*a^{18}*b^{11}*c^8*e^{12} - 1438679826432*a^{19}*b \\
& ^9*c^9*e^{12} + 2918430277632*a^{20}*b^7*c^{10}*e^{12} - 3949222428672*a^{21}*b^5*c^{11} \\
& *e^{12} + 3208340570112*a^{22}*b^3*c^{12}*e^{12}) + x*(271790899200*a^{20}*c^{14}*e^{12} \\
& - 230400*a^9*b^{22}*c^3*e^{12} + 9861120*a^{10}*b^{20}*c^4*e^{12} - 191038464*a^{11}*b \\
& ^{18}*c^5*e^{12} + 2207803392*a^{12}*b^{16}*c^6*e^{12} - 16878108672*a^{13}*b^{14}*c^7*e^{12} \\
& + 89374851072*a^{14}*b^{12}*c^8*e^{12} - 333226967040*a^{15}*b^{10}*c^9*e^{12} + 869 \\
& 815812096*a^{16}*b^8*c^{10}*e^{12} - 1543847804928*a^{17}*b^6*c^{11}*e^{12} + 174731349 \\
& 1968*a^{18}*b^4*c^{12}*e^{12} - 1101055131648*a^{19}*b^2*c^{13}*e^{12}) + 271790899200* \\
& a^{20}*c^{14}*d*e^{11} - 230400*a^9*b^{22}*c^3*d*e^{11} + 9861120*a^{10}*b^{20}*c^4*d*e^{11} \\
& - 191038464*a^{11}*b^{18}*c^5*d*e^{11} + 2207803392*a^{12}*b^{16}*c^6*d*e^{11} - 1687 \\
& 8108672*a^{13}*b^{14}*c^7*d*e^{11} + 89374851072*a^{14}*b^{12}*c^8*d*e^{11} - 333226967 \\
& 040*a^{15}*b^{10}*c^9*d*e^{11} + 869815812096*a^{16}*b^8*c^{10}*d*e^{11} - 154384780492 \\
& 8*a^{17}*b^6*c^{11}*d*e^{11} + 1747313491968*a^{18}*b^4*c^{12}*d*e^{11} - 1101055131648
\end{aligned}$$

$$\begin{aligned}
& a^{19}b^2c^{13}de^{11} * i + (- (9 * (25 * b^{21} - 25 * b^6 * (- (4 * a * c - b^2)^{15})^{1/2}) \\
& + 18923520 * a^{10} * b * c^{10} + 17794 * a^2 * b^{17} * c^2 - 188095 * a^3 * b^{15} * c^3 + 12998 \\
& 60 * a^4 * b^{13} * c^4 - 6126640 * a^5 * b^{11} * c^5 + 19905600 * a^6 * b^9 * c^6 - 43904256 * a^7 * b^7 * c^7 \\
& + 62684160 * a^8 * b^5 * c^8 - 52039680 * a^9 * b^3 * c^9 + 225 * a^3 * c^3 * (- (4 * a * c - b^2)^{15})^{1/2} \\
& - 995 * a * b^{19} * c - 694 * a^2 * b^2 * c^2 * (- (4 * a * c - b^2)^{15})^{1/2} + 245 * a * b^4 * c * (- (4 * a * c - b^2)^{15})^{1/2}) / (512 * (a^7 * b^{20} * e^2 + 1048576 \\
& * a^{17} * c^{10} * e^2 - 40 * a^8 * b^{18} * c * e^2 + 720 * a^9 * b^{16} * c^2 * e^2 - 7680 * a^{10} * b^{14} * c^3 * e^2 \\
& + 53760 * a^{11} * b^{12} * c^4 * e^2 - 258048 * a^{12} * b^{10} * c^5 * e^2 + 860160 * a^{13} * b^8 * c^6 * e^2 - 1966080 * a^{14} * b^6 * c^7 * e^2 \\
& + 2949120 * a^{15} * b^4 * c^8 * e^2 - 2621440 * a^{16} * b^2 * c^9 * e^2))^{1/2} * ((- (9 * (25 * b^{21} - 25 * b^6 * (- (4 * a * c - b^2)^{15})^{1/2}) \\
& + 18923520 * a^{10} * b * c^{10} + 17794 * a^2 * b^{17} * c^2 - 188095 * a^3 * b^{15} * c^3 + 12998 \\
& 60 * a^4 * b^{13} * c^4 - 6126640 * a^5 * b^{11} * c^5 + 19905600 * a^6 * b^9 * c^6 - 43904256 * a^7 * b^7 * c^7 \\
& + 62684160 * a^8 * b^5 * c^8 - 52039680 * a^9 * b^3 * c^9 + 225 * a^3 * c^3 * (- (4 * a * c - b^2)^{15})^{1/2} \\
& - 995 * a * b^{19} * c - 694 * a^2 * b^2 * c^2 * (- (4 * a * c - b^2)^{15})^{1/2} + 245 * a * b^4 * c * (- (4 * a * c - b^2)^{15})^{1/2}) / (512 * (a^7 * b^{20} * e^2 + 1048576 \\
& * a^{17} * c^{10} * e^2 - 40 * a^8 * b^{18} * c * e^2 + 720 * a^9 * b^{16} * c^2 * e^2 - 7680 * a^{10} * b^{14} * c^3 * e^2 \\
& + 53760 * a^{11} * b^{12} * c^4 * e^2 - 258048 * a^{12} * b^{10} * c^5 * e^2 + 860160 * a^{13} * b^8 * c^6 * e^2 - 1966080 * a^{14} * b^6 * c^7 * e^2 \\
& + 2949120 * a^{15} * b^4 * c^8 * e^2 - 2621440 * a^{16} * b^2 * c^9 * e^2))^{1/2} * (x * (1099511627776 * a^{26} * b * c^{13} * e^{14} - 262144 * a^{15} \\
& * b^{23} * c^2 * e^{14} + 11534336 * a^{16} * b^{21} * c^3 * e^{14} - 230686720 * a^{17} * b^{19} * c^4 * e^{14} \\
& + 2768240640 * a^{18} * b^{17} * c^5 * e^{14} - 22145925120 * a^{19} * b^{15} * c^6 * e^{14} + 1240171 \\
& 80672 * a^{20} * b^{13} * c^7 * e^{14} - 496068722688 * a^{21} * b^{11} * c^8 * e^{14} + 1417339207680 * \\
& a^{22} * b^9 * c^9 * e^{14} - 2834678415360 * a^{23} * b^7 * c^{10} * e^{14} + 3779571220480 * a^{24} * b^5 * c^{11} * e^{14} \\
& - 3023656976384 * a^{25} * b^3 * c^{12} * e^{14}) + 1099511627776 * a^{26} * b * c^{13} * d * e^{13} - 262144 * a^{15} * b^{23} * c^2 * d * e^{13} \\
& + 11534336 * a^{16} * b^{21} * c^3 * d * e^{13} - 230686720 * a^{17} * b^{19} * c^4 * d * e^{13} + 2768240640 * a^{18} * b^{17} * c^5 * d * e^{13} \\
& - 22145925120 * a^{19} * b^{15} * c^6 * d * e^{13} + 124017180672 * a^{20} * b^{13} * c^7 * d * e^{13} - 496068722688 * a^{21} * b^{11} * c^8 * d * e^{13} \\
& + 1417339207680 * a^{22} * b^9 * c^9 * d * e^{13} - 2834678415360 * a^{23} * b^7 * c^{10} * d * e^{13} + 3779571220480 * a^{24} * b^5 * c^{11} * d * e^{13} \\
& - 3023656976384 * a^{25} * b^3 * c^{12} * d * e^{13}) + 1185410973696 * a^{23} * b * c^{13} * e^{12} - 245760 * a^{12} * b^{23} * c^2 * e^{12} \\
& + 10911744 * a^{13} * b^{21} * c^3 * e^{12} - 220397568 * a^{14} * b^{19} * c^4 * e^{12} + 26730823 \\
& 68 * a^{15} * b^{17} * c^5 * e^{12} - 21630025728 * a^{16} * b^{15} * c^6 * e^{12} + 122607894528 * a^{17} * b^{13} * c^7 * e^{12} \\
& - 496773365760 * a^{18} * b^{11} * c^8 * e^{12} + 1438679826432 * a^{19} * b^9 * c^9 * e^{12} - 2918430277632 * a^{20} * b^7 * c^{10} * e^{12} \\
& + 3949222428672 * a^{21} * b^5 * c^{11} * e^{12} - 3208340570112 * a^{22} * b^3 * c^{12} * e^{12}) + x * (271790899200 * a^{20} * c^{14} * e^{12} - 23 \\
& 0400 * a^9 * b^{22} * c^3 * e^{12} + 9861120 * a^{10} * b^{20} * c^4 * e^{12} - 191038464 * a^{11} * b^{18} * c
\end{aligned}$$

$$\begin{aligned}
& ^5e^{12} + 2207803392a^{12}b^{16}c^6e^{12} - 16878108672a^{13}b^{14}c^7e^{12} + \\
& 89374851072a^{14}b^{12}c^8e^{12} - 333226967040a^{15}b^{10}c^9e^{12} + 86981581 \\
& 2096a^{16}b^8c^{10}e^{12} - 1543847804928a^{17}b^6c^{11}e^{12} + 1747313491968* \\
& a^{18}b^4c^{12}e^{12} - 1101055131648a^{19}b^2c^{13}e^{12}) + 271790899200a^{20} \\
& c^{14}d^1e^{11} - 230400a^9b^{22}c^3d^1e^{11} + 9861120a^{10}b^{20}c^4d^1e^{11} - 1 \\
& 91038464a^{11}b^{18}c^5d^1e^{11} + 2207803392a^{12}b^{16}c^6d^1e^{11} - 168781086 \\
& 72a^{13}b^{14}c^7d^1e^{11} + 89374851072a^{14}b^{12}c^8d^1e^{11} - 333226967040a \\
& ^{15}b^{10}c^9d^1e^{11} + 869815812096a^{16}b^8c^{10}d^1e^{11} - 1543847804928a^{1 \\
& 7}b^6c^{11}d^1e^{11} + 1747313491968a^{18}b^4c^{12}d^1e^{11} - 1101055131648a^{19} \\
& *b^2c^{13}d^1e^{11}) * 1i) / ((-(9*(25*b^{21} - 25*b^6*(-(4*a*c - b^2)^{15})^{1/2}) + 1 \\
& 8923520a^{10}b*c^{10} + 17794a^2*b^{17}c^2 - 188095a^3*b^{15}c^3 + 1299860a^4 \\
& *b^{13}c^4 - 6126640a^5*b^{11}c^5 + 19905600a^6*b^9c^6 - 43904256a^7*b^7 \\
& *c^7 + 62684160a^8*b^5c^8 - 52039680a^9*b^3c^9 + 225a^3c^3*(-(4*a*c - \\
& b^2)^{15})^{1/2} - 995*a*b^{19}c - 694*a^2*b^2c^2*(-(4*a*c - b^2)^{15})^{1/2} \\
& + 245*a*b^4c*(-(4*a*c - b^2)^{15})^{1/2}))/ (512*(a^7*b^{20}e^2 + 1048576a^{17} \\
& *c^{10}e^2 - 40a^8*b^{18}c^1e^2 + 720a^9*b^{16}c^2e^2 - 7680a^{10}b^{14}c^3e \\
& ^2 + 53760a^{11}b^{12}c^4e^2 - 258048a^{12}b^{10}c^5e^2 + 860160a^{13}b^8c \\
& ^6e^2 - 1966080a^{14}b^6c^7e^2 + 2949120a^{15}b^4c^8e^2 - 2621440a^{16} \\
& *b^2c^9e^2)))^{1/2} * ((-(9*(25*b^{21} - 25*b^6*(-(4*a*c - b^2)^{15})^{1/2}) + 1 \\
& 8923520a^{10}b*c^{10} + 17794a^2*b^{17}c^2 - 188095a^3*b^{15}c^3 + 1299860a^4 \\
& *b^{13}c^4 - 6126640a^5*b^{11}c^5 + 19905600a^6*b^9c^6 - 43904256a^7*b^7 \\
& *c^7 + 62684160a^8*b^5c^8 - 52039680a^9*b^3c^9 + 225a^3c^3*(-(4*a*c - \\
& b^2)^{15})^{1/2} - 995*a*b^{19}c - 694*a^2*b^2c^2*(-(4*a*c - b^2)^{15})^{1/2} \\
& + 245*a*b^4c*(-(4*a*c - b^2)^{15})^{1/2}))/ (512*(a^7*b^{20}e^2 + 1048576a^{17} \\
& *c^{10}e^2 - 40a^8*b^{18}c^1e^2 + 720a^9*b^{16}c^2e^2 - 7680a^{10}b^{14}c^3e \\
& ^2 + 53760a^{11}b^{12}c^4e^2 - 258048a^{12}b^{10}c^5e^2 + 860160a^{13}b^8c \\
& ^6e^2 - 1966080a^{14}b^6c^7e^2 + 2949120a^{15}b^4c^8e^2 - 2621440a^{16} \\
& *b^2c^9e^2)))^{1/2} * (x*(1099511627776a^{26}b^13c^{13}e^{14} - 262144a^{15}b^{23} \\
& *c^2e^{14} + 11534336a^{16}b^{21}c^3e^{14} - 230686720a^{17}b^{19}c^4e^{14} + 27 \\
& 68240640a^{18}b^{17}c^5e^{14} - 22145925120a^{19}b^{15}c^6e^{14} + 124017180672 \\
& *a^{20}b^{13}c^7e^{14} - 496068722688a^{21}b^{11}c^8e^{14} + 1417339207680a^{22} \\
& b^9c^9e^{14} - 2834678415360a^{23}b^7c^{10}e^{14} + 3779571220480a^{24}b^5c^ \\
& 11e^{14} - 3023656976384a^{25}b^3c^{12}e^{14}) + 1099511627776a^{26}b^13c^{13}d^1e \\
& ^{13} - 262144a^{15}b^{23}c^2d^1e^{13} + 11534336a^{16}b^{21}c^3d^1e^{13} - 2306867 \\
& 20a^{17}b^{19}c^4d^1e^{13} + 2768240640a^{18}b^{17}c^5d^1e^{13} - 22145925120a^{1 \\
& 9}b^{15}c^6d^1e^{13} + 124017180672a^{20}b^{13}c^7d^1e^{13} - 496068722688a^{21}b
\end{aligned}$$

$$\begin{aligned}
& ^{11}c^8d^3e^{13} + 1417339207680a^{22}b^9c^9d^3e^{13} - 2834678415360a^{23}b^7 \\
& *c^{10}d^3e^{13} + 3779571220480a^{24}b^5c^{11}d^3e^{13} - 3023656976384a^{25}b^3* \\
& c^{12}d^3e^{13}) + 1185410973696a^{23}b^3c^{13}e^{12} - 245760a^{12}b^{23}c^2e^{12} + \\
& 10911744a^{13}b^{21}c^3e^{12} - 220397568a^{14}b^{19}c^4e^{12} + 2673082368a^{15} \\
& b^{17}c^5e^{12} - 21630025728a^{16}b^{15}c^6e^{12} + 122607894528a^{17}b^{13} \\
& c^7e^{12} - 496773365760a^{18}b^{11}c^8e^{12} + 1438679826432a^{19}b^9c^9e^{12} \\
& - 2918430277632a^{20}b^7c^{10}e^{12} + 3949222428672a^{21}b^5c^{11}e^{12} - 3 \\
& 208340570112a^{22}b^3c^{12}e^{12}) + x*(271790899200a^{20}c^{14}e^{12} - 230400* \\
& a^9b^{22}c^3e^{12} + 9861120a^{10}b^{20}c^4e^{12} - 191038464a^{11}b^{18}c^5e^{12} \\
& + 2207803392a^{12}b^{16}c^6e^{12} - 16878108672a^{13}b^{14}c^7e^{12} + 89374 \\
& 851072a^{14}b^{12}c^8e^{12} - 333226967040a^{15}b^{10}c^9e^{12} + 869815812096* \\
& a^{16}b^8c^{10}e^{12} - 1543847804928a^{17}b^6c^{11}e^{12} + 1747313491968a^{18} \\
& b^4c^{12}e^{12} - 1101055131648a^{19}b^2c^{13}e^{12}) + 271790899200a^{20}c^{14} \\
& d^3e^{11} - 230400a^9b^{22}c^3d^3e^{11} + 9861120a^{10}b^{20}c^4d^3e^{11} - 191038 \\
& 464a^{11}b^{18}c^5d^3e^{11} + 2207803392a^{12}b^{16}c^6d^3e^{11} - 16878108672a^{13} \\
& b^{14}c^7d^3e^{11} + 89374851072a^{14}b^{12}c^8d^3e^{11} - 333226967040a^{15}b^{10} \\
& c^9d^3e^{11} + 869815812096a^{16}b^8c^{10}d^3e^{11} - 1543847804928a^{17}b^6 \\
& c^{11}d^3e^{11} + 1747313491968a^{18}b^4c^{12}d^3e^{11} - 1101055131648a^{19}b^2 \\
& c^{13}d^3e^{11}) - ((-9*(25b^{21} - 25b^6*(-(4ac - b^2)^{15})^{1/2}) + 18923520* \\
& a^{10}b^3c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 \\
& - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 6 \\
& 2684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3*(-(4ac - b^2)^{15} \\
&)^{1/2} - 995a^3b^{19}c - 694a^2b^2c^2*(-(4ac - b^2)^{15})^{1/2} + 245a^3 \\
& b^4c*(-(4ac - b^2)^{15})^{1/2}))/((512*(a^7b^{20}e^2 + 1048576a^{17}c^{10}e^2 \\
& - 40a^8b^{18}c^3e^2 + 720a^9b^{16}c^2e^2 - 7680a^{10}b^{14}c^3e^2 + 537 \\
& 60a^{11}b^{12}c^4e^2 - 258048a^{12}b^{10}c^5e^2 + 860160a^{13}b^8c^6e^2 - \\
& 1966080a^{14}b^6c^7e^2 + 2949120a^{15}b^4c^8e^2 - 2621440a^{16}b^2c^9 \\
& e^2)))^{1/2} * ((-9*(25b^{21} - 25b^6*(-(4ac - b^2)^{15})^{1/2}) + 18923520* \\
& a^{10}b^3c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 \\
& - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 6 \\
& 2684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3*(-(4ac - b^2)^{15} \\
&)^{1/2} - 995a^3b^{19}c - 694a^2b^2c^2*(-(4ac - b^2)^{15})^{1/2} + 245a^3 \\
& b^4c*(-(4ac - b^2)^{15})^{1/2}))/((512*(a^7b^{20}e^2 + 1048576a^{17}c^{10}e^2 \\
& - 40a^8b^{18}c^3e^2 + 720a^9b^{16}c^2e^2 - 7680a^{10}b^{14}c^3e^2 + 537 \\
& 60a^{11}b^{12}c^4e^2 - 258048a^{12}b^{10}c^5e^2 + 860160a^{13}b^8c^6e^2 - \\
& 1966080a^{14}b^6c^7e^2 + 2949120a^{15}b^4c^8e^2 - 2621440a^{16}b^2c^9 \\
& e^2)))^{1/2} * ((-9*(25b^{21} - 25b^6*(-(4ac - b^2)^{15})^{1/2}) + 18923520* \\
& a^{10}b^3c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 \\
& - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 6 \\
& 2684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3*(-(4ac - b^2)^{15} \\
&)^{1/2} - 995a^3b^{19}c - 694a^2b^2c^2*(-(4ac - b^2)^{15})^{1/2} + 245a^3 \\
& b^4c*(-(4ac - b^2)^{15})^{1/2}))/((512*(a^7b^{20}e^2 + 1048576a^{17}c^{10}e^2 \\
& - 40a^8b^{18}c^3e^2 + 720a^9b^{16}c^2e^2 - 7680a^{10}b^{14}c^3e^2 + 537 \\
& 60a^{11}b^{12}c^4e^2 - 258048a^{12}b^{10}c^5e^2 + 860160a^{13}b^8c^6e^2 - \\
& 1966080a^{14}b^6c^7e^2 + 2949120a^{15}b^4c^8e^2 - 2621440a^{16}b^2c^9
\end{aligned}$$

$$\begin{aligned}
& *e^2)))^{(1/2)} * (x * (1099511627776 * a^{26} * b * c^{13} * e^{14} - 262144 * a^{15} * b^{23} * c^2 * e^{14} \\
& + 11534336 * a^{16} * b^{21} * c^3 * e^{14} - 230686720 * a^{17} * b^{19} * c^4 * e^{14} + 2768240640 \\
& * a^{18} * b^{17} * c^5 * e^{14} - 22145925120 * a^{19} * b^{15} * c^6 * e^{14} + 124017180672 * a^{20} * b^{13} * c^7 * e^{14} \\
& - 496068722688 * a^{21} * b^{11} * c^8 * e^{14} + 1417339207680 * a^{22} * b^9 * c^9 * e^{14} - 2834678415360 * a^{23} * b^7 * c^{10} * e^{14} \\
& + 3779571220480 * a^{24} * b^5 * c^{11} * e^{14} - 3023656976384 * a^{25} * b^3 * c^{12} * e^{14}) + 1099511627776 * a^{26} * b * c^{13} * d * e^{13} - 26 \\
& 2144 * a^{15} * b^{23} * c^2 * d * e^{13} + 11534336 * a^{16} * b^{21} * c^3 * d * e^{13} - 230686720 * a^{17} * b^{19} * c^4 * d * e^{13} \\
& + 2768240640 * a^{18} * b^{17} * c^5 * d * e^{13} - 22145925120 * a^{19} * b^{15} * c^6 * d * e^{13} + 124017180672 * a^{20} * b^{13} * c^7 * d * e^{13} \\
& - 496068722688 * a^{21} * b^{11} * c^8 * d * e^{13} + 1417339207680 * a^{22} * b^9 * c^9 * d * e^{13} - 2834678415360 * a^{23} * b^7 * c^{10} * d * e^{13} \\
& + 3779571220480 * a^{24} * b^5 * c^{11} * d * e^{13} - 3023656976384 * a^{25} * b^3 * c^{12} * d * e^{13}) \\
& - 1185410973696 * a^{23} * b * c^{13} * e^{12} + 245760 * a^{12} * b^{23} * c^2 * e^{12} - 1091174 \\
& 4 * a^{13} * b^{21} * c^3 * e^{12} + 220397568 * a^{14} * b^{19} * c^4 * e^{12} - 2673082368 * a^{15} * b^{17} * c^5 * e^{12} \\
& + 21630025728 * a^{16} * b^{15} * c^6 * e^{12} - 122607894528 * a^{17} * b^{13} * c^7 * e^{12} \\
& + 496773365760 * a^{18} * b^{11} * c^8 * e^{12} - 1438679826432 * a^{19} * b^9 * c^9 * e^{12} + 2918 \\
& 430277632 * a^{20} * b^7 * c^{10} * e^{12} - 3949222428672 * a^{21} * b^5 * c^{11} * e^{12} + 320834057 \\
& 0112 * a^{22} * b^3 * c^{12} * e^{12}) + x * (271790899200 * a^{20} * c^{14} * e^{12} - 230400 * a^9 * b^{22} \\
& * c^3 * e^{12} + 9861120 * a^{10} * b^{20} * c^4 * e^{12} - 191038464 * a^{11} * b^{18} * c^5 * e^{12} + 220 \\
& 7803392 * a^{12} * b^{16} * c^6 * e^{12} - 16878108672 * a^{13} * b^{14} * c^7 * e^{12} + 89374851072 * a^{14} * b^{12} * c^8 * e^{12} \\
& - 333226967040 * a^{15} * b^{10} * c^9 * e^{12} + 869815812096 * a^{16} * b^8 * c^{10} * e^{12} - 1543847804928 * a^{17} * b^6 * c^{11} * e^{12} \\
& + 1747313491968 * a^{18} * b^4 * c^{12} * e^{12} - 1101055131648 * a^{19} * b^2 * c^{13} * e^{12}) + 271790899200 * a^{20} * c^{14} * d * e^{11} - \\
& 230400 * a^9 * b^{22} * c^3 * d * e^{11} + 9861120 * a^{10} * b^{20} * c^4 * d * e^{11} - 191038464 * a^{11} \\
& * b^{18} * c^5 * d * e^{11} + 2207803392 * a^{12} * b^{16} * c^6 * d * e^{11} - 16878108672 * a^{13} * b^{14} * c^7 * d * e^{11} \\
& + 89374851072 * a^{14} * b^{12} * c^8 * d * e^{11} - 333226967040 * a^{15} * b^{10} * c^9 * d * e^{11} + 869815812096 * a^{16} * b^8 * c^{10} * d * e^{11} \\
& - 1543847804928 * a^{17} * b^6 * c^{11} * d * e^{11} + 1747313491968 * a^{18} * b^4 * c^{12} * d * e^{11} - 1101055131648 * a^{19} * b^2 * c^{13} * d * e^{11}) \\
& + 191102976000 * a^{17} * c^{14} * e^{10} + 2851200 * a^9 * b^{16} * c^6 * e^{10} - 92568960 * a^{10} * b^{14} * c^7 * e^{10} \\
& + 1312630272 * a^{11} * b^{12} * c^8 * e^{10} - 10611136512 * a^{12} * b^{10} * c^9 * e^{10} + 53445353472 * a^{13} * b^8 * c^{10} * e^{10} \\
& - 171591892992 * a^{14} * b^6 * c^{11} * e^{10} + 342580396032 * a^{15} * b^4 * c^{12} * e^{10} - 388363714560 * a^{16} * b^2 * c^{13} * e^{10}) * (- (9 * \\
& (25 * b^{21} - 25 * b^6 * (- (4 * a * c - b^2)^{15})^{(1/2)} + 18923520 * a^{10} * b * c^{10} + 17794 * a^2 * b^{17} * c^2 \\
& - 188095 * a^3 * b^{15} * c^3 + 1299860 * a^4 * b^{13} * c^4 - 6126640 * a^5 * b^{11} * c^5 + 19905600 * a^6 * b^9 * c^6 \\
& - 43904256 * a^7 * b^7 * c^7 + 62684160 * a^8 * b^5 * c^8 - 52039680 * a^9 * b^3 * c^9 + 225 * a^3 * c^3 * (- (4 * a * c - b^2)^{15})^{(1/2)} \\
& - 995 * a * b^{19} * c - 694 * a^2 * b^2 * c^2 * (- (4 * a * c - b^2)^{15})^{(1/2)} + 245 * a * b^4 * c * (- (4 * a * c - b^2)^{15})^{(1/2)})) / (512 * (a^7 * b^{20} * e^2 \\
& + 1048576 * a^{17} * c^{10} * e^2 - 40 * a^8 * b^{18} * c * e^2 + 720 * a^9 * b^{16} * c^2 * e^2 - 7680 * a^{10} * b^{14} * c^3 * e^2 \\
& + 53760 * a^{11} * b^{12} * c^4 * e^2 - 258048 * a^{12} * b^{10} * c^5 * e^2 + 860160 * a^{13} * b^8 * c^6 * e^2 - 1966080 * a^{14} * b^6 * c^7 * e^2 \\
& + 2949120 * a^{15} * b^4 * c^8 * e^2 - 2621440 * a^{16} * b^2 * c^9 * e^2)))^{(1/2)} * 2i - a \\
& \tan(((- (9 * (25 * b^{21} + 25 * b^6 * (- (4 * a * c - b^2)^{15})^{(1/2)} + 18923520 * a^{10} * b * c^{10} + 17794 * a^2 * b^{17} * c^2 \\
& - 188095 * a^3 * b^{15} * c^3 + 1299860 * a^4 * b^{13} * c^4 - 6126640 * a^5 * b^{11} * c^5 + 19905600 * a^6 * b^9 * c^6 \\
& - 43904256 * a^7 * b^7 * c^7 + 62684160 * a^8 * b^5 * c^8 - 52039680 * a^9 * b^3 * c^9 - 225 * a^3 * c^3 * (- (4 * a * c - b^2)^{15})^{(1/2)} \\
& - 995 * a * b^{19} * c + 694 * a^2 * b^2 * c^2 * (- (4 * a * c - b^2)^{15})^{(1/2)} - 245 * a * b^4 * c * (- (4
\end{aligned}$$

$$\begin{aligned}
& *a*c - b^2)^{15})^{(1/2)))/(512*(a^7*b^{20}*e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8 \\
& *b^{18}*c*e^2 + 720*a^9*b^{16}*c^2*e^2 - 7680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12} \\
& *c^4*e^2 - 258048*a^{12}*b^{10}*c^5*e^2 + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14} \\
& *b^6*c^7*e^2 + 2949120*a^{15}*b^4*c^8*e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1 \\
& /2)*((-9*(25*b^{21} + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^1 \\
& 0 + 17794*a^2*b^{17}*c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 61266 \\
& 40*a^5*b^{11}*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8 \\
& *b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - \\
& 995*a*b^{19}*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4 \\
& *a*c - b^2)^{15})^{(1/2)))/(512*(a^7*b^{20}*e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8 \\
& *b^{18}*c*e^2 + 720*a^9*b^{16}*c^2*e^2 - 7680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12} \\
& *c^4*e^2 - 258048*a^{12}*b^{10}*c^5*e^2 + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14} \\
& *b^6*c^7*e^2 + 2949120*a^{15}*b^4*c^8*e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1 \\
& /2)*((-9*(25*b^{21} + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^1 \\
& 0 + 17794*a^2*b^{17}*c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 61266 \\
& 40*a^5*b^{11}*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8 \\
& *b^5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - \\
& 995*a*b^{19}*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4 \\
& *a*c - b^2)^{15})^{(1/2)))/(512*(a^7*b^{20}*e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8 \\
& *b^{18}*c*e^2 + 720*a^9*b^{16}*c^2*e^2 - 7680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12} \\
& *c^4*e^2 - 258048*a^{12}*b^{10}*c^5*e^2 + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14} \\
& *b^6*c^7*e^2 + 2949120*a^{15}*b^4*c^8*e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1 \\
& /2)*(x*(1099511627776*a^{26}*b*c^{13}*e^{14} - 262144*a^{15}*b^{23}*c^2*e^{14} + 115343 \\
& 36*a^{16}*b^{21}*c^3*e^{14} - 230686720*a^{17}*b^{19}*c^4*e^{14} + 2768240640*a^{18}*b^{17} \\
& *c^5*e^{14} - 22145925120*a^{19}*b^{15}*c^6*e^{14} + 124017180672*a^{20}*b^{13}*c^7*e^{14} \\
& 4 - 496068722688*a^{21}*b^{11}*c^8*e^{14} + 1417339207680*a^{22}*b^9*c^9*e^{14} - 283 \\
& 4678415360*a^{23}*b^7*c^{10}*e^{14} + 3779571220480*a^{24}*b^5*c^{11}*e^{14} - 30236569 \\
& 76384*a^{25}*b^3*c^{12}*e^{14}) + 1099511627776*a^{26}*b*c^{13}*d*e^{13} - 262144*a^{15} \\
& *b^{23}*c^2*d*e^{13} + 11534336*a^{16}*b^{21}*c^3*d*e^{13} - 230686720*a^{17}*b^{19}*c^4*d \\
& *e^{13} + 2768240640*a^{18}*b^{17}*c^5*d*e^{13} - 22145925120*a^{19}*b^{15}*c^6*d*e^{13} \\
& + 124017180672*a^{20}*b^{13}*c^7*d*e^{13} - 496068722688*a^{21}*b^{11}*c^8*d*e^{13} + 1 \\
& 417339207680*a^{22}*b^9*c^9*d*e^{13} - 2834678415360*a^{23}*b^7*c^{10}*d*e^{13} + 377 \\
& 9571220480*a^{24}*b^5*c^{11}*d*e^{13} - 3023656976384*a^{25}*b^3*c^{12}*d*e^{13}) - 118 \\
& 5410973696*a^{23}*b*c^{13}*e^{12} + 245760*a^{12}*b^{23}*c^2*e^{12} - 10911744*a^{13}*b^2 \\
& 1*c^3*e^{12} + 220397568*a^{14}*b^{19}*c^4*e^{12} - 2673082368*a^{15}*b^{17}*c^5*e^{12} + \\
& 21630025728*a^{16}*b^{15}*c^6*e^{12} - 122607894528*a^{17}*b^{13}*c^7*e^{12} + 4967733 \\
& 65760*a^{18}*b^{11}*c^8*e^{12} - 1438679826432*a^{19}*b^9*c^9*e^{12} + 2918430277632*a \\
& ^{20}*b^7*c^{10}*e^{12} - 3949222428672*a^{21}*b^5*c^{11}*e^{12} + 3208340570112*a^{22} \\
& *b^3*c^{12}*e^{12}) + x*(271790899200*a^{20}*c^{14}*e^{12} - 230400*a^9*b^{22}*c^3*e^{12} \\
& + 9861120*a^{10}*b^{20}*c^4*e^{12} - 191038464*a^{11}*b^{18}*c^5*e^{12} + 2207803392*a^ \\
& ^{12}*b^{16}*c^6*e^{12} - 16878108672*a^{13}*b^{14}*c^7*e^{12} + 89374851072*a^{14}*b^{12}*c \\
& ^8*e^{12} - 333226967040*a^{15}*b^{10}*c^9*e^{12} + 869815812096*a^{16}*b^8*c^{10}*e^{12} \\
& - 1543847804928*a^{17}*b^6*c^{11}*e^{12} + 1747313491968*a^{18}*b^4*c^{12}*e^{12} - 11 \\
& 01055131648*a^{19}*b^2*c^{13}*e^{12}) + 271790899200*a^{20}*c^{14}*d*e^{11} - 230400*a^ \\
& ^9*b^{22}*c^3*d*e^{11} + 9861120*a^{10}*b^{20}*c^4*d*e^{11} - 191038464*a^{11}*b^{18}*c^5*
\end{aligned}$$

$$\begin{aligned}
& d^*e^{11} + 2207803392*a^{12}*b^{16}*c^6*d^*e^{11} - 16878108672*a^{13}*b^{14}*c^7*d^*e^{11} \\
& + 89374851072*a^{14}*b^{12}*c^8*d^*e^{11} - 333226967040*a^{15}*b^{10}*c^9*d^*e^{11} + 8 \\
& 69815812096*a^{16}*b^8*c^{10}*d^*e^{11} - 1543847804928*a^{17}*b^6*c^{11}*d^*e^{11} + 174 \\
& 7313491968*a^{18}*b^4*c^{12}*d^*e^{11} - 1101055131648*a^{19}*b^2*c^{13}*d^*e^{11}) * i + \\
& (- (9*(25*b^{21} + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^{10} + 1 \\
& 7794*a^2*b^{17}*c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a^5 \\
& *b^{11}*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5 \\
& *c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a \\
& *b^{19}*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4*a*c \\
& - b^2)^{15})^{(1/2)})) / (512*(a^7*b^{20}*e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8*b^{18} \\
& *c*e^2 + 720*a^9*b^{16}*c^2*e^2 - 7680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12}*c^4 \\
& *e^2 - 258048*a^{12}*b^{10}*c^5*e^2 + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14}*b^6 \\
& *c^7*e^2 + 2949120*a^{15}*b^4*c^8*e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1/2)} * (\\
& (- (9*(25*b^{21} + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^{10} + 1 \\
& 7794*a^2*b^{17}*c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a^5 \\
& *b^{11}*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5 \\
& *c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a \\
& *b^{19}*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4*a*c \\
& - b^2)^{15})^{(1/2)})) / (512*(a^7*b^{20}*e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8*b^{18} \\
& *c*e^2 + 720*a^9*b^{16}*c^2*e^2 - 7680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12}*c^4 \\
& *e^2 - 258048*a^{12}*b^{10}*c^5*e^2 + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14}*b^6 \\
& *c^7*e^2 + 2949120*a^{15}*b^4*c^8*e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1/2)} * (\\
& (- (9*(25*b^{21} + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^{10} + 1 \\
& 7794*a^2*b^{17}*c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a^5 \\
& *b^{11}*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5 \\
& *c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a \\
& *b^{19}*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4*a*c \\
& - b^2)^{15})^{(1/2)})) / (512*(a^7*b^{20}*e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8*b^{18} \\
& *c*e^2 + 720*a^9*b^{16}*c^2*e^2 - 7680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12}*c^4 \\
& *e^2 - 258048*a^{12}*b^{10}*c^5*e^2 + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14}*b^6 \\
& *c^7*e^2 + 2949120*a^{15}*b^4*c^8*e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1/2)} * (\\
& x*(1099511627776*a^{26}*b*c^{13}*e^{14} - 262144*a^{15}*b^{23}*c^2*e^{14} + 11534336*a^{16} \\
& *b^{21}*c^3*e^{14} - 230686720*a^{17}*b^{19}*c^4*e^{14} + 2768240640*a^{18}*b^{17}*c^5* \\
& e^{14} - 22145925120*a^{19}*b^{15}*c^6*e^{14} + 124017180672*a^{20}*b^{13}*c^7*e^{14} - 4 \\
& 96068722688*a^{21}*b^{11}*c^8*e^{14} + 1417339207680*a^{22}*b^9*c^9*e^{14} - 28346784 \\
& 15360*a^{23}*b^7*c^{10}*e^{14} + 3779571220480*a^{24}*b^5*c^{11}*e^{14} - 3023656976384 \\
& *a^{25}*b^3*c^{12}*e^{14}) + 1099511627776*a^{26}*b*c^{13}*d^*e^{13} - 262144*a^{15}*b^{23}* \\
& c^2*d^*e^{13} + 11534336*a^{16}*b^{21}*c^3*d^*e^{13} - 230686720*a^{17}*b^{19}*c^4*d^*e^{13} \\
& + 2768240640*a^{18}*b^{17}*c^5*d^*e^{13} - 22145925120*a^{19}*b^{15}*c^6*d^*e^{13} + 124 \\
& 017180672*a^{20}*b^{13}*c^7*d^*e^{13} - 496068722688*a^{21}*b^{11}*c^8*d^*e^{13} + 141733 \\
& 9207680*a^{22}*b^9*c^9*d^*e^{13} - 2834678415360*a^{23}*b^7*c^{10}*d^*e^{13} + 37795712 \\
& 20480*a^{24}*b^5*c^{11}*d^*e^{13} - 3023656976384*a^{25}*b^3*c^{12}*d^*e^{13}) + 11854109 \\
& 73696*a^{23}*b*c^{13}*e^{12} - 245760*a^{12}*b^{23}*c^2*e^{12} + 10911744*a^{13}*b^{21}*c^3 \\
& *e^{12} - 220397568*a^{14}*b^{19}*c^4*e^{12} + 2673082368*a^{15}*b^{17}*c^5*e^{12} - 2163 \\
& 0025728*a^{16}*b^{15}*c^6*e^{12} + 122607894528*a^{17}*b^{13}*c^7*e^{12} - 496773365760
\end{aligned}$$

$$\begin{aligned}
& a^{18}b^{11}c^8e^{12} + 1438679826432a^{19}b^9c^9e^{12} - 2918430277632a^{20}b^7c^{10}e^{12} + 3949222428672a^{21}b^5c^{11}e^{12} - 3208340570112a^{22}b^3c^{12}e^{12} \\
& + x(271790899200a^{20}c^{14}e^{12} - 230400a^9b^{22}c^3e^{12} + 9861120a^{10}b^{20}c^4e^{12} - 191038464a^{11}b^{18}c^5e^{12} + 2207803392a^{12}b^{16}c^6e^{12} \\
& - 16878108672a^{13}b^{14}c^7e^{12} + 89374851072a^{14}b^{12}c^8e^{12} - 333226967040a^{15}b^{10}c^9e^{12} + 869815812096a^{16}b^8c^{10}e^{12} - 1543847804928a^{17}b^6c^{11}e^{12} \\
& + 1747313491968a^{18}b^4c^{12}e^{12} - 1101055131648a^{19}b^2c^{13}e^{12}) + 271790899200a^{20}c^{14}d^{11}e^{11} - 230400a^9b^2c^3d^{11}e^{11} + 9861120a^{10}b^{20}c^4d^{11}e^{11} \\
& - 191038464a^{11}b^{18}c^5d^{11}e^{11} + 2207803392a^{12}b^{16}c^6d^{11}e^{11} - 16878108672a^{13}b^{14}c^7d^{11}e^{11} + 89374851072a^{14}b^{12}c^8d^{11}e^{11} \\
& - 333226967040a^{15}b^{10}c^9d^{11}e^{11} + 869815812096a^{16}b^8c^{10}d^{11}e^{11} - 1543847804928a^{17}b^6c^{11}d^{11}e^{11} + 1747313491968a^{18}b^4c^{12}d^{11}e^{11} \\
& - 1101055131648a^{19}b^2c^{13}d^{11}e^{11}) \cdot i / \left((-9(25b^{21} + 25b^6(-4ac - b^2)^{15})^{1/2} + 18923520a^{10}b^3c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 - 225a^3c^3(-4ac - b^2)^{15})^{1/2} - 995a^2b^{19}c + 694a^2b^2c^2(-4ac - b^2)^{15})^{1/2} - 245a^2b^4c(-4ac - b^2)^{15})^{1/2} \right) \\
& \left(512(a^7b^{20}e^2 + 1048576a^{17}c^{10}e^2 - 40a^8b^{18}c^3e^2 + 720a^9b^{16}c^2e^2 - 7680a^{10}b^{14}c^3e^2 + 53760a^{11}b^{12}c^4e^2 - 258048a^{12}b^{10}c^5e^2 + 860160a^{13}b^8c^6e^2 - 1966080a^{14}b^6c^7e^2 + 2949120a^{15}b^4c^8e^2 - 2621440a^{16}b^2c^9e^2) \right)^{1/2} \cdot \left((-9(25b^{21} + 25b^6(-4ac - b^2)^{15})^{1/2} + 18923520a^{10}b^3c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 - 225a^3c^3(-4ac - b^2)^{15})^{1/2} - 995a^2b^{19}c + 694a^2b^2c^2(-4ac - b^2)^{15})^{1/2} - 245a^2b^4c(-4ac - b^2)^{15})^{1/2} \right) \\
& \left(512(a^7b^{20}e^2 + 1048576a^{17}c^{10}e^2 - 40a^8b^{18}c^3e^2 + 720a^9b^{16}c^2e^2 - 7680a^{10}b^{14}c^3e^2 + 53760a^{11}b^{12}c^4e^2 - 258048a^{12}b^{10}c^5e^2 + 860160a^{13}b^8c^6e^2 - 1966080a^{14}b^6c^7e^2 + 2949120a^{15}b^4c^8e^2 - 2621440a^{16}b^2c^9e^2) \right)^{1/2} \cdot \left((-9(25b^{21} + 25b^6(-4ac - b^2)^{15})^{1/2} + 18923520a^{10}b^3c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 - 225a^3c^3(-4ac - b^2)^{15})^{1/2} - 995a^2b^{19}c + 694a^2b^2c^2(-4ac - b^2)^{15})^{1/2} - 245a^2b^4c(-4ac - b^2)^{15})^{1/2} \right) \\
& \left(512(a^7b^{20}e^2 + 1048576a^{17}c^{10}e^2 - 40a^8b^{18}c^3e^2 + 720a^9b^{16}c^2e^2 - 7680a^{10}b^{14}c^3e^2 + 53760a^{11}b^{12}c^4e^2 - 258048a^{12}b^{10}c^5e^2 + 860160a^{13}b^8c^6e^2 - 1966080a^{14}b^6c^7e^2 + 2949120a^{15}b^4c^8e^2 - 2621440a^{16}b^2c^9e^2) \right)^{1/2} \cdot (x(1099511627776a^{26}b^3c^{13}e^{14} - 262144a^{15}b^{23}c^2e^{14} + 11534336a^{16}b^{21}c^3e^{14} - 230686720a^{17}b^{19}c^4e^{14} + 2768240640a^{18}b^{17}c^5e^{14} - 22145925120a^{19}b^{15}c^6e^{14} + 124017180672a^{20}b^{13}c^7e^{14} - 496068722688a^{21}b^{11}c^8e^{14} + 1417339207680a^{22}b^9c^9e^{14} - 2834678415360a^{23}b^7c^{10}e^{14} + 3779571220480a^{24}b^5c^{11}e^{14} - 3023656976384a^{25}
\end{aligned}$$

$$\begin{aligned}
& *b^3*c^{12}*e^{14}) + 1099511627776*a^{26}*b*c^{13}*d*e^{13} - 262144*a^{15}*b^{23}*c^2*d \\
& *e^{13} + 11534336*a^{16}*b^{21}*c^3*d*e^{13} - 230686720*a^{17}*b^{19}*c^4*d*e^{13} + 27 \\
& 68240640*a^{18}*b^{17}*c^5*d*e^{13} - 22145925120*a^{19}*b^{15}*c^6*d*e^{13} + 12401718 \\
& 0672*a^{20}*b^{13}*c^7*d*e^{13} - 496068722688*a^{21}*b^{11}*c^8*d*e^{13} + 14173392076 \\
& 80*a^{22}*b^9*c^9*d*e^{13} - 2834678415360*a^{23}*b^7*c^{10}*d*e^{13} + 3779571220480 \\
& *a^{24}*b^5*c^{11}*d*e^{13} - 3023656976384*a^{25}*b^3*c^{12}*d*e^{13}) + 1185410973696 \\
& *a^{23}*b*c^{13}*e^{12} - 245760*a^{12}*b^{23}*c^2*e^{12} + 10911744*a^{13}*b^{21}*c^3*e^{12} \\
& - 220397568*a^{14}*b^{19}*c^4*e^{12} + 2673082368*a^{15}*b^{17}*c^5*e^{12} - 216300257 \\
& 28*a^{16}*b^{15}*c^6*e^{12} + 122607894528*a^{17}*b^{13}*c^7*e^{12} - 496773365760*a^{18} \\
& *b^{11}*c^8*e^{12} + 1438679826432*a^{19}*b^9*c^9*e^{12} - 2918430277632*a^{20}*b^7*c \\
& ^{10}*e^{12} + 3949222428672*a^{21}*b^5*c^{11}*e^{12} - 3208340570112*a^{22}*b^3*c^{12}*e \\
& ^{12}) + x*(271790899200*a^{20}*c^{14}*e^{12} - 230400*a^9*b^{22}*c^3*e^{12} + 9861120* \\
& a^{10}*b^{20}*c^4*e^{12} - 191038464*a^{11}*b^{18}*c^5*e^{12} + 2207803392*a^{12}*b^{16}*c^ \\
& 6*e^{12} - 16878108672*a^{13}*b^{14}*c^7*e^{12} + 89374851072*a^{14}*b^{12}*c^8*e^{12} - \\
& 333226967040*a^{15}*b^{10}*c^9*e^{12} + 869815812096*a^{16}*b^8*c^{10}*e^{12} - 1543847 \\
& 804928*a^{17}*b^6*c^{11}*e^{12} + 1747313491968*a^{18}*b^4*c^{12}*e^{12} - 110105513164 \\
& 8*a^{19}*b^2*c^{13}*e^{12}) + 271790899200*a^{20}*c^{14}*d*e^{11} - 230400*a^9*b^{22}*c^3 \\
& *d*e^{11} + 9861120*a^{10}*b^{20}*c^4*d*e^{11} - 191038464*a^{11}*b^{18}*c^5*d*e^{11} + 2 \\
& 207803392*a^{12}*b^{16}*c^6*d*e^{11} - 16878108672*a^{13}*b^{14}*c^7*d*e^{11} + 8937485 \\
& 1072*a^{14}*b^{12}*c^8*d*e^{11} - 333226967040*a^{15}*b^{10}*c^9*d*e^{11} + 86981581209 \\
& 6*a^{16}*b^8*c^{10}*d*e^{11} - 1543847804928*a^{17}*b^6*c^{11}*d*e^{11} + 1747313491968 \\
& *a^{18}*b^4*c^{12}*d*e^{11} - 1101055131648*a^{19}*b^2*c^{13}*d*e^{11}) - ((-9*(25*b^{21} \\
& + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^{10} + 17794*a^2*b^{17} \\
& *c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a^5*b^{11}*c^5 + \\
& 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396 \\
& 80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a*b^{19}*c + 694 \\
& *a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4*a*c - b^2)^{15})^{(1 \\
& /2)))/(512*(a^7*b^{20}*e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8*b^{18}*c*e^2 + 720* \\
& a^9*b^{16}*c^2*e^2 - 7680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12}*c^4*e^2 - 25804 \\
& 8*a^{12}*b^{10}*c^5*e^2 + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14}*b^6*c^7*e^2 + \\
& 2949120*a^{15}*b^4*c^8*e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1/2)}*((-9*(25*b^{21} \\
& + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^{10} + 17794*a^2*b^{17} \\
& *c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a^5*b^{11}*c^5 + \\
& 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396 \\
& 80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a*b^{19}*c + 694 \\
& *a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4*a*c - b^2)^{15})^{(1 \\
& /2)))/(512*(a^7*b^{20}*e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8*b^{18}*c*e^2 + 720* \\
& a^9*b^{16}*c^2*e^2 - 7680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12}*c^4*e^2 - 25804 \\
& 8*a^{12}*b^{10}*c^5*e^2 + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14}*b^6*c^7*e^2 + \\
& 2949120*a^{15}*b^4*c^8*e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1/2)}*((-9*(25*b^{21} \\
& + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^{10} + 17794*a^2*b^{17} \\
& *c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a^5*b^{11}*c^5 + \\
& 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 520396 \\
& 80*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a*b^{19}*c + 694 \\
& *a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4*a*c - b^2)^{15})^{(1
\end{aligned}$$

$$\begin{aligned}
& /2)) / (512*(a^7*b^{20}*e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8*b^{18}*c*e^2 + 720* \\
& a^9*b^{16}*c^2*e^2 - 7680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12}*c^4*e^2 - 25804 \\
& 8*a^{12}*b^{10}*c^5*e^2 + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14}*b^6*c^7*e^2 + \\
& 2949120*a^{15}*b^4*c^8*e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1/2)}*(x*(1099511627 \\
& 776*a^{26}*b*c^{13}*e^{14} - 262144*a^{15}*b^{23}*c^2*e^{14} + 11534336*a^{16}*b^{21}*c^3*e \\
& ^{14} - 230686720*a^{17}*b^{19}*c^4*e^{14} + 2768240640*a^{18}*b^{17}*c^5*e^{14} - 221459 \\
& 25120*a^{19}*b^{15}*c^6*e^{14} + 124017180672*a^{20}*b^{13}*c^7*e^{14} - 496068722688*a \\
& ^{21}*b^{11}*c^8*e^{14} + 1417339207680*a^{22}*b^9*c^9*e^{14} - 2834678415360*a^{23}*b^ \\
& 7*c^{10}*e^{14} + 3779571220480*a^{24}*b^5*c^{11}*e^{14} - 3023656976384*a^{25}*b^3*c^ \\
& ^{12}*e^{14}) + 1099511627776*a^{26}*b*c^{13}*d*e^{13} - 262144*a^{15}*b^{23}*c^2*d*e^{13} + \\
& 11534336*a^{16}*b^{21}*c^3*d*e^{13} - 230686720*a^{17}*b^{19}*c^4*d*e^{13} + 2768240640 \\
& *a^{18}*b^{17}*c^5*d*e^{13} - 22145925120*a^{19}*b^{15}*c^6*d*e^{13} + 124017180672*a^{20} \\
& *b^{13}*c^7*d*e^{13} - 496068722688*a^{21}*b^{11}*c^8*d*e^{13} + 1417339207680*a^{22}* \\
& b^9*c^9*d*e^{13} - 2834678415360*a^{23}*b^7*c^{10}*d*e^{13} + 3779571220480*a^{24}*b^ \\
& 5*c^{11}*d*e^{13} - 3023656976384*a^{25}*b^3*c^{12}*d*e^{13}) - 1185410973696*a^{23}*b* \\
& c^{13}*e^{12} + 245760*a^{12}*b^{23}*c^2*e^{12} - 10911744*a^{13}*b^{21}*c^3*e^{12} + 22039 \\
& 7568*a^{14}*b^{19}*c^4*e^{12} - 2673082368*a^{15}*b^{17}*c^5*e^{12} + 21630025728*a^{16}* \\
& b^{15}*c^6*e^{12} - 122607894528*a^{17}*b^{13}*c^7*e^{12} + 496773365760*a^{18}*b^{11}*c^ \\
& 8*e^{12} - 1438679826432*a^{19}*b^9*c^9*e^{12} + 2918430277632*a^{20}*b^7*c^{10}*e^{12} \\
& - 3949222428672*a^{21}*b^5*c^{11}*e^{12} + 3208340570112*a^{22}*b^3*c^{12}*e^{12}) + x \\
& *(271790899200*a^{20}*c^{14}*e^{12} - 230400*a^9*b^{22}*c^3*e^{12} + 9861120*a^{10}*b^2 \\
& 0*c^4*e^{12} - 191038464*a^{11}*b^{18}*c^5*e^{12} + 2207803392*a^{12}*b^{16}*c^6*e^{12} - \\
& 16878108672*a^{13}*b^{14}*c^7*e^{12} + 89374851072*a^{14}*b^{12}*c^8*e^{12} - 33322696 \\
& 7040*a^{15}*b^{10}*c^9*e^{12} + 869815812096*a^{16}*b^8*c^{10}*e^{12} - 1543847804928*a \\
& ^{17}*b^6*c^{11}*e^{12} + 1747313491968*a^{18}*b^4*c^{12}*e^{12} - 1101055131648*a^{19}*b \\
& ^2*c^{13}*e^{12}) + 271790899200*a^{20}*c^{14}*d*e^{11} - 230400*a^9*b^{22}*c^3*d*e^{11} \\
& + 9861120*a^{10}*b^{20}*c^4*d*e^{11} - 191038464*a^{11}*b^{18}*c^5*d*e^{11} + 220780339 \\
& 2*a^{12}*b^{16}*c^6*d*e^{11} - 16878108672*a^{13}*b^{14}*c^7*d*e^{11} + 89374851072*a^{14} \\
& *b^{12}*c^8*d*e^{11} - 333226967040*a^{15}*b^{10}*c^9*d*e^{11} + 869815812096*a^{16}*b^ \\
& ^8*c^{10}*d*e^{11} - 1543847804928*a^{17}*b^6*c^{11}*d*e^{11} + 1747313491968*a^{18}*b^ \\
& 4*c^{12}*d*e^{11} - 1101055131648*a^{19}*b^2*c^{13}*d*e^{11}) + 191102976000*a^{17}*c^1 \\
& 4*e^{10} + 2851200*a^9*b^{16}*c^6*e^{10} - 92568960*a^{10}*b^{14}*c^7*e^{10} + 13126302 \\
& 72*a^{11}*b^{12}*c^8*e^{10} - 10611136512*a^{12}*b^{10}*c^9*e^{10} + 53445353472*a^{13}*b \\
& ^8*c^{10}*e^{10} - 171591892992*a^{14}*b^6*c^{11}*e^{10} + 342580396032*a^{15}*b^4*c^{12} \\
& *e^{10} - 388363714560*a^{16}*b^2*c^{13}*e^{10}))*(-(9*(25*b^{21} + 25*b^6*(-(4*a*c - \\
& b^2)^{15}))^{(1/2)} + 18923520*a^{10}*b*c^{10} + 17794*a^2*b^{17}*c^2 - 188095*a^3*b^ \\
& 15*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a^5*b^{11}*c^5 + 19905600*a^6*b^9*c^6 \\
& - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225 \\
& *a^3*c^3*(-(4*a*c - b^2)^{15}))^{(1/2)} - 995*a*b^{19}*c + 694*a^2*b^2*c^2*(-(4*a*c \\
& c - b^2)^{15}))^{(1/2)} - 245*a*b^4*c*(-(4*a*c - b^2)^{15}))^{(1/2)})) / (512*(a^7*b^{20} \\
& *e^2 + 1048576*a^{17}*c^{10}*e^2 - 40*a^8*b^{18}*c*e^2 + 720*a^9*b^{16}*c^2*e^2 - 7 \\
& 680*a^{10}*b^{14}*c^3*e^2 + 53760*a^{11}*b^{12}*c^4*e^2 - 258048*a^{12}*b^{10}*c^5*e^2 \\
& + 860160*a^{13}*b^8*c^6*e^2 - 1966080*a^{14}*b^6*c^7*e^2 + 2949120*a^{15}*b^4*c^8 \\
& *e^2 - 2621440*a^{16}*b^2*c^9*e^2))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out

$$3.524 \quad \int \frac{1}{(d+ex)^3(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=325

$$\frac{3b \log(a + b(d + ex)^2 + c(d + ex)^4)}{4a^4e} - \frac{3b \log(d + ex)}{a^4e} - \frac{3(b^2 - 5ac)(b^2 - 2ac)}{2a^3e(b^2 - 4ac)^2(d + ex)^2} + \frac{20a^2c^2 + 3bc(b^2 - 6ac)(d + ex)}{4a^2e(b^2 - 4ac)^2(d + ex)^2(a + b(d + ex)^2 + c(d + ex)^4)}$$

Rubi [A] time = 0.58, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2 + 3bc(b^2 - 6ac)(d + ex)^2 - 20ab^2c + 3b^4}{4a^2e(b^2 - 4ac)^2(d + ex)^2(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{3(30a^2b^2c^2 - 20a^3c^3 - 10ab^4c + b^6) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4e(b^2-4ac)^{3/2}} - \frac{3(b^2-5ac)(b^2-2ac)}{2a^3e(b^2-4ac)^2(d+ex)^2} + \frac{3b \log(a + b(d + ex)^2 + c(d + ex)^4)}{4a^4e} - \frac{3b \log(d + ex)}{a^4e} + \frac{-2ac + b^2 + bc(d + ex)^2}{4ae(b^2 - 4ac)(d + ex)^2(a + b(d + ex)^2 + c(d + ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

[Out] $(-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)^2) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)*e) - (3*b*Log[d + e*x])/(a^4*e) + (3*b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^4*e)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 740

Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 800

Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 822

Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1114

Int[(x_)^m*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^p, x_Symbol] := Dis


```
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^3} dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^3} dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} - \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^3} dx, x, (d+ex)^2\right)}{4a^2(b^2-4ac)^2} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4}{4a^2(b^2-4ac)^2} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3b^4}{4a^2(b^2-4ac)^2} \\
&= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} \\
&= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} \\
&= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2} \\
&= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2 e(d+ex)^2} + \frac{b^2-2ac+bc(d+ex)^2}{4a(b^2-4ac)e(d+ex)^2(a+b(d+ex)^2+c(d+ex)^4)^2}
\end{aligned}$$

Mathematica [A] time = 6.18, size = 491, normalized size = 1.51

$$\frac{3b \log(d+ex)}{2^2 d^2 e^2} - \frac{1}{2^2 d^2 e^2} - \frac{3b(b^2-2ac)(d+ex)^2 + b^3 + b^2(d+ex)^2}{4a^2(b^2-4ac)^2(e+bx+cx^2+c(d+ex)^2)} - \frac{4ab^2b^2-2b^2(d+ex)^2+2b^2b^2+2ab^2(d+ex)^2-4b^2(d+ex)^2}{4a^2(b^2-4ac)^2(e+bx+cx^2+c(d+ex)^2)} - \frac{1}{4a^2(b^2-4ac)^2} \left(\frac{2b^2b^2+3b^2b^2+16a^2b^2\sqrt{b^2-4ac}-10ab^2+b^2\sqrt{b^2-4ac}-8a^2\sqrt{b^2-4ac}+b^2\log(-\sqrt{b^2-4ac}+b+2(d+ex))}{4a^2(b^2-4ac)^2} - \frac{1}{4a^2(b^2-4ac)^2} \left(\frac{2b^2b^2-3b^2b^2+16a^2b^2\sqrt{b^2-4ac}+10ab^2+b^2\sqrt{b^2-4ac}-8a^2\sqrt{b^2-4ac}-b^2\log(\sqrt{b^2-4ac}+b+2(d+ex))}{4a^2(b^2-4ac)^2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out]
$$-1/2*1/(a^3*e*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c))*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2$$

$$+ (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*b^2*c^2*(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e) + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*sqrt[b^2 - 4*a*c] - 8*a*b^3*c*sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*sqrt[b^2 - 4*a*c])*Log[b - sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e) + (3*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*sqrt[b^2 - 4*a*c] - 8*a*b^3*c*sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*sqrt[b^2 - 4*a*c])*Log[b + sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

fricas [B] time = 7.53, size = 15165, normalized size = 46.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out]
$$[-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2$$

$$\begin{aligned}
& + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 7 \\
& 9*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + (9*a^2*b^7 - 104*a^3*b^ \\
& 5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3 \\
& 8*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3* \\
& b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 \\
& + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*e^2*x^2 + (9*a^2*b^7 - 104*a^3*b^5*c + \\
& 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 + \\
& 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3* \\
& b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 \\
& + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4* \\
& b^3*c^2 - 488*a^5*b*c^3)*d)*e*x + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^ \\
& ^4 - 20*a^3*c^5)*e^10*x^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - \\
& 20*a^3*c^5)*d*e^9*x^9 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b \\
& *c^4 + 45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*e^8*x \\
& ^8 + 8*(15*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(\\
& b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*e^7*x^7 + (b^8 - 8 \\
& *a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10 \\
& *a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + \\
& 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^2)*e^6*x^6 + (b^6*c^2 - 10*a*b^4*c^3 + 30* \\
& a^2*b^2*c^4 - 20*a^3*c^5)*d^10 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^ \\
& 2*c^4 - 20*a^3*c^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^ \\
& 3*b*c^4)*d^3 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^ \\
& 4*c^4)*d)*e^5*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4 \\
&)*d^8 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6* \\
& c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b \\
& ^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2* \\
& b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*e^4*x^4 + (b^8 - 8*a*b^6*c + 10 \\
& *a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^6 + 4*(30*(b^6*c^2 - 10*a*b^4 \\
& *c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2 \\
& *b^3*c^3 - 20*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3 \\
& *b^2*c^3 - 40*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20* \\
& a^4*b*c^3)*d)*e^3*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b \\
& *c^3)*d^4 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8 \\
& + a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b \\
& ^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2* \\
& b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30* \\
& a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*e^2*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4 \\
& *b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 \\
& - 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4 \\
&)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)* \\
& d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b \\
& ^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*e*x)*sqrt(b^2 - 4*a*c)* \\
& log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2* \\
& x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + \\
& 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c
\end{aligned}$$

$$\begin{aligned}
& d^4 + (6cd^2 + b)e^{2x^2} + bd^2 + 2(2cd^3 + bd)ex + a) - 3((b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)e^{10x^{10}} + 10(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^2e^{9x^9} + (2b^8c - 24ab^6c^2 + 96a^2b^4c^3 - 128a^3b^2c^4 + 45(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^2)e^{8x^8} + 8(15(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^3 + 2(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d)e^{7x^7} + (b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4 + 210(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^4 + 56(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^2)e^{6x^6} + (b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^10 + 2(126(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^5 + 56(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^3 + 3(b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4)d)e^{5x^5} + 2(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^8 + (2ab^8 - 24a^2b^6c + 96a^3b^4c^2 - 128a^4b^2c^3 + 210(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^6 + 140(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^4 + 15(b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4)d^2)e^{4x^4} + (b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4)d^6 + 4(30(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^7 + 28(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^5 + 5(b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4)d^3 + 2(ab^8 - 12a^2b^6c + 48a^3b^4c^2 - 64a^4b^2c^3)d)e^{3x^3} + 2(ab^8 - 12a^2b^6c + 48a^3b^4c^2 - 64a^4b^2c^3)d^4 + (a^2b^7 - 12a^3b^5c + 48a^4b^3c^2 - 64a^5b^2c^3 + 45(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^8 + 56(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^6 + 15(b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4)d^4 + 12(ab^8 - 12a^2b^6c + 48a^3b^4c^2 - 64a^4b^2c^3)d^2)e^{2x^2} + (a^2b^7 - 12a^3b^5c + 48a^4b^3c^2 - 64a^5b^2c^3)d^2 + 2(5(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^9 + 8(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^7 + 3(b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4)d^5 + 4(ab^8 - 12a^2b^6c + 48a^3b^4c^2 - 64a^4b^2c^3)d^3 + (a^2b^7 - 12a^3b^5c + 48a^4b^3c^2 - 64a^5b^2c^3)d)ex) * \log(c^4e^{4x^4} + 4cd^3e^{3x^3} + cd^4 + (6cd^2 + b)e^{2x^2} + bd^2 + 2(2cd^3 + bd)ex + a) + 12((b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)e^{10x^{10}} + 10(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^2e^{9x^9} + (2b^8c - 24ab^6c^2 + 96a^2b^4c^3 - 128a^3b^2c^4 + 45(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^2)e^{8x^8} + 8(15(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^3 + 2(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d)e^{7x^7} + (b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4b^2c^4 + 210(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^4 + 56(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^2)e^{6x^6} + (b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^{10} + 2(126(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3b^2c^5)d^5 + 56(
\end{aligned}$$

$$\begin{aligned}
& b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a \\
& *b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b \\
& ^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24* \\
& a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 \\
& + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b \\
& ^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a \\
& ^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c \\
& ^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + \\
& 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4* \\
& c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b \\
& ^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64 \\
& *a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^ \\
& 4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + \\
& 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c \\
& - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7 \\
& *c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12* \\
& a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a \\
& ^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^ \\
& 3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b \\
& ^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^ \\
& 3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - \\
& 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b* \\
& c^3)*d)*e*x)*log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 \\
& - 64*a^7*c^5)*e^11*x^10 + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 \\
& - 64*a^7*c^5)*d*e^10*x^9 + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*a^6*b^3*c^3 \\
& - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^ \\
& 7*c^5)*d^2)*e^9*x^8 + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 \\
& - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7 \\
& *b*c^4)*d)*e^8*x^7 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2* \\
& c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64 \\
& *a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b* \\
& c^4)*d^2)*e^7*x^6 + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - \\
& 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7 \\
& *b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - \\
& 128*a^8*c^4)*d)*e^6*x^5 + (2*a^5*b^7 - 24*a^6*b^5*c + 96*a^7*b^3*c^2 - 128 \\
& *a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^ \\
& 5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d \\
& ^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8 \\
& *c^4)*d^2)*e^5*x^4 + 4*(30*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - \\
& 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7 \\
& *b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - \\
& 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c \\
& ^3)*d)*e^4*x^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45 \\
& *(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4 \\
& *b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8
\end{aligned}$$

$$\begin{aligned}
& - 10a^5b^6c + 24a^6b^4c^2 + 32a^7b^2c^3 - 128a^8c^4)d^4 + 12*(a^5b^7 - 12a^6b^5c + 48a^7b^3c^2 - 64a^8b^2c^3)*d^2)*e^3*x^2 + 2*(5*(a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5)*d^9 + 8*(a^4b^7c - 12a^5b^5c^2 + 48a^6b^3c^3 - 64a^7b^2c^4)*d^7 + 3*(a^4b^8 - 10a^5b^6c + 24a^6b^4c^2 + 32a^7b^2c^3 - 128a^8c^4)*d^5 + 4*(a^5b^7 - 12a^6b^5c + 48a^7b^3c^2 - 64a^8b^2c^3)*d^3 + (a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)*d)*e^2*x + ((a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5)*d^10 + 2*(a^4b^7c - 12a^5b^5c^2 + 48a^6b^3c^3 - 64a^7b^2c^4)*d^8 + (a^4b^8 - 10a^5b^6c + 24a^6b^4c^2 + 32a^7b^2c^3 - 128a^8c^4)*d^6 + 2*(a^5b^7 - 12a^6b^5c + 48a^7b^3c^2 - 64a^8b^2c^3)*d^4 + (a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)*d^2)*e), -1/4*(6*(a*b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*e^8*x^8 + 48*(a*b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*d*e^7*x^7 + 3*(4*a*b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4 + 56*(a*b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*d^2)*e^6*x^6 + 6*(56*(a*b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*d^3 + 3*(4*a*b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)*d)*e^5*x^5 + 2*a^3b^6 - 24a^4b^4c + 96a^5b^2c^2 - 128a^6c^3 + 6*(a*b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*d^8 + (6*a*b^8 - 60a^2b^6c + 158a^3b^4c^2 + 44a^4b^2c^3 - 400a^5c^4 + 420*(a*b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*d^4 + 45*(4*a*b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)*d^6 + 4*(84*(a*b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*d^5 + 15*(4*a*b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)*d^3 + 2*(3*a*b^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)*d)*e^3*x^3 + 2*(3*a*b^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)*d^4 + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3 + 168*(a*b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*d^6 + 45*(4*a*b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)*d^4 + 12*(3*a*b^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)*d^2)*e^2*x^2 + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3)*d^2 + 2*(24*(a*b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)*d^7 + 9*(4*a*b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)*d^5 + 4*(3*a*b^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)*d^3 + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3)*d)*e*x + 6*((b^6c^2 - 10a*b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)*e^10*x^10 + 10*(b^6c^2 - 10a*b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)*d*e^9*x^9 + (2*b^7c - 20a*b^5c^2 + 60a^2b^3c^3 - 40a^3b^2c^4 + 45*(b^6c^2 - 10a*b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)*d^2)*e^8*x^8 + 8*(15*(b^6c^2 - 10a*b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)*d^3 + 2*(b^7c - 10a*b^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)*d)*e^7*x^7 + (b^8 - 8*a*b^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4 + 210*(b^6c^2 - 10a*b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)*d^4 + 56*(b^7c - 10a*b^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)*d^2)*e^6*x^6 + (b^6c^2 - 10a*b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)*d^10 + 2*(126*(b^6c^2 - 10a*b
\end{aligned}$$

$$\begin{aligned}
&^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^5 + 56(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3bc^4)d^3 + 3(b^8 - 8a^2b^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d)e^5x^5 + 2(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3bc^4)d^8 + (2ab^7 - 20a^2b^5c + 60a^3b^3c^2 - 40a^4b^2c^3 + 210(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^6 + 140(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3bc^4)d^4 + 15(b^8 - 8a^2b^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^2)e^4x^4 + (b^8 - 8a^2b^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^6 + 4(30(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^7 + 28(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3bc^4)d^5 + 5(b^8 - 8a^2b^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^3 + 2(ab^7 - 10a^2b^5c + 30a^3b^3c^2 - 20a^4bc^3)d)e^3x^3 + 2(ab^7 - 10a^2b^5c + 30a^3b^3c^2 - 20a^4bc^3)d^4 + (45(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^8 + a^2b^6 - 10a^3b^4c + 30a^4b^2c^2 - 20a^5c^3 + 56(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3bc^4)d^6 + 15(b^8 - 8a^2b^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^4 + 12(ab^7 - 10a^2b^5c + 30a^3b^3c^2 - 20a^4bc^3)d^2)e^2x^2 + (a^2b^6 - 10a^3b^4c + 30a^4b^2c^2 - 20a^5c^3)d^2 + 2(5(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^9 + 8(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3bc^4)d^7 + 3(b^8 - 8a^2b^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^5 + 4(ab^7 - 10a^2b^5c + 30a^3b^3c^2 - 20a^4bc^3)d^3 + (a^2b^6 - 10a^3b^4c + 30a^4b^2c^2 - 20a^5c^3)d)e)x)*\sqrt{-b^2 + 4ac})\arctan(-(2c^2e^2x^2 + 4cd^2e^2x^2 + 2cd^2 + b)\sqrt{-b^2 + 4ac})/(b^2 - 4ac)) - 3((b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3bc^5)e^{10x^{10}} + 10(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3bc^5)d^2)e^8x^8 + 8(15(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3bc^5)d^3 + 2(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d)e^7x^7 + (b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4bc^4 + 210(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3bc^5)d^4 + 56(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^2)e^6x^6 + (b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3bc^5)d^{10} + 2(126(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3bc^5)d^5 + 56(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^3 + 3(b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4bc^4)d)e^5x^5 + 2(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^8 + (2ab^8 - 24a^2b^6c + 96a^3b^4c^2 - 128a^4b^2c^3 + 210(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3bc^5)d^6 + 140(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^4 + 15(b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4bc^4)d^2)e^4x^4 + (b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4bc^4)d^6 + 4(30(b^7c^2 - 12ab^5c^3 + 48a^2b^3c^4 - 64a^3bc^5)d^7 + 28(b^8c - 12ab^6c^2 + 48a^2b^4c^3 - 64a^3b^2c^4)d^5 + 5(b^9 - 10ab^7c + 24a^2b^5c^2 + 32a^3b^3c^3 - 128a^4bc^4)d^3 + 2(ab^8 - 12a^2b^6c + 48a^3b^4c^2 - 64a^4b^2c^3)d)e^3x^3 +
\end{aligned}$$

$$\begin{aligned}
& 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 \\
& - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 \\
& + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 \\
& - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 \\
& - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2 \\
& *e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 \\
& + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 \\
& + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c \\
& + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d \\
& *e*x)*\log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d) \\
& *e*x + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^{10*x^10} + 10*(b^7*c^2 - 12*a*b^5*c^3 \\
& + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 \\
& + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 \\
& + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 \\
& - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 \\
& - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 \\
& + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c \\
& + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 \\
& + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 \\
& + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 \\
& - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 \\
& + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c \\
& + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d \\
& *e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 \\
& + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 \\
& + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2 \\
& *e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 \\
& + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 \\
& + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d \\
& *e*x)*\log(e*x + d))/((a^4*b^6*c^2
\end{aligned}$$

$$\begin{aligned}
& - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*e^{11*x^{10}} + 10*(a^4*b^6*c^2 \\
& - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d*e^{10*x^9} + (2*a^4*b^7*c \\
& - 24*a^5*b^5*c^2 + 96*a^6*b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a \\
& ^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^2)*e^9*x^8 + 8*(15*(a^4*b^6*c^2 \\
& - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^ \\
& 5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d)*e^8*x^7 + (a^4*b^8 - 10*a^5*b \\
& ^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 1 \\
& 2*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b \\
& ^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^2)*e^7*x^6 + 2*(126*(a^4*b^6*c^2 \\
& - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^ \\
& 5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c \\
& + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^6*x^5 + (2*a^5*b^7 - \\
& 24*a^6*b^5*c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b \\
& ^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 \\
& + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6 \\
& *b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^5*x^4 + 4*(30*(a^4*b^6*c^2 \\
& - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^ \\
& 5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c \\
& + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6 \\
& b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^4*x^3 + (a^6*b^6 - 12*a^7*b^4*c \\
& + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6 \\
& b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 \\
& - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7 \\
& *b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - \\
& 64*a^8*b*c^3)*d^2)*e^3*x^2 + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b \\
& ^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - \\
& 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^ \\
& 2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64 \\
& a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)* \\
& e^2*x + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^{10} \\
& + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4 \\
& *b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 + \\
& 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 - \\
& 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e)]
\end{aligned}$$

giac [A] time = 0.64, size = 377, normalized size = 1.16

$$\frac{3(b^6 - 10ab^4c + 30a^2b^2c^2 - 20a^3c^3) \arctan\left(\frac{b + 2a}{\sqrt{-b^2 + 4ac}}\right) e^{(-1)} + \frac{3bc^{(-1)} \log\left(c + \frac{b}{(x+d)^2} + \frac{a}{(x+d)}\right) e^{(-1)}}{4a^4} - \frac{e^{(-1)}}{2(xe + d)^2 a^3} + \frac{\left(5b^2c^2 - 36ab^2c^3 + 58a^2bc^4 + \frac{2(5b^2c^2 - 38ab^2c^3 + 71a^2b^2c^2 - 14a^3c^4)e^{(-1)}}{(x+d)^2} + \frac{(5b^2c^2 - 34ab^2c^3 + 41a^2b^2c^2 + 42a^3c^3)e^{(-2)}}{(x+d)^2} + \frac{6(ab^2c^2 - 8a^2b^2c^3 + 17a^2b^2c^2 - 6a^3c^3)e^{(-3)}}{(x+d)^2}\right) e^{(-1)}}{4(b^2 - 4ac)^2 a^4 \left(c + \frac{b}{(x+d)^2} + \frac{a}{(x+d)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sq

$$\text{rt}(-b^2 + 4*a*c)) + 3/4*b*e^{(-1)}*\log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^4 - 1/2*e^{(-1)}/((x*e + d)^2*a^3) + 1/4*(5*b^5*c^2 - 36*a*b^3*c^3 + 58*a^2*b*c^4 + 2*(5*b^6*c*e - 38*a*b^4*c^2*e + 71*a^2*b^2*c^3*e - 14*a^3*c^4*e)*e^{(-1)}/(x*e + d)^2 + (5*b^7*e^2 - 34*a*b^5*c*e^2 + 41*a^2*b^3*c^2*e^2 + 42*a^3*b*c^3*e^2)*e^{(-2)}/(x*e + d)^4 + 6*(a*b^6*e^3 - 8*a^2*b^4*c*e^3 + 17*a^3*b^2*c^2*e^3 - 6*a^4*c^3*e^3)*e^{(-3)}/(x*e + d)^6)*e^{(-1)}/((b^2 - 4*a*c)^2*a^4*(c + b/(x*e + d)^2 + a/(x*e + d)^4)^2)$$

maple [C] time = 0.09, size = 5575, normalized size = 17.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)`

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 22.45, size = 21465, normalized size = 66.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)`

[Out] $(\log(((27*c^4*e^{14}*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b^2)^6) - ((3*b - 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))^2/(a^8*e^2*(4*a*c - b^2)^5))^{(1/2)})*((9*c^3*e^{15}*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b - 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))^2/(a^8*e^2*(4*a*c - b^2)^5))^{(1/2)})*((6*c^2*e^{16}*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*(4*a*c - b^2)^2) + (6*c^3*e^{18}*x^2*(b^6 + 10*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*e^{16}*(3*b - 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))^2/(a^8*e^2*(4*a*c - b^2)^5))^{(1/2)})*$

$$\begin{aligned}
& a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 \\
& - 20*a*c*d*e*x)/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 \\
& - 2*a*b^4*c))/a^3*(4*a*c - b^2)^2)/((4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^8 \\
& + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/a^6*(4 \\
& *a*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 \\
& - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/a^6*(4*a*c - b^2)^4)/((4*a^4*e) + (27*c \\
& ^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/a^9*(4*a*c - b^2)^6) + (54*c \\
& ^5*d*e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/a^9*(4*a*c - b^2)^6))*((27*c \\
& ^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + \\
& 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/a^9*(4*a*c - b^2)^6) - ((3*b \\
& + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(\\
& 4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6 \\
& - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b^3*c^2*d^2 \\
& + 90*a^2*b*c^3*d^2))/a^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*e*(-(b^6 - 20*a \\
& ^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))* \\
& (6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4 \\
& *d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/a^3*(4*a*c - b \\
& ^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/ \\
& a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30 \\
& *a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2 \\
& *d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d \\
& *e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4 \\
& *c))/a^3*(4*a*c - b^2)^2)/((4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4 \\
& *c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/a^6*(4*a*c - b^2) \\
& ^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3* \\
& b^2*c^3 - 89*a*b^6*c))/a^6*(4*a*c - b^2)^4)/((4*a^4*e) + (27*c^5*e^16*x^2 \\
& *(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/a^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x \\
& *(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/a^9*(4*a*c - b^2)^6))*((6*b^11*e + 960* \\
& a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6 \\
& 144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 \\
& + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - ((x \\
& ^4*(6*b^6*e^3 + 100*a^3*c^3*e^3 + 180*b^5*c*d^2*e^3 + 14*a^2*b^2*c^2*e^3 + \\
& 4200*a^2*c^4*d^4*e^3 + 420*b^4*c^2*d^4*e^3 - 36*a*b^4*c*e^3 - 1305*a*b^3*c^ \\
& 2*d^2*e^3 + 2070*a^2*b*c^3*d^2*e^3 - 2940*a*b^2*c^3*d^4*e^3))/(4*(a^3*b^4 + \\
& 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^6*(4*b^5*c*e^5 - 29*a*b^3*c^2*e^5 + 46*a \\
& ^2*b*c^3*e^5 + 560*a^2*c^4*d^2*e^5 + 56*b^4*c^2*d^2*e^5 - 392*a*b^2*c^3*d^2 \\
& *e^5))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x*(12*b^6*d^3 + 36*b^5*c \\
& *d^5 + 200*a^3*c^3*d^3 + 240*a^2*c^4*d^7 + 24*b^4*c^2*d^7 + 9*a*b^5*d - 261 \\
& *a*b^3*c^2*d^5 + 414*a^2*b*c^3*d^5 - 168*a*b^2*c^3*d^7 + 28*a^2*b^2*c^2*d^3 \\
& - 68*a^2*b^3*c*d + 122*a^3*b*c^2*d - 72*a*b^4*c*d^3))/(2*(a^3*b^4 + 16*a^5 \\
& *c^2 - 8*a^4*b^2*c)) + (3*x^5*(560*a^2*c^4*d^3*e^4 + 56*b^4*c^2*d^3*e^4 + 1 \\
& 2*b^5*c*d*e^4 - 87*a*b^3*c^2*d*e^4 + 138*a^2*b*c^3*d*e^4 - 392*a*b^2*c^3*d^ \\
& 3*e^4))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^8*(10*a^2*c^4*e^7 + \\
& b^4*c^2*e^7 - 7*a*b^2*c^3*e^7))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + \\
& (x^2*(36*b^6*d^2*e + 9*a*b^5*e + 600*a^3*c^3*d^2*e + 1680*a^2*c^4*d^6*e +
\end{aligned}$$

$$\begin{aligned}
& 168*b^4*c^2*d^6*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e + 180*b^5*c*d^4*e - 21 \\
& 6*a*b^4*c*d^2*e - 1305*a*b^3*c^2*d^4*e + 2070*a^2*b*c^3*d^4*e - 1176*a*b^2* \\
& c^3*d^6*e + 84*a^2*b^2*c^2*d^2*e)/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) \\
& + (x^3*(6*b^6*d*e^2 + 100*a^3*c^3*d*e^2 + 60*b^5*c*d^3*e^2 + 840*a^2*c^4*d \\
& ^5*e^2 + 84*b^4*c^2*d^5*e^2 - 36*a*b^4*c*d*e^2 + 14*a^2*b^2*c^2*d*e^2 - 435 \\
& *a*b^3*c^2*d^3*e^2 + 690*a^2*b*c^3*d^3*e^2 - 588*a*b^2*c^3*d^5*e^2))/(a^3*b \\
& ^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (12*x^7*(10*a^2*c^4*d*e^6 + b^4*c^2*d*e^6 \\
& - 7*a*b^2*c^3*d*e^6))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (2*a^2*b^4 + 3 \\
& 2*a^4*c^2 + 6*b^6*d^4 - 16*a^3*b^2*c + 9*a*b^5*d^2 + 12*b^5*c*d^6 + 100*a^3 \\
& *c^3*d^4 + 60*a^2*c^4*d^8 + 6*b^4*c^2*d^8 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^ \\
& 2*d^2 - 87*a*b^3*c^2*d^6 + 138*a^2*b*c^3*d^6 - 42*a*b^2*c^3*d^8 + 14*a^2*b^ \\
& 2*c^2*d^4 - 36*a*b^4*c*d^4)/(4*e*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))/(x^ \\
& 4*(15*b^2*d^2*e^4 + 210*c^2*d^6*e^4 + 2*a*b*e^4 + 30*a*c*d^2*e^4 + 140*b*c* \\
& d^4*e^4) + x^8*(45*c^2*d^2*e^8 + 2*b*c*e^8) + x^5*(6*b^2*d*e^5 + 252*c^2*d^ \\
& 5*e^5 + 12*a*c*d*e^5 + 112*b*c*d^3*e^5) + x^3*(20*b^2*d^3*e^3 + 120*c^2*d^7 \\
& *e^3 + 8*a*b*d*e^3 + 40*a*c*d^3*e^3 + 112*b*c*d^5*e^3) + x^7*(120*c^2*d^3*e \\
& ^7 + 16*b*c*d*e^7) + x*(6*b^2*d^5*e + 10*c^2*d^9*e + 2*a^2*d*e + 8*a*b*d^3* \\
& e + 12*a*c*d^5*e + 16*b*c*d^7*e) + x^6*(b^2*e^6 + 210*c^2*d^4*e^6 + 2*a*c*e \\
& ^6 + 56*b*c*d^2*e^6) + x^2*(a^2*e^2 + 15*b^2*d^4*e^2 + 45*c^2*d^8*e^2 + 12* \\
& a*b*d^2*e^2 + 30*a*c*d^4*e^2 + 56*b*c*d^6*e^2) + a^2*d^2 + b^2*d^6 + c^2*d^ \\
& 10 + c^2*e^10*x^10 + 2*a*b*d^4 + 2*a*c*d^6 + 2*b*c*d^8 + 10*c^2*d*e^9*x^9) \\
& - (3*b*log(d + e*x))/(a^4*e) - (3*atan((x^2*(((27000*a^6*c^11*e^16 + 27*b^ \\
& 12*c^5*e^16 - 567*a*b^10*c^6*e^16 + 4779*a^2*b^8*c^7*e^16 - 20601*a^3*b^6*c \\
& ^8*e^16 + 47790*a^4*b^4*c^9*e^16 - 56700*a^5*b^2*c^10*e^16)/(a^9*b^12 + 409 \\
& 6*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a \\
& ^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((129600*a^9*b*c^10*e^17 + 54*a^3*b^13* \\
& c^4*e^17 - 1233*a^4*b^11*c^5*e^17 + 11583*a^5*b^9*c^6*e^17 - 57204*a^6*b^7* \\
& c^7*e^17 + 156276*a^7*b^5*c^8*e^17 - 223200*a^8*b^3*c^9*e^17)/(a^9*b^12 + 4 \\
& 096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840 \\
& *a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c^10*e^18 + 6*a^6*b^14* \\
& c^3*e^18 - 108*a^7*b^12*c^4*e^18 + 588*a^8*b^10*c^5*e^18 + 792*a^9*b^8*c^6* \\
& e^18 - 22272*a^10*b^6*c^7*e^18 + 100608*a^11*b^4*c^8*e^18 - 199680*a^12*b^2 \\
& *c^9*e^18)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - \\
& 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11*e + 9 \\
& 60*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e \\
& - 6144*a^5*b*c^5*e)*(163840*a^16*b*c^9*e^19 - 12*a^9*b^15*c^2*e^19 + 328*a^ \\
& 10*b^13*c^3*e^19 - 3840*a^11*b^11*c^4*e^19 + 24960*a^12*b^9*c^5*e^19 - 9728 \\
& 0*a^13*b^7*c^6*e^19 + 227328*a^14*b^5*c^7*e^19 - 294912*a^15*b^3*c^8*e^19)) \\
& / (2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2 \\
& *e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)*(a^9*b^12 + 4096*a^15*c \\
& ^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4* \\
& c^4 - 6144*a^14*b^2*c^5)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3 \\
& *e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/(2*(4*a^4*b^10 \\
& *e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7 \\
& *b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)))*(6*b^11*e + 960*a^2*b^7*c^2*e - 3840
\end{aligned}$$

$$\begin{aligned}
& e^{19} - 294912a^{15}b^3c^8e^{19}) / (2(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^2e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2) \\
& (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) \\
& (b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c)) / (4a^4e(4ac - b^2)^{(5/2)}) - (3(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c) \\
& (6b^{11}e + 960a^2b^7c^2e - 3840a^3b^5c^3e + 7680a^4b^3c^4e - 120ab^9c^5e - 6144a^5b^7c^5e) \\
& (163840a^{16}b^9c^9e^{19} - 12a^9b^{15}c^2e^{19} + 328a^{10}b^{13}c^3e^{19} - 3840a^{11}b^{11}c^4e^{19} + 24960a^{12}b^9c^5e^{19} - 97280a^{13}b^7c^6e^{19} \\
& - 227328a^{14}b^5c^7e^{19} - 294912a^{15}b^3c^8e^{19})) / (8a^4e(4ac - b^2)^{(5/2)}) \\
& (4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^2e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2) \\
& (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) \\
& (6b^{11}e + 960a^2b^7c^2e - 3840a^3b^5c^3e + 7680a^4b^3c^4e - 120ab^9c^5e - 6144a^5b^7c^5e) / (2(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^2e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2) \\
& (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) \\
& - (3((129600a^9b^3c^{10}e^{17} + 54a^3b^{13}c^4e^{17} - 1233a^4b^{11}c^5e^{17} + 11583a^5b^9c^6e^{17} - 57204a^6b^7c^7e^{17} + 156276a^7b^5c^8e^{17} - 223200a^8b^3c^9e^{17} - 792200a^9b^1c^{10}e^{17} + 100608a^{11}b^4c^8e^{18} - 199680a^{12}b^2c^9e^{18} / (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5) \\
& - ((6b^{11}e + 960a^2b^7c^2e - 3840a^3b^5c^3e + 7680a^4b^3c^4e - 120ab^9c^5e - 6144a^5b^7c^5e) \\
& (163840a^{16}b^9c^9e^{19} - 12a^9b^{15}c^2e^{19} + 328a^{10}b^{13}c^3e^{19} - 3840a^{11}b^{11}c^4e^{19} + 24960a^{12}b^9c^5e^{19} - 97280a^{13}b^7c^6e^{19} + 227328a^{14}b^5c^7e^{19} - 294912a^{15}b^3c^8e^{19})) / (2(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^2e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2) \\
& (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) \\
& (6b^{11}e + 960a^2b^7c^2e - 3840a^3b^5c^3e + 7680a^4b^3c^4e - 120ab^9c^5e - 6144a^5b^7c^5e) / (2(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^2e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2) \\
& (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) \\
& (3(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c)) / (4a^4e(4ac - b^2)^{(5/2)}) + (27(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c) \\
& (163840a^{16}b^9c^9e^{19} - 12a^9b^{15}c^2e^{19} + 328a^{10}b^{13}c^3e^{19} - 3840a^{11}b^{11}c^4e^{19} + 24960a^{12}b^9c^5e^{19} - 97280a^{13}b^7c^6e^{19} + 227328a^{14}b^5c^7e^{19} - 294912a^{15}b^3c^8e^{19})) / (64a^{12}e^3(4ac - b^2)^{(15/2)}) \\
& (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) \\
& (3b^8 + 190a^4c^4 + 180a^2b^4c^2 - 335a^3b^2c^3 - 39ab^6c)) / (8a^3c^2(4ac - b^2)^{(13/2)}) \\
& (100a^6c^6 - 6b^{12} - 960a^2b^8c^2 + 3840a^3b^6c^3 - 7675a^4b^4c^4 + 6100a^5b^3c^5)
\end{aligned}$$

$$\begin{aligned}
& b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) * (6b^{11}e + 960a^2b^7c^2e - 3840a^3b^5c^3e + 7680a^4b^3c^4e - 120ab^9c^5e - 6144a^5b^6c^5e)) / (2(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^5e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2)) - (3((2(129600a^9b^6c^{10}d^16 + 54a^3b^{13}c^4d^16 - 1233a^4b^{11}c^5d^16 + 11583a^5b^9c^6d^16 - 57204a^6b^7c^7d^16 + 156276a^7b^5c^8d^16 - 223200a^8b^3c^9d^16)) / (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5) - (((2(153600a^{13}c^{10}d^17 + 6a^6b^{14}c^3d^17 - 108a^7b^{12}c^4d^17 + 588a^8b^{10}c^5d^17 + 792a^9b^8c^6d^17 - 22272a^{10}b^6c^7d^17 + 100608a^{11}b^4c^8d^17 - 199680a^{12}b^2c^9d^17) / (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5) - ((6b^{11}e + 960a^2b^7c^2e - 3840a^3b^5c^3e + 7680a^4b^3c^4e - 120ab^9c^5e - 6144a^5b^6c^5e) * (163840a^{16}b^6c^9d^18 - 12a^9b^{15}c^2d^18 + 328a^{10}b^{13}c^3d^18 - 3840a^{11}b^{11}c^4d^18 + 24960a^{12}b^9c^5d^18 - 97280a^{13}b^7c^6d^18 + 227328a^{14}b^5c^7d^18 - 294912a^{15}b^3c^8d^18) / ((4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^5e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2)) * (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) * (6b^{11}e + 960a^2b^7c^2e - 3840a^3b^5c^3e + 7680a^4b^3c^4e - 120ab^9c^5e - 6144a^5b^6c^5e)) / (2(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^5e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2))) * (b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c) / (4a^4e * (4a^4c - b^2)^{(5/2)}) + (27(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c))^3 * (163840a^{16}b^6c^9d^18 - 12a^9b^{15}c^2d^18 + 328a^{10}b^{13}c^3d^18 - 3840a^{11}b^{11}c^4d^18 + 24960a^{12}b^9c^5d^18 - 97280a^{13}b^7c^6d^18 + 227328a^{14}b^5c^7d^18 - 294912a^{15}b^3c^8d^18) / (32a^{12}e^3 * (4a^4c - b^2)^{(15/2)} * (a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) * (3b^8 + 190a^4c^4 + 180a^2b^4c^2 - 335a^3b^2c^3 - 39ab^6c) / (8a^3c^2 * (4a^4c - b^2)^{(13/2)} * (100a^6c^6 - 6b^{12} - 960a^2b^8c^2 + 3840a^3b^6c^3 - 7675a^4b^4c^4 + 6100a^5b^2c^5 + 120ab^{10}c)) * (16a^{12}b^{12} * (4a^4c - b^2)^{(15/2)} + 65536a^{18}c^6 * (4a^4c - b^2)^{(15/2)} - 384a^{13}b^{10}c * (4a^4c - b^2)^{(15/2)} + 3840a^{14}b^8c^2 * (4a^4c - b^2)^{(15/2)} - 20480a^{15}b^6c^3 * (4a^4c - b^2)^{(15/2)} + 61440a^{16}b^4c^4 * (4a^4c - b^2)^{(15/2)} - 98304a^{17}b^2c^5 * (4a^4c - b^2)^{(15/2))) / (10800a^6c^8e^{14} + 27b^{12}c^2e^{14} - 540ab^{10}c^3e^{14} + 4320a^2b^8c^4e^{14} - 17280a^3b^6c^5e^{14} + 35100a^4b^4c^6e^{14} - 32400a^5b^2c^7e^{14}) - (((((36a^3b^{14}c^3e^{15} - 14400a^{10}c^{10}e^{15} - 837a^4b^{12}c^4e^{15} + 8046a^5b^{10}c^5e^{15} - 40941a^6b^8c^6e^{15} + 116532a^7b^6c^7e^{15} - 177588a^8b^4c^8e^{15} + 119520a^9b^2c^9e^{15} + 54a^3b^{13}c^4d^2e^{15} - 1233a^4b^{11}c^5d^2e^{15} + 11583a^5b^9c^6d^2e^{15} - 57204a^6b^7c^7d^2e^{15} + 156276a^7b^5c^8d^2e^{15} - 223200a^8b^3c^9d^2e^{15} + 129600a^9b^6c^{10}d^2e^{15}) / (a^9b^{12} + 4096
\end{aligned}$$

$$\begin{aligned}
& *d^2e^{17} - 227328a^{14}b^5c^7d^2e^{17} + 294912a^{15}b^3c^8d^2e^{17} - 1 \\
& 63840a^{16}b^2c^9d^2e^{17}))/((2*(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5* \\
& b^8c^2e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2 \\
& ^2)*(a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^ \\
& 12b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)))*(b^6 - 20a^3c^3 + 3 \\
& 0a^2b^2c^2 - 10a*b^4c))/((4a^4e*(4a*c - b^2)^{(5/2)}) + (3*(b^6 - 20a \\
& ^3c^3 + 30a^2b^2c^2 - 10a*b^4c)*(6b^{11}e + 960a^2b^7c^2e - 3840* \\
& a^3b^5c^3e + 7680a^4b^3c^4e - 120a*b^9c^5e - 6144a^5b^2c^5e)*(4a \\
& ^10b^{14}c^2e^{17} - 96a^{11}b^{12}c^3e^{17} + 960a^{12}b^{10}c^4e^{17} - 5120a \\
& ^13b^8c^5e^{17} + 15360a^{14}b^6c^6e^{17} - 24576a^{15}b^4c^7e^{17} + 1638 \\
& 4a^{16}b^2c^8e^{17} + 12a^9b^{15}c^2d^2e^{17} - 328a^{10}b^{13}c^3d^2e^{17} \\
& + 3840a^{11}b^{11}c^4d^2e^{17} - 24960a^{12}b^9c^5d^2e^{17} + 97280a^{13}b \\
& ^7c^6d^2e^{17} - 227328a^{14}b^5c^7d^2e^{17} + 294912a^{15}b^3c^8d^2e^ \\
& 17 - 163840a^{16}b^2c^9d^2e^{17}))/((8a^4e*(4a*c - b^2)^{(5/2)}*(4a^4b^{10} \\
& e^2 - 4096a^9c^5e^2 - 80a^5b^8c^2e^2 + 640a^6b^6c^2e^2 - 2560a^7* \\
& b^4c^3e^2 + 5120a^8b^2c^4e^2)*(a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10} \\
& c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14} \\
& b^2c^5)))*(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10a*b^4c))/((4a^4e*(4a* \\
& c - b^2)^{(5/2)}) + (9*(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10a*b^4c)^2*(6* \\
& b^{11}e + 960a^2b^7c^2e - 3840a^3b^5c^3e + 7680a^4b^3c^4e - 120* \\
& a*b^9c^5e - 6144a^5b^2c^5e)*(4a^{10}b^{14}c^2e^{17} - 96a^{11}b^{12}c^3e^{17} \\
& + 960a^{12}b^{10}c^4e^{17} - 5120a^{13}b^8c^5e^{17} + 15360a^{14}b^6c^6e^{17} \\
& - 24576a^{15}b^4c^7e^{17} + 16384a^{16}b^2c^8e^{17} + 12a^9b^{15}c^2d^2 \\
& e^{17} - 328a^{10}b^{13}c^3d^2e^{17} + 3840a^{11}b^{11}c^4d^2e^{17} - 24960a^ \\
& 12b^9c^5d^2e^{17} + 97280a^{13}b^7c^6d^2e^{17} - 227328a^{14}b^5c^7d^2 \\
& e^{17} + 294912a^{15}b^3c^8d^2e^{17} - 163840a^{16}b^2c^9d^2e^{17}))/((32a^8 \\
& e^2*(4a*c - b^2)^5*(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^2e^2 \\
& + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2)*(a^9b \\
& ^12 + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 \\
& + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)))*(3b^8 + 10a^4c^4 + 120a^2b \\
& ^4c^2 - 145a^3b^2c^3 - 33a*b^6c)*(16a^{12}b^{12}(4a*c - b^2)^{(15/2)} + \\
& 65536a^{18}c^6(4a*c - b^2)^{(15/2)} - 384a^{13}b^{10}c*(4a*c - b^2)^{(15/2)} \\
& + 3840a^{14}b^8c^2*(4a*c - b^2)^{(15/2)} - 20480a^{15}b^6c^3*(4a*c - b^2 \\
&)^{(15/2)} + 61440a^{16}b^4c^4*(4a*c - b^2)^{(15/2)} - 98304a^{17}b^2c^5*(4* \\
& a*c - b^2)^{(15/2}))/((8a^3c^2*(4a*c - b^2)^6*(10800a^6c^8e^{14} + 27b^{1 \\
& 2}c^2e^{14} - 540a*b^{10}c^3e^{14} + 4320a^2b^8c^4e^{14} - 17280a^3b^6c^ \\
& 5e^{14} + 35100a^4b^4c^6e^{14} - 32400a^5b^2c^7e^{14})*(100a^6c^6 - 6* \\
& b^{12} - 960a^2b^8c^2 + 3840a^3b^6c^3 - 7675a^4b^4c^4 + 6100a^5b^2 \\
& c^5 + 120a*b^{10}c)) - (b*((3*((36a^3b^{14}c^3e^{15} - 14400a^{10}c^{10}e^{1 \\
& 5} - 837a^4b^{12}c^4e^{15} + 8046a^5b^{10}c^5e^{15} - 40941a^6b^8c^6e^{15} \\
& + 116532a^7b^6c^7e^{15} - 177588a^8b^4c^8e^{15} + 119520a^9b^2c^9e \\
& ^{15} + 54a^3b^{13}c^4d^2e^{15} - 1233a^4b^{11}c^5d^2e^{15} + 11583a^5b^9 \\
& c^6d^2e^{15} - 57204a^6b^7c^7d^2e^{15} + 156276a^7b^5c^8d^2e^{15} - \\
& 223200a^8b^3c^9d^2e^{15} + 129600a^9b^2c^{10}d^2e^{15}))/((a^9b^{12} + 4096* \\
& a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}
\end{aligned}$$

$$\begin{aligned}
& 3*b^4*c^4 - 6144*a^{14}*b^2*c^5) - (((12*a^6*b^{15}*c^2*e^{16} - 30720*a^{13}*b*c^9 \\
& *e^{16} - 300*a^7*b^{13}*c^3*e^{16} + 3156*a^8*b^{11}*c^4*e^{16} - 17976*a^9*b^9*c^5* \\
& e^{16} + 59136*a^{10}*b^7*c^6*e^{16} - 109824*a^{11}*b^5*c^7*e^{16} + 101376*a^{12}*b^3 \\
& *c^8*e^{16} + 153600*a^{13}*c^{10}*d^2*e^{16} + 6*a^6*b^{14}*c^3*d^2*e^{16} - 108*a^7*b \\
& ^{12}*c^4*d^2*e^{16} + 588*a^8*b^{10}*c^5*d^2*e^{16} + 792*a^9*b^8*c^6*d^2*e^{16} - 2 \\
& 2272*a^{10}*b^6*c^7*d^2*e^{16} + 100608*a^{11}*b^4*c^8*d^2*e^{16} - 199680*a^{12}*b^2 \\
& *c^9*d^2*e^{16})/(a^9*b^{12} + 4096*a^{15}*c^6 - 24*a^{10}*b^{10}*c + 240*a^{11}*b^8*c^ \\
& 2 - 1280*a^{12}*b^6*c^3 + 3840*a^{13}*b^4*c^4 - 6144*a^{14}*b^2*c^5) + ((6*b^{11}*e \\
& + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9* \\
& c*e - 6144*a^5*b*c^5*e)*(4*a^{10}*b^{14}*c^2*e^{17} - 96*a^{11}*b^{12}*c^3*e^{17} + 960 \\
& *a^{12}*b^{10}*c^4*e^{17} - 5120*a^{13}*b^8*c^5*e^{17} + 15360*a^{14}*b^6*c^6*e^{17} - 24 \\
& 576*a^{15}*b^4*c^7*e^{17} + 16384*a^{16}*b^2*c^8*e^{17} + 12*a^9*b^{15}*c^2*d^2*e^{17} \\
& - 328*a^{10}*b^{13}*c^3*d^2*e^{17} + 3840*a^{11}*b^{11}*c^4*d^2*e^{17} - 24960*a^{12}*b^9 \\
& *c^5*d^2*e^{17} + 97280*a^{13}*b^7*c^6*d^2*e^{17} - 227328*a^{14}*b^5*c^7*d^2*e^{17} \\
& + 294912*a^{15}*b^3*c^8*d^2*e^{17} - 163840*a^{16}*b*c^9*d^2*e^{17}))/((2*(4*a^4*b^1 \\
& 0*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^ \\
& 7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2))*(a^9*b^{12} + 4096*a^{15}*c^6 - 24*a^{10}*b \\
& ^{10}*c + 240*a^{11}*b^8*c^2 - 1280*a^{12}*b^6*c^3 + 3840*a^{13}*b^4*c^4 - 6144*a^1 \\
& 4*b^2*c^5)))*(6*b^{11}*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4* \\
& b^3*c^4*e - 120*a*b^9*c*e - 6144*a^5*b*c^5*e))/((2*(4*a^4*b^{10}*e^2 - 4096*a^ \\
& 9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + \\
& 5120*a^8*b^2*c^4*e^2)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/ \\
& (4*a^4*e*(4*a*c - b^2)^{(5/2)}) - (((3*((12*a^6*b^{15}*c^2*e^{16} - 30720*a^{13}*b* \\
& c^9*e^{16} - 300*a^7*b^{13}*c^3*e^{16} + 3156*a^8*b^{11}*c^4*e^{16} - 17976*a^9*b^9*c^ \\
& ^5*e^{16} + 59136*a^{10}*b^7*c^6*e^{16} - 109824*a^{11}*b^5*c^7*e^{16} + 101376*a^{12} \\
& b^3*c^8*e^{16} + 153600*a^{13}*c^{10}*d^2*e^{16} + 6*a^6*b^{14}*c^3*d^2*e^{16} - 108*a^ \\
& 7*b^{12}*c^4*d^2*e^{16} + 588*a^8*b^{10}*c^5*d^2*e^{16} + 792*a^9*b^8*c^6*d^2*e^{16} \\
& - 22272*a^{10}*b^6*c^7*d^2*e^{16} + 100608*a^{11}*b^4*c^8*d^2*e^{16} - 199680*a^{12} \\
& b^2*c^9*d^2*e^{16})/(a^9*b^{12} + 4096*a^{15}*c^6 - 24*a^{10}*b^{10}*c + 240*a^{11}*b^8 \\
& *c^2 - 1280*a^{12}*b^6*c^3 + 3840*a^{13}*b^4*c^4 - 6144*a^{14}*b^2*c^5) + ((6*b^1 \\
& 1*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b \\
& ^9*c*e - 6144*a^5*b*c^5*e)*(4*a^{10}*b^{14}*c^2*e^{17} - 96*a^{11}*b^{12}*c^3*e^{17} + \\
& 960*a^{12}*b^{10}*c^4*e^{17} - 5120*a^{13}*b^8*c^5*e^{17} + 15360*a^{14}*b^6*c^6*e^{17} - \\
& 24576*a^{15}*b^4*c^7*e^{17} + 16384*a^{16}*b^2*c^8*e^{17} + 12*a^9*b^{15}*c^2*d^2*e^ \\
& 17 - 328*a^{10}*b^{13}*c^3*d^2*e^{17} + 3840*a^{11}*b^{11}*c^4*d^2*e^{17} - 24960*a^{12} \\
& b^9*c^5*d^2*e^{17} + 97280*a^{13}*b^7*c^6*d^2*e^{17} - 227328*a^{14}*b^5*c^7*d^2*e^ \\
& 17 + 294912*a^{15}*b^3*c^8*d^2*e^{17} - 163840*a^{16}*b*c^9*d^2*e^{17}))/((2*(4*a^4* \\
& b^{10}*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2 + 640*a^6*b^6*c^2*e^2 - 2560 \\
& *a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2))*(a^9*b^{12} + 4096*a^{15}*c^6 - 24*a^1 \\
& 0*b^{10}*c + 240*a^{11}*b^8*c^2 - 1280*a^{12}*b^6*c^3 + 3840*a^{13}*b^4*c^4 - 6144* \\
& a^{14}*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4*e* \\
& (4*a*c - b^2)^{(5/2)}) + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)* \\
& (6*b^{11}*e + 960*a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 1 \\
& 20*a*b^9*c*e - 6144*a^5*b*c^5*e)*(4*a^{10}*b^{14}*c^2*e^{17} - 96*a^{11}*b^{12}*c^3* \\
& e^{17} + 960*a^{12}*b^{10}*c^4*e^{17} - 5120*a^{13}*b^8*c^5*e^{17} + 15360*a^{14}*b^6*c^6*
\end{aligned}$$

$$\begin{aligned}
& e^{17} - 24576a^{15}b^4c^7e^{17} + 16384a^{16}b^2c^8e^{17} + 12a^9b^{15}c^2d^2e^{17} - 328a^{10}b^{13}c^3d^2e^{17} + 3840a^{11}b^{11}c^4d^2e^{17} - 24960 \\
& a^{12}b^9c^5d^2e^{17} + 97280a^{13}b^7c^6d^2e^{17} - 227328a^{14}b^5c^7d^2e^{17} + 294912a^{15}b^3c^8d^2e^{17} - 163840a^{16}b^1c^9d^2e^{17} \\
&) / (8a^4e(4ac - b^2)^{(5/2)}(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^2e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2)(a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) * (6b^{11}e + 960a^2b^7c^2e - 3840a^3b^5c^3e + 7680a^4b^3c^4e - 120ab^9c^2e - 6144a^5b^6c^5e)) / (2(4a^4b^{10}e^2 - 4096a^9c^5e^2 - 80a^5b^8c^2e^2 + 640a^6b^6c^2e^2 - 2560a^7b^4c^3e^2 + 5120a^8b^2c^4e^2)) + (27(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c))^3(4a^{10}b^{14}c^2e^{17} - 96a^{11}b^{12}c^3e^{17} + 960a^{12}b^{10}c^4e^{17} - 5120a^{13}b^8c^5e^{17} + 15360a^{14}b^6c^6e^{17} - 24576a^{15}b^4c^7e^{17} + 16384a^{16}b^2c^8e^{17} + 12a^9b^{15}c^2d^2e^{17} - 328a^{10}b^{13}c^3d^2e^{17} + 3840a^{11}b^{11}c^4d^2e^{17} - 24960a^{12}b^9c^5d^2e^{17} + 97280a^{13}b^7c^6d^2e^{17} - 227328a^{14}b^5c^7d^2e^{17} + 294912a^{15}b^3c^8d^2e^{17} - 163840a^{16}b^1c^9d^2e^{17}))/ (64a^{12}e^3(4ac - b^2)^{(15/2)}(a^9b^{12} + 4096a^{15}c^6 - 24a^{10}b^{10}c + 240a^{11}b^8c^2 - 1280a^{12}b^6c^3 + 3840a^{13}b^4c^4 - 6144a^{14}b^2c^5)) * (3b^8 + 190a^4c^4 + 180a^2b^4c^2 - 335a^3b^2c^3 - 39ab^6c) * (16a^{12}b^{12}(4ac - b^2)^{(15/2)} + 65536a^{18}c^6(4ac - b^2)^{(15/2)} - 384a^{13}b^{10}c(4ac - b^2)^{(15/2)} + 3840a^{14}b^8c^2(4ac - b^2)^{(15/2)} - 20480a^{15}b^6c^3(4ac - b^2)^{(15/2)} + 61440a^{16}b^4c^4(4ac - b^2)^{(15/2)} - 98304a^{17}b^2c^5(4ac - b^2)^{(15/2)))/ (8a^3c^2(4ac - b^2)^{(13/2)}(10800a^6c^8e^{14} + 27b^{12}c^2e^{14} - 540ab^{10}c^3e^{14} + 4320a^2b^8c^4e^{14} - 17280a^3b^6c^5e^{14} + 35100a^4b^4c^6e^{14} - 32400a^5b^2c^7e^{14})*(100a^6c^6 - 6b^{12} - 960a^2b^8c^2 + 3840a^3b^6c^3 - 7675a^4b^4c^4 + 6100a^5b^2c^5 + 120ab^{10}c)) * (b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c)) / (2a^4e(4ac - b^2)^{(5/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out

$$3.525 \quad \int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4} dx$$

Optimal. Leaf size=202

$$-\frac{f^4 \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} c^{3/2} e \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{f^4 \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} c^{3/2} e \sqrt{\sqrt{b^2 - 4ac} + b}} + \frac{f^4 x}{c}$$

Rubi [A] time = 0.36, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1142, 1122, 1166, 205}

$$-\frac{f^4 \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} c^{3/2} e \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{f^4 \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} c^{3/2} e \sqrt{\sqrt{b^2 - 4ac} + b}} + \frac{f^4 x}{c}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] (f^4*x)/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1122

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{(df + efx)^4}{a + b(d + ex)^2 + c(d + ex)^4} dx &= \frac{f^4 \text{Subst}\left(\int \frac{x^4}{a + bx^2 + cx^4} dx, x, d + ex\right)}{e} \\ &= \frac{f^4 x}{c} - \frac{f^4 \text{Subst}\left(\int \frac{a + bx^2}{a + bx^2 + cx^4} dx, x, d + ex\right)}{ce} \\ &= \frac{f^4 x}{c} - \frac{\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) f^4 \text{Subst}\left(\int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx, x, d + ex\right)}{2ce} - \frac{\left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) f^4 \text{Subst}\left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx, x, d + ex\right)}{2ce} \\ &= \frac{f^4 x}{c} - \frac{\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) f^4 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}e} - \frac{\left(b + \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) f^4 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}e} \end{aligned}$$

Mathematica [A] time = 0.05, size = 222, normalized size = 1.10

$$f^4 \left(\frac{\sqrt{2}(b\sqrt{b^2 - 4ac} + 2ac - b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2}(b\sqrt{b^2 - 4ac} - 2ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{b^2 - 4ac} + b}\right)}{\sqrt{b^2 - 4ac}\sqrt{b^2 - 4ac} + b} + 2\sqrt{c}(d + ex) \right) \frac{1}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]
```

```
[Out] (f^4*(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
```


$4*a*c]*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[2]*(b^2 - 2*a*c + b*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + e*x))/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])))/(2*c^(3/2)*e)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df + efx)^4}{a + b(d + ex)^2 + c(d + ex)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

fricas [B] time = 1.32, size = 1346, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] $\frac{1}{2}*(2*f^4*x - \text{sqrt}(1/2)*c*\text{sqrt}(-((b^3 - 3*a*b*c)*f^8 + \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*\log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 + \text{sqrt}(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^3*c^3 - 4*a*b*c^4)*e^3)*\text{sqrt}(-((b^3 - 3*a*b*c)*f^8 + \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + \text{sqrt}(1/2)*c*\text{sqrt}(-((b^3 - 3*a*b*c)*f^8 + \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*\log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - \text{sqrt}(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 - \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^3*c^3 - 4*a*b*c^4)*e^3)*\text{sqrt}(-((b^3 - 3*a*b*c)*f^8 + \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) - \text{sqrt}(1/2)*c*\text{sqrt}(-((b^3 - 3*a*b*c)*f^8 - \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*\log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 + \text{sqrt}(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 + \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^3*c^3 - 4*a*b*c^4)*e^3)*\text{sqrt}(-((b^3 - 3*a*b*c)*f^8 - \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))) + \text{sqrt}(1/2)*c*\text{sqrt}(-((b^3 - 3*a*b*c)*f^8 - \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2))*\log(-2*(a*b^2 - a^2*c)*e*f^12*x - 2*(a*b^2 - a^2*c)*d*f^12 - \text{sqrt}(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*e*f^8 + \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^3*c^3 - 4*a*b*c^4)*e^3)*\text{sqrt}(-((b^3 - 3*a*b*c)*f^8 - \text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/((b^2*c^6 - 4*a*c^7)*e^4)))*(b^2*c^3 - 4*a*c^4)*e^2)/((b^2*c^3 - 4*a*c^4)*e^2)))$

$$\frac{(e^x f^8 + \sqrt{(b^4 - 2ab^2c + a^2c^2)f^{16}/((b^2c^6 - 4a^2c^7)e^4)}) \cdot (b^3c^3 - 4ab^2c^4)e^3 \sqrt{-(b^3 - 3ab^2c)f^8 - \sqrt{(b^4 - 2ab^2c + a^2c^2)f^{16}/((b^2c^6 - 4a^2c^7)e^4)}} \cdot (b^2c^3 - 4a^2c^4)e^2)/((b^2c^3 - 4a^2c^4)e^2))}{c}$$

giac [B] time = 0.54, size = 1245, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] $f^4 x/c + 1/2 * (((d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^2 b f^4 e^6 - 2(d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) b d f^4 e^5 + b d^2 f^4 e^4 + a f^4 e^4) \log(d e^{-1} + x + \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) / (2(d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^3 c e^4 - 6(d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) + ((d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^2 b f^4 e^6 - 2(d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) b d f^4 e^5 + b d^2 f^4 e^4 + a f^4 e^4) \log(d e^{-1} + x - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) / (2(d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^3 c e^4 - 6(d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) + ((d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^2 b f^4 e^6 - 2(d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) b d f^4 e^5 + b d^2 f^4 e^4 + a f^4 e^4) \log(d e^{-1} + x + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) / (2(d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^3 c e^4 - 6(d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) + ((d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^2 b f^4 e^6 - 2(d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) b d f^4 e^5 + b d^2 f^4 e^4 + a f^4 e^4) \log(d e^{-1} + x - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c)) / (2(d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^3 c e^4 - 6(d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1} - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4}/c))))) e^{-4}/c$

$$\begin{aligned}
& *b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)}/(8*(16*a^2*c^5*e^2 + b^4*c^3 \\
& *e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*x*(b^4*e^{12*f^8} + 2*a^2*c^2*e^{12*f^8} - \\
& 4*a*b^2*c^2*e^{12*f^8}))/c*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^{(1/2)} + 12 \\
& *a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a \\
& ^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}*1i)/(((2*b^4*d*e^{11*f^8} \\
& + 4*a^2*c^2*d*e^{11*f^8} - 8*a*b^2*c*d*e^{11*f^8}))/c + ((16*a^2*c^3*e^{12*f^4} - \\
& 4*a*b^2*c^2*e^{12*f^4}))/c + ((8*b^3*c^3*d*e^{13} - 32*a*b*c^4*d*e^{13}))/c + (2*x \\
& *(4*b^3*c^3*e^{14} - 16*a*b*c^4*e^{14}))/c*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/ \\
& (8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - \\
& a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*x*(b^4*e^{12*f^8} + 2*a^2*c^2*e^{12*f^8} - 4*a*b^2*c^2*e \\
& ^{12*f^8}))/c*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 \\
& - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + \\
& b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} - ((2*b^4*d*e^{11*f^8} + 4*a^2*c^2*d* \\
& e^{11*f^8} - 8*a*b^2*c*d*e^{11*f^8}))/c - ((16*a^2*c^3*e^{12*f^4} - 4*a*b^2*c^2*e^{12*f^4}))/c - \\
& ((8*b^3*c^3*d*e^{13} - 32*a*b*c^4*d*e^{13}))/c + (2*x*(4*b^3*c^3*e^{14} - 16*a*b*c^4*e^{14}))/c*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^{(1/2)} + 12 \\
& *a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a \\
& ^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}*(-(b^5*f^8 + b^2*f^8*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a \\
& *c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*x*(b^4*e^{12*f^8} + 2*a^2*c^2*e^{12*f^8} - 4*a*b^2*c^2*e^{12*f^8}))/c*(- \\
& (b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - \\
& 8*a*b^2*c^4*e^2))^{(1/2)} + (2*a^2*b*e^{10*f^12}))/c*(-(b^5*f^8 + b^2*f^8*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c \\
& - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}*2i + \operatorname{atan}(((2*b^4*d*e^{11*f^8} + 4*a^2*c^2*d*e^{11*f^8} - 8*a*b^2*c*d*e^{11*f^8}))/c + ((16*a^2*c^3*e^{12*f^4} - 4*a*b^2*c^2*e^{12*f^4}))/c + ((8*b^3*c^3*d*e^{13} - 32*a*b*c^4*d*e^{13}))/c + (2*x*(4*b^3*c^3*e^{14} - 16*a*b*c^4*e^{14}))/c*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)} + (2*x*(b^4*e^{12*f^8} + 2*a^2*c^2*e^{12*f^8} - 4*a*b^2*c^2*e^{12*f^8}))/c*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}*1i + ((2*b^4*d*e^{11*f^8} + 4*a^2*c^2*d*e^{11*f^8} - 8*a*b^2*c*d*e^{11*f^8}))/c - ((16*a^2*c^3*e^{12*f^4} - 4*a*b^2*c^2*e^{12*f^4}))/c - ((8*b^3*c^3*d*e^{13} - 32*a*b*c^4*d*e^{13}))/c + (2*x*(4*b^3*c^3*e^{14} - 16*a*b*c^4*e^{14}))/c*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))^{(1/2)}
\end{aligned}$$

$$3.526 \quad \int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4} dx$$

Optimal. Leaf size=87

$$\frac{bf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}} + \frac{f^3 \log(a+b(d+ex)^2+c(d+ex)^4)}{4ce}$$

Rubi [A] time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1142, 1114, 634, 618, 206, 628}

$$\frac{bf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}} + \frac{f^3 \log(a+b(d+ex)^2+c(d+ex)^4)}{4ce}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]*e) + (f^3*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1114

`Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]`

Rule 1142

`Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]`

Rubi steps

$$\begin{aligned}
 \int \frac{(df + efx)^3}{a + b(d + ex)^2 + c(d + ex)^4} dx &= \frac{f^3 \operatorname{Subst}\left(\int \frac{x^3}{a + bx^2 + cx^4} dx, x, d + ex\right)}{e} \\
 &= \frac{f^3 \operatorname{Subst}\left(\int \frac{x}{a + bx + cx^2} dx, x, (d + ex)^2\right)}{2e} \\
 &= \frac{f^3 \operatorname{Subst}\left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, (d + ex)^2\right)}{4ce} - \frac{(bf^3) \operatorname{Subst}\left(\int \frac{1}{a + bx + cx^2} dx, x, (d + ex)^2\right)}{4ce} \\
 &= \frac{f^3 \log(a + b(d + ex)^2 + c(d + ex)^4)}{4ce} + \frac{(bf^3) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2c(d + ex)^2\right)}{2ce} \\
 &= \frac{bf^3 \tanh^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{b^2 - 4ac}}\right)}{2c\sqrt{b^2 - 4ac}e} + \frac{f^3 \log(a + b(d + ex)^2 + c(d + ex)^4)}{4ce}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.92

$$\frac{f^3 \left(\log(a + b(d + ex)^2 + c(d + ex)^4) - \frac{2b \tan^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} \right)}{4ce}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] $(f^3 * ((-2 * b * \text{ArcTan}[(b + 2 * c * (d + e * x)^2) / \text{Sqrt}[-b^2 + 4 * a * c]]) / \text{Sqrt}[-b^2 + 4 * a * c] + \text{Log}[a + b * (d + e * x)^2 + c * (d + e * x)^4]) / (4 * c * e)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df + efx)^3}{a + b(d + ex)^2 + c(d + ex)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

fricas [A] time = 1.19, size = 446, normalized size = 5.13

$$\frac{\sqrt{b^2 - 4ac} \log\left(\frac{2c^2 d^2 + 2c^2 d e x + 2c^2 e^2 x^2 + 2c^2 d^2 + 2c^2 d e x + 2c^2 e^2 x^2 + 2c^2 d^2 + 2c^2 d e x + 2c^2 e^2 x^2}{4(b^2 - 4ac)^2}\right) + (b^2 - 4ac) \log\left(\frac{c^2 d^2 + 4cd^2 e x + c^2 d^2 + (6cd^2 + b)^2 e^2 x^2 + b^2 + 2(2cd^2 + b)ex + a}{4(b^2 - 4ac)^2}\right)}{4(b^2 - 4ac)^2} + \frac{2\sqrt{b^2 - 4ac} \arctan\left(\frac{2cd^2 + 2cd^2 e x + 2cd^2 e^2 x^2}{2cd^2 + 4ac}\right) + (b^2 - 4ac) \log\left(\frac{c^2 d^2 + 4cd^2 e x + c^2 d^2 + (6cd^2 + b)^2 e^2 x^2 + b^2 + 2(2cd^2 + b)ex + a}{4(b^2 - 4ac)^2}\right)}{4(b^2 - 4ac)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="fricas")

[Out] $[1/4 * (\text{sqrt}(b^2 - 4 * a * c) * b * f^3 * \log((2 * c^2 * e^4 * x^4 + 8 * c^2 * d * e^3 * x^3 + 2 * c^2 * d^2 * e^2 * x^2 + 2 * (6 * c^2 * d^2 + b * c) * e^2 * x^2 + 2 * b * c * d^2 + 4 * (2 * c^2 * d^3 + b * c * d) * e * x + b^2 - 2 * a * c + (2 * c * e^2 * x^2 + 4 * c * d * e * x + 2 * c * d^2 + b) * \text{sqrt}(b^2 - 4 * a * c)) / (c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + c * d^4 + (6 * c * d^2 + b) * e^2 * x^2 + b * d^2 + 2 * (2 * c * d^3 + b * d) * e * x + a)) + (b^2 - 4 * a * c) * f^3 * \log(c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + c * d^4 + (6 * c * d^2 + b) * e^2 * x^2 + b * d^2 + 2 * (2 * c * d^3 + b * d) * e * x + a)) / ((b^2 * c - 4 * a * c^2) * e), 1/4 * (2 * \text{sqrt}(-b^2 + 4 * a * c) * b * f^3 * \arctan(-(2 * c * e^2 * x^2 + 4 * c * d * e * x + 2 * c * d^2 + b) * \text{sqrt}(-b^2 + 4 * a * c)) / (b^2 - 4 * a * c)) + (b^2 - 4 * a * c) * f^3 * \log(c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + c * d^4 + (6 * c * d^2 + b) * e^2 * x^2 + b * d^2 + 2 * (2 * c * d^3 + b * d) * e * x + a)) / ((b^2 * c - 4 * a * c^2) * e)]$

giac [B] time = 0.43, size = 162, normalized size = 1.86

$$\frac{bf^3 \arctan\left(\frac{2cd^2 f + 2(fx^2 e + 2dfx)ce + bf}{\sqrt{-b^2 + 4ac} f}\right) e^{(-1)}}{2\sqrt{-b^2 + 4ac} c} + \frac{f^3 e^{(-1)} \log\left(cd^4 f^2 + 2(fx^2 e + 2dfx)cd^2 fe + bd^2 f^2 + (fx^2 e + 2dfx)^2 ce^2 + (fx^2 e + 2dfx)bfe + af^2\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="giac")

[Out] $-1/2 * b * f^3 * \arctan((2 * c * d^2 * f + 2 * (f * x^2 * e + 2 * d * f * x) * c * e + b * f) / (\text{sqrt}(-b^2 + 4 * a * c) * f)) * e^{(-1)} / (\text{sqrt}(-b^2 + 4 * a * c) * c) + 1/4 * f^3 * e^{(-1)} * \log(c * d^4 * f^2 + 2 * (f * x^2 * e + 2 * d * f * x) * c * d^2 * f * e + b * d^2 * f^2 + (f * x^2 * e + 2 * d * f * x)^2 * c * e^2 + (f * x^2 * e + 2 * d * f * x) * b * f * e + a * f^2) / c$

maple [C] time = 0.00, size = 154, normalized size = 1.77

$$\frac{f^3 \left(\text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a)^3 + 3 \text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a)^2 d^2 + 3 \text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a) d^2 + 3 \text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a) \ln(-\text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a)) \right)}{2 \left(2 \text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a)^3 + 6 \text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a)^2 d^2 + 6 \text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a) d^2 + 6 \text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a) \ln(-\text{RootOf}(_Z^4 + 4_Z^3 d^2 + c^2 + b^2 + (4c^2 + 4b^2)_Z + (4d^2 + 2ab)_Z + a)) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4), x)

[Out] 1/2*f^3/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x), _R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(efx + df)^3}{(ex + d)^4 c + (ex + d)^2 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="maxima")

[Out] integrate((e*f*x + d*f)^3/((e*x + d)^4*c + (e*x + d)^2*b + a), x)

mupad [B] time = 0.44, size = 287, normalized size = 3.30

$$\frac{4ace^3 \ln(cd^4 + 4cd^3ex + 6cd^2e^2x^2 + bd^2 + 4cd^2e^2x^3 + 2bdex + ce^4x^4 + be^2x^2 + a)}{16a^2e^2 - 4b^2ce^2} - \frac{bf^3 \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cd^2}{\sqrt{4ac-b^2}} + \frac{2ce^2x^2}{\sqrt{4ac-b^2}} + \frac{4cdex}{\sqrt{4ac-b^2}}\right)}{2ce\sqrt{4ac-b^2}} - \frac{b^2e^3 \ln(cd^4 + 4cd^3ex + 6cd^2e^2x^2 + bd^2 + 4cd^2e^2x^3 + 2bdex + ce^4x^4 + be^2x^2 + a)}{16a^2e^2 - 4b^2ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x)

[Out] (4*a*c*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b*f^3*atan(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*e^2*x^2)/(4*a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*a*c - b^2)^(1/2)) - (b^2*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2)

sympy [B] time = 1.95, size = 332, normalized size = 3.82

$$\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)} + \frac{f^3}{4ce} \right) \log\left(\frac{2dx}{e} + x^2 + \frac{-8ace\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{f^3}{4ce}\right) + 2af^3 + 2b^2e\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{f^3}{4ce}\right) + bd^2f^3}{be^2f^3} \right) + \left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)} + \frac{f^3}{4ce} \right) \log\left(\frac{2dx}{e} + x^2 + \frac{-8ace\left(\frac{bf^3\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{f^3}{4ce}\right) + 2af^3 + 2b^2e\left(\frac{bf^3\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{f^3}{4ce}\right) + bd^2f^3}{be^2f^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)

```
[Out] (-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d
*x/e + x**2 + (-8*a*c*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2))
+ f**3/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*
(4*a*c - b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3)) + (b*f**3*sqrt
(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d*x/e + x**2
+ (-8*a*c*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*
e)) + 2*a*f**3 + 2*b**2*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)
) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3))
```

$$3.527 \quad \int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4} dx$$

Optimal. Leaf size=170

$$\frac{f^2 \sqrt{\sqrt{b^2 - 4ac} + b} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{2} \sqrt{c} e \sqrt{b^2 - 4ac}} - \frac{f^2 \sqrt{b - \sqrt{b^2 - 4ac}} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} e \sqrt{b^2 - 4ac}}$$

Rubi [A] time = 0.15, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1142, 1130, 205}

$$\frac{f^2 \sqrt{\sqrt{b^2 - 4ac} + b} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{2} \sqrt{c} e \sqrt{b^2 - 4ac}} - \frac{f^2 \sqrt{b - \sqrt{b^2 - 4ac}} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} e \sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]

[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1130

Int[((d_)*(x_))^(m_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 1142

Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned} \int \frac{(df + efx)^2}{a + b(d + ex)^2 + c(d + ex)^4} dx &= \frac{f^2 \operatorname{Subst}\left(\int \frac{x^2}{a + bx^2 + cx^4} dx, x, d + ex\right)}{e} \\ &= \frac{\left(\left(1 - \frac{b}{\sqrt{b^2 - 4ac}}\right) f^2\right) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx, x, d + ex\right)}{2e} + \frac{\left(\left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) f^2\right) \operatorname{Subst}\left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx, x, d + ex\right)}{2e} \\ &= -\frac{\sqrt{b - \sqrt{b^2 - 4ac}} f^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}e} + \frac{\sqrt{b + \sqrt{b^2 - 4ac}} f^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}e} \end{aligned}$$

Mathematica [A] time = 0.10, size = 178, normalized size = 1.05

$$\frac{f^2 \left(\left(\sqrt{b^2 - 4ac} - b \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) + \sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{\sqrt{b^2 - 4ac} + b} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right) \right)}{\sqrt{2}\sqrt{c}e\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] (f^2*((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]))/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df + efx)^2}{a + b(d + ex)^2 + c(d + ex)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

fricas [B] time = 1.10, size = 799, normalized size = 4.70

$$\frac{1}{2} \sqrt{2} \sqrt{\frac{\sqrt{b^2 - 4ac} - b}{\sqrt{b^2 - 4ac}}} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{\frac{\sqrt{b^2 - 4ac} - b}{\sqrt{b^2 - 4ac}}}} \right) + \sqrt{b - \sqrt{b^2 - 4ac}} \sqrt{\sqrt{b^2 - 4ac} + b} \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}(d + ex)}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
[Out] 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^3*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^2)/((b^2*c - 4*a*c^2)*e^2)) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^3*sqrt(-(b*f^4 + (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^2)/((b^2*c - 4*a*c^2)*e^2)) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^3*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^2)/((b^2*c - 4*a*c^2)*e^2)) + 1/2*sqrt(1/2)*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^2)/((b^2*c - 4*a*c^2)*e^2))*log(e*f^6*x + d*f^6 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^3*sqrt(-(b*f^4 - (b^2*c - 4*a*c^2)*sqrt(f^8/((b^2*c^2 - 4*a*c^3)*e^4)))*e^2)/((b^2*c - 4*a*c^2)*e^2))
```

giac [B] time = 0.47, size = 1325, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
[Out] -1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) - 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) - 1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
```

- 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) - 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e)

maple [C] time = 0.00, size = 143, normalized size = 0.84

$$\frac{\int^x \left(\text{RootOf} \left(_Z^4 + 4 _Z^3 d^2 + c^2 + b^2 + (4c^2 d^2 + b^2) _Z^2 + (4bd + 2ab) _Z + a \right)^2 + 2 \text{RootOf} \left(_Z^4 + 4 _Z^3 d^2 + c^2 + b^2 + (4c^2 d^2 + b^2) _Z^2 + (4bd + 2ab) _Z + a \right) \ln \left(\frac{\text{RootOf} \left(_Z^4 + 4 _Z^3 d^2 + c^2 + b^2 + (4c^2 d^2 + b^2) _Z^2 + (4bd + 2ab) _Z + a \right)}{2 \left(\text{RootOf} \left(_Z^4 + 4 _Z^3 d^2 + c^2 + b^2 + (4c^2 d^2 + b^2) _Z^2 + (4bd + 2ab) _Z + a \right) + \text{RootOf} \left(_Z^4 + 4 _Z^3 d^2 + c^2 + b^2 + (4c^2 d^2 + b^2) _Z^2 + (4bd + 2ab) _Z + a \right) \right)} \right)}{2 \left(\text{RootOf} \left(_Z^4 + 4 _Z^3 d^2 + c^2 + b^2 + (4c^2 d^2 + b^2) _Z^2 + (4bd + 2ab) _Z + a \right) + \text{RootOf} \left(_Z^4 + 4 _Z^3 d^2 + c^2 + b^2 + (4c^2 d^2 + b^2) _Z^2 + (4bd + 2ab) _Z + a \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4), x)
 [Out] 1/2*f^2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x), _R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(efx + df)^2}{(ex + d)^4c + (ex + d)^2b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4), x, algorithm="maxima")
 [Out] integrate((e*f*x + d*f)^2/((e*x + d)^4*c + (e*x + d)^2*b + a), x)

mupad [B] time = 1.79, size = 683, normalized size = 4.02

$$\frac{\int^x \left(\frac{f^2 \sqrt{4ac - b^2} \sqrt{4ac - b^2}}{4ac^2 d^2} \left(\frac{(4bd^2 + 2ab) \sqrt{4ac - b^2} \sqrt{4ac - b^2}}{4ac^2 d^2} + 4bd^2 d^2 f^2 - 2bd^2 d^2 f^2 \right) \right)}{4ac^2 d^2} \frac{\sqrt{4ac - b^2} \sqrt{4ac - b^2}}{4ac^2 d^2} \left(\frac{(4bd^2 + 2ab) \sqrt{4ac - b^2} \sqrt{4ac - b^2}}{4ac^2 d^2} + 4bd^2 d^2 f^2 - 2bd^2 d^2 f^2 \right)}{4ac^2 d^2} \frac{\sqrt{4ac - b^2} \sqrt{4ac - b^2}}{4ac^2 d^2} \left(\frac{(4bd^2 + 2ab) \sqrt{4ac - b^2} \sqrt{4ac - b^2}}{4ac^2 d^2} + 4bd^2 d^2 f^2 - 2bd^2 d^2 f^2 \right)}{4ac^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x)`

[Out]
$$-2 \operatorname{atanh}\left(\frac{-(b^3 f^4 + f^4(-4ac - b^2)^3)^{1/2} - 4abc f^4}{8(b^4 c e^2 + 16a^2 c^3 e^2 - 8ab^2 c^2 e^2)}\right)^{1/2} \cdot (x(4a^2 c^2 e^{12} f^4 - 2b^2 c e^{12} f^4) + ((x(8b^3 c^2 e^{14} - 32abc^3 e^{14}) + 8b^3 c^2 d e^{13} - 32abc^3 d e^{13}) \cdot (b^3 f^4 + f^4(-4ac - b^2)^3)^{1/2} - 4abc f^4)) / (8(b^4 c e^2 + 16a^2 c^3 e^2 - 8ab^2 c^2 e^2)) + 4a^2 c^2 d e^{11} f^4 - 2b^2 c d e^{11} f^4) / (a c e^{10} f^6) \cdot (- (b^3 f^4 + f^4(-4ac - b^2)^3)^{1/2} - 4abc f^4) / (8(b^4 c e^2 + 16a^2 c^3 e^2 - 8ab^2 c^2 e^2))\right)^{1/2} - 2 \operatorname{atanh}\left(\frac{(f^4(-4ac - b^2)^3)^{1/2} - b^3 f^4 + 4abc f^4}{8(b^4 c e^2 + 16a^2 c^3 e^2 - 8ab^2 c^2 e^2)}\right)^{1/2} \cdot (x(4a^2 c^2 e^{12} f^4 - 2b^2 c e^{12} f^4) - ((x(8b^3 c^2 e^{14} - 32abc^3 e^{14}) + 8b^3 c^2 d e^{13} - 32abc^3 d e^{13}) \cdot (f^4(-4ac - b^2)^3)^{1/2} - b^3 f^4 + 4abc f^4)) / (8(b^4 c e^2 + 16a^2 c^3 e^2 - 8ab^2 c^2 e^2)) + 4a^2 c^2 d e^{11} f^4 - 2b^2 c d e^{11} f^4) / (a c e^{10} f^6) \cdot ((f^4(-4ac - b^2)^3)^{1/2} - b^3 f^4 + 4abc f^4) / (8(b^4 c e^2 + 16a^2 c^3 e^2 - 8ab^2 c^2 e^2))\right)^{1/2}$$

sympy [A] time = 1.59, size = 124, normalized size = 0.73

$$\operatorname{RootSum}\left(t^4(256a^2c^3e^4 - 128ab^2c^2e^4 + 16b^4ce^4) + t^2(-16abce^2f^4 + 4b^3e^2f^4) + af^8, \left(t \mapsto t \log\left(x + \frac{64t^3ac^2e^3 - 16t^3b^2ce^3 - 2tbf^4 + df^6}{ef^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)`

[Out] `RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**2*f**4 + 4*b**3*e**2*f**4) + a*f**8, Lambda(_t, _t*log(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e*f**4 + d*f**6)/(e*f**6))))`

$$3.528 \quad \int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4} dx$$

Optimal. Leaf size=44

$$\frac{f \tanh^{-1} \left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}} \right)}{e\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1142, 1107, 618, 206}

$$\frac{f \tanh^{-1} \left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}} \right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] -((f*ArcTanh[(b + 2*c*(d + e*x)^2]/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*e))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4} dx &= \frac{f \operatorname{Subst}\left(\int \frac{x}{a + bx^2 + cx^4} dx, x, d + ex\right)}{e} \\
&= \frac{f \operatorname{Subst}\left(\int \frac{1}{a + bx + cx^2} dx, x, (d + ex)^2\right)}{2e} \\
&= \frac{f \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2c(d + ex)^2\right)}{e} \\
&= \frac{f \tanh^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{b^2 - 4ac} e}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.07

$$\frac{f \tan^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{4ac - b^2}}\right)}{e\sqrt{4ac - b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] (f*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{df + efx}{a + b(d + ex)^2 + c(d + ex)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x]

fricas [A] time = 1.22, size = 274, normalized size = 6.23

$$\left[\frac{f \log\left(\frac{2c^2e^4x^4 + 8c^2de^3x^3 + 2c^2d^4 + 2(6c^2d^2 + bc)e^2x^2 + 2bcd^2 + 4(2c^2d^3 + bcd)ex + b^2 - 2ac - (2ce^2x^2 + 4cdex + 2cd^2 + b)\sqrt{b^2 - 4ac}}{ce^4x^4 + 4cde^3x^3 + cd^4 + (6cd^2 + b)e^2x^2 + bd^2 + 2(2cd^3 + bd)ex + a}\right)}{2\sqrt{b^2 - 4ac}e}, -\frac{\sqrt{-b^2 + 4ac} f \arctan\left(\frac{(2ce^2x^2 + 4cdex + 2cd^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right)}{(b^2 - 4ac)e} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] [1/2*f*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(sqrt(b^2 - 4*a*c)*e), -sqrt(-b^2 + 4*a*c)*f*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)*e)]

giac [A] time = 0.40, size = 62, normalized size = 1.41

$$\frac{f \arctan\left(\frac{2cd^2f+2(fx^2e+2dfx)ce+bf}{\sqrt{-b^2+4ac}f}\right)e^{(-1)}}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)*f))*e^(-1)/sqrt(-b^2 + 4*a*c)

maple [C] time = 0.00, size = 130, normalized size = 2.95

$$\frac{f \operatorname{RootOf}\left(\frac{2cd^2f+2(fx^2e+2dfx)ce+bf}{\sqrt{-b^2+4ac}f}\right)e^{(-1)}}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)

[Out] 1/2*f/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx + df}{(ex + d)^4c + (ex + d)^2b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] integrate((e*f*x + d*f)/((e*x + d)^4*c + (e*x + d)^2*b + a), x)

mupad [B] time = 1.62, size = 477, normalized size = 10.84

$$f \operatorname{atan} \left(\frac{\frac{f \left(4c^2 d^2 e^7 f + 4c^2 e^9 f x^2 - \frac{f(8bc^2 d^2 e^8 + 16bc^2 d e^9 x + 8bc^2 e^{10} x^2 + 16ac^2 e^8)}{2e\sqrt{b^2 - 4ac}} + 8c^2 d e^8 f x \right)}{2e\sqrt{b^2 - 4ac}} + \frac{f \left(4c^2 d^2 e^7 f + 4c^2 e^9 f x^2 + \frac{f(8bc^2 d^2 e^8 + 16bc^2 d e^9 x + 8bc^2 e^{10} x^2 + 16ac^2 e^8)}{2e\sqrt{b^2 - 4ac}} + 8c^2 d e^8 f x \right)}{2e\sqrt{b^2 - 4ac}}}{\frac{f \left(4c^2 d^2 e^7 f + 4c^2 e^9 f x^2 - \frac{f(8bc^2 d^2 e^8 + 16bc^2 d e^9 x + 8bc^2 e^{10} x^2 + 16ac^2 e^8)}{2e\sqrt{b^2 - 4ac}} + 8c^2 d e^8 f x \right)}{2e\sqrt{b^2 - 4ac}} - \frac{f \left(4c^2 d^2 e^7 f + 4c^2 e^9 f x^2 + \frac{f(8bc^2 d^2 e^8 + 16bc^2 d e^9 x + 8bc^2 e^{10} x^2 + 16ac^2 e^8)}{2e\sqrt{b^2 - 4ac}} + 8c^2 d e^8 f x \right)}{2e\sqrt{b^2 - 4ac}}} \right) 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4), x)`

[Out]
$$\begin{aligned} & (f \operatorname{atan}(((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^{10}*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c))^{(1/2)})) \\ & + 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2 - 4*a*c))^{(1/2)} + (f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^{10}*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c))^{(1/2)})) \\ & + 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2 - 4*a*c))^{(1/2)})))/((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^{10}*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c))^{(1/2)})) \\ & + 8*c^2*d*e^8*f*x))/(2*e*(b^2 - 4*a*c))^{(1/2)} - (f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^{10}*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c))^{(1/2)})) \\ & + 8*c^2*d*e^8*f*x))/(2*e*(b^2 - 4*a*c))^{(1/2)})))*1i)/(e*(b^2 - 4*a*c))^{(1/2)} \end{aligned}$$

sympy [B] time = 1.19, size = 189, normalized size = 4.30

$$\frac{f \sqrt{-\frac{1}{4ac-b^2}} \log\left(\frac{2dx}{e} + x^2 + \frac{-4acf \sqrt{-\frac{1}{4ac-b^2}} + b^2 f \sqrt{-\frac{1}{4ac-b^2}} + bf + 2cd^2 f}{2ce^2 f}\right)}{2e} + \frac{f \sqrt{-\frac{1}{4ac-b^2}} \log\left(\frac{2dx}{e} + x^2 + \frac{4acf \sqrt{-\frac{1}{4ac-b^2}} - b^2 f \sqrt{-\frac{1}{4ac-b^2}} + bf + 2cd^2 f}{2ce^2 f}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)`

[Out]
$$\begin{aligned} & -f \sqrt{-1/(4*a*c - b**2)} * \log(2*d*x/e + x**2 + (-4*a*c*f \sqrt{-1/(4*a*c - b**2)} + b**2*f \sqrt{-1/(4*a*c - b**2)} + b*f + 2*c*d**2*f)/(2*c*e**2*f)) / \\ & (2*e) + f \sqrt{-1/(4*a*c - b**2)} * \log(2*d*x/e + x**2 + (4*a*c*f \sqrt{-1/(4*a*c - b**2)} - b**2*f \sqrt{-1/(4*a*c - b**2)} + b*f + 2*c*d**2*f)/(2*c*e**2*f)) / \\ & (2*e) \end{aligned}$$

$$3.529 \quad \int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal. Leaf size=103

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4aef} + \frac{\log(d+ex)}{aef}$$

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1142, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4aef} + \frac{\log(d+ex)}{aef}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(2*a*Sqrt[b^2 - 4*a*c]*e*f) + Log[d + e*x]/(a*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*e*f)

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1114

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(df + efx)(a + b(d + ex)^2 + c(d + ex)^4)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)} dx, x, d + ex\right)}{ef} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)} dx, x, (d + ex)^2\right)}{2ef} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, (d + ex)^2\right)}{2aef} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2} dx, x, (d + ex)^2\right)}{2aef} \\
&= \frac{\log(d + ex)}{aef} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, (d + ex)^2\right)}{4aef} - \frac{b \text{Subst}\left(\int \frac{1}{x} dx, x, (d + ex)^2\right)}{2aef} \\
&= \frac{\log(d + ex)}{aef} - \frac{\log(a + b(d + ex)^2 + c(d + ex)^4)}{4aef} + \frac{b \text{Subst}\left(\int \frac{1}{x} dx, x, (d + ex)^2\right)}{2aef} \\
&= \frac{b \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}ef} + \frac{\log(d + ex)}{aef} - \frac{\log(a + b(d + ex)^2 + c(d + ex)^4)}{4aef}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 131, normalized size = 1.27

$$\frac{4\sqrt{b^2 - 4ac} \log(d + ex) - (\sqrt{b^2 - 4ac} + b) \log(-\sqrt{b^2 - 4ac} + b + 2c(d + ex)^2) + (b - \sqrt{b^2 - 4ac}) \log(\sqrt{b^2 - 4ac} + b + 2c(d + ex)^2)}{4aef\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e*f)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)(a + b(d + ex)^2 + c(d + ex)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x
]

fricas [A] time = 1.18, size = 474, normalized size = 4.60

$$\frac{\sqrt{b^2 - 4ac} \log\left(\frac{c^2 d^2 + 4cd^2 e^2 + 4c^2 d^2 e^2 + (6cd^2 + b^2)e^2 + b^2 + 2(2cd^2 + bd)ex + a}{4(b^2 - 4ac)^2}\right) - (b^2 - 4ac) \log(c^2 e^4 + 4cd^2 e^3 + c^2 d^4 + (6cd^2 + b^2)e^2 + b^2 + 2(2cd^2 + bd)ex + a) + 4(b^2 - 4ac) \log(ex + d) - 2\sqrt{b^2 - 4ac} \arctan\left(\frac{2cd^2 + 4cd^2 e^2 + c^2 d^4}{4c^2 d^2}\right) - (b^2 - 4ac) \log(c^2 e^4 + 4cd^2 e^3 + c^2 d^4 + (6cd^2 + b^2)e^2 + b^2 + 2(2cd^2 + bd)ex + a) + 4(b^2 - 4ac) \log(ex + d)}{4(b^2 - 4ac)^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^2*4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(ex + d))/((a*b^2 - 4*a^2*c)*e*f), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^2 - 4*a*c)*log(ex + d))/((a*b^2 - 4*a^2*c)*e*f)]

giac [B] time = 1.19, size = 285, normalized size = 2.77

$$\frac{e^{-1} \log\left(\frac{c^2 x^4 + 4cdx^3 + 6cd^2x^2 + 4cd^3x + cd^4 + b^2x^2 + 2bdxe + bd^2 + a}{af}\right) + e^{-1} \log\left(\frac{c^2 x^4 + 4cdx^3 + 6cd^2x^2 + 4cd^3x + cd^4 + b^2x^2 + 2bdxe + bd^2 + a}{af}\right)}{4af} - \frac{\left(\frac{abc^2 \log\left(\frac{bx^2 + 2bdx + \sqrt{b^2 - 4ac}x^2 + 2\sqrt{b^2 - 4ac}dx + b^2 + \sqrt{b^2 - 4ac}d^2 + a}{\sqrt{b^2 - 4ac}}\right) - abc^2 \log\left(\frac{-bx^2 - 2bdx + \sqrt{b^2 - 4ac}x^2 + 2\sqrt{b^2 - 4ac}dx - b^2 + \sqrt{b^2 - 4ac}d^2 + a}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{b^2 - 4ac}}\right) e^{-4}}{4a^2 c f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] -1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a*f) + e^(-1)*log(abs(x*e + d))/(a*f) - 1/4*(a*b*c*f*e^3*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sqrt(b^2 - 4*a*c) - a*b*c*f*e^3*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^2*c*f^2)

maple [C] time = 0.01, size = 190, normalized size = 1.84

$$\frac{\ln\left(\frac{c^2 x^4 + 4cdx^3 + 6cd^2x^2 + 4cd^3x + cd^4 + b^2x^2 + 2bdxe + bd^2 + a}{af}\right) + e^{-1} \log\left(\frac{c^2 x^4 + 4cdx^3 + 6cd^2x^2 + 4cd^3x + cd^4 + b^2x^2 + 2bdxe + bd^2 + a}{af}\right)}{4af} - \frac{\left(\frac{abc^2 \log\left(\frac{bx^2 + 2bdx + \sqrt{b^2 - 4ac}x^2 + 2\sqrt{b^2 - 4ac}dx + b^2 + \sqrt{b^2 - 4ac}d^2 + a}{\sqrt{b^2 - 4ac}}\right) - abc^2 \log\left(\frac{-bx^2 - 2bdx + \sqrt{b^2 - 4ac}x^2 + 2\sqrt{b^2 - 4ac}dx - b^2 + \sqrt{b^2 - 4ac}d^2 + a}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{b^2 - 4ac}}\right) e^{-4}}{4a^2 c f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)

[Out] $\frac{1}{2} \frac{f}{a} \frac{e}{\sum} \left((-R^3 c e^3 - 3 R^2 c d e^2 - c d^3 + (-3 c d^2 - b) R e - b d) / (2 R^3 c e^3 + 6 R^2 c d e^2 + 6 R c d^2 e + 2 c d^3 + R b e + b d) \ln(-R+x), R = \text{RootOf}(Z^4 c e^4 + 4 Z^3 c d e^3 + c d^4 + b d^2 + (6 c d^2 e^2 + b e^2) Z^2 + (4 c d^3 e + 2 b d e) Z + a) \right) + \ln(e x + d) / a e / f$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 3.46, size = 2520, normalized size = 24.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)`

[Out] $\log(d + e x) / (a e f) - (\log(a + b d^2 + c d^4 + b e^2 x^2 + c e^4 x^4 + 2 b d e x + 6 c d^2 e^2 x^2 + 4 c d^3 e x + 4 c d e^3 x^3) * (2 b^2 e f - 8 a c e f)) / (2 * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2)) - (b \operatorname{atan}((16 a^3 f^3 x (4 a c - b^2)^{(3/2)} * ((3 b^3 - 8 a b c) * (b^2 * ((2 * (2 b^2 e f - 8 a c e f) * (6 b^3 c^2 d e^{18} f - 20 a b c^3 d e^{18} f)) / (f * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2 - 2)) + (20 b c^3 d e^{17}) / f)) / (16 a^2 e^2 f^2 * (4 a c - b^2)) - ((2 b^2 e f - 8 a c e f)^2 * ((2 * (2 b^2 e f - 8 a c e f) * (6 b^3 c^2 d e^{18} f - 20 a b c^3 d e^{18} f)) / (f * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2)) + (20 b c^3 d e^{17}) / f)) / (4 * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2)^2 + (b^2 * (2 b^2 e f - 8 a c e f) * (6 b^3 c^2 d e^{18} f - 20 a b c^3 d e^{18} f)) / (4 a^2 e^2 f^3 * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2) * (4 a c - b^2)))) / (8 a^3 c^2 * (25 a c - 6 b^2)) - ((b * (2 b^2 e f - 8 a c e f) * ((2 * (2 b^2 e f - 8 a c e f) * (6 b^3 c^2 d e^{18} f - 20 a b c^3 d e^{18} f)) / (f * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2)) + (20 b c^3 d e^{17}) / f)) / (4 a e f * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2) * (4 a c - b^2)^{(1/2)}) - (b^3 * (6 b^3 c^2 d e^{18} f - 20 a b c^3 d e^{18} f)) / (16 a^3 e^3 f^4 * (4 a c - b^2)^{(3/2)}) + (b * (2 b^2 e f - 8 a c e f)^2 * (6 b^3 c^2 d e^{18} f - 20 a b c^3 d e^{18} f)) / (4 a e f^2 * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2)^2 * (4 a c - b^2)^{(1/2)})) * (3 b^4 + 10 a^2 c^2 - 14 a b^2 c)) / (8 a^3 c^2 * (4 a c - b^2)^{(1/2)} * (25 a c - 6 b^2))) / (b^2 c^2 e^{14} + (2 f^3 * (3 b^3 - 8 a b c) * (4 a c - b^2)^{(3/2)} * (b^2 * ((2 * (2 b^2 c^2 e^{16} + 5 b c^3 d^2 e^{16})) / f + ((2 b^2 e f - 8 a c e f) * (2 a b^2 c^2 e^{17} f + 6 b^3 c^2 d^2 e^{17} f - 20 a b c^3 d^2 e^{17} f)) / (f * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2)))))) / (16 a^2 e^2 f^2 * (4 a c - b^2)) - ((2 b^2 e f - 8 a c e f)^2 * ((2 * (2 b^2 c^2 e^{16} + 5 b c^3 d^2 e^{16})) / f + ((2 b^2 e f - 8 a c e f) * (2 a b^2 c^2 e^{17} f + 6 b^3 c^2 d^2 e^{17} f - 20 a b c^3 d^2 e^{17} f)) / (f * (4 a b^2 e^2 f^2 - 16 a^2 c e^2 f^2)))))) / (4 * (4 a b^2$

$$\begin{aligned}
& 2e^{2f^2} - 16a^2c^2e^{2f^2})^2 + (b^2(2b^2ef - 8a^2c^2e^{2f^2}) + (b^2(2b^2ef - 8a^2c^2e^{2f^2}) + 6b^3c^2d^2e^{17f} - 20a^2b^2c^3d^2e^{17f}))/((8a^2e^{2f^3}(4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2})(4a^2c - b^2))) + (16a^3f^3x^2(4a^2c - b^2)^{(3/2)} * (((3b^3 - 8a^2b^2c) * ((b^2 * ((10b^3c^3e^{18})/f + ((2b^2ef - 8a^2c^2e^{19f} - 20a^2b^2c^3e^{19f}))/((f * (4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2})))))) / (16a^2e^{2f^2}(4a^2c - b^2)) - (((10b^3c^3e^{18})/f + ((2b^2ef - 8a^2c^2e^{19f} - 20a^2b^2c^3e^{19f}))/((f * (4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2})))) * (2b^2ef - 8a^2c^2e^{19f} - 20a^2b^2c^3e^{19f}))/((4 * (4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2})^2) + (b^2(2b^2ef - 8a^2c^2e^{19f} - 20a^2b^2c^3e^{19f}))/((8a^2e^{2f^3}(4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2})(4a^2c - b^2)))))) / (8a^3c^2(25a^2c - 6b^2)) - ((3b^4 + 10a^2c^2 - 14a^2b^2c) * ((b * ((10b^3c^3e^{18})/f + ((2b^2ef - 8a^2c^2e^{19f} - 20a^2b^2c^3e^{19f}))/((f * (4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2})))) * (2b^2ef - 8a^2c^2e^{19f} - 20a^2b^2c^3e^{19f}))/((4a^2e^{19f} * (4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2}) * (4a^2c - b^2)^{(1/2)}) - (b^3(6b^3c^2e^{19f} - 20a^2b^2c^3e^{19f}))/((32a^3e^{19f} * (4a^2c - b^2)^{(3/2)}) + (b * (2b^2ef - 8a^2c^2e^{19f} - 20a^2b^2c^3e^{19f}))/((8a^2e^{19f} * (4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2})^2 * (4a^2c - b^2)^{(1/2)}))) / (8a^3c^2(4a^2c - b^2)^{(1/2}) * (25a^2c - 6b^2)))) / (b^2c^2e^{14}) - (2f^3(4a^2c - b^2) * (3b^4 + 10a^2c^2 - 14a^2b^2c) * ((b * (2b^2ef - 8a^2c^2e^{16} + 5b^3c^3d^2e^{16}))/f + ((2b^2ef - 8a^2c^2e^{16} + 5b^3c^3d^2e^{16}))/((f * (4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2})))))) / (4a^2e^{16} * (4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2}) * (4a^2c - b^2)^{(1/2)}) - (b^3(2a^2b^2c^2e^{17f} + 6b^3c^2d^2e^{17f} - 20a^2b^2c^3d^2e^{17f}))/((32a^3e^{17f} * (4a^2c - b^2)^{(3/2)}) + (b * (2b^2ef - 8a^2c^2e^{17f} - 20a^2b^2c^3d^2e^{17f}))/((8a^2e^{17f} * (4a^2b^2e^{2f^2} - 16a^2c^2e^{2f^2})^2 * (4a^2c - b^2)^{(1/2)}))) / (b^2c^4e^{14}(25a^2c - 6b^2)))) / (2a^2e^{14} * (4a^2c - b^2)^{(1/2)})
\end{aligned}$$

sympy [B] time = 7.16, size = 348, normalized size = 3.38

$$\left(\frac{b\sqrt{-4ac+b^2}}{4acf(4ac-b^2)} - \frac{1}{4acf} \right) \log \left(\frac{2dx}{e} + x^2 + \frac{-8a^2cef \left(\frac{b\sqrt{-4ac+b^2}}{4af(4ac-b^2)} - \frac{1}{4af} \right) + 2ab^2ef \left(\frac{b\sqrt{-4ac+b^2}}{4af(4ac-b^2)} - \frac{1}{4af} \right) - 2ac + b^2 + bcd^2}{bce^2} \right) + \left(\frac{b\sqrt{-4ac+b^2}}{4acf(4ac-b^2)} - \frac{1}{4acf} \right) \log \left(\frac{2dx}{e} + x^2 + \frac{-8a^2cef \left(\frac{b\sqrt{-4ac+b^2}}{4af(4ac-b^2)} - \frac{1}{4af} \right) + 2ab^2ef \left(\frac{b\sqrt{-4ac+b^2}}{4af(4ac-b^2)} - \frac{1}{4af} \right) - 2ac + b^2 + bcd^2}{bce^2} \right) + \frac{\log \left(\frac{d}{e} + x \right)}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)

[Out] $(-b\sqrt{-4ac + b^2})/(4a^2e^2f(4a^2c - b^2)) - 1/(4a^2e^2f) * \log(2d*x/e + x^2 + (-8a^2c^2e^2f*(-b\sqrt{-4ac + b^2})/(4a^2e^2f(4a^2c - b^2)) - 1/(4a^2e^2f) + 2a^2b^2e^2f*(-b\sqrt{-4ac + b^2})/(4a^2e^2f(4a^2c - b^2)) - 1/(4a^2e^2f) - 2a^2c + b^2 + b^2cd^2)/(b^2ce^2)) + (b\sqrt{-4ac + b^2})/(4a^2e^2f(4a^2c - b^2)) - 1/(4a^2e^2f) * \log(2d*x/e + x^2 + (-8a^2c^2e^2f*(b\sqrt{-4ac + b^2})/(4a^2e^2f(4a^2c - b^2)) - 1/(4a^2e^2f) + 2a^2b^2e^2f*(b\sqrt{-4ac + b^2})/(4a^2e^2f(4a^2c - b^2)) - 1/(4a^2e^2f) - 2a^2c + b^2 + b^2cd^2)/(b^2ce^2)) + \log(d/e + x)/(a^2e^2f)$

$$3.530 \quad \int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal. Leaf size=204

$$\frac{\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} aef^2 \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} aef^2 \sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{aef^2(d+ex)}$$

Rubi [A] time = 0.27, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1142, 1123, 1166, 205}

$$\frac{\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} aef^2 \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} aef^2 \sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{aef^2(d+ex)}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] -(1/(a*e*f^2*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1123

Int[((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,

$x], x, v], x] /; \text{FreeQ}[\{a, b, c, m, p\}, x] \ \&\& \ \text{LinearPairQ}[u, v, x]$

Rule 1166

$\text{Int}[\frac{(d_.) + (e_.)(x_)^2}{(a_) + (b_.)(x_)^2 + (c_.)(x_)^4}, x_Symbol] :$
 $> \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[e/2 + (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 - q/2 + c*x^2), x], x] + \text{Dist}[e/2 - (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 + q/2 + c*x^2), x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(df + efx)^2 (a + b(d + ex)^2 + c(d + ex)^4)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)} dx, x, d + ex\right)}{ef^2} \\ &= -\frac{1}{aef^2(d + ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4} dx, x, d + ex\right)}{aef^2} \\ &= -\frac{1}{aef^2(d + ex)} - \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx, x, d + ex\right)}{2aef^2} \\ &= -\frac{1}{aef^2(d + ex)} - \frac{\sqrt{c}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}ef^2} - \frac{\sqrt{c}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}ef^2} \end{aligned}$$

Mathematica [A] time = 0.34, size = 209, normalized size = 1.02

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2}{d+ex}}{2aef^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]*Sqrt[

$b - \sqrt{b^2 - 4ac}] + (\sqrt{2}\sqrt{c}(-b + \sqrt{b^2 - 4ac}))\text{ArcTan}[\frac{(\sqrt{2}\sqrt{c}(d + ex))/\sqrt{b + \sqrt{b^2 - 4ac}}]}{(\sqrt{b^2 - 4ac})*\sqrt{b + \sqrt{b^2 - 4ac}}]})/(aef^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)^2 (a + b(d + ex)^2 + c(d + ex)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

fricas [B] time = 1.45, size = 1477, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] $\frac{1}{2} * (\sqrt{1/2} * (a * e^2 * f^2 * x + a * d * e * f^2) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2) / ((a^6 * b^2 - 4 * a^7 * c) * e^4 * f^8)) + b^3 - 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4)) * \log(-2 * (b^2 * c^2 - a * c^3) * e * x - 2 * (b^2 * c^2 - a * c^3) * d + \sqrt{1/2} * ((a^3 * b^4 - 6 * a^4 * b^2 * c + 8 * a^5 * c^2) * e^3 * f^6 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2) / ((a^6 * b^2 - 4 * a^7 * c) * e^4 * f^8)) - (b^5 - 5 * a * b^3 * c + 4 * a^2 * b * c^2) * e * f^2) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2) / ((a^6 * b^2 - 4 * a^7 * c) * e^4 * f^8)) + b^3 - 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4)) - \sqrt{1/2} * (a * e^2 * f^2 * x + a * d * e * f^2) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2) / ((a^6 * b^2 - 4 * a^7 * c) * e^4 * f^8)) + b^3 - 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4)) * \log(-2 * (b^2 * c^2 - a * c^3) * e * x - 2 * (b^2 * c^2 - a * c^3) * d - \sqrt{1/2} * ((a^3 * b^4 - 6 * a^4 * b^2 * c + 8 * a^5 * c^2) * e^3 * f^6 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2) / ((a^6 * b^2 - 4 * a^7 * c) * e^4 * f^8)) - (b^5 - 5 * a * b^3 * c + 4 * a^2 * b * c^2) * e * f^2) * \sqrt{-((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2) / ((a^6 * b^2 - 4 * a^7 * c) * e^4 * f^8)) + b^3 - 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4)) - \sqrt{1/2} * (a * e^2 * f^2 * x + a * d * e * f^2) * \sqrt{((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2) / ((a^6 * b^2 - 4 * a^7 * c) * e^4 * f^8)) - b^3 + 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4)) * \log(-2 * (b^2 * c^2 - a * c^3) * e * x - 2 * (b^2 * c^2 - a * c^3) * d + \sqrt{1/2} * ((a^3 * b^4 - 6 * a^4 * b^2 * c + 8 * a^5 * c^2) * e^3 * f^6 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2) / ((a^6 * b^2 - 4 * a^7 * c) * e^4 * f^8)) + (b^5 - 5 * a * b^3 * c + 4 * a^2 * b * c^2) * e * f^2) * \sqrt{((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4 * \sqrt{(b^4 - 2 * a * b^2 * c + a^2 * c^2) / ((a^6 * b^2 - 4 * a^7 * c) * e^4 * f^8)) + b^3 - 3 * a * b * c} / ((a^3 * b^2 - 4 * a^4 * c) * e^2 * f^4))$

$$- 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4))) + \sqrt{1/2}*(a*e^2*f^2*x + a*d*e*f^2)*\sqrt{((a^3*b^2 - 4*a^4*c)*e^2*f^4*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4)}*\log(-2*(b^2*c^2 - a*c^3)*e*x - 2*(b^2*c^2 - a*c^3)*d - \sqrt{1/2}*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^3*f^6*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)} + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e*f^2)*\sqrt{((a^3*b^2 - 4*a^4*c)*e^2*f^4*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*e^4*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*e^2*f^4)}) - 2)/(a*e^2*f^2*x + a*d*e*f^2)$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueDone

maple [C] time = 0.01, size = 174, normalized size = 0.85

$$\frac{(-\text{RootOf}(Z^2 + 4Z^2d^2 + c^2 + b^2 + (4c^2d^2 + b^2)Z^2 + (4c^2d^2 + b^2)Z + a))e^2 - 2\text{RootOf}(Z^2 + 4Z^2d^2 + c^2 + b^2 + (4c^2d^2 + b^2)Z^2 + (4c^2d^2 + b^2)Z + a)\ln(-\text{RootOf}(Z^2 + 4Z^2d^2 + c^2 + b^2 + (4c^2d^2 + b^2)Z^2 + (4c^2d^2 + b^2)Z + a))}{2af^2(2af^2\text{RootOf}(Z^2 + 4Z^2d^2 + c^2 + b^2 + (4c^2d^2 + b^2)Z^2 + (4c^2d^2 + b^2)Z + a) + 4af^2\text{RootOf}(Z^2 + 4Z^2d^2 + c^2 + b^2 + (4c^2d^2 + b^2)Z^2 + (4c^2d^2 + b^2)Z + a) + 6af^2\text{RootOf}(Z^2 + 4Z^2d^2 + c^2 + b^2 + (4c^2d^2 + b^2)Z^2 + (4c^2d^2 + b^2)Z + a) + 2af^2 + 4af^2\text{RootOf}(Z^2 + 4Z^2d^2 + c^2 + b^2 + (4c^2d^2 + b^2)Z^2 + (4c^2d^2 + b^2)Z + a))} \frac{1}{(af^2 + b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)

[Out] $1/2/f^2/a/e*\sum((-R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d))*\ln(-R+x),_R=\text{RootOf}(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/a/e/f^2/(e*x+d)$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 3.87, size = 4339, normalized size = 21.27

$$\begin{aligned}
& (6*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/2)} + 4*a^4*c^4*d*e^{11*f^6} - 2 \\
& *a^3*b^2*c^3*d*e^{11*f^6} + 2*a^3*c^4*e^{10*f^4}) * (- (b^5 + b^2 * (- (4*a*c - b^2 \\
&)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a \\
& ^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/2)} * 2i - \text{ata} \\
& n(((- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (\\
& - (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b \\
& ^2*c*e^2*f^4))^{(1/2)} * (x*(4*a^4*c^4*e^{12*f^6} - 2*a^3*b^2*c^3*e^{12*f^6}) - ((\\
& x*(8*a^5*b^3*c^2*e^{14*f^10} - 32*a^6*b*c^3*e^{14*f^10}) - 32*a^6*b*c^3*d*e^{13* \\
& f^10 + 8*a^5*b^3*c^2*d*e^{13*f^10}) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 1 \\
& 2*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^2*f^4 \\
& + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/2)} - 4*a^4*b^3*c^2*e^{12*f \\
& ^8 + 16*a^5*b*c^3*e^{12*f^8}) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b \\
& *c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^2*f^4 + 16* \\
& a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/2)} + 4*a^4*c^4*d*e^{11*f^6} - 2*a \\
& ^3*b^2*c^3*d*e^{11*f^6}) * 1i + (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b \\
& *c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^2*f^4 + 16* \\
& a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/2)} * (x*(4*a^4*c^4*e^{12*f^6} - 2*a \\
& ^3*b^2*c^3*e^{12*f^6}) - ((x*(8*a^5*b^3*c^2*e^{14*f^10} - 32*a^6*b*c^3*e^{14*f^1 \\
& 0} - 32*a^6*b*c^3*d*e^{13*f^10} + 8*a^5*b^3*c^2*d*e^{13*f^10}) * (- (b^5 - b^2 * (- (\\
& 4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(\\
& 1/2)}) / (8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/ \\
& 2)} + 4*a^4*b^3*c^2*e^{12*f^8} - 16*a^5*b*c^3*e^{12*f^8}) * (- (b^5 - b^2 * (- (4*a*c \\
& - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)}) / \\
& (8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/2)} + 4 \\
& *a^4*c^4*d*e^{11*f^6} - 2*a^3*b^2*c^3*d*e^{11*f^6}) * 1i) / (((- (b^5 - b^2 * (- (4*a*c \\
& - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)}) / \\
& (8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/2)} * (x* \\
& (4*a^4*c^4*e^{12*f^6} - 2*a^3*b^2*c^3*e^{12*f^6}) - ((x*(8*a^5*b^3*c^2*e^{14*f^1 \\
& 0} - 32*a^6*b*c^3*e^{14*f^10}) - 32*a^6*b*c^3*d*e^{13*f^10} + 8*a^5*b^3*c^2*d*e^{ \\
& 13*f^10}) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c \\
& + a*c * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8 \\
& *a^4*b^2*c*e^2*f^4))^{(1/2)} + 4*a^4*b^3*c^2*e^{12*f^8} - 16*a^5*b*c^3*e^{12*f^ \\
& 8}) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (\\
& - (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b \\
& ^2*c*e^2*f^4))^{(1/2)} + 4*a^4*c^4*d*e^{11*f^6} - 2*a^3*b^2*c^3*d*e^{11*f^6}) - \\
& (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c * (- (4 \\
& *a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2* \\
& c*e^2*f^4))^{(1/2)} * (x*(4*a^4*c^4*e^{12*f^6} - 2*a^3*b^2*c^3*e^{12*f^6}) - ((x*(\\
& 8*a^5*b^3*c^2*e^{14*f^10} - 32*a^6*b*c^3*e^{14*f^10}) - 32*a^6*b*c^3*d*e^{13*f^1 \\
& 0} + 8*a^5*b^3*c^2*d*e^{13*f^10}) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a \\
& ^2*b*c^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^2*f^4 + \\
& 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/2)} - 4*a^4*b^3*c^2*e^{12*f^8} \\
& + 16*a^5*b*c^3*e^{12*f^8}) * (- (b^5 - b^2 * (- (4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c \\
& ^2 - 7*a*b^3*c + a*c * (- (4*a*c - b^2)^3)^{(1/2)}) / (8*(a^3*b^4*e^2*f^4 + 16*a^5 \\
& *c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))^{(1/2)} + 4*a^4*c^4*d*e^{11*f^6} - 2*a^3*
\end{aligned}$$

$$b^2*c^3*d*e^{11*f^6} + 2*a^3*c^4*e^{10*f^4})*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^{(1/2)})/(8*(a^3*b^4*e^{2*f^4} + 16*a^5*c^2*e^{2*f^4} - 8*a^4*b^2*c*e^{2*f^4}))^{(1/2)*2i} - 1/(a*e*(d*f^2 + e*f^2*x)))$$

sympy [A] time = 4.90, size = 258, normalized size = 1.26

$$\text{RootSum}\left(t^4(256a^5c^2e^{11f^6} - 128a^4b^2c^4e^{11f^6} + 16a^3b^4e^{11f^6}) + t^2(48a^2b^2c^2e^{11f^6} - 28ab^3c^2e^{11f^6} + 4b^5e^{11f^6}) + c^3\left(t \mapsto t \log\left(x + \frac{-64t^3a^5c^2e^{11f^6} + 48t^3a^4b^2c^3e^{11f^6} - 8t^3a^3b^4e^{11f^6} - 10t^2b^2c^2e^{11f^6} + 10tab^3ce^{11f^6} - 2tb^5e^{11f^6} + ac^3d - b^2c^2d}{ac^3e - b^2c^2e}\right)\right)\right) - \frac{1}{ade^2 + ae^2f^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)

[Out] RootSum(_t**4*(256*a**5*c**2*e**4*f**8 - 128*a**4*b**2*c*e**4*f**8 + 16*a**3*b**4*e**4*f**8) + _t**2*(48*a**2*b*c**2*e**2*f**4 - 28*a*b**3*c*e**2*f**4 + 4*b**5*e**2*f**4) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**3*f**6 + 48*_t**3*a**4*b**2*c*e**3*f**6 - 8*_t**3*a**3*b**4*e**3*f**6 - 10*_t*a**2*b*c**2*e*f**2 + 10*_t*a*b**3*c*e*f**2 - 2*_t*b**5*e*f**2 + a*c**3*d - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e*f**2 + a*e**2*f**2*x)

$$3.531 \quad \int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal. Leaf size=133

$$\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}} + \frac{b \log(a + b(d+ex)^2 + c(d+ex)^4)}{4a^2ef^3} - \frac{b \log(d+ex)}{a^2ef^3} - \frac{1}{2aef^3(d+ex)^2}$$

Rubi [A] time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1142, 1114, 709, 800, 634, 618, 206, 628}

$$\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}} + \frac{b \log(a + b(d+ex)^2 + c(d+ex)^4)}{4a^2ef^3} - \frac{b \log(d+ex)}{a^2ef^3} - \frac{1}{2aef^3(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] -1/(2*a*e*f^3*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e*f^3) - (b*Log[d + e*x])/(a^2*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e*f^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\text{t}[(b + 2cx)/(a + bx + cx^2), x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2cd - b², 0] && NeQ[b² - 4ac, 0] && !NiceSqrtQ[b² - 4ac]

Rule 709

$\text{Int}[(d + e)x^m / (a + bx + cx^2), x_Symbol] \rightarrow \text{Simp}[(e(d + ex)^{m+1}) / ((m+1)(cd^2 - bde + ae^2)), x] + \text{Dist}[1 / (cd^2 - bde + ae^2), \text{Int}[(d + ex)^{m+1} \text{Simp}[cd - bde - cex, x] / (a + bx + cx^2), x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && NeQ[b² - 4ac, 0] && NeQ[cd² - bde + ae², 0] && NeQ[2cd - b², 0] && LtQ[m, -1]

Rule 800

$\text{Int}[(d + e)x^m (f + gx) / (a + bx + cx^2), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + ex)^m (f + gx) / (a + bx + cx^2), x], x] /;$ FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b² - 4ac, 0] && NeQ[cd² - bde + ae², 0] && IntegerQ[m]

Rule 1114

$\text{Int}[x^m (a + bx + cx^2)^p, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{(m-1)/2} (a + bx + cx^2)^p, x], x, x^2], x] /;$ FreeQ[{a, b, c, p}, x] && IntegerQ[(m-1)/2]

Rule 1142

$\text{Int}[u^m (a + bv + cv^2)^p, x_Symbol] \rightarrow \text{Dist}[u^m / (\text{Coefficient}[v, x, 1] v^m), \text{Subst}[\text{Int}[x^m (a + bx^2 + cx^{2*2})^p, x], x, v], x] /;$ FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)} dx &= \frac{\text{Subst} \left(\int \frac{1}{x^3(a+bx^2+cx^4)} dx, x, d + ex \right)}{ef^3} \\
&= \frac{\text{Subst} \left(\int \frac{1}{x^2(a+bx+cx^2)} dx, x, (d + ex)^2 \right)}{2ef^3} \\
&= -\frac{1}{2aef^3(d + ex)^2} + \frac{\text{Subst} \left(\int \frac{-b-cx}{x(a+bx+cx^2)} dx, x, (d + ex)^2 \right)}{2aef^3} \\
&= -\frac{1}{2aef^3(d + ex)^2} + \frac{\text{Subst} \left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)} \right) dx, x, (d + ex)^2 \right)}{2aef^3} \\
&= -\frac{1}{2aef^3(d + ex)^2} - \frac{b \log(d + ex)}{a^2ef^3} + \frac{\text{Subst} \left(\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx, x, (d + ex)^2 \right)}{2a^2ef^3} \\
&= -\frac{1}{2aef^3(d + ex)^2} - \frac{b \log(d + ex)}{a^2ef^3} + \frac{b \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, (d + ex)^2 \right)}{4a^2ef^3} \\
&= -\frac{1}{2aef^3(d + ex)^2} - \frac{b \log(d + ex)}{a^2ef^3} + \frac{b \log(a + b(d + ex)^2 + c(d + ex)^4)}{4a^2ef^3} \\
&= -\frac{1}{2aef^3(d + ex)^2} - \frac{(b^2 - 2ac) \tanh^{-1} \left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}} \right)}{2a^2\sqrt{b^2-4ac}ef^3} - \frac{b \log(d + ex)}{a^2ef^3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 157, normalized size = 1.18

$$\frac{\frac{(b\sqrt{b^2-4ac}-2ac+b^2)\log(-\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{\sqrt{b^2-4ac}} + \frac{(b\sqrt{b^2-4ac}+2ac-b^2)\log(\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{\sqrt{b^2-4ac}}}{4a^2ef^3} - \frac{2a}{(d+ex)^2} - 4b \log(d + ex)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c])/(4*a^2*e*f^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)), x]

fricas [B] time = 1.93, size = 828, normalized size = 6.23

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + \\ & (b^2 - 2*a*c)*d^2)*\text{sqrt}(b^2 - 4*a*c)*\log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 \\ & + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + \\ & b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*\text{sqrt}(b^2 - 4*a*c)) / (c*e^4*x^4 + \\ & 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - \\ & ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*\log(c*e^4*x^4 + \\ & 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + \\ & 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*\log(e*x + d) / \\ & ((a^2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2 - 4*a^3*c)*d^2*e*f^3) \\ & , -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*e^2*x^2 + 2*(b^2 - 2*a*c)*d*e*x + \\ & (b^2 - 2*a*c)*d^2)*\text{sqrt}(-b^2 + 4*a*c)*\arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*\text{sqrt}(-b^2 + 4*a*c) / \\ & (b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*\log \\ & (c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + \\ & 4*((b^3 - 4*a*b*c)*e^2*x^2 + 2*(b^3 - 4*a*b*c)*d*e*x + (b^3 - 4*a*b*c)*d^2)*\log(e*x + d) / \\ & ((a^2*b^2 - 4*a^3*c)*e^3*f^3*x^2 + 2*(a^2*b^2 - 4*a^3*c)*d*e^2*f^3*x + (a^2*b^2 - 4*a^3*c)*d^2*e*f^3)] \end{aligned}$$

giac [B] time = 1.11, size = 348, normalized size = 2.62

$$\frac{b^{(-1)} \log\left(\frac{c x^4 + 4 c d x^3 + 6 c d^2 x^2 + 4 c d^3 x + c d^4 + b x^2 + 2 b d x + b d^2 + d}{4 a^2 f^3}\right) - b^{(-1)} \log\left(\frac{x e + d}{a^2 f^3}\right) - \frac{e^{(-1)}}{2 (x e + d)^2 a f^3} + \left(\frac{(b^2 c f^3 - 2 a^2 c^2 f^3) \log\left(\frac{b^2 x^2 + 2 b d x + \sqrt{b^2 - 4 a c} x^2 + 2 \sqrt{b^2 - 4 a c} d x + b d^2 + \sqrt{b^2 - 4 a c} d^2 + d}{\sqrt{b^2 - 4 a c}}\right) - (b^2 c f^3 - 2 a^2 c^2 f^3) \log\left(\frac{-b^2 x^2 - 2 b d x + \sqrt{b^2 - 4 a c} x^2 + 2 \sqrt{b^2 - 4 a c} d x - b d^2 + \sqrt{b^2 - 4 a c} d^2 - d}{\sqrt{b^2 - 4 a c}}\right)}{4 a^2 c f^3}\right)}{4 a^2 c f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

[Out] $\frac{1}{4} b e^{-1} \log(\operatorname{abs}(c x^4 e^4 + 4 c d x^3 e^3 + 6 c d^2 x^2 e^2 + 4 c d^3 x e + c d^4 + b x^2 e^2 + 2 b d x e + b d^2 + a)) / (a^2 f^3) - b e^{-1} \log(\operatorname{abs}(x e + d)) / (a^2 f^3) - \frac{1}{2} e^{-1} / ((x e + d)^2 a f^3) + \frac{1}{4} ((a^2 b^2 c f^3 e^3 - 2 a^3 c^2 f^3 e^3) \log(\operatorname{abs}(b x^2 e^2 + 2 b d x e + \sqrt{b^2 - 4 a c}) x^2 e^2 + 2 \sqrt{b^2 - 4 a c} d x e + b d^2 + \sqrt{b^2 - 4 a c} d^2 + 2 a)) / \sqrt{b^2 - 4 a c} - (a^2 b^2 c f^3 e^3 - 2 a^3 c^2 f^3 e^3) \log(\operatorname{abs}(-b x^2 e^2 - 2 b d x e + \sqrt{b^2 - 4 a c}) x^2 e^2 + 2 \sqrt{b^2 - 4 a c} d x e - b d^2 + \sqrt{b^2 - 4 a c} d^2 - 2 a)) / \sqrt{b^2 - 4 a c}) e^{-4} / (a^4 c f^6)$

maple [C] time = 0.01, size = 222, normalized size = 1.67

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Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x)

[Out] $\frac{1}{2} f^3 a^2 / e \sum((_R^3 b^3 c e^3 + 3 _R^2 b^2 c d e^2 + b^2 c d^3 - a^2 c d + b^2 d + (3 b^2 c d^2 - a^2 c + b^2) _R e) / (2 _R^3 c e^3 + 6 _R^2 c d e^2 + 6 _R c d^2 e + 2 c d^3 + _R b e + b d) \ln(-_R + x), _R = \operatorname{RootOf}(_Z^4 c e^4 + 4 _Z^3 c d e^3 + c d^4 + b d^2 + (6 c d^2 e^2 + b e^2) _Z^2 + (4 c d^3 e + 2 b d e) _Z + a)) - \frac{1}{2} / a / e / f^3 / (e x + d)^2 - b \ln(e x + d) / a^2 / e / f^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 6.98, size = 5947, normalized size = 44.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)

[Out] $(\operatorname{atan}((16 a^6 f^9 x^3 ((3 b^4 + a^2 c^2 - 9 a b^2 c) (((2 b^3 e f^3 - 8 a b^2 c e f^3) * ((2 * (20 a^3 c^4 d e^{17} f^6 + 2 a^2 b^2 c^3 d e^{17} f^6)) / (a^3 f^9) + ((40 a^4 b^3 c^3 d e^{18} f^9 - 12 a^3 b^3 c^2 d e^{18} f^9) * (2 b^3 e f^3 - 8 a b^2 c e f^3)) / (a^3 f^9 * (16 a^3 c e^{2} f^6 - 4 a^2 b^2 e^{2} f^6)))))) / (2 * (16 a^6 f^9 x^3 ((3 b^4 + a^2 c^2 - 9 a b^2 c) (((2 b^3 e f^3 - 8 a b^2 c e f^3) * ((2 * (20 a^3 c^4 d e^{17} f^6 + 2 a^2 b^2 c^3 d e^{17} f^6)) / (a^3 f^9) + ((40 a^4 b^3 c^3 d e^{18} f^9 - 12 a^3 b^3 c^2 d e^{18} f^9) * (2 b^3 e f^3 - 8 a b^2 c e f^3)) / (a^3 f^9 * (16 a^3 c e^{2} f^6 - 4 a^2 b^2 e^{2} f^6))))))$

$$\begin{aligned}
& 3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6})) + (12*b*c^4*d*e^{16})/(a^2*f^6))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)/(2*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6})) + (2*c^5*d*e^{15})/(a^3*f^9) - (((((2*(20*a^3*c^4*d*e^{17*f^6} + 2*a^2*b^2*c^3*d*e^{17*f^6}))/a^3*f^9) + ((40*a^4*b*c^3*d*e^{18*f^9} - 12*a^3*b^3*c^2*d*e^{18*f^9})*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))) * (2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) + ((40*a^4*b*c^3*d*e^{18*f^9} - 12*a^3*b^3*c^2*d*e^{18*f^9})*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))/(4*a^5*e*f^{12}*(4*a*c - b^2)^{(1/2})*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))) * (2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) - ((40*a^4*b*c^3*d*e^{18*f^9} - 12*a^3*b^3*c^2*d*e^{18*f^9})*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)^2)/(16*a^7*e^{2*f^{15}}*(4*a*c - b^2)*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))) / (8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)*((((((2*(20*a^3*c^4*d*e^{17*f^6} + 2*a^2*b^2*c^3*d*e^{17*f^6}))/a^3*f^9) + ((40*a^4*b*c^3*d*e^{18*f^9} - 12*a^3*b^3*c^2*d*e^{18*f^9})*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))) * (2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) + ((40*a^4*b*c^3*d*e^{18*f^9} - 12*a^3*b^3*c^2*d*e^{18*f^9})*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))/(4*a^5*e*f^{12}*(4*a*c - b^2)^{(1/2})*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))) * (2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6})) + (((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*((2*(20*a^3*c^4*d*e^{17*f^6} + 2*a^2*b^2*c^3*d*e^{17*f^6}))/a^3*f^9) + ((40*a^4*b*c^3*d*e^{18*f^9} - 12*a^3*b^3*c^2*d*e^{18*f^9})*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6})))))/(2*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6})) + (12*b*c^4*d*e^{16})/(a^2*f^6))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) - ((40*a^4*b*c^3*d*e^{18*f^9} - 12*a^3*b^3*c^2*d*e^{18*f^9})*(2*a*c - b^2)^3)/(32*a^9*e^{3*f^{18}}*(4*a*c - b^2)^{(3/2)})))/(8*a^3*c^2*(4*a*c - b^2)^{(1/2})*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) * (4*a*c - b^2)^{(3/2)})/(4*a^2*c^4*e^{14} + b^4*c^2*e^{14} - 4*a*b^2*c^3*e^{14}) + (16*a^6*f^9*x^2*((3*b^4 + a^2*c^2 - 9*a*b^2*c)*((((((20*a^3*c^4*e^{18*f^6} + 2*a^2*b^2*c^3*e^{18*f^6}))/a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^{19*f^9} - 40*a^4*b*c^3*e^{19*f^9}))/2*a^3*f^9*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))) * (2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6})) + (6*b*c^4*e^{17})/(a^2*f^6))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6})) + (c^5*e^{16})/(a^3*f^9) - ((2*a*c - b^2)*((((((20*a^3*c^4*e^{18*f^6} + 2*a^2*b^2*c^3*e^{18*f^6}))/a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^{19*f^9} - 40*a^4*b*c^3*e^{19*f^9}))/2*a^3*f^9*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))) * (2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^{19*f^9} - 40*a^4*b*c^3*e^{19*f^9})*(2*a*c - b^2))/(8*a^5*e*f^{12}*(4*a*c - b^2)^{(1/2})*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) + ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^{19*f^9} - 40*a^4*b*c^3*e^{19*f^9})*(2*a*c - b^2)^2)/(32*a^7*e^{2*f^{15}}*(4*a*c - b^2)*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)*((((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*((((((20*a^3*c^4*e^{18*f^6} + 2*a^2*b^2*c^3*e^{18*f^6}))/a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^{19*f^9} - 40*a^4*b*c^3*e^{19*f^9}))/2*a^3*f^9*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))))))
\end{aligned}$$

$$\begin{aligned}
&))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2))/(8*a^5*e*f^12*(4*a*c - b^2)^{(1/2)}*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (((12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2)^3)/(64*a^9*e^3*f^18*(4*a*c - b^2)^{(3/2)}) + (((((20*a^3*c^4*e^18*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (6*b*c^4*e^17)/(a^2*f^6))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)})))/(8*a^3*c^2*(4*a*c - b^2)^{(1/2)}*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^{(3/2)})/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14) + (2*a^3*f^9*(4*a*c - b^2)^{(3/2)}*(3*b^4 + a^2*c^2 - 9*a*b^2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/(a^3*f^9) + (((((4*a^2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (4*a*b^2*c^3*e^15*f^3 - a^2*c^4*e^15*f^3 + 6*a*b*c^4*d^2*e^15*f^3)/(a^3*f^9))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) - ((2*a*c - b^2)*(((4*a^2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(8*a^5*e*f^12*(4*a*c - b^2)^{(1/2)}*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) + ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)^2*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(32*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e^14)) + (2*a^3*f^9*(4*a*c - b^2)*(((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(((4*a^2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(8*a^5*e*f^12*(4*a*c - b^2)^{(1/2)}*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (((((4*a^2*b^3*c^2*e^16*f^6 + 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*
\end{aligned}$$

$$\begin{aligned}
& e^f)^3)/(2*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6})) + (4*a*b^2*c^3*e^{15*f^3} \\
& - a^2*c^4*e^{15*f^3} + 6*a*b*c^4*d^2*e^{15*f^3})/(a^3*f^9)*(2*a*c - b^2))/(4*a \\
& ^2*e*f^3*(4*a*c - b^2)^{(1/2)}) + ((2*a*c - b^2)^3*(4*a^4*b^2*c^2*e^{17*f^9} - \\
& 40*a^4*b*c^3*d^2*e^{17*f^9} + 12*a^3*b^3*c^2*d^2*e^{17*f^9}))/((64*a^9*e^3*f^{18} \\
& (4*a*c - b^2)^{(3/2)}))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(c^2*(a^2*c^2 - \\
& 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4*e^{14} + b^4*c^2*e^{14} - 4*a*b^2*c^3*e^{14}))* (2 \\
& *a*c - b^2))/(2*a^2*e*f^3*(4*a*c - b^2)^{(1/2)}) - 1/(2*a*e*(d^2*f^3 + e^2*f^ \\
& 3*x^2 + 2*d*e*f^3*x)) - (b*log(d + e*x))/(a^2*e*f^3) - (log(((c^5*e^{16*x^2}) \\
& / (a^3*f^9) - ((b + a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2))) \\
& ^{(1/2)}))*((c^3*e^{15*(4*b^2 - a*c + 6*b*c*d^2)))/(a^2*f^6) - ((b + a^2*e*f^3*(- \\
& -(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^{(1/2)}))*((2*c^2*e^{16*(2*b^3 + \\
& 10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c)))/(a*f^3) + (2*c^3*e^{18*x^2*(10*a*c + b^ \\
& 2)))/(a*f^3) + (b*c^2*e^{16*(b + a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4 \\
& a*c - b^2)))^{(1/2)}))* (a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d \\
& *e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f^3) + (4*c^3*d*e^{17*x*(10*a*c \\
& + b^2)))/(a*f^3)))/(4*a^2*e*f^3) + (6*b*c^4*e^{17*x^2})/(a^2*f^6) + (12*b*c^4* \\
& d*e^{16*x})/(a^2*f^6)))/(4*a^2*e*f^3) + (c^4*e^{14*(b + c*d^2)})/(a^3*f^9) + (2 \\
& *c^5*d*e^{15*x})/(a^3*f^9))*((c^5*e^{16*x^2})/(a^3*f^9) - ((b - a^2*e*f^3*(-(2* \\
& a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^{(1/2)}))*((c^3*e^{15*(4*b^2 - a*c + \\
& 6*b*c*d^2)))/(a^2*f^6) - ((b - a^2*e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a \\
& *c - b^2)))^{(1/2)}))*((2*c^2*e^{16*(2*b^3 + 10*a*c^2*d^2 + b^2*c*d^2 - 2*a*b*c \\
&)))/(a*f^3) + (2*c^3*e^{18*x^2*(10*a*c + b^2)))/(a*f^3) + (b*c^2*e^{16*(b - a^2 \\
& *e*f^3*(-(2*a*c - b^2)^2/(a^4*e^2*f^6*(4*a*c - b^2)))^{(1/2)}))* (a*b + 3*b^2*d \\
& ^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e \\
& *x))/(a^2*f^3) + (4*c^3*d*e^{17*x*(10*a*c + b^2)))/(a*f^3)))/(4*a^2*e*f^3) + \\
& (6*b*c^4*e^{17*x^2})/(a^2*f^6) + (12*b*c^4*d*e^{16*x})/(a^2*f^6)))/(4*a^2*e*f^3 \\
&) + (c^4*e^{14*(b + c*d^2)})/(a^3*f^9) + (2*c^5*d*e^{15*x})/(a^3*f^9))* (2*b^3* \\
& e^f^3 - 8*a*b*c*e^f^3))/(2*(16*a^3*c*e^{2*f^6} - 4*a^2*b^2*e^{2*f^6}))
\end{aligned}$$

sympy [B] time = 156.63, size = 532, normalized size = 4.00

$$\left(\frac{1}{4a^2f^3} \frac{\sqrt{4ac-b^2}(2ac-b^2)}{4a^2f^3(4ac-b^2)} \right) \log \left(\frac{2dx}{a^2} + x^2 + \frac{-8a^2cf^3 \left(\frac{b}{4a^2f^3} \frac{\sqrt{4ac-b^2}(2ac-b^2)}{4a^2f^3(4ac-b^2)} + 2a^2b^2f^3 \left(\frac{b}{4a^2f^3} \frac{\sqrt{4ac-b^2}(2ac-b^2)}{4a^2f^3(4ac-b^2)} + 3abc + 2a^2d^2 - b^2 - b^2d^2 \right) \right)}{2a^2c^2 - b^2c} \right) + \left(\frac{b}{4a^2f^3} + \frac{\sqrt{4ac-b^2}(2ac-b^2)}{4a^2f^3(4ac-b^2)} \right) \log \left(\frac{2dx}{a^2} + x^2 + \frac{-8a^2cf^3 \left(\frac{b}{4a^2f^3} \frac{\sqrt{4ac-b^2}(2ac-b^2)}{4a^2f^3(4ac-b^2)} + 2a^2b^2f^3 \left(\frac{b}{4a^2f^3} \frac{\sqrt{4ac-b^2}(2ac-b^2)}{4a^2f^3(4ac-b^2)} + 3abc + 2a^2d^2 - b^2 - b^2d^2 \right) \right)}{2a^2c^2 - b^2c} \right) + \frac{1}{2a^2cf^3 + 4a^2d^2f^3 + 2a^2f^3c^2} \frac{b \log \left(\frac{2}{a} + x \right)}{a^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)

[Out] (b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*e*f**3*(b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2)))) + 2*a**2*b**2*e*f**3*(b/(4*a**2*e*f**3) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2*d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2) + (b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2)))*log(2*d*x/e + x**2 + (-8*a**3*c*e*f**3*(b/(4*a**2*e*f**3) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))))

$$\begin{aligned}
& *3*(4*a*c - b**2))) + 2*a**2*b**2*e*f**3*(b/(4*a**2*e*f**3) + \text{sqrt}(-4*a*c + \\
& b**2)*(2*a*c - b**2)/(4*a**2*e*f**3*(4*a*c - b**2))) + 3*a*b*c + 2*a*c**2* \\
& d**2 - b**3 - b**2*c*d**2)/(2*a*c**2*e**2 - b**2*c*e**2)) - 1/(2*a*d**2*e*f \\
& **3 + 4*a*d*e**2*f**3*x + 2*a*e**3*f**3*x**2) - b*\log(d/e + x)/(a**2*e*f**3 \\
&)
\end{aligned}$$

$$3.532 \quad \int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal. Leaf size=236

$$\frac{\sqrt{c} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} a^2 e f^4 \sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c} \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} a^2 e f^4 \sqrt{\sqrt{b^2-4ac}+b}} + \frac{b}{a^2 e f^4 (d+ex)} - \frac{1}{3 a e f^4 (d+ex)^3}$$

Rubi [A] time = 0.49, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1142, 1123, 1281, 1166, 205}

$$\frac{\sqrt{c} \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} a^2 e f^4 \sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c} \left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} a^2 e f^4 \sqrt{\sqrt{b^2-4ac}+b}} + \frac{b}{a^2 e f^4 (d+ex)} - \frac{1}{3 a e f^4 (d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] -1/(3*a*e*f^4*(d + e*x)^3) + b/(a^2*e*f^4*(d + e*x)) + (Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1123

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dist[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1281

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned} \int \frac{1}{(df + efx)^4 (a + b(d + ex)^2 + c(d + ex)^4)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)} dx, x, d + ex\right)}{ef^4} \\ &= -\frac{1}{3aef^4(d + ex)^3} + \frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2(a+bx^2+cx^4)} dx, x, d + ex\right)}{3aef^4} \\ &= -\frac{1}{3aef^4(d + ex)^3} + \frac{b}{a^2ef^4(d + ex)} - \frac{\text{Subst}\left(\int \frac{-3(b^2-ac)-3bcx^2}{a+bx^2+cx^4} dx, x, d + ex\right)}{3a^2ef^4} \\ &= -\frac{1}{3aef^4(d + ex)^3} + \frac{b}{a^2ef^4(d + ex)} + \frac{\left(c\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{1}{x} dx, x, \frac{b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}}{\sqrt{2}a^2\sqrt{b - \sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b - \sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 238, normalized size = 1.01

$$\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{2a}{(d+ex)^3} + \frac{6b}{d+ex}$$

$$6a^2ef^4$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(6*a^2*e*f^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)^4 (a + b(d + ex)^2 + c(d + ex)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]

fricas [B] time = 1.32, size = 2212, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] 1/6*(6*b*e^2*x^2 + 12*b*d*e*x + 6*b*d^2 + 3*sqrt(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*sqrt(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)*e^4*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8)*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^

$$\begin{aligned}
& 2*c^3 + a^4*c^4)/((a^{10}*b^2 - 4*a^{11}*c)*e^4*f^16)) - (b^8 - 8*a*b^6*c + 20* \\
& a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*\text{sqrt}(-((a^5*b^2 - 4*a^6*c) \\
& *e^2*f^8*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/ \\
& ((a^{10}*b^2 - 4*a^{11}*c)*e^4*f^16))) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^ \\
& 2 - 4*a^6*c)*e^2*f^8))) - 3*\text{sqrt}(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^ \\
& 2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)*\text{sqrt}(-((a^5*b^2 - 4*a^6*c)*e^2*f^8 \\
& *\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^{10}*b \\
& ^2 - 4*a^{11}*c)*e^4*f^16))) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^ \\
& 6*c)*e^2*f^8))*\text{log}(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3 \\
& *a*b^2*c^4 + a^2*c^5)*d - \text{sqrt}(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2) \\
& *e^3*f^12*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4) \\
& /((a^{10}*b^2 - 4*a^{11}*c)*e^4*f^16))) - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17 \\
& *a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*\text{sqrt}(-((a^5*b^2 - 4*a^6*c)*e^2*f^8*\text{sqrt}((b \\
& ^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^{10}*b^2 - 4*a \\
& ^11*c)*e^4*f^16))) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2 \\
& *f^8))) - 3*\text{sqrt}(1/2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^ \\
& 2*f^4*x + a^2*d^3*e*f^4)*\text{sqrt}(((a^5*b^2 - 4*a^6*c)*e^2*f^8*\text{sqrt}((b^8 - 6*a* \\
& b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^{10}*b^2 - 4*a^{11}*c)*e^ \\
& 4*f^16))) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*\text{lo} \\
& \text{g}(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2* \\
& c^5)*d + \text{sqrt}(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*\text{sqrt}((b \\
& ^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^{10}*b^2 - 4*a \\
& ^11*c)*e^4*f^16))) + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4* \\
& a^4*c^4)*e*f^4)*\text{sqrt}(((a^5*b^2 - 4*a^6*c)*e^2*f^8*\text{sqrt}((b^8 - 6*a*b^6*c + 1 \\
& 1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^{10}*b^2 - 4*a^{11}*c)*e^4*f^16))) \\
& - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) + 3*\text{sqrt}(1 \\
& /2)*(a^2*e^4*f^4*x^3 + 3*a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3* \\
& e*f^4)*\text{sqrt}(((a^5*b^2 - 4*a^6*c)*e^2*f^8*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4 \\
& *c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^{10}*b^2 - 4*a^{11}*c)*e^4*f^16))) - b^5 + 5 \\
& *a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))*\text{log}(2*(b^4*c^3 - 3*a \\
& *b^2*c^4 + a^2*c^5)*e*x + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - \text{sqrt}(1/2) \\
& *((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^3*f^12*\text{sqrt}((b^8 - 6*a*b^6*c + 1 \\
& 1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^{10}*b^2 - 4*a^{11}*c)*e^4*f^16))) \\
& + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e*f^4)*\text{sq} \\
& \text{rt}(((a^5*b^2 - 4*a^6*c)*e^2*f^8*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6* \\
& a^3*b^2*c^3 + a^4*c^4)/((a^{10}*b^2 - 4*a^{11}*c)*e^4*f^16))) - b^5 + 5*a*b^3*c \\
& - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*e^2*f^8))) - 2*a)/(a^2*e^4*f^4*x^3 + 3* \\
& a^2*d*e^3*f^4*x^2 + 3*a^2*d^2*e^2*f^4*x + a^2*d^3*e*f^4)
\end{aligned}$$

giac [B] time = 0.57, size = 1249, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")

```
[Out] -1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
- b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c))*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-
1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))*b*c*d*e +
b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c))*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c))*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4
)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4
)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*
e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c))*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c))*e^2)*e^(-4)/c))))/(a^2*f^4) + 1/3*(3*b*x^2*e^2 + 6*b*d*x*e
+ 3*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^2*f^4)
```

maple [C] time = 0.01, size = 197, normalized size = 0.83

$$\frac{1}{3(a^2 + d)^3} \frac{b}{(a + d)^2 f^4} + \frac{\left(\text{RootOf}(2x^2 + 4x\sqrt{d^2 + c^2} + c^2 + b^2 + (a^2 d^2 + b^2)^2 + (4d^2 + 2ab)Z + a^2) \sqrt{b^2 + 2\text{RootOf}(2x^2 + 4x\sqrt{d^2 + c^2} + c^2 + b^2 + (a^2 d^2 + b^2)^2 + (4d^2 + 2ab)Z + a^2)}(b^2 + b^2 - ac + b^2) \ln(-\text{RootOf}(2x^2 + 4x\sqrt{d^2 + c^2} + c^2 + b^2 + (a^2 d^2 + b^2)^2 + (4d^2 + 2ab)Z + a^2)) \right)}{2a^2 f^4 \left(2 \sqrt{b^2 + 2\text{RootOf}(2x^2 + 4x\sqrt{d^2 + c^2} + c^2 + b^2 + (a^2 d^2 + b^2)^2 + (4d^2 + 2ab)Z + a^2)} \sqrt{b^2 + 2\text{RootOf}(2x^2 + 4x\sqrt{d^2 + c^2} + c^2 + b^2 + (a^2 d^2 + b^2)^2 + (4d^2 + 2ab)Z + a^2)} + b^2 + 2\text{RootOf}(2x^2 + 4x\sqrt{d^2 + c^2} + c^2 + b^2 + (a^2 d^2 + b^2)^2 + (4d^2 + 2ab)Z + a^2) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4), x)
```

```
[Out] 1/2/f^4/a^2/e*sum(( _R^2*b*c*e^2+2*_R*b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3
+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x), _R=RootOf(_Z^4*c
*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e
)*_Z+a))-1/3/a/e/f^4/(e*x+d)^3+b/a^2/e/f^4/(e*x+d)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 3.17, size = 5771, normalized size = 24.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)

[Out]
$$\begin{aligned} & ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3*f^4 + e^3*f^4*x^3 + 3*d*e^2*f^4*x^2 + 3*d^2*e*f^4*x) - \operatorname{atan}\left(\frac{(b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})}{(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8))^{(1/2)}}\right) \\ & * \left(\frac{(b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})}{(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8))^{(1/2)}}\right) \\ & * \left(\frac{(b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})}{(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8))^{(1/2)}}\right) \\ & * (x*(8*a^{10}*b^3*c^2*e^{14}*f^{20} - 32*a^{11}*b*c^3*e^{14}*f^{20}) - 32*a^{11}*b*c^3*d*e^{13}*f^{20} + 8*a^{10}*b^3*c^2*d*e^{13}*f^{20}) - 16*a^{10}*c^4*e^{12}*f^{16} - 4*a^8*b^4*c^2*e^{12}*f^{16} + 20*a^9*b^2*c^3*e^{12}*f^{16}) \\ & + x*(4*a^8*c^5*e^{12}*f^{12} + 2*a^6*b^4*c^3*e^{12}*f^{12} - 8*a^7*b^2*c^4*e^{12}*f^{12}) + 4*a^8*c^5*d*e^{11}*f^{12} + 2*a^6*b^4*c^3*d*e^{11}*f^{12} - 8*a^7*b^2*c^4*d*e^{11}*f^{12}) * i + \left(\frac{(b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})}{(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8))^{(1/2)}}\right) \\ & * \left(\frac{(b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})}{(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8))^{(1/2)}}\right) \\ & * \left(\frac{(b^4*(-(4*a*c - b^2)^3)^{(1/2)} - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})}{(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8))^{(1/2)}}\right) \\ & * (x*(8*a^{10}*b^3*c^2*e^{14}*f^{20} - 32*a^{11}*b*c^3*e^{14}*f^{20}) - 32*a^{11}*b*c^3*d*e^{13}*f^{20} + 8*a^{10}*b^3*c^2*d*e^{13}*f^{20}) + 16*a^{10}*c^4*e^{12}*f^{16} + 4*a^8*b^4*c^2*e^{12}*f^{16} - 20*a^9*b^2*c^3*e^{12}*f^{16}) \\ & + x*(4*a^8*c^5*e^{12}*f^{12} + 2*a^6*b^4*c^3*e^{12}*f^{12} - 8*a^7*b^2*c^4*e^{12}*f^{12}) \end{aligned}$$

$$\begin{aligned}
& ^{12}f^{12}) + 4a^8c^5d^5e^{11}f^{12} + 2a^6b^4c^3d^5e^{11}f^{12} - 8a^7b^2c^4d^5e^{11}f^{12}) \\
& \cdot i) / (((b^4(-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^3c^3 - 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot (((b^4(-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^3c^3 - 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot (((b^4(-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^3c^3 - 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot (((b^4(-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^3c^3 - 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot (x(8a^{10}b^3c^2e^{14}f^{20} - 32a^{11}b^3c^3e^{14}f^{20}) - 32a^{11}b^3c^3d^5e^{13}f^{20} + 8a^{10}b^3c^2d^5e^{13}f^{20}) - 16a^{10}c^4e^{12}f^{16} - 4a^8b^4c^2e^{12}f^{16} + 20a^9b^2c^3e^{12}f^{16}) + x(4a^8c^5e^{12}f^{12} + 2a^6b^4c^3e^{12}f^{12} - 8a^7b^2c^4e^{12}f^{12}) + 4a^8c^5d^5e^{11}f^{12} + 2a^6b^4c^3d^5e^{11}f^{12} - 8a^7b^2c^4d^5e^{11}f^{12}) \\
& - ((b^4(-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^3c^3 - 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot (((b^4(-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^3c^3 - 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot (((b^4(-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^3c^3 - 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot (x(8a^{10}b^3c^2e^{14}f^{20} - 32a^{11}b^3c^3e^{14}f^{20}) - 32a^{11}b^3c^3d^5e^{13}f^{20} + 8a^{10}b^3c^2d^5e^{13}f^{20}) + 16a^{10}c^4e^{12}f^{16} + 4a^8b^4c^2e^{12}f^{16} - 20a^9b^2c^3e^{12}f^{16}) + x(4a^8c^5e^{12}f^{12} + 2a^6b^4c^3e^{12}f^{12} - 8a^7b^2c^4e^{12}f^{12}) + 4a^8c^5d^5e^{11}f^{12} + 2a^6b^4c^3d^5e^{11}f^{12} - 8a^7b^2c^4d^5e^{11}f^{12}) + 2a^6b^3c^5e^{10}f^8) \cdot ((b^4(-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^3c^3 - 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot 2i - \operatorname{atan}(((b^7 + b^4(-4ac - b^2)^3)^{1/2} - 20a^3b^3c^3 + 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot ((b^7 + b^4(-4ac - b^2)^3)^{1/2} - 20a^3b^3c^3 + 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} \cdot ((x(8a^{10}b^3c^2e^{14}f^{20} - 32a^{11}b^3c^3e^{14}f^{20}) - 32a^{11}b^3c^3d^5e^{13}f^{20} + 8a^{10}b^3c^2d^5e^{13}f^{20}) \cdot (-b^7 + b^4(-4ac - b^2)^3)^{1/2} - 20a^3b^3c^3 + 25a^2b^3c^2 + a^2c^2(-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c(-4ac - b^2)^3)^{1/2} \\
& / (8(a^5b^4e^{2f^8} + 16a^7c^2e^{2f^8} - 8a^6b^2ce^{2f^8}))^{1/2} - 16a^{10}c^4e^{12}f^{16} - 4a^8b^4c^2e^{12}f^{16} + 20a^9b^2c^3e^{12}f^{16}) + x(4a^8c^5e^{12}f^{12} + 2a^6b^4c^3e^{12}f^{12} - 8a^7b^2c^4e^{12}f^{12})
\end{aligned}$$

$(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8))^{(1/2)*2i}$
sympy [A] time = 14.01, size = 411, normalized size = 1.74

$\frac{-a + 3b^2 + 6bd + 3b^2d}{3a^2b^2c^2 + 9a^2b^2cd + 9a^2b^2f^2 + 3a^2cd^2} + \text{RootSum}\left(t^4(256a^7c^2e^4f^{16} - 128a^6b^2c^2e^4f^{16} + 16a^5b^4c^2e^4f^{16}) + t^2(-80a^3b^3c^2e^2f^8 + 100a^2b^3c^2e^2f^8 - 36a^2b^2c^2e^2f^8 + 4b^2c^2f^8) + c^5, \left(1 + t \log\left(x + \frac{-96t^3a^7b^3c^2e^3f^{12} + 56t^3a^6b^3c^2e^3f^{12} - 8t^3a^5b^3c^2e^3f^{12} - 40t^3a^4c^4e^3f^{12} + 32a^3b^2c^2e^3f^4 - 40a^2b^4c^2e^3f^4 + 16a^2b^2c^2e^3f^4 - 2a^2b^2c^2e^3f^4 - 3a^2b^2c^2e^3f^4 - 3a^2b^2c^2e^3f^4}{a^2c^5 - 3a^2c^5 + 14c^5}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4), x)

[Out] $(-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e*f**4 + 9*a**2*d**2*e**2*f**4*x + 9*a**2*d*e**3*f**4*x**2 + 3*a**2*e**4*f**4*x**3) + \text{RootSum}(_t**4*(256*a**7*c**2*e**4*f**16 - 128*a**6*b**2*c*e**4*f**16 + 16*a**5*b**4*e**4*f**16) + _t**2*(-80*a**3*b**3*c**3*e**2*f**8 + 100*a**2*b**3*c**2*e**2*f**8 - 36*a*b**5*c*e**2*f**8 + 4*b**7*e**2*f**8) + c**5, \text{Lambda}(_t, _t*\log(x + (-96*_t**3*a**7*b**3*c**2*e**3*f**12 + 56*_t**3*a**6*b**3*c*e**3*f**12 - 8*_t**3*a**5*b**5*e**3*f**12 - 4*_t*a**4*c**4*e*f**4 + 32*_t*a**3*b**2*c**3*e*f**4 - 40*_t*a**2*b**4*c**2*e*f**4 + 16*_t*a*b**6*c*e*f**4 - 2*_t*b**8*e*f**4 + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2*c**4*e + b**4*c**3*e))))$

$$3.533 \quad \int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=279

$$\frac{f^4(d+ex)(2a+b(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)}{2\sqrt{2}\sqrt{c}e(b^2-4ac)^{3/2}}$$

Rubi [A] time = 0.54, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1142, 1120, 1166, 205}

$$\frac{f^4(d+ex)(2a+b(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}e(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}e(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1120

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(d^3*(d*x)^(m-3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^(m-4)*(2*a*(m-3) + b*(m+4*p+3)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{(df + efx)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx &= \frac{f^4 \text{Subst}\left(\int \frac{x^4}{(a + bx^2 + cx^4)^2} dx, x, d + ex\right)}{e} \\ &= \frac{f^4(d + ex)(2a + b(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{f^4 \text{Subst}\left(\int \frac{2a - bx^2}{a + bx^2 + cx^4} dx, x, d + ex\right)}{2(b^2 - 4ac)e} \\ &= \frac{f^4(d + ex)(2a + b(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{\left((b^2 + 4ac - b\sqrt{b^2 - 4ac})f\right)}{4} \\ &= \frac{f^4(d + ex)(2a + b(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{\left(b^2 + 4ac - b\sqrt{b^2 - 4ac}\right)f^4}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{a + b(d + ex)^2 + c(d + ex)^4}} \end{aligned}$$

Mathematica [A] time = 0.46, size = 266, normalized size = 0.95

$$\frac{f^4 \left(-\frac{2(-2a(d+ex) - b(d+ex)^3)}{(b^2 - 4ac)(a + b(d+ex)^2 + c(d+ex)^4)} + \frac{\sqrt{2}(b\sqrt{b^2 - 4ac} - 4ac - b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}(b\sqrt{b^2 - 4ac} + 4ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{4e}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(4*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df + efx)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [B] time = 1.26, size = 2578, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*e^3*f^4*x^3 + 6*b*d*e^2*f^4*x^2 + 2*(3*b*d^2 + 2*a)*e*f^4*x + 2*(b*d^3 + 2*a*d)*f^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*log(((3*b^2 + 4*a*c)*e*f^12*x + (3*b^2 + 4*a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 + 2*sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))
```

$$\begin{aligned}
& e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*\log((3*b^2 \\
& + 4*a*c)*e*f^{12}*x + (3*b^2 + 4*a*c)*d*f^{12} - \text{sqrt}(1/2)*((b^4 - 8*a*b^2*c + \\
& 16*a^2*c^2)*e*f^8 + 2*\text{sqrt}(f^{16}/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 \\
& - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)* \\
& e^3)*\text{sqrt}(-((b^3 + 12*a*b*c)*f^8 + \text{sqrt}(f^{16}/((b^6*c^2 - 12*a*b^4*c^3 + 48* \\
& a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64 \\
& *a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))) \\
& + \text{sqrt}(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (\\
& b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d \\
& ^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + \\
& (b^3 - 4*a*b*c)*d^2)*e)*\text{sqrt}(-((b^3 + 12*a*b*c)*f^8 - \text{sqrt}(f^{16}/((b^6*c^2 - \\
& 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + \\
& 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - \\
& 64*a^3*c^4)*e^2))*\log((3*b^2 + 4*a*c)*e*f^{12}*x + (3*b^2 + 4*a*c)*d*f^{12} + \\
& \text{sqrt}(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^8 - 2*\text{sqrt}(f^{16}/((b^6*c^2 - 1 \\
& 2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^7*c - 12*a*b^5*c^2 + 48 \\
& *a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*\text{sqrt}(-((b^3 + 12*a*b*c)*f^8 - \text{sqrt}(f^{16}/(\\
& (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4))*(b^6*c - 12*a* \\
& b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12*a*b^4*c^2 + 48*a^2 \\
& *b^2*c^3 - 64*a^3*c^4)*e^2))) - \text{sqrt}(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b \\
& ^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x \\
& ^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a* \\
& c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*\text{sqrt}(-((b^3 + 12*a*b*c \\
&)*f^8 - \text{sqrt}(f^{16}/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e \\
& ^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((b^6*c - 12 \\
& *a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))*\log((3*b^2 + 4*a*c)*e*f^{12}* \\
& x + (3*b^2 + 4*a*c)*d*f^{12} - \text{sqrt}(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e*f^ \\
& 8 - 2*\text{sqrt}(f^{16}/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^4 \\
&))*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e^3)*\text{sqrt}(-((b^3 \\
& + 12*a*b*c)*f^8 - \text{sqrt}(f^{16}/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64* \\
& a^3*c^5)*e^4))*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2)/((\\
& b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*e^2))))/((b^2*c - 4*a*c \\
& ^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4 \\
& *a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2* \\
& x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
\end{aligned}$$

giac [B] time = 0.60, size = 1370, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^(-4)/c \\
))^-2*b*f^4*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e

$$\begin{aligned} &^2) * e^{-4} / c)) * b * d * f^4 * e + b * d^2 * f^4 - 2 * a * f^4) * \log(d * e^{-1} + x + \sqrt{1/2} \\ & * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c)) / (2 * (d * e^{-1} + \sqrt{1/2} \\ & * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))^3 * c * e^4 - 6 * (d * e^{-1} + \sqrt{1/2} \\ & * \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))^2 * c * d * e^3 - 2 * c * d \\ & ^3 * e - b * d * e + (6 * c * d^2 * e^2 + b * e^2) * (d * e^{-1} + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{1/2} \\ & * \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))) + ((d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{1/2} \\ & * \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))^2 * b * f^4 * e^2 - 2 * (d * e^{-1} - \sqrt{1/2} * \sqrt{1/2} * \sqrt{ \\ & -(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c)) * b * d * f^4 * e + b * d^2 * f^4 - 2 * a * f^4 \\ & * \log(d * e^{-1} + x - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} \\ & / c)) / (2 * (d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} \\ & / c))^3 * c * e^4 - 6 * (d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2} \\ &) * e^{-4} / c))^2 * c * d * e^3 - 2 * c * d^3 * e - b * d * e + (6 * c * d^2 * e^2 + b * e^2) * (d * e^{-1} \\ &) - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))) + ((d * e^{-1} \\ &) + \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))^2 * b * f^4 * e^2 \\ & - 2 * (d * e^{-1} + \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c)) * \\ & b * d * f^4 * e + b * d^2 * f^4 - 2 * a * f^4) * \log(d * e^{-1} + x + \sqrt{1/2} * \sqrt{-(b * e^2 \\ & - \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c)) / (2 * (d * e^{-1} + \sqrt{1/2} * \sqrt{-(b * e^2 - \\ & \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))^3 * c * e^4 - 6 * (d * e^{-1} + \sqrt{1/2} * \sqrt{-(b * e^2 - \\ & (b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))^2 * c * d * e^3 - 2 * c * d^3 * e - b * d * e + \\ & (6 * c * d^2 * e^2 + b * e^2) * (d * e^{-1} + \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c} \\ &) * e^2} * e^{-4} / c))) + ((d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c} \\ &) * e^2} * e^{-4} / c))^2 * b * f^4 * e^2 - 2 * (d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{ \\ & (b^2 - 4 * a * c) * e^2} * e^{-4} / c)) * b * d * f^4 * e + b * d^2 * f^4 - 2 * a * f^4) * \log(d * e^{-1} \\ & + x - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c)) / (2 * (d * e^{-1} \\ & - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))^3 * c * e^4 - \\ & 6 * (d * e^{-1} - \sqrt{1/2} * \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))^2 * \\ & c * d * e^3 - 2 * c * d^3 * e - b * d * e + (6 * c * d^2 * e^2 + b * e^2) * (d * e^{-1} - \sqrt{1/2} * \sqrt{ \\ & \sqrt{-(b * e^2 - \sqrt{b^2 - 4 * a * c}) * e^2} * e^{-4} / c))))) / (b^2 - 4 * a * c) + 1/2 * (b * f^4 * x^3 * e^3 + \\ & 3 * b * d * f^4 * x^2 * e^2 + 3 * b * d^2 * f^4 * x * e + b * d^3 * f^4 + 2 * a * f^4 * x * e + \\ & 2 * a * d * f^4) / ((c * x^4 * e^4 + 4 * c * d * x^3 * e^3 + 6 * c * d^2 * x^2 * e^2 + 4 * c * d^3 * x * e + c \\ & * d^4 + b * x^2 * e^2 + 2 * b * d * x * e + b * d^2 + a) * (b^2 * e - 4 * a * c * e)) \end{aligned}$$

maple [C] time = 0.02, size = 695, normalized size = 2.49

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2, x)$

[Out]
$$\begin{aligned} & -1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d \\ & ^4+2*b*d*e*x+b*d^2+a)*b*e^2/(4*a*c-b^2)*x^3-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^ \\ & 3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*b*e/(4*a \\ & *c-b^2)*x^2-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b \\ & e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*d^2-f^4/(c*e^4*x^4+4*c*d*e \\ & ^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a* \end{aligned}$$

$$\frac{c-b^2}{c} \int \frac{1}{(c^2 e^{4x} + 4c^2 d e^{3x} + 6c^2 d^2 e^{2x} + 4c^2 d^3 e^x + b^2 e^{2x} + c^2 d^4 + 2b^2 d e^x + b^2 d^2 + a)} \frac{dx}{(4ac - b^2) d^3 / e^{bf^4} / (c^2 e^{4x} + 4c^2 d e^{3x} + 6c^2 d^2 e^{2x} + 4c^2 d^3 e^x + b^2 e^{2x} + c^2 d^4 + 2b^2 d e^x + b^2 d^2 + a)} / (4ac - b^2) d / e^{a + 1/4 f^4} / (4ac - b^2) / e^{\sum((-R^2 b e^{-2} - 2R b d e^{-b d^2 + 2a}) / (2R^3 c e^3 + 6R^2 c d e^2 + 6R c d^2 e + 2c d^3 + R b e + b d) \ln(-R+x), R = \text{RootOf}(_Z^4 c e^4 + 4_Z^3 c d e^3 + c d^4 + b d^2 + (6c d^2 e^2 + b e^2) _Z^2 + (4c d^3 e + 2b d e) _Z + a)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \int \frac{b^2 x^2 + 2bdx + b^2 - 2a}{(b^2 c - 4ac^2) e^{4x} + 4(b^2 c - 4ac^2) d e^{3x} + (b^2 c - 4ac^2) d^2 e^{2x} + (b^2 c - 4ac^2) d^3 e^x + (b^2 c - 4ac^2) d^4 + (b^2 c - 4ac^2) a} dx + \frac{b^2 f^4 + 3bd^2 f^2 + (3bd^2 + 2a) f^2 x + (bd^3 + 2ad) f^4}{2((b^2 c - 4ac^2) e^{4x} + 4(b^2 c - 4ac^2) d e^{3x} + (b^2 c - 4ac^2) d^2 e^{2x} + 2(2(b^2 c - 4ac^2) d^3 + (b^2 c - 4ac^2) d^4 + (b^2 c - 4ac^2) a) e^{a + 1/4 f^4} / (4ac - b^2) d / e^{a + 1/4 f^4} / (4ac - b^2) / e^{\sum((-R^2 b e^{-2} - 2R b d e^{-b d^2 + 2a}) / (2R^3 c e^3 + 6R^2 c d e^2 + 6R c d^2 e + 2c d^3 + R b e + b d) \ln(-R+x), R = \text{RootOf}(_Z^4 c e^4 + 4_Z^3 c d e^3 + c d^4 + b d^2 + (6c d^2 e^2 + b e^2) _Z^2 + (4c d^3 e + 2b d e) _Z + a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out]
$$-1/2 f^4 \int \frac{-(b^2 c - 4ac^2) e^{4x} + 4(b^2 c - 4ac^2) d e^{3x} + (b^2 c - 4ac^2) d^2 e^{2x} + a b^2 - 4a^2 c + (b^3 - 4a^2 b c) d^2 + 2(2(b^2 c - 4ac^2) d^3 + (b^3 - 4a^2 b c) d) e^x}{(b^2 c - 4ac^2) e^{5x} + 4(b^2 c - 4ac^2) d e^{4x} + (b^3 - 4a^2 b c + 6(b^2 c - 4ac^2) d^2) e^{3x} + 2(2(b^2 c - 4ac^2) d^3 + (b^3 - 4a^2 b c) d) e^{2x} + ((b^2 c - 4ac^2) d^4 + a b^2 - 4a^2 c + (b^3 - 4a^2 b c) d) e} dx + 1/2 (b^2 c - 4ac^2) e^{4x} + 4(b^2 c - 4ac^2) d e^{3x} + (b^2 c - 4ac^2) d^2 e^{2x} + a b^2 - 4a^2 c + (b^3 - 4a^2 b c) d^2 + 2(2(b^2 c - 4ac^2) d^3 + (b^3 - 4a^2 b c) d) e^x$$

mupad [B] time = 5.09, size = 8025, normalized size = 28.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)

[Out]
$$\text{atan}\left(\frac{(2048a^4c^5e^{12f^4} + 384a^2b^4c^3e^{12f^4} - 1536a^3b^2c^4e^{12f^4} - 32a^2b^6c^2e^{12f^4}) / (8(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12a^2b^4c)) + ((64b^9c^2de^{13} - 1024a^2b^7c^3de^{13} + 16384a^4b^5c^4de^{13} + 6144a^2b^5c^4de^{13} - 16384a^3b^3c^5de^{13}) / (8(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12a^2b^4c)) + (x(16b^7c^2e^{14} - 192a^2b^5c^3e^{14} - 1024a^3b^3c^5e^{14} + 768a^2b^3c^4e^{14})) / (2(b^4 + 16a^2c^2 - 8a^2b^2c))}{(-b^9f^8 + f^8(-4ac - b^2)^9)^{1/2} - 768a^4b^3c^4f^8 - 96a^2b^5c^2f^8 + 512a^3b^3c^3f^8} / (32(b^{12}c^2e^2 + 4096a^6c^7e^2 - 24a^2b^{10}c^2e^2 + 240a^2b^8c^3e^2 - 1280a^3b^6c^4e^2 + 3840a^4b^4c^5e^2 - 6144a^5b^2c^6e^2))^{1/2} \cdot (-b^9f^8 + f^8(-4ac - b^2)^9)^{1/2} - 768a^4b^3c^4f^8 - 96a^2b^5c^2f^8 + 512a^3b^3c^3f^8} / (32(b^{12}c^2e^2 + 4096a^6c^7e^2 - 24a^2b^{10}c^2e^2 + 240a^2b^8c^3e^2 - 1280a^3b^6c^4e^2 + 3840a^4b^4c^5e^2 - 6144a^5b^2c^6e^2))^{1/2}\right)$$

$$\begin{aligned}
& *b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2))^{(1/2)} + ((128*a^3*c^4*d*e^{11}*f^8 - 4*b^6*c*d*e^{11}*f^8 + 8*a*b^4*c^2*d*e^{11}*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^{12}*f^4 + 384*a^2*b^4*c^3*e^{12}*f^4 - 1536*a^3*b^2*c^4*e^{12}*f^4 - 32*a*b^6*c^2*e^{12}*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^{13} - 1024*a*b^7*c^3*d*e^{13} + 16384*a^4*b*c^6*d*e^{13} + 6144*a^2*b^5*c^4*d*e^{13} - 16384*a^3*b^3*c^5*d*e^{13})/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^{14} - 192*a*b^5*c^3*e^{14} - 1024*a^3*b*c^5*e^{14} + 768*a^2*b^3*c^4*e^{14}))/((2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^{(1/2)} * (- (b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^{(1/2)} - (x*(b^4*c*e^{12}*f^8 + 8*a^2*c^3*e^{12}*f^8 + 2*a*b^2*c^2*e^{12}*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^{(1/2)} - (3*a*b^3*c*e^{10}*f^{12} + 4*a^2*b*c^2*e^{10}*f^{12})/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) * (- (b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^{(1/2)} * 2i - ((x*(2*a*f^4 + 3*b*d^2*f^4))/(2*(4*a*c - b^2)) + (b*d^3*f^4 + 2*a*d*f^4)/(2*e*(4*a*c - b^2)) + (b*e^2*f^4*x^3)/(2*(4*a*c - b^2)) + (3*b*d*e*f^4*x^2)/(2*(4*a*c - b^2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan((((2048*a^4*c^5*e^{12}*f^4 + 384*a^2*b^4*c^3*e^{12}*f^4 - 1536*a^3*b^2*c^4*e^{12}*f^4 - 32*a*b^6*c^2*e^{12}*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^{13} - 1024*a*b^7*c^3*d*e^{13} + 16384*a^4*b*c^6*d*e^{13} + 6144*a^2*b^5*c^4*d*e^{13} - 16384*a^3*b^3*c^5*d*e^{13})/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^{14} - 192*a*b^5*c^3*e^{14} - 1024*a^3*b*c^5*e^{14} + 768*a^2*b^3*c^4*e^{14}))/((2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * ((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^{(1/2)} * ((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^{(1/2)} - (128*a^3*c^4*d*e^{11}*f^8 - 4*b^6*c*d*e^{11}*f^8 + 8*a*b^4*c^2*d*e^{11}*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^{12}*f^8 + 8*a^2*c^3*e^{12}*f^8 + 2*a*b^2*c^2*e^{12}*f^8)
\end{aligned}$$

$$\begin{aligned} & \cdot c^2 e^{12f^8}))/2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9) \\ &)^{(1/2)} - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3 \\ & *f^8)/(32*(b^{12}*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^{10}*c^2*e^2 + 240*a^2*b^8 \\ & *c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e \\ & ^2))))^{(1/2)}*i - ((128*a^3*c^4*d*e^{11}*f^8 - 4*b^6*c*d*e^{11}*f^8 + 8*a*b^4*c^2 \\ & *d*e^{11}*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((2048 \\ & *a^4*c^5*e^{12}*f^4 + 384*a^2*b^4*c^3*e^{12}*f^4 - 1536*a^3*b^2*c^4*e^{12}*f^4 - \\ & 32*a*b^6*c^2*e^{12}*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) \\ & - ((64*b^9*c^2*d*e^{13} - 1024*a*b^7*c^3*d*e^{13} + 16384*a^4*b*c^6*d*e^{13} + 6 \\ & 144*a^2*b^5*c^4*d*e^{13} - 16384*a^3*b^3*c^5*d*e^{13))/(8*(b^6 - 64*a^3*c^3 + 4 \\ & 8*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^{14} - 192*a*b^5*c^3*e^{14} - 1 \\ & 024*a^3*b*c^5*e^{14} + 768*a^2*b^3*c^4*e^{14}))/2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c) \\ &)))*((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5 \\ & *c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^{12}*c*e^2 + 4096*a^6*c^7*e^2 - 24*a \\ & *b^{10}*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5 \\ & *e^2 - 6144*a^5*b^2*c^6*e^2))))^{(1/2)})*((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - b \\ & ^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32* \\ & (b^{12}*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^{10}*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - \\ & 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2))))^{(1/2)} \\ & - (x*(b^4*c*e^{12}*f^8 + 8*a^2*c^3*e^{12}*f^8 + 2*a*b^2*c^2*e^{12}*f^8))/2*(b^4 \\ & + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*f^8 + 768 \\ & *a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^{12}*c*e^2 \\ & + 4096*a^6*c^7*e^2 - 24*a*b^{10}*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6 \\ & *c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2))))^{(1/2)}*i)/(((204 \\ & 8*a^4*c^5*e^{12}*f^4 + 384*a^2*b^4*c^3*e^{12}*f^4 - 1536*a^3*b^2*c^4*e^{12}*f^4 - \\ & 32*a*b^6*c^2*e^{12}*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) \\ &) + ((64*b^9*c^2*d*e^{13} - 1024*a*b^7*c^3*d*e^{13} + 16384*a^4*b*c^6*d*e^{13} + \\ & 6144*a^2*b^5*c^4*d*e^{13} - 16384*a^3*b^3*c^5*d*e^{13))/(8*(b^6 - 64*a^3*c^3 + \\ & 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^{14} - 192*a*b^5*c^3*e^{14} - \\ & 1024*a^3*b*c^5*e^{14} + 768*a^2*b^3*c^4*e^{14}))/2*(b^4 + 16*a^2*c^2 - 8*a*b^2 \\ & *c)))*((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2 \\ & *b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^{12}*c*e^2 + 4096*a^6*c^7*e^2 - 24 \\ & *a*b^{10}*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4 \\ & *c^5*e^2 - 6144*a^5*b^2*c^6*e^2))))^{(1/2)})*((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - \\ & b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32 \\ & *(b^{12}*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^{10}*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - \\ & 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2))))^{(1/2)} \\ &) - (128*a^3*c^4*d*e^{11}*f^8 - 4*b^6*c*d*e^{11}*f^8 + 8*a*b^4*c^2*d*e^{11}*f^8)/ \\ & (8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^{12}*f^8 + \\ & 8*a^2*c^3*e^{12}*f^8 + 2*a*b^2*c^2*e^{12}*f^8))/2*(b^4 + 16*a^2*c^2 - 8*a*b^2 \\ & *c)))*((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2 \\ & *b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^{12}*c*e^2 + 4096*a^6*c^7*e^2 - 24 \\ & *a*b^{10}*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4 \\ & *c^5*e^2 - 6144*a^5*b^2*c^6*e^2))))^{(1/2)} + ((128*a^3*c^4*d*e^{11}*f^8 - 4*b^6 \\ & *c*d*e^{11}*f^8 + 8*a*b^4*c^2*d*e^{11}*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c \end{aligned}$$

$$\begin{aligned} &^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^{12*f^4} + 384*a^2*b^4*c^3*e^{12*f^4} - 15 \\ &36*a^3*b^2*c^4*e^{12*f^4} - 32*a*b^6*c^2*e^{12*f^4})/(8*(b^6 - 64*a^3*c^3 + 48* \\ &a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^{13} - 1024*a*b^7*c^3*d*e^{13} + \\ &16384*a^4*b*c^6*d*e^{13} + 6144*a^2*b^5*c^4*d*e^{13} - 16384*a^3*b^3*c^5*d*e^{13} \\ &))/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^{14} \\ &- 192*a*b^5*c^3*e^{14} - 1024*a^3*b*c^5*e^{14} + 768*a^2*b^3*c^4*e^{14}))/((2*(b \\ &^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - b^9*f^8 + 7 \\ &68*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(32*(b^{12}*c*e^2 \\ &+ 4096*a^6*c^7*e^2 - 24*a*b^{10}*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b \\ &^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^{(1/2)})*((f^8*(- \\ &(4*a*c - b^2)^9)^{(1/2)} - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - \\ &512*a^3*b^3*c^3*f^8)/(32*(b^{12}*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^{10}*c^2*e^2 \\ &+ 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 614 \\ &4*a^5*b^2*c^6*e^2)))^{(1/2)} - (x*(b^4*c*e^{12*f^8} + 8*a^2*c^3*e^{12*f^8} + 2*a* \\ &b^2*c^2*e^{12*f^8}))/((2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((f^8*(-(4*a*c - b^2 \\ &)^9)^{(1/2)} - b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3 \\ &*c^3*f^8)/(32*(b^{12}*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^{10}*c^2*e^2 + 240*a^2* \\ &b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^ \\ &6*e^2)))^{(1/2)} - (3*a*b^3*c*e^{10*f^{12}} + 4*a^2*b*c^2*e^{10*f^{12}})/(4*(b^6 - 64 \\ &*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*((f^8*(-(4*a*c - b^2)^9)^{(1/2)} - \\ &b^9*f^8 + 768*a^4*b*c^4*f^8 + 96*a^2*b^5*c^2*f^8 - 512*a^3*b^3*c^3*f^8)/(3 \\ &2*(b^{12}*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^{10}*c^2*e^2 + 240*a^2*b^8*c^3*e^2 \\ &- 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^{(1/ \\ &2)}*2i \end{aligned}$$

sympy [B] time = 9.82, size = 641, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] $(-2*a*d*f**4 - b*d**3*f**4 - 3*b*d*e**2*f**4*x**2 - b*e**3*f**4*x**3 + x*(-$
 $2*a*e*f**4 - 3*b*d**2*e*f**4))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e +$
 $8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2$
 $*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e$
 $**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*$
 $c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + Roo$
 $tSum(_t**4*(1048576*a**6*c**7*e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a$
 $**4*b**4*c**5*e**4 - 327680*a**3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4$
 $- 6144*a*b**10*c**2*e**4 + 256*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e$
 $**2*f**8 + 8192*a**3*b**3*c**3*e**2*f**8 - 1536*a**2*b**5*c**2*e**2*f**8 +$
 $16*b**9*e**2*f**8) + 16*a**3*c**2*f**16 + 24*a**2*b**2*c*f**16 + 9*a*b**4*f$
 $**16, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t**3*a**$
 $2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e**3 + 64$

$$\frac{t^2 a^2 c^2 e^{f^8} - 128 t a b^2 c e^{f^8} - 4 t b^4 e^{f^8} + 4 a c d f^{12} + 3 b^2 d f^{12}}{(4 a c e^{f^{12}} + 3 b^2 e^{f^{12}})}$$

$$3.534 \quad \int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=103

$$\frac{f^3 (2a + b(d + ex)^2)}{2e(b^2 - 4ac)(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{bf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1142, 1114, 638, 618, 206}

$$\frac{f^3 (2a + b(d + ex)^2)}{2e(b^2 - 4ac)(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{bf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] (f^3*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1142

Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(df + efx)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx &= \frac{f^3 \operatorname{Subst}\left(\int \frac{x^3}{(a + bx^2 + cx^4)^2} dx, x, d + ex\right)}{e} \\
 &= \frac{f^3 \operatorname{Subst}\left(\int \frac{x}{(a + bx + cx^2)^2} dx, x, (d + ex)^2\right)}{2e} \\
 &= \frac{f^3 (2a + b(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{(bf^3) \operatorname{Subst}\left(\int \frac{1}{a + bx + cx^2} dx\right)}{2(b^2 - 4ac)e} \\
 &= \frac{f^3 (2a + b(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{(bf^3) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx\right)}{(b^2 - 4ac)} \\
 &= \frac{f^3 (2a + b(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{bf^3 \tanh^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2} e}
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 103, normalized size = 1.00

$$\frac{f^3 \left(\frac{2a + b(d + ex)^2}{(b^2 - 4ac)(a + (d + ex)^2(b + c(d + ex)^2))} - \frac{2b \tan^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{3/2}} \right)}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] (f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2]/Sqrt[-b^2 + 4*a*c]))/(-b^2 + 4*a*c)^(3/2)))/(2*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df + efx)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x
]

fricas [B] time = 1.21, size = 1077, normalized size = 10.46

[f^3*(2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2)) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2]/Sqrt[-b^2 + 4*a*c]))/(-b^2 + 4*a*c)^(3/2))/(2*e)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out] [1/2*((b^3 - 4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2)*f^3 - (b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2*d^2 + a*b)*f^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c)))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*e^2*f^3*x^2 + 2*(b^3 - 4*a*b*c)*d*e*f^3*x + (2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2)*f^3 - 2*(b*c*e^4*f^3*x^4 + 4*b*c*d*e^3*f^3*x^3 + (6*b*c*d^2 + b^2)*e^2*f^3*x^2 + 2*(2*b*c*d^3 + b^2*d)*e*f^3*x + (b*c*d^4 + b^2*d^2 + a*b)*f^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)

$$*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]$$

giac [B] time = 0.52, size = 211, normalized size = 2.05

$$\frac{bf^3 \arctan\left(\frac{2cd^2f+2(fx^2e+2dfx)ce+bf}{\sqrt{-b^2+4ac}f}\right)e^{(-1)}}{(b^2-4ac)\sqrt{-b^2+4ac}} + \frac{bd^2f^5 + (fx^2e+2dfx)bf^4e + 2af^5}{2\left(cd^4f^2 + 2(fx^2e+2dfx)cd^2fe + bd^2f^2 + (fx^2e+2dfx)^2ce^2 + (fx^2e+2dfx)bfe + af^2\right)(b^2e-4ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] b*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)*f))*e^(-1)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*d^2*f^5 + (f*x^2*e + 2*d*f*x)*b*f^4*e + 2*a*f^5)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)*(b^2*e - 4*a*c*e)

maple [C] time = 0.02, size = 500, normalized size = 4.85

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out] -1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*e/(4*a*c-b^2)*x^2-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*b*d/(4*a*c-b^2)*x-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)/e*b*d^2-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)/e*a+1/2*f^3/(4*a*c-b^2)*b/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x), _R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b^3 \int \frac{cx+d}{(b^2-4ac)^2e^{4x}+4(b^2-4ac)d^2e^{3x}+(b^2-4ac)^2e^{2x}+6(b^2-4ac)d^2e^{2x}+6b^2-4ac+(b^2-4ac)d^2+2(2(b^2-4ac)d^2+(b^2-4ac)d^2)} dx + \frac{bd^2f^5 + 2bdf^3x + (bf^2 + 2a)f^5}{2((b^2-4ac)^2e^{4x} + 4(b^2-4ac)d^2e^{3x} + (b^2-4ac+6(b^2-4ac)d^2)e^{2x} + 2(2(b^2-4ac)d^2+(b^2-4ac)d^2))e^{2x} + a(b^2-4ac)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] $-b*f^3*\integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) + 1/2*(b*e^2*f^3*x^2 + 2*b*d*e*f^3*x + (b*d^2 + 2*a)*f^3)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)$

mupad [B] time = 1.90, size = 460, normalized size = 4.47

$$b f^3 \operatorname{atan}\left(\frac{(4 a c-b^2)^4\left(\frac{b^3 f^6\left(2 d^2 d^2 d^2 d^2 d^2\right)}{a^2(4 a c-b^2)^{11 / 2}}-\frac{2 d^2 d^2 f^6}{a(4 a c-b^2)^{9 / 2}}\right)+\frac{b^3 f^6\left(2 d^2 d^2 d^2 d^2 d^2\right)}{2 a^2(4 a c-b^2)^{11 / 2}}-\frac{2 d^2 d^2 f^6}{a(4 a c-b^2)^{9 / 2}}}{2 d^2 d^2 f^6}\right)}{e(4 a c-b^2)^{3 / 2}}-\frac{\frac{f^3(b d^2+2 a)}{2 e(4 a c-b^2)}+\frac{b d f^3 x}{4 a c-b^2}+\frac{b e f^3 x^2}{2(4 a c-b^2)}}{a+x^2\left(6 c d^2 e^2+b e^2\right)+b d^2+c d^4+x\left(4 c e d^3+2 b e d\right)+c e^4 x^4+4 c d e^3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x)$

[Out] $(b*f^3*\operatorname{atan}(((4*a*c - b^2)^4*(x*((b^3*f^6*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^9)))/(a*e^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7*f^6)/(a*(4*a*c - b^2)^(7/2))) + x^2*((b^3*f^6*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c - b^2)^(11/2)) - (b^2*c^2*e^8*f^6)/(a*(4*a*c - b^2)^(7/2))) - (b^3*f^6*(16*a^2*c^3*e^8 - 4*a*b^2*c^2*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(11/2)) - (b^2*c^2*d^2*e^6*f^6)/(a*(4*a*c - b^2)^(7/2)))/(2*b^2*c^2*e^6*f^6)))/(e*(4*a*c - b^2)^(3/2)) - ((f^3*(2*a + b*d^2))/(2*e*(4*a*c - b^2)) + (b*d*f^3*x)/(4*a*c - b^2) + (b*e*f^3*x^2)/(2*(4*a*c - b^2)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)$

sympy [B] time = 5.29, size = 556, normalized size = 5.40

$$\frac{b^3 \sqrt{\frac{1}{(4 a c-b^2)}} \log\left(\frac{2 d^2 d^2 e^2+2 a^2+\frac{2 a d^2 d^2 f^6 \sqrt{\frac{1}{(4 a c-b^2)}} \sqrt{\frac{1}{(4 a c-b^2)}} \sqrt{\frac{1}{(4 a c-b^2)}} \sqrt{\frac{1}{(4 a c-b^2)}}}{2 a^2 f^6}}{\frac{2 d^2 d^2 f^6}{a(4 a c-b^2)^{9 / 2}}}\right)+\frac{b^3 \sqrt{\frac{1}{(4 a c-b^2)}} \log\left(\frac{2 d^2 d^2 e^2+2 a^2+\frac{2 a d^2 d^2 f^6 \sqrt{\frac{1}{(4 a c-b^2)}} \sqrt{\frac{1}{(4 a c-b^2)}} \sqrt{\frac{1}{(4 a c-b^2)}} \sqrt{\frac{1}{(4 a c-b^2)}}}{2 a^2 f^6}}{\frac{2 d^2 d^2 f^6}{a(4 a c-b^2)^{9 / 2}}}\right)}{2 e(4 a c-b^2)^{3 / 2}}-\frac{-2 a^2-b^2 f^2-2 b d f^2 x-b^2 f^2 x^2}{8 a^2 c^2-2 a b^2 c+8 a b c^2 e+8 a^2 c^2 e^2-2 b^3 d^2 e-2 b^2 d^2 e^2+x^2\left(8 a c^2 d^2-2 b^2 d^2\right)+x^2\left(2 a c^2 d^4-8 a^2 d^4\right)+x^2\left(8 a b c^2 e+4 8 a c^2 d^2 e-2 b^3 d^2 e-12 b^2 d^2 e^2\right)+x^2\left(8 a b c^2 e^2+32 a c^2 d^2 e^2-4 b^3 d^2 e^2-8 b^2 d^2 e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2, x)$

[Out] $b*f**3*\sqrt{-1/(4*a*c - b**2)**3}*\log(2*d*x/e + x**2 + (-16*a**2*b*c**2*f**3*\sqrt{-1/(4*a*c - b**2)**3} + 8*a*b**3*c*f**3*\sqrt{-1/(4*a*c - b**2)**3} - b**5*f**3*\sqrt{-1/(4*a*c - b**2)**3} + b**2*f**3 + 2*b*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) - b*f**3*\sqrt{-1/(4*a*c - b**2)**3}*\log(2*d*x/e + x**2 + (16*a**2*b*c**2*f**3*\sqrt{-1/(4*a*c - b**2)**3} - 8*a*b**3*c*f**3*\sqrt{-1/(4*a*c - b**2)**3} + b**5*f**3*\sqrt{-1/(4*a*c - b**2)**3} + b**2*f**3 + 2*b*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) + (-2*a*f**3 - b*d**2*f**3 - 2*b*d*e*f**3*x - b*e**2*f**3*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a$

$$\begin{aligned} & *c^{**2}*d^{**4}*e - 2*b^{**3}*d^{**2}*e - 2*b^{**2}*c*d^{**4}*e + x^{**4}*(8*a*c^{**2}*e^{**5} - 2*b^{**2}*c*e^{**5}) + x^{**3}*(32*a*c^{**2}*d*e^{**4} - 8*b^{**2}*c*d*e^{**4}) + x^{**2}*(8*a*b*c*e^{**3} \\ & + 48*a*c^{**2}*d^{**2}*e^{**3} - 2*b^{**3}*e^{**3} - 12*b^{**2}*c*d^{**2}*e^{**3}) + x*(16*a*b*c*d \\ & *e^{**2} + 32*a*c^{**2}*d^{**3}*e^{**2} - 4*b^{**3}*d*e^{**2} - 8*b^{**2}*c*d^{**3}*e^{**2}) \end{aligned}$$

$$3.535 \quad \int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=263

$$\frac{f^2(d+ex)(b+2c(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c}f^2(2b-\sqrt{b^2-4ac})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}f^2(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.37, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1142, 1119, 1166, 205}

$$\frac{f^2(d+ex)(b+2c(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{c}f^2(2b-\sqrt{b^2-4ac})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}e(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}f^2(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}e(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] -(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1119

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(d*x)^(m-1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^(m-2)*(b*(m-1) + 2*c*(m+4*p+5)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx = \frac{f^2 \operatorname{Subst}\left(\int \frac{x^2}{(a + bx^2 + cx^4)^2} dx, x, d + ex\right)}{e}$$

$$= -\frac{f^2(d + ex)(b + 2c(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{f^2 \operatorname{Subst}\left(\int \frac{b - 2cx^2}{a + bx^2 + cx^4} dx, x, d + ex\right)}{2(b^2 - 4ac)e}$$

$$= -\frac{f^2(d + ex)(b + 2c(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{(c(2b - \sqrt{b^2 - 4ac})f^2)}{2(b^2 - 4ac)e}$$

$$= -\frac{f^2(d + ex)(b + 2c(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{\sqrt{c}(2b - \sqrt{b^2 - 4ac})f^2}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.94, size = 250, normalized size = 0.95

$$\frac{f^2 \left(\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}(\sqrt{b^2-4ac}-2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} \right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f^2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2))*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2))*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/e
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [B] time = 1.32, size = 2600, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*e^3*f^2*x^3 + 12*c*d*e^2*f^2*x^2 + 2*(6*c*d^2 + b)*e*f^2*x + 2*(2*c*d^3 + b*d)*f^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)))*e^3)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4)))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2)))
```

$$\begin{aligned}
&^4))e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*\log((\\
&3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*\sqrt{1/2)*((b^ \\
&5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^ \\
&3 - 256*a^5*c^4)*\sqrt{f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^ \\
&5*c^3)*e^4))*e^3)*\sqrt{-((b^3 + 12*a*b*c)*f^4 + (a*b^6 - 12*a^2*b^4*c + 48* \\
&a^3*b^2*c^2 - 64*a^4*c^3)*\sqrt{f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^ \\
&2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4 \\
&*c^3)*e^2))) + \sqrt{1/2)*((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d \\
&*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c \\
&- 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 \\
&- 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*\sqrt{-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - \\
&12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*\sqrt{f^8/((a^2*b^6 - 12*a^3*b^4 \\
&*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^ \\
&3*b^2*c^2 - 64*a^4*c^3)*e^2))*\log((3*b^2*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + \\
&4*a*c^2)*d*f^6 + 1/2*\sqrt{1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a \\
&*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*\sqrt{f^8/((a^2*b^6 - 12 \\
&*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^3)*\sqrt{-((b^3 + 12*a*b*c \\
&)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*\sqrt{f^8/((a^2 \\
&*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12* \\
&a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))) - \sqrt{1/2)*((b^2*c - 4*a*c \\
&^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4 \\
&*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2* \\
&x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*\sqrt \\
&-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c \\
&^3)*\sqrt{f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*e^4))* \\
&e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*e^2))*\log((3*b^2 \\
&*c + 4*a*c^2)*e*f^6*x + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*\sqrt{1/2)*((b^5 - 8 \\
&*a*b^3*c + 16*a^2*b*c^2)*e*f^4 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 2 \\
&56*a^5*c^4)*\sqrt{f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 \\
&)*e^4))*e^3)*\sqrt{-((b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b \\
&^2*c^2 - 64*a^4*c^3)*\sqrt{f^8/((a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6 \\
&4*a^5*c^3)*e^4))*e^2)/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3) \\
&*e^2)))/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 \\
&- 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + \\
&(b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 \\
&- 4*a*b*c)*d^2)*e)
\end{aligned}$$

giac [B] time = 0.67, size = 1378, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] 1/4*((2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c))*e^2))*e^(-4)/

```

c))2*c*f2*e2 - 4*(d*e(-1) + sqrt(1/2)*sqrt(-(b*e2 + sqrt(b2 - 4*a*c)*
e2)*e(-4)/c))*c*d*f2*e + 2*c*d2*f2 - b*f2)*log(d*e(-1) + x + sqrt(1/
2)*sqrt(-(b*e2 + sqrt(b2 - 4*a*c)*e2)*e(-4)/c))/(2*(d*e(-1) + sqrt(1/2
)*sqrt(-(b*e2 + sqrt(b2 - 4*a*c)*e2)*e(-4)/c)))3*c*e4 - 6*(d*e(-1) +
sqrt(1/2)*sqrt(-(b*e2 + sqrt(b2 - 4*a*c)*e2)*e(-4)/c))2*c*d*e3 - 2*c*
d3*e - b*d*e + (6*c*d2*e2 + b*e2)*(d*e(-1) + sqrt(1/2)*sqrt(-(b*e2 +
sqrt(b2 - 4*a*c)*e2)*e(-4)/c))) + (2*(d*e(-1) - sqrt(1/2)*sqrt(-(b*e2
+ sqrt(b2 - 4*a*c)*e2)*e(-4)/c))2*c*f2*e2 - 4*(d*e(-1) - sqrt(1/2)*s
qrt(-(b*e2 + sqrt(b2 - 4*a*c)*e2)*e(-4)/c))*c*d*f2*e + 2*c*d2*f2 - b
*f2)*log(d*e(-1) + x - sqrt(1/2)*sqrt(-(b*e2 + sqrt(b2 - 4*a*c)*e2)*e(
-4)/c))/(2*(d*e(-1) - sqrt(1/2)*sqrt(-(b*e2 + sqrt(b2 - 4*a*c)*e2)*e(
-4)/c)))3*c*e4 - 6*(d*e(-1) - sqrt(1/2)*sqrt(-(b*e2 + sqrt(b2 - 4*a*c)*
e2)*e(-4)/c))2*c*d*e3 - 2*c*d3*e - b*d*e + (6*c*d2*e2 + b*e2)*(d*e(
-1) - sqrt(1/2)*sqrt(-(b*e2 + sqrt(b2 - 4*a*c)*e2)*e(-4)/c))) + (2*(d*
e(-1) + sqrt(1/2)*sqrt(-(b*e2 - sqrt(b2 - 4*a*c)*e2)*e(-4)/c))2*c*f2
*e2 - 4*(d*e(-1) + sqrt(1/2)*sqrt(-(b*e2 - sqrt(b2 - 4*a*c)*e2)*e(-4)
/c))*c*d*f2*e + 2*c*d2*f2 - b*f2)*log(d*e(-1) + x + sqrt(1/2)*sqrt(-(b
*e2 - sqrt(b2 - 4*a*c)*e2)*e(-4)/c))/(2*(d*e(-1) + sqrt(1/2)*sqrt(-(b*
e2 - sqrt(b2 - 4*a*c)*e2)*e(-4)/c)))3*c*e4 - 6*(d*e(-1) + sqrt(1/2)*s
qrt(-(b*e2 - sqrt(b2 - 4*a*c)*e2)*e(-4)/c))2*c*d*e3 - 2*c*d3*e - b*d
*e + (6*c*d2*e2 + b*e2)*(d*e(-1) + sqrt(1/2)*sqrt(-(b*e2 - sqrt(b2 -
4*a*c)*e2)*e(-4)/c))) + (2*(d*e(-1) - sqrt(1/2)*sqrt(-(b*e2 - sqrt(b2
- 4*a*c)*e2)*e(-4)/c))2*c*f2*e2 - 4*(d*e(-1) - sqrt(1/2)*sqrt(-(b*e2
- sqrt(b2 - 4*a*c)*e2)*e(-4)/c))*c*d*f2*e + 2*c*d2*f2 - b*f2)*log(d
*e(-1) + x - sqrt(1/2)*sqrt(-(b*e2 - sqrt(b2 - 4*a*c)*e2)*e(-4)/c))/(2
*(d*e(-1) - sqrt(1/2)*sqrt(-(b*e2 - sqrt(b2 - 4*a*c)*e2)*e(-4)/c)))3*c
*e4 - 6*(d*e(-1) - sqrt(1/2)*sqrt(-(b*e2 - sqrt(b2 - 4*a*c)*e2)*e(-4)
/c))2*c*d*e3 - 2*c*d3*e - b*d*e + (6*c*d2*e2 + b*e2)*(d*e(-1) - sqrt
(1/2)*sqrt(-(b*e2 - sqrt(b2 - 4*a*c)*e2)*e(-4)/c))))/(b2 - 4*a*c) - 1/
2*(2*c*f2*x3*e3 + 6*c*d*f2*x2*e2 + 6*c*d2*f2*x*e + 2*c*d3*f2 + b*
f2*x*e + b*d*f2)/((c*x4*e4 + 4*c*d*x3*e3 + 6*c*d2*x2*e2 + 4*c*d3*
x*e + c*d4 + b*x2*e2 + 2*b*d*x*e + b*d2 + a)*(b2*e - 4*a*c*e))

```

maple [C] time = 0.02, size = 693, normalized size = 2.63

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int ((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2, x)$

[Out] $f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e^2/(4*a*c-b^2)*x^3+3*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d*c*e/(4*a*c-b^2)*x^2+3*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c*d^2+1/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^$

$$\frac{3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a}{(4*a*c-b^2)*x*b+f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*d^3/e*c+1/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*d/e*b+1/4*f^2/(4*a*c-b^2)/e*sum((2*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \int \frac{2a^2x^2 + 4adex + 2af - b}{(b^2c - 4a^2c^2)x^4 + 4(b^2c - 4a^2c^2)d^3e^3x^3 + (b^2c - 4a^2c^2)d^4 + (b^3 - 4a^2bc + 6(b^2c - 4a^2c^2)d^2)e^2x^2 + ab^2 - 4a^2c + (b^3 - 4a^2bc)\beta + 2((b^2c - 4a^2c^2)\beta + (b^3 - 4a^2bc)\beta)}{2((b^2c - 4a^2c^2)\beta^2 + 4(b^2c - 4a^2c^2)\beta^3 + (b^3 - 4a^2bc + 6(b^2c - 4a^2c^2)\beta^2 + 2((b^2c - 4a^2c^2)\beta + (b^3 - 4a^2bc)\beta))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] 1/2*f^2*integrate(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 - b)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^3*f^2*x^3 + 6*c*d*e^2*f^2*x^2 + (6*c*d^2 + b)*e*f^2*x + (2*c*d^3 + b*d)*f^2)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)

mupad [B] time = 4.40, size = 7835, normalized size = 29.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)

[Out] ((x*(b*f^2 + 6*c*d^2*f^2))/(2*(4*a*c - b^2)) + (2*c*d^3*f^2 + b*d*f^2)/(2*e*(4*a*c - b^2)) + (c*e^2*f^2*x^3)/(4*a*c - b^2) + (3*c*d*e*f^2*x^2)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^3*d*e^13

$$\begin{aligned}
& b^2c^5e^2)^{(1/2)} + (2b^7c^2e^{12}f^2 + 96a^2b^3c^4e^{12}f^2 - 24a^* \\
& b^5c^3e^{12}f^2 - 128a^3b^3c^5e^{12}f^2)/(b^6 - 64a^3c^3 + 48a^2b^2c^ \\
& ^2 - 12a^*b^4c) + (16a^2c^5d^*e^{11}f^4 + 5b^4c^3d^*e^{11}f^4 - 24a^*b^ \\
& ^2c^4d^*e^{11}f^4)/(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12a^*b^4c) - (x*(4^* \\
& a^c^4e^{12}f^4 - 5b^2c^3e^{12}f^4))/(b^4 + 16a^2c^2 - 8a^*b^2c) - (((\\
& f^4*(-(4a^*c - b^2)^9)^{(1/2)})/32 - (b^9f^4)/32 + 24a^4b^*c^4f^4 + 3a^2^* \\
& b^5c^2f^4 - 16a^3b^3c^3f^4)/(a^*b^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^ \\
& ^10c^*e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^ \\
& ^2 - 6144a^6b^2c^5e^2))^{(1/2)}*(((f^4*(-(4a^*c - b^2)^9)^{(1/2)})/32 - (b \\
& ^9f^4)/32 + 24a^4b^*c^4f^4 + 3a^2b^5c^2f^4 - 16a^3b^3c^3f^4)/(a^* \\
& b^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^10c^*e^2 + 240a^3b^8c^2e^2 - 128 \\
& 0a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{(1/2)}*(((\\
& 8b^9c^2d^*e^{13} - 128a^*b^7c^3d^*e^{13} + 2048a^4b^*c^6d^*e^{13} + 768a^2b^ \\
& ^5c^4d^*e^{13} - 2048a^3b^3c^5d^*e^{13}))/ (b^6 - 64a^3c^3 + 48a^2b^2c^2 \\
& - 12a^*b^4c) + (x*(8b^7c^2e^{14} - 96a^*b^5c^3e^{14} - 512a^3b^*c^5e^{1 \\
& 4 + 384a^2b^3c^4e^{14}))/ (b^4 + 16a^2c^2 - 8a^*b^2c))*(((f^4*(-(4a^*c \\
& - b^2)^9)^{(1/2)})/32 - (b^9f^4)/32 + 24a^4b^*c^4f^4 + 3a^2b^5c^2f^4 - \\
& 16a^3b^3c^3f^4)/(a^*b^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^10c^*e^2 + 2 \\
& 40a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6 \\
& *b^2c^5e^2))^{(1/2)} - (2b^7c^2e^{12}f^2 + 96a^2b^3c^4e^{12}f^2 - 24a^* \\
& *b^5c^3e^{12}f^2 - 128a^3b^3c^5e^{12}f^2)/(b^6 - 64a^3c^3 + 48a^2b^2c^* \\
& ^2 - 12a^*b^4c) + (16a^2c^5d^*e^{11}f^4 + 5b^4c^3d^*e^{11}f^4 - 24a^*b^ \\
& ^2c^4d^*e^{11}f^4)/(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12a^*b^4c) - (x*(4^* \\
& *a^c^4e^{12}f^4 - 5b^2c^3e^{12}f^4))/(b^4 + 16a^2c^2 - 8a^*b^2c)))*((\\
& f^4*(-(4a^*c - b^2)^9)^{(1/2)} - b^9f^4 + 768a^4b^*c^4f^4 + 96a^2b^5c^2 \\
& *f^4 - 512a^3b^3c^3f^4)/(32*(a^*b^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^1 \\
& 0c^*e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 \\
& - 6144a^6b^2c^5e^2))^{(1/2)}*2i + \operatorname{atan}(((b^9f^4)/32 + (f^4*(-(4a^*c \\
& - b^2)^9)^{(1/2)})/32 - 24a^4b^*c^4f^4 - 3a^2b^5c^2f^4 + 16a^3b^3c^ \\
& ^3f^4)/(a^*b^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^10c^*e^2 + 240a^3b^8c^2 \\
& *e^2 - 1280a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2)) \\
& ^{(1/2)}*((-(b^9f^4)/32 + (f^4*(-(4a^*c - b^2)^9)^{(1/2)})/32 - 24a^4b^*c^4 \\
& f^4 - 3a^2b^5c^2f^4 + 16a^3b^3c^3f^4)/(a^*b^{12}e^2 + 4096a^7c^6e^ \\
& ^2 - 24a^2b^10c^*e^2 + 240a^3b^8c^2e^2 - 1280a^4b^6c^3e^2 + 3840a^ \\
& ^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{(1/2)}*(((8b^9c^2d^*e^{13} - 128a^*b^ \\
& ^7c^3d^*e^{13} + 2048a^4b^*c^6d^*e^{13} + 768a^2b^5c^4d^*e^{13} - 2048a^3b^ \\
& ^3c^5d^*e^{13}))/ (b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12a^*b^4c) + (x*(8b^7 \\
& *c^2e^{14} - 96a^*b^5c^3e^{14} - 512a^3b^*c^5e^{14} + 384a^2b^3c^4e^{14}))/ \\
& / (b^4 + 16a^2c^2 - 8a^*b^2c))*(-(b^9f^4)/32 + (f^4*(-(4a^*c - b^2)^9)^ \\
& ^{(1/2)})/32 - 24a^4b^*c^4f^4 - 3a^2b^5c^2f^4 + 16a^3b^3c^3f^4)/(a^*b \\
& ^{12}e^2 + 4096a^7c^6e^2 - 24a^2b^10c^*e^2 + 240a^3b^8c^2e^2 - 1280 \\
& *a^4b^6c^3e^2 + 3840a^5b^4c^4e^2 - 6144a^6b^2c^5e^2))^{(1/2)} + (2 \\
& *b^7c^2e^{12}f^2 + 96a^2b^3c^4e^{12}f^2 - 24a^*b^5c^3e^{12}f^2 - 128a^ \\
& ^3b^3c^5e^{12}f^2)/(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12a^*b^4c) + (16^* \\
& a^2c^5d^*e^{11}f^4 + 5b^4c^3d^*e^{11}f^4 - 24a^*b^2c^4d^*e^{11}f^4)/(b^6 -
\end{aligned}$$

$$\begin{aligned} & *c - b^2)^9)^{(1/2)}/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2) \\ &)^{(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-((b^9*f^4)/32 + (f^4*(-(4*a*c - b^2)^9)^{(1/2)}/32 - 24*a^4*b*c^4*f^4 - 3*a^2*b^5*c^2*f^4 + 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^{(1/2)} - (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^4 + f^4*(-(4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4*f^4 - 96*a^2*b^5*c^2*f^4 + 512*a^3*b^3*c^3*f^4)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^{(1/2)}*2i \end{aligned}$$

sympy [B] time = 19.76, size = 646, normalized size = 2.46

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] (b*d*f**2 + 2*c*d**3*f**2 + 6*c*d*e**2*f**2*x**2 + 2*c*e**3*f**2*x**3 + x*(b*e*f**2 + 6*c*d**2*e*f**2))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**7*c**6*e**4 - 1572864*a**6*b**2*c**5*e**4 + 983040*a**5*b**4*c**4*e**4 - 327680*a**4*b**6*c**3*e**4 + 61440*a**3*b**8*c**2*e**4 - 6144*a**2*b**10*c*e**4 + 256*a*b**12*e**4) + _t**2*(-12288*a**4*b*c**4*e**2*f**4 + 8192*a**3*b**3*c**3*e**2*f**4 - 1536*a**2*b**5*c**2*e**2*f**4 + 16*b**9*e**2*f**4) + 16*a**2*c**3*f**8 + 24*a*b**2*c**2*f**8 + 9*b**4*c*f**8, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4*e**3 - 8192*_t**3*a**4*b**2*c**3*e**3 + 512*_t**3*a**2*b**6*c*e**3 - 64*_t**3*a*b**8*e**3 - 128*_t*a**2*b*c**2*e*f**4 - 16*_t*a*b**3*c*e*f**4 - 4*_t*b**5*e*f**4 + 4*a*c**2*d*f**6 + 3*b**2*c*d*f**6)/(4*a*c**2*e*f**6 + 3*b**2*c*e*f**6))))

$$3.536 \quad \int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=98

$$\frac{2cf \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{3/2}} - \frac{f(b+2c(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}$$

Rubi [A] time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{2cf \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{3/2}} - \frac{f(b+2c(d+ex)^2)}{2e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] -(f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (2*c*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

`Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]`

Rule 1142

`Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]`

Rubi steps

$$\begin{aligned}
 \int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx &= \frac{f \operatorname{Subst}\left(\int \frac{x}{(a + bx^2 + cx^4)^2} dx, x, d + ex\right)}{e} \\
 &= \frac{f \operatorname{Subst}\left(\int \frac{1}{(a + bx + cx^2)^2} dx, x, (d + ex)^2\right)}{2e} \\
 &= -\frac{f(b + 2c(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{(cf) \operatorname{Subst}\left(\int \frac{1}{a + bx + cx^2} dx, x, (d + ex)^2\right)}{(b^2 - 4ac)e} \\
 &= -\frac{f(b + 2c(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{(2cf) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, (d + ex)^2\right)}{(b^2 - 4ac)e} \\
 &= -\frac{f(b + 2c(d + ex)^2)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{2cf \tanh^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}e}
 \end{aligned}$$

Mathematica [A] time = 0.12, size = 99, normalized size = 1.01

$$-\frac{f\left(\frac{4c \tan^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} + \frac{b + 2c(d + ex)^2}{a + b(d + ex)^2 + c(d + ex)^4}\right)}{2e(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]

```
[Out] -1/2*(f*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/(b^2 - 4*a*c)*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

```
[Out] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2, x]
```

fricas [B] time = 1.11, size = 1066, normalized size = 10.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x + 2*(c^2*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*e*f*x + (c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c)))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + (b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*e^2*f*x^2 + 4*(b^2*c - 4*a*c^2)*d*e*f*x - 4*(c^2*e^4*f*x^4 + 4*c^2*d*e^3*f*x^3 + (6*c^2*d^2 + b*c)*e^2*f*x^2 + 2*(2*c^2*d^3 + b*c*d)*e*f*x + (c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^5*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^4*x^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^3*x^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e^2*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```


giac [B] time = 0.44, size = 211, normalized size = 2.15

$$\frac{2cf \arctan\left(\frac{2cd^2f + 2(fx^2e + 2dfx)ce + bf}{\sqrt{-b^2 + 4ac}}\right)e^{-1}}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{2cd^2f^3 + 2(fx^2e + 2dfx)cf^2e + bf^3}{2(cd^4f^2 + 2(fx^2e + 2dfx)cd^2fe + bd^2f^2 + (fx^2e + 2dfx)^2ce^2 + (fx^2e + 2dfx)bfe + af^2)(b^2e - 4ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] $-2*c*f*\arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(\sqrt{-b^2 + 4*a*c})*f)*e^{-1}/((b^2 - 4*a*c)*\sqrt{-b^2 + 4*a*c}) - 1/2*(2*c*d^2*f^3 + 2*(f*x^2*e + 2*d*f*x)*c*f^2*e + b*f^3)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)*(b^2*e - 4*a*c*e)$

maple [C] time = 0.02, size = 484, normalized size = 4.94

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out] $f/(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + b*e^2*x^2 + c*d^4 + 2*b*d*e*x + b*d^2 + a)*c*e/(4*a*c - b^2)*x^2 + 2*f/(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + b*e^2*x^2 + c*d^4 + 2*b*d*e*x + b*d^2 + a)*c*d/(4*a*c - b^2)*x + f/(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + b*e^2*x^2 + c*d^4 + 2*b*d*e*x + b*d^2 + a)/(4*a*c - b^2)/e*c*d^2 + 1/2*f/(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + b*e^2*x^2 + c*d^4 + 2*b*d*e*x + b*d^2 + a)/(4*a*c - b^2)/e*b*f*c/(4*a*c - b^2)/e*sum((R*e+d)/(2*_R^3*c*e^3 + 6*_R^2*c*d*e^2 + 6*_R*c*d^2*e + 2*c*d^3 + _R*b*e + b*d)*ln(-R+x), _R=RootOf(_Z^4*c*e^4 + 4*_Z^3*c*d*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*_Z^2 + (4*c*d^3*e + 2*b*d*e)*_Z + a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2cf \int \frac{e^x + d}{(b^2c - 4ac^2)^4x^4 + 4(b^2c - 4ac^2)d^3x^3 + (b^2c - 4ac^2)d^2x^2 + (b^2c - 4ac^2)d^2x^2 + ab^2 - 4c^2 + (b^2 - 4abc)d^2 + 2(2(b^2c - 4ac^2)d^2 + (b^2 - 4abc)d)x + 2((b^2c - 4ac^2)d^3 + 4(b^2c - 4ac^2)d^2x + (b^2c - 4ac^2)d^2x + (b^2c - 4ac^2)d^2x + ab^2 - 4c^2 + (b^2 - 4abc)d^2)} dx - \frac{2c^2f^2 + 4cdefx + (2cd^2 + b)f}{2((b^2c - 4ac^2)d^3 + 4(b^2c - 4ac^2)d^2x + (b^2c - 4ac^2)d^2x + (b^2c - 4ac^2)d^2x + ab^2 - 4c^2 + (b^2 - 4abc)d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] $2*c*f*\integrate(-(e*x + d)/((b^2*c - 4*a*c^2)*e^4*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^3*x^3 + (b^2*c - 4*a*c^2)*d^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^2*x^2 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e*x), x) - 1/2*(2*c*e^2*f*x^2 + 4*c*d*e*f*x + (2*c*d^2 + b)*f)/((b^2*c - 4*a*c^2)*e^5*x^4 + 4*(b^2*c - 4*a*c^2)*d*e^4*x^3 +$

$$(b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*e^3*x^2 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*e^2*x + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e$$

mupad [B] time = 1.91, size = 442, normalized size = 4.51

$$\frac{\frac{f(2cd^2+ab)}{2c(4ac-b^2)} + \frac{2cdfx}{4ac-b^2} + \frac{cef^2}{4ac-b^2}}{a+x^2(6cd^2e^2+be^2)+bd^2+cd^4+x(4ced^3+2bed)+ce^4x^4+4cd^3x^3} + \frac{2cf \operatorname{atan}\left(\frac{(4ac-b^2)^2 \sqrt{x\left(\frac{8c^4d^2f^2}{a(4ac-b^2)^{7/2}} - \frac{8b^2d^2(b^3d^2-4ab^2cd^2)}{a^2(4ac-b^2)^{11/2}}\right)}{a(4ac-b^2)^{7/2}} + x^2\left(\frac{4c^4d^2}{a(4ac-b^2)^{7/2}} - \frac{4b^2d^2(b^3d^2-4ab^2cd^2)}{a^2(4ac-b^2)^{11/2}}\right)}{8c^4d^2f^2} + \frac{4c^4d^2f^2}{a(4ac-b^2)^{7/2}} - \frac{4b^2d^2(8c^2d^3-2ab^2cd^2-4ab^2cd^2-4ab^2cd^2)}{a^2(4ac-b^2)^{11/2}}\right)}{c(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)`

[Out] $((f*(b + 2*c*d^2))/(2*e*(4*a*c - b^2)) + (2*c*d*f*x)/(4*a*c - b^2) + (c*e*f*x^2)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*f*\operatorname{atan}(((4*a*c - b^2)^4*(x*((8*c^4*d*e^7*f^2)/(a*(4*a*c - b^2)^{(7/2)}) - (8*b*c^2*f^2*(b^3*c^2*d*e^9 - 4*a*b*c^3*d*e^9))/(a*e^2*(4*a*c - b^2)^{(11/2)})) + x^2*((4*c^4*e^8*f^2)/(a*(4*a*c - b^2)^{(7/2)}) - (4*b*c^2*f^2*(b^3*c^2*e^{10} - 4*a*b*c^3*e^{10}))/ (a*e^2*(4*a*c - b^2)^{(11/2)})) + (4*c^4*d^2*e^6*f^2)/(a*(4*a*c - b^2)^{(7/2)}) + (4*b*c^2*f^2*(8*a^2*c^3*e^8 - 2*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)^{(11/2)})))/(8*c^4*e^6*f^2)))/(e*(4*a*c - b^2)^{(3/2)})$

sympy [B] time = 5.07, size = 525, normalized size = 5.36

$$\frac{cf \sqrt{\frac{a}{4ac-b^2}} \log\left(\frac{2dx}{e} + x^2 + \frac{8a^2c^2 \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}}}{2c^2f}\right)}{c} + \frac{cf \sqrt{\frac{a}{4ac-b^2}} \log\left(\frac{2dx}{e} + x^2 + \frac{8a^2c^2 \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}} \sqrt{\frac{a}{4ac-b^2}}}{2c^2f}\right)}{c} + \frac{bf + 2cdf + 4cdfx + 2cdfx^2}{8c^4e - 2ab^2 + 8ab^2c + 8ac^2d^2 - 2b^3d^2 - 2b^3cd^2 + x^2(32a^2cd^2 - 8b^2cd^2) + x^2(8ab^2c^2 + 48ac^2d^2 - 2b^3d^2 - 12b^3cd^2) + x^2(16ab^2cd^2 + 32ac^2d^2 - 4b^3d^2 - 8b^3cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)`

[Out] $-c*f*\sqrt{-1/(4*a*c - b**2)**3}*\log(2*d*x/e + x**2 + (-16*a**2*c**3*f*\sqrt{-1/(4*a*c - b**2)**3} + 8*a*b**2*c**2*f*\sqrt{-1/(4*a*c - b**2)**3} - b**4*c*f*\sqrt{-1/(4*a*c - b**2)**3} + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e + c*f*\sqrt{-1/(4*a*c - b**2)**3}*\log(2*d*x/e + x**2 + (16*a**2*c**3*f*\sqrt{-1/(4*a*c - b**2)**3} - 8*a*b**2*c**2*f*\sqrt{-1/(4*a*c - b**2)**3} + b**4*c*f*\sqrt{-1/(4*a*c - b**2)**3} + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e + (b*f + 2*c*d**2*f + 4*c*d*e*f*x + 2*c*e**2*f*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))$

$$3.537 \quad \int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=174

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef(b^2 - 4ac)^{3/2}} - \frac{\log(a + b(d+ex)^2 + c(d+ex)^4)}{4a^2ef} + \frac{\log(d+ex)}{a^2ef} + \frac{-2ac + b^2 + bc(d+ex)^2}{2aef(b^2 - 4ac)(a + b(d+ex)^2 + c(d+ex)^4)}$$

Rubi [A] time = 0.30, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef(b^2 - 4ac)^{3/2}} - \frac{\log(a + b(d+ex)^2 + c(d+ex)^4)}{4a^2ef} + \frac{\log(d+ex)}{a^2ef} + \frac{-2ac + b^2 + bc(d+ex)^2}{2aef(b^2 - 4ac)(a + b(d+ex)^2 + c(d+ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2]/Sqrt[b^2 - 4*a*c])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e*f) + Log[d + e*x]/(a^2*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e*f)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(df + efx)(a + b(d + ex)^2 + c(d + ex)^4)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^2} dx, x, d + ex\right)}{ef} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^2} dx, x, (d + ex)^2\right)}{2ef} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{\text{Subst}\left(\int \frac{-b^2}{x(a+bx+cx^2)} dx, x, (d + ex)^2\right)}{2a} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{\text{Subst}\left(\int \left(\frac{-b}{a+bx+cx^2}\right) dx, x, (d + ex)^2\right)}{2a} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{\log(d + ex)}{a^2ef} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{\log(d + ex)}{a^2ef} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{\log(d + ex)}{a^2ef} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)} + \frac{b(b^2 - 6ac)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 238, normalized size = 1.37

$$\frac{2a(-2ac+b^2+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} - \frac{(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3)\log(-\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{(b^2-4ac)^{3/2}} + \frac{(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3)\log(\sqrt{b^2-4ac}+b+2c(d+ex)^2)}{(b^2-4ac)^{3/2}} + 4\log(d+ex)}{4a^2ef}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*sqrt[b^2 - 4*a*c

$$\frac{-4ac\sqrt{b^2 - 4ac} \log[b - \sqrt{b^2 - 4ac}] + 2c(d + ex)^2}{(b^2 - 4ac)^{3/2}} + \frac{(b^3 - 6abc - b^2\sqrt{b^2 - 4ac} + 4ac\sqrt{b^2 - 4ac}) \log[b + \sqrt{b^2 - 4ac}] + 2c(d + ex)^2}{(b^2 - 4ac)^{3/2}} \Big/ (4a^2ef)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

fricas [B] time = 2.34, size = 2486, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & \left[\frac{1}{4} * (2 * a * b^4 - 12 * a^2 * b^2 * c + 16 * a^3 * c^2 + 2 * (a * b^3 * c - 4 * a^2 * b * c^2) * e^2 * x^2 + 4 * (a * b^3 * c - 4 * a^2 * b * c^2) * d * e * x + 2 * (a * b^3 * c - 4 * a^2 * b * c^2) * d^2 + ((b^3 * c - 6 * a * b * c^2) * e^4 * x^4 + 4 * (b^3 * c - 6 * a * b * c^2) * d * e^3 * x^3 + (b^3 * c - 6 * a * b * c^2) * d^4 + (b^4 - 6 * a * b^2 * c + 6 * (b^3 * c - 6 * a * b * c^2) * d^2) * e^2 * x^2 + a * b^3 - 6 * a^2 * b * c + (b^4 - 6 * a * b^2 * c) * d^2 + 2 * (2 * (b^3 * c - 6 * a * b * c^2) * d^3 + (b^4 - 6 * a * b^2 * c) * d) * e * x) * \text{sqrt}(b^2 - 4 * a * c) * \log((2 * c^2 * e^4 * x^4 + 8 * c^2 * d * e^3 * x^3 + 2 * c^2 * d^4 + 2 * (6 * c^2 * d^2 + b * c) * e^2 * x^2 + 2 * b * c * d^2 + 4 * (2 * c^2 * d^3 + b * c * d) * e * x + b^2 - 2 * a * c + (2 * c * e^2 * x^2 + 4 * c * d * e * x + 2 * c * d^2 + b) * \text{sqrt}(b^2 - 4 * a * c)) / (c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + c * d^4 + (6 * c * d^2 + b) * e^2 * x^2 + b * d^2 + 2 * (2 * c * d^3 + b * d) * e * x + a)) - ((b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) * e^4 * x^4 + 4 * (b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) * d * e^3 * x^3 + a * b^4 - 8 * a^2 * b^2 * c + 16 * a^3 * c^2 + (b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) * d^4 + (b^5 - 8 * a * b^3 * c + 16 * a^2 * b * c^2 + 6 * (b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) * d^2) * e^2 * x^2 + (b^5 - 8 * a * b^3 * c + 16 * a^2 * b * c^2) * d^2 + 2 * (2 * (b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) * d^3 + (b^5 - 8 * a * b^3 * c + 16 * a^2 * b * c^2) * d) * e * x) * \log(c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + c * d^4 + (6 * c * d^2 + b) * e^2 * x^2 + b * d^2 + 2 * (2 * c * d^3 + b * d) * e * x + a) + 4 * ((b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) * e^4 * x^4 + 4 * (b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) * d * e^3 * x^3 + a * b^4 - 8 * a^2 * b^2 * c + 16 * a^3 * c^2 + (b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) * d^4 + (b^5 - 8 * a * b^3 * c + 16 * a^2 * b * c^2 + 6 * (b^4 * c - 8 * a * b^2 * c^2 + 16 * a^2 * c^3) * d^2) * e^2 * x^2 + (b^5 - 8 * a * b^3 * c + 16 * a^2 * b * c^2) * d^2 + 2 * (2 * (b^4 * c - \end{aligned} \right.$$

$$\begin{aligned}
& 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*1 \\
& \log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b \\
& ^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 1 \\
& 6*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2 \\
& *(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + \\
& 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c \\
& - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 \\
&)*d^2)*e*f), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2* \\
& b*c^2)*e^2*x^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e*x + 2*(a*b^3*c - 4*a^2*b*c^2 \\
&)*d^2 + 2*((b^3*c - 6*a*b*c^2)*e^4*x^4 + 4*(b^3*c - 6*a*b*c^2)*d*e^3*x^3 + \\
& (b^3*c - 6*a*b*c^2)*d^4 + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^2)*e^2 \\
& *x^2 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2) \\
& *d^3 + (b^4 - 6*a*b^2*c)*d)*e*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + \\
& 4*c*d*e*x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a* \\
& b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3* \\
& x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3) \\
& *d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^ \\
& 3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a* \\
& b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(c* \\
& e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^ \\
& 3 + b*d)*e*x + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e^4*x^4 + 4*(b^4*c \\
& - 8*a*b^2*c^2 + 16*a^2*c^3)*d*e^3*x^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 \\
& + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 \\
& + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*e^2*x^2 + (b^5 - 8*a*b^3*c + 16 \\
& *a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a* \\
& b^3*c + 16*a^2*b*c^2)*d)*e*x)*log(e*x + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 1 \\
& 6*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x \\
& ^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + \\
& 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3) \\
& *d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4 \\
& *b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b \\
& ^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e*f)]
\end{aligned}$$

giac [B] time = 1.43, size = 476, normalized size = 2.74

$$\frac{(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e*f)}{4*a^2*c^2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*e^5*f*x^4 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*e^4*f*x^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*e^3*f*x^2 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*f*x + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e*f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] -1/4*((a^2*b^3*c*f*e^3 - 6*a^3*b*c^2*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2 *e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*f*e^3 - 6*a^3*b*c^2*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2 *e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/((

$$a^4 b^4 c f^2 e^4 - 8 a^5 b^2 c^2 f^2 e^4 + 16 a^6 c^3 f^2 e^4) - 1/4 e^{(-1)} * \log(\text{abs}(c x^4 e^4 + 4 c d x^3 e^3 + 6 c d^2 x^2 e^2 + 4 c d^3 x e + c d^4 + b x^2 e^2 + 2 b d x e + b d^2 + a)) / (a^2 f) + e^{(-1)} * \log(\text{abs}(x e + d)) / (a^2 f) + 1/2 * (a b c x^2 e^2 + 2 a b c d x e + a b c d^2 + a b^2 - 2 a^2 c) * e^{(-1)} / ((c x^4 e^4 + 4 c d x^3 e^3 + 6 c d^2 x^2 e^2 + 4 c d^3 x e + c d^4 + b x^2 e^2 + 2 b d x e + b d^2 + a) * (b^2 - 4 a c) * a^2 f)$$

maple [C] time = 0.03, size = 714, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out]
$$-1/2/f/a/(c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) * b c e / (4 a c - b^2) * x^2 - 1/f/a/(c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) * b c d / (4 a c - b^2) * x - 1/2/f/a/(c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) / e / (4 a c - b^2) * b c d^2 + 1/f/(c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) / e / (4 a c - b^2) * c - 1/2/f/a/(c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) / e / (4 a c - b^2) * b^2 - 1/2/f/a^2 / (4 a c - b^2) / e * \text{sum}(((4 a c - b^2) * _R^3 c e^3 + 3 * (4 a c - b^2) * _R^2 c d e^2 + 4 a c^2 d^3 - b^2 c d^3 + 5 a b c d - b^3 d + (12 a c^2 d^2 - 3 b^2 c d^2 + 5 a b c - b^3) * _R e) / (2 * _R^3 c e^3 + 6 * _R^2 c d e^2 + 6 * _R c d^2 e + 2 c d^3 + _R b e + b d) * \ln(-_R + x), _R = \text{RootOf}(_Z^4 c e^4 + 4 * _Z^3 c d e^3 + c d^4 + b d^2 + (6 c d^2 e^2 + b e^2) * _Z^2 + (4 c d^3 e + 2 b d e) * _Z + a)) + \ln(e x + d) / a^2 / e / f$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 11.69, size = 13434, normalized size = 77.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)

[Out]
$$((b^2 - 2 a c + b c d^2) / (2 e (a b^2 - 4 a^2 c)) + (b c e x^2) / (2 (a b^2 - 4 a^2 c))) + (b c d x) / (a b^2 - 4 a^2 c) / (a f + x^2 (b e^2 f + 6 c d^2 e^2))$$

$$\begin{aligned}
& f) + x(4c*d^3*e*f + 2*b*d*e*f) + b*d^2*f + c*d^4*f + c*e^4*f*x^4 + 4*c*d* \\
& e^3*f*x^3) - (\log(\frac{((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - \\
& b^2)^3))^{(1/2)} - 1)*((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c \\
& - b^2)^3))^{(1/2)} - 1)*((2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10 \\
& *a*b*c))/(a*f*(4*a*c - b^2)) + (b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a*c - b^2)^2) \\
&)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^{(1/2)} - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 \\
& - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f) - (2* \\
& b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10* \\
& a*c - b^2))/(a*f*(4*a*c - b^2)))/(4*a^2*e*f) - (b*c^3*e^15*(4*b^3 - 20*a*c \\
& ^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17 \\
& *x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a* \\
& c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (b^3*c^5*e^16*x^2)/(a \\
& ^3*f^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(\\
& 4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3))*((b^3*c^5 \\
& *e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) - ((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a \\
& ^4*e^2*f^2*(4*a*c - b^2)^3))^{(1/2)} + 1)*((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a \\
& ^4*e^2*f^2*(4*a*c - b^2)^3))^{(1/2)} + 1)*((b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a \\
& *c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^{(1/2)} + 1)*(a*b + 3*b^2*d^2 + 3 \\
& *b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(\\
& a^2*f) - (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*f* \\
& (4*a*c - b^2)) + (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) + (4 \\
& *b*c^3*d*e^17*x*(10*a*c - b^2))/(a*f*(4*a*c - b^2)))/(4*a^2*e*f) - (b*c^3* \\
& e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2 \\
&)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b \\
& *c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (\\
& b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(4*a*c - b^2)^3) + (2*b^3*c^ \\
& 5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3))*((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a \\
& ^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f \\
& ^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + \log(d + e*x)/(a^2*e \\
& *f) + (b*atan((x^2*(((b*(6*a*c - b^2))*((6*a*b^5*c^4*e^17*f + 80*a^3*b*c^6* \\
& e^17*f - 44*a^2*b^3*c^5*e^17*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4* \\
& c*f^3 + 48*a^5*b^2*c^2*f^3) - (((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^ \\
& 18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - \\
& 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128 \\
& *a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^ \\
& 3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5 \\
& *e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3)))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12 \\
& *a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f \\
& ^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3 \\
& *c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 25 \\
& 6*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))/(4*a \\
& ^2*e*f*(4*a*c - b^2)^{(3/2)}) - (((b*((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^ \\
& 4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^ \\
& 3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - \\
& 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^1
\end{aligned}$$

$$\begin{aligned}
& 9*f^3 - 184*a^4*b^7*c^3*e^{19*f^3} + 1056*a^5*b^5*c^4*e^{19*f^3} - 2688*a^6*b^3*c^5*e^{19*f^3} \\
& *c^5*e^{19*f^3} + 2560*a^7*b*c^6*e^{19*f^3})/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 \\
& - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 \\
& + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(\\
& 4*a^2*e*f*(4*a*c - b^2)^{(3/2)}) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3* \\
& e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^{19*f^3} - 184*a \\
& ^4*b^7*c^3*e^{19*f^3} + 1056*a^5*b^5*c^4*e^{19*f^3} - 2688*a^6*b^3*c^5*e^{19*f^3} \\
& + 2560*a^7*b*c^6*e^{19*f^3}))/((8*a^2*e*f*(4*a*c - b^2)^{(3/2)}*(a^3*b^6*f^3 - \\
& 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 \\
& - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(\\
& 2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a \\
& ^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4 \\
& *c*e^2*f^2)) + (b^3*(6*a*c - b^2)^3*(12*a^3*b^9*c^2*e^{19*f^3} - 184*a^4*b^7* \\
& c^3*e^{19*f^3} + 1056*a^5*b^5*c^4*e^{19*f^3} - 2688*a^6*b^3*c^5*e^{19*f^3} + 2560 \\
& *a^7*b*c^6*e^{19*f^3}))/((64*a^6*e^3*f^3*(4*a*c - b^2)^{(9/2)}*(a^3*b^6*f^3 - 64 \\
& *a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(3*b^6 - 40*a^3*c^3 \\
& + 69*a^2*b^2*c^2 - 27*a*b^4*c))/((8*a^3*c^2*(4*a*c - b^2)^{(7/2)}*(6*b^6 - 40 \\
& 0*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b \\
& ^2*c)*(((6*a*b^5*c^4*e^{17*f} + 80*a^3*b*c^6*e^{17*f} - 44*a^2*b^3*c^5*e^{17*f}) \\
& /((a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (\\
& ((2*a^2*b^7*c^3*e^{18*f^2} - 36*a^3*b^5*c^4*e^{18*f^2} + 192*a^4*b^3*c^5*e^{18*f \\
& ^2} - 320*a^5*b*c^6*e^{18*f^2})/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f \\
& ^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e \\
& *f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^{19*f^3} - 184*a^4*b^7*c^3*e^{19*f^3} + \\
& 1056*a^5*b^5*c^4*e^{19*f^3} - 2688*a^6*b^3*c^5*e^{19*f^3} + 2560*a^7*b*c^6*e^{19 \\
& *f^3)))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2 \\
& *f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - \\
& 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - \\
& 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2 \\
& *c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^ \\
& 2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^ \\
& 2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (b^3*c^5*e^{16})/(a^3* \\
& b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (b*((b* \\
& ((2*a^2*b^7*c^3*e^{18*f^2} - 36*a^3*b^5*c^4*e^{18*f^2} + 192*a^4*b^3*c^5*e^{18*f \\
& ^2} - 320*a^5*b*c^6*e^{18*f^2})/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f \\
& ^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e \\
& *f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^{19*f^3} - 184*a^4*b^7*c^3*e^{19*f^3} + \\
& 1056*a^5*b^5*c^4*e^{19*f^3} - 2688*a^6*b^3*c^5*e^{19*f^3} + 2560*a^7*b*c^6*e^{19 \\
& *f^3)))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2 \\
& *f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - \\
& 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^{(3/2)}) + (b \\
& *(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4 \\
& *c*e*f)*(12*a^3*b^9*c^2*e^{19*f^3} - 184*a^4*b^7*c^3*e^{19*f^3} + 1056*a^5*b^5* \\
& c^4*e^{19*f^3} - 2688*a^6*b^3*c^5*e^{19*f^3} + 2560*a^7*b*c^6*e^{19*f^3}))/((8*a^2 \\
& *e*f*(4*a*c - b^2)^{(3/2)}*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 +
\end{aligned}$$

$$\begin{aligned}
& 48a^5b^2c^2f^3)(4a^2b^6e^2f^2 - 256a^5c^3e^2f^2 + 192a^4b^2 \\
& c^2e^2f^2 - 48a^3b^4c^2e^2f^2)))(6ac - b^2)/(4a^2ef(4ac - b \\
& ^2)^{(3/2)} + (b^2(6ac - b^2)^2(2b^6ef - 128a^3c^3ef + 96a^2b^2 \\
& c^2ef - 24ab^4cef))(12a^3b^9c^2e^{19}f^3 - 184a^4b^7c^3e^{19} \\
& f^3 + 1056a^5b^5c^4e^{19}f^3 - 2688a^6b^3c^5e^{19}f^3 + 2560a^7b^2c^6 \\
& e^{19}f^3))/(32a^4e^2f^2(4ac - b^2)^3(a^3b^6f^3 - 64a^6c^3f^3 \\
& - 12a^4b^4cf^3 + 48a^5b^2c^2f^3)(4a^2b^6e^2f^2 - 256a^5c^3e \\
& ^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4c^2e^2f^2)))/(8a^3c^2(4a \\
& ac - b^2)^3(6b^6 - 400a^3c^3 + 291a^2b^2c^2 - 72ab^4c)))(16a^6 \\
& b^6f^3(4ac - b^2)^{(9/2)} - 1024a^9c^3f^3(4ac - b^2)^{(9/2)} - 192a^ \\
& 7b^4cf^3(4ac - b^2)^{(9/2)} + 768a^8b^2c^2f^3(4ac - b^2)^{(9/2))) \\
& / (b^6c^2e^{14} - 12ab^4c^3e^{14} + 36a^2b^2c^4e^{14}) + (x((((b(6a \\
& ac - b^2)((2(320a^5b^2c^6de^{17}f^2 - 2a^2b^7c^3de^{17}f^2 + 36a^3 \\
& b^5c^4de^{17}f^2 - 192a^4b^3c^5de^{17}f^2))/(a^3b^6f^3 - 64a^6c^ \\
& ^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) - ((2b^6ef - 128a^3c^3 \\
& ef + 96a^2b^2c^2ef - 24ab^4cef))(2560a^7b^2c^6de^{18}f^3 + 12 \\
& a^3b^9c^2de^{18}f^3 - 184a^4b^7c^3de^{18}f^3 + 1056a^5b^5c^4de^{18} \\
& f^3 - 2688a^6b^3c^5de^{18}f^3)))/((a^3b^6f^3 - 64a^6c^3f^3 - 12 \\
& a^4b^4cf^3 + 48a^5b^2c^2f^3)(4a^2b^6e^2f^2 - 256a^5c^3e^2f^ \\
& ^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4c^2e^2f^2)))/((4a^2ef(4ac - \\
& b^2)^{(3/2)} - (b(6ac - b^2)(2b^6ef - 128a^3c^3ef + 96a^2b^2c^ \\
& ^2ef - 24ab^4cef))(2560a^7b^2c^6de^{18}f^3 + 12a^3b^9c^2de^{18} \\
& f^3 - 184a^4b^7c^3de^{18}f^3 + 1056a^5b^5c^4de^{18}f^3 - 2688a^6b \\
& ^3c^5de^{18}f^3))/(4a^2ef(4ac - b^2)^{(3/2)}(a^3b^6f^3 - 64a^6c^ \\
& ^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3)(4a^2b^6e^2f^2 - 256a^5 \\
& c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4c^2e^2f^2)))(2b^6ef \\
& - 128a^3c^3ef + 96a^2b^2c^2ef - 24ab^4cef))/(2(4a^2b^6e^ \\
& ^2f^2 - 256a^5c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4c^2e^2f^ \\
& ^2)) + (b((2(6ab^5c^4de^{16}f - 44a^2b^3c^5de^{16}f + 80a^3b^2c^6 \\
& de^{16}f))/(a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^ \\
& ^2f^3) + (((2(320a^5b^2c^6de^{17}f^2 - 2a^2b^7c^3de^{17}f^2 + 36a^ \\
& 3b^5c^4de^{17}f^2 - 192a^4b^3c^5de^{17}f^2))/(a^3b^6f^3 - 64a^6c^ \\
& ^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) - ((2b^6ef - 128a^3c^3 \\
& ef + 96a^2b^2c^2ef - 24ab^4cef))(2560a^7b^2c^6de^{18}f^3 + 12 \\
& a^3b^9c^2de^{18}f^3 - 184a^4b^7c^3de^{18}f^3 + 1056a^5b^5c^4de^{18} \\
& f^3 - 2688a^6b^3c^5de^{18}f^3)))/((a^3b^6f^3 - 64a^6c^3f^3 - 12 \\
& a^4b^4cf^3 + 48a^5b^2c^2f^3)(4a^2b^6e^2f^2 - 256a^5c^3e^2f^ \\
& ^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4c^2e^2f^2)))(2b^6ef - 128a^3 \\
& c^3ef + 96a^2b^2c^2ef - 24ab^4cef))/(2(4a^2b^6e^2f^2 - 25 \\
& 6a^5c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4c^2e^2f^2)))(6ac \\
& - b^2)/(4a^2ef(4ac - b^2)^{(3/2)} + (b^3(6ac - b^2)^3(2560a^7 \\
& b^2c^6de^{18}f^3 + 12a^3b^9c^2de^{18}f^3 - 184a^4b^7c^3de^{18}f^3 + \\
& 1056a^5b^5c^4de^{18}f^3 - 2688a^6b^3c^5de^{18}f^3))/(32a^6e^3f^ \\
& ^3(4ac - b^2)^{(9/2)}(a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48 \\
& a^5b^2c^2f^3)))(3b^6 - 40a^3c^3 + 69a^2b^2c^2 - 27ab^4c))/(8
\end{aligned}$$

$$\begin{aligned}
& a^3c^2(4ac - b^2)^{7/2}(6b^6 - 400a^3c^3 + 291a^2b^2c^2 - 72ab^4c) + (3b(b^4 + 11a^2c^2 - 7ab^2c) * (((2(6ab^5c^4de^{16f} - 44a^2b^3c^5de^{16f} + 80a^3b^3c^6de^{16f}))/ (a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) + (((2(320a^5b^3c^6de^{17f^2} - 2a^2b^7c^3de^{17f^2} + 36a^3b^5c^4de^{17f^2} - 192a^4b^3c^5de^{17f^2}))/ (a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) - ((2b^6ef - 128a^3c^3ef + 96a^2b^2c^2ef - 24ab^4cef) * (2560a^7b^3c^6de^{18f^3} + 12a^3b^9c^2de^{18f^3} - 184a^4b^7c^3de^{18f^3} + 1056a^5b^5c^4de^{18f^3} - 2688a^6b^3c^5de^{18f^3}))) / ((a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) * (4a^2b^6e^2f^2 - 256a^5c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4cef^2))) * (2b^6ef - 128a^3c^3ef + 96a^2b^2c^2ef - 24ab^4cef) / (2(4a^2b^6e^2f^2 - 256a^5c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4cef^2))) * (2b^6ef - 128a^3c^3ef + 96a^2b^2c^2ef - 24ab^4cef) / (2(4a^2b^6e^2f^2 - 256a^5c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4cef^2))) - (2b^3c^5de^{15}) / (a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) - (b(6ac - b^2) * ((b(6ac - b^2) * ((2(320a^5b^3c^6de^{17f^2} - 2a^2b^7c^3de^{17f^2} + 36a^3b^5c^4de^{17f^2} - 192a^4b^3c^5de^{17f^2}))/ (a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) - ((2b^6ef - 128a^3c^3ef + 96a^2b^2c^2ef - 24ab^4cef) * (2560a^7b^3c^6de^{18f^3} + 12a^3b^9c^2de^{18f^3} - 184a^4b^7c^3de^{18f^3} + 1056a^5b^5c^4de^{18f^3} - 2688a^6b^3c^5de^{18f^3}))) / ((a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) * (4a^2b^6e^2f^2 - 256a^5c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4cef^2)))))) / (4a^2ef(4ac - b^2)^{3/2}) - (b(6ac - b^2) * (2b^6ef - 128a^3c^3ef + 96a^2b^2c^2ef - 24ab^4cef) * (2560a^7b^3c^6de^{18f^3} + 12a^3b^9c^2de^{18f^3} - 184a^4b^7c^3de^{18f^3} + 1056a^5b^5c^4de^{18f^3} - 2688a^6b^3c^5de^{18f^3}))) / (4a^2ef(4ac - b^2)^{3/2}) * (a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) * (4a^2b^6e^2f^2 - 256a^5c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4cef^2))) / (4a^2ef(4ac - b^2)^{3/2}) + (b^2(6ac - b^2)^2(2b^6ef - 128a^3c^3ef + 96a^2b^2c^2ef - 24ab^4cef) * (2560a^7b^3c^6de^{18f^3} + 12a^3b^9c^2de^{18f^3} - 184a^4b^7c^3de^{18f^3} + 1056a^5b^5c^4de^{18f^3} - 2688a^6b^3c^5de^{18f^3}))) / (16a^4e^2f^2(4ac - b^2)^3(a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) * (4a^2b^6e^2f^2 - 256a^5c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4cef^2))) / (8a^3c^2(4ac - b^2)^3(6b^6 - 400a^3c^3 + 291a^2b^2c^2 - 72ab^4c)) * (16a^6b^6f^3(4ac - b^2)^{9/2} - 1024a^9c^3f^3(4ac - b^2)^{9/2} - 192a^7b^4cf^3(4ac - b^2)^{9/2} + 768a^8b^2c^2f^3(4ac - b^2)^{9/2}) / (b^6c^2e^{14} - 12ab^4c^3e^{14} + 36a^2b^2c^4e^{14}) + (((b((4ab^6c^3e^{15f} - 33a^2b^4c^4e^{15f} + 68a^3b^2c^5e^{15f} + 6ab^5c^4d^2e^{15f} + 80a^3b^3c^6d^2e^{15f} - 44a^2b^3c^5d^2e^{15f}) / (a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4cf^3 + 48a^5b^2c^2f^3) - (((4a^2b^8c^2e^{16f^2} - 52a^3b^6c^3e^{16f^2}
\end{aligned}$$

$$\begin{aligned}
& 92*a^7*b^4*c*f^3*(4*a*c - b^2)^{(9/2)} + 768*a^8*b^2*c^2*f^3*(4*a*c - b^2)^{(9/2)} \\
& *(((4*a*b^6*c^3*e^{15*f} - 33*a^2*b^4*c^4*e^{15*f} + 68*a^3*b^2*c^5*e^{15*f} \\
& + 6*a*b^5*c^4*d^2*e^{15*f} + 80*a^3*b*c^6*d^2*e^{15*f} - 44*a^2*b^3*c^5*d^2*e^{15*f}) \\
& / (a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) \\
&) - (((4*a^2*b^8*c^2*e^{16*f^2} - 52*a^3*b^6*c^3*e^{16*f^2} + 224*a^4*b^4*c^4*e^{16*f^2} \\
& - 320*a^5*b^2*c^5*e^{16*f^2} - 320*a^5*b*c^6*d^2*e^{16*f^2} + 2*a^2*b^7*c^3*d^2*e^{16*f^2} \\
& - 36*a^3*b^5*c^4*d^2*e^{16*f^2} + 192*a^4*b^3*c^5*d^2*e^{16*f^2}) \\
& / (a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) \\
& + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^{17*f^3} \\
& - 48*a^5*b^6*c^3*e^{17*f^3} + 192*a^6*b^4*c^4*e^{17*f^3} - 256*a^7*b^2*c^5*e^{17*f^3} \\
& + 2560*a^7*b*c^6*d^2*e^{17*f^3} + 12*a^3*b^9*c^2*d^2*e^{17*f^3} - 184*a^4*b^7*c^3*d^2*e^{17*f^3} \\
& + 1056*a^5*b^5*c^4*d^2*e^{17*f^3} - 2688*a^6*b^3*c^5*d^2*e^{17*f^3}))/ (2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 \\
& - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 \\
& - 48*a^3*b^4*c*e^2*f^2))) * (2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f) \\
& / (2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))) * (2*b^6*e*f \\
& - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f) / (2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 \\
& + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) - (b^4*c^4*e^{14} - 4*a*b^2*c^5*e^{14} + b^3*c^5*d^2*e^{14}) \\
& / (a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + (b*((b*(6*a*c - b^2) \\
& *((4*a^2*b^8*c^2*e^{16*f^2} - 52*a^3*b^6*c^3*e^{16*f^2} + 224*a^4*b^4*c^4*e^{16*f^2} - 320*a^5*b^2*c^5*e^{16*f^2} \\
& - 320*a^5*b*c^6*d^2*e^{16*f^2} + 2*a^2*b^7*c^3*d^2*e^{16*f^2} - 36*a^3*b^5*c^4*d^2*e^{16*f^2} \\
& + 192*a^4*b^3*c^5*d^2*e^{16*f^2}) / (a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) \\
& + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(4*a^4*b^8*c^2*e^{17*f^3} \\
& - 48*a^5*b^6*c^3*e^{17*f^3} + 192*a^6*b^4*c^4*e^{17*f^3} - 256*a^7*b^2*c^5*e^{17*f^3} \\
& + 2560*a^7*b*c^6*d^2*e^{17*f^3} + 12*a^3*b^9*c^2*d^2*e^{17*f^3} - 184*a^4*b^7*c^3*d^2*e^{17*f^3} \\
& + 1056*a^5*b^5*c^4*d^2*e^{17*f^3} - 2688*a^6*b^3*c^5*d^2*e^{17*f^3}))/ (2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 \\
& - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 \\
& - 48*a^3*b^4*c*e^2*f^2)))) / (4*a^2*e*f*(4*a*c - b^2)^{(3/2)}) + (b*(6*a*c - b^2) * (2*b^6*e*f \\
& - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f) * (4*a^4*b^8*c^2*e^{17*f^3} \\
& - 48*a^5*b^6*c^3*e^{17*f^3} + 192*a^6*b^4*c^4*e^{17*f^3} - 256*a^7*b^2*c^5*e^{17*f^3} \\
& + 2560*a^7*b*c^6*d^2*e^{17*f^3} + 12*a^3*b^9*c^2*d^2*e^{17*f^3} - 184*a^4*b^7*c^3*d^2*e^{17*f^3} \\
& + 1056*a^5*b^5*c^4*d^2*e^{17*f^3} - 2688*a^6*b^3*c^5*d^2*e^{17*f^3}))/ (8*a^2*e*f*(4*a*c - b^2)^{(3/2)} \\
& * (a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) * (4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 \\
& + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))) * (6*a*c - b^2) / (4*a^2*e*f*(4*a*c - b^2)^{(3/2)}) + (b^2 \\
& * (6*a*c - b^2)^2 * (2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f) * (4*a^4*b^8*c^2*e^{17*f^3} \\
& - 48*a^5*b^6*c^3*e^{17*f^3} + 192*a^6*b^4*c^4*e^{17*f^3} - 256*a^7*b^2*c^5*e^{17*f^3} \\
& + 2560*a^7*b*c^6*d^2*e^{17*f^3} + 12*a^3*b^9*c^2*d^2*e^{17*f^3} - 184*a^4*b^7*c^3*d^2*e^{17*f^3} \\
& + 1056*a^5*b^5*c^4*d^2*e^{17*f^3} - 2688*a^6*b^3*c^5*d^2*e^{17*f^3}))/ (32*a^4*e^2*f^2*(4*a*c - b^2)^{(3/2)})
\end{aligned}$$

$$2)^3(a^3b^6f^3 - 64a^6c^3f^3 - 12a^4b^4c^3f^3 + 48a^5b^2c^2f^3) \\ * (4a^2b^6e^2f^2 - 256a^5c^3e^2f^2 + 192a^4b^2c^2e^2f^2 - 48a^3b^4c^3e^2f^2) \\ / (8a^3c^2(4ac - b^2)^3(b^6c^2e^{14} - 12ab^4c^3e^{14} + 36a^2b^2c^4e^{14}) \\ * (6b^6 - 400a^3c^3 + 291a^2b^2c^2 - 72ab^4c)) * (6ac - b^2) / (2a^2ef(4ac - b^2)^{3/2})$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out

$$3.538 \quad \int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=360

$$\frac{3b^2 - 10ac}{2a^2ef^2(b^2 - 4ac)(d + ex)} - \frac{\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2ef^2 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(- (3b^2 - 10ac) \sqrt{b^2 - 4ac} + 16abc - 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2ef^2 (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 1.60, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{3b^2 - 10ac}{2a^2ef^2(b^2 - 4ac)(d + ex)} - \frac{\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2ef^2 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(- (3b^2 - 10ac) \sqrt{b^2 - 4ac} + 16abc - 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2ef^2 (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{-2ac + b^2 + bc(d + ex)^2}{2aef^2(b^2 - 4ac)(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] $-(3b^2 - 10ac)/(2a^2(b^2 - 4ac)*ef^2(d + ex)) + (b^2 - 2ac + bc(d + ex)^2)/(2a(b^2 - 4ac)*ef^2(d + ex)*(a + b(d + ex)^2 + c(d + ex)^4)) - (\text{Sqrt}[c]*(3b^3 - 16abc + (3b^2 - 10ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + ex))/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]]])/(2*\text{Sqrt}[2]*a^2*(b^2 - 4ac)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]]*ef^2) + (\text{Sqrt}[c]*(3b^3 - 16abc - (3b^2 - 10ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + ex))/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]]])/(2*\text{Sqrt}[2]*a^2*(b^2 - 4ac)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]]*ef^2)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1121

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[((d*x)^(m + 1)*(b^2 - 2ac + bc*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2a*d*(p + 1)*(b^2 - 4ac)), x] + Dist[1/(2a*(p + 1)*(b^2 - 4ac)), Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2ac*(m + 4*p + 5) + bc*(m + 4*p + 7)*x^2, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4ac, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(df + efx)^2 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx &= \frac{\text{Subst} \left(\int \frac{1}{x^2(a+bx^2+cx^4)^2} dx, x, d + ex \right)}{ef^2} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^2(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{\dots}{\dots} \\
&= -\frac{3b^2 - 10ac}{2a^2(b^2 - 4ac)ef^2(d + ex)} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^2(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{3b^2 - 10ac}{2a^2(b^2 - 4ac)ef^2(d + ex)} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^2(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{3b^2 - 10ac}{2a^2(b^2 - 4ac)ef^2(d + ex)} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^2(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)}
\end{aligned}$$

Mathematica [A] time = 1.47, size = 342, normalized size = 0.95

$$\frac{2(d+ex)(-3abc-2ac^2(d+ex)^2+b^3+c(d+ex)^2)}{(4ac-b^2)(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{4}{d+ex}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]

[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])/(4*a^2*e*f^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)^2 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

fricas [B] time = 1.76, size = 4520, normalized size = 12.56

result too large to display

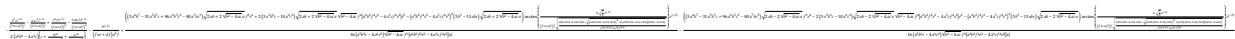
Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(2*(3*b^2*c - 10*a*c^2)*e^4*x^4 + 8*(3*b^2*c - 10*a*c^2)*d*e^3*x^3 + 2 \\ & *(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^2) \\ & *e^2*x^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c - 10*a*c^2)*d^3 \\ & + (3*b^3 - 11*a*b*c)*d)*e*x + \sqrt{1/2}*((a^2*b^2*c - 4*a^3*c^2)*e^6*f^2*x^5 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*e^5*f^2*x^4 \\ & + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*e^4*f^2*x^3 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 \\ & + 3*(a^2*b^3 - 4*a^3*b*c)*d)*e^3*f^2*x^2 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 \\ & + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*e^2*f^2*x + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 \\ & + (a^3*b^2 - 4*a^4*c)*d)*e*f^2)*\sqrt{-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)} \\ & /((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4)} \\ & *\log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*e*x - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*\sqrt{1/2}*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*e^3*f^6*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)} \\ & /((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e*f^2)*\sqrt{-((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)} \\ & /((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*e^4*f^8)) + 9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*e^2*f^4)} \end{aligned}$$

$$\begin{aligned}
& 5f^2x^4 + (a^2b^3 - 4a^3bc + 10(a^2b^2c - 4a^3c^2)d^2)e^4f^2x^3 + (10(a^2b^2c - 4a^3c^2)d^3 + 3(a^2b^3 - 4a^3bc)d)e^3f^2x^2 + (a^3b^2 - 4a^4c + 5(a^2b^2c - 4a^3c^2)d^4 + 3(a^2b^3 - 4a^3bc)d^2)e^2f^2x + ((a^2b^2c - 4a^3c^2)d^5 + (a^2b^3 - 4a^3bc)d^3 + (a^3b^2 - 4a^4c)d)e^2f^2x + ((a^2b^2c - 4a^3c^2)d^5 + (a^2b^3 - 4a^3bc)d^3 + (a^3b^2 - 4a^4c)d)e^2f^2x + ((a^2b^2c - 4a^3c^2)d^5 + (a^2b^3 - 4a^3bc)d^3 + (a^3b^2 - 4a^4c)d)e^2f^2x \\
& + ((a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)e^2f^4\sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / ((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)e^4f^8)) - 9b^7 + 105ab^5c - 385a^2b^3c^2 + 420a^3b^2c^3 / ((a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)e^2f^4)) * \log(- (189b^6c^3 - 1971ab^4c^4 + 5625a^2b^2c^5 - 2500a^3c^6) * e^x - (189b^6c^3 - 1971ab^4c^4 + 5625a^2b^2c^5 - 2500a^3c^6) * d - 1/2\sqrt{1/2} * ((3a^5b^{10} - 55a^6b^8c + 392a^7b^6c^2 - 1344a^8b^4c^3 + 2176a^9b^2c^4 - 1280a^{10}c^5) * e^3f^6\sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / ((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)e^4f^8)) + (27b^{11} - 486ab^9c + 3330a^2b^7c^2 - 10549a^3b^5c^3 + 14408a^4b^3c^4 - 5200a^5b^2c^5) * e^2f^2\sqrt{(a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)e^2f^4\sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / ((a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)e^4f^8)) - 9b^7 + 105ab^5c - 385a^2b^3c^2 + 420a^3b^2c^3 / ((a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)e^2f^4)) / ((a^2b^2c - 4a^3c^2) * e^6f^2x^5 + 5(a^2b^2c - 4a^3c^2)d * e^5f^2x^4 + (a^2b^3 - 4a^3bc + 10(a^2b^2c - 4a^3c^2)d^2) * e^4f^2x^3 + (10(a^2b^2c - 4a^3c^2)d^3 + 3(a^2b^3 - 4a^3bc)d) * e^3f^2x^2 + (a^3b^2 - 4a^4c + 5(a^2b^2c - 4a^3c^2)d^4 + 3(a^2b^3 - 4a^3bc)d^2) * e^2f^2x + ((a^2b^2c - 4a^3c^2)d^5 + (a^2b^3 - 4a^3bc)d^3 + (a^3b^2 - 4a^4c)d) * e^2f^2)
\end{aligned}$$

giac [B] time = 1.02, size = 999, normalized size = 2.78



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/2*(b^2*c*e^{-1})/((f*x*e + d*f)*f) - 2*a*c^2*e^{-1}/((f*x*e + d*f)*f) + b^3*f*e^{-1}/(f*x*e + d*f)^3 - 3*a*b*c*f*e^{-1}/(f*x*e + d*f)^3 / ((a^2*b^2 - 4a^3c)*(c + b*f^2/(f*x*e + d*f)^2 + a*f^4/(f*x*e + d*f)^4)) - e^{-1}/((f*x*e + d*f)*a^2*f) + 1/16*((3a^4b^7 - 31a^5b^5c + 96a^6b^3c^2 - 80a^7b^2c^3)*\sqrt{2*a*b + 2*\sqrt{b^2 - 4*a*c}*a}*f^8*e^4 + 2*(3a^3b^2c - 10a^4c^2)*\sqrt{2*a*b + 2*\sqrt{b^2 - 4*a*c}*a}*\sqrt{b^2 - 4*a*c}*f^4*abs(a^2*b^2*f^4*e^2 - 4a^3c*f^4*e^2) * e^2 - (a^2*b^2*f^4*e^2 - 4a^3c*f^4*e^2)^2*(3b^3 - 13a*b*c)*\sqrt{2*a*b + 2*\sqrt{b^2 - 4*a*c}*a})*\arctan(2*\sqrt{1/2}) * e^{-1} / ((f*x*e + d*f)*f*\sqrt{(a^2*b^3*f^4*e^2 - 4a^3b*c*f^4*e^2 + \sqrt{
\end{aligned}$$

$$\frac{(a^2 b^3 f^4 e^2 - 4 a^3 b^2 c f^4 e^2)^2 - 4(a^3 b^2 f^8 e^4 - 4 a^4 c f^8 e^4) e^{-3}}{(a^2 b^2 c - 4 a^3 c^2)} \frac{1}{(a^3 b^2 f^8 e^4 - 4 a^4 c f^8 e^4)} e^{-3} \frac{1}{((a^5 b^2 c - 4 a^6 c^2) \sqrt{b^2 - 4 a c} f^6 \text{abs}(a^2 b^2 f^4 e^2 - 4 a^3 c f^4 e^2) \text{abs}(a)) - 1/16((3 a^4 b^7 - 31 a^5 b^5 c + 96 a^6 b^3 c^2 - 80 a^7 b c^3) \sqrt{2 a b - 2 \sqrt{b^2 - 4 a c}} a) f^8 e^4 - 2(3 a^3 b^2 c - 10 a^4 c^2) \sqrt{2 a b - 2 \sqrt{b^2 - 4 a c}} a) \sqrt{b^2 - 4 a c} f^4 \text{abs}(a^2 b^2 f^4 e^2 - 4 a^3 c f^4 e^2) e^2 - (a^2 b^2 f^4 e^2 - 4 a^3 c f^4 e^2)^2 (3 b^3 - 13 a b c) \sqrt{2 a b - 2 \sqrt{b^2 - 4 a c}} a)} \arctan(2 \sqrt{1/2} e^{-1}) / ((f x e + d f) f \sqrt{(a^2 b^3 f^4 e^2 - 4 a^3 b^2 c f^4 e^2 - \sqrt{(a^2 b^3 f^4 e^2 - 4 a^3 b^2 c f^4 e^2)^2 - 4(a^3 b^2 f^8 e^4 - 4 a^4 c f^8 e^4)} (a^2 b^2 c - 4 a^3 c^2)) / (a^3 b^2 f^8 e^4 - 4 a^4 c f^8 e^4))} e^{-3} / ((a^5 b^2 c - 4 a^6 c^2) \sqrt{b^2 - 4 a c} f^6 \text{abs}(a^2 b^2 f^4 e^2 - 4 a^3 c f^4 e^2) \text{abs}(a))$$

maple [C] time = 0.03, size = 1346, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e f x + d f)^2 / (a + b(e x + d)^2 + c(e x + d)^4)^2, x)$

[Out]
$$\begin{aligned} & -1/f^2/a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) c^2 e^2 / (4 a c - b^2) x^3 + 1/2/f^2/a^2 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) c e^2 / (4 a c - b^2) x^3 b^2 - 3/f^2/a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) d c^2 e / (4 a c - b^2) x^2 + 3/2/f^2/a^2 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) d c e / (4 a c - b^2) x^2 b^2 - 3/f^2/a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) / (4 a c - b^2) x c^2 d^2 + 3/2/f^2/a^2 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) / (4 a c - b^2) x b^2 c d^2 - 3/2/f^2/a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) / (4 a c - b^2) x b^3 - 1/f^2/a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) d^3 e / (4 a c - b^2) c^2 + 1/2/f^2/a^2 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) d^3 e / (4 a c - b^2) b^2 c - 3/2/f^2/a / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) d e / (4 a c - b^2) b c + 1/2/f^2/a^2 / (c e^4 x^4 + 4 c d e^3 x^3 + 6 c d^2 e^2 x^2 + 4 c d^3 e x + b e^2 x^2 + c d^4 + 2 b d e x + b d^2 + a) d e / (4 a c - b^2) b^3 - 1/4/f^2/a^2 / (4 a c - b^2) e \text{sum}(((10 a c - 3 b^2) * _R^2 c e^2 + 10 a c^2 d^2 - 3 b^2 c d^2 + 2(10 a c - 3 b^2) * _R c d e + 13 a b c - 3 b^3) / (2 * _R^3 c e^3 + 6 * _R^2 c d e^2 + 6 * _R c d^2 e + 2 c d^3 + _R b e + b d) * \ln(-_R + x), _R = \text{RootOf}(_Z^4 c e^4 + 4 * _Z^3 c d e^3 + c d^4 + b d^2 + (6 c d^2 e^2 + b e^2) * _Z^2 + (4 c d^3 e + 2 b d e) * _Z + a)) - 1/f^2/a^2 e / (e x + d) \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 7.29, size = 12008, normalized size = 33.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)

[Out] - atan(((9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 1261568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*

$$\begin{aligned}
& c^7 e^{12} f^6 - 458752 a^{11} b^2 c^8 e^{12} f^6) + 204800 a^{12} c^9 d e^{11} f^6 + \\
& 144 a^6 b^{12} c^3 d e^{11} f^6 - 3264 a^7 b^{10} c^4 d e^{11} f^6 + 30112 a^8 b^8 \\
& c^5 d e^{11} f^6 - 143360 a^9 b^6 c^6 d e^{11} f^6 + 365568 a^{10} b^4 c^7 d e^{11} \\
& f^6 - 458752 a^{11} b^2 c^8 d e^{11} f^6) * i + (- (9 b^{13} - 9 b^4 (- (4 a c - b \\
& ^2)^9)^{(1/2)} + 26880 a^6 b^6 c^6 + 2077 a^2 b^9 c^2 - 10656 a^3 b^7 c^3 + 302 \\
& 40 a^4 b^5 c^4 - 44800 a^5 b^3 c^5 - 25 a^2 c^2 (- (4 a c - b^2)^9)^{(1/2)} - \\
& 213 a b^{11} c + 51 a b^2 c (- (4 a c - b^2)^9)^{(1/2)}) / (32 (a^5 b^{12} e^{2} f^4 + \\
& 4096 a^{11} c^6 e^2 f^4 + 240 a^7 b^8 c^2 e^2 f^4 - 1280 a^8 b^6 c^3 e^2 f^4 \\
& + 3840 a^9 b^4 c^4 e^2 f^4 - 6144 a^{10} b^2 c^5 e^2 f^4 - 24 a^6 b^{10} c^3 e^2 \\
& f^4))^{(1/2)} * ((- (9 b^{13} - 9 b^4 (- (4 a c - b^2)^9)^{(1/2)} + 26880 a^6 b^6 c^6 \\
& + 2077 a^2 b^9 c^2 - 10656 a^3 b^7 c^3 + 30240 a^4 b^5 c^4 - 44800 a^5 b^3 \\
& c^5 - 25 a^2 c^2 (- (4 a c - b^2)^9)^{(1/2)} - 213 a b^{11} c + 51 a b^2 c (- (4 \\
& a c - b^2)^9)^{(1/2)}) / (32 (a^5 b^{12} e^{2} f^4 + 4096 a^{11} c^6 e^2 f^4 + 240 a \\
& ^7 b^8 c^2 e^2 f^4 - 1280 a^8 b^6 c^3 e^2 f^4 + 3840 a^9 b^4 c^4 e^2 f^4 - \\
& 6144 a^{10} b^2 c^5 e^2 f^4 - 24 a^6 b^{10} c^3 e^2 f^4))^{(1/2)} * ((- (9 b^{13} - 9 b \\
& ^4 (- (4 a c - b^2)^9)^{(1/2)} + 26880 a^6 b^6 c^6 + 2077 a^2 b^9 c^2 - 10656 a^ \\
& 3 b^7 c^3 + 30240 a^4 b^5 c^4 - 44800 a^5 b^3 c^5 - 25 a^2 c^2 (- (4 a c - b \\
& ^2)^9)^{(1/2)} - 213 a b^{11} c + 51 a b^2 c (- (4 a c - b^2)^9)^{(1/2)}) / (32 (a^5 \\
& b^{12} e^{2} f^4 + 4096 a^{11} c^6 e^2 f^4 + 240 a^7 b^8 c^2 e^2 f^4 - 1280 a^8 b^6 \\
& c^3 e^2 f^4 + 3840 a^9 b^4 c^4 e^2 f^4 - 6144 a^{10} b^2 c^5 e^2 f^4 - 24 \\
& a^6 b^{10} c^3 e^2 f^4))^{(1/2)} * (x * (256 a^{10} b^{13} c^2 e^{14} f^{10} - 6144 a^{11} b^ \\
& 11 c^3 e^{14} f^{10} + 61440 a^{12} b^9 c^4 e^{14} f^{10} - 327680 a^{13} b^7 c^5 e^{14} \\
& f^{10} + 983040 a^{14} b^5 c^6 e^{14} f^{10} - 1572864 a^{15} b^3 c^7 e^{14} f^{10} + 104 \\
& 8576 a^{16} b^3 c^8 e^{14} f^{10}) + 1048576 a^{16} b^3 c^8 d e^{13} f^{10} + 256 a^{10} b^{13} \\
& c^2 d e^{13} f^{10} - 6144 a^{11} b^{11} c^3 d e^{13} f^{10} + 61440 a^{12} b^9 c^4 d e^{13} \\
& f^{10} - 327680 a^{13} b^7 c^5 d e^{13} f^{10} + 983040 a^{14} b^5 c^6 d e^{13} f^{10} \\
& - 1572864 a^{15} b^3 c^7 d e^{13} f^{10}) + 192 a^8 b^{13} c^2 e^{12} f^8 - 4672 a^9 \\
& b^{11} c^3 e^{12} f^8 + 47360 a^{10} b^9 c^4 e^{12} f^8 - 256000 a^{11} b^7 c^5 e^{12} \\
& f^8 + 778240 a^{12} b^5 c^6 e^{12} f^8 - 1261568 a^{13} b^3 c^7 e^{12} f^8 + 85196 \\
& 8 a^{14} b^3 c^8 e^{12} f^8) + x * (204800 a^{12} c^9 e^{12} f^6 + 144 a^6 b^{12} c^3 e^{12} \\
& f^6 - 3264 a^7 b^{10} c^4 e^{12} f^6 + 30112 a^8 b^8 c^5 e^{12} f^6 - 143360 a^9 \\
& b^6 c^6 e^{12} f^6 + 365568 a^{10} b^4 c^7 e^{12} f^6 - 458752 a^{11} b^2 c^8 e^{12} \\
& f^6) + 204800 a^{12} c^9 d e^{11} f^6 + 144 a^6 b^{12} c^3 d e^{11} f^6 - 3264 a^7 \\
& b^{10} c^4 d e^{11} f^6 + 30112 a^8 b^8 c^5 d e^{11} f^6 - 143360 a^9 b^6 c^6 d \\
& e^{11} f^6 + 365568 a^{10} b^4 c^7 d e^{11} f^6 - 458752 a^{11} b^2 c^8 d e^{11} f^6 \\
&) * i) / ((- (9 b^{13} - 9 b^4 (- (4 a c - b^2)^9)^{(1/2)} + 26880 a^6 b^6 c^6 + 2077 \\
& a^2 b^9 c^2 - 10656 a^3 b^7 c^3 + 30240 a^4 b^5 c^4 - 44800 a^5 b^3 c^5 - 2 \\
& 5 a^2 c^2 (- (4 a c - b^2)^9)^{(1/2)} - 213 a b^{11} c + 51 a b^2 c (- (4 a c - b \\
& ^2)^9)^{(1/2)}) / (32 (a^5 b^{12} e^{2} f^4 + 4096 a^{11} c^6 e^2 f^4 + 240 a^7 b^8 c^2 \\
& e^2 f^4 - 1280 a^8 b^6 c^3 e^2 f^4 + 3840 a^9 b^4 c^4 e^2 f^4 - 6144 a^{10} b^2 c^5 \\
& e^2 f^4 - 24 a^6 b^{10} c^3 e^2 f^4))^{(1/2)} * ((- (9 b^{13} - 9 b^4 (- (4 a \\
& c - b^2)^9)^{(1/2)} + 26880 a^6 b^6 c^6 + 2077 a^2 b^9 c^2 - 10656 a^3 b^7 c^ \\
& 3 + 30240 a^4 b^5 c^4 - 44800 a^5 b^3 c^5 - 25 a^2 c^2 (- (4 a c - b^2)^9)^{(\\
& 1/2)} - 213 a b^{11} c + 51 a b^2 c (- (4 a c - b^2)^9)^{(1/2)}) / (32 (a^5 b^{12} e^ \\
& 2 f^4 + 4096 a^{11} c^6 e^2 f^4 + 240 a^7 b^8 c^2 e^2 f^4 - 1280 a^8 b^6 c^3 e^2
\end{aligned}$$

$$\begin{aligned}
& e^{2f^4} + 3840a^9b^4c^4e^{2f^4} - 6144a^{10}b^2c^5e^{2f^4} - 24a^6b^1 \\
& 0c^6e^{2f^4}))^{(1/2)} * ((-9b^{13} - 9b^4(-4ac - b^2)^9)^{(1/2)} + 26880a^6b^1 \\
& 6b^1c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 \\
& - 25a^2c^2(-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c + 51ab^2 \\
& *c(-4ac - b^2)^9)^{(1/2)}) / (32(a^5b^{12}e^{2f^4} + 4096a^{11}c^6e^{2f^4} \\
& + 240a^7b^8c^2e^{2f^4} - 1280a^8b^6c^3e^{2f^4} + 3840a^9b^4c^4e^{2f^4} \\
& *f^4 - 6144a^{10}b^2c^5e^{2f^4} - 24a^6b^{10}c^6e^{2f^4}))^{(1/2)} * (x(256a \\
& ^{10}b^{13}c^2e^{14f^{10}} - 6144a^{11}b^{11}c^3e^{14f^{10}} + 61440a^{12}b^9c^4e^{14f^{10}} \\
& e^{14f^{10}} - 327680a^{13}b^7c^5e^{14f^{10}} + 983040a^{14}b^5c^6e^{14f^{10}} - \\
& 1572864a^{15}b^3c^7e^{14f^{10}} + 1048576a^{16}b^1c^8e^{14f^{10}}) + 1048576a \\
& ^{16}b^1c^8d^13f^{10} + 256a^{10}b^{13}c^2d^13f^{10} - 6144a^{11}b^{11}c^3 \\
& d^13f^{10} + 61440a^{12}b^9c^4d^13f^{10} - 327680a^{13}b^7c^5d^13f^{10} \\
& ^{10} + 983040a^{14}b^5c^6d^13f^{10} - 1572864a^{15}b^3c^7d^13f^{10}) + \\
& 192a^8b^{13}c^2e^{12f^8} - 4672a^9b^{11}c^3e^{12f^8} + 47360a^{10}b^9c^4e^{12f^8} \\
& - 256000a^{11}b^7c^5e^{12f^8} + 778240a^{12}b^5c^6e^{12f^8} - \\
& 1261568a^{13}b^3c^7e^{12f^8} + 851968a^{14}b^1c^8e^{12f^8}) + x(204800a^1 \\
& 2c^9e^{12f^6} + 144a^6b^{12}c^3e^{12f^6} - 3264a^7b^{10}c^4e^{12f^6} + 3 \\
& 0112a^8b^8c^5e^{12f^6} - 143360a^9b^6c^6e^{12f^6} + 365568a^{10}b^4c^7e^{12f^6} \\
& - 458752a^{11}b^2c^8e^{12f^6}) + 204800a^{12}c^9d^11f^6 + \\
& 144a^6b^{12}c^3d^11f^6 - 3264a^7b^{10}c^4d^11f^6 + 30112a^8b^8c^5 \\
& d^11f^6 - 143360a^9b^6c^6d^11f^6 + 365568a^{10}b^4c^7d^11 \\
& *f^6 - 458752a^{11}b^2c^8d^11f^6) - ((-9b^{13} - 9b^4(-4ac - b^2)^9)^{(1/2)} \\
& + 26880a^6b^1c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - \\
& 44800a^5b^3c^5 - 25a^2c^2(-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c + 51ab^2 \\
& *c(-4ac - b^2)^9)^{(1/2)}) / (32(a^5b^{12}e^{2f^4} + 4096a^{11}c^6e^{2f^4} + 240a^7b^8c^2e^{2f^4} \\
& - 1280a^8b^6c^3e^{2f^4} + 3840a^9b^4c^4e^{2f^4} - 6144a^{10}b^2c^5e^{2f^4} - 24a^6b^{10}c^6e^{2f^4} \\
&))^{(1/2)} * ((-9b^{13} - 9b^4(-4ac - b^2)^9)^{(1/2)} + 26880a^6b^1c^6 + 2 \\
& 077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 \\
& - 25a^2c^2(-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c + 51ab^2 *c(-4ac - \\
& b^2)^9)^{(1/2)}) / (32(a^5b^{12}e^{2f^4} + 4096a^{11}c^6e^{2f^4} + 240a^7b^8c^2e^{2f^4} \\
& - 1280a^8b^6c^3e^{2f^4} + 3840a^9b^4c^4e^{2f^4} - 6144a^{10}b^2c^5e^{2f^4} - 24a^6b^{10}c^6e^{2f^4} \\
&))^{(1/2)} * ((-9b^{13} - 9b^4(-4ac - b^2)^9)^{(1/2)} + 26880a^6b^1c^6 + 2077a^2b^9c^2 - \\
& 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2(-4ac - b^2)^9)^{(1/2)} \\
& - 213ab^{11}c + 51ab^2 *c(-4ac - b^2)^9)^{(1/2)}) / (32(a^5b^{12}e^{2f^4} + 4096a^{11}c^6e^{2f^4} \\
& + 240a^7b^8c^2e^{2f^4} - 1280a^8b^6c^3e^{2f^4} + 3840a^9b^4c^4e^{2f^4} - 6144a^{10}b^2c^5e^{2f^4} - \\
& 24a^6b^{10}c^6e^{2f^4}))^{(1/2)} * (x(256a^{10}b^{13}c^2e^{14f^{10}} - 6144a^{11}b^{11}c^3e^{14f^{10}} \\
& + 61440a^{12}b^9c^4e^{14f^{10}} - 327680a^{13}b^7c^5e^{14f^{10}} + 983040a^{14}b^5c^6e^{14f^{10}} - \\
& 1572864a^{15}b^3c^7e^{14f^{10}} + 1048576a^{16}b^1c^8e^{14f^{10}}) + 1048576a^{16}b^1c^8d^13f^{10} \\
& + 256a^{10}b^{13}c^2d^13f^{10} - 6144a^{11}b^{11}c^3d^13f^{10} + 61440a^{12}b^9c^4d^13f^{10} \\
& - 327680a^{13}b^7c^5d^13f^{10} + 983040a^{14}b^5c^6d^13f^{10} - 1 \\
& 572864a^{15}b^3c^7d^13f^{10}) - 192a^8b^{13}c^2e^{12f^8} + 4672a^9b^1
\end{aligned}$$

$$\begin{aligned}
& 1*c^3*e^{12*f^8} - 47360*a^{10}*b^9*c^4*e^{12*f^8} + 256000*a^{11}*b^7*c^5*e^{12*f^8} \\
& - 778240*a^{12}*b^5*c^6*e^{12*f^8} + 1261568*a^{13}*b^3*c^7*e^{12*f^8} - 851968*a^{14}*b*c^8*e^{12*f^8} + x*(204800*a^{12}*c^9*e^{12*f^6} + 144*a^6*b^{12}*c^3*e^{12*f^6} \\
& - 3264*a^7*b^{10}*c^4*e^{12*f^6} + 30112*a^8*b^8*c^5*e^{12*f^6} - 143360*a^9*b^6*c^6*e^{12*f^6} + 365568*a^{10}*b^4*c^7*e^{12*f^6} - 458752*a^{11}*b^2*c^8*e^{12*f^6} \\
& + 204800*a^{12}*c^9*d*e^{11*f^6} + 144*a^6*b^{12}*c^3*d*e^{11*f^6} - 3264*a^7*b^{10}*c^4*d*e^{11*f^6} + 30112*a^8*b^8*c^5*d*e^{11*f^6} - 143360*a^9*b^6*c^6*d*e^{11*f^6} \\
& + 365568*a^{10}*b^4*c^7*d*e^{11*f^6} - 458752*a^{11}*b^2*c^8*d*e^{11*f^6}) + 128000*a^{10}*c^9*e^{10*f^4} + 504*a^6*b^8*c^5*e^{10*f^4} - 8112*a^7*b^6*c^6*e^{10*f^4} \\
& *f^4 + 48704*a^8*b^4*c^7*e^{10*f^4} - 129280*a^9*b^2*c^8*e^{10*f^4})) * (- (9*b^{13} - 9*b^4 * (- (4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2 * (- (4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c + 51*a*b^2*c * (- (4*a*c - b^2)^9)^{(1/2)}) / (32*(a^5*b^{12}*e^{2*f^4} + 4096*a^{11}*c^6*e^{2*f^4} + 240*a^7*b^8*c^2*e^{2*f^4} - 1280*a^8*b^6*c^3*e^{2*f^4} + 3840*a^9*b^4*c^4*e^{2*f^4} - 6144*a^{10}*b^2*c^5*e^{2*f^4} - 24*a^6*b^{10}*c*e^{2*f^4}))^{(1/2)} * 2i - \operatorname{atan}(((- (9*b^{13} + 9*b^4 * (- (4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2 * (- (4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c * (- (4*a*c - b^2)^9)^{(1/2)}) / (32*(a^5*b^{12}*e^{2*f^4} + 4096*a^{11}*c^6*e^{2*f^4} + 240*a^7*b^8*c^2*e^{2*f^4} - 1280*a^8*b^6*c^3*e^{2*f^4} + 3840*a^9*b^4*c^4*e^{2*f^4} - 6144*a^{10}*b^2*c^5*e^{2*f^4} - 24*a^6*b^{10}*c*e^{2*f^4}))^{(1/2)} * ((- (9*b^{13} + 9*b^4 * (- (4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2 * (- (4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c * (- (4*a*c - b^2)^9)^{(1/2)}) / (32*(a^5*b^{12}*e^{2*f^4} + 4096*a^{11}*c^6*e^{2*f^4} + 240*a^7*b^8*c^2*e^{2*f^4} - 1280*a^8*b^6*c^3*e^{2*f^4} + 3840*a^9*b^4*c^4*e^{2*f^4} - 6144*a^{10}*b^2*c^5*e^{2*f^4} - 24*a^6*b^{10}*c*e^{2*f^4}))^{(1/2)} * ((- (9*b^{13} + 9*b^4 * (- (4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2 * (- (4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c * (- (4*a*c - b^2)^9)^{(1/2)}) / (32*(a^5*b^{12}*e^{2*f^4} + 4096*a^{11}*c^6*e^{2*f^4} + 240*a^7*b^8*c^2*e^{2*f^4} - 1280*a^8*b^6*c^3*e^{2*f^4} + 3840*a^9*b^4*c^4*e^{2*f^4} - 6144*a^{10}*b^2*c^5*e^{2*f^4} - 24*a^6*b^{10}*c*e^{2*f^4}))^{(1/2)} * (x*(256*a^{10}*b^{13}*c^2*e^{14*f^10} - 6144*a^{11}*b^{11}*c^3*e^{14*f^10} + 61440*a^{12}*b^9*c^4*e^{14*f^10} - 327680*a^{13}*b^7*c^5*e^{14*f^10} + 983040*a^{14}*b^5*c^6*e^{14*f^10} - 1572864*a^{15}*b^3*c^7*e^{14*f^10} + 1048576*a^{16}*b*c^8*e^{14*f^10} + 1048576*a^{16}*b*c^8*d*e^{13*f^10} + 256*a^{10}*b^{13}*c^2*d*e^{13*f^10} - 6144*a^{11}*b^{11}*c^3*d*e^{13*f^10} + 61440*a^{12}*b^9*c^4*d*e^{13*f^10} - 327680*a^{13}*b^7*c^5*d*e^{13*f^10} + 983040*a^{14}*b^5*c^6*d*e^{13*f^10} - 1572864*a^{15}*b^3*c^7*d*e^{13*f^10} - 192*a^8*b^{13}*c^2*e^{12*f^8} + 4672*a^9*b^{11}*c^3*e^{12*f^8} - 47360*a^{10}*b^9*c^4*e^{12*f^8} + 256000*a^{11}*b^7*c^5*e^{12*f^8} - 778240*a^{12}*b^5*c^6*e^{12*f^8} + 1261568*a^{13}*b^3*c^7*e^{12*f^8} - 851968*a^{14}*b*c^8*e^{12*f^8} + x*(204800*a^{12}*c^9*e^{12*f^6} + 144*a^6*b^{12}*c^3*e^{12*f^6} - 3264*a^7*b^{10}*c^4*e^{12*f^6} + 30112*a^8*b^8*c^5*e^{12*f^6} - 143360*a^9*b^6*c^6*e^{12*f^6} + 365568*a^{10}*b^4*c^7*e^{12*f^6} - 458752*a^{11}*b^2*c^8*e^{12*f^6} + 204800*a^{12}*c^9*d*e^{11*f^6} + 144*a^6*b^{12}*c^3*d*e^{11*f^6} - 3264*a
\end{aligned}$$

$$\begin{aligned}
& 7*b^{10}*c^4*d*e^{11}*f^6 + 30112*a^8*b^8*c^5*d*e^{11}*f^6 - 143360*a^9*b^6*c^6* \\
& d*e^{11}*f^6 + 365568*a^{10}*b^4*c^7*d*e^{11}*f^6 - 458752*a^{11}*b^2*c^8*d*e^{11}*f^ \\
& 6)*i + (- (9*b^{13} + 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077 \\
& *a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + \\
& 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - \\
& b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^2*f^4 + 4096*a^{11}*c^6*e^2*f^4 + 240*a^7*b^8* \\
& c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^ \\
& 10*b^2*c^5*e^2*f^4 - 24*a^6*b^{10}*c*e^2*f^4))^{(1/2)}*((-(9*b^{13} + 9*b^4*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c \\
& ^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e \\
& ^2*f^4 + 4096*a^{11}*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3 \\
& *e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^{10}*b^2*c^5*e^2*f^4 - 24*a^6*b^ \\
& 10*c*e^2*f^4))^{(1/2)}*((-(9*b^{13} + 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a \\
& ^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800 \\
& *a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^ \\
& 2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^2*f^4 + 4096*a^{11}*c^6*e^2*f^4 \\
& + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^ \\
& 2*f^4 - 6144*a^{10}*b^2*c^5*e^2*f^4 - 24*a^6*b^{10}*c*e^2*f^4))^{(1/2)}*(x*(256* \\
& a^{10}*b^{13}*c^2*e^{14}*f^{10} - 6144*a^{11}*b^{11}*c^3*e^{14}*f^{10} + 61440*a^{12}*b^9*c^4 \\
& *e^{14}*f^{10} - 327680*a^{13}*b^7*c^5*e^{14}*f^{10} + 983040*a^{14}*b^5*c^6*e^{14}*f^{10} \\
& - 1572864*a^{15}*b^3*c^7*e^{14}*f^{10} + 1048576*a^{16}*b*c^8*e^{14}*f^{10}) + 1048576* \\
& a^{16}*b*c^8*d*e^{13}*f^{10} + 256*a^{10}*b^{13}*c^2*d*e^{13}*f^{10} - 6144*a^{11}*b^{11}*c^3 \\
& *d*e^{13}*f^{10} + 61440*a^{12}*b^9*c^4*d*e^{13}*f^{10} - 327680*a^{13}*b^7*c^5*d*e^{13} \\
& f^{10} + 983040*a^{14}*b^5*c^6*d*e^{13}*f^{10} - 1572864*a^{15}*b^3*c^7*d*e^{13}*f^{10}) \\
& + 192*a^8*b^{13}*c^2*e^{12}*f^8 - 4672*a^9*b^{11}*c^3*e^{12}*f^8 + 47360*a^{10}*b^9*c \\
& ^4*e^{12}*f^8 - 256000*a^{11}*b^7*c^5*e^{12}*f^8 + 778240*a^{12}*b^5*c^6*e^{12}*f^8 - \\
& 1261568*a^{13}*b^3*c^7*e^{12}*f^8 + 851968*a^{14}*b*c^8*e^{12}*f^8) + x*(204800*a^ \\
& 12*c^9*e^{12}*f^6 + 144*a^6*b^{12}*c^3*e^{12}*f^6 - 3264*a^7*b^{10}*c^4*e^{12}*f^6 + \\
& 30112*a^8*b^8*c^5*e^{12}*f^6 - 143360*a^9*b^6*c^6*e^{12}*f^6 + 365568*a^{10}*b^4* \\
& c^7*e^{12}*f^6 - 458752*a^{11}*b^2*c^8*e^{12}*f^6) + 204800*a^{12}*c^9*d*e^{11}*f^6 + \\
& 144*a^6*b^{12}*c^3*d*e^{11}*f^6 - 3264*a^7*b^{10}*c^4*d*e^{11}*f^6 + 30112*a^8*b^8 \\
& *c^5*d*e^{11}*f^6 - 143360*a^9*b^6*c^6*d*e^{11}*f^6 + 365568*a^{10}*b^4*c^7*d*e^{11} \\
& *f^6 - 458752*a^{11}*b^2*c^8*d*e^{11}*f^6)*i)/((- (9*b^{13} + 9*b^4*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302 \\
& 40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^2*f^4 + \\
& 4096*a^{11}*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 \\
& + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^{10}*b^2*c^5*e^2*f^4 - 24*a^6*b^{10}*c*e^2 \\
& *f^4))^{(1/2)}*((-(9*b^{13} + 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 \\
& + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3 \\
& *c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c*(-(4 \\
& *a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^2*f^4 + 4096*a^{11}*c^6*e^2*f^4 + 240*a \\
& ^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - \\
& 6144*a^{10}*b^2*c^5*e^2*f^4 - 24*a^6*b^{10}*c*e^2*f^4))^{(1/2)}*((-(9*b^{13} + 9*b
\end{aligned}$$

$$\begin{aligned}
&^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^{2*f^4} + 4096*a^{11}*c^6*e^{2*f^4} + 240*a^7*b^8*c^2*e^{2*f^4} - 1280*a^8*b^6*c^3*e^{2*f^4} + 3840*a^9*b^4*c^4*e^{2*f^4} - 6144*a^{10}*b^2*c^5*e^{2*f^4} - 24*a^6*b^{10}*c*e^{2*f^4}))^{(1/2)}*(x*(256*a^{10}*b^{13}*c^2*e^{14*f^10} - 6144*a^{11}*b^{11}*c^3*e^{14*f^10} + 61440*a^{12}*b^9*c^4*e^{14*f^10} - 327680*a^{13}*b^7*c^5*e^{14*f^10} + 983040*a^{14}*b^5*c^6*e^{14*f^10} - 1572864*a^{15}*b^3*c^7*e^{14*f^10} + 1048576*a^{16}*b*c^8*e^{14*f^10}) + 1048576*a^{16}*b*c^8*d*e^{13*f^10} + 256*a^{10}*b^{13}*c^2*d*e^{13*f^10} - 6144*a^{11}*b^{11}*c^3*d*e^{13*f^10} + 61440*a^{12}*b^9*c^4*d*e^{13*f^10} - 327680*a^{13}*b^7*c^5*d*e^{13*f^10} + 983040*a^{14}*b^5*c^6*d*e^{13*f^10} - 1572864*a^{15}*b^3*c^7*d*e^{13*f^10}) + 192*a^8*b^{13}*c^2*e^{12*f^8} - 4672*a^9*b^{11}*c^3*e^{12*f^8} + 47360*a^{10}*b^9*c^4*e^{12*f^8} - 256000*a^{11}*b^7*c^5*e^{12*f^8} + 778240*a^{12}*b^5*c^6*e^{12*f^8} - 1261568*a^{13}*b^3*c^7*e^{12*f^8} + 851968*a^{14}*b*c^8*e^{12*f^8}) + x*(204800*a^{12}*c^9*e^{12*f^6} + 144*a^6*b^{12}*c^3*e^{12*f^6} - 3264*a^7*b^{10}*c^4*e^{12*f^6} + 30112*a^8*b^8*c^5*e^{12*f^6} - 143360*a^9*b^6*c^6*e^{12*f^6} + 365568*a^{10}*b^4*c^7*e^{12*f^6} - 458752*a^{11}*b^2*c^8*e^{12*f^6}) + 204800*a^{12}*c^9*d*e^{11*f^6} + 144*a^6*b^{12}*c^3*d*e^{11*f^6} - 3264*a^7*b^{10}*c^4*d*e^{11*f^6} + 30112*a^8*b^8*c^5*d*e^{11*f^6} - 143360*a^9*b^6*c^6*d*e^{11*f^6} + 365568*a^{10}*b^4*c^7*d*e^{11*f^6} - 458752*a^{11}*b^2*c^8*d*e^{11*f^6}) - ((-9*b^{13} + 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^{2*f^4} + 4096*a^{11}*c^6*e^{2*f^4} + 240*a^7*b^8*c^2*e^{2*f^4} - 1280*a^8*b^6*c^3*e^{2*f^4} + 3840*a^9*b^4*c^4*e^{2*f^4} - 6144*a^{10}*b^2*c^5*e^{2*f^4} - 24*a^6*b^{10}*c*e^{2*f^4}))^{(1/2)}*((-9*b^{13} + 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12}*e^{2*f^4} + 4096*a^{11}*c^6*e^{2*f^4} + 240*a^7*b^8*c^2*e^{2*f^4} - 1280*a^8*b^6*c^3*e^{2*f^4} + 3840*a^9*b^4*c^4*e^{2*f^4} - 6144*a^{10}*b^2*c^5*e^{2*f^4} - 24*a^6*b^{10}*c*e^{2*f^4}))^{(1/2)}*(x*(256*a^{10}*b^{13}*c^2*e^{14*f^10} - 6144*a^{11}*b^{11}*c^3*e^{14*f^10} + 61440*a^{12}*b^9*c^4*e^{14*f^10} - 327680*a^{13}*b^7*c^5*e^{14*f^10} + 983040*a^{14}*b^5*c^6*e^{14*f^10} - 1572864*a^{15}*b^3*c^7*e^{14*f^10} + 1048576*a^{16}*b*c^8*e^{14*f^10}) + 1048576*a^{16}*b*c^8*d*e^{13*f^10} + 256*a^{10}*b^{13}*c^2*d*e^{13*f^10} - 6144*a^{11}*b^{11}*c^3*d*e^{13*f^10} + 61440*a^{12}*b^9*c^4*d*e^{13*f^10} - 327680*a^{13}*b^7*c^5*d*e^{13*f^10} + 983040*a^{14}*b^5*c^6*d*e^{13*f^10} - 1572864*a^{15}*b^3*c^7*d*e^{13*f^10}) - 192*a^8*b^{13}*c^2*e^{12*f^8} + 4672*a^9*b^{11}*c^3*e^{12*f^8} - 47360*a^{10}*b^9*c^4*e^{12*f^8} + 256000*a^{11}*b^7*c^5*e^{12*f^8} - 778240*a^{12}*b^5*c^6*e^{12*f^8} + 1261568*a^{13}*b^3*c^7*e^{12*f^8} - 851968*a^{14}*b*c^8*e^{12*f^8})
\end{aligned}$$

$$\begin{aligned}
& 1568*a^{13}*b^3*c^7*e^{12*f^8} - 851968*a^{14}*b*c^8*e^{12*f^8}) + x*(204800*a^{12}*c^9*e^{12*f^6} + 144*a^6*b^{12}*c^3*e^{12*f^6} - 3264*a^7*b^{10}*c^4*e^{12*f^6} + 30112*a^8*b^8*c^5*e^{12*f^6} - 143360*a^9*b^6*c^6*e^{12*f^6} + 365568*a^{10}*b^4*c^7*e^{12*f^6} - 458752*a^{11}*b^2*c^8*e^{12*f^6}) + 204800*a^{12}*c^9*d*e^{11*f^6} + 144*a^6*b^{12}*c^3*d*e^{11*f^6} - 3264*a^7*b^{10}*c^4*d*e^{11*f^6} + 30112*a^8*b^8*c^5*d*e^{11*f^6} - 143360*a^9*b^6*c^6*d*e^{11*f^6} + 365568*a^{10}*b^4*c^7*d*e^{11*f^6} - 458752*a^{11}*b^2*c^8*d*e^{11*f^6}) + 128000*a^{10}*c^9*e^{10*f^4} + 504*a^6*b^8*c^5*e^{10*f^4} - 8112*a^7*b^6*c^6*e^{10*f^4} + 48704*a^8*b^4*c^7*e^{10*f^4} - 129280*a^9*b^2*c^8*e^{10*f^4})) * (- (9*b^{13} + 9*b^4 * (- (4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2 * (- (4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c - 51*a*b^2*c * (- (4*a*c - b^2)^9)^{(1/2)}) / (32*(a^5*b^{12}*e^{2*f^4} + 4096*a^{11}*c^6*e^{2*f^4} + 240*a^7*b^8*c^2*e^{2*f^4} - 1280*a^8*b^6*c^3*e^{2*f^4} + 3840*a^9*b^4*c^4*e^{2*f^4} - 6144*a^{10}*b^2*c^5*e^{2*f^4} - 24*a^6*b^{10}*c*e^{2*f^4}))^{(1/2)} * i - ((x*(3*b^3*d - 20*a*c^2*d^3 + 6*b^2*c*d^3 - 11*a*b*c*d)) / (a*(a*b^2 - 4*a^2*c)) - (x^4*(10*a*c^2*e^3 - 3*b^2*c*e^3)) / (2*a*(a*b^2 - 4*a^2*c)) - (2*x^3*(10*a*c^2*d*e^2 - 3*b^2*c*d*e^2)) / (a*(a*b^2 - 4*a^2*c)) + (2*a*b^2 - 8*a^2*c + 3*b^3*d^2 - 10*a*c^2*d^4 + 3*b^2*c*d^4 - 11*a*b*c*d^2) / (2*a*e*(a*b^2 - 4*a^2*c)) + (x^2*(3*b^3*e - 60*a*c^2*d^2*e + 18*b^2*c*d^2*e - 11*a*b*c*e)) / (2*a*(a*b^2 - 4*a^2*c))) / (x^2*(10*c*d^3*e^2*f^2 + 3*b*d*e^2*f^2) + x*(a*e*f^2 + 3*b*d^2*e*f^2 + 5*c*d^4*e*f^2) + x^3*(b*e^3*f^2 + 10*c*d^2*e^3*f^2) + b*d^3*f^2 + c*d^5*f^2 + a*d*f^2 + c*e^5*f^2*x^5 + 5*c*d*e^4*f^2*x^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out

$$3.539 \quad \int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=228

$$\frac{b \log(a + b(d + ex)^2 + c(d + ex)^4)}{2a^3ef^3} - \frac{2b \log(d + ex)}{a^3ef^3} - \frac{b^2 - 3ac}{a^2ef^3(b^2 - 4ac)(d + ex)^2} - \frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2c}{\sqrt{b^2-4ac}}\right)}{a^3ef^3(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.37, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1142, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3ef^3(b^2 - 4ac)^{3/2}} - \frac{b^2 - 3ac}{a^2ef^3(b^2 - 4ac)(d + ex)^2} + \frac{b \log(a + b(d + ex)^2 + c(d + ex)^4)}{2a^3ef^3} - \frac{2b \log(d + ex)}{a^3ef^3} + \frac{-2ac + b^2 + bc(d + ex)^2}{2aef^3(b^2 - 4ac)(d + ex)^2(a + b(d + ex)^2 + c(d + ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] $-\frac{(b^2 - 3ac)}{a^2(b^2 - 4ac)*ef^3(d + ex)^2} + \frac{(b^2 - 2ac + b*c*(d + ex)^2)}{(2a*(b^2 - 4ac)*ef^3(d + ex)^2*(a + b*(d + ex)^2 + c*(d + ex)^4))} - \frac{((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2]/Sqrt[b^2 - 4*a*c])}{(a^3*(b^2 - 4*a*c)^{(3/2)}*ef^3)} - \frac{(2*b*Log[d + e*x])}{(a^3*ef^3)} + \frac{(b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])}{(2*a^3*ef^3)}$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx &= \frac{\text{Subst} \left(\int \frac{1}{x^3 (a + bx^2 + cx^4)^2} dx, x, d + ex \right)}{ef^3} \\
&= \frac{\text{Subst} \left(\int \frac{1}{x^2 (a + bx + cx^2)^2} dx, x, (d + ex)^2 \right)}{2ef^3} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^3(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)} - \frac{bc(d + ex)}{2a(b^2 - 4ac)ef^3(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^3(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)} - \frac{bc(d + ex)}{2a(b^2 - 4ac)ef^3(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)ef^3(d + ex)^2} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^3(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)ef^3(d + ex)^2} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^3(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)ef^3(d + ex)^2} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^3(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)ef^3(d + ex)^2} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a(b^2 - 4ac)ef^3(d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 287, normalized size = 1.26

$$\frac{\frac{(6a^2c^2 - 6ab^2c - 4abc\sqrt{b^2 - 4ac} + b^3\sqrt{b^2 - 4ac} + b^4)\log(-\sqrt{b^2 - 4ac} + b + 2c(d + ex)^2)}{(b^2 - 4ac)^{3/2}} + \frac{(-6a^2c^2 + 6ab^2c - 4abc\sqrt{b^2 - 4ac} + b^3\sqrt{b^2 - 4ac} - b^4)\log(\sqrt{b^2 - 4ac} + b + 2c(d + ex)^2)}{(b^2 - 4ac)^{3/2}} + \frac{a(-3abc - 2ac^2(d + ex)^2 + b^3 + b^2c(d + ex)^2)}{(4ac - b^2)(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{a}{(d + ex)^2} - 4b\log(d + ex)}{2a^3ef^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] (-a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e

$*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*\text{Sqrt}[b^2 - 4*a*c] - 4*a*b*c*\text{Sqrt}[b^2 - 4*a*c])*\text{Log}[b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*\text{Sqrt}[b^2 - 4*a*c] - 4*a*b*c*\text{Sqrt}[b^2 - 4*a*c])*\text{Log}[b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(3/2))/(2*a^3*e*f^3)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

fricas [B] time = 5.90, size = 4604, normalized size = 20.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out] $[-1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3$

$$\begin{aligned}
&)e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 \\
& + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 \\
& + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 \\
& + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*\log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*\log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3*x^6 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*f^3*x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*e*f^3), -1/2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*e^4*x^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^3*x^3 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*e^2*x^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*e*x + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^6*x^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*e^5*x^5 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*e^4*x^4 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*e^3*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*e^2*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*e*x)*\sqrt{-b^2 + 4*a*c}*\arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*\sqrt{-b^2
\end{aligned}$$

$$\begin{aligned}
& + 4*a*c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6 \\
& *(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2 \\
& *b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - \\
& 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3) \\
& *d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 1 \\
& 6*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*a \\
& b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2 \\
& *x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 \\
& + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - \\
& 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*\log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + \\
& (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*((b^5*c - 8 \\
& *a*b^3*c^2 + 16*a^2*b*c^3)*e^6*x^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3) \\
& *d*e^5*x^5 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + \\
& 16*a^2*b*c^3)*d^2)*e^4*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + 4*(\\
& 5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c \\
& ^2)*d)*e^3*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + (a*b^5 - 8*a^2*b \\
& ^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 \\
& - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*e^2*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3 \\
& *b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b \\
& ^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e*x)*\log(e*x + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e^7*f^3*x^6 + 6*(a^3 \\
& *b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*e^6*f^3*x^5 + (a^3*b^5 - 8*a^4*b^3*c \\
& + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*e^5*f^3* \\
& x^4 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3 \\
& *c + 16*a^5*b*c^2)*d)*e^4*f^3*x^3 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 \\
& + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c \\
& + 16*a^5*b*c^2)*d^2)*e^3*f^3*x^2 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a \\
& ^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a \\
& ^5*b^2*c + 16*a^6*c^2)*d)*e^2*f^3*x + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5 \\
& *c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^ \\
& 2*c + 16*a^6*c^2)*d^2)*e*f^3)]
\end{aligned}$$

giac [B] time = 1.30, size = 687, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out] 1/2*((a^3*b^4*c*f^3*e^3 - 6*a^4*b^2*c^2*f^3*e^3 + 6*a^5*c^3*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^3*b^4*c*f^3*e^3 - 6*a^4*b^2*c^2*f^3*e^3 + 6*a^5*c^3*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))

$$c)*d*x*e - b*d^2 + \sqrt{b^2 - 4*a*c}*d^2 - 2*a)))/(a^6*b^4*c*f^6*e^4 - 8*a^7*b^2*c^2*f^6*e^4 + 16*a^8*c^3*f^6*e^4) + 1/2*b*e^{(-1)}*\log(\text{abs}(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^3*f^3) - 2*b*e^{(-1)}*\log(\text{abs}(x*e + d))/(a^3*f^3) - 1/2*(2*a*b^2*c*d^4 - 6*a^2*c^2*d^4 + 2*a*b^3*d^2 - 7*a^2*b*c*d^2 + 2*(a*b^2*c*e^4 - 3*a^2*c^2*e^4)*x^4 + a^2*b^2 - 4*a^3*c + 8*(a*b^2*c*d*e^3 - 3*a^2*c^2*d*e^3)*x^3 + (12*a*b^2*c*d^2*e^2 - 36*a^2*c^2*d^2*e^2 + 2*a*b^3*e^2 - 7*a^2*b*c*e^2)*x^2 + 2*(4*a*b^2*c*d^3*e - 12*a^2*c^2*d^3*e + 2*a*b^3*d*e - 7*a^2*b*c*d*e)*x)*e^{(-1)}/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*(x*e + d)^2*a^3*f^3)$$

maple [C] time = 0.03, size = 1047, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x)

[Out]
$$-1/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*e/(4*a*c-b^2)*x^2+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*e/(4*a*c-b^2)*x^2*b^2-2/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c^2*d/(4*a*c-b^2)*x+1/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*c*d/(4*a*c-b^2)*x*b^2-1/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*c^2*d^2+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^2*c*d^2-3/2/f^3/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b*c+1/2/f^3/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/e/(4*a*c-b^2)*b^3+1/f^3/a^3/(4*a*c-b^2)/e*sum(((4*a*c-b^2)*_R^3*b*c*e^3+3*(4*a*c-b^2)*_R^2*b*c*d*e^2+4*a*b*c^2*d^3-b^3*c*d^3-3*a^2*c^2*d+5*a*b^2*c*d-b^4*d+(12*a*b*c^2*d^2-3*b^3*c*d^2-3*a^2*c^2+5*a*b^2*c-b^4)*_R*e)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/2/f^3/a^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e/f^3$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 13.52, size = 14830, normalized size = 65.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x)$

[Out]
$$\begin{aligned} & ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2) \\ & - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d* \\ & e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6* \\ & a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2* \\ & b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2 \\ &))/(x^3*(20*c*d^3*e^3*f^3 + 4*b*d*e^3*f^3) + x*(2*a*d*e*f^3 + 4*b*d^3*e*f^ \\ & 3 + 6*c*d^5*e*f^3) + x^4*(b*e^4*f^3 + 15*c*d^2*e^4*f^3) + x^2*(a*e^2*f^3 + \\ & 6*b*d^2*e^2*f^3 + 15*c*d^4*e^2*f^3) + a*d^2*f^3 + b*d^4*f^3 + c*d^6*f^3 + c \\ & *e^6*f^3*x^6 + 6*c*d*e^5*f^3*x^5) + (\log((((b + a^3*e*f^3*(-(b^4 + 6*a^2*c^ \\ & 2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((b + a^3*e*f^3*(-(\\ & b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^ \\ & 2*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a \\ & *b^2*c^2*d^2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 \\ & + 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*f^3*(-(b^4 \\ & + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3* \\ & b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a* \\ & c*d*e*x))/(a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f \\ & ^3*(4*a*c - b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a \\ & ^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2) \\ & ^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c \\ & - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(\\ & 4*a*c - b^2)^2))/(2*a^3*e*f^3) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*f^9 \\ & *(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c* \\ & d^2 - 4*a*b*c))/(a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a*c - b^2)^ \\ & 3)/(a^6*f^9*(4*a*c - b^2)^3))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2 \\ & *c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2 \\ & *c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b \\ & ^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^ \\ & 2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c \\ &))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b - a^3*e*f^3*(-(b^4 + 6*a^2*c^ \\ & 2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3 \\ & *b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(\\ & a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c \\ & - b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6* \\ & b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2)^2) + (4*b* \\ & c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2)^2) + \end{aligned}$$

$$\begin{aligned}
& \left((8*b*c^4*d*e^{16}*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c)) / (a^4*f^6*(4*a*c - b^2)^2) \right) / (2*a^3*e*f^3) - (8*c^5*e^{16}*x^2*(3*a*c - b^2)^3) / (a^6*f^9*(4*a*c - b^2)^3) + (8*c^4*e^{14}*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b*c)) / (a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^{15}*x*(3*a*c - b^2)^3) / (a^6*f^9*(4*a*c - b^2)^3)) * (b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3) / (2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (2*b*log(d + e*x)) / (a^3*e*f^3) - (atan(((2*a^9*b^6*f^9*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*f^9*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*f^9*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*f^9*(4*a*c - b^2)^(9/2)) * (x * (((8*(54*a^3*c^8*d*e^15 - 2*b^6*c^5*d*e^15 + 18*a*b^4*c^6*d*e^15 - 54*a^2*b^2*c^7*d*e^15)) / (a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(276*a^5*b*c^7*d*e^16*f^3 - 6*a^2*b^7*c^4*d*e^16*f^3 + 65*a^3*b^5*c^5*d*e^16*f^3 - 233*a^4*b^3*c^6*d*e^16*f^3)) / (a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^17*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*c^6*d*e^17*f^6)) / (a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3) * (640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9)) / ((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) * (a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6))) * (b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)) / (2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6))) * (b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3) / (2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (((((8*(480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^17*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*c^6*d*e^17*f^6)) / (a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3) * (640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9)) / ((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) * (a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6))) * (b^4 + 6*a^2*c^2 - 6*a*b^2*c)) / (2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c) * (b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3) * (640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9)) / (a^3*e*f^3*(4*a*c - b^2)^(3/2) * (a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) * (a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6))) * (b^4 + 6*a^2*c^2 - 6*a*b^2*c)) / (2*a^3*e*f^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2 * (b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3) * (640*a^10*b*c^6*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9)) / (a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) * (a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)
\end{aligned}$$

$$\begin{aligned}
& b^3c^5d^18e^{18f^9})/(a^6e^{2f^6}(4ac - b^2)^3(a^6b^6f^9 - 64a^9c^3 \\
& *f^9 - 12a^7b^4c^2f^9 + 48a^8b^2c^2f^9)*(a^3b^6e^{2f^6} - 64a^6c^3 \\
& *e^{2f^6} + 48a^5b^2c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6}))) * (3b^6 - 3a^3c^3 \\
& + 36a^2b^2c^2 - 21ab^4c)) / ((8a^3c^2(4ac - b^2)^3(9a^4c^4 - \\
& 6b^8 - 288a^2b^4c^2 + 382a^3b^2c^3 + 72ab^6c)) - (b * ((((((8*(480 \\
& *a^8c^7d^17f^6 - a^4b^8c^3d^17f^6 + 6a^5b^6c^4d^17f^6 + 3 \\
& 0a^6b^4c^5d^17f^6 - 272a^7b^2c^6d^17f^6)) / (a^6b^6f^9 - 64a^9c^3 \\
& *f^9 - 12a^7b^4c^2f^9 + 48a^8b^2c^2f^9) - (4*(b^7e^{f^3} - 12a^5 \\
& b^5c^2e^{f^3} - 64a^3b^3c^2e^{f^3} + 48a^2b^3c^2e^{f^3})*(640a^{10}b^6c^6d^ \\
& e^{18f^9} + 3a^6b^9c^2d^18f^9 - 46a^7b^7c^3d^18f^9 + 264a^8b^5 \\
& c^4d^18f^9 - 672a^9b^3c^5d^18f^9)) / ((a^6b^6f^9 - 64a^9c^3 \\
& *f^9 - 12a^7b^4c^2f^9 + 48a^8b^2c^2f^9)*(a^3b^6e^{2f^6} - 64a^6c^3 \\
& *e^{2f^6} + 48a^5b^2c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6}))) * (b^4 + 6a^2c^2 \\
& - 6ab^2c)) / (2a^3e^{f^3}(4ac - b^2)^{(3/2)}) - (2*(b^4 + 6a^2c^2 - 6 \\
& *ab^2c)*(b^7e^{f^3} - 12a^5b^5c^2e^{f^3} - 64a^3b^3c^2e^{f^3} + 48a^2b^3c^2 \\
& e^{f^3})*(640a^{10}b^6c^6d^18f^9 + 3a^6b^9c^2d^18f^9 - 46a^7b^7 \\
& c^3d^18f^9 + 264a^8b^5c^4d^18f^9 - 672a^9b^3c^5d^18f^9) \\
&) / (a^3e^{f^3}(4ac - b^2)^{(3/2)} * (a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4 \\
& *c^2f^9 + 48a^8b^2c^2f^9) * (a^3b^6e^{2f^6} - 64a^6c^3e^{2f^6} + 48a^5 \\
& *b^2c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6}))) * (b^7e^{f^3} - 12a^5b^5c^2e^{f^3} - \\
& 64a^3b^3c^2e^{f^3} + 48a^2b^3c^2e^{f^3})) / (2*(a^3b^6e^{2f^6} - 64a^6c^3 \\
& e^{2f^6} + 48a^5b^2c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6})) - (((8*(276a^5 \\
& *b^7c^4d^16f^3 - 6a^2b^7c^4d^16f^3 + 65a^3b^5c^5d^16f^3 - \\
& 233a^4b^3c^6d^16f^3)) / (a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^2f^9 \\
& + 48a^8b^2c^2f^9) - (((8*(480a^8c^7d^17f^6 - a^4b^8c^3d^17 \\
& f^6 + 6a^5b^6c^4d^17f^6 + 30a^6b^4c^5d^17f^6 - 272a^7b^2c^6d^17 \\
& f^6)) / (a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^2f^9 + 48a^8b^2c^2f^9) \\
& - (4*(b^7e^{f^3} - 12a^5b^5c^2e^{f^3} - 64a^3b^3c^2e^{f^3} + 48a^2 \\
& b^3c^2e^{f^3})*(640a^{10}b^6c^6d^18f^9 + 3a^6b^9c^2d^18f^9 - 4 \\
& 6a^7b^7c^3d^18f^9 + 264a^8b^5c^4d^18f^9 - 672a^9b^3c^5d^18f^9) \\
&)) / ((a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^2f^9 + 48a^8b^2c^2 \\
& f^9) * (a^3b^6e^{2f^6} - 64a^6c^3e^{2f^6} + 48a^5b^2c^2e^{2f^6} - 12 \\
& a^4b^4c^2e^{2f^6}))) * (b^7e^{f^3} - 12a^5b^5c^2e^{f^3} - 64a^3b^3c^2e^{f^3} + 4 \\
& 8a^2b^3c^2e^{f^3})) / (2*(a^3b^6e^{2f^6} - 64a^6c^3e^{2f^6} + 48a^5b^2 \\
& *c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6}))) * (b^4 + 6a^2c^2 - 6ab^2c)) / (2a^ \\
& 3e^{f^3}(4ac - b^2)^{(3/2)}) + ((b^4 + 6a^2c^2 - 6ab^2c)^3 * (640a^{10}b^6 \\
& c^6d^18f^9 + 3a^6b^9c^2d^18f^9 - 46a^7b^7c^3d^18f^9 + 26 \\
& 4a^8b^5c^4d^18f^9 - 672a^9b^3c^5d^18f^9)) / (a^9e^{3f^9}(4ac \\
& - b^2)^{(9/2)} * (a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^2f^9 + 48a^8b^2 \\
& *c^2f^9))) * (3b^6 - 49a^3c^3 + 72a^2b^2c^2 - 27ab^4c)) / (8a^3c^2 * \\
& (4ac - b^2)^{(7/2)} * (9a^4c^4 - 6b^8 - 288a^2b^4c^2 + 382a^3b^2c^3 \\
& + 72ab^6c))) + x^2 * (((4*(54a^3c^8e^{16} - 2b^6c^5e^{16} + 18ab^4c^6 \\
& e^{16} - 54a^2b^2c^7e^{16})) / (a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^2 \\
& *f^9 + 48a^8b^2c^2f^9) + (((4*(6a^2b^7c^4e^{17f^3} - 65a^3b^5c^5 \\
& e^{17f^3} + 233a^4b^3c^6e^{17f^3} - 276a^5b^3c^7e^{17f^3})) / (a^6b^6f^9
\end{aligned}$$

$$\begin{aligned}
& e^3 f^3 - 12 a^2 b^5 c^2 e^3 f^3 - 64 a^3 b^3 c^3 e^3 f^3 + 48 a^2 b^3 c^2 e^3 f^3) \cdot (3 a^6 b^9 c^2 e^{19} f^9 - 46 a^7 b^7 c^3 e^{19} f^9 + 264 a^8 b^5 c^4 e^{19} f^9 - 672 a^9 b^3 c^5 e^{19} f^9 + 640 a^{10} b^2 c^6 e^{19} f^9) / (a^3 e^3 f^3 (4 a^2 c - b^2)^{(3/2)} (a^6 b^6 f^9 - 64 a^9 c^3 f^9 - 12 a^7 b^4 c f^9 + 48 a^8 b^2 c^2 f^9) \cdot (a^3 b^6 e^2 f^6 - 64 a^6 c^3 e^2 f^6 + 48 a^5 b^2 c^2 e^2 f^6 - 12 a^4 b^4 c e^2 f^6)) \cdot (b^7 e^3 f^3 - 12 a^2 b^5 c^2 e^3 f^3 - 64 a^3 b^3 c^3 e^3 f^3 + 48 a^2 b^3 c^2 e^3 f^3) / (2 (a^3 b^6 e^2 f^6 - 64 a^6 c^3 e^2 f^6 + 48 a^5 b^2 c^2 e^2 f^6 - 12 a^4 b^4 c e^2 f^6)) + (((4 (6 a^2 b^7 c^4 e^{17} f^3 - 65 a^3 b^5 c^5 e^{17} f^3 + 233 a^4 b^3 c^6 e^{17} f^3 - 276 a^5 b^2 c^7 e^{17} f^3)) / (a^6 b^6 f^9 - 64 a^9 c^3 f^9 - 12 a^7 b^4 c f^9 + 48 a^8 b^2 c^2 f^9) + ((4 (480 a^8 c^7 e^{18} f^6 - a^4 b^8 c^3 e^{18} f^6 + 6 a^5 b^6 c^4 e^{18} f^6 + 30 a^6 b^4 c^5 e^{18} f^6 - 272 a^7 b^2 c^6 e^{18} f^6)) / (a^6 b^6 f^9 - 64 a^9 c^3 f^9 - 12 a^7 b^4 c f^9 + 48 a^8 b^2 c^2 f^9) - (2 (b^7 e^3 f^3 - 12 a^2 b^5 c^2 e^3 f^3 - 64 a^3 b^3 c^3 e^3 f^3 + 48 a^2 b^3 c^2 e^3 f^3) \cdot (3 a^6 b^9 c^2 e^{19} f^9 - 46 a^7 b^7 c^3 e^{19} f^9 + 264 a^8 b^5 c^4 e^{19} f^9 - 672 a^9 b^3 c^5 e^{19} f^9 + 640 a^{10} b^2 c^6 e^{19} f^9)) / ((a^6 b^6 f^9 - 64 a^9 c^3 f^9 - 12 a^7 b^4 c f^9 + 48 a^8 b^2 c^2 f^9) \cdot (a^3 b^6 e^2 f^6 - 64 a^6 c^3 e^2 f^6 + 48 a^5 b^2 c^2 e^2 f^6 - 12 a^4 b^4 c e^2 f^6))) \cdot (b^7 e^3 f^3 - 12 a^2 b^5 c^2 e^3 f^3 - 64 a^3 b^3 c^3 e^3 f^3 + 48 a^2 b^3 c^2 e^3 f^3) / (2 (a^3 b^6 e^2 f^6 - 64 a^6 c^3 e^2 f^6 + 48 a^5 b^2 c^2 e^2 f^6 - 12 a^4 b^4 c e^2 f^6)) \cdot (b^4 + 6 a^2 c^2 - 6 a b^2 c) / (2 a^3 e^3 f^3 (4 a^2 c - b^2)^{(3/2)}) + ((b^4 + 6 a^2 c^2 - 6 a b^2 c)^3 (3 a^6 b^9 c^2 e^{19} f^9 - 46 a^7 b^7 c^3 e^{19} f^9 + 264 a^8 b^5 c^4 e^{19} f^9 - 672 a^9 b^3 c^5 e^{19} f^9 + 640 a^{10} b^2 c^6 e^{19} f^9)) / (2 a^9 e^3 f^9 (4 a^2 c - b^2)^{(9/2)} (a^6 b^6 f^9 - 64 a^9 c^3 f^9 - 12 a^7 b^4 c f^9 + 48 a^8 b^2 c^2 f^9)) \cdot (3 b^6 - 49 a^3 c^3 + 72 a^2 b^2 c^2 - 27 a b^4 c) / (8 a^3 c^2 (4 a^2 c - b^2)^{(7/2)} (9 a^4 c^4 - 6 b^8 - 288 a^2 b^4 c^2 + 382 a^3 b^2 c^3 + 72 a b^6 c)) + (((((4 (36 a^6 c^7 e^{15} f^3 + 4 a^2 b^8 c^3 e^{15} f^3 - 45 a^3 b^6 c^4 e^{15} f^3 + 170 a^4 b^4 c^5 e^{15} f^3 - 225 a^5 b^2 c^6 e^{15} f^3 - 276 a^5 b^2 c^7 d^2 e^{15} f^3 + 6 a^2 b^7 c^4 d^2 e^{15} f^3 - 65 a^3 b^5 c^5 d^2 e^{15} f^3 + 233 a^4 b^3 c^6 d^2 e^{15} f^3)) / (a^6 b^6 f^9 - 64 a^9 c^3 f^9 - 12 a^7 b^4 c f^9 + 48 a^8 b^2 c^2 f^9) - (((4 (2 a^4 b^9 c^2 e^{16} f^6 - 26 a^5 b^7 c^3 e^{16} f^6 + 118 a^6 b^5 c^4 e^{16} f^6 - 208 a^7 b^3 c^5 e^{16} f^6 - 480 a^8 c^7 d^2 e^{16} f^6 + 96 a^8 b^3 c^6 e^{16} f^6 + a^4 b^8 c^3 d^2 e^{16} f^6 - 6 a^5 b^6 c^4 d^2 e^{16} f^6 - 30 a^6 b^4 c^5 d^2 e^{16} f^6 + 272 a^7 b^2 c^6 d^2 e^{16} f^6)) / (a^6 b^6 f^9 - 64 a^9 c^3 f^9 - 12 a^7 b^4 c f^9 + 48 a^8 b^2 c^2 f^9) + (2 (b^7 e^3 f^3 - 12 a^2 b^5 c^2 e^3 f^3 - 64 a^3 b^3 c^3 e^3 f^3 + 48 a^2 b^3 c^2 e^3 f^3) \cdot (a^7 b^8 c^2 e^{17} f^9 - 12 a^8 b^6 c^3 e^{17} f^9 + 48 a^9 b^4 c^4 e^{17} f^9 - 64 a^{10} b^2 c^5 e^{17} f^9 + 640 a^{10} b^2 c^5 d^2 e^{17} f^9 + 3 a^6 b^9 c^2 d^2 e^{17} f^9 - 46 a^7 b^7 c^3 d^2 e^{17} f^9 + 264 a^8 b^5 c^4 d^2 e^{17} f^9 - 672 a^9 b^3 c^5 d^2 e^{17} f^9)) / ((a^6 b^6 f^9 - 64 a^9 c^3 f^9 - 12 a^7 b^4 c f^9 + 48 a^8 b^2 c^2 f^9) \cdot (a^3 b^6 e^2 f^6 - 64 a^6 c^3 e^2 f^6 + 48 a^5 b^2 c^2 e^2 f^6 - 12 a^4 b^4 c e^2 f^6))) \cdot (b^7 e^3 f^3 - 12 a^2 b^5 c^2 e^3 f^3 - 64 a^3 b^3 c^3 e^3 f^3 + 48 a^2 b^3 c^2 e^3 f^3) / (2 (a^3 b^6 e^2 f^6 - 64 a^6 c^3 e^2 f^6 + 48 a^5 b^2 c^2 e^2 f^6 - 12 a^4 b^4 c e^2 f^6)) \cdot (b^7 e^3 f^3 - 12 a^2 b^5 c^2 e^3 f^3 - 64 a^3 b^3 c^3 e^3 f^3 +
\end{aligned}$$

$$\begin{aligned}
& 48*a^2*b^3*c^2*e*f^3)/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (4*(2*b^7*c^4*e^14 - 20*a*b^5*c^5* \\
& e^14 - 72*a^3*b*c^7*e^14 + 66*a^2*b^3*c^6*e^14 - 54*a^3*c^8*d^2*e^14 + 2*b^6*c^5*d^2*e^14 + 54*a^2*b^2*c^7*d^2*e^14 - 18*a*b^4*c^6*d^2*e^14))/(a^6*b^6 \\
& *f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (((((4*(2* \\
& a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4*e^16*f^6 - \\
& 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b*c^6*e^16*f^6 \\
& + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30*a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6)))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 \\
& - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 \\
& 3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + \\
& 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9)) \\
& /((a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c \\
& *e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(4*a*c - b^2)^(3/2) \\
&) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b \\
& *c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17*f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 \\
& + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9))/(a^3*e*f^3* \\
& (4*a*c - b^2)^(3/2)*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2* \\
& f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*e*f^3*(\\
& 4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7*e*f^3 - 12*a*b^5 \\
& *c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17*f^9 - 12*a^8*b^6*c^3*e^17*f^9 + 48*a^9*b^4*c^4*e^17*f^9 - 64*a^10*b^2*c^5*e^17 \\
& *f^9 + 640*a^10*b*c^6*d^2*e^17*f^9 + 3*a^6*b^9*c^2*d^2*e^17*f^9 - 46*a^7*b^7*c^3*d^2*e^17*f^9 + 264*a^8*b^5*c^4*d^2*e^17*f^9 - 672*a^9*b^3*c^5*d^2*e^17*f^9)) \\
& /((2*a^6*e^2*f^6*(4*a*c - b^2)^3*(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12* \\
& a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + \\
& 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 2 \\
& 88*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*(((4*(36*a^6*c^7*e^15 \\
& *f^3 + 4*a^2*b^8*c^3*e^15*f^3 - 45*a^3*b^6*c^4*e^15*f^3 + 170*a^4*b^4*c^5*e^15*f^3 - 225*a^5*b^2*c^6*e^15*f^3 - 276*a^5*b*c^7*d^2*e^15*f^3 + 6*a^2*b^7 \\
& *c^4*d^2*e^15*f^3 - 65*a^3*b^5*c^5*d^2*e^15*f^3 + 233*a^4*b^3*c^6*d^2*e^15* \\
& f^3)))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) \\
&) - (((4*(2*a^4*b^9*c^2*e^16*f^6 - 26*a^5*b^7*c^3*e^16*f^6 + 118*a^6*b^5*c^4 \\
& *e^16*f^6 - 208*a^7*b^3*c^5*e^16*f^6 - 480*a^8*c^7*d^2*e^16*f^6 + 96*a^8*b \\
& *c^6*e^16*f^6 + a^4*b^8*c^3*d^2*e^16*f^6 - 6*a^5*b^6*c^4*d^2*e^16*f^6 - 30* \\
& a^6*b^4*c^5*d^2*e^16*f^6 + 272*a^7*b^2*c^6*d^2*e^16*f^6)))/(a^6*b^6*f^9 - 64 \\
& *a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) + (2*(b^7*e*f^3 - 12* \\
& a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(a^7*b^8*c^2*e^17
\end{aligned}$$

$$\begin{aligned}
& 7f^9 - 12a^8b^6c^3e^{17f^9} + 48a^9b^4c^4e^{17f^9} - 64a^{10}b^2c^5e^{17f^9} + 640a^{10}b^6c^6d^2e^{17f^9} + 3a^6b^9c^2d^2e^{17f^9} - 46a^7b^7c^3d^2e^{17f^9} + 264a^8b^5c^4d^2e^{17f^9} - 672a^9b^3c^5d^2e^{17f^9}) / ((a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^3f^9 + 48a^8b^2c^2f^9) * (a^3b^6e^{2f^6} - 64a^6c^3e^{2f^6} + 48a^5b^2c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6})) * (b^7e^{f^3} - 12a^5c^3e^{f^3} - 64a^3b^3c^3e^{f^3} + 48a^2b^3c^2e^{f^3})) / (2 * (a^3b^6e^{2f^6} - 64a^6c^3e^{2f^6} + 48a^5b^2c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6})) * (b^4 + 6a^2c^2 - 6a^2b^2c)) / (2 * a^3e^{f^3} * (4a^2c - b^2)^{(3/2)}) - (((((4 * (2a^4b^9c^2e^{16f^6} - 26a^5b^7c^3e^{16f^6} + 118a^6b^5c^4e^{16f^6} - 208a^7b^3c^5e^{16f^6} - 480a^8c^7d^2e^{16f^6} + 96a^8b^6c^6e^{16f^6} + a^4b^8c^3d^2e^{16f^6} - 6a^5b^6c^4d^2e^{16f^6} - 30a^6b^4c^5d^2e^{16f^6} + 272a^7b^2c^6d^2e^{16f^6)) / (a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^3f^9 + 48a^8b^2c^2f^9) + (2 * (b^7e^{f^3} - 12a^5c^3e^{f^3} - 64a^3b^3c^3e^{f^3} + 48a^2b^3c^2e^{f^3})) * (a^7b^8c^2e^{17f^9} - 12a^8b^6c^3e^{17f^9} + 48a^9b^4c^4e^{17f^9} - 64a^{10}b^2c^5e^{17f^9} + 640a^{10}b^6c^6d^2e^{17f^9} + 3a^6b^9c^2d^2e^{17f^9} - 46a^7b^7c^3d^2e^{17f^9} + 264a^8b^5c^4d^2e^{17f^9} - 672a^9b^3c^5d^2e^{17f^9})) / ((a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^3f^9 + 48a^8b^2c^2f^9) * (a^3b^6e^{2f^6} - 64a^6c^3e^{2f^6} + 48a^5b^2c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6})) * (b^4 + 6a^2c^2 - 6a^2b^2c)) / (2 * a^3e^{f^3} * (4a^2c - b^2)^{(3/2)}) + ((b^4 + 6a^2c^2 - 6a^2b^2c) * (b^7e^{f^3} - 12a^5c^3e^{f^3} - 64a^3b^3c^3e^{f^3} + 48a^2b^3c^2e^{f^3})) * (a^7b^8c^2e^{17f^9} - 12a^8b^6c^3e^{17f^9} + 48a^9b^4c^4e^{17f^9} - 64a^{10}b^2c^5e^{17f^9} + 640a^{10}b^6c^6d^2e^{17f^9} + 3a^6b^9c^2d^2e^{17f^9} - 46a^7b^7c^3d^2e^{17f^9} + 264a^8b^5c^4d^2e^{17f^9} - 672a^9b^3c^5d^2e^{17f^9})) / (a^3e^{f^3} * (4a^2c - b^2)^{(3/2)} * (a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^3f^9 + 48a^8b^2c^2f^9) * (a^3b^6e^{2f^6} - 64a^6c^3e^{2f^6} + 48a^5b^2c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6})) * (b^7e^{f^3} - 12a^5c^3e^{f^3} - 64a^3b^3c^3e^{f^3} + 48a^2b^3c^2e^{f^3})) / (2 * (a^3b^6e^{2f^6} - 64a^6c^3e^{2f^6} + 48a^5b^2c^2e^{2f^6} - 12a^4b^4c^2e^{2f^6})) + ((b^4 + 6a^2c^2 - 6a^2b^2c)^3 * (a^7b^8c^2e^{17f^9} - 12a^8b^6c^3e^{17f^9} + 48a^9b^4c^4e^{17f^9} - 64a^{10}b^2c^5e^{17f^9} + 640a^{10}b^6c^6d^2e^{17f^9} + 3a^6b^9c^2d^2e^{17f^9} - 46a^7b^7c^3d^2e^{17f^9} + 264a^8b^5c^4d^2e^{17f^9} - 672a^9b^3c^5d^2e^{17f^9})) / (2 * a^9e^{3f^9} * (4a^2c - b^2)^{(9/2)} * (a^6b^6f^9 - 64a^9c^3f^9 - 12a^7b^4c^3f^9 + 48a^8b^2c^2f^9)) * (3b^6 - 49a^3c^3 + 72a^2b^2c^2 - 27a^2b^4c)) / (8 * a^3c^2 * (4a^2c - b^2)^{(7/2)} * (9a^4c^4 - 6b^8 - 288a^2b^4c^2 + 382a^3b^2c^3 + 72a^2b^6c)) / ((36a^4c^6e^{14} + b^8c^2e^{14} - 12a^6b^6c^3e^{14} + 48a^2b^4c^4e^{14} - 72a^3b^2c^5e^{14})) * (b^4 + 6a^2c^2 - 6a^2b^2c)) / (a^3e^{f^3} * (4a^2c - b^2)^{(3/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

$$3.540 \quad \int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal. Leaf size=423

$$\frac{b(5b^2 - 19ac)}{2a^3ef^4(b^2 - 4ac)(d + ex)} - \frac{5b^2 - 14ac}{6a^2ef^4(b^2 - 4ac)(d + ex)^3} + \frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c + b(5b^2 - 19ac) \sqrt{b^2 - 4ac} + 5b \right)}{2\sqrt{2}a^3ef^4(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 3.55, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1142, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c + b(5b^2 - 19ac) \sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b^2 - 4ac}} \right)}{2\sqrt{2}a^3ef^4(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c - b(5b^2 - 19ac) \sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b^2 - 4ac} + b} \right)}{2\sqrt{2}a^3ef^4(b^2 - 4ac)^{3/2} \sqrt{b^2 - 4ac} + b} + \frac{b(5b^2 - 19ac)}{2a^3ef^4(b^2 - 4ac)(d + ex)} - \frac{5b^2 - 14ac}{6a^2ef^4(b^2 - 4ac)(d + ex)^3} + \frac{-2ac + b^2 + bc(d + ex)^2}{2aef^4(b^2 - 4ac)(d + ex)^3(a + b(d + ex)^2 + c(d + ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] $-(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3) + (b*(5*b^2 - 19*a*c))/(2*a^3*(b^2 - 4*a*c)*e*f^4*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (\text{Sqrt}[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + e*x))/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a^3*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]*e*f^4) - (\text{Sqrt}[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + e*x))/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a^3*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]*e*f^4)$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1121

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(df + efx)^4 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx &= \frac{\text{Subst} \left(\int \frac{1}{x^4 (a + bx^2 + cx^4)^2} dx, x, d + ex \right)}{ef^4} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{2a (b^2 - 4ac) ef^4 (d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{5b^2 - 14ac}{6a^2 (b^2 - 4ac) ef^4 (d + ex)^3} + \frac{b^2 - 2ac + bc(d + ex)^2}{2a (b^2 - 4ac) ef^4 (d + ex)^3 (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{5b^2 - 14ac}{6a^2 (b^2 - 4ac) ef^4 (d + ex)^3} + \frac{b(5b^2 - 19ac)}{2a^3 (b^2 - 4ac) ef^4 (d + ex)^3} \\
&= -\frac{5b^2 - 14ac}{6a^2 (b^2 - 4ac) ef^4 (d + ex)^3} + \frac{b(5b^2 - 19ac)}{2a^3 (b^2 - 4ac) ef^4 (d + ex)^3} \\
&= -\frac{5b^2 - 14ac}{6a^2 (b^2 - 4ac) ef^4 (d + ex)^3} + \frac{b(5b^2 - 19ac)}{2a^3 (b^2 - 4ac) ef^4 (d + ex)^3}
\end{aligned}$$

Mathematica [A] time = 3.02, size = 387, normalized size = 0.91

$$\frac{6(d+ex)(2a^2c^2-4ab^2c-3abc^2(d+ex)^2+b^4+b^3c(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} + \frac{3\sqrt{2}\sqrt{c}(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}(-28a^2c^2+29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}-5b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{4a}{(d+ex)^3} + \frac{24b}{d+ex}$$

$12a^3ef^4$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]

[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) + (3*sqrt[2]*sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*sqrt[b^2 - 4*a*c] - 19*a*b*c*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) + (3*sqrt[2]*sqrt[c]*(-5*b^4 + 29*a*b^2*c - 28*a^2*c^2 + 5*b^3*sqrt[b^2 - 4*a*c] - 19*a*b*c*sqrt[b^2 - 4*a*c])*ArcTan

$$\frac{(\sqrt{2} \sqrt{c} (d + ex)) / \sqrt{b + \sqrt{b^2 - 4ac}}}{(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} / (12a^3 e f^4)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)^4 (a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2), x]

fricas [B] time = 2.13, size = 5954, normalized size = 14.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/12*(6*(5*b^3*c - 19*a*b*c^2)*e^6*x^6 + 36*(5*b^3*c - 19*a*b*c^2)*d*e^5*x^5 \\ & + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45*(5*b^3*c - 19*a*b*c^2)*d^2)*e^4*x^4 \\ & + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 8*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (15*b^4 \\ & - 62*a*b^2*c + 14*a^2*c^2)*d)*e^3*x^3 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^4 \\ & + 2*(45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10*a*b^3 - 40*a^2*b*c + 6*(15*b^4 - 62*a*b^2*c \\ & + 14*a^2*c^2)*d^2)*e^2*x^2 - 4*a^2*b^2 + 16*a^3*c + 20*(a*b^3 - 4*a^2*b*c)*d^2 \\ & + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^3 \\ & + 10*(a*b^3 - 4*a^2*b*c)*d)*e*x - 3*\sqrt{1/2}*(a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 \\ & + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 \\ & + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 \\ & - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 \\ & + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3*f^4*x^2 \\ & + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x \\ & + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4 \\ & * \sqrt{-(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8 * \sqrt{(625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)} / ((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)} + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4) / ((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8) * \log((1125*b^8*c^4 \end{aligned}$$

$$\begin{aligned}
& - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)* \\
& e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c \\
& ^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9* \\
& b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12 \\
& *sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 7 \\
& 6686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b \\
& ^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) - (125*b^14 - 2425*a*b^12* \\
& c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^ \\
& 5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12* \\
& a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b \\
& ^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a \\
& ^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 6 \\
& 4*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3 \\
& *b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a \\
& ^10*c^3)*e^2*f^8)) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7* \\
& (a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c \\
& - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b \\
& ^3 - 4*a^4*b*c)*d)*e^5*f^4*x^4 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4 \\
& *c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*e^4*f^4*x^3 + (21*(a^3*b^2*c - 4* \\
& a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*e^3* \\
& f^4*x^2 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3* \\
& (a^4*b^2 - 4*a^5*c)*d^2)*e^2*f^4*x + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^ \\
& 3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e*f^4)*sqrt(-((a^7*b^6 - 12*a \\
& ^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8*sqrt((625*b^12 - 8250*a*b \\
& ^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^ \\
& 5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64 \\
& *a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3* \\
& b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^ \\
& 10*c^3)*e^2*f^8))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - \\
& 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*e*x + (1125*b^8*c^4 - 12325*a*b^6*c^5 + \\
& 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5 \\
& *a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11* \\
& b^3*c^4 - 3328*a^12*b*c^5)*e^3*f^12*sqrt((625*b^12 - 8250*a*b^10*c + 39525* \\
& a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2 \\
& 401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^ \\
& 4*f^16)) - (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c \\
& ^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^ \\
& 7*c^7)*e*f^4)*sqrt(-((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3 \\
&)*e^2*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^ \\
& 6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - \\
& 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*e^4*f^16)) + 25*b^9 - 315*a* \\
& b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)/((a^7*b^6 - 1 \\
& 2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*e^2*f^8)) + 3*sqrt(1/2)*((a^3* \\
& b^2*c - 4*a^4*c^2)*e^8*f^4*x^7 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*e^7*f^4*x^6 + \\
& (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*e^6*f^4*x^5 + 5*(7*(
\end{aligned}$$

$$\begin{aligned}
& a^3 b^2 c - 4 a^4 c^2) d^3 + (a^3 b^3 - 4 a^4 b c) d) e^5 f^4 x^4 + (a^4 b^2 - 4 a^5 c + 35(a^3 b^2 c - 4 a^4 c^2) d^4 + 10(a^3 b^3 - 4 a^4 b c) d^2) \\
& e^4 f^4 x^3 + (21(a^3 b^2 c - 4 a^4 c^2) d^5 + 10(a^3 b^3 - 4 a^4 b c) d^3 + 3(a^4 b^2 - 4 a^5 c) d) e^3 f^4 x^2 + (7(a^3 b^2 c - 4 a^4 c^2) d^6 \\
& + 5(a^3 b^3 - 4 a^4 b c) d^4 + 3(a^4 b^2 - 4 a^5 c) d^2) e^2 f^4 x + ((a^3 b^2 c - 4 a^4 c^2) d^7 + (a^3 b^3 - 4 a^4 b c) d^5 + (a^4 b^2 - 4 a^5 c) \\
& d^3) e f^4) \sqrt{((a^7 b^6 - 12 a^8 b^4 c + 48 a^9 b^2 c^2 - 64 a^{10} c^3) e^2 f^8 \sqrt{(625 b^{12} - 8250 a b^{10} c + 39525 a^2 b^8 c^2 - 83630 a^3 b^6 c^3 + 76686 a^4 b^4 c^4 - 24108 a^5 b^2 c^5 + 2401 a^6 c^6) / ((a^{14} b^6 - 12 \\
& a^{15} b^4 c + 48 a^{16} b^2 c^2 - 64 a^{17} c^3) e^4 f^{16})) - 25 b^9 + 315 a b^7 c - 1386 a^2 b^5 c^2 + 2415 a^3 b^3 c^3 - 1260 a^4 b c^4) / ((a^7 b^6 - 12 a^8 b^4 c + 48 a^9 b^2 c^2 - 64 a^{10} c^3) e^2 f^8)) * \log((1125 b^8 c^4 - 123 \\
& 25 a b^6 c^5 + 43410 a^2 b^4 c^6 - 50421 a^3 b^2 c^7 + 9604 a^4 c^8) e^x + (1125 b^8 c^4 - 12325 a b^6 c^5 + 43410 a^2 b^4 c^6 - 50421 a^3 b^2 c^7 + 9 \\
& 604 a^4 c^8) d + 1/2 \sqrt{1/2} * ((5 a^7 b^{11} - 94 a^8 b^9 c + 700 a^9 b^7 c^2 - 2576 a^{10} b^5 c^3 + 4672 a^{11} b^3 c^4 - 3328 a^{12} b c^5) e^3 f^{12} \sqrt{(625 b^{12} - 8250 a b^{10} c + 39525 a^2 b^8 c^2 - 83630 a^3 b^6 c^3 + 76686 a^4 b^4 c^4 - 24108 a^5 b^2 c^5 + 2401 a^6 c^6) / ((a^{14} b^6 - 12 a^{15} b^4 c + 48 a^{16} b^2 c^2 - 64 a^{17} c^3) e^4 f^{16})) + (125 b^{14} - 2425 a b^{12} c + 18 \\
& 940 a^2 b^{10} c^2 - 75579 a^3 b^8 c^3 + 160932 a^4 b^6 c^4 - 172990 a^5 b^4 c^5 + 79408 a^6 b^2 c^6 - 10976 a^7 c^7) e f^4) \sqrt{((a^7 b^6 - 12 a^8 b^4 c + 48 a^9 b^2 c^2 - 64 a^{10} c^3) e^2 f^8 \sqrt{(625 b^{12} - 8250 a b^{10} c + 39525 a^2 b^8 c^2 - 83630 a^3 b^6 c^3 + 76686 a^4 b^4 c^4 - 24108 a^5 b^2 c^5 + 2401 a^6 c^6) / ((a^{14} b^6 - 12 a^{15} b^4 c + 48 a^{16} b^2 c^2 - 64 a^{17} c^3) e^4 f^{16})) - 25 b^9 + 315 a b^7 c - 1386 a^2 b^5 c^2 + 2415 a^3 b^3 c^3 - 1260 a^4 b c^4) / ((a^7 b^6 - 12 a^8 b^4 c + 48 a^9 b^2 c^2 - 64 a^{10} c^3) e^2 f^8))} - 3 \sqrt{1/2} * ((a^3 b^2 c - 4 a^4 c^2) e^8 f^4 x^7 + 7(a^3 b^2 c - 4 a^4 c^2) d^2) e^6 f^4 x^5 + 5(7(a^3 b^2 c - 4 a^4 c^2) d^3 + (a^3 b^3 - 4 a^4 b c) d) e^5 f^4 x^4 + (a^4 b^2 - 4 a^5 c + 35(a^3 b^2 c - 4 a^4 c^2) d^4 + 10(a^3 b^3 - 4 a^4 b c) d^2) e^4 f^4 x^3 + (21(a^3 b^2 c - 4 a^4 c^2) d^5 + 10(a^3 b^3 - 4 a^4 b c) d^3 + 3(a^4 b^2 - 4 a^5 c) d) e^3 f^4 x^2 + (7(a^3 b^2 c - 4 a^4 c^2) d^6 + 5(a^3 b^3 - 4 a^4 b c) d^4 + 3(a^4 b^2 - 4 a^5 c) d^2) e^2 f^4 x + ((a^3 b^2 c - 4 a^4 c^2) d^7 + (a^3 b^3 - 4 a^4 b c) d^5 + (a^4 b^2 - 4 a^5 c) d^3) e f^4) \sqrt{((a^7 b^6 - 12 a^8 b^4 c + 48 a^9 b^2 c^2 - 64 a^{10} c^3) e^2 f^8 \sqrt{(625 b^{12} - 8250 a b^{10} c + 39525 a^2 b^8 c^2 - 83630 a^3 b^6 c^3 + 76686 a^4 b^4 c^4 - 24108 a^5 b^2 c^5 + 2401 a^6 c^6) / ((a^{14} b^6 - 12 a^{15} b^4 c + 48 a^{16} b^2 c^2 - 64 a^{17} c^3) e^4 f^{16})) - 25 b^9 + 315 a b^7 c - 1386 a^2 b^5 c^2 + 2415 a^3 b^3 c^3 - 1260 a^4 b c^4) / ((a^7 b^6 - 12 a^8 b^4 c + 48 a^9 b^2 c^2 - 64 a^{10} c^3) e^2 f^8)) * \log((1125 b^8 c^4 - 12325 a b^6 c^5 + 43410 a^2 b^4 c^6 - 50421 a^3 b^2 c^7 + 9604 a^4 c^8) e^x + (1125 b^8 c^4 - 12325 a b^6 c^5 + 43410 a^2 b^4 c^6 - 50421 a^3 b^2 c^7 + 9604 a^4 c^8) d - 1/2 \sqrt{1/2} * ((5 a^7 b^{11} - 94 a^8 b^9 c + 700 a^9 b^7 c^2 - 2576 a^{10} b^5 c^3 + 4672 a^{11} b^3 c^4 - 3328 a^{12} b c^5) e^3 f^{12} \sqrt{(625 b^{12} - 8250 a b^{10} c + 39525 a^2 b^8 c^2 - 83630 a^3 b^6 c^3 + 76686 a^4 b^4 c^4 - 24108 a^5 b^2 c^5 + 2401 a^6 c^6) / ((a^{14} b^6 - 12 a^{15} b^4 c + 48 a^{16} b^2 c^2 - 64 a^{17} c^3) e^4 f^{16})) - 25 b^9 + 315 a b^7 c - 1386 a^2 b^5 c^2 + 2415 a^3 b^3 c^3 - 1260 a^4 b c^4) / ((a^7 b^6 - 12 a^8 b^4 c + 48 a^9 b^2 c^2 - 64 a^{10} c^3) e^2 f^8))}
\end{aligned}$$

$$\begin{aligned} & c^2 - 83630a^3b^6c^3 + 76686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6) / ((a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3)e^{4f^{16}}) \\ & + (125b^{14} - 2425a^2b^{12}c + 18940a^2b^{10}c^2 - 75579a^3b^8c^3 + 160932a^4b^6c^4 - 172990a^5b^4c^5 + 79408a^6b^2c^6 - 10976a^7c^7)e \\ & * f^4) * \sqrt{((a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3)e^{2f^8} \\ & * \sqrt{(625b^{12} - 8250a^2b^{10}c + 39525a^2b^8c^2 - 83630a^3b^6c^3 + 7 \\ & 6686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6) / ((a^{14}b^6 - 12a^{15}b^4c \\ & + 48a^{16}b^2c^2 - 64a^{17}c^3)e^{4f^{16}}) - 25b^9 + 315a^2b^7c - 1 \\ & 386a^2b^5c^2 + 2415a^3b^3c^3 - 1260a^4b^2c^4) / ((a^7b^6 - 12a^8b^4c \\ & + 48a^9b^2c^2 - 64a^{10}c^3)e^{2f^8}))} / ((a^3b^2c - 4a^4c^2)e^8 \\ & * f^4 * x^7 + 7(a^3b^2c - 4a^4c^2) * d * e^7 * f^4 * x^6 + (a^3b^3 - 4a^4b^2c + \\ & 21(a^3b^2c - 4a^4c^2) * d^2) * e^6 * f^4 * x^5 + 5(7(a^3b^2c - 4a^4c^2) \\ & * d^3 + (a^3b^3 - 4a^4b^2c) * d) * e^5 * f^4 * x^4 + (a^4b^2 - 4a^5c + 35(a^3b^2c \\ & - 4a^4c^2) * d^4 + 10(a^3b^3 - 4a^4b^2c) * d^2) * e^4 * f^4 * x^3 + (21(a^3b^2c \\ & - 4a^4c^2) * d^5 + 10(a^3b^3 - 4a^4b^2c) * d^3 + 3(a^4b^2 - 4a^5c) * d) * e^3 * f^4 * x^2 + \\ & (7(a^3b^2c - 4a^4c^2) * d^6 + 5(a^3b^3 - 4a^4b^2c) * d^4 + 3(a^4b^2 - 4a^5c) * d^2) * e^2 * f^4 * x + \\ & ((a^3b^2c - 4a^4c^2) * d^7 + (a^3b^3 - 4a^4b^2c) * d^5 + (a^4b^2 - 4a^5c) * d^3) * e * f^4) \end{aligned}$$

giac [B] time = 0.71, size = 2002, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/4 * ((5 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & ^2 * b^3 * c * e^2 - 19 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & ^2 * a * b * c^2 * e^2 - 10 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & * b^3 * c * d * e + 38 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & * a * b * c^2 * d * e + 5 * b^3 * c * d^2 - 19 * a * b * c^2 * d^2 + 5 * b^4 - 24 * a * b^2 * c + 14 * a^2 * c^2) * \log(d * e^{-1} + x + \sqrt{1/2} \\ & * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) / (2 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & ^3 * c * e^4 - 6 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & ^2 * c * d * e^3 - 2 * c * d^3 * e - b * d * e + (6 * c * d^2 * e^2 + b * e^2) * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ &) + (5 * (d * e^{-1}) - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & ^2 * b^3 * c * e^2 - 19 * (d * e^{-1}) - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & ^2 * a * b * c^2 * e^2 - 10 * (d * e^{-1}) - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & * b^3 * c * d * e + 38 * (d * e^{-1}) - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \\ & * a * b * c^2 * d * e + 5 * b^3 * c * d^2 - 19 * a * b * c^2 * d^2 + 5 * b^4 - 24 * a * b^2 * c + 14 * a^2 * c^2) \\ & * \log(d * e^{-1} + x - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) / (2 * (d * e^{-1}) - \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) \end{aligned}$$

$$\frac{4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b*c^2*d^2-3/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^3*c*d^2-1/f^4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^2*c-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)/(4*a*c-b^2)*x*b^4+3/2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b*c^2-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d^3/e/(4*a*c-b^2)*b^3*c-1/f^4/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*c^2+2/f^4/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^2*c-1/2/f^4/a^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)*d/e/(4*a*c-b^2)*b^4+1/4/f^4/a^3/(4*a*c-b^2)/e*sum(((19*a*c-5*b^2)*_R^2*b*c*e^2+19*a*b*c^2*d^2-5*b^3*c*d^2+2*(19*a*c-5*b^2)*_R*b*c*d*e-14*a^2*c^2+24*a*b^2*c-5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))-1/3/f^4/a^2/e/(e*x+d)^3+2/f^4/a^3*b/e/(e*x+d)$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 10.45, size = 13781, normalized size = 32.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)

[Out] atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)

$$\begin{aligned}
& 9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a \\
& *b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12}*e^2*f^8 + 4096*a^{13}*c^6*e^2* \\
& f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^{10}*b^6*c^3*e^2*f^8 + 3840*a^{11}*b^4*c \\
& ^4*e^2*f^8 - 6144*a^{12}*b^2*c^5*e^2*f^8 - 24*a^8*b^{10}*c*e^2*f^8)))^{(1/2)}*(x* \\
& (256*a^{15}*b^{13}*c^2*e^{14}*f^{20} - 6144*a^{16}*b^{11}*c^3*e^{14}*f^{20} + 61440*a^{17}*b^ \\
& 9*c^4*e^{14}*f^{20} - 327680*a^{18}*b^7*c^5*e^{14}*f^{20} + 983040*a^{19}*b^5*c^6*e^{14}* \\
& f^{20} - 1572864*a^{20}*b^3*c^7*e^{14}*f^{20} + 1048576*a^{21}*b*c^8*e^{14}*f^{20}) + 104 \\
& 8576*a^{21}*b*c^8*d*e^{13}*f^{20} + 256*a^{15}*b^{13}*c^2*d*e^{13}*f^{20} - 6144*a^{16}*b^1 \\
& 1*c^3*d*e^{13}*f^{20} + 61440*a^{17}*b^9*c^4*d*e^{13}*f^{20} - 327680*a^{18}*b^7*c^5*d* \\
& e^{13}*f^{20} + 983040*a^{19}*b^5*c^6*d*e^{13}*f^{20} - 1572864*a^{20}*b^3*c^7*d*e^{13}*f \\
& ^{20}) + 917504*a^{19}*c^9*e^{12}*f^{16} - 320*a^{12}*b^{14}*c^2*e^{12}*f^{16} + 7936*a^{13}* \\
& b^{12}*c^3*e^{12}*f^{16} - 82816*a^{14}*b^{10}*c^4*e^{12}*f^{16} + 468480*a^{15}*b^8*c^5*e^ \\
& 12*f^{16} - 1536000*a^{16}*b^6*c^6*e^{12}*f^{16} + 2867200*a^{17}*b^4*c^7*e^{12}*f^{16} - \\
& 2719744*a^{18}*b^2*c^8*e^{12}*f^{16}) - x*(401408*a^{16}*c^{10}*e^{12}*f^{12} - 400*a^9* \\
& b^{14}*c^3*e^{12}*f^{12} + 9440*a^{10}*b^{12}*c^4*e^{12}*f^{12} - 92816*a^{11}*b^{10}*c^5*e^1 \\
& 2*f^{12} + 488096*a^{12}*b^8*c^6*e^{12}*f^{12} - 1458688*a^{13}*b^6*c^7*e^{12}*f^{12} + 2 \\
& 401280*a^{14}*b^4*c^8*e^{12}*f^{12} - 1871872*a^{15}*b^2*c^9*e^{12}*f^{12}) - 401408*a^ \\
& 16*c^{10}*d*e^{11}*f^{12} + 400*a^9*b^{14}*c^3*d*e^{11}*f^{12} - 9440*a^{10}*b^{12}*c^4*d*e \\
& ^{11}*f^{12} + 92816*a^{11}*b^{10}*c^5*d*e^{11}*f^{12} - 488096*a^{12}*b^8*c^6*d*e^{11}*f^1 \\
& 2 + 1458688*a^{13}*b^6*c^7*d*e^{11}*f^{12} - 2401280*a^{14}*b^4*c^8*d*e^{11}*f^{12} + 1 \\
& 871872*a^{15}*b^2*c^9*d*e^{11}*f^{12})*i)/((- (25*b^{15} - 25*b^6*(-(4*a*c - b^2)^9) \\
&)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a \\
& a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12}*e^2*f^8 + 4096*a^{13}*c^6*e^2*f^8 \\
& + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^{10}*b^6*c^3*e^2*f^8 + 3840*a^{11}*b^4*c^4*e^2*f^8 \\
& - 6144*a^{12}*b^2*c^5*e^2*f^8 - 24*a^8*b^{10}*c*e^2*f^8)))^{(1/2)}*((-(25*b^{15} - 25*b^6*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 \\
& + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3 \\
& *(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9) \\
&)^{(1/2)} + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12}*e^2*f^8 + 40 \\
& 96*a^{13}*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^{10}*b^6*c^3*e^2*f^8 + \\
& 3840*a^{11}*b^4*c^4*e^2*f^8 - 6144*a^{12}*b^2*c^5*e^2*f^8 - 24*a^8*b^{10}*c*e^2* \\
& f^8)))^{(1/2)}*(x*(256*a^{15}*b^{13}*c^2*e^{14}*f^{20} - 6144*a^{16}*b^{11}*c^3*e^{14}*f^{20} \\
& + 61440*a^{17}*b^9*c^4*e^{14}*f^{20} - 327680*a^{18}*b^7*c^5*e^{14}*f^{20} + 983040*a^ \\
& 19*b^5*c^6*e^{14}*f^{20} - 1572864*a^{20}*b^3*c^7*e^{14}*f^{20} + 1048576*a^{21}*b*c^8* \\
& e^{14}*f^{20}) + 1048576*a^{21}*b*c^8*d*e^{13}*f^{20} + 256*a^{15}*b^{13}*c^2*d*e^{13}*f^{20} \\
& - 6144*a^{16}*b^{11}*c^3*d*e^{13}*f^{20} + 61440*a^{17}*b^9*c^4*d*e^{13}*f^{20} - 327680
\end{aligned}$$

$$\begin{aligned}
& *a^{18}b^7c^5d^5e^{13}f^{20} + 983040a^{19}b^5c^6d^5e^{13}f^{20} - 1572864a^{20}b^3c^7d^5e^{13}f^{20}) - 917504a^{19}c^9e^{12}f^{16} + 320a^{12}b^{14}c^2e^{12}f^{16} \\
& - 7936a^{13}b^{12}c^3e^{12}f^{16} + 82816a^{14}b^{10}c^4e^{12}f^{16} - 468480a^{15}b^8c^5e^{12}f^{16} + 1536000a^{16}b^6c^6e^{12}f^{16} - 2867200a^{17}b^4c^7e^{12}f^{16} \\
& + 2719744a^{18}b^2c^8e^{12}f^{16}) - x(401408a^{16}c^{10}e^{12}f^{12} - 400a^9b^{14}c^3e^{12}f^{12} + 9440a^{10}b^{12}c^4e^{12}f^{12} - 92816a^{11}b^{10}c^5e^{12}f^{12} \\
& + 488096a^{12}b^8c^6e^{12}f^{12} - 1458688a^{13}b^6c^7e^{12}f^{12} + 2401280a^{14}b^4c^8e^{12}f^{12} - 1871872a^{15}b^2c^9e^{12}f^{12}) - 401408a^{16}c^{10}d^5e^{11}f^{12} + 400a^9b^{14}c^3d^5e^{11}f^{12} - 9440a^{10}b^{12}c^4d^5e^{11}f^{12} \\
& + 92816a^{11}b^{10}c^5d^5e^{11}f^{12} - 488096a^{12}b^8c^6d^5e^{11}f^{12} + 1458688a^{13}b^6c^7d^5e^{11}f^{12} - 2401280a^{14}b^4c^8d^5e^{11}f^{12} + 1871872a^{15}b^2c^9d^5e^{11}f^{12}) - ((25b^{15} - 25b^6(-4ac - b^2)^9)^{(1/2)} - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 + 49a^3c^3(-4ac - b^2)^9)^{(1/2)} - 615a^2b^{13}c - 246a^2b^2c^2(-4ac - b^2)^9)^{(1/2)} + 165a^4b^4c^4(-4ac - b^2)^9)^{(1/2)})/(32(a^7b^{12}e^{2}f^8 + 4096a^{13}c^6e^{2}f^8 + 240a^9b^8c^2e^{2}f^8 - 1280a^{10}b^6c^3e^{2}f^8 + 3840a^{11}b^4c^4e^{2}f^8 - 6144a^{12}b^2c^5e^{2}f^8 - 24a^8b^{10}c^8e^{2}f^8)))^{(1/2)} * (((-25b^{15} - 25b^6(-4ac - b^2)^9)^{(1/2)} - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 + 49a^3c^3(-4ac - b^2)^9)^{(1/2)} - 615a^2b^{13}c - 246a^2b^2c^2(-4ac - b^2)^9)^{(1/2)} + 165a^4b^4c^4(-4ac - b^2)^9)^{(1/2)})/(32(a^7b^{12}e^{2}f^8 + 4096a^{13}c^6e^{2}f^8 + 240a^9b^8c^2e^{2}f^8 - 1280a^{10}b^6c^3e^{2}f^8 + 3840a^{11}b^4c^4e^{2}f^8 - 6144a^{12}b^2c^5e^{2}f^8 - 24a^8b^{10}c^8e^{2}f^8)))^{(1/2)} * (((-25b^{15} - 25b^6(-4ac - b^2)^9)^{(1/2)} - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 + 49a^3c^3(-4ac - b^2)^9)^{(1/2)} - 615a^2b^{13}c - 246a^2b^2c^2(-4ac - b^2)^9)^{(1/2)} + 165a^4b^4c^4(-4ac - b^2)^9)^{(1/2)})/(32(a^7b^{12}e^{2}f^8 + 4096a^{13}c^6e^{2}f^8 + 240a^9b^8c^2e^{2}f^8 - 1280a^{10}b^6c^3e^{2}f^8 + 3840a^{11}b^4c^4e^{2}f^8 - 6144a^{12}b^2c^5e^{2}f^8 - 24a^8b^{10}c^8e^{2}f^8)))^{(1/2)} * (x(256a^{15}b^{13}c^2e^{14}f^{20} - 6144a^{16}b^{11}c^3e^{14}f^{20} + 61440a^{17}b^9c^4e^{14}f^{20} - 327680a^{18}b^7c^5e^{14}f^{20} - 0 + 983040a^{19}b^5c^6e^{14}f^{20} - 1572864a^{20}b^3c^7e^{14}f^{20} + 1048576a^{21}b^1c^8e^{14}f^{20}) + 1048576a^{21}b^1c^8d^5e^{13}f^{20} + 256a^{15}b^{13}c^2d^5e^{13}f^{20} - 6144a^{16}b^{11}c^3d^5e^{13}f^{20} + 61440a^{17}b^9c^4d^5e^{13}f^{20} - 327680a^{18}b^7c^5d^5e^{13}f^{20} + 983040a^{19}b^5c^6d^5e^{13}f^{20} - 1572864a^{20}b^3c^7d^5e^{13}f^{20}) + 917504a^{19}c^9e^{12}f^{16} - 320a^{12}b^{14}c^2e^{12}f^{16} + 7936a^{13}b^{12}c^3e^{12}f^{16} - 82816a^{14}b^{10}c^4e^{12}f^{16} + 468480a^{15}b^8c^5e^{12}f^{16} - 1536000a^{16}b^6c^6e^{12}f^{16} + 2867200a^{17}b^4c^7e^{12}f^{16} - 2719744a^{18}b^2c^8e^{12}f^{16}) - x(401408a^{16}c^{10}e^{12}f^{12} - 400a^9b^{14}c^3e^{12}f^{12} + 9440a^{10}b^{12}c^4e^{12}f^{12} - 92816a^{11}b^{10}c^5e^{12}f^{12} + 488096a^{12}b^8c^6e^{12}f^{12} - 1458688a^{13}b^6c^7e^{12}f^{12} + 2401280a^{14}b^4c^8e^{12}f^{12} - 1871872a^{15}b^2c^9e^{12}f^{12}) - 401408a^{16}c^{10}d^5e^{11}f^{12} + 400a^9b^{14}c^3d^5e^{11}f^{12} - 9440a^{10}b^{12}c^4d^5e^{11}f^{12} + 92816a^{11}b^{10}c^5d^5e^{11}f^{12} - 488096a^{12}b^8c^6d^5e^{11}f^{12} + 1458688a^{13}b^6c^7d^5e^{11}f^{12} - 2401280a^{14}b^4c^8d^5e^{11}f^{12} + 1871872a^{15}b^2c^9d^5e^{11}f^{12})
\end{aligned}$$

$$\begin{aligned}
& f^{12} - 9440a^{10}b^{12}c^4d^4e^{11}f^{12} + 92816a^{11}b^{10}c^5d^4e^{11}f^{12} - 4 \\
& 88096a^{12}b^8c^6d^4e^{11}f^{12} + 1458688a^{13}b^6c^7d^4e^{11}f^{12} - 2401280 \\
& a^{14}b^4c^8d^4e^{11}f^{12} + 1871872a^{15}b^2c^9d^4e^{11}f^{12} + 1800a^9b^9 \\
& c^6e^{10}f^8 - 29080a^{10}b^7c^7e^{10}f^8 + 176032a^{11}b^5c^8e^{10}f^8 \\
& - 473216a^{12}b^3c^9e^{10}f^8 + 476672a^{13}b^3c^{10}e^{10}f^8) * (- (25b^{15} \\
& - 25b^6 * (- (4ac - b^2)^9)^{1/2} - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 3 \\
& 5767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3 \\
& c^6 + 49a^3c^3 * (- (4ac - b^2)^9)^{1/2} - 615a^2b^{13}c - 246a^2b^2c^2 \\
& * (- (4ac - b^2)^9)^{1/2} + 165a^2b^4c * (- (4ac - b^2)^9)^{1/2}) / (32 * (a^7b^{12} \\
& e^2f^8 + 4096a^{13}c^6e^2f^8 + 240a^9b^8c^2e^2f^8 - 1280a^{10}b^6 \\
& c^3e^2f^8 + 3840a^{11}b^4c^4e^2f^8 - 6144a^{12}b^2c^5e^2f^8 - 2 \\
& 4a^8b^{10}c^2e^2f^8))^{1/2} * 2i - ((x^4 * (15b^4e^3 + 14a^2c^2e^3 + 225 \\
& b^3cd^2e^3 - 62a^2b^2c^2e^3 - 855a^2b^2cd^2e^3)) / (6a^2(4a^3c - a^2b^2)) \\
& + (3x^5 * (5b^3cd^2e^4 - 19a^2b^2cd^2e^4)) / (a^2(4a^3c - a^2b^2)) \\
& + (2x^3 * (15b^4d^2e^2 + 14a^2c^2d^2e^2 + 75b^3cd^3e^2 - 62a^2b^2cd^2 \\
& e^2 - 285a^2b^2cd^3e^2)) / (3a^2(4a^3c - a^2b^2)) + (x * (30b^4d^3 + \\
& 45b^3cd^5 + 28a^2c^2d^3 + 10a^2b^3d - 40a^2b^2cd - 124a^2b^2cd^3 \\
& - 171a^2b^2cd^5)) / (3a^2(4a^3c - a^2b^2)) + (x^6 * (5b^3c^2e^5 - 19a^2b^2 \\
& c^2e^5)) / (2a^2(4a^3c - a^2b^2)) + (x^2 * (90b^4d^2e + 10a^2b^3e + 84 \\
& a^2c^2d^2e - 40a^2b^2c^2e + 225b^3cd^4e - 372a^2b^2cd^2e - 855a^2 \\
& b^2cd^4e)) / (6a^2(4a^3c - a^2b^2)) + (8a^3c - 2a^2b^2 + 15b^4d^4 \\
& + 10a^2b^3d^2 + 15b^3cd^6 + 14a^2c^2d^4 - 40a^2b^2cd^2 - 62a^2b^2 \\
& cd^4 - 57a^2b^2cd^6) / (6a^2 * (- (4ac - b^2)^9)^{1/2}) / (x * (3a^2d^2ef^4 + \\
& 5b^2d^4ef^4 + 7cd^6ef^4) + x^4 * (35cd^3e^4f^4 + 5b^2d^4ef^4) + x \\
& ^2 * (10b^2d^3e^2f^4 + 21cd^5e^2f^4 + 3a^2d^2ef^4) + x^5 * (b^2e^5f^4 + \\
& 21cd^2e^5f^4) + x^3 * (a^2e^3f^4 + 10b^2d^2e^3f^4 + 35cd^4e^3f^4) \\
& + a^2d^3f^4 + b^2d^5f^4 + cd^7f^4 + c^2e^7f^4 * x^7 + 7cd^6e^6f^4 * x^6) + \\
& \operatorname{atan}(((- (25b^{15} + 25b^6 * (- (4ac - b^2)^9)^{1/2} - 80640a^7b^7c^7 + 6366 \\
& a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 \\
& + 215040a^6b^3c^6 - 49a^3c^3 * (- (4ac - b^2)^9)^{1/2} - 615a^2b^{13}c \\
& + 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2} - 165a^2b^4c * (- (4ac - b^2)^9)^{1/2} \\
& ^{1/2}) / (32 * (a^7b^{12}e^2f^8 + 4096a^{13}c^6e^2f^8 + 240a^9b^8c^2e^2 \\
& f^8 - 1280a^{10}b^6c^3e^2f^8 + 3840a^{11}b^4c^4e^2f^8 - 6144a^{12}b^2 \\
& c^5e^2f^8 - 24a^8b^{10}c^2e^2f^8))^{1/2} * ((- (25b^{15} + 25b^6 * (- (4ac - \\
& b^2)^9)^{1/2} - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 \\
& + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 - 49a^3c^3 * \\
& (- (4ac - b^2)^9)^{1/2} - 615a^2b^{13}c + 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2} \\
& - 165a^2b^4c * (- (4ac - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^2f^8 + 4 \\
& 096a^{13}c^6e^2f^8 + 240a^9b^8c^2e^2f^8 - 1280a^{10}b^6c^3e^2f^8 \\
& + 3840a^{11}b^4c^4e^2f^8 - 6144a^{12}b^2c^5e^2f^8 - 24a^8b^{10}c^2 \\
& e^2f^8))^{1/2} * ((- (25b^{15} + 25b^6 * (- (4ac - b^2)^9)^{1/2} - 80640a^7b^7c^ \\
& ^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5 \\
& b^5c^5 + 215040a^6b^3c^6 - 49a^3c^3 * (- (4ac - b^2)^9)^{1/2} - 615a^2 \\
& b^{13}c + 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2} - 165a^2b^4c * (- (4ac - \\
& b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^2f^8 + 4096a^{13}c^6e^2f^8 + 240a^9b^8
\end{aligned}$$

$$\begin{aligned}
& 8*c^2*e^2*f^8 - 1280*a^{10}*b^6*c^3*e^2*f^8 + 3840*a^{11}*b^4*c^4*e^2*f^8 - 614 \\
& 4*a^{12}*b^2*c^5*e^2*f^8 - 24*a^8*b^{10}*c*e^2*f^8))^{(1/2)}*(x*(256*a^{15}*b^{13}*c \\
& ^2*e^{14}*f^{20} - 6144*a^{16}*b^{11}*c^3*e^{14}*f^{20} + 61440*a^{17}*b^9*c^4*e^{14}*f^{20} \\
& - 327680*a^{18}*b^7*c^5*e^{14}*f^{20} + 983040*a^{19}*b^5*c^6*e^{14}*f^{20} - 1572864*a \\
& ^{20}*b^3*c^7*e^{14}*f^{20} + 1048576*a^{21}*b*c^8*e^{14}*f^{20}) + 1048576*a^{21}*b*c^8* \\
& d*e^{13}*f^{20} + 256*a^{15}*b^{13}*c^2*d*e^{13}*f^{20} - 6144*a^{16}*b^{11}*c^3*d*e^{13}*f^{20} \\
& 0 + 61440*a^{17}*b^9*c^4*d*e^{13}*f^{20} - 327680*a^{18}*b^7*c^5*d*e^{13}*f^{20} + 9830 \\
& 40*a^{19}*b^5*c^6*d*e^{13}*f^{20} - 1572864*a^{20}*b^3*c^7*d*e^{13}*f^{20}) - 917504*a^{19} \\
& *c^9*e^{12}*f^{16} + 320*a^{12}*b^{14}*c^2*e^{12}*f^{16} - 7936*a^{13}*b^{12}*c^3*e^{12}*f^{16} \\
& + 82816*a^{14}*b^{10}*c^4*e^{12}*f^{16} - 468480*a^{15}*b^8*c^5*e^{12}*f^{16} + 153600 \\
& 0*a^{16}*b^6*c^6*e^{12}*f^{16} - 2867200*a^{17}*b^4*c^7*e^{12}*f^{16} + 2719744*a^{18}*b^2 \\
& *c^8*e^{12}*f^{16}) - x*(401408*a^{16}*c^{10}*e^{12}*f^{12} - 400*a^9*b^{14}*c^3*e^{12}*f^{12} \\
& + 9440*a^{10}*b^{12}*c^4*e^{12}*f^{12} - 92816*a^{11}*b^{10}*c^5*e^{12}*f^{12} + 488096*a \\
& ^{12}*b^8*c^6*e^{12}*f^{12} - 1458688*a^{13}*b^6*c^7*e^{12}*f^{12} + 2401280*a^{14}*b^4* \\
& c^8*e^{12}*f^{12} - 1871872*a^{15}*b^2*c^9*e^{12}*f^{12}) - 401408*a^{16}*c^{10}*d*e^{11}*f \\
& ^{12} + 400*a^9*b^{14}*c^3*d*e^{11}*f^{12} - 9440*a^{10}*b^{12}*c^4*d*e^{11}*f^{12} + 92816 \\
& *a^{11}*b^{10}*c^5*d*e^{11}*f^{12} - 488096*a^{12}*b^8*c^6*d*e^{11}*f^{12} + 1458688*a^{13} \\
& *b^6*c^7*d*e^{11}*f^{12} - 2401280*a^{14}*b^4*c^8*d*e^{11}*f^{12} + 1871872*a^{15}*b^2* \\
& c^9*d*e^{11}*f^{12})*i + ((- (25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640* \\
& a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 21 \\
& 9744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(- \\
& (4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12}*e^2*f^8 + 4096*a^{13}*c^6*e^2*f^8 + 240 \\
& *a^9*b^8*c^2*e^2*f^8 - 1280*a^{10}*b^6*c^3*e^2*f^8 + 3840*a^{11}*b^4*c^4*e^2*f^8 \\
& 8 - 6144*a^{12}*b^2*c^5*e^2*f^8 - 24*a^8*b^{10}*c*e^2*f^8))^{(1/2)}*((-(25*b^{15} \\
& + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 3 \\
& 5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3 \\
& *c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2 \\
& *(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7* \\
& b^{12}*e^2*f^8 + 4096*a^{13}*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^{10}* \\
& b^6*c^3*e^2*f^8 + 3840*a^{11}*b^4*c^4*e^2*f^8 - 6144*a^{12}*b^2*c^5*e^2*f^8 - 2 \\
& 4*a^8*b^{10}*c*e^2*f^8))^{(1/2)}*((-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7 \\
& *c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a \\
& *b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12}*e^2*f^8 + 4096*a^{13}*c^6*e^2* \\
& f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^{10}*b^6*c^3*e^2*f^8 + 3840*a^{11}*b^4*c \\
& ^4*e^2*f^8 - 6144*a^{12}*b^2*c^5*e^2*f^8 - 24*a^8*b^{10}*c*e^2*f^8))^{(1/2)}*(x* \\
& (256*a^{15}*b^{13}*c^2*e^{14}*f^{20} - 6144*a^{16}*b^{11}*c^3*e^{14}*f^{20} + 61440*a^{17}*b^ \\
& 9*c^4*e^{14}*f^{20} - 327680*a^{18}*b^7*c^5*e^{14}*f^{20} + 983040*a^{19}*b^5*c^6*e^{14}* \\
& f^{20} - 1572864*a^{20}*b^3*c^7*e^{14}*f^{20} + 1048576*a^{21}*b*c^8*e^{14}*f^{20}) + 104 \\
& 8576*a^{21}*b*c^8*d*e^{13}*f^{20} + 256*a^{15}*b^{13}*c^2*d*e^{13}*f^{20} - 6144*a^{16}*b^1 \\
& 1*c^3*d*e^{13}*f^{20} + 61440*a^{17}*b^9*c^4*d*e^{13}*f^{20} - 327680*a^{18}*b^7*c^5*d* \\
& e^{13}*f^{20} + 983040*a^{19}*b^5*c^6*d*e^{13}*f^{20} - 1572864*a^{20}*b^3*c^7*d*e^{13}*f \\
& ^{20}) + 917504*a^{19}*c^9*e^{12}*f^{16} - 320*a^{12}*b^{14}*c^2*e^{12}*f^{16} + 7936*a^{13}
\end{aligned}$$

$$\begin{aligned}
& b^{12}c^3e^{12}f^{16} - 82816a^{14}b^{10}c^4e^{12}f^{16} + 468480a^{15}b^8c^5e^{12}f^{16} - 1536000a^{16}b^6c^6e^{12}f^{16} + 2867200a^{17}b^4c^7e^{12}f^{16} - \\
& 2719744a^{18}b^2c^8e^{12}f^{16}) - x(401408a^{16}c^{10}e^{12}f^{12} - 400a^9b^{14}c^3e^{12}f^{12} + 9440a^{10}b^{12}c^4e^{12}f^{12} - 92816a^{11}b^{10}c^5e^{12}f^{12} - \\
& 2f^{12} + 488096a^{12}b^8c^6e^{12}f^{12} - 1458688a^{13}b^6c^7e^{12}f^{12} + 2401280a^{14}b^4c^8e^{12}f^{12} - 1871872a^{15}b^2c^9e^{12}f^{12}) - 401408a^{16}c^{10}d^{11}e^{11}f^{12} + \\
& 400a^9b^{14}c^3d^{11}e^{11}f^{12} - 9440a^{10}b^{12}c^4d^{11}e^{11}f^{12} + 92816a^{11}b^{10}c^5d^{11}e^{11}f^{12} - 488096a^{12}b^8c^6d^{11}e^{11}f^{12} + 1458688a^{13}b^6c^7d^{11}e^{11}f^{12} - \\
& 2401280a^{14}b^4c^8d^{11}e^{11}f^{12} + 1871872a^{15}b^2c^9d^{11}e^{11}f^{12}) * i) / (((- (25b^{15} + 25b^6 * (- (4ac - b^2)^9)^{1/2}) - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + 116928a^4b^7c^4 - \\
& 219744a^5b^5c^5 + 215040a^6b^3c^6 - 49a^3c^3 * (- (4ac - b^2)^9)^{1/2}) - 615ab^{13}c + 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2}) - 165ab^4c * (- (4ac - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^{2f^8} + 4096a^{13}c^6e^{2f^8} + \\
& 240a^9b^8c^2e^{2f^8} - 1280a^{10}b^6c^3e^{2f^8} + 3840a^{11}b^4c^4e^{2f^8} - 6144a^{12}b^2c^5e^{2f^8} - 24a^8b^{10}c^2e^{2f^8}))^{1/2}) * (((- (25b^{15} + 25b^6 * (- (4ac - b^2)^9)^{1/2}) - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - \\
& 35767a^3b^9c^3 + 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 - 49a^3c^3 * (- (4ac - b^2)^9)^{1/2}) - 615ab^{13}c + 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2}) - 165ab^4c * (- (4ac - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^{2f^8} + 4096a^{13}c^6e^{2f^8} + \\
& 240a^9b^8c^2e^{2f^8} - 1280a^{10}b^6c^3e^{2f^8} + 3840a^{11}b^4c^4e^{2f^8} - 6144a^{12}b^2c^5e^{2f^8} - 24a^8b^{10}c^2e^{2f^8}))^{1/2}) * (((- (25b^{15} + 25b^6 * (- (4ac - b^2)^9)^{1/2}) - 80640a^7b^7c^7 + 6366a^2b^{11}c^2 - 35767a^3b^9c^3 + \\
& 116928a^4b^7c^4 - 219744a^5b^5c^5 + 215040a^6b^3c^6 - 49a^3c^3 * (- (4ac - b^2)^9)^{1/2}) - 615ab^{13}c + 246a^2b^2c^2 * (- (4ac - b^2)^9)^{1/2}) - 165ab^4c * (- (4ac - b^2)^9)^{1/2}) / (32 * (a^7b^{12}e^{2f^8} + 4096a^{13}c^6e^{2f^8} + \\
& 240a^9b^8c^2e^{2f^8} - 1280a^{10}b^6c^3e^{2f^8} + 3840a^{11}b^4c^4e^{2f^8} - 6144a^{12}b^2c^5e^{2f^8} - 24a^8b^{10}c^2e^{2f^8}))^{1/2}) * (x * (256a^{15}b^{13}c^2e^{14f^{20}} - 6144a^{16}b^{11}c^3e^{14f^{20}} + 61440a^{17}b^9c^4e^{14f^{20}} - 327680a^{18}b^7c^5e^{14f^{20}} + 983040a^{19}b^5c^6e^{14f^{20}} - \\
& 1572864a^{20}b^3c^7e^{14f^{20}} + 1048576a^{21}b^1c^8e^{14f^{20}}) + 1048576a^{21}b^1c^8d^{13}e^{13f^{20}} + 256a^{15}b^{13}c^2d^{13}e^{13f^{20}} - 6144a^{16}b^{11}c^3d^{13}e^{13f^{20}} + 61440a^{17}b^9c^4d^{13}e^{13f^{20}} - 327680a^{18}b^7c^5d^{13}e^{13f^{20}} + 983040a^{19}b^5c^6d^{13}e^{13f^{20}} - 1572864a^{20}b^3c^7d^{13}e^{13f^{20}} - \\
& 917504a^{19}c^9e^{12}f^{16} + 320a^{12}b^{14}c^2e^{12}f^{16} - 7936a^{13}b^{12}c^3e^{12}f^{16} + 82816a^{14}b^{10}c^4e^{12}f^{16} - 468480a^{15}b^8c^5e^{12}f^{16} + 1536000a^{16}b^6c^6e^{12}f^{16} - 2867200a^{17}b^4c^7e^{12}f^{16} + 2719744a^{18}b^2c^8e^{12}f^{16}) - x(401408a^{16}c^{10}e^{12}f^{12} - 400a^9b^{14}c^3e^{12}f^{12} + 9440a^{10}b^{12}c^4e^{12}f^{12} - 92816a^{11}b^{10}c^5e^{12}f^{12} + 488096a^{12}b^8c^6e^{12}f^{12} - 1458688a^{13}b^6c^7e^{12}f^{12} + 2401280a^{14}b^4c^8e^{12}f^{12} - 1871872a^{15}b^2c^9e^{12}f^{12}) - 401408a^{16}c^{10}d^{11}e^{11}f^{12} + 400a^9b^{14}c^3d^{11}e^{11}f^{12} - 9440a^{10}b^{12}c^4d^{11}e^{11}f^{12} + 92816a^{11}b^{10}c^5d^{11}e^{11}f^{12} - 488096a^{12}b^8c^6d^{11}e^{11}f^{12} + 1458688a^{13}b^6c^7d^{11}e^{11}f^{12} - 2401280a^{14}b^4c^8d^{11}e^{11}f^{12}
\end{aligned}$$

$$\begin{aligned}
& *d*e^{11}f^{12} + 1871872*a^{15}b^2*c^9*d*e^{11}f^{12}) - ((-25*b^{15} + 25*b^6*(-(4 \\
& *a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}c^2 - 35767*a^3*b^9* \\
& c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3 \\
& *c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}c + 246*a^2*b^2*c^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12}*e^{2}f^8 \\
& + 4096*a^{13}c^6*e^{2}f^8 + 240*a^9*b^8*c^2*e^{2}f^8 - 1280*a^{10}*b^6*c^3*e^{2}f \\
& ^8 + 3840*a^{11}*b^4*c^4*e^{2}f^8 - 6144*a^{12}*b^2*c^5*e^{2}f^8 - 24*a^8*b^{10}*c \\
& *e^{2}f^8)))^{(1/2)}*((-25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7* \\
& b*c^7 + 6366*a^2*b^{11}c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744 \\
& *a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 6 \\
& 15*a*b^{13}c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a \\
& *c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12}*e^{2}f^8 + 4096*a^{13}c^6*e^{2}f^8 + 240*a^9 \\
& *b^8*c^2*e^{2}f^8 - 1280*a^{10}*b^6*c^3*e^{2}f^8 + 3840*a^{11}*b^4*c^4*e^{2}f^8 - \\
& 6144*a^{12}*b^2*c^5*e^{2}f^8 - 24*a^8*b^{10}*c*e^{2}f^8)))^{(1/2)}*((-25*b^{15} + 25 \\
& *b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}c^2 - 35767 \\
& *a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 \\
& - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}c + 246*a^2*b^2*c^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12} \\
& *e^{2}f^8 + 4096*a^{13}c^6*e^{2}f^8 + 240*a^9*b^8*c^2*e^{2}f^8 - 1280*a^{10}*b^6* \\
& c^3*e^{2}f^8 + 3840*a^{11}*b^4*c^4*e^{2}f^8 - 6144*a^{12}*b^2*c^5*e^{2}f^8 - 24*a^ \\
& 8*b^{10}*c*e^{2}f^8)))^{(1/2)}*(x*(256*a^{15}b^{13}c^2*e^{14}f^{20} - 6144*a^{16}b^{11} \\
& c^3*e^{14}f^{20} + 61440*a^{17}b^9*c^4*e^{14}f^{20} - 327680*a^{18}b^7*c^5*e^{14}f^{20} \\
& 0 + 983040*a^{19}b^5*c^6*e^{14}f^{20} - 1572864*a^{20}b^3*c^7*e^{14}f^{20} + 104857 \\
& 6*a^{21}b*c^8*e^{14}f^{20}) + 1048576*a^{21}b*c^8*d*e^{13}f^{20} + 256*a^{15}b^{13}c^ \\
& 2*d*e^{13}f^{20} - 6144*a^{16}b^{11}c^3*d*e^{13}f^{20} + 61440*a^{17}b^9*c^4*d*e^{13} \\
& f^{20} - 327680*a^{18}b^7*c^5*d*e^{13}f^{20} + 983040*a^{19}b^5*c^6*d*e^{13}f^{20} - \\
& 1572864*a^{20}b^3*c^7*d*e^{13}f^{20}) + 917504*a^{19}c^9*e^{12}f^{16} - 320*a^{12}b^ \\
& 14*c^2*e^{12}f^{16} + 7936*a^{13}b^{12}c^3*e^{12}f^{16} - 82816*a^{14}b^{10}c^4*e^{12} \\
& f^{16} + 468480*a^{15}b^8*c^5*e^{12}f^{16} - 1536000*a^{16}b^6*c^6*e^{12}f^{16} + 286 \\
& 7200*a^{17}b^4*c^7*e^{12}f^{16} - 2719744*a^{18}b^2*c^8*e^{12}f^{16}) - x*(401408*a \\
& ^{16}c^{10}e^{12}f^{12} - 400*a^9*b^{14}c^3*e^{12}f^{12} + 9440*a^{10}b^{12}c^4*e^{12}f \\
& ^{12} - 92816*a^{11}b^{10}c^5*e^{12}f^{12} + 488096*a^{12}b^8*c^6*e^{12}f^{12} - 14586 \\
& 88*a^{13}b^6*c^7*e^{12}f^{12} + 2401280*a^{14}b^4*c^8*e^{12}f^{12} - 1871872*a^{15}b \\
& ^2*c^9*e^{12}f^{12}) - 401408*a^{16}c^{10}d*e^{11}f^{12} + 400*a^9*b^{14}c^3*d*e^{11} \\
& f^{12} - 9440*a^{10}b^{12}c^4*d*e^{11}f^{12} + 92816*a^{11}b^{10}c^5*d*e^{11}f^{12} - 4 \\
& 88096*a^{12}b^8*c^6*d*e^{11}f^{12} + 1458688*a^{13}b^6*c^7*d*e^{11}f^{12} - 2401280 \\
& *a^{14}b^4*c^8*d*e^{11}f^{12} + 1871872*a^{15}b^2*c^9*d*e^{11}f^{12}) + 1800*a^9*b^ \\
& 9*c^6*e^{10}f^8 - 29080*a^{10}b^7*c^7*e^{10}f^8 + 176032*a^{11}b^5*c^8*e^{10}f^8 \\
& - 473216*a^{12}b^3*c^9*e^{10}f^8 + 476672*a^{13}b*c^{10}e^{10}f^8)))*((-25*b^{15} \\
& + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}c^2 - 3 \\
& 5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3 \\
& *c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}c + 246*a^2*b^2*c^2 \\
& *(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7* \\
& b^{12}*e^{2}f^8 + 4096*a^{13}c^6*e^{2}f^8 + 240*a^9*b^8*c^2*e^{2}f^8 - 1280*a^{10} \\
& b^6*c^3*e^{2}f^8 + 3840*a^{11}*b^4*c^4*e^{2}f^8 - 6144*a^{12}*b^2*c^5*e^{2}f^8 - 2
\end{aligned}$$

$4*a^8*b^{10}*c*e^{2*f^8}))^{(1/2)*2i}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)

[Out] Timed out

$$3.541 \quad \int \frac{(df+efx)^4}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=353

$$\frac{f^4(d+ex)(-4ac+7b^2+12bc(d+ex)^2)}{8e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{f^4(d+ex)(2a+b(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3\sqrt{c}f^4(-2b\sqrt{b^2-4ac})}{4\sqrt{2}e(b^2-4ac)}$$

Rubi [A] time = 0.87, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1142, 1120, 1178, 1166, 205}

$$\frac{f^4(d+ex)(-4ac+7b^2+12bc(d+ex)^2)}{8e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{f^4(d+ex)(2a+b(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3\sqrt{c}f^4(-2b\sqrt{b^2-4ac}+4ac+3b^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{c}f^4(2b\sqrt{b^2-4ac}+4ac+3b^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2 - (f^4*(d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*sqrt[c]*(3*b^2 + 4*a*c - 2*b*sqrt[b^2 - 4*a*c])*f^4*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]]])/(4*sqrt[2]*(b^2 - 4*a*c)^(5/2)*sqrt[b - sqrt[b^2 - 4*a*c]]*e) - (3*sqrt[c]*(3*b^2 + 4*a*c + 2*b*sqrt[b^2 - 4*a*c])*f^4*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b + sqrt[b^2 - 4*a*c]]])/(4*sqrt[2]*(b^2 - 4*a*c)^(5/2)*sqrt[b + sqrt[b^2 - 4*a*c]]*e)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1120

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(d^3*(d*x)^(m-3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^(m-4)*(2*a*(m-3) + b*(m+4*p+3)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(df + efx)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx &= \frac{f^4 \text{Subst} \left(\int \frac{x^4}{(a + bx^2 + cx^4)^3} dx, x, d + ex \right)}{e} \\
&= \frac{f^4 (d + ex) (2a + b(d + ex)^2)}{4 (b^2 - 4ac) e (a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{f^4 \text{Subst} \left(\int \frac{2a - 5bx^2}{(a + bx^2 + cx^4)^2} dx, x, d + ex \right)}{4 (b^2 - 4ac) e} \\
&= \frac{f^4 (d + ex) (2a + b(d + ex)^2)}{4 (b^2 - 4ac) e (a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{f^4 (d + ex) (7b^2 - 4ac + 1)}{8 (b^2 - 4ac)^2 e (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= \frac{f^4 (d + ex) (2a + b(d + ex)^2)}{4 (b^2 - 4ac) e (a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{f^4 (d + ex) (7b^2 - 4ac + 1)}{8 (b^2 - 4ac)^2 e (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= \frac{f^4 (d + ex) (2a + b(d + ex)^2)}{4 (b^2 - 4ac) e (a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{f^4 (d + ex) (7b^2 - 4ac + 1)}{8 (b^2 - 4ac)^2 e (a + b(d + ex)^2 + c(d + ex)^4)}
\end{aligned}$$

Mathematica [A] time = 4.45, size = 331, normalized size = 0.94

$$\frac{f^4 \left(\frac{(d+ex)(4ac-7b^2-12bc(d+ex)^2)}{(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{2(-2a(d+ex)-b(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3\sqrt{2}\sqrt{c}(-2b\sqrt{b^2-4ac}+4ac+3b^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}(2b\sqrt{b^2-4ac}+4ac+3b^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} \right)}{8e}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(8*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df + efx)^4}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x
]

fricas [B] time = 1.41, size = 6770, normalized size = 19.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/16*(24*b*c^2*e^7*f^4*x^7 + 168*b*c^2*d*e^6*f^4*x^6 + 2*(252*b*c^2*d^2 + \\ & 19*b^2*c - 4*a*c^2)*e^5*f^4*x^5 + 10*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d \\ &)*e^4*f^4*x^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^2) \\ &)*d^2)*e^3*f^4*x^3 + 2*(252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5 \\ & *b^3 + 16*a*b*c)*d)*e^2*f^4*x^2 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)* \\ & d^4 + 3*a*b^2 + 12*a^2*c + 3*(5*b^3 + 16*a*b*c)*d^2)*e*f^4*x + 2*(12*b*c^2* \\ & d^7 + (19*b^2*c - 4*a*c^2)*d^5 + (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4*a^2*c \\ &)*d)*f^4 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^9*x^8 + 8*(\\ & b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e^8*x^7 + 2*(b^5*c - 8*a*b^3*c^2 + 16 \\ & *a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*e^7*x^6 + 4*(14*(\\ & b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b \\ & *c^3)*d)*e^6*x^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 \\ & + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*e^5*x^4 \\ & + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 \\ & + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*e^4*x^3 + 2*(14*(b^ \\ & 4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 \\ & + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3 \\ & *c^3)*d^2)*e^3*x^2 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5 \\ & *c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + \\ & (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*e^2*x + ((b^4*c^2 - 8*a*b^2*c^3 + \\ & 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8* \\ & a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8* \\ & a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)* \\ & f^8 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5 \\ & *b^2*c^4 - 1024*a^6*c^5)*sqrt(f^16/((a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - \end{aligned}$$

$$\begin{aligned}
& 2 - 640a^4b^4c^3 + 1280a^5b^2c^4 - 1024a^6c^5) \sqrt{f^{16}/((a^2b^{10} \\
& - 20a^3b^8c + 160a^4b^6c^2 - 640a^5b^4c^3 + 1280a^6b^2c^4 - 10 \\
& 24a^7c^5)e^4)} e^2)/((a^2b^{10} - 20a^3b^8c + 160a^4b^6c^2 - 640a^4b \\
& b^4c^3 + 1280a^5b^2c^4 - 1024a^6c^5)e^2)) * \log(27(5b^4c + 40ab^2 \\
& c^2 + 16a^2c^3) * e^{f^{12}x} + 27(5b^4c + 40ab^2c^2 + 16a^2c^3) * d * f^{ \\
& 12 - 27/2 \sqrt{1/2} * ((b^8 - 8ab^6c + 128a^3b^2c^3 - 256a^4c^4) * e^{f^ \\
& 8 + (ab^{13} - 8a^2b^{11}c - 80a^3b^9c^2 + 1280a^4b^7c^3 - 6400a^5b \\
& ^5c^4 + 14336a^6b^3c^5 - 12288a^7b^2c^6) * \sqrt{f^{16}/((a^2b^{10} - 20a^3 \\
& * b^8c + 160a^4b^6c^2 - 640a^5b^4c^3 + 1280a^6b^2c^4 - 1024a^7c^ \\
& 5)e^4)}) * e^3) * \sqrt{-((b^5 + 40ab^3c + 80a^2b^2c^2) * f^8 - (a^2b^{10} - 20a \\
& ^2b^8c + 160a^3b^6c^2 - 640a^4b^4c^3 + 1280a^5b^2c^4 - 1024a^6c \\
& ^5) * \sqrt{f^{16}/((a^2b^{10} - 20a^3b^8c + 160a^4b^6c^2 - 640a^5b^4c^ \\
& 3 + 1280a^6b^2c^4 - 1024a^7c^5)e^4)}) * e^2)/((a^2b^{10} - 20a^3b^8c + 1 \\
& 60a^3b^6c^2 - 640a^4b^4c^3 + 1280a^5b^2c^4 - 1024a^6c^5)e^2))))) \\
& /((b^4c^2 - 8ab^2c^3 + 16a^2c^4) * e^{9x^8} + 8(b^4c^2 - 8ab^2c^3 + \\
& 16a^2c^4) * d * e^{8x^7} + 2(b^5c - 8ab^3c^2 + 16a^2b^2c^3 + 14(b^4c^ \\
& 2 - 8ab^2c^3 + 16a^2c^4) * d^2) * e^{7x^6} + 4(14(b^4c^2 - 8ab^2c^3 + \\
& 16a^2c^4) * d^3 + 3(b^5c - 8ab^3c^2 + 16a^2b^2c^3) * d) * e^{6x^5} + (b^6 \\
& - 6ab^4c + 32a^3c^3 + 70(b^4c^2 - 8ab^2c^3 + 16a^2c^4) * d^4 + 3 \\
& 0(b^5c - 8ab^3c^2 + 16a^2b^2c^3) * d^2) * e^{5x^4} + 4(14(b^4c^2 - 8ab \\
& b^2c^3 + 16a^2c^4) * d^5 + 10(b^5c - 8ab^3c^2 + 16a^2b^2c^3) * d^3 + (\\
& b^6 - 6ab^4c + 32a^3c^3) * d) * e^{4x^3} + 2(14(b^4c^2 - 8ab^2c^3 + 1 \\
& 6a^2c^4) * d^6 + ab^5 - 8a^2b^3c + 16a^3b^2c^2 + 15(b^5c - 8ab^3c \\
& ^2 + 16a^2b^2c^3) * d^4 + 3(b^6 - 6ab^4c + 32a^3c^3) * d^2) * e^{3x^2} + 4 \\
& (2(b^4c^2 - 8ab^2c^3 + 16a^2c^4) * d^7 + 3(b^5c - 8ab^3c^2 + 16a \\
& ^2b^2c^3) * d^5 + (b^6 - 6ab^4c + 32a^3c^3) * d^3 + (ab^5 - 8a^2b^3c + \\
& 16a^3b^2c^2) * d) * e^{2x} + ((b^4c^2 - 8ab^2c^3 + 16a^2c^4) * d^8 + 2(b^ \\
& 5c - 8ab^3c^2 + 16a^2b^2c^3) * d^6 + a^2b^4 - 8a^3b^2c + 16a^4c^2 \\
& + (b^6 - 6ab^4c + 32a^3c^3) * d^4 + 2(ab^5 - 8a^2b^3c + 16a^3b^2c^ \\
& 2) * d^2) * e)
\end{aligned}$$

giac [B] time = 0.80, size = 1844, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] $3/16 * ((4 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4}) / c)^2 * b * c * f^4 * e^2 - 8 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) * b * c * d * f^4 * e + 4 * b * c * d^2 * f^4 - b^2 * f^4 - 4 * a * c * f^4) * \log(d * e^{-1} + x + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c) / (2 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c)^3 * c * e^4 - 6 * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c)^2 * c * d * e^3 - 2 * c * d^3 * e - b * d * e + (6 * c * d^2 * e^2 + b * e^2) * (d * e^{-1}) + \sqrt{1/2} * \sqrt{-(b * e^2 + \sqrt{b^2 - 4 * a * c}) * e^2}) * e^{-4} / c$

$$\begin{aligned}
& \left(\frac{1}{2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c} \right) + (4 (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 b c f^4 e^2 - 8 (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) b c d f^4 e + 4 b c d^2 f^4 - b^2 f^4 - 4 a c f^4) \log(d e^{-1} + x - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) / (2 (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^3 c e^4 - 6 (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 + \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) \\
& + (4 (d e^{-1}) + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 b c f^4 e^2 - 8 (d e^{-1}) + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) b c d f^4 e + 4 b c d^2 f^4 - b^2 f^4 - 4 a c f^4) \log(d e^{-1} + x + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) / (2 (d e^{-1}) + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^3 c e^4 - 6 (d e^{-1}) + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1}) + \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) \\
& + (4 (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 b c f^4 e^2 - 8 (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) b c d f^4 e + 4 b c d^2 f^4 - b^2 f^4 - 4 a c f^4) \log(d e^{-1} + x - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) / (2 (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^3 c e^4 - 6 (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c})^2 c d e^3 - 2 c d^3 e - b d e + (6 c d^2 e^2 + b e^2) (d e^{-1}) - \sqrt{1/2} \sqrt{-(b e^2 - \sqrt{b^2 - 4 a c}) e^2} e^{-4/c}) \\
&) / (b^4 - 8 a b^2 c + 16 a^2 c^2) - 1/8 (12 b c^2 f^4 x^7 e^7 + 84 b c^2 d f^4 x^6 e^6 + 252 b c^2 d^2 f^4 x^5 e^5 + 420 b c^2 d^3 f^4 x^4 e^4 + 420 b c^2 d^4 f^4 x^3 e^3 + 252 b c^2 d^5 f^4 x^2 e^2 + 84 b c^2 d^6 f^4 x e + 12 b c^2 d^7 f^4 + 19 b^2 c f^4 x^5 e^5 - 4 a c^2 f^4 x^5 e^5 + 95 b^2 c d f^4 x^4 e^4 - 20 a c^2 d f^4 x^4 e^4 + 190 b^2 c d^2 f^4 x^3 e^3 - 40 a c^2 d^2 f^4 x^3 e^3 + 190 b^2 c d^3 f^4 x^2 e^2 - 40 a c^2 d^3 f^4 x^2 e^2 + 95 b^2 c d^4 f^4 x e - 20 a c^2 d^4 f^4 x e + 19 b^2 c d^5 f^4 - 4 a c^2 d^5 f^4 + 5 b^3 f^4 x^3 e^3 + 16 a b c f^4 x^3 e^3 + 15 b^3 d f^4 x^2 e^2 + 48 a b c d f^4 x^2 e^2 + 15 b^3 d^2 f^4 x e + 48 a b c d^2 f^4 x e + 5 b^3 d^3 f^4 + 16 a b c d^3 f^4 + 3 a b^2 f^4 x e + 12 a^2 c f^4 x e + 3 a b^2 d f^4 + 12 a^2 c d f^4) / ((c x^4 e^4 + 4 c d x^3 e^3 + 6 c d^2 x^2 e^2 + 4 c d^3 x e + c d^4 + b x^2 e^2 + 2 b d x e + b d^2 + a)^2 (b^4 e - 8 a b^2 c e + 16 a^2 c^2 e))
\end{aligned}$$

maple [C] time = 0.05, size = 3432, normalized size = 9.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e f x + d f)^4 / (a + b (e x + d)^2 + c (e x + d)^4)^3, x)$

[Out] $-95/4 f^4 / (c e^4 x^4 + 4 c d x^3 e^3 + 6 c d^2 x^2 e^2 + 4 c d^3 x e + c d^4 + b x^2 e^2 + 2 b d x e + b d^2 + a)^2 (b^4 e - 8 a b^2 c e + 16 a^2 c^2 e)$

$$\begin{aligned}
& d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c*d^2-2*f^4 \\
& / (c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d \\
& *e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*a*b*c-63/2*f^4/(c*e^4*x^4 \\
& +4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2 \\
& +a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+5*f^4/(c*e^4*x^4+4*c*d*e^3 \\
& *x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e \\
& / (16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*c^2-95/4*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c \\
& *d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^2 \\
& *c^2-8*a*b^2*c+b^4)*x^2*b^2*c-6*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^ \\
& 2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^ \\
& 4)*x*a*b*c*d^2-2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b \\
& *e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^3/e*a*b*c- \\
& 105/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c \\
& d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b+5/2*f \\
& ^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b \\
& *d*e*x+b*d^2+a)^2*c^2*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*a-95/8*f^4/(c*e^ \\
& 4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b \\
& *d^2+a)^2*c*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2-6*f^4/(c*e^4*x^4+4*c*d \\
& *e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d \\
& *e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*b*c+3/16*f^4/(16*a^2*c^2-8*a*b^2*c+b^4) \\
& /e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c*d^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_ \\
& R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4 \\
& +4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z \\
& +a))-15/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^ \\
& 2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^3-21/2*f^ \\
& 4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b* \\
& d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2*d^6+5/2*f^4/(c*e^4*x^4+ \\
& 4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a \\
&)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a*c^2*d^4-95/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^ \\
& 3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2* \\
& c^2-8*a*b^2*c+b^4)*x*b^2*c*d^4-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2 \\
& *x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c \\
& +b^4)*d^7/e*b*c^2+1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3* \\
& e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^5/e*a \\
& *c^2-19/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^ \\
& 2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^5/e*b^2*c-3/2*f^4 \\
& / (c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d \\
& *e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d/e*a^2*c-3/8*f^4/(c*e^4*x^4+4*c \\
& *d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2 \\
& / (16*a^2*c^2-8*a*b^2*c+b^4)*d/e*a*b^2+1/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c* \\
& d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^ \\
& 2*c^2-8*a*b^2*c+b^4)*x^5*a-19/8*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^ \\
& 2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e^4/(16*a^2*c^2-8*a*b^ \\
& 2*c+b^4)*x^5*b^2-3/2*f^4/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e \\
& *x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)
\end{aligned}$$

$$\begin{aligned} &) * x^7 - 5/8 * f^4 / (c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + 6 * c * d^2 * e^2 * x^2 + 4 * c * d^3 * e * x + b * e^2 * x^2 \\ & + c * d^4 + 2 * b * d * e * x + b * d^2 * a)^2 * e^2 / (16 * a^2 * c^2 - 8 * a * b^2 * c + b^4) * x^3 * b^3 - 15/8 * f^4 / \\ & (c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + 6 * c * d^2 * e^2 * x^2 + 4 * c * d^3 * e * x + b * e^2 * x^2 + c * d^4 + 2 * b * \\ & d * e * x + b * d^2 * a)^2 / (16 * a^2 * c^2 - 8 * a * b^2 * c + b^4) * x * b^3 * d^2 - 3/2 * f^4 / (c * e^4 * x^4 + 4 * \\ & c * d * e^3 * x^3 + 6 * c * d^2 * e^2 * x^2 + 4 * c * d^3 * e * x + b * e^2 * x^2 + c * d^4 + 2 * b * d * e * x + b * d^2 * a)^2 \\ & / (16 * a^2 * c^2 - 8 * a * b^2 * c + b^4) * x * a^2 * c - 3/8 * f^4 / (c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + 6 * c * d^2 * \\ & e^2 * x^2 + 4 * c * d^3 * e * x + b * e^2 * x^2 + c * d^4 + 2 * b * d * e * x + b * d^2 * a)^2 / (16 * a^2 * c^2 - 8 * a * \\ & b^2 * c + b^4) * x * a * b^2 - 5/8 * f^4 / (c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + 6 * c * d^2 * e^2 * x^2 + 4 * c * d^3 * \\ & e * x + b * e^2 * x^2 + c * d^4 + 2 * b * d * e * x + b * d^2 * a)^2 / (16 * a^2 * c^2 - 8 * a * b^2 * c + b^4) * d^3 / e \\ & * b^3 - 105/2 * f^4 / (c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + 6 * c * d^2 * e^2 * x^2 + 4 * c * d^3 * e * x + b * e^2 * x^2 + \\ & c * d^4 + 2 * b * d * e * x + b * d^2 * a)^2 * e^2 / (16 * a^2 * c^2 - 8 * a * b^2 * c + b^4) * x^3 * b * c^2 * d^4 + \\ & 5 * f^4 / (c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + 6 * c * d^2 * e^2 * x^2 + 4 * c * d^3 * e * x + b * e^2 * x^2 + c * d^4 + \\ & 2 * b * d * e * x + b * d^2 * a)^2 * e^2 / (16 * a^2 * c^2 - 8 * a * b^2 * c + b^4) * x^3 * a * c^2 * d^2 - 21/2 * f^4 / \\ & (c * e^4 * x^4 + 4 * c * d * e^3 * x^3 + 6 * c * d^2 * e^2 * x^2 + 4 * c * d^3 * e * x + b * e^2 * x^2 + c * d^4 + 2 * b * d * \\ & e * x + b * d^2 * a)^2 * c^2 * d * e^5 * b / (16 * a^2 * c^2 - 8 * a * b^2 * c + b^4) * x^6 - 63/2 * f^4 / (c * e^4 * x^4 + \\ & 4 * c * d * e^3 * x^3 + 6 * c * d^2 * e^2 * x^2 + 4 * c * d^3 * e * x + b * e^2 * x^2 + c * d^4 + 2 * b * d * e * x + b * d^2 \\ & + a)^2 * c^2 * e^4 / (16 * a^2 * c^2 - 8 * a * b^2 * c + b^4) * x^5 * b * d^2 \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 7.52, size = 13840, normalized size = 39.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)

[Out] atan((((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8)))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c

$$\begin{aligned}
&)) + (x*(128*b^{11}*c^2*e^{14} - 2560*a*b^9*c^3*e^{14} - 131072*a^5*b*c^7*e^{14} + \\
&20480*a^2*b^7*c^4*e^{14} - 81920*a^3*b^5*c^5*e^{14} + 163840*a^4*b^3*c^6*e^{14})) \\
&/((16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) * \\
&((-9*(b^{15}*f^8 + f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7*f^8 - 560* \\
&a^2*b^{11}*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5* \\
&b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + \\
&1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4 \\
&*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a \\
&^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 262144 \\
&0*a^{10}*b^2*c^9*e^2)))^{(1/2)} - (786432*a^6*c^8*e^{12}*f^4 - 192*b^{12}*c^2*e^{12}* \\
&f^4 - 15360*a^2*b^8*c^4*e^{12}*f^4 + 245760*a^4*b^4*c^6*e^{12}*f^4 - 786432*a^5 \\
&*b^2*c^7*e^{12}*f^4 + 3072*a*b^{10}*c^3*e^{12}*f^4)/(128*(b^{12} + 4096*a^6*c^6 + 2 \\
&40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2 \\
&4*a*b^{10}*c))) * ((-9*(b^{15}*f^8 + f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b* \\
&c^7*f^8 - 560*a^2*b^{11}*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f \\
&^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^{13}*c*f^8))/(512* \\
&(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2* \\
&e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5* \\
&e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8 \\
&*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} + (18432*a^4*c^7*d*e^{11}*f^8 + 936 \\
&*b^8*c^3*d*e^{11}*f^8 - 6912*a*b^6*c^4*d*e^{11}*f^8 + 11520*a^2*b^4*c^5*d*e^{11}* \\
&f^8)/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840* \\
&a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c))) + (x*(144*a^2*c^5*e^{12}*f^8 + \\
&117*b^4*c^3*e^{12}*f^8 + 72*a*b^2*c^4*e^{12}*f^8))/(16*(b^8 + 256*a^4*c^4 + 96 \\
&*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) * 1i + ((-9*(b^{15}*f^8 + f^8*(- \\
&(4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7*f^8 - 560*a^2*b^{11}*c^2*f^8 + 4160 \\
&*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6 \\
&*b^3*c^6*f^8 + 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - \\
&40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a \\
&^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 196608 \\
&0*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(\\
&1/2)} * (((1024*b^{15}*c^2*d*e^{13} - 28672*a*b^{13}*c^3*d*e^{13} - 16777216*a^7*b*c^ \\
&9*d*e^{13} + 344064*a^2*b^{11}*c^4*d*e^{13} - 2293760*a^3*b^9*c^5*d*e^{13} + 917504 \\
&0*a^4*b^7*c^6*d*e^{13} - 22020096*a^5*b^5*c^7*d*e^{13} + 29360128*a^6*b^3*c^8*d \\
&*e^{13}))/((128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 384 \\
&0*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c))) + (x*(128*b^{11}*c^2*e^{14} - \\
&2560*a*b^9*c^3*e^{14} - 131072*a^5*b*c^7*e^{14} + 20480*a^2*b^7*c^4*e^{14} - 8192 \\
&0*a^3*b^5*c^5*e^{14} + 163840*a^4*b^3*c^6*e^{14}))/((16*(b^8 + 256*a^4*c^4 + 96* \\
&a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))) * ((-9*(b^{15}*f^8 + f^8*(-(4*a*c \\
&- b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7*f^8 - 560*a^2*b^{11}*c^2*f^8 + 4160*a^3*b \\
&^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c \\
&^6*f^8 + 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^ \\
&2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^1 \\
&2*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8* \\
&b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} +
\end{aligned}$$

$$\begin{aligned}
& (786432a^6c^8e^{12f^4} - 192b^{12}c^2e^{12f^4} - 15360a^2b^8c^4e^{12f^4} + 245760a^4b^4c^6e^{12f^4} - 786432a^5b^2c^7e^{12f^4} + 3072ab^{10}c^3e^{12f^4}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) * (-9(b^{15}f^8 + f^8(-4ac - b^2)^{15})^{1/2} - 81920a^7b^7c^7f^8 - 560a^2b^{11}c^2f^8 + 4160a^3b^9c^3f^8 - 11520a^4b^7c^4f^8 - 1024a^5b^5c^5f^8 + 61440a^6b^3c^6f^8 + 20ab^{13}cf^8) / (512(ab^{20}e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^6e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2))^{1/2} + (18432a^4c^7de^{11f^8} + 936b^8c^3de^{11f^8} - 6912ab^6c^4de^{11f^8} + 11520a^2b^4c^5de^{11f^8}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) + (x(144a^2c^5e^{12f^8} + 117b^4c^3e^{12f^8} + 72ab^2c^4e^{12f^8})) / (16(b^8 + 256a^4c^4 + 96a^2b^4c^2 - 256a^3b^2c^3 - 16ab^6c)) * 1i) / ((-9(b^{15}f^8 + f^8(-4ac - b^2)^{15})^{1/2} - 81920a^7b^7c^7f^8 - 560a^2b^{11}c^2f^8 + 4160a^3b^9c^3f^8 - 11520a^4b^7c^4f^8 - 1024a^5b^5c^5f^8 + 61440a^6b^3c^6f^8 + 20ab^{13}cf^8) / (512(ab^{20}e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^6e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2))^{1/2} * (((1024b^{15}c^2de^{13} - 28672ab^{13}c^3de^{13} - 16777216a^7b^7c^9de^{13} + 344064a^2b^{11}c^4de^{13} - 2293760a^3b^9c^5de^{13} + 9175040a^4b^7c^6de^{13} - 22020096a^5b^5c^7de^{13} + 29360128a^6b^3c^8de^{13}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) + (x(128b^{11}c^2e^{14} - 2560ab^9c^3e^{14} - 131072a^5b^7c^7e^{14} + 20480a^2b^7c^4e^{14} - 81920a^3b^5c^5e^{14} + 163840a^4b^3c^6e^{14})) / (16(b^8 + 256a^4c^4 + 96a^2b^4c^2 - 256a^3b^2c^3 - 16ab^6c))) * (-9(b^{15}f^8 + f^8(-4ac - b^2)^{15})^{1/2} - 81920a^7b^7c^7f^8 - 560a^2b^{11}c^2f^8 + 4160a^3b^9c^3f^8 - 11520a^4b^7c^4f^8 - 1024a^5b^5c^5f^8 + 61440a^6b^3c^6f^8 + 20ab^{13}cf^8) / (512(ab^{20}e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^6e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2))^{1/2} - (786432a^6c^8e^{12f^4} - 192b^{12}c^2e^{12f^4} - 15360a^2b^8c^4e^{12f^4} + 245760a^4b^4c^6e^{12f^4} - 786432a^5b^2c^7e^{12f^4} + 3072ab^{10}c^3e^{12f^4}) / (128(b^{12} + 4096a^6c^6 + 240a^2b^8c^2 - 1280a^3b^6c^3 + 3840a^4b^4c^4 - 6144a^5b^2c^5 - 24ab^{10}c)) * (-9(b^{15}f^8 + f^8(-4ac - b^2)^{15})^{1/2} - 81920a^7b^7c^7f^8 - 560a^2b^{11}c^2f^8 + 4160a^3b^9c^3f^8 - 11520a^4b^7c^4f^8 - 1024a^5b^5c^5f^8 + 61440a^6b^3c^6f^8 + 20ab^{13}cf^8) / (512(ab^{20}e^2 + 1048576a^{11}c^{10}e^2 - 40a^2b^{18}c^6e^2 + 720a^3b^{16}c^2e^2 - 7680a^4b^{14}c^3e^2 + 53760a^5b^{12}c^4e^2 - 258048a^6b^{10}c^5e^2 + 860160a^7b^8c^6e^2 - 1966080a^8b^6c^7e^2 + 2949120a^9b^4c^8e^2 - 2621440a^{10}b^2c^9e^2) + 2
\end{aligned}$$

$$\begin{aligned}
& (949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2))^{(1/2)} + (18432*a^4*c^7 \\
& *d*e^{11}*f^8 + 936*b^8*c^3*d*e^{11}*f^8 - 6912*a*b^6*c^4*d*e^{11}*f^8 + 11520*a^ \\
& 2*b^4*c^5*d*e^{11}*f^8)/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^ \\
& 3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(144*a \\
& ^2*c^5*e^{12}*f^8 + 117*b^4*c^3*e^{12}*f^8 + 72*a*b^2*c^4*e^{12}*f^8))/(16*(b^8 + \\
& 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - ((-9*(b^{15} \\
& *f^8 + f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7*f^8 - 560*a^2*b^{11} \\
& *c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^ \\
& ^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a \\
& ^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3 \\
& *e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6 \\
& *e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2 \\
& *c^9*e^2))^{(1/2)}*(((1024*b^{15}*c^2*d*e^{13} - 28672*a*b^{13}*c^3*d*e^{13} - 167 \\
& 77216*a^7*b*c^9*d*e^{13} + 344064*a^2*b^{11}*c^4*d*e^{13} - 2293760*a^3*b^9*c^5*d \\
& *e^{13} + 9175040*a^4*b^7*c^6*d*e^{13} - 22020096*a^5*b^5*c^7*d*e^{13} + 29360128 \\
& *a^6*b^3*c^8*d*e^{13}))/((128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3 \\
& *b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(128*b^ \\
& 11*c^2*e^{14} - 2560*a*b^9*c^3*e^{14} - 131072*a^5*b*c^7*e^{14} + 20480*a^2*b^7*c \\
& ^4*e^{14} - 81920*a^3*b^5*c^5*e^{14} + 163840*a^4*b^3*c^6*e^{14}))/((16*(b^8 + 256 \\
& *a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-9*(b^{15}*f^8 \\
& + f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7*f^8 - 560*a^2*b^{11}*c^2*f^ \\
& 8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 6 \\
& 1440*a^6*b^3*c^6*f^8 + 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^ \\
& 10*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + \\
& 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 \\
& - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9* \\
& e^2))^{(1/2)} + (786432*a^6*c^8*e^{12}*f^4 - 192*b^{12}*c^2*e^{12}*f^4 - 15360*a^2 \\
& *b^8*c^4*e^{12}*f^4 + 245760*a^4*b^4*c^6*e^{12}*f^4 - 786432*a^5*b^2*c^7*e^{12}*f^ \\
& ^4 + 3072*a*b^{10}*c^3*e^{12}*f^4)/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 \\
& - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)))*(- \\
& (9*(b^{15}*f^8 + f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7*f^8 - 560*a \\
& ^2*b^{11}*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^ \\
& ^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1 \\
& 048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4* \\
& b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^ \\
& 7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440 \\
& *a^{10}*b^2*c^9*e^2))^{(1/2)} + (18432*a^4*c^7*d*e^{11}*f^8 + 936*b^8*c^3*d*e^{11} \\
& *f^8 - 6912*a*b^6*c^4*d*e^{11}*f^8 + 11520*a^2*b^4*c^5*d*e^{11}*f^8)/(128*(b^{12} \\
& + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6 \\
& 144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(144*a^2*c^5*e^{12}*f^8 + 117*b^4*c^3*e^ \\
& 12*f^8 + 72*a*b^2*c^4*e^{12}*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - \\
& 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (135*b^5*c^3*e^{10}*f^{12} + 1080*a*b^3*c^4*e \\
& ^{10}*f^{12} + 432*a^2*b*c^5*e^{10}*f^{12}))/((64*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8* \\
& c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c) \\
&)))*(-9*(b^{15}*f^8 + f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - 81920*a^7*b*c^7*f^8 -
\end{aligned}$$

$$\begin{aligned}
& 560*a^2*b^{11}*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024* \\
& a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^{13}*c*f^8)/(512*(a*b^{20}*e^ \\
& 2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680 \\
& *a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 8601 \\
& 60*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 26 \\
& 21440*a^{10}*b^2*c^9*e^2)))^{(1/2)}*2i - ((x^3*(5*b^3*e^2*f^4 + 16*a*b*c*e^2*f^ \\
& 4 - 40*a*c^2*d^2*e^2*f^4 + 190*b^2*c*d^2*e^2*f^4 + 420*b*c^2*d^4*e^2*f^4))/ \\
& (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(19*b^2*c*d*e^3*f^4 - 4*a*c^2*d \\
& *e^3*f^4 + 84*b*c^2*d^3*e^3*f^4))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5 \\
& *(19*b^2*c*e^4*f^4 - 4*a*c^2*e^4*f^4 + 252*b*c^2*d^2*e^4*f^4))/(8*(b^4 + 16 \\
& *a^2*c^2 - 8*a*b^2*c)) + (x*(3*a*b^2*f^4 + 12*a^2*c*f^4 + 15*b^3*d^2*f^4 - \\
& 20*a*c^2*d^4*f^4 + 95*b^2*c*d^4*f^4 + 84*b*c^2*d^6*f^4 + 48*a*b*c*d^2*f^4)) \\
& / (8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(15*b^3*d*e*f^4 - 40*a*c^2*d^3*e \\
& *f^4 + 190*b^2*c*d^3*e*f^4 + 252*b*c^2*d^5*e*f^4 + 48*a*b*c*d*e*f^4))/(8*(b \\
& ^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*b^3*d^3*f^4 - 4*a*c^2*d^5*f^4 + 19*b^2*c \\
& *d^5*f^4 + 12*b*c^2*d^7*f^4 + 3*a*b^2*d*f^4 + 12*a^2*c*d*f^4 + 16*a*b*c*d^3 \\
& *f^4)/(8*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*e^6*f^4*x^7)/(2*(b^4 \\
& + 16*a^2*c^2 - 8*a*b^2*c)) + (21*b*c^2*d*e^5*f^4*x^6)/(2*(b^4 + 16*a^2*c^2 \\
& - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^ \\
& 2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e \\
& + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 \\
& + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 1 \\
& 2*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) \\
& + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 \\
& + 8*c^2*d*e^7*x^7) + atan((((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + \\
& 81920*a^7*b*c^7*f^8 + 560*a^2*b^{11}*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520* \\
& a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^{13}* \\
& c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720* \\
& a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048* \\
& a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 29491 \\
& 20*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)}*(((1024*b^{15}*c^2*d* \\
& e^{13} - 28672*a*b^{13}*c^3*d*e^{13} - 16777216*a^7*b*c^9*d*e^{13} + 344064*a^2*b^1 \\
& 1*c^4*d*e^{13} - 2293760*a^3*b^9*c^5*d*e^{13} + 9175040*a^4*b^7*c^6*d*e^{13} - 22 \\
& 020096*a^5*b^5*c^7*d*e^{13} + 29360128*a^6*b^3*c^8*d*e^{13))/(128*(b^{12} + 4096* \\
& a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5* \\
& b^2*c^5 - 24*a*b^{10}*c)) + (x*(128*b^{11}*c^2*e^{14} - 2560*a*b^9*c^3*e^{14} - 131 \\
& 072*a^5*b*c^7*e^{14} + 20480*a^2*b^7*c^4*e^{14} - 81920*a^3*b^5*c^5*e^{14} + 1638 \\
& 40*a^4*b^3*c^6*e^{14}))/ (16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2 \\
& *c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^15)^(1/2) - b^15*f^8 + 81920* \\
& a^7*b*c^7*f^8 + 560*a^2*b^{11}*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7 \\
& *c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^{13}*c*f^8)) \\
& / (512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^1 \\
& 6*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^1 \\
& 0*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9* \\
& b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} - (786432*a^6*c^8*e^{12}*f^4
\end{aligned}$$

$$\begin{aligned}
& - 192*b^{12}*c^2*e^{12}*f^4 - 15360*a^2*b^8*c^4*e^{12}*f^4 + 245760*a^4*b^4*c^6*e^{12}*f^4 - 786432*a^5*b^2*c^7*e^{12}*f^4 + 3072*a*b^{10}*c^3*e^{12}*f^4)/(128*(b^{12} \\
& + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)))*((9*(f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{15} \\
& *f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^{11}*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a \\
& *b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 2 \\
& 58048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} + (18432*a^4*c \\
& ^7*d*e^{11}*f^8 + 936*b^8*c^3*d*e^{11}*f^8 - 6912*a*b^6*c^4*d*e^{11}*f^8 + 11520*a^2*b^4*c^5*d*e^{11}*f^8)/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280* \\
& a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(144*a^2*c^5*e^{12}*f^8 + 117*b^4*c^3*e^{12}*f^8 + 72*a*b^2*c^4*e^{12}*f^8))/(16*(b^8 \\
& + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i + ((9*(f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{15}*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b \\
& ^{11}*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 10485 \\
& 76*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^ \\
& 8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)}*(((1024*b^{15}*c^2*d*e^{13} - 28672*a*b^{13}*c^3*d*e^{13} - \\
& 16777216*a^7*b*c^9*d*e^{13} + 344064*a^2*b^{11}*c^4*d*e^{13} - 2293760*a^3*b^9*c^5*d*e^{13} + 9175040*a^4*b^7*c^6*d*e^{13} - 22020096*a^5*b^5*c^7*d*e^{13} + 2936 \\
& 0128*a^6*b^3*c^8*d*e^{13}))/128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(12 \\
& 8*b^{11}*c^2*e^{14} - 2560*a*b^9*c^3*e^{14} - 131072*a^5*b*c^7*e^{14} + 20480*a^2*b^7*c^4*e^{14} - 81920*a^3*b^5*c^5*e^{14} + 163840*a^4*b^3*c^6*e^{14}))/16*(b^8 + \\
& 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*(f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{15}*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^{11}*c^2 \\
& *f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11} \\
& *c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 \\
& - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} + (786432*a^6*c^8*e^{12}*f^4 - 192*b^{12}*c^2*e^{12}*f^4 - 15360* \\
& a^2*b^8*c^4*e^{12}*f^4 + 245760*a^4*b^4*c^6*e^{12}*f^4 - 786432*a^5*b^2*c^7*e^{12}*f^4 + 3072*a*b^{10}*c^3*e^{12}*f^4)/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 \\
& + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)))*((9*(f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{15}*f^8 + 81920*a^7*b*c^7*f^8 + 560 \\
& *a^2*b^{11}*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + \\
& 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160* \\
& a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 26214
\end{aligned}$$

$$\begin{aligned}
& 40*a^{10}*b^2*c^9*e^2))^{(1/2)} + (18432*a^4*c^7*d*e^{11}*f^8 + 936*b^8*c^3*d*e^{11}*f^8 - 6912*a*b^6*c^4*d*e^{11}*f^8 + 11520*a^2*b^4*c^5*d*e^{11}*f^8)/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(144*a^2*c^5*e^{12}*f^8 + 117*b^4*c^3*e^{12}*f^8 + 72*a*b^2*c^4*e^{12}*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) * i) / (((9*(f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{15}*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^{11}*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} * (((1024*b^{15}*c^2*d*e^{13} - 28672*a*b^{13}*c^3*d*e^{13} - 16777216*a^7*b*c^9*d*e^{13} + 344064*a^2*b^{11}*c^4*d*e^{13} - 2293760*a^3*b^9*c^5*d*e^{13} + 9175040*a^4*b^7*c^6*d*e^{13} - 22020096*a^5*b^5*c^7*d*e^{13} + 29360128*a^6*b^3*c^8*d*e^{13} / (128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(128*b^{11}*c^2*e^{14} - 2560*a*b^9*c^3*e^{14} - 131072*a^5*b*c^7*e^{14} + 20480*a^2*b^7*c^4*e^{14} - 81920*a^3*b^5*c^5*e^{14} + 163840*a^4*b^3*c^6*e^{14}))) / (16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) * ((9*(f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{15}*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^{11}*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} - (786432*a^6*c^8*e^{12}*f^4 - 192*b^{12}*c^2*e^{12}*f^4 - 15360*a^2*b^8*c^4*e^{12}*f^4 + 245760*a^4*b^4*c^6*e^{12}*f^4 - 786432*a^5*b^2*c^7*e^{12}*f^4 + 3072*a*b^{10}*c^3*e^{12}*f^4) / (128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c))) * ((9*(f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{15}*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^{11}*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*a^4*b^{14}*c^3*e^2 + 53760*a^5*b^{12}*c^4*e^2 - 258048*a^6*b^{10}*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^{10}*b^2*c^9*e^2)))^{(1/2)} + (18432*a^4*c^7*d*e^{11}*f^8 + 936*b^8*c^3*d*e^{11}*f^8 - 6912*a*b^6*c^4*d*e^{11}*f^8 + 11520*a^2*b^4*c^5*d*e^{11}*f^8)/(128*(b^{12} + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^{10}*c)) + (x*(144*a^2*c^5*e^{12}*f^8 + 117*b^4*c^3*e^{12}*f^8 + 72*a*b^2*c^4*e^{12}*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - ((9*(f^8*(-(4*a*c - b^2)^{15})^{(1/2)} - b^{15}*f^8 + 81920*a^7*b*c^7*f^8 + 560*a^2*b^{11}*c^2*f^8 - 4160*a^3*b^9*c^3*f^8 + 11520*a^4*b^7*c^4*f^8 + 1024*a^5*b^5*c^5*f^8 - 61440*a^6*b^3*c^6*f^8 - 20*a*b^{13}*c*f^8))/(512*(a*b^{20}*e^2 + 1048576*a^{11}*c^{10}*e^2 - 40*a^2*b^{18}*c*e^2 + 720*a^3*b^{16}*c^2*e^2 - 7680*
\end{aligned}$$

$$\begin{aligned}
& a^4 b^{14} c^3 e^2 + 53760 a^5 b^{12} c^4 e^2 - 258048 a^6 b^{10} c^5 e^2 + 860160 a^7 b^8 c^6 e^2 - 1966080 a^8 b^6 c^7 e^2 + 2949120 a^9 b^4 c^8 e^2 - 2621440 a^{10} b^2 c^9 e^2) \Big)^{(1/2)} * (((1024 b^{15} c^2 d e^{13} - 28672 a b^{13} c^3 d e^{13} - 16777216 a^7 b c^9 d e^{13} + 344064 a^2 b^{11} c^4 d e^{13} - 2293760 a^3 b^9 c^5 d e^{13} + 9175040 a^4 b^7 c^6 d e^{13} - 22020096 a^5 b^5 c^7 d e^{13} + 29360128 a^6 b^3 c^8 d e^{13}) / (128 (b^{12} + 4096 a^6 c^6 + 240 a^2 b^8 c^2 - 1280 a^3 b^6 c^3 + 3840 a^4 b^4 c^4 - 6144 a^5 b^2 c^5 - 24 a a b^{10} c))) \\
& + (x (128 b^{11} c^2 e^{14} - 2560 a b^9 c^3 e^{14} - 131072 a^5 b c^7 e^{14} + 20480 a^2 b^7 c^4 e^{14} - 81920 a^3 b^5 c^5 e^{14} + 163840 a^4 b^3 c^6 e^{14})) / (16 (b^8 + 256 a^4 c^4 + 96 a^2 b^4 c^2 - 256 a^3 b^2 c^3 - 16 a a b^6 c))) * ((9 * (f^8 * (-4 a c - b^2)^{15})^{(1/2)} - b^{15} f^8 + 81920 a^7 b c^7 f^8 + 560 a^2 b^{11} c^2 f^8 - 4160 a^3 b^9 c^3 f^8 + 11520 a^4 b^7 c^4 f^8 + 1024 a^5 b^5 c^5 f^8 - 61440 a^6 b^3 c^6 f^8 - 20 a a b^{13} c f^8)) / (512 * (a b^{20} e^2 + 1048576 a^{11} c^{10} e^2 - 40 a^2 b^{18} c e^2 + 720 a^3 b^{16} c^2 e^2 - 7680 a^4 b^{14} c^3 e^2 + 53760 a^5 b^{12} c^4 e^2 - 258048 a^6 b^{10} c^5 e^2 + 860160 a^7 b^8 c^6 e^2 - 1966080 a^8 b^6 c^7 e^2 + 2949120 a^9 b^4 c^8 e^2 - 2621440 a^{10} b^2 c^9 e^2)) \Big)^{(1/2)} + (786432 a^6 c^8 e^{12} f^4 - 192 b^{12} c^2 e^{12} f^4 - 15360 a^2 b^8 c^4 e^{12} f^4 + 245760 a^4 b^4 c^6 e^{12} f^4 - 786432 a^5 b^2 c^7 e^{12} f^4 + 3072 a a b^{10} c^3 e^{12} f^4) / (128 (b^{12} + 4096 a^6 c^6 + 240 a^2 b^8 c^2 - 1280 a^3 b^6 c^3 + 3840 a^4 b^4 c^4 - 6144 a^5 b^2 c^5 - 24 a a b^{10} c))) * ((9 * (f^8 * (-4 a c - b^2)^{15})^{(1/2)} - b^{15} f^8 + 81920 a^7 b c^7 f^8 + 560 a^2 b^{11} c^2 f^8 - 4160 a^3 b^9 c^3 f^8 + 11520 a^4 b^7 c^4 f^8 + 1024 a^5 b^5 c^5 f^8 - 61440 a^6 b^3 c^6 f^8 - 20 a a b^{13} c f^8)) / (512 * (a b^{20} e^2 + 1048576 a^{11} c^{10} e^2 - 40 a^2 b^{18} c e^2 + 720 a^3 b^{16} c^2 e^2 - 7680 a^4 b^{14} c^3 e^2 + 53760 a^5 b^{12} c^4 e^2 - 258048 a^6 b^{10} c^5 e^2 + 860160 a^7 b^8 c^6 e^2 - 1966080 a^8 b^6 c^7 e^2 + 2949120 a^9 b^4 c^8 e^2 - 2621440 a^{10} b^2 c^9 e^2)) \Big)^{(1/2)} + (18432 a^4 c^7 d e^{11} f^8 + 936 b^8 c^3 d e^{11} f^8 - 6912 a a b^6 c^4 d e^{11} f^8 + 11520 a^2 b^4 c^5 d e^{11} f^8) / (128 (b^{12} + 4096 a^6 c^6 + 240 a^2 b^8 c^2 - 1280 a^3 b^6 c^3 + 3840 a^4 b^4 c^4 - 6144 a^5 b^2 c^5 - 24 a a b^{10} c)) + (x (144 a^2 c^5 e^{12} f^8 + 117 b^4 c^3 e^{12} f^8 + 72 a a b^2 c^4 e^{12} f^8)) / (16 (b^8 + 256 a^4 c^4 + 96 a^2 b^4 c^2 - 256 a^3 b^2 c^3 - 16 a a b^6 c))) + (135 b^5 c^3 e^{10} f^{12} + 1080 a a b^3 c^4 e^{10} f^{12} + 432 a^2 b c^5 e^{10} f^{12}) / (64 (b^{12} + 4096 a^6 c^6 + 240 a^2 b^8 c^2 - 1280 a^3 b^6 c^3 + 3840 a^4 b^4 c^4 - 6144 a^5 b^2 c^5 - 24 a a b^{10} c))) * ((9 * (f^8 * (-4 a c - b^2)^{15})^{(1/2)} - b^{15} f^8 + 81920 a^7 b c^7 f^8 + 560 a^2 b^{11} c^2 f^8 - 4160 a^3 b^9 c^3 f^8 + 11520 a^4 b^7 c^4 f^8 + 1024 a^5 b^5 c^5 f^8 - 61440 a^6 b^3 c^6 f^8 - 20 a a b^{13} c f^8)) / (512 * (a b^{20} e^2 + 1048576 a^{11} c^{10} e^2 - 40 a^2 b^{18} c e^2 + 720 a^3 b^{16} c^2 e^2 - 7680 a^4 b^{14} c^3 e^2 + 53760 a^5 b^{12} c^4 e^2 - 258048 a^6 b^{10} c^5 e^2 + 860160 a^7 b^8 c^6 e^2 - 1966080 a^8 b^6 c^7 e^2 + 2949120 a^9 b^4 c^8 e^2 - 2621440 a^{10} b^2 c^9 e^2)) \Big)^{(1/2)} * 2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

$$3.542 \quad \int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=159

$$\frac{3bf^3(b+2c(d+ex)^2)}{4e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{f^3(2a+b(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3bcf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}}$$

Rubi [A] time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1142, 1114, 638, 614, 618, 206}

$$\frac{3bf^3(b+2c(d+ex)^2)}{4e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{f^3(2a+b(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{3bcf^3 \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (f^3*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2 - (3*b*f^3*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(df + efx)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx &= \frac{f^3 \operatorname{Subst} \left(\int \frac{x^3}{(a+bx^2+cx^4)^3} dx, x, d + ex \right)}{e} \\
&= \frac{f^3 \operatorname{Subst} \left(\int \frac{x}{(a+bx+cx^2)^3} dx, x, (d + ex)^2 \right)}{2e} \\
&= \frac{f^3 (2a + b(d + ex)^2)}{4(b^2 - 4ac) e (a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{(3bf^3) \operatorname{Subst} \left(\int \frac{1}{(a+bx+cx^2)^2} dx, x, (d + ex)^2 \right)}{4(b^2 - 4ac)} \\
&= \frac{f^3 (2a + b(d + ex)^2)}{4(b^2 - 4ac) e (a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{3bf^3 (b + 2c(d + ex)^2)}{4(b^2 - 4ac)^2 e (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= \frac{f^3 (2a + b(d + ex)^2)}{4(b^2 - 4ac) e (a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{3bf^3 (b + 2c(d + ex)^2)}{4(b^2 - 4ac)^2 e (a + b(d + ex)^2 + c(d + ex)^4)} \\
&= \frac{f^3 (2a + b(d + ex)^2)}{4(b^2 - 4ac) e (a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{3bf^3 (b + 2c(d + ex)^2)}{4(b^2 - 4ac)^2 e (a + b(d + ex)^2 + c(d + ex)^4)}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 149, normalized size = 0.94

$$\frac{f^3 \left(\frac{(b^2-4ac)(2a+b(d+ex)^2)}{(a+(d+ex)^2(b+c(d+ex)^2))^2} - \frac{12bc \tan^{-1} \left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}} - \frac{3b(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4} \right)}{4e(b^2-4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (f^3*((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/(4*(b^2 - 4*a*c)^2*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(df + efx)^3}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x
]

fricas [B] time = 1.44, size = 3843, normalized size = 24.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*e^5*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^4*f^3*x^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*e^3*f^3*x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*e*f^3*x + (6*(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*f^3 - 6*(b*c^3*e^8*f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*e^6*f^3*x^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3*d^8 + 2*b^2*c^2*d^6 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a))/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -

$$\begin{aligned}
& 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^5*x^4 \\
& + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c \\
& + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + \\
& 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 \\
& - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b \\
& *c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + \\
& ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12 \\
& *a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2 \\
& *b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64* \\
& a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*e^6*f^3*x^6 + 36*(b^3*c^2 \\
& - 4*a*b*c^3)*d*e^5*f^3*x^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b \\
& c^3)*d^2)*e^4*f^3*x^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b \\
& ^2*c^2)*d)*e^3*f^3*x^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4* \\
& a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*e^2*f^3*x^2 + 4*(9*(b^3*c^2 - \\
& 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2 \\
&)*d)*e*f^3*x + (6*(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3 \\
& *c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)* \\
& f^3 - 12*(b*c^3*e^8*f^3*x^8 + 8*b*c^3*d*e^7*f^3*x^7 + 2*(14*b*c^3*d^2 + b^2 \\
& *c^2)*e^6*f^3*x^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*e^5*f^3*x^5 + (70*b*c^3* \\
& d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*e^4*f^3*x^4 + 4*(14*b*c^3*d^5 + 1 \\
& 0*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*e^3*f^3*x^3 + 2*(14*b*c^3*d^6 + 15*b \\
& ^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*e^2*f^3*x^2 + 4*(2*b*c^3* \\
& d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*e*f^3*x + (b*c^3 \\
& *d^8 + 2*b^2*c^2*d^6 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f \\
& ^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt \\
& (-b^2 + 4*a*c))/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - \\
& 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) \\
& *d*e^8*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 1 \\
& 4*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(\\
& 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - \\
& 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c \\
& + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 \\
& + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - \\
& 64*a^3*b*c^4)*d^2)*e^5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - \\
& 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4) \\
& *d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d) \\
& *e^4*x^3 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - \\
& 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*
\end{aligned}$$

$$(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e)]$$

giac [B] time = 0.78, size = 447, normalized size = 2.81

$$\frac{3bcf^3 \arctan\left(\frac{2d^2f+2df+2d}{\sqrt{3b^2+4ac}}\right)^{d-1}}{(d^4-8ab^2c+16a^2c^2)\sqrt{-b^2+4ac}} - \frac{6bc^2d^6f^7 + 18(f^2e+2dfx)bc^2d^4f^6e + 9b^2cd^4f^7 + 18(f^2e+2dfx)^2bc^2d^2f^6e + 18(f^2e+2dfx)^2cd^2f^6e + 2b^3d^4f^7 + 10abcd^4f^7 + 6(f^2e+2dfx)^2bc^2f^6e + 9(f^2e+2dfx)^2d^2c^2f^6e + 2(f^2e+2dfx)^2d^2f^6e + 10(f^2e+2dfx)abc^2f^6e + ab^2f^7 + 8a^2c^2f^7}{4(d^4f^2 + 2(f^2e+2dfx)cd^2f^6e + b^2f^6e + (f^2e+2dfx)^2cd^2 + (f^2e+2dfx)^2d^2f^6e + af^2)^2(b^4e - 8a^2b^2c^2e + 16a^2c^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] $-3*b*c*f^3*\arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(\sqrt{-b^2 + 4*a*c})*f))e^{-1}/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*\sqrt{-b^2 + 4*a*c}) - 1/4*(6*b*c^2*d^6*f^7 + 18*(f*x^2*e + 2*d*f*x)*b*c^2*d^4*f^6*e + 9*b^2*c*d^4*f^7 + 18*(f*x^2*e + 2*d*f*x)^2*b*c^2*d^2*f^5*e^2 + 18*(f*x^2*e + 2*d*f*x)*b^2*c*d^2*f^6*e + 2*b^3*d^2*f^7 + 10*a*b*c*d^2*f^7 + 6*(f*x^2*e + 2*d*f*x)^3*b*c^2*f^4*e^3 + 9*(f*x^2*e + 2*d*f*x)^2*b^2*c*f^5*e^2 + 2*(f*x^2*e + 2*d*f*x)*b^3*f^6*e + 10*(f*x^2*e + 2*d*f*x)*a*b*c*f^6*e + a*b^2*f^7 + 8*a^2*c*f^7)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))$

maple [C] time = 0.05, size = 2181, normalized size = 13.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)

[Out] $-3/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^4*b*c^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-45/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*c^2*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*d^2-9/4*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^2*c*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-30*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2*b/(1$

$$\begin{aligned}
& 6*a^2*c^2-8*a*b^2*c+b^4)*x^3-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2 \\
& +4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2*b^2/(16*a^2*c^2-8 \\
& *a*b^2*c+b^4)*x^3-45/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3 \\
& *e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^ \\
& 2*c^2*d^4-27/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e \\
& ^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*b^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c*d^ \\
& 2-5/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c* \\
& d^4+2*b*d*e*x+b*d^2+a)^2*b*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*a*c-1/2*f^3/(c* \\
& e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x \\
& +b*d^2+a)^2*b^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-9*f^3/(c*e^4*x^4+4*c*d*e^3 \\
& *x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*b \\
& /(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^2-9*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^ \\
& 2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*b^2/(16*a^2*c^2- \\
& 8*a*b^2*c+b^4)*x*c-5*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e \\
& *x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x*a* \\
& c-f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+ \\
& 2*b*d*e*x+b*d^2+a)^2*d*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*f^3/(c*e^4*x^4+ \\
& 4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a \\
&)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b*c^2*d^6-9/4*f^3/(c*e^4*x^4+4*c*d*e^3*x^3 \\
& +6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c \\
& ^2-8*a*b^2*c+b^4)/e*b^2*c*d^4-5/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x \\
& ^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+ \\
& b^4)/e*a*b*c*d^2-1/2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e \\
& *x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*b^3*d^ \\
& 2-2*f^3/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^ \\
& 4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*a^2*c-1/4*f^3/(c*e^4*x^ \\
& 4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2 \\
& +a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*a*b^2+3/2*f^3*b*c/(16*a^2*c^2-8*a*b^2*c+ \\
& b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b \\
& *e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2* \\
& e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 4.02, size = 1267, normalized size = 7.97

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x)$

[Out]
$$- \left(\frac{(a*b^2*f^3 + 8*a^2*c*f^3 + 2*b^3*d^2*f^3 + 9*b^2*c*d^4*f^3 + 6*b*c^2*d^6*f^3 + 10*a*b*c*d^2*f^3)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e*f^3 + 27*b^2*c*d^2*e*f^3 + 45*b*c^2*d^4*e*f^3 + 5*a*b*c*e*f^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*x^4*(b^2*c*e^3*f^3 + 10*b*c^2*d^2*e^3*f^3))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*b^2*c*e^2*f^3 + 10*b*c^2*d^2*e^2*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3*f^3 + 9*b^2*c*d^2*f^3 + 9*b*c^2*d^4*f^3 + 5*a*b*c*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*b*c^2*e^5*f^3*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*f^3*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) \right) / (x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*f^3*\text{atan}(((b^4*(4*a*c - b^2))^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*c^4*e^8*f^6)/(a*(4*a*c - b^2))^{9/2}*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*f^6*(2*b^5*c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^2*(4*a*c - b^2))^{15/2}*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((9*b^3*c^2*f^6*(2*b^5*c^2*d*e^9 - 16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2))^{15/2}*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (18*b^2*c^4*d*e^7*f^6)/(a*(4*a*c - b^2))^{9/2}*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c^4*d^2*e^6*f^6)/(a*(4*a*c - b^2))^{9/2}*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*f^6*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8 - 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2))^{15/2}*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(18*b^2*c^4*e^6*f^6)))/(e*(4*a*c - b^2))^{5/2})$$

sympy [B] time = 14.58, size = 1794, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3, x)$

[Out]
$$3*b*c*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5*\log(2*d*x/e + x**2 + (-192*a**3*b*c**4*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5) + 144*a**2*b**3*c**3*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5) - 36*a*b**5*c**2*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5) + 3*b**7*c*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5) + 3*b**2*c*f**3 + 6*b*c**2*d**2*f**3)/(6*b*c**2*e**2*f**3)/(2*e) - 3*b*c*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5*\log(2*d*x/e + x**2 + (192*a**3*b*c**4*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5) - 144*a**2*b**3*c**3*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5) + 36*a*b**5*c**2*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5) + 36*a*b**5*c**2*f**3*\text{sqrt}(-1/(4*a*c - b**2))**5)$$

$$\begin{aligned}
& 2)^{**5}) - 3*b^{**7}*c*f^{**3}*sqrt(-1/(4*a*c - b^{**2}))^{**5}) + 3*b^{**2}*c*f^{**3} + 6*b*c^{**2} \\
& *d^{**2}*f^{**3})/(6*b*c^{**2}*e^{**2}*f^{**3})/(2*e) + (-8*a^{**2}*c*f^{**3} - a*b^{**2}*f^{**3} - \\
& 10*a*b*c*d^{**2}*f^{**3} - 2*b^{**3}*d^{**2}*f^{**3} - 9*b^{**2}*c*d^{**4}*f^{**3} - 6*b*c^{**2}*d^{**6}* \\
& f^{**3} - 36*b*c^{**2}*d*e^{**5}*f^{**3}*x^{**5} - 6*b*c^{**2}*e^{**6}*f^{**3}*x^{**6} + x^{**4}*(-9*b^{**2} \\
& *c*e^{**4}*f^{**3} - 90*b*c^{**2}*d^{**2}*e^{**4}*f^{**3}) + x^{**3}*(-36*b^{**2}*c*d*e^{**3}*f^{**3} - 1 \\
& 20*b*c^{**2}*d^{**3}*e^{**3}*f^{**3}) + x^{**2}*(-10*a*b*c*e^{**2}*f^{**3} - 2*b^{**3}*e^{**2}*f^{**3} - \\
& 54*b^{**2}*c*d^{**2}*e^{**2}*f^{**3} - 90*b*c^{**2}*d^{**4}*e^{**2}*f^{**3}) + x*(-20*a*b*c*d*e*f^{**3} \\
& - 4*b^{**3}*d*e*f^{**3} - 36*b^{**2}*c*d^{**3}*e*f^{**3} - 36*b*c^{**2}*d^{**5}*e*f^{**3}))/ (64*a \\
& **4*c^{**2}*e - 32*a**3*b^{**2}*c*e + 128*a**3*b*c^{**2}*d^{**2}*e + 128*a**3*c**3*d^{**4} \\
& *e + 4*a**2*b**4*e - 64*a**2*b**3*c*d^{**2}*e + 128*a**2*b*c**3*d**6*e + 64*a* \\
& *2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6 \\
& *e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2* \\
& d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) \\
& + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) \\
&) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2* \\
& e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7) \\
& + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c** \\
& 2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d* \\
& *3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a** \\
& 2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b* \\
& *2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4 \\
& *e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a** \\
& 2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a \\
& *b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c** \\
& 2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a* \\
& *2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8* \\
& a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b* \\
& *2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c** \\
& 2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a* \\
& *2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16 \\
& *a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a* \\
& b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c** \\
& 2*d**7*e**2))
\end{aligned}$$

$$3.543 \quad \int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=375

$$\frac{f^2(d+ex)(c(20ac+b^2)(d+ex)^2+b(8ac+b^2))}{8ae(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{f^2(d+ex)(b+2c(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{\sqrt{c} f^2 \left(\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}} \right)}{8\sqrt{2} ae}$$

Rubi [A] time = 0.97, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1142, 1119, 1178, 1166, 205}

$$\frac{f^2(d+ex)(c(20ac+b^2)(d+ex)^2+b(8ac+b^2))}{8ae(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{f^2(d+ex)(b+2c(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{\sqrt{c} f^2 \left(\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}} + 20ac + b^2 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{8\sqrt{2} ae (b^2-4ac)^2 \sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c} f^2 \left(-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}} + 20ac + b^2 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} (d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{8\sqrt{2} ae (b^2-4ac)^2 \sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] $-(f^2(d+ex)(b+2c(d+ex)^2))/(4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2) + (f^2(d+ex)(b(b^2+8ac)+c(b^2+20ac)(d+ex)^2))/(8a(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)) + (\text{Sqrt}[c](b^2+20ac+(b(b^2-52ac))/\text{Sqrt}[b^2-4ac]))/\text{Sqrt}[b^2-4ac]*f^2*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c](d+ex))/\text{Sqrt}[b-\text{Sqrt}[b^2-4ac]])]/(8*\text{Sqrt}[2]*a*(b^2-4ac)^2*\text{Sqrt}[b-\text{Sqrt}[b^2-4ac]])*e) + (\text{Sqrt}[c](b^2+20ac-(b(b^2-52ac))/\text{Sqrt}[b^2-4ac]))/\text{Sqrt}[b^2-4ac]*f^2*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c](d+ex))/\text{Sqrt}[b+\text{Sqrt}[b^2-4ac]])]/(8*\text{Sqrt}[2]*a*(b^2-4ac)^2*\text{Sqrt}[b+\text{Sqrt}[b^2-4ac]])*e)$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1119

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(d*x)^(m-1)*(b+2*c*x^2)*(a+b*x^2+c*x^4)^(p+1))/(2*(p+1)*(b^2-4*a*c)), x] - Dist[d^2/(2*(p+1)*(b^2-4*a*c)), Int[(d*x)^(m-2)*(b*(m-1)+2*c*(m+4*p+5)*x^2)*(a+b*x^2+c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2-4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1142

```
Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx &= \frac{f^2 \operatorname{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^3} dx, x, d + ex\right)}{e} \\
&= -\frac{f^2(d + ex)(b + 2c(d + ex)^2)}{4(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{f^2 \operatorname{Subst}\left(\int \frac{b-10cx^2}{(a+bx^2+cx^4)^2} dx, x, d + ex\right)}{4(b^2 - 4ac)e} \\
&= -\frac{f^2(d + ex)(b + 2c(d + ex)^2)}{4(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{f^2(d + ex)(b(b^2 + 8ac))}{8a(b^2 - 4ac)^2 e(a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{f^2(d + ex)(b + 2c(d + ex)^2)}{4(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{f^2(d + ex)(b(b^2 + 8ac))}{8a(b^2 - 4ac)^2 e(a + b(d + ex)^2 + c(d + ex)^4)} \\
&= -\frac{f^2(d + ex)(b + 2c(d + ex)^2)}{4(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{f^2(d + ex)(b(b^2 + 8ac))}{8a(b^2 - 4ac)^2 e(a + b(d + ex)^2 + c(d + ex)^4)}
\end{aligned}$$

Mathematica [A] time = 4.48, size = 385, normalized size = 1.03

$$\frac{f^2 \left(-\frac{4(b(d+ex)+2c(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{2(d+ex)(8abc+20ac^2(d+ex)^2+b^3+b^2c(d+ex)^2)}{a(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} \right)}{16e}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (f^2*((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 20*a*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/((16*e)

$$\begin{aligned}
& 4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*\text{sqrt}(-((b^7 - \\
& 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)* \\
& \text{sqrt}((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))*e^2 \\
&)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*\log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2* \\
& b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2* \\
& *b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*\text{sqrt}(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e*f^4 \\
& - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*\text{sqrt}((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))*e^3)*\text{sqrt}(- \\
& ((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*\text{sqrt}((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))) - \text{sqrt}(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*e^9*x^8 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*e^8*x^7 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*e^7*x^6 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*e^6*x^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*e^5*x^4 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*e^4*x^3 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*\text{sqrt}(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*\text{sqrt}((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/((a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)*e^4)))*e^2)/((a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*e^2))*\log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*e*f^6*x + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 - 1/2*\text{sqrt}(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*
\end{aligned}$$

$$\begin{aligned}
& b^5 c^5) * e * f^4 - (a^3 b^{14} - 38 a^4 b^{12} c + 480 a^5 b^{10} c^2 - 2720 a^6 b^8 c^3 + 6400 a^7 b^6 c^4 + 1536 a^8 b^4 c^5 - 32768 a^9 b^2 c^6 + 40960 a^{10} c^7) * \sqrt{(b^4 - 50 a b^2 c + 625 a^2 c^2) * f^8 / ((a^6 b^{10} - 20 a^7 b^8 c + 160 a^8 b^6 c^2 - 640 a^9 b^4 c^3 + 1280 a^{10} b^2 c^4 - 1024 a^{11} c^5) * e^4)} \\
&) * e^3) * \sqrt{(-(b^7 - 35 a b^5 c + 280 a^2 b^3 c^2 + 1680 a^3 b c^3) * f^4 + (a^3 b^{10} - 20 a^4 b^8 c + 160 a^5 b^6 c^2 - 640 a^6 b^4 c^3 + 1280 a^7 b^2 c^4 - 1024 a^8 c^5) * \sqrt{(b^4 - 50 a b^2 c + 625 a^2 c^2) * f^8 / ((a^6 b^{10} - 20 a^7 b^8 c + 160 a^8 b^6 c^2 - 640 a^9 b^4 c^3 + 1280 a^{10} b^2 c^4 - 1024 a^{11} c^5) * e^4)}) * e^2) / ((a^3 b^{10} - 20 a^4 b^8 c + 160 a^5 b^6 c^2 - 640 a^6 b^4 c^3 + 1280 a^7 b^2 c^4 - 1024 a^8 c^5) * e^2))} + \sqrt{1/2} * ((a b^4 c^2 - 8 a^2 b^2 c^3 + 16 a^3 c^4) * e^9 x^8 + 8 * (a b^4 c^2 - 8 a^2 b^2 c^3 + 16 a^3 c^4) * d * e^8 x^7 + 2 * (a b^5 c - 8 a^2 b^3 c^2 + 16 a^3 b c^3 + 14 * (a b^4 c^2 - 8 a^2 b^2 c^3 + 16 a^3 c^4) * d^2) * e^7 x^6 + 4 * (14 * (a b^4 c^2 - 8 a^2 b^2 c^3 + 16 a^3 c^4) * d^3 + 3 * (a b^5 c - 8 a^2 b^3 c^2 + 16 a^3 b c^3) * d) * e^6 x^5 + (a b^6 - 6 a^2 b^4 c + 32 a^4 c^3 + 70 * (a b^4 c^2 - 8 a^2 b^2 c^3 + 16 a^3 c^4) * d^4 + 30 * (a b^5 c - 8 a^2 b^3 c^2 + 16 a^3 b c^3) * d^2) * e^5 x^4 + 4 * (14 * (a b^4 c^2 - 8 a^2 b^2 c^3 + 16 a^3 c^4) * d^5 + 10 * (a b^5 c - 8 a^2 b^3 c^2 + 16 a^3 b c^3) * d^3 + (a b^6 - 6 a^2 b^4 c + 32 a^4 c^3) * d) * e^4 x^3 + 2 * (a^2 b^5 - 8 a^3 b^3 c + 16 a^4 b c^2 + 14 * (a b^4 c^2 - 8 a^2 b^2 c^3 + 16 a^3 c^4) * d^6 + 15 * (a b^5 c - 8 a^2 b^3 c^2 + 16 a^3 b c^3) * d^4 + 3 * (a b^6 - 6 a^2 b^4 c + 32 a^4 c^3) * d^2) * e^3 x^2 + 4 * (2 * (a b^4 c^2 - 8 a^2 b^2 c^3 + 16 a^3 c^4) * d^7 + 3 * (a b^5 c - 8 a^2 b^3 c^2 + 16 a^3 b c^3) * d^5 + (a b^6 - 6 a^2 b^4 c + 32 a^4 c^3) * d^3 + (a^2 b^5 - 8 a^3 b^3 c + 16 a^4 b c^2) * d) * e^2 x + ((a b^4 c^2 - 8 a^2 b^2 c^3 + 16 a^3 c^4) * d^8 + a^3 b^4 - 8 a^4 b^2 c + 16 a^5 c^2 + 2 * (a b^5 c - 8 a^2 b^3 c^2 + 16 a^3 b c^3) * d^6 + (a b^6 - 6 a^2 b^4 c + 32 a^4 c^3) * d^4 + 2 * (a^2 b^5 - 8 a^3 b^3 c + 16 a^4 b c^2) * d^2) * e) * \sqrt{(-(b^7 - 35 a b^5 c + 280 a^2 b^3 c^2 + 1680 a^3 b c^3) * f^4 - (a^3 b^{10} - 20 a^4 b^8 c + 160 a^5 b^6 c^2 - 640 a^6 b^4 c^3 + 1280 a^7 b^2 c^4 - 1024 a^8 c^5) * \sqrt{(b^4 - 50 a b^2 c + 625 a^2 c^2) * f^8 / ((a^6 b^{10} - 20 a^7 b^8 c + 160 a^8 b^6 c^2 - 640 a^9 b^4 c^3 + 1280 a^{10} b^2 c^4 - 1024 a^{11} c^5) * e^4)}) * e^2) / ((a^3 b^{10} - 20 a^4 b^8 c + 160 a^5 b^6 c^2 - 640 a^6 b^4 c^3 + 1280 a^7 b^2 c^4 - 1024 a^8 c^5) * e^2)) * \log((35 b^6 c^2 - 1491 a b^4 c^3 + 15000 a^2 b^2 c^4 + 10000 a^3 c^5) * e * f^6 x + (35 b^6 c^2 - 1491 a b^4 c^3 + 15000 a^2 b^2 c^4 + 10000 a^3 c^5) * d * f^6 + 1/2 * \sqrt{1/2} * (b^{11} - 53 a b^9 c + 940 a^2 b^7 c^2 - 6832 a^3 b^5 c^3 + 21824 a^4 b^3 c^4 - 25600 a^5 b c^5) * e * f^4 + (a^3 b^{14} - 38 a^4 b^{12} c + 480 a^5 b^{10} c^2 - 2720 a^6 b^8 c^3 + 6400 a^7 b^6 c^4 + 1536 a^8 b^4 c^5 - 32768 a^9 b^2 c^6 + 40960 a^{10} c^7) * \sqrt{(b^4 - 50 a b^2 c + 625 a^2 c^2) * f^8 / ((a^6 b^{10} - 20 a^7 b^8 c + 160 a^8 b^6 c^2 - 640 a^9 b^4 c^3 + 1280 a^{10} b^2 c^4 - 1024 a^{11} c^5) * e^4)}) * e^3) * \sqrt{(-(b^7 - 35 a b^5 c + 280 a^2 b^3 c^2 + 1680 a^3 b c^3) * f^4 - (a^3 b^{10} - 20 a^4 b^8 c + 160 a^5 b^6 c^2 - 640 a^6 b^4 c^3 + 1280 a^7 b^2 c^4 - 1024 a^8 c^5) * \sqrt{(b^4 - 50 a b^2 c + 625 a^2 c^2) * f^8 / ((a^6 b^{10} - 20 a^7 b^8 c + 160 a^8 b^6 c^2 - 640 a^9 b^4 c^3 + 1280 a^{10} b^2 c^4 - 1024 a^{11} c^5) * e^4)}) * e^2) / ((a^3 b^{10} - 20 a^4 b^8 c + 160 a^5 b^6 c^2 - 640 a^6 b^4 c^3 + 1280 a^7 b^2 c^4 - 1024 a^8 c^5) * e^2))} - \sqrt{1/2}
\end{aligned}$$

$$\begin{aligned} &^3*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*e^3*x^2 + 4*(2*(a*b^4*c \\ &^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3* \\ &b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3* \\ &c + 16*a^4*b*c^2)*d)*e^2*x + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 \\ &+ a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3* \\ &b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^ \\ &3*c + 16*a^4*b*c^2)*d^2)*e) \end{aligned}$$

giac [B] time = 0.79, size = 2527, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} &-1/16*(((d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/} \\ &c))^2*b^2*c*f^2*e^2 + 20*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4* \\ &a*c})*e^2})*e^{(-4)/c})^2*a*c^2*f^2*e^2 - 2*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 \\ &+ \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})*b^2*c*d*f^2*e - 40*(d*e^{(-1)} + \sqrt{1/} \\ &2)*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})*a*c^2*d*f^2*e + b^2*c*d \\ &^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*\log(d*e^{(-1)} + x + \sqrt{ \\ &1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})/(2*(d*e^{(-1)} + \sqrt{ \\ &1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} \\ &+ \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*c*d*e^3 - 2 \\ &*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 \\ &+ \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})) + ((d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 \\ &+ \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*b^2*c*f^2*e^2 + 20*(d*e^{(-1)} - \sqrt{ \\ &1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*a*c^2*f^2*e^2 - 2*(\\ &d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})*b^2*c \\ &*d*f^2*e - 40*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e \\ &^{(-4)/c})*a*c^2*d*f^2*e + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a \\ &*b*c*f^2)*\log(d*e^{(-1)} + x - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2} \\ &)*e^{(-4)/c})/(2*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2} \\ &)*e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a} \\ &*c)*e^2})*e^{(-4)/c})^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(\\ &d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 + \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})) + ((\\ &d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*b^2 \\ &*c*f^2*e^2 + 20*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2} \\ &)*e^{(-4)/c})^2*a*c^2*f^2*e^2 - 2*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b} \\ &^2 - 4*a*c)*e^2})*e^{(-4)/c})*b^2*c*d*f^2*e - 40*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(\\ &b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})*a*c^2*d*f^2*e + b^2*c*d^2*f^2 + \\ &20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*\log(d*e^{(-1)} + x + \sqrt{1/2}*\sqrt{ \\ &-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})/(2*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{ \\ &-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} + \sqrt{1/} \\ &2)*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2})*e^{(-4)/c})^2*c*d*e^3 - 2*c*d^3*e \end{aligned}$$

$$\begin{aligned}
& - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} + \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}*e^{(-4)/c})) + ((d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}*e^{(-4)/c})^2*b^2*c*f^2*e^2 + 20*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}*e^{(-4)/c})^2*a*c^2*f^2*e^2 - 2*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}*e^{(-4)/c})*b^2*c*d*f^2*e - 40*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}*e^{(-4)/c}))*a*c^2*d*f^2*e + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2) \\
& * \log(d*e^{(-1)} + x - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}*e^{(-4)/c}) / (2*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}*e^{(-4)/c})^3*c*e^4 - 6*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}*e^{(-4)/c})^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^{(-1)} - \sqrt{1/2}*\sqrt{-(b*e^2 - \sqrt{b^2 - 4*a*c})*e^2}*e^{(-4)/c}))) / (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*(b^2*c^2*f^2*x^7*e^7 + 20*a*c^3*f^2*x^7*e^7 + 7*b^2*c^2*d*f^2*x^6*e^6 + 140*a*c^3*d*f^2*x^6*e^6 + 21*b^2*c^2*d^2*f^2*x^5*e^5 + 420*a*c^3*d^2*f^2*x^5*e^5 + 35*b^2*c^2*d^3*f^2*x^4*e^4 + 700*a*c^3*d^3*f^2*x^4*e^4 + 35*b^2*c^2*d^4*f^2*x^3*e^3 + 700*a*c^3*d^4*f^2*x^3*e^3 + 21*b^2*c^2*d^5*f^2*x^2*e^2 + 420*a*c^3*d^5*f^2*x^2*e^2 + 7*b^2*c^2*d^6*f^2*x*e + 140*a*c^3*d^6*f^2*x*e + b^2*c^2*d^7*f^2 + 20*a*c^3*d^7*f^2 + 2*b^3*c*f^2*x^5*e^5 + 28*a*b*c^2*f^2*x^5*e^5 + 10*b^3*c*d*f^2*x^4*e^4 + 140*a*b*c^2*d*f^2*x^4*e^4 + 20*b^3*c*d^2*f^2*x^3*e^3 + 280*a*b*c^2*d^2*f^2*x^3*e^3 + 20*b^3*c*d^3*f^2*x^2*e^2 + 280*a*b*c^2*d^3*f^2*x^2*e^2 + 10*b^3*c*d^4*f^2*x*e + 140*a*b*c^2*d^4*f^2*x*e + 2*b^3*c*d^5*f^2 + 28*a*b*c^2*d^5*f^2 + b^4*f^2*x^3*e^3 + 5*a*b^2*c*f^2*x^3*e^3 + 36*a^2*c^2*f^2*x^3*e^3 + 3*b^4*d*f^2*x^2*e^2 + 15*a*b^2*c*d*f^2*x^2*e^2 + 108*a^2*c^2*d*f^2*x^2*e^2 + 3*b^4*d^2*f^2*x*e + 15*a*b^2*c*d^2*f^2*x*e + 108*a^2*c^2*d^2*f^2*x*e + b^4*d^3*f^2 + 5*a*b^2*c*d^3*f^2 + 36*a^2*c^2*d^3*f^2 - a*b^3*f^2*x*e + 16*a^2*b*c*f^2*x*e - a*b^3*d*f^2 + 16*a^2*b*c*d*f^2) / ((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(a*b^4*e - 8*a^2*b^2*c*e + 16*a^3*c^2*e))
\end{aligned}$$

maple [C] time = 0.05, size = 4751, normalized size = 12.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int ((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3, x)$

[Out] $7/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6*b^2+21/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5*b^2*d^2+35/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4*b^2+5/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4*b^3+35/8*f^2/(c*e^4*$

$$\begin{aligned}
& x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3*b^2*c^2*d^4+3/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^4*d^2+105/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3+35/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c^2*d^4+15/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^2*c*d^2+7/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^5/e*b*c^2+5/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^3/e*b^2*c+35/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x*b*c+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*d^3/e*c^2+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*d^3/e*b^4+175/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*d^3*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+175/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*c^3*d^4+5/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2*c+105/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*d^2+7/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5*b+9/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x^3*c^2+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3*b^4+27/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x*c^2*d^2+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+35/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3*d^6+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d^7/e*c^3-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*d/e*b^3+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*d^7/e*b^2*c^2+1/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)
\end{aligned}$$

$$2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*d^5/e*b^3*c+2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*a*d/e*b*c+35/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b+35*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b*c^2*d^2+35*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2+15/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c+1/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5*b^3+1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^6/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7*b^2+27/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)*a*x^2*c^2+3/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^4+7/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x*b^2*c^2*d^6+5/4*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3*b^3*c*d^2+21/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^5*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^2*c^2+5/2*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^2*b^3*c-1/8*f^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^3+1/16*f^2/(16*a^2*c^2-8*a*b^2*c+b^4)/a/e*sum(((20*a*c+b^2)*_R^2*c*e^2+20*a*c^2*d^2+b^2*c*d^2+2*(20*a*c+b^2)*_R*c*d*e-16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(-_R+x),_R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 7.94, size = 16025, normalized size = 42.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x)$

[Out] $\text{atan}\left(\frac{\left(\frac{67108864a^9b^9c^9d^9e^{13} - 4096a^2b^{15}c^2d^9e^{13} + 114688a^3b^{13}c^3d^9e^{13} - 1376256a^4b^{11}c^4d^9e^{13} + 9175040a^5b^9c^5d^9e^{13} - 36700160a^6b^7c^6d^9e^{13} + 88080384a^7b^5c^7d^9e^{13} - 117440512a^8b^3c^8d^9e^{13}}{512(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)} + (x(262144a^7b^9c^7e^{14} - 256a^2b^{11}c^2e^{14} + 5120a^3b^9c^3e^{14} - 40960a^4b^7c^4e^{14} + 163840a^5b^5c^5e^{14} - 327680a^6b^3c^6e^{14}))}{32(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)}\right) * \left(-b^{17}f^4 + b^2f^4 * \left(-4ac - b^2\right)^{15}\right)^{1/2} - 1720320a^8b^8c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^8b^{15}c^8f^4 - 25a^8c^8f^4 * \left(-4ac - b^2\right)^{15}\right)^{1/2} / \left(512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^8e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2)\right)^{1/2} - \left(122880a^3b^9c^4e^{12}f^2 - 9216a^2b^{11}c^3e^{12}f^2 - 819200a^4b^7c^5e^{12}f^2 + 2949120a^5b^5c^6e^{12}f^2 - 5505024a^6b^3c^7e^{12}f^2 + 256a^8b^{13}c^2e^{12}f^2 + 4194304a^7b^8c^8e^{12}f^2\right) / \left(512(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)\right) * \left(-b^{17}f^4 + b^2f^4 * \left(-4ac - b^2\right)^{15}\right)^{1/2} - 1720320a^8b^8c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^8b^{15}c^8f^4 - 25a^8c^8f^4 * \left(-4ac - b^2\right)^{15}\right)^{1/2} / \left(512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^8e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2)\right)^{1/2} + \left(204800a^5c^8d^9e^{11}f^4 - 16b^{10}c^3d^9e^{11}f^4 + 672a^8b^8c^4d^9e^{11}f^4 - 28160a^2b^6c^5d^9e^{11}f^4 + 209920a^3b^4c^6d^9e^{11}f^4 - 479232a^4b^2c^7d^9e^{11}f^4\right) / \left(512(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)\right) + \left(x(800a^3c^6e^{12}f^4 - b^6c^3e^{12}f^4 - 1472a^2b^2c^5e^{12}f^4 + 34a^8b^4c^4e^{12}f^4)\right) / \left(32(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)\right) * \left(-b^{17}f^4 + b^2f^4 * \left(-4ac - b^2\right)^{15}\right)^{1/2} - 1720320a^8b^8c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^8b^{15}c^8f^4 - 25a^8c^8f^4 * \left(-4ac - b^2\right)^{15}\right)^{1/2} / \left(512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^8e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2)\right)^{1/2}$

$$\begin{aligned}
& b^6 c^7 e^2 + 2949120 a^{11} b^4 c^8 e^2 - 2621440 a^{12} b^2 c^9 e^2))^{(1/2)} * \\
& 1i + (((((67108864 a^9 b^3 c^9 d e^{13} - 4096 a^2 b^{15} c^2 d e^{13} + 114688 a^3 b^{13} c^3 d e^{13} - 1376256 a^4 b^{11} c^4 d e^{13} + 9175040 a^5 b^9 c^5 d e^{13} \\
& - 36700160 a^6 b^7 c^6 d e^{13} + 88080384 a^7 b^5 c^7 d e^{13} - 117440512 a^8 \\
& * b^3 c^8 d e^{13}) / (512 (a^2 b^{12} + 4096 a^8 c^6 - 24 a^3 b^{10} c + 240 a^4 b^8 c^2 - 1280 a^5 b^6 c^3 + 3840 a^6 b^4 c^4 - 6144 a^7 b^2 c^5)) + (x * (2621 \\
& 44 a^7 b^3 c^7 e^{14} - 256 a^2 b^{11} c^2 e^{14} + 5120 a^3 b^9 c^3 e^{14} - 40960 a^4 b^7 c^4 e^{14} + 163840 a^5 b^5 c^5 e^{14} - 327680 a^6 b^3 c^6 e^{14})) / (32 * (\\
& a^2 b^8 + 256 a^6 c^4 - 16 a^3 b^6 c + 96 a^4 b^4 c^2 - 256 a^5 b^2 c^3))) * \\
& (- (b^{17} f^4 + b^2 f^4 * (- (4 a c - b^2)^{15})^{(1/2)} - 1720320 a^8 b^3 c^8 f^4 + 1 \\
& 140 a^2 b^{13} c^2 f^4 - 10160 a^3 b^{11} c^3 f^4 + 34880 a^4 b^9 c^4 f^4 + 437 \\
& 76 a^5 b^7 c^5 f^4 - 680960 a^6 b^5 c^6 f^4 + 1863680 a^7 b^3 c^7 f^4 - 55 a \\
& a b^{15} c f^4 - 25 a c f^4 * (- (4 a c - b^2)^{15})^{(1/2)}) / (512 (a^3 b^{20} e^2 + 1 \\
& 048576 a^{13} c^{10} e^2 - 40 a^4 b^{18} c e^2 + 720 a^5 b^{16} c^2 e^2 - 7680 a^6 b^{14} c^3 e^2 + 53760 a^7 b^{12} c^4 e^2 - 258048 a^8 b^{10} c^5 e^2 + 860160 a^9 b^8 c^6 e^2 \\
& - 1966080 a^{10} b^6 c^7 e^2 + 2949120 a^{11} b^4 c^8 e^2 - 26214 \\
& 40 a^{12} b^2 c^9 e^2)))^{(1/2)} + (122880 a^3 b^9 c^4 e^{12} f^2 - 9216 a^2 b^{11} \\
& c^3 e^{12} f^2 - 819200 a^4 b^7 c^5 e^{12} f^2 + 2949120 a^5 b^5 c^6 e^{12} f^2 \\
& - 5505024 a^6 b^3 c^7 e^{12} f^2 + 256 a a b^{13} c^2 e^{12} f^2 + 4194304 a^7 b^3 c^8 e^{12} f^2) / (512 (a^2 b^{12} + 4096 a^8 c^6 - 24 a^3 b^{10} c + 240 a^4 b^8 c^2 \\
& - 1280 a^5 b^6 c^3 + 3840 a^6 b^4 c^4 - 6144 a^7 b^2 c^5))) * (- (b^{17} f^4 + \\
& b^2 f^4 * (- (4 a c - b^2)^{15})^{(1/2)} - 1720320 a^8 b^3 c^8 f^4 + 1140 a^2 b^{13} c^2 f^4 - 10160 a^3 b^{11} c^3 f^4 + 34880 a^4 b^9 c^4 f^4 + 43776 a^5 b^7 c^5 \\
& f^4 - 680960 a^6 b^5 c^6 f^4 + 1863680 a^7 b^3 c^7 f^4 - 55 a b^{15} c f^4 - \\
& 25 a c f^4 * (- (4 a c - b^2)^{15})^{(1/2)}) / (512 (a^3 b^{20} e^2 + 1048576 a^{13} c^{10} e^2 - 40 a^4 b^{18} c e^2 + 720 a^5 b^{16} c^2 e^2 - 7680 a^6 b^{14} c^3 e^2 + \\
& 53760 a^7 b^{12} c^4 e^2 - 258048 a^8 b^{10} c^5 e^2 + 860160 a^9 b^8 c^6 e^2 \\
& - 1966080 a^{10} b^6 c^7 e^2 + 2949120 a^{11} b^4 c^8 e^2 - 2621440 a^{12} b^2 c^9 e^2)))^{(1/2)} + (204800 a^5 c^8 d e^{11} f^4 - 16 b^{10} c^3 d e^{11} f^4 + 672 a \\
& a b^8 c^4 d e^{11} f^4 - 28160 a^2 b^6 c^5 d e^{11} f^4 + 209920 a^3 b^4 c^6 d e^{11} f^4 - 479232 a^4 b^2 c^7 d e^{11} f^4) / (512 (a^2 b^{12} + 4096 a^8 c^6 - 2 \\
& 4 a^3 b^{10} c + 240 a^4 b^8 c^2 - 1280 a^5 b^6 c^3 + 3840 a^6 b^4 c^4 - 6144 \\
& a^7 b^2 c^5)) + (x * (800 a^3 c^6 e^{12} f^4 - b^6 c^3 e^{12} f^4 - 1472 a^2 b^2 \\
& c^5 e^{12} f^4 + 34 a a b^4 c^4 e^{12} f^4)) / (32 (a^2 b^8 + 256 a^6 c^4 - 16 a^3 \\
& b^6 c + 96 a^4 b^4 c^2 - 256 a^5 b^2 c^3))) * (- (b^{17} f^4 + b^2 f^4 * (- (4 a c \\
& - b^2)^{15})^{(1/2)} - 1720320 a^8 b^3 c^8 f^4 + 1140 a^2 b^{13} c^2 f^4 - 10160 a^3 b^{11} c^3 f^4 + 34880 a^4 b^9 c^4 f^4 + 43776 a^5 b^7 c^5 f^4 - 680960 a^6 b^5 c^6 f^4 + 1863680 a^7 b^3 c^7 f^4 - 55 a a b^{15} c f^4 - 25 a c f^4 * (- (4 \\
& a c - b^2)^{15})^{(1/2)}) / (512 (a^3 b^{20} e^2 + 1048576 a^{13} c^{10} e^2 - 40 a^4 b^{18} c e^2 + 720 a^5 b^{16} c^2 e^2 - 7680 a^6 b^{14} c^3 e^2 + 53760 a^7 b^{12} c^4 e^2 - 258048 a^8 b^{10} c^5 e^2 + 860160 a^9 b^8 c^6 e^2 - 1966080 a^{10} b^6 c^7 e^2 + 2949120 a^{11} b^4 c^8 e^2 - 2621440 a^{12} b^2 c^9 e^2)))^{(1/2)} * 1 \\
& i) / ((((((67108864 a^9 b^3 c^9 d e^{13} - 4096 a^2 b^{15} c^2 d e^{13} + 114688 a^3 b^{13} c^3 d e^{13} - 1376256 a^4 b^{11} c^4 d e^{13} + 9175040 a^5 b^9 c^5 d e^{13} - \\
& 36700160 a^6 b^7 c^6 d e^{13} + 88080384 a^7 b^5 c^7 d e^{13} - 117440512 a^8
\end{aligned}$$

$$\begin{aligned}
& b^3c^8de^{13})/(512*(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 \\
& *c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + (x*(26214 \\
& 4a^7b^7c^7e^{14} - 256a^2b^{11}c^2e^{14} + 5120a^3b^9c^3e^{14} - 40960a^4 \\
& 4b^7c^4e^{14} + 163840a^5b^5c^5e^{14} - 327680a^6b^3c^6e^{14}))/((32*(a \\
& ^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)))* \\
& -(b^{17}f^4 + b^2f^4*(-(4ac - b^2)^{15})^{1/2} - 1720320a^8b^8c^8f^4 + 11 \\
& 40a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 4377 \\
& 6a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a \\
& *b^{15}c^5f^4 - 25a^2c^4f^4*(-(4ac - b^2)^{15})^{1/2}))/((512*(a^3b^{20}e^2 + 10 \\
& 48576a^{13}c^{10}e^2 - 40a^4b^{18}c^4e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b \\
& ^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9 \\
& *b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 262144 \\
& 0a^{12}b^2c^9e^2)))^{1/2} - (122880a^3b^9c^4e^{12}f^2 - 9216a^2b^{11} \\
& c^3e^{12}f^2 - 819200a^4b^7c^5e^{12}f^2 + 2949120a^5b^5c^6e^{12}f^2 - \\
& 5505024a^6b^3c^7e^{12}f^2 + 256a^2b^{13}c^2e^{12}f^2 + 4194304a^7b^7c^8 \\
& *e^{12}f^2))/((512*(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 \\
& - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)))*-(b^{17}f^4 + b \\
& ^2f^4*(-(4ac - b^2)^{15})^{1/2} - 1720320a^8b^8c^8f^4 + 1140a^2b^{13}c^ \\
& ^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5 \\
& f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^2b^{15}c^5f^4 - \\
& 25a^2c^4f^4*(-(4ac - b^2)^{15})^{1/2}))/((512*(a^3b^{20}e^2 + 1048576a^{13}c^ \\
& ^10e^2 - 40a^4b^{18}c^4e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + \\
& 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - \\
& 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9 \\
& *e^2)))^{1/2} + (204800a^5c^8de^{11}f^4 - 16b^{10}c^3de^{11}f^4 + 672a \\
& *b^8c^4de^{11}f^4 - 28160a^2b^6c^5de^{11}f^4 + 209920a^3b^4c^6de \\
& ^{11}f^4 - 479232a^4b^2c^7de^{11}f^4)/(512*(a^2b^{12} + 4096a^8c^6 - 24 \\
& *a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a \\
& ^7b^2c^5)) + (x*(800a^3c^6e^{12}f^4 - b^6c^3e^{12}f^4 - 1472a^2b^2c^ \\
& ^5e^{12}f^4 + 34a^2b^4c^4e^{12}f^4))/((32*(a^2b^8 + 256a^6c^4 - 16a^3b^ \\
& ^6c + 96a^4b^4c^2 - 256a^5b^2c^3)))*-(b^{17}f^4 + b^2f^4*(-(4ac \\
& - b^2)^{15})^{1/2} - 1720320a^8b^8c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^ \\
& ^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6 \\
& *b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^2b^{15}c^5f^4 - 25a^2c^4f^4*(-(4 \\
& ac - b^2)^{15})^{1/2}))/((512*(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b \\
& ^{18}c^4e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^ \\
& ^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^ \\
& ^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2)))^{1/2} - \\
& (((67108864a^9b^9c^9de^{13} - 4096a^2b^{15}c^2de^{13} + 114688a^3b^{13} \\
& c^3de^{13} - 1376256a^4b^{11}c^4de^{13} + 9175040a^5b^9c^5de^{13} - 367 \\
& 00160a^6b^7c^6de^{13} + 88080384a^7b^5c^7de^{13} - 117440512a^8b^3c^ \\
& ^8de^{13}))/((512*(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 \\
& - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + (x*(262144a^ \\
& ^7b^7c^7e^{14} - 256a^2b^{11}c^2e^{14} + 5120a^3b^9c^3e^{14} - 40960a^4b^ \\
& ^7c^4e^{14} + 163840a^5b^5c^5e^{14} - 327680a^6b^3c^6e^{14}))/((32*(a^2b
\end{aligned}$$

$$\begin{aligned}
&^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)) * (- (b^17*f^4 + b^2*f^4 * (- (4*a*c - b^2)^15)^{1/2} - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4 * (- (4*a*c - b^2)^15)^{1/2}) / (512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^{1/2} + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2) / (512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))) * (- (b^17*f^4 + b^2*f^4 * (- (4*a*c - b^2)^15)^{1/2} - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4 * (- (4*a*c - b^2)^15)^{1/2}) / (512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^{1/2} + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4) / (512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4)) / (32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3))) * (- (b^17*f^4 + b^2*f^4 * (- (4*a*c - b^2)^15)^{1/2} - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4 * (- (4*a*c - b^2)^15)^{1/2}) / (512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^{1/2} + (800*a^3*c^7*e^10*f^6 - 35*b^6*c^4*e^10*f^6 + 12720*a^2*b^2*c^6*e^10*f^6 - 84*a*b^4*c^5*e^10*f^6) / (256*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5))) * (- (b^17*f^4 + b^2*f^4 * (- (4*a*c - b^2)^15)^{1/2} - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4 * (- (4*a*c - b^2)^15)^{1/2}) / (512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^{1/2} * 2i + \operatorname{atan}((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2
\end{aligned}$$

$$\begin{aligned}
& *b^{15}c^2d^2e^{13} + 114688a^3b^{13}c^3d^2e^{13} - 1376256a^4b^{11}c^4d^2e^{13} \\
& + 9175040a^5b^9c^5d^2e^{13} - 36700160a^6b^7c^6d^2e^{13} + 88080384a^7b^5c^7d^2e^{13} - 117440512a^8b^3c^8d^2e^{13}) / (512(a^2b^{12} + 4096a^8c^6 \\
& - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + (x*(262144a^7b^7c^7e^{14} - 256a^2b^{11}c^2e^{14} + 5 \\
& 120a^3b^9c^3e^{14} - 40960a^4b^7c^4e^{14} + 163840a^5b^5c^5e^{14} - 3 \\
& 27680a^6b^3c^6e^{14})) / (32(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3))) * (- (b^{17}f^4 - b^2f^4 * (- (4ac - b^2)^{15})^{1/2} \\
& - 1720320a^8b^7c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + \\
& 1863680a^7b^3c^7f^4 - 55a^2b^{15}c^2f^4 + 25a^2c^2f^4 * (- (4ac - b^2)^{15})^{1/2})) / (512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^2e^2 + 72 \\
& 0a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 29 \\
& 49120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2)))^{1/2} - (122880a^3b^9c^4e^{12}f^2 - 9216a^2b^{11}c^3e^{12}f^2 - 819200a^4b^7c^5e^{12}f^2 + \\
& 2949120a^5b^5c^6e^{12}f^2 - 5505024a^6b^3c^7e^{12}f^2 + 256a^2b^{13}c^2e^{12}f^2 + 4194304a^7b^7c^8e^{12}f^2) / (512(a^2b^{12} + 4096a^8c^6 - 2 \\
& 4a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5))) * (- (b^{17}f^4 - b^2f^4 * (- (4ac - b^2)^{15})^{1/2} - 1720320a^8b^7c^8f^4 \\
& + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^2b^{15}c^2f^4 \\
& + 25a^2c^2f^4 * (- (4ac - b^2)^{15})^{1/2})) / (512(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^2e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5 \\
& e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2)))^{1/2} + (204800a^5c^8d^2e^{11}f^4 - \\
& 16b^{10}c^3d^2e^{11}f^4 + 672a^2b^8c^4d^2e^{11}f^4 - 28160a^2b^6c^5d^2e^{11}f^4 + 209920a^3b^4c^6d^2e^{11}f^4 - 479232a^4b^2c^7d^2e^{11}f^4) / (51 \\
& 2(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + (x*(800a^3c^6e^{12}f^4 - b^6c^3e^{12}f^4 - 1472a^2b^2c^5e^{12}f^4 + 34a^2b^4c^4e^{12}f^4)) / (32(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3))) * \\
& (- (b^{17}f^4 - b^2f^4 * (- (4ac - b^2)^{15})^{1/2} - 1720320a^8b^7c^8f^4 + 1 \\
& 140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 437 \\
& 76a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^2b^{15}c^2f^4 + 25a^2c^2f^4 * (- (4ac - b^2)^{15})^{1/2})) / (512(a^3b^{20}e^2 + 1 \\
& 048576a^{13}c^{10}e^2 - 40a^4b^{18}c^2e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 26214 \\
& 40a^{12}b^2c^9e^2)))^{1/2} * i + (((67108864a^9b^7c^9d^2e^{13} - 4096a^2b^{15}c^2d^2e^{13} + 114688a^3b^{13}c^3d^2e^{13} - 1376256a^4b^{11}c^4d^2e^{13} \\
& + 9175040a^5b^9c^5d^2e^{13} - 36700160a^6b^7c^6d^2e^{13} + 88080384a^7b^5c^7d^2e^{13} - 117440512a^8b^3c^8d^2e^{13}) / (512(a^2b^{12} + 4096a^8c^6 \\
& - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 -
\end{aligned}$$

$$\begin{aligned}
& 6144a^7b^2c^5)) + (x*(262144a^7b^7c^7e^{14} - 256a^2b^{11}c^2e^{14} + 51 \\
& 20a^3b^9c^3e^{14} - 40960a^4b^7c^4e^{14} + 163840a^5b^5c^5e^{14} - 32 \\
& 7680a^6b^3c^6e^{14}))/((32*(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)))*(- (b^{17}f^4 - b^2f^4*(-(4ac - b^2)^{15})^{1/2} \\
&) - 1720320a^8b^8c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 \\
& + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + \\
& 1863680a^7b^3c^7f^4 - 55a^*b^{15}c^*f^4 + 25a^*c^*f^4*(-(4ac - b^2)^{15})^{1/2} \\
& (1/2))/((512*(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^*e^2 + 720 \\
& *a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048 \\
& *a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 294 \\
& 9120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2)))^{1/2} + (122880a^3b^9 \\
& *c^4e^{12}f^2 - 9216a^2b^{11}c^3e^{12}f^2 - 819200a^4b^7c^5e^{12}f^2 + \\
& 2949120a^5b^5c^6e^{12}f^2 - 5505024a^6b^3c^7e^{12}f^2 + 256a^*b^{13}c^ \\
& 2e^{12}f^2 + 4194304a^7b^8c^8e^{12}f^2))/((512*(a^2b^{12} + 4096a^8c^6 - 24 \\
& *a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^ \\
& a^7b^2c^5)))*(- (b^{17}f^4 - b^2f^4*(-(4ac - b^2)^{15})^{1/2} - 1720320a^ \\
& 8b^8c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^ \\
& 9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^ \\
& 3c^7f^4 - 55a^*b^{15}c^*f^4 + 25a^*c^*f^4*(-(4ac - b^2)^{15})^{1/2}))/((512*(a \\
& ^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^*e^2 + 720a^5b^{16}c^2 \\
& e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5 \\
& e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^ \\
& c^8e^2 - 2621440a^{12}b^2c^9e^2)))^{1/2} + (204800a^5c^8d^*e^{11}f^4 - \\
& 16b^{10}c^3d^*e^{11}f^4 + 672a^*b^8c^4d^*e^{11}f^4 - 28160a^2b^6c^5d^*e^{11} \\
& 1f^4 + 209920a^3b^4c^6d^*e^{11}f^4 - 479232a^4b^2c^7d^*e^{11}f^4))/((512 \\
& *(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + (x*(800a^3c^6e^{12}f^4 - b^ \\
& 6c^3e^{12}f^4 - 1472a^2b^2c^5e^{12}f^4 + 34a^*b^4c^4e^{12}f^4))/((32*(a \\
& ^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)))*(- (b^{17}f^4 - b^2f^4*(-(4ac - b^2)^{15})^{1/2} - 1720320a^8b^8c^8f^4 + 11 \\
& 40a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 4377 \\
& 6a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^ \\
& *b^{15}c^*f^4 + 25a^*c^*f^4*(-(4ac - b^2)^{15})^{1/2}))/((512*(a^3b^{20}e^2 + 10 \\
& 48576a^{13}c^{10}e^2 - 40a^4b^{18}c^*e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b \\
& ^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9 \\
& *b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 262144 \\
& 0a^{12}b^2c^9e^2)))^{1/2}*i)/((((67108864a^9b^9c^9d^*e^{13} - 4096a^2b \\
& ^{15}c^2d^*e^{13} + 114688a^3b^{13}c^3d^*e^{13} - 1376256a^4b^{11}c^4d^*e^{13} + \\
& 9175040a^5b^9c^5d^*e^{13} - 36700160a^6b^7c^6d^*e^{13} + 88080384a^7b^ \\
& 5c^7d^*e^{13} - 117440512a^8b^3c^8d^*e^{13}))/((512*(a^2b^{12} + 4096a^8c^6 \\
& - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6 \\
& 144a^7b^2c^5)) + (x*(262144a^7b^7c^7e^{14} - 256a^2b^{11}c^2e^{14} + 512 \\
& 0a^3b^9c^3e^{14} - 40960a^4b^7c^4e^{14} + 163840a^5b^5c^5e^{14} - 327 \\
& 680a^6b^3c^6e^{14}))/((32*(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3)))*(- (b^{17}f^4 - b^2f^4*(-(4ac - b^2)^{15})^{1/2}
\end{aligned}$$

$$\begin{aligned}
& - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + \\
& 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1 \\
& 863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(\\
& 1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720* \\
& a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048* \\
& a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949 \\
& 120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9* \\
& c^4*e^12*f^2 - 9216*a^2*b^11*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2 \\
& 949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2 \\
& *e^12*f^2 + 4194304*a^7*b*c^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24* \\
& a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a \\
& ^7*b^2*c^5)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8 \\
& *b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9 \\
& *c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3 \\
& *c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^ \\
& 3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e \\
& ^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e \\
& ^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c \\
& ^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 1 \\
& 6*b^10*c^3*d*e^11*f^4 + 672*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11 \\
& *f^4 + 209920*a^3*b^4*c^6*d*e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512* \\
& (a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c \\
& ^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6 \\
& *c^3*e^12*f^4 - 1472*a^2*b^2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^ \\
& 2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(- \\
& (b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 114 \\
& 0*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776 \\
& *a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a* \\
& b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 104 \\
& 8576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^ \\
& 14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9* \\
& b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440 \\
& *a^12*b^2*c^9*e^2)))^(1/2) - (((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15* \\
& c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 917 \\
& 5040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^ \\
& 7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24 \\
& *a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144* \\
& a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^ \\
& 3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680* \\
& a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c \\
& ^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 - b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1 \\
& 720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 348 \\
& 80*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 18636 \\
& 80*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 + 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2) \\
&))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*
\end{aligned}$$

$$\begin{aligned}
& b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{(1/2)} + (122880a^3b^9c^4e^{12}f^2 - 9216a^2b^{11}c^3e^{12}f^2 - 819200a^4b^7c^5e^{12}f^2 + 2949120a^5b^5c^6e^{12}f^2 - 5505024a^6b^3c^7e^{12}f^2 + 256a^8b^{13}c^2e^{12}f^2 + 4194304a^7b^8c^8e^{12}f^2)/(512*(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5))) * (-(b^{17}f^4 - b^2f^4 * (-(4ac - b^2)^{15})^{(1/2)} - 1720320a^8b^8c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^8b^{15}c^8f^4 + 25a^9c^9f^4 * (-(4ac - b^2)^{15})^{(1/2)}))/(512*(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^8e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{(1/2)} + (204800a^5c^8d^8e^{11}f^4 - 16b^{10}c^3d^8e^{11}f^4 + 672a^8b^8c^4d^8e^{11}f^4 - 28160a^2b^6c^5d^8e^{11}f^4 + 209920a^3b^4c^6d^8e^{11}f^4 - 479232a^4b^2c^7d^8e^{11}f^4)/(512*(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5)) + (x*(800a^3c^6e^{12}f^4 - b^6c^3e^{12}f^4 - 1472a^2b^2c^5e^{12}f^4 + 34a^4b^4c^4e^{12}f^4))/(32*(a^2b^8 + 256a^6c^4 - 16a^3b^6c + 96a^4b^4c^2 - 256a^5b^2c^3))) * (-(b^{17}f^4 - b^2f^4 * (-(4ac - b^2)^{15})^{(1/2)} - 1720320a^8b^8c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^8b^{15}c^8f^4 + 25a^9c^9f^4 * (-(4ac - b^2)^{15})^{(1/2)}))/(512*(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^8e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{(1/2)} + (8000a^3c^7e^{10}f^6 - 35b^6c^4e^{10}f^6 + 12720a^2b^2c^6e^{10}f^6 - 84a^4b^4c^5e^{10}f^6)/(256*(a^2b^{12} + 4096a^8c^6 - 24a^3b^{10}c + 240a^4b^8c^2 - 1280a^5b^6c^3 + 3840a^6b^4c^4 - 6144a^7b^2c^5))) * (-(b^{17}f^4 - b^2f^4 * (-(4ac - b^2)^{15})^{(1/2)} - 1720320a^8b^8c^8f^4 + 1140a^2b^{13}c^2f^4 - 10160a^3b^{11}c^3f^4 + 34880a^4b^9c^4f^4 + 43776a^5b^7c^5f^4 - 680960a^6b^5c^6f^4 + 1863680a^7b^3c^7f^4 - 55a^8b^{15}c^8f^4 + 25a^9c^9f^4 * (-(4ac - b^2)^{15})^{(1/2)}))/(512*(a^3b^{20}e^2 + 1048576a^{13}c^{10}e^2 - 40a^4b^{18}c^8e^2 + 720a^5b^{16}c^2e^2 - 7680a^6b^{14}c^3e^2 + 53760a^7b^{12}c^4e^2 - 258048a^8b^{10}c^5e^2 + 860160a^9b^8c^6e^2 - 1966080a^{10}b^6c^7e^2 + 2949120a^{11}b^4c^8e^2 - 2621440a^{12}b^2c^9e^2))^{(1/2)} * 2i + ((x^5*(2b^3c^8e^4f^2 + 21b^2c^2d^2e^4f^2 + 28a^2b^2c^2e^4f^2 + 420a^3c^3d^2e^4f^2))/(8a*(b^4 + 16a^2c^2 - 8a^2b^2c)) + (x^3*(b^4e^2f^2 + 36a^2c^2e^2f^2 + 35b^2c^2d^4e^2f^2 + 5a^2b^2c^2e^2f^2 + 700a^3c^3d^4e^2f^2 + 20b^3c^3d^2e^2f^2 + 280a^2b^2c^2d^2e^2f^2))/(8a*(b^4 + 16a^2c^2 - 8a^2b^2c)) + (x*(3b^4d^2f^2 - a^2b^3f^2 + 140a^3c^3d^6f^2 + 10b^3c^3d^4f^2 + 108a^2c^2d^2f^2 + 7b^2c^2d^6f^2 + 16a^2b^2c^2f^2 + 15a^2
\end{aligned}$$

$$\begin{aligned}
& b^2*c*d^2*f^2 + 140*a*b*c^2*d^4*f^2) / (8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) \\
& + (x^2*(3*b^4*d*e*f^2 + 108*a^2*c^2*d*e*f^2 + 420*a*c^3*d^5*e*f^2 + 20*b^3* \\
& c*d^3*e*f^2 + 21*b^2*c^2*d^5*e*f^2 + 15*a*b^2*c*d*e*f^2 + 280*a*b*c^2*d^3*e \\
& *f^2)) / (8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^4*d^3*f^2 + 20*a*c^3*d^7*f \\
& ^2 + 2*b^3*c*d^5*f^2 + 36*a^2*c^2*d^3*f^2 + b^2*c^2*d^7*f^2 - a*b^3*d*f^2 + \\
& 16*a^2*b*c*d*f^2 + 5*a*b^2*c*d^3*f^2 + 28*a*b*c^2*d^5*f^2) / (8*a*e*(b^4 + 1 \\
& 6*a^2*c^2 - 8*a*b^2*c)) + (7*x^6*(20*a*c^3*d*e^5*f^2 + b^2*c^2*d*e^5*f^2)) / \\
& (8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(7*b^2*c^2*d^3*e^3*f^2 + 2*b^ \\
& 3*c*d*e^3*f^2 + 140*a*c^3*d^3*e^3*f^2 + 28*a*b*c^2*d*e^3*f^2)) / (8*a*(b^4 + \\
& 16*a^2*c^2 - 8*a*b^2*c)) + (f^2*x^7*(20*a*c^3*e^6 + b^2*c^2*e^6)) / (8*a*(b^4 \\
& + 16*a^2*c^2 - 8*a*b^2*c)) / (x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e \\
& ^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + \\
& x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^ \\
& 3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c \\
& ^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30 \\
& *b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d \\
& ^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out

$$3.544 \quad \int \frac{df+efx}{(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=153

$$\frac{6c^2 f \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}} + \frac{3cf(b+2c(d+ex)^2)}{2e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{f(b+2c(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}$$

Rubi [A] time = 0.19, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1142, 1107, 614, 618, 206}

$$\frac{6c^2 f \tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}} + \frac{3cf(b+2c(d+ex)^2)}{2e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} - \frac{f(b+2c(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]

[Out] -(f*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*c*f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (6*c^2*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2,
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1142

Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
 \int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx &= \frac{f \operatorname{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^3} dx, x, d + ex\right)}{e} \\
 &= \frac{f \operatorname{Subst}\left(\int \frac{1}{(a+bx+cx^2)^3} dx, x, (d + ex)^2\right)}{2e} \\
 &= -\frac{f(b + 2c(d + ex)^2)}{4(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{(3cf) \operatorname{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2} dx, x, (d + ex)^2\right)}{2(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)} \\
 &= -\frac{f(b + 2c(d + ex)^2)}{4(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{3cf(b + 2c(d + ex)^2)}{2(b^2 - 4ac)^2 e(a + b(d + ex)^2 + c(d + ex)^4)} \\
 &= -\frac{f(b + 2c(d + ex)^2)}{4(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{3cf(b + 2c(d + ex)^2)}{2(b^2 - 4ac)^2 e(a + b(d + ex)^2 + c(d + ex)^4)} \\
 &= -\frac{f(b + 2c(d + ex)^2)}{4(b^2 - 4ac)e(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{3cf(b + 2c(d + ex)^2)}{2(b^2 - 4ac)^2 e(a + b(d + ex)^2 + c(d + ex)^4)}
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 148, normalized size = 0.97

$$\frac{f \left(\frac{24c^2 \tan^{-1} \left(\frac{b+2c(d+ex)^2}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}} + \frac{(b^2-4ac)(-b-2c(d+ex)^2)}{(a+b(d+ex)^2+c(d+ex)^4)^2} + \frac{6c(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4} \right)}{4e(b^2-4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] (f*((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/ (4*(b^2 - 4*a*c)^2*e)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{df + efx}{(a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] IntegrateAlgebraic[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3, x]

fricas [B] time = 1.59, size = 3748, normalized size = 24.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c^3 - 4*a*c^4)*d*e^5*f*x^5 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*e^4*f*x^4 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*e^3*f*x^3 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x + 12*(c^4*e^8*f*x^8 + 8*c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6 + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*e^4*f*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*e^3*f*x^3 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*e^2*f*x^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*e*f*x + (c^

$$\begin{aligned}
& 4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + a^2*c^2)*f) \\
& *sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6* \\
& c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a* \\
& c - (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*e^4*x^4 + \\
& 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)* \\
& e*x + a)) + (12*(b^2*c^3 - 4*a*c^4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + \\
& 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/ \\
& ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^9*x^8 + 8*(b^6*c^2 \\
& - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^8*x^7 + 2*(b^7*c - 12*a \\
& *b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48* \\
& a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^7*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48 \\
& *a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - \\
& 64*a^3*b*c^4)*d)*e^6*x^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2 \\
& *c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) \\
& *d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^ \\
& 5*x^4 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + \\
& 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a \\
& *b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^4*x^3 + 2*(a*b \\
& ^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4* \\
& c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2* \\
& b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3 \\
& *b^2*c^3 - 128*a^4*c^4)*d^2)*e^3*x^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^ \\
& 2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64 \\
& *a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128 \\
& *a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e \\
& ^2*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 \\
& - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + \\
& 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 3 \\
& 2*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 \\
& - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b^2*c^3 - 4*a*c^4)*e^6*f*x^6 + 72*(b^2*c \\
& ^3 - 4*a*c^4)*d*e^5*f*x^5 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4) \\
&)*d^2)*e^4*f*x^4 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3) \\
& *d)*e^3*f*x^3 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)* \\
& d^4 + 27*(b^3*c^2 - 4*a*b*c^3)*d^2)*e^2*f*x^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^ \\
& 5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*e*f*x \\
& - 24*(c^4*e^8*f*x^8 + 8*c^4*d*e^7*f*x^7 + 2*(14*c^4*d^2 + b*c^3)*e^6*f*x^6 \\
& + 4*(14*c^4*d^3 + 3*b*c^3*d)*e^5*f*x^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2* \\
& c^2 + 2*a*c^3)*e^4*f*x^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^ \\
& 3)*d)*e^3*f*x^3 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a \\
& *c^3)*d^2)*e^2*f*x^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + \\
& 2*a*c^3)*d^3)*e*f*x + (c^4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2 \\
& *a*c^3)*d^4 + a^2*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*e^2*x^2 + 4*c*d*e \\
& *x + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (12*(b^2*c^3 - 4*a*c^ \\
& 4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4 \\
& *(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2
\end{aligned}$$

$$\begin{aligned}
& b^2c^4 - 64a^3c^5)e^9x^8 + 8(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 \\
& - 64a^3c^5)d^8x^7 + 2(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3b^2c^4 \\
& + 14(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)d^2)e^7x^6 + 4(14(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)d^3 + \\
& 3(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3b^2c^4)d)e^6x^5 + (b^8 \\
& - 10ab^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4 + 70(b^6c^2 \\
& - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)d^4 + 30(b^7c - 12ab^5c^2 \\
& + 48a^2b^3c^3 - 64a^3b^2c^4)d^2)e^5x^4 + 4(14(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)d^5 + 10(b^7c - 12ab^5c^2 + 48a^2 \\
& b^3c^3 - 64a^3b^2c^4)d^3 + (b^8 - 10ab^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4)d)e^4x^3 + 2(ab^7 - 12a^2b^5c + 48a^3b^3c^2 \\
& - 64a^4b^2c^3 + 14(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)d^6 + 15(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3b^2c^4)d^4 + 3(\\
& b^8 - 10ab^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4)d^2)e^3x^2 + 4(2(b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5)d^7 + 3(\\
& b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3b^2c^4)d^5 + (b^8 - 10ab^6c \\
& + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4)d^3 + (ab^7 - 12a^2b^5c \\
& + 48a^3b^3c^2 - 64a^4b^2c^3)d)e^2x + ((b^6c^2 - 12ab^4c^3 + \\
& 48a^2b^2c^4 - 64a^3c^5)d^8 + a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 \\
& - 64a^5c^3 + 2(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3b^2c^4)d^6 + (b^8 - 10ab^6c + 24a^2b^4c^2 + 32a^3b^2c^3 - 128a^4c^4)d^4 \\
& + 2(ab^7 - 12a^2b^5c + 48a^3b^3c^2 - 64a^4b^2c^3)d^2)e]
\end{aligned}$$

giac [B] time = 0.69, size = 445, normalized size = 2.91

$$\frac{6c^2f \arctan\left(\frac{2c^2d^2(f^2+2dfx+ab^2)}{\sqrt{-b^2+4ac}}\right)d^{-1}}{(b^2-8ab^2c+16a^2c^2)\sqrt{-b^2+4ac}} \cdot \frac{12c^2d^6f^5+36(f^2e+2dfx)c^2d^4f^4e+18b^2d^4f^4+36(f^2e+2dfx)^2c^2d^2f^3e+36(f^2e+2dfx)cb^2d^2f^3e+4b^2cd^2f^3+20ac^2d^2f^3+12(f^2e+2dfx)^3c^2f^2e^3+18(f^2e+2dfx)^2bc^2f^2e^3+4(f^2e+2dfx)^2c^2f^2e+20(f^2e+2dfx)ac^2f^2e-b^3f^5+10abc^2f^5}{4(ad^2f^2+2(f^2e+2dfx)cd^2fe+bd^2f^2+(f^2e+2dfx)^2c^2+(f^2e+2dfx)fe+af^2)(b^2e-8ab^2ce+16a^2c^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

[Out] $6c^2f \arctan\left(\frac{2c^2d^2f + 2(fx^2e + 2d*fx)x}{\sqrt{-b^2 + 4ac}}\right) e^{-1} / ((b^4 - 8ab^2c + 16a^2c^2) \sqrt{-b^2 + 4ac}) + 1/4 (12c^3d^6f^5 + 36(fx^2e + 2d*fx)x^3d^4f^4e + 18b^2c^2d^4f^5 + 36(fx^2e + 2d*fx)x^2c^3d^2f^3e^2 + 36(fx^2e + 2d*fx) * b^2c^2d^2f^4e + 4b^2c^2d^2f^5 + 20ac^2d^2f^5 + 12(fx^2e + 2d*fx)^3c^3f^2e^3 + 18(fx^2e + 2d*fx)^2b^2c^2f^3e^2 + 4(fx^2e + 2d*fx) * b^2c^2f^4e + 20(fx^2e + 2d*fx) * ac^2f^4e - b^3f^5 + 10ab^2c^2f^5) / ((c^2d^4f^2 + 2(fx^2e + 2d*fx) * c^2d^2f^2e + b^2d^2f^2 + (fx^2e + 2d*fx) * x)^2c^2e^2 + (fx^2e + 2d*fx) * b^2f^2e + a^2f^2)^2(b^4e - 8ab^2ce + 16a^2c^2e)$

maple [C] time = 0.05, size = 2132, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e^f x + d^f)/(a + b(e^f x + d)^2 + c(e^f x + d)^4)^3, x)$

[Out]
$$\frac{3f/(c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^3 e^5 / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^6 + 18f/(c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 e^4 c^3 d / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^5 + 45f/(c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^3 e^3 / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^4 d^2 + 9/2 f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^2 e^3 / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^4 b + 60f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^3 d^3 e^2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^3 + 18f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^2 d e^2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^3 b + 45f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^3 e / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^2 d^4 + 27f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^2 e / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^2 b d^2 + 5f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^2 e / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^2 a + f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c e / (16a^2 c^2 - 8a^2 b^2 c + b^4) x^2 b^2 + 18f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^3 d^5 / (16a^2 c^2 - 8a^2 b^2 c + b^4) x + 18f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^2 d^3 / (16a^2 c^2 - 8a^2 b^2 c + b^4) x b + 10f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c^2 d / (16a^2 c^2 - 8a^2 b^2 c + b^4) x a + 2f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 c d / (16a^2 c^2 - 8a^2 b^2 c + b^4) x b^2 + 3f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / e^3 c^3 d^6 + 9/2 f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / e^3 c^2 d^4 + 5f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / e^3 a c^2 d^2 + f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / e^3 b^2 c^2 d^2 + 5/2 f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / e^3 a b c - 1/4 f / (c^4 e^4 x^4 + 4c^3 d e^3 x^3 + 6c^2 d^2 e^2 x^2 + 4c d^3 e x + b e^2 x^2 + c d^4 + 2b d e x + b d^2 + a)^2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / e^3 b^3 + 3f c^2 / (16a^2 c^2 - 8a^2 b^2 c + b^4) / e^3 \text{sum}((_R e + d) / (2 _R^3 c e^3 + 6 _R^2 c d e^2 + 6 _R c d^2 e + 2 c d^3 + _R b e + b d) * \ln(-_R + x), _R = \text{RootOf}(_Z^4 c e^4 + 4 _Z^3 c d e^3 + c d^4 + b d^2 + (6 c d^2 e^2 + b e^2) * _Z^2 + (4 c d^3 e + 2 b d e) * _Z + a))$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 3.99, size = 1199, normalized size = 7.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((x^2*(5*a*c^2*e*f + b^2*c*e*f + 45*c^3*d^4*e*f + 27*b*c^2*d^2*e*f))/(b^4 +
16*a^2*c^2 - 8*a*b^2*c) + (12*c^3*d^6*f - b^3*f + 20*a*c^2*d^2*f + 4*b^2*c
*d^2*f + 18*b*c^2*d^4*f + 10*a*b*c*f)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (9*x^4*(10*c^3*d^2*e^3*f + b*c^2*e^3*f))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (2*d*x*(9*c^3*d^4*f + 5*a*c^2*f + b^2*c*f + 9*b*c^2*d^2*f))/(b^4 + 16*
a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(10*c^3*d^2*e^2*f + 3*b*c^2*e^2*f))/(b^4 +
16*a^2*c^2 - 8*a*b^2*c) + (3*c^3*e^5*f*x^6)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
+ (18*c^3*d*e^4*f*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2
+ 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c
^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b
*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 4
0*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^
2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8
*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + (6*c^2*f*atan
(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^
2)^5)*(x^2*((36*c^6*e^8*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (36*b*c^4*f^2*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*c^4*e^
10)))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((72*
c^6*d*e^7*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72
*b*c^4*f^2*(b^5*c^2*d*e^9 - 8*a*b^3*c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2
*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (36*c^6*d^2*e^6*f^
2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*f^2*(
32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^2*e^8 - 8
*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))))/(72*c^6*e^6*f^2))/(e*(4*a*c - b^2)^(5/2))
```

sympy [B] time = 13.97, size = 1707, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*c**2*f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*f
*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**
5) - 36*a*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*f*sqrt(-1/(4
*a*c - b**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + 3*c**2*
f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*f*sqrt(-1/
(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**5) + 36*a
*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*f*sqrt(-1/(4*a*c - b
**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + (10*a*b*c*f + 20
*a*c**2*d**2*f - b**3*f + 4*b**2*c*d**2*f + 18*b*c**2*d**4*f + 12*c**3*d**6
*f + 72*c**3*d*e**5*f*x**5 + 12*c**3*e**6*f*x**6 + x**4*(18*b*c**2*e**4*f +
180*c**3*d**2*e**4*f) + x**3*(72*b*c**2*d*e**3*f + 240*c**3*d**3*e**3*f) +
x**2*(20*a*c**2*e**2*f + 4*b**2*c*e**2*f + 108*b*c**2*d**2*e**2*f + 180*c
**3*d**4*e**2*f) + x*(40*a*c**2*d*e*f + 8*b**2*c*d*e*f + 72*b*c**2*d**3*e*f
+ 72*c**3*d**5*e*f))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d
**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*
a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d
**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b
**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3
*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d
e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d
**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7
+ 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*
d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c
*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b
*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*
c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2
*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b
*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b
**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*
c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a
**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 179
2*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3
*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*
c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a*
**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 51
2*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b
**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5
*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

$$3.545 \quad \int \frac{1}{(df+efx)(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=270

$$\frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^3ef} + \frac{\log(d+ex)}{a^3ef} + \frac{16a^2c^2+2bc(b^2-7ac)(d+ex)^2-15ab^2c+2b^4}{4a^2ef(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{b(30a^2c^2-10ab^2c+b^4)\tanh^{-1}\left(\frac{b+2c(d+ex)}{\sqrt{b^2-4ac}}\right)}{2a^3ef(b^2-4ac)^{5/2}} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^3ef} + \frac{\log(d+ex)}{a^3ef} + \frac{-2ac+b^2+bc(d+ex)^2}{4aef(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Rubi [A] time = 0.50, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, number of rules / integrand size = 0.273, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{16a^2c^2+2bc(b^2-7ac)(d+ex)^2-15ab^2c+2b^4}{4a^2ef(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)} + \frac{b(30a^2c^2-10ab^2c+b^4)\tanh^{-1}\left(\frac{b+2c(d+ex)}{\sqrt{b^2-4ac}}\right)}{2a^3ef(b^2-4ac)^{5/2}} - \frac{\log(a+b(d+ex)^2+c(d+ex)^4)}{4a^3ef} + \frac{\log(d+ex)}{a^3ef} + \frac{-2ac+b^2+bc(d+ex)^2}{4aef(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2 + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e*f) + Log[d + e*x]/(a^3*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e*f)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1114

```
Int[(x_)^m*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^p, x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1142

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] := Di  
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,  
x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(df + efx)(a + b(d + ex)^2 + c(d + ex)^4)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^3} dx, x, d + ex\right)}{ef} \\
 &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^3} dx, x, (d + ex)^2\right)}{2ef} \\
 &= \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)^2} - \frac{\text{Subst}\left(\int \frac{-2(t)}{x(a+bx+cx^2)^3} dx, t, (d + ex)^2\right)}{4a} \\
 &= \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{2b^4 - 15ab^2c}{4a^2(b^2 - 4ac)} \\
 &= \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{2b^4 - 15ab^2c}{4a^2(b^2 - 4ac)} \\
 &= \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{2b^4 - 15ab^2c}{4a^2(b^2 - 4ac)} \\
 &= \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{2b^4 - 15ab^2c}{4a^2(b^2 - 4ac)} \\
 &= \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{2b^4 - 15ab^2c}{4a^2(b^2 - 4ac)} \\
 &= \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{2b^4 - 15ab^2c}{4a^2(b^2 - 4ac)}
 \end{aligned}$$

Mathematica [A] time = 3.93, size = 394, normalized size = 1.46

$$\frac{\frac{a^2(2ac - b^2 - bc(d + ex)^2)}{(4ac - b^2)(a + b(d + ex)^2 + c(d + ex)^4)^2} + \frac{a(16a^2c^2 - 15ab^2c - 14ab^2(d + ex)^2 + 2b^4 + 2b^3c(d + ex)^2)}{(b^2 - 4ac)^2(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{(16a^2c^2\sqrt{b^2 - 4ac} + 30a^2bc^2 - 10ab^3c - 8ab^2c\sqrt{b^2 - 4ac} + b^4\sqrt{b^2 - 4ac} + b^5)\log(\sqrt{b^2 - 4ac} + b + 2c(d + ex)^2)}{(b^2 - 4ac)^{5/2}} + \frac{(-16a^2c^2\sqrt{b^2 - 4ac} + 30a^2bc^2 - 10ab^3c + 8ab^2c\sqrt{b^2 - 4ac} - b^4\sqrt{b^2 - 4ac} - b^5)\log(\sqrt{b^2 - 4ac} + b + 2c(d + ex)^2)}{(b^2 - 4ac)^{5/2}} + 4\log(d + ex)}{4a^3ef}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out]
$$\frac{((a^2(-b^2 + 2ac - bc(d + ex)^2))/((-b^2 + 4ac)(a + b(d + ex)^2 + c(d + ex)^4)^2) + (a(2b^4 - 15ab^2c + 16a^2c^2 + 2b^3c(d + ex)^2 - 14ab^2c^2(d + ex)^2))/((b^2 - 4ac)^2(a + (d + ex)^2(b + c(d + ex)^2))) + 4\text{Log}[d + ex] - ((b^5 - 10ab^3c + 30a^2b^2c^2 + b^4\text{Sqrt}[b^2 - 4ac] - 8ab^2c\text{Sqrt}[b^2 - 4ac] + 16a^2c^2\text{Sqrt}[b^2 - 4ac])\text{Log}[b - \text{Sqrt}[b^2 - 4ac] + 2c(d + ex)^2])/(b^2 - 4ac)^{5/2} + ((b^5 - 10ab^3c + 30a^2b^2c^2 - b^4\text{Sqrt}[b^2 - 4ac] + 8ab^2c\text{Sqrt}[b^2 - 4ac] - 16a^2c^2\text{Sqrt}[b^2 - 4ac])\text{Log}[b + \text{Sqrt}[b^2 - 4ac] + 2c(d + ex)^2])/(b^2 - 4ac)^{5/2})/(4a^3ef)}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)(a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

fricas [B] time = 7.39, size = 9926, normalized size = 36.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 + 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a \end{aligned}$$

$$\begin{aligned}
& *b^3*c^3 + 30*a^2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*e^4*x^4 + 8*c^2*d*e^3*x^3 + 2*c^2*d^4 + 2*(6*c^2*d^2 + b*c)*e^2*x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c)))/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 4*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
\end{aligned}$$

$$\begin{aligned}
& - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*e^7*x^7 + 2*(b^7*c - 12*a* \\
& b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a \\
& ^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48* \\
& a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - \\
& 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^ \\
& 3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4* \\
& c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b \\
& ^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*e^4*x^4 + a^2*b^6 \\
& - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 4 \\
& 8*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2* \\
& b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64* \\
& a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128* \\
& a^4*c^4)*d)*e^3*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - \\
& 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 \\
& + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c \\
& - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c \\
& + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - \\
& 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2 + 4*(2*(b^6*c^2 - 12*a*b \\
& ^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^ \\
& 2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3 \\
& *b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a \\
& ^4*b*c^3)*d)*e*x)*\log(e*x + d))/((a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2 \\
& *c^4 - 64*a^6*c^5)*e^9*f*x^8 + 8*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2 \\
& *c^4 - 64*a^6*c^5)*d*e^8*f*x^7 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3 \\
& *c^3 - 64*a^6*b*c^4 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 6 \\
& 4*a^6*c^5)*d^2)*e^7*f*x^6 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^ \\
& 2*c^4 - 64*a^6*c^5)*d^3 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - \\
& 64*a^6*b*c^4)*d)*e^6*f*x^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32* \\
& a^6*b^2*c^3 - 128*a^7*c^4 + 70*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c \\
& ^4 - 64*a^6*c^5)*d^4 + 30*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64 \\
& *a^6*b*c^4)*d^2)*e^5*f*x^4 + 4*(14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b \\
& ^2*c^4 - 64*a^6*c^5)*d^5 + 10*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 \\
& - 64*a^6*b*c^4)*d^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2 \\
& *c^3 - 128*a^7*c^4)*d)*e^4*f*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c \\
& ^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64* \\
& a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c \\
& ^4)*d^4 + 3*(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128 \\
& *a^7*c^4)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2* \\
& c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64 \\
& *a^6*b*c^4)*d^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 \\
& - 128*a^7*c^4)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c \\
& ^3)*d)*e^2*f*x + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a \\
& ^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7 \\
& *c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^ \\
& 4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 -
\end{aligned}$$

$$\begin{aligned}
& 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e*f), 1/4*(2*(a*b^5*c^2 \\
& - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*e^6*x^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3 \\
& + 28*a^3*b*c^4)*d*e^5*x^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - \\
& 64*a^4*c^4 + 30*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*e^4*x^4 + \\
& 3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^2 - 1 \\
& 1*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^6 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28* \\
& a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4 \\
&)*d)*e^3*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)* \\
& d^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3 + 15*(a*b^5*c^2 \\
& - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 45*a^2*b^4*c^2 + 13 \\
& 2*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*e^2*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3 \\
& *b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c \\
& ^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^3 + \\
& (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)*e*x + 2*((b^5*c^2 \\
& - 10*a*b^3*c^3 + 30*a^2*b*c^4)*e^8*x^8 + 8*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^ \\
& 2*b*c^4)*d*e^7*x^7 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 \\
& - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^2)*e^6*x^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^ \\
& 3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*e^5*x^ \\
& 5 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^8 + (b^7 - 8*a*b^5*c + 10*a^2 \\
& *b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^4 + \\
& 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2)*e^4*x^4 + a^2*b^5 - 10*a^3* \\
& b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(1 \\
& 4*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + \\
& 30*a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)* \\
& e^3*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 2*(a*b^6 \\
& - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4 \\
&)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*(b^7 - 8*a*b^5*c \\
& + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*e^2*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + \\
& 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^7 + 3* \\
& (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3 \\
& *c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d)*e*x)* \\
& \text{sqrt}(-b^2 + 4*a*c)*\arctan(-(2*c*e^2*x^2 + 4*c*d*e*x + 2*c*d^2 + b)*\text{sqrt}(-b^ \\
& 2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64* \\
& a^3*c^5)*e^8*x^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) \\
& *d*e^7*x^7 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(\\
& b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*e^6*x^6 + 4*(14* \\
& (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12* \\
& a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*e^5*x^5 + (b^6*c^2 - 12*a*b^4 \\
& *c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^ \\
& 2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2* \\
& c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3* \\
& b*c^4)*d^2)*e^4*x^4 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 \\
& + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*(b^6 \\
& *c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b \\
& ^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^
\end{aligned}$$

$$\begin{aligned}
& 2*c^3 - 128*a^7*c^4)*d)*e^4*f*x^3 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3 + 14*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^6 + 15*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^4 + 3*(a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^2)*e^3*f*x^2 + 4*(2*(a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^7 + 3*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^5 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^3 + (a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d)*e^2*f*x + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*d^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*d^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*d^2)*e*f)]
\end{aligned}$$

giac [B] time = 1.70, size = 1044, normalized size = 3.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
[Out] -1/4*((a^3*b^7*c*f*e^3 - 14*a^4*b^5*c^2*f*e^3 + 70*a^5*b^3*c^3*f*e^3 - 120*a^6*b*c^4*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^3*b^7*c*f*e^3 - 14*a^4*b^5*c^2*f*e^3 + 70*a^5*b^3*c^3*f*e^3 - 120*a^6*b*c^4*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^6*b^8*c*f^2*e^4 - 16*a^7*b^6*c^2*f^2*e^4 + 96*a^8*b^4*c^3*f^2*e^4 - 256*a^9*b^2*c^4*f^2*e^4 + 256*a^10*c^5*f^2*e^4) - 1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^3*f) + e^(-1)*log(abs(x*e + d))/(a^3*f) + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e^5)*x^5 + (30*a*b^3*c^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4 + 16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4*c*d^3*e - 29*a^2*b^2*c^2*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^2*b^3*c*d*e - a^3*b*c^2*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2 - 4*a*c)^2*a^3*f)
```

maple [C] time = 0.08, size = 4606, normalized size = 17.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3, x)$

[Out]
$$-1/2/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b*c^2-1/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b*c^2-1/2/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b*c^2*d^2+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^5+1/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d/(16*a^2*c^2-8*a*b^2*c+b^4)*x*b^5+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^5*d^2+\ln(e*x+d)/a^3/e/f-7/2/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^3*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*e^5*b^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+6/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2+3/4/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4+4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*e^3*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+16/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*d^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x*c^3+4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*d^4-21/4/f/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2/e/(16*a^2*c^2-8*a*b^2*c+b^4)*b^2*c-1/2/f/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^3*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*c*d*e^2*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(4*8*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c^2*d^3+b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*\ln(-_R+x), _R=RootOf(_Z^4*c*e^4+4*_Z^3*c*d*e^3+c*d^4+b*d^2+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+a))+10/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d^3*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^3-29/f/a/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c^2*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^2+4/f/a^2/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2*c*d*e^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3*b^4-$$

$$\begin{aligned}
& 105/2/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)x^2b^3c^3d^4+15/2/f/a^2/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)x^2b^3c^2d^4-87/2/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)x^2b^2c^2d^2+6/f/a^2/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)x^2b^4c^2d^2-21/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2b^3c^3d^4/(16a^2c^2-8a^2b^2c+b^4)x^5+3/f/a^2/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2b^3c^2d^4/(16a^2c^2-8a^2b^2c+b^4)x^5-105/2/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e^3c^3/(16a^2c^2-8a^2b^2c+b^4)x^4b^2d^2+15/2/f/a^2/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e^3c^2/(16a^2c^2-8a^2b^2c+b^4)x^4b^3d^2-70/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2c^3d^3e^2/(16a^2c^2-8a^2b^2c+b^4)x^3b+16/f/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2c^3d^2e^2/(16a^2c^2-8a^2b^2c+b^4)x^3+24/f/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)x^2c^3d^2+1/f/a^2/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e^3c/(16a^2c^2-8a^2b^2c+b^4)x^4b^4-3/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)x^2b^3c-21/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2d^5/(16a^2c^2-8a^2b^2c+b^4)x^2b^3c+3/f/a^2/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2d^5/(16a^2c^2-8a^2b^2c+b^4)x^2b^3c^2-29/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2d^3/(16a^2c^2-8a^2b^2c+b^4)x^2b^2c^2+4/f/a^2/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2d^3/(16a^2c^2-8a^2b^2c+b^4)x^2b^4c-6/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2d/(16a^2c^2-8a^2b^2c+b^4)x^2b^3c-7/2/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)b^3c^2d^6+1/2/f/a^2/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)b^3c^2d^6-29/4/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)b^2c^2d^4+1/f/a^2/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)b^4c^2d^4-3/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)b^3c^2d^2-29/4/f/a/(c^4x^4+4c^3d^2e^2x^2+4c^3d^3e^2x^2+c^4d^4+2b^2d^2e^2x^2+a)^2e/(16a^2c^2-8a^2b^2c+b^4)x^4b^2
\end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 18.49, size = 22621, normalized size = 83.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)

[Out]
$$\begin{aligned} & ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 - 21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(56*c^2*d^5*e^3*f + 4*b^2*d*e^3*f + 40*b*c*d^3*e^3*f + 8*a*c*d*e^3*f) + x^2*(6*b^2*d^2*e^2*f + 28*c^2*d^6*e^2*f + 2*a*b*e^2*f + 12*a*c*d^2*e^2*f + 30*b*c*d^4*e^2*f) + x*(4*b^2*d^3*e*f + 8*c^2*d^7*e*f + 4*a*b*d*e*f + 8*a*c*d^3*e*f + 12*b*c*d^5*e*f) + x^4*(b^2*e^4*f + 70*c^2*d^4*e^4*f + 2*a*c*e^4*f + 30*b*c*d^2*e^4*f) + x^5*(56*c^2*d^3*e^5*f + 12*b*c*d*e^5*f) + a^2*f + x^6*(28*c^2*d^2*e^6*f + 2*b*c*e^6*f) + b^2*d^4*f + c^2*d^8*f + c^2*e^8*f*x^8 + 2*a*b*d^2*f + 2*a*c*d^4*f + 2*b*c*d^6*f + 8*c^2*d^8*f*x^7) - (log((((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) + (b*c^2*e^16*(a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^$$

$$\begin{aligned}
& 2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^{17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c)))/(a \\
& ^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) + (b*c^3*e^{15*(7*a*c - b^2)*(4*b^5 + 71 \\
& *a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2)) \\
& /(a^4*f^2*(4*a*c - b^2)^4) - (b*c^4*e^{17*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2 \\
& *b^2*c^2 - 89*a*b^4*c)))/(a^4*f^2*(4*a*c - b^2)^4) - (2*b*c^4*d*e^{16*x*(6*b^6 \\
& - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c)))/(a^4*f^2*(4*a*c - b^2)^4)) \\
&)/(4*a^3*e*f) - (b^3*c^5*e^{16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6 \\
&) + (b^2*c^4*e^{14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c \\
& - 7*a*b*c^2*d^2)))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^{15*x*(7*a*c - \\
& b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6))*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 1 \\
& 0*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*e*f*(-(b^2*(\\
& b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1) \\
& *((2*b*c^2*e^{16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b \\
& ^3*c - 2*a*b^2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) - (b*c^2*e^{16*(a^3*e*f*(-(\\
& b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) \\
& - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c* \\
& e^2*x^2 - 20*a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^{18*x^2*(b^4 + 10*a^2*c^2 - 2* \\
& a*b^2*c)))/(a^2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^{17*x*(b^4 + 10*a^2*c^2 - 2 \\
& *a*b^2*c)))/(a^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) - (b*c^3*e^{15*(7*a*c - b^2 \\
&)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a* \\
& b^2*c^2*d^2))/(a^4*f^2*(4*a*c - b^2)^4) + (b*c^4*e^{17*x^2*(6*b^6 - 560*a^3* \\
& c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c)))/(a^4*f^2*(4*a*c - b^2)^4) + (2*b*c^4*d \\
& *e^{16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c)))/(a^4*f^2*(4*a \\
& *c - b^2)^4)))/(4*a^3*e*f) - (b^3*c^5*e^{16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4 \\
& *a*c - b^2)^6) + (b^2*c^4*e^{14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^ \\
& 2 - 8*a*b^2*c - 7*a*b*c^2*d^2)))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^ \\
& 15*x*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6)))*(2*b^10*e*f - 2048*a^5*c^ \\
& 5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - \\
& 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b \\
& ^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a \\
& ^4*b^8*c*e^2*f^2)) + log(d + e*x)/(a^3*e*f) - (b*atan((x((((((b*((2*(5120* \\
& a^10*b*c^9*d*e^{17*f^2 + 2*a^4*b^13*c^3*d*e^{17*f^2 - 36*a^5*b^11*c^4*d*e^{17* \\
& f^2 + 276*a^6*b^9*c^5*d*e^{17*f^2 - 1216*a^7*b^7*c^6*d*e^{17*f^2 + 3456*a^8*b \\
& ^5*c^7*d*e^{17*f^2 - 6144*a^9*b^3*c^8*d*e^{17*f^2)))/(a^6*b^12*f^3 + 4096*a^12 \\
& *c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + \\
& 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c \\
& ^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f \\
& - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^{18*f^3 - 12*a^6*b^15*c^2*d*e^{18*f^ \\
& 3 + 328*a^7*b^13*c^3*d*e^{18*f^3 - 3840*a^8*b^11*c^4*d*e^{18*f^3 + 24960*a^9* \\
& b^9*c^5*d*e^{18*f^3 - 97280*a^10*b^7*c^6*d*e^{18*f^3 + 227328*a^11*b^5*c^7*d* \\
& e^{18*f^3 - 294912*a^12*b^3*c^8*d*e^{18*f^3)))/((4*a^3*b^10*e^2*f^2 - 4096*a^8 \\
& *c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^ \\
& 7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 \\
& - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^ \\
& 10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/
\end{aligned}$$

$$\begin{aligned}
& (4a^3ef(4ac - b^2)^{(5/2)}) - (b(b^4 + 30a^2c^2 - 10ab^2c))(2b^1 \\
& 0ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 256 \\
& 0a^4b^2c^4ef - 40ab^8c^3ef)(16384a^{13}b^9d^{18}f^3 - 12a^6 \\
& b^{15}c^2d^{18}f^3 + 328a^7b^{13}c^3d^{18}f^3 - 3840a^8b^{11}c^4d^{18}f^3 \\
& + 24960a^9b^9c^5d^{18}f^3 - 97280a^{10}b^7c^6d^{18}f^3 + 2273 \\
& 28a^{11}b^5c^7d^{18}f^3 - 294912a^{12}b^3c^8d^{18}f^3)) / (4a^3ef(4 \\
& ac - b^2)^{(5/2)}(4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2 \\
& e^{2f^2} - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^3e^{2f^2} \\
& (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 \\
& - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3)) \\
& (2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef \\
& - 40ab^8c^3ef)) / (2(4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} \\
& - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^3e^{2f^2})) - (b((2(6a^2 \\
& b^{11}c^4d^{16}f - 137a^3b^9c^5d^{16}f + 1217a^4b^7c^6d^{16}f - 5256a^5b^5c^7d^{16}f \\
& + 11024a^6b^3c^8d^{16}f - 8960a^7b^1c^9d^{16}f)) / (a^6b^{12}f^3 + 4096a^{12}c^6f^3 \\
& - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 \\
& - 6144a^{11}b^2c^5f^3) - (((2(5120a^{10}b^9c^9d^{17}f^2 + 2a^4b^{13}c^3d^{17}f^2 - 36a^5 \\
& b^{11}c^4d^{17}f^2 + 276a^6b^9c^5d^{17}f^2 - 1216a^7b^7c^6d^{17}f^2 + 3456a^8b^5c^7d^{17}f^2 \\
& - 6144a^9b^3c^8d^{17}f^2)) / (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 \\
& - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3) - ((2b^{10}ef \\
& - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^3ef) \\
& (16384a^{13}b^9d^{18}f^3 - 12a^6b^{15}c^2d^{18}f^3 + 328a^7b^{13}c^3d^{18}f^3 - 3840a^8b^{11}c^4d^{18}f^3 \\
& + 24960a^9b^9c^5d^{18}f^3 - 97280a^{10}b^7c^6d^{18}f^3 + 227328a^{11}b^5c^7d^{18}f^3 \\
& - 294912a^{12}b^3c^8d^{18}f^3)) / ((4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} \\
& - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^3e^{2f^2})) (a^6b^{12}f^3 \\
& + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 \\
& - 6144a^{11}b^2c^5f^3)) (2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef \\
& + 2560a^4b^2c^4ef - 40ab^8c^3ef)) / (2(4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} \\
& - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^3e^{2f^2})) (b^4 + 30a^2c^2 - 10ab^2c) / (4a^3ef(4 \\
& ac - b^2)^{(5/2)}) + (b^3(b^4 + 30a^2c^2 - 10ab^2c))^3(16384a^{13}b^9d^{18}f^3 - 12a^6 \\
& b^{15}c^2d^{18}f^3 + 328a^7b^{13}c^3d^{18}f^3 - 3840a^8b^{11}c^4d^{18}f^3 + 24960a^9b^9c^5d^{18}f^3 \\
& - 97280a^{10}b^7c^6d^{18}f^3 + 227328a^{11}b^5c^7d^{18}f^3 - 294912a^{12}b^3c^8d^{18}f^3) / (32a^9e^3f^3(4ac - b^2)^{(15/2)}(a^6b^{12}f^3 + 4096a^{12}c^6f^3 \\
& - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3)) \\
& (3b^8 + 160a^4c^4 + 180a^2b^4c^2 - 325a^3b^2c^3 - 39ab^6c)) / (8a^3c^2(4ac - b^2)^{(13/2)}(6b^{10} - 6400a^5c^5 + 960a^2b^6c^2 - 3850a^3b^4c^3 + 7775a^4
\end{aligned}$$

$$\begin{aligned}
& b^2c^4 - 120ab^8c) + (3b*((2*(b^9c^5d^15 - 21ab^7c^6d^15 + \\
& 147a^2b^5c^7d^15 - 343a^3b^3c^8d^15)))/(a^6b^12f^3 + 4096a^12 \\
& *c^6f^3 - 24a^7b^10c^4f^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + \\
& 3840a^10b^4c^4f^3 - 6144a^11b^2c^5f^3) - (((2*(6a^2b^11c^4d^16 \\
& *f - 137a^3b^9c^5d^16f + 1217a^4b^7c^6d^16f - 5256a^5b^5c^7 \\
& *d^16f + 11024a^6b^3c^8d^16f - 8960a^7b^1c^9d^16f)))/(a^6* \\
& b^12f^3 + 4096a^12c^6f^3 - 24a^7b^10c^4f^3 + 240a^8b^8c^2f^3 - 12 \\
& 80a^9b^6c^3f^3 + 3840a^10b^4c^4f^3 - 6144a^11b^2c^5f^3) - (((2* \\
& (5120a^10b^1c^9d^17f^2 + 2a^4b^13c^3d^17f^2 - 36a^5b^11c^4d \\
& *e^17f^2 + 276a^6b^9c^5d^17f^2 - 1216a^7b^7c^6d^17f^2 + 3456 \\
& *a^8b^5c^7d^17f^2 - 6144a^9b^3c^8d^17f^2)))/(a^6b^12f^3 + 409 \\
& 6a^12c^6f^3 - 24a^7b^10c^4f^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3 \\
& *f^3 + 3840a^10b^4c^4f^3 - 6144a^11b^2c^5f^3) - ((2b^10e*f - 2048 \\
& *a^5c^5e*f + 320a^2b^6c^2e*f - 1280a^3b^4c^3e*f + 2560a^4b^2c^4 \\
& *e*f - 40ab^8c^3e*f)*(163840a^13b^1c^9d^18f^3 - 12a^6b^15c^2d^18 \\
& *f^3 + 328a^7b^13c^3d^18f^3 - 3840a^8b^11c^4d^18f^3 + 2496 \\
& 0a^9b^9c^5d^18f^3 - 97280a^10b^7c^6d^18f^3 + 227328a^11b^5c^7 \\
& *d^18f^3 - 294912a^12b^3c^8d^18f^3)))/((4a^3b^10e^2f^2 - 40 \\
& 96a^8c^5e^2f^2 + 640a^5b^6c^2e^2f^2 - 2560a^6b^4c^3e^2f^2 + 5 \\
& 120a^7b^2c^4e^2f^2 - 80a^4b^8c^3e^2f^2)*(a^6b^12f^3 + 4096a^12c \\
& ^6f^3 - 24a^7b^10c^4f^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3 \\
& 840a^10b^4c^4f^3 - 6144a^11b^2c^5f^3)))*(2b^10e*f - 2048a^5c^5 \\
& *e*f + 320a^2b^6c^2e*f - 1280a^3b^4c^3e*f + 2560a^4b^2c^4e*f - 4 \\
& 0ab^8c^3e*f))/(2*(4a^3b^10e^2f^2 - 4096a^8c^5e^2f^2 + 640a^5b^6 \\
& *c^2e^2f^2 - 2560a^6b^4c^3e^2f^2 + 5120a^7b^2c^4e^2f^2 - 80a^4 \\
& *b^8c^3e^2f^2)))*(2b^10e*f - 2048a^5c^5e*f + 320a^2b^6c^2e*f - 12 \\
& 80a^3b^4c^3e*f + 2560a^4b^2c^4e*f - 40ab^8c^3e*f))/(2*(4a^3b^10 \\
& *e^2f^2 - 4096a^8c^5e^2f^2 + 640a^5b^6c^2e^2f^2 - 2560a^6b^4c^3 \\
& *e^2f^2 + 5120a^7b^2c^4e^2f^2 - 80a^4b^8c^3e^2f^2) - (b*((b*((2* \\
& (5120a^10b^1c^9d^17f^2 + 2a^4b^13c^3d^17f^2 - 36a^5b^11c^4d \\
& *e^17f^2 + 276a^6b^9c^5d^17f^2 - 1216a^7b^7c^6d^17f^2 + 3456 \\
& *a^8b^5c^7d^17f^2 - 6144a^9b^3c^8d^17f^2)))/(a^6b^12f^3 + 409 \\
& 6a^12c^6f^3 - 24a^7b^10c^4f^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3 \\
& *f^3 + 3840a^10b^4c^4f^3 - 6144a^11b^2c^5f^3) - ((2b^10e*f - 2048 \\
& *a^5c^5e*f + 320a^2b^6c^2e*f - 1280a^3b^4c^3e*f + 2560a^4b^2c^4 \\
& *e*f - 40ab^8c^3e*f)*(163840a^13b^1c^9d^18f^3 - 12a^6b^15c^2d^18 \\
& *f^3 + 328a^7b^13c^3d^18f^3 - 3840a^8b^11c^4d^18f^3 + 2496 \\
& 0a^9b^9c^5d^18f^3 - 97280a^10b^7c^6d^18f^3 + 227328a^11b^5c^7 \\
& *d^18f^3 - 294912a^12b^3c^8d^18f^3)))/((4a^3b^10e^2f^2 - 40 \\
& 96a^8c^5e^2f^2 + 640a^5b^6c^2e^2f^2 - 2560a^6b^4c^3e^2f^2 + 5 \\
& 120a^7b^2c^4e^2f^2 - 80a^4b^8c^3e^2f^2)*(a^6b^12f^3 + 4096a^12c \\
& ^6f^3 - 24a^7b^10c^4f^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3 \\
& 840a^10b^4c^4f^3 - 6144a^11b^2c^5f^3)))*(b^4 + 30a^2c^2 - 10ab^2 \\
& *c))/(4a^3e*f*(4a*c - b^2)^(5/2)) - (b*(b^4 + 30a^2c^2 - 10ab^2*c) * \\
& (2b^10e*f - 2048a^5c^5e*f + 320a^2b^6c^2e*f - 1280a^3b^4c^3e*f
\end{aligned}$$

$$\begin{aligned}
& + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 1 \\
& 2*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4 \\
& *d*e^18*f^3 + 24960*a^9*b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 \\
& + 227328*a^11*b^5*c^7*d*e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/((4*a^3* \\
& e*f*(4*a*c - b^2)^(5/2)*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^ \\
& 5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 8 \\
& 0*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 \\
& + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144 \\
& *a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b \\
& ^2)^(5/2)) + (b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2*(2*b^10*e*f - 2048*a^5* \\
& c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f \\
& - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f \\
& ^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11*c^4*d*e^18*f^3 + 24960*a^9 \\
& *b^9*c^5*d*e^18*f^3 - 97280*a^10*b^7*c^6*d*e^18*f^3 + 227328*a^11*b^5*c^7*d \\
& *e^18*f^3 - 294912*a^12*b^3*c^8*d*e^18*f^3))/(16*a^6*e^2*f^2*(4*a*c - b^2)^(5 \\
& * (4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 25 \\
& 60*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(\\
& a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 \\
& - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(\\
& b^6 - 45*a^3*c^3 + 40*a^2*b^2*c^2 - 11*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^6 \\
& *(6*b^10 - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2 \\
& *c^4 - 120*a*b^8*c)))*(16*a^9*b^12*f^3*(4*a*c - b^2)^(15/2) + 65536*a^15*c^ \\
& 6*f^3*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*f^3*(4*a*c - b^2)^(15/2) + 384 \\
& 0*a^11*b^8*c^2*f^3*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*f^3*(4*a*c - b \\
& ^2)^(15/2) + 61440*a^13*b^4*c^4*f^3*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c \\
& ^5*f^3*(4*a*c - b^2)^(15/2)))/(b^10*c^2*e^14 - 20*a*b^8*c^3*e^14 + 160*a^2* \\
& b^6*c^4*e^14 - 600*a^3*b^4*c^5*e^14 + 900*a^4*b^2*c^6*e^14) + (x^2*(((b* \\
& ((2*a^4*b^13*c^3*e^18*f^2 - 36*a^5*b^11*c^4*e^18*f^2 + 276*a^6*b^9*c^5*e^18 \\
& *f^2 - 1216*a^7*b^7*c^6*e^18*f^2 + 3456*a^8*b^5*c^7*e^18*f^2 - 6144*a^9*b^3 \\
& *c^8*e^18*f^2 + 5120*a^10*b*c^9*e^18*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 \\
& - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^ \\
& 10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5*e*f + \\
& 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b \\
& ^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840*a^8* \\
& b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^19*f^ \\
& 3 - 227328*a^11*b^5*c^7*e^19*f^3 + 294912*a^12*b^3*c^8*e^19*f^3 - 163840*a^ \\
& 13*b*c^9*e^19*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5 \\
& *b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80 \\
& *a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + \\
& 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144* \\
& a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^ \\
& 2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 2048*a^5*c^5*e \\
& *f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40 \\
& *a*b^8*c*e*f)*(12*a^6*b^15*c^2*e^19*f^3 - 328*a^7*b^13*c^3*e^19*f^3 + 3840* \\
& a^8*b^11*c^4*e^19*f^3 - 24960*a^9*b^9*c^5*e^19*f^3 + 97280*a^10*b^7*c^6*e^1
\end{aligned}$$

$$\begin{aligned}
& 9f^3 - 227328a^{11}b^5c^7e^{19f^3} + 294912a^{12}b^3c^8e^{19f^3} - 16384 \\
& 0a^{13}b^9c^9e^{19f^3}) / (8a^3e^f(4ac - b^2)^{(5/2)}(4a^3b^{10}e^{2f^2} \\
& - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} - 2560a^6b^4c^3e^{2f^2} \\
& + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^2e^{2f^2}))(a^6b^{12}f^3 + 4096a^{12} \\
& c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 \\
& + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3)))(2b^{10}ef - 2048a^5c^5 \\
& ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef \\
& - 40ab^8c^2ef)) / (2(4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5 \\
& b^6c^2e^{2f^2} - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80 \\
& a^4b^8c^2e^{2f^2})) + (b((8960a^7b^9c^5e^{17f} - 6a^2b^{11}c^4e^{17f} + \\
& 137a^3b^9c^5e^{17f} - 1217a^4b^7c^6e^{17f} + 5256a^5b^5c^7e^{17f} \\
& - 11024a^6b^3c^8e^{17f}) / (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10} \\
& cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 \\
& - 6144a^{11}b^2c^5f^3) + (((2a^4b^{13}c^3e^{18f^2} - 36a^5b^{11}c^4e^{18f^2} \\
& + 276a^6b^9c^5e^{18f^2} - 1216a^7b^7c^6e^{18f^2} + 3456a^8b^5c^7e^{18f^2} \\
& - 6144a^9b^3c^8e^{18f^2} + 5120a^{10}b^1c^9e^{18f^2}) / (a^6b^{12}f^3 + \\
& 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 \\
& + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3) + ((2 \\
& b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + \\
& 2560a^4b^2c^4ef - 40ab^8c^2ef))(12a^6b^{15}c^2e^{19f^3} - 328a^7 \\
& b^{13}c^3e^{19f^3} + 3840a^8b^{11}c^4e^{19f^3} - 24960a^9b^9c^5e^{19f^3} \\
& + 97280a^{10}b^7c^6e^{19f^3} - 227328a^{11}b^5c^7e^{19f^3} + 294912a^{12} \\
& b^3c^8e^{19f^3} - 163840a^{13}b^9c^9e^{19f^3})) / (2(4a^3b^{10}e^{2f^2} - \\
& 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} - 2560a^6b^4c^3e^{2f^2} + \\
& 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^2e^{2f^2}))(a^6b^{12}f^3 + 4096a^{12} \\
& c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + \\
& 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3)))(2b^{10}ef - 2048a^5c^5 \\
& ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - \\
& 40ab^8c^2ef)) / (2(4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6 \\
& c^2e^{2f^2} - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4 \\
& b^8c^2e^{2f^2}))(b^4 + 30a^2c^2 - 10ab^2c)) / (4a^3e^f(4ac - b^2)^{(5/2)} \\
& - (b^3(b^4 + 30a^2c^2 - 10ab^2c))^3(12a^6b^{15}c^2e^{19f^3} \\
& - 328a^7b^{13}c^3e^{19f^3} + 3840a^8b^{11}c^4e^{19f^3} - 24960a^9b^9c^5 \\
& e^{19f^3} + 97280a^{10}b^7c^6e^{19f^3} - 227328a^{11}b^5c^7e^{19f^3} + \\
& 294912a^{12}b^3c^8e^{19f^3} - 163840a^{13}b^9c^9e^{19f^3})) / (64a^9e^3f^3 \\
& (4ac - b^2)^{(15/2)}(a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 \\
& + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 61 \\
& 44a^{11}b^2c^5f^3)))(3b^8 + 160a^4c^4 + 180a^2b^4c^2 - 325a^3b^2 \\
& c^3 - 39ab^6c)) / (8a^3c^2(4ac - b^2)^{(13/2)}(6b^{10} - 6400a^5c^5 \\
& + 960a^2b^6c^2 - 3850a^3b^4c^3 + 7775a^4b^2c^4 - 120ab^8c)) + (\\
& 3b((b^9c^5e^{16} - 21ab^7c^6e^{16} + 147a^2b^5c^7e^{16} - 343a^3b^3 \\
& c^8e^{16}) / (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8 \\
& b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2 \\
& c^5f^3) + (((8960a^7b^9c^5e^{17f} - 6a^2b^{11}c^4e^{17f} + 137a^3b^9c^5 \\
& e^{17f} - 1217a^4b^7c^6e^{17f} + 5256a^5b^5c^7e^{17f} - 11024a^6b^3
\end{aligned}$$

$$\begin{aligned}
&^3c^8e^{17f})/(a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3) + (((2a^4b^{13}c^3e^{18f^2} - 36a^5b^{11}c^4e^{18f^2} + 276a^6b^9c^5e^{18f^2} - 1216a^7b^7c^6e^{18f^2} + 3456a^8b^5c^7e^{18f^2} - 6144a^9b^3c^8e^{18f^2} + 5120a^{10}b^1c^9e^{18f^2})/(a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3) + ((2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^2ef) * (12a^6b^{15}c^2e^{19f^3} - 328a^7b^{13}c^3e^{19f^3} + 3840a^8b^{11}c^4e^{19f^3} - 24960a^9b^9c^5e^{19f^3} + 97280a^{10}b^7c^6e^{19f^3} - 227328a^{11}b^5c^7e^{19f^3} + 294912a^{12}b^3c^8e^{19f^3} - 163840a^{13}b^1c^9e^{19f^3}))/((2(4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^2e^{2f^2})) * (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3))) * (2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^2ef) / (2(4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^2e^{2f^2}))) * (2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^2ef) / (2(4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^2e^{2f^2})) - (b * ((b * ((2a^4b^{13}c^3e^{18f^2} - 36a^5b^{11}c^4e^{18f^2} + 276a^6b^9c^5e^{18f^2} - 1216a^7b^7c^6e^{18f^2} + 3456a^8b^5c^7e^{18f^2} - 6144a^9b^3c^8e^{18f^2} + 5120a^{10}b^1c^9e^{18f^2})/(a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3) + ((2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^2ef) * (12a^6b^{15}c^2e^{19f^3} - 328a^7b^{13}c^3e^{19f^3} + 3840a^8b^{11}c^4e^{19f^3} - 24960a^9b^9c^5e^{19f^3} + 97280a^{10}b^7c^6e^{19f^3} - 227328a^{11}b^5c^7e^{19f^3} + 294912a^{12}b^3c^8e^{19f^3} - 163840a^{13}b^1c^9e^{19f^3}))/((2(4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^2e^{2f^2})) * (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3))) * (b^4 + 30a^2c^2 - 10ab^2c)) / (4a^3ef * (4ac - b^2)^{(5/2)}) + (b * (b^4 + 30a^2c^2 - 10ab^2c)) * (2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^2ef) * (12a^6b^{15}c^2e^{19f^3} - 328a^7b^{13}c^3e^{19f^3} + 3840a^8b^{11}c^4e^{19f^3} - 24960a^9b^9c^5e^{19f^3} + 97280a^{10}b^7c^6e^{19f^3} - 227328a^{11}b^5c^7e^{19f^3} + 294912a^{12}b^3c^8e^{19f^3} - 163840a^{13}b^1c^9e^{19f^3}))/((8a^3ef * (4ac - b^2)^{(5/2)}) * (4a^3b^{10}e^{2f^2} - 4096a^8c^5e^{2f^2} + 640a^5b^6c^2e^{2f^2} - 2560a^6b^4c^3e^{2f^2} + 5120a^7b^2c^4e^{2f^2} - 80a^4b^8c^2e^{2f^2})) * (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}cf^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3)
\end{aligned}$$

$$\begin{aligned}
& a^7 b^{10} c^3 f^3 + 240 a^8 b^8 c^2 f^3 - 1280 a^9 b^6 c^3 f^3 + 3840 a^{10} b^4 c^4 f^3 - 6144 a^{11} b^2 c^5 f^3) (b^4 + 30 a^2 c^2 - 10 a b^2 c) / (4 a^3 e f (4 a^3 c - b^2)^{(5/2)}) - (b^2 (b^4 + 30 a^2 c^2 - 10 a b^2 c)^2 (2 b^{10} e f - 2048 a^5 c^5 e f + 320 a^2 b^6 c^2 e f - 1280 a^3 b^4 c^3 e f + 2560 a^4 b^2 c^4 e f - 40 a b^8 c e f) * (12 a^6 b^{15} c^2 e^{19} f^3 - 328 a^7 b^{13} c^3 e^{19} f^3 + 3840 a^8 b^{11} c^4 e^{19} f^3 - 24960 a^9 b^9 c^5 e^{19} f^3 + 97280 a^{10} b^7 c^6 e^{19} f^3 - 227328 a^{11} b^5 c^7 e^{19} f^3 + 294912 a^{12} b^3 c^8 e^{19} f^3 - 163840 a^{13} b c^9 e^{19} f^3)) / (32 a^6 e^2 f^2 (4 a^3 c - b^2)^5 * (4 a^3 b^{10} e^2 f^2 - 4096 a^8 c^5 e^2 f^2 + 640 a^5 b^6 c^2 e^2 f^2 - 2560 a^6 b^4 c^3 e^2 f^2 + 5120 a^7 b^2 c^4 e^2 f^2 - 80 a^4 b^8 c e^2 f^2) * (a^6 b^{12} f^3 + 4096 a^{12} c^6 f^3 - 24 a^7 b^{10} c f^3 + 240 a^8 b^8 c^2 f^3 - 1280 a^9 b^6 c^3 f^3 + 3840 a^{10} b^4 c^4 f^3 - 6144 a^{11} b^2 c^5 f^3)) * (b^6 - 45 a^3 c^3 + 40 a^2 b^2 c^2 - 11 a b^4 c) / (8 a^3 c^2 (4 a^3 c - b^2)^6 (6 b^{10} - 6400 a^5 c^5 + 960 a^2 b^6 c^2 - 3850 a^3 b^4 c^3 + 7775 a^4 b^2 c^4 - 120 a b^8 c)) * (16 a^9 b^{12} f^3 (4 a^3 c - b^2)^{(15/2)} + 65536 a^{15} c^6 f^3 (4 a^3 c - b^2)^{(15/2)} - 384 a^{10} b^{10} c f^3 (4 a^3 c - b^2)^{(15/2)} + 3840 a^{11} b^8 c^2 f^3 (4 a^3 c - b^2)^{(15/2)} - 20480 a^{12} b^6 c^3 f^3 (4 a^3 c - b^2)^{(15/2)} + 61440 a^{13} b^4 c^4 f^3 (4 a^3 c - b^2)^{(15/2)} - 98304 a^{14} b^2 c^5 f^3 (4 a^3 c - b^2)^{(15/2)}) / (b^{10} c^2 e^{14} - 20 a b^8 c^3 e^{14} + 160 a^2 b^6 c^4 e^{14} - 600 a^3 b^4 c^5 e^{14} + 900 a^4 b^2 c^6 e^{14}) - (((b^4 (4 a^2 b^{12} c^3 e^{15} f - 93 a^3 b^{10} c^4 e^{15} f + 854 a^4 b^8 c^5 e^{15} f - 3889 a^5 b^6 c^6 e^{15} f + 8808 a^6 b^4 c^7 e^{15} f - 7952 a^7 b^2 c^8 e^{15} f - 8960 a^7 b c^9 d^2 e^{15} f + 6 a^2 b^{11} c^4 d^2 e^{15} f - 137 a^3 b^9 c^5 d^2 e^{15} f + 1217 a^4 b^7 c^6 d^2 e^{15} f - 5256 a^5 b^5 c^7 d^2 e^{15} f + 11024 a^6 b^3 c^8 d^2 e^{15} f) / (a^6 b^{12} f^3 + 4096 a^{12} c^6 f^3 - 24 a^7 b^{10} c f^3 + 240 a^8 b^8 c^2 f^3 - 1280 a^9 b^6 c^3 f^3 + 3840 a^{10} b^4 c^4 f^3 - 6144 a^{11} b^2 c^5 f^3) - (((4 a^4 b^{14} c^2 e^{16} f^2 - 100 a^5 b^{12} c^3 e^{16} f^2 + 1052 a^6 b^{10} c^4 e^{16} f^2 - 5952 a^7 b^8 c^5 e^{16} f^2 + 19072 a^8 b^6 c^6 e^{16} f^2 - 32768 a^9 b^4 c^7 e^{16} f^2 + 23552 a^{10} b^2 c^8 e^{16} f^2 + 5120 a^{10} b c^9 d^2 e^{16} f^2 + 2 a^4 b^{13} c^3 d^2 e^{16} f^2 - 36 a^5 b^{11} c^4 d^2 e^{16} f^2 + 276 a^6 b^9 c^5 d^2 e^{16} f^2 - 1216 a^7 b^7 c^6 d^2 e^{16} f^2 + 3456 a^8 b^5 c^7 d^2 e^{16} f^2 - 6144 a^9 b^3 c^8 d^2 e^{16} f^2) / (a^6 b^{12} f^3 + 4096 a^{12} c^6 f^3 - 24 a^7 b^{10} c f^3 + 240 a^8 b^8 c^2 f^3 - 1280 a^9 b^6 c^3 f^3 + 3840 a^{10} b^4 c^4 f^3 - 6144 a^{11} b^2 c^5 f^3) + ((2 b^{10} e f - 2048 a^5 c^5 e f + 320 a^2 b^6 c^2 e f - 1280 a^3 b^4 c^3 e f + 2560 a^4 b^2 c^4 e f - 40 a b^8 c e f) * (4 a^7 b^{14} c^2 e^{17} f^3 - 96 a^8 b^{12} c^3 e^{17} f^3 + 960 a^9 b^{10} c^4 e^{17} f^3 - 5120 a^{10} b^8 c^5 e^{17} f^3 + 15360 a^{11} b^6 c^6 e^{17} f^3 - 24576 a^{12} b^4 c^7 e^{17} f^3 + 16384 a^{13} b^2 c^8 e^{17} f^3 - 163840 a^{13} b c^9 d^2 e^{17} f^3 + 12 a^6 b^{15} c^2 d^2 e^{17} f^3 - 328 a^7 b^{13} c^3 d^2 e^{17} f^3 + 3840 a^8 b^{11} c^4 d^2 e^{17} f^3 - 24960 a^9 b^9 c^5 d^2 e^{17} f^3 + 97280 a^{10} b^7 c^6 d^2 e^{17} f^3 - 227328 a^{11} b^5 c^7 d^2 e^{17} f^3 + 294912 a^{12} b^3 c^8 d^2 e^{17} f^3)) / (2 (4 a^3 b^{10} e^2 f^2 - 4096 a^8 c^5 e^2 f^2 + 640 a^5 b^6 c^2 e^2 f^2 - 2560 a^6 b^4 c^3 e^2 f^2 + 5120 a^7 b^2 c^4 e^2 f^2 - 80 a^4 b^8 c e^2 f^2) * (a^6 b^{12} f^3 + 4096 a^{12} c^6 f^3 - 24 a^7 b^{10} c f^3 + 240 a^8 b^8 c^2 f^3 - 1280 a^9 b^6 c^3 f^3 +
\end{aligned}$$

$$\begin{aligned}
& (3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3)) * (2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^2ef) / (2(4a^3b^{10}e^2f^2 - 4096a^8c^5e^2f^2 + 640a^5b^6c^2e^2f^2 - 2560a^6b^4c^3e^2f^2 + 5120a^7b^2c^4e^2f^2 - 80a^4b^8c^2e^2f^2)) * (b^4 + 30a^2c^2 - 10ab^2c) / (4a^3ef(4ac - b^2)^{(5/2)}) - (((b((4a^4b^{14}c^2e^{16}f^2 - 100a^5b^{12}c^3e^{16}f^2 + 1052a^6b^{10}c^4e^{16}f^2 - 5952a^7b^8c^5e^{16}f^2 + 19072a^8b^6c^6e^{16}f^2 - 32768a^9b^4c^7e^{16}f^2 + 23552a^{10}b^2c^8e^{16}f^2 + 5120a^{10}b^2c^9d^2e^{16}f^2 + 2a^4b^{13}c^3d^2e^{16}f^2 - 36a^5b^{11}c^4d^2e^{16}f^2 + 276a^6b^9c^5d^2e^{16}f^2 - 1216a^7b^7c^6d^2e^{16}f^2 + 3456a^8b^5c^7d^2e^{16}f^2 - 6144a^9b^3c^8d^2e^{16}f^2) / (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}c^2f^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3) + ((2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^2ef) * (4a^7b^{14}c^2e^{17}f^3 - 96a^8b^{12}c^3e^{17}f^3 + 960a^9b^{10}c^4e^{17}f^3 - 5120a^{10}b^8c^5e^{17}f^3 + 15360a^{11}b^6c^6e^{17}f^3 - 24576a^{12}b^4c^7e^{17}f^3 + 16384a^{13}b^2c^8e^{17}f^3 - 163840a^{13}b^2c^9d^2e^{17}f^3 + 12a^6b^{15}c^2d^2e^{17}f^3 - 328a^7b^{13}c^3d^2e^{17}f^3 + 3840a^8b^{11}c^4d^2e^{17}f^3 - 24960a^9b^9c^5d^2e^{17}f^3 + 97280a^{10}b^7c^6d^2e^{17}f^3 - 227328a^{11}b^5c^7d^2e^{17}f^3 + 294912a^{12}b^3c^8d^2e^{17}f^3)) / (2(4a^3b^{10}e^2f^2 - 4096a^8c^5e^2f^2 + 640a^5b^6c^2e^2f^2 - 2560a^6b^4c^3e^2f^2 + 5120a^7b^2c^4e^2f^2 - 80a^4b^8c^2e^2f^2) * (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}c^2f^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3))) * (b^4 + 30a^2c^2 - 10ab^2c) / (4a^3ef(4ac - b^2)^{(5/2)}) + (b(b^4 + 30a^2c^2 - 10ab^2c) * (2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^2ef) * (4a^7b^{14}c^2e^{17}f^3 - 96a^8b^{12}c^3e^{17}f^3 + 960a^9b^{10}c^4e^{17}f^3 - 5120a^{10}b^8c^5e^{17}f^3 + 15360a^{11}b^6c^6e^{17}f^3 - 24576a^{12}b^4c^7e^{17}f^3 + 16384a^{13}b^2c^8e^{17}f^3 - 163840a^{13}b^2c^9d^2e^{17}f^3 + 12a^6b^{15}c^2d^2e^{17}f^3 - 328a^7b^{13}c^3d^2e^{17}f^3 + 3840a^8b^{11}c^4d^2e^{17}f^3 - 24960a^9b^9c^5d^2e^{17}f^3 + 97280a^{10}b^7c^6d^2e^{17}f^3 - 227328a^{11}b^5c^7d^2e^{17}f^3 + 294912a^{12}b^3c^8d^2e^{17}f^3)) / (8a^3ef(4ac - b^2)^{(5/2)} * (4a^3b^{10}e^2f^2 - 4096a^8c^5e^2f^2 + 640a^5b^6c^2e^2f^2 - 2560a^6b^4c^3e^2f^2 + 5120a^7b^2c^4e^2f^2 - 80a^4b^8c^2e^2f^2) * (a^6b^{12}f^3 + 4096a^{12}c^6f^3 - 24a^7b^{10}c^2f^3 + 240a^8b^8c^2f^3 - 1280a^9b^6c^3f^3 + 3840a^{10}b^4c^4f^3 - 6144a^{11}b^2c^5f^3))) * (2b^{10}ef - 2048a^5c^5ef + 320a^2b^6c^2ef - 1280a^3b^4c^3ef + 2560a^4b^2c^4ef - 40ab^8c^2ef) / (2(4a^3b^{10}e^2f^2 - 4096a^8c^5e^2f^2 + 640a^5b^6c^2e^2f^2 - 2560a^6b^4c^3e^2f^2 + 5120a^7b^2c^4e^2f^2 - 80a^4b^8c^2e^2f^2)) + (b^3(b^4 + 30a^2c^2 - 10ab^2c))^3 * (4a^7b^{14}c^2e^{17}f^3 - 96a^8b^{12}c^3e^{17}f^3 + 960a^9b^{10}c^4e^{17}f^3 - 5120a^{10}b^8c^5e^{17}f^3 + 15360a^{11}b^6c^6e^{17}f^3 - 24576a^{12}b^4c^7e^{17}f^3 + 16384a^{13}b^2c^8e^{17}f^3 - 1638
\end{aligned}$$

$$\begin{aligned}
& 40*a^{13}*b^9*d^2*e^{17}*f^3 + 12*a^6*b^{15}*c^2*d^2*e^{17}*f^3 - 328*a^7*b^{13}*c^3*d^2*e^{17}*f^3 + 3840*a^8*b^{11}*c^4*d^2*e^{17}*f^3 - 24960*a^9*b^9*c^5*d^2*e^{17}*f^3 + 97280*a^{10}*b^7*c^6*d^2*e^{17}*f^3 - 227328*a^{11}*b^5*c^7*d^2*e^{17}*f^3 \\
& + 294912*a^{12}*b^3*c^8*d^2*e^{17}*f^3)/(64*a^9*e^3*f^3*(4*a*c - b^2)^{(15/2)}*(a^6*b^{12}*f^3 + 4096*a^{12}*c^6*f^3 - 24*a^7*b^{10}*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^{10}*b^4*c^4*f^3 - 6144*a^{11}*b^2*c^5*f^3)))* \\
& (3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 325*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^9*b^{12}*f^3*(4*a*c - b^2)^{(15/2)} + 65536*a^{15}*c^6*f^3*(4*a*c - b^2)^{(15/2)} - 384*a^{10}*b^{10}*c*f^3*(4*a*c - b^2)^{(15/2)} + 3840*a^{11}*b^8*c^2*f^3*(4*a*c - b^2)^{(15/2)} - 20480*a^{12}*b^6*c^3*f^3*(4*a*c - b^2)^{(15/2)} + 61440*a^{13}*b^4*c^4*f^3*(4*a*c - b^2)^{(15/2)} - 98304*a^{14}*b^2*c^5*f^3*(4*a*c - b^2)^{(15/2)})))/ \\
& (8*a^3*c^2*(4*a*c - b^2)^{(13/2)}*(b^{10}*c^2*e^{14} - 20*a*b^8*c^3*e^{14} + 160*a^2*b^6*c^4*e^{14} - 600*a^3*b^4*c^5*e^{14} + 900*a^4*b^2*c^6*e^{14})*(6*b^{10} - 6400*a^5*c^5 + 960*a^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^4 - 120*a*b^8*c)) - (3*b*(((4*a^2*b^{12}*c^3*e^{15}*f - 93*a^3*b^{10}*c^4*e^{15}*f + 854*a^4*b^8*c^5*e^{15}*f - 3889*a^5*b^6*c^6*e^{15}*f + 8808*a^6*b^4*c^7*e^{15}*f - 7952*a^7*b^2*c^8*e^{15}*f - 8960*a^7*b*c^9*d^2*e^{15}*f + 6*a^2*b^{11}*c^4*d^2*e^{15}*f - 137*a^3*b^9*c^5*d^2*e^{15}*f + 1217*a^4*b^7*c^6*d^2*e^{15}*f - 5256*a^5*b^5*c^7*d^2*e^{15}*f + 11024*a^6*b^3*c^8*d^2*e^{15}*f)/(a^6*b^{12}*f^3 + 4096*a^{12}*c^6*f^3 - 24*a^7*b^{10}*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^{10}*b^4*c^4*f^3 - 6144*a^{11}*b^2*c^5*f^3) - (((4*a^4*b^{14}*c^2*e^{16}*f^2 - 100*a^5*b^{12}*c^3*e^{16}*f^2 + 1052*a^6*b^{10}*c^4*e^{16}*f^2 - 5952*a^7*b^8*c^5*e^{16}*f^2 + 19072*a^8*b^6*c^6*e^{16}*f^2 - 32768*a^9*b^4*c^7*e^{16}*f^2 + 23552*a^{10}*b^2*c^8*e^{16}*f^2 + 5120*a^{10}*b*c^9*d^2*e^{16}*f^2 + 2*a^4*b^{13}*c^3*d^2*e^{16}*f^2 - 36*a^5*b^{11}*c^4*d^2*e^{16}*f^2 + 276*a^6*b^9*c^5*d^2*e^{16}*f^2 - 1216*a^7*b^7*c^6*d^2*e^{16}*f^2 + 3456*a^8*b^5*c^7*d^2*e^{16}*f^2 - 6144*a^9*b^3*c^8*d^2*e^{16}*f^2)/(a^6*b^{12}*f^3 + 4096*a^{12}*c^6*f^3 - 24*a^7*b^{10}*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^{10}*b^4*c^4*f^3 - 6144*a^{11}*b^2*c^5*f^3) + ((2*b^{10}*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^{14}*c^2*e^{17}*f^3 - 96*a^8*b^{12}*c^3*e^{17}*f^3 + 960*a^9*b^{10}*c^4*e^{17}*f^3 - 5120*a^{10}*b^8*c^5*e^{17}*f^3 + 15360*a^{11}*b^6*c^6*e^{17}*f^3 - 24576*a^{12}*b^4*c^7*e^{17}*f^3 + 16384*a^{13}*b^2*c^8*e^{17}*f^3 - 163840*a^{13}*b*c^9*d^2*e^{17}*f^3 + 12*a^6*b^{15}*c^2*d^2*e^{17}*f^3 - 328*a^7*b^{13}*c^3*d^2*e^{17}*f^3 + 3840*a^8*b^{11}*c^4*d^2*e^{17}*f^3 - 24960*a^9*b^9*c^5*d^2*e^{17}*f^3 + 97280*a^{10}*b^7*c^6*d^2*e^{17}*f^3 - 227328*a^{11}*b^5*c^7*d^2*e^{17}*f^3 + 294912*a^{12}*b^3*c^8*d^2*e^{17}*f^3)))/(2*(4*a^3*b^{10}*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^{12}*f^3 + 4096*a^{12}*c^6*f^3 - 24*a^7*b^{10}*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^{10}*b^4*c^4*f^3 - 6144*a^{11}*b^2*c^5*f^3)))* \\
& (2*b^{10}*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^{10}*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)))*(2*b^{10}*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)
\end{aligned}$$

$$\begin{aligned}
& 0*a*b^8*c*e*f)/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6 \\
& *c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4 \\
& *b^8*c*e^2*f^2)) - (b^10*c^4*e^14 - 22*a*b^8*c^5*e^14 + 177*a^2*b^6*c^6*e^1 \\
& 4 - 616*a^3*b^4*c^7*e^14 + 784*a^4*b^2*c^8*e^14 + b^9*c^5*d^2*e^14 + 147*a^ \\
& 2*b^5*c^7*d^2*e^14 - 343*a^3*b^3*c^8*d^2*e^14 - 21*a*b^7*c^6*d^2*e^14)/(a^6 \\
& *b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1 \\
& 280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + (b*(\\
& (b*((4*a^4*b^14*c^2*e^16*f^2 - 100*a^5*b^12*c^3*e^16*f^2 + 1052*a^6*b^10*c^ \\
& 4*e^16*f^2 - 5952*a^7*b^8*c^5*e^16*f^2 + 19072*a^8*b^6*c^6*e^16*f^2 - 32768 \\
& *a^9*b^4*c^7*e^16*f^2 + 23552*a^10*b^2*c^8*e^16*f^2 + 5120*a^10*b*c^9*d^2*e \\
& ^16*f^2 + 2*a^4*b^13*c^3*d^2*e^16*f^2 - 36*a^5*b^11*c^4*d^2*e^16*f^2 + 276* \\
& a^6*b^9*c^5*d^2*e^16*f^2 - 1216*a^7*b^7*c^6*d^2*e^16*f^2 + 3456*a^8*b^5*c^7 \\
& *d^2*e^16*f^2 - 6144*a^9*b^3*c^8*d^2*e^16*f^2)/(a^6*b^12*f^3 + 4096*a^12*c^ \\
& 6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 38 \\
& 40*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) + ((2*b^10*e*f - 2048*a^5*c^5* \\
& e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 4 \\
& 0*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^ \\
& 9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17* \\
& f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 - 163840*a^ \\
& 13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^13*c^3*d^2 \\
& *e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d^2*e^17*f^3 \\
& + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^17*f^3 + 294 \\
& 912*a^12*b^3*c^8*d^2*e^17*f^3))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f \\
& ^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4* \\
& e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7* \\
& b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4 \\
& *f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f \\
& *(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10*e*f - 20 \\
& 48*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2* \\
& c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7*b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f \\
& ^3 + 960*a^9*b^10*c^4*e^17*f^3 - 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^ \\
& 6*c^6*e^17*f^3 - 24576*a^12*b^4*c^7*e^17*f^3 + 16384*a^13*b^2*c^8*e^17*f^3 \\
& - 163840*a^13*b*c^9*d^2*e^17*f^3 + 12*a^6*b^15*c^2*d^2*e^17*f^3 - 328*a^7*b^ \\
& ^13*c^3*d^2*e^17*f^3 + 3840*a^8*b^11*c^4*d^2*e^17*f^3 - 24960*a^9*b^9*c^5*d \\
& ^2*e^17*f^3 + 97280*a^10*b^7*c^6*d^2*e^17*f^3 - 227328*a^11*b^5*c^7*d^2*e^1 \\
& 7*f^3 + 294912*a^12*b^3*c^8*d^2*e^17*f^3))/(8*a^3*e*f*(4*a*c - b^2)^(5/2))*(\\
& 4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560* \\
& a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)*(a^6 \\
& *b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1 \\
& 280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3)))*(b^4 \\
& + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*e*f*(4*a*c - b^2)^(5/2)) + (b^2*(b^4 + \\
& 30*a^2*c^2 - 10*a*b^2*c))^2*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2 \\
& *e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(4*a^7 \\
& *b^14*c^2*e^17*f^3 - 96*a^8*b^12*c^3*e^17*f^3 + 960*a^9*b^10*c^4*e^17*f^3 - \\
& 5120*a^10*b^8*c^5*e^17*f^3 + 15360*a^11*b^6*c^6*e^17*f^3 - 24576*a^12*b^4*
\end{aligned}$$

$$3.546 \quad \int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=499

$$\frac{3(5b^2 - 12ac)(b^2 - 5ac)}{8a^3ef^2(b^2 - 4ac)^2(d+ex)} + \frac{36a^2c^2 + bc(5b^2 - 32ac)(d+ex)^2 - 35ab^2c + 5b^4}{8a^2ef^2(b^2 - 4ac)^2(d+ex)(a+b(d+ex)^2+c(d+ex)^4)} - \frac{3\sqrt{c} \left(\frac{b(124a^2c^2 - 47ab^2c + 5b^4)}{\sqrt{b^2 - 4ac}} \right)}{8\sqrt{2} a^3}$$

Rubi [A] time = 1.09, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1142, 1121, 1277, 1281, 1166, 205}

$$\frac{36a^2c^2 + bc(5b^2 - 32ac)(d+ex)^2 - 35ab^2c + 5b^4}{8a^3ef^2(b^2 - 4ac)^2(d+ex)} - \frac{3\sqrt{c} \left(\frac{b(124a^2c^2 - 47ab^2c + 5b^4)}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{d+ex}}{\sqrt{b^2 - 4ac}} \right)}{8\sqrt{2}a^3ef^2(b^2 - 4ac)^2\sqrt{b^2 - 4ac}} - \frac{3\sqrt{c} \left((5b^2 - 12ac)(b^2 - 5ac) - \frac{124a^2bc^2 - 47ab^3c + 5b^4}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{d+ex}}{\sqrt{b^2 - 4ac}} \right)}{8\sqrt{2}a^3ef^2(b^2 - 4ac)^2\sqrt{b^2 - 4ac} + b} - \frac{3(5b^2 - 12ac)(b^2 - 5ac)}{8a^3ef^2(b^2 - 4ac)^2(d+ex)} + \frac{-2ac + b^2 + bc(d+ex)^2}{4ae^2(b^2 - 4ac)(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2)))/sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]]]/(8*sqrt[2]*a^3*(b^2 - 4*a*c)^2*sqrt[b - sqrt[b^2 - 4*a*c]]*e*f^2) - (3*sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b + sqrt[b^2 - 4*a*c]]]/(8*sqrt[2]*a^3*(b^2 - 4*a*c)^2*sqrt[b + sqrt[b^2 - 4*a*c]]*e*f^2)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]]/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1121

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[((d*x)^(m+1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*a*d*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p+1)*Simp[b^2*(m+2*p+3) - 2*a*c*(m+4*p+5) + b*c*(m+4*p+7)*x^2, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In

tegerQ[m])

Rule 1142

Int[(u_)^(m_)*((a_) + (b_)*(v_)^2 + (c_)*(v_)^4)^(p_), x_Symbol] := Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p, x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1277

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1281

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(df + efx)^2 (a + b(d + ex)^2 + c(d + ex)^4)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^3} dx, x, d + ex\right)}{ef^2} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef^2(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)^2} - \dots \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef^2(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)^2} + \dots \\
&= -\frac{3(5b^2 - 12ac)(b^2 - 5ac)}{8a^3(b^2 - 4ac)^2 ef^2(d + ex)} + \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef^2(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)^2} \\
&= -\frac{3(5b^2 - 12ac)(b^2 - 5ac)}{8a^3(b^2 - 4ac)^2 ef^2(d + ex)} + \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef^2(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)^2} \\
&= -\frac{3(5b^2 - 12ac)(b^2 - 5ac)}{8a^3(b^2 - 4ac)^2 ef^2(d + ex)} + \frac{b^2 - 2ac + bc(d + ex)^2}{4a(b^2 - 4ac)ef^2(d + ex)(a + b(d + ex)^2 + c(d + ex)^4)^2}
\end{aligned}$$

Mathematica [A] time = 6.21, size = 575, normalized size = 1.15

$$\frac{1}{2\sqrt{2}\sqrt{d+ex}} + \frac{-3ab(d+ex) - 2ac^2(d+ex)^2 + b^2(d+ex) + b^2cd + c^2d^2}{4ef^2(4ac-b^2)(a+bd+cx^2+cd+cx^4)^2} - \frac{3\sqrt{c}\left(60a^2c\sqrt{b^2-4ac} + 124a^2b^2c - 47ab^2c - 37ab^2c\sqrt{b^2-4ac} + 50a^4\sqrt{b^2-4ac} + 50b^4\right)\text{atan}^{-1}\left(\frac{\sqrt{2}\sqrt{d+ex}}{\sqrt{b^2-4ac}}\right)}{8\sqrt{2}ef^2(b-4ac)^2\sqrt{b^2-4ac}} - \frac{3\sqrt{c}\left(60a^2c\sqrt{b^2-4ac} - 124a^2b^2c + 47ab^2c - 37ab^2c\sqrt{b^2-4ac} + 50a^4\sqrt{b^2-4ac} + 50b^4\right)\text{atan}^{-1}\left(\frac{\sqrt{2}\sqrt{d+ex}}{\sqrt{b^2-4ac}}\right)}{8\sqrt{2}ef^2(b-4ac)^2\sqrt{b^2-4ac}} + \frac{-84a^2b^2(d+ex) - 52ab^2(d+cx) + 47ab^2(d+cx) - 2b^2(d+cx) - 7b^2(d+cx)^2}{8b^2ef^2(4ac-b^2)(a+bd+cx^2+cd+cx^4)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out] $-(1/(a^3*ef^2*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d + e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*ef^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*c^3*(d + e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*ef^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*sqrt[b^2 - 4*a*c] - 37*a*b^2*c*sqrt[b^2 - 4*a*c] + 60*a^2*c^2*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*(d + e*x))/sqrt[b - sqrt[b^2 - 4*a*c]]]/(8*sqrt[2]*a^$

$3*(b^2 - 4*a*c)^{(5/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]*e*f^2) - (3*\text{Sqrt}[c]*(-5*b^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*\text{Sqrt}[b^2 - 4*a*c] - 37*a*b^2*c*\text{Sqrt}[b^2 - 4*a*c] + 60*a^2*c^2*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*(d + e*x))/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(8*\text{Sqrt}[2]*a^3*(b^2 - 4*a*c)^{(5/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]*e*f^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)^2 (a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

fricas [B] time = 3.61, size = 10518, normalized size = 21.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] $-1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*e^8*x^8 + 48*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d*e^7*x^7 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3 + 84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^2)*e^6*x^6 + 12*(28*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d)*e^5*x^5 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 + 210*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^2)*e^4*x^4 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 + 8*(42*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d)*e^3*x^3 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^2)*e^2*x^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*e*x - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a$

$$\begin{aligned}
&^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d * e^9 * f^2 * x^8 + 2 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3 + 18 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^2) * e^8 * f^2 * x^7 + 14 * (6 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^3 + (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3) * d) * e^7 * f^2 * x^6 + (a^3b^6 - 6a^4b^4c + 32a^6c^3 + 126 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^4 + 42 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3) * d^2) * e^6 * f^2 * x^5 + (126 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^5 + 70 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3) * d^3 + 5 * (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d) * e^5 * f^2 * x^4 + 2 * (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2 + 42 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^6 + 35 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3) * d^4 + 5 * (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d^2) * e^4 * f^2 * x^3 + 2 * (18 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^7 + 21 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3) * d^5 + 5 * (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d^3 + 3 * (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2) * d) * e^3 * f^2 * x^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2 + 9 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^8 + 14 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3) * d^6 + 5 * (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d^4 + 6 * (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2) * d^2) * e^2 * f^2 * x + ((a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^9 + 2 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3) * d^7 + (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d^5 + 2 * (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2) * d^3 + (a^5b^4 - 8a^6b^2c + 16a^7c^2) * d) * e * f^2) * \sqrt{-(25b^{11} - 495ab^9c + 3894a^2b^7c^2 - 15015a^3b^5c^3 + 27720a^4b^3c^4 - 18480a^5b^2c^5 + (a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5) * e^2 * f^4 * \sqrt{(625b^{12} - 12250a * b^{10}c + 94725a^2 * b^8c^2 - 351310a^3 * b^6c^3 + 591886a^4 * b^4c^4 - 312300a^5 * b^2c^5 + 50625a^6 * c^6) / ((a^{14}b^{10} - 20a^{15}b^8c + 160a^{16}b^6c^2 - 640a^{17}b^4c^3 + 1280a^{18}b^2c^4 - 1024a^{19}c^5) * e^4 * f^8))} / ((a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5) * e^2 * f^4)) * \log(-27 * (4125b^{10}c^4 - 77825a * b^8c^5 + 571030a^2 * b^6c^6 - 1957349a^3 * b^4c^7 + 2835000a^4 * b^2c^8 - 810000a^5 * c^9) * e * x - 27 * (4125b^{10}c^4 - 77825a * b^8c^5 + 571030a^2 * b^6c^6 - 1957349a^3 * b^4c^7 + 2835000a^4 * b^2c^8 - 810000a^5 * c^9) * d + 27/2 * \sqrt{1/2} * ((5a^7b^{16} - 152a^8b^{14}c + 2006a^9b^{12}c^2 - 14960a^{10}b^{10}c^3 + 68640a^{11}b^8c^4 - 197120a^{12}b^6c^5 + 342528a^{13}b^4c^6 - 323584a^{14}b^2c^7 + 122880a^{15}c^8) * e^3 * f^6 * \sqrt{(625b^{12} - 12250a * b^{10}c + 94725a^2 * b^8c^2 - 351310a^3 * b^6c^3 + 591886a^4 * b^4c^4 - 312300a^5 * b^2c^5 + 50625a^6 * c^6) / ((a^{14}b^{10} - 20a^{15}b^8c + 160a^{16}b^6c^2 - 640a^{17}b^4c^3 + 1280a^{18}b^2c^4 - 1024a^{19}c^5) * e^4 * f^8)) - (125b^{17} - 3775a * b^{15}c + 49360a^2 * b^{13}c^2 - 362733a^3 * b^{11}c^3 + 1623534a^4 * b^9c^4 - 4463140a^5 * b^7c^5 + 7146736a^6 * b^5c^6 - 5684672a^7 * b^3c^7 + 1324800a^8 * b^2c^8) * e * f^2) * \sqrt{-(25b^{11} - 495a * b^9c + 3894a^2 * b^7c^2 - 15015a^3 * b^5c^3 + 27720a^4 * b^3c^4 - 18480a^5 * b^2c^5 + (a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5) * e^2 * f^4 * \sqrt{(625b^{12} - 12250a * b^{10}c + 94725a^2 * b^8c^2 - 351310a^3 * b^6c^3 + 591886a^4 * b^4c^4 - 312300a^5 * b^2c^5 + 50625a^6 * c^6) / ((a^{14}b^{10} - 20a^{15}b^8c + 160a^{16}b^6c^2 - 640a^{17}b^4c^3 + 1280a^{18}b^2c^4 - 1024a^{19}c^5) * e^4 * f^8))}
\end{aligned}$$

$$\begin{aligned}
& 024*a^{12}*c^5)*e^{2*f^4*\sqrt{(625*b^{12} - 12250*a*b^{10}*c + 94725*a^2*b^8*c^2 - \\
& 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)}}/((a^{14}*b^{10} - 20*a^{15}*b^8*c + 160*a^{16}*b^6*c^2 - 640*a^{17}*b^4*c^3 + 128 \\
& 0*a^{18}*b^2*c^4 - 1024*a^{19}*c^5)*e^{4*f^8}))/((a^7*b^{10} - 20*a^8*b^8*c + 160* \\
& a^9*b^6*c^2 - 640*a^{10}*b^4*c^3 + 1280*a^{11}*b^2*c^4 - 1024*a^{12}*c^5)*e^{2*f^4} \\
&)) + 3*\sqrt{1/2}*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^{10*f^2*x^9} \\
& + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c \\
& - 8*a^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5* \\
& c^4)*d^2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^ \\
& 3 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - \\
& 6*a^4*b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d \\
& ^4 + 42*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126* \\
& (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3* \\
& c^2 + 16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2 \\
& *x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^ \\
& 2*c^3 + 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 \\
& + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4 \\
& *c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16 \\
& *a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - \\
& 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a \\
& ^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - \\
& 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3) \\
& *d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^{2*f^2*x} + ((a^3*b^4* \\
& c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a \\
& ^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a \\
& ^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^ \\
& 2)*\sqrt{-(25*b^{11} - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27 \\
& 720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^{10} - 20*a^8*b^8*c + 160*a^9*b^6* \\
& c^2 - 640*a^{10}*b^4*c^3 + 1280*a^{11}*b^2*c^4 - 1024*a^{12}*c^5)*e^{2*f^4*\sqrt{(6 \\
& 25*b^{12} - 12250*a*b^{10}*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886* \\
& a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)}}/((a^{14}*b^{10} - 20*a^{15}*b^8 \\
& *c + 160*a^{16}*b^6*c^2 - 640*a^{17}*b^4*c^3 + 1280*a^{18}*b^2*c^4 - 1024*a^{19}*c^ \\
& 5)*e^{4*f^8}))/((a^7*b^{10} - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^{10}*b^4*c^ \\
& 3 + 1280*a^{11}*b^2*c^4 - 1024*a^{12}*c^5)*e^{2*f^4}))*\log(-27*(4125*b^{10}*c^4 - 7 \\
& 7825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2 \\
& *c^8 - 810000*a^5*c^9)*e*x - 27*(4125*b^{10}*c^4 - 77825*a*b^8*c^5 + 571030*a \\
& ^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d \\
& + 27/2*\sqrt{1/2}*((5*a^7*b^{16} - 152*a^8*b^{14}*c + 2006*a^9*b^{12}*c^2 - 14960* \\
& a^{10}*b^{10}*c^3 + 68640*a^{11}*b^8*c^4 - 197120*a^{12}*b^6*c^5 + 342528*a^{13}*b^4* \\
& c^6 - 323584*a^{14}*b^2*c^7 + 122880*a^{15}*c^8)*e^3*f^6*\sqrt{(625*b^{12} - 12250 \\
& *a*b^{10}*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 3 \\
& 12300*a^5*b^2*c^5 + 50625*a^6*c^6)}}/((a^{14}*b^{10} - 20*a^{15}*b^8*c + 160*a^{16}*b \\
& ^6*c^2 - 640*a^{17}*b^4*c^3 + 1280*a^{18}*b^2*c^4 - 1024*a^{19}*c^5)*e^{4*f^8})) + \\
& (125*b^{17} - 3775*a*b^{15}*c + 49360*a^2*b^{13}*c^2 - 362733*a^3*b^{11}*c^3 + 1623 \\
& 534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b
\end{aligned}$$

$$\begin{aligned}
& ^3c^7 + 1324800a^8b^8c^8) * e * f^2) * \sqrt{-(25b^{11} - 495a * b^9c + 3894a^2 * \\
& b^7c^2 - 15015a^3b^5c^3 + 27720a^4b^3c^4 - 18480a^5b^8c^5 - (a^7b^{10} - 20a^8b^8c \\
& + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5) * e^2 * f^4 * \sqrt{((625b^{12} - 12250a * b^{10}c + 94725a^2 * b^8c^2 \\
& - 351310a^3b^6c^3 + 591886a^4b^4c^4 - 312300a^5b^2c^5 + 50625a^6c^6) / ((a^{14}b^{10} - 20a^{15}b^8c + 160a^{16}b^6c^2 - 640a^{17}b^4c^3 + \\
& 1280a^{18}b^2c^4 - 1024a^{19}c^5) * e^4 * f^8)) / ((a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5) * e^2 * \\
& f^4)) - 3 * \sqrt{1/2} * ((a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * e^{10} * f^2 * x^9 + 9 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d * e^9 * f^2 * x^8 + 2 * (a^3b^5c \\
& - 8a^4b^3c^2 + 16a^5b^8c^3 + 18 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^2) * e^8 * f^2 * x^7 + 14 * (6 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) \\
& * d^3 + (a^3b^5c - 8a^4b^3c^2 + 16a^5b^8c^3) * d) * e^7 * f^2 * x^6 + (a^3b^6 \\
& - 6a^4b^4c + 32a^6c^3 + 126 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^4 + 42 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^8c^3) * d^2) * e^6 * f^2 * x^5 + (1 \\
& 26 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^5 + 70 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^8c^3) * d^3 + 5 * (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d) * e^5 * \\
& f^2 * x^4 + 2 * (a^4b^5 - 8a^5b^3c + 16a^6b^8c^2 + 42 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^6 + 35 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^8c^3) * \\
& d^4 + 5 * (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d^2) * e^4 * f^2 * x^3 + 2 * (18 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^7 + 21 * (a^3b^5c - 8a^4b^3c^2 + \\
& 16a^5b^8c^3) * d^5 + 5 * (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d^3 + 3 * (a^4b^5 \\
& - 8a^5b^3c + 16a^6b^8c^2) * d) * e^3 * f^2 * x^2 + (a^5b^4 - 8a^6b^2c + 16a^7c^2 + 9 * (a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^8 + 14 * (a^3b^5c \\
& - 8a^4b^3c^2 + 16a^5b^8c^3) * d^6 + 5 * (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d^4 + 6 * (a^4b^5 - 8a^5b^3c + 16a^6b^8c^2) * d^2) * e^2 * f^2 * x + ((a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4) * d^9 + 2 * (a^3b^5c - 8a^4b^3c^2 + 16a^5b^8c^3) * d^7 + (a^3b^6 - 6a^4b^4c + 32a^6c^3) * d^5 + 2 * (a^4b^5 - 8a^5b^3c + 16a^6b^8c^2) * d^3 + (a^5b^4 - 8a^6b^2c + 16a^7c^2) * d) * e * f^2) * \sqrt{-(25b^{11} - 495a * b^9c + 3894a^2 * b^7c^2 - 15015a^3b^5c^3 + 27720a^4b^3c^4 - 18480a^5b^8c^5 - (a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5) * e^2 * f^4 * \sqrt{((625b^{12} - 12250a * b^{10}c + 94725a^2 * b^8c^2 - 351310a^3b^6c^3 + 591886a^4b^4c^4 - 312300a^5b^2c^5 + 50625a^6c^6) / ((a^{14}b^{10} - 20a^{15}b^8c + 160a^{16}b^6c^2 - 640a^{17}b^4c^3 + 1280a^{18}b^2c^4 - 1024a^{19}c^5) * e^4 * f^8)) / ((a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5) * e^2 * f^4)) * \log(-27 * (4125b^{10}c^4 - 77825a * b^8c^5 + 571030a^2 * b^6c^6 - 1957349a^3 * b^4c^7 + 2835000a^4 * b^2c^8 - 810000a^5c^9) * e * x - 27 * (4125b^{10}c^4 - 77825a * b^8c^5 + 571030a^2 * b^6c^6 - 1957349a^3 * b^4c^7 + 2835000a^4 * b^2c^8 - 810000a^5c^9) * d - 27/2 * \sqrt{1/2} * ((5a^7b^{16} - 152a^8b^{14}c + 2006a^9b^{12}c^2 - 14960a^{10}b^{10}c^3 + 68640a^{11}b^8c^4 - 197120a^{12}b^6c^5 + 342528a^{13}b^4c^6 - 323584a^{14}b^2c^7 + 122880a^{15}c^8) * e^3 * f^6 * \sqrt{((625b^{12} - 12250a * b^{10}c + 94725a^2 * b^8c^2 - 351310a^3b^6c^3 + 591886a^4b^4c^4 - 312300a^5b^2c^5 + 50625a^6c^6) / ((a^{14}b^{10} - 20a^{15}b^8c + 160a^{16}b^6c^2 - 640a^{17}b^4c^3 + 1280a^{18}b^2c^4 - 1024a^{19}c^5) * e^4 * f^8)) / ((a^7b^{10} - 20a^8b^8c + 160a^9b^6c^2 - 640a^{10}b^4c^3 + 1280a^{11}b^2c^4 - 1024a^{12}c^5) * e^2 * f^4))}
\end{aligned}$$

$$\begin{aligned}
& 6*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)) \\
& + (125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1 \\
& 623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^ \\
& 7*b^3*c^7 + 1324800*a^8*b*c^8)*e*f^2)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a \\
& ^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7 \\
& *b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c \\
& ^4 - 1024*a^12*c^5)*e^2*f^4)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8 \\
& *c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625 \\
& *a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 \\
& + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*e^4*f^8)))/((a^7*b^10 - 20*a^8*b^8*c \\
& + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*e \\
& ^2*f^4)))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^10*f^2*x^9 + 9*(a^ \\
& 3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*e^9*f^2*x^8 + 2*(a^3*b^5*c - 8*a^ \\
& 4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^ \\
& 2)*e^8*f^2*x^7 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^ \\
& 3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*e^7*f^2*x^6 + (a^3*b^6 - 6*a^4*b \\
& ^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42 \\
& *(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*e^6*f^2*x^5 + (126*(a^3*b^ \\
& 4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 1 \\
& 6*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*e^5*f^2*x^4 + \\
& 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + \\
& 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a \\
& ^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*e^4*f^2*x^3 + 2*(18*(a^3*b^4*c^2 - \\
& 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b \\
& c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5 \\
& b^3*c + 16*a^6*b*c^2)*d)*e^3*f^2*x^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 \\
& + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4 \\
& b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + \\
& 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*e^2*f^2*x + ((a^3*b^4*c^2 - 8 \\
& *a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^ \\
& 3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3 \\
& c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*e*f^2)
\end{aligned}$$

giac [B] time = 1.42, size = 1658, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/8*(7*b^4*c^2*e^(-1)/((f*x*e + d*f)*f) - 47*a*b^2*c^3*e^(-1)/((f*x*e + d*f)*f) + 52*a^2*c^4*e^(-1)/((f*x*e + d*f)*f) + 14*b^5*c*f*e^(-1)/(f*x*e + d*f)^3 - 99*a*b^3*c^2*f*e^(-1)/(f*x*e + d*f)^3 + 136*a^2*b*c^3*f*e^(-1)/(f*x*e + d*f)^3 + 7*b^6*f^3*e^(-1)/(f*x*e + d*f)^5 - 43*a*b^4*c*f^3*e^(-1)/(f*x
```

$$\begin{aligned}
& e + d*f)^5 + 25*a^2*b^2*c^2*f^3*e^{(-1)}/(f*x*e + d*f)^5 + 68*a^3*c^3*f^3*e^{(-1)}/(f*x*e + d*f)^5 + 9*a*b^5*f^5*e^{(-1)}/(f*x*e + d*f)^7 - 66*a^2*b^3*c*f^5 \\
& *e^{(-1)}/(f*x*e + d*f)^7 + 108*a^3*b*c^2*f^5*e^{(-1)}/(f*x*e + d*f)^7)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b*f^2/(f*x*e + d*f)^2 + a*f^4/(f*x*e + \\
& d*f)^4)^2) - e^{(-1)}/((f*x*e + d*f)*a^3*f) + 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11 \\
& *b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c))*a)*f^8*e^4 + 2 \\
& *(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b \\
& + 2*sqrt(b^2 - 4*a*c))*a)*sqrt(b^2 - 4*a*c)*f^4*abs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4 \\
& *e^2 + 16*a^5*c^2*f^4*e^2)*e^2 - (a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4 \\
& *e^2 + 16*a^5*c^2*f^4*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a* \\
& b + 2*sqrt(b^2 - 4*a*c))*a))*arctan(2*sqrt(1/2)*e^{(-1)}/((f*x*e + d*f)*f*sqrt \\
& ((a^3*b^5*f^4*e^2 - 8*a^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2 + sqrt((a^3*b^5*f^4*e^2 - 8*a^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2)^2 - 4*(a^4*b^4*f^8 \\
& *e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e^4))*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*b^4*f^8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e^4 \\
& ^4))))*e^{(-3)}/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)* \\
& sqrt(b^2 - 4*a*c)*f^6*abs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^2 \\
& *f^4*e^2)*abs(a)) - 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 \\
& - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b* \\
& c^6)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c))*a)*f^8*e^4 - 2*(5*a^4*b^6*c - 57*a^5* \\
& b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c))*a \\
&)*sqrt(b^2 - 4*a*c)*f^4*abs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5* \\
& c^2*f^4*e^2)*e^2 - (a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^2*f^4* \\
& e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c) \\
& *a))*arctan(2*sqrt(1/2)*e^{(-1)}/((f*x*e + d*f)*f*sqrt((a^3*b^5*f^4*e^2 - 8*a \\
& ^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2 - sqrt((a^3*b^5*f^4*e^2 - 8*a^4*b^3 \\
& *c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2)^2 - 4*(a^4*b^4*f^8*e^4 - 8*a^5*b^2*c*f^8 \\
& *e^4 + 16*a^6*c^2*f^8*e^4))*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4* \\
& b^4*f^8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e^4))))*e^{(-3)}/((a^7*b^6 \\
& *c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*f^6*a \\
& bs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*e^2)*abs(a))
\end{aligned}$$

maple [C] time = 0.07, size = 7019, normalized size = 14.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 15.40, size = 20580, normalized size = 41.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] - atan((( -(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4))^(1/2)*(x*(271790899200*a^20*c^14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 16878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 333226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6 - 1101055131648*a^19*b^2*c^13*e^12*f^6) - ( -(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4))^(1/2)*(( -(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4))^(1/2)
```

$$\begin{aligned}
& b^{12}c^4e^{2f^4} - 258048a^{12}b^{10}c^5e^{2f^4} + 860160a^{13}b^8c^6e^{2f^4} - 1966080a^{14}b^6c^7e^{2f^4} + 2949120a^{15}b^4c^8e^{2f^4} - 2621440a^{16}b^2c^9e^{2f^4} - 40a^8b^{18}c^*e^{2f^4}))^{(1/2)}(x*(262144a^{15}b^{23}c^2e^{14}f^{10} - 11534336a^{16}b^{21}c^3e^{14}f^{10} + 230686720a^{17}b^{19}c^4e^{14}f^{10} - 2768240640a^{18}b^{17}c^5e^{14}f^{10} + 22145925120a^{19}b^{15}c^6e^{14}f^{10} - 124017180672a^{20}b^{13}c^7e^{14}f^{10} + 496068722688a^{21}b^{11}c^8e^{14}f^{10} - 1417339207680a^{22}b^9c^9e^{14}f^{10} + 2834678415360a^{23}b^7c^{10}e^{14}f^{10} - 3779571220480a^{24}b^5c^{11}e^{14}f^{10} + 3023656976384a^{25}b^3c^{12}e^{14}f^{10} - 1099511627776a^{26}b^*c^{13}e^{14}f^{10}) - 1099511627776a^{26}b^*c^{13}d^*e^{13}f^{10} + 262144a^{15}b^{23}c^2d^*e^{13}f^{10} - 11534336a^{16}b^{21}c^3d^*e^{13}f^{10} + 230686720a^{17}b^{19}c^4d^*e^{13}f^{10} - 2768240640a^{18}b^{17}c^5d^*e^{13}f^{10} + 22145925120a^{19}b^{15}c^6d^*e^{13}f^{10} - 124017180672a^{20}b^{13}c^7d^*e^{13}f^{10} + 496068722688a^{21}b^{11}c^8d^*e^{13}f^{10} - 1417339207680a^{22}b^9c^9d^*e^{13}f^{10} + 2834678415360a^{23}b^7c^{10}d^*e^{13}f^{10} - 3779571220480a^{24}b^5c^{11}d^*e^{13}f^{10} + 3023656976384a^{25}b^3c^{12}d^*e^{13}f^{10}) - 245760a^{12}b^{23}c^2e^{12}f^8 + 10911744a^{13}b^{21}c^3e^{12}f^8 - 220397568a^{14}b^{19}c^4e^{12}f^8 + 2673082368a^{15}b^{17}c^5e^{12}f^8 - 21630025728a^{16}b^{15}c^6e^{12}f^8 + 122607894528a^{17}b^{13}c^7e^{12}f^8 - 496773365760a^{18}b^{11}c^8e^{12}f^8 + 1438679826432a^{19}b^9c^9e^{12}f^8 - 2918430277632a^{20}b^7c^{10}e^{12}f^8 + 3949222428672a^{21}b^5c^{11}e^{12}f^8 - 3208340570112a^{22}b^3c^{12}e^{12}f^8 + 1185410973696a^{23}b^*c^{13}e^{12}f^8) + 271790899200a^{20}c^{14}d^*e^{11}f^6 - 230400a^9b^{22}c^3d^*e^{11}f^6 + 9861120a^{10}b^{20}c^4d^*e^{11}f^6 - 191038464a^{11}b^{18}c^5d^*e^{11}f^6 + 2207803392a^{12}b^{16}c^6d^*e^{11}f^6 - 16878108672a^{13}b^{14}c^7d^*e^{11}f^6 + 89374851072a^{14}b^{12}c^8d^*e^{11}f^6 - 333226967040a^{15}b^{10}c^9d^*e^{11}f^6 + 869815812096a^{16}b^8c^{10}d^*e^{11}f^6 - 1543847804928a^{17}b^6c^{11}d^*e^{11}f^6 + 1747313491968a^{18}b^4c^{12}d^*e^{11}f^6 - 1101055131648a^{19}b^2c^{13}d^*e^{11}f^6)*i + (-(9*(25b^{21} - 25b^6*(-(4ac - b^2)^{15})^{(1/2)} + 18923520a^{10}b^*c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3*(-(4ac - b^2)^{15})^{(1/2)} - 995a*b^{19}c - 694a^2b^2c^2*(-(4ac - b^2)^{15})^{(1/2)} + 245a*b^4c*(-(4ac - b^2)^{15})^{(1/2)}))/(512*(a^7b^{20}e^{2f^4} + 1048576a^{17}c^{10}e^{2f^4} + 720a^9b^{16}c^2e^{2f^4} - 7680a^{10}b^{14}c^3e^{2f^4} + 53760a^{11}b^{12}c^4e^{2f^4} - 258048a^{12}b^{10}c^5e^{2f^4} + 860160a^{13}b^8c^6e^{2f^4} - 1966080a^{14}b^6c^7e^{2f^4} + 2949120a^{15}b^4c^8e^{2f^4} - 2621440a^{16}b^2c^9e^{2f^4} - 40a^8b^{18}c^*e^{2f^4}))^{(1/2)}(x*(271790899200a^{20}c^{14}e^{12}f^6 - 230400a^9b^{22}c^3e^{12}f^6 + 9861120a^{10}b^{20}c^4e^{12}f^6 - 191038464a^{11}b^{18}c^5e^{12}f^6 + 2207803392a^{12}b^{16}c^6e^{12}f^6 - 16878108672a^{13}b^{14}c^7e^{12}f^6 + 89374851072a^{14}b^{12}c^8e^{12}f^6 - 333226967040a^{15}b^{10}c^9e^{12}f^6 + 869815812096a^{16}b^8c^{10}e^{12}f^6 - 1543847804928a^{17}b^6c^{11}e^{12}f^6 + 1747313491968a^{18}b^4c^{12}e^{12}f^6 - 1101055131648a^{19}b^2c^{13}e^{12}f^6) - (-(9*(25b^{21} - 25b^6*(-(4ac - b^2)^{15})^{(1/2)} + 18923520a^{10}b^*c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 1990
\end{aligned}$$

$$\begin{aligned}
& 5600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3(-4ac - b^2)^{15} \\
& \left(\frac{1}{2} - 995ab^{19}c - 694a^2b^2c^2(-4ac - b^2)^{15} \right)^{1/2} + 245a^4c^4(-4ac - b^2)^{15} \\
& \left(\frac{1}{2} + 245a^4c^4(-4ac - b^2)^{15} \right)^{1/2} \Big/ (512(a^7b^{20}e^{2f^4} + 1048576a^{17}c^{10}e^{2f^4} + 720a^9b^{16}c^2e^{2f^4} \\
& - 7680a^{10}b^{14}c^3e^{2f^4} + 53760a^{11}b^{12}c^4e^{2f^4} - 258048a^{12}b^{10}c^5e^{2f^4} + 860160a^{13}b^8c^6e^{2f^4} \\
& - 1966080a^{14}b^6c^7e^{2f^4} + 2949120a^{15}b^4c^8e^{2f^4} - 2621440a^{16}b^2c^9e^{2f^4} - 40a^8b^{18}c^8e^{2f^4})) \\
& \left(\frac{1}{2} * ((-9(25b^{21} - 25b^6(-4ac - b^2)^{15}))^{1/2} + 18923520a^{10}b^3c^3 + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3(-4ac - b^2)^{15})^{1/2} - 995ab^{19}c - 694a^2b^2c^2(-4ac - b^2)^{15})^{1/2} + 245a^4c^4(-4ac - b^2)^{15})^{1/2} \Big/ (512(a^7b^{20}e^{2f^4} + 1048576a^{17}c^{10}e^{2f^4} + 720a^9b^{16}c^2e^{2f^4} - 7680a^{10}b^{14}c^3e^{2f^4} + 53760a^{11}b^{12}c^4e^{2f^4} - 258048a^{12}b^{10}c^5e^{2f^4} + 860160a^{13}b^8c^6e^{2f^4} - 1966080a^{14}b^6c^7e^{2f^4} + 2949120a^{15}b^4c^8e^{2f^4} - 2621440a^{16}b^2c^9e^{2f^4} - 40a^8b^{18}c^8e^{2f^4})) \\
& \left(\frac{1}{2} * (x^2(62144a^{15}b^{23}c^2e^{14f^{10}} - 11534336a^{16}b^{21}c^3e^{14f^{10}} + 230686720a^{17}b^{19}c^4e^{14f^{10}} - 2768240640a^{18}b^{17}c^5e^{14f^{10}} + 22145925120a^{19}b^{15}c^6e^{14f^{10}} - 124017180672a^{20}b^{13}c^7e^{14f^{10}} + 496068722688a^{21}b^{11}c^8e^{14f^{10}} - 1417339207680a^{22}b^9c^9e^{14f^{10}} + 2834678415360a^{23}b^7c^{10}e^{14f^{10}} - 3779571220480a^{24}b^5c^{11}e^{14f^{10}} + 3023656976384a^{25}b^3c^{12}e^{14f^{10}} - 1099511627776a^{26}b^1c^{13}e^{14f^{10}}) - 1099511627776a^{26}b^1c^{13}d^{13}e^{13f^{10}} + 262144a^{15}b^{23}c^2d^{13}e^{13f^{10}} - 11534336a^{16}b^{21}c^3d^{13}e^{13f^{10}} + 230686720a^{17}b^{19}c^4d^{13}e^{13f^{10}} - 2768240640a^{18}b^{17}c^5d^{13}e^{13f^{10}} + 22145925120a^{19}b^{15}c^6d^{13}e^{13f^{10}} - 124017180672a^{20}b^{13}c^7d^{13}e^{13f^{10}} + 496068722688a^{21}b^{11}c^8d^{13}e^{13f^{10}} - 1417339207680a^{22}b^9c^9d^{13}e^{13f^{10}} + 2834678415360a^{23}b^7c^{10}d^{13}e^{13f^{10}} - 3779571220480a^{24}b^5c^{11}d^{13}e^{13f^{10}} + 3023656976384a^{25}b^3c^{12}d^{13}e^{13f^{10}}) + 245760a^{12}b^{23}c^2e^{12f^8} - 10911744a^{13}b^{21}c^3e^{12f^8} + 220397568a^{14}b^{19}c^4e^{12f^8} - 2673082368a^{15}b^{17}c^5e^{12f^8} + 21630025728a^{16}b^{15}c^6e^{12f^8} - 122607894528a^{17}b^{13}c^7e^{12f^8} + 496773365760a^{18}b^{11}c^8e^{12f^8} - 1438679826432a^{19}b^9c^9e^{12f^8} + 2918430277632a^{20}b^7c^{10}e^{12f^8} - 3949222428672a^{21}b^5c^{11}e^{12f^8} + 3208340570112a^{22}b^3c^{12}e^{12f^8} - 1185410973696a^{23}b^1c^{13}e^{12f^8}) + 271790899200a^{20}c^{14}d^{11}e^{11f^6} - 230400a^9b^{22}c^3d^{11}e^{11f^6} + 9861120a^{10}b^{20}c^4d^{11}e^{11f^6} - 191038464a^{11}b^{18}c^5d^{11}e^{11f^6} + 2207803392a^{12}b^{16}c^6d^{11}e^{11f^6} - 16878108672a^{13}b^{14}c^7d^{11}e^{11f^6} + 89374851072a^{14}b^{12}c^8d^{11}e^{11f^6} - 333226967040a^{15}b^{10}c^9d^{11}e^{11f^6} + 869815812096a^{16}b^8c^{10}d^{11}e^{11f^6} - 1543847804928a^{17}b^6c^{11}d^{11}e^{11f^6} + 1747313491968a^{18}b^4c^{12}d^{11}e^{11f^6} - 1101055131648a^{19}b^2c^{13}d^{11}e^{11f^6}) * i) \Big/ ((-9(25b^{21} - 25b^6(-4ac - b^2)^{15}))^{1/2} + 18923520a^{10}b^3c^3 + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a
\end{aligned}$$

$$\begin{aligned}
& ^3c^3(-4ac - b^2)^{15})^{1/2} - 995ab^{19}c - 694a^2b^2c^2(-4ac - b^2)^{15})^{1/2} + 245ab^4c(-4ac - b^2)^{15})^{1/2}) / (512(a^7b^{20}e^{2f^4} + 1048576a^{17}c^{10}e^{2f^4} + 720a^9b^{16}c^2e^{2f^4} - 7680a^{10}b^{14}c^3e^{2f^4} + 53760a^{11}b^{12}c^4e^{2f^4} - 258048a^{12}b^{10}c^5e^{2f^4} + 860160a^{13}b^8c^6e^{2f^4} - 1966080a^{14}b^6c^7e^{2f^4} + 2949120a^{15}b^4c^8e^{2f^4} - 2621440a^{16}b^2c^9e^{2f^4} - 40a^8b^{18}c^2e^{2f^4})) \\
&)^{1/2} * (x(271790899200a^{20}c^{14}e^{12f^6} - 230400a^9b^{22}c^3e^{12f^6} + 9861120a^{10}b^{20}c^4e^{12f^6} - 191038464a^{11}b^{18}c^5e^{12f^6} + 2207803392a^{12}b^{16}c^6e^{12f^6} - 16878108672a^{13}b^{14}c^7e^{12f^6} + 89374851072a^{14}b^{12}c^8e^{12f^6} - 333226967040a^{15}b^{10}c^9e^{12f^6} + 869815812096a^{16}b^8c^{10}e^{12f^6} - 1543847804928a^{17}b^6c^{11}e^{12f^6} + 1747313491968a^{18}b^4c^{12}e^{12f^6} - 1101055131648a^{19}b^2c^{13}e^{12f^6}) - (-9(25b^{21} - 25b^6(-4ac - b^2)^{15})^{1/2} + 18923520a^{10}b^2c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3(-4ac - b^2)^{15})^{1/2} - 995ab^{19}c - 694a^2b^2c^2(-4ac - b^2)^{15})^{1/2} + 245ab^4c(-4ac - b^2)^{15})^{1/2}) / (512(a^7b^{20}e^{2f^4} + 1048576a^{17}c^{10}e^{2f^4} + 720a^9b^{16}c^2e^{2f^4} - 7680a^{10}b^{14}c^3e^{2f^4} + 53760a^{11}b^{12}c^4e^{2f^4} - 258048a^{12}b^{10}c^5e^{2f^4} + 860160a^{13}b^8c^6e^{2f^4} - 1966080a^{14}b^6c^7e^{2f^4} + 2949120a^{15}b^4c^8e^{2f^4} - 2621440a^{16}b^2c^9e^{2f^4} - 40a^8b^{18}c^2e^{2f^4})) \\
&)^{1/2} * ((-9(25b^{21} - 25b^6(-4ac - b^2)^{15})^{1/2} + 18923520a^{10}b^2c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3(-4ac - b^2)^{15})^{1/2} - 995ab^{19}c - 694a^2b^2c^2(-4ac - b^2)^{15})^{1/2} + 245ab^4c(-4ac - b^2)^{15})^{1/2}) / (512(a^7b^{20}e^{2f^4} + 1048576a^{17}c^{10}e^{2f^4} + 720a^9b^{16}c^2e^{2f^4} - 7680a^{10}b^{14}c^3e^{2f^4} + 53760a^{11}b^{12}c^4e^{2f^4} - 258048a^{12}b^{10}c^5e^{2f^4} + 860160a^{13}b^8c^6e^{2f^4} - 1966080a^{14}b^6c^7e^{2f^4} + 2949120a^{15}b^4c^8e^{2f^4} - 2621440a^{16}b^2c^9e^{2f^4} - 40a^8b^{18}c^2e^{2f^4})) \\
&)^{1/2} * (x(262144a^{15}b^{23}c^2e^{14f^{10}} - 11534336a^{16}b^{21}c^3e^{14f^{10}} + 230686720a^{17}b^{19}c^4e^{14f^{10}} - 2768240640a^{18}b^{17}c^5e^{14f^{10}} + 22145925120a^{19}b^{15}c^6e^{14f^{10}} - 124017180672a^{20}b^{13}c^7e^{14f^{10}} + 496068722688a^{21}b^{11}c^8e^{14f^{10}} - 1417339207680a^{22}b^9c^9e^{14f^{10}} + 2834678415360a^{23}b^7c^{10}e^{14f^{10}} - 3779571220480a^{24}b^5c^{11}e^{14f^{10}} + 3023656976384a^{25}b^3c^{12}e^{14f^{10}} - 1099511627776a^{26}b^2c^{13}e^{14f^{10}} - 1099511627776a^{26}b^2c^{13}d^2e^{13f^{10}} + 262144a^{15}b^{23}c^2d^2e^{13f^{10}} - 11534336a^{16}b^{21}c^3d^2e^{13f^{10}} + 230686720a^{17}b^{19}c^4d^2e^{13f^{10}} - 2768240640a^{18}b^{17}c^5d^2e^{13f^{10}} + 22145925120a^{19}b^{15}c^6d^2e^{13f^{10}} - 124017180672a^{20}b^{13}c^7d^2e^{13f^{10}} + 496068722688a^{21}b^{11}c^8d^2e^{13f^{10}} - 1417339207680a^{22}b^9c^9d^2e^{13f^{10}} + 2834678415360a^{23}b^7c^{10}d^2e^{13f^{10}} - 3779571220480a^{24}b^5c^{11}d^2e^{13f^{10}} + 3023656976384a^{25}b^3c^{12}d^2e^{13f^{10}}) + 245760a^{12}b^{23}c^2e^{12f^8} - 10911744a^{13}b^{21}c^3e^{12f^8} + 220397568a^{14}b^{19}c^4e^{12f^8} -
\end{aligned}$$

$$\begin{aligned}
& 2673082368a^{15}b^{17}c^5e^{12}f^8 + 21630025728a^{16}b^{15}c^6e^{12}f^8 - 12 \\
& 2607894528a^{17}b^{13}c^7e^{12}f^8 + 496773365760a^{18}b^{11}c^8e^{12}f^8 - 1 \\
& 438679826432a^{19}b^9c^9e^{12}f^8 + 2918430277632a^{20}b^7c^{10}e^{12}f^8 - \\
& 3949222428672a^{21}b^5c^{11}e^{12}f^8 + 3208340570112a^{22}b^3c^{12}e^{12}f^8 \\
& 8 - 1185410973696a^{23}b^3c^{13}e^{12}f^8) + 271790899200a^{20}c^{14}d^5e^{11}f^6 \\
& - 230400a^9b^{22}c^3d^5e^{11}f^6 + 9861120a^{10}b^{20}c^4d^5e^{11}f^6 - 1910 \\
& 38464a^{11}b^{18}c^5d^5e^{11}f^6 + 2207803392a^{12}b^{16}c^6d^5e^{11}f^6 - 1687 \\
& 8108672a^{13}b^{14}c^7d^5e^{11}f^6 + 89374851072a^{14}b^{12}c^8d^5e^{11}f^6 - 3 \\
& 33226967040a^{15}b^{10}c^9d^5e^{11}f^6 + 869815812096a^{16}b^8c^{10}d^5e^{11}f^6 \\
& 6 - 1543847804928a^{17}b^6c^{11}d^5e^{11}f^6 + 1747313491968a^{18}b^4c^{12}d^5 \\
& e^{11}f^6 - 1101055131648a^{19}b^2c^{13}d^5e^{11}f^6) - ((-9(25b^{21} - 25b^6 \\
& *(-4ac - b^2)^{15})^{1/2} + 18923520a^{10}b^3c^{10} + 17794a^2b^{17}c^2 - 18 \\
& 8095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6 \\
& b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3 \\
& c^3*(-4ac - b^2)^{15})^{1/2} - 995ab^{19}c - 694a^2b^2c^2*(-4ac - b^2)^{15})^{1/2} \\
& + 245a^4b^4c^4*(-4ac - b^2)^{15})^{1/2}))/512*(a^7b^{20}e^{2}f^4 + 1048576a^{17}c^{10}e^{2}f^4 + 720a^9b^{16}c^2e^{2}f^4 \\
& - 7680a^{10}b^{14}c^3e^{2}f^4 + 53760a^{11}b^{12}c^4e^{2}f^4 - 258048a^{12}b^{10}c^5e^{2}f^4 \\
& + 860160a^{13}b^8c^6e^{2}f^4 - 1966080a^{14}b^6c^7e^{2}f^4 + 2949120a^{15}b^4c^8e^{2}f^4 \\
& - 2621440a^{16}b^2c^9e^{2}f^4 - 40a^8b^{18}c^5e^{2}f^4))^{1/2}*(x*(271790899200a^{20}c^{14}e^{12}f^6 - 230400a^9b^{22} \\
& c^3e^{12}f^6 + 9861120a^{10}b^{20}c^4e^{12}f^6 - 191038464a^{11}b^{18}c^5e^{12}f^6 \\
& + 2207803392a^{12}b^{16}c^6e^{12}f^6 - 16878108672a^{13}b^{14}c^7e^{12}f^6 \\
& + 89374851072a^{14}b^{12}c^8e^{12}f^6 - 333226967040a^{15}b^{10}c^9e^{12}f^6 \\
& + 869815812096a^{16}b^8c^{10}e^{12}f^6 - 1543847804928a^{17}b^6c^{11}e^{12}f^6 \\
& + 1747313491968a^{18}b^4c^{12}e^{12}f^6 - 1101055131648a^{19}b^2c^{13}e^{12}f^6) - \\
& (-9(25b^{21} - 25b^6*(-4ac - b^2)^{15})^{1/2} + 18923520a^{10}b^3c^{10} + 17794a^2b^{17}c^2 \\
& - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 \\
& - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3*(-4ac - b^2)^{15})^{1/2} \\
& - 995ab^{19}c - 694a^2b^2c^2*(-4ac - b^2)^{15})^{1/2} + 245a^4b^4c^4*(-4ac - b^2)^{15})^{1/2}))/ \\
& (512*(a^7b^{20}e^{2}f^4 + 1048576a^{17}c^{10}e^{2}f^4 + 720a^9b^{16}c^2e^{2}f^4 - 7680a^{10}b^{14}c^3e^{2}f^4 \\
& + 53760a^{11}b^{12}c^4e^{2}f^4 - 258048a^{12}b^{10}c^5e^{2}f^4 + 860160a^{13}b^8c^6e^{2}f^4 \\
& - 1966080a^{14}b^6c^7e^{2}f^4 + 2949120a^{15}b^4c^8e^{2}f^4 - 2621440a^{16}b^2c^9e^{2}f^4 - 40a^8b^{18} \\
& c^5e^{2}f^4))^{1/2}*((-9(25b^{21} - 25b^6*(-4ac - b^2)^{15})^{1/2} + 18923520a^{10}b^3c^{10} \\
& + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600 \\
& a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3*(-4ac - b^2)^{15})^{1/2} \\
& - 995ab^{19}c - 694a^2b^2c^2*(-4ac - b^2)^{15})^{1/2} + 245a^4b^4c^4*(-4ac - b^2)^{15})^{1/2}))/ \\
& (512*(a^7b^{20}e^{2}f^4 + 1048576a^{17}c^{10}e^{2}f^4 + 720a^9b^{16}c^2e^{2}f^4 - 7680a^{10}b^{14}c^3e^{2}f^4 \\
& + 53760a^{11}b^{12}c^4e^{2}f^4 - 258048a^{12}b^{10}c^5e^{2}f^4 + 860160a^{13}b^8c^6e^{2}f^4 - 1966080a^{14}b^6c^7e^{2}f^4 \\
& + 2949120a^{15}b^4c^8e^{2}f^4 - 2621440a^{16}b^2c^9e^{2}f^4 - 40a^8b^{18}c^5e^{2}f^4)
\end{aligned}$$

$$\begin{aligned}
& b^{18}c^2e^{2f^4}))^{(1/2)} * (x * (262144a^{15}b^{23}c^2e^{14}f^{10} - 11534336a^{16} \\
& b^{21}c^3e^{14}f^{10} + 230686720a^{17}b^{19}c^4e^{14}f^{10} - 2768240640a^{18}b^{17} \\
& c^5e^{14}f^{10} + 22145925120a^{19}b^{15}c^6e^{14}f^{10} - 124017180672a^{20} \\
& b^{13}c^7e^{14}f^{10} + 496068722688a^{21}b^{11}c^8e^{14}f^{10} - 1417339207680a \\
& ^{22}b^9c^9e^{14}f^{10} + 2834678415360a^{23}b^7c^{10}e^{14}f^{10} - 37795712204 \\
& 80a^{24}b^5c^{11}e^{14}f^{10} + 3023656976384a^{25}b^3c^{12}e^{14}f^{10} - 109951 \\
& 1627776a^{26}b^2c^{13}e^{14}f^{10}) - 1099511627776a^{26}b^2c^{13}d^2e^{13}f^{10} + 26 \\
& 2144a^{15}b^{23}c^2d^2e^{13}f^{10} - 11534336a^{16}b^{21}c^3d^2e^{13}f^{10} + 23068 \\
& 6720a^{17}b^{19}c^4d^2e^{13}f^{10} - 2768240640a^{18}b^{17}c^5d^2e^{13}f^{10} + 221 \\
& 45925120a^{19}b^{15}c^6d^2e^{13}f^{10} - 124017180672a^{20}b^{13}c^7d^2e^{13}f^{10} \\
& + 496068722688a^{21}b^{11}c^8d^2e^{13}f^{10} - 1417339207680a^{22}b^9c^9d^2e^{13} \\
& f^{10} + 2834678415360a^{23}b^7c^{10}d^2e^{13}f^{10} - 3779571220480a^{24}b^5 \\
& c^{11}d^2e^{13}f^{10} + 3023656976384a^{25}b^3c^{12}d^2e^{13}f^{10}) - 245760a^{12}b \\
& ^{23}c^2e^{12}f^8 + 10911744a^{13}b^{21}c^3e^{12}f^8 - 220397568a^{14}b^{19}c^4 \\
& e^{12}f^8 + 2673082368a^{15}b^{17}c^5e^{12}f^8 - 21630025728a^{16}b^{15}c^6e^{12} \\
& f^8 + 122607894528a^{17}b^{13}c^7e^{12}f^8 - 496773365760a^{18}b^{11}c^8e^{12} \\
& f^8 + 1438679826432a^{19}b^9c^9e^{12}f^8 - 2918430277632a^{20}b^7c^{10}e^{12} \\
& f^8 + 3949222428672a^{21}b^5c^{11}e^{12}f^8 - 3208340570112a^{22}b^3 \\
& c^{12}e^{12}f^8 + 1185410973696a^{23}b^2c^{13}e^{12}f^8) + 271790899200a^{20}c^{14} \\
& d^2e^{11}f^6 - 230400a^9b^{22}c^3d^2e^{11}f^6 + 9861120a^{10}b^{20}c^4d^2e^{11} \\
& f^6 - 191038464a^{11}b^{18}c^5d^2e^{11}f^6 + 2207803392a^{12}b^{16}c^6d^2e^{11} \\
& f^6 - 16878108672a^{13}b^{14}c^7d^2e^{11}f^6 + 89374851072a^{14}b^{12}c^8d^2e^{11} \\
& f^6 - 333226967040a^{15}b^{10}c^9d^2e^{11}f^6 + 869815812096a^{16}b^8c^{10} \\
& d^2e^{11}f^6 - 1543847804928a^{17}b^6c^{11}d^2e^{11}f^6 + 1747313491968a^{18} \\
& b^4c^{12}d^2e^{11}f^6 - 1101055131648a^{19}b^2c^{13}d^2e^{11}f^6) + 191102976 \\
& 000a^{17}c^{14}e^{10}f^4 + 2851200a^9b^{16}c^6e^{10}f^4 - 92568960a^{10}b^{14} \\
& c^7e^{10}f^4 + 1312630272a^{11}b^{12}c^8e^{10}f^4 - 10611136512a^{12}b^{10}c^9 \\
& e^{10}f^4 + 53445353472a^{13}b^8c^{10}e^{10}f^4 - 171591892992a^{14}b^6c^{11} \\
& e^{10}f^4 + 342580396032a^{15}b^4c^{12}e^{10}f^4 - 388363714560a^{16}b^2c^{13} \\
& e^{10}f^4)) * ((-9*(25*b^{21} - 25*b^6*(-(4*a*c - b^2)^15))^{(1/2)} + 18923520* \\
& a^{10}b^2c^{10} + 17794a^{22}b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 \\
& - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 6 \\
& 2684160a^8b^5c^8 - 52039680a^9b^3c^9 + 225a^3c^3*(-(4*a*c - b^2)^15) \\
&)^{(1/2)} - 995a*b^{19}c - 694a^2b^2c^2*(-(4*a*c - b^2)^15)^{(1/2)} + 245a* \\
& b^4c*(-(4*a*c - b^2)^15)^{(1/2)))/(512*(a^7*b^{20}*e^{2*f^4} + 1048576*a^{17}*c^{10} \\
& *e^{2*f^4} + 720*a^9*b^{16}*c^2*e^{2*f^4} - 7680*a^{10}*b^{14}*c^3*e^{2*f^4} + 53760*a \\
& ^{11}*b^{12}*c^4*e^{2*f^4} - 258048*a^{12}*b^{10}*c^5*e^{2*f^4} + 860160*a^{13}*b^8*c^6 \\
& *e^{2*f^4} - 1966080*a^{14}*b^6*c^7*e^{2*f^4} + 2949120*a^{15}*b^4*c^8*e^{2*f^4} - 2621 \\
& 440*a^{16}*b^2*c^9*e^{2*f^4} - 40*a^8*b^{18}*c^2*e^{2*f^4}))^{(1/2)} * 2i - \operatorname{atan}(((-9*(\\
& 25*b^{21} + 25*b^6*(-(4*a*c - b^2)^15))^{(1/2)} + 18923520*a^{10}b^2c^{10} + 17794a \\
& ^{22}b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11} \\
& *c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - \\
& 52039680a^9b^3c^9 - 225a^3c^3*(-(4*a*c - b^2)^15)^{(1/2)} - 995a*b^{19} \\
& c + 694a^2b^2c^2*(-(4*a*c - b^2)^15)^{(1/2)} - 245a*b^4c*(-(4*a*c - b^2) \\
& ^15)^{(1/2)))/(512*(a^7*b^{20}*e^{2*f^4} + 1048576*a^{17}*c^{10}*e^{2*f^4} + 720*a^9*b
\end{aligned}$$

$$\begin{aligned}
& ^{16}c^2e^2f^4 - 7680a^{10}b^{14}c^3e^2f^4 + 53760a^{11}b^{12}c^4e^2f^4 \\
& - 258048a^{12}b^{10}c^5e^2f^4 + 860160a^{13}b^8c^6e^2f^4 - 1966080a^{14} \\
& *b^6c^7e^2f^4 + 2949120a^{15}b^4c^8e^2f^4 - 2621440a^{16}b^2c^9e^2f^4 - 40a^8b^{18}c^5e^2f^4))^{(1/2)}*(x*(271790899200a^{20}c^{14}e^{12}f^6 - \\
& 230400a^9b^{22}c^3e^{12}f^6 + 9861120a^{10}b^{20}c^4e^{12}f^6 - 191038464a \\
& ^{11}b^{18}c^5e^{12}f^6 + 2207803392a^{12}b^{16}c^6e^{12}f^6 - 16878108672a^{13} \\
& b^{14}c^7e^{12}f^6 + 89374851072a^{14}b^{12}c^8e^{12}f^6 - 333226967040a^{15} \\
& b^{10}c^9e^{12}f^6 + 869815812096a^{16}b^8c^{10}e^{12}f^6 - 1543847804928a \\
& ^{17}b^6c^{11}e^{12}f^6 + 1747313491968a^{18}b^4c^{12}e^{12}f^6 - 110105513164 \\
& 8a^{19}b^2c^{13}e^{12}f^6) - ((-9*(25*b^{21} + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} \\
&) + 18923520a^{10}b*c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 12998 \\
& 60a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7 \\
& b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 - 225a^3c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a*b^{19}c + 694*a^2b^2c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4c*(-(4*a*c - b^2)^{15})^{(1/2)))/(512*(a^7b^{20}e^2f^4 + 104 \\
& 8576a^{17}c^{10}e^2f^4 + 720a^9b^{16}c^2e^2f^4 - 7680a^{10}b^{14}c^3e^2f^4 + 53760a^{11}b^{12}c^4e^2f^4 - 258048a^{12}b^{10}c^5e^2f^4 + 860160a^{13}b^8c^6e^2f^4 - 1966080a^{14}b^6c^7e^2f^4 + 2949120a^{15}b^4c^8e^2f^4 - 2621440a^{16}b^2c^9e^2f^4 - 40a^8b^{18}c^5e^2f^4))^{(1/2)}*((-9*(25*b^{21} + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520a^{10}b*c^{10} + 1779 \\
& 4a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 - 225a^3c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a*b^{19}c + 694*a^2b^2c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4c*(-(4*a*c - b^2)^{15})^{(1/2)))/(512*(a^7b^{20}e^2f^4 + 1048576a^{17}c^{10}e^2f^4 + 720a^9b^{16}c^2e^2f^4 - 7680a^{10}b^{14}c^3e^2f^4 + 53760a^{11}b^{12}c^4e^2f^4 - 258048a^{12}b^{10}c^5e^2f^4 + 860160a^{13}b^8c^6e^2f^4 - 1966080a^{14}b^6c^7e^2f^4 + 2949120a^{15}b^4c^8e^2f^4 - 2621440a^{16}b^2c^9e^2f^4 - 40a^8b^{18}c^5e^2f^4))^{(1/2)}*(x*(262144a^{15}b^{23}c^2e^{14}f^{10} \\
& - 11534336a^{16}b^{21}c^3e^{14}f^{10} + 230686720a^{17}b^{19}c^4e^{14}f^{10} - 27 \\
& 68240640a^{18}b^{17}c^5e^{14}f^{10} + 22145925120a^{19}b^{15}c^6e^{14}f^{10} - 12 \\
& 4017180672a^{20}b^{13}c^7e^{14}f^{10} + 496068722688a^{21}b^{11}c^8e^{14}f^{10} - \\
& 1417339207680a^{22}b^9c^9e^{14}f^{10} + 2834678415360a^{23}b^7c^{10}e^{14}f^{10} \\
& - 3779571220480a^{24}b^5c^{11}e^{14}f^{10} + 3023656976384a^{25}b^3c^{12}e^{14}f^{10} - 1099511627776a^{26}b*c^{13}e^{14}f^{10}) - 1099511627776a^{26}b*c^{13} \\
& d*e^{13}f^{10} + 262144a^{15}b^{23}c^2*d*e^{13}f^{10} - 11534336a^{16}b^{21}c^3*d*e^{13}f^{10} + 230686720a^{17}b^{19}c^4*d*e^{13}f^{10} - 2768240640a^{18}b^{17}c^5*d \\
& *e^{13}f^{10} + 22145925120a^{19}b^{15}c^6*d*e^{13}f^{10} - 124017180672a^{20}b^{13} \\
& *c^7*d*e^{13}f^{10} + 496068722688a^{21}b^{11}c^8*d*e^{13}f^{10} - 1417339207680a^{22}b^9c^9*d*e^{13}f^{10} + 2834678415360a^{23}b^7c^{10}*d*e^{13}f^{10} - 3779571 \\
& 220480a^{24}b^5c^{11}*d*e^{13}f^{10} + 3023656976384a^{25}b^3c^{12}*d*e^{13}f^{10}) \\
& - 245760a^{12}b^{23}c^2e^{12}f^8 + 10911744a^{13}b^{21}c^3e^{12}f^8 - 220397 \\
& 568a^{14}b^{19}c^4e^{12}f^8 + 2673082368a^{15}b^{17}c^5e^{12}f^8 - 2163002572 \\
& 8a^{16}b^{15}c^6e^{12}f^8 + 122607894528a^{17}b^{13}c^7e^{12}f^8 - 4967733657 \\
& 60a^{18}b^{11}c^8e^{12}f^8 + 1438679826432a^{19}b^9c^9e^{12}f^8 - 291843027
\end{aligned}$$

$$\begin{aligned}
& 7632a^{20}b^7c^{10}e^{12}f^8 + 3949222428672a^{21}b^5c^{11}e^{12}f^8 - 320834 \\
& 0570112a^{22}b^3c^{12}e^{12}f^8 + 1185410973696a^{23}b^1c^{13}e^{12}f^8 + 2717 \\
& 90899200a^{20}c^{14}d^2e^{11}f^6 - 230400a^9b^{22}c^3d^2e^{11}f^6 + 9861120a^ \\
& 10b^{20}c^4d^2e^{11}f^6 - 191038464a^{11}b^{18}c^5d^2e^{11}f^6 + 2207803392a^ \\
& 12b^{16}c^6d^2e^{11}f^6 - 16878108672a^{13}b^{14}c^7d^2e^{11}f^6 + 89374851072 \\
& a^{14}b^{12}c^8d^2e^{11}f^6 - 333226967040a^{15}b^{10}c^9d^2e^{11}f^6 + 8698158 \\
& 12096a^{16}b^8c^{10}d^2e^{11}f^6 - 1543847804928a^{17}b^6c^{11}d^2e^{11}f^6 + 1 \\
& 747313491968a^{18}b^4c^{12}d^2e^{11}f^6 - 1101055131648a^{19}b^2c^{13}d^2e^{11}f^6 \\
& f^6) * i + \left(-(9(25b^{21} + 25b^6(-4ac - b^2)^{15})^{1/2}) + 18923520a^{10}b^ \\
& c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - \\
& 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 626841 \\
& 60a^8b^5c^8 - 52039680a^9b^3c^9 - 225a^3c^3(-4ac - b^2)^{15})^{1/2} - 995a^ \\
& b^{19}c + 694a^2b^2c^2(-4ac - b^2)^{15})^{1/2} - 245a^4b^4c^4 \\
& (-4ac - b^2)^{15})^{1/2} \right) / (512(a^7b^{20}e^{2}f^4 + 1048576a^{17}c^{10}e^2 \\
& f^4 + 720a^9b^{16}c^2e^2f^4 - 7680a^{10}b^{14}c^3e^2f^4 + 53760a^{11}b^{12}c^4e^2 \\
& f^4 - 258048a^{12}b^{10}c^5e^2f^4 + 860160a^{13}b^8c^6e^2f^4 - 1966080a^{14}b^6c^7e^2 \\
& f^4 + 2949120a^{15}b^4c^8e^2f^4 - 2621440a^{16}b^2c^9e^2f^4 - 40a^8b^{18}c^2e^2f^4)) \\
&)^{1/2} * (x(271790899200a^{20}c^{14}e^{12}f^6 - 230400a^9b^{22}c^3e^{12}f^6 + 9861120a^{10}b^{20}c^4e^{12}f^6 \\
& - 191038464a^{11}b^{18}c^5e^{12}f^6 + 2207803392a^{12}b^{16}c^6e^{12}f^6 - \\
& 16878108672a^{13}b^{14}c^7e^{12}f^6 + 89374851072a^{14}b^{12}c^8e^{12}f^6 - \\
& 333226967040a^{15}b^{10}c^9e^{12}f^6 + 869815812096a^{16}b^8c^{10}e^{12}f^6 - \\
& 1543847804928a^{17}b^6c^{11}e^{12}f^6 + 1747313491968a^{18}b^4c^{12}e^{12}f^6 \\
& - 1101055131648a^{19}b^2c^{13}e^{12}f^6) - \left(-(9(25b^{21} + 25b^6(-4ac - b^2)^{15})^{1/2}) + 18923520a^{10}b^ \\
& c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - \\
& 43904256a^7b^7c^7 + 62684160a^8b^5c^8 - 52039680a^9b^3c^9 - 2 \\
& 25a^3c^3(-4ac - b^2)^{15})^{1/2} - 995a^4b^4c^4(-4ac - b^2)^{15})^{1/2} - 245a^4b^4c^4 \\
& (-4ac - b^2)^{15})^{1/2} \right) / (512(a^7b^{20}e^2f^4 + 1048576a^{17}c^{10}e^2f^4 + 720a^9b^{16}c^2e^2f^4 - 7680a^{10}b^{14}c^3e^2f^4 + 53760a^{11}b^{12}c^4e^2f^4 - 258048a^{12}b^{10}c^5e^2f^4 + 860160a^{13}b^8c^6e^2f^4 - 1966080a^{14}b^6c^7e^2f^4 + 2949120a^{15}b^4c^8e^2f^4 - 2621440a^{16}b^2c^9e^2f^4 - 40a^8b^{18}c^2e^2f^4)) \\
&)^{1/2} * \left(-(9(25b^{21} + 25b^6(-4ac - b^2)^{15})^{1/2}) + 18923520a^{10}b^ \\
& c^{10} + 17794a^2b^{17}c^2 - 188095a^3b^{15}c^3 + 1299860a^4b^{13}c^4 - 6126640a^5b^{11}c^5 + 19905600a^6b^9c^6 - 43904256a^7b^7c^7 + 626 \\
& 84160a^8b^5c^8 - 52039680a^9b^3c^9 - 225a^3c^3(-4ac - b^2)^{15})^{1/2} - 995a^ \\
& b^{19}c + 694a^2b^2c^2(-4ac - b^2)^{15})^{1/2} - 245a^4b^4c^4 \\
& (-4ac - b^2)^{15})^{1/2} \right) / (512(a^7b^{20}e^2f^4 + 1048576a^{17}c^{10}e^2f^4 + 720a^9b^{16}c^2e^2f^4 - 7680a^{10}b^{14}c^3e^2f^4 + 53760a^{11}b^{12}c^4e^2f^4 - 258048a^{12}b^{10}c^5e^2f^4 + 860160a^{13}b^8c^6e^2f^4 - 1966080a^{14}b^6c^7e^2f^4 + 2949120a^{15}b^4c^8e^2f^4 - 2621440a^{16}b^2c^9e^2f^4 - 40a^8b^{18}c^2e^2f^4)) \\
&)^{1/2} * (x(262144a^{15}b^2 \\
& 3c^2e^{14}f^{10} - 11534336a^{16}b^{21}c^3e^{14}f^{10} + 230686720a^{17}b^{19}c^4e^{14}f^{10} - 2768240640a^{18}b^{17}c^5e^{14}f^{10} + 22145925120a^{19}b^{15}c^7e^{14}f^{10} - 11534336a^{20}b^{13}c^7e^{14}f^{10} + 11534336a^{21}b^{11}c^9e^{14}f^{10} - 11534336a^{22}b^9c^{11}e^{14}f^{10} + 11534336a^{23}b^7c^{13}e^{14}f^{10} - 11534336a^{24}b^5c^{15}e^{14}f^{10} + 11534336a^{25}b^3c^{17}e^{14}f^{10} - 11534336a^{26}b^1c^{19}e^{14}f^{10} + 11534336a^{27}c^{21}e^{14}f^{10} - 11534336a^{28}e^{14}f^{10}))
\end{aligned}$$

$$\begin{aligned}
& 6e^{14}f^{10} - 124017180672a^{20}b^{13}c^7e^{14}f^{10} + 496068722688a^{21}b^{11} \\
& c^8e^{14}f^{10} - 1417339207680a^{22}b^9c^9e^{14}f^{10} + 2834678415360a^{23} \\
& b^7c^{10}e^{14}f^{10} - 3779571220480a^{24}b^5c^{11}e^{14}f^{10} + 3023656976384 \\
& a^{25}b^3c^{12}e^{14}f^{10} - 1099511627776a^{26}b^1c^{13}e^{14}f^{10} - 1099511627 \\
& 776a^{26}b^1c^{13}d^1e^{13}f^{10} + 262144a^{15}b^{23}c^2d^1e^{13}f^{10} - 11534336a \\
& ^{16}b^{21}c^3d^1e^{13}f^{10} + 230686720a^{17}b^{19}c^4d^1e^{13}f^{10} - 2768240640 \\
& a^{18}b^{17}c^5d^1e^{13}f^{10} + 22145925120a^{19}b^{15}c^6d^1e^{13}f^{10} - 124017 \\
& 180672a^{20}b^{13}c^7d^1e^{13}f^{10} + 496068722688a^{21}b^{11}c^8d^1e^{13}f^{10} - \\
& 1417339207680a^{22}b^9c^9d^1e^{13}f^{10} + 2834678415360a^{23}b^7c^{10}d^1e^{13} \\
& f^{10} - 3779571220480a^{24}b^5c^{11}d^1e^{13}f^{10} + 3023656976384a^{25}b^3c^{12} \\
& d^1e^{13}f^{10} + 245760a^{12}b^{23}c^2e^{12}f^8 - 10911744a^{13}b^{21}c^3e^{12} \\
& f^8 + 220397568a^{14}b^{19}c^4e^{12}f^8 - 2673082368a^{15}b^{17}c^5e^{12} \\
& f^8 + 21630025728a^{16}b^{15}c^6e^{12}f^8 - 122607894528a^{17}b^{13}c^7e^{12} \\
& f^8 + 496773365760a^{18}b^{11}c^8e^{12}f^8 - 1438679826432a^{19}b^9c^9e^{12} \\
& f^8 + 2918430277632a^{20}b^7c^{10}e^{12}f^8 - 3949222428672a^{21}b^5c^{11}e^{12} \\
& f^8 + 3208340570112a^{22}b^3c^{12}e^{12}f^8 - 1185410973696a^{23}b^1c^{13}e^{12} \\
& f^8 + 271790899200a^{20}c^{14}d^1e^{11}f^6 - 230400a^9b^{22}c^3d^1e^{11} \\
& f^6 + 9861120a^{10}b^{20}c^4d^1e^{11}f^6 - 191038464a^{11}b^{18}c^5d^1e^{11}f^6 \\
& + 2207803392a^{12}b^{16}c^6d^1e^{11}f^6 - 16878108672a^{13}b^{14}c^7d^1e^{11}f^6 \\
& + 89374851072a^{14}b^{12}c^8d^1e^{11}f^6 - 333226967040a^{15}b^{10}c^9d^1e^{11} \\
& f^6 + 869815812096a^{16}b^8c^{10}d^1e^{11}f^6 - 1543847804928a^{17}b^6c^{11} \\
& d^1e^{11}f^6 + 1747313491968a^{18}b^4c^{12}d^1e^{11}f^6 - 1101055131648a^{19} \\
& b^2c^{13}d^1e^{11}f^6) * i) / ((- (9 * (25 * b^{21} + 25 * b^6 * (- (4 * a * c - b^2)^{15})^{1/2}) \\
& + 18923520 * a^{10} * b * c^{10} + 17794 * a^2 * b^{17} * c^2 - 188095 * a^3 * b^{15} * c^3 + 1299860 \\
& * a^4 * b^{13} * c^4 - 6126640 * a^5 * b^{11} * c^5 + 19905600 * a^6 * b^9 * c^6 - 43904256 * a^7 * \\
& b^7 * c^7 + 62684160 * a^8 * b^5 * c^8 - 52039680 * a^9 * b^3 * c^9 - 225 * a^3 * c^3 * (- (4 * a * \\
& c - b^2)^{15})^{1/2} - 995 * a * b^{19} * c + 694 * a^2 * b^2 * c^2 * (- (4 * a * c - b^2)^{15})^{1/2} \\
& - 245 * a * b^4 * c * (- (4 * a * c - b^2)^{15})^{1/2})) / (512 * (a^7 * b^{20} * e^{2 * f^4} + 10485 \\
& 76 * a^{17} * c^{10} * e^{2 * f^4} + 720 * a^9 * b^{16} * c^2 * e^{2 * f^4} - 7680 * a^{10} * b^{14} * c^3 * e^{2 * f^4} \\
& + 53760 * a^{11} * b^{12} * c^4 * e^{2 * f^4} - 258048 * a^{12} * b^{10} * c^5 * e^{2 * f^4} + 860160 * a^{13} * \\
& b^8 * c^6 * e^{2 * f^4} - 1966080 * a^{14} * b^6 * c^7 * e^{2 * f^4} + 2949120 * a^{15} * b^4 * c^8 * e^{2 * \\
& f^4} - 2621440 * a^{16} * b^2 * c^9 * e^{2 * f^4} - 40 * a^8 * b^{18} * c * e^{2 * f^4}))^{1/2} * (x * (27 \\
& 1790899200 * a^{20} * c^{14} * e^{12} * f^6 - 230400 * a^9 * b^{22} * c^3 * e^{12} * f^6 + 9861120 * a^{10} \\
& * b^{20} * c^4 * e^{12} * f^6 - 191038464 * a^{11} * b^{18} * c^5 * e^{12} * f^6 + 2207803392 * a^{12} * b^{16} * \\
& c^6 * e^{12} * f^6 - 16878108672 * a^{13} * b^{14} * c^7 * e^{12} * f^6 + 89374851072 * a^{14} * b^{12} * \\
& c^8 * e^{12} * f^6 - 333226967040 * a^{15} * b^{10} * c^9 * e^{12} * f^6 + 869815812096 * a^{16} * b^8 * \\
& c^{10} * e^{12} * f^6 - 1543847804928 * a^{17} * b^6 * c^{11} * e^{12} * f^6 + 1747313491968 * a^{18} * \\
& b^4 * c^{12} * e^{12} * f^6 - 1101055131648 * a^{19} * b^2 * c^{13} * e^{12} * f^6) - (- (9 * (25 * b^{21} + \\
& 25 * b^6 * (- (4 * a * c - b^2)^{15})^{1/2}) + 18923520 * a^{10} * b * c^{10} + 17794 * a^2 * b^{17} * c^2 \\
& - 188095 * a^3 * b^{15} * c^3 + 1299860 * a^4 * b^{13} * c^4 - 6126640 * a^5 * b^{11} * c^5 + 19 \\
& 905600 * a^6 * b^9 * c^6 - 43904256 * a^7 * b^7 * c^7 + 62684160 * a^8 * b^5 * c^8 - 52039680 \\
& * a^9 * b^3 * c^9 - 225 * a^3 * c^3 * (- (4 * a * c - b^2)^{15})^{1/2} - 995 * a * b^{19} * c + 694 * a^2 * \\
& b^2 * c^2 * (- (4 * a * c - b^2)^{15})^{1/2} - 245 * a * b^4 * c * (- (4 * a * c - b^2)^{15})^{1/2} \\
&)) / (512 * (a^7 * b^{20} * e^{2 * f^4} + 1048576 * a^{17} * c^{10} * e^{2 * f^4} + 720 * a^9 * b^{16} * c^2 * e^{2 * \\
& f^4} - 7680 * a^{10} * b^{14} * c^3 * e^{2 * f^4} + 53760 * a^{11} * b^{12} * c^4 * e^{2 * f^4} - 258048 *
\end{aligned}$$

$$\begin{aligned}
& 5*b^4*c^8*e^{2*f^4} - 2621440*a^{16}*b^2*c^9*e^{2*f^4} - 40*a^8*b^{18}*c*e^{2*f^4})) \\
& ^{(1/2)}*(x*(271790899200*a^{20}*c^{14}*e^{12*f^6} - 230400*a^9*b^{22}*c^3*e^{12*f^6} + \\
& 9861120*a^{10}*b^{20}*c^4*e^{12*f^6} - 191038464*a^{11}*b^{18}*c^5*e^{12*f^6} + 220780 \\
& 3392*a^{12}*b^{16}*c^6*e^{12*f^6} - 16878108672*a^{13}*b^{14}*c^7*e^{12*f^6} + 89374851 \\
& 072*a^{14}*b^{12}*c^8*e^{12*f^6} - 333226967040*a^{15}*b^{10}*c^9*e^{12*f^6} + 86981581 \\
& 2096*a^{16}*b^8*c^{10}*e^{12*f^6} - 1543847804928*a^{17}*b^6*c^{11}*e^{12*f^6} + 174731 \\
& 3491968*a^{18}*b^4*c^{12}*e^{12*f^6} - 1101055131648*a^{19}*b^2*c^{13}*e^{12*f^6}) - (- \\
& (9*(25*b^{21} + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^{10} + 177 \\
& 94*a^2*b^{17}*c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a^5* \\
& b^{11}*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^ \\
& ^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a*b \\
& ^{19}*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4*a*c - \\
& b^2)^{15})^{(1/2)})))/(512*(a^7*b^{20}*e^{2*f^4} + 1048576*a^{17}*c^{10}*e^{2*f^4} + 720*a \\
& ^9*b^{16}*c^2*e^{2*f^4} - 7680*a^{10}*b^{14}*c^3*e^{2*f^4} + 53760*a^{11}*b^{12}*c^4*e^{2* \\
& f^4} - 258048*a^{12}*b^{10}*c^5*e^{2*f^4} + 860160*a^{13}*b^8*c^6*e^{2*f^4} - 1966080* \\
& a^{14}*b^6*c^7*e^{2*f^4} + 2949120*a^{15}*b^4*c^8*e^{2*f^4} - 2621440*a^{16}*b^2*c^9* \\
& e^{2*f^4} - 40*a^8*b^{18}*c*e^{2*f^4}))^{(1/2)}*((-(9*(25*b^{21} + 25*b^6*(-(4*a*c - \\
& b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^{10} + 17794*a^2*b^{17}*c^2 - 188095*a^3*b^ \\
& 15*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a^5*b^{11}*c^5 + 19905600*a^6*b^9*c^6 \\
& - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 - 225 \\
& *a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995*a*b^{19}*c + 694*a^2*b^2*c^2*(-(4*a* \\
& c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4*a*c - b^2)^{15})^{(1/2)})))/(512*(a^7*b^{20} \\
& *e^{2*f^4} + 1048576*a^{17}*c^{10}*e^{2*f^4} + 720*a^9*b^{16}*c^2*e^{2*f^4} - 7680*a^{10} \\
& *b^{14}*c^3*e^{2*f^4} + 53760*a^{11}*b^{12}*c^4*e^{2*f^4} - 258048*a^{12}*b^{10}*c^5*e^{2* \\
& f^4} + 860160*a^{13}*b^8*c^6*e^{2*f^4} - 1966080*a^{14}*b^6*c^7*e^{2*f^4} + 2949120* \\
& a^{15}*b^4*c^8*e^{2*f^4} - 2621440*a^{16}*b^2*c^9*e^{2*f^4} - 40*a^8*b^{18}*c*e^{2*f^4} \\
&))^{(1/2)}*(x*(262144*a^{15}*b^{23}*c^2*e^{14*f^{10}} - 11534336*a^{16}*b^{21}*c^3*e^{14* \\
& f^{10}} + 230686720*a^{17}*b^{19}*c^4*e^{14*f^{10}} - 2768240640*a^{18}*b^{17}*c^5*e^{14*f^{10}} \\
& + 22145925120*a^{19}*b^{15}*c^6*e^{14*f^{10}} - 124017180672*a^{20}*b^{13}*c^7*e^{14* \\
& f^{10}} + 496068722688*a^{21}*b^{11}*c^8*e^{14*f^{10}} - 1417339207680*a^{22}*b^9*c^9*e^{ \\
& 14*f^{10}} + 2834678415360*a^{23}*b^7*c^{10}*e^{14*f^{10}} - 3779571220480*a^{24}*b^5*c^{ \\
& 11}*e^{14*f^{10}} + 3023656976384*a^{25}*b^3*c^{12}*e^{14*f^{10}} - 1099511627776*a^{26}*b \\
& *c^{13}*e^{14*f^{10}}) - 1099511627776*a^{26}*b*c^{13}*d*e^{13*f^{10}} + 262144*a^{15}*b^{23} \\
& *c^2*d*e^{13*f^{10}} - 11534336*a^{16}*b^{21}*c^3*d*e^{13*f^{10}} + 230686720*a^{17}*b^{19} \\
& *c^4*d*e^{13*f^{10}} - 2768240640*a^{18}*b^{17}*c^5*d*e^{13*f^{10}} + 22145925120*a^{19} \\
& *b^{15}*c^6*d*e^{13*f^{10}} - 124017180672*a^{20}*b^{13}*c^7*d*e^{13*f^{10}} + 49606872268 \\
& 8*a^{21}*b^{11}*c^8*d*e^{13*f^{10}} - 1417339207680*a^{22}*b^9*c^9*d*e^{13*f^{10}} + 2834 \\
& 678415360*a^{23}*b^7*c^{10}*d*e^{13*f^{10}} - 3779571220480*a^{24}*b^5*c^{11}*d*e^{13*f^{10}} \\
& + 3023656976384*a^{25}*b^3*c^{12}*d*e^{13*f^{10}}) - 245760*a^{12}*b^{23}*c^2*e^{12*f^ \\
& ^8} + 10911744*a^{13}*b^{21}*c^3*e^{12*f^8} - 220397568*a^{14}*b^{19}*c^4*e^{12*f^8} + 2 \\
& 673082368*a^{15}*b^{17}*c^5*e^{12*f^8} - 21630025728*a^{16}*b^{15}*c^6*e^{12*f^8} + 122 \\
& 607894528*a^{17}*b^{13}*c^7*e^{12*f^8} - 496773365760*a^{18}*b^{11}*c^8*e^{12*f^8} + 14 \\
& 38679826432*a^{19}*b^9*c^9*e^{12*f^8} - 2918430277632*a^{20}*b^7*c^{10}*e^{12*f^8} + \\
& 3949222428672*a^{21}*b^5*c^{11}*e^{12*f^8} - 3208340570112*a^{22}*b^3*c^{12}*e^{12*f^8} \\
& + 1185410973696*a^{23}*b*c^{13}*e^{12*f^8}) + 271790899200*a^{20}*c^{14}*d*e^{11*f^6}
\end{aligned}$$

$$\begin{aligned}
& - 230400*a^9*b^{22}*c^3*d*e^{11}*f^6 + 9861120*a^{10}*b^{20}*c^4*d*e^{11}*f^6 - 19103 \\
& 8464*a^{11}*b^{18}*c^5*d*e^{11}*f^6 + 2207803392*a^{12}*b^{16}*c^6*d*e^{11}*f^6 - 16878 \\
& 108672*a^{13}*b^{14}*c^7*d*e^{11}*f^6 + 89374851072*a^{14}*b^{12}*c^8*d*e^{11}*f^6 - 33 \\
& 3226967040*a^{15}*b^{10}*c^9*d*e^{11}*f^6 + 869815812096*a^{16}*b^8*c^{10}*d*e^{11}*f^6 \\
& - 1543847804928*a^{17}*b^6*c^{11}*d*e^{11}*f^6 + 1747313491968*a^{18}*b^4*c^{12}*d*e \\
& ^{11}*f^6 - 1101055131648*a^{19}*b^2*c^{13}*d*e^{11}*f^6) + 191102976000*a^{17}*c^{14} \\
& e^{10}*f^4 + 2851200*a^9*b^{16}*c^6*e^{10}*f^4 - 92568960*a^{10}*b^{14}*c^7*e^{10}*f^4 \\
& + 1312630272*a^{11}*b^{12}*c^8*e^{10}*f^4 - 10611136512*a^{12}*b^{10}*c^9*e^{10}*f^4 + \\
& 53445353472*a^{13}*b^8*c^{10}*e^{10}*f^4 - 171591892992*a^{14}*b^6*c^{11}*e^{10}*f^4 + \\
& 342580396032*a^{15}*b^4*c^{12}*e^{10}*f^4 - 388363714560*a^{16}*b^2*c^{13}*e^{10}*f^4)) \\
& *(- (9*(25*b^{21} + 25*b^6*(-(4*a*c - b^2)^{15})^{(1/2)} + 18923520*a^{10}*b*c^{10} + \\
& 17794*a^2*b^{17}*c^2 - 188095*a^3*b^{15}*c^3 + 1299860*a^4*b^{13}*c^4 - 6126640*a \\
& ^5*b^{11}*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^ \\
& 5*c^8 - 52039680*a^9*b^3*c^9 - 225*a^3*c^3*(-(4*a*c - b^2)^{15})^{(1/2)} - 995* \\
& a*b^{19}*c + 694*a^2*b^2*c^2*(-(4*a*c - b^2)^{15})^{(1/2)} - 245*a*b^4*c*(-(4*a*c \\
& - b^2)^{15})^{(1/2)}))/ (512*(a^7*b^{20}*e^{2}*f^4 + 1048576*a^{17}*c^{10}*e^{2}*f^4 + 72 \\
& 0*a^9*b^{16}*c^2*e^{2}*f^4 - 7680*a^{10}*b^{14}*c^3*e^{2}*f^4 + 53760*a^{11}*b^{12}*c^4*e \\
& ^2*f^4 - 258048*a^{12}*b^{10}*c^5*e^{2}*f^4 + 860160*a^{13}*b^8*c^6*e^{2}*f^4 - 19660 \\
& 80*a^{14}*b^6*c^7*e^{2}*f^4 + 2949120*a^{15}*b^4*c^8*e^{2}*f^4 - 2621440*a^{16}*b^2*c \\
& ^9*e^{2}*f^4 - 40*a^8*b^{18}*c*e^{2}*f^4))^{(1/2)}*2i - ((x^4*(15*b^6*e^3 + 324*a^ \\
& 3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2*e^3 + 12600*a^2*c^4*d^4*e^3 \\
& + 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 3405*a*b^3*c^2*d^2*e^3 + 5880*a^2 \\
& *b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/ (8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^ \\
& 3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*c^2*e^5 + 392*a^2*b*c^3*e^5 + 50 \\
& 40*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 - 3108*a*b^2*c^3*d^2*e^5))/ (8*a*(a \\
& ^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(30*b^6*d^3 + 90*b^5*c*d^5 + 648*a \\
& ^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^2*d^7 + 25*a*b^5*d - 681*a*b^3*c^2* \\
& d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^3*d^7 + 50*a^2*b^2*c^2*d^3 - 194*a^2 \\
& *b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4*c*d^3))/ (4*a*(a^2*b^4 + 16*a^4*c^2 - \\
& 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4*d^3*e^4 + 140*b^4*c^2*d^3*e^4 + 30*b^ \\
& 5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 392*a^2*b*c^3*d*e^4 - 1036*a*b^2*c^3*d^3* \\
& e^4))/ (4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^8*(60*a^2*c^4*e^7 + \\
& 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7))/ (8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2* \\
& c)) + (x^2*(90*b^6*d^2*e + 25*a*b^5*e + 1944*a^3*c^3*d^2*e + 5040*a^2*c^4*d \\
& ^6*e + 420*b^4*c^2*d^6*e - 194*a^2*b^3*c*e + 364*a^3*b*c^2*e + 450*b^5*c*d^ \\
& 4*e - 546*a*b^4*c*d^2*e - 3405*a*b^3*c^2*d^4*e + 5880*a^2*b*c^3*d^4*e - 310 \\
& 8*a*b^2*c^3*d^6*e + 150*a^2*b^2*c^2*d^2*e))/ (8*a*(a^2*b^4 + 16*a^4*c^2 - 8* \\
& a^3*b^2*c)) + (x^3*(15*b^6*d*e^2 + 324*a^3*c^3*d*e^2 + 150*b^5*c*d^3*e^2 + \\
& 2520*a^2*c^4*d^5*e^2 + 210*b^4*c^2*d^5*e^2 - 91*a*b^4*c*d*e^2 + 25*a^2*b^2* \\
& c^2*d*e^2 - 1135*a*b^3*c^2*d^3*e^2 + 1960*a^2*b*c^3*d^3*e^2 - 1554*a*b^2*c^ \\
& 3*d^5*e^2))/ (2*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^7*(60*a^2*c^4 \\
& *d*e^6 + 5*b^4*c^2*d*e^6 - 37*a*b^2*c^3*d*e^6))/ (a*(a^2*b^4 + 16*a^4*c^2 - \\
& 8*a^3*b^2*c)) + (8*a^2*b^4 + 128*a^4*c^2 + 15*b^6*d^4 - 64*a^3*b^2*c + 25*a \\
& *b^5*d^2 + 30*b^5*c*d^6 + 324*a^3*c^3*d^4 + 180*a^2*c^4*d^8 + 15*b^4*c^2*d^ \\
& 8 - 194*a^2*b^3*c*d^2 + 364*a^3*b*c^2*d^2 - 227*a*b^3*c^2*d^6 + 392*a^2*b*c
\end{aligned}$$

$$\frac{d^3 - 111ab^2c^3d^8 + 25a^2b^2c^2d^4 - 91ab^4cd^4}{(8ae(a^2b^4 + 16a^4c^2 - 8a^3b^2c))} \cdot \frac{1}{(x^3(10b^2d^2e^3f^2 + 84c^2d^6e^3f^2 + 2ab^2e^3f^2 + 20ac^2d^2e^3f^2 + 70b^2cd^4e^3f^2) + x^6(84c^2d^3e^6f^2 + 14b^2cd^4e^6f^2) + x^2(10b^2d^3e^2f^2 + 36c^2d^7e^2f^2 + 6ab^2d^2e^2f^2 + 20ac^2d^3e^2f^2 + 42b^2cd^5e^2f^2) + x^4(5b^2d^4e^4f^2 + 126c^2d^5e^4f^2 + 10ac^2d^4e^4f^2 + 70b^2cd^3e^4f^2) + x^7(36c^2d^2e^7f^2 + 2b^2c^2e^7f^2) + x^5(b^2e^5f^2 + 126c^2d^4e^5f^2 + 2ac^2e^5f^2 + 42b^2cd^2e^5f^2) + x(a^2e^5f^2 + 5b^2d^4e^5f^2 + 9c^2d^8e^5f^2 + 6ab^2d^2e^5f^2 + 10ac^2d^4e^5f^2 + 14b^2cd^6e^5f^2) + a^2d^5f^2 + b^2d^5f^2 + c^2d^9f^2 + c^2e^9f^2x^9 + 2ab^2d^3f^2 + 2ac^2d^5f^2 + 2b^2cd^7f^2 + 9c^2d^8e^8f^2x^8)}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)

[Out] Timed out

$$3.547 \quad \int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)^3} dx$$

Optimal. Leaf size=343

$$\frac{3b \log(a + b(d + ex)^2 + c(d + ex)^4)}{4a^4ef^3} - \frac{3b \log(d + ex)}{a^4ef^3} - \frac{3(b^2 - 5ac)(b^2 - 2ac)}{2a^3ef^3(b^2 - 4ac)^2(d + ex)^2} + \frac{20a^2c^2 + 3bc(b^2 - 6ac)}{4a^2ef^3(b^2 - 4ac)^2(d + ex)^2}$$

Rubi [A] time = 0.59, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1142, 1114, 740, 822, 800, 634, 618, 206, 628}

$$\frac{20a^2c^2 + 3bc(b^2 - 6ac)(d + ex)^2 - 20ab^2c + 3b^4}{4a^2ef^3(b^2 - 4ac)^2(d + ex)^2(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{3(30a^2b^2c^2 - 20a^3c^3 - 10ab^4c + b^6) \tanh^{-1}\left(\frac{b + 2c(d + ex)^2}{\sqrt{b^2 - 4ac}}\right)}{2a^4ef^3(b^2 - 4ac)^{3/2}} - \frac{3(b^2 - 5ac)(b^2 - 2ac)}{2a^3ef^3(b^2 - 4ac)^2(d + ex)^2} + \frac{3b \log(a + b(d + ex)^2 + c(d + ex)^4)}{4a^4ef^3} - \frac{3b \log(d + ex)}{a^4ef^3} + \frac{-2ac + b^2 + bc(d + ex)^2}{4ae f^3(b^2 - 4ac)(d + ex)^2(a + b(d + ex)^2 + c(d + ex)^4)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(2*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^4*e*f^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 822

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1114

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dis

$t[1/2, \text{Subst}[\text{Int}[x^{(m-1)/2}(a + b*x + c*x^2)^p, x], x, x^2], x] /;$ FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1142

$\text{Int}[(u_)^{(m_*)}((a_*) + (b_*)*(v_)^2 + (c_*)*(v_)^4)^{(p_*)}, x_Symbol] :> \text{Dist}[u^m/(\text{Coefficient}[v, x, 1]*v^m), \text{Subst}[\text{Int}[x^m*(a + b*x^2 + c*x^{(2*2)})^p, x], x, v], x] /;$ FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^3} dx &= \frac{\text{Subst} \left(\int \frac{1}{x^3(a+bx^2+cx^4)^3} dx, x, d + ex \right)}{ef^3} \\
&= \frac{\text{Subst} \left(\int \frac{1}{x^2(a+bx+cx^2)^3} dx, x, (d + ex)^2 \right)}{2ef^3} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{4a (b^2 - 4ac) ef^3 (d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)^2} \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{4a (b^2 - 4ac) ef^3 (d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)^2} + \dots \\
&= \frac{b^2 - 2ac + bc(d + ex)^2}{4a (b^2 - 4ac) ef^3 (d + ex)^2 (a + b(d + ex)^2 + c(d + ex)^4)^2} + \dots \\
&= -\frac{3(b^2 - 5ac)(b^2 - 2ac)}{2a^3 (b^2 - 4ac)^2 ef^3 (d + ex)^2} + \frac{b^2 - 2ac}{4a (b^2 - 4ac) ef^3 (d + ex)^2} \\
&= -\frac{3(b^2 - 5ac)(b^2 - 2ac)}{2a^3 (b^2 - 4ac)^2 ef^3 (d + ex)^2} + \frac{b^2 - 2ac}{4a (b^2 - 4ac) ef^3 (d + ex)^2} \\
&= -\frac{3(b^2 - 5ac)(b^2 - 2ac)}{2a^3 (b^2 - 4ac)^2 ef^3 (d + ex)^2} + \frac{b^2 - 2ac}{4a (b^2 - 4ac) ef^3 (d + ex)^2} \\
&= -\frac{3(b^2 - 5ac)(b^2 - 2ac)}{2a^3 (b^2 - 4ac)^2 ef^3 (d + ex)^2} + \frac{b^2 - 2ac}{4a (b^2 - 4ac) ef^3 (d + ex)^2}
\end{aligned}$$

Mathematica [A] time = 6.16, size = 509, normalized size = 1.48

$$\frac{3b \log(d + ex)}{4ef^3} - \frac{1}{2a^3(b^2 + 4ac)} - \frac{-3b(b^2 - 2ac)d + ac^2 + b^3 + b^2(d + ex)^2}{4a^2(b^2 + 4ac)(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{-4ab^2b^2 - 2b^2c^2d + ac^2 + 2ab^2b^2 + 2ab^2c^2(d + ex)^2 - ab^2c^2 + ac^2}{4a^2(b^2 + 4ac)(a + b(d + ex)^2 + c(d + ex)^4)} - \frac{3 \left(2b^2c^2 + 3ab^2b^2 + 16a^2b^2c^2\sqrt{4ac} - 10ab^2c + b^2\sqrt{4ac} - ab^2c\sqrt{4ac} + b^2 \right) \log(\sqrt{4ac} - b + 2(d + ex)^2)}{4a^2(b^2 + 4ac)^2} - \frac{3 \left(2b^2c^2 - 3ab^2b^2 + 16a^2b^2c^2\sqrt{4ac} + 10ab^2c + b^2\sqrt{4ac} - ab^2c\sqrt{4ac} - b^2 \right) \log(\sqrt{4ac} + b + 2(d + ex)^2)}{4a^2(b^2 + 4ac)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out]
$$-1/2*1/(a^3*e*f^3*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*f^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*b^2*c^2*(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*f^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3) + (3*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(df + efx)^3 (a + b(d + ex)^2 + c(d + ex)^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out] IntegrateAlgebraic[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3), x]

fricas [B] time = 22.61, size = 15231, normalized size = 44.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out]
$$[-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*e^8*x^8 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*e^7*x^7 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*e^6*x^6 + 6*(56*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d)*e^5*x^5 + 2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + (6*a*b^8 - 60*a^2*b^6*c + 158*a^3*b^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^2)*e^4*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4*c^3 +$$

$$\begin{aligned}
& 38a^3b^2c^4 - 40a^4c^5)d^5 + 15(4ab^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)d^3 + 2(3ab^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)d)e^3x^3 + 2(3ab^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)d^4 + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3 + 168(ab^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^6 + 45(4ab^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)d^4 + 12(3ab^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)d^2)e^2x^2 + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3)d^2 + 2(24(ab^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^7 + 9(4ab^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)d^5 + 4(3ab^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)d^3 + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3)d)e^2x + 3((b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)e^{10}x^{10} + 10(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^2e^9x^9 + (2b^7c - 20ab^5c^2 + 60a^2b^3c^3 - 40a^3b^2c^4 + 45(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^2)e^8x^8 + 8(15(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^3 + 2(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)d)e^7x^7 + (b^8 - 8ab^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4 + 210(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^4 + 56(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)d^2)e^6x^6 + (b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^{10} + 2(126(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^5 + 56(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)d^3 + 3(b^8 - 8ab^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d)e^5x^5 + 2(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)d^8 + (2ab^7 - 20a^2b^5c + 60a^3b^3c^2 - 40a^4b^2c^3 + 210(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^6 + 140(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)d^4 + 15(b^8 - 8ab^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^2)e^4x^4 + (b^8 - 8ab^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^6 + 4(30(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^7 + 28(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)d^5 + 5(b^8 - 8ab^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^3 + 2(ab^7 - 10a^2b^5c + 30a^3b^3c^2 - 20a^4b^2c^3)d)e^3x^3 + 2(ab^7 - 10a^2b^5c + 30a^3b^3c^2 - 20a^4b^2c^3)d^4 + (45(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^8 + a^2b^6 - 10a^3b^4c + 30a^4b^2c^2 - 20a^5c^3 + 56(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)d^6 + 15(b^8 - 8ab^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^4 + 12(a^2b^6 - 10a^3b^4c + 30a^4b^2c^2 - 20a^5c^3)d^2 + 2(5(b^6c^2 - 10ab^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^9 + 8(b^7c - 10ab^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)d^7 + 3(b^8 - 8ab^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4)d^5 + 4(ab^7 - 10a^2b^5c + 30a^3b^3c^2 - 20a^4b^2c^3)d^3 + (a^2b^6 - 10a^3b^4c + 30a^4b^2c^2 - 20a^5c^3)d)e^2x) * sqrt(b^2 - 4ac) * log((2c^2e^4x^4 + 8c^2de^3x^3 + 2c^2d^4 + 2(6c^2d^2 + bc)e^2
\end{aligned}$$

$$\begin{aligned}
& x^2 + 2*b*c*d^2 + 4*(2*c^2*d^3 + b*c*d)*e*x + b^2 - 2*a*c + (2*c*e^2*x^2 + \\
& 4*c*d*e*x + 2*c*d^2 + b)*\text{sqrt}(b^2 - 4*a*c)/(c*e^4*x^4 + 4*c*d*e^3*x^3 + c* \\
& d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) - 3*((b^7 \\
& *c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^{10}*x^{10} + 10*(b^7*c^2 \\
& - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24 \\
& *a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 \\
& + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c \\
& ^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2* \\
& b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + \\
& 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c \\
& ^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^ \\
& 3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3 \\
& *b*c^5)*d^{10} + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c \\
& ^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + \\
& 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)* \\
& e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + \\
& (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - \\
& 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c \\
& ^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2* \\
& b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c \\
& + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 1 \\
& 2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 \\
& + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c \\
& ^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^ \\
& 3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b \\
& ^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6 \\
& 4*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d \\
& ^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(\\
& b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 1 \\
& 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (\\
& a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 \\
& - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c \\
& ^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b \\
& ^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48 \\
& *a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c \\
& ^2 - 64*a^5*b*c^3)*d)*e*x)*\log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 \\
& + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^ \\
& 5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^{10}*x^{10} + 10*(b^7*c^2 - 12*a*b^5*c \\
& ^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 9 \\
& 6*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c \\
& ^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^ \\
& 3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a \\
& ^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^ \\
& 3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b \\
& *c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2
\end{aligned}$$

$$\begin{aligned}
&) * e^6 * x^6 + (b^7 * c^2 - 12 * a * b^5 * c^3 + 48 * a^2 * b^3 * c^4 - 64 * a^3 * b * c^5) * d^{10} + \\
& 2 * (126 * (b^7 * c^2 - 12 * a * b^5 * c^3 + 48 * a^2 * b^3 * c^4 - 64 * a^3 * b * c^5) * d^5 + 56 * (\\
& b^8 * c - 12 * a * b^6 * c^2 + 48 * a^2 * b^4 * c^3 - 64 * a^3 * b^2 * c^4) * d^3 + 3 * (b^9 - 10 * a \\
& * b^7 * c + 24 * a^2 * b^5 * c^2 + 32 * a^3 * b^3 * c^3 - 128 * a^4 * b * c^4) * d) * e^5 * x^5 + 2 * (b \\
& ^8 * c - 12 * a * b^6 * c^2 + 48 * a^2 * b^4 * c^3 - 64 * a^3 * b^2 * c^4) * d^8 + (2 * a * b^8 - 24 * \\
& a^2 * b^6 * c + 96 * a^3 * b^4 * c^2 - 128 * a^4 * b^2 * c^3 + 210 * (b^7 * c^2 - 12 * a * b^5 * c^3 \\
& + 48 * a^2 * b^3 * c^4 - 64 * a^3 * b * c^5) * d^6 + 140 * (b^8 * c - 12 * a * b^6 * c^2 + 48 * a^2 * b \\
& ^4 * c^3 - 64 * a^3 * b^2 * c^4) * d^4 + 15 * (b^9 - 10 * a * b^7 * c + 24 * a^2 * b^5 * c^2 + 32 * a \\
& ^3 * b^3 * c^3 - 128 * a^4 * b * c^4) * d^2) * e^4 * x^4 + (b^9 - 10 * a * b^7 * c + 24 * a^2 * b^5 * c \\
& ^2 + 32 * a^3 * b^3 * c^3 - 128 * a^4 * b * c^4) * d^6 + 4 * (30 * (b^7 * c^2 - 12 * a * b^5 * c^3 + \\
& 48 * a^2 * b^3 * c^4 - 64 * a^3 * b * c^5) * d^7 + 28 * (b^8 * c - 12 * a * b^6 * c^2 + 48 * a^2 * b^4 * \\
& c^3 - 64 * a^3 * b^2 * c^4) * d^5 + 5 * (b^9 - 10 * a * b^7 * c + 24 * a^2 * b^5 * c^2 + 32 * a^3 * b \\
& ^3 * c^3 - 128 * a^4 * b * c^4) * d^3 + 2 * (a * b^8 - 12 * a^2 * b^6 * c + 48 * a^3 * b^4 * c^2 - 64 \\
& * a^4 * b^2 * c^3) * d) * e^3 * x^3 + 2 * (a * b^8 - 12 * a^2 * b^6 * c + 48 * a^3 * b^4 * c^2 - 64 * a^ \\
& 4 * b^2 * c^3) * d^4 + (a^2 * b^7 - 12 * a^3 * b^5 * c + 48 * a^4 * b^3 * c^2 - 64 * a^5 * b * c^3 + \\
& 45 * (b^7 * c^2 - 12 * a * b^5 * c^3 + 48 * a^2 * b^3 * c^4 - 64 * a^3 * b * c^5) * d^8 + 56 * (b^8 * c \\
& - 12 * a * b^6 * c^2 + 48 * a^2 * b^4 * c^3 - 64 * a^3 * b^2 * c^4) * d^6 + 15 * (b^9 - 10 * a * b^7 \\
& * c + 24 * a^2 * b^5 * c^2 + 32 * a^3 * b^3 * c^3 - 128 * a^4 * b * c^4) * d^4 + 12 * (a * b^8 - 12 * \\
& a^2 * b^6 * c + 48 * a^3 * b^4 * c^2 - 64 * a^4 * b^2 * c^3) * d^2) * e^2 * x^2 + (a^2 * b^7 - 12 * a \\
& ^3 * b^5 * c + 48 * a^4 * b^3 * c^2 - 64 * a^5 * b * c^3) * d^2 + 2 * (5 * (b^7 * c^2 - 12 * a * b^5 * c \\
& ^3 + 48 * a^2 * b^3 * c^4 - 64 * a^3 * b * c^5) * d^9 + 8 * (b^8 * c - 12 * a * b^6 * c^2 + 48 * a^2 * b \\
& ^4 * c^3 - 64 * a^3 * b^2 * c^4) * d^7 + 3 * (b^9 - 10 * a * b^7 * c + 24 * a^2 * b^5 * c^2 + 32 * a^ \\
& 3 * b^3 * c^3 - 128 * a^4 * b * c^4) * d^5 + 4 * (a * b^8 - 12 * a^2 * b^6 * c + 48 * a^3 * b^4 * c^2 - \\
& 64 * a^4 * b^2 * c^3) * d^3 + (a^2 * b^7 - 12 * a^3 * b^5 * c + 48 * a^4 * b^3 * c^2 - 64 * a^5 * b * \\
& c^3) * d) * e * x) * \log(e * x + d) / ((a^4 * b^6 * c^2 - 12 * a^5 * b^4 * c^3 + 48 * a^6 * b^2 * c^4 \\
& - 64 * a^7 * c^5) * e^{11} * f^3 * x^{10} + 10 * (a^4 * b^6 * c^2 - 12 * a^5 * b^4 * c^3 + 48 * a^6 * b^2 \\
& * c^4 - 64 * a^7 * c^5) * d * e^{10} * f^3 * x^9 + (2 * a^4 * b^7 * c - 24 * a^5 * b^5 * c^2 + 96 * a^6 * \\
& b^3 * c^3 - 128 * a^7 * b * c^4 + 45 * (a^4 * b^6 * c^2 - 12 * a^5 * b^4 * c^3 + 48 * a^6 * b^2 * c^4 \\
& - 64 * a^7 * c^5) * d^2) * e^9 * f^3 * x^8 + 8 * (15 * (a^4 * b^6 * c^2 - 12 * a^5 * b^4 * c^3 + 48 * \\
& a^6 * b^2 * c^4 - 64 * a^7 * c^5) * d^3 + 2 * (a^4 * b^7 * c - 12 * a^5 * b^5 * c^2 + 48 * a^6 * b^3 * \\
& c^3 - 64 * a^7 * b * c^4) * d) * e^8 * f^3 * x^7 + (a^4 * b^8 - 10 * a^5 * b^6 * c + 24 * a^6 * b^4 * c \\
& ^2 + 32 * a^7 * b^2 * c^3 - 128 * a^8 * c^4 + 210 * (a^4 * b^6 * c^2 - 12 * a^5 * b^4 * c^3 + 48 * \\
& a^6 * b^2 * c^4 - 64 * a^7 * c^5) * d^4 + 56 * (a^4 * b^7 * c - 12 * a^5 * b^5 * c^2 + 48 * a^6 * b^3 \\
& * c^3 - 64 * a^7 * b * c^4) * d^2) * e^7 * f^3 * x^6 + 2 * (126 * (a^4 * b^6 * c^2 - 12 * a^5 * b^4 * c^ \\
& 3 + 48 * a^6 * b^2 * c^4 - 64 * a^7 * c^5) * d^5 + 56 * (a^4 * b^7 * c - 12 * a^5 * b^5 * c^2 + 48 * \\
& a^6 * b^3 * c^3 - 64 * a^7 * b * c^4) * d^3 + 3 * (a^4 * b^8 - 10 * a^5 * b^6 * c + 24 * a^6 * b^4 * c^ \\
& 2 + 32 * a^7 * b^2 * c^3 - 128 * a^8 * c^4) * d) * e^6 * f^3 * x^5 + (2 * a^5 * b^7 - 24 * a^6 * b^5 * \\
& c + 96 * a^7 * b^3 * c^2 - 128 * a^8 * b * c^3 + 210 * (a^4 * b^6 * c^2 - 12 * a^5 * b^4 * c^3 + 48 \\
& * a^6 * b^2 * c^4 - 64 * a^7 * c^5) * d^6 + 140 * (a^4 * b^7 * c - 12 * a^5 * b^5 * c^2 + 48 * a^6 * b \\
& ^3 * c^3 - 64 * a^7 * b * c^4) * d^4 + 15 * (a^4 * b^8 - 10 * a^5 * b^6 * c + 24 * a^6 * b^4 * c^2 + \\
& 32 * a^7 * b^2 * c^3 - 128 * a^8 * c^4) * d^2) * e^5 * f^3 * x^4 + 4 * (30 * (a^4 * b^6 * c^2 - 12 * a^ \\
& 5 * b^4 * c^3 + 48 * a^6 * b^2 * c^4 - 64 * a^7 * c^5) * d^7 + 28 * (a^4 * b^7 * c - 12 * a^5 * b^5 * c \\
& ^2 + 48 * a^6 * b^3 * c^3 - 64 * a^7 * b * c^4) * d^5 + 5 * (a^4 * b^8 - 10 * a^5 * b^6 * c + 24 * a^ \\
& 6 * b^4 * c^2 + 32 * a^7 * b^2 * c^3 - 128 * a^8 * c^4) * d^3 + 2 * (a^5 * b^7 - 12 * a^6 * b^5 * c + \\
& 48 * a^7 * b^3 * c^2 - 64 * a^8 * b * c^3) * d) * e^4 * f^3 * x^3 + (a^6 * b^6 - 12 * a^7 * b^4 * c +
\end{aligned}$$

$$\begin{aligned}
& 48a^8b^2c^2 - 64a^9c^3 + 45(a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5)d^8 + 56(a^4b^7c - 12a^5b^5c^2 + 48a^6b^3c^3 - \\
& 64a^7b^2c^4)d^6 + 15(a^4b^8 - 10a^5b^6c + 24a^6b^4c^2 + 32a^7b^2c^3 - 128a^8c^4)d^4 + 12(a^5b^7 - 12a^6b^5c + 48a^7b^3c^2 - 64 \\
& a^8b^2c^3)d^2)e^3f^3x^2 + 2(5(a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5)d^9 + 8(a^4b^7c - 12a^5b^5c^2 + 48a^6b^3c^3 \\
& - 64a^7b^2c^4)d^7 + 3(a^4b^8 - 10a^5b^6c + 24a^6b^4c^2 + 32a^7b^2c^3 - 128a^8c^4)d^5 + 4(a^5b^7 - 12a^6b^5c + 48a^7b^3c^2 - 64 \\
& a^8b^2c^3)d^3 + (a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)d)e^2f^3x + ((a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5)d \\
& d^{10} + 2(a^4b^7c - 12a^5b^5c^2 + 48a^6b^3c^3 - 64a^7b^2c^4)d^8 + (a^4b^8 - 10a^5b^6c + 24a^6b^4c^2 + 32a^7b^2c^3 - 128a^8c^4)d \\
& ^6 + 2(a^5b^7 - 12a^6b^5c + 48a^7b^3c^2 - 64a^8b^2c^3)d^4 + (a^6b^6 - 12a^7b^4c + 48a^8b^2c^2 - 64a^9c^3)d^2)e^2f^3), -1/4(6(a^6b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)e^8x^8 + 48(a^6b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^2)e^7x^7 + 3(4a^6b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4 + 56(a^6b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^2)e^6x^6 + 6(56(a^6b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^3 + 3(4a^6b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)d^2)e^5x^5 + 2a^3b^6 - 24a^4b^4c + 96a^5b^2c^2 - 128a^6c^3 + 6(a^6b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^8 + (6a^6b^8 - 60a^2b^6c + 158a^3b^4c^2 + 44a^4b^2c^3 - 400a^5c^4 + 420(a^6b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^4 + 45(4a^6b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)d^2)e^4x^4 + 3(4a^6b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)d^6 + 4(84(a^6b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^5 + 15(4a^6b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)d^3 + 2(3a^6b^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)d^2)e^3x^3 + 2(3a^6b^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)d^4 + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3 + 168(a^6b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^6 + 45(4a^6b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)d^4 + 12(3a^6b^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)d^2)e^2x^2 + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3)d^2 + 2(24(a^6b^6c^2 - 11a^2b^4c^3 + 38a^3b^2c^4 - 40a^4c^5)d^7 + 9(4a^6b^7c - 45a^2b^5c^2 + 162a^3b^3c^3 - 184a^4b^2c^4)d^5 + 4(3a^6b^8 - 30a^2b^6c + 79a^3b^4c^2 + 22a^4b^2c^3 - 200a^5c^4)d^3 + (9a^2b^7 - 104a^3b^5c + 394a^4b^3c^2 - 488a^5b^2c^3)d)e^2x + 6((b^6c^2 - 10a^2b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)e^10x^10 + 10(b^6c^2 - 10a^2b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^2)e^9x^9 + (2b^7c - 20a^2b^5c^2 + 60a^2b^3c^3 - 40a^3b^2c^4 + 45(b^6c^2 - 10a^2b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^2)e^8x^8 + 8(15(b^6c^2 - 10a^2b^4c^3 + 30a^2b^2c^4 - 20a^3c^5)d^3 + 2(b^7c - 10a^2b^5c^2 + 30a^2b^3c^3 - 20a^3b^2c^4)d^2)e^7x^7 + (b^8 - 8a^2b^6c + 10a^2b^4c^2 + 40a^3b^2c^3 - 40a^4c^4 + 210(b^6c^2 - 10a^2b^4c^3 +
\end{aligned}$$

$$\begin{aligned}
& *c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*log(c*e^4*x^4 + 4*c*d*e^3*x^3 + c*d^4 + (6*c*d^2 + b)*e^2*x^2 + b*d^2 + 2*(2*c*d^3 + b*d)*e*x + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*e^10*x^10 + 10*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*e^9*x^9 + (2*b^8*c - 24*a*b^6*c^2 + 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^2)*e^8*x^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d)*e^7*x^7 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^4 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^2)*e^6*x^6 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + 2*(126*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^5 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^3 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d)*e^5*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^8 + (2*a*b^8 - 24*a^2*b^6*c + 96*a^3*b^4*c^2 - 128*a^4*b^2*c^3 + 210*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^6 + 140*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^4 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^2)*e^4*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^6 + 4*(30*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^7 + 28*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^5 + 5*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d)*e^3*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^8 + 56*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^6 + 15*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^4 + 12*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^2)*e^2*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d^2 + 2*(5*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^9 + 8*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*d^7 + 3*(b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 3
\end{aligned}$$

$$\begin{aligned}
& 2*a^3*b^3*c^3 - 128*a^4*b*c^4)*d^5 + 4*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*d^3 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*d)*e*x)*\log(e*x + d))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*e^{11*f^3*x^{10}} + 10*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d*e^{10*f^3*x^9} + (2*a^4*b^7*c - 24*a^5*b^5*c^2 + 96*a^6*b^3*c^3 - 128*a^7*b*c^4 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^2)*e^{9*f^3*x^8} + 8*(15*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^3 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d)*e^{8*f^3*x^7} + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^4 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^2)*e^{7*f^3*x^6} + 2*(126*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^5 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^3 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d)*e^{6*f^3*x^5} + (2*a^5*b^7 - 24*a^6*b^5*c + 96*a^7*b^3*c^2 - 128*a^8*b*c^3 + 210*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^6 + 140*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^4 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^2)*e^{5*f^3*x^4} + 4*(30*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^7 + 28*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^5 + 5*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d)*e^{4*f^3*x^3} + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + 45*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^8 + 56*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^6 + 15*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^4 + 12*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^2)*e^{3*f^3*x^2} + 2*(5*(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^9 + 8*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^7 + 3*(a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^5 + 4*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^3 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d)*e^{2*f^3*x} + ((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*d^{10} + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*d^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*d^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*d^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^2)*e*f^3)]
\end{aligned}$$

giac [B] time = 1.63, size = 1735, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")

```
[Out] 3/4*((a^4*b^8*c*f^3*e^3 - 14*a^5*b^6*c^2*f^3*e^3 + 70*a^6*b^4*c^3*f^3*e^3 -
140*a^7*b^2*c^4*f^3*e^3 + 80*a^8*c^5*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*
x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c))*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e
+ b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^4*b^8*c*f^3*e^3 - 14*a^5*b^6*
c^2*f^3*e^3 + 70*a^6*b^4*c^3*f^3*e^3 - 140*a^7*b^2*c^4*f^3*e^3 + 80*a^8*c^5
*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a
*c))*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2
*a)))/(a^8*b^8*c*f^6*e^4 - 16*a^9*b^6*c^2*f^6*e^4 + 96*a^10*b^4*c^3*f^6*e^4
- 256*a^11*b^2*c^4*f^6*e^4 + 256*a^12*c^5*f^6*e^4) - 1/4*(6*b^4*c^2*x^8*e^
8 - 42*a*b^2*c^3*x^8*e^8 + 60*a^2*c^4*x^8*e^8 + 48*b^4*c^2*d*x^7*e^7 - 336*
a*b^2*c^3*d*x^7*e^7 + 480*a^2*c^4*d*x^7*e^7 + 168*b^4*c^2*d^2*x^6*e^6 - 117
6*a*b^2*c^3*d^2*x^6*e^6 + 1680*a^2*c^4*d^2*x^6*e^6 + 336*b^4*c^2*d^3*x^5*e^
5 - 2352*a*b^2*c^3*d^3*x^5*e^5 + 3360*a^2*c^4*d^3*x^5*e^5 + 420*b^4*c^2*d^4
*x^4*e^4 - 2940*a*b^2*c^3*d^4*x^4*e^4 + 4200*a^2*c^4*d^4*x^4*e^4 + 336*b^4*c
^2*d^5*x^3*e^3 - 2352*a*b^2*c^3*d^5*x^3*e^3 + 3360*a^2*c^4*d^5*x^3*e^3 + 1
68*b^4*c^2*d^6*x^2*e^2 - 1176*a*b^2*c^3*d^6*x^2*e^2 + 1680*a^2*c^4*d^6*x^2*
e^2 + 48*b^4*c^2*d^7*x*e - 336*a*b^2*c^3*d^7*x*e + 480*a^2*c^4*d^7*x*e + 6*
b^4*c^2*d^8 - 42*a*b^2*c^3*d^8 + 60*a^2*c^4*d^8 + 12*b^5*c*x^6*e^6 - 87*a*b
^3*c^2*x^6*e^6 + 138*a^2*b*c^3*x^6*e^6 + 72*b^5*c*d*x^5*e^5 - 522*a*b^3*c^2
*d*x^5*e^5 + 828*a^2*b*c^3*d*x^5*e^5 + 180*b^5*c*d^2*x^4*e^4 - 1305*a*b^3*c
^2*d^2*x^4*e^4 + 2070*a^2*b*c^3*d^2*x^4*e^4 + 240*b^5*c*d^3*x^3*e^3 - 1740*
a*b^3*c^2*d^3*x^3*e^3 + 2760*a^2*b*c^3*d^3*x^3*e^3 + 180*b^5*c*d^4*x^2*e^2
- 1305*a*b^3*c^2*d^4*x^2*e^2 + 2070*a^2*b*c^3*d^4*x^2*e^2 + 72*b^5*c*d^5*x*
e - 522*a*b^3*c^2*d^5*x*e + 828*a^2*b*c^3*d^5*x*e + 12*b^5*c*d^6 - 87*a*b^3
*c^2*d^6 + 138*a^2*b*c^3*d^6 + 6*b^6*x^4*e^4 - 36*a*b^4*c*x^4*e^4 + 14*a^2*
b^2*c^2*x^4*e^4 + 100*a^3*c^3*x^4*e^4 + 24*b^6*d*x^3*e^3 - 144*a*b^4*c*d*x^
3*e^3 + 56*a^2*b^2*c^2*d*x^3*e^3 + 400*a^3*c^3*d*x^3*e^3 + 36*b^6*d^2*x^2*
e^2 - 216*a*b^4*c*d^2*x^2*e^2 + 84*a^2*b^2*c^2*d^2*x^2*e^2 + 600*a^3*c^3*d^2
*x^2*e^2 + 24*b^6*d^3*x*e - 144*a*b^4*c*d^3*x*e + 56*a^2*b^2*c^2*d^3*x*e +
400*a^3*c^3*d^3*x*e + 6*b^6*d^4 - 36*a*b^4*c*d^4 + 14*a^2*b^2*c^2*d^4 + 100
*a^3*c^3*d^4 + 9*a*b^5*x^2*e^2 - 68*a^2*b^3*c*x^2*e^2 + 122*a^3*b*c^2*x^2*
e^2 + 18*a*b^5*d*x*e - 136*a^2*b^3*c*d*x*e + 244*a^3*b*c^2*d*x*e + 9*a*b^5*d
^2 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a
^4*c^2)/(a^3*b^4*f^3*e - 8*a^4*b^2*c*f^3*e + 16*a^5*c^2*f^3*e)*(c*x^5*e^5
+ 5*c*d*x^4*e^4 + 10*c*d^2*x^3*e^3 + 10*c*d^3*x^2*e^2 + 5*c*d^4*x*e + c*d^5
+ b*x^3*e^3 + 3*b*d*x^2*e^2 + 3*b*d^2*x*e + b*d^3 + a*x*e + a*d)^2) + 3/4*
b*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e
+ c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^4*f^3) - 3*b*e^(-1)*log(ab
s(x*e + d))/(a^4*f^3)
```

maple [C] time = 0.08, size = 5737, normalized size = 16.73

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x)$

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, \text{algorithm}=\text{"maxima"})$

[Out] Timed out

mupad [B] time = 24.91, size = 25334, normalized size = 73.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)$

[Out]
$$\left(\log\left(\frac{(27*c^5*e^{16}*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3}{(a^9*f^9*(4*a*c - b^2)^6) - ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^{(1/2)})*((9*c^3*e^{15}*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4) - ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^{(1/2)})*((6*c^2*e^{16}*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^{16}*(3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^{(1/2)})*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^{18}*x^2*(b^6 + 10*0*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2) + (12*c^3*d*e^{17}*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2)}\right)/(4*a^4*e*f^3) + (9*b*c^4*e^{17}*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4) + (18*b*c^4*d*e^{16}*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4))/(4*a^4*e*f^3) + (27*c^4*e^{14}*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c - b^2)^6) + (54*c^5*d*e^{15}*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6))*((27*c^5*e^{16}*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6) - ((3*b + 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^{(1/2)})*((9*c^3*e^{15}*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b$$

$$\begin{aligned}
& \left(3c^2d^2 + 90a^2b^3c^3d^2 \right) / (a^6f^6(4ac - b^2)^4) - \left((3b + 3a^4e \right. \\
& \left. * f^3(-b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c)^2 / (a^8e^2f^6(4a \right. \\
& \left. * c - b^2)^5) \right)^{(1/2)} * \left((6c^2e^{16}(2b^7 - 20a^3b^3c^3 + b^6cd^2 + 46a^ \right. \\
& \left. 2b^3c^2 + 100a^3c^4d^2 - 18ab^5c - 2ab^4c^2d^2 - 30a^2b^2c^3 \right. \\
& \left. * d^2) \right) / (a^3f^3(4ac - b^2)^2) + (b^2e^{16}(3b + 3a^4e * f^3(-b^6 - \right. \\
& \left. 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c)^2 / (a^8e^2f^6(4ac - b^2)^5) \right)^{(1/2)} \\
& * (ab + 3b^2d^2 + 3b^2e^2x^2 - 10acd^2 + 6b^2d * e * x - 10ac \right. \\
& \left. * e^2x^2 - 20ac * d * e * x) \right) / (a^4f^3) + (6c^3e^{18}x^2(b^6 + 100a^3c^3 - \right. \\
& \left. 30a^2b^2c^2 - 2ab^4c) \right) / (a^3f^3(4ac - b^2)^2) + (12c^3d * e^{17} * x * \left(\right. \\
& \left. b^6 + 100a^3c^3 - 30a^2b^2c^2 - 2ab^4c) \right) / (a^3f^3(4ac - b^2)^2) \right) \\
& \left. \right) / (4a^4e * f^3) + (9 * b^4c^4e^{17} * x^2 * (6b^8 + 900a^4c^4 + 479a^2b^4c^2 \right. \\
& \left. - 1100a^3b^2c^3 - 89ab^6c) \right) / (a^6f^6(4ac - b^2)^4) + (18 * b^4c^4d * e \right. \\
& \left. ^{16} * x * (6b^8 + 900a^4c^4 + 479a^2b^4c^2 - 1100a^3b^2c^3 - 89ab^6c) \right) / \left(a^6f^6(4ac - b^2)^4 \right) \right) / (4a^4e * f^3) + (27 * c^4e^{14} * (b^4 + 10a^2 * \right. \\
& \left. c^2 - 7ab^2c) \right)^2 * (b^5 + 16a^2b^3c^2 + b^4cd^2 + 10a^2c^3d^2 - 8ab \right. \\
& \left. ^3c - 7ab^2c^2d^2) \right) / (a^9f^9(4ac - b^2)^6) + (54 * c^5d * e^{15} * x * (b^4 \right. \\
& \left. + 10a^2c^2 - 7ab^2c) \right)^3 / (a^9f^9(4ac - b^2)^6) \right) * (6b^{11} * e * f^3 - 12 \right. \\
& \left. 0 * ab^9 * c * e * f^3 - 6144 * a^5 * b * c^5 * e * f^3 + 960 * a^2 * b^7 * c^2 * e * f^3 - 3840 * a^3 * b \right. \\
& \left. ^5 * c^3 * e * f^3 + 7680 * a^4 * b^3 * c^4 * e * f^3) \right) / (2 * (4a^4 * b^{10} * e^2 * f^6 - 4096 * a^9 * c \right. \\
& \left. ^5 * e^2 * f^6 + 640 * a^6 * b^6 * c^2 * e^2 * f^6 - 2560 * a^7 * b^4 * c^3 * e^2 * f^6 + 5120 * a^8 * \right. \\
& \left. b^2 * c^4 * e^2 * f^6 - 80 * a^5 * b^8 * c * e^2 * f^6) - ((x^4 * (6b^6 * e^3 + 100a^3 * c^3 * e \right. \\
& \left. ^3 + 180 * b^5 * c * d^2 * e^3 + 14a^2 * b^2 * c^2 * e^3 + 4200 * a^2 * c^4 * d^4 * e^3 + 420 * b \right. \\
& \left. ^4 * c^2 * d^4 * e^3 - 36 * a * b^4 * c * e^3 - 1305 * a * b^3 * c^2 * d^2 * e^3 + 2070 * a^2 * b * c^3 * d \right. \\
& \left. ^2 * e^3 - 2940 * a * b^2 * c^3 * d^4 * e^3) \right) / (4 * (a^3 * b^4 + 16a^5 * c^2 - 8a^4 * b^2 * c)) + \left(\right. \\
& \left. 3 * x^6 * (4b^5 * c * e^5 - 29 * a * b^3 * c^2 * e^5 + 46 * a^2 * b * c^3 * e^5 + 560 * a^2 * c^4 * d \right. \\
& \left. ^2 * e^5 + 56 * b^4 * c^2 * d^2 * e^5 - 392 * a * b^2 * c^3 * d^2 * e^5) \right) / (4 * (a^3 * b^4 + 16a^5 * c \right. \\
& \left. ^2 - 8a^4 * b^2 * c)) + (x * (12 * b^6 * d^3 + 36 * b^5 * c * d^5 + 200 * a^3 * c^3 * d^3 + 240 * \right. \\
& \left. a^2 * c^4 * d^7 + 24 * b^4 * c^2 * d^7 + 9 * a * b^5 * d - 261 * a * b^3 * c^2 * d^5 + 414 * a^2 * b * c \right. \\
& \left. ^3 * d^5 - 168 * a * b^2 * c^3 * d^7 + 28 * a^2 * b^2 * c^2 * d^3 - 68 * a^2 * b^3 * c * d + 122 * a^3 * b \right. \\
& \left. * c^2 * d - 72 * a * b^4 * c * d^3) \right) / (2 * (a^3 * b^4 + 16a^5 * c^2 - 8a^4 * b^2 * c)) + (3 * x^5 \right. \\
& \left. * (560 * a^2 * c^4 * d^3 * e^4 + 56 * b^4 * c^2 * d^3 * e^4 + 12 * b^5 * c * d * e^4 - 87 * a * b^3 * c^2 * \right. \\
& \left. d * e^4 + 138 * a^2 * b * c^3 * d * e^4 - 392 * a * b^2 * c^3 * d^3 * e^4) \right) / (2 * (a^3 * b^4 + 16a^5 * \right. \\
& \left. c^2 - 8a^4 * b^2 * c)) + (3 * x^8 * (10 * a^2 * c^4 * e^7 + b^4 * c^2 * e^7 - 7 * a * b^2 * c^3 * e \right. \\
& \left. ^7) \right) / (2 * (a^3 * b^4 + 16a^5 * c^2 - 8a^4 * b^2 * c)) + (x^2 * (36 * b^6 * d^2 * e + 9 * a * b^5 \right. \\
& \left. * e + 600 * a^3 * c^3 * d^2 * e + 1680 * a^2 * c^4 * d^6 * e + 168 * b^4 * c^2 * d^6 * e - 68 * a^2 * b \right. \\
& \left. ^3 * c * e + 122 * a^3 * b * c^2 * e + 180 * b^5 * c * d^4 * e - 216 * a * b^4 * c * d^2 * e - 1305 * a * b^3 * \right. \\
& \left. c^2 * d^4 * e + 2070 * a^2 * b * c^3 * d^4 * e - 1176 * a * b^2 * c^3 * d^6 * e + 84 * a^2 * b^2 * c^2 * d \right. \\
& \left. ^2 * e) \right) / (4 * (a^3 * b^4 + 16a^5 * c^2 - 8a^4 * b^2 * c)) + (x^3 * (6 * b^6 * d * e^2 + 100 * a \right. \\
& \left. ^3 * c^3 * d * e^2 + 60 * b^5 * c * d^3 * e^2 + 840 * a^2 * c^4 * d^5 * e^2 + 84 * b^4 * c^2 * d^5 * e^2 - \right. \\
& \left. 36 * a * b^4 * c * d * e^2 + 14 * a^2 * b^2 * c^2 * d * e^2 - 435 * a * b^3 * c^2 * d^3 * e^2 + 690 * a^2 * \right. \\
& \left. b * c^3 * d^3 * e^2 - 588 * a * b^2 * c^3 * d^5 * e^2) \right) / (a^3 * b^4 + 16a^5 * c^2 - 8a^4 * b^2 * c \right. \\
& \left.) + (12 * x^7 * (10 * a^2 * c^4 * d * e^6 + b^4 * c^2 * d * e^6 - 7 * a * b^2 * c^3 * d * e^6) \right) / (a^3 * b \right. \\
& \left. ^4 + 16a^5 * c^2 - 8a^4 * b^2 * c) + (2 * a^2 * b^4 + 32 * a^4 * c^2 + 6 * b^6 * d^4 - 16 * a \right. \\
& \left. ^3 * b^2 * c + 9 * a * b^5 * d^2 + 12 * b^5 * c * d^6 + 100 * a^3 * c^3 * d^4 + 60 * a^2 * c^4 * d^8 + 6 \right. \\
& \left. * b^4 * c^2 * d^8 - 68 * a^2 * b^3 * c * d^2 + 122 * a^3 * b * c^2 * d^2 - 87 * a * b^3 * c^2 * d^6 + 13 \right.
\end{aligned}$$

$$\begin{aligned}
& 8*a^2*b*c^3*d^6 - 42*a*b^2*c^3*d^8 + 14*a^2*b^2*c^2*d^4 - 36*a*b^4*c*d^4)/ \\
& (4*e*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(x^4*(15*b^2*d^2*e^4*f^3 + 210*c \\
& ^2*d^6*e^4*f^3 + 2*a*b*e^4*f^3 + 30*a*c*d^2*e^4*f^3 + 140*b*c*d^4*e^4*f^3) \\
& + x^7*(120*c^2*d^3*e^7*f^3 + 16*b*c*d*e^7*f^3) + x*(6*b^2*d^5*e*f^3 + 10*c^ \\
& 2*d^9*e*f^3 + 2*a^2*d*e*f^3 + 8*a*b*d^3*e*f^3 + 12*a*c*d^5*e*f^3 + 16*b*c*d \\
& ^7*e*f^3) + x^3*(20*b^2*d^3*e^3*f^3 + 120*c^2*d^7*e^3*f^3 + 8*a*b*d*e^3*f^3 \\
& + 40*a*c*d^3*e^3*f^3 + 112*b*c*d^5*e^3*f^3) + x^2*(a^2*e^2*f^3 + 15*b^2*d^ \\
& 4*e^2*f^3 + 45*c^2*d^8*e^2*f^3 + 12*a*b*d^2*e^2*f^3 + 30*a*c*d^4*e^2*f^3 + \\
& 56*b*c*d^6*e^2*f^3) + x^5*(6*b^2*d*e^5*f^3 + 252*c^2*d^5*e^5*f^3 + 12*a*c*d \\
& *e^5*f^3 + 112*b*c*d^3*e^5*f^3) + x^8*(45*c^2*d^2*e^8*f^3 + 2*b*c*e^8*f^3) \\
& + x^6*(b^2*e^6*f^3 + 210*c^2*d^4*e^6*f^3 + 2*a*c*e^6*f^3 + 56*b*c*d^2*e^6*f \\
& ^3) + a^2*d^2*f^3 + b^2*d^6*f^3 + c^2*d^10*f^3 + c^2*e^10*f^3*x^10 + 2*a*b* \\
& d^4*f^3 + 2*a*c*d^6*f^3 + 2*b*c*d^8*f^3 + 10*c^2*d*e^9*f^3*x^9) - (3*b*log(\\
& d + e*x))/(a^4*e*f^3) + (3*atan((x^2*(((54*a^3*b^13*c^4*e^17*f^3 - 1233* \\
& a^4*b^11*c^5*e^17*f^3 + 11583*a^5*b^9*c^6*e^17*f^3 - 57204*a^6*b^7*c^7*e^17 \\
& *f^3 + 156276*a^7*b^5*c^8*e^17*f^3 - 223200*a^8*b^3*c^9*e^17*f^3 + 129600*a \\
& ^9*b*c^10*e^17*f^3)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 \\
& + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 61 \\
& 44*a^14*b^2*c^5*f^9) - (((153600*a^13*c^10*e^18*f^6 + 6*a^6*b^14*c^3*e^18*f \\
& ^6 - 108*a^7*b^12*c^4*e^18*f^6 + 588*a^8*b^10*c^5*e^18*f^6 + 792*a^9*b^8*c^ \\
& 6*e^18*f^6 - 22272*a^10*b^6*c^7*e^18*f^6 + 100608*a^11*b^4*c^8*e^18*f^6 - 1 \\
& 99680*a^12*b^2*c^9*e^18*f^6)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^ \\
& 10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4 \\
& *f^9 - 6144*a^14*b^2*c^5*f^9) + ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a \\
& ^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4* \\
& b^3*c^4*e*f^3)*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^10*b^13*c^3*e^19*f^9 + 384 \\
& 0*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^19*f^9 + 97280*a^13*b^7*c^6 \\
& *e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912*a^15*b^3*c^8*e^19*f^9 - 1 \\
& 63840*a^16*b*c^9*e^19*f^9))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + \\
& 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2* \\
& f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^1 \\
& 0*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4* \\
& f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5 \\
& *b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^ \\
& 3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c \\
& ^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b \\
& ^8*c*e^2*f^6)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + \\
& 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(\\
& 2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 25 \\
& 60*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) \\
& - (27000*a^6*c^11*e^16 + 27*b^12*c^5*e^16 - 567*a*b^10*c^6*e^16 + 4779*a^2* \\
& b^8*c^7*e^16 - 20601*a^3*b^6*c^8*e^16 + 47790*a^4*b^4*c^9*e^16 - 56700*a^5* \\
& b^2*c^10*e^16)/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240 \\
& *a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^ \\
& 14*b^2*c^5*f^9) + (3*((3*((153600*a^13*c^10*e^18*f^6 + 6*a^6*b^14*c^3*e^18*
\end{aligned}$$

$$\begin{aligned}
& f^6 - 108a^7b^{12}c^4e^{18}f^6 + 588a^8b^{10}c^5e^{18}f^6 + 792a^9b^8c^6e^{18}f^6 - 22272a^{10}b^6c^7e^{18}f^6 + 100608a^{11}b^4c^8e^{18}f^6 - \\
& 199680a^{12}b^2c^9e^{18}f^6)/(a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^4f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - \\
& 6144a^{14}b^2c^5f^9) + ((6b^{11}e^3f^3 - 120ab^9c^3e^3f^3 - 6144a^5b^3c^4e^3f^3 + 960a^2b^7c^2e^3f^3 - 3840a^3b^5c^3e^3f^3 + 7680a^4 \\
& b^3c^4e^3f^3)*(12a^9b^{15}c^2e^{19}f^9 - 328a^{10}b^{13}c^3e^{19}f^9 + 3840a^{11}b^{11}c^4e^{19}f^9 - 24960a^{12}b^9c^5e^{19}f^9 + 97280a^{13}b^7c^6e^{19}f^9 - \\
& 227328a^{14}b^5c^7e^{19}f^9 + 294912a^{15}b^3c^8e^{19}f^9 - 163840a^{16}b^1c^9e^{19}f^9))/(2*(4a^4b^{10}e^2f^6 - 4096a^9c^5e^2f^6 + 640a^6b^6c^2e^2f^6 - \\
& 2560a^7b^4c^3e^2f^6 + 5120a^8b^2c^4e^2f^6 - 80a^5b^8c^3e^2f^6)*(a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^4f^9 + 240a^{11}b^8c^2f^9 - \\
& 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9))*(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c))/((4a^4e^3f^3*(4ac - b^2)^{(5/2)} + (3*(b^6 - 20a^3c^3 + 30a^2b^2c^2 - \\
& 10ab^4c)*(6b^{11}e^3f^3 - 120ab^9c^3e^3f^3 - 6144a^5b^3c^4e^3f^3 + 960a^2b^7c^2e^3f^3 - 3840a^3b^5c^3e^3f^3 + 7680a^4b^3c^4e^3f^3) \\
& *(12a^9b^{15}c^2e^{19}f^9 - 328a^{10}b^{13}c^3e^{19}f^9 + 3840a^{11}b^{11}c^4e^{19}f^9 - 24960a^{12}b^9c^5e^{19}f^9 + 97280a^{13}b^7c^6e^{19}f^9 - 227328a^{14}b^5c^7e^{19}f^9 + \\
& 294912a^{15}b^3c^8e^{19}f^9 - 163840a^{16}b^1c^9e^{19}f^9)))/(8a^4e^3f^3*(4ac - b^2)^{(5/2)}*(4a^4b^{10}e^2f^6 - 4096a^9c^5e^2f^6 + 640a^6b^6c^2e^2f^6 - \\
& 2560a^7b^4c^3e^2f^6 + 5120a^8b^2c^4e^2f^6 - 80a^5b^8c^3e^2f^6)*(a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^4f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3 \\
& 840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9))*(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c))/((4a^4e^3f^3*(4ac - b^2)^{(5/2)} + (9*(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c) \\
& ^2*(6b^{11}e^3f^3 - 120ab^9c^3e^3f^3 - 6144a^5b^3c^4e^3f^3 + 960a^2b^7c^2e^3f^3 - 3840a^3b^5c^3e^3f^3 + 7680a^4b^3c^4e^3f^3) \\
& *(12a^9b^{15}c^2e^{19}f^9 - 328a^{10}b^{13}c^3e^{19}f^9 + 3840a^{11}b^{11}c^4e^{19}f^9 - 24960a^{12}b^9c^5e^{19}f^9 + 97280a^{13}b^7c^6e^{19}f^9 - 227328a^{14}b^5c^7e^{19}f^9 + \\
& 294912a^{15}b^3c^8e^{19}f^9 - 163840a^{16}b^1c^9e^{19}f^9)))/(32a^8e^2f^6*(4ac - b^2)^5*(4a^4b^{10}e^2f^6 - 4096a^9c^5e^2f^6 + 640a^6b^6c^2e^2f^6 - 2560a^7b^4c^3e^2f^6 + \\
& 5120a^8b^2c^4e^2f^6 - 80a^5b^8c^3e^2f^6)*(a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^4f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - \\
& 6144a^{14}b^2c^5f^9))*(3b^8 + 10a^4c^4 + 120a^2b^4c^2 - 145a^3b^2c^3 - 33ab^6c))/(8a^3c^2*(4ac - b^2)^6*(100a^6c^6 - 6b^{12} - 960a^2b^8c^2 + 3840a^3b^6c^3 - 7 \\
& 675a^4b^4c^4 + 6100a^5b^2c^5 + 120ab^{10}c)) + (b*((3*((54a^3b^{13}c^4e^{17}f^3 - 1233a^4b^{11}c^5e^{17}f^3 + 11583a^5b^9c^6e^{17}f^3 - 57204a^6b^7c^7e^{17}f^3 + \\
& 156276a^7b^5c^8e^{17}f^3 - 223200a^8b^3c^9e^{17}f^3 + 129600a^9b^1c^{10}e^{17}f^3)/(a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^4f^9 + 240a^{11}b^8c^2f^9 - \\
& 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) - (((153600a^{13}c^{10}e^{18}f^6 + 6a^6b^{14}c^3e^{18}f^6 - 108a^7b^{12}c^4e^{18}f^6 + 588a^8b^{10}c^5e^{18}f^6 -
\end{aligned}$$

$$\begin{aligned}
& 2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6) + (27*(b^6 - 20* \\
& a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^3*(12*a^9*b^15*c^2*e^19*f^9 - 328*a^ \\
& 10*b^13*c^3*e^19*f^9 + 3840*a^11*b^11*c^4*e^19*f^9 - 24960*a^12*b^9*c^5*e^1 \\
& 9*f^9 + 97280*a^13*b^7*c^6*e^19*f^9 - 227328*a^14*b^5*c^7*e^19*f^9 + 294912 \\
& *a^15*b^3*c^8*e^19*f^9 - 163840*a^16*b*c^9*e^19*f^9))/(64*a^12*e^3*f^9*(4*a \\
& *c - b^2)^(15/2)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 2 \\
& 40*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144* \\
& a^14*b^2*c^5*f^9)))*(3*b^8 + 190*a^4*c^4 + 180*a^2*b^4*c^2 - 335*a^3*b^2*c^ \\
& 3 - 39*a*b^6*c))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(100*a^6*c^6 - 6*b^12 - 96 \\
& 0*a^2*b^8*c^2 + 3840*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 12 \\
& 0*a*b^10*c)))*(16*a^12*b^12*f^9*(4*a*c - b^2)^(15/2) + 65536*a^18*c^6*f^9*(\\
& 4*a*c - b^2)^(15/2) - 384*a^13*b^10*c*f^9*(4*a*c - b^2)^(15/2) + 3840*a^14* \\
& b^8*c^2*f^9*(4*a*c - b^2)^(15/2) - 20480*a^15*b^6*c^3*f^9*(4*a*c - b^2)^(15 \\
& /2) + 61440*a^16*b^4*c^4*f^9*(4*a*c - b^2)^(15/2) - 98304*a^17*b^2*c^5*f^9* \\
& (4*a*c - b^2)^(15/2)))/(10800*a^6*c^8*e^14 + 27*b^12*c^2*e^14 - 540*a*b^10* \\
& c^3*e^14 + 4320*a^2*b^8*c^4*e^14 - 17280*a^3*b^6*c^5*e^14 + 35100*a^4*b^4*c \\
& ^6*e^14 - 32400*a^5*b^2*c^7*e^14) - (x*(((2*(27000*a^6*c^11*d*e^15 + 27*b^ \\
& 12*c^5*d*e^15 - 567*a*b^10*c^6*d*e^15 + 4779*a^2*b^8*c^7*d*e^15 - 20601*a^3 \\
& *b^6*c^8*d*e^15 + 47790*a^4*b^4*c^9*d*e^15 - 56700*a^5*b^2*c^10*d*e^15)))/(a \\
& ^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 \\
& - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) - \\
& (((2*(129600*a^9*b*c^10*d*e^16*f^3 + 54*a^3*b^13*c^4*d*e^16*f^3 - 1233*a^4 \\
& *b^11*c^5*d*e^16*f^3 + 11583*a^5*b^9*c^6*d*e^16*f^3 - 57204*a^6*b^7*c^7*d*e \\
& ^16*f^3 + 156276*a^7*b^5*c^8*d*e^16*f^3 - 223200*a^8*b^3*c^9*d*e^16*f^3)))/(\\
& a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^ \\
& 9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9) \\
& - (((2*(153600*a^13*c^10*d*e^17*f^6 + 6*a^6*b^14*c^3*d*e^17*f^6 - 108*a^7*b \\
& ^12*c^4*d*e^17*f^6 + 588*a^8*b^10*c^5*d*e^17*f^6 + 792*a^9*b^8*c^6*d*e^17*f \\
& ^6 - 22272*a^10*b^6*c^7*d*e^17*f^6 + 100608*a^11*b^4*c^8*d*e^17*f^6 - 19968 \\
& 0*a^12*b^2*c^9*d*e^17*f^6)))/(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 - 24*a^10*b^1 \\
& 0*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840*a^13*b^4*c^4* \\
& f^9 - 6144*a^14*b^2*c^5*f^9) - ((6*b^11*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^ \\
& 5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b \\
& ^3*c^4*e*f^3)*(163840*a^16*b*c^9*d*e^18*f^9 - 12*a^9*b^15*c^2*d*e^18*f^9 + \\
& 328*a^10*b^13*c^3*d*e^18*f^9 - 3840*a^11*b^11*c^4*d*e^18*f^9 + 24960*a^12*b \\
& ^9*c^5*d*e^18*f^9 - 97280*a^13*b^7*c^6*d*e^18*f^9 + 227328*a^14*b^5*c^7*d*e \\
& ^18*f^9 - 294912*a^15*b^3*c^8*d*e^18*f^9)))/((4*a^4*b^10*e^2*f^6 - 4096*a^9* \\
& c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8 \\
& *b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)*(a^9*b^12*f^9 + 4096*a^15*c^6*f^9 \\
& - 24*a^10*b^10*c*f^9 + 240*a^11*b^8*c^2*f^9 - 1280*a^12*b^6*c^3*f^9 + 3840* \\
& a^13*b^4*c^4*f^9 - 6144*a^14*b^2*c^5*f^9)))*(6*b^11*e*f^3 - 120*a*b^9*c*e*f \\
& ^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 \\
& + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + \\
& 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6
\end{aligned}$$

$$\begin{aligned}
& e^{17f^6} + 6a^6b^{14}c^3d^3e^{17f^6} - 108a^7b^{12}c^4d^3e^{17f^6} + 588a^8b^{10}c^5d^3e^{17f^6} + 792a^9b^8c^6d^3e^{17f^6} - 22272a^{10}b^6c^7d^3e^{17f^6} \\
& + 100608a^{11}b^4c^8d^3e^{17f^6} - 199680a^{12}b^2c^9d^3e^{17f^6}) / (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 \\
& - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) - ((6b^{11}e^{17f^6} - 120a^9b^9c^3e^{17f^6} - 6144a^5b^7c^5e^{17f^6} + 960a^2b^7 \\
& c^2e^{17f^6} - 3840a^3b^5c^3e^{17f^6} + 7680a^4b^3c^4e^{17f^6}) * (163840a^{16}b^9c^9d^3e^{18f^9} - 12a^9b^{15}c^2d^3e^{18f^9} + 328a^{10}b^{13}c^3d^3e^{18f^9} \\
& - 3840a^{11}b^{11}c^4d^3e^{18f^9} + 24960a^{12}b^9c^5d^3e^{18f^9} - 97280a^{13}b^7c^6d^3e^{18f^9} + 227328a^{14}b^5c^7d^3e^{18f^9} - 294912a^{15}b^3c^8d^3e^{18f^9}) \\
&) / ((4a^4b^{10}e^{2f^6} - 4096a^9c^5e^{2f^6} + 640a^6b^6c^2e^{2f^6} - 2560a^7b^4c^3e^{2f^6} + 5120a^8b^2c^4e^{2f^6} - 80a^5b^8c^3e^{2f^6}) * (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 \\
& - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9)) * (b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c) / (4a^4e^{17f^6} * (4a^3c - b^2)^{(5/2)}) - (3(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c) * (6b^{11}e^{17f^6} - 120a^9b^9c^3e^{17f^6} - 6144a^5b^7c^5e^{17f^6} + 960a^2b^7c^2e^{17f^6} - 3840a^3b^5c^3e^{17f^6} + 7680a^4b^3c^4e^{17f^6}) * (163840a^{16}b^9c^9d^3e^{18f^9} - 12a^9b^{15}c^2d^3e^{18f^9} + 328a^{10}b^{13}c^3d^3e^{18f^9} - 3840a^{11}b^{11}c^4d^3e^{18f^9} + 24960a^{12}b^9c^5d^3e^{18f^9} - 97280a^{13}b^7c^6d^3e^{18f^9} + 227328a^{14}b^5c^7d^3e^{18f^9} - 294912a^{15}b^3c^8d^3e^{18f^9}) / (4a^4e^{17f^6} * (4a^3c - b^2)^{(5/2)}) * (4a^4b^{10}e^{2f^6} - 4096a^9c^5e^{2f^6} + 640a^6b^6c^2e^{2f^6} - 2560a^7b^4c^3e^{2f^6} + 5120a^8b^2c^4e^{2f^6} - 80a^5b^8c^3e^{2f^6}) * (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9)) * (6b^{11}e^{17f^6} - 120a^9b^9c^3e^{17f^6} - 6144a^5b^7c^5e^{17f^6} + 960a^2b^7c^2e^{17f^6} - 3840a^3b^5c^3e^{17f^6} + 7680a^4b^3c^4e^{17f^6}) / (2(4a^4b^{10}e^{2f^6} - 4096a^9c^5e^{2f^6} + 640a^6b^6c^2e^{2f^6} - 2560a^7b^4c^3e^{2f^6} + 5120a^8b^2c^4e^{2f^6} - 80a^5b^8c^3e^{2f^6})) - (3((2(129600a^9b^9c^10d^3e^{16f^3} + 54a^3b^{13}c^4d^3e^{16f^3} - 1233a^4b^{11}c^5d^3e^{16f^3} + 11583a^5b^9c^6d^3e^{16f^3} - 57204a^6b^7c^7d^3e^{16f^3} + 156276a^7b^5c^8d^3e^{16f^3} - 223200a^8b^3c^9d^3e^{16f^3})) / (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) - ((2(153600a^{13}c^{10}d^3e^{17f^6} + 6a^6b^{14}c^3d^3e^{17f^6} - 108a^7b^{12}c^4d^3e^{17f^6} + 588a^8b^{10}c^5d^3e^{17f^6} + 792a^9b^8c^6d^3e^{17f^6} - 22272a^{10}b^6c^7d^3e^{17f^6} + 100608a^{11}b^4c^8d^3e^{17f^6} - 199680a^{12}b^2c^9d^3e^{17f^6})) / (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) - ((6b^{11}e^{17f^6} - 120a^9b^9c^3e^{17f^6} - 6144a^5b^7c^5e^{17f^6} + 960a^2b^7c^2e^{17f^6} - 3840a^3b^5c^3e^{17f^6} + 7680a^4b^3c^4e^{17f^6}) * (163840a^{16}b^9c^9d^3e^{18f^9} - 12a^9b^{15}c^2d^3e^{18f^9} + 328a^{10}b^{13}c^3d^3e^{18f^9} - 3840a^{11}b^{11}c^4d^3e^{18f^9} + 24960a^{12}b^9c^5d^3e^{18f^9} - 97280a^{13}b^7c^6d^3e^{18f^9} + 227328a^{14}b^5c^7d^3e^{18f^9} - 294912a^{15}b^3c^8d^3e^{18f^9}
\end{aligned}$$

$$\begin{aligned}
& 9)) / ((4a^4b^{10}e^{2f^6} - 4096a^9c^5e^{2f^6} + 640a^6b^6c^2e^{2f^6} - \\
& 2560a^7b^4c^3e^{2f^6} + 5120a^8b^2c^4e^{2f^6} - 80a^5b^8c^2e^{2f^6} \\
&) * (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2 \\
& * f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9 \\
& 9))) * (6b^{11}e^{f^3} - 120a^9c^5e^{f^3} - 6144a^5b^3c^5e^{f^3} + 960a^2b^7 \\
& * c^2e^{f^3} - 3840a^3b^5c^3e^{f^3} + 7680a^4b^3c^4e^{f^3}) / (2 * (4a^4b^{10} \\
& e^{2f^6} - 4096a^9c^5e^{2f^6} + 640a^6b^6c^2e^{2f^6} - 2560a^7b^4c^3e^{2f^6} \\
& + 5120a^8b^2c^4e^{2f^6} - 80a^5b^8c^2e^{2f^6})) * (b^6 - 20a^3c^3 + 30a^2b^2c^2 \\
& - 10a^2b^4c) / (4a^4e^{f^3} * (4a^3c - b^2)^{(5/2)}) + \\
& (27 * (b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10a^2b^4c)^3 * (163840a^{16}b^6c^9d \\
& e^{18f^9} - 12a^9b^{15}c^2d^2e^{18f^9} + 328a^{10}b^{13}c^3d^2e^{18f^9} - 38 \\
& 40a^{11}b^{11}c^4d^2e^{18f^9} + 24960a^{12}b^9c^5d^2e^{18f^9} - 97280a^{13}b^7 \\
& c^6d^2e^{18f^9} + 227328a^{14}b^5c^7d^2e^{18f^9} - 294912a^{15}b^3c^8d^2e^{18f^9} \\
&)) / (32a^{12}e^3f^9 * (4a^3c - b^2)^{(15/2)} * (a^9b^{12}f^9 + 4096a^{15}c^6f^9 \\
& - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 \\
& + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9))) * (3b^8 + 190a^4c^4 + 1 \\
& 80a^2b^4c^2 - 335a^3b^2c^3 - 39a^2b^6c) / (8a^3c^2 * (4a^3c - b^2)^{(1 \\
& 3/2)} * (100a^6c^6 - 6b^{12} - 960a^2b^8c^2 + 3840a^3b^6c^3 - 7675a^4b^4 \\
& c^4 + 6100a^5b^2c^5 + 120a^2b^{10}c))) * (16a^{12}b^{12}f^9 * (4a^3c - b^2)^{(15/2)} \\
& + 65536a^{18}c^6f^9 * (4a^3c - b^2)^{(15/2)} - 384a^{13}b^{10}c^2f^9 * (4 \\
& a^3c - b^2)^{(15/2)} + 3840a^{14}b^8c^2f^9 * (4a^3c - b^2)^{(15/2)} - 20480a^{15} \\
& b^6c^3f^9 * (4a^3c - b^2)^{(15/2)} + 61440a^{16}b^4c^4f^9 * (4a^3c - b^2)^{(15/2)} \\
& - 98304a^{17}b^2c^5f^9 * (4a^3c - b^2)^{(15/2}))) / (10800a^6c^8e^{14} + \\
& 27b^{12}c^2e^{14} - 540a^2b^{10}c^3e^{14} + 4320a^2b^8c^4e^{14} - 17280a^3b^6c^5e^{14} \\
& + 35100a^4b^4c^6e^{14} - 32400a^5b^2c^7e^{14}) + (((((36a^3b^{14}c^3e^{15f^3} - 14400a^{10}c^{10}e^{15f^3} \\
& - 837a^4b^{12}c^4e^{15f^3} + 8046a^5b^{10}c^5e^{15f^3} - 40941a^6b^8c^6e^{15f^3} + 116532a^7b^6 \\
& c^7e^{15f^3} - 177588a^8b^4c^8e^{15f^3} + 119520a^9b^2c^9e^{15f^3} + 129600a^9b^3c^{10} \\
& d^2e^{15f^3} + 54a^3b^{13}c^4d^2e^{15f^3} - 1233a^4b^{11}c^5d^2e^{15f^3} + 11583a^5b^9c^6d^2e^{15f^3} \\
& - 57204a^6b^7c^7d^2e^{15f^3} + 156276a^7b^5c^8d^2e^{15f^3} - 223200a^8b^3c^9d^2e^{15f^3} \\
& - 1233a^4b^{11}c^5d^2e^{15f^3}))) / (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8 \\
& c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) - (((12a^6b^{15}c^2e^{16f^6} \\
& - 300a^7b^{13}c^3e^{16f^6} + 3156a^8b^{11}c^4e^{16f^6} - 17976a^9b^9c^5e^{16f^6} + 59136a^{10}b^7c^6e^{16f^6} \\
& - 109824a^{11}b^5c^7e^{16f^6} + 101376a^{12}b^3c^8e^{16f^6} + 153600a^{13}c^{10}d^2e^{16f^6} - 30720a^{13}b^6c^9e^{16f^6} \\
& + 6a^6b^{14}c^3d^2e^{16f^6} - 108a^7b^{12}c^4d^2e^{16f^6} + 588a^8b^{10}c^5d^2e^{16f^6} + 792a^9b^8c^6d^2e^{16f^6} \\
& - 22272a^{10}b^6c^7d^2e^{16f^6} + 100608a^{11}b^4c^8d^2e^{16f^6} - 199680a^{12}b^2c^9d^2e^{16f^6}))) / (a^9b^{12}f^9 + 409 \\
& 6a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) + ((6b^{11}e^{f^3} - \\
& 120a^9c^5e^{f^3} - 6144a^5b^3c^5e^{f^3} + 960a^2b^7c^2e^{f^3} - 3840a^3b^5c^3e^{f^3} + 7680a^4b^3c^4e^{f^3}) * (4a^{10}b^{14}c^2e^{17f^9} - 96a^{11} \\
& b^{12}c^3e^{17f^9} + 960a^{12}b^{10}c^4e^{17f^9} - 5120a^{13}b^8c^5e^{17f^9} - 5120a^{13}b^8c^5e^{17f^9}
\end{aligned}$$

$$\begin{aligned}
& f^9 + 15360a^{14}b^6c^6e^{17}f^9 - 24576a^{15}b^4c^7e^{17}f^9 + 16384a^{16}b^2c^8e^{17}f^9 - 163840a^{16}b^2c^9d^2e^{17}f^9 + 12a^9b^{15}c^2d^2e^{17}f^9 - 328a^{10}b^{13}c^3d^2e^{17}f^9 + 3840a^{11}b^{11}c^4d^2e^{17}f^9 \\
& - 24960a^{12}b^9c^5d^2e^{17}f^9 + 97280a^{13}b^7c^6d^2e^{17}f^9 - 227328a^{14}b^5c^7d^2e^{17}f^9 + 294912a^{15}b^3c^8d^2e^{17}f^9) / (2(4a^4b^{10}e^{2}f^6 - 4096a^9c^5e^{2}f^6 + 640a^6b^6c^2e^{2}f^6 - 2560a^7b^4c^3e^{2}f^6 + 5120a^8b^2c^4e^{2}f^6 - 80a^5b^8c^2e^{2}f^6) * (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9)) * (6b^{11}e^3f^3 - 120a^9c^5e^3f^3 - 6144a^5b^3c^4e^3f^3 + 960a^2b^7c^2e^3f^3 - 3840a^3b^5c^3e^3f^3 + 7680a^4b^3c^4e^3f^3) / (2(4a^4b^{10}e^{2}f^6 - 4096a^9c^5e^{2}f^6 + 640a^6b^6c^2e^{2}f^6 - 2560a^7b^4c^3e^{2}f^6 + 5120a^8b^2c^4e^{2}f^6 - 80a^5b^8c^2e^{2}f^6)) * (6b^{11}e^3f^3 - 120a^9c^5e^3f^3 - 6144a^5b^3c^4e^3f^3 + 960a^2b^7c^2e^3f^3 - 3840a^3b^5c^3e^3f^3 + 7680a^4b^3c^4e^3f^3) / (2(4a^4b^{10}e^{2}f^6 - 4096a^9c^5e^{2}f^6 + 640a^6b^6c^2e^{2}f^6 - 2560a^7b^4c^3e^{2}f^6 + 5120a^8b^2c^4e^{2}f^6 - 80a^5b^8c^2e^{2}f^6)) - (27b^{13}c^4e^{14} - 594a^3b^{11}c^5e^{14} + 43200a^6b^3c^{10}e^{14} + 5319a^2b^9c^6e^{14} - 24732a^3b^7c^7e^{14} + 62748a^4b^5c^8e^{14} - 82080a^5b^3c^9e^{14} + 27000a^6c^{11}d^2e^{14} + 27b^{12}c^5d^2e^{14} + 4779a^2b^8c^7d^2e^{14} - 20601a^3b^6c^8d^2e^{14} + 47790a^4b^4c^9d^2e^{14} - 56700a^5b^2c^{10}d^2e^{14} - 567a^6b^{10}c^6d^2e^{14}) / (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) + (3((3((12a^6b^{15}c^2e^{16}f^6 - 300a^7b^{13}c^3e^{16}f^6 + 3156a^8b^{11}c^4e^{16}f^6 - 17976a^9b^9c^5e^{16}f^6 + 59136a^{10}b^7c^6e^{16}f^6 - 109824a^{11}b^5c^7e^{16}f^6 + 101376a^{12}b^3c^8e^{16}f^6 + 153600a^{13}c^{10}d^2e^{16}f^6 - 30720a^{13}b^2c^9e^{16}f^6 + 6a^{16}b^{14}c^3d^2e^{16}f^6 - 108a^7b^{12}c^4d^2e^{16}f^6 + 588a^8b^{10}c^5d^2e^{16}f^6 + 792a^9b^8c^6d^2e^{16}f^6 - 22272a^{10}b^6c^7d^2e^{16}f^6 + 100608a^{11}b^4c^8d^2e^{16}f^6 - 199680a^{12}b^2c^9d^2e^{16}f^6) / (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) + ((6b^{11}e^3f^3 - 120a^9c^5e^3f^3 - 6144a^5b^3c^4e^3f^3 + 960a^2b^7c^2e^3f^3 - 3840a^3b^5c^3e^3f^3 + 7680a^4b^3c^4e^3f^3) * (4a^{10}b^{14}c^2e^{17}f^9 - 96a^{11}b^{12}c^3e^{17}f^9 + 960a^{12}b^{10}c^4e^{17}f^9 - 5120a^{13}b^8c^5e^{17}f^9 + 15360a^{14}b^6c^6e^{17}f^9 - 24576a^{15}b^4c^7e^{17}f^9 + 16384a^{16}b^2c^8e^{17}f^9 - 163840a^{16}b^2c^9d^2e^{17}f^9 + 12a^9b^{15}c^2d^2e^{17}f^9 - 328a^{10}b^{13}c^3d^2e^{17}f^9 + 3840a^{11}b^{11}c^4d^2e^{17}f^9 - 24960a^{12}b^9c^5d^2e^{17}f^9 + 97280a^{13}b^7c^6d^2e^{17}f^9 - 227328a^{14}b^5c^7d^2e^{17}f^9 + 294912a^{15}b^3c^8d^2e^{17}f^9) / (2(4a^4b^{10}e^{2}f^6 - 4096a^9c^5e^{2}f^6 + 640a^6b^6c^2e^{2}f^6 - 2560a^7b^4c^3e^{2}f^6 + 5120a^8b^2c^4e^{2}f^6 - 80a^5b^8c^2e^{2}f^6) * (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^2f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9))) * (b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10a^4c) / (4a^4e^3f^3 * (
\end{aligned}$$

$$\begin{aligned}
& 4*a*c - b^2)^{(5/2)} + (3*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c) * (\\
& 6*b^{11}*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e \\
& *f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3) * (4*a^{10}*b^{14}*c^2*e^ \\
& 17*f^9 - 96*a^{11}*b^{12}*c^3*e^{17*f^9} + 960*a^{12}*b^{10}*c^4*e^{17*f^9} - 5120*a^{13} \\
& *b^8*c^5*e^{17*f^9} + 15360*a^{14}*b^6*c^6*e^{17*f^9} - 24576*a^{15}*b^4*c^7*e^{17*f^9} \\
& ^9 + 16384*a^{16}*b^2*c^8*e^{17*f^9} - 163840*a^{16}*b*c^9*d^2*e^{17*f^9} + 12*a^9* \\
& b^{15}*c^2*d^2*e^{17*f^9} - 328*a^{10}*b^{13}*c^3*d^2*e^{17*f^9} + 3840*a^{11}*b^{11}*c^4 \\
& *d^2*e^{17*f^9} - 24960*a^{12}*b^9*c^5*d^2*e^{17*f^9} + 97280*a^{13}*b^7*c^6*d^2*e^ \\
& 17*f^9 - 227328*a^{14}*b^5*c^7*d^2*e^{17*f^9} + 294912*a^{15}*b^3*c^8*d^2*e^{17*f^9} \\
& 9)) / (8*a^4*e*f^3 * (4*a*c - b^2)^{(5/2)} * (4*a^4*b^{10}*e^2*f^6 - 4096*a^9*c^5*e^2 \\
& *f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*b^2*c^ \\
& 4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6) * (a^9*b^{12}*f^9 + 4096*a^{15}*c^6*f^9 - 24*a^ \\
& 10*b^{10}*c*f^9 + 240*a^{11}*b^8*c^2*f^9 - 1280*a^{12}*b^6*c^3*f^9 + 3840*a^{13}*b^ \\
& 4*c^4*f^9 - 6144*a^{14}*b^2*c^5*f^9)) * (b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 1 \\
& 0*a*b^4*c) / (4*a^4*e*f^3 * (4*a*c - b^2)^{(5/2)} + (9*(b^6 - 20*a^3*c^3 + 30*a \\
& ^2*b^2*c^2 - 10*a*b^4*c)^2 * (6*b^{11}*e*f^3 - 120*a*b^9*c*e*f^3 - 6144*a^5*b*c \\
& ^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b^5*c^3*e*f^3 + 7680*a^4*b^3*c^ \\
& 4*e*f^3) * (4*a^{10}*b^{14}*c^2*e^{17*f^9} - 96*a^{11}*b^{12}*c^3*e^{17*f^9} + 960*a^{12}*b \\
& ^{10}*c^4*e^{17*f^9} - 5120*a^{13}*b^8*c^5*e^{17*f^9} + 15360*a^{14}*b^6*c^6*e^{17*f^9} \\
& - 24576*a^{15}*b^4*c^7*e^{17*f^9} + 16384*a^{16}*b^2*c^8*e^{17*f^9} - 163840*a^{16} \\
& *b*c^9*d^2*e^{17*f^9} + 12*a^9*b^{15}*c^2*d^2*e^{17*f^9} - 328*a^{10}*b^{13}*c^3*d^2*e \\
& ^{17*f^9} + 3840*a^{11}*b^{11}*c^4*d^2*e^{17*f^9} - 24960*a^{12}*b^9*c^5*d^2*e^{17*f^9} \\
& + 97280*a^{13}*b^7*c^6*d^2*e^{17*f^9} - 227328*a^{14}*b^5*c^7*d^2*e^{17*f^9} + 294 \\
& 912*a^{15}*b^3*c^8*d^2*e^{17*f^9})) / (32*a^8*e^2*f^6 * (4*a*c - b^2)^5 * (4*a^4*b^{10} \\
& *e^2*f^6 - 4096*a^9*c^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^ \\
& 3*e^2*f^6 + 5120*a^8*b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6) * (a^9*b^{12}*f^9 \\
& + 4096*a^{15}*c^6*f^9 - 24*a^{10}*b^{10}*c*f^9 + 240*a^{11}*b^8*c^2*f^9 - 1280*a^{12} \\
& *b^6*c^3*f^9 + 3840*a^{13}*b^4*c^4*f^9 - 6144*a^{14}*b^2*c^5*f^9)) * (3*b^8 + 10 \\
& *a^4*c^4 + 120*a^2*b^4*c^2 - 145*a^3*b^2*c^3 - 33*a*b^6*c) * (16*a^{12}*b^{12}*f^ \\
& 9 * (4*a*c - b^2)^{(15/2)} + 65536*a^{18}*c^6*f^9 * (4*a*c - b^2)^{(15/2)} - 384*a^{13} \\
& *b^{10}*c*f^9 * (4*a*c - b^2)^{(15/2)} + 3840*a^{14}*b^8*c^2*f^9 * (4*a*c - b^2)^{(15/ \\
& 2)} - 20480*a^{15}*b^6*c^3*f^9 * (4*a*c - b^2)^{(15/2)} + 61440*a^{16}*b^4*c^4*f^9 * (\\
& 4*a*c - b^2)^{(15/2)} - 98304*a^{17}*b^2*c^5*f^9 * (4*a*c - b^2)^{(15/2)})) / (8*a^3* \\
& c^2 * (4*a*c - b^2)^6 * (10800*a^6*c^8*e^{14} + 27*b^{12}*c^2*e^{14} - 540*a*b^{10}*c^3 \\
& *e^{14} + 4320*a^2*b^8*c^4*e^{14} - 17280*a^3*b^6*c^5*e^{14} + 35100*a^4*b^4*c^6* \\
& e^{14} - 32400*a^5*b^2*c^7*e^{14}) * (100*a^6*c^6 - 6*b^{12} - 960*a^2*b^8*c^2 + 38 \\
& 40*a^3*b^6*c^3 - 7675*a^4*b^4*c^4 + 6100*a^5*b^2*c^5 + 120*a*b^{10}*c)) + (b * \\
& ((3 * ((36*a^3*b^{14}*c^3*e^{15*f^3} - 14400*a^{10}*c^{10}*e^{15*f^3} - 837*a^4*b^{12}*c^ \\
& 4*e^{15*f^3} + 8046*a^5*b^{10}*c^5*e^{15*f^3} - 40941*a^6*b^8*c^6*e^{15*f^3} + 1165 \\
& 32*a^7*b^6*c^7*e^{15*f^3} - 177588*a^8*b^4*c^8*e^{15*f^3} + 119520*a^9*b^2*c^9* \\
& e^{15*f^3} + 129600*a^9*b*c^{10}*d^2*e^{15*f^3} + 54*a^3*b^{13}*c^4*d^2*e^{15*f^3} - \\
& 1233*a^4*b^{11}*c^5*d^2*e^{15*f^3} + 11583*a^5*b^9*c^6*d^2*e^{15*f^3} - 57204*a^6 \\
& *b^7*c^7*d^2*e^{15*f^3} + 156276*a^7*b^5*c^8*d^2*e^{15*f^3} - 223200*a^8*b^3*c^ \\
& 9*d^2*e^{15*f^3}) / (a^9*b^{12}*f^9 + 4096*a^{15}*c^6*f^9 - 24*a^{10}*b^{10}*c*f^9 + 24 \\
& 0*a^{11}*b^8*c^2*f^9 - 1280*a^{12}*b^6*c^3*f^9 + 3840*a^{13}*b^4*c^4*f^9 - 6144*a
\end{aligned}$$

$$\begin{aligned}
& ^{14}b^2c^5f^9) - (((12a^6b^{15}c^2e^{16}f^6 - 300a^7b^{13}c^3e^{16}f^6 \\
& + 3156a^8b^{11}c^4e^{16}f^6 - 17976a^9b^9c^5e^{16}f^6 + 59136a^{10}b^7c^6e^{16}f^6 - 109824a^{11}b^5c^7e^{16}f^6 + 101376a^{12}b^3c^8e^{16}f^6 \\
& + 153600a^{13}c^{10}d^2e^{16}f^6 - 30720a^{13}b^9c^9e^{16}f^6 + 6a^6b^{14}c^3d^2e^{16}f^6 - 108a^7b^{12}c^4d^2e^{16}f^6 + 588a^8b^{10}c^5d^2e^{16}f^6 \\
& + 792a^9b^8c^6d^2e^{16}f^6 - 22272a^{10}b^6c^7d^2e^{16}f^6 + 100608a^{11}b^4c^8d^2e^{16}f^6 - 199680a^{12}b^2c^9d^2e^{16}f^6)/(a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^6f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) + ((6b^{11}ef^3 - 120ab^9c^3ef^3 - 6144a^5b^3c^4ef^3 + 960a^2b^7c^2ef^3 - 3840a^3b^5c^3ef^3 + 7680a^4b^3c^4ef^3)*(4a^{10}b^{14}c^2e^{17}f^9 - 96a^{11}b^{12}c^3e^{17}f^9 + 960a^{12}b^{10}c^4e^{17}f^9 - 5120a^{13}b^8c^5e^{17}f^9 + 15360a^{14}b^6c^6e^{17}f^9 - 24576a^{15}b^4c^7e^{17}f^9 + 16384a^{16}b^2c^8e^{17}f^9 - 163840a^{16}b^9c^9d^2e^{17}f^9 + 12a^9b^{15}c^2d^2e^{17}f^9 - 328a^{10}b^{13}c^3d^2e^{17}f^9 + 3840a^{11}b^{11}c^4d^2e^{17}f^9 - 24960a^{12}b^9c^5d^2e^{17}f^9 + 97280a^{13}b^7c^6d^2e^{17}f^9 - 227328a^{14}b^5c^7d^2e^{17}f^9 + 294912a^{15}b^3c^8d^2e^{17}f^9))/(2*(4a^4b^{10}e^2f^6 - 4096a^9c^5e^2f^6 + 640a^6b^6c^2e^2f^6 - 2560a^7b^4c^3e^2f^6 + 5120a^8b^2c^4e^2f^6 - 80a^5b^8c^3e^2f^6)*(a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^6f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9)))*(6b^{11}ef^3 - 120ab^9c^3ef^3 - 6144a^5b^3c^4ef^3 + 960a^2b^7c^2ef^3 - 3840a^3b^5c^3ef^3 + 7680a^4b^3c^4ef^3))/(2*(4a^4b^{10}e^2f^6 - 4096a^9c^5e^2f^6 + 640a^6b^6c^2e^2f^6 - 2560a^7b^4c^3e^2f^6 + 5120a^8b^2c^4e^2f^6 - 80a^5b^8c^3e^2f^6)))*(b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c))/(4a^4ef^3*(4ac - b^2)^{(5/2)}) - ((3*((12a^6b^{15}c^2e^{16}f^6 - 300a^7b^{13}c^3e^{16}f^6 + 3156a^8b^{11}c^4e^{16}f^6 - 17976a^9b^9c^5e^{16}f^6 + 59136a^{10}b^7c^6e^{16}f^6 - 109824a^{11}b^5c^7e^{16}f^6 + 101376a^{12}b^3c^8e^{16}f^6 + 153600a^{13}c^{10}d^2e^{16}f^6 - 30720a^{13}b^9c^9e^{16}f^6 + 6a^6b^{14}c^3d^2e^{16}f^6 - 108a^7b^{12}c^4d^2e^{16}f^6 + 588a^8b^{10}c^5d^2e^{16}f^6 + 792a^9b^8c^6d^2e^{16}f^6 - 22272a^{10}b^6c^7d^2e^{16}f^6 + 100608a^{11}b^4c^8d^2e^{16}f^6 - 199680a^{12}b^2c^9d^2e^{16}f^6)/(a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^6f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9) + ((6b^{11}ef^3 - 120ab^9c^3ef^3 - 6144a^5b^3c^4ef^3 + 960a^2b^7c^2ef^3 - 3840a^3b^5c^3ef^3 + 7680a^4b^3c^4ef^3)*(4a^{10}b^{14}c^2e^{17}f^9 - 96a^{11}b^{12}c^3e^{17}f^9 + 960a^{12}b^{10}c^4e^{17}f^9 - 5120a^{13}b^8c^5e^{17}f^9 + 15360a^{14}b^6c^6e^{17}f^9 - 24576a^{15}b^4c^7e^{17}f^9 + 16384a^{16}b^2c^8e^{17}f^9 - 163840a^{16}b^9c^9d^2e^{17}f^9 + 12a^9b^{15}c^2d^2e^{17}f^9 - 328a^{10}b^{13}c^3d^2e^{17}f^9 + 3840a^{11}b^{11}c^4d^2e^{17}f^9 - 24960a^{12}b^9c^5d^2e^{17}f^9 + 97280a^{13}b^7c^6d^2e^{17}f^9 - 227328a^{14}b^5c^7d^2e^{17}f^9 + 294912a^{15}b^3c^8d^2e^{17}f^9))/(2*(4a^4b^{10}e^2f^6 - 4096a^9c^5e^2f^6 + 640a^6b^6c^2e^2f^6 - 2560a^7b^4c^3e^2f^6 + 5120a^8b^2c^4e^2f^6 - 80a^5b^8c^3e^2f^6)*(a^9b^{12}f^9 + 40
\end{aligned}$$

$$\begin{aligned}
& 96a^{15}c^6f^9 - 24a^{10}b^{10}c^4f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6 \\
& *c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5f^9)) * (b^6 - 20a^3c^3 \\
& + 30a^2b^2c^2 - 10ab^4c)) / (4a^4ef^3(4ac - b^2)^{(5/2)}) + (3 * \\
& (b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c) * (6b^{11}ef^3 - 120ab^9c \\
& *ef^3 - 6144a^5b^3c^5ef^3 + 960a^2b^7c^2ef^3 - 3840a^3b^5c^3ef^3 \\
& + 7680a^4b^3c^4ef^3) * (4a^{10}b^{14}c^2e^{17}f^9 - 96a^{11}b^{12}c^3e \\
& ^{17}f^9 + 960a^{12}b^{10}c^4e^{17}f^9 - 5120a^{13}b^8c^5e^{17}f^9 + 15360a \\
& ^{14}b^6c^6e^{17}f^9 - 24576a^{15}b^4c^7e^{17}f^9 + 16384a^{16}b^2c^8e^{17} \\
& f^9 - 163840a^{16}b^3c^9d^2e^{17}f^9 + 12a^9b^{15}c^2d^2e^{17}f^9 - 32 \\
& 8a^{10}b^{13}c^3d^2e^{17}f^9 + 3840a^{11}b^{11}c^4d^2e^{17}f^9 - 24960a^{12} \\
& b^9c^5d^2e^{17}f^9 + 97280a^{13}b^7c^6d^2e^{17}f^9 - 227328a^{14}b^5c^7 \\
& d^2e^{17}f^9 + 294912a^{15}b^3c^8d^2e^{17}f^9)) / (8a^4ef^3(4ac - \\
& b^2)^{(5/2)} * (4a^4b^{10}e^2f^6 - 4096a^9c^5e^2f^6 + 640a^6b^6c^2e^2 \\
& *f^6 - 2560a^7b^4c^3e^2f^6 + 5120a^8b^2c^4e^2f^6 - 80a^5b^8c^2e^2 \\
& *f^6) * (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 24a^{10}b^{10}c^4f^9 + 240a^{11}b \\
& ^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13}b^4c^4f^9 - 6144a^{14}b^2c^5 \\
& f^9)) * (6b^{11}ef^3 - 120ab^9c^2ef^3 - 6144a^5b^3c^5ef^3 + 960a^2 \\
& b^7c^2ef^3 - 3840a^3b^5c^3ef^3 + 7680a^4b^3c^4ef^3)) / (2 * (4a^4 \\
& b^{10}e^2f^6 - 4096a^9c^5e^2f^6 + 640a^6b^6c^2e^2f^6 - 2560a^7b^4c^3 \\
& e^2f^6 + 5120a^8b^2c^4e^2f^6 - 80a^5b^8c^2e^2f^6)) + (27 \\
& * (b^6 - 20a^3c^3 + 30a^2b^2c^2 - 10ab^4c)^3 * (4a^{10}b^{14}c^2e^{17}f^9 \\
& - 96a^{11}b^{12}c^3e^{17}f^9 + 960a^{12}b^{10}c^4e^{17}f^9 - 5120a^{13}b^8c^5e^{17} \\
& f^9 + 15360a^{14}b^6c^6e^{17}f^9 - 24576a^{15}b^4c^7e^{17}f^9 + \\
& 16384a^{16}b^2c^8e^{17}f^9 - 163840a^{16}b^3c^9d^2e^{17}f^9 + 12a^9b^{15} \\
& c^2d^2e^{17}f^9 - 328a^{10}b^{13}c^3d^2e^{17}f^9 + 3840a^{11}b^{11}c^4d^2 \\
& *e^{17}f^9 - 24960a^{12}b^9c^5d^2e^{17}f^9 + 97280a^{13}b^7c^6d^2e^{17}f^9 \\
& - 227328a^{14}b^5c^7d^2e^{17}f^9 + 294912a^{15}b^3c^8d^2e^{17}f^9)) / \\
& (64a^{12}e^3f^9(4ac - b^2)^{(15/2)} * (a^9b^{12}f^9 + 4096a^{15}c^6f^9 - 2 \\
& 4a^{10}b^{10}c^4f^9 + 240a^{11}b^8c^2f^9 - 1280a^{12}b^6c^3f^9 + 3840a^{13} \\
& b^4c^4f^9 - 6144a^{14}b^2c^5f^9)) * (3b^8 + 190a^4c^4 + 180a^2b^4 \\
& *c^2 - 335a^3b^2c^3 - 39ab^6c) * (16a^{12}b^{12}f^9 * (4ac - b^2)^{(15/2)} \\
& + 65536a^{18}c^6f^9 * (4ac - b^2)^{(15/2)} - 384a^{13}b^{10}c^4f^9 * (4ac - b \\
& ^2)^{(15/2)} + 3840a^{14}b^8c^2f^9 * (4ac - b^2)^{(15/2)} - 20480a^{15}b^6c^3 \\
& f^9 * (4ac - b^2)^{(15/2)} + 61440a^{16}b^4c^4f^9 * (4ac - b^2)^{(15/2)} - \\
& 98304a^{17}b^2c^5f^9 * (4ac - b^2)^{(15/2))} / (8a^3c^2 * (4ac - b^2)^{(13/ \\
& 2)} * (10800a^6c^8e^{14} + 27b^{12}c^2e^{14} - 540ab^{10}c^3e^{14} + 4320a^2b^8 \\
& c^4e^{14} - 17280a^3b^6c^5e^{14} + 35100a^4b^4c^6e^{14} - 32400a^5b^2c^7 \\
& e^{14}) * (100a^6c^6 - 6b^{12} - 960a^2b^8c^2 + 3840a^3b^6c^3 - 7675a^4b^4 \\
& c^4 + 6100a^5b^2c^5 + 120ab^{10}c)) * (b^6 - 20a^3c^3 + 30a^2b^2c^2 - \\
& 10ab^4c)) / (2a^4ef^3(4ac - b^2)^{(5/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

$$3.548 \quad \int (2 + 3x)^6 \left(1 + (2 + 3x)^7 + (2 + 3x)^{14} \right) dx$$

Optimal. Leaf size=34

$$\frac{1}{63}(3x + 2)^{21} + \frac{1}{42}(3x + 2)^{14} + \frac{1}{21}(3x + 2)^7$$

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1390, 14}

$$\frac{1}{63}(3x + 2)^{21} + \frac{1}{42}(3x + 2)^{14} + \frac{1}{21}(3x + 2)^7$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14), x]

[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1390

Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_.))^(p_.), x_Symbol] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned} \int (2 + 3x)^6 \left(1 + (2 + 3x)^7 + (2 + 3x)^{14} \right) dx &= \frac{1}{3} \text{Subst} \left(\int x^6 (1 + x^7 + x^{14}) dx, x, 2 + 3x \right) \\ &= \frac{1}{3} \text{Subst} \left(\int (x^6 + x^{13} + x^{20}) dx, x, 2 + 3x \right) \\ &= \frac{1}{21}(2 + 3x)^7 + \frac{1}{42}(2 + 3x)^{14} + \frac{1}{63}(2 + 3x)^{21} \end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{1}{63}(3x+2)^{21} + \frac{1}{42}(3x+2)^{14} + \frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14), x]

[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 + 3x)^6 (1 + (2 + 3x)^7 + (2 + 3x)^{14}) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14), x]

[Out] IntegrateAlgebraic[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14), x]

fricas [B] time = 1.17, size = 104, normalized size = 3.06

$\frac{1162261467}{7}x^{21} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203}{14}x^{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^9 + 173635132896x^8 + 66158154783x^7 + 20588764518x^6 + 5149786572x^5 + 1010576952x^4 + 149902032x^3 + 15808800x^2 + 1056832x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14), x, algorithm="fricas")

[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 + 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 677082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 + 20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15808800*x^2 + 1056832*x

giac [A] time = 0.35, size = 28, normalized size = 0.82

$$\frac{1}{63}(3x+2)^{21} + \frac{1}{42}(3x+2)^{14} + \frac{1}{21}(3x+2)^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14), x, algorithm="giac")

[Out] 1/63*(3*x + 2)^21 + 1/42*(3*x + 2)^14 + 1/21*(3*x + 2)^7

maple [B] time = 0.00, size = 105, normalized size = 3.09

$\frac{1162261467}{7}x^{21} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203}{14}x^{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^9 + 173635132896x^8 + 66158154783x^7 + 20588764518x^6 + 5149786572x^5 + 1010576952x^4 + 149902032x^3 + 15808800x^2 + 1056832x$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14), x)

[Out] 1162261467/7*x^21+2324522934*x^20+15496819560*x^19+65431015920*x^18+196293047760*x^17+444930908256*x^16+790988281344*x^15+15819767221203/14*x^14+1318314865122*x^13+1269491970942*x^12+1015602174288*x^11+677082445416*x^10+376174427616*x^9+173635132896*x^8+66158154783*x^7+20588764518*x^6+5149786572*x^5+1010576952*x^4+149902032*x^3+15808800*x^2+1056832*x

maxima [B] time = 0.54, size = 104, normalized size = 3.06

$\frac{1162261467}{7}x^{21} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203}{14}x^{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^9 + 173635132896x^8 + 66158154783x^7 + 20588764518x^6 + 5149786572x^5 + 1010576952x^4 + 149902032x^3 + 15808800x^2 + 1056832x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14), x, algorithm="maxima")

[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 + 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 677082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 + 20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 15808800*x^2 + 1056832*x

mupad [B] time = 1.58, size = 29, normalized size = 0.85

$$\frac{(3x + 2)^7 (3(3x + 2)^7 + 2(3x + 2)^{14} + 6)}{126}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1), x)

[Out] ((3*x + 2)^7*(3*(3*x + 2)^7 + 2*(3*x + 2)^14 + 6))/126

sympy [B] time = 0.10, size = 107, normalized size = 3.15

$\frac{1162261467}{7}x^{21} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203}{14}x^{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^9 + 173635132896x^8 + 66158154783x^7 + 20588764518x^6 + 5149786572x^5 + 1010576952x^4 + 149902032x^3 + 15808800x^2 + 1056832x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14), x)

[Out] $1162261467x^{21/7} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + 15819767221203x^{14/14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^9 + 173635132896x^8 + 66158154783x^7 + 20588764518x^6 + 5149786572x^5 + 1010576952x^4 + 149902032x^3 + 15808800x^2 + 1056832x$

$$3.549 \quad \int (2 + 3x)^6 \left(1 + (2 + 3x)^7 + (2 + 3x)^{14}\right)^2 dx$$

Optimal. Leaf size=56

$$\frac{1}{105}(3x+2)^{35} + \frac{1}{42}(3x+2)^{28} + \frac{1}{21}(3x+2)^{21} + \frac{1}{21}(3x+2)^{14} + \frac{1}{21}(3x+2)^7$$

Rubi [A] time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1390, 1352, 611}

$$\frac{1}{105}(3x+2)^{35} + \frac{1}{42}(3x+2)^{28} + \frac{1}{21}(3x+2)^{21} + \frac{1}{21}(3x+2)^{14} + \frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]

[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/21 + (2 + 3*x)^21/21 + (2 + 3*x)^28/42 + (2 + 3*x)^35/105

Rule 611

Int[((a_.) + (b_.)*(x_.) + (c_.)*(x_)^2)^p, x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && (EqQ[a, 0] || !PerfectSquareQ[b^2 - 4*a*c])

Rule 1352

Int[(x_)^m*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rule 1390

Int[(u_)^m*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && LinearPairQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int (2+3x)^6 (1+(2+3x)^7+(2+3x)^{14})^2 dx &= \frac{1}{3} \text{Subst} \left(\int x^6 (1+x^7+x^{14})^2 dx, x, 2+3x \right) \\
&= \frac{1}{21} \text{Subst} \left(\int (1+x+x^2)^2 dx, x, (2+3x)^7 \right) \\
&= \frac{1}{21} \text{Subst} \left(\int (1+2x+3x^2+2x^3+x^4) dx, x, (2+3x)^7 \right) \\
&= \frac{1}{21}(2+3x)^7 + \frac{1}{21}(2+3x)^{14} + \frac{1}{21}(2+3x)^{21} + \frac{1}{42}(2+3x)^{28} + \frac{1}{105}(2+3x)^{35}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 188, normalized size = 3.36

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]

[Out] 17451466816*x + 443569828128*x^2 + 7299544818384*x^3 + 87406679578680*x^4 + (4057390785756924*x^5)/5 + 6077684727888102*x^6 + 37727143432895007*x^7 + 197897276851452864*x^8 + 889942562270387136*x^9 + (17344958593049772048*x^10)/5 + 11821487501620716192*x^11 + 35454069480572048124*x^12 + 94069263918929616324*x^13 + 221699757548270194389*x^14 + 465517091041681015296*x^15 + 872775774067455498528*x^16 + 1463104032160519033200*x^17 + 2194577166014752240080*x^18 + 2945285062308448290360*x^19 + 3534290697929473864098*x^20 + (26506949038858918036881*x^21)/7 + 3614565944605222108800*x^22 + 3064515076512846852480*x^23 + 2298383223254096766840*x^24 + (7584660010542711771792*x^25)/5 + 875152864622814086340*x^26 + 437576396725285446564*x^27 + (2625458326972530284475*x^28)/14 + 67899784121041365504*x^29 + (101849676181562048256*x^30)/5 + 4928210137817518464*x^31 + 924039400840784712*x^32 + 126005372841925188*x^33 + 11118121133111046*x^34 + (16677181699666569*x^35)/35

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2+3x)^6 (1+(2+3x)^7+(2+3x)^{14})^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]

[Out] IntegrateAlgebraic[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2, x]

fricas [B] time = 1.00, size = 174, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="fricas")

[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^33 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28 + 437576396725285446564*x^27 + 875152864622814086340*x^26 + 7584660010542711771792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18 + 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 465517091041681015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 35454069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 + 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x

giac [A] time = 0.42, size = 46, normalized size = 0.82

$$\frac{1}{105} (3x + 2)^{35} + \frac{1}{42} (3x + 2)^{28} + \frac{1}{21} (3x + 2)^{21} + \frac{1}{21} (3x + 2)^{14} + \frac{1}{21} (3x + 2)^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="giac")

[Out] 1/105*(3*x + 2)^35 + 1/42*(3*x + 2)^28 + 1/21*(3*x + 2)^21 + 1/21*(3*x + 2)^14 + 1/21*(3*x + 2)^7

maple [B] time = 0.00, size = 175, normalized size = 3.12

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x+2)^6*(1+(3*x+2)^7+(3*x+2)^14)^2,x)

[Out] 17451466816*x+16677181699666569/35*x^35+67899784121041365504*x^29+2625458326972530284475/14*x^28+875152864622814086340*x^26+2298383223254096766840*x^24+3614565944605222108800*x^22+3534290697929473864098*x^20+2194577166014752240080*x^18+872775774067455498528*x^16+11118121133111046*x^34+126005372841925188*x^33+924039400840784712*x^32+4928210137817518464*x^31+101849676181562048256/5*x^30+437576396725285446564*x^27+889942562270387136*x^9+11821487501620716192*x^11+94069263918929616324*x^13+465517091041681015296*x^15+1463104032160519033200*x^17+2945285062308448290360*x^19+26506949038858918036881/7*x^21+3064515076512846852480*x^23+7584660010542711771792/5*x^25+8740667957868

0*x^4+443569828128*x^2+6077684727888102*x^6+197897276851452864*x^8+17344958593049772048/5*x^10+35454069480572048124*x^12+221699757548270194389*x^14+7299544818384*x^3+4057390785756924/5*x^5+37727143432895007*x^7

maxima [B] time = 0.77, size = 174, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="maxima")

[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^33 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28 + 437576396725285446564*x^27 + 875152864622814086340*x^26 + 7584660010542711771792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18 + 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 465517091041681015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 35454069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 + 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x

mupad [B] time = 1.60, size = 46, normalized size = 0.82

$$\frac{(3x+2)^7}{21} + \frac{(3x+2)^{14}}{21} + \frac{(3x+2)^{21}}{21} + \frac{(3x+2)^{28}}{42} + \frac{(3x+2)^{35}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1)^2,x)

[Out] (3*x + 2)^7/21 + (3*x + 2)^14/21 + (3*x + 2)^21/21 + (3*x + 2)^28/42 + (3*x + 2)^35/105

sympy [B] time = 0.15, size = 187, normalized size = 3.34

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)

[Out] 16677181699666569*x**35/35 + 11118121133111046*x**34 + 126005372841925188*x**33 + 924039400840784712*x**32 + 4928210137817518464*x**31 + 1018496761815

62048256*x**30/5 + 67899784121041365504*x**29 + 2625458326972530284475*x**28/14 + 437576396725285446564*x**27 + 875152864622814086340*x**26 + 7584660010542711771792*x**25/5 + 2298383223254096766840*x**24 + 3064515076512846852480*x**23 + 3614565944605222108800*x**22 + 26506949038858918036881*x**21/7 + 3534290697929473864098*x**20 + 2945285062308448290360*x**19 + 2194577166014752240080*x**18 + 1463104032160519033200*x**17 + 872775774067455498528*x**16 + 465517091041681015296*x**15 + 221699757548270194389*x**14 + 94069263918929616324*x**13 + 35454069480572048124*x**12 + 11821487501620716192*x**11 + 17344958593049772048*x**10/5 + 889942562270387136*x**9 + 197897276851452864*x**8 + 37727143432895007*x**7 + 6077684727888102*x**6 + 4057390785756924*x**5/5 + 87406679578680*x**4 + 7299544818384*x**3 + 443569828128*x**2 + 17451466816*x

Chapter 4

Appendix

Local contents

4.1	Download section3154
4.2	Listing of Grading functions3154

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
```

```

If[ExpnType[result]<=ExpnType[optimal],
  If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
    If[LeafCount[result]<=2*LeafCount[optimal],
      "A",
      "B"],
    "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],

```

```

    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
            ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```



```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc;
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`') or type
(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(6,m1) #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```